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UNIVERSITY OF STAVANGER

MASTER THESIS

Comparative study of deterministic and probabilistic fatigue assessment methods

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Abstract

The subject of this thesis is to compare the use of a deterministic and probabilistic assessment methods in fatigue analysis. The thesis focuses on the nominal stress method in the stress-life approach. Three different assessment methods are presented and compared, the traditional method, the closed form method and the probabilistic method. The traditional method is the one described in DNV-RP-C203 (DNV, 2014). The closed form method involves using a Weibull distribution to represent the stress spectrum. The probabilistic method involves the use of Monte Carlo simulations to obtain the probability of failure P_f.

The associated procedures of the three assessment methods above were verified by considering a simple example. The assessment methods were then used for fatigue analysis of two case studies for traffic loads on a bridge. A single-span and a three-span bridge is evaluated. An attempt on improving the traffic load model presented in the Eurocode (ECS, 2010a) is also proposed.

Based on the research and the obtained results, possible limitations and shortcomings are discussed.

Preface

This thesis is submitted in fulfilment of the requirements for the Masters degree in constructions and materials, at the University of Stavanger, Faculty of Science and Technology, Norway. The research presented has been carried out at the University of Stavanger in the period from January 2015 to June 2015.

I would like to thank my to advisers at the University of Stavanger, Associate Professor S.A.Sudath C Siriwardane and Adjunct Professor Gerhard Ersdal for their guidance, help and critique during this work. I would also like to thank Statens Vegvesen for providing me with information about possible bridges to be used in this thesis.

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Abbreviations

AADT	Annual average density traffic
ATLM	Alternative traffic load model
CDF	Cumulative density function
COV	Coefficient of variation
DNV	Det Norske Veritas
ECS	European Committee for Standardisation
FLM4	Fatigue load model 4
NORSOK	Standards developed by the Norwegian petroleum industry
PDF	Probability density function (or frequency function)
SD	Standard deviation

Chapter 1

Introduction

1.1 Background and previous work on the subject

A good understanding of fatigue is very important to any structural engineer, as it is the single largest cause of failure in metals. The deterministic method of fatigue assessment is used in the majority of current fatigue assessment standards. A deterministic method means that given a particular input, the fatigue calculations will always produce the same output. The fatigue behaviour in metal is random by nature, which means the deterministic fatigue method has many uncertainties that are accounted for by using characteristic values and safety factors in rules and regulations. These uncertainties are among others in the fatigue load, material properties, geometries and in human influences. Probabilistic fatigue models are an alternative method to model these uncertainties as they take the variation in the variables into account. The probabilistic method of fatigue assessment is increasingly being used by the industry. In recent years at least two standards involving probabilistic analysis have been introduced. These two are NORSOK N-006 (NORSOK, 2015) and DNVGL-RP-0001 (DNV, 2015). However, these standards are only defined for offshore structures, and focus more on probabilistic inspection planning for fatigue cracks in existing structures, structural integrity and life extension. Any standard covering probabilistic fatigue analysis on land-based structures has yet to be made.

The lack of such a standard is the main reason for this thesis. The goal is to establish a good understanding of the different approaches and implement them into an analysis of a land-based structure. Bridges are land-based structures that are heavily exposed to cyclic loading and serves as perfect structures to examine the differences in deterministic and probabilistic fatigue assessment. If the reader wants to read more about the use of probabilistic approaches in fatigue analysis, than is described here, these two textbooks are recommended:

- Lassen, T. and Recho, N., *Fatigue Life Analyses of Welded Structures*, ISTE Ltd, 2006
- Bai, Y, *Marine Structural Design*, Elsevier Ltd, 2003

1.2 Problem statement

What are the major differences between the deterministic and probabilistic approaches of fatigue assessment?

- What shortcomings, limitations, advantages or disadvantages do the different approaches have?
- Is one approach better than the other?

How are the established fatigue analysis procedures for traffic loads over a bridge in rules and regulations?

- Can these procedures be improved in some way?

1.3 Limitations

The focus in thesis is on the stress-life or S-N approach and not fracture mechanics. More specifically, the nominal stress method in the S-N approach is used in the fatigue assessments.

In the case studies only the fatigue effects of the traffic and dead loads are analyzed. Other loads like wind, centrifugal, braking forces etc. are not included in the calculations.

The use of eye-fitting of Weibull distribution to stress spectrum to determine Weibull parameters is assumed to be sufficient.

1.4 Overview of thesis

Chapter 2 is the theory chapter of this thesis. It consists of a small summary of the history of fatigue analysis and provides a basic understanding of the general fatigue damage process. It also provides background theory on the approaches to fatigue analysis.

Chapter 3 describes the methods that have been used in this thesis.

Chapter 4 gives an example of fatigue assessment using both deterministic and probabilistic approach.

Chapter 5 is the Case Study which consists of two different examples. The focus of the case study is fatigue effects of traffic loads on a bridge.

Chapter 6 is where the results gathered throughout the thesis is discussed.

Chapter 7 includes the conclusions and recommendations for further work on the subject.

Chapter 2

Fatigue theory

2.1 History of fatigue analysis

The first article published about fatigue was written by William Albert in 1837. He was a German mining administrator who observed and studied the failure of mining hoist chains. He discovered that the failure was not associated with an accidental overload, but was dependent on load and the number of repetitions of load cycles. Two years later in 1839, Jean-Victor Poncelet, designer of cast iron axles for mill wheels, officially used the term fatigue for the first time in a book on mechanics. In 1842, one of the worst rail disasters of the 19th century occurred near Versailles in which a locomotive broke an axle. Examination of the broken axle by William John Macquorn Rankine of the British Railways showed that it had failed by brittle cracking across its diameter.

In 1860 August Wöhler, a technologist in the German railroad system conducted the first systematic study of fatigue. Wöhler was concerned by the failure of axles after various service lives, at loads considerably less than expected. His experiments simulated the service life situation of the axles by rotating the axles and exposing them to a constant moment. This resulted in a cyclic loading around a zero mean stress. The results of these tests were presented in diagrams where fatigue strength was given as a function of the number of load cycles prior to failure. The same form of curves are still in use today and are called S-N curves or Wöhler curves. He also introduced the endurance or fatigue limit of metal, which represents the stress level below which the component would have infinite or very high fatigue life. His experiments also showed that fatigue life is dramatically reduced by the presence of a notch in the material. In 1903 Sir James Alfred Ewing demonstrated the origin of fatigue failure in microscopic cracks. In 1910, O.H. Baskin defined the shape of a typical S-N curve by using Wöhler's test data and proposed a log-log relationship.

In 1945 M.A. Miner popularised a rule that had first been proposed by A. Palmgren in 1924. The rule, either called *Miner's rule* or *Palmgren-Miner rule*, is used to calculate the cumulative damage of the material. In 1963 P.C. Paris and F. Erdogan proposed a method for predicting the rate of growth of individual fatigue cracks, called *Paris law*.

2.2 Basic mechanisms of metal fatigue

Fatigue is a form of failure that occurs when a structural material is subjected to cyclic loading over time. Over time the damage accumulated from each cycles reaches a critical level, causing failure. The result is that fatigue may cause failure at loads significantly lower than the maximum value, different from most of the failure modes. Fatigue is a three-stage process that involves the following stages:

- **Crack initiation** - a small crack forms at some point of high stress concentration.
- **Crack propagation** - crack grows with each stress cycle.
- **Final fracture failure** - occurs when the crack reaches a critical level.

Crack initiation

Cyclic loading can produce microscopic surface discontinuities resulting from dislocation slip steps that may also act as stress raisers and therefore as crack initiation sites. This is only valid for components that does not have any other material defects. In most cases the crack is initiated at some point of stress concentration, because of defects in the material. Such defects may be scratches, dents or in welds. To a certain degree weld defects always exist both internally and on the weld surface. These weld defects may trigger the cracks to grow (typically from the weld surface).

Crack propagation

Compared to the crack initiation, the crack propagation stage is better understood and different theories exist to model the crack growth, i.e. fracture mechanics. The major parameter governing crack propagation is the stress range to which the structural detail is subjected to. Also, the welding geometry and initial crack size have a large impact on the fatigue life of the structural detail. In welded structures, fatigue cracks almost always start at a weld defect and the propagation period accounts for more than 90% of the fatigue life.

Final fracture

Fracture failure of the structural details will eventually occur when the crack size propagates to a critical size. The final fracture depend upon a couple of parameters, such as stress level, crack size and material toughness. Similar to crack initiation, the fatigue life during the final fracture is a small part and is usually negligible compared to the crack propagation stage.

2.2.1 Important parameters to the fatigue process

The following conditions and parameters are important to the fatigue process:

- External cyclic loading
- Geometry of the item
- Material characteristics
- Residual stresses

- Production quality in general
- Surface finish in particular
- Environmental condition during service
- Endurance limits

External loads and stresses

"The external forces may create normal, bending or torsion effects on a structural item with associated stress situations near a potential crack location. These loading and response situations are often referred to as loading and stress modes. The latter concept is defined by the stress direction relative to the crack planes. The normal and bending mode will give rise to normal stresses that will act as the main reasons for the crack initiation and growth. In this case, the crack planes will be moved directly apart by the normal stresses." (Lassen and Recho, 2006)

The most important part of the cyclic loading is the variation of the force, or the stress range $\Delta\sigma$, and the number of cycles.

Geometry, stress and strain concentrations

Most structural members contain some form of geometrical or micro-structural discontinuities, often referred to as a notches. These discontinuities often result in high local stresses σ_l that are many times greater than the nominal stress σ_n in the component or member. This ratio is defined as the stress concentration factor K_t. In ideally elastic members, the theoretical stress concentration factor K_t is defined as:

$$K_t = \frac{\sigma_l}{\sigma_n} \tag{2.1}$$

The stresses can be reduced by increasing the dimensions of the item or improving the local geometry of the notch, typically the notch radius. The latter option is preferred since it can be achieved without any additional weight and costs.

Material properties

The common material parameters, such as yield strength, tensile strength, and module of elasticity, have an impact on the fatigue strength of the metal. Fatigue resistance is determined by experimental testing of specimens of the material. The resistance is determined by applying a constant amplitude stress range to a smooth specimen of the material and find the number of cycles until failure.

Residual stress

"Residual stresses are defined as the static inherent stresses present in the structural item before the external forces are applied. They are often created by the fabrication procedure." (Lassen and Recho, 2006) Areas subjected to tensile residual stresses are more vulnerable to fatigue.

Residual stresses in structures can be separated into two types:

- Short-ranged stresses exist only in and close to a weld, and are self-balanced over the cross section of one member. The cause of these stresses is the thermal contraction of parts of the cross section, under restraint from the cooler portions. Stress will generally be large and with large through-thickness gradients.

- Long-range stresses are uniform throughout a structural member, and are self-balanced within the structure. The origin is from the procedure of assembling a structure from pre-fabricated components, whereby welding shrinkage and the use of local heating, mechanical restraints, brute force etc. in the process of fitting the pieces together may cause significant locked-in stresses.

Hence, in large structures residual stresses needs to be accounted for.

Fabrication quality and surface finish

How a component or structure is actually built compared to the drawings will, in the end, decide the fatigue strength.

”Dimension control must be carried out to check that the dimensions are within the given tolerances. If misalignment occurs it may introduce secondary bending for an axial loading mode. Sharp flaws may act as starters for fatigue crack growth and in the worst cases the crack initiation phase is lost. Also a smooth surface will increase time until crack initiation.” (Lassen and Recho, 2006)

To ensure quality regarding these matters, dimension control and non-destructive testing (NDT) should be carried out.

Influence of the environment

”The environment that the structural part is exposed to has an influence on its fatigue life. When welded joints are subjected to repetitive loading in a corrosive environment there is a synergy effect between the mechanical-fatigue damage process and the electro-chemical corrosion process. The corrosion may result in surface pits that shorten the crack initiation period. Furthermore, the corrosion process aggravates the condition within a crack near the crack front and may therefore significantly speed up the growth rate. Hence, welded structures in seawater and other corrosive environments should always have some sort of corrosion protection. This is usually provided by cathodic protection and/or protective coating.

The principal differences in fatigue resistance between the in-air environment, cathodic protection and free corrosion are shown in Fig. 2.1. At high stress levels fatigue life under cathodic protection is close to 2,5 shorter than fatigue life in dry air, whereas for low stress ranges the cathodic protection is very efficient. At small stress ranges, fatigue life is very close to the life found in dry air and the assumption of a fatigue limit for corrosion protection is acceptable. The reason for this behavior is that the corrosion process may blunt the crack front at low stress ranges and inflict calcareous deposits in the wake of the crack front. The last effect may lead to crack closure. In free corrosion environment, the curve gives significant shorter fatigue lives at all stress levels.” (Lassen and Recho, 2006)

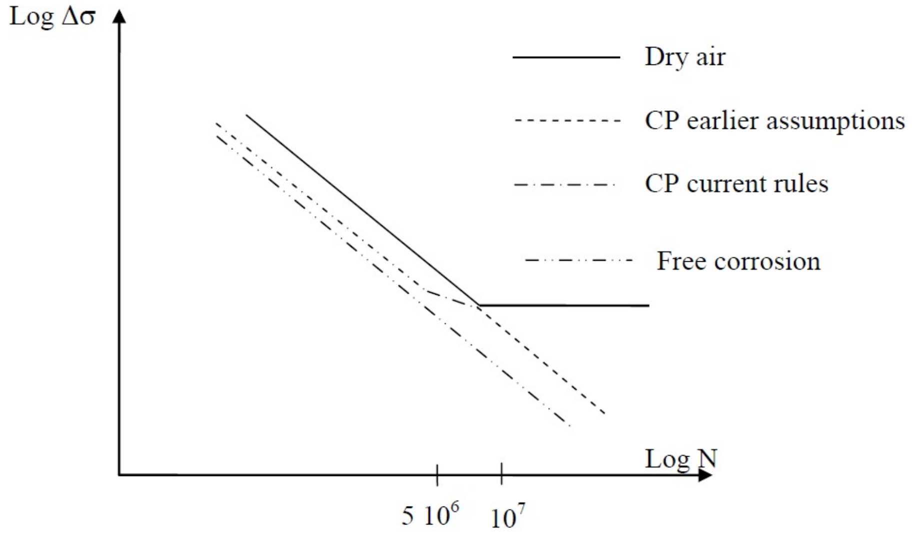


Figure 2.1: Corrosion effect on S-N curves in seawater. (Lassen and Recho, 2006, Figure 5.5)

Endurance limits

Certain materials have a fatigue limit or endurance limit which represents a stress level where the material does not fail and can be cycled infinitely. If the applied stress level is below the endurance limit of the material, the structure is said to have an infinite life. This is characteristic of steel and titanium in benign environmental conditions. Many non-ferrous metals and alloys, such as aluminum, magnesium, and copper alloys, do not exhibit well-defined endurance limits. These materials instead display a continuously decreasing S-N response.

2.3 Fracture mechanics

Fracture mechanics focuses on the study of the propagation of cracks in materials. It is based on the relation of crack growth and a single load parameter such as the stress-intensity factor *K*.

It is convenient to express the functional relationship for crack growth in the following form:

$$\frac{da}{dN} = f_1(\Delta K, R) \tag{2.2}$$

where
$\Delta K = (K_{max} - K_{min})$
$R = K_{min}/K_{max}$
da/dN = crack growth per cycle.

Eq. 2.2 can be integrated to estimate fatigue life. The number of cycles required to propagate a crack from an initial length a_0 to a final length a_f is given by:

$$N = \int_{a_0}^{a_f} \frac{da}{f_1(\Delta K, R)} \tag{2.3}$$

2.3.1 Stress intensity factor *K*

The stress intensity factor *K* completely characterizes the crack-tip conditions in a linear elastic material. If *K* is known, the entire stress distribution at the crack tip can be computed. It was G.R. Irwin who in the late 1950s discovered this with the help of a paper published by H.M. Westergaard in 1938. If one assumes that the material fails locally at some critical combination of stress and strain, then it follows that fracture must occur at a critical stress intensity K_{cr}. Thus, K_{cr} is an alternative measure of fracture toughness.

The stress intensity factor is given by:

$$K = Y\sigma\sqrt{\pi a} \tag{2.4}$$

where

σ = characteristic stress

a = characteristic crack dimension

Y = dimensionless constant that depends on the geometry and the mode of loading

For a plate subject to remote tensile stress with a width *W* much larger than the characteristic crack dimension a, Y = 1.

2.3.2 Paris Law

Figure 2.2 is a schematic log-log plot of *da/dN* vs. ΔK, which illustrates typical fatigue crack growth behavior in metals. The sigmoidal curve contains three distinct regions. At intermediate ΔK values or region II, the curve is linear, but the crack growth rate deviates from the linear trend at high and low ΔK levels. At the low end or region I, *da/dN* approaches zero at a threshold ΔK_{th}, below which the crack will not grow. At the high end or region III, as the ΔK values approaches a critical level K_{cr} the crack growth accelerates and fracture will occur.

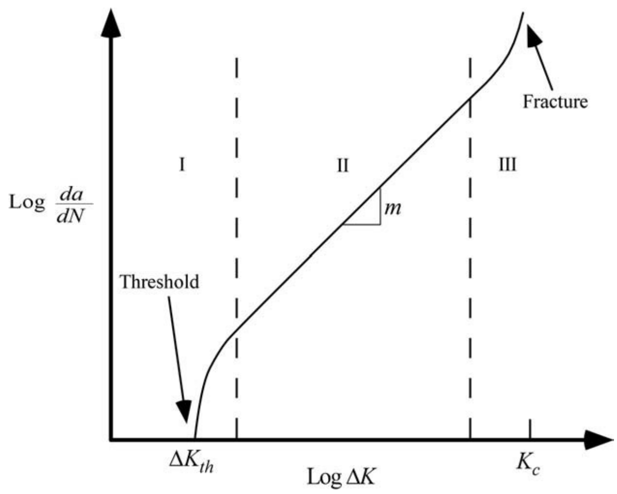


Figure 2.2: Typical fatigue crack growth behavior in metals. (Anderson, 2005, Figure 10.2)

The linear region of the log-log plot in fig. 2.2 can be described by the following power law, also known as the *Paris Law*:

$$\frac{da}{dN} = C\Delta K^m \tag{2.5}$$

where *C* and *m* are material constants that are determined experimentally. According to Eq. (2.5), the fatigue crack growth rate depends only on ΔK; *da/dN* is insensitive to the *R* ratio in region II. Studies over the past four decades have shown that the exponent *m* can range from 2 to 4 for most metals in the absence of a corrosive environment.

2.4 Stress-life approach

There are three well-known methodologies when using S-N curves for calculating the fatigue life of welded joints:

- Nominal stress method
- Structural hot spot stress (SHSS) method
- Effective notch stress method

A short description of the methods is provided below. Since the nominal stress method is the one used in this thesis, a more detailed description of the assessment method is provided in section. 2.6.

All three methods follow the same step-by-step method to some degree, with the major difference being how you determine the stress range.

2.4.1 Nominal stress method

The nominal stress method is a non-local fatigue assessment method. It is based on the notion that the fatigue life of a welded joint can be sufficiently specified by the characteristic global geometry of the joint and the history of nominal stresses at specified locations.

Nominal stress method step-by-step:

1. Choose detail class (and hence respective S-N curve type).
2. Evaluate environment (in-air, cathodic protected, free corrosion) for selection of S-N curve.
3. Calculate nominal stress range.
4. Determine cycles to failure from S-N curve.
5. Use Palmgren-Miner law to calculate damage and life.

Nominal stress is the stress calculated in the sectional area under consideration, disregarding the local stress raising effects of the welded joint, but including the stress raising effects of the macro-geometric shape of the component in the vicinity of the joint, such as e.g. large cut-outs. Overall elastic behaviour is assumed.

All types of fluctuating load acting on the component and the resulting stresses at potential sites for fatigue have to be considered. Stresses or stress intensity factors then have to be determined according to the fatigue assessment procedure applied. The actions originate from live loads, dead weights, snow, wind, waves, pressure, accelerations, dynamic response etc. Actions due to transient temperature changes should be considered. Improper knowledge of fatigue actions is one of the major sources of fatigue problems. Tensile residual stresses due to welding decrease the fatigue resistance, however, the influence of residual weld stresses is already included in the fatigue resistance data given in S-N curves.

Effects of macro-geometric features of the component as well as stress fields in the vicinity of concentrated loads must be included in the nominal stress.

2.4.2 Structural hot spot stress (SHSS) method

The structural hot spot method is based on detail categories and detail category numbers, very much in the same sense as the nominal stress method is. The only difference is that the hot-spot method incorporates the more detailed measure of the geometrical stress, as opposed to the more limited measure nominal stress in step 3. As a consequence, the structural hot spot approach requires fewer detail categories than the nominal stress in order to provide comparable versatility.

2.4.3 Effective notch stress methods

The effective notch stress method, proposed by Radaj (1990), is more complex and time-consuming, i.e. uses more computational time as well as it requires a more detailed modelling than the ones mentioned above, but is able to cover effective stresses in the weld root as well as in the weld toe. Due to the high complexity of this method, it is not efficient to apply it on large structures consisting of numerous welded members.

2.5 Uncertainties in fatigue analysis

In this section some of the major uncertainties in fatigue analysis are briefly discussed. The uncertainties discussed are:

- The scatter in test results for S-N curves.
- The fatigue/endurance limit of the constant amplitude S-N curve.
- Modelling variable stress spectrum as a histogram.
- The assumption that the accumulated damage is linear.

2.5.1 S-N curves

Scatter

The design S-N curves used in Eurocode, DNV, NORSOK etc. are based on experimental results from samples exposed to constant amplitude. The results of the experiments returns a scatter of different fatigue lives at a given stress ranges (fig.2.3). To account for this variation in strength it is assumed that the fatigue life at a given stress range is a stochastic variable which is log normally distributed. A mean curve is defined using linear regression analysis and is defined by a 50% probability of survival. The design curves used in codes equals this mean curve minus two standard deviations as mentioned in eq. 2.7. This design curve corresponds to a probability of failure of 2.3%.

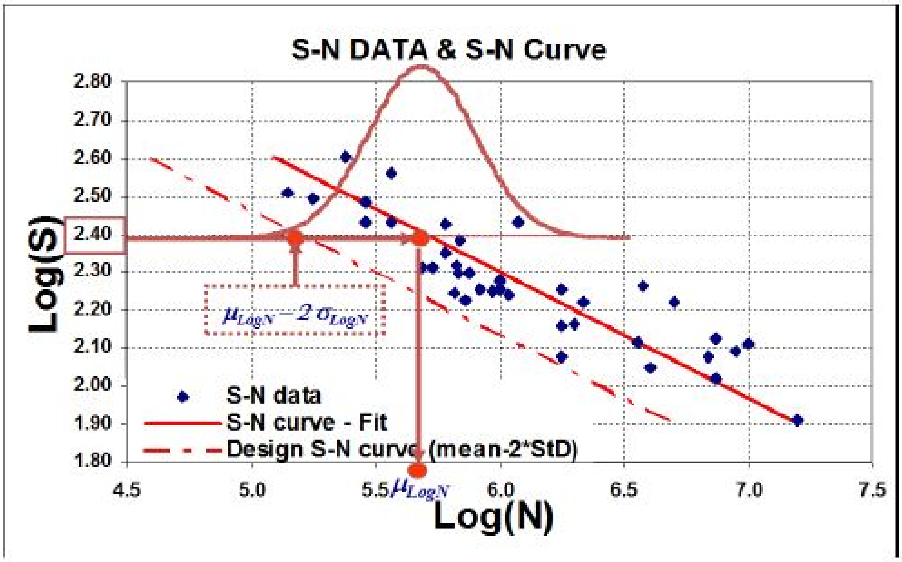


Figure 2.3: Scatter in S-N curves.

The endurance/threshold limit

As mentioned before, some materials under constant amplitude loading are said to have a fatigue limit or endurance limit which represents a stress level where the material supposedly does not fail and can be cycled infinitely.

The concept of an endurance limit is used in infinite-life or safe stress designs. It is due to interstitial elements (such as carbon or nitrogen in iron) that pin dislocations, thus preventing the slip mechanism that leads to the formation of microcracks. Care must be taken when using an endurance limit in design applications because it can disappear due to:

- Periodic overloads (unpin dislocations)
- Corrosive environments (due to fatigue corrosion interaction)
- High temperatures (mobilize dislocations)

The endurance limit is not a true property of a material, since other significant influences such as surface finish cannot be entirely eliminated. However, a test values obtained from polished specimens provide a baseline to which other factors can be applied. Influences that can affect the endurance limit include:

- Surface finish
- Temperature
- Stress concentration

- Notch sensitivity
- Size
- Environment

2.5.2 Stress modelling

As mentioned above, most S-N curves in rules and regulations are based on tests using constant amplitude. A welded detail in a structure will usually be subjected to a variable amplitude loading.

Stress spectrum to histogram

It is often possible to present the variable stress spectrum on a histogram format i.e. in terms of stress blocks where each block is defined by its stress range $\Delta\sigma_i$ and corresponding number of cycles n_i. Since we do not know the fatigue strength for a detail when subjected to a variable load spectrum, we have to use the S-N curves that are based on constant amplitude. In order to make life predictions for variable loads we assume that each individual stress block contributes to the fatigue damage according to its damage ratio n_i/N_i. The nominator n_i is the number of cycles to failure actually occurring, whereas the denominator N_i is the number of cycles to failure according to the S-N curve for the actual stress range. It is further assumed that the total damage caused by all stress blocks accumulates linearly.

2.5.3 Linear damage accumulation

All damage calculations in this thesis is based on the assumption of linear damage accumulation. The validity of this assumption has often been questioned. One of the consequences of this assumption is that the order of the stress blocks does not matter. As more variable amplitude testing data has become available it has been shown that the chronological order of the stress blocks is important. The standard case is that a stress block with a low stress range may, in the beginning, be inferior to the constant amplitude fatigue limit, and therefore not contribute to fatigue damage. However, if this stress block appears after several of the other more severe stress blocks, these blocks may have created a crack and the fatigue limit is no longer valid. The detail has become more vulnerable to fatigue damage and the stress block may now contribute to the fatigue damage. As a consequence, a constant amplitude S-N curve with a fatigue limit cannot be used in the fatigue limit area for variable amplitude loading. A conservative approach is to neglect the fatigue limit all together and draw one line from the finite-life area down towards the zero stress range without changing the inverse slope *m*. The approach used in most rules and regulations is to change the inverse slope *m* to 5 from the finite area and down.

2.6 Deterministic assessment method - Nominal stress

This chapter describes the general assessment method using nominal stresses. The first method is based on procedures described in standards like DNV-RP-C203 (DNV, 2014) (also known as DNVGL-RP-0005) and NS-EN 1993-1-9 (ECS, 2010b). In this thesis this method will be called the ”Traditional approach”.

The second method is called the ”Closed form approach” and involves using a Weibull distribution to represent the stress spectrum, instead of the stress blocks used in the traditional approach.

2.6.1 Traditional approach

1. Detail class

For fatigue analysis based on the nominal stress approach, welded joints are divided into several classes. Each class has a designated S-N curve. The classification of S-N curves depends on the geometry of the detail, the direction of the fluctuating stress relative to the detail, and the method of fabrication and inspection of the detail. The types of joint, including plate-to-plate, tube-to-plate, and tube-to-tube connections have alphabetical classification types, where each type relates to a particular S-N relationship as determined by experimental fatigue tests.

The design S-N curves are based on characteristic values (the mean-minus-two-standard-deviation curves) for relevant experimental data similar to what is used for characteristic strength of the material. The S-N curves are thus associated with a 97.6% probability of survival. These S-N curves are based on constant amplitude loading.

For example, Norwegian and British codes reference the D curve for simple plate connections with the load transverse to the direction of the weld, and the T curve for tubular brace to chord connections.

Each construction detail, at which fatigue cracks may potentially develop, should be placed in its relevant joint class in accordance with criteria given in the codes. Fatigue cracks may develop in several locations, e.g. at the weld toe in each of the parts joined, at the weld root, and in the weld itself. Each location should be classified separately.

2. Evaluate environment

The reason we need to consider the effect of the environment on fatigue life has already been discussed in chapter 2.2.1.

3. Calculation of nominal stress

In simple components the nominal stress can be determined using elementary theories of structural mechanics based on linear-elastic behaviour.

