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Event-triggered fault-tolerant leader-following control for homogeneous multi-agent systems

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Abstract: In this paper, the design of an event-triggered fault-tolerant control for leader-following consensus in multi-agent systems subject to actuator faults is presented. The problem under consideration is to reduce the control update rate and to guarantee that all the agents follow the trajectories of a leader when one or all the agents are subject to actuators faults. The proposed fault-tolerant strategy is based on distributed virtual actuators without re-tuning the nominal leader-following control. Linear matrix inequalities-based conditions are synthesized to guarantee the stability of the synchronization error and estimation error. The effectiveness of the event-triggered fault-tolerant strategy is illustrated through numerical examples.

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Keywords: Fault-tolerant control, leader-following consensus, multi-agent systems, event-triggered control, virtual actuator, LMI.

1. INTRODUCTION

The increasing demand for safe and reliable dynamic systems has been an important concern in the last years. Modern control systems are becoming more complex and sophisticated, and as a consequence, the issues of availability, cost efficiency, reliability, operating safety, and environmental protection are of major importance. Faults can be defined as an unexpected change of system function (Blanke et al., 2006). A fault-tolerant control (FTC) system is designed to maintain some portion of control integrity in the case of a specified set of possible faults or large changes. This can be done if the control system has a built-in element of automatic reconfiguration, once a fault has been detected and isolated (Chen and Patton, 2012). One of the important issues in FTC systems is to maintain the system performance close to the desired one and to preserve stability conditions after a fault occurrence (Youmin and Jin, 2008). FTC systems can be classified into two types: passive and active. Passive approaches use fixed controllers designed to be robust against faults, and they are very restrictive because all the expected faults cannot be known *a priori* (Noura et al., 2009). Active approaches react actively to the faults by reconfiguring the controller so that stability and an acceptable performance can be ensured (Youmin and Jin, 2008). These controllers consist of adjusting the controllers on-line according to estimation and isolation of the fault by a fault diagnosis module in order to retain the closed-loop system performance (Noura et al., 2009).

Research works have increased the focus on multi-agent systems (MASs) due to their potential in accomplishing missions that a single agent cannot perform (Li and Duan, 2014). However, due to the higher complexity and the increasing number of components, MASs are particularly sensitive to faults, which can happen with a higher probability and result in performance degradation or breakdown of all the agents (Chen et al., 2020).

In event-triggered (E-T) control, the controller is not updated unless it is required (Aström, 2008; Lunze, 2014; Xu et al., 2020; Wang et al., 2019). The E-T control is a control methodology often used in MASs for reducing the communication load of a digital network and the control update rate. Unlike Ren et al. (2021); Xu et al. (2021); Zheng et al. (2021); Wang et al. (2020), where mostly FTC adaptive strategies are used, the main contribution in this paper is the design of an E-T FTC for MASs under actuator faults based on the virtual actuator approach. Virtual actuators have been proposed as a fault accommodation approach in which the faulty system is

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modified adding a virtual actuator block which allows to reconfigure the controller (Blanke et al., 2006). The main advantage of virtual actuators is that the nominal control could be used without re-tuning it (Rotondo et al., 2015). The leader-following (L-F) consensus is solved in order for all the agents to follow the trajectories of a virtual agent. Inspired by the reconfiguration method presented in Rotondo et al. (2015) and the E-T strategy in Chen and Hao (2012), linear matrix inequalities (LMI)s-based conditions are obtained in order to guarantee the stability of the synchronization error, the virtual actuators, and the estimation error in spite of actuator faults. Not only does the proposed strategy guarantee the stability of the consensus in spite of faults, but it also reduces the number of control updates.

The remaining of the paper is organized as follows. The problem statement is provided in Section 2. The design of the proposed strategy is presented in Section 3. Numerical simulations are given in Section 4. Finally, conclusions are drawn in Section 5.

2. PRELIMINARIES

Given a matrix X, X^T denotes its transpose, $X > 0 (< 0)$ denotes a symmetric positive (negative) definite matrix. $\|.\|$ is the Euclidean norm. The symbol $*$ within a symmetric matrix represents the symmetric entries. The Hermitian part of a square matrix X is denoted by $\text{He}\{X\} := X + X^T$. The dependency on time of signals will be omitted when needed to decrease the notation burden. The symbol $\otimes$ denotes the Kronecker product, which for real matrices A, B, C, and D with appropriate dimensions, satisfies the following properties (Langville and Stewart, 2004):

1) $(A + B) \otimes C = A \otimes C + B \otimes C$;
2) $(A \otimes B)^T = A^T \otimes B^T$;
3) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$.

