

Geometrical Modeling for Material Flow Management

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ABSTRACT

Theory of Connection is based on geometry, and has been used to solve many problems in engineering, economics, and in management. In this paper, firstly, the theory of connection is introduced. Secondly, a toolbox of functions for modeling and simulation is presented; the toolbox is developed for MATLAB platform. Thirdly, a case study is presented on the application of the theory to solve material flow problems. The scope of this paper is limited to introducing and applying the theory of connection. The major contribution of the paper is the introduction of the toolbox with which mathematical modeling and simulations can be done efficiently on an operating platform like MATLAB.

Keywords: *Theory of connection; geometrical modeling, MATLAB toolbox, material flow management.*

INTRODUCTION

In this paper, we present the theory of connection (ToC) based approach for formulation of mathematical model of material flow processes. The major advantage in using the theory of connection is that, the same set of functions (or procedures) can be used for formulation of mathematical models of many engineering and management processes (Bjørke, 1995; Davidrajuh, 2000).

Theory of connection (ToC), previously known as manufacturing system theory, in the form it is presented here, is due to the work of the Scandinavian School of Systems Theory for the past 30 years; for detailed study of ToC, interested readers are referred to Bjørke (1995). The idea behind ToC is to bring geometry and algebra together: first, geometric modeling is used to model physical phenomena, and then a set of algebraic equations are drawn out of the geometric model, so that by using a computer these equations could be solved. The usefulness of ToC is that, different subsystems of different disciplines can be modeled and integrated by performing the same procedure; this is very important for fields like e-commerce as e-commerce involves diverse disciplines like manufacturing, business management, supplier selection, etc.

MODELING APPROACH BASED ON ToC

First, the concept of *system model* is introduced; this is done with the help of a simple electrical network known as the “inductor-resistor-capacitor (LRC)” network. Then, the simple LRC system model is mapped into the geometrical space with the help of properties matrix.

System Model

A system consists of three fundamental components, such as *elements*, *connections*, and *sources*. The elements carry all the physical or economical *properties* of the system. Elements are the building blocks of the physical system. For example, in a LRC network, the resistors, capacitors and inductors are the elements; the property of a resistor is its admittance, whereas a machine element's property could be its processing time, ratio between input items and output items, scrap percentage etc.

When there is no connection between the elements, the set of isolated elements is called the primitive system. The connections reflect how the elements influence each other and it represents the structure of the system. The set of connected elements is called the connected system. Finally, the sources reflect the influence between the total system and the environment. Sources are the environment's influence on the system; in an electrical circuit, source *s* could be current or voltage sources; in production planning, demand of products, startup-setup times, costs involved are some of the sources.

Geometrical Spaces, Vectors, and Matrices

ToC is based on the use of continuous geometrical 3-space or more typically an n-space volume (a Euclidean space R^n). A *vector* (called a *contravariant vector*) represents a point in the Euclidean space, or *primary space*, from the origin to the point; A vector in a 3-space is represented by:

$x = x^1 \varepsilon_1 + x^2 \varepsilon_2 + x^3 \varepsilon_3$, where $(\varepsilon_1, \varepsilon_2, \varepsilon_3)$ is the basis vector along the three axes. Whereas, a point (called a *covariant vector*) in the corresponding *dual space* of the vector space, defines another vector which can be represented by:

$$a = a_1 \varepsilon^1 + a_2 \varepsilon^2 + a_3 \varepsilon^3, \text{ Where } (\varepsilon^1, \varepsilon^2, \varepsilon^3) \text{ is the basis co-vector along the three axes.}$$

A *property matrix* keeps properties (or characteristics) of primitive elements, whereas a *connection matrix* represents the connections between the primitive elements.