In other cases, finite element method (FEM) modelling may be used. This is primarily the case in:

1. Complicated statically in-determined (redundant) structures
2. Structural components incorporating macro-geometric discontinuities, for which no analytical solutions are available

Using FEM, meshing can be simple and coarse. However, care must be taken to ensure that all stress raising effects of the structural detail of the welded joint are excluded when calculating the modified (local) nominal stress.

For variable amplitude loading, the different stresses can be modelled as a histogram with stress blocks where each block is defined by its stress range $\Delta\sigma_i$ and corresponding number of cycles n_i, see figure 2.4.

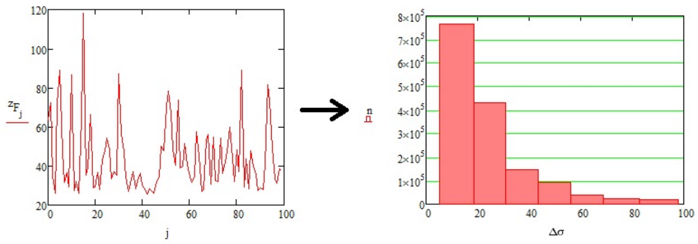


Figure 2.4: Variable amplitude stress spectrum to stress blocks in hisogram.

4. Determining cycles to failure from S-N curve

The basic design S-N curve is given as:

$$\log N = \log \bar{a} - m\ \log \Delta\sigma \tag{2.6}$$

Where $\Delta\sigma$ is the stress range, *N* is the predicted number of cycles to failure for stress range *S* and *m* is the negative inverse slope of the S-N curve. $log\bar{a}$ is the intercept of *log N*-axis by the S-N curve minus two standard deviations of *log N*:

$$\log \bar{a} = \log a - 2\, s_{logN} \tag{2.7}$$

5. Calculate damage and life

Many structures are subjected to a range of load fluctuations and frequencies. In order to predict the fatigue life of a structural detail subjected to a variable load history based on constant amplitude test data, a number of cumulative damage theories have been proposed. For instance, the Palmgren-Miner cumulative damage law (Miner, 1945) states that:

$$D = \sum_{i=1}^{k} \frac{n_i}{N_i} \leq 1 \tag{2.8}$$

Where *D* is the fatigue damage, *k* is the number of stress range levels in the block of load spectrum, n_i is the number of stress cycles with the stress level $\Delta\sigma_i$, N_i is the fatigue life at stress level $\Delta\sigma_i$ according to the formula $N_i = A \times \Delta\sigma_i^{-m}$.

The hypothesis of Miner is based on several assumptions:

- Sinusoidal load cycles

- Purely alternating load
- Crack initiation as the failure mode
- No contribution to damage by load cycles below the endurance limit
- Sequence of load cycles not considered

The Palmgren-Miner law is still widely applied in engineering due to its simplicity.

2.6.2 Closed form fatigue approach

The Weibull distribution can be fitted to a stress histogram with stress blocks and use this to modify the Palmgren-Miner cumulative damage law (2.8) to this:

$$D = \int_{\Delta\sigma=0}^{\infty} \frac{nf(\Delta\sigma)d\Delta\sigma}{\bar{a}/\Delta\sigma^m} \tag{2.9}$$

where $f(\Delta\sigma)$ is the frequency function (PDF) to the histogram which reads:

$$f(\Delta\sigma) = \frac{h}{q}\left(\frac{\Delta\sigma}{q}\right)^{h-1} e^{(-\frac{\Delta\sigma}{q})^h} \tag{2.10}$$

where *h* is the shape parameter and *q* is the scale parameter in the distribution. The integral of the damage can be solved by introducing the auxiliary variable $t = (\Delta\sigma/q)^h$. The integral can then be determined by using the well-known Gamma function:

$$\Gamma(\alpha) = \int_0^{\infty} t^{\alpha-1}e^{-t}dt \tag{2.11}$$

This function can be found in standard tables. Using equation ((2.11)) to solve equation (2.9) we get:

$$D = \frac{n}{\bar{a}}q^m\Gamma\left(1 + \frac{m}{h}\right) \tag{2.12}$$

This equation is valid for single slope S-N curves. In case of a bi-linear S-N curve we have to replace the Gamma function with the complementary Gamma function (2.13) and incomplete Gamma function ((2.14)):

$$\Gamma(\alpha; x) = \int_x^{\infty} t^{\alpha-1}e^{-t}dt \tag{2.13}$$

$$\gamma(\alpha; x) = \int_{\infty}^{x} t^{\alpha-1}e^{-t}dt \tag{2.14}$$

The damage ratio for a bi-linear S-N curve will read:

$$D = n\left[\frac{q^{m_1}}{\bar{a}_1}\Gamma\left(1+\frac{m_1}{h},\left(\frac{S_1}{q}\right)^h\right)+\frac{q^{m_2}}{\bar{a}_2}\gamma\left(1+\frac{m_2}{h},\left(\frac{S_1}{q}\right)^h\right)\right] \tag{2.15}$$

where

n = the total number of applied loading cycles

$\bar{a}_1, \bar{a}_2$ = is the intercept of *log N*-axis by the S-N curve for the upper and lower S-N line segment, respectively

m_1, m_2 = the slope of the upper and lower S-N line segment, respectively

S_1 = is the stress level at the change in slope of S-N curves (point of discontinuity)

The Weibull scale parameter q may be related to the most likely maximum stress range S_{max} occurring during a given number of cycles n:

$$q = \frac{S_{max}}{ln(n)^{1/h}} \tag{2.16}$$

The most likely maximum stress range has by definition a probability of exceedance equal to *1/n*. The number of cycles n must be large enough to characterize the loading process so that the scale parameter q becomes constant.

To get the cumulative distribution function (CDF) of the Weibull distribution, we have to integrate the frequency function ($f(\Delta\sigma)$):

$$F(\Delta\sigma) = \int_0^{\Delta\sigma} f(t)dt = 1 - e^{\left(\frac{t}{q}\right)^h} \tag{2.17}$$

The two parameters h and q can be related to the mean and the variance of $\Delta\sigma$ by the equations:

$$\mu_{\Delta\sigma} = q\,\Gamma(1+\frac{1}{h}) \tag{2.18}$$

$$sd^2_{\Delta\sigma} = q^2\left(\Gamma\left(1+\frac{1}{h}\right)+\left[\Gamma\left(1+\frac{1}{h}\right)\right]^2\right) \tag{2.19}$$

An approximation of the connection between the shape parameter h and the covariance of the distribution reads:

$$h \approx COV_t^{-1.08} \tag{2.20}$$

The method for fitting the Weibull distribution to the stress histogram used in this thesis is described in section 3.2.2.

2.7 Probabilistic assessment method

2.7.1 Fatigue reliability models

The calculation of the fatigue damage for a structural detail is based on several variables. Most of these variables are to some extent uncertain. In conventional fatigue analysis, characteristic values and safety factors are widely used to account for these uncertainties. The safety factors are rather subjective measures that are calibrated based on past experience. Information about the degree of uncertainty in the different variables cannot be accounted for effectively.

Reliability theory offers a way to include uncertainty information in the fatigue damage calculation. It makes it possible to calculate the component reliability, i.e. the probability that a detail has failed at the end of the specified lifetime. Using system reliability it is possible to evaluate the reliability of a system of structural details.

A probabilistic approach to fatigue life prediction consists of probabilistic methods applied in combination with either S-N approach or fracture mechanics approach. Probabilistic analysis in combination with the S-N approach is usually carried out at the structural design stage, while the probabilistic analysis of remaining life after inspection is usually based on fracture mechanics (FM) techniques. (Bai, 2003, 27.3)

Limit state function

The limit state function is defined as the function dividing the event space for the basic random variables into a failure zone and safe zone. The limit state function for fatigue reliability may simply be written:

$$g = \Delta - D \tag{2.21}$$

where Δ is the Miner sum at failure and *D* is the calculated damage. Failure is defined by the event given by $g \leq 0$, whereas $g > 0$ is considered the safe zone.

Inserting the simplest form of closed form damage calculation from eq. 2.12, the limit state function will be:

$$g = \Delta - \frac{n}{\bar{a}} q^m \Gamma\left(1 + \frac{m}{h}\right) \tag{2.22}$$

The objective when using the limit state function is to determine the probability of limit state failure (P_f) of the structure, usually by some form of simulation (Monte Carlo) or approximate analytical solution (FORM/SORM).

Because we are only interested in whether the limit state function is above zero or below zero (and not how much above or below zero it is), the limit state may also be written as:

$$g = \log \Delta - \log D \tag{2.23}$$

The simplest form of fatigue S-N curve representation is:

$$N = \frac{a}{\sigma^m} \tag{2.24}$$

which gives the following damage:

$$D = \frac{n}{N} = \frac{n\,\sigma^m}{a} \tag{2.25}$$

The limit state may then be written as:

$$g = \log \Delta - \log D = \log \Delta - \log n - m\ \log \sigma + \log a \tag{2.26}$$

If the random variables σ and a are independent log-normally distributed, then $\log \sigma, \log a$ and g are normal distributed and a simple numerical solution can be found to the safety index β and the respective probability of limit state failure P_f.

This is achieved by finding the mean and standard deviation of each individual variable *i*:

$$sd_{lni} = \sqrt{ln(1 + COV_i^2)} \tag{2.27}$$

$$\mu_{lni} = ln(\mu_i) - 0.5\,sd_{lni}^2 \tag{2.28}$$

Which gives the following mean and standard deviation of the limit state funtion *g*:

$$sd_g = \sqrt{m^2\,sd_{ln\sigma}^2 + sd_{lna}^2} \tag{2.29}$$

$$\mu_g = -m\,\mu_{ln\sigma} + \mu_{lnA} - ln(n) \tag{2.30}$$

The safety index and probability of limit state failure are then:

$$\beta = \frac{\mu_g}{sd_g} \tag{2.31}$$

$$P_f = \Phi(-\beta) \tag{2.32}$$

where $\Phi()$ is cumulative distribution function found in tables.

2.7.2 Target reliability index

In order to have a criteria to compare the probability of limit state failure found in calculations a target reliability index is defined.

Based on survival probability of 95%, a target reliability index (β) of 1.65 is assumed implying a failure probability of approximately 0.05. It is noted that a target reliability level may be determined according to the importance levels of respective structural details. (Frangopol and Kwon, 2010)

2.7.3 Simulation using Monte Carlo

In most cases the probability of limit state failure cannot be found using a simple numerical solution like the one described for the simplest form of S-N curves above. In those cases the random aspect of the problem can be dealt with by a simulations technique. This is done by repeating the calculation for various sets of the variables in i.e. the limit state function. Each set of these variables will contain values that are in accordance with the frequency functions (PDF) for the variables. This means that most of the sets will have values of the variables near the peak of each frequency function. Values found out on the tails of the frequency functions will appear less frequent, but these events can be even more important as they may lead to accelerated fatigue damage and reduced fatigue life. The calculation can practically be carried out by the method of Monte Carlo simulation.

Call one of these random variables z. If we know the frequency function (PDF) and cumulative density function (CDF) for this random variable, we can simulate realizations of this variable by generating a random number r having a continuous uniform distribution with possible outcomes in the range [0, 1]. It can then be shown that a number r defined by:

$$F(z) = r \ or \ z = F^{-1}(r) \tag{2.33}$$

is a random realization of the variable z according to its frequency function. $F(z)$ is the CDF function of the distribution.

The principle is illustrated in fig. 2.5. As can be seen from the sketch, the rectangular distributed r is generated on the vertical axis between 0 and 1 and shot into the back of the CDF curve. The realization of z is then found if we proceed vertically down the z-axis. If several realizations are carried out, most of the values will be close to the mean value of the z variable.

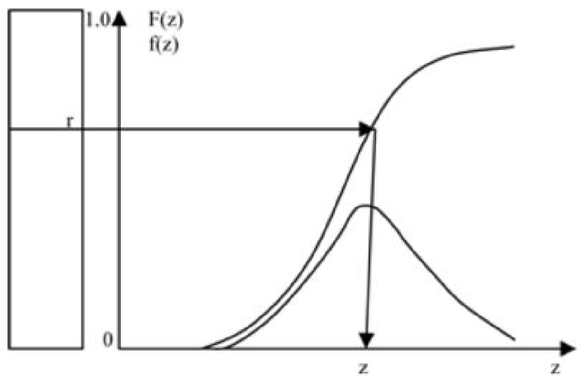


Figure 2.5: Random number realization for a given variable according to Monte Carlo method. (Lassen and Recho, 2006, Figure 7.7)

The purpose of these realizations of z is to apply the resulting values in calculations to obtain histograms of a result variables, such as fatigue life or the limit state function. Hence, the known simulated variables will enter into a calculation scheme, a main result of which is to give the calculated realization for this unknown result variable. The principle is shown in fig. 2.6

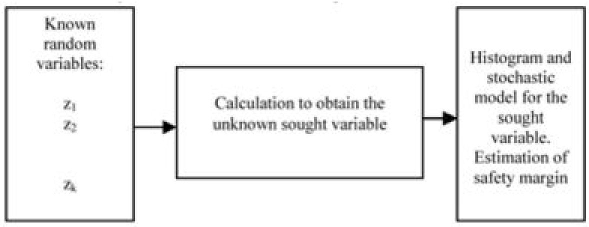


Figure 2.6: General procedure for determining the result variable.
(Lassen and Recho, 2006, Figure 7.8)

For a Weibull distributed variable, equation 2.33 can be inverted directly:

$$z = [q(-ln(1-r))^{1/h}] \tag{2.34}$$

For normal and log-normal distributed variables, the inverted equations are:

$$z = (\Phi(r)\, sd) + \mu \tag{2.35}$$

$$z = exp[(\Phi(r)\, sd_{ln}) + \mu_{ln}] \tag{2.36}$$

The probability failure is defined by:

$$P_f = \frac{N_f}{N} \tag{2.37}$$

where N is the number of simulations, and N_f is the number of times the result variable is less than a desired value. The desired value can be the design fatigue life or when the limit state function is below zero. The confidence in the probability of failure will vary depending on the number of simulations performed. The standard error of P_f is estimated by:

$$s = \sqrt{\frac{P_f\,(1-P_f)}{N}} \tag{2.38}$$

Chapter 3

Proposed assessment methods

3.1 Introduction

This chapter describes the methods that have been used to solve the method validation example and the two problems in the case study. These methods are:

- Traditional deterministic assessment method
- Closed form deterministic assessment method
- Probabilistic assessment method

A short description of the different software that has been used throughout the thesis is also provided.

Structural analysis using SAP2000

SAP2000 is an integrated software for structural analysis and design made by Computers & Software, Inc. (CSi). This software was used to draw and determine the stresses on the bridges in the case study, and was chosen because of its ability to use moving loads in the analysis. It also has an integrated design check in ultimate limit state, using the rules and regulations from Eurocode 3 (ECS, 2008). The process of drawing and analyzing the bridges is described for each bridge in chapter 5.

Calculation using Mathcad

Mathcad is a engineering calculation software made by Parametric Technology Corporation (PTC). This software was used for all the calculations and some simulations in both the validation example and the Case Study. It was chosen because of the writers knowledge in using it.

Simulation using R

R is a free software environment for statistical computing and graphics made by R Development Core Team. R was used to do simulations that Mathcad was not able to do.

3.2 Deterministic approach

3.2.1 Traditional approach

1. Determining detail class

According to the procedure for nominal stress method described in section 2.6, the first thing to decide is the detail class to find the correct S-N curve. The appropriate class is decided depending on what code you are using. In this thesis the S-N curves are taken from DNV-RP-C203, and the detail class for the detail is decided in Appendix A.

2. Evaluate environment and determine S-N curve

After determining the detail class for the detail, the environment around the structure needs to be evaluated. Depending on if the detail is in air, in seawater with cathodic protection or in seawater with free corrosion we select the appropriate S-N curve from the code. The different S-N curves in this thesis are found in (DNV, 2014, 2.4). The difference between i.e. in air and seawater with cathodic protection can be seen in the figures below:

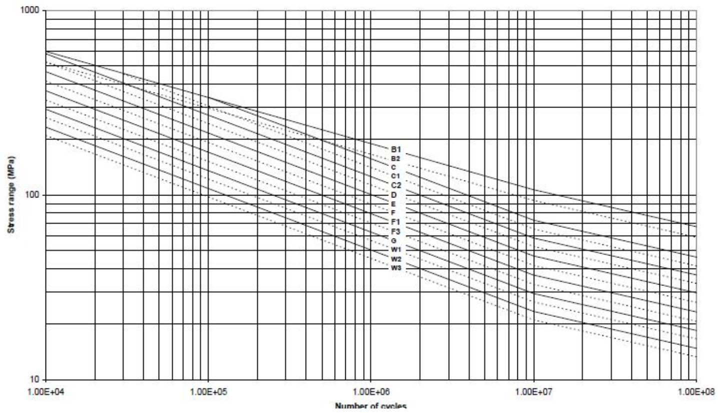


Figure 3.1: S-N curves in air. (DNV, 2014, Figure 2-8)

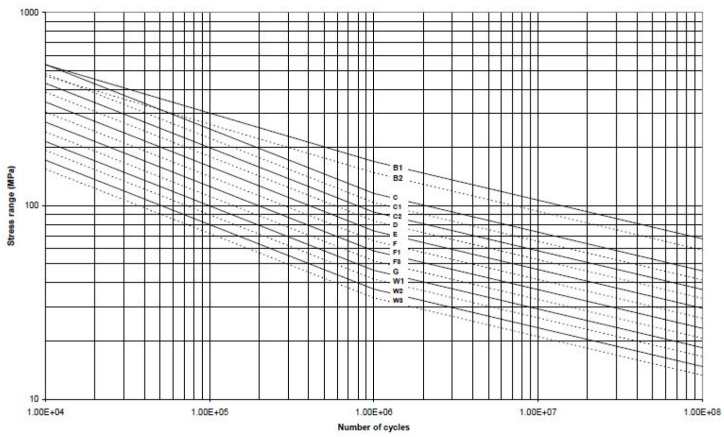


Figure 3.2: S-N curves in seawater with cathodic protection. (DNV, 2014, Figure 2-9)

S-N curve	$N \leq 10^7$ cycles		$N > 10^7$ cycles $\log \bar{a}_2$	Fatigue limit at 10^7 cycles *)	Thickness exponent k	Structural stress concentration embedded in the detail (S-N class), ref. also equation (2.3.2)
	m_1	$\log \bar{a}_1$	$m_2 = 5.0$			
B1	4.0	15.117	17.146	106.97	0	
B2	4.0	14.885	16.856	93.59	0	
C	3.0	12.592	16.320	73.10	0.05	
C1	3.0	12.449	16.081	65.50	0.10	
C2	3.0	12.301	15.835	58.48	0.15	
D	3.0	12.164	15.606	52.63	0.20	1.00
E	3.0	12.010	15.350	46.78	0.20	1.13
F	3.0	11.855	15.091	41.52	0.25	1.27
F1	3.0	11.699	14.832	36.84	0.25	1.43
F3	3.0	11.546	14.576	32.75	0.25	1.61
G	3.0	11.398	14.330	29.24	0.25	1.80
W1	3.0	11.261	14.101	26.32	0.25	2.00
W2	3.0	11.107	13.845	23.39	0.25	2.25
W3	3.0	10.970	13.617	21.05	0.25	2.50
T	3.0	12.164	15.606	52.63	0.25 for SCF ≤ 10.0 0.30 for SCF >10.0	1.00

Figure 3.3: S-N data in air. (DNV, 2014, Table 2-1)

S-N curve	$N \leq 10^6$ cycles		$N > 10^6$ cycles $\log \bar{a}_2$	Fatigue limit at 10^7 cycles*)	Thickness exponent k	Stress concentration in the S-N detail as derived by the hot spot method
	m_1	$\log \bar{a}_1$	$m_2 = 5.0$			
B1	4.0	14.917	17.146	106.97	0	
B2	4.0	14.685	16.856	93.59	0	
C	3.0	12.192	16.320	73.10	0.05	
C1	3.0	12.049	16.081	65.50	0.10	
C2	3.0	11.901	15.835	58.48	0.15	
D	3.0	11.764	15.606	52.63	0.20	1.00
E	3.0	11.610	15.350	46.78	0.20	1.13
F	3.0	11.455	15.091	41.52	0.25	1.27
F1	3.0	11.299	14.832	36.84	0.25	1.43
F3	3.0	11.146	14.576	32.75	0.25	1.61
G	3.0	10.998	14.330	29.24	0.25	1.80
W1	3.0	10.861	14.101	26.32	0.25	2.00
W2	3.0	10.707	13.845	23.39	0.25	2.25
W3	3.0	10.570	13.617	21.05	0.25	2.50
T	3.0	11.764	15.606	52.63	0.25 for SCF ≤ 10.0 0.30 for SCF >10.0	1.00
*) see also [2.11]						

Figure 3.4: S-N data in seawater with cathodic protection. (DNV, 2014, Table 2-2)

3. Calculate nominal stresses

The calculation of the nominal stresses is dependent on the scenario being evaluated. In the method validation example the stress ranges are already listed, and the two different scenarios in the case study uses the same procedure. Calculating the nominal stresses and the stress range in the case study is done the following way:

1. Determine location of detail to be evaluated.
2. Determine maximum and minimum moment at location of that detail. If axial forces are significant, they will also have to be determined.
3. Divide the moment range ΔM and eventually the axial force range ΔP by the section modulus W and section area A for the cross section to determine the stress range $\Delta\sigma$, see eq. 3.1 .

$$\Delta\sigma = \frac{\Delta P}{A} + \frac{\Delta M}{W} \tag{3.1}$$

4. Calculate cycles to failure

Next up is determining cycles to failure and the equation for this according to a basic S-N curve is already mentioned in Eq. (2.6). In DNV the S-N curves for in air and in seawater with CP are bi-linear, so we need to modify the equation as follows:

$$\log N = \log \bar{a_1} - m_1 \log \Delta\sigma \quad \text{if} \quad \Delta\sigma > S_1 \tag{3.2}$$

$$\log N = \log \bar{a_2} - m_2 \log \Delta\sigma \quad \text{if} \quad \Delta\sigma \leq S_1 \tag{3.3}$$

where $\bar{a_1}, \bar{a_2}, m_1, m_2$ and the point of discontinuity S_1 can be found in fig. 3.3 for details in air. S_1 is the value in the column called *"Fatiuge limit at* 10^7 *cycles"*. For details in seawater with CP it can be determined by the following equation:

$$S_1 = \left(\frac{N_{kp}}{\bar{a_1}}\right)^{-1/m_1} \tag{3.4}$$

where N_{kp} is the knee-point of the S-N curve where the $\bar{a_1}$ changes to $\bar{a_2}$. N_{kp} is 10^7 for in air and 10^6 for in seawater with CP.

5. Calculate damage

To find the damage accumulation we use the Palmgren-Miner cumaltive damage law already defined in eq.2.8:

Most designs are made using some sort of safety factor. According to (DNV, 2014, 2.2) the accumulated damage *D* needs to be smaller or equal to 1/Design Fatigue Factor (DFF) which can be obtained from (DNV, 2011, Section 6). This gives the new equation to find the damage:

$$D = \sum_{i=0}^{k} \frac{n_i}{N_i} \leq \frac{1}{DFF} \tag{3.5}$$

3.2.2 Closed form approach

As already mentioned in section 2.6.2, we can modify the cumulative damage law by fitting a Weibull distribution to the stress histogram. This can be achieved using different methods, but in this thesis eye-fitting is being used. First one has to find the cumulative distribution function (CDF) of the stress histogram. Then one creates the CDF for the Weibull distribution (2.17) using random values of the shape parameter *h* and scale parameter *q*. The next step is to adjust the two parameters until the two CDF's match to a satisfying degree.

After the two parameters *h* and *q* are determined insert them into eq.2.15. The variables $\bar{a_1}, \bar{a_2}, m_1, m_2$ and S_1 are the same as determined above in section 3.2. Also here, we need to check the damage using a safety factor. We use the same DFF as in eq. 3.5 and the damage with a Weibull distribution becomes:

$$D = n\left[\frac{q^{m_1}}{\bar{a_1}}\Gamma\left(1+\frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right) + \frac{q^{m_2}}{\bar{a_2}}\gamma\left(1+\frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right)\right] \leq \frac{1}{DFF} \tag{3.6}$$

3.3 Probabilistic approach

Before the probability of limit state failure can be found using Monte Carlo simulations, the first thing we need to do is to get the real mean *a* for the S-N curve. The mean *a* can be determined with the following equation:

$$\log a = \log \bar{a} + 2\, s_{logN} \tag{3.7}$$

where according to (DNV, 2014, D.5) the standard deviation s_{logN} is equal to 0.200 for the S-N curves in air and in seawater with CP. This will also move the knee-point of the curve by two standard deviations, changing the equation for the point of discontinuity from eq. 3.4 to:

$$S_{1m} = \left(\frac{\log N_{kp} + 2\, s_{logN}}{a_1} \right)^{-1/m_1} \tag{3.8}$$

3.3.1 Using Monte Carlo simulations

To find the probability of limit state failure P_f using Monte Carlo simulations, we solve the eq. 3.9 , N_{sim} times, until P_f converges to a finite value.

$$g = \Delta - n_0 \left[\frac{(B\,q)^{m_1}}{a_1} \Gamma\left(1 + \frac{m_1}{h}, \left(\frac{S_1}{B\,q}\right)^h\right) + \frac{(B\,q)^{m_2}}{a_2} \gamma\left(1 + \frac{m_2}{h}, \left(\frac{S_1}{B\,q}\right)^h\right)\right] \tag{3.9}$$

using random realizations of the variables a_1, a_2, S_1, B and Δ. Where *B* is a factor to account for uncertainties in the stress modelling. *B* is equal to 1 with a COV depending on how accurate and detailed the stress modelling is (Table 3.1). For the calculations in this thesis a COV of 0,3 is used.

Table 3.1: Levels of confidence in stress modelling. (Bai, 2003, Table 27.1)

Level of confidence	COV
Little	0,3
Reasonable	0,25
Moderate	0,20
Comprehensive	0,15

The realization of the random variables a_1, B and Δ can be determined as described in section 2.7.3 using the inverted CDF function for a log-normal distribution (2.36) and random number *r*. In this thesis it has been determined using a function integrated in the calculation software that has been used (Mathcad and R). This function does the same as above and returns a random value from the log-normal distribution and is defined as:

$$z = rlnorm(N_{sim}, \mu_{ln}, sd_{ln}) \tag{3.10}$$

where μ_{ln} is the mean and sd_{ln} is the standard deviation of the distribution.

To find a value of a_2 that gives a continuous curve with the random a_1, we need to find the correlation between them, which is:

$$a_{2i} = S_{1i}^2 \, a_{1i} \quad \text{if} \quad m_1 = 3 \tag{3.11}$$

$$a_{2i} = S_{1i} \, a_{1i} \quad \text{if} \quad m_1 = 4 \tag{3.12}$$

When functions for all the random variables are determined and eq. 3.9 has been simulated N_{sim} times, the final task to find P_f is to determine the number of times the limit state function g is below or equal to zero. The probability of limit state failure P_f is then:

$$P_f = \frac{SUM(g \leq 0)}{N_{sim}} \tag{3.13}$$

The standard error of P_f is estimated by:

$$s = \sqrt{\frac{P_f \, (1 - P_f)}{N_{sim}}} \tag{3.14}$$

A target reliability index β of 1,65 (Frangopol and Kwon, 2010) is used to compare with the probability of limit state failure. This gives a target probability of failure P_t equal to:

$$P_t = \Phi(-\beta) = 0,05 \tag{3.15}$$

Chapter 4

Method Validation Example

4.1 Introduction

This is a simple example that has been chosen to validate the chosen assessment methods in practice. It also provides another case to be used when comparing the different approaches.

4.1.1 Description of problem

A member of a railway bridge is connected by fillet welded lap joint. It is in air and the joint is external and accessible for inspection and repair in dry and clean conditions, which gives a DFF of 1. The stress ranges with related number of cycles are listed in Table 4.1. The number of years in service the number of cycles represent is not known.

Table 4.1: Stress range and cycles for Example.

Block no.	**Stress range** $(\Delta\sigma_i)$	**Cycles** (n_i)
1	12	765000
2	25	432000
3	37	145000
4	50	93000
5	62	39000
6	75	25000
7	90	20000

4.2 Calculation and results

4.2.1 Deterministic calculation

According to (DNV, 2014, Appendix A) the detail class for a fillet welded lap joint is W1, see Fig. 4.1

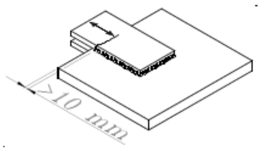
W1	4. 	4. Fillet welded overlap joint. Crack in overlapping plate.	4. — Stress to be calculated in the overlapping plate elements — Weld termination more than 10 mm from plate edge. — Shear cracking in the weld should be verified using detail 7.