A directed graph $\mathcal{G}$ is a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{\mathbf{v}_1, \ldots, \mathbf{v}_N\}$ is a non-empty finite node set and $\mathcal{E} = \{(i,j) : i,j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$ is an edge set of ordered pairs of N nodes (total agents). The neighbors of node i are denoted as $j \in \mathcal{N}_i$. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N}$ associated with the graph $\mathcal{G}$ is defined such that $a_{ii} = 0$, $a_{ij} > 0$ if and only if $(i,j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. The Laplacian matrix $\mathcal{L} = [\mathcal{L}_{ij}] \in \mathbb{R}^{N \times N}$ of the graph $\mathcal{G}$ is defined as $\mathcal{L}_{ii} = \sum_{j \neq i} a_{ij}$ and $\mathcal{L}_{ij} = -a_{ij}, i \neq j$. This paper considers an undirected graph where $(i,j) = (j,i)$ if and only if $(i,j) \in \mathcal{E}$. When a leader agent is considered in the graph, its relationship between each follower is described by α_i where $\alpha_i > 0$ if there is communication with the *i-th* agent, and $\alpha_i = 0$ otherwise. Then, the following matrix is obtained $\bar{\mathcal{L}} = \mathcal{L} + \Lambda$, where $\Lambda = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N)$.

Lemma 1. (Ni and Cheng (2010)). The matrix $\bar{\mathcal{L}}$ has nonnegative eigenvalues. The matrix $\bar{\mathcal{L}}$ is positive definite if and only if the graph $\mathcal{G}$ is connected.

3. PROBLEM STATEMENT

Consider the following healthy homogeneous MAS:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + Bu_{n_i}(t), \\ y_i(t) = Cx_i(t), \end{cases} \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_x}$, $u_{n_i}(t) \in \mathbb{R}^{n_u}$, and $y_i(t) \in \mathbb{R}^{n_y}$ are the *i-th* state vector, the *i-th* nominal control input vector, and the *i-th* measurement output vector with appropriate matrices. The agents must follow the following leader agent:

$$\dot{x}_r(t) = Ax_r(t), \tag{2}$$

where $x_r(t) \in \mathbb{R}^{n_x}$ is the leader's state vector. Let us define the synchronization error and obtain its dynamics as follows:

$$\begin{aligned} \delta_i(t) &= x_i(t) - x_r(t), \\ \dot{\delta}_i(t) &= A\delta_i(t) + Bu_{n_i}(t). \end{aligned} \tag{3}$$

According to Ni and Cheng (2010), the nominal control is given by the following equation:

$$u_{n_i}(t) = K_c \left[\sum_{j \in \mathcal{N}_i} a_{ij} \left(\hat{x}_i(t) - \hat{x}_j(t) \right) + \alpha_i \left(\hat{x}_i(t) - x_r(t) \right) \right], \tag{4}$$

where $K_c \in \mathbb{R}^{n_u \times n_x}$ is the controller gain to be designed, α_i is the leader adjacency, $\alpha_i > 0$ if there is communication with the *i-th* agent, and $\alpha_i = 0$ otherwise, $\hat{x}_i(t)$ and $\hat{x}_j(t)$ are the estimated state vectors of i and its neighbors j. The control uses the estimation provided by the following observer:

$$\begin{cases} \dot{\hat{x}}_i(t) = A\hat{x}_i(t) + Bu_{n_i}(t) + L_o \left(y_i(t) - \hat{y}_i(t) \right), \\ \hat{y}_i(t) = C\hat{x}_i(t), \end{cases} \tag{5}$$

where $\hat{x}_i(t) \in \mathbb{R}^{n_x}$ is the estimated state vector, $\hat{y}_i(t) \in \mathbb{R}^{n_y}$ is the estimated output vector, and the matrix $L_o \in \mathbb{R}^{n_x \times n_y}$ is the observer gain to be designed. The nominal controller and observer gains can be calculated as reported in Vazquez Trejo et al. (2020).

