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Value at Risk analysis with Monte Carlo simulation

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June 5, 2009

Abstract

Value at Risk (VaR) is a commonly used measurement for financial institutions and investment parties to valuate the risk in a portfolio, and defines the worst case scenario within a certain confidence level over a specified time horizon. Although it is a popular tool it has been criticized for being procyclical and underestimating the market risk. Recently the Turner Review concluded that these characteristics of VaR had been one of the factors which fuelled the financial crisis seen in 2008.

Statoil Hydro ASA experienced underestimation during the extremely volatile period after the summer of 2008. The oil market itself is characterised by shifts between periods of low volatility and periods of high volatility. It is therefore important to find a VaR method which can adjust to these changes rapidly.

As this thesis will show the last years until the summer of 2008 was a period of relatively calm and stable market development, after which it has become more volatile with bigger fluctuations from day to day. These sudden shifts in market are difficult to include in a VaR estimation method and Statoil Hydro ASA's historical simulation failed to do this in the fall of 2008.

The thesis will therefore investigate other options in estimating VaR and examine Monte Carlo methods. One of the important flaws of VaR estimates is that it often does not include the fat tails and high peaks which most market factors experience. This was also pointed out by the Turner Review, as VaR is highly dependant on the tails of a distribution.

The Monte Carlo methods in this thesis therefore try two alternatives which takes the fat tails more into consideration. Firstly the change in portfolio (and VaR) is defined as a multivariate student t distribution with the underlying risk factors being student t distributed. Secondly a delta-gamma approximation is used in order to achieve an Importance Sampling where the fat tails are considered important regions of outcome.

The two methods provide results which in the more volatile period (01.06.2008 – 31.12.2008) are better than the historical simulation. However the multivariate student t is time consuming as it converges slowly and needs much calculation for each estimate. The delta-gamma method is more efficient, but seems to overestimate VaR, although this overestimation is reduced when the portfolio value decreases.

The thesis include an estimation tool which lets users at Statoil Hydro ASA select portfolio based on price history in Excel and estimate VaR with all implemented methods. The tool also includes several variables which ensure many alternatives for the methods, as well as the possibility to run back-tests to compare the methods.

Preface

This report completes my education for a M.Sc. degree in Industrial Economy at the University of Stavanger with Computer Science and Contract management as field of study.

I was introduced to Value at Risk by Øystein Håland and Andreas Kirkerød at Statoil Hydro ASA as a potential master thesis in the fall of 2008. During the recent financial crisis they had experienced a change in volatility which was difficult to capture in their VaR estimate with historical simulation. Therefore they wanted to investigate alternative methods with Monte Carlo simulation.

The thesis has been a great opportunity for me to utilize my diverse background from my education. With the combination of econometrics, finance, statistics and software development the work has been inspiring and challenging.

Microsoft Excel 2003 was selected as a programming platform for the developed tool as well as for analysis of data and presentation of results. With Visual Basic for Application (VBA) Excel offers the opportunity to program user defined functions which can be utilized in data analysis and processing. In addition the familiarity with Excel at Statoil Hydro ASA made the program a natural choice.

The CD included with this thesis contains the Excel tool developed for the thesis, as well as this report in PDF-format.

I am very thankful to my instructor Ragnar Tveterås at the University of Stavanger, whose initiatives, advice and feedback have helped me produce this thesis. I am equally thankful to Øystein Håland and Andreas Kirkerød at Statoil Hydro OTS whom have given constructive feedback, enthusiasm, suggestions and guidance during the entire project. In addition I would like to thank Odd Bjarte Nilsen for feedback and help on the statistical matters and discussions about the delta-gamma approximation.

Stavanger, June 5, 2009

Roy Endré Dahl

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1. Introduction

This chapter will outline the scope of the thesis including an introduction of Value at Risk (VaR), its features and the most common methods for solving VaR. In addition this chapter will present the motivation for this thesis before giving an overview of the organization of this report.

1.1 Scope of the thesis

This thesis will compare several methods for estimating Value at Risk (VaR) which has been a widely used estimate for companies and institutions to valuate the risk in a portfolio of assets. It provides the answer to the question: How much is it possible to lose within a certain time period at a certain confidence level?

The estimate is therefore dependant on two inputs to provide the outcome (VaR). Firstly it depends on the time horizon of the evaluation, which is often a day. Secondly it depends on the desired confidence level, which is often set to either 95% or 99%.

Thus the answer to the question can be:

- for the next day (period) 1% (significance level) of losses will be bigger than 500k (VaR), or
- I am 99% (confidence level) certain that my losses will not be bigger than 500k (VaR) the next day (period).

Value at Risk (VaR) is defined with respect to a desired significance level p identifying the wanted percentile. The VaR of a portfolio is the lowest amount x_p such that with probability p the loss L will not exceed x_p

$$P(L > x_p) = p$$

There are 3 methods for estimating Value at Risk (VaR):

- Analytical approach
- Historical simulation
- Monte Carlo simulation

The historical simulation uses historical scenarios to estimate tomorrow's VaR. This approach has been efficient and is used widely in combination with other methods such as the exponentially weighted moving average. However through changes of regimes when a relatively good (bad) market becomes the opposite this approach has proved inadequate as it underestimates VaR.

The majority of analytical approaches to VaR rely on linear approximation of the portfolio risks. By assuming a joint normal (or lognormal) distribution of the underlying market parameters VaR can be easily calculated by setting the probability distribution of the change in portfolio to be normal with mean zero and standard deviation a sum of standard deviation and correlation between the underlying risk factors.

However, these assumptions are seldom correct as the distributions of the underlying risk factors often experience positive kurtosis which means that the distributions have fat tails and high peaks.

This master's thesis will give an alternative to the historical and analytical approach by estimating VaR with Monte Carlo (MC) simulation. Theoretically MC will give a more accurate estimation, but due to its nature and the size of a portfolio it might be time consuming.

The most recent research on MC simulation has tried to reduce the variance of trial output in order to reduce number of trials necessary to produce an accurate VaR. This thesis will implement a partial simulation developed by Glasserman, Heidelberger and Shahabuddin (Glasserman, et al., 2000) which utilizes delta-gamma approximation in a sampling method which emphasize the heavy tails in VaR (Importance Sampling).

The thesis will develop a tool using Microsoft Excel and its native Visual Basic for Application (VBA) for programming. This tool will include several methods to estimate VaR and make a comparison of their estimates.

In addition to the comparison of VaR methods the thesis will provide an insight in the oil market and do basic statistical analysis of the underlying risk factors in the thesis portfolio. This is important to get a firm understanding of the development of VaR and its significance as a risk tool.

1.2 Background

Trading more than 2 million barrels of crude and condensate (light oil) oil per day Statoil Hydro ASA is one of the biggest oil and gas companies of the world. It trades in petroleum products, methanol, power and emission allowances and is ranked as the third largest net seller of crude oil in the world. Represented in 40 countries it is a global company operating oil and gas fields in countries ranging from Angola, Brazil and Canada to Libya, China and Venezuela. Still its largest activities takes place in Norway and the operations on the Norwegian continental shelf make Statoil Hydro a leading offshore operator.

With refineries in Norway (Mongstad) and Denmark (Kalundborg) crude oil and condensate (light oil) is refined into petrol, jet fuel, diesel oil, propane, heating oil and fuel oil. The principal market of the two refineries is Europe.

The main trading activities are controlled from the company's head office situated in Stavanger, Norway. Here the Oil Trading and & Supply department is placed where crude oil, refined products, NGLs, electricity and carbon emission allowance are traded. Furthermore VaR is calculated on a daily basis in order to capture the shifts in the market and the risk in Statoil Hydro's portfolio. In addition Statoil Hydro has trading offices in London (UK), Stamford (USA), Singapore, Riga (Latvia) and Oslo (Norway).

StatoilHydro ASA uses historical simulation to estimate VaR and experienced in the fall of 2008 problems of including the regime shift in volatility. This caused underestimating of VaR and coverage dropped below the wanted percentile. As a consequence the link to the University of Stavanger was established and the scope of this thesis identified to examine optional estimation methods for VaR.

1.3 Organization of thesis

This first chapter has introduced Value at Risk and the scope and motivation of the thesis.

The second chapter gives an introduction to the oil market, emphasizing the different products, volatility and demand and supply in the market.

The third chapter discusses the fundamental theoretical background including basic statistics and develops important statistical notations used in the thesis. Different methods for estimating correlation will be discussed; and the Greeks will be presented and explained. In addition this chapter will include a presentation of Value at Risk and methods used to estimate it, including Monte

Carlo methods and a discussion on possible variance reduction techniques. The chapter will compare the methods and lead to a recommendation for the method used in this thesis.

Analysis of the historical data will be carried out in the fourth chapter. This part will also present the portfolio used in this thesis. The analysis will include a discussion about distribution as well as a test for kurtosis. In addition the correlation between the products will be examined.

The fifth chapter will present the estimation tool developed for this thesis. In addition to a presentation of the graphical user interface (GUI) the key functions will be presented.

Results comparing today's historical approach with this thesis' Monte Carlo approaches will be detailed in chapter six. This chapter includes a presentation of the back-test used for the simulation. Furthermore it displays the results on accuracy, relative bias, tail size and correlation, as well as a discussion about the underlying distribution of the different methods.

A summary and the conclusion are given in chapter seven. In addition some thoughts about further work are included.

The bibliography is included at the end of the thesis followed by the appendices. The appendices include the source code for the estimation tool developed in Excel and more results obtained during the back-test.

2. Oil market

This chapter will give a short introduction to the oil industry and paper contracts utilized to reduce short term risk, before moving on to a presentation of market fundamentals. This includes an examination of demand and supply and the elasticity of them. In addition possible exogenous factors in the oil market are identified. The main focus will be on short term changes since the thesis' agenda is to estimate changes in small time frames. Still some comparisons and notes will be made about the long term changes.

This introduction is followed by an empirical walkthrough comparing the market fundamentals with historical evidence. Finally a glimpse of the future is provided with a discussion on price forecast and long term considerations about short term factors.

2.1 Oil industry

The oil market is divided in three sectors:

- Upstream (search and recovery of oil and gas fields)
- Midstream (processing, storing, trading and transportation of oil)
- Downstream (marketing, selling and distribution)

Midstream is often included in downstream. The upstream oil sector is also known as the exploration and production (E&P) sector, while the downstream includes processing, refinery and distribution of oil to consumers. A fully integrated petroleum company is involved in all three areas of the oil market.

2.1.1 Refinery process

An oil refinery maximizes profits by diversifying the products. The different qualities provide commodities for different use which are then sold at different prices according to the demand.

As seen in Figure 2-1 the process of an oil refinery is complex and delivers a variety of products, represented in blue text.

The distillation of crude oil is done in an oil refinery where crude oil is separated into fractions by a fractional distillation. Crude oil consists of hundreds of different hydrocarbon molecules which have different boiling points. This difference makes it possible to separate them by distillation. The higher fractions have lower boiling points compared to the fractions at the bottom.

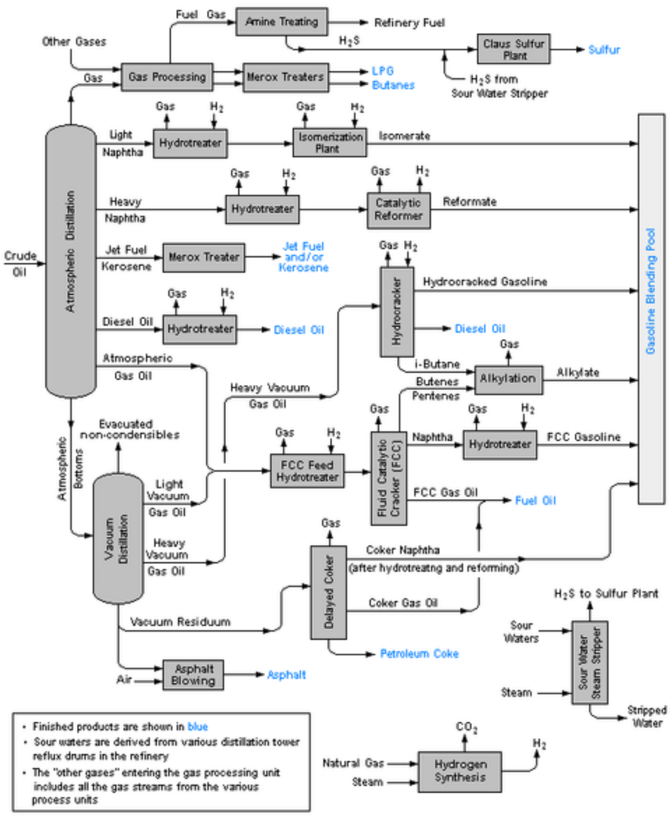

Figure 2-1 - Flow diagram oil refinery[1]

[1] http://en.wikipedia.org/wiki/File:RefineryFlow.png

The differences in the hydrocarbon molecules (consisting of varying complexity and lengths of hydrogen and carbon molecules, as well as some oxygen) are also what give them their diverse properties. As a result the distillation process produces a range of outputs from paraffin, naphtenes and alkenes to alkynes and dienes.

After the separation the fuel or lubricant can be sold without any further processing. However different techniques can be used to further refine the outputs into more valuable products. Octane grades and requirements can be achieved in processes like alkylation or catalytic reforming. Gasoils might also be reprocessed by cracking which produces lighter short-chained oil.

Based on the way crude oil is distilled and separated into fractions the products can be grouped into three categories:

- light distillates (LPG, gasoline, naphtha)
- middle distillates (kerosene, diesel)
- heavy distillates and residuum (fuel oil, lubricating oils, wax, tar)

The most common products of an oil refinery are:

- Liquid petroleum gas (LPG)
- Gasoline (also known as petrol)
- Naphtha
- Kerosene and related jet aircraft fuels
- Diesel fuel
- Fuel oils
- Lubricating oils
- Paraffin wax
- Asphalt and Tar
- Petroleum coke

2.1.2 Transportation and storage

Since consumers of oil products do not live in the same area where the production takes place, an important part of the oil market concerns transport. This creates exporting regions (like the Middle East) where supply is greater than demand, and importing regions (like the USA) where the opposite is true.

Moreover the storage of is oil another link between the producer, refiners, marketers and consumers. Costs associated with transportation and storage is therefore crucial in determining the trading pattern of importers and exporters.

In general the highest value produces the highest price. If quality is set aside the proximity of the goods to the consumer defines the price, as oil moves to the nearest market first. This ensures low transportation and storage cost and the seller therefore gets most profit.

These considerations can be seen in the major region’s trade, as most of the import to USA comes from its neighbouring countries Canada and Mexico, while the great economies in Asia gets most of its oil from the Middle East.

Transportation and storage do not only concern long term prices as the dependence on oil from certain areas can create short term price jumps due to trade embargoes or wars. This is further discussed in section 2.4.2 Exogenous factors.

2.2 Paper contracts

The trade in the oil market uses regular instruments, including derivatives like futures, forwards, options, CFDs and swaps. For an oil company these instruments are utilized in order to reduce their risk in the physical position of oil they hold. This risk reduction is known as hedging as risk is transferred between two parties and a perfect hedge would completely eliminate risk.

There are two possible hedges: short and long. A short hedge is incorporated when the investor needs to offset their risk of a negative price development for an asset. E.g. an oil company who knows it will sell an amount of oil within a time frame can use a short hedge to offset the possible risk of a decrease in the oil price. Whenever the oil price decreases the reduction in oil value is offset by the increased value of the short hedge.

A long position is mostly used when a company knows it will purchase an asset at a future date. By going long price fluctuations in this asset will be offset by the long hedge as an increase in the price will provide a profit which balances the extra cost of the future purchase.

Futures contracts are agreements to buy or sell an asset for a certain price at a certain future time. It is a standardized contract traded on an exchange and settled daily. A standard future contract will define the asset for sale, amount and price and are offered with a range of date delivery dates as well as the grade of deliverable. This grade specifies for instance how high sulphur content is acceptable.

In most cases the futures contract is closed out before the maturity date. This is done by entering into an opposite position to the original futures contracts. E.g. an investor who is short on Brent oil can buy a long position before the short position matures. The gain or loss of the original futures contract is then decided by the change in the futures price between the original purchase and the date that contract is closed out.

Daily settlements incur virtually no credit risk as the balance is settled daily via a margin account. The margin account is organised by the exchange responsible for the trade and at the end of each day the investor’s profit (or loss) is added to (or subtracted from) the margin account. This ensures only small daily payments as the futures price develops towards the closing date. The only cost for the trade parties is the maintenance margins which trading parties at the exchange must guarantee at their margin account. Some exchanges do however offer interest on the margin account, thus making the cost of this trade virtually zero.

On the other hand, a forward contract is a private contract between two parties to buy or sell an asset for a certain price at a certain future time. It is not standardized and usually only has one specified delivery date. The settlement is done at the end of the contract when delivery or final cash settlement takes place.

This agreement suffers some credit risk as the settlement is not done before the end of the contract. By that time either the will or ability to fulfil the agreement of one of the parties can endanger the forward contract; the buyer or seller may regret the original agreement or have problems meeting the agreement due to financial problems.

The profit or loss of an investment in the same asset for the same period of time is nevertheless independent of what type of contract is used. The difference lies in how the settlement is carried out, and while a futures contract spreads the profit over the whole period this is settled in one lump sum at the end of a forward contract.

A third alternative is a swap which is an agreement between two parties to exchange cash flows in the future. The exchanges are carried out on several dates and are usually adjusted by the future value of an interest rate, exchange rate or other market variables. There are especially two common swaps: plain vanilla interest rate swaps and fixed-for-fixed currency swaps.

The difference between the forward contract and a swap is that the latter contains several exchanges. A swap therefore combines the frequency of the futures contract with the settlements and delivery of a forward contract.

A swap does involve some credit risk as it is an agreement between two companies. As with a forward contract one of the parties might endanger the fulfilment of the contract by having financial difficulties before the end of the contract and default.

Options are a fourth instrument utilized in the oil market. An option differs from a future, forward and swap as the holder of an option does not need to exercise the right to fulfil the option. As opposed to the other instruments an option incurs an up-front payment in order to attain this possibility.

There are two types of options: a call option which gives the holder of the option a right to buy an asset to a certain price, and a put option which gives the holder a right to sell an asset to a certain price called the exercise or strike price. The option must be used within a certain date (or at a certain date) known as the expiration or maturity date.

For each option there are two positions. One is for the buyer of the option which has entered into a long position. The second is for the seller (or writer) of the options which has entered into a short position. There are therefore four types of option positions.

One last possible instrument are the CFDs (Contract for Difference). A CFD is an agreement between two parties to settle a future development in the price of an asset. If the asset increases in value the seller must pay the difference to the buyer. If there is a negative development the buyer must pay the seller. These contracts are therefore traded on margin, and it is possible for a company to make the same profit without owning the asset. For a company this creates another possibility to go short or long.

Unlike a futures or forward contract the CFD have no fixed expiry date and it does not have any standardized contract. Since settlements are only done when the buyer ends the investment the amount of profit or loss may build up and risk connected to CFD is therefore considered high.

Among the most important derivate exchanges trading in crude oil are the International Petroleum Exchange and the New York Mercantile Exchange.

2.3 Oil prices

The oil market is characterized by periods of relatively calm and periods of dramatic changes in volatility. The change of regimes is difficult to predict and recent developments confirmed this in the summer and fall of 2008 as oil prices dropped from $140 pr bbl to below $40 in just 6 months time. The last year has seen high volatility as a new price regime has yet to find its foundation.

The price of oil and its by-products/refinements are dependant on many factors. Its demand is firstly dependant on the overall business cycles. As the world economy experiences steady growth this leads to an increase in demand and the opposite. Still this is true only until the price reaches some limit when alternative energy sources become competitive. These hypothesis have been confirmed the last years and recently by the fall in demand in the aftermath of a global recession.

The supply side depends on factors like investment, decisions of oil production level by the major producers and exogenous factors like politics and wars. Although the supply side is dependant on long term decisions these decisions also affect the supply side's ability to adjust to market demand in short term.

Hamilton (2009) concludes that the changes in the real price of oil have historically been:

- Permanent
- Difficult to predict
- Governed by very different regimes at different points in time

In this section the fundamental theory of demand and supply of the oil market is presented, which is presented with recent market developments in mind. In addition a presentation of other exogenous factors is covered. This is then compared to the historical development, before the section is ended with some thoughts on future development.

2.3.1 Demand and supply

The microeconomic model for demand and supply is used to describe the relationship between price and quantity in a market. The goal is to find the price and quantity equilibrium between the demand of goods by consumers and supply of goods by producers. By using the aggregate supply and demand curves it is possible to find the market equilibrium.

The supply curve is an upward sloping curve as the production is increased when the price offered increases. A perfect market will have a supply curve where the supply is based on the marginal cost of the suppliers without any mark-up. In the oil market this means that only a few suppliers can offer the oil at low prices (e.g. at $10 per barrel), but considerably more can offer it at a medium price (around $30-$40). However if output reaches the short term capacity cost and thus prices will increase exponentially as consumers now demands a very scarce good. See Figure 2-2.

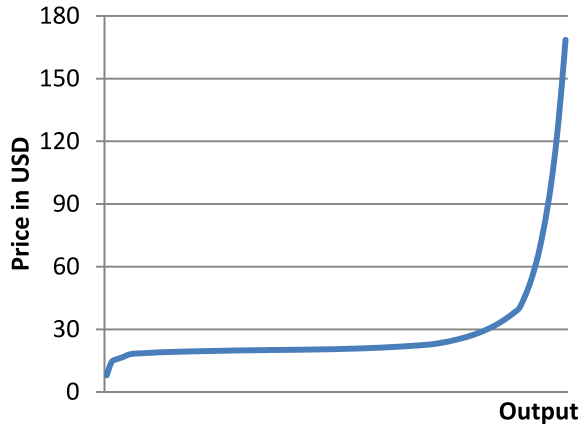

Figure 2-2 Aggregate supply curve

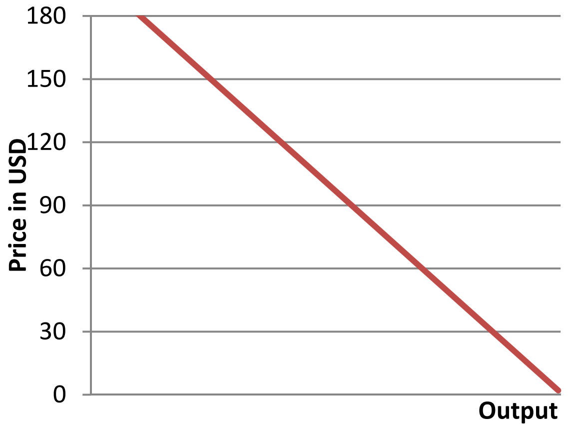

Figure 2-3 Aggregate demand curve

The demand curve slopes downwards as it is inversely proportional to price and is determined by the reservation prices of the buyers. When the price increases the consumers demand less. Together with the supply curve the demand curve intersects at the equilibrium point. This point decides a price p* where the consumers are willing to purchase a quantity q* units. At the same point the suppliers are willing to sell q* items for p*. See Figure 2-4.

A shift in demand occurs when more consumers want to purchase the goods. This creates a higher reservation price and the demand curve therefore switches outwards. As a result more goods are produced and sold at a higher price. As seen in Figure 2-5 a shift outwards because of an increase in demand of oil can lead to a relatively big increase in price. This is especially true in a short horizon because of the inelasticity in both demand and supply.

A similar shift can happen for the supply side as a result of change in marginal cost (e.g. due to new technology). A shift downwards occurs as a result of a reduction in marginal cost, and result in more goods produced at lower prices. Consequently more consumers will purchase the good.

The slope of the supply and demand curve together with the size of the shift decides the degree of volatility for a product. In periods of high volatility the shifts are bigger and it occurs at an inelastic point of the demand-and-supply curve.

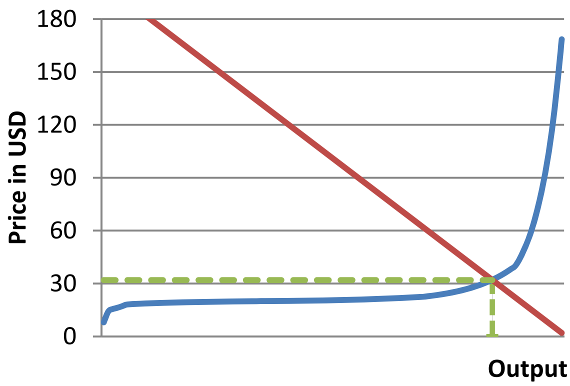

Figure 2-4 Demand and supply equilibrium

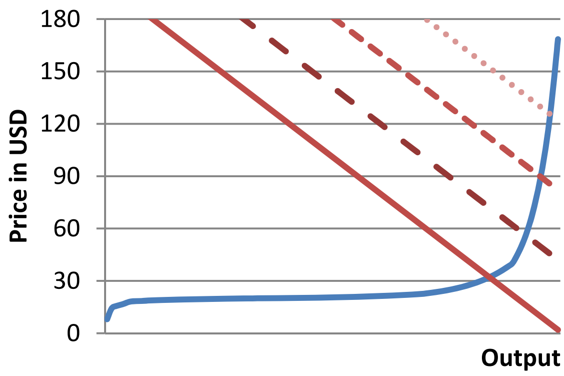

Figure 2-5 New equilibrium points as demand shift outwards

2.3.2 Elasticity

Price volatility is closely linked to price elasticises of demand and supply. The price elasticity measures the relationship between the quantity of demand and supply and their responsiveness to price changes. If demand is elastic it will adjust to price changes rapidly, while an inelastic demand will stay unaffected by changes in price. Likewise an elastic supply will adjust supply levels according to the changes in price, while an inelastic supply will stay constant regardless of price level.

Price elasticity is defined as:

$$\varepsilon_p = \frac{\partial Q}{\partial p}\frac{p}{Q}$$

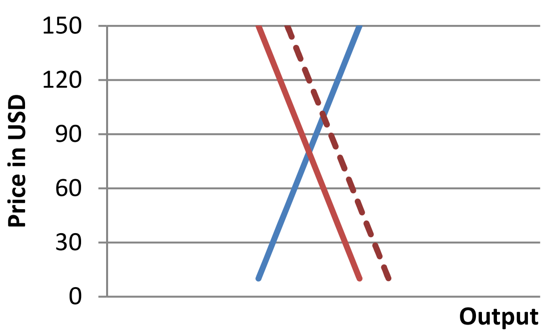

Figure 2-6 Inelastic demand and supply curve. A change in demand results in a relatively big change in price.

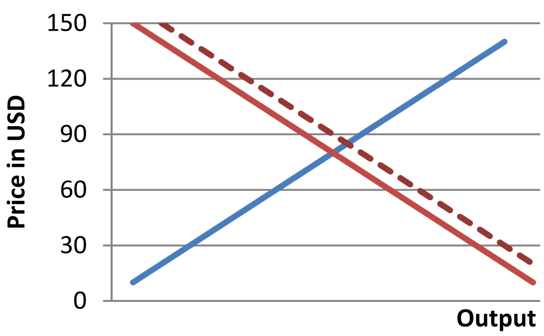

Figure 2-7 Elastic demand and supply curve. A change in demand results in a relatively small change in price.

Elasticity can graphically be interpreted as the slope of the demand and supply curve. A steep curve is a characteristic of an inelastic demand as the demand is minimal affected by changes in price. Similarly a steep supply curve indicates an inelastic supply as changes in the price do not affect the output of suppliers, see Figure 2-6. Thus a small change in demand will create a big jump in price.

These characteristics can be easily seen for the supply curve of oil in Figure 2-5 at the end of the output-axis. At a short time horizon the supply side is highly inelastic due to problems of adjusting the production level according to the quantity demanded.

2.3.3 Exogenous factors

An exogenous event is characterized as a change that occurs outside the market model and which is unaffected by the model. For an important commodity like oil most of the exogenous factors are a consequence of international politics.

Wars have been fought to ensure a steady flow of oil as disagreements of boundaries between countries and the ownership of natural resources have been an important reason for war. Furthermore trade embargoes has been used to establish political power as oil is now the most coveted resource on earth which has lead to both. Such exogenous events might only create uncertainty in the supply of oil but can also create a temporarily stop in supply. The result is nevertheless an increase in oil price.

Furthermore environmental issues have recently been fuelled with arguments for a reduction in oil as the global warming continues to mount. Consumer preferences might be changed as a result of the threat to the environment in addition with governmental regulations which can further deter the consumption of carbon energy sources.

Other exogenous factors are the scarcity of oil and the development of new technology to provide alternative energy sources which in many aspects are a perfect substitute for oil related energy. These factors are both related to concerns about the environment and together they may provide a considerably change in the oil market in the years to come.

2.4 Empirical evidence

This section will compare the market fundamentals with historical price development. Firstly the demand and supply and its elasticity will be considered, before exogenous factors are discussed.

2.4.1 Demand and supply

The world's population is dependant on energy to maintain its consumption and production of goods and has been so since the industrial revolution in the 18th century. With the great expansion in the world economy over the last decades, spurred by the growth in countries like China and India, this has led to an enormous demand for oil. In addition consumption is rising worldwide as the newly industrialized countries require the same standard of living as the western world has enjoyed for decades. This increase in demand has led to several shifts outwards in demand therefore increasing output and price. Furthermore; since this way of living is highly committed to consumption of oil and gas in both mass production, transportation and user consumption the demand side is highly inelastic in the short run and consumption therefore stays stable even at higher prices.

The increase in demand the last decade led to a record high oil price in the summer of 2008 when oil was selling at 147.27 dollars per barrel. It has been discussed if the sharp rise was fuelled by speculations in the futures market. Although an investigation by the U.S. Commodity Futures Trading Commission (CFTC) has lead to an interim conclusion that this was not the case, several indicators suggest otherwise. CFTC has therefore coupled with the United Kingdom Financial Services Authority and ICE Futures Europe in order to expand surveillance and information sharing of various futures contracts.

The interim report gives a good understanding of several key properties of the oil market. Firstly the oil market is dependant on business cycles. And because of a great expansion in the world economy during the last years the demand for oil had outrun the supply of oil. Although this dependency on business cycles is more a long term source for price changes, the change of business cycle causes short term volatility which affect the price fluctuations.

Secondly both demand and supply of oil are inelastic in the short run, which ultimately resulted in the record high price in 2008. The higher price did not deter the consumer as they had commitments and habits that fuelled their consumption. These habits and commitments take time to adjust, thus creating an inelastic demand. This was also confirmed statistically by Cooper (2003) who calculated the short-run price elasticity of oil to be -0.05, while the long-run price elasticity was -0.21. Although these numbers are debated by Hamilton, he also concludes that the elasticity must be low.

In addition, estimates by Hughes, Knittel, and Sperling (2008) indicate that the elasticity is far lower now than in the 80s. Their calculations show that short-run gasoline demand elasticity was between -0.21 to -0.34 over 1975-1980 compared to only -0.034 and -0.077 for the 2001-06 period. There are several reasons to this, and the most important being the increased buying power of Americans.

Studies also conclude that the income elasticity has declined as GDP per person has increased. As a consequence the share of oil consumption in GDP has been reduced from 8.3% in 1980 to 1.1% of total GDP in 1998.

The supply side is equally inelastic as it needs time to adjust to the high demand since increasing production is costly and time consuming. When both supply and demand sides are inelastic in the short run this creates the possibility for big price shocks.

The decrease many of the oil producing countries experienced in this period further fuelled the gap between supply and demand. Indeed, Norway is among the countries which have seen a decline in oil production over the last years due to the scarcity of new findings. Other countries like Mexico, Venezuela and USA have also seen a downturn in oil production. See Figure 2-8 below.

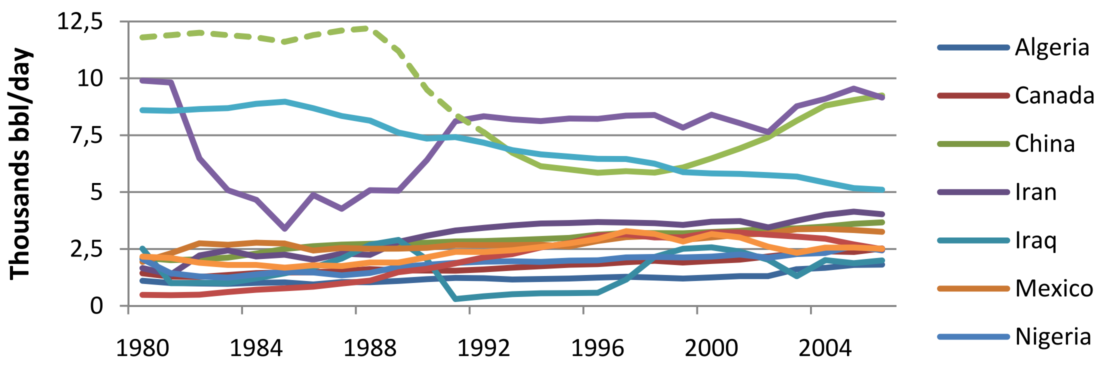

Figure 2-8 – Crude oil production in selected countries from 1980 to 2006 in barrels/day in thousands

The official numbers from USSR is not published due to governmental regulations. The figure is therefore based on numbers found in several sources[2,3,4,5].

[2]Energy Information Administration (EIA)

In the long term the supply-demand will eventually balance out as consumers find alternative energy sources in response to the high prices. In addition the high prices spur investments in the oil companies in order to produce more oil, thus raising the supply. This together ensures a mean reversion where the market players adjust demand and supply to a stable level.

The latest years have seen such an increase in investments as the prices have been extraordinarily high and the predictions for future prices have been optimistic. This has also lead to the development of oil fields which were thought to be non-profitable due to their high production costs. Examples are the Canadian oil sand and the continued utilization of marginal oil fields.

This investment plan is however debated as it also produces an increase in cost for the oil company, thus increasing the marginal cost of an oil field. A simultaneous increase in use of oil rigs and other equipment necessary for research, seismic and development of oil fields has seen cost climb sharply, thus turning these marginal fields into more risky projects.

Although these investment plans are carried out in a long term horizon, the effects can also be measured by increased or decreased elasticity on the supply side. Years of increase in investment in research and production of oil fields increases the elasticity as more oil is available. Thus the output-axis in Figure 2-5 is expanded to the right and the exponentially growth in price will not happen until more oil is demanded.

As the financial crisis unfolded in the fall of 2008, the demand for oil plummeted. This resulted in big inventories and the prices dropped to under $40 in just 6 months time from the peak of $147 per barrel. When the supply exceeds the demand microeconomic theory says the price should collapse to the marginal cost of production. For the greatest oil fields the marginal cost is around $10 pr barrel. But for many projects undertaken in the most optimistic part of the expansion period the marginal cost might be as high as $40-50, or even higher. Indeed, this is the case for many projects undertaken in Canadian oil sand where companies are postponing production due to the fall in oil prices. If the price drops further the most expensive wells become uneconomical and are shut down, at least temporarily. Therefore price equilibrium is set somewhere near the production cost of the most expensive source needed to meet the global demand.

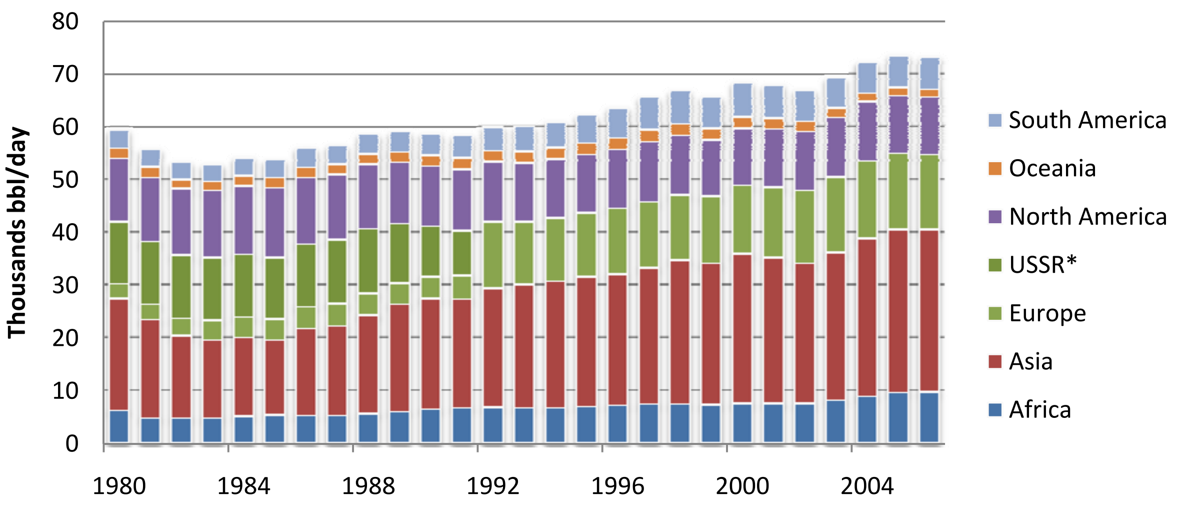

Figure 2-9 - Total world crude oil production from 1980 to 2006 in barrels/day in thousands

[3]Russia Energy Survey 2002 published by International Energy Agency (IEA)
[4]Monthly Oil Market Report, July 1992, published by IEA
[5]Index Mundi

In the latest 20 years oil production has still increased as seen in the figure below. This is despite the reductions seen in the United States which has been covered by increase in production in other areas. This figure is also based on approximations for oil production in the former Soviet Union. From 1992 oil production in Russia is included in the numbers for Europe.

2.4.2 Exogenous factors

Aside from the general demand and supply discussion there are other factors which affects the rise and fall of the oil prices. These are related to international politics, as oil has already generated embargoes, boundary politics and wars.

One evident example of this is the results of the Iranian revolution in 1978 when Iran dropped their production by 5.4 million barrels per day. The production was additionally reduced by 3.1 mb/d as a consequence of the war between Iraq and Iran commencing in 1980, creating a solid loss of oil supply. The result was inevitably a large increase in oil price, and between January 1979 and April 1980 the price soared by 81.1% (logarithmically).

As seen in Figure 2-10 conflicts including oil producing nations have had imminent impact on the oil price. From the Yom Kippur conflict in 1973 which was followed by an OPEC embargo, the Iran-Iraq conflict in 1979-80, the Kuwait invasion in 1990 until today's war in Iraq. The declines in oil price has either come as a result of an imbalance in supply and demand, as in the 1980s when demand had adjusted to the supply levels during the Iran-Iraq conflict and during the 1990s, or as result of a financial crisis like the crisis in Asia in 1997 which brought the price down to $16 per barrel.

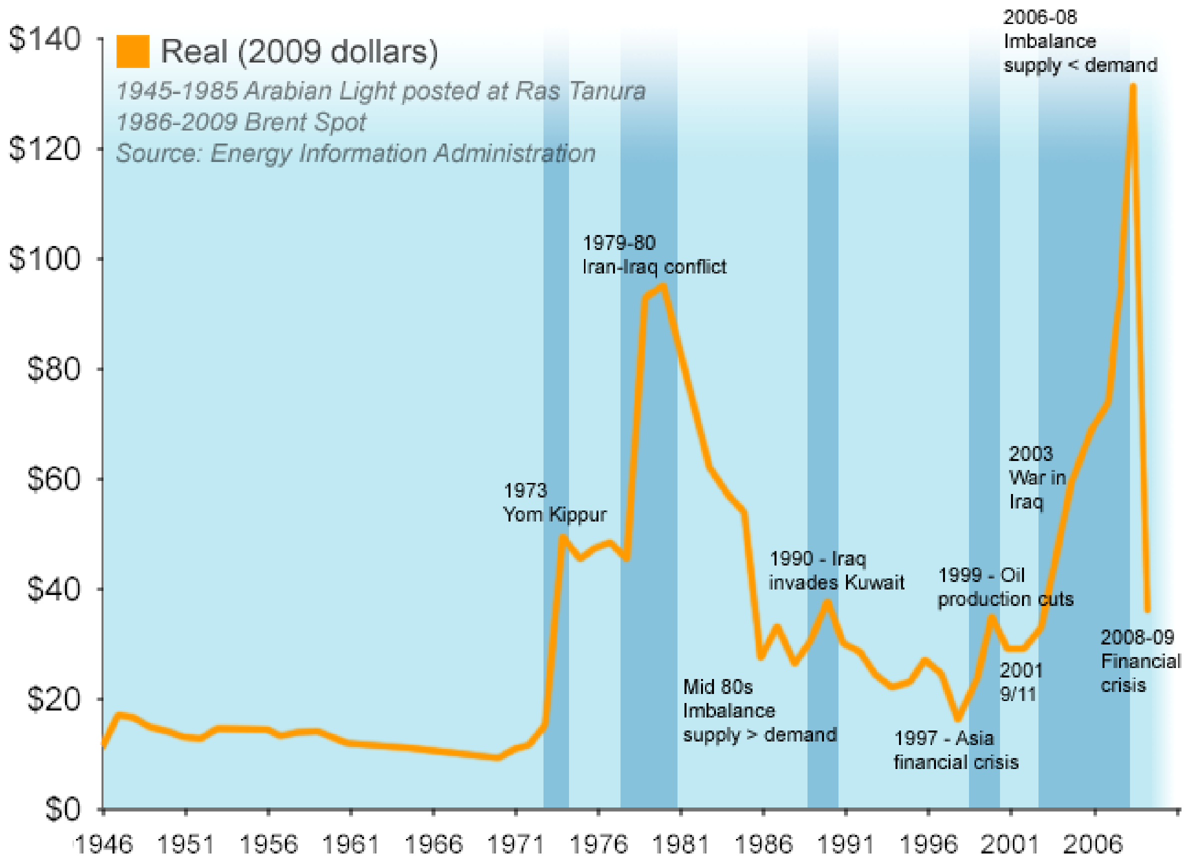

Figure 2-10 Oil price development since World War II with major events affecting the oil price.

The imbalance between supply and demand combined with the inelasticity on both supply and demand sides created the record high prices seen in 2008. The following financial crisis has since reduced worldwide demand and the oil price has thus plummeted, as already discussed in section 2.4.1 Demand and supply.

Both Iran and Iraq are members of the Organization of Petroleum Exporting Countries (OPEC), which controls two-thirds of the world's oil reserves and 35.6% of the oil production. Despite its power the cartel has had problems controlling the price, and since the 80s the increase in oil production in the North Sea, Canada, Gulf of Mexico and Russia has challenged their superiority. Recent fall in oil price has however been accepted by the cartel to ensure healthy growth conditions for companies worldwide in the aftermath of the financial crisis.

The major challenge in OPEC is the members' incentives to cheat on the collaboration in order to maximize its own profit. Due to the fact that the marginal revenue for one individual member is bigger than then marginal revenue of the whole group, the members will earn more by producing more than the group has agreed upon. History has demonstrated that OPEC countries do not follow the quotas set, and of the recent measures set to balance the financial crisis only Saudi Arabia has followed up by cutting their production.

The latest decade has seen Russia become a superpower again, mostly because of their vast energy resources of oil and gas. Their trading with China has furthermore increased this impression as well as their stoppage of gas supply to Ukraine in the fall of 2008 proving their position as a major energy supplier.

Furthermore politics is sure to become tense when the potential oil reserves below the North Pole are to be explored. Already the debate is between the neighbouring countries Canada, Norway, Russia, USA and Denmark (Greenland) on the definition of sea dominion.

The concern of the environment is another exogenous factor that has affected the development of new oil fields and tried to affect our demand for oil. Firstly the major output of carbon dioxide from the fossil fuel energy has created an environmental crisis, deeming the earth to a fever with warm and chaotic weather resulting in melting poles and further global warming. Secondly there are examples where production of oil has been set aside (at least temporarily) due to other environmental issues like the natural habitat for fish and birds outside of Lofoten in the Norwegian Sea.

Many actions have been taken in order to reduce the dependency on oil as leaders all around the world (now including USA) are encouraging development of environmental energy resources to become more independent of oil and revitalize the planet. It has been argued that the financial crisis might even lead to a more environmental way of living as people need to decrease their consumption, thus decreasing the demand for fossil fuel, and by giving more economical support to research and production of environmental products.

The high oil prices occurring in 2008 has already resulted in increased investments in alternative energy. Bio-fuel, solar and wind power are among the top renewable energy alternatives to fossil fuel which all saw an increase in research and development in the last years.

The sharp decrease in oil price has however made solar and wind power less competitive. Even with high subsidiaries the price of oil is comparatively lower. The alternatives are dependant on more research in order to become more cost efficient.

2.4.3 Final thoughts on historical development

History has shown that prices in the oil market have peaked due to political events more than fluctuations in demand and supply. This is in part true due to the fact that supply until the 70s was adequate enough to provide the world with low cost energy. The jump in price in the late 70s and early 80s was a result of the Iranian revolution as well as the war between Iran and Iraq. The small peak in price in the beginning of the 90s was a direct result of the Gulf war. Since then the prices have had some small adjustments according to international politics.

The last decade has however seen a higher dependence between prices and the supply and demand balance. The record low of only $16 per barrel was set in January 1999, when Iran decided to increase oil production at the same time the Asian financial crisis occurred and decreased demand.

The latest development with the colossal increase and the following decrease in 2008 was also a result of imbalance between supply and demand, as concluded in the interim report by CFTC. As the world economy was going at a high gear the oil suppliers did not manage (or wanted) to increase oil production to meet the demand. Therefore the price jumped to a record in the summer of 2008.