Figure 4.1: Detail class for fillet welded lap joint. (DNV, 2014, Table A-8)

The bridge is in air and according to fig. 3.3 we get the following values for detail class W1:

$$a_1 := 10^{11.261} \qquad m_1 := 3$$

$$a_2 := 10^{14.101} \qquad m_2 := 5$$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 26.323$$

Figure 4.2: Variables from fig.3.3 for Example defined in Mathcad.

The calculation of number of cycles until failure and damage is shown below:

$$N_i := \begin{vmatrix} 10^{\log(a_1)-m_1\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2)-m_2\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{vmatrix}$$

$$D_d := \sum_{i=0}^{6} \frac{n_i}{N_i} \qquad D_d = 0.328$$

Figure 4.3: Mathcad calculation of the number of cycles and damage for Example.

4.2.2 Closed form calculation

Below is the CDF of stress spectrum and the eye-fitted CDF of the Weibull distribution, together with its respective shape parameter *h* and scale parameter *q*:

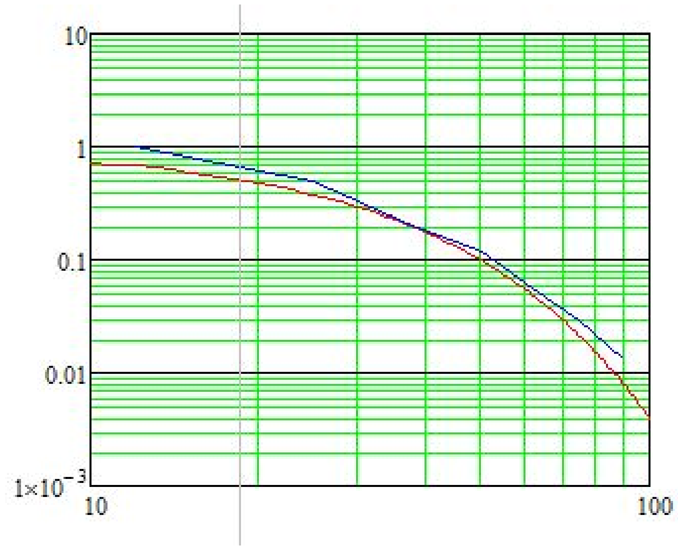


Figure 4.4: CDF of stress spectrum (blue line) and Weibull distribution (red line) for Example.

The corresponding closed form damage using the Weibull distribution is:

$$n_0 := \sum_{i=0}^{6} n_i = 1.519 \times 10^{6}$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right] \right]$$

$$D_{cf} = 0.404$$

Figure 4.5: Mathcad calculation of closed form damage for Example.

4.2.3 Probabilistic calculation

The mean S-N curve gives the following a_1 and a_2 together with the new formula for S_1:

$$sd_a := 0.2$$

$$a_{m1} := 10^{\log(a_1)+2 \cdot sd_a} = 4.581 \times 10^{11} \qquad a_{m2} := 10^{\log(a_2)+2sd_a} = 3.17 \times 10^{14}$$

$$S_{1m} := \left(\frac{10^{7+2 \cdot sd_a}}{a_{m1}} \right)^{\frac{-1}{m_1}} = 26.323$$

Figure 4.6: Mathcad calculation of mean values of S-N variables for Example.

The limit state function and the variables in it are listed below:

$$g = \Delta - n_0 \left[\frac{(B\,q)^{m_1}}{a_1} \Gamma\left(1 + \frac{m_1}{h}, \left(\frac{S_1}{B\,q}\right)^h\right) + \frac{(B\,q)^{m_2}}{a_2} \gamma\left(1 + \frac{m_2}{h}, \left(\frac{S_1}{B\,q}\right)^h\right)\right]$$

Table 4.2: Variables in Example.

Variable	**Value**	
a_1	Random	Log-normal
a_2	Eq. 3.11	Log-normal
S_1	Eq. 3.8	Deterministic
m_1	3	Deterministic
m_2	5	Deterministic
h	1,25	Deterministic
q	25,5	Deterministic
B	Random	Log-normal
Δ	Random	Log-normal
n_0	$1,519 \times 10^6$	Deterministic

where the mean (μ), standard deviation (sd) and covariance (COV) for the random variables are listed in table 4.3

Table 4.3: Random variables in Example.

Variable	**Mean**	**SD**	**COV**	**Distribution**
a_1	26,744	0,461	0,487	Log-normal
B	1	0,294	0,300	Log-normal
Δ	1	0,294	0,300	Log-normal

To make sure the different random S-N curves are continuous, a sample of the first 4 simulations has been checked.

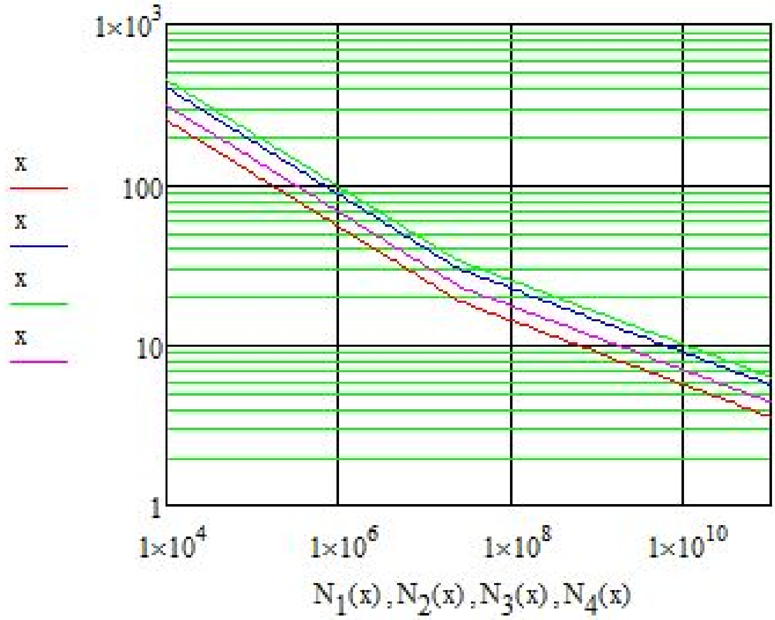


Figure 4.7: Check of random S-N curves for Example.

The limit state function is simulated N_{sim} times in R and the following results are obtained:

Table 4.4: Probabilities of limit state failure (P_f) for Example.

N_{sim}	$SUM(g \leq 0)$	P_f	s
10^4	531	$5,311 \times 10^{-2}$	$2,242 \times 10^{-3}$
10^7	502556	$5,026 \times 10^{-2}$	$6,913 \times 10^{-5}$

The probability of failure (P_f) is approximately the same as the target probability of failure ($P_t = 5,0 \times 10^{-2}$).

The script used in R to find P_f for $N_{sim} = 10^7$ is:

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.519*10^6

q = 25.5
h = 1.25
m1 = 3
m2 = 5

mu_a1 = 26.744
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z^2*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))

g1 = z_delta - D

g0 = sum(g1<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Figure 4.8: Script in R for Example.

The factor for uncertainties in the stress modelling *B* is assumed to have a COV of 0,3 based on the confidence of the stress modelling. To check what effect a better stress modelling would have on the probability of failure, the other COV listen in table 3.1 are compared to a COV of 0,3:

Table 4.5: Probability of limit state failure for different COV of *B* in Example. $N_{sim} = 10^7$

COV	Probability of failure
0,30	$5,026 \times 10^{-2}$
0,25	$3,183 \times 10^{-2}$
0,20	$1,732 \times 10^{-2}$
0,15	$0,794 \times 10^{-2}$

Chapter 5

Case Studies: Fatigue assessment of Road Bridges

5.1 Introduction

This case study will examine the fatigue effects of traffic loads on a bridge. As mentioned in the introduction, only the dead load and traffic loads are considered. The case study is divided into two different cases; 1) a single-span bridge and 2) a three-span bridge. Both cases will be evaluated using both the deterministic and probabilistic approach. The goal with Case 1 is to get a better understanding of the basic effects of a moving load on a bridge and to establish a load model for a simple model. Case 2 is more complex and based on a real bridge. This real bridge is provided by Statens Vegvesen, but has been simplified to make the stress analysis easier. Both bridges are properly described in separate sections later. Both bridges are considered to be in air and the considered detail is external and accessible for inspection and repair in dry and clean conditions, which gives a DFF of 1.

5.2 Traffic load model

This thesis will examine two different load models for traffic:

- Fatigue load model 4 (FLM4) from NS-EN 1991-2 [4.6.5] (ECS, 2010a).

- An alternative version that tries to take the entire traffic into account.

5.2.1 Fatigue load model 4

Fatigue load model 4 consists of sets of standard trucks which together produce effects equivalent to those of typical traffic on European roads. The set of trucks with their axle loads, axle distance and probability of occurrence is defined in the tables below (ECS, 2010a, Table 4.7):

Table 5.1: Vehicle loads in FLM4.

Vechicle class	Axle 1 (kN)	Axle 2 (kN)	Axle 3 (kN)	Axle 4 (kN)	Axle 5 (kN)
Truck1 in FLM4	70	130	-	-	-
Truck2 in FLM4	70	120	120	-	-
Truck3 in FLM4	70	150	90	90	90
Truck4 in FLM4	70	140	90	90	-
Truck5 in FLM4	70	130	90	80	80

Table 5.2: Axle distance for vehicles classes in FLM4.

Vechicle class	1-2 (m)	2-3 (m)	3-4 (m)	4-5 (m)
Truck1 in FLM4	4,5	-	-	-
Truck2 in FLM4	4,2	1,3	-	-
Truck3 in FLM4	3,2	5,2	1,3	1,3
Truck4 in FLM4	3,4	6,0	1,8	-
Truck5 in FLM4	4,8	3,6	4,4	1,3

The traffic type in question is considered to be ”Medium distance” giving the following distribution:

Table 5.3: Distribution of the five trucks in FLM4.

Vechicle class	Probability (%)
Truck1	40
Truck2	10
Truck3	30
Truck4	15
Truck5	5

We consider the road in question to be a ”Main road with low flow rate of trucks” which gives an expected number of trucks per year and per slow lane to be (ECS, 2010a, Table 4.5):

$$N_{obs} = 0,125 \times 10^6$$

5.2.2 Alternative traffic load model

The alternative traffic load model is based on fatigue load model 4 from the Eurocode, but has been modified to include vehicles classes that represent a more realistic traffic. Six new categories that represent the rest of the traffic have been added. The number of vehicles expected to cross the bridge is determined based on the annual average density traffic (AADT) in the area of the bridges, which is assumed to be 8000. The expected number of vehicles per year and per slow lane is then:

$$N_{alt} = 0,5 \times 365 \times \text{AADT}$$

The different load categories are listed below. All axle loads, except trucks from FLM4, have been multiplied with a dynamic amplification factor ($\Delta\phi_{fat}$) of 1,3 as required by NS-EN 1991-2 [4.6.1 (6)] (ECS (2010a)). The trucks from FLM4 have already been multiplied with the dynamic amplification factor ϕ_{fat}.

Table 5.4: Vehicle loads in alternative model.

Vechicle class	**Axle 1 (kN)**	**Axle 2 (kN)**	**Axle 3 (kN)**	**Axle 4 (kN)**	**Axle 5 (kN)**
Kombi	12	12	-	-	-
Sedan	16	16	-	-	-
Stationwagon	19	19	-	-	-
SUV/Minivan	21	21	-	-	-
Pickup/Van	29	29	-	-	-
Tractor/Smaller trucks	64	64	-	-	-
Truck1 in FLM4	70	130	-	-	-
Truck2 in FLM4	70	120	120	-	-
Truck3 in FLM4	70	150	90	90	90
Truck4 in FLM4	70	140	90	90	-
Truck5 in FLM4	70	130	90	80	80

Table 5.5: Axle distance for vehicles classes in alternative model.

Vechicle class	**1-2 (m)**	**2-3 (m)**	**3-4 (m)**	**4-5 (m)**
Kombi	2,5	-	-	-
Sedan	2,9	-	-	-
Stationwagon	3,0	-	-	-
SUV/Minivan	3,1	-	-	-
Pickup/Van	3,2	-	-	-
Tractor/Smaller trucks	3,5	-	-	-
Truck1 in FLM4	4,5	-	-	-
Truck2 in FLM4	4,2	1,3	-	-
Truck3 in FLM4	3,2	5,2	1,3	1,3
Truck4 in FLM4	3,4	6,0	1,8	-
Truck5 in FLM4	4,8	3,6	4,4	1,3

According to traffic data from Statens Vegvesen (Vegvesen (2015b)) the percentage of heavy vehicles varies from 5% to 20% depending on the AADT and location of the road. Based on the assumed AADT on the bridge an estimated representation of the five vehicles from FLM4 would be in the region of 8-15%. Based on the assumed AADT and using the expected number of trucks in FLM4 above, the percentage of heavy vehicles in FLM4 is 8,56%. Three different scenarios will be examined to compare the two load models and to check how an increase or decrease of heavy vehicles affect the fatigue damage:

- **Scenario 1**: 4% (Decrease)
- **Scenario 2**: 8,56% (Comparison with FLM4)
- **Scenario 3**: 15% (Increase)

The probabilities of the six other vehicle categories have been roughly estimated based on a statistic of the distribution of registered vehicles in Norway according to Statens Vegvesen (Vegvesen (2015a)), and has the following distribution:

Table 5.6: Distribution of the six first vehicle categories.

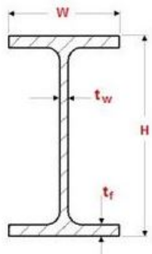
Vechicle class	Probability (%)
Kombi	22
Sedan	22
Stationwagon	19
SUV/Minivan	16,5
Pickup/Van	12,5
Tractor/Smaller trucks	8

5.3 Case 1 - Single-span bridge

5.3.1 Description

The single-span bridge in this case is for simplicity assumed to be just a single-span beam. This is not a realistic representation of a bridge, but the method of analyzing the stress is similar to a realistic bridge, just simpler. The bridge is also assumed to have only one lane.

The single-span beam used in this case is assumed to be continuous with a length of 34 meters. It was drawn in SAP2000 as a simply supported beam divided into 34 frames of 1 m, to provide a more accurate calculation. The beam is laterally braced at $L_c = 8$ m and ultimate limit state capacity has been checked using Load Model 1 from NS-EN 1991-2 [4.3.2] (ECS (2010a)). The beam is class 3 and has the following cross section:



H	1350 mm
W	650 mm
t_f	40 mm
t_w	20 mm
W_{el}	$3{,}71 \times 10^7$ mm^3

Figure 5.1: Cross section for beam in Case 1.

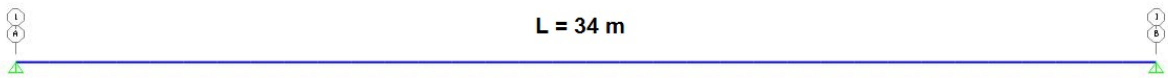


Figure 5.2: Span for beam in Case 1.

5.3.2 Load model

The traffic load vehicles in section 5.2 is defined in SAP2000 as moving loads crossing the bridge in combination with the dead load of the beam. It is assumed that just one vehicle crosses the bridge at a time.

Since the bridge is considered to have only one lane, we only use half of the original AADT. The design life considered is 100 years, which gives a total number of vehicles passing over the bridge equal to:

Table 5.7: Number of cycles for each load model in Case 1.

Load model	Cycles
FLM4	$1,25 \times 10^7$
Alternative	$1,46 \times 10^8$

The combined probability of occurrence for the four different scenarios are:

Table 5.8: Probability of occurrence for different scenarios in Case 1.

Vechicle class	Scenario 1 (%)	Scenario 2 (%)	Scenario 3 (%)	FLM4 (%)
Kombi	21,12	20,1168	18,70	-
Sedan	21,12	20,1168	18,70	-
Stationwagon	18,24	17,3736	16,15	-
SUV/Minivan	15,84	15,0876	14,025	-
Pickup/Van	12,00	11,43	10,625	-
Tractor/Smaller trucks	7,68	7,3152	6,80	-
Truck1 in FLM4	1,60	3,424	6,00	40
Truck2 in FLM4	0,40	0,856	1,50	10
Truck3 in FLM4	1,20	2,568	4,50	30
Truck4 in FLM4	0,60	1,284	2,25	15
Truck5 in FLM4	0,20	0,428	0,75	5

5.3.3 Calculations and results

Determining stress range

Since the bridge is one continuous simply supported beam, there is no effect from axial forces. Therefore, the point of interest for fatigue analysis will be where the moment range is the highest. This is as fig. 5.3 shows at its mid-span and using SAP2000, the minimum and maximum moments from the moving loads are determined at this location. The moment range is divided by the section of modulus (W_{el}) of the cross section to obtain the stress range. W_{el} is listed in fig. 5.1

Table 5.9: Moment and stress range for Case 1.

Vechicle class	M_{max} (kNm)	M_{min}(kNm)	$\Delta\sigma$(MPa)
Kombi	1335,45	1162,20	4,547
Sedan	1395,45	1162,20	6,122
Stationwagon	1441,20	1162,20	7,323
SUV/Minivan	1471,20	1162,20	8,110
Pickup/Van	1578,00	1162,20	10,913
Tractor/Smaller trucks	1955,19	1162,20	20,813
Truck1 in FLM4	2704,66	1162,20	40,485
Truck2 in FLM4	3572,15	1162,20	63,253
Truck3 in FLM4	4467,62	1162,20	86,756
Truck4 in FLM4	3737,14	1162,20	67,584
Truck5 in FLM4	4055,12	1162,20	75,930

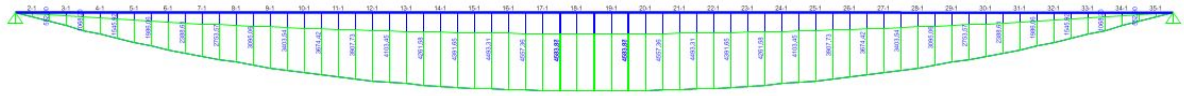


Figure 5.3: Moment diagram for bridge in Case 1.

Deterministic approach (traditional and closed form)

According to (DNV, 2014, Appendix A) the detail class for a rolled section is B1, see Fig. 5.4

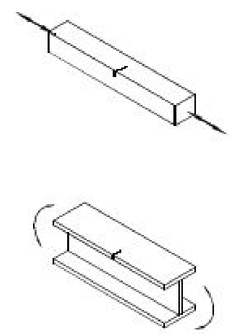
B1	1.  2.	1. Rolled or extruded plates and flats 2. Rolled sections	1. to 2. — Sharp edges, surface and rolling flaws to be improved by grinding. — For members that can acquire stress concentrations due to rust pitting etc. curve C is required.

Figure 5.4: Detail class for rolled section. (DNV, 2014, Table A-1)

The bridge is in air and according to fig. 3.3 we get the following values for detail class B1:

$$a_1 := 10^{15.117} = 1.309 \times 10^{15} \qquad m_1 := 4$$

$$a_2 := 10^{17.146} = 1.4 \times 10^{17} \qquad m_2 := 5$$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 106.967$$

Figure 5.5: Variables from fig.3.3 for Case 1 defined in Mathcad.

Below is the CDF of stress spectrum for the three scenarios and their eye-fitted CDF of the Weibull distribution. To establish some consistency with the eye-fitting of each scenario, the target has been to find a shape parameter h and a scale parameter q that gives a closed form damage equal 1,25 times larger than that acquired using the traditional method.

$$D_{cf} \approx 1,25\, D_t$$

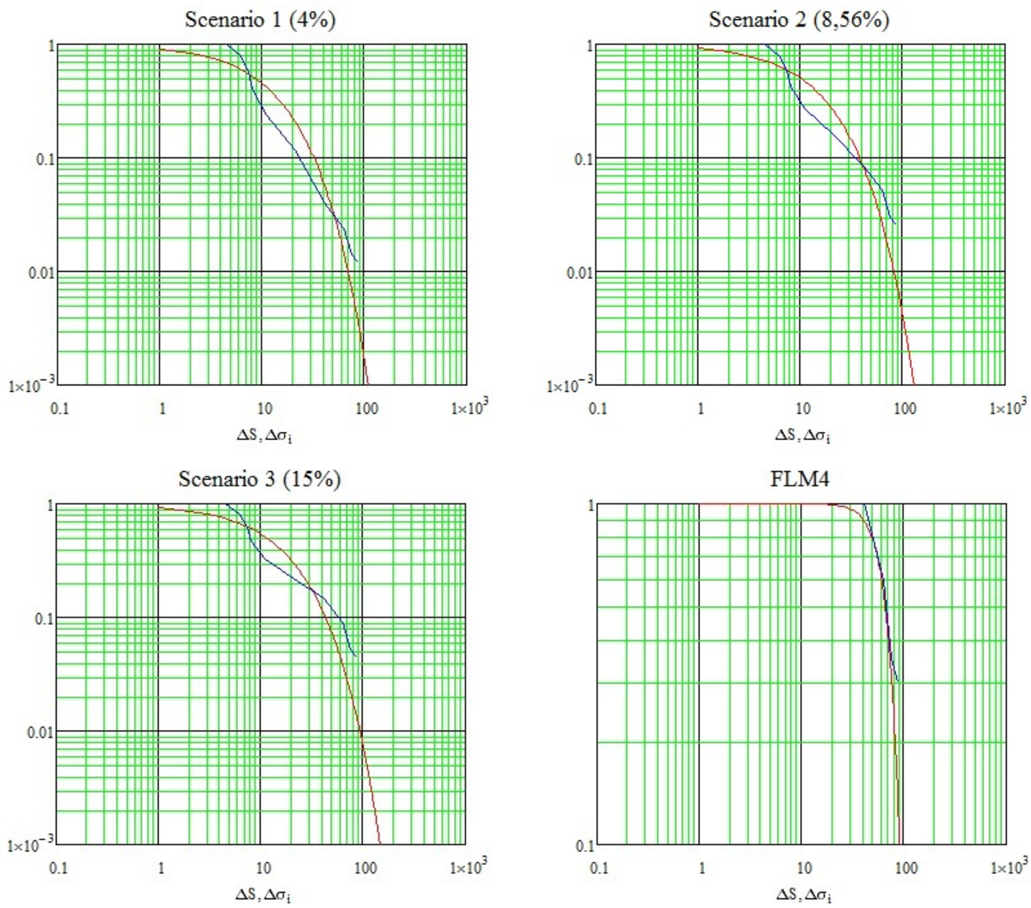


Figure 5.6: CDF of stress spectrum (blue lines) and Weibull distribution (red lines) for Case 1.

Table 5.10: Shape (*h*) and scale (*q*) parameter for each scenario in Case 1.

Parameter	Scenario 1	Scenario 2	Scenario 3	FLM4
h	0,90	0,90	0,90	3,75
q	12,75	15,10	17,10	73,00

The calculated damage for each scenario is:

Table 5.11: Accumulated damage for each scenario in Case 1.

Scenario	Traditional	Closed form
1 (4%)	0,082	0,101
2 (8,56%)	0,1751	0,219
3 (15%)	0,307	0,382
FLM4	0,1748	0,219

Probabilistic calculation

The mean S-N curve gives the following a_1 and a_2 together with the new formula for S_1:

$$sd_a := 0.2$$

$$a_{m1} := 10^{\log(a_1)+0.4} = 3.289 \times 10^{15} \qquad a_{m2} := 10^{\log(a_2)+0.4} = 3.516 \times 10^{17}$$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 106.967$$

Figure 5.7: Mathcad calculation of mean values for S-N variables for Case 1.

The limit state function and the variables in it are listed below:

$$g = \Delta - n_0 \left[\frac{(B\,q)^{m_1}}{a_1} \Gamma\left(1 + \frac{m_1}{h}, \left(\frac{S_1}{B\,q}\right)^h\right) + \frac{(B\,q)^{m_2}}{a_2} \gamma\left(1 + \frac{m_2}{h}, \left(\frac{S_1}{B\,q}\right)^h\right)\right]$$

Table 5.12: Variables in Case 1.

Variable	Value	
a_1	Random	Log-normal
a_2	Eq. 3.12	Log-normal
S_1	Eq. 3.8	Deterministic
m_1	4	Deterministic
m_2	5	Deterministic
h	Tab. 5.10	Deterministic
q	Tab. 5.10	Deterministic
B	Random	Log-normal
Δ	Random	Log-normal
n_0	Tab. 5.7	Deterministic

where the mean (μ), standard deviation (sd) and covariance (COV) for the random variables are listed in table 5.13

Table 5.13: Random variables in Case 1.

Variable	Mean	SD	COV	Distribution
a_1	35,623	0,461	0,487	Log-normal
B	1	0,294	0,300	Log-normal
Δ	1	0,294	0,300	Log-normal

To make sure the different random S-N curves are continuous, a sample of the first 4 simulations has been checked.

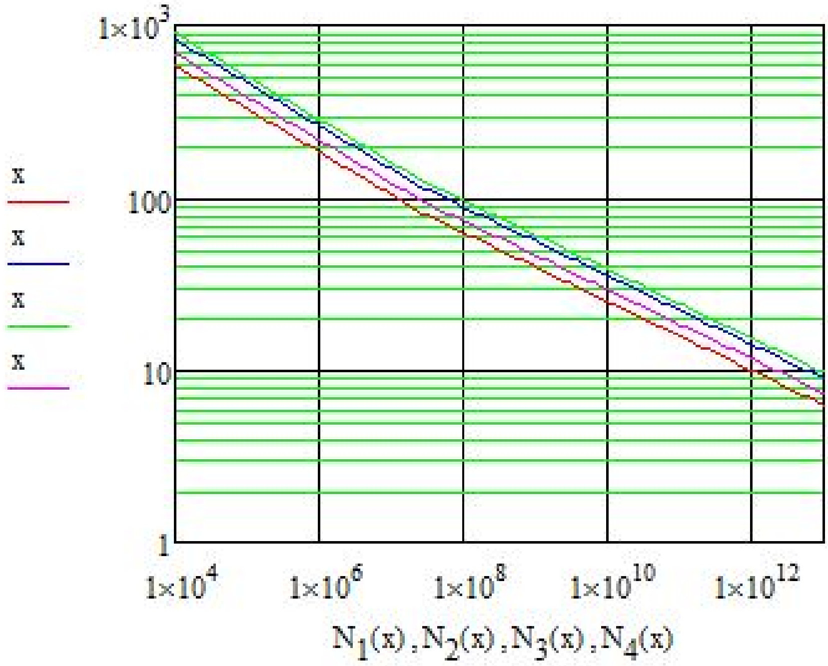


Figure 5.8: Check of random S-N curves Case 1.

The limit state function is simulated N_{sim} times and the following results are obtained:

Table 5.14: Probabilities of limit state failure (P_f) Case 1.