TOOLBOX OF FUNCTIONS

The process of mathematical modeling and simulation approach based on ToC is traditionally done by a toolbox of functions implemented in APL (A Programming Language). APL is a symbolic language that was developed by IBM in the 1960s. This paper introduces a toolbox of functions for the industry-standard MATLAB platform. MATLAB platform for implementing ToC toolbox brings many benefits such as ease of use (programming), access to hundreds of functions from other MATLAB toolboxes like fuzzy toolbox, control systems toolbox, statistical analysis toolbox, optimization toolbox, etc., and facilities for high quality graphical outputs.

Using the Toolbox

The toolbox of function for ToC based approach is divided into four groups:

- Creating elements, and grouping of elements (primitive system)
- Extracting topological structure (connected system)
- Solving the equations
- Saving/Loading/Printing System (utility system)
- The functions are explained with the help of the following simple example.

Example: A Simple LCR Network

Figure 1 shows a simple LCR network consisting of three primitive elements—a resistor R, a capacitor C, and an inductor L. Conventions used in **Figure 1**: In **1a**, nodes are identified with block letters **A** and **B**; elements are identified with numerals **1** (L), **2** (C), and **3** (R). In **1b**, there are three branches connecting the nodes **A** and **B**: the branch-1, consisting of the element **1**, the branch-2 consisting of the element **2**, and the branch-3 consisting of the element **3**.

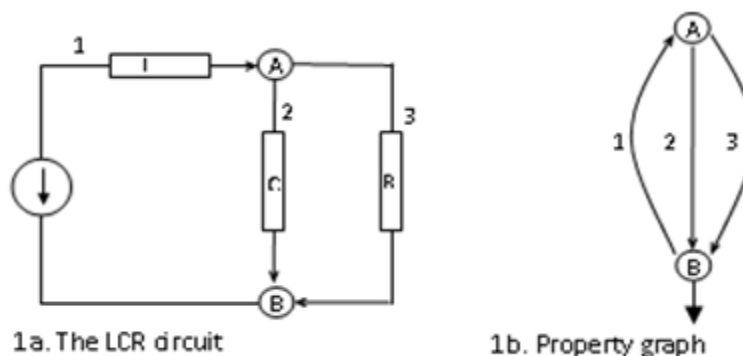


Figure 1: LCR network

Phase 1: Identifying the Primitive System.

First we create the three elements with the help of the function *element*. Function *element* takes four inputs: element number, name of the element, property value, and comment. Only the first (element number) and the third (property value) are compulsory inputs. For example, for the resistor, $R = \text{element}(3, 20)$. Meaning, the resistor is the second element and the value is 20 ohms (for other elements, property value is the respective impedance value).

After creating the other elements, all the elements are grouped as a primitive system using the function *primitive_system*: $> PS1 = \text{primitive_system}(L, C, R)$.

Phase 2: Making the Connected System

The connection between the three elements through the two nodes has to be established in this phase. This is done with the help of the function *connect_system* and a matrix known as the Branin table; the first row of the Branin table consists of the nodes that are tail-end to the branches, and the second row consists of the nodes that are head-end to the branches. For example, considering the three branches 1, 2 and 3 (or elements L, C, and R) the tail-ends are B, A, and A. The head-ends are A, B, and B. Together with these two rows of the Branin table, and with the primitive system, calling the function *connect_system* return the matrices VP, Y, and VN, where Y is the property matrix that includes the property value of all the elements in the primitive system, VP and VN are the connection (incidence) matrices for direct connections, the tail part and the head part respectively. The function *remove_reference* removes the reference node from the VP, VN matrices and returns the resultant incidence matrix V.

The Roth's diagram (shown in **Figure 2**) is used to solve the system. The Roth's diagram shows the involved matrices and tensors, and how they are connected. **Figure 3** shows all the functions used for solving. **Figure 3** shows that the data structure for an element contains two obligatory items, element number and the property value; these two items must be given at the element creation time. Also shown in the figure that the data structure for the set of primitive elements (the primitive system) contains multiple primitive elements, and how the vectors and tensors identified in the Roth's diagram are extracted from the primitive system.