It is speculated that Saudi Arabia further increased this imbalance between supply and demand by cutting its production in 2006 and 2007 despite the increase in demand. Even though an oil price above $100 gave other producers the chance to increase their productions at high profits, the long lead times from initial discovery of an oil field to the production of oil made it difficult to meet demands. This further demonstrates that the supply side in the short run is inelastic as discussed before.

However due to the financial crisis and restrictions on credit demand turned down thus creating the opposite imbalance and prices dropped below $40 in December 2008.

2.5 Price forecast

As is evident from Figure 2-10 the oil price has seen many shifts since the 70s. Hamilton (2009) investigates the changes in oil price and defines p as 100 times the natural log of the real oil price and Δp as the quarterly percentage change. Although Hamilton estimates the average change to be 1.12 percentages pr quarter over 1970:Q1-2008:Q1, the result is not statistically significant and cannot reject the hypothesis that the expected oil price change could be zero or even negative.

Through a series of tests Hamilton concludes that the oil price is not easy to forecast and that the real price of oil seems to follow a random walk without drift. While the price has increased since the 70s, the result might as well have been the opposite.

"To predict the price of oil one quarter, one year, or one decade ahead, it is not at all naive to offer as a forecast whatever the price currently happens to be."

Hamilton, 2009 p. 181

Although Hamilton suggests taking today's price as the prediction for future prices he emphasizes that this would be a prediction with much uncertainty. Over the sample period the standard deviation for change in p was 15.28%. Hamilton further examines this by setting 2008 Q1 as the origin with an average price of $115 per barrel and assuming a Gaussian distribution for the change in p. This would produce a 95% confidence interval for the price in the next quarter to be between $85 and $156.

By Table 2-1 we can now conclude that even though Hamilton included this summer's big increase (upper limit of $177 is above the record at $147 that summer), this wide confidence interval did not include the latest drop to $34 in December 2008 (2008 Q4 between $68 and $195). This proves how difficult making predictions of future oil price are.

The reason why the standard deviation of price change in oil is big (above 15%) is that the big changes dominate the data from 1970 until 2008. Thus the oil prices are very unpredictable.

date	forecast	lower	upper
2008:Q1	115		
2008:Q2	115	85	156
2008:Q3	115	75	177
2008:Q4	115	68	195
2009:Q1	115	62	212
2010:Q1	115	48	273
2011:Q1	115	40	332
2012:Q1	115	34	391

Table 2-1 Forecast model with 95% upper and lower bounds for oil price (Hamilton, 2009)

2.6 Future development

When considering future development there are two important features of the oil market that cannot be ignored. First of all the population's dependency on energy will continue to be strong and thus the demand for oil will be high. Secondly the limitations in supply and availability; both as a result of politics and as a consequence of it being a limited resource.

The demand for oil will continue to be high and inevitably keep growing in the nearest future. The reason is the extreme growth in China, Middle East and India who is expected to continue their growth despite the financial crisis. China had an increase in demand at 7.2% annually between 1991 and 2006, and at this rate they will consume the same amount of oil as USA by 2020. Within 2040 they will have doubled the consumption again. And even at those levels their consumption will be less per capita than the USA, see Figure 2-11.

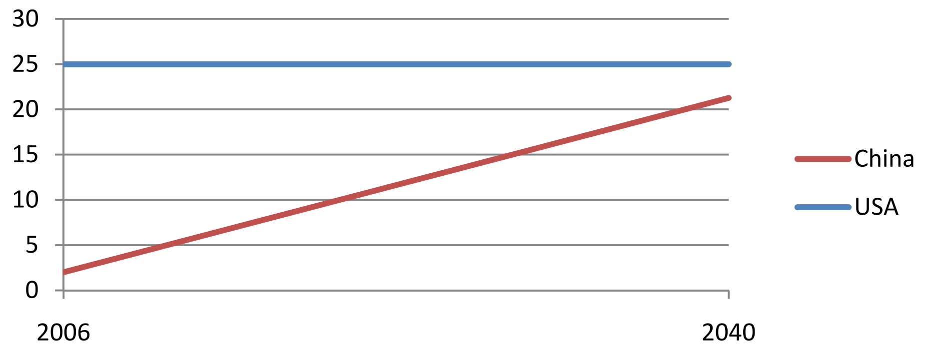

Figure 2-11 Estimated oil consumption in China 2040 with a stable growth of 7.2% from 2 bbl/capita in 2006, compared to consumption in USA in 2006 at 25 bbl/capita.

Supply is and will be affected by the drainage of oil reserves as oil is a limited resource. The moment of peak oil production has been widely discussed as some researchers believe we have already seen the peak, while others estimates the opposite and believe the peak is yet to come in many years.

However as mentioned before Norway, USA, Canada and Mexico have already experienced difficulties in keeping the same oil production levels.

When the decline in oil production is a direct result of the shortage in oil reserves (after oil peak), more of the oil production will take place at oil fields with a higher cost. The demand for oil will likely encourage development of high cost oil sources like deep water sites, oil sand and oil shale. This will result in a steeper supply curve, and shifts in demand will cause larger swings in market price.

In light of the fact that oil production many places in the world are declining the imbalance between demand and supply might see even more extremes in the near future.

This chapter has provided some insight in demand and supply of oil and based on historical evidence several key issues have been presented regarding oil price development. The low price elasticity of demand, the strong growth in demand from China and other newly industrialized economies and the failure of global production to increase were major contributors to the extreme increase in oil price seen in 2008.

The financial crisis has put a stop to the high oil prices as demands are temporarily down worldwide. However the factors just presented will become evident again as the financial system bounce back.

Politics will continue to play a significant part as dialogs between suppliers and purchasers is entangled in other political disputes. With the world leaders trying to balance out the financial crisis with the environmental challenges ahead the sum might have huge influence on the development of the oil price in the near future.

The short term dynamics will be dependant on the same factors as discussed but these might become strengthened as increased scarcity will increase the inelasticity of the supply side. Likewise the consumer power of more of the world’s population creates higher inelasticity as much of the energy consumption is fuelled by oil. The short term volatility might therefore be increased.

3. Theoretical background

This chapter will introduce statistical notation and terms necessary to develop an understanding of the model used in this thesis. These include covariance, correlation, skewness and kurtosis. In addition the relationship between the portfolio and its underlying risk factors as measured by different types of rate of change collectively called the Greeks (delta, gamma and theta) are presented.

Value at Risk (VaR) will then be defined accompanied by the different methods to estimate VaR used in this thesis. In addition to a comparison of the methods the chapter includes a discussion about variance reduction techniques used with Monte Carlo.

Finally this chapter include an introduction to several methods used to compare the estimation methods. The methods gives a better understanding as to what challenges exist when estimating VaR.

3.1 Covariance and correlation

Covariance occurs when two or more distributions relates to each other by either sharing phases to some degree or by having opposite phases. This can be seen as the two distributions either develops in the same direction or in the opposite direction.

If *X* and *Y* are two random variates with sample size N, the covariance between the two distributions is defined by;

$$Cov(X,Y) = E((X - \mu_X)(Y - \mu_Y))$$

$$Cov(X,Y) = \sum_{i=1}^{N} \frac{(x_i - \bar{x})(y_i - \bar{y})}{N}$$

where *X* and *Y* are two real random variables with expected value $E[X] = \mu_X$ and $E[Y] = \mu_Y$. Here $\bar{x}$ and $\bar{y}$ can be found as the mean of the empirically drawn samples:

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x_i$$

If the two distributions are independent of each other, their covariance is 0.

Covariance is a general form of correlation. While covariance can have values ranging from –infinity to +infinity, a correlation will be in the range of -1 to 1. The correlation coefficient ρ_{XY} between two random variables *X* and *Y* with expected values μ_X and μ_Y and standard deviations σ_X and σ_Y is defined as;

$$\rho_{x,y} = \frac{Cov(X,Y)}{\sigma_x \sigma_y} = \frac{E((X - \mu_X)(Y - \mu_Y))}{\sigma_x \sigma_y}$$

where the standard deviation is defined as:

$$Var(x) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - \overline{x})^2$$

$$Var(x) = \sigma^2$$

With a high positive correlation the linear relationship between the two distributions are high. If the opposite is true and the correlation is highly negative the linear relationship is still strong but now in the opposite direction. If the correlation is 0 the two distributions are independent as before.

3.2 Kurtosis

The Law of Large Numbers and the Central Limit Theorem both envisage that the distributions of the change of a market factor should converge towards a normal distribution with enough historical data. However it has been proved in several studies (Mandelbrot (1963), Praetz(1972) and Huisman et.al. (1998)) that this is not the case of market factors as they include heavy tails and high peaks. This character is known as a positive kurtosis and can be found as the fourth moment of a distribution;

$$K_X = \left\{\frac{N^2 - 2N + 3}{(N-1)(N-2)(N-3)}\sum_{i=!}^{N}\left(\frac{X_i - \overline{x}}{\sigma_x}\right)^4\right\} - 3\frac{(N-1)(N-3)}{N(N-2)(N-3)}$$

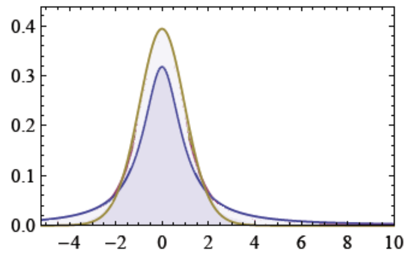

Figure 3-1 Comparing 0 kurtosis in normal distribution (green line) and positive kurtosis in stutdent t (blue line)

The kurtosis is significantly different from a normal distribution if it is bigger than 2 standard errors of kurtosis. The standard error of kurtosis can be found by:

$$sek = \sqrt{\frac{6}{N}}$$

Kurtosis can also be used to find the degree of freedom for a student t distribution. The excess kurtosis for a student t distribution is found by;

$$K_X = \frac{6}{v-4}, \quad v > 4$$

where v is the degree of freedom.

3.3 Skewness

A distribution might also be asymmetric around its mean. This character is measured by its skewness and is found by the third moment of a distribution;

$$S_x = \frac{N}{(N-1)(N-2)} \sum_{i=1}^{N} \left(\frac{X_i - \overline{x}}{\sigma_x} \right)^3$$

A positive skew indicates that the distribution is asymmetric towards the positive side with a heavy right side. A negative skew indicates the opposite, as displayed in Figure 3-3.

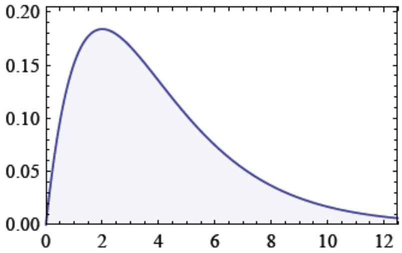

Figure 3-2 Positive skew

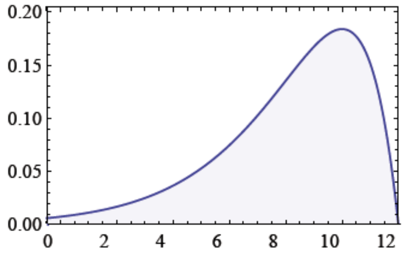

Figure 3-3 Negative skew

The skewness is significantly different from a normal distribution if it is bigger than 2 standard errors of skewness. The standard error of skewenss can be found by:

$$ses = \sqrt{\frac{24}{N}}$$

3.3 The Greeks

The Greeks presented in this part are important to the method developed later in this thesis. Delta, gamma and theta together comprise the delta-gamma approximation used in the Monte Carlo method.

3.3.1 Delta (δ)

The rate of change of the portfolio with respect to an underlying asset is defined as:

$$\delta = \frac{\partial P}{\partial S_i}$$

Thus delta tells about the slope of the portfolio according to changes in the underlying asset. If delta of the portfolio is 0.2 then a change in S_i is reflected in the portfolio with a 20% change of that amount.

One method often used to calculate delta is the explicit finite differencing approach as proposed by Jäckel (2002):

$$\delta = \frac{\partial v}{\partial S_0} \approx \frac{v(S_0 + \Delta S_0) - v(S_0)}{\Delta S_0} \qquad (1)$$

Here S_0 is yesterday's position and ΔS0 is the change in this position for today. The estimate is therefore prone to error as it is only dependant on one point in the data series. Today's slope only equals yesterday's slope if there is a continuous growth which is seldom (or never) the case. In order to make this estimate more robust it can be recalculated twice including an estimate for both tomorrow's and yesterday's slope:

$$\delta = \frac{\partial v}{\partial S_0} \approx \frac{v(S_0 + \Delta S_0) - v(S_0 - \Delta S_0)}{2\Delta S_0}$$

This is called the centre differencing approach. Since this estimate is dependant on tomorrow's price we could solve this by using an iterative method which is run until some convergence.

This thesis defines ΔS0 as the percentage change in the position's value, and (1) therefore becomes

$$\delta \approx \frac{v(S_0(1 + \Delta S_0)) - v(S_0)}{\Delta S_0}$$

Because of the linearity in the thesis' portfolio it is possible to calculate the delta directly by defining:

$$v(S_0(1 + \Delta S_0)) = v(S_0 + S_0\Delta S_0) = v(S_0) + S_0\Delta S_0$$

Which concludes:

$$\delta \approx \frac{v(S_0) + S_0\Delta S_0 - v(S_0)}{\Delta S_0} = S_0$$

An equal result can be produced when considering the centre differencing approach. Delta for an underlying risk factor is therefore defined (for a linear portfolio) to be the position of the underlying factor.

3.3.2 Gamma (Γ)

The rate of change of delta with respect to the price of the underlying asset is found by:

$$\Gamma = \frac{\partial^2 P}{\partial S_i^2}$$

Gamma therefore tells by what degree the delta changes dependant on changes in the underlying risk factor. If gamma is small, delta changes slowly.

As with delta the estimate of gamma can be calculated by using the centre differencing approach:

$$\Gamma = \frac{\partial^2 v}{\partial S_0^2} \approx \frac{v(S_0 + \Delta S_0) - 2v(S_0) + v(S_0 - \Delta S_0)}{\Delta S_0^2}$$

For a linear portfolio the second derivative is always zero since delta is constant.

3.3.3 Theta (θ)

The rate of change of the portfolio with respect to time is defined by:

$$\theta = \frac{\partial P}{\partial t}$$

which is often referred to as the time decay of the portfolio.

3.4 Value at Risk (VaR)

Value at Risk (VaR) defines the worst case scenario within a certain confidence level over a specified time horizon. This introduces the two parameters of a VaR: its confidence level and time horizon.

The confidence level defines at what rate the portfolio managers want the estimate to be within the true change. A usual confidence level is 95% which means that the estimated VaR will not be underrating the true change in 95% of the time.

Time horizon is often referred to as the holding period, the time which the assets in the portfolio are constant and thus the portfolio is unchanged. A regular time horizon is 1 day.

VaR can be defined by asking a simple question: How much is it possible to lose within a certain time period at a certain significance level?

$$P(L > x_p) = p$$

E.g.: For the next day (period) 1% (significance level) of losses will be bigger than 500k (VaR). Or: I am 99% (confidence level) certain that my losses will not be bigger than 500k (VaR) the next day (period).

Most VaR approaches use historical data to estimate potential changes. There are 3 commonly used approaches to estimating VaR which will be tested in this thesis:

- Historical simulation
- Analytical estimation
- Monte Carlo simulation

Figure 3-4 compares the methods which will be further developed later in this section.

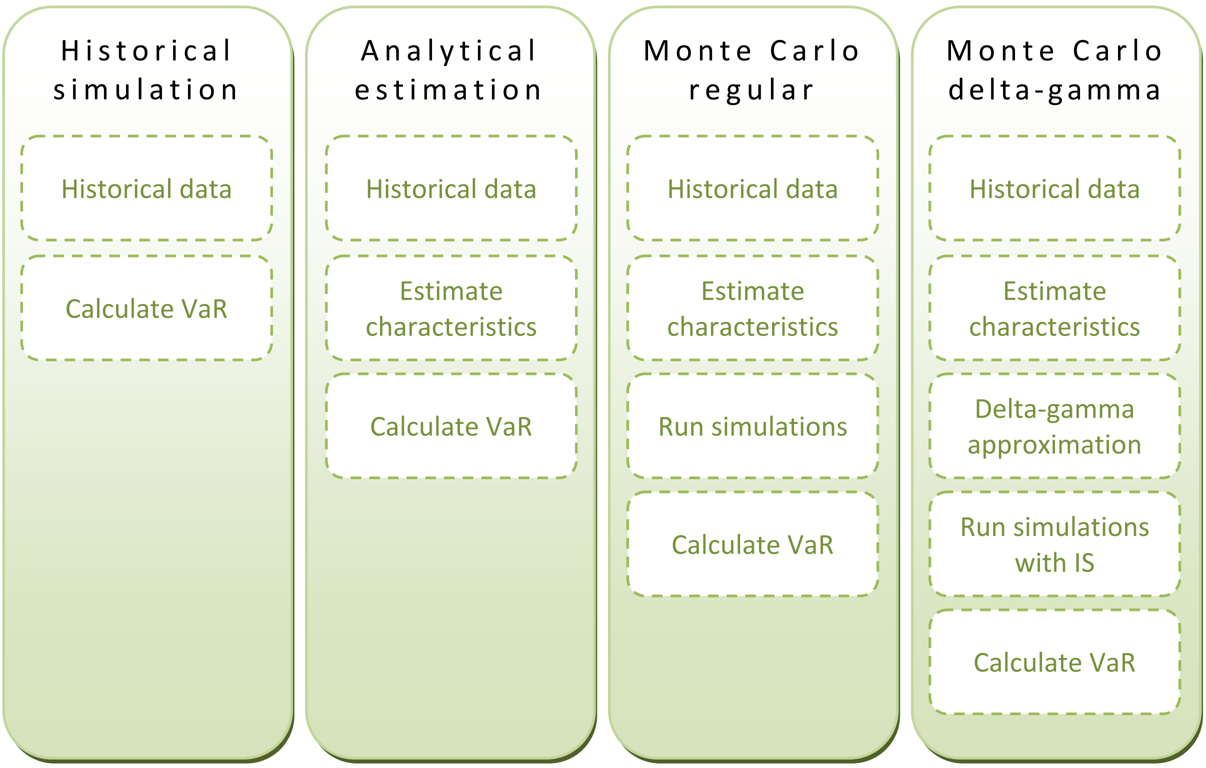

Figure 3-4 Comparison of the 4 methods used to estimate VaR in this thesis.

3.4.1 Historical simulation

By assuming that the historical development of the risk factors is a good model for tomorrow's development, the historical approach can give a good approximation for tomorrow's VaR. This approach does not make any assumptions on the distribution of data.

The historical approach uses the historical data directly by using historical changes as the possible outcomes of the coming change. If the historical data comprise of 501 days, the 500 possible changes together constitute the distribution of tomorrow's change. By sorting the outcomes VaR can be easily found as the 5th worst scenario for a 99% confidence level.

The estimate can be easily updated day by day as the newest 501 days are used as the historical data set.

3.4.2 Analytical estimation

Most analytical approaches assume normality and often serial independence in order to develop an analytical solution to a VaR estimate. By assuming normality the wanted percentile is a multiple of the distributions standard deviation of the portfolio's change. Furthermore by assuming serial independence the change one day will not affect the next day, which makes it easy to calculate VaR for longer horizons by the square root of the number of days.

The change in the portfolio value consisting of n products can be defined as:

$$\Delta P = \sum_{i=1}^{n} \propto_i \Delta x_i$$

Where α_i is the amount invested in product i and Δx_i is the return on asset i measured in percentage.

When assuming that the Δx_i are multivariate normal, ΔP is also normally distributed. By definition the expected change of Δx_i are 0, thus meaning that the expected change of ΔP is also 0.

The key to the analytical estimation is therefore in the estimation of the standard deviation. It can be estimated by the historical correlation and standard deviation of the portfolio's products which is its underlying risk factors. The variance of ΔP can be found by;

$$\sigma_P^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \rho_{ij}\alpha_i\alpha_j\sigma_i\sigma_j$$

$$\sigma_P^2 = \sum_{i=1}^{n} \alpha_i^2\sigma_i^2 + 2\sum_{i=1}^{n}\sum_{j<i} \rho_{ij}\alpha_i\alpha_j\sigma_i\sigma_j$$

As a more complex approach it is possible to calculate the standard deviation of the risk factors by emphasising the most recent data. This is made possible by the exponentially weighted moving average (EWMA) which adjusts the standard deviation of a risk factor t to be:

$$\sigma_t = \sqrt{(1-\lambda)\sum_{s=t-k}^{t-1} \lambda^{t-s-1}(x_s-\mu)^2}$$

Here the parameter λ is called the decay ratio or the smoothing factor and decides at what pace the importance of data should decrease with time.

EWMA can be further generalised with GARCH (generalized autoregressive conditional heteroskedasticity). More specified EWMA equals GARCH(1,1) where the sum equals to one as with EWMA.

3.4.3 Monte Carlo simulation

Basic Monte Carlo is based on repeated random sampling in order to produce a distribution of possible outcomes. Due to the often enormous amount of simulations and recalculations a computer is necessary to carry out the Monte Carlo simulation.

The method has a wide area of applications, beginning with the estimation of physical mass in the 30s and further utilized for military purposes like the Manhattan Project and during the Cold War. Because of the amount of simulations needed the method was researched more in depth with the raise of computational power. This also led to the development of pseudorandom numbers to further increase the efficiency.

While the computational power has increased since the late 40s when they needed 6 weeks to generate 1 million random numbers (Jäckel, 2002), the complexity of the tasks have also increased. Today the method is used for simulating mathematical, financial and physical systems. It is most often used when there is no analytical solution to the problem as the resulting distribution of a Monte Carlo simulation will produce a close estimate of the true result.

In the financial world the method is widely used for estimating risk when there are several underlying risk factors and significant uncertainty in inputs. This is the situation when evaluating Value at Risk which is the purpose of this thesis.

3.4.3.1 Monte Carlo simulation methods

There are 3 approaches to a Monte Carlo simulation:

- Brute force
- Scenario simulation
- Partial simulation by variance reduction

While the brute force approach can give accurate results dependant on the assumptions made in the model, it also requires the most time. The two alternatives were developed in order to meet the 2 difficulties concerned with the computational cost of Monte Carlo simulation:

1. Portfolio consists of large number of financial instruments.
2. Large number of runs required to obtain an accurate simulation.

The most common variance reduction techniques are presented and compared before the method utilized in this thesis is further examined.

3.4.3.2 Brute force MC

The brute force approach utilizes the information of the underlying risk factors like their distribution and correlation, in order to create random outputs. This approach therefore gives accurate estimates but to achieve a high level of confidence the number of simulations needed is high thus resulting in a time consuming approach.

The method for a brute force Monte Carlo simulation is given in Figure 3-5.

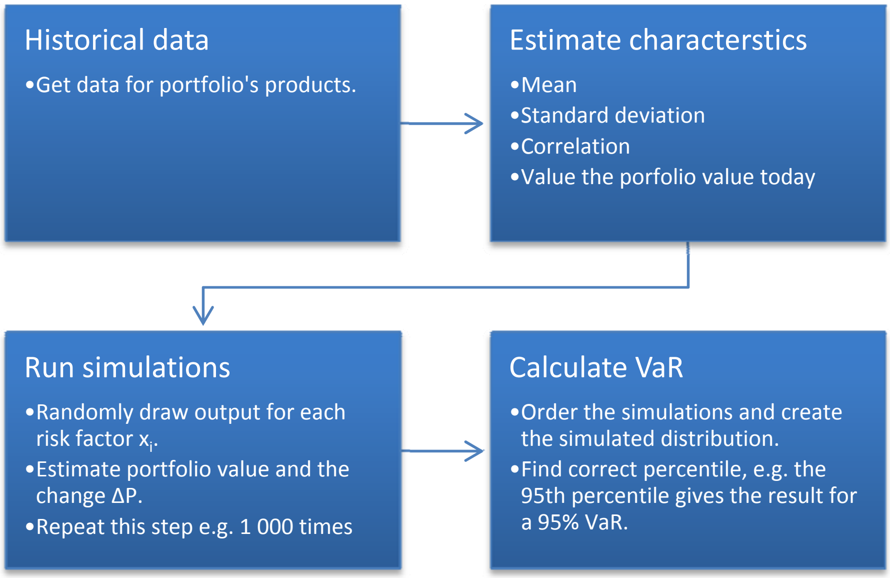

Figure 3-5 Monte Carlo Brute Force method outlined

When assuming a normal distribution for every underlying risk factor the change in the portfolio's value ΔP can be found by the same distribution as in the analytical estimate. However due to randomness in the sample drawing this does not necessarily mean that the brute force Monte Carlo method with normality assumption and the analytical estimate will give the same output. Still it will converge to the same result as the number of draws is increased due to the central limit theorem and the law of large numbers.

Since estimates of VaR are especially concerned with the fat tails of the distribution the student t-distribution can provide better estimates. A key characteristic of a student t-distribution is its ability to include uncertainty at the end of the distribution by setting the degree of freedom. As the degree of freedom (df) is increased the distribution converges to a normal distribution, and this is evident already from df > 10 as seen in Figure 3-6.

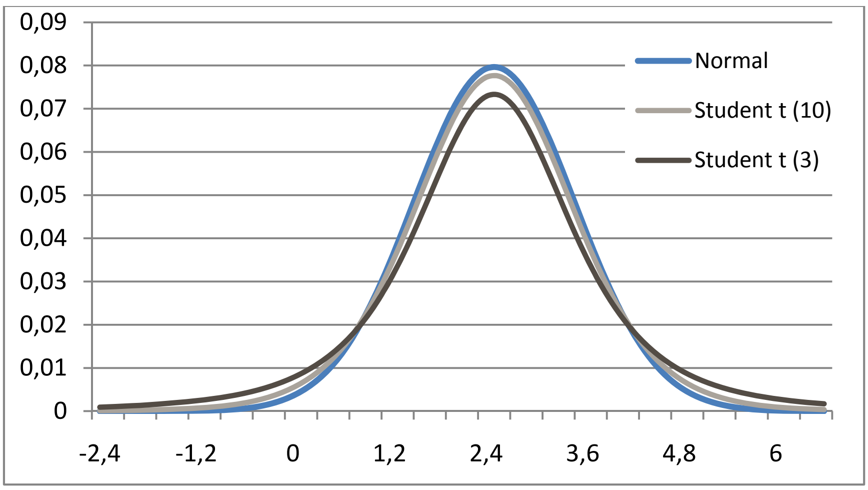

Figure 3-6 Normal distribution compared to Student t distribution with 3 (red line) and 10 (green line) degrees of freedom. Student t distribution converges to normal as the degree of freedom increases.

For market factors it is recommended by Glasserman, Heidelberger and Shahabuddin (2000) and others to use a degree of freedom between 3 and 7, depending on the risk factors at hand. This will then increase the probability of drawing outcomes at the end of the tail.

A portfolio which consists of underlying risk factors with student t-distribution can be calculated as a multivariate student t-distribution. This is accomplished by:

$$\boldsymbol{T_{df}} = \frac{\boldsymbol{V}}{\sqrt{\boldsymbol{X}^2_{df} / df}} = \boldsymbol{V} * \sqrt{df \;/\; \boldsymbol{X}^2_{df}}$$

where $\mathbf{X}^2$ is Chi-distributed with df degrees of freedom and;

$$\boldsymbol{V} = \boldsymbol{C} * \boldsymbol{Z}$$

Z is a standard normal distribution and **C** is found by the Cholesky decomposition with the correlation matrix **∑**.

$$\boldsymbol{CC^T} = \sum$$

3.4.3.3 Scenario Simulation

Scenario simulation was introduced by Jamshidan and Zhu (1997) as a faster approach than the brute force method. The key issue was to separate the portfolio revaluations from the simulation step in VaR by Monte Carlo by defining possible scenarios in advantage. The scenarios are later referred to in a lookup table which promptly produces the outcome of a simulation. However, the results have not been all positive as the number of scenarios might get out of hand and problems with finding the extreme results. This becomes evident in the fat tails of a VaR, where scenario simulation has a tendency to underestimate the 99th percentile more often than the 95th percentile (Abken, 2000). Also see Rockafellar and Uryasev (2000).

The method for scenario simulation is given in Figure 3-7.

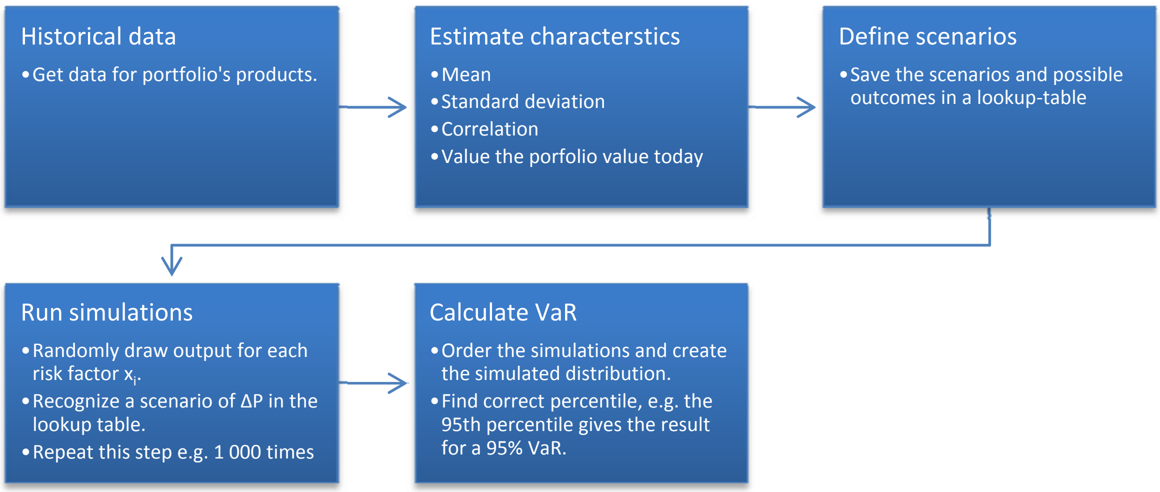

Figure 3-7 Monte Carlo Scenario Simulation method outlined

3.4.3.4 Variance reduction techniques

To decrease the necessary number of trials several methods have been developed in order to reduce the variance. With a lower variance the simulation will be more precise, and thus fewer runs are required.

The algorithms for a Monte Carlo simulation utilizing a variance reduction technique are similar to the brute force approach. In order to achieve the variance reduction the algorithm must however include an analytical part before the simulation. See method outlined in Figure 3-8.

There are several variance reduction techniques and the following text relies heavily on Glasserman (2004) and Hull (2008), which describes the techniques in more detail. The techniques discussed are:

- Antithetic variates
- Control variates
- Stratified Sampling (SS)
- Latin Hypercube (generalization of SS into more dimensions)
- Quasi random sampling
- Importance Sampling (IS) (also known as delta-gamma or quadratic)

The first two techniques draw random outcomes and try to correct the error of the outcome by adjusting it. The last four use methods to draw outcomes with more precision by trying to approach the true distribution.

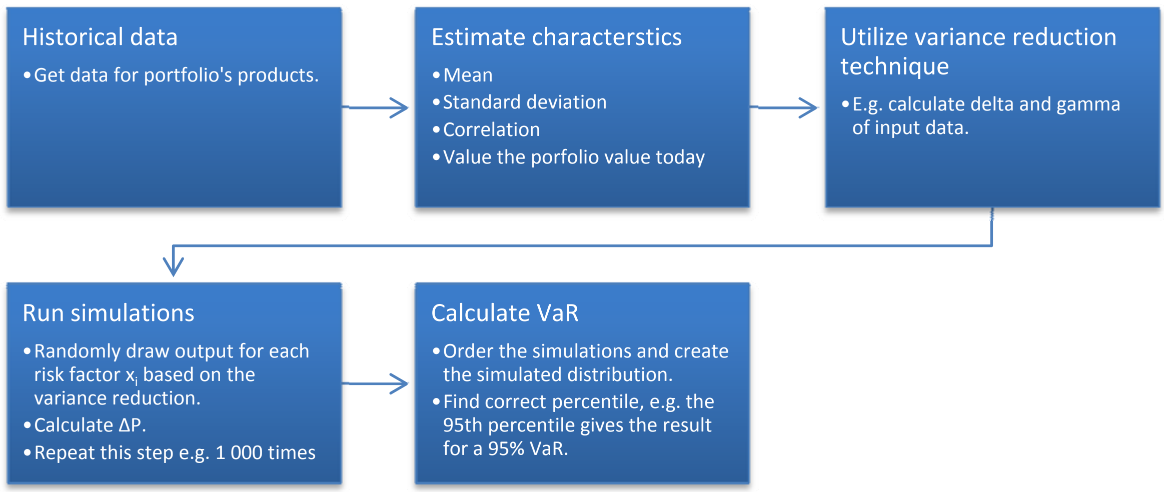

Figure 3-8 Monte Carlo with Variance Reduction outlined

An antithetic variable technique sample averages Fs of two opposite outcomes by changing the sign of the first outcome for the second. If the first outcome f1 is positive the second outcome is therefore negative. This provides a lower variance due to the fact that the two outcomes often will be on each side of the true value. The estimate of the price is then the average of all Fs.

$$F = (f1 + f2) / 2$$

A control variate uses an observed error to control the outcomes of trials. The observed error is calculated by a second and similar commodity where there is an analytical solution. An example is a commodity A with a stochastic volatility and a similar commodity B with a constant volatility. By utilizing the correlation between the outcomes of the two commodities in the trials it is possible to correct the estimates for the wanted commodity.

$$fA = f^*A - f^*B + fB$$

In stratified sampling (SS) the outputs are divided into fractions (strata) with a set possibility for an outcome to appear in a given fraction. Random sampling does not take into account any probability for which fractions the outcomes will reside in (other than the probability distribution). When the number of trials increases the random sampling should however approach the stratified sampling. SS therefore eliminates sampling variability across strata but keeps sampling variability within strata.

Latin hypercube is a generalization of stratified sampling to include more dimensions. Compared to a random sampling Latin hypercube sampling can guarantee that the ensemble of random numbers is representative of the real variability. A further improvement is the orthogonal sampling which gives a very good representative of the real variability.

Quasi random sampling (low-discrepancy sampling) also resembles stratified sampling but is more flexible as we do not need to know how many samples will be taken in advantage. The samples in a quasi random sampling are always filling in the gaps between the existing samples, thus putting the samples evenly spaced in the probability space.

Through a delta-gamma approximation importance sampling (IS) changes the measurement in order to give more weight to important outcomes. This increases the sampling efficiency. For a VaR estimator this will give more weight to outcomes at the tails of a distribution, thus imitating the fat tails. The measurement change is made with a likelihood ratio to determine the likelihood of outcomes in the important area.

Glasserman, Heidelberger and Shahabuddin (GHS) have in their papers concerning Monte Carlo simulation of VaR tried to minimize the variance in their models. With variance reduction techniques such as Importance Sampling they have proved a reduction in variance for a portfolio which also reduces the number of runs required in MC (GHS, 1999). The resulting model uses a delta-gamma approach in their partial simulation.

This method uses the structure of a basic brute force Monte Carlo and includes an analytical part before the simulation where delta and gamma is calculated. The analytical part results in a more effective sampling method and thus requires less simulation runs. This effectiveness must however be considered in conjunction with the extra time needed to calculate the likelihood of each drawn estimate.

GHS (1999) concluded that the delta-gamma method with importance sampling would reduce variance by 14-52 times compared to regular brute force Monte Carlo simulation. The most effective scenarios included portfolios where the underlying assets are not correlated and long options. Still for the portfolios with correlated assets the variance was reduced by 14-28 times.

As concluded by Figure 3-9; Importance Sampling is the most complex method but also has most potential and produces the best variance reduction. The IS approach can however produce the adverse as the efficiency is dependant on calculation time of delta, gamma and the covariance matrix as well as the assumptions made in the model (e.g. the distribution of the risk factors). The wrong assumptions might even create a worse estimate.

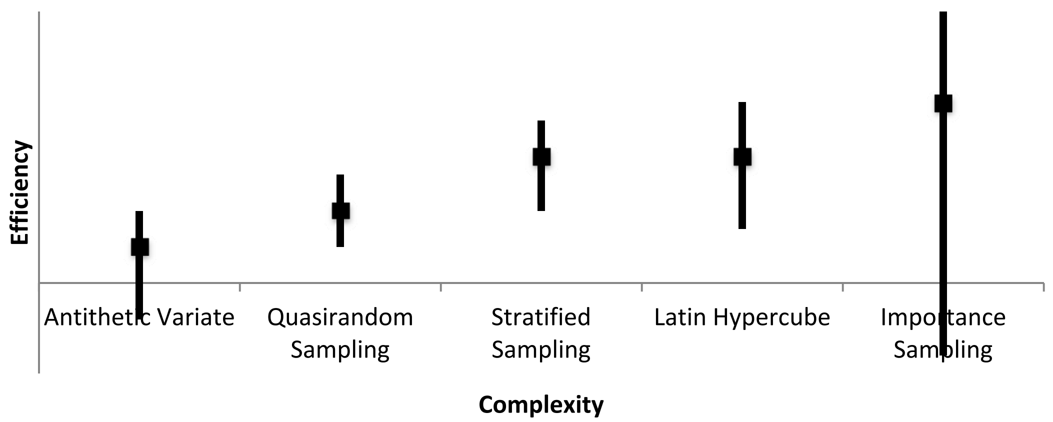

Figure 3-9 Comparison of variance reduction techniques

This Importance Sampling (delta-gamma) method is implemented in this master's thesis. The implementation is based on the method described in GHS 2000. In this method the portfolio is a multivariate normal distribution which does not directly correct for the heavy tails in the commodities' distribution. However the IS will emphasize the important areas which are at the fat tails of the distribution of ΔP.

3.4.3.5 Monte Carlo delta-gamma

The Monte Carlo delta-gamma uses the delta-gamma approximation described by Glasserman, Heidelberger and Shahabuddin (2000) in order to accomplish an importance sampling process as described in the previous part. By reducing variance and generating more precise random outcomes the number of runs in the simulation can be reduced, thus minimizing the amount of time needed for the simulation.

However, as described in section 3.4.3.4 Variance Reduction Techniques this method requires some calculations before commencing the simulation. It therefore occur a trade-off between the time won by reducing variance and the time cost in order to produce this reduction. In the evaluation of GHS they argue that most financial institutions have delta and gamma available from other tools, and this part therefore does not entail any time consume on their methods.

The Monte Carlo delta-gamma method is outlined in Figure 3-10.

The first two parts are identical for all Monte Carlo models as they are dependant on the same characteristics of the historical distributions. In addition to the regular characteristics this model needs to calculate the delta and gamma of the underlying risk assets. This is accomplished as described earlier in this chapter.

The correlation ($\sum_S$), delta (δ) and gamma (Γ) matrices complete the inputs for the Importance Sampling method developed by GHS. These inputs are then further employed in order to carry out the process.

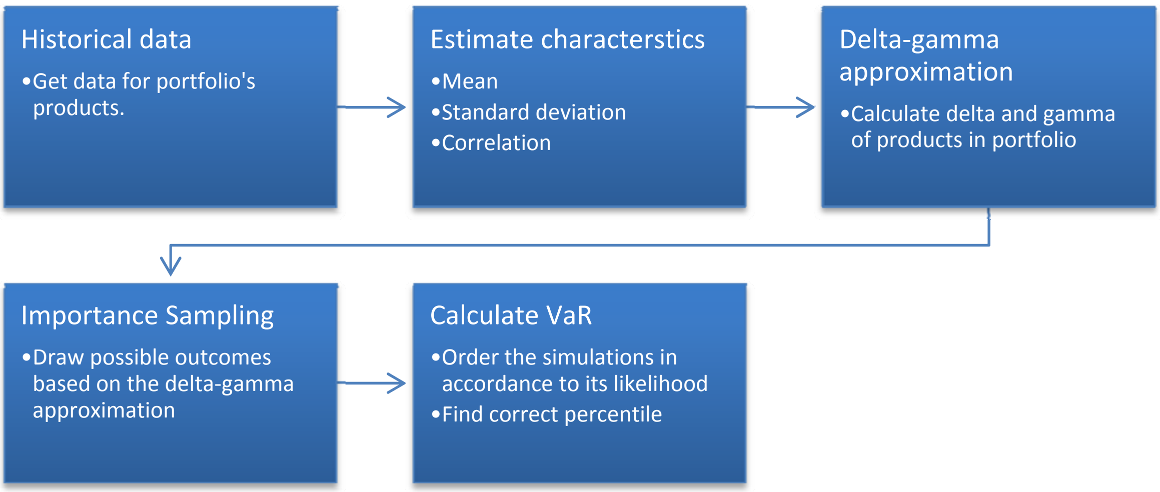

Figure 3-10 Monte Carlo delta-gamma method outlined

A change in the portfolio's value is approximated by a Taylor series;

$$L \approx -\theta \Delta t - \boldsymbol{\delta}^T \Delta \boldsymbol{S} - \frac{1}{2} \Delta \boldsymbol{S}^T \boldsymbol{\Gamma} \Delta \boldsymbol{S}$$

where a bold symbol denotes a matrix and ^{T} notates a transposed matrix. Further the **ΔS** is a matrix of changes in the underlying risk factors over the time period Δt and θ is the calculated theta. Δt is defined to be 1 day in the model.

Θ is argued by Hull (2008) to be 0 as the equation is dominated by the delta and gamma parts since Θ is very close to zero. This assumption is also used in this thesis and L is therefore defined as:

$$L \approx -\boldsymbol{\delta}^T \Delta\boldsymbol{S} - \frac{1}{2}\Delta\boldsymbol{S}^T \boldsymbol{\Gamma} \Delta\boldsymbol{S}$$

ΔS is defined to be a multivariate normal with mean 0 and the correlation matrix as its standard deviation. That is:

$$\Delta\boldsymbol{S} \sim N(0, \textstyle\sum_S)$$

ΔS is found by:

$$\Delta\boldsymbol{S} = \boldsymbol{CZ}$$

Where **Z** is a multivariate standard normal distribution (**Z**~N(0,1)) and **C** is found by solving the following equation by Cholesky decomposition:

$$\boldsymbol{CC}^T = \textstyle\sum_S$$

By defining

$$\boldsymbol{b}^T = -\boldsymbol{\delta}^T \boldsymbol{C}$$

and

$$\boldsymbol{\Lambda} = \begin{bmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_m \end{bmatrix}$$

with $\lambda_1 > \lambda_2 > \cdots > \lambda_m$ the eigenvalues of $-\frac{1}{2}\boldsymbol{\Gamma}\sum_S$, and m number of products. This can be found with the **C** found in the Cholesky decomposition:

$$\boldsymbol{\Lambda} = -\frac{1}{2}\boldsymbol{C}^T \textstyle\sum_S \boldsymbol{C}$$

The definitions of **b** and **Λ** together with the implicit definition of **Z** in **ΔS** achieve a rewrite of the approximation of L:

$$L \approx \boldsymbol{b}^T \boldsymbol{Z} + \boldsymbol{Z}^T \boldsymbol{\Lambda} \boldsymbol{Z}$$

Here **Z** is based on the correlated changes in **ΔS**, the **b** is dependant on **δ** and **Λ** is dependant on **Γ**. As a result of this revision the characteristic function of L is now be defined by:

$$\psi(\theta) = \sum_{i=1}^{m} \frac{1}{2}\left[\frac{(\theta b_i)^2}{1 - 2\theta\lambda_i} - log(1 - 2\theta\lambda_i)\right]$$

A key characteristic of the Importance Sampling method is the change in the distribution from which the underlying risk factors are generated from. This change creates more samples from the important areas of the distribution and is accomplished by changing Z from a standard normal distribution into:

$$Z_i \sim N(\mu(\theta), \sum(\theta))$$

Here μ and ∑ are defined by the parameter θ which is found by an iterative process where θ is solved for:

$$\frac{d}{d\theta}\psi(\theta_x) = E_{\mu(\theta_x),\sum(\theta_x)}[Q] = x$$

For any $\theta > 0$ and $\theta < 1/2\theta\lambda_i$ we have:

$$\sum(\theta) = (\boldsymbol{I} - 2\theta\boldsymbol{\Lambda})^T \quad and \quad \mu(\theta) = \theta\sum(\theta)\boldsymbol{b}$$

Now Z_i becomes normal with mean and variance:

$$\mu_i(\theta) = \frac{\theta b_i}{1 - 2\theta\lambda_i} \quad and \quad \sigma_i^2(\theta) = \frac{1}{1 - 2\theta\lambda_i}$$

The key identity for the importance sampling is the following probability:

$$P(L > x) = E_{\mu,\sum}[l(Z)I(L > x)]$$

This corrects for the change in distribution with a likelihood ratio:

$$l(Z) = |Z|^{0.5} e^{-0.5\mu^T\sum^{-1}\mu} e^{-0.5[Z^T(I-Z^{-1})Z-2\mu^T\sum^{-1}Z]}$$

This expression is challenging but thanks to the use of a parameter θ it can be simplified to;

$$l(Z) = e^{-\theta Q + \psi(\theta)}$$

where $\boldsymbol{Q} = \boldsymbol{b}^T\boldsymbol{Z} + \boldsymbol{Z}^T\boldsymbol{\Lambda}\boldsymbol{Z}$.