Actuator faults are considered such that the healthy MAS (1) is changed as follows:

$$\begin{cases} \dot{x}_i(t) = Ax_i(t) + B_f(\sigma_i(t))u_i(t), \\ y_i(t) = Cx_i(t), \end{cases} \tag{6}$$

where $u_i(t)$ is the reconfigured control input and the multiplicative actuator faults are represented in the matrix $B_f(\sigma_i(t))$ defined as follows:

$$B_f(\sigma_i(t)) = B\Upsilon(\sigma_i(t)) \tag{7}$$

where

$$\begin{aligned} &\Upsilon(\sigma_i(t)) = diag\left(\sigma_{i_1}(t), \sigma_{i_2}(t), \ldots, \sigma_{i_m}(t)\right), \\ &\text{with } 0 \leq \sigma_{i_k}(t) \leq 1, \end{aligned} \tag{8}$$

$\sigma_{i_k}(t)$ represents the *k-th* actuator effectiveness ($\sigma_{i_k}(t) = 0$ is a total failure, $\sigma_{i_k}(t) = 1$ is the healthy *k-th* actuator). The faulty synchronization error dynamics are obtained as follows:

$$\begin{cases} \dot{\delta}_i(t) = A\delta_i(t) + B_f(\sigma_i(t))u_i(t), \\ y_{\delta_i}(t) = C\delta_i(t). \end{cases} \tag{9}$$

The following assumptions hold in this paper:

Assumption 1. The graph $\mathcal{G}$ is undirected and connected.

Assumption 2. The magnitude of the faults are considered known, this is, $\hat{\sigma}_i(t) \simeq \sigma_i(t)$ based on an efficient fault detection and isolation (FDI) module in each agent.

The problem under consideration in this paper is to design an event-triggered fault-tolerant control strategy for multi-agent systems (9) under actuator faults such that all the agents achieve the leader's trajectories (2) in a consensus in spite of the actuator faults.

4. EVENT-TRIGGERED FAULT TOLERANT CONTROL DESIGN

In event-triggered control, the control action is updated depending on a threshold. At the same time, this threshold depends on an event error. When this event error exceeds the threshold, the value of the control law is updated, otherwise the control action keeps the last calculated value. For this reason, the nominal control (4) is modified as follows:

$$u_{n_i}(t) = K_c\left[\sum_{j\in\mathcal{N}_i} a_{ij}\left(\hat{x}_i(t_l)-\hat{x}_j(t_l)\right)+\alpha_i\left(\hat{x}_i(t_l)-x_r(t_l)\right)\right], \quad t\in[t_l,t_{l+1}),\ l\in\mathbb{Z}_0^+, \tag{10}$$

where t_l denotes the *l-th* the time when the event is triggered.

In order to reconfigure the event-triggered nominal control for the faulty MAS (6), the tolerance can be obtained by either a proportional or a dynamical compensation depending on whether the following rank condition is satisfied:

$$rank\left(B_f\left(\sigma_i(t)\right)\right) = rank\left(B\right)\neq 0. \tag{11}$$

If (11) holds, the compensation can be expressed as a linear combination of the remaining actuators. In this case, the reconfiguration structure is defined as follows:

$$u_i(t) = N_v(\hat{\sigma}_i(t))u_{n_i}(t) \tag{12}$$

where $\hat{\sigma}_i(t)$ is the estimation of $\sigma_i(t)$ and $N_v(\hat{\sigma}_i(t))$ is given by:

$$N_v(\hat{\sigma}_i(t)) = B_f(\hat{\sigma}_i(t))^{\dagger}B, \tag{13}$$

where $\dagger$ denotes the pseudo inverse. In cases where (11) is not satisfied, the fault tolerance is achieved through the following matrix:

$$B^* = B_f(\sigma_i(t))N_v(\hat{\sigma}_i(t)), \tag{14}$$

and the following virtual actuator:

$$u_i(t) = N_v(\hat{\sigma}_i(t))\left(u_{n_i}(t)-M_v x_{v_i}(t)\right), \tag{15}$$

where $M_v\in\mathbb{R}^{n_u\times n_x}$ is the virtual actuator gain. The virtual actuator state vector $x_{v_i}(t)$ is calculated as follows:

$$\begin{cases}\dot{x}_{v_i}(t) = \left(A+B^*M_v\right)x_{v_i}(t)+\left(B-B^*\right)u_{n_i}(t),\\ y_{v_i}(t) = Cx_{v_i}(t).\end{cases} \tag{16}$$

The observer (5) is modified as follows:

$$\begin{cases}\dot{\hat{x}}_i(t) = A\hat{x}_i(t)+Bu_{n_i}(t)+L_o\left(y_i(t)+y_{v_i}(t)-\hat{y}_i(t)\right),\\ \hat{y}_i(t) = C\hat{x}_i(t).\end{cases} \tag{17}$$