Scenario	N_{sim}	$SUM(g \leq 0)$	P_f	s
1	10^4	147	$1,47 \times 10^{-2}$	$1,204 \times 10^{-3}$
1	10^7	143795	$1,44 \times 10^{-2}$	$3,765 \times 10^{-5}$
2	10^4	452	$4,52 \times 10^{-2}$	$2,078 \times 10^{-3}$
2	10^7	480255	$4,80 \times 10^{-2}$	$6,761 \times 10^{-5}$
3	10^4	997	$9,97 \times 10^{-2}$	$2,983 \times 10^{-3}$
3	10^7	1004634	$10,05 \times 10^{-2}$	$9,506 \times 10^{-5}$
FLM4	10^4	530	$5,30 \times 10^{-2}$	$2,240 \times 10^{-3}$
FLM4	10^7	523634	$5,24 \times 10^{-2}$	$7,044 \times 10^{-5}$

5.3.4 Comparison

The estimated fatigue life for the closed form damage calculations can be found when D is equal to 1. A graph with the accumulated damage versus years in service for each scenario is defined in fig. 5.9. The estimated fatigue life for each scenario is listed in table 5.15

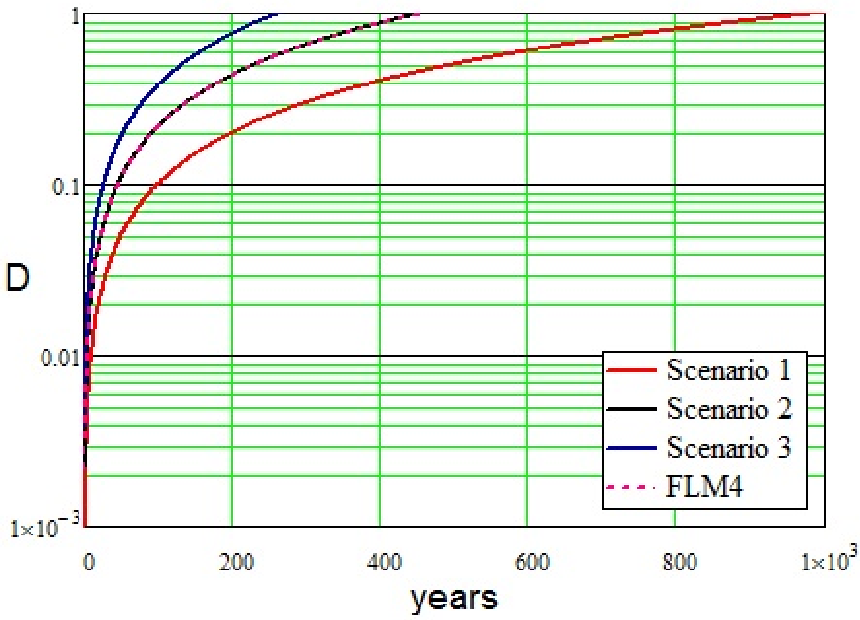


Figure 5.9: Accumulated damage versus time in Case 1.

Table 5.15: Deterministic estimated fatigue life for scenarios in Case 1.

Scenario	Estimated fatigue life (years)
1	991
2	457
3	262
FLM4	457

The estimated fatigue life for the probabilistic approach is where the probability of failure P_f is equal to the target probability of failure P_t. The probability of failure versus years in service for each scenario is defined in fig. 5.10. The probabilistic estimated fatigue life with the corresponding deterministic damage for each scenario is listed in table 5.16.

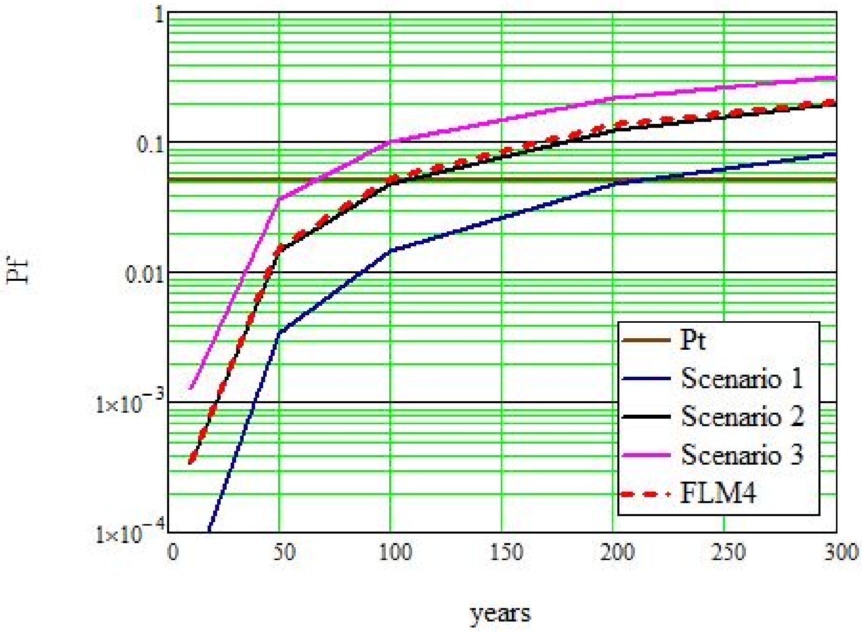


Figure 5.10: Probability of failure versus time in Case 1.

Table 5.16: Probabilistic estimated fatigue life and corresponding deterministic damage for scenarios in Case 1. $N_{sim} = 10^7$

Scenario	**Estimated fatigue life (years)**	**Corresponding damage**
1	211	0,213
2	103	0,226
3	61	0,233
FLM4	98	0,214

The factor for uncertainties in the stress modelling *B* is assumed to have a COV of 0,3 based on the confidence of the stress modelling. To check what effect a better stress modelling would have on the probability of failure, the other values of the COV, listed in table 3.1, are compared to a COV of 0,3 for Scenario 2:

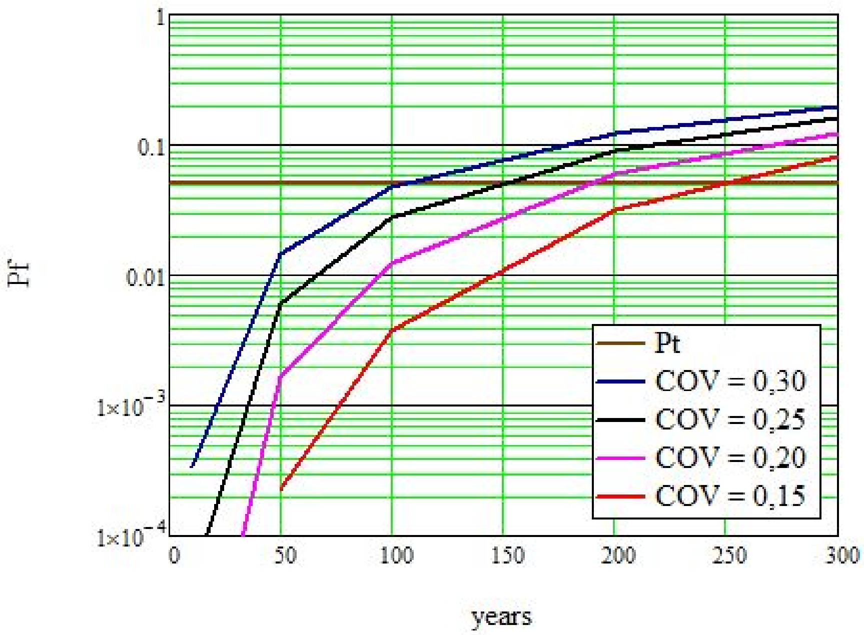


Figure 5.11: Effect of change in COV of factor *B* for Scenario 2 in Case 1.

Table 5.17: Probabilistic estimated fatigue life and corresponding deterministic damage for different COV of *B* for Scenario 2 in Case 1. $N_{sim} = 10^7$

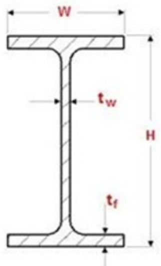
COV	Estimated fatigue life (years)	Corresponding damage
0,30	103	0,226
0,25	139	0,304
0,20	184	0,403
0,15	241	0,528

5.4 Case 2 - Three-span bridge

5.4.1 Description

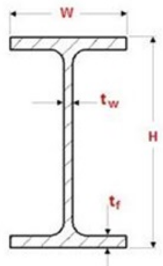
The bridge in Case 2 is a three-span bridge with a concrete deck on top of two steel girders. The bridge is 90 meters long, where the first and third spans are 28 meters long, while the mid-span is 34 meters long. The concrete deck is 10 meters wide with a 0,5 meter wide railing on each side. The distance between the two steel girders is 5,7 meters. Lateral bracing between the two girders is located at every 7 meters for span one and three, and at every 6,8 meters for the mid-span. The bridge is divided into two lanes that are 3 meters wide, with 1,5 meter wide road shoulders on each side of the lanes. The bridge consists of two different cross sections and their locations on the bridge, as well as their properties is shown below:

Cross section 1



H	1500 mm
W	600 mm
t_f	40 mm
t_w	20 mm
A	$7{,}64 \times 10^4$ mm^2
W_{el}	$4{,}05 \times 10^7$ mm^3

Cross section 2



H	1500 mm
W	800 mm
t_f	40 mm
t_w	20 mm
A	$9{,}24 \times 10^4$ mm^2
W_{el}	$5{,}18 \times 10^7$ mm^3

Figure 5.12: Cross section for beams in Case 2.

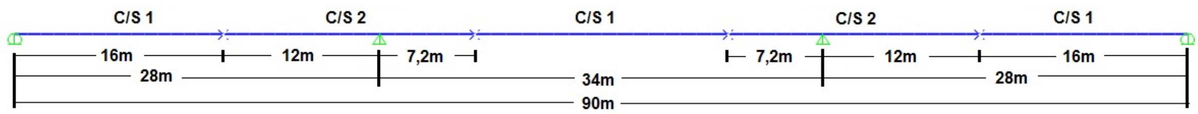


Figure 5.13: Dimensions of the bridge in Case 2.

The center line of the two lanes is 1,35m from the girders as shown below:

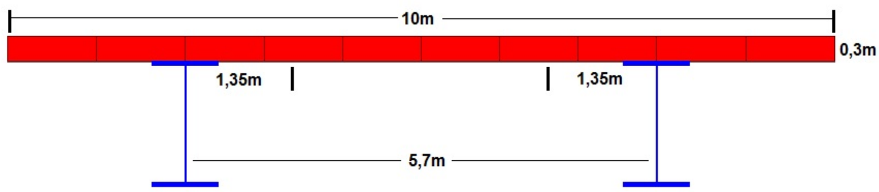


Figure 5.14: Section of the cross section of the girders and deck in Case 2.

5.4.2 Load model

The traffic load vehicles from section 5.2 are again defined in SAP2000 as moving loads crossing the bridge in combination with the dead load of the bridge. Because of symmetry in the section of the bridge (fig. 5.14) we only need to examine one of the girders.

Fatigue load model 4 is performed in the same manner as in Case 1, with the same distribution and axle loads. Each truck in FLM4 is considered to cross the bridge in the absence of any other vehicle. (ECS, 2010a, 4.6.5 (3))

For the alternative load model there are some modifications. On this bridge, if we assume a vehicle is crossing it in the right lane according to fig. 5.14, we get the following distribution of the load onto the two girders:

Table 5.18: Distribution of the load on girders.

Girder	Formula	Distribution (%)
Left	$1,35/5,7$	≈ 25
Right	$(5,7 - 1,35)/5,7$	≈ 75

Since almost 25% of the load will go to the left girder, we need to account for the load effect a vehicle crossing in the opposite lane of the girder we are examining. To account for this effect, we add eleven more load cases where both lanes are loaded with the same vehicle type, resulting in a total of twenty-two load cases.

The AADT is 8000 and we assume the number of vehicles crossing the bridge in one of the lanes is half of this. It is also assumed that half of these are with just one lane loaded, while the other half is with both lanes loaded. The design life considered is 100 years, which gives a total number of vehicles passing over the bridge equal to:

Table 5.19: Number of cycles for each load model in Case 2.

Load model	Cycles
FLM4	$1,25 \times 10^7$
Alternative	$1,46 \times 10^8$

The combined probability of occurrence for the four different scenarios are:

Table 5.20: Probability of occurrence for different scenarios in Case 2.
(1) One lane loaded (2) Both lanes loaded.

Vechicle class	Scenario 1 (%)	Scenario 2 (%)	Scenario 3 (%)	FLM4 (%)
Kombi (1)	10,56	10,0584	9,35	-
Sedan (1)	10,56	10,0584	9,35	-
Stationwagon (1)	9,12	8,6868	8,075	-
SUV/Minivan (1)	7,92	7,5438	7,0125	-
Pickup/Van (1)	6,00	5,715	5,3125	-
Tractor/Smaller trucks (1)	3,84	3,6576	3,40	-
Truck1 in FLM4 (1)	0,80	1,712	3,00	40
Truck2 in FLM4 (1)	0,20	0,428	0,75	10
Truck3 in FLM4 (1)	0,60	1,284	2,25	30
Truck4 in FLM4 (1)	0,30	0,642	1,125	15
Truck5 in FLM4 (1)	0,10	0,214	0,375	5
Kombi (2)	10,56	10,0584	9,35	-
Sedan (2)	10,56	10,0584	9,35	-
Stationwagon (2)	9,12	8,6868	8,075	-
SUV/Minivan (2)	7,92	7,5438	7,0125	-
Pickup/Van (2)	6,00	5,715	5,3125	-
Tractor/Smaller trucks (2)	3,84	3,6576	3,40	-
Truck1 in FLM4 (2)	0,80	1,712	3,00	-
Truck2 in FLM4 (2)	0,20	0,428	0,75	-
Truck3 in FLM4 (2)	0,60	1,284	2,25	-
Truck4 in FLM4 (2)	0,30	0,642	1,125	-
Truck5 in FLM4 (2)	0,10	0,214	0,375	-

5.4.3 Calculations and results

Determining stress range

To determine which point on the bridge most exposed to fatigue, several different points were examined. The points of interest and their respective stress range when exposed to the highest load case (Truck 3 in FLM4 (2)) is listed below. These points are either places where the moment/axial forces are at its highest or where a weld is located.

Table 5.21: List of the points of interest for fatigue effects.

Position x (m)	Detail class	Stress range (MPa)
11	B1	46,9
16	**E**	**49,0**
28	B1	25,9
35,2	E	36,0
45	B1	44,5

Table 5.21 shows that the point most exposed to fatigue is at x=16m, where a weld between

the two cross-sections is located. Using SAP2000, the minimum and maximum moments and axial forces from the moving loads are determined at this location. The stress range is determined by inserting the moment and axial force range into equation 3.1 using the section of modulus (W_{el}) and area (A) of the smallest cross section (C/S 1). W_{el} and (A) is listed in fig. 5.12.

Table 5.22: Moment and stress range for Case 2.

Vechicle class	ΔP **(kN)**	ΔM**(kNm)**	$\Delta\sigma$(MPa)
Kombi (1)	71,80	43,30	2,009
Sedan (1)	94,24	57,35	2,650
Stationwagon (1)	111,46	67,83	3,134
SUV/Minivan (1)	122,73	74,71	3,451
Pickup/Van (1)	168,85	102,82	4,749
Tractor/Smaller trucks (1)	368,41	224,54	10,366
Truck1 in FLM4 (1)	576,34	361,34	16,466
Truck2 in FLM4 (1)	905,22	551,95	25,477
Truck3 in FLM4 (1)	1210,58	721,45	33,659
Truck4 in FLM4 (1)	935,67	570,92	26,344
Truck5 in FLM4 (1)	1019,38	611,87	28,451
Kombi (2)	102,06	61,83	2,863
Sedan (2)	134,40	81,40	3,769
Stationwagon (2)	159,10	96,36	4,462
SUV/Minivan (2)	175,30	106,19	4,916
Pickup/Van (2)	241,31	146,23	6,769
Tractor/Smaller trucks (2)	527,51	319,94	14,804
Truck1 in FLM4 (2)	822,16	508,56	23,318
Truck2 in FLM4 (2)	1287,48	780,94	36,134
Truck3 in FLM4 (2)	1761,77	1052,30	49,043
Truck4 in FLM4 (2)	1365,31	828,93	38,338
Truck5 in FLM4 (2)	1497,05	898,91	41,790

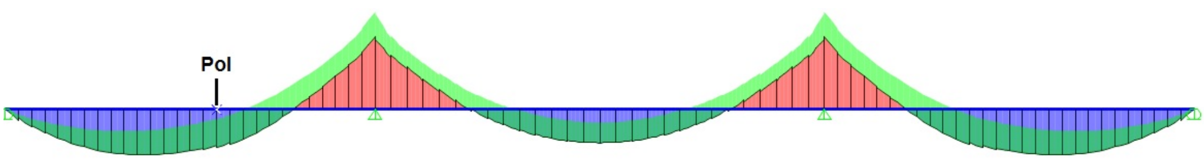


Figure 5.15: Moment diagram for bridge with point of interest in Case 2.

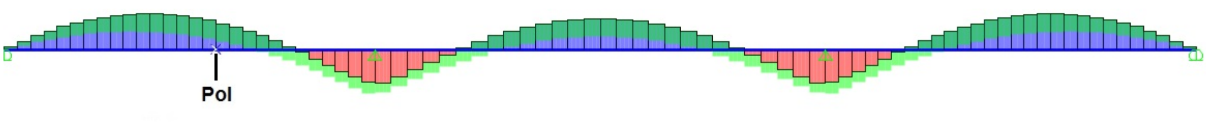


Figure 5.16: Axial force diagram for bridge with point of interest in Case 2.

Deterministic approach (traditional and closed form)

According to (DNV, 2014, Appendix A) the detail class for a rolled section is E, see Fig. 5.17

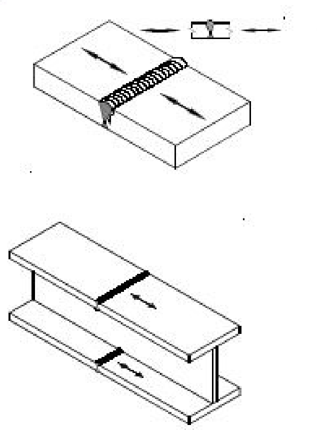
E	7. 	7. Transverse splices in plates, flats, rolled sections or plate girders made at site. (Detail category D may be used for welds made in flat position at site meeting the requirements under 4., 5. and 6 and when 100% MPI of the weld is performed.)	7. — The height of the weld convexity not to be greater than 20% of the weld width. — Weld run-off pieces to be used and subsequently removed. Plate edges to be ground flush in direction of stress.

Figure 5.17: Detail class for rolled section with transverse welds. (DNV, 2014, Table A-5)

The bridge is in air and according to fig. 3.3 we get the following values for detail class B1:

$$a_1 := 10^{12.01} = 1.023 \times 10^{12} \qquad m_1 := 3$$

$$a_2 := 10^{15.35} = 2.239 \times 10^{15} \qquad m_2 := 5$$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 46.774$$

Figure 5.18: Variables from fig.3.3 for Case 2 defined in Mathcad.

Below are the CDF of stress spectrums for the three scenarios and their eye-fitted CDF of the Weibull distribution. To establish some consistency with the eye-fitting of each scenario, the target has been to find a shape parameter *h* and a scale parameter *q* that gives a closed form damage equal 1,15 times larger than that acquired using the traditional method.

$$D_{cf} \approx 1,15\,D_t$$

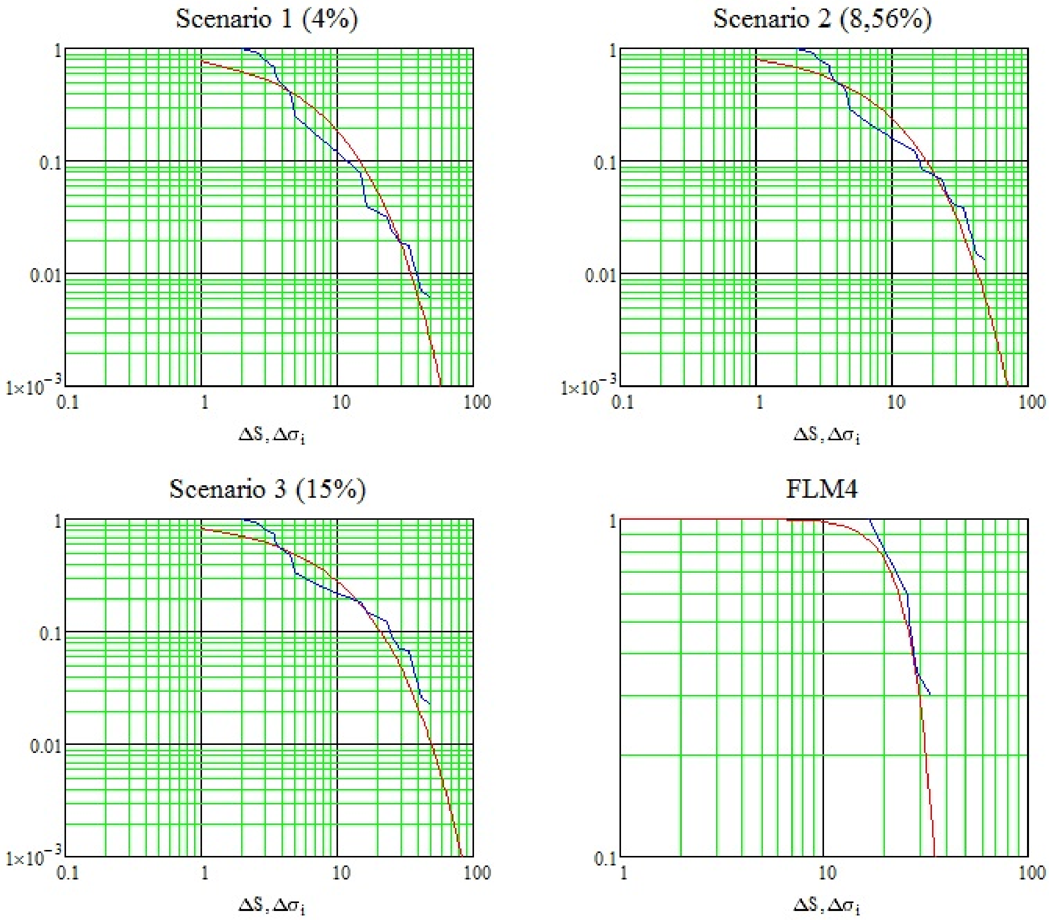


Figure 5.19: CDF of stress spectrum (blue lines) and Weibull distribution (red lines) for Case 2.

Table 5.23: Shape (h) and scale (q) parameters for each scenario in Case 2.

Parameter	Scenario 1	Scenario 2	Scenario 3	FLM4
h	0,8	0,8	0,8	3,75
q	5,21	6,33	7,36	27,85

The calculated damage for each scenario is:

Table 5.24: Accumulated damage for each scenario in Case 2.

Scenario	Traditional	Closed form
1 (4%)	0,162	0,186
2 (8,56%)	0,344	0,396
3 (15%)	0,601	0,692
FLM4	0,097	0,111

Probabilistic approach

The mean S-N curve gives the following a_1 and a_2 together with the new formula for S_1:

$$sd_a := 0.2$$

$$a_{m1} := 10^{\log(a_1)+0.4} = 2.57 \times 10^{12} \qquad a_{m2} := 10^{\log(a_2)+0.4} = 5.623 \times 10^{15}$$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 46.774$$

Figure 5.20: Mathcad calculation of mean values for S-N variables for Case 2.

The limit state function and the variables in it are listed below:

$$g = \Delta - n_0 \left[\frac{(B\,q)^{m_1}}{a_1} \Gamma\left(1 + \frac{m_1}{h}, \left(\frac{S_1}{B\,q}\right)^h\right) + \frac{(B\,q)^{m_2}}{a_2} \gamma\left(1 + \frac{m_2}{h}, \left(\frac{S_1}{B\,q}\right)^h\right)\right]$$

Table 5.25: Variables in Case 2.

Variable	Value	
a_1	Random	Log-normal
a_2	Eq. 3.11	Log-normal
S_1	Eq. 3.8	Deterministic
m_1	3	Deterministic
m_2	5	Deterministic
h	Tab. 5.23	Deterministic
q	Tab. 5.23	Deterministic
B	Random	Log-normal
Δ	Random	Log-normal
n_0	Tab. 5.19	Deterministic

where the mean (μ), standard deviation (sd) and covariance (COV) for the random variables are listed in table 5.13

To make sure the different random S-N curves are continuous, a sample of the first 4 simulations has been checked.

Table 5.26: Random variables in Case 2.

Variable	**Mean**	**SD**	**COV**	**Distribution**
a_1	28,469	0,461	0,487	Log-normal
B	1	0,294	0,300	Log-normal
Δ	1	0,294	0,300	Log-normal

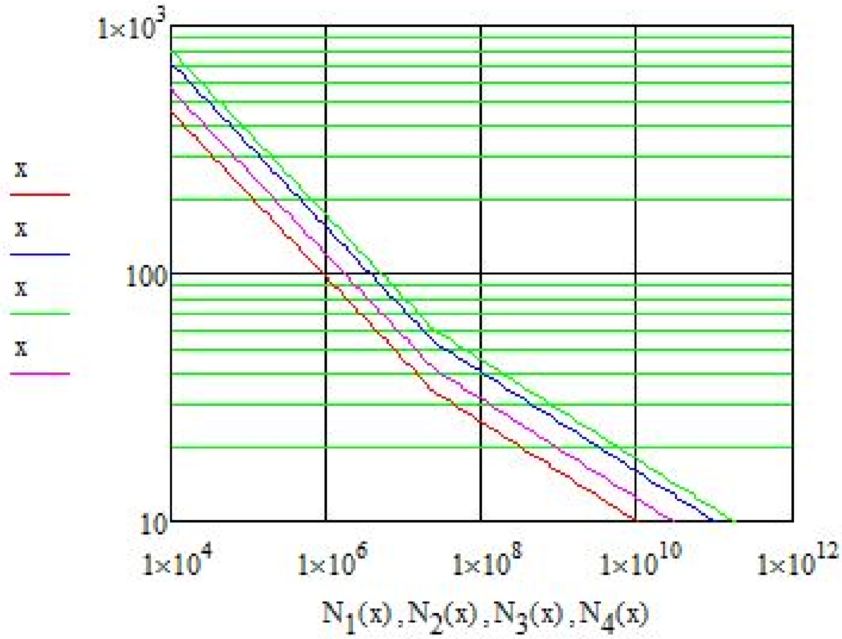


Figure 5.21: Check of random S-N curves Case 2.

The limit state function is simulated N_sim times and the following results are obtained:

Table 5.27: Probabilities of limit state failure (P_f) Case 2.

Scenario	N_{sim}	$SUM(g \leq 0)$	P_f	**s**
1	10^4	232	$2,32 \times 10^{-2}$	$1,505 \times 10^{-3}$
1	10^7	232477	$2,32 \times 10^{-2}$	$4,765 \times 10^{-5}$
2	10^4	749	$7,49 \times 10^{-2}$	$2,632 \times 10^{-3}$
2	10^7	772008	$7,72 \times 10^{-2}$	$8,440 \times 10^{-5}$
3	10^4	1615	0,162	$3,680 \times 10^{-3}$
3	10^7	1621893	0,162	$1,166 \times 10^{-4}$
FLM4	10^4	150	$1,50 \times 10^{-2}$	$1,216 \times 10^{-3}$
FLM4	10^7	150387	$1,50 \times 10^{-2}$	$3,849 \times 10^{-5}$

5.4.4 Comparison

The estimated fatigue life for the closed form damage calculations can be found when D is equal to 1. A graph with the accumulated damage versus years in service for each scenario is defined in fig. 5.22. The estimated fatigue life for each scenario is listed in table 5.28

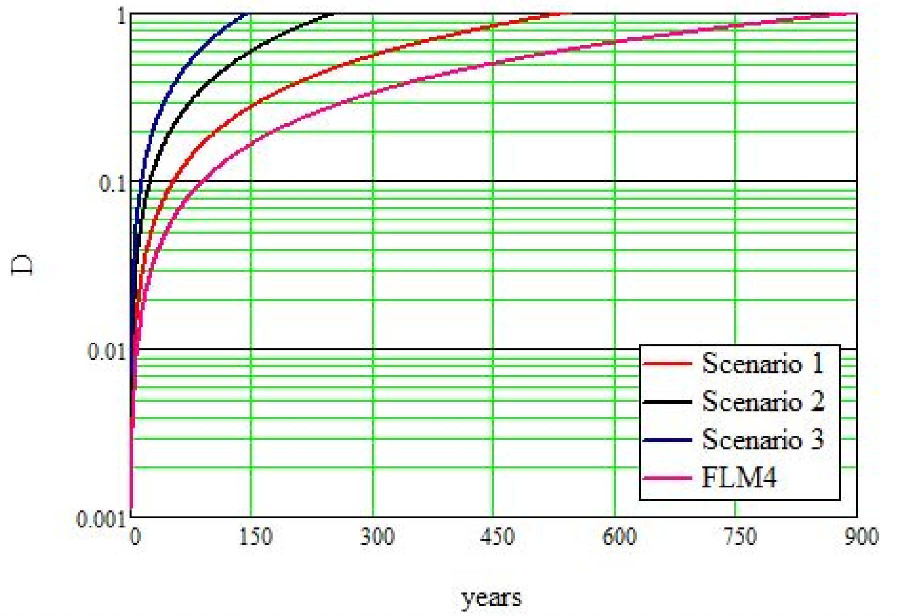


Figure 5.22: Accumulated damage versus time in Case 2.

Table 5.28: Deterministic estimated fatigue life for scenarios in Case 1.