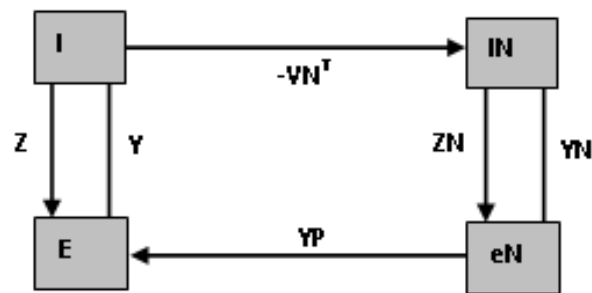


Figure 2: Roth's Diagram

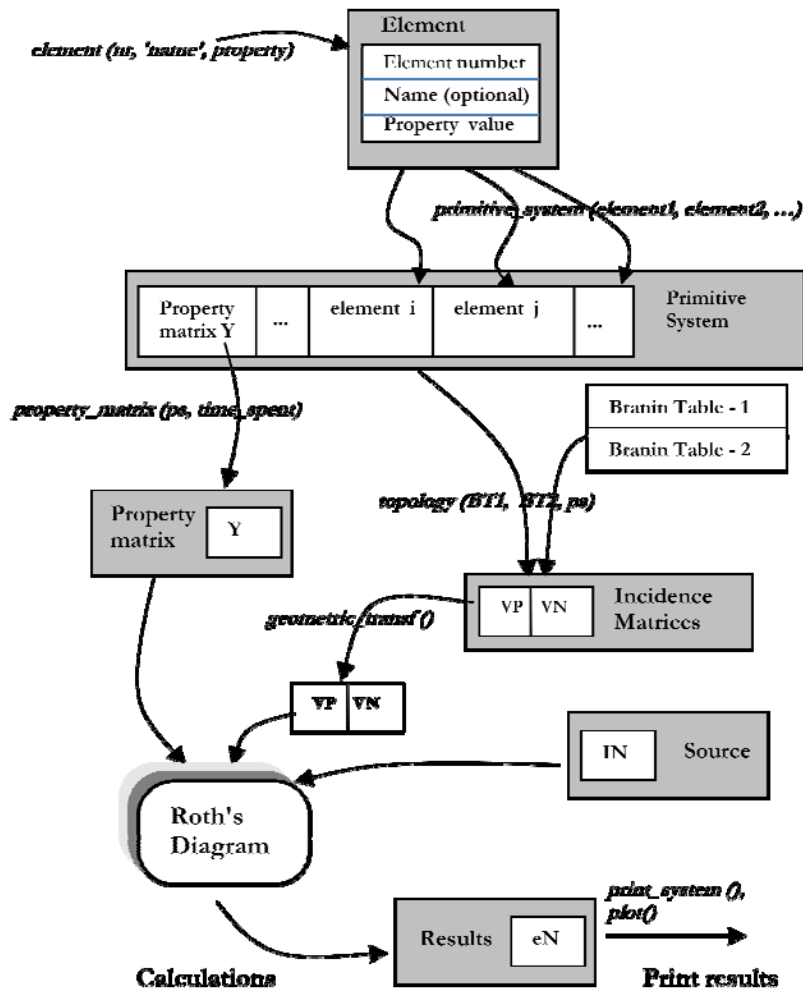


Figure 3: Overview of the Main Functions of the Toolbox

Phase 3: Applying the Sources, Solving the Connected System and Displaying the Results

There is only one source applied to the branch-1, whereas branches-2 and -3 do not have applied sources. Thus, in tensor form, the sources are represented by $[220; 0; 0]$, where 220 (volts) is the applied source.

CASE STUDY

A case study is presented in this section as a proof of concept for modeling and simulation using ToC. This case study shows that by using ToC for modeling and simulation of physical systems, the modeler will be aware of the fundamental geometric transformations and algebraic computations all the time; this is unlike modeling and simulations using graphical GUI based software which treats the models as a black box thus hides or abstracts away all the mathematical details.

The case study is about modeling and analysis of material flows and productivity of a batch-processing engineering company which produces a large variety of electrical motors.