Let us define the event error $\xi_i(t) = \hat{\delta}_i(t_l)-\hat{\delta}_i(t)$, and the following new vectors $\delta(t) = \left[\delta_1(t)^T,\delta_2(t)^T,\ldots,\delta_N(t)^T\right]^T$, $\xi(t) = \left[\xi_1(t)^T,\xi_2(t)^T,\ldots,\xi_N(t)^T\right]^T$, $x_v(t) = \left[x_{v_1}(t)^T,x_{v_2}(t)^T,\ldots,x_{v_N}(t)^T\right]^T$, $\hat{x}(t) = \left[\hat{x}_1(t)^T,\hat{x}_2(t)^T,\ldots,\hat{x}_N(t)^T\right]^T$, then based on (10) and using the Kronecker product, the faulty synchronization error dynamics (9), the virtual actuator (16), and the observer (17) become:

$$\begin{cases}\dot{x}_v = \left(I_N\otimes\left(A+B^*M_v\right)\right)x_v+\left(\bar{\mathcal{L}}\otimes\left(B-B^*\right)K_c\right)\left(\hat{\delta}+\xi\right),\\ \dot{\delta} = \left(I_N\otimes A\right)\delta+\left(\bar{\mathcal{L}}\otimes B^*K_c\right)\left(\hat{\delta}+\xi\right)-\left(I_N\otimes B^*M_v\right)x_v,\\ \dot{\hat{x}} = \left(I_N\otimes A\right)\hat{x}+\left(\bar{\mathcal{L}}\otimes BK_c\right)\left(\hat{\delta}+\xi\right)+\left(I_N\otimes L_oC\right)\left(x+x_v-\hat{x}\right)\end{cases} \tag{18}$$

Let us define a state transformation in order to introduce new states $z_1 = x_v$, $z_2 = \delta+x_v$, and $z_3 = \delta+x_v-\hat{\delta} = x+x_v-\hat{x}$, then (18) can be rewritten as follows:

$$\begin{cases}\dot{z}_1 = \left(I_N\otimes\left(A+B^*M_v\right)\right)z_1+\left(\bar{\mathcal{L}}\otimes\left(B-B^*\right)K_c\right)\left(z_2-z_3+\xi\right),\\ \dot{z}_2 = \left(I_N\otimes A\right)z_2+\left(\bar{\mathcal{L}}\otimes BK_c\right)\left(z_2-z_3+\xi\right),\\ \dot{z}_3 = \left(I_N\otimes\left(A-L_oC\right)\right)z_3.\end{cases} \tag{19}$$

In order to calculate the controller K_c, observer L_o, and virtual actuator M_v gains, the following theorem provides LMIs-based conditions which guarantee the stability of the synchronization error, the virtual actuator, and the observer using an event-triggered approach.

Theorem 1. Given eigenvalues of $\bar{\mathcal{L}}$ $(\lambda_j(\bar{\mathcal{L}}))$, if there exist matrices K_c, L_o, M_v, and symmetric matrices $P_1>0$, $P_2>0$, $P_3>0$, $M_\delta>0$ and $M_\xi>0$, such that the following inequalities hold:

$$\begin{bmatrix}\Theta_{11} & \Theta_{12_j} & -\Theta_{12_j} & \Theta_{12_j}\\ * & \Theta_{22_j} & -\Theta_{23_j}-M_\delta & \Theta_{23_j}\\ * & * & \Theta_{33} & 0\\ * & * & * & -M_\xi\end{bmatrix}<0 \tag{20}$$

where

$$\begin{aligned}\Theta_{11} &= \mathrm{He}\{P_1\left(A+B^*M_v\right)\},\\ \Theta_{12_j} &= \lambda_jP_1\left(B-B^*\right)K_c,\\ \Theta_{22_j} &= \mathrm{He}\{P_2\left(A+\lambda_jBK_c\right)\}+M_\delta,\\ \Theta_{23_j} &= \lambda_jP_2BK_c,\\ \Theta_{33} &= \mathrm{He}\{P_3\left(A-L_oC\right)\}+M_\delta,\end{aligned}$$

then the synchronization error, the virtual actuator, and the estimation error are stable under the event condition

$$\xi(t)^T\left(I_N\otimes M_\xi\right)\xi(t)<\hat{\delta}(t)^T\left(I_N\otimes M_\delta\right)\hat{\delta}(t). \tag{21}$$