By sampling **Z** from $N(\mu(\theta_x), \sum(\theta_x))$ the scenarios which were rare now are typical, thus increasing the chances of drawing from the important areas.

The estimate returned after completing the simulation with importance sampling is given by:

$$\frac{1}{N}\sum_{i=1}^{N} e^{-\theta Q + \psi(\theta)}\, I(L^i > x)$$

3.5 Comparison methods

As already described VaR is dependant on two factors; the time horizon and level of confidence. Moreover the estimates also depend on the historical time horizon, which is how much of the historical data will be used to calculate possible scenarios in the historical simulation and be the foundation for the data analysis in the analytical approach and Monte Carlo simulations. Finally the Monte Carlo simulation is dependant on the number of runs. A higher number of runs creates a more precise estimate but consumes more time.

These variables will together create many alternatives for the methods presented earlier and need several comparison methods to separate their accuracy and efficiency. The methods presented in this are described in the paper by Hendricks (1996) who introduces several methods in order to compare the historical simulation with two analytical approaches based on equally weighted moving average and EWMA.

Accuracy is tested by checking the fraction of outcomes covered by a chosen method. An optimal method will cover 95% of the daily changes over a given timeframe when testing for a 95% level of confidence. The magnitude of a loss bigger than VaR is not of importance in this comparison.

Mean relative bias controls the mutual distance between the methods estimates. By comparing a method's estimate to the average of the tested methods deviations can be easily found.

The size of the error in an estimate can be captured by looking at the multiple an actual loss outdoes the VaR estimated by the method when it fails to cover the actual loss. Although a method covers 95% of the actual changes at 95% VaR it can still cause less or more damage as indicated by this measurement.

A similar comparison method tests the average multiple of tail event to risk measure and thus indicates the size of the tail of the method. This examines by what factor the average loss is bigger if the estimate for VaR does not cover the actual loss. A further expansion of this test can be carried out by inspecting the maximum multiple of tail event, which checks what the biggest multiple of VaR has occurred.

Finally Hendricks suggests a test which examines the correlation between risk measure and the absolute value of outcome. If the correlation is highly positive between the method's estimate and the actual changes in the portfolio value this means that the estimate follows the market changes closely.

These tests are accomplished by a back-test which simulates VaR estimates over a historical time period. It is therefore easy to test if the methods meet the standards needed for a good estimation method.

4. Analysis of data

The historical price data for each product provides an important link between the theory presented and the model developed. The goal of analysing data is to get a better understanding of the market factors including their distribution and correlation.

Firstly the portfolio will be presented. The products have different characteristics as described in chapter 2, but their price is still dependant on many of the same factors as described in chapter 2 Oil market. This presentation will also give a brief discussion about their price and volatility history.

Secondly the statistical attributes of the products will be analyzed, and their probable distributions mapped.

As already discussed many of the risk factors do not have a normal distribution, but tend to have higher peaks and fat tails. This is perhaps more true when considering short horizons, as the regularly small changes are intercepted by occasionally very large changes. The products in this thesis will be tested for kurtosis by a Jarque Bera test.

Finally the products correlation will be calculated.

All analysis is carried out on the logarithmic percentage change, that is:

$$X_i = \frac{x_i - x_{i-1}}{x_i}$$

Here X_i is the ith element of the distribution, while x_i is the historical data for the ith day.

4.1 The portfolio

The portfolio consists of products covering a wide range of Statoil Hydro ASAs portfolio of refined products. The thesis' portfolio contains the following 9 products:

- Brent NWE CIF ARA Platts Mid As Quoted(US$:F/T)
- Propane NWE FOB Seagoing Platts Mid As Quoted(US$:F/T)
- Gasoil 0.2% NWE CIF ARA Platts Mid As Quoted(US$:F/T)
- Jet/Kerosene NWE CIF ARA Platts Mid As Quoted(US$:F/T)
- HSFO 3.5% NWE CIF ARA Platts Mid As Quoted(US$:F/T)
- No.6 1%/LSFO NWE CIF ARA Platts Mid As Quoted(US$:F/T)
- Naphtha NWE FOB Barges Platts Mid As Quoted(US$:F/T)
- ULSD 10ppm NWE FOB Barges Platts Mid As Quoted(US$:F/T)
- Unleaded NWE FOB Barges Platts Mid As Quoted(US$:F/T)

Platts is the supplier of market data and their glossary can help clarify the terms used in the product descriptions as presented below:

- All products are NWE, which means that they are traded from the Northwest Europe oil and petrochemicals market.
- In addition the products are MID which means that the price is an arithmetic average between high and low quotations of the day.
- Products termed ARA are used in shipping when discharge or loading occur in one of the three ports in Amsterdam-Rotterdam-Antwerp.

- CIF and FOB are two types of insurance and decides the responsibility for risk of the cargo during freight. Generally Cost, Insurance and Freight (CIF) are more expensive as the goods are effectively priced at the delivery port, compared to Free on Board (FOB) where the goods are priced at the loading port and the buyer must pay for shipping and insurance of the goods in transport.

As shown in Figure 4-1 the price increased steadily for all products until 3rd of July 2008. Since then the positive trend was broken and decreased for the rest of the data period. However it is still to early to say if this is a structural break or if the reduction in prices are a result of mean reversion. The financial crisis has made the downfall even steeper, but as the market normalizes it is possible that the price will settle somewhere in between the high and lows of 2008, and perhaps continue the positive trend.

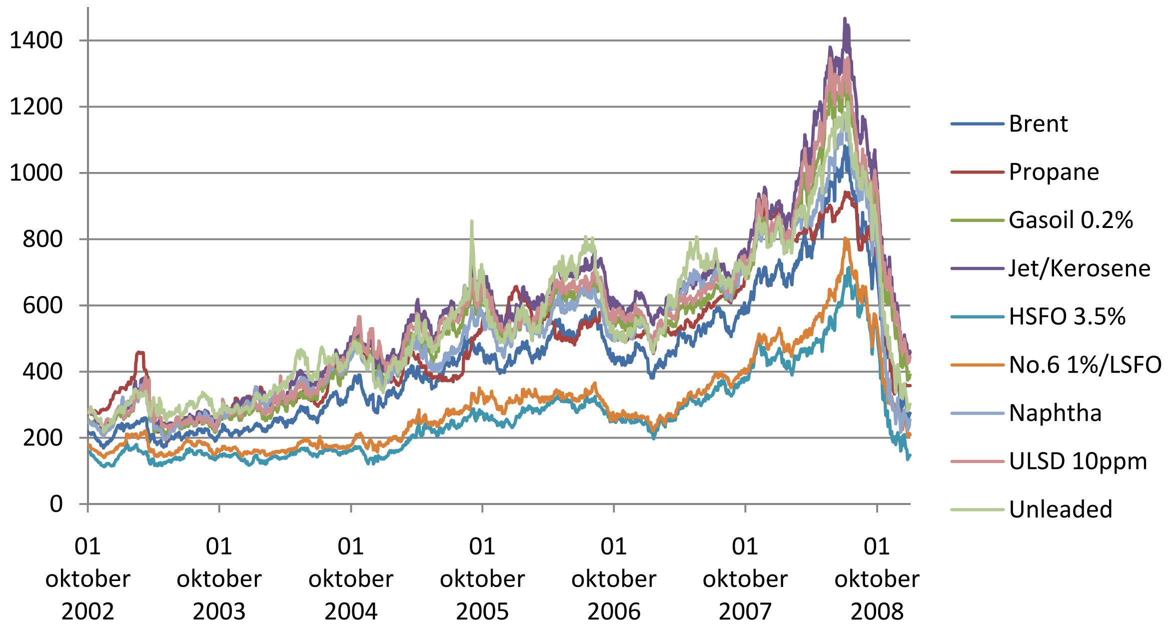

Figure 4-1 Price development in USD/ton for products in portfolio from 1st October 2002 to 31st December 2008.

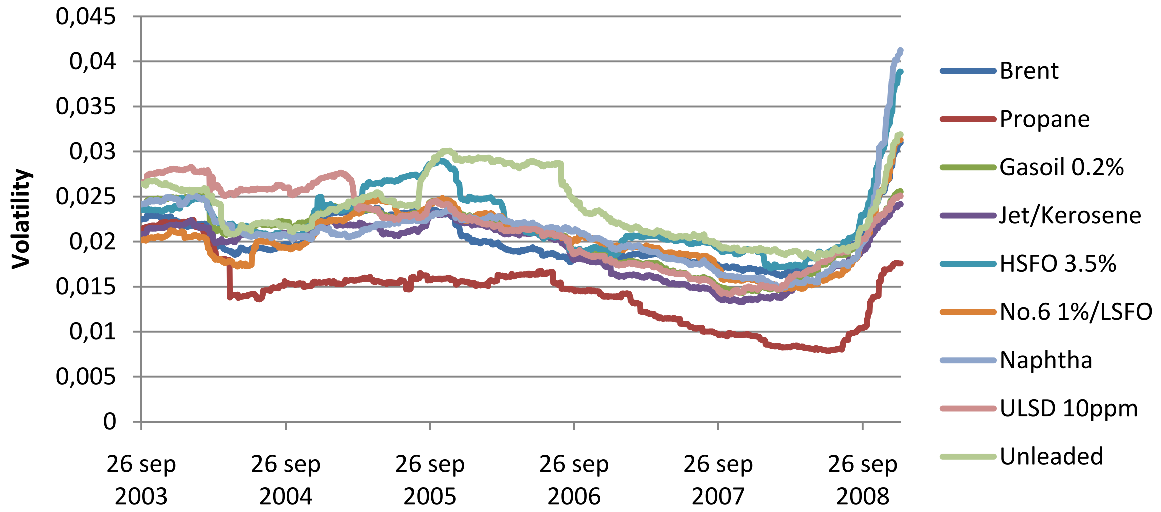

Figure 4-2 Volatility in data period as measured by standard deviation over the last 250 days.

At the same time as the prices has been falling the volatility of the products has increased. This is evident from the Figure 4-2 which shows the volatility development in the data period based on the last 250 days at each point. A shift like this makes the estimation of VaR very difficult as it is based on historical data. In the beginning of a volatility switch the majority of the historical data is based on the historical volatility level. As illustrated by Figure 4-2 the oil market went from low volatility to high, but this effect is not captured immediately as a trend needs some time to settle. When going from low to high volatility the VaR estimate is likely to be underestimating the risk in the market, and the opposite is true when going from high to low volatility.

As shown in Table 4-1 the mean of the price change of every product is very close to 0. This is further confirmed by a hypothesis test, where Z lies inside the interval of a two-sided test for significance level at both 0.01 and 0.05. In addition all confidence interval at a 5% level contains 0.

	Brent	Propane	Gasoil 0.2%	Jet/ Kerosene	HSFO 3.5%	No.6 1% LSFO	Naphtha	ULSD 10ppm	Unleaded
Mean	5.19E-04	2.99E-04	5.18E-04	5.20E-04	2.86E-04	3.46E-04	3.38E-04	5.39E-04	3.82E-04
St.Error	5.61E-04	4.02E-04	5.47E-04	5.09E-04	6.46E-04	5.60E-04	6.36E-04	5.73E-04	6.41E-04
Median	1.42E-03	0.00E+00	9.29E-04	6.16E-04	0.00E+00	0.00E+00	1.15E-03	8.63E-04	5.66E-04
Mode	0	0	0	0	0	0	0	0	0
St.Dev	0.0223	0.0160	0.0218	0.0202	0.0257	0.0223	0.0253	0.0228	0.0255
Variance	4.96E-04	2.55E-04	4.73E-04	4.09E-04	6.58E-04	4.96E-04	6.39E-04	5.19E-04	6.50E-04
Excess kurtosis	2.342	16.088	3.199	1.071	4.966	3.064	7.836	4.128	3.044
Skewness	-0.148	0.514	-0.282	0.016	0.170	-0.081	-0.008	-0.320	0.187
Jarque-Bera	34.300	11339.80	23.477	244.846	261.897	1.986	1538.907	110.760	9.314
Z	0.925	0.744	0.946	1.022	0.443	0.618	0.531	0.940	0.595
p-value	1.645	1.543	1.656	1.693	1.342	1.463	1.405	1.653	1.448
Range	0.235	0.287	0.254	0.168	0.297	0.231	0.377	0.271	0.269
Minimum	-0.120	-0.110	-0.169	-0.084	-0.124	-0.115	-0.189	-0.180	-0.108
Maximum	0.115	0.176	0.084	0.085	0.173	0.116	0.188	0.091	0.160
Observations	1579	1579	1579	1579	1579	1579	1579	1579	1579
Conf.int high	1.62E-03	1.09E-03	1.59E-03	1.52E-03	1.55E-03	1.44E-03	1.58E-03	1.66E-03	1.64E-03
Conf.int low	-5.8E-04	-4.9E-04	-5.5E-04	-4.8E-04	-9.8E-04	-7.5E-04	-9.1E-04	-5.8E-04	-8.8E-04

Table 4-1 Summary of key statistical characteristics of products in portfolio. Based on price changes during 02. Oct 2002 - 31. Dec 2008

Skewness measures how asymmetric a distribution is around its mean as defined in section 3.2. As is evident in Table 4-1 the majority of the distributions has some skewness. A distribution is often regarded as significantly skewed if the skewness is bigger than 2 standard errors of skewness. Apart from Naphta, Jet/Kerosene and No. 6.1% LSFO the products therefore appears to be significantly skewed.

Kurtosis measures the peakness of a distribution compared to a normal distribution as defined in section 3.2. A positive excess kurtosis indicates that the distribution has a higher peak, which is evident for every product. The lowest excess kurtosis is measured for Jet/Kerosene at 1.071; this is still significant as a normal distribution would return an excess kurtosis of 0. A comparison with 2 standard errors of skewness returns the same conclusion.

A positive kurtosis confirms that the distribution is leptokurtic which indicates simultaneously high peaks and flat tails.

Based on the kurtosis and skewness of a distribution the Jarque-Bera test examines any departure from a normal distribution. As a goodness-of-fit test it checks how well the distribution fits the statistical model. The Jarque-Bera test is carried out as a hypothesis test where the hypotheses are defined as:

H0: The distribution is a normal distribution
HA: The distribution is not a normal distribution

As the sample size increases Jarque-Bera converges to a chi-square distribution with two degrees of freedom. Thus the test can be compared to the table of chi-square distribution, e.g. at a 5% significance level the null hypotheses is rejected if the Jarque-Bera test is bigger than $X^2_{0.05,2} = 5.991$. Compared to the Jarque-Bera values in Table 4-1 the null hypothesis is rejected for every product as it is clear that their distribution is not normal. The only exception is LSFO which keeps the null hypothesis with a distribution close to normal.

4.2 Distribution

The strong law of large numbers together with the central limit theorem suggests that over a large period the distribution of price change for any product and any market should converge to a normal distribution. However, many studies have concluded otherwise as the empirical returns show higher peaks and fat tails, especially over short horizons.

Glasserman, Heidelberger and Shahabuddin (2002) introduces many of these studies, ranging from the early studies by Mandelbrot (1963) and Praetz (1972) to more recent work by Huisman et. al. (1998) and Embrechts, McNeil and Straumann (2001). They all conclude that liquid markets have high kurtosis and fat tails.

The products' distributions are presented together with a normal distribution for easy comparison; see Figure 4-3 to Figure 4-11. Every product shows proof of high peaks and heavy tails and thus confirms the studies mentioned earlier and the conclusion of the kurtosis and Jarque Bera test. LSFO being the only exception as mentioned in the Jarque Bera test.

The propane graph differs from the other products due to many days of no change in price. The graph therefore has an especially high peak.

As proven by the data analysis of the product's distribution and key characteristics the price change of the products are not normally distributed. As an alternative the student t distribution is often mentioned, as it more closely resembles a symmetric leptokurcic distribution.

Furthermore, the student t distribution can have more or less kurtosis by adjusting the degree of freedom. Glasserman, Heidelberger and Shahabuddin (2002) conclude that by setting the degree of freedom between 3 -7 most of the real world market factors will be more correctly modelled. In addition they recommend using the t-Copula if it is necessary to use different degrees of freedom for different risk factors in the portfolio.

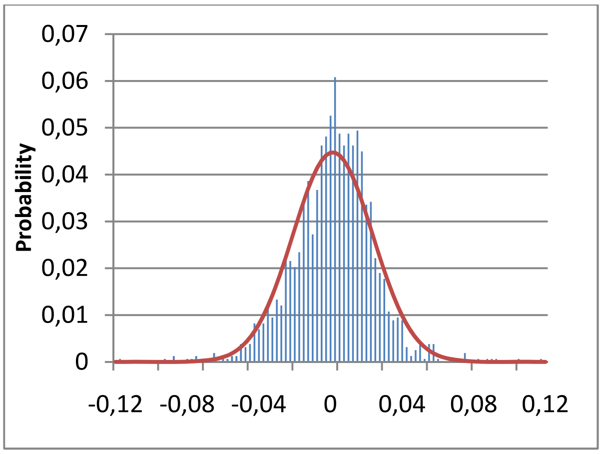

Figure 4-3 Histogram of price changes Brent

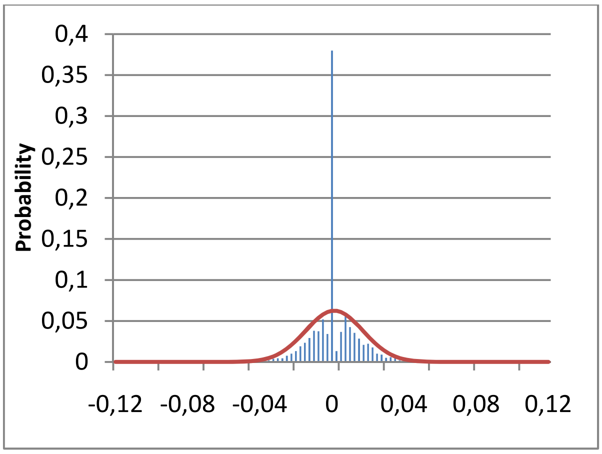

Figure 4-4 Histogram of price changes Propane

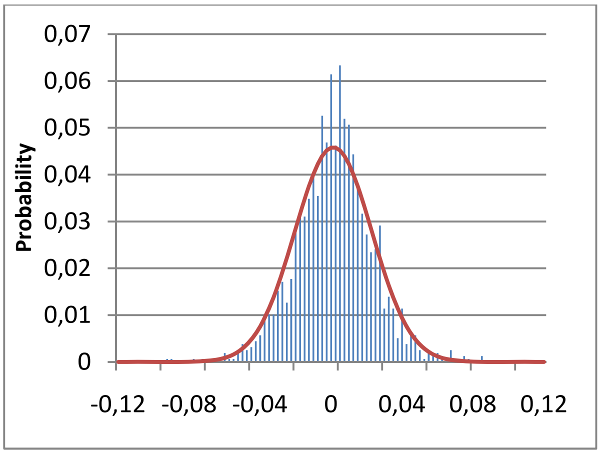

Figure 4-5 Histogram of price changes Gasoil 0.2%

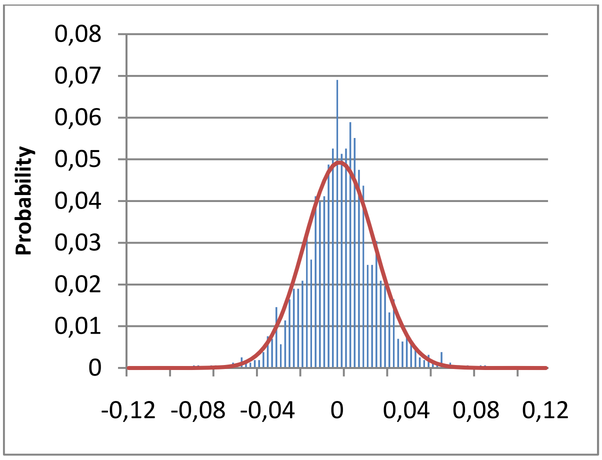

Figure 4-6 Histogram of price changes Jet/Kerosene

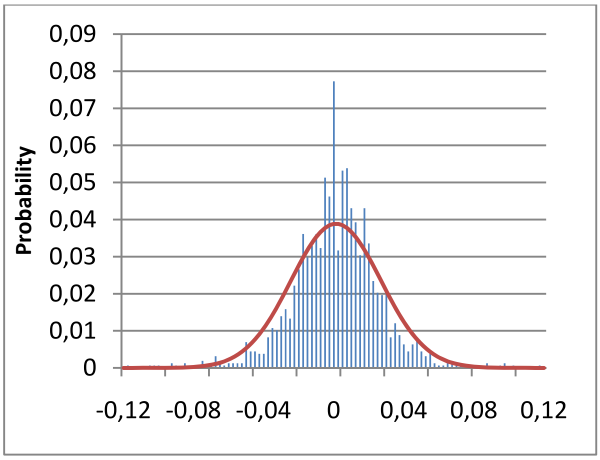

Figure 4-7 Histogram of price changes HSFO 3.5%

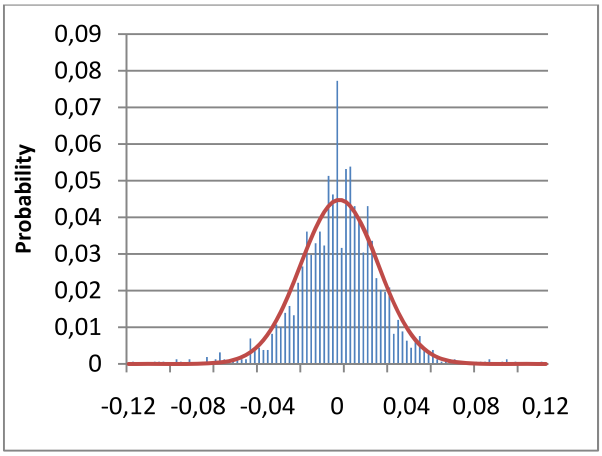

Figure 4-8 Histogram of price changes No.6 1% LSFO

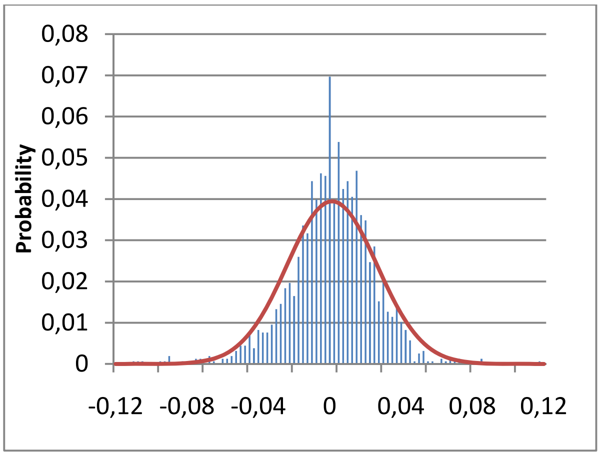

Figure 4-9 Histogram of price changes Naphta

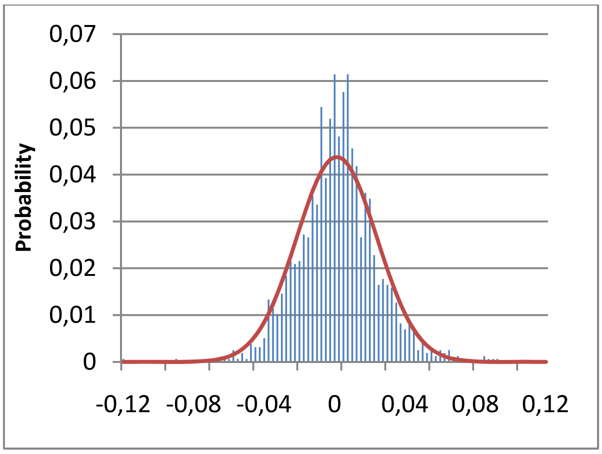

Figure 4-10 Histogram of price changes ULSD 10ppm

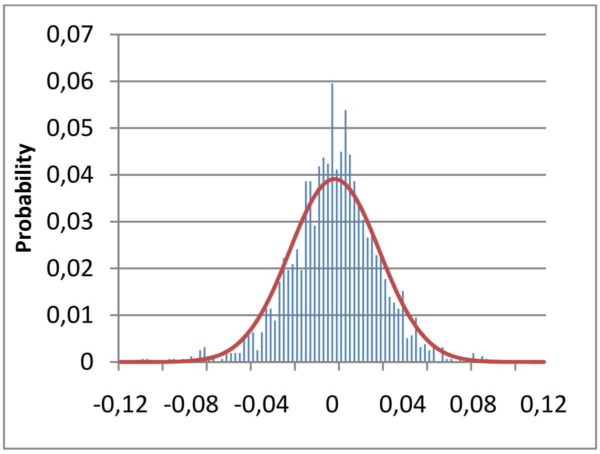

Figure 4-11 Histogram of price changes Unleaded

By utilizing the formula for kurtosis in section 3.2 Kurtosis it is possible to find the degree of freedom for each product. Table 4-2 presents the degree of freedom for each product based on the complete historical period.

	Brent	*Propane*	*Gasoil 0.2%*	*Jet/Kerosene*	*HSFO 3.5%*	*No.6 1%/LSFO*	*Naphtha*	*ULSD 10ppm*	*Unleaded*
Degree of freedom	7	4	6	10	5	6	5	5	6

Table 4-2 Degree of freedom calculated for each product based on the complete data period.

By assuming that the degree of freedom for the multivariate student t distributed portfolio is dependant on the underlying risk factors this suggests that the degree of freedom can be set to 6. However as is evident from Figure 4-12, by generating the degree of freedom based on the last 250 days of data the product's degree of freedom fluctuates more. Some periods the degree of freedom is even high enough to assume a normal distribution. However as the data is clearly student t distributed the limit is set to 15.

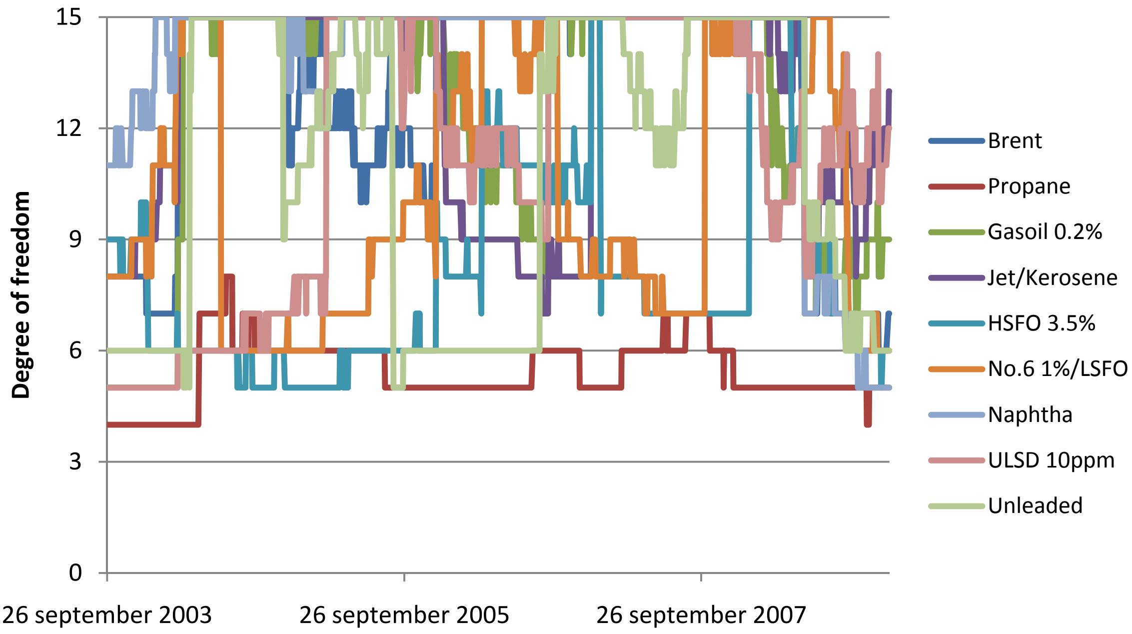

Figure 4-12 Degree of freedom moving estimate (250 days data period).

A similar analysis of the portfolio where the portfolio's degree of freedom is the average of the underlying risk factor's degree of freedom is shown in Figure 4-13. The low degree of freedom at the beginning and the end of this period indicates that the data in these two periods are more volatile than in the period between.

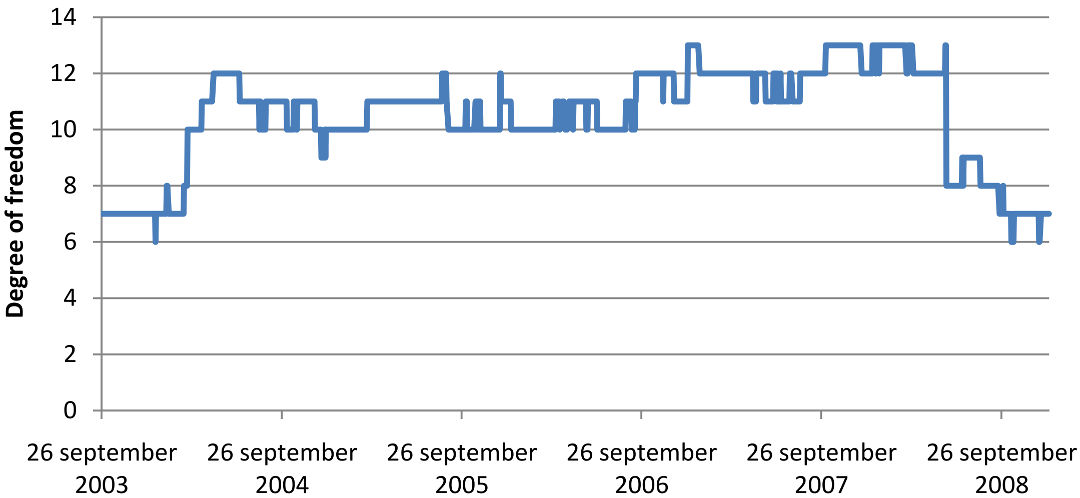

Figure 4-13 Degree of freedom for portfolio as the average of the underlying risk factor's degree of freedom (250 days data period)

4.3 Correlation

A diversified portfolio reduces the risk for the owner because of the correlation between its assets. This is less true if the products are all highly and positively correlated, since the assets then develops in the same way. Two products with correlation 1 are perfectly correlated and their value will move synchronous. If the opposite is true and the correlation is -1 the two products will move in opposite direction and a portfolio consisting of equal positions in these two assets would have zero risk. If the correlation is 0 the products are considered uncorrelated and there are no dependency between the products.

The correlation matrix between the portfolio's products is presented in Table 4-3 and is calculated as defined in section 3.1. The correlations are calculated based on the whole data period, and every product combination has a strong positive correlation.

	Brent	*Propane*	*Gasoil 0.2%*	*Jet/Kerosene*	*HSFO 3.5%*	*No.6 1%/LSFO*	*Naphtha*	*ULSD 10ppm*	*Unleaded*
Brent	1	0.9496	0.9939	0.9928	0.9759	0.9764	0.9832	0.9901	0.9805
Propane	0.9496	1	0.9422	0.9381	0.9349	0.9355	0.9610	0.9365	0.9193
Gasoil 0.2%	0.9939	0.9422	1	0.9987	0.9649	0.9719	0.9697	0.9986	0.9711
Jet/Kerosene	0.9928	0.9381	0.9987	1	0.9642	0.9707	0.9682	0.9977	0.9709
HSFO 3.5%	0.9759	0.9349	0.9649	0.9642	1	0.9913	0.9610	0.9585	0.9539
No.6 1%/LSFO	0.9764	0.9355	0.9719	0.9707	0.9913	1	0.9511	0.9667	0.9462
Naphtha	0.9832	0.9610	0.9697	0.9682	0.9610	0.9511	1	0.9639	0.9816
ULSD 10ppm	0.9901	0.9365	0.9986	0.9977	0.9585	0.9667	0.9639	1	0.9664
Unleaded	0.9805	0.9193	0.9711	0.9709	0.9539	0.9462	0.9816	0.9664	1

Table 4-3 Correlation matrix between portfolio's products for the entire data period.

This strong correlation can be further confirmed by considering the pattern of the price development in Figure 4-1. However this figure does not take into consideration the different price levels. Figure 4-14 displays the relative change from the beginning of the period by dividing every price point with the price at the first date of the data period. Most of the products move in parallel as only LSFO and HSFO deviate some from the majority. At the end of the graph all plots are drawn together, and thus confirming the findings in the correlation matrix

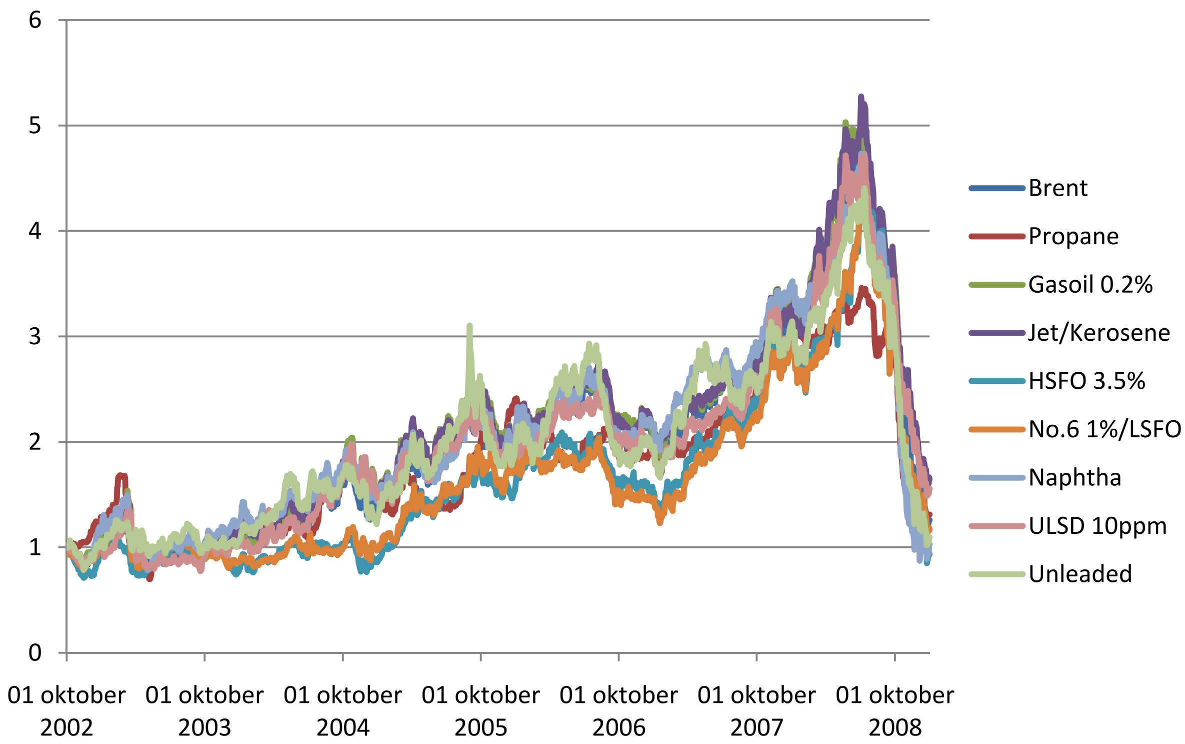

Figure 4-14 Relative price development for products in data period.

However when the data period is changed the correlation is not as apparent as Table 4-3 presumes. The correlation matrix of a randomly selected date period is presented in Table 4-4. The data period consists of 124 trading days from the first 6 months of 2006.

	Brent	*Propane*	*Gasoil 0.2%*	*Jet/Kerosene*	*HSFO 3.5%*	*No.6 1%/LSFO*	*Naphtha*	*ULSD 10ppm*	*Unleaded*
Brent	1	0.3878	0.8820	0.8647	0.7452	0.6517	0.8599	0.8481	0.8199
Propane	0.3878	1	0.4226	0.4020	0.3031	0.3458	0.3481	0.3992	0.3495
Gasoil 0.2%	0.8820	0.4226	1	0.9567	0.7542	0.6622	0.8078	0.9483	0.8288
Jet/Kerosene	0.8647	0.4020	0.9567	1	0.7831	0.6690	0.8111	0.9312	0.8213
HSFO 3.5%	0.7452	0.3031	0.7542	0.7831	1	0.7661	0.6987	0.7143	0.7028
No.6 1%/LSFO	0.6517	0.3458	0.6622	0.6690	0.7661	1	0.6265	0.6166	0.5703
Naphtha	0.8599	0.3481	0.8078	0.8111	0.6987	0.6265	1	0.7778	0.7697
ULSD 10ppm	0.8481	0.3992	0.9483	0.9312	0.7143	0.6166	0.7778	1	0.8141
Unleaded	0.8199	0.3495	0.8288	0.8213	0.7028	0.5703	0.7697	0.8141	1

Table 4-4 Correlation matrix between portfolio's products for data between 01.01.2006 and 01.07.2006

Although the correlations are all positive the coefficients are considerably lower in many cases. The correlation matrix must therefore be updated for each new estimate based on the historical period used for the estimation.

5. Estimation tool

This chapter will present the estimation tool developed to compare different methods for estimating Value at Risk. The chapter will first discuss why Microsoft Excel 2003 was chosen as the programming platform. Next the basic classes and structure of the code will be introduced, before the graphical user interface (GUI) is presented.

The full code is presented in Appendix A –Source Code.

5.1 Programming platform

The tool used for estimating VaR is developed using Microsoft Excel 2003.This was chosen due to several reasons:

- Availability and familiarity with Excel 2003 at Statoil Hydro ASA
- Visual Basic for Application (VBA)
- Flexible presentation tool

Statoil Hydro ASA uses Excel 2003 for both analysis and calculation of VaR today. This availability and familiarity is important for the tool to be utilized frequently. Knowledge about VBA also ensures that it can be easily improved by the users both by making adjustments or further development.

Excel 2003 use Visual Basic for Application (VBA) as its programming language. While the program includes many essential mathematical functions, VBA ensures complicated calculations to be carried out by user defined functions. In addition VBA enables classes to be utilized for object oriented programming.

VBA also gives the opportunity to create sub routines which can combine user defined functions, built-in function and worksheet data to carry out standard procedure. The results can then be directly presented in an Excel worksheet, which is another key feature of Excel. The subroutines programmed with VBA can output data directly to worksheets in Excel and directly utilize functions available from Excel on the newly output data. This output to worksheets also makes further processing of the results easy.

The alternatives considered were Matlab and Excel 2007. Matlab is stronger for mathematical calculations like linear algebra and has the ability of object oriented programming, but the knowledge and presentation of results (including later processing of the results) are not as readily available compared to Excel. And although the 2007 version of Excel has undergone improvements like the implementation of VBA .NET and the expansion in number of rows and columns, the older version was chosen to ensure better availability for the user.

5.2 Classes and structure

The structure and classes used when programming needs to be efficient in order to make the tool easily editable as well as ensuring simulation speed. This is carried out with object oriented programming (OOP) which utilizes reusable code and classes.

5.2.1 Classes

The product class gives an important foundation to the estimation. To avoid several readings of data and calculations of key characteristics of a distribution the data is saved as a product instance. An instance of the product is instantiated with the call setInitValues which uses a data range as input. The data is then read and the characteristics calculated.

Table 5-1 displays variables as well as the most important functions and parameters for the product class.

When CalculateDelta is called the delta of the product is calculated as defined in section 3.3.1 Delta (δ).

GetRangeOfData returns a range of data as specified by the start and end parameter. This function uses the getDataAt to retrieve data from the Data variable.

To achieve an efficient back-test and comparison of methods, several key features of the VaR-estimates need to be calculated by one subroutine. This is made possible by the VarEstimate class which includes a confidence interval (high and low) as well as the maximum and mean excess loss. Table 5-2 displays VarEstimates variables and functions.

Each class' parameter is set and retrieved through the properties Set, Get and Let. Set and Let are used to write to the parameter while Get is used to retrieve data.

Product
Name As String
Weight As Double
WeightUSD As Double
Avg As Double
standardDev As Double
Max As Double
Min As Double
Median As Double
Quartile1 As Double
Quartile3 As Double
Observations As Integer
Data As Range
PrcChange() As Double
Public Sub setInitValues(Value As Range, first As Boolean, nameInput As String, weightsInput As Double)
Property Get CalculateDelta() As Double
Property Get getRangeOfData(start As Integer, sEnd As Integer) As Variant
Property Get getDataAt(ind As Integer) As Double

Table 5-1The Product class and its variables and functions

VarEstimate
VaR As Double
ConfUpper As Double
ConfLower As Double
MeanExcessLoss As Double
MaxLoss As Double

Table 5-2 The VarEstimate class and its variables and functions

5.2.2 Model

The model for the VaR estimation tool is divided into 3 parts:

- Input
- Processing
- Output

As seen in Figure 5-1 input consists of the historical data and parameters as defined by the user. The parameters include significance level for the VaR estimate (alpha) and historical period in days which sets the amount of data history to use in the estimation. In addition there are several optional parameters used with different methods. Runs must be defined when estimating VaR with a Monte Carlo simulation. Degree of freedom is a parameter only applicable to methods utilizing the student t distribution. The back-test period defines the number of days used in a back-test.

The default values for the parameters are:

- Alpha = 0.01
- Historical period = 250
- Runs = 100
- Degree of freedom = 3
- Back-test period = 0, which equals no back-test and estimates next day's VaR.

The last input defines what methods to use for the estimation. It is possible to choose several methods for comparison.

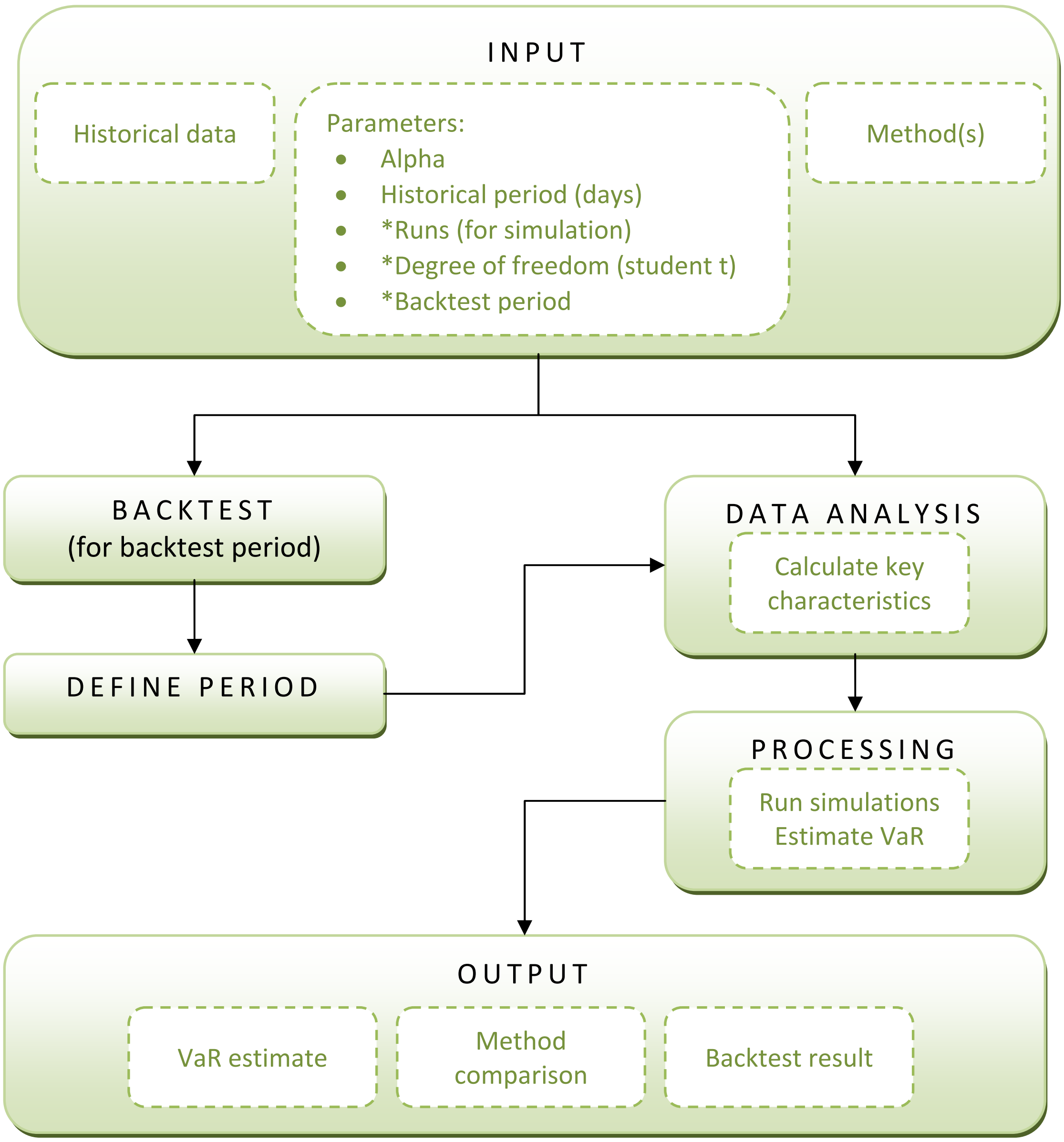

Figure 5-1 Model data flow with inputs, processing and output.

The processing part is divided into two parts dependant on the use of back-test. A back-test lets the user define a number of historical days to compare estimated VaR with actual change in portfolio. If used the period needs to be redefined for each day in the back-test before the data is analysed and processed in order to estimate VaR.