Scenario	Estimated fatigue life (years)
1	538
2	252
3	144
FLM4	898

The estimated fatigue life for the probabilistic approach is where the probability of failure P_f is equal to the target probability of failure P_t. The probability of failure versus years in service for each scenario is defined in fig. 5.23. The probabilistic estimated fatigue life with the corresponding deterministic damage for each scenario is listed in table 5.29.

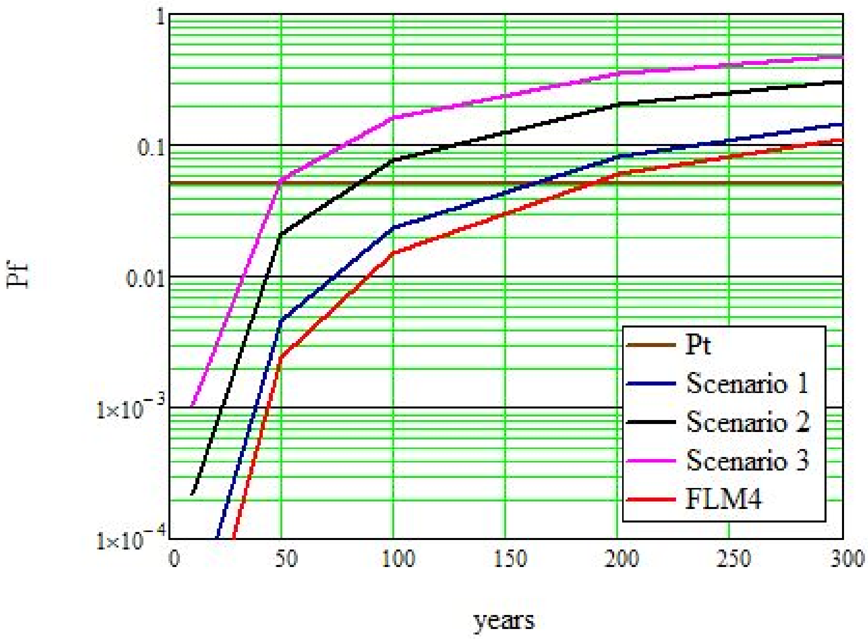


Figure 5.23: Probability of failure versus time in Case 1.

Table 5.29: Probabilistic estimated fatigue life and corresponding deterministic damage for scenarios in Case 2. $N_{sim} = 10^7$

Scenario	Estimated fatigue life (years)	Corresponding damage
1	150	0,279
2	78	0,309
3	48	0,332
FLM4	181	0,201

The factor for uncertainties in the stress modelling *B* is assumed to have a COV of 0,3 based on the confidence of the stress modelling. To check what effect a better stress modelling would have on the probability of failure, the other COV listen in table 3.1 are compared to a COV of 0,3 for Scenario 2:

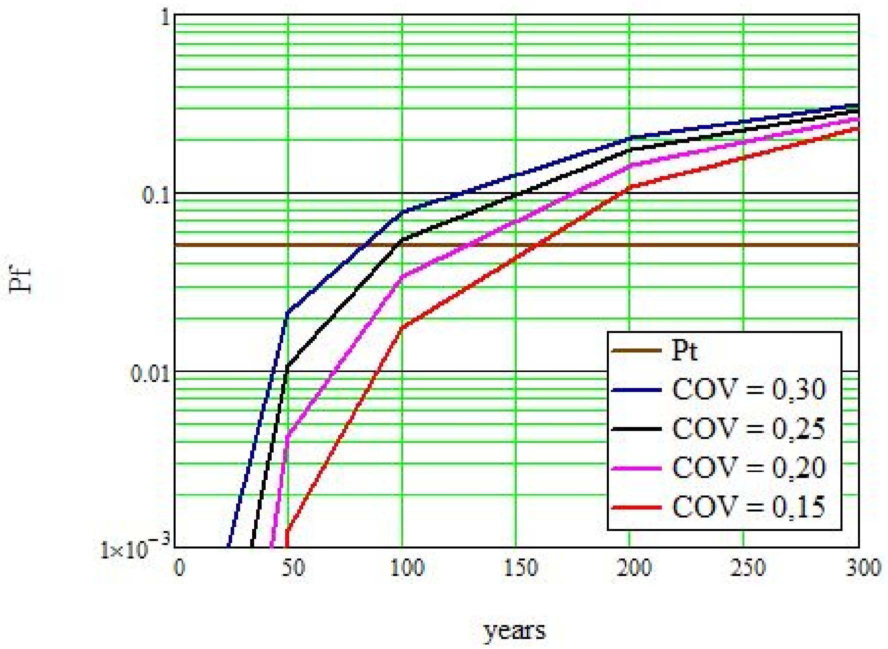


Figure 5.24: Effect of change in COV of factor *B* for Scenario 2 in Case 2.

Table 5.30: Probabilistic estimated fatigue life and corresponding deterministic damage for different COV of *B* for Scenario 2 in Case 2. $N_{sim} = 10^7$

COV	Estimated fatigue life (years)	Corresponding damage
0,30	78	0,309
0,25	97	0,384
0,20	119	0,471
0,15	144	0,571

Chapter 6

Discussion

This chapter will start with a small summary and discussion of the results obtained by three fatigue assessments considered in this thesis. The alternative traffic load model compared to the one in the Eurocode will be discussed, based on the results in the Case Studies. Finally, the shortcomings, limitations, advantages or disadvantages for different approaches will be discussed based on the observations made from solving the validation example and Case Studies.

6.1 Results

6.1.1 Method Validation Example

This example involved examining a railway bridge member connected by a fillet welded lap joint. It was solved to validate that the assessment methods established in Chapter 3 would work in practice.

Deterministic approach

The traditional approach resulted in an accumulated damage of 0,328 which, assuming a damage of 1,0 would result in failure, means that about one third of the members capacity has been utilized. The second deterministic approach, the closed form, where a Weibull distribution has been fitted to the stress histogram resulted an accumulated damage of 0,404. This represents an increase of about 23% in the accumulated damage compared to the traditional approach. The reason the accumulated damage from the closed form approach is higher than from the traditional approach could have a couple of explanations.

One explanation could be that the parameters of the Weibull distribution are incorrect and does not represent the original stress spectrum. These parameters are in this thesis decided by the method called eye-fitting, which is a subjective method, relying on the knowledge and opinions of the person performing it.

Another possible explanation for the difference in the damage could be that the closed form approach represents a more realistic stress spectrum than in the traditional approach. In the traditional approach the calculations are based on a stress histogram of i=7 stress blocks with a constant stress range for each block $\Delta\sigma_i$, all with their respective number of cycles n_i. This means that there are only 7 different stress ranges between 12 and 90 MPa that the joint is assumed to be exposed to during its service life. In the closed form approach, these 7 stress blocks are represented by a Weibull distribution. This changes the possible stress range from the 7 possible values to an infinite number of values within the range of 0 to 90 MPa, and also values above it. This is shown in figure 4.4, where one can see that the CDF of the Weibull distribution (red line) continues beyond 90 MPa. One can also see that the probability for a stress range of 90 MPa in the Weibull distribution is lower than for the original stress spectrum. One might say this is a misrepresentation of the original stress spectrum, or one could argue this is a way to compensate for the higher stress ranges. I.e. if the probability for a stress range of 90 MPa was the same for both representations of the stress spectrum, that would also include an increased probability for all higher values as well.

Probabilistic approach

The probabilistic approach resulted in a probability of limit state failure P_f of $5,026 \times 10^-2$ (Table 4.4), which is approximately the same as the target probability P_t used in this thesis. This result would suggest that the joint is considered safe in the fatigue limit state. The calculations to obtain P_f are based on Weibull parameters used in the closed form calculations. This would indicate that if the parameters do not represent the real stress spectrum in a satisfying manner, the obtained P_f would be misleading. On the other hand, within the formula used to determine P_f there is a factor *B* that aims to account for the uncertainty in the stress modelling, more specifically, the scale parameter q for the Weibull distribution. As described in section 3.3.1, the value of *B* is assumed to be 1 with a covariance based on the level of confidence in the stress modelling. The conservative value of 0,30 for the COV was used in this example, which would suggest that there is little confidence in the stress modelling.

When the probability of limit state failure in the example was checked for its sensitivity to the COV of the factor *B* (Table 4.5), a significant change in the P_f for each COV was obtained. A change in the COV from 0,30 to 0,25, resulted in a decrease over 50% in the P_f. The P_f for a COV of 0,15 for the factor *B* was over 6 times lower than with a COV of 0,30.

6.1.2 Case Studies: Traffic load models

The case study involved examining the fatigue effects of traffic loads on a bridge. To represent the traffic loads, two different models were presented:

- Fatigue load model 4 (FLM4) from NS-EN 1991-2 [4.6.5] (ECS, 2010a).
- An alternative traffic load model (ATLM) that tries to take the entire traffic into account.

The aim of the alternative model was to see if representing the entire traffic would provide different results than that proposed in the Eurocode. It also provided a way to see what effects an increase or decrease in the number of heavy vehicles have on fatigue damage. That resulted in four scenarios being solved:

- **Scenario 1**: The alternative traffic load model with 4% heavy vehicles
- **Scenario 2**: The alternative traffic load model with 8,56% heavy vehicles
- **Scenario 3**: The alternative traffic load model with 15% heavy vehicles
- **Scenario 4**: Fatigue load model 4 (FLM4)

8,56% heavy vehicles were chosen in scenario 2 in order to compare the alternative traffic load model with fatigue load model 4, as 8,56% is also the percentage of heavy vehicles in FLM4 based on the assumed AADT in the area. The effect of ATLM will be discussed further in the Case Studies.

6.1.3 Case Studies: Case 1 - Single-span bridge

The first case in the Case Studies involved examining the fatigue effects of traffic loads on a one lane single-span bridge. The single-span bridge was assumed to be represented by one simply supported beam. One could argue that this is not a realistic representation of a bridge, but that was not the purpose of the case either. In the same way the first example was solved in order to validate the proposed assessment methods, this case was solved in order to validate that the traffic load models gave reasonable results.

Deterministic approach

The results from the traditional approach, listed in table 5.11, provided a couple of interesting remarks worth discussing:

- The results from the three scenarios in ATLM indicate that the accumulated damage is, to some degree, proportional to the percentage of heavy traffic.

$$\text{Scenario 1} = \frac{0,082}{4\%} = 0,0205 \qquad \text{Scenario 2} = \frac{0,175}{8,56\%} = 0,0204$$

$$\text{Scenario 3} = \frac{0,307}{15\%} = 0,0205$$

  this might be explained by the fact that the distribution within the five trucks from FLM4 is the same for all percentages of heavy vehicles, see table 5.3. If you go from 1% of heavy vehicles to 2%, the number of times each truck passes doubles, and the accumulated damage seems to do the same. This would also indicate that the six other vehicles in the traffic load model do not contribute much to the damage.

- The results also show that Scenario 2 gives almost the same damage as for FLM4. The small difference of about 3×10^{-4} between them is the contribution from the six other vehicles in the alternative model.

For the closed form approach in this case, besides the use of eye-fitting, the scale parameter q for the Weibull distribution is adjusted to give a closed form accumulated damage approximately 25% higher than the damage for the traditional approach. There is a possibility that a number as high as 25% overestimates the closed form damage, but if that is the case, the result would be an underestimated fatigue capacity and higher safety margin.

Since the number of cycles for each year is known, it is possible to estimate the fatigue life for each scenario. The different estimations for the closed form calculations are listed in table 5.15. The results show a major difference in estimated life between the scenarios. This big difference is likely linked to the assumed proportionality between the percentage of heavy traffic and the accumulated damage. For every percent you add, the damage increases a value of x, while the estimated life decrease by a value of y.

Probabilistic approach

The results for the probabilistic approach for a design life of 100 years listed in table 5.14, shows that the probability of limit state failure P_f is below the target probability P_t for Scenario 1 and 2 and over for Scenario 3 and FLM4. The P_f for FLM4 is just $0,24 \times 10^{-2}$ above P_t, while it is almost twice as high for Scenario 3. Even tough Scenario 2 and FLM4 have the same values for the accumulated damage in the deterministic approach, the P_f is higher for FLM4 for the probabilistic approach. The reason for that could be because of the sensitivity of the scale parameter q in the simulations. I.e. if q is increased by 25% in each scenario, FLM4 would experience a higher increase accumulated damage than Scenario 2. The reason FLM4's scale parameter is more sensitive to change seems to be because of the range of stresses available in the distribution. While Scenario 2 may experience stresses in a range between i.e. 1 and 90 MPa, FLM4s range is between i.e. 30 and 90 MPa.

It is also possible to estimate the fatigue life for the probabilistic approach, by finding the number of years it takes until P_f is equal to P_t. The estimated life, listed in table 5.16, shows a significant reduction compared to the estimates from the deterministic approach. It also shows the corresponding closed form damage for the same number of years. Why the probabilistic estimated fatigue life is so much lower than the deterministic could be because the probabilistic calculations include the uncertainties.

In order to check the effect the degree of uncertainty has on the estimated fatigue life, Scenario 2 was checked with different values for the COV of the stress modelling factor *B*. The results, listed in table 5.17, show an increase in the estimated life as the COV decreases. With a COV equal to 0,15, the estimated life increases from 103 to 241 years, while the corresponding damage also increases from 0,226 to 0,528.

6.1.4 Case Studies: Case 2 - Three-span bridge

The second case in the Case Studies involved examining the fatigue effects of traffic loads on a two lane three-span bridge. Because of symmetry in the section of the bridge (fig. 5.14), only one of the girders were examined. The scenario using fatigue load model 4 was solved according to the Eurocode, same as in Case 1. The code states that each truck in the model is considered to cross the bridge in the absence of any other vehicle. As a result, the trucks were modelled to cross the bridge in the lane closest to the girder being examined. Because of an eccentricity between the center-line of the lane, where the load was placed, and the location of the girder, the effect that the load had on the opposite girder was checked. The results showed that the opposite girder experienced approximately 25% of the total load. The effect of a vehicle in the opposite lane of the girder in question was considered to be too big to ignore in ATLM. In order to account for the effect of a vehicle in the opposite lane, to a certain degree, half of the crossings over the bridge were assumed to be with an equal vehicle in each lane. One could argue that it is not realistic to assume two equal vehicles cross a bridge side-by-side, but in that case, neither would assuming that all vehicles cross by themselves.

Deterministic approach

The results from the traditional approach, listed in table 5.24 show, among other things, quite the difference in accumulated damage between Scenario 2 and FLM4, which has the same percentage of heavy vehicles. This is most likely because of the inclusion of loads from vehicles in the opposite lane. Another point worth mentioning, is the indication that the accumulated damage seems to be less proportional with the percentage of heavy vehicles, compared to Case 1:

$$\text{Scenario 1} = \frac{0,162}{4\%} = 0,0405 \qquad \text{Scenario 2} = \frac{0,344}{8,56\%} = 0,0402$$

$$\text{Scenario 3} = \frac{0,601}{15\%} = 0,0401$$

with the reason probably being that the six other vehicles in ATLM have a slightly higher influence on the accumulated damage in this case, compared to Case 1.
For the closed form approach in this case, besides the use of eye-fitting, the scale parameter *q* for the Weibull distribution is adjusted to give a closed form accumulated damage approximately 15% higher than the damage for the traditional approach.

The different closed form estimations for fatigue life are listed in table 5.15. The results indicate the same as in Case 1: higher percentage of heavy traffic results in a lower fatigue life. They also show the effect neglecting the loads in the opposite lane has, with estimated fatigue life for FLM4 over 3,5 times larger than for Scenario 2.

Probabilistic approach

The results for the probabilistic approach for a design life of 100 years listed in table 5.14, shows that the probability of limit state failure P_f is below the target probability P_t for Scenario 1 and FLM4 and over for Scenario 2 and 3. These probabilities of failure provide no surprises based on the accumulated damage determined with the deterministic approach and the results in the prior problems.

The probabilistic estimated fatigue life for Case 2 is listed in table 5.16. Same as in Case 1, they indicate a significant reduction in the estimated life compared to the deterministic approach. More interesting is the fact that the size of the reduction is different for the each scenario. Where Scenario 3 has the lowest reduction, at 3 times lower, and FLM4 has the highest with an estimated life almost 5 times lower than estimated using the deterministic approach.

P_f sensitivity for the COV of *B* was also checked for this case using Scenario 2 and the different estimated fatigue lives and corresponding damage is listed in table 5.30. Same as in Case 1, both the estimated fatigue life and corresponding damage increase as the COV decreases. The difference between a COV of 0,30 and 0,15, gives an increase in the estimated life from 78 to 144 years, while the corresponding damage also increases from 0,309 to 0,571.

6.1.5 Alternative traffic load model

An alternative traffic load model was proposed in order to compare fatigue load model 4 proposed by the Eurocode (ECS, 2010a) with a traffic load model that describes the entire traffic spectrum, instead of just the heavy vehicles. The primary goal was to see which effect the addition of more vehicle classes would have on the accumulated damage. In Case 2, the effect of both lanes being loaded simultaneously was also checked and compared to FLM4.

The results from both cases indicate that the effect of including six smaller vehicles classes into the traffic model, just contributes a small part to the total damage. The highest contribution for both cases is in Scenario 1, where the smaller vehicles classes contribute to $3,44 \times 10^{-4}$ in Case 1 and $2,18 \times 10^{-3}$ in Case 2. The most likely explanation for the small contribution is because the stress ranges created by the smaller vehicle classes are too small. It seems that as long as the damage is assumed to be accumulating linearly, the lower stress ranges will not contribute in any significant form to the total damage.

The results from Case 2, where the contribution of a vehicle in the opposite lane is included in half the cycles, show a significant difference in the accumulated damage and probability of failure between Scenario 2 and FLM4. These results indicate that the contribution of a vehicle in the opposite lane is to large to neglect, and that just using FLM4 when analyzing for fatigue would overestimate the capacity. One could argue that the proposed method for including this effect is unrealistic, as vehicles in opposite lanes, also would move in the opposite direction. The number of cycles with both lanes loaded might also be an overstatement. On the other hand, the results show strong indications that not accounting for loads in the opposite lane would also be unrealistic.

6.2 Shortcomings, limitations, advantages or disadvantages

6.2.1 Traditional approach

The traditional assessment method seems to be a, more or less, straightforward procedure once the stresses are determined. It provides an end result that states if the detail in question is okay or not in the fatigue limit state. On the other hand, as long as the stress spectrum is defined by i stress blocks, each with a constant stress range, the variation in the spectrum is not accounted for. The uncertainty in the material strength is taken into consideration by the use of characteristic values for $\bar{a}$. The uncertainty in the stress modelling and in the assumption of linear accumulated damage are not directly considered in the model. These uncertainties seem to be accounted for by the use of design fatigue factors in the model. These safety factors are usually defined in the rules and regulations, and not considered towards the specific problem in question.

6.2.2 Closed form approach

The closed form assessment method is an extension of the traditional method, where the stress blocks are replaced by a Weibull distribution to represent the stress spectrum. It seems to require a bit more time and effort, than the traditional approach. Same as the traditional method, it gives an end result that states if the detail in question is okay or not. Determining the correct shape h and scale q parameter for the Weibull distribution is the most challenging part of this method. If there is little knowledge available about what reasonable values for the parameters are, it is difficult to clearly state that one has a good representation of the stress spectrum. This method does not account for the uncertainty in the stress modelling and in the assumption of linear accumulated damage either, and relies on safety factors as well.

6.2.3 Probabilistic approach

The probabilistic approach used in this thesis could be described as an evolution of the closed form approach since it is based on the same formula for determining the accumulated damage. The difference is that the probabilistic approach includes the uncertainty and the variation of the random variables, like those listed in table 5.26 and 5.13. Another difference is the output, where the deterministic approaches give the accumulated damage, the probabilistic approach gives the probability of failure.

The probability of failure has in this thesis been compared to a target probability of failure (P_t) in order to determine if the detail in question is considered safe or not. Based on what the results in this thesis indicate, a few points on what the probability of failure really means is proposed:

- Finding P_f involves simulating the limit state function with realizations of the random variables N_{sim} times, where the P_f is defined as the number of times the limit state function fails and divided by N_{sim}. One could argue that P_f defines the probability that 1 out of N_{sim} identical details fail, when exposed to an identical stress spectrum.
- Another possible way of looking at the probability of failure is that it describes the quality of the analysis, the confidence one has in the stress modelling. This argument is backed up by the results where the P_f is reduced when the covariance of factor *B* is reduced.

Chapter 7

Conclusion

Throughout this thesis three cases have been solved using three different assessment methods, two deterministic and one probabilistic. The purpose of the first example was to validate that the different proposed assessment methods produced appropriate results. The other two cases were part of the Case Study, where the fatigue effects on bridges exposed to traffic loads were examined. As a part of the Case Study an alternative traffic load model was proposed in an attempt to improve the current procedure in rules and regulations. Based on the results obtained in this thesis the following conclusions have been drawn:

- None of the assessment methods appear to be without its limitations. Both the closed form and probabilistic approach is, to some degree, dependent on another assessment method. The closed form approach will in most cases be based on the stress histogram determined in the traditional approach, and the probabilistic approach is usually based on the Weibull distribution determined in the closed form method. The traditional approach is independent from the other methods, but it is based on a limited representation of a realistic stress spectrum.

- The traditional approach should be used only as a way to examine if the detail is exposed to significant fatigue damage and if further analysis is needed. I.e. if the accumulated damage for a 100 year design life is lower than i.e. 0,100, the detail is assumed safe and no further analysis is needed. If it is higher than 0,100, further analysis is required.

- The probabilistic approach can be used to define the confidence in the analysis and used define design fatigue factors to be used in the deterministic approach. I.e. for Scenario 2 in Case 2, the design fatigue factor would be $1/0,309 \approx 3,25$ for a COV equal to 0,30 for *B*. For a COV equal to 0,15, the design fatigue factor would be $1/0,571 \approx 1,75$.

- If the traffic load model proposed in the rules and regulations to check for fatigue damage should be modified, should be considered for each separate bridge. This is because the results in this thesis indicate that the proposed methods work well for some bridges (i.e. Case 1), while they could underestimate the fatigue damage for other bridges (Case 2). For the bridge in Case 2, the effect from a vehicle load in the opposite lane of a bridge should be included in the traffic load model.

- The number of heavy vehicles crossing the bridge should be determined specifically for each separate case. The suggested numbers in the Eurocode (ECS, 2010a) should not be used unless they are checked to be representative.

Based on the results gathered throughout this thesis, the following procedure for fatigue analysis of similar cases is proposed:

1. Define the stress spectrum in terms of a stress histogram with stress blocks. As mentioned in the second point above, check if the detail is exposed significant fatigue damage using the traditional approach after a desired design fatigue life.

2. If it is determined that further analysis is needed; Fit a Weibull distribution to represent the stress spectrum and solve using the closed form approach.

3. Define the factor for errors in the stress modelling *B* with COV that represents the confidence one has in the stress modelling. Find the probability of failure (P_f) after the design fatigue life using the probabilistic method.

4. Compare P_f with a desired target probability of failure P_t. If P_f is less than P_t, the detail is assumed safe for the design fatigue life. If P_f is higher than P_t, two different solutions are proposed:

   - Improve the quality of the stress analysis in order to reduce the COV of *B*.
   - Or increase the dimensions of the cross section in order to increase the capacity.

7.1 Recommendations for future work

- Perform similar comparison as in this thesis, but with a focus on fracture mechanics.
- Compare the recommended procedures in rules and regulations for fatigue analysis of traffic loads on a bridge (i.e. FLM4) with measured data from a real bridge.
- Expand the analysis to include other forces acting on the bridge, like wind.

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Appendix

A Mathcad sheets

Method Validation Example

The calculations done in Mathcad for this problem is on the next page.

Deterministic approach

Weld class W1 - In open air

$a_1 := 10^{11.261}$ $\quad m_1 := 3$

$a_2 := 10^{14.101}$ $\quad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 26.323$$

$$n := \begin{pmatrix} 765000 \\ 432000 \\ 145000 \\ 93000 \\ 39000 \\ 25000 \\ 20000 \end{pmatrix} \qquad \Delta\sigma := \begin{pmatrix} 12 \\ 25 \\ 37 \\ 50 \\ 62 \\ 75 \\ 90 \end{pmatrix} \qquad i := 0..6$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{6} \frac{n_i}{N_i} \qquad D_d = 0.328$$

With Weibull distribution

Higher incomplete gamma function. $\quad \Gamma(\alpha, x) := \int_x^{\infty} t^{\alpha-1} \cdot e^{-t}\,dt$

Lower incomplete gamma function. $\quad \gamma(\alpha, x) := \int_0^{x} t^{\alpha-1} \cdot e^{-t}\,dt$

$h := 1.25$

$q := 25.5$

$a_1 = 1.824 \times 10^{11}$ $\quad m_1 = 3$

$a_2 = 1.262 \times 10^{14}$ $\quad m_2 = 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 26.323$$

$$n_0 := \sum_{i=0}^{6} n_i = 1.519 \times 10^6$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.404$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1) + 2 \cdot sd_a} = 4.581 \times 10^{11}$ $\qquad a_{m2} := 10^{\log(a_2) + 2sd_a} = 3.17 \times 10^{14}$

$$S_{1m} := \left(\frac{10^{7 + 2 \cdot sd_a}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 26.323$$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188 \ln\left(1 + COV^2\right)$

$$COV_{am1} := \sqrt{e^{5.319 \cdot sd_a^2} - 1} = 0.487$$

$$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2 + 1}}\right) = 26.744 \qquad sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2 + 1\right)} = 0.461$$

$\Delta := 1$ $\qquad COV_\Delta := 0.3$ $\qquad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2 + 1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1$ $\qquad COV_B := 0.3$ $\qquad sd_{lnB} := \sqrt{\ln\left(COV_B^2 + 1\right)} = 0.294$

$g = \Delta - D$ $\qquad N_{sim} := 10^4$ $\qquad i := 0..N_{sim} - 1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \left| \begin{array}{l} \text{for } i \in 0..N_{sim} \\ \quad z_{B_i} \leftarrow \left|\text{rlnorm}\left(1, \log(B), sd_{lnB}\right)\right| \\ z_B \end{array} \right.$$

$$z_{a1} := \left| \begin{array}{l} \text{for } i \in 0..N_{sim} \\ \quad z_{a1_i} \leftarrow \left|\text{rlnorm}\left(1, \mu_{lna1}, sd_{lna1}\right)\right| \\ z_{a1} \end{array} \right. \qquad z_\Delta := \left| \begin{array}{l} \text{for } i \in 0..N_{sim} \\ \quad z_{\Delta_i} \leftarrow \left|\text{rlnorm}\left(1, \log(\Delta), sd_{ln\Delta}\right)\right| \\ z_\Delta \end{array} \right.$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 691.831$ $\quad$ this is almost equal to: $\quad S_{1m}^2 = 692.894$

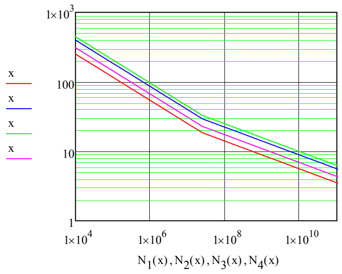
This means the correlation between z_{a1} and z_{a2} is: $\quad z_{a2} = S_z^2 \cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := \left(S_{z_i}\right)^2 \cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \frac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \frac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \frac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \frac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \frac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \frac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \frac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \frac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

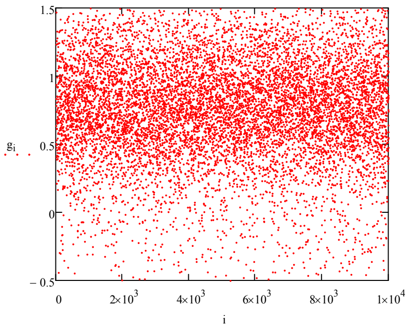


Running simulations on the limit state function:

$$D_i := n_0 \cdot \left[\frac{\left(z_{B_i} \cdot q\right)^{m_1}}{z_{a1_i}} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] + \frac{\left(z_{B_i} \cdot q\right)^{m_2}}{z_{a2_i}} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] \right]$$

$$g_i := z_{\Delta_i} - D_i$$

Plotting results:



Define a vector for how many times g < 0:

```
How_many_between_k_and_l(vektor, k, l) := | v ← vektor
                                          | nrow ← rows(v)
                                          | nn ← 0
                                          | for t.1 ∈ 0.. nrow − 1
                                          |   nn ← nn + 1  if l ≥ v_t.1 > k
                                          | nn
```

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 5.311 \times 10^{-2}$$

$$s := \sqrt{\frac{P_f \cdot \left(1 - P_f\right)}{N_{sim}}} = 2.242 \times 10^{-3}$$

Case 1 - Single-span bridge

The calculations done in Mathcad for this problem is on the next page.