Proof: Let choose a candidate Lyapunov function for (19) as follows:

$$V = z_1^T\left(I_N\otimes P_1\right)z_1+z_2^T\left(I_N\otimes P_2\right)z_2+z_3^T\left(I_N\otimes P_3\right)z_3, \tag{22}$$

where $P_1>0$, $P_2>0$, and $P_3>0$. The derivative along the trajectories of (19) is given by:

$$\begin{aligned}\dot{V} =& 2z_1^T\left(I_N\otimes P_1\right)\dot{z}_1+2z_2^T\left(I_N\otimes P_2\right)\dot{z}_2+2z_3^T\left(I_N\otimes P_3\right)\dot{z}_3,\\ =& 2z_1^T\left(I_N\otimes P_1\left(A+B^*M_v\right)\right)z_1\\ &+2z_1^T\left(\bar{\mathcal{L}}\otimes P_1\left(B-B^*\right)K_c\right)\left(z_2-z_3+\xi\right)\\ &+2z_2^T\left(I_N\otimes P_2A\right)z_2+2z_2^T\left(\bar{\mathcal{L}}\otimes P_2BK_c\right)\left(z_2-z_3+\xi\right)\\ &+2z_3^T\left(I_N\otimes P_3\left(A-L_oC\right)\right)z_3.\end{aligned} \tag{23}$$

In view of $\hat{\delta}^T\left(I_N\otimes M_\delta\right)\hat{\delta}-\xi^T\left(I_N\otimes M_\xi\right)\xi>0$, which leads to $\left(z_2-z_3\right)^T\left(I_N\otimes M_\delta\right)\left(z_2-z_3\right)-\xi^T\left(I_N\otimes M_\xi\right)\xi>0$, then (23) becomes:

$$
\begin{aligned}
\dot{V} <& 2z_1^T \left(I_N \otimes P_1 \left(A + B^* M_v\right)\right) z_1 \\
&+ 2z_1^T \left(\bar{\mathcal{L}} \otimes P_1 \left(B - B^*\right) K_c\right) \left(z_2 - z_3 + \xi\right) \\
&+ 2z_2^T \left(I_N \otimes P_2 A\right) z_2 + z_2^T \left(I_N \otimes M_\delta\right) z_2 \\
&+ 2z_2^T \left(\bar{\mathcal{L}} \otimes P_2 B K_c\right) \left(z_2 - z_3 + \xi\right) - 2z_2^T \left(I_N \otimes M_\delta\right) z_3 \\
&+ 2z_3^T \left(I_N \otimes P_3 \left(A - L_o C\right)\right) z_3 + z_3^T \left(I_N \otimes M_\delta\right) z_3 \\
&- \xi^T \left(I_N \otimes M_\xi\right) \xi.
\end{aligned} \tag{24}
$$

Let us perform a decomposition of $\bar{\mathcal{L}}$ such that $\bar{\mathcal{L}} = T\bar{\mathcal{J}}T^{-1}$ with an invertible matrix $T \in \mathbb{R}^{N\times N}$ and a diagonal matrix $\bar{\mathcal{J}} = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_N)$ where λ_j, $j = 1, 2, \ldots, N$, are the eigenvalues of $\bar{\mathcal{L}}$, in this light, let us define the following change of coordinates:

$$
\begin{aligned}
\varphi_1 &= \left(T^{-1} \otimes I_N\right) z_1 \\
\varphi_2 &= \left(T^{-1} \otimes I_N\right) z_2 \\
\varphi_3 &= \left(T^{-1} \otimes I_N\right) z_3 \\
\varphi_4 &= \left(T^{-1} \otimes I_N\right) \xi
\end{aligned} \tag{25}
$$

Replacing (25) in (23) leads to:

$$
\begin{aligned}
\dot{V} <& 2\varphi_1^T \left(I_N \otimes P_1 \left(A + B^* M_v\right)\right) \varphi_1 \\
&+ 2\varphi_1^T \left(\bar{\mathcal{J}} \otimes P_1 \left(B - B^*\right) K_c\right) \left(\varphi_2 - \varphi_3 + \varphi_4\right) \\
&+ 2\varphi_2^T \left(I_N \otimes P_2 A\right) \varphi_2 + \varphi_2^T \left(I_N \otimes M_\delta\right) \varphi_2 \\
&+ 2\varphi_2^T \left(\bar{\mathcal{J}} \otimes P_2 B K_c\right) \left(\varphi_2 - \varphi_3 + \varphi_4\right) \\
&- 2\varphi_2^T \left(I_N \otimes M_\delta\right) \varphi_3 + 2\varphi_3^T \left(I_N \otimes P_3 \left(A - L_o C\right)\right) \varphi_3 \\
&+ \varphi_3^T \left(I_N \otimes M_\delta\right) \varphi_3 - \varphi_4^T \left(I_N \otimes M_\xi\right) \varphi_4.
\end{aligned} \tag{26}
$$