A back-test therefore utilizes the second component of the processing part. It consists of the data analysis which calculates the key characteristics of the products and portfolio before the processing which estimates the VaR.

Output presents the VaR estimates as well as a method comparison if several methods are chosen. In addition the back-test results are displayed if this option is selected.

5.3 User defined functions

User defined functions allow processing of worksheet data as well as internal data in form of arrays and classes and return values depending on the processing. In the estimation tool created for this thesis the functions fall into two types of categories: linear algebra and VaR methods.

In addition to mathematical functions and the VaR methods a function for data cleaning is programmed. This function checks if any of the risk factors misses data at a certain date. If this is the case then that date is excluded (and deleted) from the historical data. The data is therefore read as a forward feed.

Key functions are presented in Table 5-3 and Table 5-4 and the code can be viewed in the appendix.

5.3.1 Linear algebra

Matrix functionality had to be programmed in VBA in order to certify that the dimensions where correct. The functions include transpose, multiplication and inverse as well as addition and subtraction of both matrices and constants. In addition the Cholesky decomposition, eigenvectors and eigenvalues had to be programmed.

The inputs and outputs of the functions are in most cases Variants or Objects. A Variant in VBA is an open data type which can hold any data type. This is necessary as the dimensions of the matrices are unknown both for input and output matrices.

The inputs and outputs are listed in Table 5-3.

Function name	Inputs	Output	Comment
GaussianElimination	index As Integer A As Object b As Object	Object	Solves Ax = b
MultiplyScalar	A As Object c As Double	Object	Vector A multiplied with scalar c
MatrixSqrt	A As Object	Object	Square root of vector A
MatrixPower	A As Object p As Integer	Object	A^p, where A is vector
Transpose	A As Object	Object	Transpose of vector A
AddOrSubtract	index As Boolean A As Object b As Object	Object	Vector A+B or A-B according to index
MultiplyTwoMatrices	index As Boolean A As Object b As Object	Object	Vector A*B or B*a according to index
Cholesky	Mat As Object	Object	CC’ = Mat, returns C
MatEigenvalue_Jacobi	A As Variant	Variant	Eigenvalue of vector A
MatEigenvector_Jacobi	A As Variant	Variant	Eigenvector of vector A

Table 5-3 Matrix algebra in VBA with input and output.

5.3.2 VaR methods

The VaR methods are all functions which return a VarEstimate. This ensures that calculation of other characteristics than VaR is carried out in parallel. The inputs of each method are listed in Table 5-4 and are further described in the following text.

The historical simulation follows the technique described in section 3.3.1 Historical simulation. The inputs for HistoricalSim are the significance level (alpha) and a collection of products with their price data and other characteristics as described in Table 5-1. These two inputs are common for every VaR method.

The AnalyticalVaR function estimates VaR analytically as described in section 3.3.2 Analytical estimation. In addition to significance level and the portfolio's price data it takes a correlation matrix (CorrelM) as an input. This correlation matrix is also input for Monte Carlo and DeltaGamma.

The MonteCarlo function is based on the technique discussed in section 3.3.3.2 Brute Force MC. It includes several alternatives in order to differentiate between a normal assumption (bNormal = true), a student t assumption (bNormal = false) and a student t with automatically calculated degrees of freedom (bDfAuto = true). The degree of freedom is then calculated as described in section 3.2 Kurtosis.

Function name	Inputs	Output	Section
HistoricalSim	alpha as Double ProductsTmp As Variant	VarEstimate	3.3.1 Historical simulation
AnalyticalVaR	alpha as Double CorrelM As Variant ProductsTmp As Variant cLevel As Double*	VarEstimate	3.3.2 Analytical estimation
MonteCarlo	alpha As Double runs As Integer CorrelM As Variant ProductsTmp As Variant df As Integer bNormal As Boolean bDfAuto As Boolean	VarEstimate	3.3.3.2 Brute Force MC
DeltaGamma	alpha As Double runs As Integer CorrelM As Variant ProductsTmp As Variant	VarEstimate	3.3.3.4 Variance reduction techniques
MonteCarloDGNormal	VaRdelta As Variant VaRgamma As Variant VaRcovmatrix As Variant smethod As String* alpha As Double days As Integer runst As Integer ProductsTmp As Variant	VarEstimate	3.3.3.4 Variance reduction techniques

Table 5-4 VaR methods in VBA with overview of inputs and outputs.

The DeltaGamma function prepares the MonteCarloDGNormal as it computes delta and gamma of the input ProductsTmp. The delta and gamma matrices are then input into MonteCarloDGNormal, in addition to the covariance matrix, number of runs (runs) and time horizon (days). This function is based on section 3.3.3.4 Variance reduction techniques and the XploRe tool developed by Härdle was used as a reference program.

5.4 GUI

This section will examine the graphical user interface (GUI) of the estimation tool and give a presentation of how the results are displayed.

5.4.1 User form

The GUI is created with Excel's user form and necessary interaction with worksheet is achieved with built-in functions.

The data used needs to be arranged as data in Table 5-5. This is due to the organization of the data reading in the data analysis.

	A	B	C		#
1	Name	Risk Factor #1	Risk Factor #1	…	Risk Factor #m
2	Position	2000	-450		1000
3	Day #1	123.20	140.12	…	71.23
4	Day #2	123.60	140.90	…	74.24
	⋮	⋮	⋮	⋮	⋮
n-2	Day #n	170.98	158.90	…	102.90

Table 5-5 Data arrangement for input of historical data and position in risk factors.

The user form is initialised by pressing CTRL + SHIFT + f. The user is then presented with the form as in Figure 5-2.

The first form asks the user to select risk factors. By clicking select the btnSelectAreas_Click sub routine is called and Excel's InputBox prompts the user to select a range. The InputBox allows the user to interact with the workbook. The selection only needs the area of the risk factor's names (B1:#1), as the rest of the data (position and price history) is selected by the sub routine.

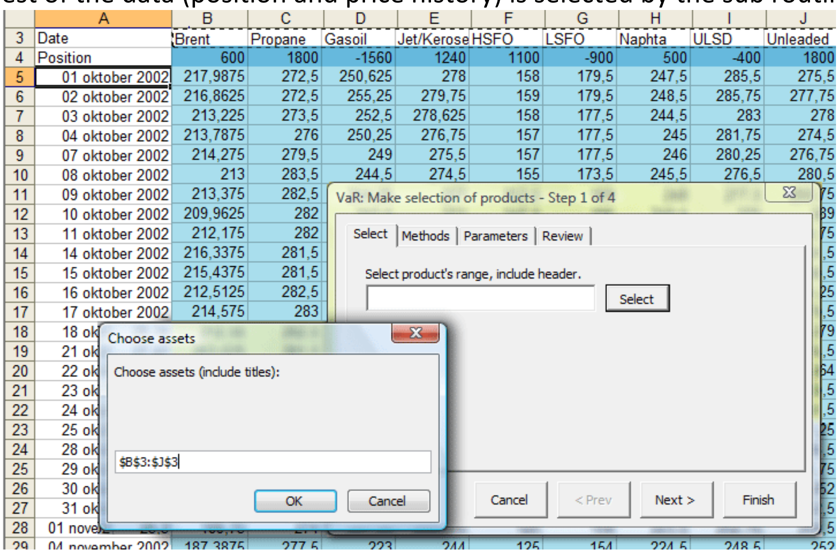

Figure 5-2 Estimation tool GUI: Make selection of products - Step 1 of 4

When the selection is done the process first erases dates with empty data and automatically starts to calculate basic key characteristics on the data input. This is a part of the setInitValue of the Product class and includes the calculation of an array with the daily change in percent. The risk factors are then gathered in a collection of Products before the correlation matrix is calculated.

With the portfolio selected and saved the next step lets the user decide what methods to use for the estimation of VaR. As seen in Figure 5-3 it is possible to select multiple methods for comparison of the outputs.

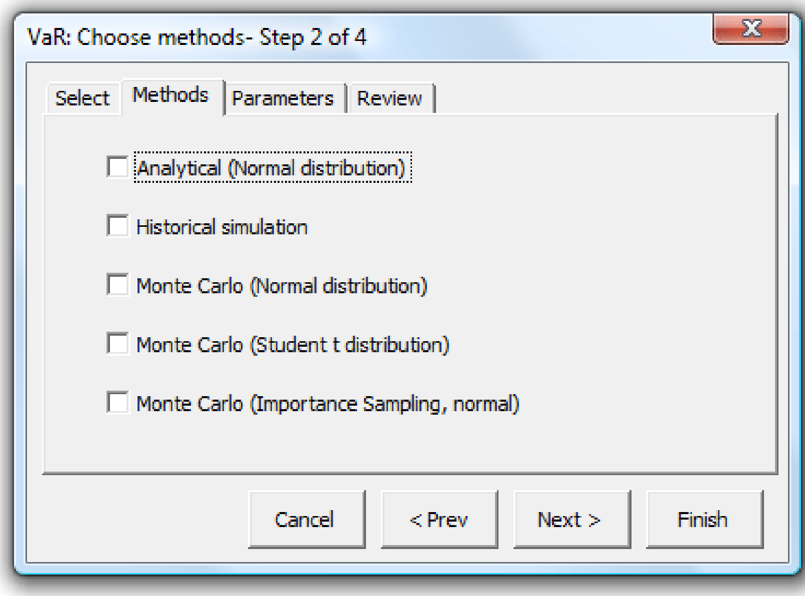

Figure 5-3 Estimation tool GUI: Choose methods - Step 2 of 4

The third step includes setting the different parameters necessary to perform the estimation, see Figure 5-4. Some of the parameters are only used in combination with some of the methods as explained in section 5.2.2 Model. The user form has the default value visible in the form if it has not been edited.

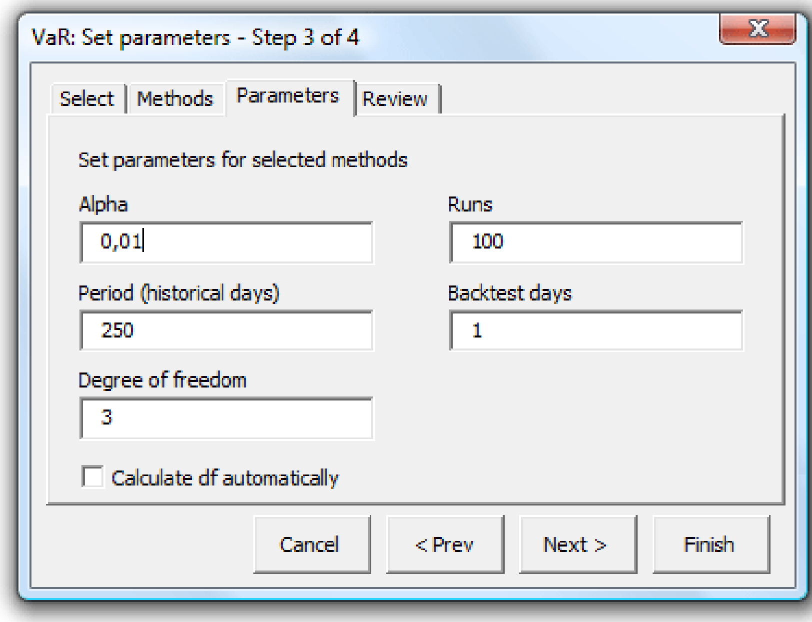

Figure 5-4 Estimation tool GUI: Set parameters - Step 3 of 4

The last step displays a review of the methods and parameters chosen see Figure 5-5.

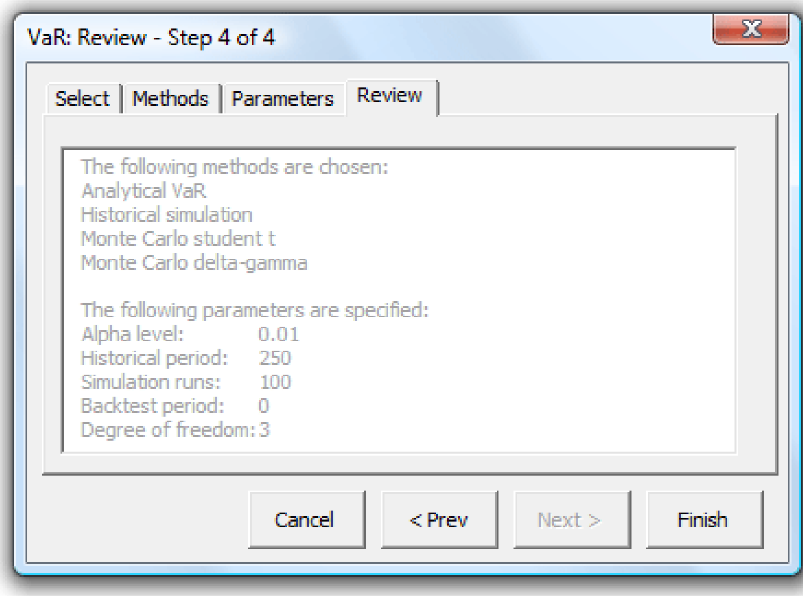

Figure 5-5 Estimation tool GUI: Review - Step 4 of 4

5.4.2 Presentation of results

The results are divided into two sections. The first gives an overview of all methods used and their estimates for VaR in a worksheet called “Backtest” or “VaR” depending on the method used. The back-test will include estimates of every day included in the back-test period, while the VaR worksheet will include an estimate for mean and max excess loss in addition to the VaR-estimate.

The second section displays the estimation of each method. For a historical simulation the scenarios are printed out in a worksheet called “Historical simulation” where the scenarios are sorted by change in portfolio. This therefore has the opportunity to create a histogram of the scenarios to examine the distribution.

For a Monte Carlo estimation each run in the simulation is printed. For a regular Monte Carlo simulation the data is sorted by change in portfolio, while for a Monte Carlo delta gamma the data is sorted by the likelihood ratio. The output is printed in worksheets “MC student t” or “MC normal” and “MC delta-gamma” respectively. Both therefore have the opportunity to create a histogram to get a better understanding of the distribution of the portfolio change.

The analytical does not have any individual output as it only provides an estimate for VaR.

Note that if the test is a backtest only estimate for the last date is displayed in the second section of outputs.

In addition to these simulations related outputs the key characteristics of each risk factor as well as the correlation between them are printed in the worksheet “Summary”.

6. Empirical results

This chapter presents the results of an extensive back-test where multiple combinations of the presented methods and alternatives are tested. The back-test setup is presented first in this chapter along with the different portfolios tested. The back-test included several key characteristics according to the comparison methods mentioned in section 3.4 Comparison methods.

As proved by the testing the methods performed similarly on every portfolio. Therefore every result for every portfolio test is not included in this chapter, but can be viewed in the appendix. Furthermore due to the amount of combinations section 6.2 Considerations about the alternatives reduces the number of methods compared in this chapter.

In addition to the results comparing the methods the chapter includes a discussion about the different methods' distribution. This tries to explain why the methods provide different estimates for VaR.

The chapter also includes a discussion about time concerns.

6.1 Back-test setup

Figure 6-1 displays the back-test setup used for the comprehensive comparison of methods and different alternatives. Both Monte Carlo methods have 3 alternatives tested as the number of runs for each estimate varies between 100, 500 and 1000 runs. In addition the student t method varies in 3 and 7 degree of freedom, in addition to an automatically calculated degree of freedom which calculates the degree of freedom as an average of the underlying risk factor's degree of freedom. This method calculates the degree of freedom as explained in 3.2 Kurtosis and uses this as suggested in 4.2 Distribution. In total this gives 14 methods and variations.

Two levels of alpha and 3 levels of historical periods are tested. This gives a total of 84 (14x3x2) estimates for each portfolio tested. The test for one portfolio was estimated to take approximately 25.5 hours.

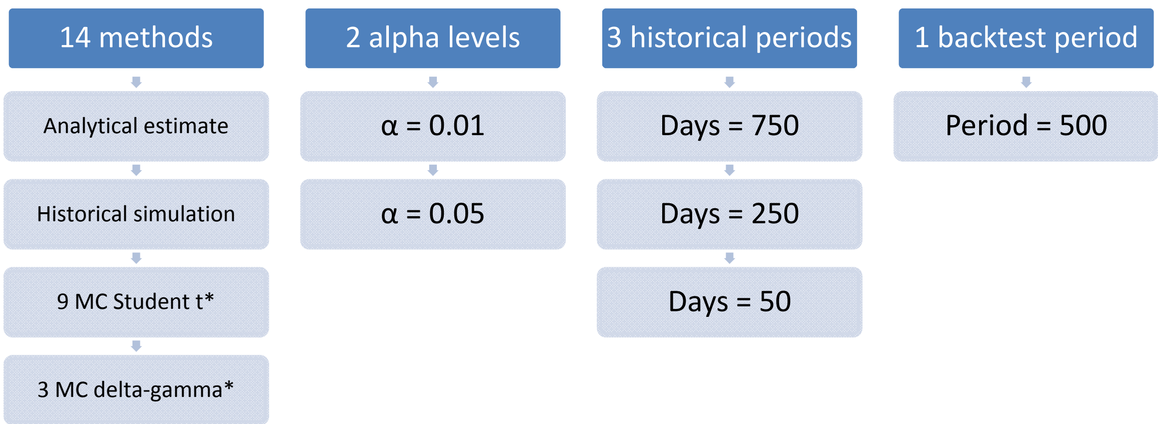

Figure 6-1 Backtest setup consisting of 14 methods, 2 alpha levels and 3 historical periods for back-test period.

The back-test was setup to run for one portfolio at a time and automatically output the different estimates for each combination of historical period and alpha level. Thus 6 worksheets of output were automatically generated for each portfolio estimate consisting of 14 estimates pr worksheet.

The methods were tested on 5 portfolios which are presented in Table 6-1. The portfolios were all combinations of the 9 products selected for this thesis. Except for portfolio A the value of the positions was close to equal.

	Brent	Propane	Gasoil 0.2%	Jet/ Kerosene	HSFO 3.5%	No.6 1% LSFO	Naphtha	ULSD 10ppm	Unleaded	Value
A	600	1 800	-1 560	1 240	1 100	-900	500	-400	1 800	2 032 407,50
B	580	450	460	400	1 000	930	450	450	450	2 036 798,25
C	-420	420	420	-300	-750	-700	420	420	-340	-111,75
D	-580	-450	-460	-400	-1 000	-930	-450	-450	-450	-2 036 798,25
E	-	2 000	2 000	1 740	-4 200	-	-	-	-	2 033 745,00

Table 6-1 Portfolios used in back-test: Their combination of positions and value at 10.01.2007.

Portfolio A is a random portfolio where the amount invested in each product is not of the same size. In portfolio B every position is long and since the underlying risk factors are correlated this should create higher VaR than portfolio A.

Portfolio C is a balanced portfolio where the sum at the beginning of the back-test period is close to 0. This portfolio should therefore have a very low VaR as the correlated underlying products should balance each other out.

For portfolio D every position is short. The positions are of the same size as portfolio B and the value of this portfolio at the first date of the back-test is therefore same as for portfolio B. The two should consequently have approximately the same VaR.

Portfolio E only consists of 4 out of the 9 products (randomly selected), but has the same amount invested as portfolio A, B and D at the beginning of the period. Because of the reduced number of underlying risk factors this should provide higher VaR due to a less diversified portfolio.

6.2 Considerations about the alternatives

Because of the high number of combinations of methods and alternatives we will first do some early considerations about the methods in order to reduce the options being compared. This will include a discussion about convergence for the Monte Carlo methods and a short comparison of the different student t methods. In addition some thoughts about the historical period are presented.

Portfolio A is the basis of the discussion in this section. For brevity the considerations made in this section will exclude the majority of the combinations from being presented in the later sections of this chapter. For a full display of every method compared, please see the appendix.

6.2.1 Monte Carlo convergence

One of the major concerns considering Monte Carlo simulation is the great number of runs needed to achieve a precise estimate because of the high variance. As the delta-gamma approximation method is a variance reduction technique leading to an importance sampling it should therefore converge earlier towards the VaR-estimate.

As displayed in Figure 6-2 this is also true when comparing the MC delta-gamma to the MC student t methods. Already at 100 runs the delta-gamma method has less variance and is more accurate than a student t method has at 1000 runs. So the delta-gamma method is at least 10 times as accurate.

This becomes even more evident as we consider the degree of freedom of the student t method. A lower degree of freedom (e.g. 3) for the student t method will increase the variance and thus require more runs to be efficient.

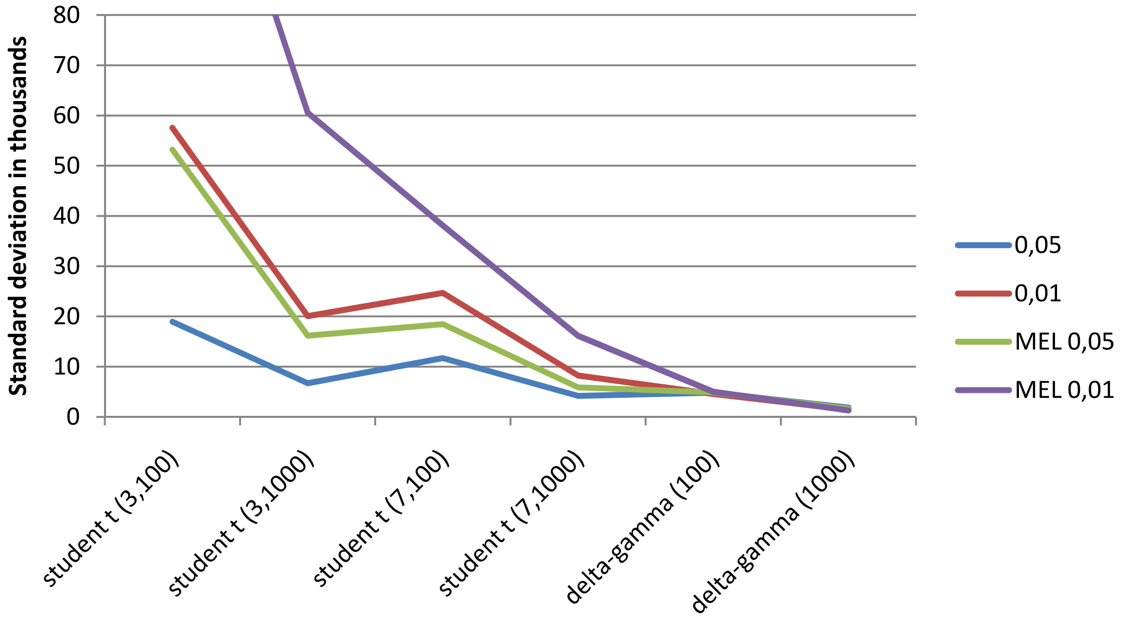

Figure 6-2 Variance test for Monte Carlo methods comparing variance of 95% and 99% VaR-estimate, as well as the 95% and 99% mean excess loss-estimate (MEL).

The differences in variance become bigger when considering the longer end of the tails. Thus the relative accuracy of delta-gamma at 99% VaR is even more advantageous than at 95% VaR.

This is further confirmed when considering the mean excess loss (average estimated loss bigger than VaR-estimate) where the variance of the student t is even more volatile than for a regular VaR-estimate, see section 6.6 Tail size.

Due to this the student t methods will only be tested with the estimates approximated with 1000 runs.

6.2.2 Student t comparison

3 levels of degree of freedom are included in the back-test for the Monte Carlo student t method. The recommendations of Glasserman, Heidelberger and Shahabuddin (2002) are followed by testing with 3 and 7 degrees of freedom (df). Furthermore an automatically student t method as presented in section 4.2 Distribution is included as the third alternative.

The convergence issues seen in the previous section are evident also in Figure 6-3. Because of the high variance in the VaR-estimate of student t with 3 df it has a bigger spread in its estimates and is more volatile.

Student t with 7 df and the automatically calculated student t method estimates VaR equally. This is not surprisingly as the calculated student t often is between 6 and 10 (see Figure 4-13).

Student t with 3 degrees of freedom estimates VaR considerably bigger than the two other methods, and therefore has a higher coverage, see Figure 6-4 and Figure 6-5. This is particularly evident in the 99% VaR-test.

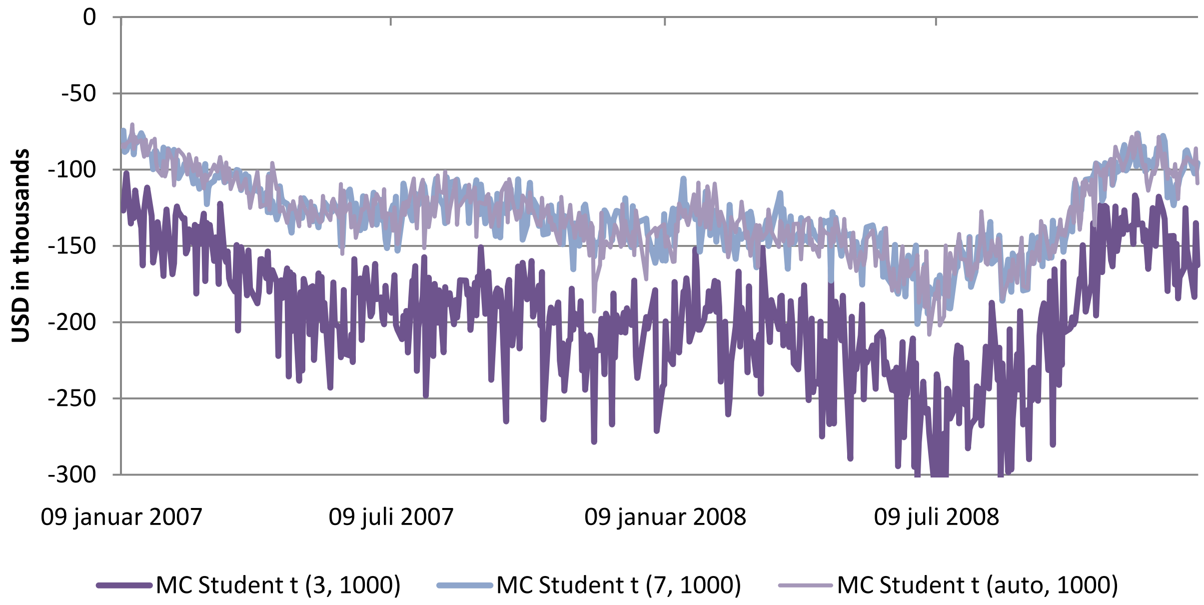

Figure 6-3 Student t comparison of 99% VaR-estimates (500 days back-test).

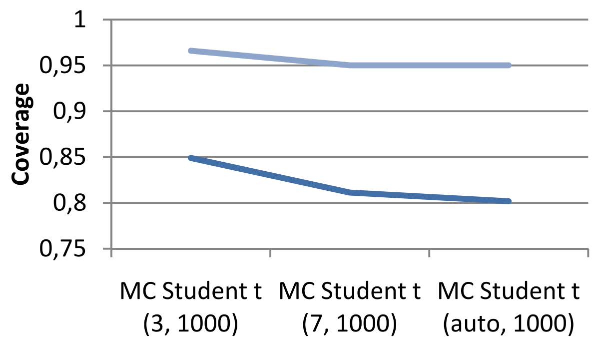

Figure 6-4 Student t comparison of coverage of 95% VaR. (500 days: Light blue line, 106 days dark blue line)

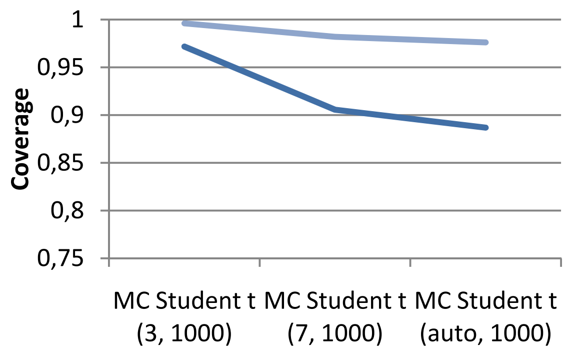

Figure 6-5 Student t comparison of coverage of 99% VaR. (500 days: Light blue line, 106 days dark blue line)

6.2.3 Historical period

The historical period decides how many dates should be included in the calculation of the correlation matrix as well as how many scenarios are included in the historical simulation. Since the methods implemented in this method weight every historical data equally (as opposed to EWMA as described in section 3.4.2 Analytical estimation), the length of the historical period can decide important key characteristics for the underlying risk factors.

Generally the more data points in a calculation the less volatile the output will be. This is true also for the estimates carried out in this thesis, and although it is apparent for all methods tested it is most obvious in the delta-gamma method, see Figure 6-6. When considering only the latest 50 days the estimation of VaR is more sensible to short term changes. These short term changes do not affect the longer term by the same factor and both the 250 days and 750 days estimate therefore are smoother.

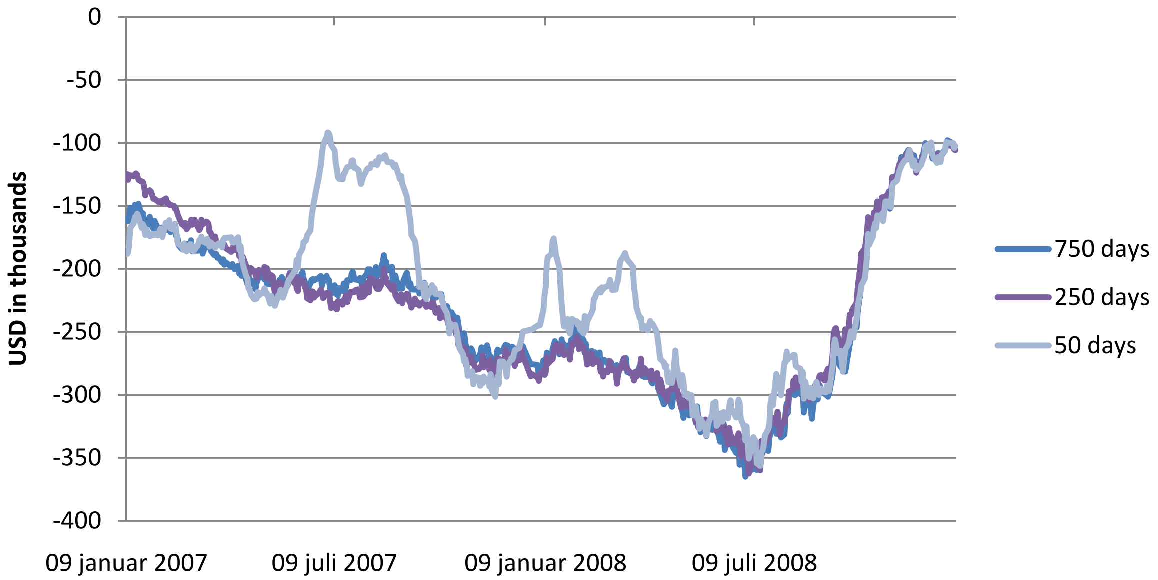

Figure 6-6 MC delta-gamma comparison of historical period, set to either 50, 250 or 750 days.

Because of the little difference in 750 and 250 days the latter will be presented in the coming analysis. The 50 days alternative will be presented where relevant.

6.3 Accuracy

A perfect method for VaR estimation would cover exactly the wanted percentile over time. Over a period of 500 days the perfect method would fail to predict the actual loss in only 25 (5) days and thus cover 95% (99%).

In addition to the 500 days the comparisons is done by only looking at the latest 106 days (from 01.06.2008 to 31.12.2008). This period was characterised by high volatility (as seen in Figure 4-2), and it may therefore hold as a stress-test were the changes in the market were rough.

As seen in Figure 6-7 the MC student t with 3 degrees of freedom performs best when considering the 95% VaR. Although slightly over the 95% mark over the full 500 days period it has the best performance on the stress-test over the last 106 days of 2008 with when it averages at 0.949 and has minimum at 0.934 and maximum at 0.981 with a historical period of 50 days. What is more important is that it performs equally well on every portfolio with VaR coverage around 0.95.The other variations of student t performance similarly but has bigger differences between maximum and minimum.

The delta-gamma method is the only method which averages at around 0.95 for each historical period for the stress-test. On the basis of the numbers for each portfolio it is clear however that it overestimates VaR for portfolios A, B and E (average coverage at 0.99) and underestimate VaR for portfolio C (average coverage 0.90), see appendix.

The historical simulation does not reach the 0.95 mark for the full 500 period tests but performs well on every historical period for this timeframe and equally on every portfolio. On the stress-test the historical simulation performs worst when using a longer period of historical data, which is natural since this would neglect the changes in volatility which occurred during this period. Still the 50 days stress test only averages at 0.90.

Just worse than the historical simulation is the analytical approach which uses the assumption of normal distribution for each underlying risk factor. Although it averages reasonably well at the 500 days back-test it falls through at the 106 days stress-test in the same way as the historical simulation.

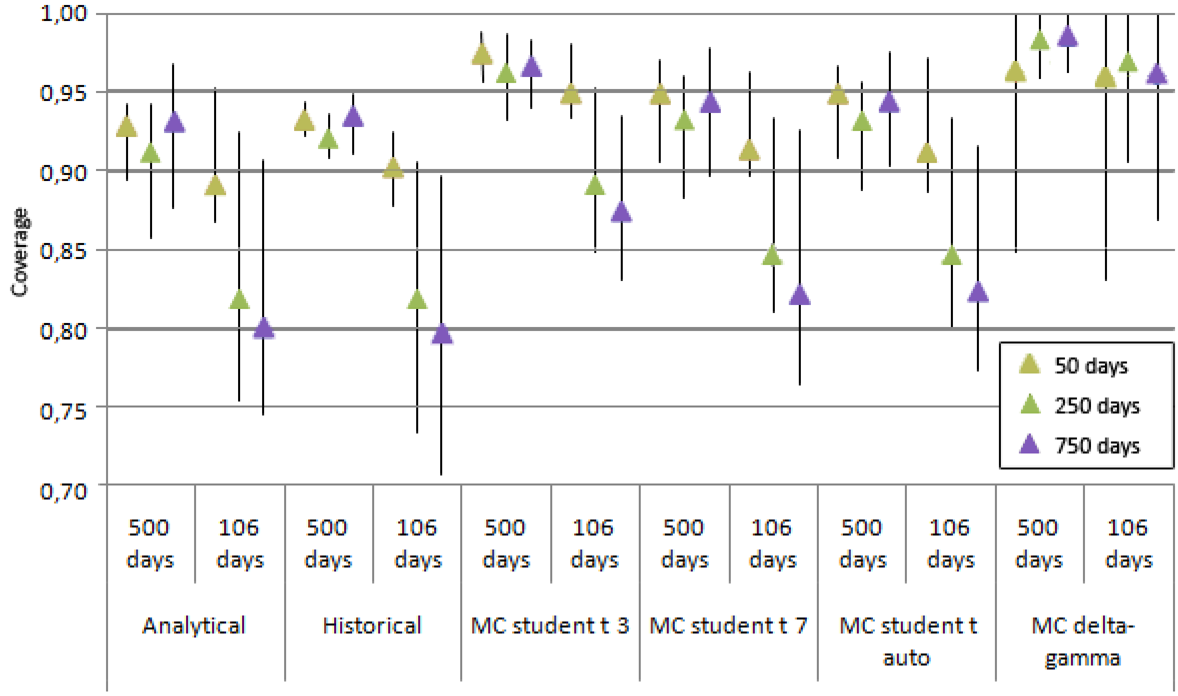

Figure 6-7 Coverage at 95% VaR for both back-test and stress-test period.

The performance at the 99% VaR clearly favours the MC student t with 3 degrees of freedom as it performs just around 0.99 even in the stress-test period. Although it seems to overestimate when only using 50 days as the historical period, with 250 days it performs very well averaging at 0.99 in the stress-test period with minimum at 0.972 (portfolio C). Still when considering the numbers it seems the method overestimates 3 of 5 portfolios (B, D, E) as it has a 100% coverage.

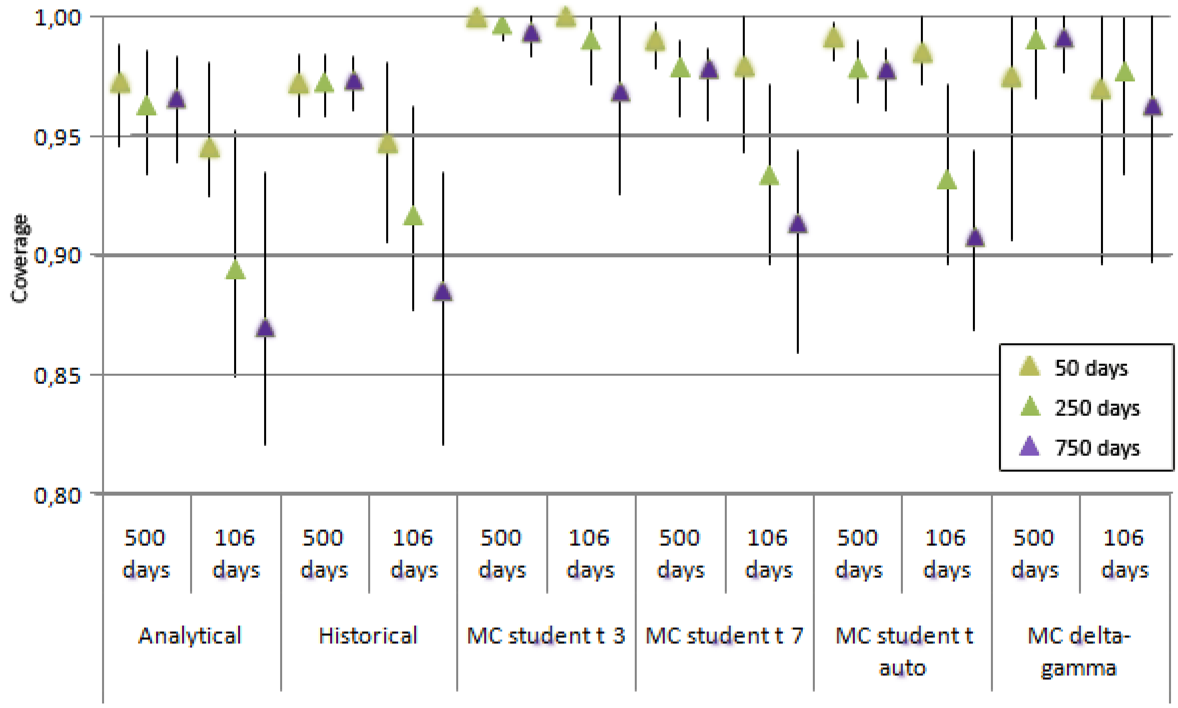

Figure 6-8 Coverage at 99% VaR for both back-test and stress-test period.

This overestimation is however corrected by the other alternatives of MC student t by their higher degree of freedom. With a historical period of 50 days both MC student t 7 and MC student t auto performs well with coverage averaging at 0.99 (minimum 0.982 and maximum 0.998). Both also achieve good results for the stress-test period.

MC delta-gamma also has a high coverage at 99% VaR and performs best with 250 days as the historical period. Similar to the student t with 3 degrees of freedom it overestimates VaR for portfolio B and E, but performs strongly on the 3 other portfolios.

Both the analytical and the historical estimate perform fairly well at the full 500 back-test averaging at 0.97. Nevertheless they fall through for the stress-test period where their best performance (with 50 historical days) average at 0.94.

Figure 6-9 to Figure 6-13 display the back-test and the stress-test for every portfolio at 95% VaR when used with a historical period of 250 days. The appendix also includes figures for every method compared as well as for the historical period of 50 days.

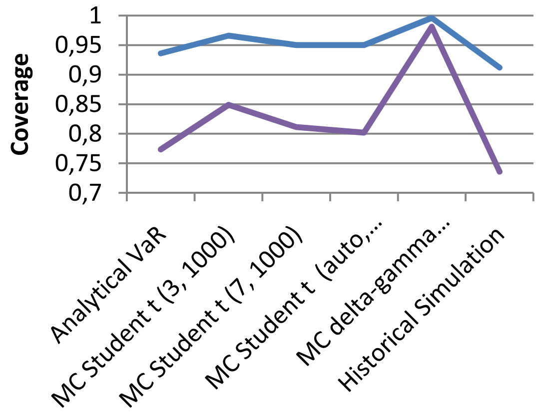

Figure 6-9 Coverage at 95% VaR for portfolio A estimated with 250 historical days.

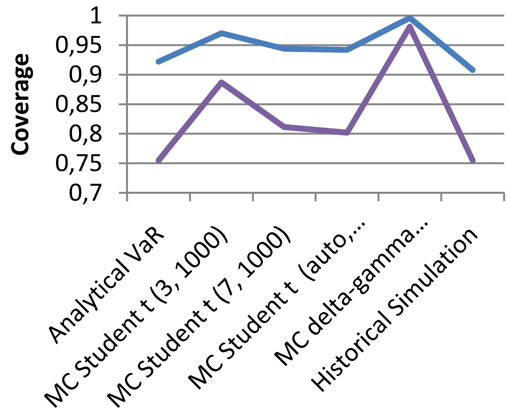

Figure 6-10 Coverage at 95% VaR for portfolio B estimated with 250 historical days.

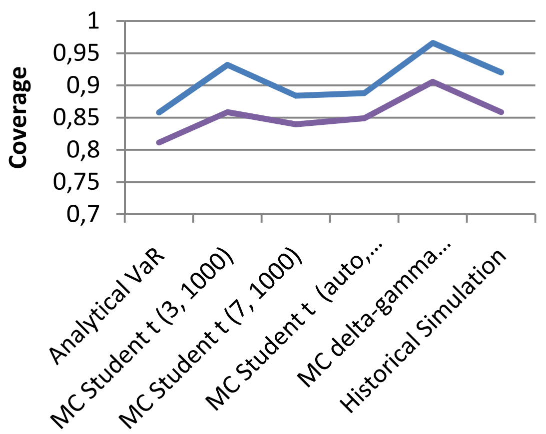

Figure 6-11 Coverage at 95% VaR for portfolio C estimated with 250 historical days.

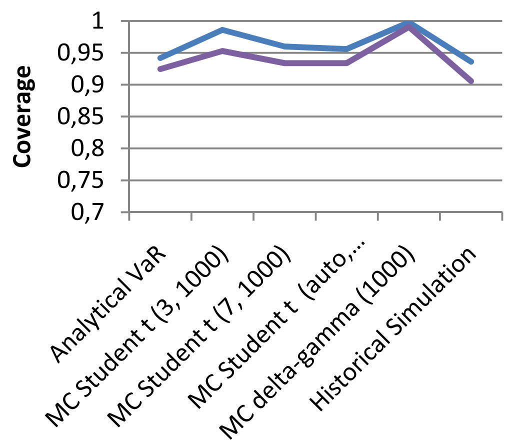

Figure 6-12 Coverage at 95% VaR for portfolio D estimated with 250 historical days.

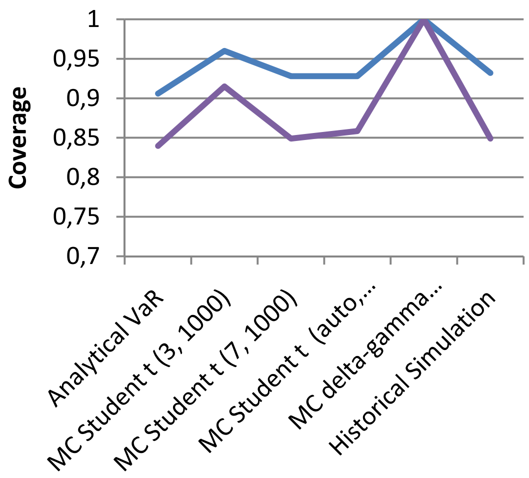

Figure 6-13 Coverage at 95% VaR for portfolio E estimated with 250 historical days.

6.4 Relative bias

To examine how the different VaR-methods perform relative to each other the relative bias compares the VaR-estimate of a method to the average VaR-estimate of all methods. If a method's VaR-estimate equals the average of the VaR-estimates it will have a factor of 0. A positive factor indicates that the average is bigger (with a higher loss) than the VaR-estimate and a negative factor indicate the opposite (the VaR-estimate indicates a higher loss than the average).

The box-and-whisker graph displays the median, first and third quartile as the boxes, with the whiskers marking the lower and higher limits of the interval at a maximum of 1.5 times the interquartile range. Furthermore the average is marked by a diamond.

As seen in Figure 6-14 to Figure 6-17 the MC student t with 3 degrees of freedom is closest to the average estimate. In addition the VaR-estimate of MC delta-gamma seems to be overestimated as it is on average 1.1 times as big as the average estimate. The other methods are slightly lower than the average estimate.

This is also confirmed in the majority of the portfolios as the MC delta-gamma is in many cases overestimating VaR, and MC student t 3 df always is somewhere between the delta-gamma method and the other estimates. Eventually this also leads to the MC student t 3 df being negative (and thus estimating VaR to be bigger than the average), like in portfolio D, see Figure 6-16 and Figure 6-17.

The only deviating portfolio was portfolio C and this was especially evident when using only 50 days of historical data. Here both historical simulation and student t 3 df were estimating VaR higher than the average, and both student t 7 df and student t auto were estimating at the average.

Also evident in these figures are the spread of estimates compared to the average. Both historical simulation, delta-gamma and student t 3 df has bigger spreads than the other methods. Still the graph does not include outliers due to high number of outliers.