The case study is already solved by Stecke and Solberg (1985) using queuing networks, and by Wang (1995) using APL language. However, the case study presented in this paper uses the modeling approach by ToC and the new toolbox of function developed in MATLAB language for simulations. Thus, interested readers are encouraged to compare the solutions by the three works, in order to judge the simplicity and elegance behind the approach presented in this paper.

The production system consists of 12 production stages, divided into 2 production lines: one for producing rotors and the other for producing stators. The components are finally assembled into motor (See **Figure 4**).

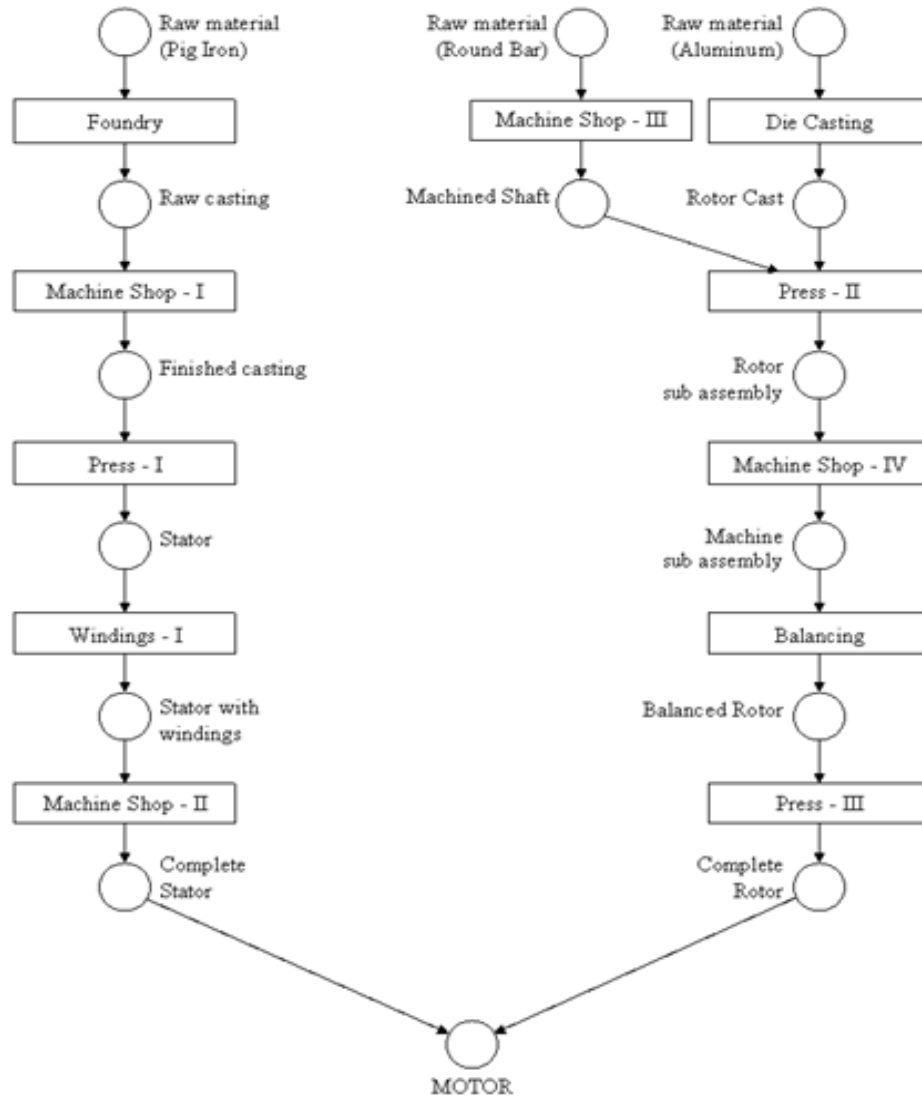


Figure 4: The Simplified Material Flow

Figure 5 shows the program code that uses ToC functions. First of all, the primitive elements (e1...e29) are defined; the primitive elements are defined with a name (label) and the material needed to make one unit of output. Second, the primitive elements are grouped together as a primitive system (ps). The property matrix is extracted from the primitive system as **Y** matrix.

```

% CREATE ELEMENTS
e01=element(01,'FOUNDRY :material input', 1.176); e02=element(02,'FOUNDRY :wastage', 0.176);
e03=element(03,'MSHOP-I :material input', 1.250); e04=element(04,'MSHOP-I :wastage', 0.250);
e05=element(05,'PRESS-I :material input', 0.670); e06=element(06,'PRESS-I :wastage', 0.010);
e07=element(07,'PRESS-I :wastage', 0.340); e08=element(08,'WINDING-I:material input', 0.893);
e09=element(09,'WINDING-I:wastage', 0.020); e10=element(10,'WINDING-I:wastage', 0.127);
e11=element(11,'MSHOP-II :material input', 1.220); e12=element(12,'MSHOP-II :wastage', 0.220);
e13=element(13,'ASSEMBLY :', 0.705); e14=element(14,'ASSEMBLY :', 0.385);
e15=element(15,'PRESS-III :wastage', 0.031); e16=element(16,'PRESS-III :wastage', 0.005);
e17=element(17,'PRESS-III:material input', 0.974); e18=element(18,'BALANCING:wastage', 0.001);
e19=element(19,'BALANCING:material input', 0.001); e20=element(20,'MSHOP-IV :wastage', 0.053);