Let us define $\phi_j = \left[\varphi_1^T, \varphi_2^T, \varphi_3^T, \varphi_4^T\right]^T$ so that:

$$
\begin{aligned}
\dot{V} &< \sum_{j=1}^{N} \phi_j^T \Theta_j \phi_j \\
\Theta_j &= \begin{bmatrix} \Theta_{11} & \Theta_{12_j} & -\Theta_{12_j} & \Theta_{12_j} \\ * & \Theta_{22_j} & -\Theta_{23_j} - M_\delta & \Theta_{23_j} \\ * & * & \Theta_{33} & 0 \\ * & * & * & -M_\xi \end{bmatrix}, \\
\Theta_{11} &= \text{He}\{P_1 \left(A + B^* M_v\right)\}, \\
\Theta_{12_j} &= \lambda_j P_1 \left(B - B^*\right) K_c, \\
\Theta_{22_j} &= \text{He}\{P_2 \left(A + \lambda_j B K_c\right)\} + M_\delta, \\
\Theta_{23_j} &= \lambda_j P_2 B K_c, \\
\Theta_{33} &= \text{He}\{P_3 \left(A - L_o C\right)\} + M_\delta.
\end{aligned} \tag{27}
$$

If $\Theta_j < 0$, $\forall j = 1, 2, \ldots, N$, then $\dot{V} < 0$ and the synchronization error, the virtual actuator, and the estimation error are asymptotically stable, which completes the proof. □

Remark 1. In order to solve the bilinear conditions (20), an LMI formulation can be obtained by pre- and post-multiplying Θ_j by $\text{diag}\left(P_1^{-1}, I, I, I\right)$ and then defining $N_v = M_v \bar{P}_1$, $N_o = P_3 L_o$ and $\bar{P}_1 = P_1^{-1}$, $P_2 = I$.

Remark 2. It should be mentioned that the next event happens at the time t_{l+1}, which is the first time instant after t_l if $\xi^T \left(I_N \otimes M\xi\right) \xi \geq \hat{\delta}^T \left(I_N \otimes M_\delta\right) \hat{\delta}$.

Remark 3. The LMIs-based conditions obtained by Theorem 1 guarantee stability of the synchronization error. Nevertheless, in order to modify the transient behavior, LMIs-based conditions based on D-stability can be applied (Chilali and Gahinet, 1996).

In the following section, a numerical example is presented in order to illustrate the effectiveness of the proposed event-triggered fault-tolerant strategy based on virtual actuators.

5. NUMERICAL EXAMPLE

In order to show the effectiveness of the proposed event-triggered approach, the following numerical example considers the same dynamic system, topology network, and fault scenario as reported in Vazquez Trejo et al. (2021). Consider five agents described by:

$$
A = \begin{bmatrix} -0.05 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & -3 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ -1 & -1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.
$$

with the communication topology as shown in Fig. 1. The agents' initial conditions are presented as follows: $x_1(0) = [0, 0, 0, 0]^T$, $x_2(0) = [1, 3, 0, 0]^T$, $x_3(0) = [-2, -2, 0, 0]^T$, $x_4(0) = [4, -2, 0, 0]^T$, and $x_5(0) = [5, 3, 0, 0]^T$.

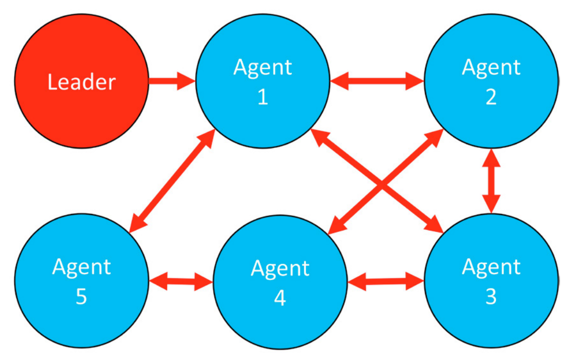

Fig. 1. Communication topology between agents.

Fig. 2 shows the performance of the synchronization error between the agents and the leader in a free fault case.