Scenario 1 - 4% AADTT

Cross section properties:

$f_y := 355MPa \qquad W_{el} := 0.0381m^3 \qquad i := 0..10$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 1335.45 \\ 1395.45 \\ 1441.20 \\ 1471.20 \\ 1578.00 \\ 1955.19 \\ 2704.66 \\ 3572.15 \\ 4467.62 \\ 3737.14 \\ 4055.12 \end{pmatrix} \cdot kN \cdot m \qquad M_{min} := 1162.20 \cdot kN \cdot m$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$$\Delta\sigma_a := \frac{M_{max} - M_{min}}{W_{el}} =$$

	0
0	4.547
1	6.122
2	7.323
3	8.11
4	10.913
5	20.813
6	40.485
7	63.253
8	86.756
9	67.584
10	75.93

$\cdot MPa$

$$\Delta\sigma := \begin{pmatrix} 4.547 \\ 6.122 \\ 7.323 \\ 8.110 \\ 10.913 \\ 20.813 \\ 40.485 \\ 63.253 \\ 67.584 \\ 75.930 \\ 86.756 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.2112 \\ 0.2112 \\ 0.1824 \\ 0.1584 \\ 0.12 \\ 0.0768 \\ 0.016 \\ 0.004 \\ 0.006 \\ 0.002 \\ 0.012 \end{pmatrix}$$

AADT in the region is: $AADT := 8000$

Number of cycles in one lane for a design life of 100 years is:

$N_{100} := 0.5 \cdot 365 \cdot AADT \cdot 100 = 146000000$

Define cycles to each stress range:

$n := N_{100} \cdot pn$

$$n_0 := \sum_{i=0}^{10} n_i = 1.46 \times 10^8$$

Cumulative distribution for the stress range:

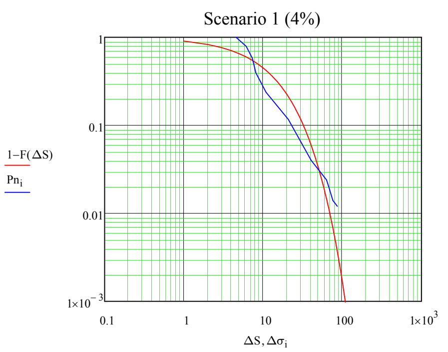
$$Pn_i := \sum_{j=0+i}^{10} pn_j \qquad Pn =$$

	0
0	1
1	0.789
2	0.578
3	0.395
4	0.237
5	0.117
6	0.04
7	0.024
8	0.02
9	0.014
10	0.012

Fitting Weibull to the stress range:

$h := 0.9 \qquad q := 12.75 \qquad \Delta S := 0..350$

$$f(\Delta S) := \frac{h}{q} \cdot \left(\frac{\Delta S}{q}\right)^{h-1} \cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^h\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$



From DNV Class B1

$a_1 := 10^{15.117} = 1.309 \times 10^{15} \qquad m_1 := 4$

$a_2 := 10^{17.146} = 1.4 \times 10^{17} \qquad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 106.967$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{10} \frac{n_i}{N_i} \qquad D_d = 0.082$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\,dt$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.101$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1)+0.4} = 3.289 \times 10^{15}$ $\qquad$ $a_{m2} := 10^{\log(a_2)+0.4} = 3.516 \times 10^{17}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 106.967$$

Convert the following equation to find COV for a_m: $\quad sd_{\log}{}^2 = 0.188\ln\left(1 + COV^2\right)$

$$COV_{am1} := \sqrt{e^{5.319 \cdot sd_a{}^2} - 1} = 0.487$$

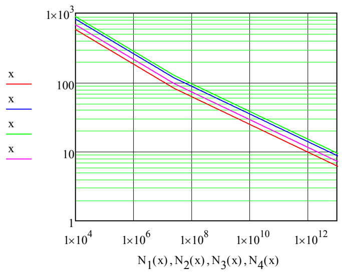
$$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}{}^2 + 1}}\right) = 35.623$$

$$sd_{lna1} := \sqrt{\ln\left(COV_{am1}{}^2 + 1\right)} = 0.461$$

$\Delta := 1$ $\qquad$ $COV_\Delta := 0.3$ $\qquad$ $sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta{}^2 + 1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1$ $\qquad$ $COV_B := 0.3$ $\qquad$ $sd_{lnB} := \sqrt{\ln\left(COV_B{}^2 + 1\right)} = 0.294$

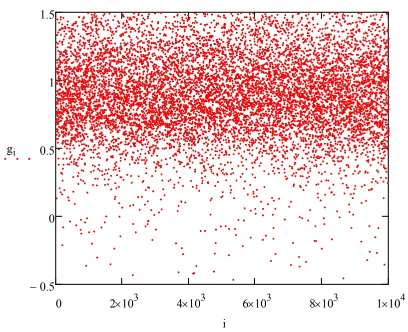


Running simulations on the limit state function:

$$D_i := n_0 \cdot \left[\frac{\left(z_{B_i} \cdot q\right)^{m_1}}{z_{a1_i}} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] + \frac{\left(z_{B_i} \cdot q\right)^{m_2}}{z_{a2_i}} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right]\right]$$

$g_i := z_{\Delta_i} - D_i$

Plotting results:



$g = \Delta - D$ $\qquad$ $N_{sim} := 10^4$ $\qquad$ $i := 0 .. N_{sim} - 1$

Create random realizations for a_1, a_2, B and Δ

```
z_B :=  | for i ∈ 0..N_sim
        |    z_B_i ← |rlnorm(1, log(B), sd_lnB)|
        | z_B
```

```
z_a1 := | for i ∈ 0..N_sim
        |    z_a1_i ← |rlnorm(1, μ_lna1, sd_lna1)|
        | z_a1

z_Δ :=  | for i ∈ 0..N_sim
        |    z_Δ_i ← |rlnorm(1, log(Δ), sd_lnΔ)|
        | z_Δ
```

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 106.905$ $\qquad$ this is equal to: $\qquad$ $S_{1m} = 106.967$

This means the correlation between z_{a1} and z_{a2} is: $\quad z_{a2} = S_z \cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := S_{z_i} \cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \dfrac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \dfrac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \dfrac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \dfrac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \dfrac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \dfrac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \dfrac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \dfrac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

Define a vector for how many times g < 0:

```
How_many_between_k_and_l(vektor, k, l) :=  | v ← vektor
                                           | nrow ← rows(v)
                                           | nn ← 0
                                           | for t1 ∈ 0..nrow − 1
                                           |    nn ← nn + 1  if l ≥ v_t1 > k
                                           | nn
```

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.015$$

Standard error:

$$s := \sqrt{\frac{P_f \cdot \left(1 - P_f\right)}{N_{sim}}} = 1.204 \times 10^{-3}$$

Scenario 2 - 8,56% AADTT

Cross section properties:

$f_y := 355MPa \qquad W_{el} := 0.0381m^3 \qquad i := 0..10$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 1335.45 \\ 1395.45 \\ 1441.20 \\ 1471.20 \\ 1578.00 \\ 1955.19 \\ 2704.66 \\ 3572.15 \\ 4467.62 \\ 3737.14 \\ 4055.12 \end{pmatrix} \cdot kN \cdot m \qquad M_{min} := 1162.20 \cdot kN \cdot m$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$$\Delta\sigma_a := \frac{M_{max} - M_{min}}{W_{el}} \cdot MPa$$

	0
0	4.547
1	6.122
2	7.323
3	8.11
4	10.913
5	20.813
6	40.485
7	63.253
8	86.756
9	67.584
10	75.93

$$\Delta\sigma := \begin{pmatrix} 4.547 \\ 6.122 \\ 7.323 \\ 8.110 \\ 10.913 \\ 20.813 \\ 40.485 \\ 63.253 \\ 67.584 \\ 75.930 \\ 86.756 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.201168 \\ 0.201168 \\ 0.173736 \\ 0.150876 \\ 0.1143 \\ 0.073152 \\ 0.03424 \\ 0.00856 \\ 0.01284 \\ 0.00428 \\ 0.02568 \end{pmatrix}$$

AADT in the region is: $ADT := 4000$

Number of cycles in one lane for a design life of 100 years is:

$N_{100} := 365 \cdot ADT \cdot 100 = 146000000$

Define cycles to each stress range:

$n := N_{100} \cdot pn$

$$n_0 := \sum_{i=0}^{10} n_i = 1.46 \times 10^8$$

Cumulative distribution for the stress range:

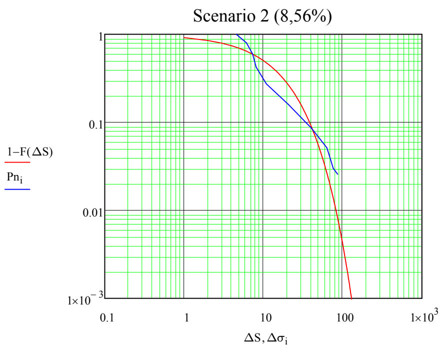
$$Pn_i := \sum_{j=0+i}^{10} pn_j \qquad Pn =$$

	0
0	1
1	0.798832
2	0.597664
3	0.423928
4	0.273052
5	0.158752
6	0.0856
7	0.05136
8	0.0428
9	0.02996
10	0.02568

Fitting Weibull to the stress range:

$h := 0.9 \qquad q := 15.1 \qquad \Delta S := 0..350$

$$f(\Delta S) := \frac{h}{q} \cdot \left(\frac{\Delta S}{q}\right)^{h-1} \cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^h\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$



From DNV Class B1

$a_1 := 10^{15.117} = 1.309 \times 10^{15} \qquad m_1 := 4$

$a_2 := 10^{17.146} = 1.4 \times 10^{17} \qquad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 106.967$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{10} \frac{n_i}{N_i} \qquad D_d = 0.175$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\,dt$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.219$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1)+0.4} = 3.289 \times 10^{15}$ $\qquad a_{m2} := 10^{\log(a_2)+0.4} = 3.516 \times 10^{17}$

$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 106.967$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188\ln(1 + COV^2)$

$COV_{am1} := \sqrt{e^{5.319 \cdot sd_a^2} - 1} = 0.487$

$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2 + 1}}\right) = 35.623$

$sd_{lna1} := \sqrt{\ln(COV_{am1}^2 + 1)} = 0.461$

$\Delta := 1 \qquad COV_\Delta := 0.3 \qquad sd_{ln\Delta} := \sqrt{\ln(COV_\Delta^2 + 1)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1 \qquad COV_B := 0.3 \qquad sd_{lnB} := \sqrt{\ln(COV_B^2 + 1)} = 0.294$

$g = \Delta - D \qquad N_{sim} := 10^4 \qquad i := 0..N_{sim} - 1$

Create random realizations for a_1, a_2, B and Δ

```
zB := | for i ∈ 0..Nsim
      |    zB_i ← |rlnorm(1, log(B), sdlnB)|
      | zB

za1 := | for i ∈ 0..Nsim
       |    za1_i ← |rlnorm(1, μlna1, sdlna1)|
       | za1

zΔ := | for i ∈ 0..Nsim
      |    zΔ_i ← |rlnorm(1, log(Δ), sdlnΔ)|
      | zΔ
```

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 106.905$ this is equal to: $S_{1m} = 106.967$

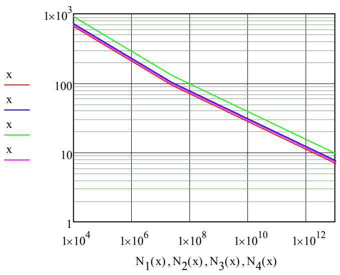
This means the correlation between z_{a1} and z_{a2} is: $z_{a2} = S_z \cdot z_{a1}$

$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := S_{z_i} \cdot z_{a1_i}$

Visual check that z_{a1} and z_{a2} correlate:

$N_1(x) := \begin{cases} \frac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \frac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \frac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \frac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$

$N_2(x) := \begin{cases} \frac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \frac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \frac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \frac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$

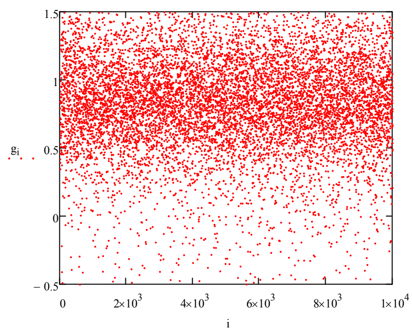


Running simulations on the limit state function:

$$D_i := n_0 \cdot \left[\frac{(z_{B_i} \cdot q)^{m_1}}{z_{a1_i}} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] + \frac{(z_{B_i} \cdot q)^{m_2}}{z_{a2_i}} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right]\right]$$

$g_i := z_{\Delta_i} - D_i$

Plotting results:



Define a vector for how many times g < 0:

```
How_many_between_k_and_l(vektor, k, l) := | v ← vektor
                                          | nrow ← rows(v)
                                          | nn ← 0
                                          | for t1 ∈ 0..nrow − 1
                                          |    nn ← nn + 1  if l ≥ v_t1 > k
                                          | nn
```

Probability of failure:

$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.045$

Standard error:

$s := \sqrt{\frac{P_f \cdot (1 - P_f)}{N_{sim}}} = 2.078 \times 10^{-3}$

Scenario 3 - 15% AADTT

Cross section properties:

$f_y := 355MPa \qquad W_{el} := 0.0381m^3 \qquad i := 0..10$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 1335.45 \\ 1395.45 \\ 1441.20 \\ 1471.20 \\ 1578.00 \\ 1955.19 \\ 2704.66 \\ 3572.15 \\ 4467.62 \\ 3737.14 \\ 4055.12 \end{pmatrix} \cdot kN \cdot m \qquad M_{min} := 1162.20 \cdot kN \cdot m$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$\Delta\sigma_a := \frac{M_{max} - M_{min}}{W_{el}} =$

	0
0	4.547
1	6.122
2	7.323
3	8.11
4	10.913
5	20.813
6	40.485
7	63.253
8	86.756
9	67.584
10	75.93

$\cdot MPa$

$$\Delta\sigma := \begin{pmatrix} 4.547 \\ 6.122 \\ 7.323 \\ 8.110 \\ 10.913 \\ 20.813 \\ 40.485 \\ 63.253 \\ 67.584 \\ 75.930 \\ 86.756 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.187 \\ 0.187 \\ 0.1615 \\ 0.14025 \\ 0.10625 \\ 0.068 \\ 0.06 \\ 0.015 \\ 0.0225 \\ 0.0075 \\ 0.045 \end{pmatrix}$$

AADT in the region is: $AADT := 8000$

Number of cycles in one lane for a design life of 100 years is:

$N_{100} := 0.5 \cdot 365 \cdot AADT \cdot 100 = 146000000$

Define cycles to each stress range:

$n := N_{100} \cdot pn$

$$n_0 := \sum_{i=0}^{10} n_i = 1.46 \times 10^8$$

Cumulative distribution for the stress range:

$$Pn_i := \sum_{j=0+i}^{10} pn_j$$

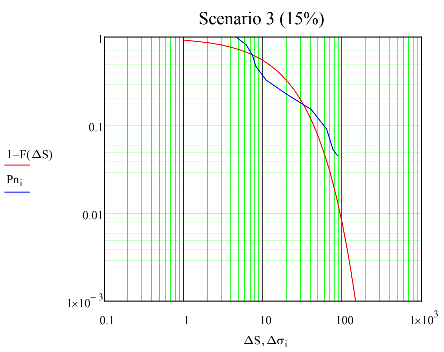
Pn =

	0
0	1
1	0.813
2	0.626
3	0.465
4	0.324
5	0.218
6	0.15
7	0.09
8	0.075
9	0.053
10	0.045

Fitting Weibull to the stress range:

$h := 0.9 \qquad q := 17.1 \qquad \Delta S := 0..350$

$$f(\Delta S) := \frac{h}{q} \cdot \left(\frac{\Delta S}{q}\right)^{h-1} \cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^h\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$



From DNV Class B1

$a_1 := 10^{15.117} = 1.309 \times 10^{15} \qquad m_1 := 4$

$a_2 := 10^{17.146} = 1.4 \times 10^{17} \qquad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 106.967$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{10} \frac{n_i}{N_i} \qquad D_d = 0.307$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\,dt$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.382$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1)+0.4} = 3.289\times10^{15}$ $\qquad a_{m2} := 10^{\log(a_2)+0.4} = 3.516\times10^{17}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 106.967$$

Convert the following equation to find COV for a_m: $sd_{log}^2 = 0.188\ln\left(1+COV^2\right)$

$$COV_{am1} := \sqrt{e^{5.319\cdot sd_a^2}-1} = 0.487$$

$$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2+1}}\right) = 35.623$$

$$sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2+1\right)} = 0.461$$

$\Delta := 1 \qquad COV_\Delta := 0.3 \qquad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2+1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1 \qquad COV_B := 0.3 \qquad sd_{lnB} := \sqrt{\ln\left(COV_B^2+1\right)} = 0.294$

$g = \Delta - D \qquad N_{sim} := 10^4 \qquad i := 0..N_{sim}-1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \left|\begin{aligned} & \text{for } i \in 0..N_{sim} \\ & \quad z_{B_i} \leftarrow \left|\text{rlnorm}(1, \log(B), sd_{lnB})\right| \\ & z_B \end{aligned}\right.$$

$$z_{a1} := \left|\begin{aligned} & \text{for } i \in 0..N_{sim} \\ & \quad z_{a1_i} \leftarrow \left|\text{rlnorm}(1, \mu_{lna1}, sd_{lna1})\right| \\ & z_{a1} \end{aligned}\right. \qquad z_\Delta := \left|\begin{aligned} & \text{for } i \in 0..N_{sim} \\ & \quad z_{\Delta_i} \leftarrow \left|\text{rlnorm}(1, \log(\Delta), sd_{ln\Delta})\right| \\ & z_\Delta \end{aligned}\right.$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 106.905$ this is equal to: $S_{1m} = 106.967$

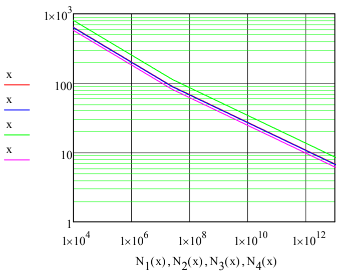
This means the correlation between z_{a1} and z_{a2} is: $z_{a2} = S_z\cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := S_{z_i}\cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \left|\begin{aligned} & \frac{z_{a1_1}}{x^{m_1}} \text{ if } x > S_{z_1} \\ & \frac{z_{a2_1}}{x^{m_2}} \text{ if } x \le S_{z_1} \end{aligned}\right. \qquad N_3(x) := \left|\begin{aligned} & \frac{z_{a1_3}}{x^{m_1}} \text{ if } x > S_{z_3} \\ & \frac{z_{a2_3}}{x^{m_2}} \text{ if } x \le S_{z_3} \end{aligned}\right.$$

$$N_2(x) := \left|\begin{aligned} & \frac{z_{a1_2}}{x^{m_1}} \text{ if } x > S_{z_2} \\ & \frac{z_{a2_2}}{x^{m_2}} \text{ if } x \le S_{z_2} \end{aligned}\right. \qquad N_4(x) := \left|\begin{aligned} & \frac{z_{a1_4}}{x^{m_1}} \text{ if } x > S_{z_4} \\ & \frac{z_{a2_4}}{x^{m_2}} \text{ if } x \le S_{z_4} \end{aligned}\right.$$

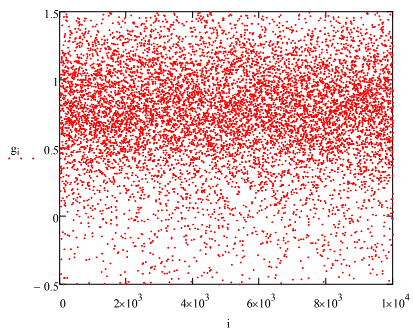


Running simulations on the limit state function:

$$D_i := n_0\cdot\left[\frac{\left(z_{B_i}\cdot q\right)^{m_1}}{z_{a1_i}}\cdot\Gamma\left[1+\frac{m_1}{h},\left(\frac{S_{z_i}}{z_{B_i}\cdot q}\right)^h\right] + \frac{\left(z_{B_i}\cdot q\right)^{m_2}}{z_{a2_i}}\cdot\gamma\left[1+\frac{m_2}{h},\left(\frac{S_{z_i}}{z_{B_i}\cdot q}\right)^h\right]\right]$$

$g_i := z_{\Delta_i} - D_i$

Plotting results:



Define a vector for how many times g < 0:

```
How_many_between_k_and_l(vektor, k, l) := | v ← vektor
                                           | nrow ← rows(v)
                                           | nn ← 0
                                           | for t1 ∈ 0..nrow − 1
                                           |   nn ← nn + 1  if  l ≥ v_t1 > k
                                           | nn
```

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g,-100,0)}{N_{sim}-1} = 0.101$$

Standard error:

$$s := \sqrt{\frac{P_f\cdot\left(1-P_f\right)}{N_{sim}}} = 3.009\times10^{-3}$$

FLM4

Cross section properties:

$f_y := 355\text{MPa} \qquad W_{el} := 0.0381\text{m}^3 \qquad i := 0..4$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 2704.66 \\ 3572.15 \\ 4467.62 \\ 3737.14 \\ 4055.12 \end{pmatrix} \text{kN}\cdot\text{m} \qquad M_{min} := 1162.20\cdot\text{kN}\cdot\text{m}$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$$\Delta\sigma_a := \frac{M_{max} - M_{min}}{W_{el}} = \begin{pmatrix} 40.485 \\ 63.253 \\ 86.756 \\ 67.584 \\ 75.93 \end{pmatrix}\cdot\text{MPa} \qquad \Delta\sigma := \begin{pmatrix} 40.485 \\ 63.253 \\ 67.584 \\ 75.930 \\ 86.756 \end{pmatrix} \quad pn := \begin{pmatrix} 0.4 \\ 0.1 \\ 0.15 \\ 0.05 \\ 0.3 \end{pmatrix}$$

Number of cycles in one lane for a design life of 100 years is from table 4.5, row 2 in NS-EN 1991-2:

$N_{100} := 0.125\cdot 10^6\cdot 100 = 12500000$

Define cycles to each stress range:

$n := N_{100}\cdot pn$

$$n_0 := \sum_{i=0}^{4} n_i = 1.25\times 10^7$$

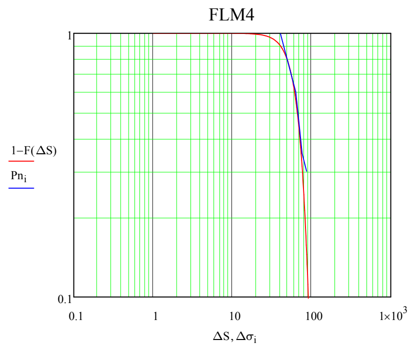
Cumulative distribution for the stress range:

$$Pn_i := \sum_{j=0+i}^{4} pn_j \qquad Pn = \begin{pmatrix} 1 \\ 0.6 \\ 0.5 \\ 0.35 \\ 0.3 \end{pmatrix}$$

Fitting Weibull to the stress range:

$h := 3.75 \qquad q := 73 \qquad \Delta S := 0..350$

$$f(\Delta S) := \frac{h}{q}\cdot\left(\frac{\Delta S}{q}\right)^{h-1}\cdot\exp\left[-\left(\frac{\Delta S}{q}\right)^{h}\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$



From DNV Class B1

$a_1 := 10^{15.117} = 1.309\times 10^{15} \qquad m_1 := 4$

$a_2 := 10^{17.146} = 1.4\times 10^{17} \qquad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 106.967$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{4} \frac{n_i}{N_i} \qquad D_d = 0.175$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1}\cdot e^{-t}\,dt$$

$$D_{cf} := n_0\cdot\left[\frac{q^{m_1}}{a_1}\cdot\Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2}\cdot\gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.219$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1)+0.4} = 3.289\times 10^{15} \qquad a_{m2} := 10^{\log(a_2)+0.4} = 3.516\times 10^{17}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 106.967$$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188\ln(1 + COV^2)$

$$COV_{am1} := \sqrt{e^{5.319\cdot sd_a^2} - 1} = 0.487$$

$$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2 + 1}}\right) = 35.623$$

$$sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2 + 1\right)} = 0.461$$

$\Delta := 1 \qquad COV_\Delta := 0.3 \qquad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2 + 1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1 \qquad COV_B := 0.3 \qquad sd_{lnB} := \sqrt{\ln\left(COV_B^2 + 1\right)} = 0.294$

$g = \Delta - D \qquad N_{sim} := 10^4 \qquad i := 0..N_{sim} - 1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{B_i} \leftarrow \left|\text{rlnorm}(1, \log(B), sd_{lnB})\right| \\ z_B \end{array}$$

$$z_{a1} := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{a1_i} \leftarrow \left|\text{rlnorm}(1, \mu_{lna1}, sd_{lna1})\right| \\ z_{a1} \end{array} \qquad z_\Delta := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{\Delta_i} \leftarrow \left|\text{rlnorm}(1, \log(\Delta), sd_{ln\Delta})\right| \\ z_\Delta \end{array}$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$$\frac{a_{m2}}{a_{m1}} = 106.905 \qquad \text{this is equal to:} \qquad S_{1m} = 106.967$$

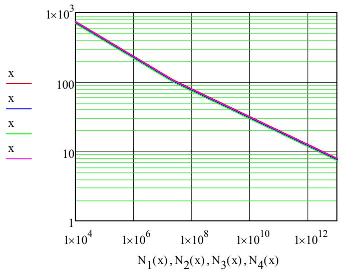
This means the correlation between z_{a1} and z_{a2} is: $z_{a2} = S_z \cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad\qquad z_{a2_i} := S_{z_i} \cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \dfrac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \dfrac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \dfrac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \dfrac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \dfrac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \dfrac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \dfrac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \dfrac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

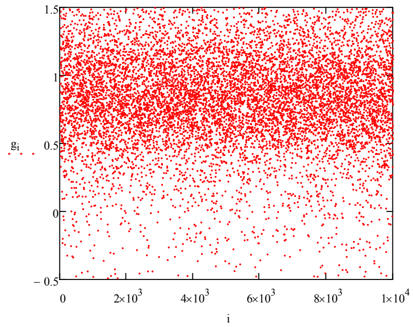


Running simulations on the limit state function:

$$D_i := n_0 \cdot \left[\frac{(z_{B_i} \cdot q)^{m_1}}{z_{a1_i}} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] + \frac{(z_{B_i} \cdot q)^{m_2}}{z_{a2_i}} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right]\right]$$

$$g_i := z_{\Delta_i} - D_i$$

Plotting results:



Define a vector for how many times g < 0:

```
How_many_between_k_and_l(vektor, k, l) := | v ← vektor
                                          | nrow ← rows(v)
                                          | nn ← 0
                                          | for t1 ∈ 0 .. nrow − 1
                                          |   nn ← nn + 1  if l ≥ v_t1 > k
                                          | nn
```

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.05$$

Standard error:

$$s := \sqrt{\frac{P_f \cdot (1 - P_f)}{N_{sim}}} = 2.184 \times 10^{-3}$$

$a_1 := 10^{15.117} = 1.309 \times 10^{15}$ $\quad m_1 := 4$ $\quad n_0 := 0.5 \cdot 365 \cdot 8000 = 1460000$

$a_2 := 10^{17.146} = 1.4 \times 10^{17}$ $\quad m_2 := 5$ $\quad n_1 := 0.125 \cdot 10^6 = 125000$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 106.967 \qquad \gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\, dt$$

$h_1 := 0.9$ $\quad q_1 := 12.75$ $\quad h_2 := 0.9$ $\quad q_2 := 15.1$ $\quad h_3 := 0.9$ $\quad q_3 := 17.1$