These differences are also clearly evident from a graph of the estimated VaR by the methods over the back-test period. As seen in Figure 6-18 the MC delta-gamma method is situated below the other methods in the first 400 days before it gradually reduces towards the other methods. The very high VaR and increase in VaR during these first 400 days is a result of the high and increasing value of the portfolio (see Figure 4-1) combined with a high correlation between the underlying risk factors (see Table 4-3). Since the fall in portfolio value in the latest 100 days the VaR-estimate has decreased.

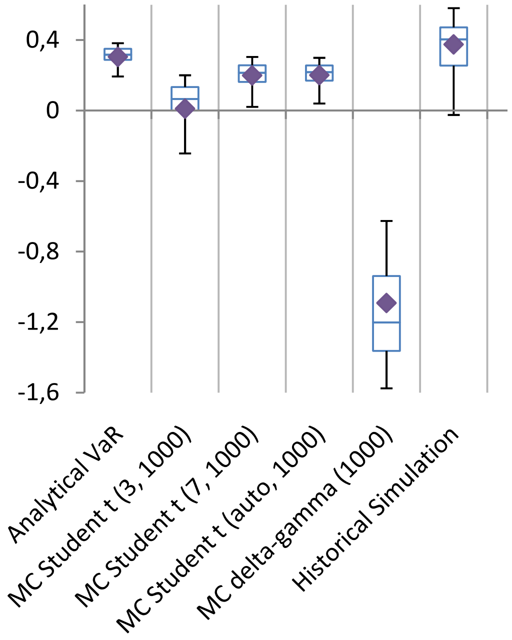

Figure 6-14 Box-and-whiskers diagram: Deviation from average for 95% VaR-estimates in portfolio A at 50 historical days.

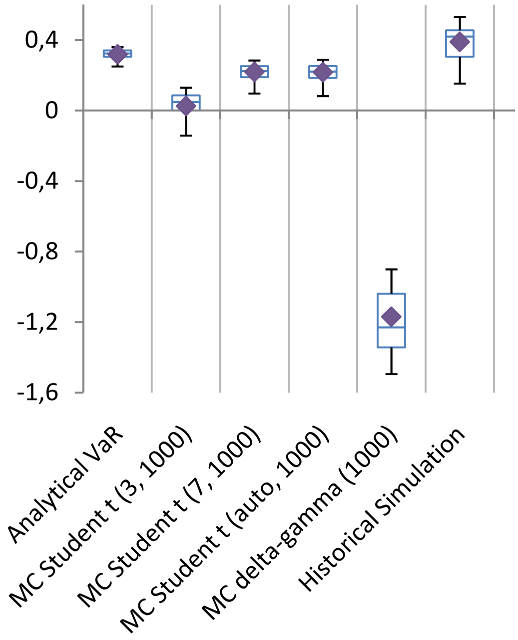

Figure 6-15 Box-and-whiskers diagram: Deviation from average for 95% VaR-estimates in portfolio A at 250 historical days.

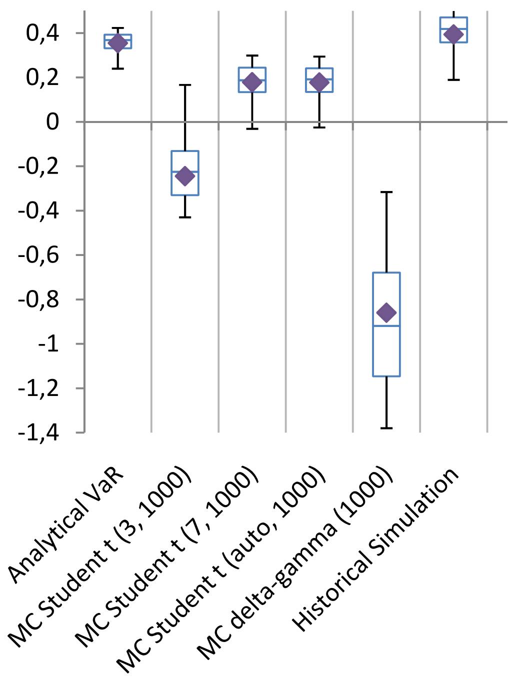

Figure 6-16 Box-and-whiskers diagram: Deviation from average for 99% VaR-estimates in portfolio D at 50 historical days.

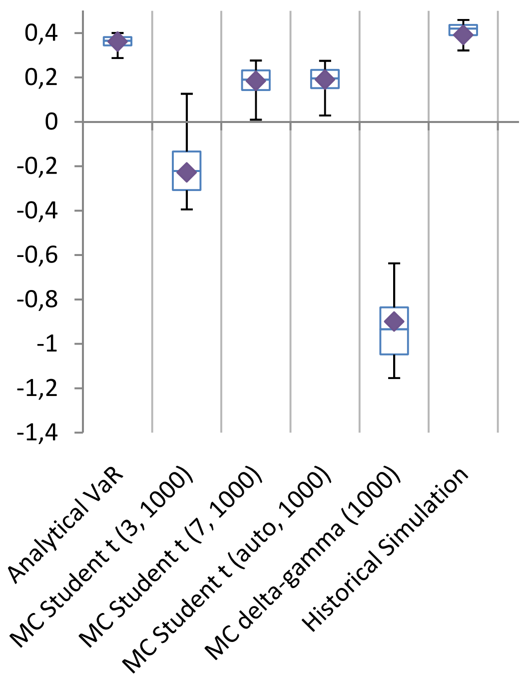

Figure 6-17 Box-and-whiskers diagram: Deviation from average for 99% VaR-estimates in portfolio D at 250 historical days.

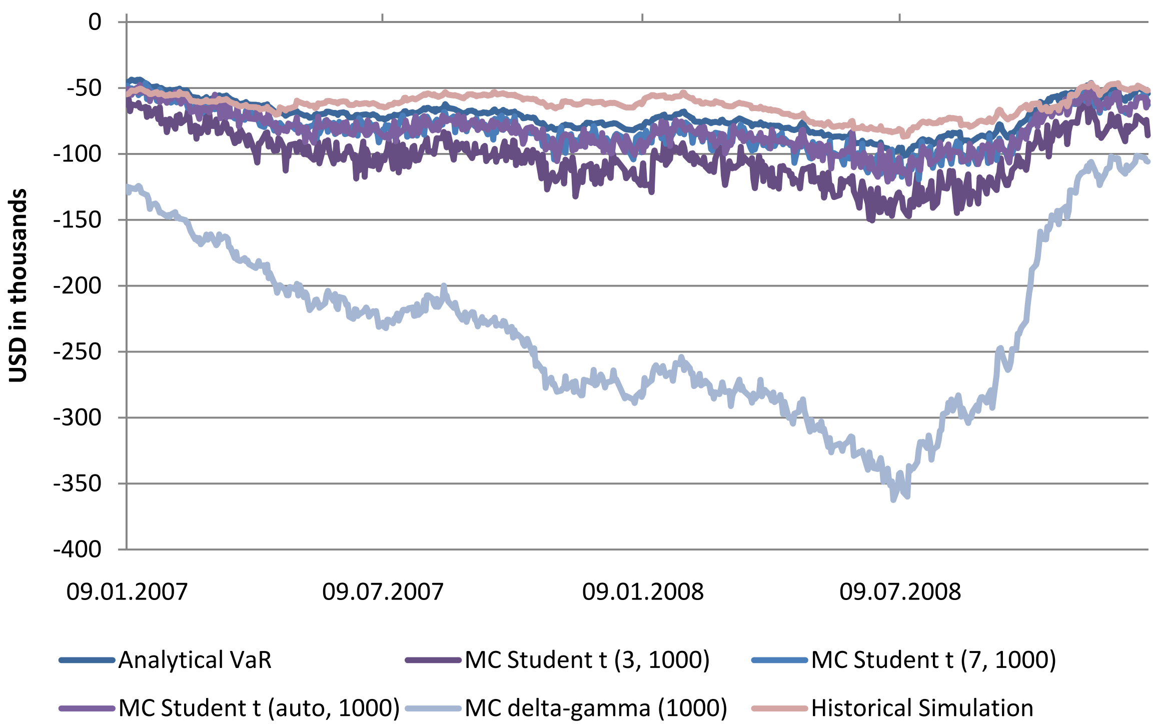

Figure 6-18 VaR-estimates over back-test period.

6.5 Size of error

It is important to know by what factor the actual change in the portfolio can outdo the VaR-estimate. This is done by comparing the actual change to the VaR-estimate for the times the VaR-estimate failed to cover the real change.

The box-and-whiskers diagrams presented in Figure 6-19 to Figure 6-23 review this for each portfolio at a 95% VaR-estimate with a 250 historical data period. Here the outliers are included and marked by dots.

The averages of the excess loss are for all methods practically the same in every portfolio. The only deviation comes from MC delta-gamma which for portfolio D and E only has small excess loss.

The boxes (measuring the difference between 3rd and 1st quartile) are all of equal size, except for the MC delta-gamma which generally has smaller boxes. Some deviations in the size of the boxes occur from portfolio to portfolio but these are only minor.

The biggest differences when comparing these numbers are the maximum excess loss, which follows the observations seen earlier. MC delta-gamma has very low maximum excess loss due to its high coverage. The student t 3 which performs best of the other methods has a lower maximum excess loss, while both the analytical approach and the historical simulation performs worst with the highest excess loss for most portfolios. This can be seen by the outliers of the figures.

The appendix also includes graph of 99% VaR with 250 historical days, as well as 95% and 99% VaR with 50 days of historical data. These show similar trends, although 99% VaR for portfolio C with 50 historical days deviated from the rest as MC delta-gamma performs worst with most and biggest outliers. However at 250 days MC delta-gamma performs as normal.

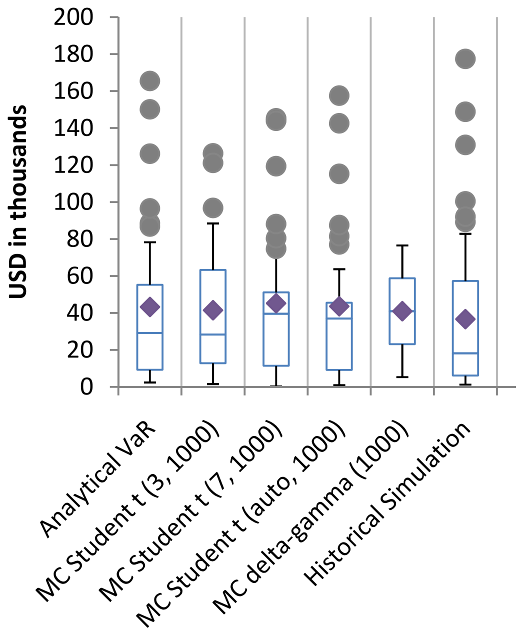

Figure 6-19 Box-and-whisker diagram: Excess loss when 95% VaR-estimate fails for portfolio A at 250 days of historical data.

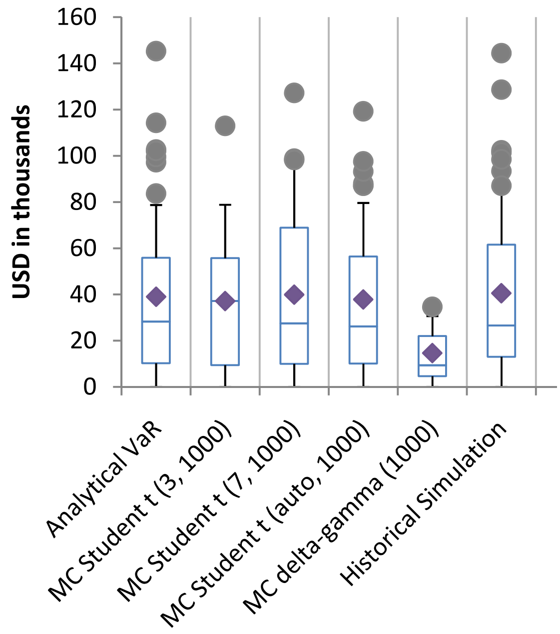

Figure 6-20 Box-and-whisker diagram: Excess loss when 95% VaR-estimate fails for portfolio B at 250 days of historical data.

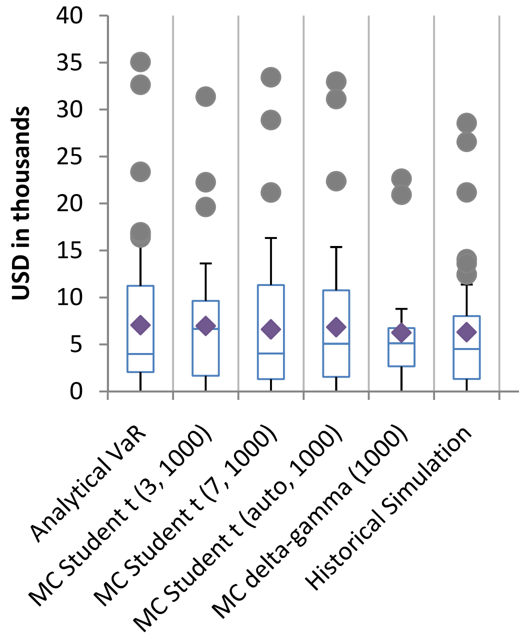

Figure 6-21 Box-and-whisker diagram: Excess loss when 95% VaR-estimate fails for portfolio C at 250 days of historical data.

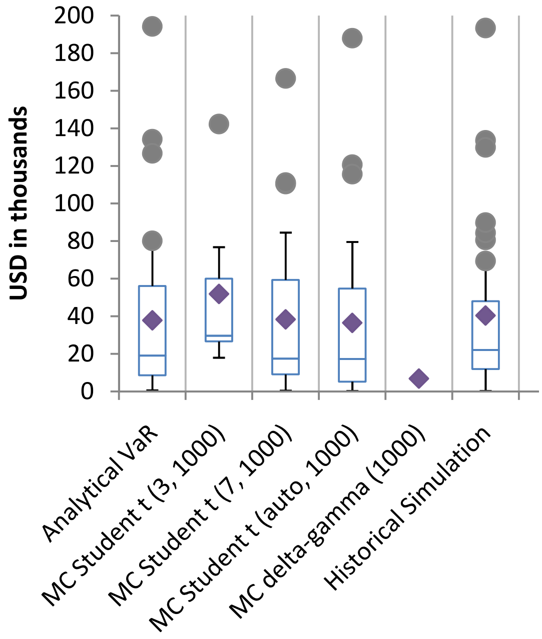

Figure 6-22 Box-and-whisker diagram: Excess loss when 95% VaR-estimate fails for portfolio D at 250 days of historical data.

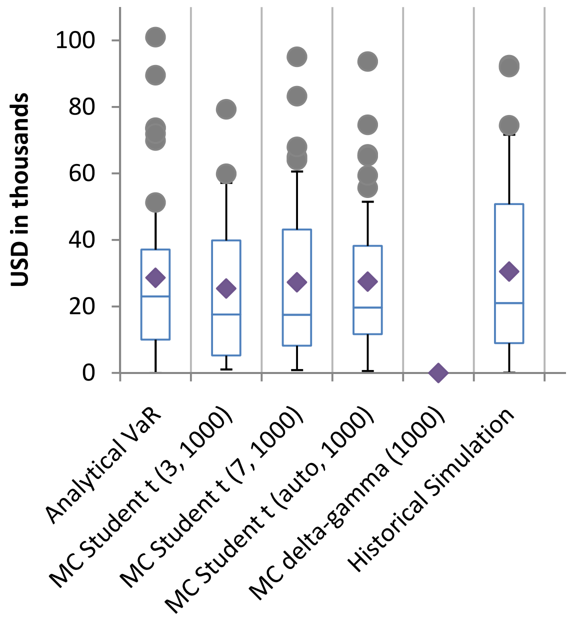

Figure 6-23 Box-and-whisker diagram: Excess loss when 95% VaR-estimate fails for portfolio E at 250 days of historical data.

6.6 Tail size

Another measure to test the tail is to compare the mean excess loss with the estimated VaR. This is done by calculating the average estimate bigger than the estimated VaR. This test was performed on the Monte Carlo methods as well as the historical simulation.

Figure 6-24 identifies several key characteristics about the tail events simulated by the different methods. Firstly the MC student t 3 has the highest spread, meaning that its estimate is the most volatile and has the highest uncertainty. The figure therefore confirms the findings in section 6.2.1 Monte Carlo Convergence and section 6.2.2 Student t comparison (see Figure 6-3).

Furthermore the MC delta-gamma has the smallest spread which is a result of the high collection of samples around the estimated VaR as one of the key characteristics of the delta-gamma approximation and its importance sampling.

Another important note is that the historical simulation has the smallest multiple average meaning that its tail is thin. This is a confirmation of the thin tail of the historical simulation's distribution, and may lead to underestimation of VaR, especially at 99%.

Figure 6-25 displays the same result but for 95% VaR. The figure shows the same trend, but with a slightly higher factor due to the fact that there is more of the tail left for a 95% estimate compared to the 99% estimate.

The same results appear regardless of the number of historical days used in the estimation. Also the trend is confirmed for the same test for the other portfolios.

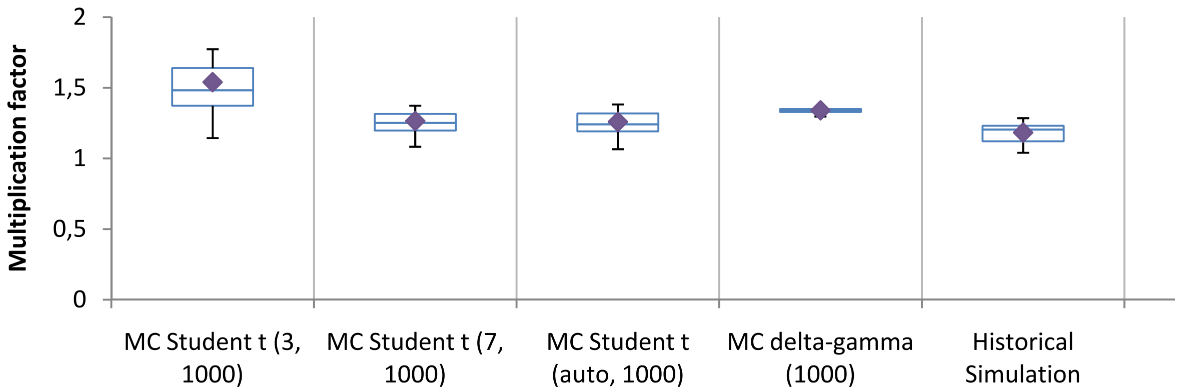

Figure 6-24 Box-and-whisker diagram: Mean excess loss for portfolio A at 99% VaR.

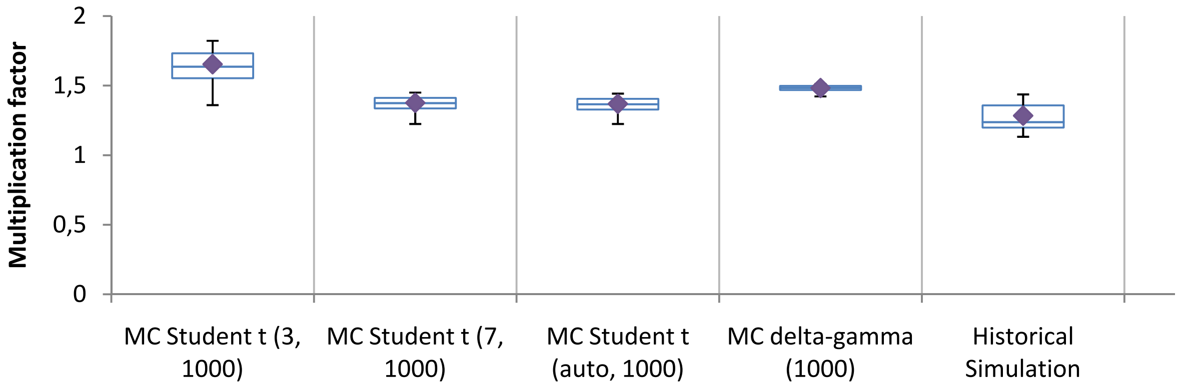

Figure 6-25 Box-and-whisker diagram: Mean excess loss for portfolio A at 95% VaR.

The maximum excess loss compares the worst outcome estimated by the method to the estimated VaR. This measures the multiple of the far end of the distribution's tail, and as seen in Figure 6-26 and Figure 6-27 it varies most for MC student t 3 due to its low convergence and high volatility in the estimation. Additionally MC delta-gamma has the biggest multiple when considering the 99% VaR, but is outdone by the MC student t 3 at the 95% VaR.

As with the mean excess loss the MC delta-gamma method has the lowest spread for the maximum excess loss. This underlines the method's good features due to its importance sampling.

The historical simulation further confirms its small tail as it has the lowest maximum excess loss multiple for both the 95% VaR and the 99% VaR.

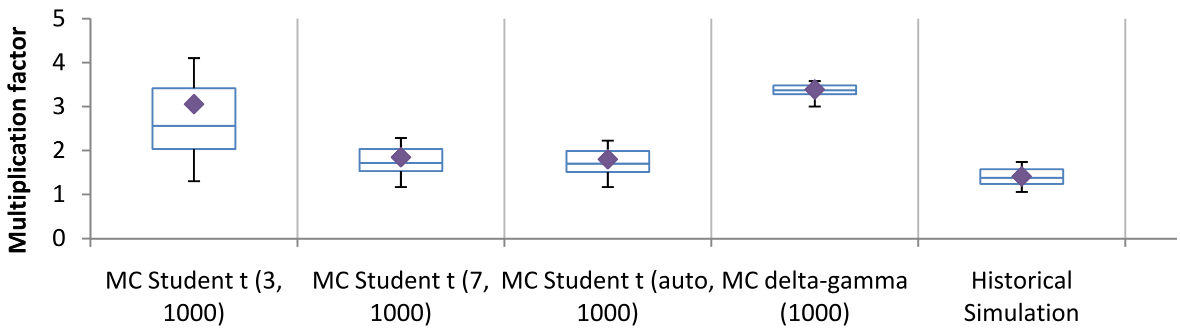

Figure 6-26 Box-and-whisker diagram: Max excess loss for portfolio A at 99% VaR.

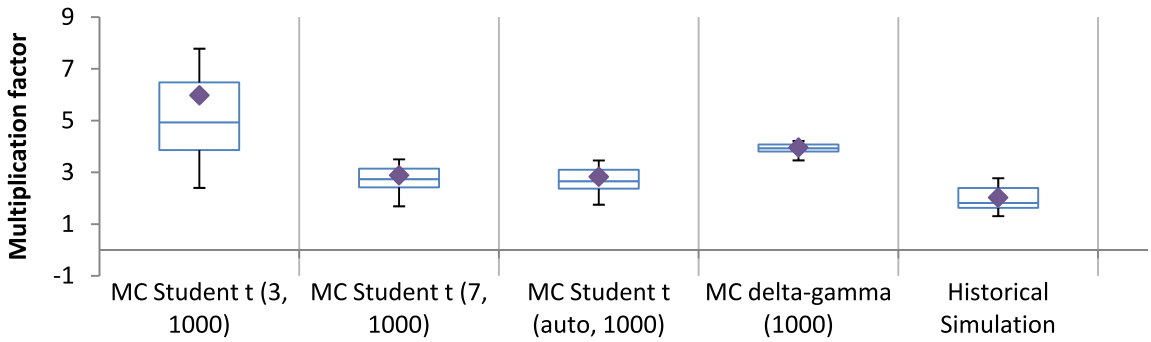

Figure 6-27 Box-and-whisker diagram: Max excess loss for portfolio A at 95% VaR.

The figures are representative for all portfolios as they show similar trends.

6.7 Correlation with portfolio

By examining the correlation between the VaR-estimate of the methods and the actual portfolio change it is possible to say something about the method's ability to adjust to changes in risk over time. The correlation is independent of the portfolio's scale and can easily be interpreted.

As seen in Figure 6-28 the correlation tend to be bigger when used with a short historical period as the estimates based on 50 historical days all produces a higher correlation than when using either 250 or 750 days. Still only the historical simulation produces a somewhat significant correlation as it averages at 0.15 with a maximum correlation of 0.25.

The differences between the correlation of 99% VaR and 95% Var are insignificant and therefore only 95% VaR is displayed.

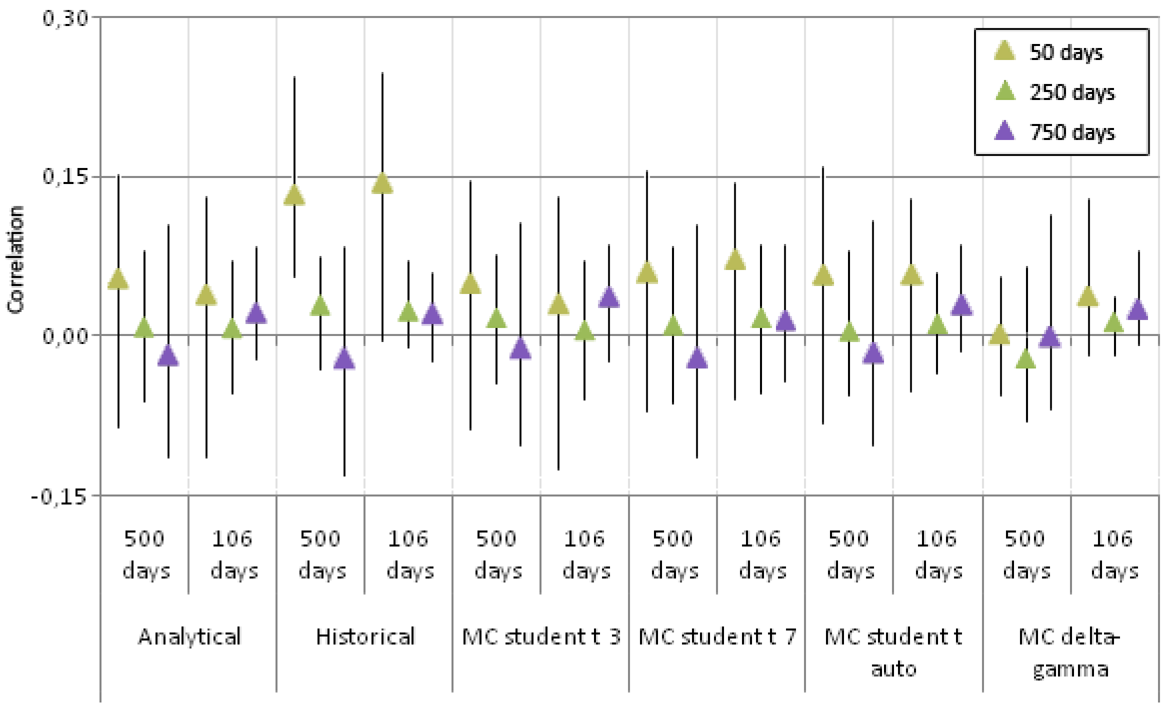

Figure 6-28 Correlation between actual portfolio change and VaR-estimates.

More interestingly are the strong correlation between the scale of the portfolio's value and the estimate of VaR. As already noted in Figure 6-18 the estimate by MC delta-gamma is reduced significantly when the portfolio's value is reduced. This correlation is confirmed by the correlation test between the size of the VaR-estimate and the size of the portfolio, see Figure 6-29.

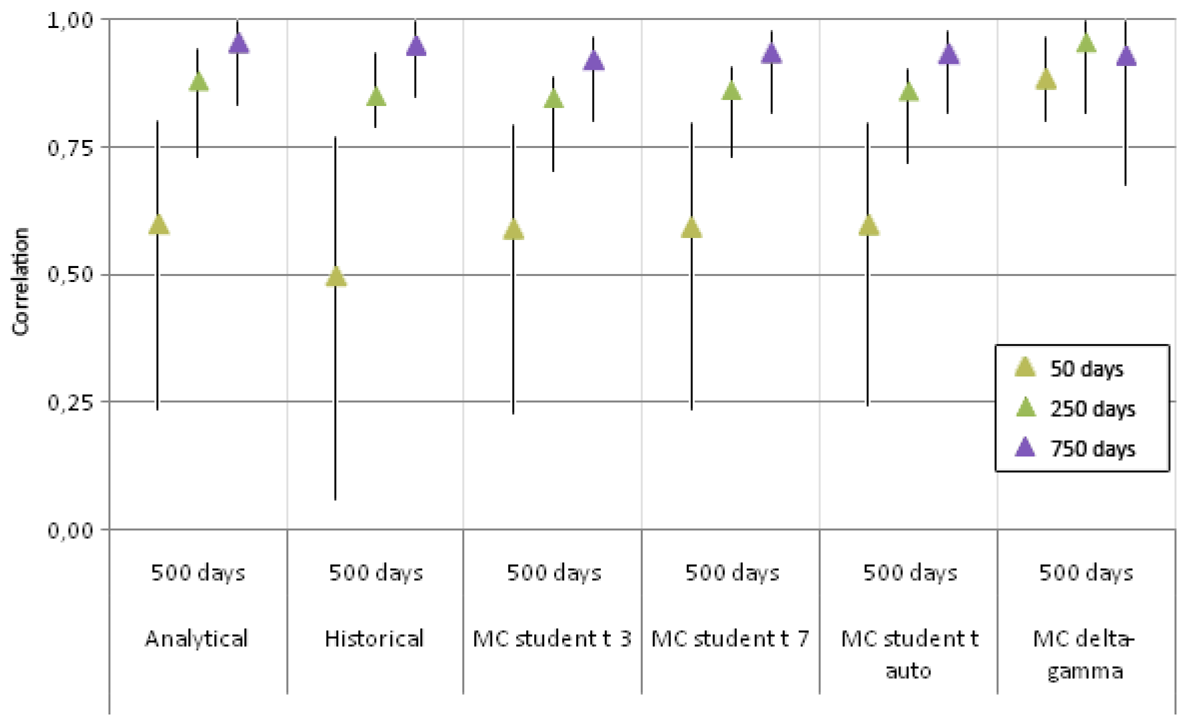

Figure 6-29 Correlation between portfolio value and VaR-estimates.

The correlation between portfolio value and size of VaR-estimate is especially clear when using a long historical period. However for MC delta-gamma it is strong for every size of the historical period.

6.8 Time concerns

One of the major challenges for the Monte Carlo methods is the number of runs needed to create an accurate simulation and estimate. These issues are tried solved with the delta-gamma approximation in order to achieve an importance sampling which will lead to faster convergence (as proved in 6.2.1 Monte Carlo convergence), and thus require less runs for an accurate estimate.

The time test is performed by doing 100 estimates of the same VaR and Figure 6-30 displays the average of these 100 estimates.

As is clearly evident the Monte Carlo student t (with a 1000 runs) needs by far most time to achieve a result. What is furthermore disturbing is that even at a 1000 runs the student t method does not seem to converge (see section 6.2.1).

The delta-gamma method performs much faster than the student t method and also has a faster convergence. Still at only a 100 runs the convergence would be better than for the student t method, and at this low number of runs the delta-gamma would be as quick as the historical simulation method.

The fastest method is the analytical approach which uses virtually no time to estimate VaR even at 72 underlying risk factors.

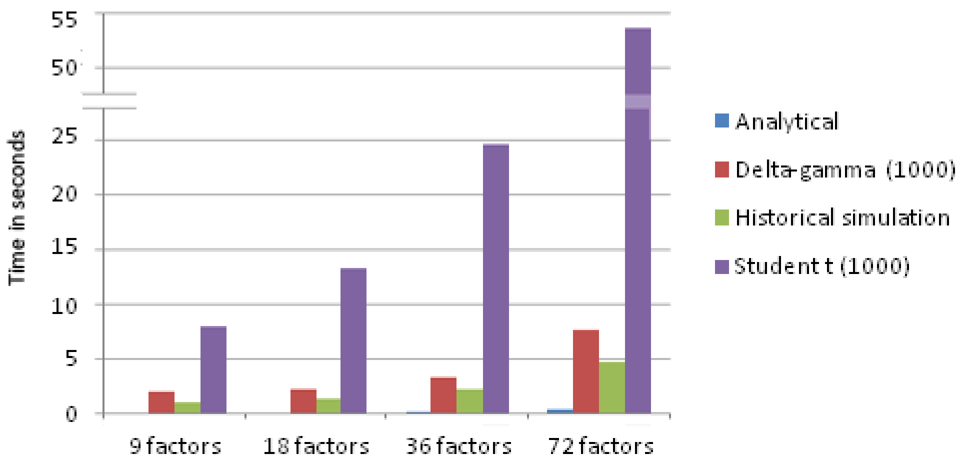

Figure 6-30 Time test of the methods displaying average time to calculate VaR at different sizes of the portfolio.

6.9 Underlying distributions

One of the key advantages with Monte Carlo simulation of VaR is the possibility to produce a plot for the simulated distribution. Figure 6-31 displays a typically scenario for a simulation with Monte Carlo with delta-gamma (dark violet: 95% VaR, violet: 99% VaR), student t (red: 3 degrees of freedom (df) and green: 7 df,) historical simulation (blue) and the analytical or normal distribution (cyan).

Due to the importance sampling achieved with the delta-gamma approximation the simulated scenarios created by MC delta-gamma have higher collection of outcomes close to the wanted percentile. This ensures as shown in section 6.2.1 Monte Carlo convergence a higher precision due to its low variance. Furthermore, as described in section 3.3.3.5 Monte Carlo delta-gamma the desired percentile is now the expected value as it is the 50th percentile. While the estimated VaR for every other method is found by examining the 5% cumulative percentile, the delta-gamma method uses the likelihood ratio and produces outputs around VaR as its most common output.

Figure 6-31 presents the 95% VaR estimates by each method by a dotted line. Both the analytical estimate and the estimate by the historical simulation are comparatively low just over 50 000. The two student t methods follow with the student t with 3 degrees having the highest VaR of the two. The biggest VaR at 95% is estimated by MC delta-gamma at around 90 000.

Figure 6-31 also displays where the 99% VaR is found for MC delta-gamma. This can be compared to where the 99% VaR is found for the alternative methods at the 1st percentile of their cumulative graphs. While delta-gamma had the biggest VaR estimate at 95% it is outdone by the student t with 3 degrees of freedom at the 99% VaR estimate. Student t with 7 degrees of freedom ends up between delta-gamma and the historical simulation. The historical simulation and the analytical approach are still very close at 99% VaR.

As noted in section 6.6 Tail size the historical simulation has the thinnest tail of the distributions. This is also confirmed by Figure 6-31 as the curve of the historical simulation is steepest around 0 change in portfolio. Although the sharpness in this estimate is not general for every estimate, in this estimate even the analytical approach has a fatter tail than the historical simulation.

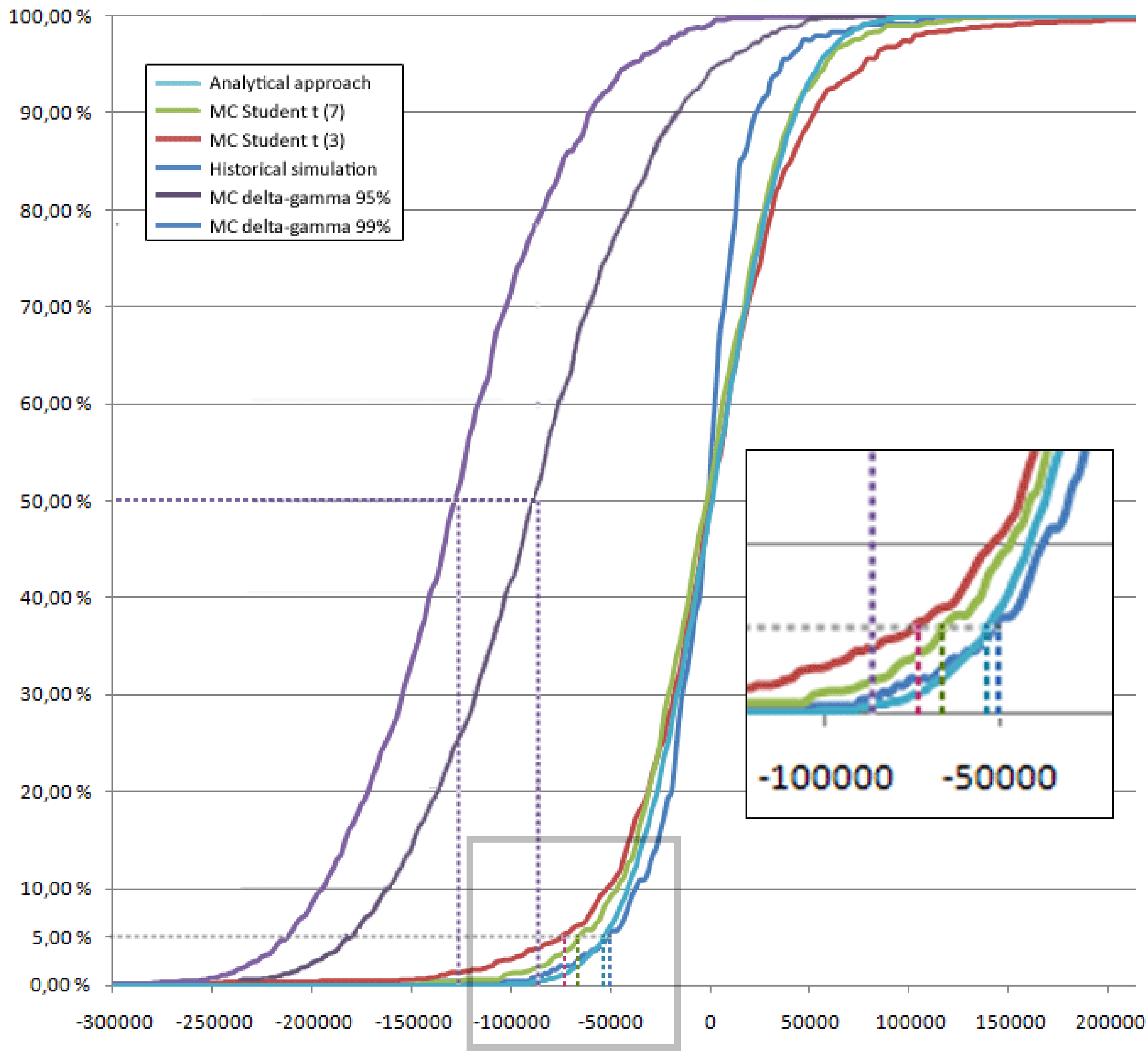

Figure 6-31 Distributions of VaR-estimates for 31.12.2008 for portfolio A estimated with 250 historical days.

7. Summary and conclusion

The objective of this thesis is to consider Monte Carlo estimation of Value at Risk (VaR) and compare it to the historical simulation and analytical estimation. VaR has been a widely used estimate for companies and institutions to valuate the risk in a portfolio of assets and this thesis compares the methods empirically with a portfolio of oil products.

The estimation of Value at Risk (VaR) is complicated and involves many factors and their correlation. It defines the worst case scenario within a certain confidence level over a specified time horizon. This introduces the two parameters of a VaR: its confidence level and time horizon.

VaR can be further defined by asking a simple question: How much is it possible to lose within a certain time period at a certain significance level?

$$P\left(L > x_p\right) = p$$

The methods for estimating VaR has been criticised recently by the Turner Review which evaluates the reasons for the financial crisis. One of the deficiencies used to explain the risk positive financial market in the booming years was the reliance on a deceitful Value at Risk (VaR) estimate. The flaws of VaR helped create more risk as years of low volatility encouraged more risk due to its procyclical character. In addition the assumption of normal distributed products used in most VaR models underestimated the fat tails empirically confirmed.

Statoil Hydro ASA also experienced underestimation during the extremely volatile period after the summer of 2008. Such periods of high volatility is not uncommon in the oil market as it shifts between periods of relatively calm and periods of high volatility. It is therefore important to find a VaR method which can adjust to these changes rapidly.

This thesis has discussed why the oil market and oil price plummeted during the fall of 2008. While much of the historical price shifts has happened due to some exogenous incident, this shift came as a result of a sharp reduction in demand as a consequence of the financial crisis.

To incorporate such shifts it was decided to examine a Monte Carlo method as an alternative to today's historical simulation for VaR estimation. Thus this thesis includes two Monte Carlo methods: One based on student t for the underlying risk factors while the second uses delta-gamma approximation to achieve importance sampling. In addition the historical simulation which calculates VaR on the basis of historically scenarios and an analytical approach which uses normality assumption for the underlying risk factors were implemented for comparison.

7.1 Conclusion

Throughout this thesis several alternatives to estimate VaR has been implemented and tested. As presented in section 6.3 Coverage the analytical approach and historical simulation tends to underestimate VaR (especially in the stress-test), Monte Carlo delta-gamma tends to overestimate VaR (although not in the stress-test). The best performance was seen by the Monte Carlo student t which in most cases performed best with 3 degrees of freedom at 95% VaR, while for the 99% VaR both student t 7 and the student t auto performed better. The added flexibility of adjusting the degree of freedom according to the risk in a portfolio this method therefore ensures added value.

Both the historical simulation and analytical approach failed the stress-test averaging below 0.90 for the 95% VaR and below 0.95 for the 99% VaR even with the smallest historical period.

The student t 3 was the estimate closest to the average of the compared methods and thus had the lowest relative bias, see section 6.4. For this criterion the overestimation of delta-gamma compared to the other methods was clear as it had by far the biggest relative bias. The other methods were slightly underestimating VaR compared to the average.

Furthermore the student t with 3 degrees of freedom performed best when comparing the size of error for each method. While the delta-gamma method had the lowest average due to its abnormally high coverage, the student t 3 had a healthy coverage and the size of error were kept at a small rate with few outliers (see section 6.5 Size of error).

In the majority of the cases the shorter historical period demonstrated the best performance. However for the hedged portfolio C the best historical period for Monte Carlo delta-gamma and student t 3 was 250 days.

As further concluded in section 6.3 Coverage the Monte Carlo student t with 3 degrees of freedom together with Monte Carlo delta-gamma has the best coverage for the 95 % VaR. Furthermore for the 95% VaR these methods should be combined with a short historic period, e.g. 50 days, in order to get an optimal coverage. However for 99% VaR they should be implemented with a longer historical period, e.g. 250 days. For student t with 3 degrees of freedom this will avoid overestimating VaR to some degree, while for delta-gamma this will ensure better and more balanced coverage.

While the Monte Carlo student t performed very well it has a major issue concerning both time and convergence. Due to the numerous calculations necessary before a simulation the method consumes more time than the other methods. Furthermore as proved in section 6.2.1 Monte Carlo convergence, due to its variance MC student t needs at least 10 times the runs to converge compared to the delta-gamma method. This can prove a problem for large portfolios with hundreds of underlying factors, as seen in section 6.8 Time concerns.

The underlying distribution displayed by the delta-gamma method proves that it is an efficient method to estimate VaR. Even though the method seems to overestimate VaR, the historical problem according to the Turner Review has been the underestimating of VaR. In this thesis the underlying risk factors were highly correlated (see section 4.3 Correlation) which caused delta-gamma to estimate a relatively high VaR. Still the reduction in the portfolio value seen in 2008 caused the estimate of VaR by delta-gamma to decrease as a result of its high correlation with the portfolio value (see section 6.7 Correlation with portfolio).

7.2 Further work

The results seen in this thesis further advocates the development of efficient and accurate Monte Carlo methods. Both the delta-gamma method and the student t method have proved its advantages as the delta-gamma is highly correlated with portfolio value, adjusts well to the correlation between the underlying risk factors and converges fast to an accurate estimate. The student t method has proved flexible with its degree of freedom and had the best coverage of all methods in this thesis as VaR estimates with student t 3/7 covered just above 95% for 95% VaR and around 99% at 99% VaR. Even in the most difficult period of the last 6 months of 2008 the estimate where effective for all portfolios.

As seen in section 4.2 the underlying risk factors all have different degrees of freedom when calculated based on the historical data. A further improvement could therefore be achieved by utilizing a t copula which can differentiate the degree of freedom used for each risk factor.

However as section 6.8 Time concerns concludes the amount of time needed to calculate necessary inputs for the multivariate student t distribution can be a factor when the number of underlying risk factors is increased. Furthermore Monte Carlo student t has proven to be more inefficient as it needs more runs to converge to an estimate. Compared to the delta-gamma method which already converges at around 100 runs, the Monte Carlo student t needs at least 10 times more runs to converge.

It has been proven by Glasserman, Heidelberger and Shahabuddin (2002) that the heavy weighted tails of the underlying risk factors can be modelled by a multivariate student t distribution combined with the importance sampling achieved with the delta-gamma approximation. By combining the good characters of the delta-gamma method and the flexibility and accuracy of student t this can prove a more efficient and accurate method.

Moreover GHS (2002) recommends combining importance sampling with stratified sampling (presented in section 3.3.3.4 Variance reduction techniques). This further increases efficiency by 3 to 4 times in their portfolios.

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Appendix A – Source code

The source code appendix is divided in three parts:

- Classes
- User Form
- Matrix algebra

The first part includes the Product class used for the underlying risk factors and the VaRestimate class used to estimate VaR and other key characteristics simultaneously. The user form includes every method implemented in this thesis as well as necessary code to interact with the user. Finally some of the necessary matrix algebra is presented.

All code is written in Visual Basic for Applications (VBA).