e21=element(21,'MSHOP-IV :material input', 1.053); e22=element(22,'PRESS-III:from MShop-III', 0.320);
e23=element(23,'PRESS-III :wastage', 0.010); e24=element(24,'PRESS-III:material input', 0.690);
e25=element(25,'MSHOP-II :material input', 1.540); e26=element(26,'MSHOP-II :wastage', 0.540);
e27=element(27,'PRESS-III :wastage', 0.280); e28=element(28,'PRESS-III :wastage', 0.220);
e29=element(29,'PRESS-III:material input', 0.940);

% THE PRIMITIVE SYSTEM (GROUP ELEMENTS)
ps=primitive_system(e01,e02,e03,e04,e05,e06,e07,e08,e09, e10,e11,e12,e13,e14,e15,e16, ...
e17,e18,e19,e20,e21, e22,e23,e24,e25,e26,e27,e28, e29);
YG = ps.Y;

% ESTABLISH THE CONNECTION
Branin_row_1 =[19,19,20,20,21, 21,21,22,22,22,23,23,24,24,25, 25,25,26,26,27,...
27,28,28,28,30, 30,29,29,29];
Branin_row_2 =[01,02,19,03,20, 04,05,21,06,07,22,08,23,25,09, 10,26,11,27,12,...
28,30,15,29,13, 14,16,17,18];

% FORM THE CONNECTED SYSTEM
[VP, VN] = connect_system(Branin_row_1, Branin_row_2, ps);

% SOLVE THE CONNECTED SYSTEM
YNG = -transpose(VN) * YG * VP;
YDIG = eye(30);
YNNG = YNG + YDIG;
ZNBG = inv(YNNG);

INB = zeros(size(ZNBG,1),1); INB(24,1) = 1;
eNBG = ZNBG * INB

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Figure 5: The Program

Third, with the help of the Branin tables, the incidence matrix **V** is obtained by the function *connected_system*. Finally, by feeding raw materials to the input elements, the connected system is solved.

CONCLUDING REMARKS

This paper presents a Theory of Connection (ToC) based approach for modeling engineering systems. The main benefit using ToC based approach is that the approach emphasizes the connection between the primitive elements that compose the system; in another words, geometry is used for system modeling. The originality of this paper is the application of a toolbox of functions for modeling and simulation. The toolbox is implemented on MATLAB platform. From the case study given in the paper, it will become apparent that with ToC based approach diverse system modeling problems can be solved efficiently, using the same set of functions.

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