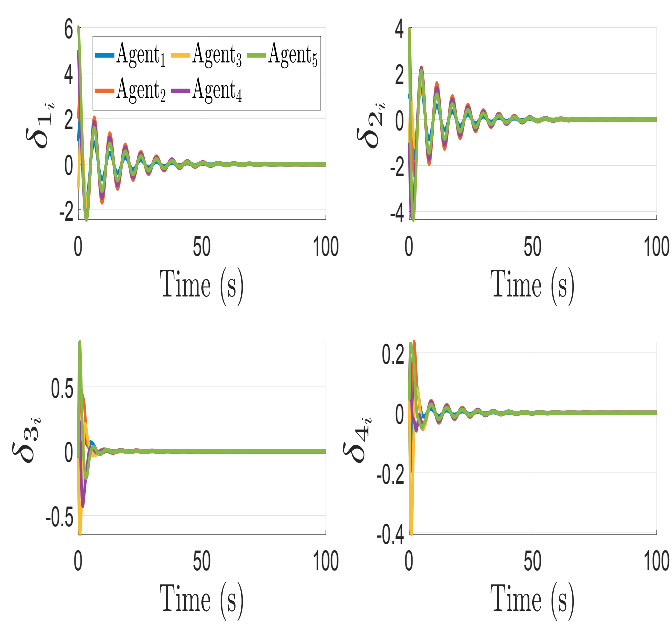

Fig. 2. Performance of the synchronization error between the leader and the agents free fault case.

The fault scenario is considered as follows: agent 3 with a partial fault $diag\left(\sigma_{3_1} = 1, \sigma_{3_2} = 0.5\right)$ starting from $t =$

1s, agent 5 with a lost actuator $diag\left(\sigma_{5_1}=1,\sigma_{5_2}=0\right)$ starting from $t=1.5$s, and agent 2 with a lost actuator $diag\left(\sigma_{2_1}=0,\sigma_{2_2}=1\right)$ starting from $t=2$s. Fig. 3 shows the performance of the synchronization error without the proposed reconfiguration module. The consensus diverges due to the faults in δ_{3_i} and δ_{4_i}.

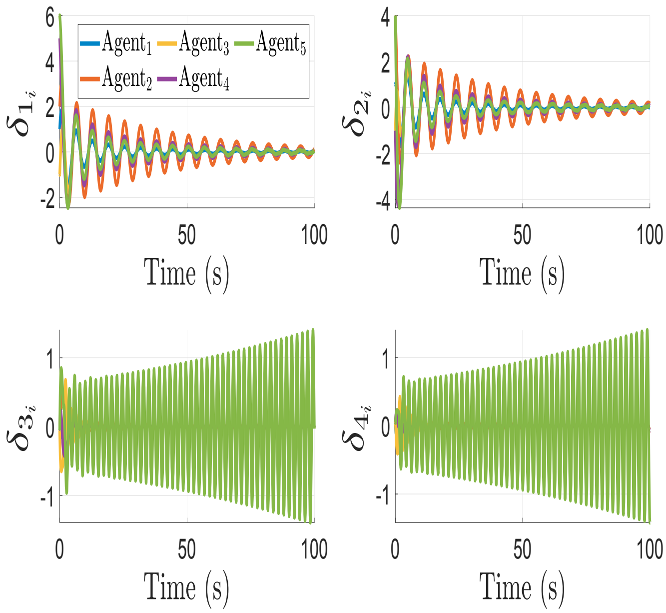

Fig. 3. Synchronization error between the leader and the agents without the reconfiguration module.

Fig. 4 shows the evolution of the synchronization error between the *i-th* agent and the leader under the assumptions that the control system gets reconfigured after one second from the fault occurrence and the magnitude of the faults is known.

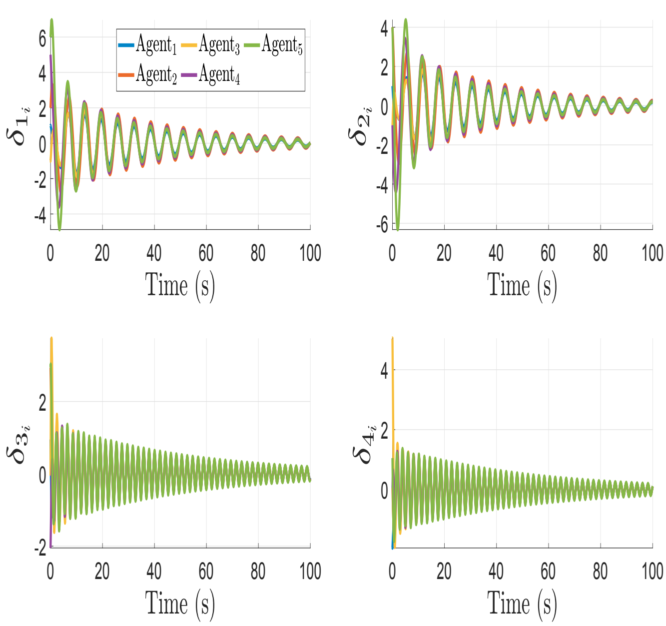

Fig. 4. Synchronization error between the leader and the agents using the reconfiguration module based on virtual actuators.