A.1 Classes

Product class

```
Option Base 1

Private pName As String
Private pWeight As Double
Private pWeightUSD As Double
Private pAvg As Double
Private pstandardDev As Double
Private pMax As Double
Private pMin As Double
Private pMedian As Double
Private pQuartile1 As Double
Private pQuartile3 As Double
Private pObservations As Integer
Private pKurtosis As Double
Private pData As Range
Private pPrcChange() As Double
Private pPrc As Range

''''''''''''''''''''''
' Set initial values with range
' calculate mean, averages, etc when inputing Range.
''''''''''''''''''''''
Public Sub setInitValues(Value As Range, first As Boolean, nameInput As String,
weightsInput As Double)
    Dim X As Range

    If first Then
        name = Value.Cells(1, 1)
        weight = Value.Cells(2, 1)
        Set X = Value.Offset(2, 0).Resize(Value.Rows.count - 2, _
        Value.Columns.count)
    Else
        name = nameInput
        weight = weightsInput
        Set X = Value
    End If
    Set Data = X ''.Offset(1, 0).Address(0, 0)  ''.Resize(-1, 0)
    ReDim pPrcChange(1 To (Data.Rows.count) - 1)  ''-1
    setPrcChange X
    Average = Application.WorksheetFunction.Average(pPrcChange)
    standardDev = Application.WorksheetFunction.StDev(pPrcChange)
    Min = Application.WorksheetFunction.Min(pPrcChange)
```

```
    Max = Application.WorksheetFunction.Max(pPrcChange)
    Quartile1 = Application.WorksheetFunction.Quartile(pPrcChange, 1)
    Quartile3 = Application.WorksheetFunction.Quartile(pPrcChange, 3)
    WeightUSD = weight * Data(Data.Rows.count)
    Observations = Data.Rows.count
    Kurtosis = Application.WorksheetFunction.Kurt(pPrcChange)
End Sub

''''''''''''''''''''''
' Provide a summary of key data for the assets
''''''''''''''''''''''
Public Sub addToSummary(pCounter As Integer)

    Sheets("Summary").Select
    Range("A1").Select
    If pCounter = 0 Then
        ActiveCell.FormulaR1C1 = "Products"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "Weight"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "Average"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "Stdev"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "Max"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "Min"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "1st Quartile"
        ActiveCell.Offset(0, 1).Range("A1").Select
        ActiveCell.FormulaR1C1 = "3rd Quartile"
        Range("A1").Select
    End If
    ActiveCell.Offset(pCounter + 1, 0).Range("A1").Select
    ActiveCell.FormulaR1C1 = name
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = weight
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = Average
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = standardDev
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = Max
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = Min
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = Quartile1
    ActiveCell.Offset(0, 1).Range("A1").Select
    ActiveCell.FormulaR1C1 = Quartile3
    pCounter = pCounter + 1
End Sub


''''''''''''''''''''''
' Name property
''''''''''''''''''''''
Public Property Get name() As String
    name = pName
End Property
Public Property Let name(Value As String)
    pName = Value
End Property

''''''''''''''''''''''
' Weight property
''''''''''''''''''''''
Public Property Get weight() As Double
    weight = pWeight
```

```
End Property
Public Property Let weight(Value As Double)
    pWeight = Value
End Property

''''''''''''''''''''''
' Weight in USD property
''''''''''''''''''''''
Public Property Get WeightUSD() As Double
    WeightUSD = pWeightUSD
End Property
Public Property Let WeightUSD(Value As Double)
    pWeightUSD = Value
End Property

''''''''''''''''''''''
' Average property
''''''''''''''''''''''
Public Property Get Average() As Double
    Average = pAvg
End Property
Public Property Let Average(Value As Double)
    pAvg = Value
End Property

''''''''''''''''''''''
' Standard deviation property
''''''''''''''''''''''
Public Property Get standardDev() As Double
    standardDev = pstandardDev
End Property
Public Property Let standardDev(Value As Double)
    pstandardDev = Value
End Property

''''''''''''''''''''''
' Max property
''''''''''''''''''''''
Public Property Get Max() As Double
    Max = pMax
End Property
Public Property Let Max(Value As Double)
    pMax = Value
End Property

''''''''''''''''''''''
' Min property
''''''''''''''''''''''
Public Property Get Min() As Double
    Min = pMin
End Property
Public Property Let Min(Value As Double)
    pMin = Value
End Property

''''''''''''''''''''''
' Median property
''''''''''''''''''''''
Public Property Get Median() As Double
    Median = pMedian
End Property
Public Property Let Median(Value As Double)
    pMedian = Value
End Property

''''''''''''''''''''''
' 1st Quartile property
''''''''''''''''''''''
```

```
Public Property Get Quartile1() As Double
    Quartile1 = pQuartile1
End Property
Public Property Let Quartile1(Value As Double)
    pQuartile1 = Value
End Property

''''''''''''''''''''''
' 3rd Quartile property
''''''''''''''''''''''
Public Property Get Quartile3() As Double
    Quartile3 = pQuartile3
End Property
Public Property Let Quartile3(Value As Double)
    pQuartile3 = Value
End Property

''''''''''''''''''''''
' Observations property
''''''''''''''''''''''
Public Property Get Observations() As Integer
    Observations = pObservations
End Property
Public Property Let Observations(Value As Integer)
    pObservations = Value
End Property

''''''''''''''''''''''
' Kurtosis property
''''''''''''''''''''''
Public Property Get Kurtosis() As Double
    Kurtosis = pKurtosis
End Property
Public Property Let Kurtosis(Value As Double)
    pKurtosis = Value
End Property

''''''''''''''''''''''
' Data property
''''''''''''''''''''''
Public Property Get Data() As Range
    Set Data = pData
End Property
Public Property Set Data(Value As Range)
    Set pData = Value
End Property

''''''''''''''''''''''
' Prc property
''''''''''''''''''''''
Public Property Get Prc() As Range
    Set Prc = Range(Data.Offset(-1, 0)).Resize(Data.Rows.count - 1, 1) -
     Range(Data.Offset(1, 0)).Resize(Data.Rows.count - 1, 1) / Range(Data.Offset(-
     1, 0)).Resize(Data.Rows.count - 1, 1)
End Property
Public Property Set Prc(Value As Range)
    Set pPrc = Value
End Property

Public Sub setPrcChange(Value As Range)
    Dim celX As Range
    Dim lastX As Double
    Dim nextX As Double
    Dim counter As Integer
    lastX = 0
    nextX = 0
    counter = 0
    For Each celX In Value.Cells
```

```
        nextX = celX.Value
        If counter <> 0 Then
            If Not lastX = 0 Then
                pPrcChange(counter) = (nextX - lastX) / lastX
            Else
                pPrcChange(counter) = 0
            End If
        End If
        lastX = nextX
        counter = counter + 1
    Next celX
End Sub

Property Get PrcChange(index As Long) As Double
    PrcChange = pPrcChange(index)
End Property

Property Let PrcChange(index As Long, inValue As Double)
    pPrcChange(index) = inValue
End Property

Property Get getDataAt(ind As Integer) As Double
    Dim tmp As Variant
    ReDim tmp(Data.Rows.count - 1)
    tmp = Data
    X = tmp(ind, 1)
    getDataAt = X
End Property

Property Get getRangeOfData(start As Integer, sEnd As Integer) As Variant
    Dim tmp As Variant
    ReDim tmp(sEnd - start, 1)
    For i = start To sEnd - 1
        tmp(i - start + 1, 1) = Data(i, 1)
    Next i
    getRangeOfData = tmp
End Property

Property Get CalculateDelta() As Double
    CalculateDelta = getDataAt(Observations) * weight
End Property
```

VarEstimate class

```
Option Base 1

Private pVaR As Double
Private pConfHigh As Double
Private pConfLow As Double
Private pMeanExcessLoss As Double
Private pMaxLoss As Double

''''''''''''''''''''''
' VaR property
''''''''''''''''''''''
Public Property Get VaR() As Double
    If (IsNumeric(VaR) = False) Then
        VaR = 0
    Else
        VaR = pVaR
    End If
End Property
Public Property Let VaR(Value As Double)
    pVaR = Value
End Property

''''''''''''''''''''''
```

```
' Confidence high property
''''''''''''''''''''''
Public Property Get ConfHigh() As Double
    ConfHigh = pConfHigh
End Property
Public Property Let ConfHigh(Value As Double)
    pConfHigh = Value
End Property

''''''''''''''''''''''
' Confidence low property
''''''''''''''''''''''
Public Property Get ConfLow() As Double
    ConfLow = pConfLow
End Property
Public Property Let ConfLow(Value As Double)
    pConfLow = Value
End Property

''''''''''''''''''''''
' Mean excess loss property
''''''''''''''''''''''
Public Property Get MeanExcessLoss() As Double
    MeanExcessLoss = pMeanExcessLoss
End Property
Public Property Let MeanExcessLoss(Value As Double)
    pMeanExcessLoss = Value
End Property

''''''''''''''''''''''
' Max loss property
''''''''''''''''''''''
Public Property Get MaxLoss() As Double
    MaxLoss = pMaxLoss
End Property
Public Property Let MaxLoss(Value As Double)
    pMaxLoss = Value
End Property

Public Sub Copy(ByVal X As VarEstimate)
    ConfLow = X.ConfLow
    ConfHigh = X.ConfHigh
    MaxLoss = X.MaxLoss
    MeanExcessLoss = X.MeanExcessLoss
    VaR = X.VaR
End Sub
```

A.2 UserForm

Declarations and interaction with user

```
Option Base 1

Public Products As Collection
Public ProductsBack As Collection
Public CorrelM As Object

Const MAXROWS = 65536   ''USE 1048576 if Excel 2007
Const MAXDF = 15    ''defines maximum degree of freedom used in student t-
                        distributions
Const CONSTALPHA = 0.01
Const CONSTDF = 3   ''degree of freedom
Const CONSTBACKTEST = 0
Const CONSTRUNS = 100
Const CONSTDAYS = 250   ''historical period

'' Cancel button
```

```
Private Sub CommandButton1_Click()
    End
End Sub

''Previous button
Private Sub CommandButton2_Click()
    Dim i As Long

    i = MultiPage1.Value - 1

    If i >= 0 Then
        MultiPage1.Value = i

    End If
End Sub

''Next button
Private Sub CommandButton3_Click()
    Dim i As Long

    i = MultiPage1.Value + 1

    If i < MultiPage1.Pages.count Then
        MultiPage1.Value = i

    End If
End Sub

'' Finish button
Private Sub CommandButton4_Click()
    On Error Resume Next
    If (Products.count = 0) Then
        UserForm1.Hide()
        Exit Sub
    End If
    Application.ScreenUpdating = False
    ''set default values:
    Dim alpha As Double, numberOfRuns As Integer
    Dim numberOfDays As Integer, numberOfBacktest As Integer, df As Integer
    Dim cLevel As Double

    If IsNumeric(txtAlpha.Value) Then
        alpha = txtAlpha.Value
    Else
        alpha = CONSTALPHA
    End If
    If IsNumeric(txtRuns.Value) Then
        numberOfRuns = txtRuns.Value
    Else
        numberOfRuns = CONSTRUNS
    End If
    If IsNumeric(txtPeriod.Value) Then
        numberOfDays = txtPeriod.Value
    Else
        numberOfDays = CONSTDAYS
    End If
    If IsNumeric(txtBacktest.Value) Then
        numberOfBacktest = txtBacktest.Value
    Else
        numberOfBacktest = CONSTBACKTEST
    End If
    If IsNumeric(txtDF.Value) Then
        df = txtDF.Value
    Else
        df = CONSTDF
    End If

    ''First check if it is a backtest
```

```
        If Not numberOfBacktest = 0 Then
            Call BackTest(numberOfDays, numberOfRuns, alpha, numberOfBacktest, df,
              cLevel)
        Else
            Dim anVar As VarEstimate, MCNormal As VarEstimate, MCStudentT As
              VarEstimate
            Dim MCDeltaGamma As VarEstimate, HisSim As VarEstimate
            anVar = New VarEstimate
            MCNormal = New VarEstimate
            MCStudentT = New VarEstimate
            MCDeltaGamma = New VarEstimate
            HisSim = New VarEstimate

            ''check what methods to run
            If chkAnalytical.Value = True Then
                Call anVar.Copy(AnalyticalVaR(alpha, CorrelM, Products, cLevel))
            End If
            If chkHistoric.Value = True Then
                Call HisSim.Copy(HistoricalSim(alpha, Products))
            End If
            If chkMCNormal.Value = True Then
                Call MCNormal.Copy(MonteCarlo(alpha, numberOfRuns, CorrelM, Products,
                  df, True, chkAutoDF.Value))
            End If
            If chkMCStudentT.Value = True Then
                Call MCStudentT.Copy(MonteCarlo(alpha, numberOfRuns, CorrelM, Products,
                  df, False, chkAutoDF.Value))
            End If
            If chkMCNormalIS.Value = True Then
                Call MCDeltaGamma.Copy(DeltaGamma(alpha, numberOfRuns, CorrelM,
                  Products))
            End If

            ''print data to VaR-sheet.
            AddSheet("VaR")
            ClearSheet("VaR")
            Sheets("VaR").Select()
            Range("A1").Select()
            ActiveCell.FormulaR1C1 = ""
            ActiveCell.Offset(1, 0).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "VaR"
            ActiveCell.Offset(1, 0).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "Mean excess loss"
            ActiveCell.Offset(1, 0).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "Max loss"
            Range("A1").Select()
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "Analytical VaR"
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "MC Normal (" & numberOfRuns & ")"
            ActiveCell.Offset(0, 1).Range("A1").Select()
            If (chkAutoDF.Value = True) Then
                ActiveCell.FormulaR1C1 = "MC Student t (auto, " & numberOfRuns & ")"
            Else
                ActiveCell.FormulaR1C1 = "MC Student t (" & df & ", " & numberOfRuns &
                 ")"
            End If
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "MC Delta-gamma (" & numberOfRuns & ")"
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "Historical simulation (" & numberOfDays & ")"

            ActiveCell.Offset(1, -4).Range("A1").Select()
            ActiveCell.FormulaR1C1 = anVar.VaR
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = MCNormal.VaR
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = MCStudentT.VaR
```

```
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCDeltaGamma.VaR
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = HisSim.VaR

        ActiveCell.Offset(1, -4).Range("A1").Select()
        ActiveCell.FormulaR1C1 = anVar.MeanExcessLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCNormal.MeanExcessLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCStudentT.MeanExcessLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCDeltaGamma.MeanExcessLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = HisSim.MeanExcessLoss

        ActiveCell.Offset(1, -4).Range("A1").Select()
        ActiveCell.FormulaR1C1 = anVar.MaxLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCNormal.MaxLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCStudentT.MaxLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = MCDeltaGamma.MaxLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        ActiveCell.FormulaR1C1 = HisSim.MaxLoss
        ActiveCell.Offset(0, 1).Range("A1").Select()
        Range("A1").Select()
    End If
    UserForm1.Hide()
End Sub

''select button
Private Sub btnSelectAreas_Click()

    Application.ScreenUpdating = True
    Dim VarRange As Range, subArea As Range, AreasStr As String
    '-- initial selection area(s) will be used as suggestion
    On Error Resume Next
    VarRange = _
        Application.InputBox("Choose assets (include titles and data):", _
           "Choose assets", Selection.Address(0, 0), Type:=8)
    On Error GoTo 0
    If VarRange Is Nothing Then Exit Sub
    VarRange = Range(ColumnLetter(VarRange.Column) & VarRange.Row,
     Range(ColumnLetter(VarRange.Column + VarRange.Columns.count - 1) &
     MAXROWS).End(xlUp))

    Application.ScreenUpdating = False

    Dim c As Integer
    c = 0

    DeleteEmptyRows(VarRange)

    Dim p As Product

    Products = New Collection

    Dim cStart, cEnd As Integer
    cStart = VarRange.Column
    cEnd = cStart + VarRange.Columns.count

    StatusLabel = "Adding products and calculating properties."

    AddSheet("Summary")
    ClearSheet("Summary")
    ''iterate columns and add as product.
```

```
        While (cStart < cEnd)
            p = New Product
            Dim X As Range
            X = VarRange.Offset(0, cStart -
             VarRange.Column).Resize(VarRange.Rows.count, 1)

            Dim s As String
            s = "nada"
            Dim dd As Double
            dd = 1.0#
            Dim bABC As Boolean
            bABC = True
            Call p.setInitValues(X, bABC, s, dd)
            Products.Add(p)
            p.addToSummary(cStart - VarRange.Column)
            cStart = cStart + 1
        End While

        StatusLabel = "Calculating correlation matrix."

        ''add the correlation matrix
        Sheets("Summary").Select()
        Range("A1").Select()
        Selection.End(xlDown).Select()
        ActiveCell.Offset(4, 0).Range("A1").Select()
        ActiveCell.FormulaR1C1 = "Correlation matrix"
        For i = 1 To Products.count
            ActiveCell.Offset(1, 0).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "" & i
        Next i
        Range("A1").Select()
        Selection.End(xlDown).Select()
        ActiveCell.Offset(4, 0).Range("A1").Select()
        ActiveCell.FormulaR1C1 = "Correlation matrix"
        For i = 1 To Products.count
            ActiveCell.Offset(0, 1).Range("A1").Select()
            ActiveCell.FormulaR1C1 = "" & i
        Next i

        Range("A1").Select()
        Selection.End(xlDown).Select()
        ActiveCell.Offset(4, 0).Range("A1").Select()

        ReDim CorrelM(1 To Products.count, 1 To Products.count)
        For i = 1 To Products.count
            Dim p1 As Product
            p1 = Products(i)
            For j = i To Products.count
                If i = j Then
                    ''return 1
                    CorrelM(i, j) = 1
                Else
                    Dim p2 As Product
                    p2 = Products(j)
                    ''return correlation between ranges
                    CorrelM(i, j) = Application.WorksheetFunction.Correl(p1.Data,
                      p2.Data)
                End If
                ActiveCell.Offset(i, j).Range("A1").Select()
                ActiveCell.FormulaR1C1 = CorrelM(i, j)
                ActiveCell.Offset(-i, -j).Range("A1").Select()
                ActiveCell.Offset(j, i).Range("A1").Select()
                ActiveCell.FormulaR1C1 = CorrelM(i, j)
                ActiveCell.Offset(-j, -i).Range("A1").Select()
            Next j
        Next i

        StatusLabel = "Finished selecting products."
```

```

    For Each subRange In VarRange.Areas
        If c > 0 Then
            AreasStr = AreasStr & ";" & subRange.Address(0, 0)
        Else
            AreasStr = AreasStr & subRange.Address(0, 0)
        End If
        c = c + 1
    Next subRange

    TextBox1.Text = AreasStr

End Sub

Private Sub MultiPage1_Change()
    If MultiPage1.Value = 0 Then
        CommandButton2.Enabled = False
        CommandButton3.Enabled = True
        UserForm1.Caption = "VaR: Make selection of products - Step 1 of 4"

    ElseIf MultiPage1.Value = 1 Then
        CommandButton2.Enabled = True
        CommandButton3.Enabled = True
        UserForm1.Caption = "VaR: Choose methods- Step 2 of 4"

    ElseIf MultiPage1.Value = 2 Then
        CommandButton2.Enabled = True
        CommandButton3.Enabled = True
        UserForm1.Caption = "VaR: Set parameters - Step 3 of 4"

    ElseIf MultiPage1.Value = 3 Then
        CommandButton2.Enabled = True
        CommandButton3.Enabled = False
        UserForm1.Caption = "VaR: Review - Step 4 of 4"
        GenerateReview()

    Else
        MsgBox("Error: invalid page value")

    End If
End Sub


Private Sub UserForm_Initialize()
    UserForm1.Caption = "VaR: Make selection of products - Step 1 of 4"
    txtAlpha.Value = CONSTALPHA
    txtPeriod.Value = CONSTDAYS
    txtBacktest.Value = CONSTBACKTEST
    txtRuns.Value = CONSTRUNS
    txtDF.Value = CONSTDF
    CommandButton2.Enabled = False
    MultiPage1.Value = 0
End Sub

Private Sub GenerateReview()
    Review.Text = "The following methods are chosen:" & vbCrLf
    Dim count As Integer
    count = 0

    If chkAnalytical.Value = True Then
        Review.Text = Review.Text & "Analytical VaR"
        count = 1
    End If
    If chkHistoric.Value = True Then
        If (count = 0) Then
            Review.Text = Review.Text & "Historical simulation"
        Else
            Review.Text = Review.Text & vbCrLf & "Historical simulation"
```

```
            End If
            count = 1
        End If
        If chkMCNormal.Value = True Then
            If (count = 0) Then
                Review.Text = Review.Text & "Monte Carlo normal"
            Else
                Review.Text = Review.Text & vbCrLf & "Monte Carlo normal"
            End If
            count = 1
        End If
        If chkMCStudentT.Value = True Then
            If (count = 0) Then
                Review.Text = Review.Text & "Monte Carlo student t"
            Else
                Review.Text = Review.Text & vbCrLf & "Monte Carlo student t"
            End If
            count = 1
        End If
        If chkMCNormalIS.Value = True Then
            If (count = 0) Then
                Review.Text = Review.Text & "Monte Carlo delta-gamma"
            Else
                Review.Text = Review.Text & vbCrLf & "Monte Carlo delta-gamma"
            End If
            count = 1
        End If

        Review.Text = Review.Text & vbCrLf & vbCrLf & "The following parameters are
         specified:" & vbCrLf

        Review.Text = Review.Text & "Alpha level:" & vbTab & txtAlpha.Text & vbCrLf
        Review.Text = Review.Text & "Historical period:" & vbTab & txtPeriod.Text &
         vbCrLf
        Review.Text = Review.Text & "Simulation runs:" & vbTab & txtRuns.Text & vbCrLf
        Review.Text = Review.Text & "Backtest period:" & vbTab & txtBacktest.Text &
         vbCrLf
        If (chkAutoDF.Value = True) Then
            Review.Text = Review.Text & "Degree of freedom:" & vbTab & "Auto"
        Else
            Review.Text = Review.Text & "Degree of freedom:" & vbTab & txtDF.Text
        End If

    End Sub
```

Methods for estimating VaR

```
''calculate analytical VaR
''get standard deviation for the multinormal distr.
Public Function AnalyticalVaR(ByVal alpha As Double, ByVal CorrelM As Object, ByVal
ProductsTmp As Object, ByVal cLevel As Double) As VarEstimate
    Dim anVar, anVar2 As Double
    anVar = 0
    For Each p In ProductsTmp
        Dim a1, var1 As Double
        a1 = p.WeightUSD
        var1 = p.standardDev
        anVar = anVar + ((a1 * a1) * (var1 * var1))
    Next p

    anVar2 = 0
    For ii = 1 To ProductsTmp.count
        For jj = 1 To ii - 1
            Dim ai, vari, rho, aj, varj As Double
            ai = ProductsTmp(ii).WeightUSD
            aj = ProductsTmp(jj).WeightUSD
            vari = ProductsTmp(ii).standardDev
```

```
                varj = ProductsTmp(jj).standardDev
                rho = CorrelM(jj, ii)
                anVar2 = anVar2 + ai * aj * vari * varj * rho
            Next jj
        Next ii

        anVar = anVar + (2 * anVar2)
        anVar = Sqr(anVar)

        If Not anVar = 0 Then
            anVar = Application.WorksheetFunction.NormInv((1 - alpha), 0, anVar)
        End If

        AnalyticalVaR = New VarEstimate
        AnalyticalVaR.VaR = -anVar
    End Function

    ''calculate historical simulation
    ''first copy all pricehistory into a new sheet
    ''then calculate every possible scenario for a day
    ''sort the data and find the correct alpha-level VaR
    Private Function HistoricalSim(ByVal alpha As Double, ByVal ProductsTmp As Object)
    As VarEstimate
        AddSheet("Historical simulation")
        ClearSheet("Historical simulation ")

        Sheets("Historical simulation ").Select()
        Range("A1").Select()

        ActiveCell.FormulaR1C1 = "Day"
        ActiveCell.Offset(0, 1).Range("A1").Select()
        For Each p In Products
            ActiveCell.FormulaR1C1 = p.name
            ActiveCell.Offset(0, 1).Range("A1").Select()
        Next p
        ActiveCell.FormulaR1C1 = "Portfolio"
        ActiveCell.Offset(1, -(ProductsTmp.count + 1)).Range("A1").Select()

        Dim pTmp As Product
        pTmp = ProductsTmp(1)

        Dim ii As Long
        For ii = 1 To pTmp.Observations - 1
            Dim Sum As Double
            Dim chn As Double
            Sum = 0
            ActiveCell.FormulaR1C1 = ii
            ActiveCell.Offset(0, 1).Range("A1").Select()
            For Each p In ProductsTmp
                chn = p.PrcChange(ii) * p.WeightUSD
                Sum = Sum + chn
                ActiveCell.FormulaR1C1 = p.PrcChange(ii)
                ActiveCell.Offset(0, 1).Range("A1").Select()
            Next p
            ActiveCell.FormulaR1C1 = Sum
            ActiveCell.Offset(1, -(ProductsTmp.count + 1)).Range("A1").Select()
        Next ii

        Dim SortCell
        Range("B1").Select()
        Selection.End(xlToRight).Select()
        SortCell = ActiveCell.Address
        Range("B2").Select()
        Range(Selection, Selection.End(xlToRight)).Select()
        Range(Selection, Selection.End(xlDown)).Select()
        Selection.Sort(Key1:=Range(SortCell), Order1:=xlDescending, Header:=xlGuess, _
            OrderCustom:=1, MatchCase:=False, Orientation:=xlTopToBottom, _
            DataOption1:=xlSortNormal)
```

```

    ''Display VaR HisSim
    Dim VaR As Double
    Dim VaRDay As Double
    Dim A, b As Double
    Dim bWeight As Double
    VaRDay = (pTmp.Observations - 1) * (1 - alpha)
    Range(SortCell).Select()
    bWeight = VaRDay - Application.WorksheetFunction.Floor(VaRDay, 1)
    ActiveCell.Offset(Application.WorksheetFunction.Floor(VaRDay, 1),
     0).Range("A1").Select()
    A = ActiveCell.Value
    ActiveCell.Offset(1, 0).Range("A1").Select()
    b = ActiveCell.Value
    BCell = ActiveCell.Address
    On Error Resume Next
    VaR = (A * (1 - bWeight)) + (b * bWeight)
    If Err.Number > 0 Then
        VaR = b
    End If

    HistoricalSim = New VarEstimate
    HistoricalSim.VaR = VaR

    ''find maximum loss:
    Range(SortCell).Select()
    Selection.End(xlDown).Select()
    HistoricalSim.MaxLoss = ActiveCell.Value2
    HistoricalSim.MeanExcessLoss =
Application.WorksheetFunction.Average(Range(BCell, ActiveCell.Address))
End Function

''call Monte Carlo with simulation of every product
''change in portfolio = sum of changes in products
''use normal assumption with bNormal = True
''use student t distr. with bNormal = False
''use copula-t distr with bDfAuto = true
Private Function MonteCarlo(ByVal alpha As Double, ByVal runs As Integer, ByVal
CorrelM As Object, ByVal ProductsTmp As Object, ByVal df As Integer, ByVal bNormal
As Boolean, ByVal bDfAuto As Boolean) As VarEstimate
    Randomize()
    Dim p As Product

    If (bNormal = True) Then
        AddSheet("MCNormal")
        ClearSheet("MCNormal")
        Sheets("MCNormal").Select()
    Else
        AddSheet("MC Student t")
        ClearSheet("MC Student t")
        Sheets("MC Student t").Select()
    End If
    Range("A1").Select()
    ActiveCell.FormulaR1C1 = "Sim#"
    ActiveCell.Offset(0, 1).Range("A1").Select()

    Dim pHistory() As Double
    ReDim pHistory(1 To ProductsTmp(1).Observations - 1)

    If bDfAuto = True Then
        last = sumProductValueAtTmp(1, ProductsTmp)
        For i = 1 To ProductsTmp(1).Observations - 2
            ''calculate portfolio history
            nxt = sumProductValueAtTmp(i + 1, ProductsTmp)
            pHistory(i) = (nxt - last) / last
            last = nxt
        Next i
    End If
```

```
Dim copulaT() As Double
ReDim copulaT(1 To ProductsTmp.count)
Dim copulaT2() As Double
ReDim copulaT2(1 To ProductsTmp.count)
cnt = 1
avgdf = 0
For Each p In ProductsTmp
    ActiveCell.FormulaR1C1 = p.name
    ActiveCell.Offset(0, 1).Range("A1").Select()

    ''create copulaT-matrix
    If bDfAuto = True Then
        Dim abcd() As Double
        ReDim abcd(1 To p.Observations - 1)
        Dim iii As Long
        For iii = 1.0# To p.Observations - 1
            abcd(iii) = ProductsTmp(cnt).PrcChange(iii)
        Next iii
        copulaT(cnt) = Application.WorksheetFunction.Min(MAXDF,
          Application.WorksheetFunction.Round(4 + (6 / p.Kurtosis), 0))
        avgdf = avgdf + copulaT(cnt)
        cnt = cnt + 1
    End If
Next p

If (bDfAuto = True) Then
    avgdf = Application.WorksheetFunction.Floor((avgdf / ProductsTmp.count), 1)
    df = avgdf
End If
ActiveCell.FormulaR1C1 = "Portfolio"
ActiveCell.Offset(1, -(ProductsTmp.count + 1)).Range("A1").Select()

m = runs
N = ProductsTmp.count
ReDim zz(1 To N, 1 To m)
ReDim s(1, 1 To m)
For j = 1 To m
    For i = 1 To N
        argx = Rnd()
        While (argx = 0)
            argx = Rnd()
        End While
        zz(i, j) = Application.WorksheetFunction.NormSInv(argx)
    Next i
    argz = Rnd()
    While (argz < 0.000001)
        argz = Rnd()
    End While
    s(1, j) = Application.WorksheetFunction.ChiInv(argz, df) ''df
Next j

X = MultiplyTwoMatrices(True, Cholesky(CorrelM), zz)

Dim ii As Long, abc As Integer
For ii = 1 To runs
    Dim Sum As Double
    Dim chn As Double
    Sum = 0
    ActiveCell.FormulaR1C1 = ii
    ActiveCell.Offset(0, 1).Range("A1").Select()
    abc = 1
    For Each p In ProductsTmp
        If bNormal = False Then
            If bDfAuto = True Then
                chn = 0 + p.WeightUSD * p.standardDev * TInvCdf(TCdf(X(abc, ii)
                    * Sqr(df / s(1, ii)), df), df)
            Else
```

```
                    chn = 0 + p.WeightUSD * p.standardDev * TInvCdf(TCdf(X(abc, ii)
                        * Sqr(df / s(1, ii)), df), df)
                End If
            Else
                chn = 0 + p.WeightUSD * p.standardDev * X(abc, ii)
            End If
            Sum = Sum + chn
            ActiveCell.FormulaR1C1 = chn
            ActiveCell.Offset(0, 1).Range("A1").Select()
            abc = abc + 1
        Next p
        ActiveCell.FormulaR1C1 = Sum
        ActiveCell.Offset(1, -(ProductsTmp.count + 1)).Range("A1").Select()
    Next ii

    Dim SortCell
    Range("B1").Select()
    Selection.End(xlToRight).Select()
    SortCell = ActiveCell.Address
    Range("B2").Select()
    Range(Selection, Selection.End(xlToRight)).Select()
    Range(Selection, Selection.End(xlDown)).Select()
    Selection.Sort(Key1:=Range(SortCell), Order1:=xlDescending, Header:=xlGuess, _
        OrderCustom:=1, MatchCase:=False, Orientation:=xlTopToBottom, _
        DataOption1:=xlSortNormal)

    ''Display VaR MC Normal
    Dim VaR As Double
    Dim VaRDay As Double
    Dim A, b As Double
    Dim bWeight As Double
    VaRDay = (runs) * (1 - alpha)
    Range(SortCell).Select()
    bWeight = VaRDay - Application.WorksheetFunction.Floor(VaRDay, 1)
    ActiveCell.Offset(Application.WorksheetFunction.Floor(VaRDay, 1),
     0).Range("A1").Select()
    A = ActiveCell.Value
    ActiveCell.Offset(1, 0).Range("A1").Select()
    b = ActiveCell.Value
    BCell = ActiveCell.Address
    VaR = (A * (1 - bWeight)) + (b * bWeight)
    MonteCarlo = New VarEstimate
    MonteCarlo.VaR = VaR

    ''find maximum loss:
    Range(SortCell).Select()
    Selection.End(xlDown).Select()
    MonteCarlo.MaxLoss = ActiveCell.Value2
    MonteCarlo.MeanExcessLoss = Application.WorksheetFunction.Average(Range(BCell,
ActiveCell.Address))
End Function

Private Function DeltaGamma(ByVal alpha As Double, ByVal runs As Integer, ByVal
CorrelM As Object, ByVal ProductsTmp As Object) As VarEstimate
    ''create necessities for ImportanceSampling
    Dim VaRdelta, VaRgamma, VaRcovmatrix As Object
    ReDim VaRcovmatrix(ProductsTmp.count, ProductsTmp.count)
    ReDim VaRdelta(ProductsTmp.count)
    ReDim VaRgamma(ProductsTmp.count, ProductsTmp.count)
    VaRcovmatrix = CorrelM
    ''this is due to linearity
    For i = 1 To ProductsTmp.count
        For j = 1 To ProductsTmp.count
            VaRgamma(i, j) = 0
        Next j
        VaRdelta(i) = ProductsTmp(i).CalculateDelta()
    Next i

```

```
    DeltaGamma = New VarEstimate
    Call DeltaGamma.Copy(MonteCarloDGNormal(VaRdelta, VaRgamma, VaRcovmatrix, "IS",
alpha, 1, runs, ProductsTmp))
End Function

Private Function MonteCarloDGNormal(ByVal VaRdelta As Object, ByVal VaRgamma As
Object, _
    ByVal VaRcovmatrix As Object, ByVal smethod As String, ByVal optAlpha As
Double, ByVal optDays As Integer, _
    ByVal optRuns As Integer, ByVal ProductsTmp As Object) As VarEstimate
    Randomize()

    Dim p As Product
    Dim N As Integer
    printCorrel(VaRcovmatrix)
    N = ProductsTmp.count

    ''create a matrix of random numbers according to a standard normal
distribution.
    Dim z As Object
    ReDim z(N, optRuns)
    z = normal(N, optRuns)

    ''cholesky, send covarianceMatrix as input
    ''"expand" the covarianceMatrix first
    For i = 1 To ProductsTmp.count
        For j = i + 1 To ProductsTmp.count
            VaRcovmatrix(j, i) = VaRcovmatrix(i, j)
        Next j
    Next i
    'divide by days:
    VaRcovmatrix = MultiplyScalar(VaRcovmatrix, optDays / 365)
    A = Cholesky(VaRcovmatrix)

    ''Importance Sampling!
    ''1. Decompostion Process
    tem = MultiplyScalar(MultiplyTwoMatrices(True, Transpose(A),
     MultiplyTwoMatrices(True, VaRgamma, A)), -0.5)
    'vx = eigsm(tem)
    v = MatEigenvector_Jacobi(tem)
    lamda = MatEigenvalue_Jacobi(tem)
    c = MultiplyTwoMatrices(True, A, v)
    b = Transpose(MultiplyTwoMatrices(True, MultiplyScalar(Transpose(VaRdelta), -
     1), c))

    ''2. Use Newton-Raphson method to find solution for theta
    Dim theta As Double
    theta = 0
    ac = 1
    i = 1
    di = 1

    ''use Delta normal method to set initial guess of x
    X = -qfn(optAlpha) * Sqr(columnSum(theta, lamda, b, "B2"))
    While ((ac <= -0.0001 Or ac >= 0.0001) And i <= 1000)
        ac = columnSum(theta, lamda, b, "AC") - X
        di = columnSum(theta, lamda, b, "DI")
        theta = theta - ac / di
        i = i + 1
    End While

    ''3. set sig and mu
    sig = GaussianElimination(1, AddOrSubtract(False, diag(matrix(N, 1)),
     MultiplyScalar(diag(lamda), 2 * theta)), 0)
    mu = MultiplyTwoMatrices(True, MultiplyScalar(sig, theta), b)

    ''4. Simulation
```

```
z = AddOrSubtractSpecial(True, mu, MultiplyTwoMatrices(True, sqrtMatrice(sig),
 z))
cs = MultiplyTwoMatrices(True, c, z)
L = MultiplyScalar(matrix(optRuns, 1), 0)
w = MultiplyScalar(matrix(optRuns, 1), 0)

''To calculate the value for moment generating function
psi = 0.5 * columnSum(theta, lamda, b, "psi")
i = 1
tmpDelta = MultiplyScalar(Transpose(VaRdelta), -1)
tmpGamma = MultiplyScalar(VaRgamma, -1)
While (i <= optRuns)
    ''sum vector calculations manually:
    Dim tmpQ1, tmpQ2 As Double
    tmpQ1 = 0
    tmpQ2 = 0
    For jkl = 1 To MatrixDim(tmpDelta)
        tmpQ1 = tmpQ1 + (tmpDelta(1, jkl) * cs(jkl, i))
        tmpQ2 = tmpQ2 + (tmpGamma(jkl, 1) * cs(jkl, i) * cs(jkl, i))
    Next jkl
    L(i, 1) = tmpQ1 + tmpQ2
    w(i, 1) = Exp(-theta * L(i, 1) + psi)
    i = i + 1
End While
Dim WL As Object
WL = concatArrays(w, L)
Wtem = MultiplyScalar(cumsum(Sort(w, 1)), 1 / optRuns)
WL = Sort(WL, 1)
On Error Resume Next
nrow = UBound(paf(Wtem, optAlpha), 1)
If Err.Number > 0 Then
    nrow = optRuns
End If
VaRMC = WL(nrow, 2)

AddSheet("MCDeltaGamma")
ClearSheet("MCDeltaGamma")
Sheets("MCDeltaGamma").Select()
Range("A1").Select()
ActiveCell.FormulaR1C1 = "Sim#"
ActiveCell.Offset(0, 1).Range("A1").Select()
ActiveCell.FormulaR1C1 = "Likelihood"
ActiveCell.Offset(0, 1).Range("A1").Select()
ActiveCell.FormulaR1C1 = "Portfolio"
ActiveCell.Offset(1, -2).Range("A1").Select()
'
Dim ii As Long
For ii = 1 To optRuns
    ActiveCell.FormulaR1C1 = ii
    ActiveCell.Offset(0, 1).Range("A1").Select()
    ActiveCell.FormulaR1C1 = WL(ii, 1)
    ActiveCell.Offset(0, 1).Range("A1").Select()
    ActiveCell.FormulaR1C1 = WL(ii, 2)
    ActiveCell.Offset(1, -2).Range("A1").Select()
Next ii
'
Dim SortCell
Range("B1").Select()
SortCell = ActiveCell.Address
Range("B2").Select()
Range(Selection, Selection.End(xlToRight)).Select()
Range(Selection, Selection.End(xlDown)).Select()
Selection.Sort(Key1:=Range(SortCell), Order1:=xlDescending, Header:=xlGuess, _
    OrderCustom:=1, MatchCase:=False, Orientation:=xlTopToBottom, _
    DataOption1:=xlSortNormal)

MonteCarloDGNormal = New VarEstimate
MonteCarloDGNormal.VaR = VaRMC
```

```

    ''find maximum loss:
    Range(SortCell).Select()
    ActiveCell.Offset(optRuns - nrow, 1).Range("A1").Select()
    BCell = ActiveCell.Address
    Range(SortCell).Select()
    Selection.End(xlDown).Select()
    ActiveCell.Offset(0, 1).Range("A1").Select()
    MonteCarloDGNormal.MaxLoss = ActiveCell.Value2
    MonteCarloDGNormal.MeanExcessLoss =
Application.WorksheetFunction.Average(Range(BCell, ActiveCell.Address))
End Function

```

A.3 Matrix and linear algebra

The Gaussian eliminiation code to solve Ax = b was provided by Urroz (see http://www.neng.usu.edu/cee/faculty/gurro/). The linear algebra with calculation of eigenvector and eigenvalue is from the free source code at http://digilander.libero.it/foxes/SoftwareDownload.htm.