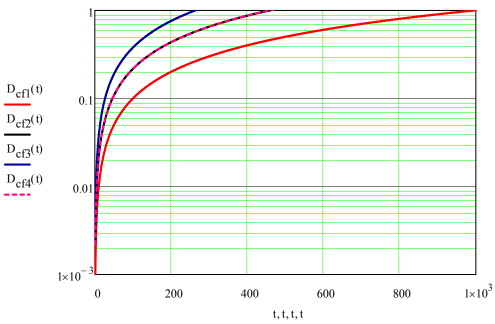
$h_4 := 3.75$ $\quad q_4 := 73$

$$D_{cf1}(t) := n_0 \cdot t \cdot \left[\frac{q_1^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_1}, \left(\frac{S_1}{q_1}\right)^{h_1}\right] + \frac{q_1^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_1}, \left(\frac{S_1}{q_1}\right)^{h_1}\right]\right]$$

$$D_{cf2}(t) := n_0 \cdot t \cdot \left[\frac{q_2^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_2}, \left(\frac{S_1}{q_2}\right)^{h_2}\right] + \frac{q_2^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_2}, \left(\frac{S_1}{q_2}\right)^{h_2}\right]\right]$$

$$D_{cf3}(t) := n_0 \cdot t \cdot \left[\frac{q_3^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_3}, \left(\frac{S_1}{q_3}\right)^{h_3}\right] + \frac{q_3^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_3}, \left(\frac{S_1}{q_3}\right)^{h_3}\right]\right]$$

$$D_{cf4}(t) := n_1 \cdot t \cdot \left[\frac{q_4^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_4}, \left(\frac{S_1}{q_4}\right)^{h_4}\right] + \frac{q_4^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_4}, \left(\frac{S_1}{q_4}\right)^{h_4}\right]\right]$$



Predicted fatigue life:

$$PFL := 1 \cdot \begin{pmatrix} D_{cf1}(1) \\ D_{cf2}(1) \\ D_{cf3}(1) \\ D_{cf4}(1) \end{pmatrix}^{-1} = \begin{pmatrix} 991.171 \\ 456.534 \\ 261.883 \\ 456.893 \end{pmatrix}$$

Probabilistic Estimated years:

Pf1 = 211

Pf2 = 103

Pf3 = 61

Pf4 = 98

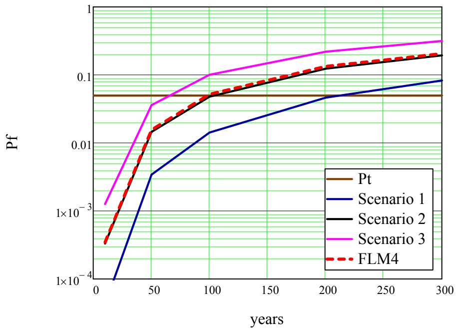
Fatigue life sensitivity to COV of B:

COV30 = 103	$D_{cf2}(103) = 0.226$
COV25 = 139	$D_{cf2}(139) = 0.304$
COV20 = 184	$D_{cf2}(184) = 0.403$
COV15 = 241	$D_{cf2}(241) = 0.528$

Case 1: Comparison of P_f

$$t := \begin{pmatrix} 10 \\ 50 \\ 100 \\ 200 \\ 300 \end{pmatrix} \qquad P_t := 0.05$$

$$P_{f1} := \begin{pmatrix} 4.30\cdot10^{-5} \\ 3.43\cdot10^{-3} \\ 1.43\cdot10^{-2} \\ 4.67\cdot10^{-2} \\ 8.30\cdot10^{-2} \end{pmatrix} \quad P_{f2} := \begin{pmatrix} 3.33\cdot10^{-4} \\ 1.46\cdot10^{-2} \\ 4.81\cdot10^{-2} \\ 0.124 \\ 0.195 \end{pmatrix} \quad P_{f3} := \begin{pmatrix} 1.27\cdot10^{-3} \\ 3.62\cdot10^{-2} \\ 0.101 \\ 0.221 \\ 0.317 \end{pmatrix} \quad P_{f4} := \begin{pmatrix} 3.47\cdot10^{-4} \\ 1.57\cdot10^{-2} \\ 5.23\cdot10^{-2} \\ 0.134 \\ 0.207 \end{pmatrix}$$



Estimated years:

Pf1 = 211

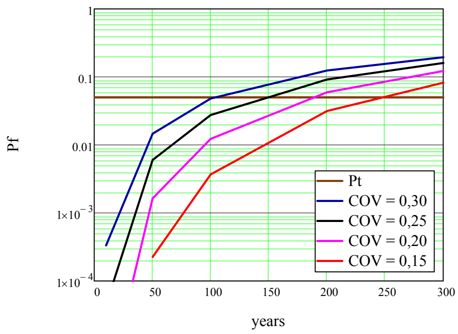
Pf2 = 103

Pf3 = 61

Pf4 = 98

Check Pf2 for sensitivity to COV of B $\qquad P_{COV30} := P_{f2}$

$$P_{COV25} := \begin{pmatrix} 4.46\cdot10^{-5} \\ 6.02\cdot10^{-3} \\ 2.76\cdot10^{-2} \\ 9.16\cdot10^{-2} \\ 0.160 \end{pmatrix} \quad P_{COV20} := \begin{pmatrix} 2.30\cdot10^{-6} \\ 1.64\cdot10^{-3} \\ 1.23\cdot10^{-2} \\ 5.95\cdot10^{-2} \\ 0.122 \end{pmatrix} \quad P_{COV15} := \begin{pmatrix} 0 \\ 2.25\cdot10^{-4} \\ 3.70\cdot10^{-3} \\ 3.15\cdot10^{-2} \\ 8.25\cdot10^{-2} \end{pmatrix}$$



Estimated years

COV30 = 103

COV25 = 139

COV20 = 184

COV15 = 241

Case 2 - Three-span bridge

The calculations done in Mathcad for this problem is on the next page.

Scenario 1 - 4% AADTT

Defining load model and stresses:

Cross section properties:

$f_y := 355MPa \qquad W_{el} := 0.0405m^3 \qquad A := 0.0764m^2 \qquad i := 0..21$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 934.12 \\ 944.43 \\ 952.39 \\ 957.60 \\ 978.98 \\ 1071.36 \\ 1178.47 \\ 1323.01 \\ 1427.71 \\ 1318.78 \\ 1340.80 \\ 946.17 \\ 960.39 \\ 971.32 \\ 978.48 \\ 1007.77 \\ 1134.56 \\ 1275.74 \\ 1474.47 \\ 1642.49 \\ 1485.61 \\ 1525.47 \end{pmatrix} \cdot kN \cdot m \qquad M_{min} := \begin{pmatrix} 890.82 \\ 887.08 \\ 884.56 \\ 882.89 \\ 876.16 \\ 846.82 \\ 817.13 \\ 771.06 \\ 706.26 \\ 747.86 \\ 728.93 \\ 884.34 \\ 878.99 \\ 874.96 \\ 872.29 \\ 861.54 \\ 814.62 \\ 767.18 \\ 693.53 \\ 590.19 \\ 656.68 \\ 626.56 \end{pmatrix} \cdot kN \cdot m$$

P_{max} and P_{min} from SAP2000 model:

$$P_{max} := \begin{pmatrix} 1656.34 \\ 1673.12 \\ 1686.08 \\ 1694.52 \\ 1729.28 \\ 1879.22 \\ 2036.95 \\ 2287.94 \\ 2483.60 \\ 2279.00 \\ 2330.62 \\ 1676.89 \\ 1700.37 \\ 1718.40 \\ 1730.17 \\ 1778.39 \\ 1986.91 \\ 2203.01 \\ 2546.38 \\ 2849.57 \\ 2563.18 \\ 2645.03 \end{pmatrix} \cdot kN \qquad P_{min} := \begin{pmatrix} 1584.54 \\ 1578.88 \\ 1574.62 \\ 1571.79 \\ 1560.43 \\ 1510.81 \\ 1460.61 \\ 1382.72 \\ 1273.02 \\ 1343.33 \\ 1311.24 \\ 1574.83 \\ 1565.97 \\ 1559.30 \\ 1554.87 \\ 1537.08 \\ 1459.40 \\ 1380.85 \\ 1258.90 \\ 1087.80 \\ 1197.87 \\ 1147.98 \end{pmatrix} \cdot kN$$

Convert moment range and axial force range to stress range:

$$\Delta\sigma_a := \frac{P_{max} - P_{min}}{A} + \frac{M_{max} - M_{min}}{W_{el}}$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$\Delta\sigma_a =$ (·MPa)

	0
0	2.009
1	2.65
2	3.134
3	3.451
4	4.749
5	10.366
6	16.466
7	25.477
8	33.659
9	26.344
10	28.451
11	2.863
12	3.769
13	4.462
14	4.916
15	6.769
16	14.804
17	23.318
18	36.134
19	49.043
20	...

$$\Delta\sigma := \begin{pmatrix} 2.009 \\ 2.650 \\ 2.863 \\ 3.451 \\ 3.451 \\ 3.769 \\ 4.462 \\ 4.749 \\ 4.916 \\ 6.769 \\ 10.366 \\ 14.804 \\ 16.466 \\ 23.318 \\ 25.477 \\ 26.344 \\ 28.451 \\ 33.659 \\ 36.134 \\ 38.338 \\ 41.790 \\ 49.043 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 12 \\ 3 \\ 4 \\ 13 \\ 14 \\ 5 \\ 15 \\ 16 \\ 6 \\ 17 \\ 7 \\ 18 \\ 8 \\ 10 \\ 11 \\ 9 \\ 19 \\ 21 \\ 22 \\ 20 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.1056 \\ 0.1056 \\ 0.1056 \\ 0.0912 \\ 0.0792 \\ 0.1056 \\ 0.0912 \\ 0.06 \\ 0.0792 \\ 0.06 \\ 0.0384 \\ 0.0384 \\ 0.008 \\ 0.008 \\ 0.002 \\ 0.003 \\ 0.001 \\ 0.006 \\ 0.002 \\ 0.003 \\ 0.001 \\ 0.006 \end{pmatrix}$$

AADT in the region is: $AADT := 8000$

Number of cycles in one lane for a design life of 100 years is:

$N_{100} := 365 \cdot AADT \cdot 100 \cdot 0.5 = 146000000$

Define cycles to each stress range:

$n := N_{100} \cdot pn$

$$n_0 := \sum_{i=0}^{21} n_i = 1.46 \times 10^8$$

Cumulative distribution for the stress range:

$$Pn_i := \sum_{j=0+i}^{21} pn_j$$

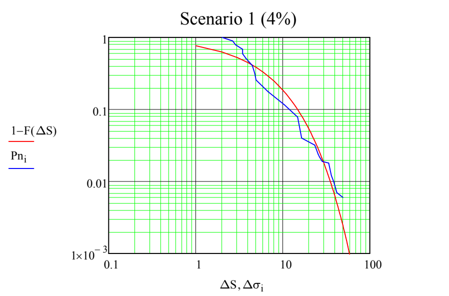
$Pn =$

	0
0	1
1	0.8944
2	0.7888
3	0.6832
4	0.592
5	0.5128
6	0.4072
7	0.316
8	0.256
9	0.1768
10	0.1168
11	0.0784
12	0.04
13	0.032
14	0.024
15	...

Fitting Weibull to the stress range:

$h := 0.8 \qquad q := 5.21 \qquad \Delta S := 0..160$

$$f(\Delta S) := \frac{h}{q} \cdot \left(\frac{\Delta S}{q}\right)^{h-1} \cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^h\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$



Deterministic and closed form calculation:

Weld class E from DNV

$a_1 := 10^{12.01} = 1.023\times 10^{12}$ $\quad m_1 := 3$

$a_2 := 10^{15.35} = 2.239\times 10^{15}$ $\quad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 46.774$$

$$N_i := \begin{cases} 10^{\log(a_1)-m_1\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2)-m_2\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{21} \frac{n_i}{N_i} \qquad D_d = 0.1619$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1}\cdot e^{-t}\,dt$$

$$D_{cf} := n_0\cdot\left[\frac{q^{m_1}}{a_1}\cdot\Gamma\left[1+\frac{m_1}{h},\left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2}\cdot\gamma\left[1+\frac{m_2}{h},\left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.186$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1)+0.4} = 2.57\times 10^{12}$ $\quad a_{m2} := 10^{\log(a_2)+0.4} = 5.623\times 10^{15}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 46.774$$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188\ln\left(1+COV^2\right)$

$COV_{am1} := \sqrt{e^{5.319\cdot sd_a^2} - 1} = 0.487$ $\quad COV_{am2} := \sqrt{e^{5.319\cdot sd_a^2} - 1} = 0.487$

$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2+1}}\right) = 28.469$ $\quad \mu_{lna2} := \ln\left(\frac{a_{m2}}{\sqrt{COV_{am2}^2+1}}\right) = 36.159$

$sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2+1\right)} = 0.461$ $\quad sd_{lna2} := \sqrt{\ln\left(COV_{am2}^2+1\right)} = 0.461$

$\Delta := 1$ $\quad COV_\Delta := 0.3$ $\quad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2+1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1$ $\quad COV_B := 0.3$ $\quad sd_{lnB} := \sqrt{\ln\left(COV_B^2+1\right)} = 0.294$

$g = \Delta - D$ $\quad N_{sim} := 10^4$ $\quad i := 0..N_{sim}-1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \left| \begin{array}{l} \text{for } i \in 0..N_{sim} \\ \quad z_{B_i} \leftarrow \left|\text{rlnorm}\left(1, \log(B), sd_{lnB}\right)\right| \\ z_B \end{array}\right.$$

$$z_{a1} := \left| \begin{array}{l} \text{for } i \in 0..N_{sim} \\ \quad z_{a1_i} \leftarrow \left|\text{rlnorm}\left(1, \mu_{lna1}, sd_{lna1}\right)\right| \\ z_{a1} \end{array}\right. \qquad z_\Delta := \left| \begin{array}{l} \text{for } i \in 0..N_{sim} \\ \quad z_{\Delta_i} \leftarrow \left|\text{rlnorm}\left(1, \log(\Delta), sd_{ln\Delta}\right)\right| \\ z_\Delta \end{array}\right.$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 2187.762$ $\quad$ this is equal to: $\quad S_{1m}^2 = 2187.762$

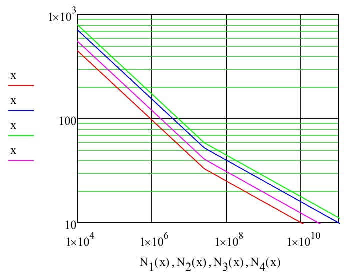
This means the correlation between z_{a1} and z_{a2} is: $\quad z_{a2} = S_z^2\cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := \left(S_{z_i}\right)^2\cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \frac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \frac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \frac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \frac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \frac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \frac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \frac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \frac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

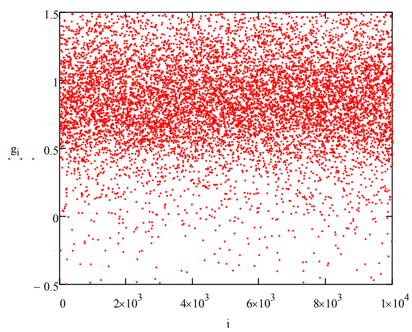


Running simulations on the limit state function:

$$D_i := n_0\cdot\left[\frac{\left(z_{B_i}\cdot q\right)^{m_1}}{z_{a1_i}}\cdot\Gamma\left[1+\frac{m_1}{h},\left(\frac{S_{z_i}}{z_{B_i}\cdot q}\right)^h\right] + \frac{\left(z_{B_i}\cdot q\right)^{m_2}}{z_{a2_i}}\cdot\gamma\left[1+\frac{m_2}{h},\left(\frac{S_{z_i}}{z_{B_i}\cdot q}\right)^h\right]\right]$$

$g_i := z_{\Delta_i} - D_i$

Plotting results:



Define a vector for how many times g < 0:

$$\text{How\_many\_between\_k\_and\_l}(\text{vektor}, k, l) := \left| \begin{array}{l} v \leftarrow \text{vektor} \\ \text{nrow} \leftarrow \text{rows}(v) \\ \text{nn} \leftarrow 0 \\ \text{for } t_1 \in 0..\text{nrow} - 1 \\ \quad \text{nn} \leftarrow \text{nn} + 1 \ \text{ if } l \geq v_{t_1} > k \\ \text{nn} \end{array} \right.$$

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.023$$

Standard error:

$$s := \sqrt{\frac{P_f \cdot (1 - P_f)}{N_{sim}}} = 1.505 \times 10^{-3}$$

Scenario 2 - 8,56% AADTT

Defining load model and stresses:

Cross section properties:

$f_y := 355MPa$ $\quad W_{el} := 0.0405m^3$ $\quad A := 0.0764m^2$ $\quad i := 0..21$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 934.12 \\ 944.43 \\ 952.39 \\ 957.60 \\ 978.98 \\ 1071.36 \\ 1178.47 \\ 1323.01 \\ 1427.71 \\ 1318.78 \\ 1340.80 \\ 946.17 \\ 960.39 \\ 971.32 \\ 978.48 \\ 1007.77 \\ 1134.56 \\ 1275.74 \\ 1474.47 \\ 1642.49 \\ 1485.61 \\ 1525.47 \end{pmatrix} \cdot kN \cdot m \qquad M_{min} := \begin{pmatrix} 890.82 \\ 887.08 \\ 884.56 \\ 882.89 \\ 876.16 \\ 846.82 \\ 817.13 \\ 771.06 \\ 706.26 \\ 747.86 \\ 728.93 \\ 884.34 \\ 878.99 \\ 874.96 \\ 872.29 \\ 861.54 \\ 814.62 \\ 767.18 \\ 693.53 \\ 590.19 \\ 656.68 \\ 626.56 \end{pmatrix} \cdot kN \cdot m$$

P_{max} and P_{min} from SAP2000 model:

$$P_{max} := \begin{pmatrix} 1656.34 \\ 1673.12 \\ 1686.08 \\ 1694.52 \\ 1729.28 \\ 1879.22 \\ 2036.95 \\ 2287.94 \\ 2483.60 \\ 2279.00 \\ 2330.62 \\ 1676.89 \\ 1700.37 \\ 1718.40 \\ 1730.17 \\ 1778.39 \\ 1986.91 \\ 2203.01 \\ 2546.38 \\ 2849.57 \\ 2563.18 \\ 2645.03 \end{pmatrix} \cdot kN \qquad P_{min} := \begin{pmatrix} 1584.54 \\ 1578.88 \\ 1574.62 \\ 1571.79 \\ 1560.43 \\ 1510.81 \\ 1460.61 \\ 1382.72 \\ 1273.02 \\ 1343.33 \\ 1311.24 \\ 1574.83 \\ 1565.97 \\ 1559.30 \\ 1554.87 \\ 1537.08 \\ 1459.40 \\ 1380.85 \\ 1258.90 \\ 1087.80 \\ 1197.87 \\ 1147.98 \end{pmatrix} \cdot kN$$

Convert moment range and axial force range to stress range:

$$\Delta\sigma_a := \frac{P_{max} - P_{min}}{A} + \frac{M_{max} - M_{min}}{W_{el}}$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$\Delta\sigma_a =$

	0
0	2.009
1	2.65
2	3.134
3	3.451
4	4.749
5	10.366
6	16.466
7	25.477
8	33.659
9	26.344
10	28.451
11	2.863
12	3.769
13	4.462
14	4.916
15	...

$\cdot MPa$

$$\Delta\sigma := \begin{pmatrix} 2.009 & 1 \\ 2.650 & 2 \\ 2.863 & 12 \\ 3.451 & 3 \\ 3.451 & 4 \\ 3.769 & 13 \\ 4.462 & 14 \\ 4.749 & 5 \\ 4.916 & 15 \\ 6.769 & 16 \\ 10.366 & 6 \\ 14.804 & 17 \\ 16.466 & 7 \\ 23.318 & 18 \\ 25.477 & 8 \\ 26.344 & 10 \\ 28.451 & 11 \\ 33.659 & 9 \\ 36.134 & 19 \\ 38.338 & 21 \\ 41.790 & 22 \\ 49.043 & 20 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.100584 \\ 0.100584 \\ 0.100584 \\ 0.086868 \\ 0.075438 \\ 0.100584 \\ 0.086868 \\ 0.05715 \\ 0.075438 \\ 0.05715 \\ 0.036576 \\ 0.036576 \\ 0.01712 \\ 0.01712 \\ 0.00428 \\ 0.00642 \\ 0.00214 \\ 0.01284 \\ 0.00428 \\ 0.00642 \\ 0.00214 \\ 0.01284 \end{pmatrix}$$

AADT in the region is: $ADT := 8000$

Number of cycles in one lane for a design life of 100 years is:

$N_{100} := 365 \cdot ADT \cdot 100 \cdot 0.5 = 146000000$

Define cycles to each stress range:

$n := N_{100} \cdot pn$

$$n_0 := \sum_{i=0}^{21} n_i = 1.46 \times 10^8$$

Cumulative distribution for the stress range:

$$Pn_i := \sum_{j=0+i}^{21} pn_j$$

$Pn =$

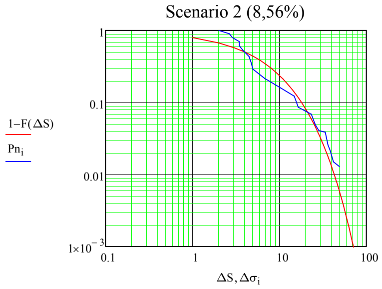
	0
0	1
1	0.899
2	0.799
3	0.698
4	0.611
5	0.536
6	0.435
7	0.348
8	0.291
9	0.216
10	0.159
11	0.122
12	0.086
13	0.068
14	0.051
15	...

Fitting Weibull to the stress range:

$h := 0.8$ $\quad q := 6.33$ $\quad \Delta S := 0..160$

$$f(\Delta S) := \frac{h}{q} \cdot \left(\frac{\Delta S}{q}\right)^{h-1} \cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^h\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\, dx$$

Scenario 2 (8,56%)



Deterministic and closed form calculation:

Weld class E from DNV

$a_1 := 10^{12.01} = 1.023 \times 10^{12}$ $\quad m_1 := 3$

$a_2 := 10^{15.35} = 2.239 \times 10^{15}$ $\quad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 46.774$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{21} \frac{n_i}{N_i} \qquad D_d = 0.344$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\, dt$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.396$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1) + 0.4} = 2.57 \times 10^{12}$ $\quad a_{m2} := 10^{\log(a_2) + 0.4} = 5.623 \times 10^{15}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 46.774$$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188 \ln(1 + COV^2)$

$COV_{am1} := \sqrt{e^{5.319 \cdot sd_a^2} - 1} = 0.487$ $\quad COV_{am2} := \sqrt{e^{5.319 \cdot sd_a^2} - 1} = 0.487$

$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2 + 1}}\right) = 28.469$ $\quad \mu_{lna2} := \ln\left(\frac{a_{m2}}{\sqrt{COV_{am2}^2 + 1}}\right) = 36.159$

$sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2 + 1\right)} = 0.461$ $\quad sd_{lna2} := \sqrt{\ln\left(COV_{am2}^2 + 1\right)} = 0.461$

$\Delta := 1$ $\quad COV_\Delta := 0.3$ $\quad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2 + 1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1$ $\quad COV_B := 0.3$ $\quad sd_{lnB} := \sqrt{\ln\left(COV_B^2 + 1\right)} = 0.294$

$g = \Delta - D$ $\quad N_{sim} := 10^4$ $\quad i := 0 .. N_{sim} - 1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{B_i} \leftarrow |\text{rlnorm}(1, \log(B), sd_{lnB})| \\ z_B \end{array}$$

$$z_{a1} := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{a1_i} \leftarrow |\text{rlnorm}(1, \mu_{lna1}, sd_{lna1})| \\ z_{a1} \end{array} \qquad z_\Delta := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{\Delta_i} \leftarrow |\text{rlnorm}(1, \log(\Delta), sd_{ln\Delta})| \\ z_\Delta \end{array}$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 2187.762$ this is equal to: $\quad S_{1m}^2 = 2187.762$

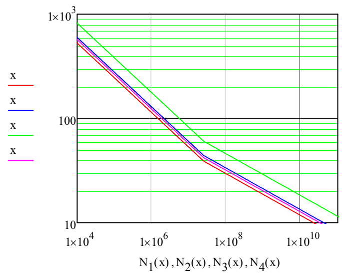
This means the correlation between z_{a1} and z_{a2} is: $\quad z_{a2} = S_z^2 \cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := \left(S_{z_i}\right)^2 \cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \frac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \frac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \frac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \frac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \frac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \frac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \frac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \frac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

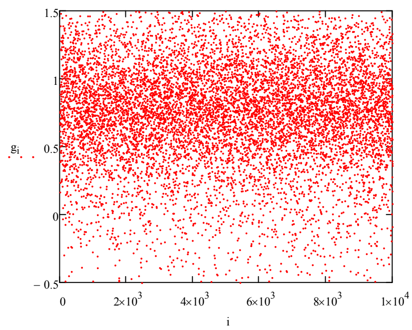


Running simulations on the limit state function:

$$D_i := n_0 \cdot \left[\frac{(z_{B_i} \cdot q)^{m_1}}{z_{a1_i}} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] + \frac{(z_{B_i} \cdot q)^{m_2}}{z_{a2_i}} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right]\right]$$

$g_i := z_{\Delta_i} - D_i$

Plotting results:



Define a vector for how many times g < 0:

$$\text{How\_many\_between\_k\_and\_l}(\text{vektor}, k, l) := \left| \begin{array}{l} v \leftarrow \text{vektor} \\ \text{nrow} \leftarrow \text{rows}(v) \\ \text{nn} \leftarrow 0 \\ \text{for } t_1 \in 0..\text{nrow} - 1 \\ \quad \text{nn} \leftarrow \text{nn} + 1 \ \text{ if } l \geq v_{t_1} > k \\ \text{nn} \end{array} \right.$$

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.075$$

Standard error:

$$s := \sqrt{\frac{P_f \cdot (1 - P_f)}{N_{sim}}} = 2.632 \times 10^{-3}$$

Scenario 3 - 15% AADTT

Defining load model and stresses:

Cross section properties:

$f_y := 355\text{MPa}$ $\quad W_{el} := 0.0405\text{m}^3$ $\quad A := 0.0764\text{m}^2$ $\quad i := 0..21$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 934.12 \\ 944.43 \\ 952.39 \\ 957.60 \\ 978.98 \\ 1071.36 \\ 1178.47 \\ 1323.01 \\ 1427.71 \\ 1318.78 \\ 1340.80 \\ 946.17 \\ 960.39 \\ 971.32 \\ 978.48 \\ 1007.77 \\ 1134.56 \\ 1275.74 \\ 1474.47 \\ 1642.49 \\ 1485.61 \\ 1525.47 \end{pmatrix} \cdot \text{kN}\cdot\text{m} \qquad M_{min} := \begin{pmatrix} 890.82 \\ 887.08 \\ 884.56 \\ 882.89 \\ 876.16 \\ 846.82 \\ 817.13 \\ 771.06 \\ 706.26 \\ 747.86 \\ 728.93 \\ 884.34 \\ 878.99 \\ 874.96 \\ 872.29 \\ 861.54 \\ 814.62 \\ 767.18 \\ 693.53 \\ 590.19 \\ 656.68 \\ 626.56 \end{pmatrix} \cdot \text{kN}\cdot\text{m}$$

P_{max} and P_{min} from SAP2000 model:

$$P_{max} := \begin{pmatrix} 1656.34 \\ 1673.12 \\ 1686.08 \\ 1694.52 \\ 1729.28 \\ 1879.22 \\ 2036.95 \\ 2287.94 \\ 2483.60 \\ 2279.00 \\ 2330.62 \\ 1676.89 \\ 1700.37 \\ 1718.40 \\ 1730.17 \\ 1778.39 \\ 1986.91 \\ 2203.01 \\ 2546.38 \\ 2849.57 \\ 2563.18 \\ 2645.03 \end{pmatrix} \cdot \text{kN} \qquad P_{min} := \begin{pmatrix} 1584.54 \\ 1578.88 \\ 1574.62 \\ 1571.79 \\ 1560.43 \\ 1510.81 \\ 1460.61 \\ 1382.72 \\ 1273.02 \\ 1343.33 \\ 1311.24 \\ 1574.83 \\ 1565.97 \\ 1559.30 \\ 1554.87 \\ 1537.08 \\ 1459.40 \\ 1380.85 \\ 1258.90 \\ 1087.80 \\ 1197.87 \\ 1147.98 \end{pmatrix} \cdot \text{kN}$$

Convert moment range and axial force range to stress range:

$$\Delta\sigma_a := \frac{P_{max} - P_{min}}{A} + \frac{M_{max} - M_{min}}{W_{el}}$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$\Delta\sigma_a =$ (MPa)

	0
0	2.009
1	2.65
2	3.134
3	3.451
4	4.749
5	10.366
6	16.466
7	25.477
8	33.659
9	26.344
10	28.451
11	2.863
12	3.769
13	4.462
14	4.916
15	...

$$\Delta\sigma := \begin{pmatrix} 2.009 \\ 2.650 \\ 2.863 \\ 3.451 \\ 3.451 \\ 3.769 \\ 4.462 \\ 4.749 \\ 4.916 \\ 6.769 \\ 10.366 \\ 14.804 \\ 16.466 \\ 23.318 \\ 25.477 \\ 26.344 \\ 28.451 \\ 33.659 \\ 36.134 \\ 38.338 \\ 41.790 \\ 49.043 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 12 \\ 3 \\ 4 \\ 13 \\ 14 \\ 5 \\ 15 \\ 16 \\ 6 \\ 17 \\ 7 \\ 18 \\ 8 \\ 10 \\ 11 \\ 9 \\ 19 \\ 21 \\ 22 \\ 20 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.0935 \\ 0.0935 \\ 0.0935 \\ 0.08075 \\ 0.070125 \\ 0.0935 \\ 0.08075 \\ 0.053125 \\ 0.070125 \\ 0.053125 \\ 0.034 \\ 0.034 \\ 0.03 \\ 0.03 \\ 0.0075 \\ 0.01125 \\ 0.00375 \\ 0.0225 \\ 0.0075 \\ 0.01125 \\ 0.00375 \\ 0.0225 \end{pmatrix}$$

AADT in the region is: $\quad ADT := 8000$

Number of cycles in one lane for a design life of 100 years is:

$$N_{100} := 365 \cdot ADT \cdot 100 \cdot 0.5 = 146000000$$

Define cycles to each stress range:

$$n := N_{100} \cdot pn$$

$$n_0 := \sum_{i=0}^{21} n_i = 1.46 \times 10^8$$

Cumulative distribution for the stress range:

$$Pn_i := \sum_{j=0+i}^{21} pn_j$$

$Pn =$

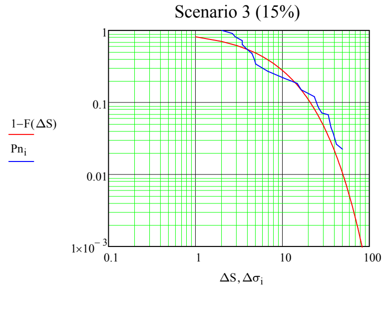
	0
0	1
1	0.906
2	0.813
3	0.719
4	0.639
5	0.569
6	0.475
7	0.394
8	0.341
9	0.271
10	0.218
11	0.184
12	0.15
13	0.12
14	0.09
15	...