The performance of the reconfiguration using the event-triggered mechanism is similar as reported in Vazquez Trejo et al. (2021), but the number of updates in the control law has been reduced as shown by Fig. 5.

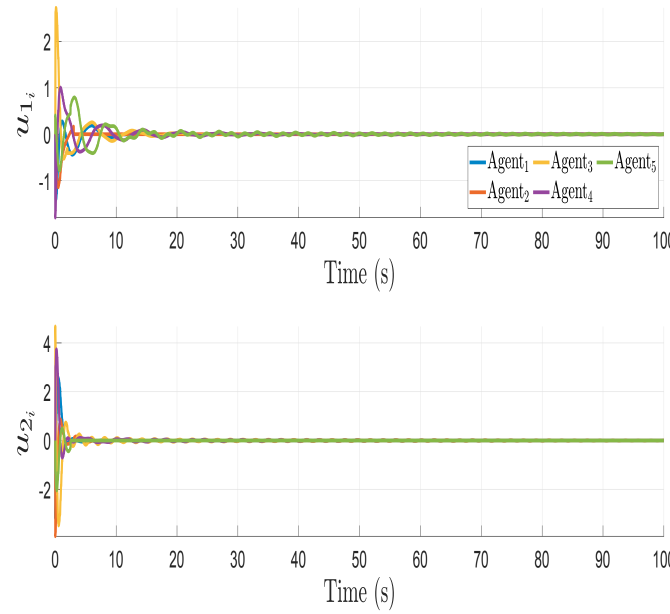

Fig. 5. Control law of the agents.

Fig. 6 presents a zoom in the control law of the agents in order to show when the control law keeps the last value before an event. The update of the control law has been triggered 0.1573% times.

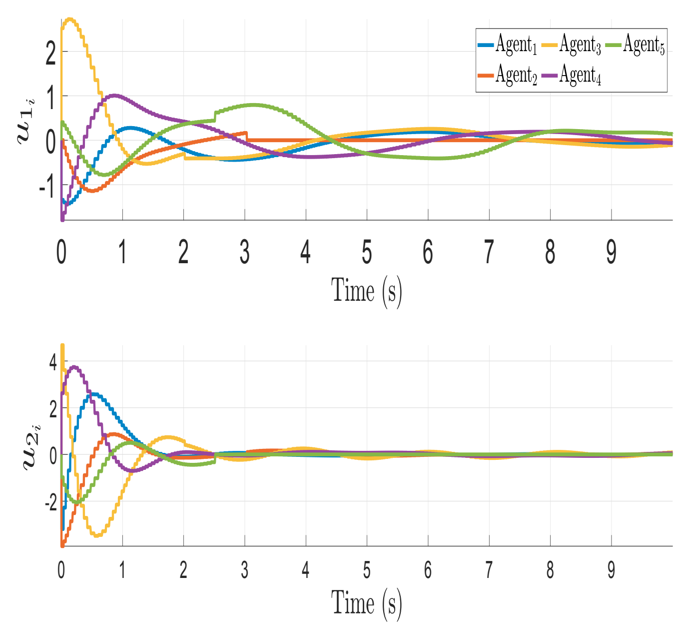

Fig. 6. Zoom in the control law of the agents.

In Table 1, root mean square error in the synchronization error is used to quantify the performance of the consensus comparing the free fault case performance and the reconfiguration strategy.

Table 1. Performance of the reconfiguration strategy.

Agent	Free fault case	Reconfiguration strategy
1	0.4428	0.6454
2	0.8648	1.4490
3	0.5742	0.9785
4	0.9433	1.2403
5	0.9606	1.4457

6. CONCLUSION

This paper proposes an event-triggered fault-tolerant leader-following control design for multi-agent systems subject to actuator faults. Sufficient design conditions have been obtained to guarantee that the synchronization error and the estimation error are asymptotically stable in spite of the actuator fault using an event-triggered mechanism and distributed virtual actuators. Not only does the proposed strategy guarantee the stability of the consensus in spite of faults, but it also reduces the number of control updates. Simulation results have shown the effectiveness of the proposed event-triggered strategy based on virtual actuators.

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