Matrix calculations

```
Option Explicit On
Option Base 1

Public Function GaussianElimination(ByVal index As Integer, ByVal A As Object,
ByVal b As Object) As Object
    '***************************************************************************
    ' This subroutine calculates the matrix X with values X = [x1 x2.. xnB]
    ' where the xi's are the right-hand side vectors of the matrix equations
    ' A*x1 = b1; A*x2 = b2; ...; A*xnB = bnB.
    ' The subroutine uses Gaussian elimination, and can be used to calculate
    ' a matrix inverse too.
    ' The particular operation performed by the subroutine depends on the value
    ' of the parameter "index", as follows:
    '   index = 1  --> inverse
    '   index = 2  --> determinant
    '   index = 3  --> linear system solution
    '***************************************************************************
    '
    'Declaration of variables
    Dim i, j, k As Integer
    Dim nA As Integer, mA As Integer
    Dim nB As Integer, mB As Integer
    Dim temp As Object
    nA = UBound(A, 1)   ''rows A
    If Not (index = 1 Or index = 2) Then
        nB = UBound(b, 1)   ''rows B
        mB = MatrixDim(b)   ''columns B
    Else
        nB = 0
        mB = 0
    End If
    mA = MatrixDim(A)   ''columns A
    ReDim temp(nA, mA)

    Dim X, xR, aMax, s As Object
    ReDim X(nA, mA)
    ReDim xR(nA, mA)
    ReDim aMax(nA)
    ReDim s(nA)
    Dim detA As Double, factor As Double
    Dim Order As Object, Otemp As Integer
    ReDim Order(nA)
    Dim Sum As Double, epsilon As Double
    Dim nExchanges As Integer
    Dim scaleFlag As Boolean
```

```
        'Making sure matrix A is a square matrix
        If nA <> mA Then
            Error = "Matrix A must be a square matrix to calculate"
            If index = 1 Then
                Error = Error + " the inverse matrix."
            ElseIf index = 2 Then
                Error = Error + " the determinant."
            Else
                Error = Error + " x from A*x = B."
            End If
            Exit Function
        End If
        '
        'Read Matrix B if needed, otherwise load identity matrix
        If index = 1 Or index = 2 Then
            ReDim b(nA, mA)
            If index = 1 Then
                nB = nA : mB = mA
            Else
                nB = nA : mB = 1
            End If
            For i = 1 To nA
                For j = 1 To mB
                    If i = j Then
                        b(i, j) = 1.0#
                    Else
                        b(i, j) = 0.0#
                    End If
                Next j
            Next i
        Else
            b = b
        End If

        ' Check that matrices are compatible
        If nA <> nB Then
            Error = "Matrices A and B must have the same number of"
            Error = Error + "rows in order to solve for the linear system Ax=B."
            Exit Function
        End If

        'Create the Order vector and load default values of aMax(i)
        For i = 1 To nA
            Order(i) = i
            aMax(i) = 1.0#
        Next i

        'Augmenting Matrix A
        ReDim Preserve A(nA, nA + mB)
        For i = 1 To nA
            For j = 1 To mB
                A(i, j + nA) = b(i, j)
            Next j
        Next i
        nA = UBound(A, 1)

        'Scaling
        ReDim s(nA)
        For i = 1 To nA
            s(i) = 0.0#
            For j = 1 To nA
                s(i) = s(i) + Abs(A(i, j))
            Next j
        Next i

        scaleFlag = False
```

```
        For i = 1 To (nA - 1)
            For j = (i + 1) To (nA)
                If s(i) > factor * s(j) Or s(j) > factor * s(i) Then
                    scaleFlag = True
                End If
            Next j
        Next i

        If scaleFlag Then
            For i = 1 To nA
                aMax(i) = 0.0000000001
                For j = 1 To (nA + mB)
                    If (A(i, j) > aMax(i)) Then
                        aMax(i) = A(i, j)
                    End If
                Next j
            Next i

            For i = 1 To nA
                For j = 1 To (nA + mB)
                    A(i, j) = A(i, j) / aMax(i)
                Next j
            Next i
        End If

        'Elimination procedure applied to rows 2 to n to fill with
        'zeros lower triangular part of matrix
        nExchanges = 0
        For k = 1 To (nA - 1)

            'Pivoting implemented in the next If statement
            For i = (k + 1) To nA
                If Abs(A(Order(i), k)) > Abs(A(Order(k), k)) Then
                    nExchanges = nExchanges + 1
                    Otemp = Order(i)
                    Order(i) = Order(k)
                    Order(k) = Otemp
                End If
            Next i

            For i = (k + 1) To nA
                For j = (k + 1) To (nA + mB)
                    A(Order(i), j) = A(Order(i), j) - A(Order(i), k) * A(Order(k), j) /
                      A(Order(k), k)
                Next j
            Next i
        Next k

        'Calculate and print determinant

        detA = 1.0#
        For i = 1 To nA
            detA = detA * A(Order(i), i) * aMax(i)
        Next i
        detA = (-1) ^ nExchanges * detA
        If index = 2 Then
            GaussianElimination = detA
            Exit Function
        End If

        'If the determinant is small, then the matrix is singular
        If Abs(detA) < epsilon Then
            MsgBox("Matrix is singular.")
            If index = 3 Then
                Error = " No solution is possible."
            ElseIf index = 1 Then
                Error = " Inverse does not exist."
            End If
```

```
        Exit Function
    End If

    'Calculating solutions
    For j = 1 To mB
        X(Order(nA), j) = A(Order(nA), j + nA) / A(Order(nA), nA)
        For i = (nA - 1) To 1 Step -1
            Sum = 0.0#
            For k = (i + 1) To nA
                Sum = Sum + A(Order(i), k) * X(Order(k), j)
            Next k
            X(Order(i), j) = (A(Order(i), j + nA) - Sum) / A(Order(i), i)
        Next i
    Next j

    GaussianElimination = X

End Function

Public Function MultiplyScalar(ByVal A As Object, ByVal c As Double) As Object
    '**************************************************************************
'This function calculates the multiplication of Matrix A with a scalar c
'**************************************************************************

    Dim nA As Integer, mA As Integer
    Dim cA As Object
    nA = UBound(A, 1)   'A.Count
    mA = MatrixDim(A)   'A.Columns.Count
    If (mA = 0) Then
        ReDim cA(nA)
    Else
        ReDim cA(nA, mA)
    End If
    '    Dim c As Double
    Dim i As Integer, j As Integer

    'Multiply all elements of the matrix by c
    For i = 1 To nA
        For j = 1 To mA
            cA(i, j) = c * A(i, j)
        Next j
        If (mA = 0) Then
            cA(i) = c * A(i)
        End If
    Next i

    MultiplyScalar = cA
End Function

Public Function MatrixSqrt(ByVal A As Object) As Object
    '**************************************************************************
    'This function takes the square root of Matrix
    '**************************************************************************
    '
    'Declaration of variables
    Dim nA As Integer, mA As Integer
    Dim cA As Object
    nA = UBound(A, 1)   'A.Count
    mA = MatrixDim(A)   'A.Columns.Count
    ReDim cA(nA, mA)
    Dim i As Integer, j As Integer

    'Multiply all elements of the matrix by c
    For i = 1 To nA
        For j = 1 To mA
            cA(i, j) = Sqr(A(i, j))
        Next j
    Next i
```

```vb

    MatrixSqrt = cA
End Function

Public Function MatrixPower(ByVal A As Object, ByVal p As Integer) As Object
    '***************************************************************************
    'This function takes the power of Matrix
    '***************************************************************************
    '
    'Declaration of variables
    Dim nA As Integer, mA As Integer
    Dim cA As Object
    nA = UBound(A, 1)    'A.Count
    mA = MatrixDim(A)    'A.Columns.Count
    If (mA = 0) Then
        ReDim cA(nA)
    Else
        ReDim cA(nA, mA)
    End If
    Dim i As Integer, j As Integer

    'Multiply all elements of the matrix with the power p
    For i = 1 To nA
        If (mA = 0) Then
            cA(i) = A(i) ^ p
        Else
            For j = 1 To mA
                cA(i, j) = A(i, j) ^ p
            Next j
        End If
    Next i

    MatrixPower = cA
End Function

Public Function Transpose(ByVal A As Object) As Object
    '***************************************************************************
    'This function returns the transposed matrix of the input matrix
    '***************************************************************************

    'Declaration of variables
    Dim nA As Integer, mA As Integer
    Dim cA As Object
    nA = UBound(A, 1)    'A.Count
    mA = MatrixDim(A)    'A.Columns.Count
    Dim AT As Object
    If mA = 0 Then
        ReDim AT(1, nA)
    Else
        ReDim AT(mA, nA)
    End If
    Dim i As Integer, j As Integer

    '
    'Produce transpose
    If mA = 0 Then
        For i = 1 To nA
            AT(1, i) = A(i)
        Next i
    Else
        For i = 1 To nA
            For j = 1 To mA
                AT(j, i) = A(i, j)
            Next j
        Next i
    End If
    Transpose = AT

```

```
End Function

Public Function AddOrSubtract(ByVal index As Boolean, ByVal A As Object, ByVal b As
Object) As Object
    '***************************************************************************
    ' This subroutine adds two matrices A and B
    ' If index = True, then perform addition.
    ' If index = False, then perform subtraction.
    '***************************************************************************
    '
    'Declaration of variables
    Dim c As Object
    Dim nA As Integer, mA As Integer
    On Error Resume Next
    nA = UBound(A, 1)    'A.Count
    If (Err.Number > 0) Then
        AddOrSubtract = -1
        Exit Function
    End If

    mA = MatrixDim(A)    'A.Columns.Count
    Dim nB As Integer, mB As Integer
    nB = UBound(b, 1)    'B.Count
    mB = MatrixDim(b)    'B.Columns.Count
    If (mB = 0) Then
        ReDim c(nA)
    Else
        ReDim c(nA, mB)
    End If
    Dim i As Integer, j As Integer
    '
    ' Check that matrices are compatible

    If nA <> nB Or mA <> mB Then
        MsgBox("Matrices A and B are not compatible for addition or subtraction.")
        Exit Function
    End If

    'Calculate sum/subtraction of matrices
    For i = 1 To nA
        For j = 1 To mA
            If index Then
                c(i, j) = A(i, j) + b(i, j)
            Else
                c(i, j) = A(i, j) - b(i, j)
            End If
        Next j
        If mA = 0 Then
            If index Then
                c(i) = A(i) + b(i)
            Else
                c(i) = A(i) - b(i)
            End If
        End If
    Next i
    '

    AddOrSubtract = c
End Function

Public Function AddOrSubtractSpecial(ByVal index As Boolean, ByVal A As Object,
ByVal b As Object) As Object
    '***************************************************************************
    ' This subroutine adds two matrices A and B of different sizes
    ' number of rows must be the same and matrix A is only one dimension
    ' If index = True, then perform addition.
    ' If index = False, then perform subtraction.
'***************************************************************************
```

```
    '
    'Declaration of variables
    Dim c As Object
    Dim nA As Integer, mA As Integer
    nA = UBound(A, 1)    'A.Count
    mA = MatrixDim(A)    'A.Columns.Count
    Dim nB As Integer, mB As Integer
    nB = UBound(b, 1)    'B.Count
    mB = MatrixDim(b)    'B.Columns.Count
    ReDim c(nA, mB)
    Dim i As Integer, j As Integer
    '
    ' Check that matrices are compatible

    If nA <> nB Or mA > 1 Then
        MsgBox("Matrices A and B are not compatible for this special addition or
          special subtraction.")
        Exit Function
    End If

    'Calculate sum/subtraction of matrices
    For i = 1 To nA
        For j = 1 To mB
            If index Then
                c(i, j) = A(i, 1) + b(i, j)
            Else
                c(i, j) = A(i, 1) - b(i, j)
            End If
        Next j
    Next i

    AddOrSubtractSpecial = c
End Function

Public Function MultiplyTwoMatrices(ByVal index As Boolean, ByVal A As Object,
ByVal b As Object) As Object
'***************************************************************************
    ' This subroutine adds two matrices A and B
    ' If index = True, then calculate A * B.
    ' If index = False, then calculate B * A.
'***************************************************************************
    '
    'Declaration of variables
    Dim matrixA As String, matrixB As String
    Dim c As Object
    Dim nA As Integer, mA As Integer, mAix As Integer
    Dim nB As Integer, mB As Integer, mBix As Integer
    Dim i As Integer, j As Integer, k As Integer
    '
    nA = UBound(A, 1)    ''rows A
    nB = UBound(b, 1)    ''rows B
    mB = MatrixDim(b)    ''columns B
    mA = MatrixDim(A)    ''columns A
    If (mB = 0) Then
        mB = 1
    End If
    If (mA = 0) Then
        mA = 1
    End If
    ' Check that matrices are compatible
    If index Then
        ReDim c(nA, mB)
        If mA <> nB Then
            MsgBox("Matrices A and B are not compatible for multiplication A*B.")
            Exit Function
        End If
    Else
        ReDim c(nB, mA)
```

```
        If mB <> nA Then
            MsgBox("Matrices A and B are not compatible for multiplication B*A.")
            Exit Function
        End If
    End If

    'Calculate multiplication of matrices
    mBix = MatrixDim(b)   ''columns B
    mAix = MatrixDim(A)   ''columns A
    If index Then
        If mBix = 0 Then
            For i = 1 To nA
                For j = 1 To mB
                    c(i, j) = 0.0#
                    For k = 1 To mA
                        c(i, j) = c(i, j) + A(i, k) * b(k)
                    Next k
                Next j
            Next i
        Else
            For i = 1 To nA
                For j = 1 To mB
                    c(i, j) = 0.0#
                    For k = 1 To mA
                        c(i, j) = c(i, j) + A(i, k) * b(k, j)
                    Next k
                Next j
            Next i
        End If
    Else
        If mAix = 0 Then
            For i = 1 To nB
                For j = 1 To mA
                    c(i, j) = 0.0#
                    For k = 1 To mB
                        c(i, j) = c(i, j) + b(i, k) * A(k)
                    Next k
                Next j
            Next i
        Else
            For i = 1 To nB
                For j = 1 To mA
                    c(i, j) = 0.0#
                    For k = 1 To mB
                        c(i, j) = c(i, j) + b(i, k) * A(k, j)
                    Next k
                Next j
            Next i
        End If
    End If

    MultiplyTwoMatrices = c
End Function
Public Function MultiplyTwoMatricesByRows(ByVal index As Boolean, ByVal A As
Object, ByVal b As Object) As Object
'***************************************************************************
    ' This subroutine adds two matrices A and B
    ' If index = True, then calculate A * B.
    ' If index = False, then calculate B * A.
'***************************************************************************
    '
    'Declaration of variables
    Dim matrixA As String, matrixB As String
    Dim c As Object
    Dim nA As Integer, mA As Integer, mAix As Integer
    Dim nB As Integer, mB As Integer, mBix As Integer
    Dim i As Integer, j As Integer, k As Integer

```

```
    nA = UBound(A, 1)   ''rows A
    nB = UBound(b, 1)   ''rows B
    mB = MatrixDim(b)   ''columns B
    mA = MatrixDim(A)   ''columns A
    If (mB = 0) Then
        ReDim c(nB)
    Else
        ReDim c(nB, mB)
    End If

    If (mB <> mA) Or (nB <> nA) Then
        MsgBox("Matrices A and B are not compatible for multiplication B*A.")
        Exit Function
    End If

    For i = 1 To nA
        If (mB = 0) Then
            c(i) = A(i) * b(i)
        Else
            For j = 1 To mA
                c(i, j) = A(i, j) * b(i, j)
            Next j
        End If
    Next i
    MultiplyTwoMatricesByRows = c
End Function

'http://puremis.net/excel/code/076.shtml
Function MatrixDim(ByVal VariantArray As Object) As Integer
    Dim i As Integer, X As Long
    On Error GoTo tooManyDims
    i = 1
    Do
        X = VariantArray(1, i)
        i = i + 1
    Loop
tooManyDims:
    MatrixDim = i - 1
End Function

'Cholesky decomposition: CC’ = T :: input T, output C
Function Cholesky(ByVal Mat As Object) As Object
    Dim A, L() As Double, s As Double
    A = Mat
    N = UBound(Mat, 1)  'mat.Rows.Count
    m = MatrixDim(Mat)    'mat.Columns.Count
    If N <> m Then
        Cholesky = "?"
        Exit Function
    End If

    ReDim L(1 To N, 1 To N)
    For j = 1 To N
        s = 0
        For k = 1 To j - 1
            s = s + L(j, k) ^ 2
        Next k
        L(j, j) = A(j, j) - s
        If L(j, j) <= 0 Then Exit For
        L(j, j) = Sqr(L(j, j))

        For i = j + 1 To N
            s = 0
            For k = 1 To j - 1
                s = s + L(i, k) * L(j, k)
            Next k
            L(i, j) = (A(i, j) - s) / L(j, j)
        Next i
```

```
    Next j
    Cholesky = L
End Function
```

Eigenvalue and eigenvector

```


Function MatEigenvalue_Jacobi(Mat, Optional MaxLoops)
    'returns all eigenvalues of symmetric matrix
    'uses the fast-Jacobi rotation algorithm
    'mod 11-1-07  VL
    Dim A, t As Double, Loops As Integer, si As Double, co As Double, dpq As
Double, tol As Double
    Dim N As Long, p As Object, q As Object, X As Object

    If IsMissing(MaxLoops) Then MaxLoops = 200
    A = Mat
    N = UBound(A, 1)
    Loops = 1
    tol = 2 * 10 ^ -14
    Do Until Loops > MaxLoops
        Loops = Loops + 1
        Jacobi_Find_Max(A, p, q)
        If p = 0 Then Exit Do
        dpq = A(q, q) - A(p, p)
        If dpq = 0 Then
            t = 1
        Else
            X = dpq / A(p, q) / 2
            t = Sgn(X) / (Abs(X) + Sqr(X ^ 2 + 1))
        End If
        co = 1 / Sqr(t ^ 2 + 1)
        si = t * co
        FastRotation_Jacobi(A, si, co, p, q)
    Loop
    MatEigenvalue_Jacobi = MatMopUp(A, tol)
End Function

Private Sub Jacobi_Find_Max(ByVal A, ByVal i, ByVal j)
    'search for max value out of the first diagonal
    'modified 23-6-02
    Dim i_ As Integer, j_ As Integer, N As Integer, big As Double
    N = UBound(A, 1)
    big = 0
    i = 0 : j = 0
    For i_ = 1 To N
        For j_ = 1 To N
            If i_ <> j_ And Abs(A(i_, j_)) > big Then
                big = Abs(A(i_, j_)) : i = i_ : j = j_
            End If
        Next j_
    Next i_
End Sub

Private Sub FastRotation_Jacobi(ByVal A, ByVal si, ByVal co, ByVal p, ByVal q)
    'fast rotation 11-6-2005 VL
    'co = Cos(teta)
    'si = Sin(teta)
    Dim i&, j&, N&, Ap#, aq#
    N = UBound(A)
    For i = 1 To N
        Ap = A(i, p)
        aq = A(i, q)
        A(i, p) = co * Ap - si * aq
        A(i, q) = si * Ap + co * aq
    Next

```

```
        For j = 1 To N
            Ap = A(p, j)
            aq = A(q, j)
            A(p, j) = co * Ap - si * aq
            A(q, j) = si * Ap + co * aq
        Next
    End Sub

    Function MatRotation_Jacobi(ByVal Mat)
        'returns the Jacobi rotation matrix
        'only for symmetric matrix
        Dim A, t, b
        Dim u() As Double, w() As Double
        Dim p, q
        A = Mat
        N = UBound(A, 1)
        Jacobi_Find_Max(A, p, q)
        d = A(q, q) - A(p, p)
        If d = 0 Then
            t = 1
        Else
            X = d / A(p, q) / 2
            t = Sgn(X) / (Abs(X) + Sqr(X ^ 2 + 1))
        End If
        c = 1 / Sqr(t ^ 2 + 1)
        s = t * c
        ReDim u(1 To N, 1 To N), w(1 To N, 1 To N)

        For i = 1 To N
            u(i, i) = 1
            w(i, i) = 1
        Next
        u(p, p) = c : u(p, q) = s
        u(q, p) = -s : u(q, q) = c
        w(p, p) = c : w(p, q) = -s
        w(q, p) = s : w(q, q) = c

        b = Application.WorksheetFunction.MMult(A, u)
        b = Application.WorksheetFunction.MMult(w, b)
        MatRotation_Jacobi = u
    End Function

    Function MatEigenvector_Jacobi(Mat, Optional MaxLoops)
        'returns the approx eigenvectors of a symmetric matrix
        'uses the fast Jacobi iterative algorithm
        'mod. 11-6-05 VL
        Dim A, t As Double, Loops As Integer, si As Double, co As Double, dpq As
    Double, N As Long
        Dim u() As Double, v As Object, p As Object, q As Object, X As Object, i As
    Integer
        A = Mat
        v = Mat
        N = UBound(A, 1)
        Loops = 1
        If IsMissing(MaxLoops) Then MaxLoops = 100
        'initialize v with unit matrix
        v = M_ID(N)
        'Jacobi algorithm start
        Do Until Loops > MaxLoops
            Loops = Loops + 1
            Jacobi_Find_Max(A, p, q)
            If p = 0 Then Exit Do 'fix bug 2.1.2007, thanks to David Schwartz
            dpq = A(q, q) - A(p, p)
            If dpq = 0 Then
                t = 1
            Else
                X = dpq / A(p, q) / 2
                t = Sgn(X) / (Abs(X) + Sqr(X ^ 2 + 1))
```

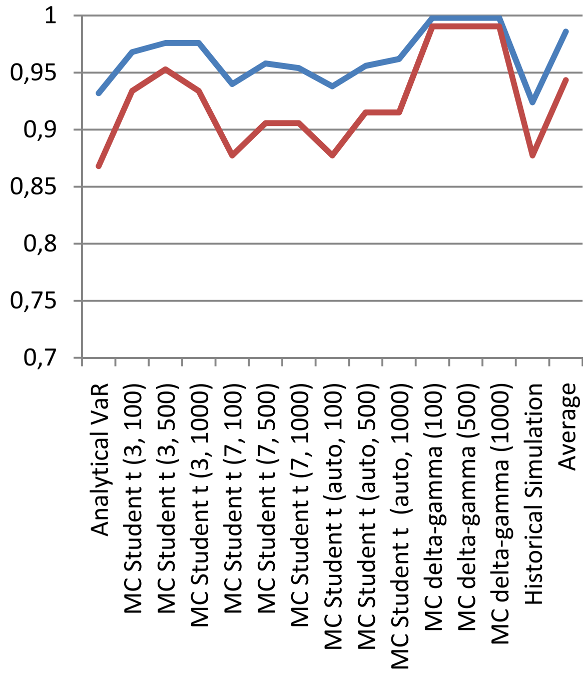
```
        End If
        co = 1 / Sqr(t ^ 2 + 1)    'cosine
        si = t * co                'sine

        FastRotation_Jacobi(A, si, co, p, q)

    ReDim u(1 To N, 1 To N), w(1 To N, 1 To N)

        For i = 1 To N : u(i, i) = 1 : Next i
        u(p, p) = co : u(p, q) = si
        u(q, p) = -si : u(q, q) = co
        v = Application.WorksheetFunction.MMult(v, u)
    Loop
    MatEigenvector_Jacobi = v
End Function

Function M_ID(ByVal N)
    'Identity Matrix
    Dim i As Integer, j As Integer
    Dim A()
    ReDim A(1 To N, 1 To N)
    For i = 1 To N
        For j = 1 To N
            A(i, j) = 0
            If i = j Then A(i, j) = 1
        Next j
    Next i
    M_ID = A
End Function

Function MatMopUp(Mat, Optional ErrMin)
    'eliminates values too small
    Dim A, i As Integer, j As Integer
    If IsMissing(ErrMin) Then ErrMin = 10 ^ -14
    A = Mat

    For i = 1 To UBound(A, 1)
        For j = 1 To UBound(A, 2)
            If IsNumeric(A(i, j)) Then
                If Abs(A(i, j)) < ErrMin Then A(i, j) = 0
            End If
        Next j
    Next i
    MatMopUp = A
End Function
```

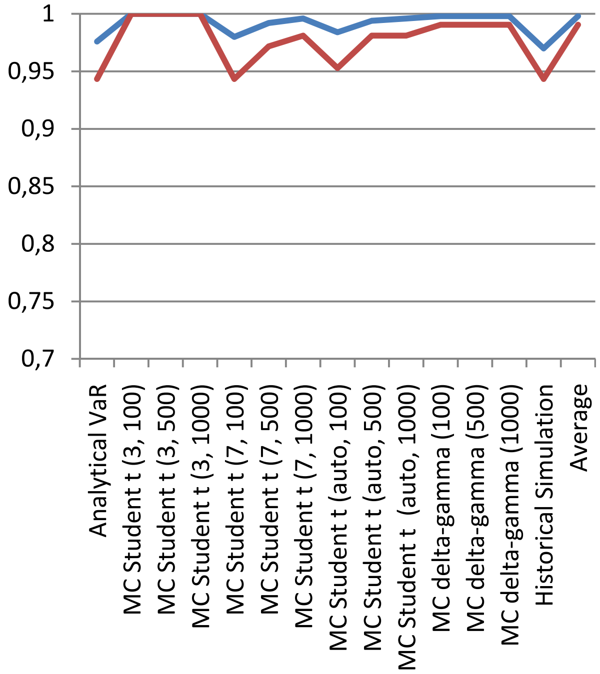
Appendix B – Coverage

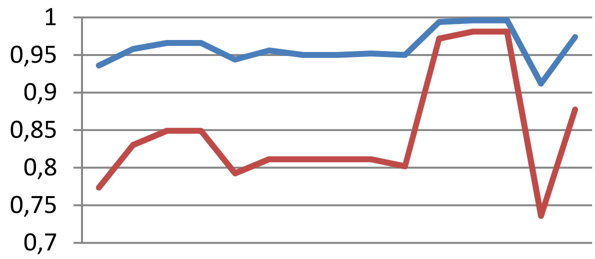
This appendix presents results that were ignored for brevity in the report. Each portfolio is presented with every combination of method and parameters tested in the back-test with a coverage graph in this appendix.

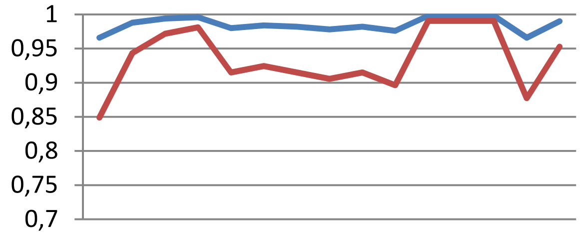
In general blue line is the back-test (for 500 days) and red line is the stress-test (106 days period.) The graphs are presented in order by 50, 250 and 750 historical days, and the 95% VaR is presented to the left and 99% VaR presented to the right.

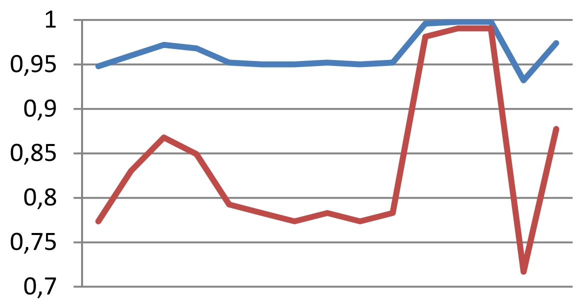
B.1 Portfolio A

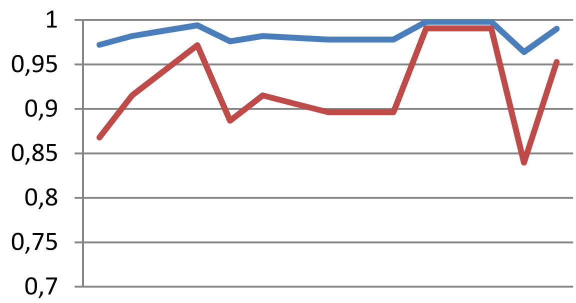












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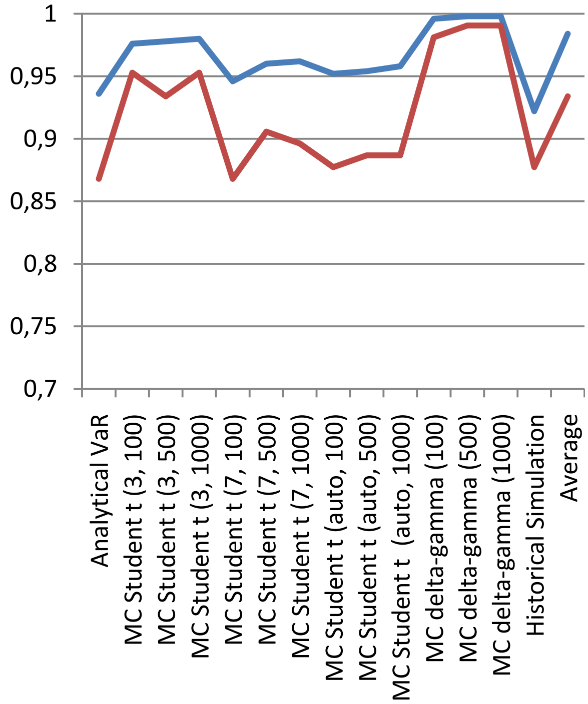

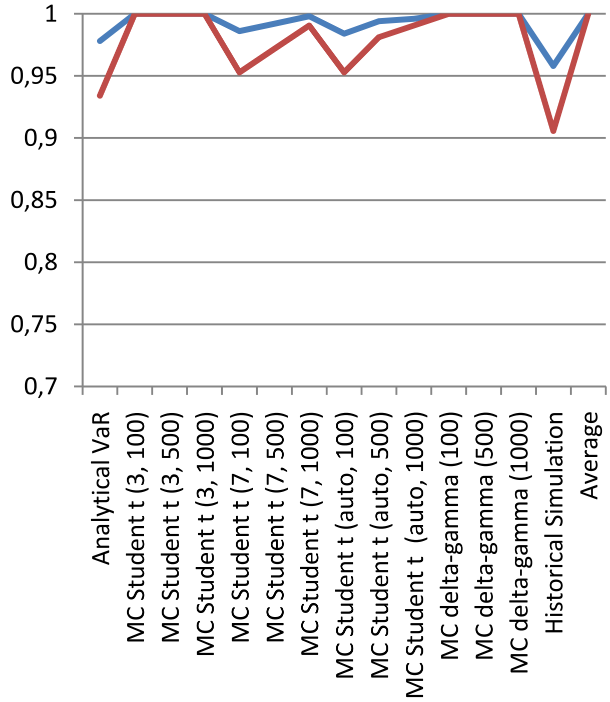

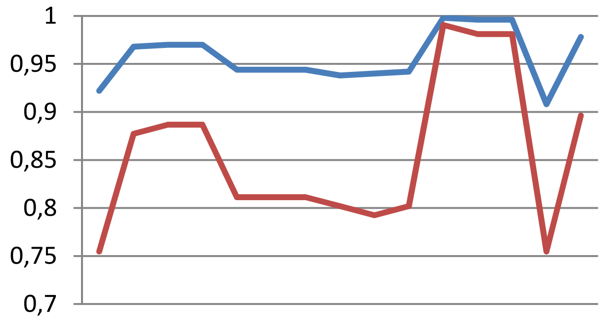

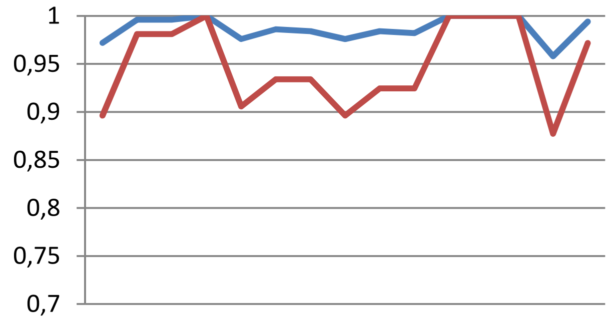

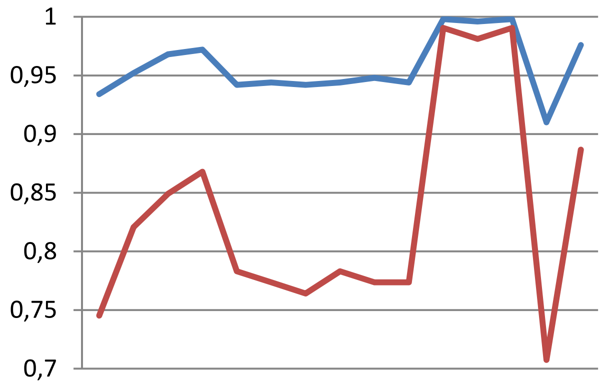

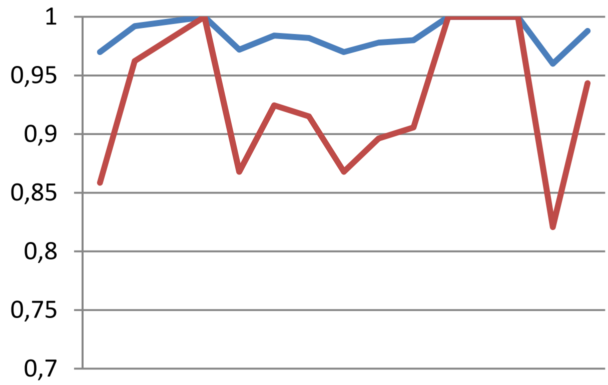

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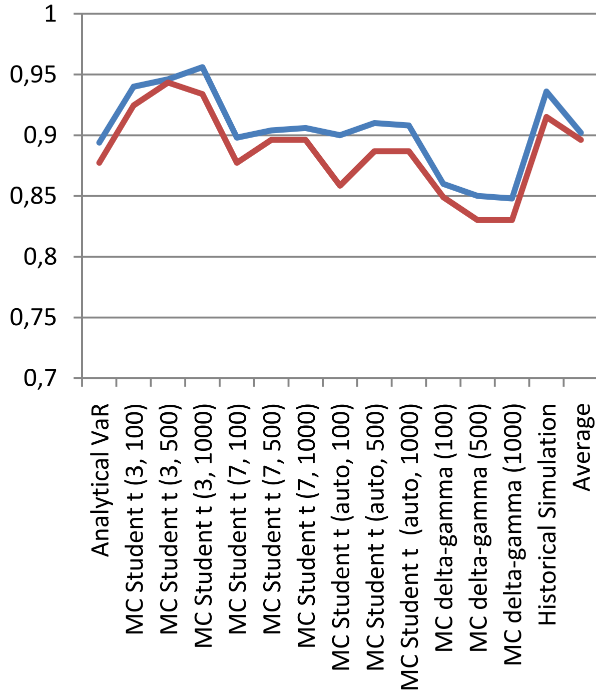

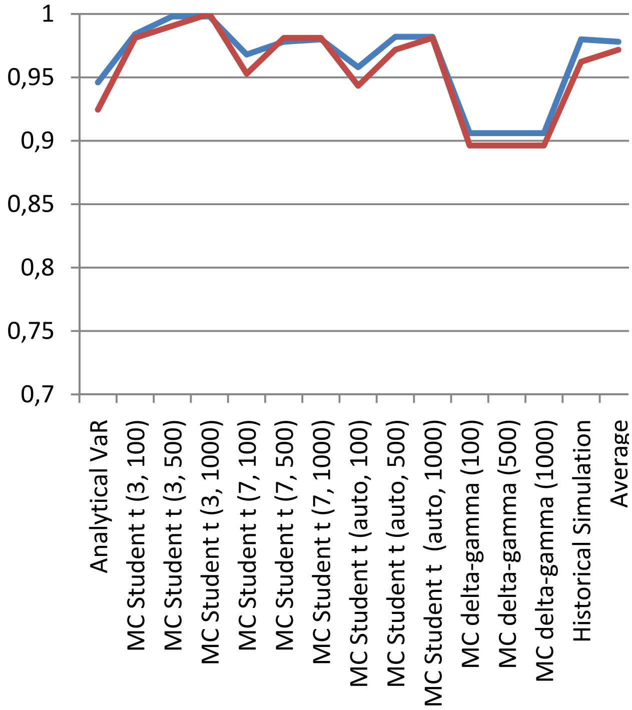

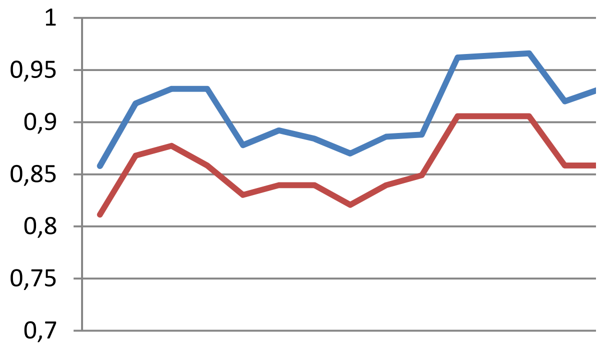

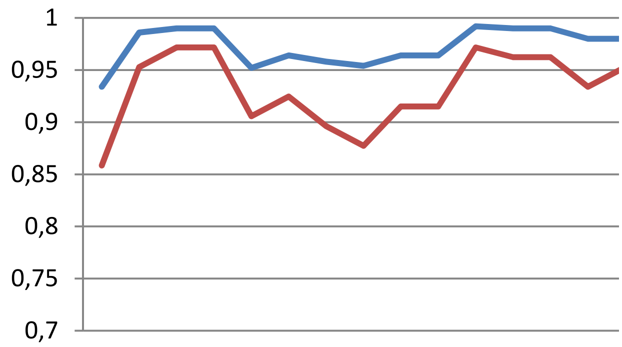

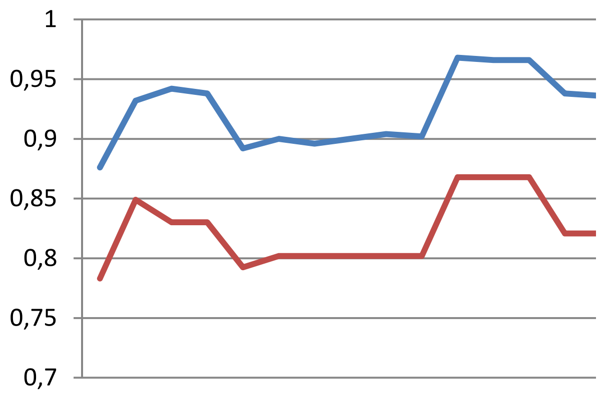

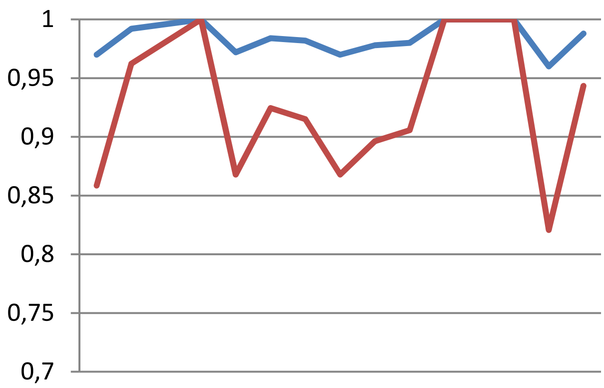

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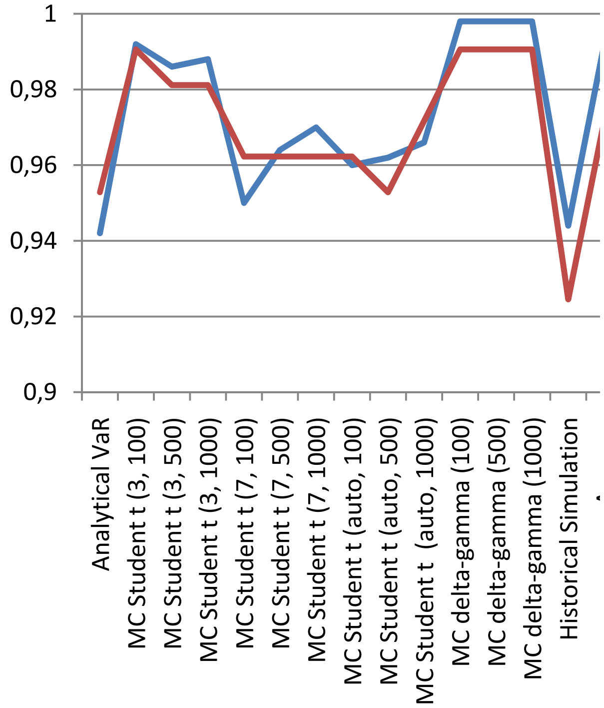

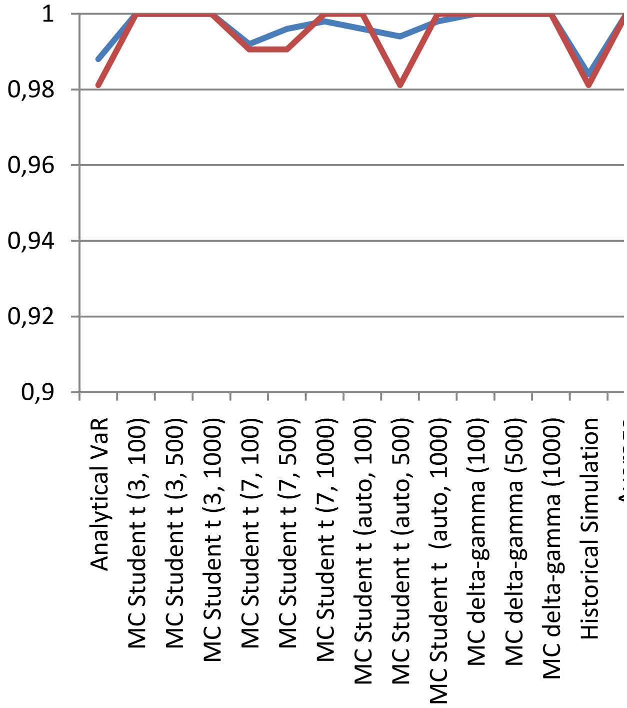

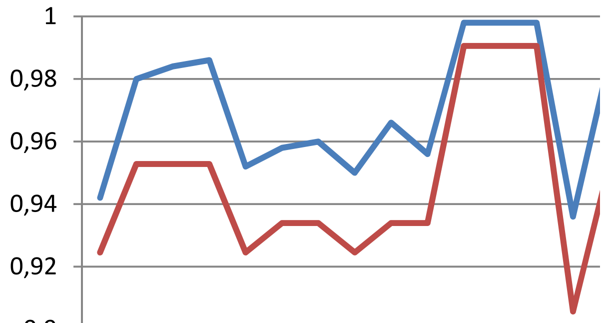

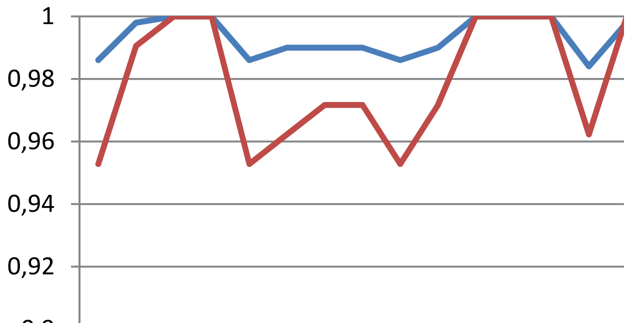

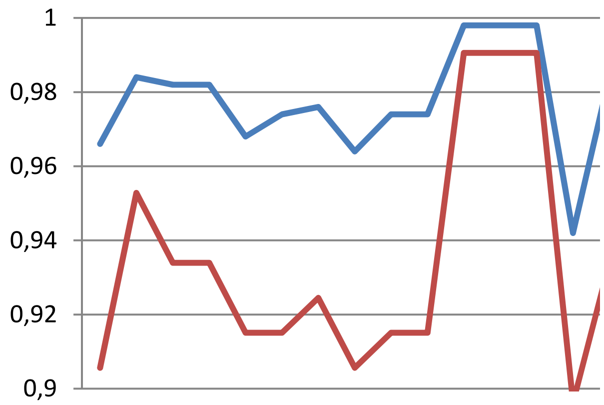

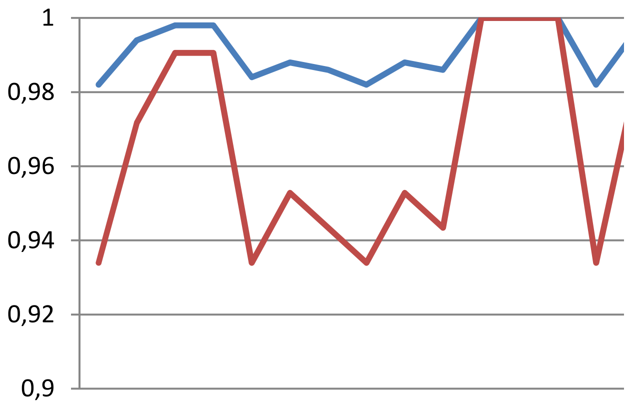

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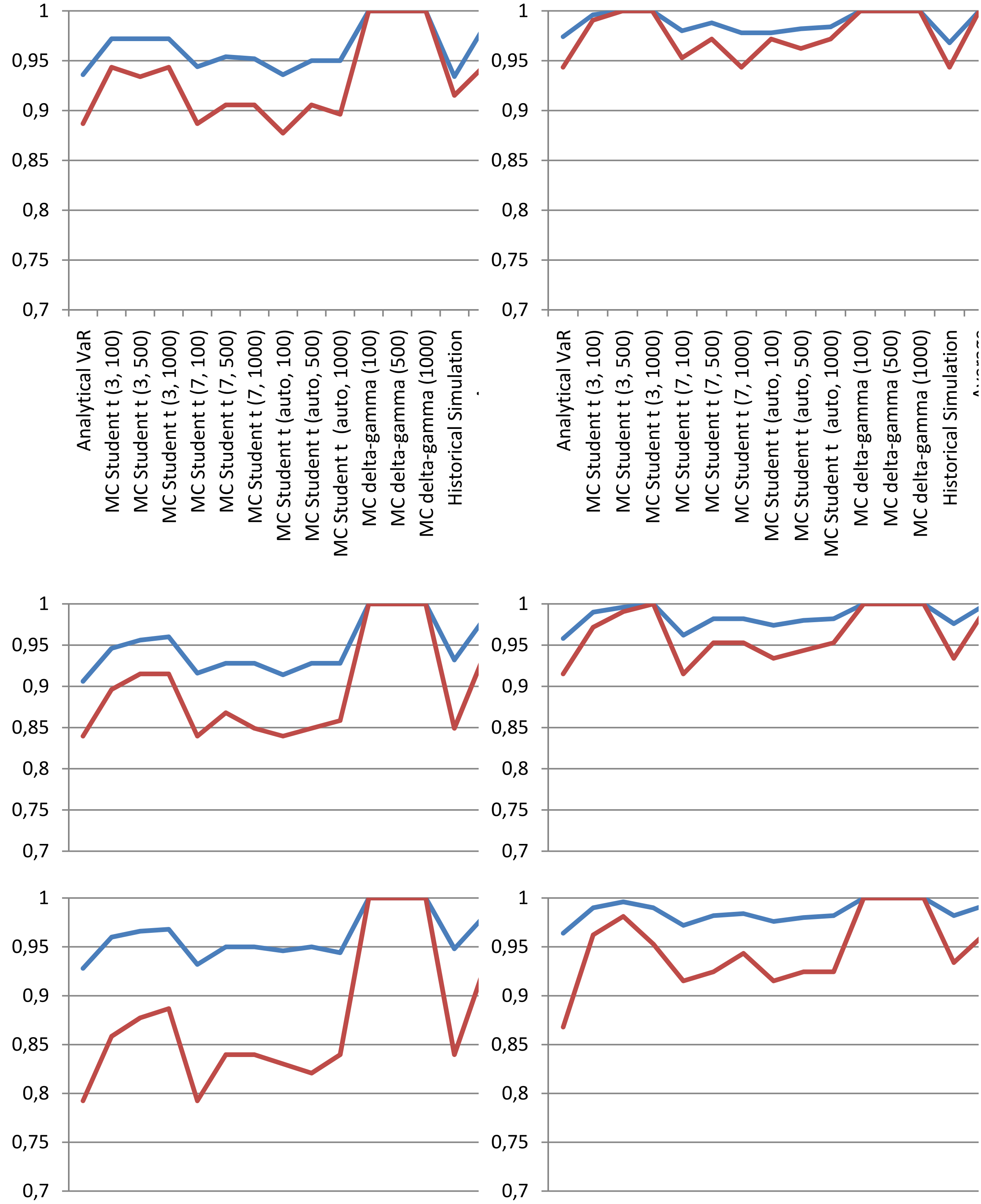


Appendix C – High-mean-low

This appendix presents the numbers behind the coverage graphs displayed in section 6.3 Accuracy and the graphs in 6.7 Correlation with portfolio.