Fitting Weibull to the stress range:

$h := 0.8 \quad q := 7.36 \quad \Delta S := 0..160$

$$f(\Delta S) := \frac{h}{q}\cdot\left(\frac{\Delta S}{q}\right)^{h-1}\cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^{h}\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$

Scenario 3 (15%)



Deterministic and closed form calculation:

Weld class E from DNV

$a_1 := 10^{12.01} = 1.023\times 10^{12}$ $\quad m_1 := 3$

$a_2 := 10^{15.35} = 2.239\times 10^{15}$ $\quad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 46.774$$

$$N_i := \begin{cases} 10^{\log(a_1)-m_1\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2)-m_2\cdot\log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{21} \frac{n_i}{N_i} \qquad D_d = 0.601$$

$$\gamma(\alpha,x) := \int_0^x t^{\alpha-1}\cdot e^{-t}\,dt$$

$$D_{cf} := n_0\cdot\left[\frac{q^{m_1}}{a_1}\cdot\Gamma\left[1+\frac{m_1}{h},\left(\frac{S_1}{q}\right)^h\right]+\frac{q^{m_2}}{a_2}\cdot\gamma\left[1+\frac{m_2}{h},\left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.692$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1)+0.4} = 2.57\times 10^{12}$ $\quad a_{m2} := 10^{\log(a_2)+0.4} = 5.623\times 10^{15}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 46.774$$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188\ln\left(1+COV^2\right)$

$COV_{am1} := \sqrt{e^{5.319\cdot sd_a^2}-1} = 0.487$ $\quad COV_{am2} := \sqrt{e^{5.319\cdot sd_a^2}-1} = 0.487$

$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2+1}}\right) = 28.469$ $\quad \mu_{lna2} := \ln\left(\frac{a_{m2}}{\sqrt{COV_{am2}^2+1}}\right) = 36.159$

$sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2+1\right)} = 0.461$ $\quad sd_{lna2} := \sqrt{\ln\left(COV_{am2}^2+1\right)} = 0.461$

$\Delta := 1$ $\quad COV_\Delta := 0.3$ $\quad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2+1\right)} = 0.294$

B is a correction factor accounting for the uncertainties in the stress modelling.

$B := 1$ $\quad COV_B := 0.3$ $\quad sd_{lnB} := \sqrt{\ln\left(COV_B^2+1\right)} = 0.294$

$g = \Delta - D$ $\quad N_{sim} := 10^4$ $\quad i := 0..N_{sim}-1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \begin{array}{|l} \text{for } i\in 0..N_{sim} \\ \quad z_{B_i} \leftarrow \left|\text{rlnorm}\left(1,\log(B),sd_{lnB}\right)\right| \\ z_B \end{array}$$

$$z_{a1} := \begin{array}{|l} \text{for } i\in 0..N_{sim} \\ \quad z_{a1_i} \leftarrow \left|\text{rlnorm}\left(1,\mu_{lna1},sd_{lna1}\right)\right| \\ z_{a1} \end{array} \qquad z_\Delta := \begin{array}{|l} \text{for } i\in 0..N_{sim} \\ \quad z_{\Delta_i} \leftarrow \left|\text{rlnorm}\left(1,\log(\Delta),sd_{ln\Delta}\right)\right| \\ z_\Delta \end{array}$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 2187.762$ $\quad$ this is equal to: $\quad S_{1m}^2 = 2187.762$

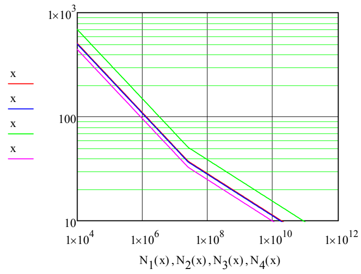
This means the correlation between z_{a1} and z_{a2} is: $\quad z_{a2} = S_z^2\cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := \left(S_{z_i}\right)^2\cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \frac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \frac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \frac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \frac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \frac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \frac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \frac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \frac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

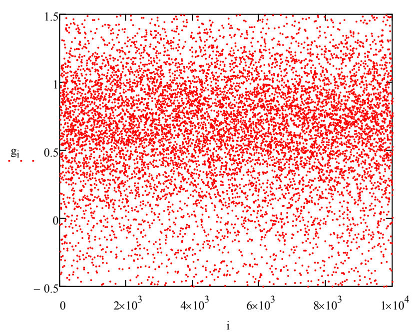


Running simulations on the limit state function:

$$D_i := n_0\cdot\left[\frac{\left(z_{B_i}\cdot q\right)^{m_1}}{z_{a1_i}}\cdot\Gamma\left[1+\frac{m_1}{h},\left(\frac{S_{z_i}}{z_{B_i}\cdot q}\right)^h\right]+\frac{\left(z_{B_i}\cdot q\right)^{m_2}}{z_{a2_i}}\cdot\gamma\left[1+\frac{m_2}{h},\left(\frac{S_{z_i}}{z_{B_i}\cdot q}\right)^h\right]\right]$$

$g_i := z_{\Delta_i} - D_i$

Plotting results:



Define a vector for how many times g < 0:

$$\text{How\_many\_between\_k\_and\_l}(\text{vektor}, k, l) := \left| \begin{array}{l} v \leftarrow \text{vektor} \\ \text{nrow} \leftarrow \text{rows}(v) \\ \text{nn} \leftarrow 0 \\ \text{for } t_1 \in 0 .. \text{nrow} - 1 \\ \quad \text{nn} \leftarrow \text{nn} + 1 \;\; \text{if } l \geq v_{t_1} > k \\ \text{nn} \end{array} \right.$$

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.165$$

Standard error:

$$s := \sqrt{\frac{P_f \cdot (1 - P_f)}{N_{sim}}} = 3.713 \times 10^{-3}$$

FLM4

Defining load model and stresses:

Cross section properties:

$f_y := 355MPa \qquad W_{el} := 0.0405m^3 \qquad A := 0.0764m^2 \qquad i := 0..4$

M_{max} and M_{min} from SAP2000 model:

$$M_{max} := \begin{pmatrix} 1178.47 \\ 1323.01 \\ 1427.71 \\ 1318.78 \\ 1340.80 \end{pmatrix} \cdot kN \cdot m \qquad M_{min} := \begin{pmatrix} 817.13 \\ 771.06 \\ 706.26 \\ 747.86 \\ 728.93 \end{pmatrix} \cdot kN \cdot m$$

P_{max} and P_{min} from SAP2000 model:

$$P_{max} := \begin{pmatrix} 2036.95 \\ 2287.94 \\ 2483.60 \\ 2279.00 \\ 2330.62 \end{pmatrix} \cdot kN \qquad P_{min} := \begin{pmatrix} 1460.61 \\ 1382.72 \\ 1273.02 \\ 1343.33 \\ 1311.24 \end{pmatrix} \cdot kN$$

Convert moment range and axial force range to stress range:

$$\Delta\sigma_a := \frac{P_{max} - P_{min}}{A} + \frac{M_{max} - M_{min}}{W_{el}}$$

Sorting the stress range $\Delta\sigma_a$ from minimum to maximum in $\Delta\sigma$.
Column next to $\Delta\sigma$ defines the corresponding load case from SAP2000.
pn defines the corresponding probability of occurance for $\Delta\sigma$.

$$\Delta\sigma_a = \begin{pmatrix} 16.466 \\ 25.477 \\ 33.659 \\ 26.344 \\ 28.451 \end{pmatrix} \cdot MPa \qquad \Delta\sigma := \begin{pmatrix} 16.466 \\ 25.477 \\ 26.344 \\ 28.451 \\ 33.659 \end{pmatrix} \qquad pn := \begin{pmatrix} 0.4 \\ 0.1 \\ 0.15 \\ 0.05 \\ 0.3 \end{pmatrix}$$

Number of cycles in one lane for a design life of 100 years is from table 4.5, row 2 in NS-EN 1991-2:

$N_{100} := 0.125 \cdot 10^6 \cdot 100 = 12500000$

Define cycles to each stress range:

$n := N_{100} \cdot pn$

$$n_0 := \sum_{i=0}^{4} n_i = 1.25 \times 10^7$$

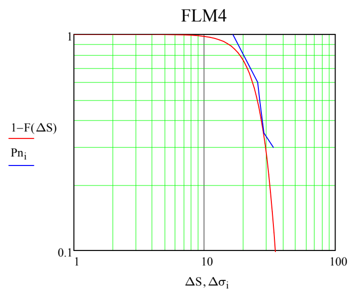
Cumulative distribution for the stress range:

$$Pn_i := \sum_{j=0+i}^{4} pn_j \qquad Pn = \begin{pmatrix} 1 \\ 0.6 \\ 0.5 \\ 0.35 \\ 0.3 \end{pmatrix}$$

Fitting Weibull to the stress range:

$h := 3.75 \qquad q := 27.85 \qquad \Delta S := 0..160$

$$f(\Delta S) := \frac{h}{q} \cdot \left(\frac{\Delta S}{q}\right)^{h-1} \cdot \exp\left[-\left(\frac{\Delta S}{q}\right)^{h}\right] \qquad F(\Delta S) := \int_0^{\Delta S} f(x)\,dx$$



Deterministic and closed form calculation:

Weld class E from DNV

$a_1 := 10^{12.01} = 1.023 \times 10^{12} \qquad m_1 := 3$

$a_2 := 10^{15.35} = 2.239 \times 10^{15} \qquad m_2 := 5$

$$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 46.774$$

$$N_i := \begin{cases} 10^{\log(a_1) - m_1 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i > S_1 \\ 10^{\log(a_2) - m_2 \cdot \log(\Delta\sigma_i)} & \text{if } \Delta\sigma_i \le S_1 \end{cases}$$

$$D_d := \sum_{i=0}^{4} \frac{n_i}{N_i} \qquad D_d = 0.097$$

$$\gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\,dt$$

$$D_{cf} := n_0 \cdot \left[\frac{q^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_1}{q}\right)^h\right] + \frac{q^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_1}{q}\right)^h\right]\right]$$

$D_{cf} = 0.111$

Probabilistic

Add 2 standard deviations to a to find the mean:

$sd_a := 0.2$

$a_{m1} := 10^{\log(a_1) + 0.4} = 2.57 \times 10^{12} \qquad a_{m2} := 10^{\log(a_2) + 0.4} = 5.623 \times 10^{15}$

$$S_{1m} := \left(\frac{10^{7.4}}{a_{m1}}\right)^{\frac{-1}{m_1}} = 46.774$$

Convert the following equation to find COV for a_m: $\quad sd_{log}^2 = 0.188 \ln(1 + COV^2)$

$$COV_{am1} := \sqrt{e^{5.319 \cdot sd_a^2} - 1} = 0.487 \qquad COV_{am2} := \sqrt{e^{5.319 \cdot sd_a^2} - 1} = 0.487$$

$$\mu_{lna1} := \ln\left(\frac{a_{m1}}{\sqrt{COV_{am1}^2 + 1}}\right) = 28.469 \qquad \mu_{lna2} := \ln\left(\frac{a_{m2}}{\sqrt{COV_{am2}^2 + 1}}\right) = 36.159$$

$$sd_{lna1} := \sqrt{\ln\left(COV_{am1}^2 + 1\right)} = 0.461 \qquad sd_{lna2} := \sqrt{\ln\left(COV_{am2}^2 + 1\right)} = 0.461$$

$$\Delta := 1 \qquad COV_\Delta := 0.3 \qquad sd_{ln\Delta} := \sqrt{\ln\left(COV_\Delta^2 + 1\right)} = 0.294$$

B is a correction factor accounting for the uncertainties in the stress modelling.

$$B := 1 \qquad COV_B := 0.3 \qquad sd_{lnB} := \sqrt{\ln\left(COV_B^2 + 1\right)} = 0.294$$

$g = \Delta - D \qquad N_{sim} := 10^4 \qquad i := 0..N_{sim} - 1$

Create random realizations for a_1, a_2, B and Δ

$$z_B := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{B_i} \leftarrow \left|\text{rlnorm}(1, \log(B), sd_{lnB})\right| \\ z_B \end{array}$$

$$z_{a1} := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{a1_i} \leftarrow \left|\text{rlnorm}(1, \mu_{lna1}, sd_{lna1})\right| \\ z_{a1} \end{array} \qquad z_\Delta := \begin{array}{|l} \text{for } i \in 0..N_{sim} \\ \quad z_{\Delta_i} \leftarrow \left|\text{rlnorm}(1, \log(\Delta), sd_{ln\Delta})\right| \\ z_\Delta \end{array}$$

To obtain a z_{a2} that correlate to a random z_{a1}, we find the relation between them.

$\frac{a_{m2}}{a_{m1}} = 2187.762$ this is equal to: $S_{1m}{}^{2} = 2187.762$

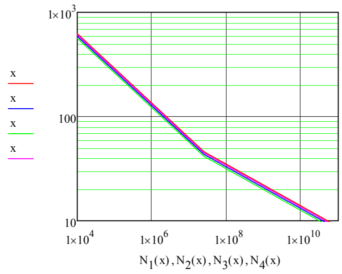
This means the correlation between z_{a1} and z_{a2} is: $z_{a2} = S_z{}^{2} \cdot z_{a1}$

$$S_{z_i} := \left(\frac{10^{7.4}}{z_{a1_i}}\right)^{\frac{-1}{m_1}} \qquad z_{a2_i} := \left(S_{z_i}\right)^2 \cdot z_{a1_i}$$

Visual check that z_{a1} and z_{a2} correlate:

$$N_1(x) := \begin{cases} \dfrac{z_{a1_1}}{x^{m_1}} & \text{if } x > S_{z_1} \\ \dfrac{z_{a2_1}}{x^{m_2}} & \text{if } x \le S_{z_1} \end{cases} \qquad N_3(x) := \begin{cases} \dfrac{z_{a1_3}}{x^{m_1}} & \text{if } x > S_{z_3} \\ \dfrac{z_{a2_3}}{x^{m_2}} & \text{if } x \le S_{z_3} \end{cases}$$

$$N_2(x) := \begin{cases} \dfrac{z_{a1_2}}{x^{m_1}} & \text{if } x > S_{z_2} \\ \dfrac{z_{a2_2}}{x^{m_2}} & \text{if } x \le S_{z_2} \end{cases} \qquad N_4(x) := \begin{cases} \dfrac{z_{a1_4}}{x^{m_1}} & \text{if } x > S_{z_4} \\ \dfrac{z_{a2_4}}{x^{m_2}} & \text{if } x \le S_{z_4} \end{cases}$$

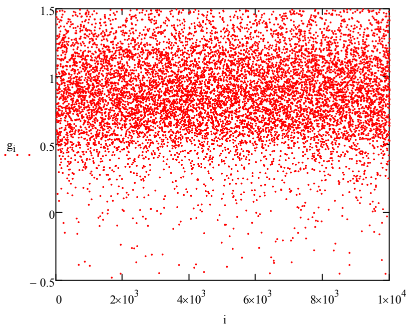


Running simulations on the limit state function:

$$D_i := n_0 \cdot \left[\frac{\left(z_{B_i} \cdot q\right)^{m_1}}{z_{a1_i}} \cdot \Gamma\left[1 + \frac{m_1}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right] + \frac{\left(z_{B_i} \cdot q\right)^{m_2}}{z_{a2_i}} \cdot \gamma\left[1 + \frac{m_2}{h}, \left(\frac{S_{z_i}}{z_{B_i} \cdot q}\right)^h\right]\right]$$

$$g_i := z_{\Delta_i} - D_i$$

Plotting results:



Define a vector for how many times g < 0:

$$\text{How\_many\_between\_k\_and\_l}(\text{vektor}, k, l) := \begin{array}{|l} v \leftarrow \text{vektor} \\ \text{nrow} \leftarrow \text{rows}(v) \\ \text{nn} \leftarrow 0 \\ \text{for } t_1 \in 0..\text{nrow} - 1 \\ \quad \text{nn} \leftarrow \text{nn} + 1 \ \text{ if } l \ge v_{t_1} > k \\ \text{nn} \end{array}$$

Probability of failure:

$$P_f := \frac{\text{How\_many\_between\_k\_and\_l}(g, -100, 0)}{N_{sim} - 1} = 0.014$$

Standard error:

$$s := \sqrt{\frac{P_f \cdot \left(1 - P_f\right)}{N_{sim}}} = 1.162 \times 10^{-3}$$

$a_1 := 10^{12.01} = 1.023 \times 10^{12}$ $\quad m_1 := 3$ $\quad n_0 := 0.5 \cdot 365 \cdot 8000 = 1460000$

$a_2 := 10^{15.35} = 2.239 \times 10^{15}$ $\quad m_2 := 5$ $\quad n_1 := 0.125 \cdot 10^6 = 125000$

$S_1 := \left(\frac{10^7}{a_1}\right)^{\frac{-1}{m_1}} = 46.774$ $\qquad \gamma(\alpha, x) := \int_0^x t^{\alpha-1} \cdot e^{-t}\, dt$

$h_1 := 0.8$ $\quad q_1 := 5.21$ $\quad h_2 := 0.8$ $\quad q_2 := 6.33$ $\quad h_3 := 0.8$ $\quad q_3 := 7.36$

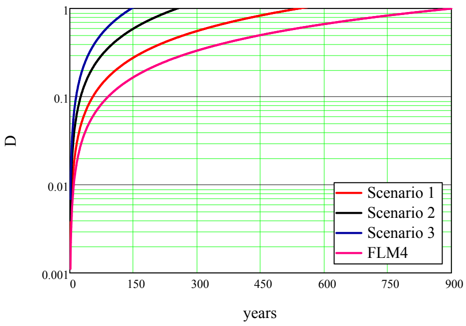
$h_4 := 3.75$ $\quad q_4 := 27.85$

$$D_{cf1}(t) := n_0 \cdot t \cdot \left[\frac{q_1^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_1}, \left(\frac{S_1}{q_1}\right)^{h_1}\right] + \frac{q_1^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_1}, \left(\frac{S_1}{q_1}\right)^{h_1}\right] \right]$$

$$D_{cf2}(t) := n_0 \cdot t \cdot \left[\frac{q_2^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_2}, \left(\frac{S_1}{q_2}\right)^{h_2}\right] + \frac{q_2^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_2}, \left(\frac{S_1}{q_2}\right)^{h_2}\right] \right]$$

$$D_{cf3}(t) := n_0 \cdot t \cdot \left[\frac{q_3^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_3}, \left(\frac{S_1}{q_3}\right)^{h_3}\right] + \frac{q_3^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_3}, \left(\frac{S_1}{q_3}\right)^{h_3}\right] \right]$$

$$D_{cf4}(t) := n_1 \cdot t \cdot \left[\frac{q_4^{m_1}}{a_1} \cdot \Gamma\left[1 + \frac{m_1}{h_4}, \left(\frac{S_1}{q_4}\right)^{h_4}\right] + \frac{q_4^{m_2}}{a_2} \cdot \gamma\left[1 + \frac{m_2}{h_4}, \left(\frac{S_1}{q_4}\right)^{h_4}\right] \right]$$



Predicted fatigue life:

$$PFL := 1 \cdot \begin{pmatrix} D_{cf1}(1) \\ D_{cf2}(1) \\ D_{cf3}(1) \\ D_{cf4}(1) \end{pmatrix}^{-1} = \begin{pmatrix} 538.428 \\ 252.4 \\ 144.542 \\ 898.623 \end{pmatrix}$$

Estimated probabilistic fatigue life:

Pf1 = 150	$D_{cf1}(150) = 0.279$
Pf2 = 78	$D_{cf2}(78) = 0.309$
Pf3 = 48	$D_{cf3}(48) = 0.332$
Pf4 = 181	$D_{cf4}(181) = 0.201$

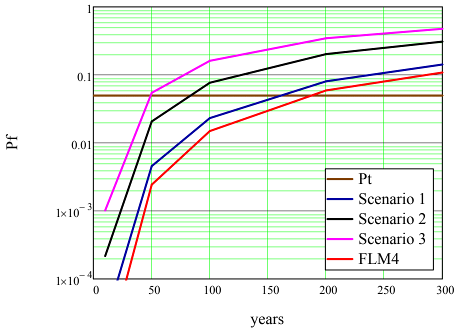
Fatigue life sensitivity to COV for B:

COV30 = 78	$D_{cf2}(78) = 0.309$
COV25 = 97	$D_{cf2}(97) = 0.384$
COV20 = 119	$D_{cf2}(119) = 0.471$
COV15 = 144	$D_{cf2}(144) = 0.571$

Case 2: Comparison of P_f

$$t := \begin{pmatrix} 10 \\ 50 \\ 100 \\ 200 \\ 300 \end{pmatrix} \qquad P_t := 0.05$$

$$P_{f1} := \begin{pmatrix} 2.39 \cdot 10^{-5} \\ 4.58 \cdot 10^{-3} \\ 2.33 \cdot 10^{-2} \\ 8.08 \cdot 10^{-2} \\ 0.143 \end{pmatrix} \quad P_{f2} := \begin{pmatrix} 2.18 \cdot 10^{-4} \\ 2.08 \cdot 10^{-2} \\ 7.72 \cdot 10^{-2} \\ 0.203 \\ 0.310 \end{pmatrix} \quad P_{f3} := \begin{pmatrix} 1.02 \cdot 10^{-3} \\ 5.48 \cdot 10^{-2} \\ 0.162 \\ 0.348 \\ 0.478 \end{pmatrix} \quad P_{f4} := \begin{pmatrix} 6.80 \cdot 10^{-6} \\ 2.44 \cdot 10^{-3} \\ 1.50 \cdot 10^{-2} \\ 5.95 \cdot 10^{-2} \\ 0.109 \end{pmatrix}$$



Estimated years:

Pf1 = 150

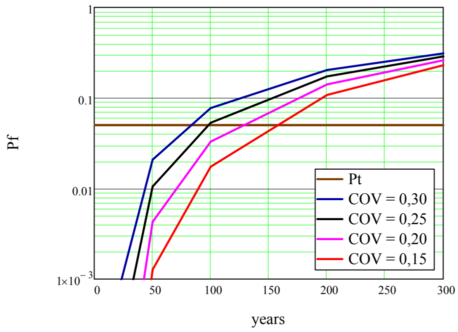
Pf2 = 78

Pf3 = 48

Pf4 = 181

Check Pf2 for sensitivity to COV of B $\qquad P_{COV30} := P_{f2}$

$$P_{COV25} := \begin{pmatrix} 3.54 \cdot 10^{-5} \\ 1.05 \cdot 10^{-2} \\ 5.33 \cdot 10^{-2} \\ 0.173 \\ 0.287 \end{pmatrix} \quad P_{COV20} := \begin{pmatrix} 1.80 \cdot 10^{-6} \\ 4.28 \cdot 10^{-3} \\ 3.29 \cdot 10^{-2} \\ 0.141 \\ 0.261 \end{pmatrix} \quad P_{COV15} := \begin{pmatrix} 1.00 \cdot 10^{-7} \\ 1.27 \cdot 10^{-3} \\ 1.74 \cdot 10^{-2} \\ 0.108 \\ 0.230 \end{pmatrix}$$



Estimated years

COV30 = 78

COV25 = 97

COV20 = 119

COV15 = 144

B R programming script

Method Validation Example

The scripts from R used in the Monte Carlo simulations for this problem is on the next page.

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.519*10^6

q = 25.5
h = 1.25
m1 = 3
m2 = 5

mu_a1 = 26.744
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z^2*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))

g1 = z_delta - D

g0 = sum(g1<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Case 1 - Single-span bridge

The scripts from R used in the Monte Carlo simulations for this problem is on the next page.

Scenario 1

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.46*10^8

q = 12.75
h = 0.9

m1 = 4
m2 = 5

mu_a1 = 35.623
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))

g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Scenario 2

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.46*10^8

q = 15.1
h = 0.9

m1 = 4
m2 = 5

mu_a1 = 35.623
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))

g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Scenario 3

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.46*10^8

q = 17.1
h = 0.9

m1 = 4
m2 = 5

mu_a1 = 35.623
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))

g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

FLM4

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.25*10^7

q = 73
h = 3.75

m1 = 4
m2 = 5

mu_a1 = 35.623
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))

g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Case 2 - Three-span bridge

The scripts from R used in the Monte Carlo simulations for this problem is on the next page.

Scenario 1

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.46*10^8

q = 5.21
h = 0.8

m1 = 3
m2 = 5

mu_a1 = 28.469
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z^2*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))


g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Scenario 2

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.46*10^8

q = 6.33
h = 0.8

m1 = 3
m2 = 5

mu_a1 = 28.469
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z^2*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))


g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

Scenario 3

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.46*10^8

q = 7.36
h = 0.8

m1 = 3
m2 = 5

mu_a1 = 28.469
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z^2*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))


g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```

FLM4

```
library(zipfR)
G = function(a,x){Igamma(a,x,lower=FALSE)}
g = function(a,x){Igamma(a,x)}

n0 = 1.25*10^7

q = 27.85
h = 3.75

m1 = 3
m2 = 5

mu_a1 = 28.469
sd_a1 = 0.461
mu_delta = log(1)
sd_delta = 0.294
mu_B = log(1)
sd_B = 0.294

N_sim = 10^7

z_B = rlnorm(N_sim,mu_B,sd_B)
z_delta = rlnorm(N_sim,mu_delta,sd_delta)
z_a1 = rlnorm(N_sim,mu_a1,sd_a1)
S_z = ((10^7.4)/z_a1)^(-1/m1)
z_a2 = S_z^2*z_a1

D = n0*((((z_B*q)^m1)/z_a1)*G(1+(m1/h),(S_z/(z_B*q))^h)+
(((z_B*q)^m2)/z_a2)*g(1+(m2/h),(S_z/(z_B*q))^h))


g = z_delta - D

g0 = sum(g<=0)

p_f = g0/N_sim

s = sqrt((p_f*(1-p_f))/N_sim)

p_f
s
```