C.1 High-mean-low coverage

50 days historical period

	Backtest Period	95% VaR high	95% VaR low	95% VaR mean	99% VaR high	99% VaR low	99% VaR mean
Analytical	500 days	0.942	0.894	0.928	0.988	0.946	0.972
	106 days	0.953	0.868	0.891	0.981	0.925	0.945
Historical	500 days	0.944	0.922	0.932	0.984	0.958	0.972
	106 days	0.925	0.877	0.902	0.981	0.906	0.947
MC student t 3	500 days	0.988	0.956	0.974	1.000	0.998	1.000
	106 days	0.981	0.934	0.949	1.000	1.000	1.000
MC student t 7	500 days	0.970	0.906	0.949	0.998	0.978	0.990
	106 days	0.962	0.896	0.913	1.000	0.943	0.979
MC student t auto	500 days	0.966	0.908	0.949	0.998	0.982	0.991
	106 days	0.972	0.887	0.911	1.000	0.972	0.985
MC delta-gamma	500 days	1.000	0.848	0.950	1.000	0.906	0.981
	106 days	1.000	0.830	0.951	1.000	0.896	0.977

250 days historical period

	Backtest Period	95% VaR high	95% VaR low	95% VaR mean	99% VaR high	99% VaR low	99% VaR mean
Analytical	500 days	0.942	0.858	0.913	0.986	0.934	0.963
	106 days	0.925	0.755	0.821	0.953	0.849	0.894
Historical	500 days	0.936	0.908	0.922	0.984	0.958	0.973
	106 days	0.906	0.736	0.821	0.962	0.877	0.917
MC student t 3	500 days	0.986	0.932	0.963	1.000	0.990	0.997
	106 days	0.953	0.849	0.892	1.000	0.972	0.991
MC student t 7	500 days	0.960	0.884	0.933	0.990	0.958	0.979
	106 days	0.934	0.811	0.849	0.972	0.896	0.934
MC student t auto	500 days	0.956	0.888	0.933	0.990	0.964	0.979
	106 days	0.934	0.802	0.849	0.972	0.896	0.932
MC delta-gamma	500 days	1.000	0.916	0.975	1.000	0.990	0.998
	106 days	1.000	0.906	0.955	1.000	0.962	0.991

750 days historical period

	Backtest Period	95% VaR high	95% VaR low	95% VaR mean	99% VaR high	99% VaR low	99% VaR mean
Analytical	500 days	0.966	0.876	0.930	0.982	0.938	0.965
	106 days	0.906	0.745	0.800	0.934	0.821	0.870
Historical	500 days	0.948	0.910	0.934	0.982	0.960	0.973
	106 days	0.896	0.708	0.796	0.934	0.821	0.885
MC student t 3	500 days	0.982	0.938	0.966	1.000	0.982	0.993
	106 days	0.934	0.830	0.874	1.000	0.925	0.968
MC student t 7	500 days	0.976	0.896	0.943	0.986	0.956	0.978
	106 days	0.925	0.764	0.821	0.943	0.858	0.913
MC student t auto	500 days	0.974	0.902	0.943	0.986	0.960	0.977
	106 days	0.915	0.774	0.823	0.943	0.868	0.908
MC delta-gamma	500 days	1.000	0.938	0.980	1.000	0.976	0.995
	106 days	1.000	0.868	0.951	1.000	0.896	0.977

C.2 High-mean-low correlation

50 days historical period

	high	low	mean
Analytical	0.801	0.236	0.599
Historical	0.769	0.060	0.497
MC student t 3	0.793	0.227	0.590
MC student t 7	0.798	0.235	0.593
MC student t auto	0.795	0.242	0.598
MC delta-gamma	0.968	0.801	0.886

250 days historical period

	high	low	mean
Analytical	0.940	0.731	0.881
Historical	0.935	0.787	0.850
MC student t 3	0.886	0.703	0.846
MC student t 7	0.905	0.731	0.862
MC student t auto	0.904	0.717	0.860
MC delta-gamma	0.998	0.817	0.957

750 days historical period

	high	low	mean
Analytical	0.997	0.833	0.957
Historical	0.996	0.847	0.952
MC student t 3	0.966	0.799	0.923
MC student t 7	0.977	0.817	0.937
MC student t auto	0.976	0.817	0.936
MC delta-gamma	0.998	0.674	0.932

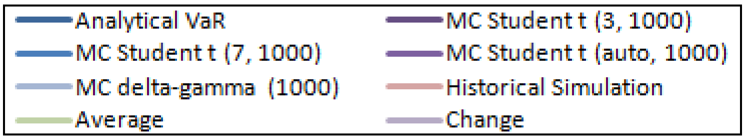
Appendix D – VaR-estimates

This appendix summarizes the VaR-estimates for the methods presented in the report. Included in the graphs is also the average of the methods as well as the actual change in portfolio value.

The graph covers the whole back-test period (09. January 2007 – 31. December 2008). Portfolio A, B and C are included and each portfolio is presented with graphs in the following order:

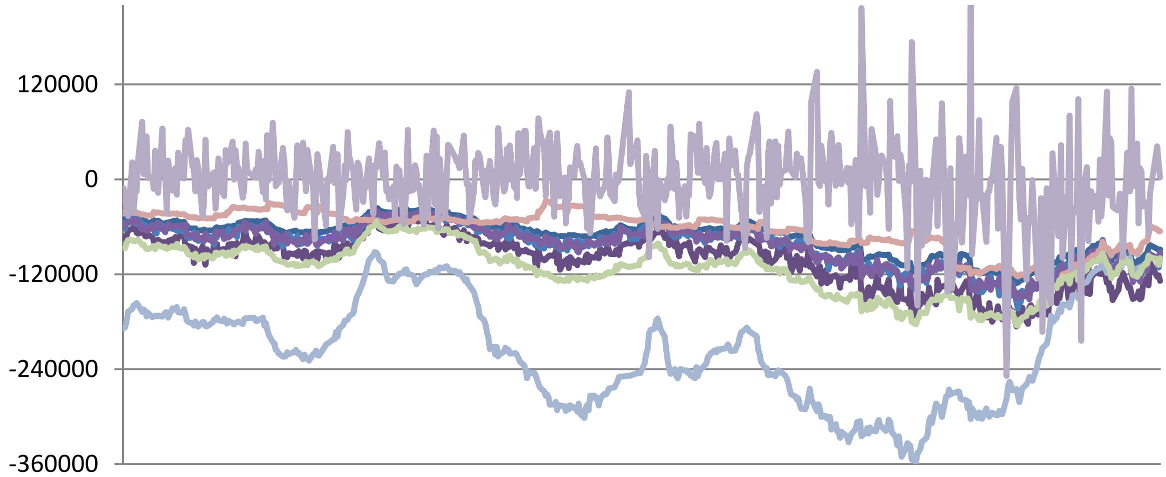
- 95% VaR estimated with 50 days historical period
- 95%VaR estimated with 250 days historical period.
- 99% VaR estimated with 50 days historical period
- 99%VaR estimated with 250 days historical period.

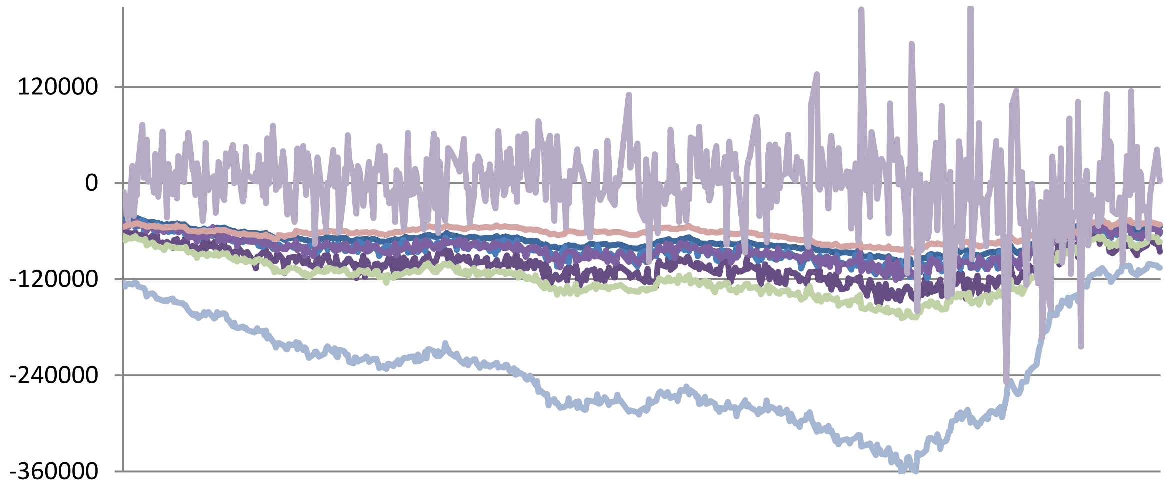
All graphs follow the legend in figure below:

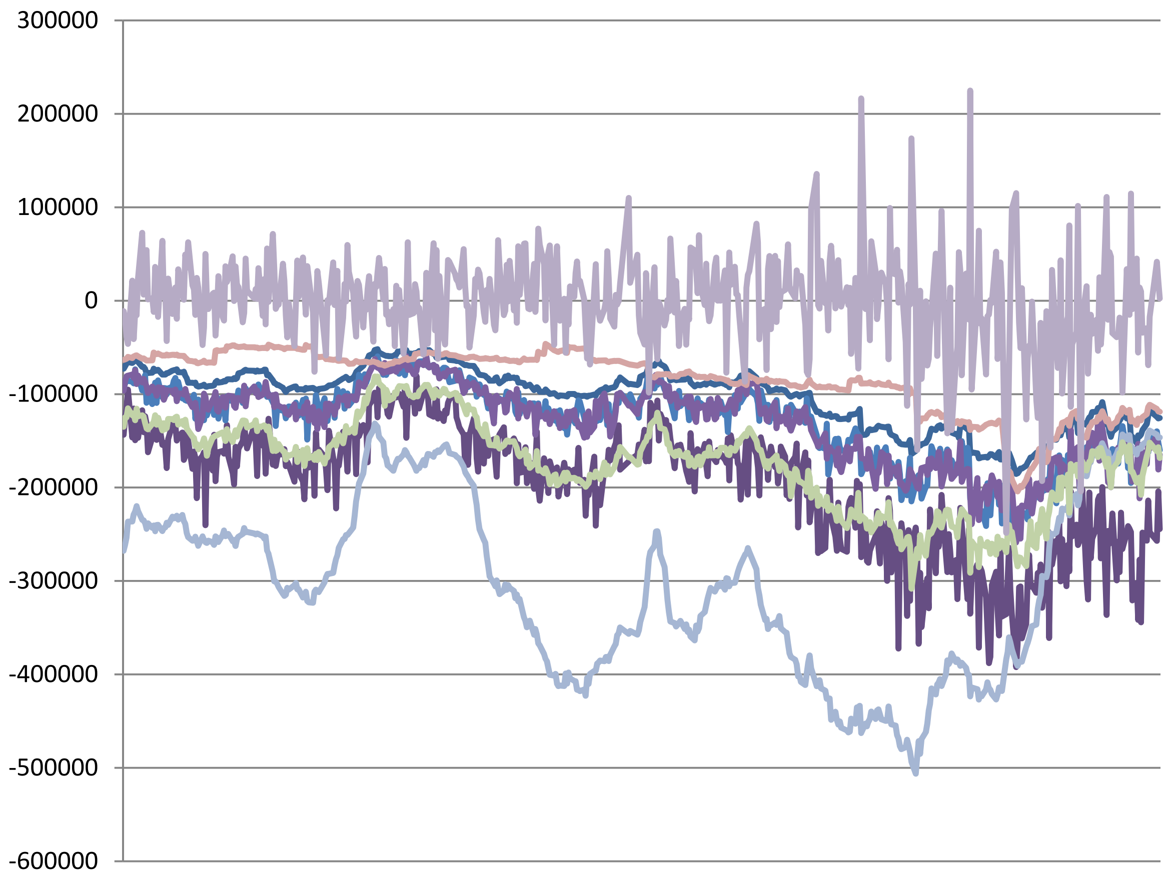


Portfolio D and E followed the same trends seen in portfolio B and A respectively.

D.1 Portfolio A



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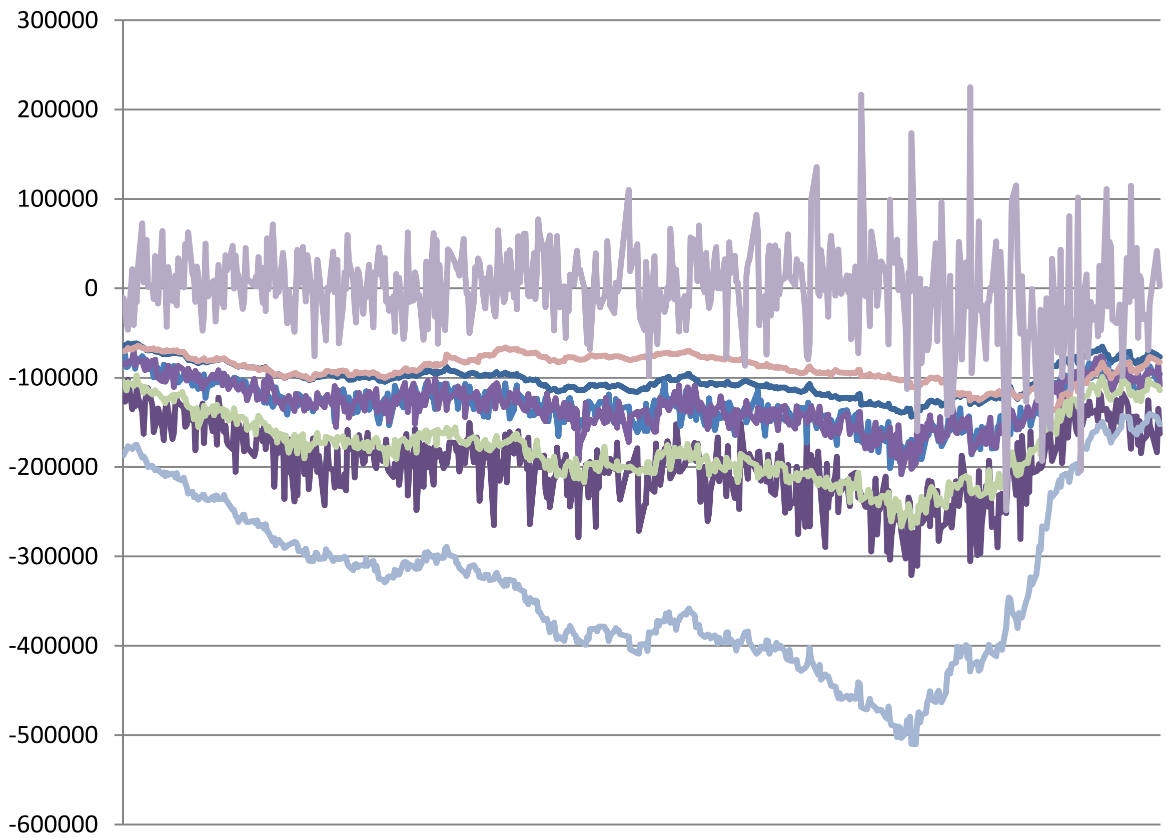
-200000
-300000
-400000
-500000
-600000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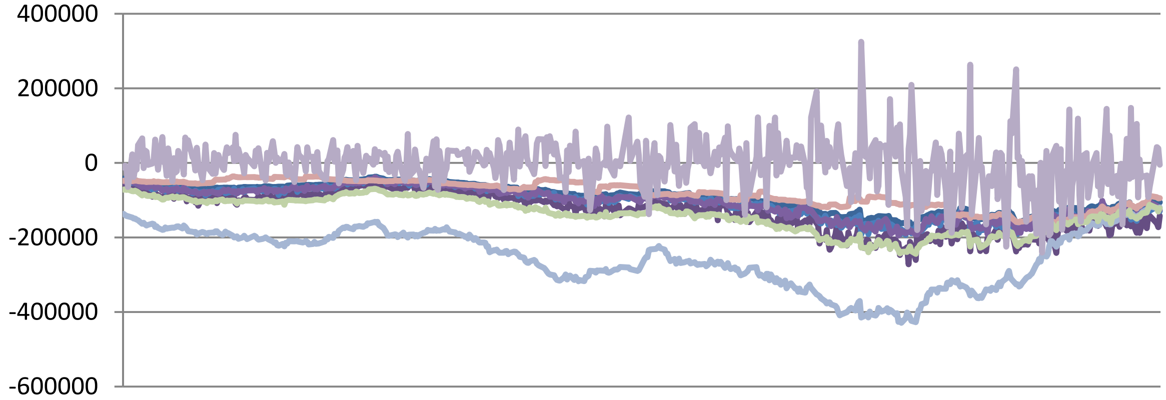

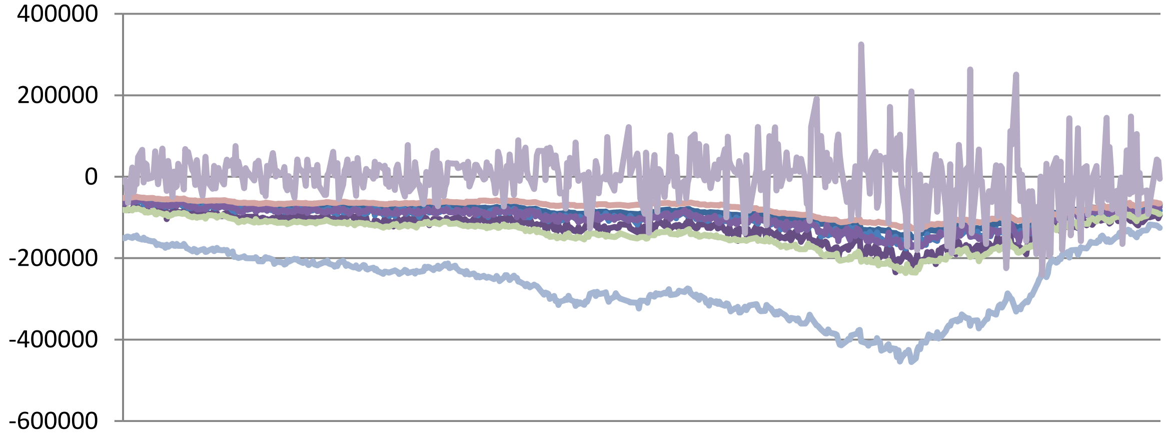

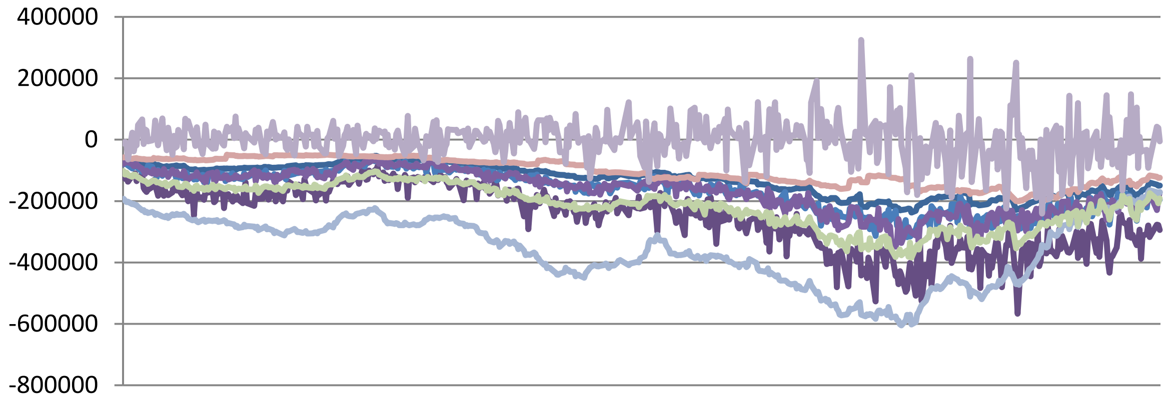

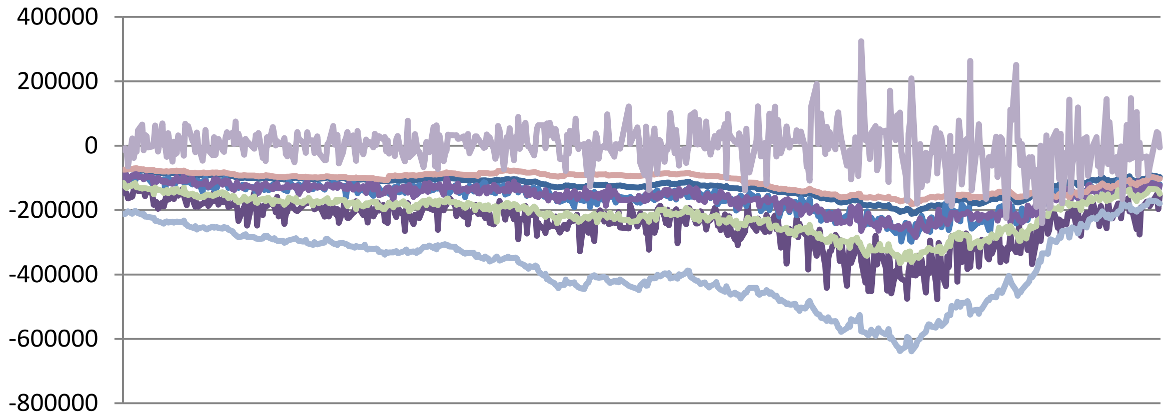

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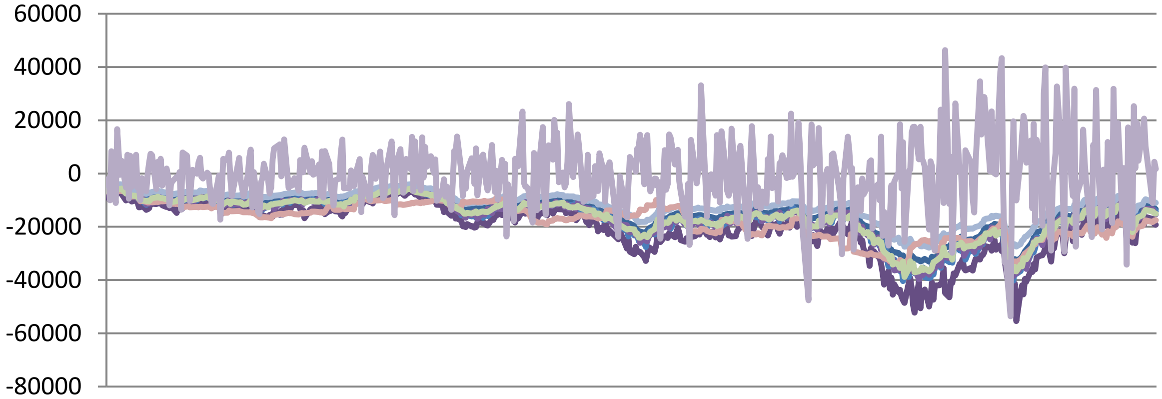
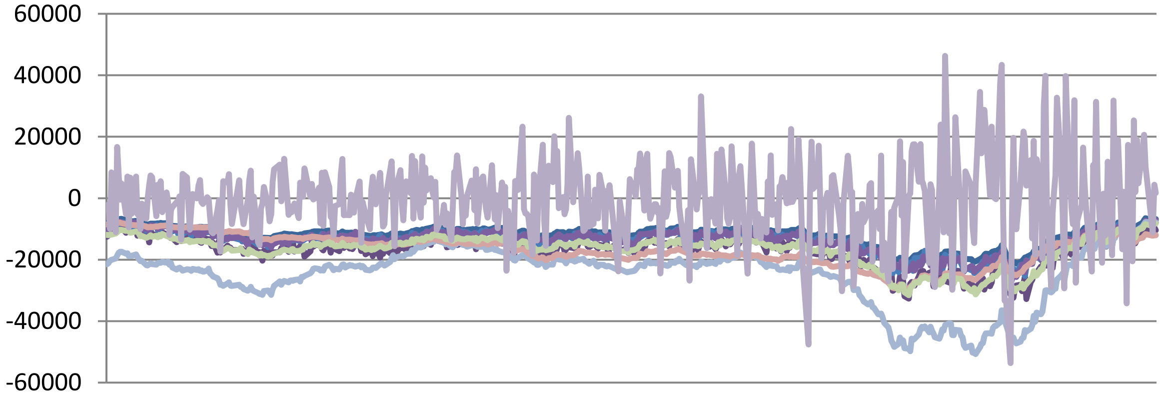
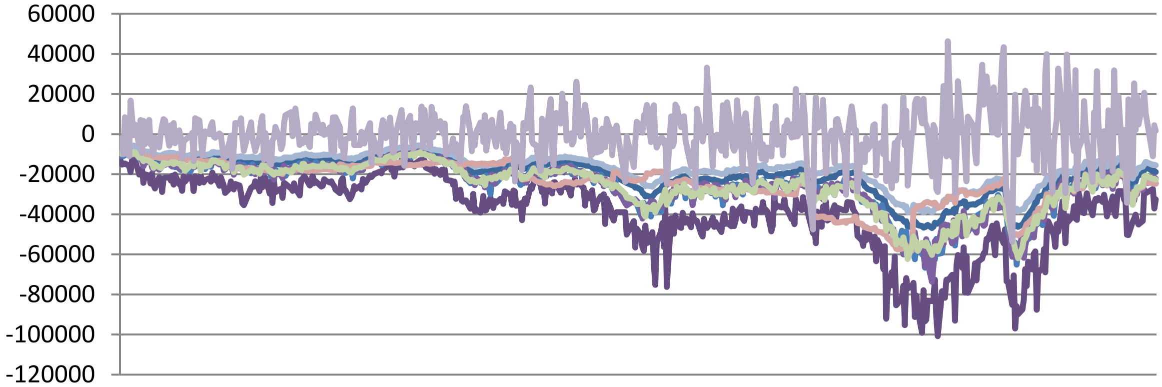
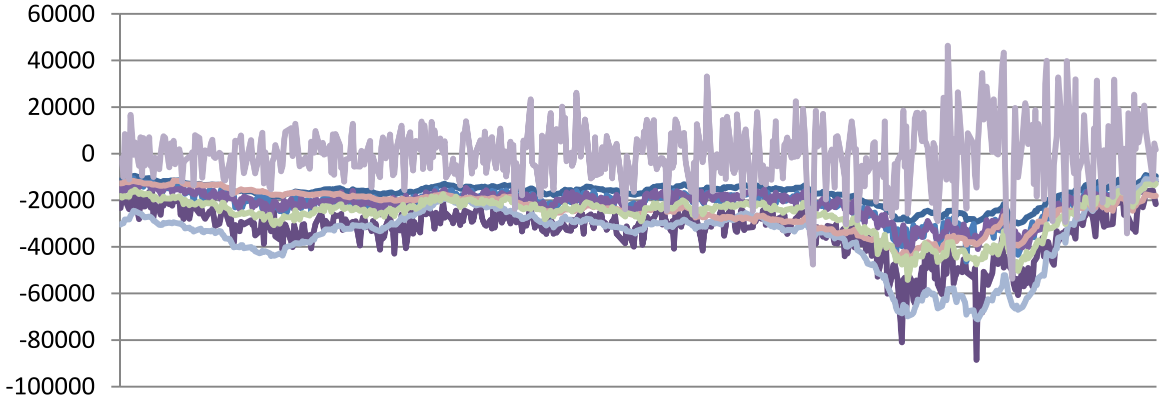






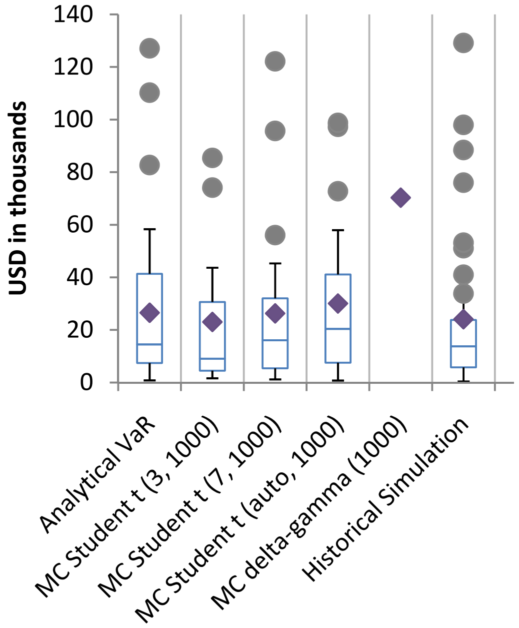


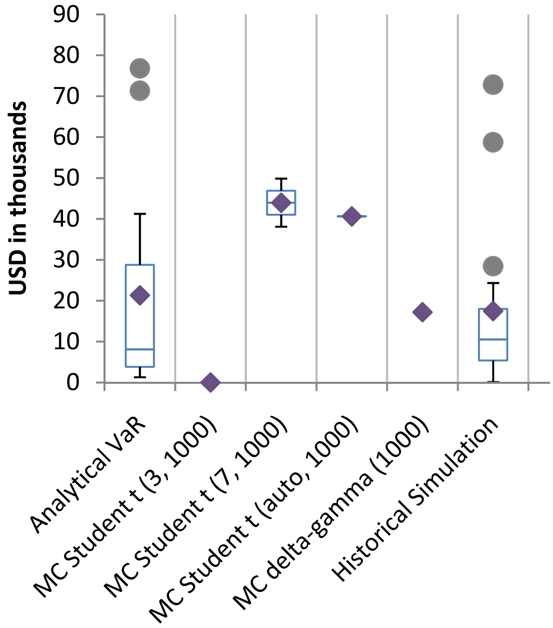
Appendix E – Size of error

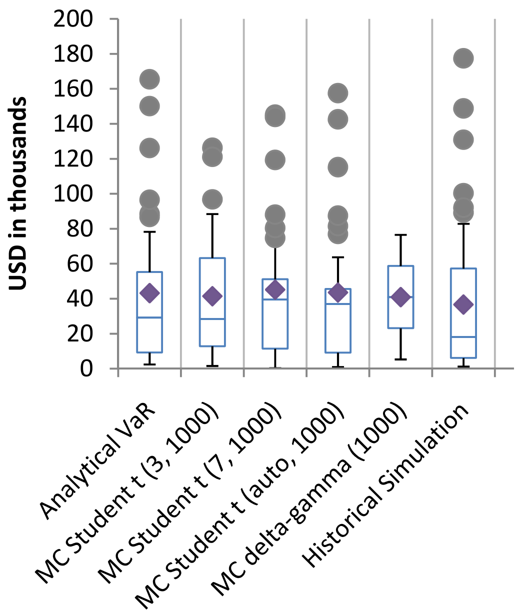
This appendix presents every method's mean excess loss for portfolio A – E. The box-and-whisker graph displays the median, first and third quartile as the boxes, with the whiskers marking the lower and higher limits of the interval at a maximum of 1.5 times the interquartile range. Furthermore the average is marked by a diamond and outliers by a grey dot.

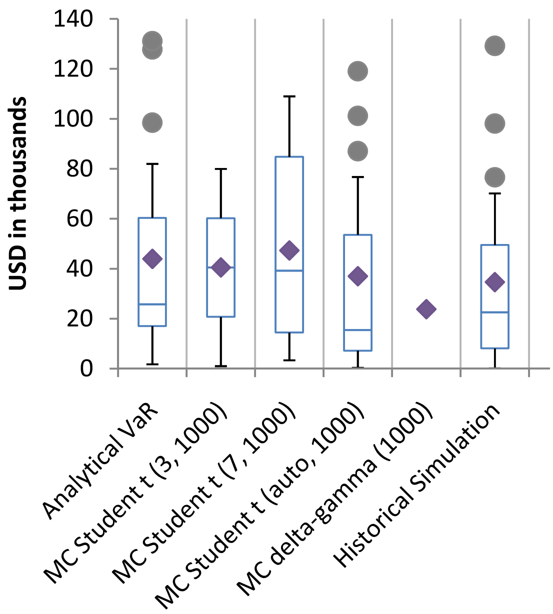
Every portfolio is presented with 50 days and 250 days, with the 95% VaR-estimate to the left and 99% VaR-estimate to the right.

E.1 Portfolio A









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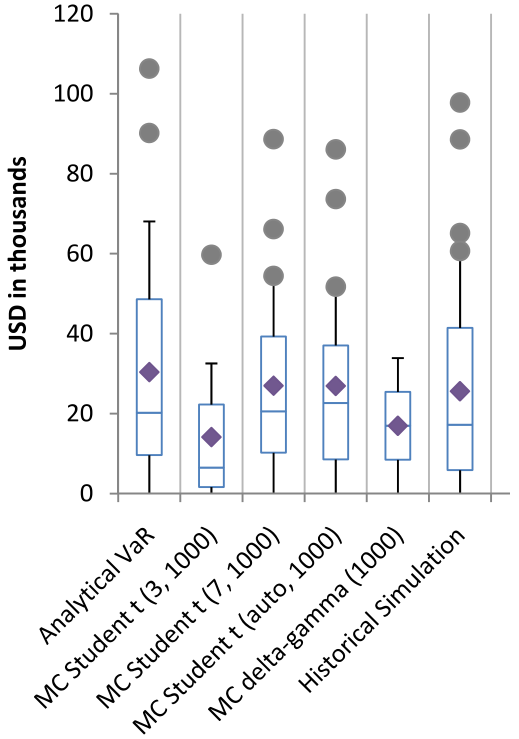

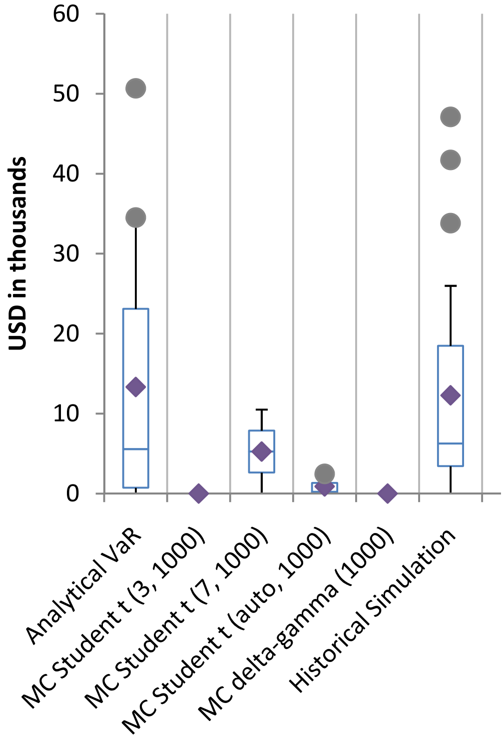

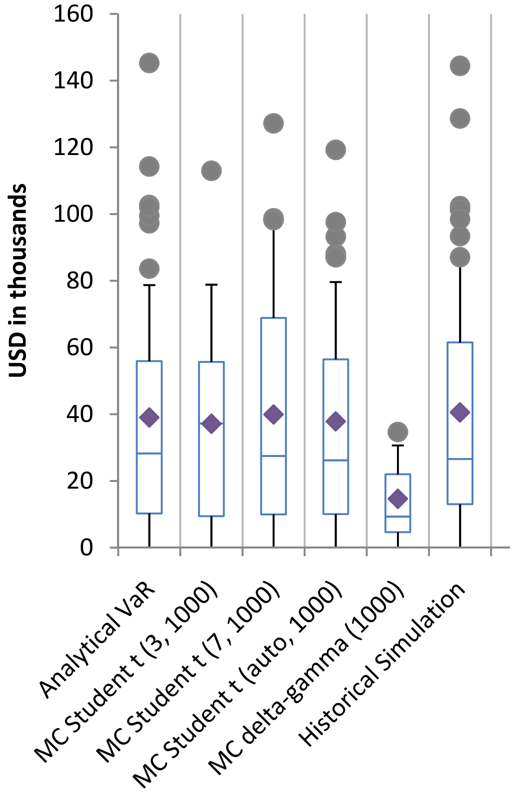

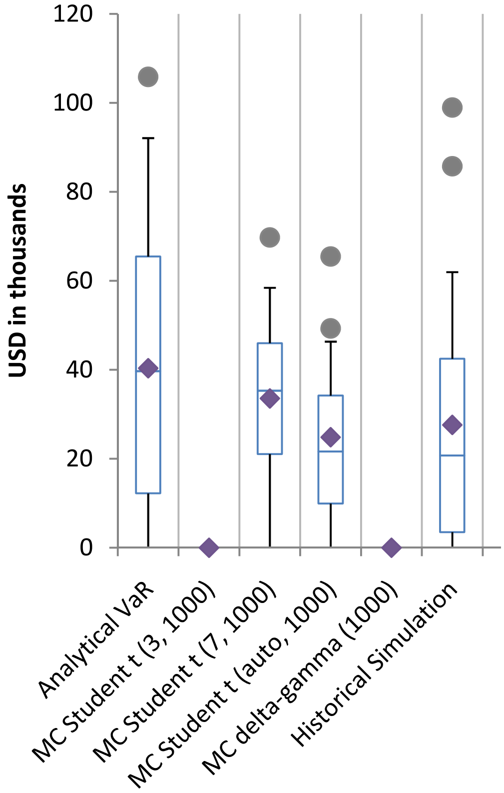

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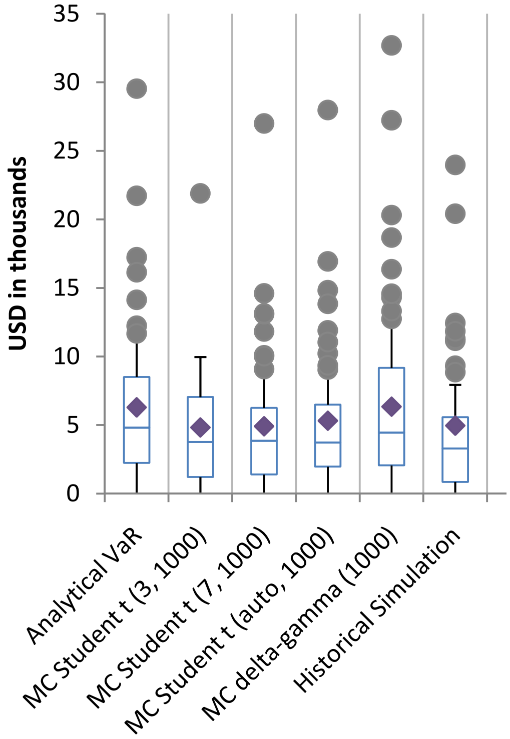

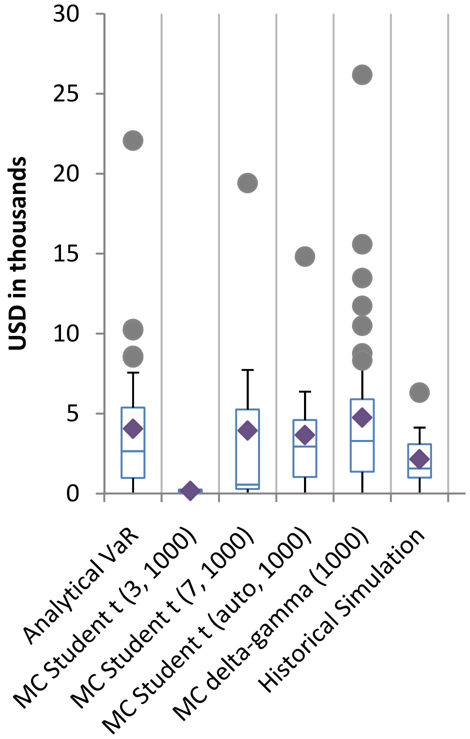

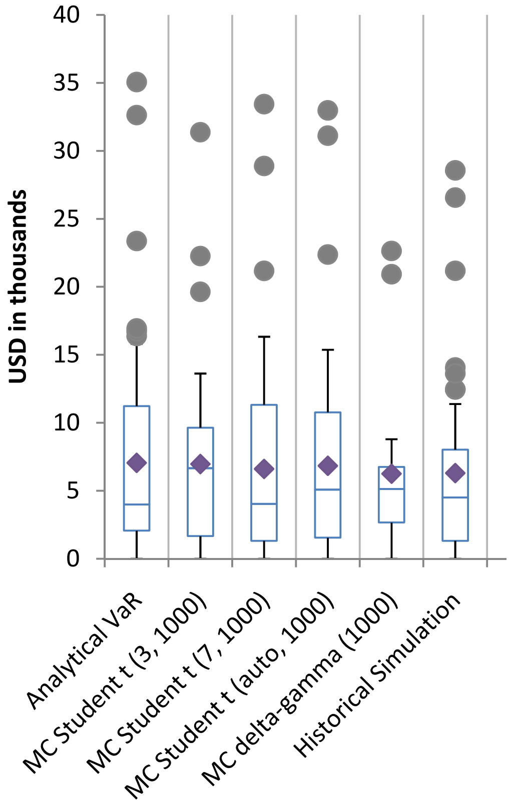

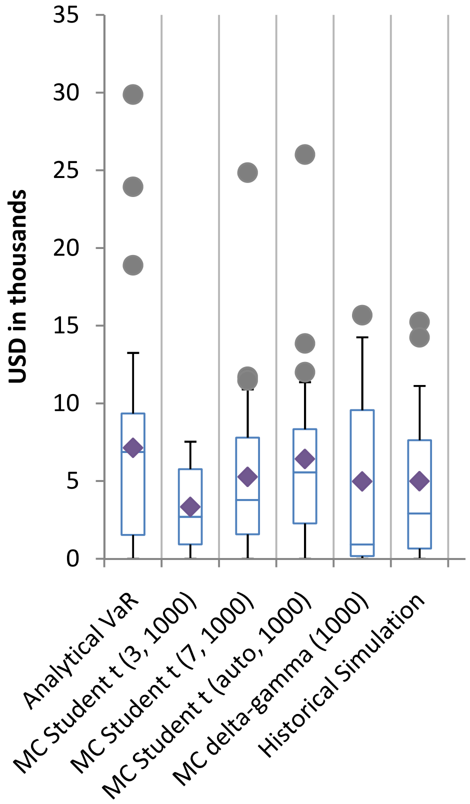

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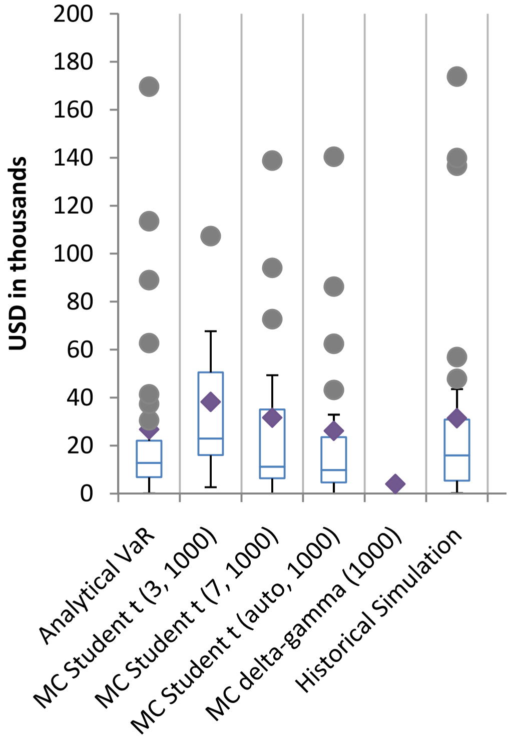

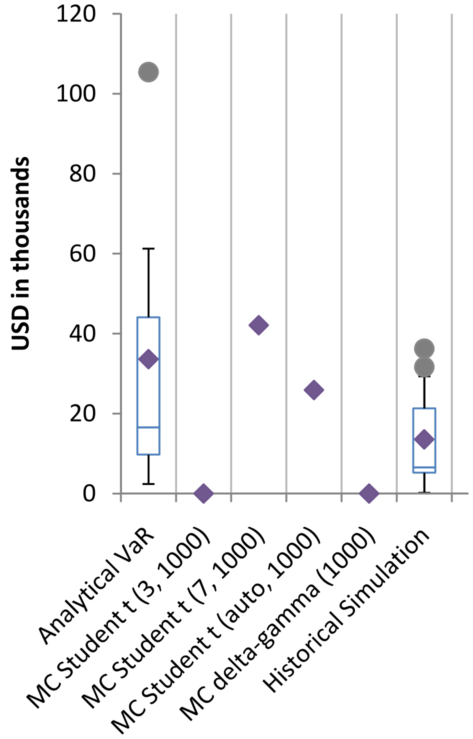

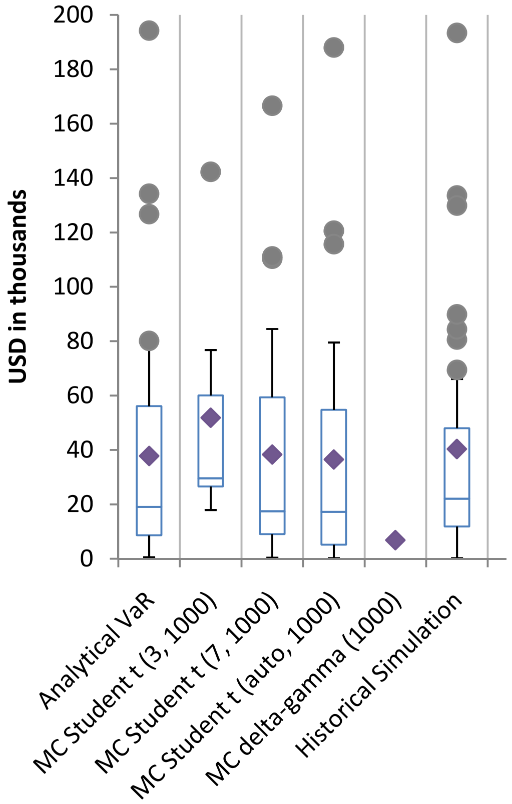

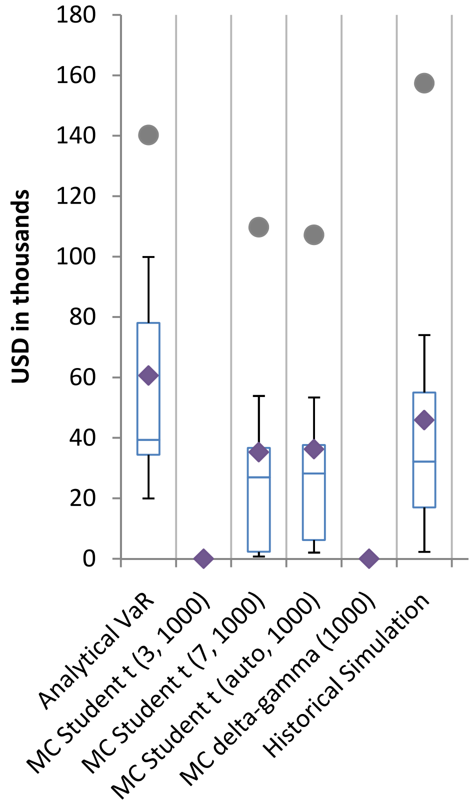

E.5 Portfolio E