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University of Stavanger

DEPARTMENT OF INDUSTRIAL ECONOMICS, RISK MANAGEMENT AND PLANNING

- MASTER THESIS -

Applications of a new framework for analyzing model output uncertainty in risk assessment

by

Torbjørn Bjerga

supervised by

Terje Aven

2012

Preface

This thesis is the final part of my study for a Master's degree in Industrial Economics at the University of Stavanger. The thesis comprises two articles written in collaboration with my supervisor, Professor Terje Aven, and co-supervisor Professor Enrico Zio from Politecnico di Milano/ Ecole Centrale Paris and Supelec. Paper I is a conference article which is to be presented at the PSAM'11/ESREL 2012 conference in Helsinki, and which will be published in the conference proceedings. Paper II will be attempted published in a scientific journal.

Acknowledgements

I will foremost give a sincere and genuine gratitude to my supervisor Professor Terje Aven for introducing me to a whole new level of the academic world – writing and presenting ideas to a scientific society. Professor Aven has been a light in an unknown world, leading way and sharing generously from his extensive knowledge in a most supporting and inspiring manner. The papers would never have reached the surface without his guidance. Thank you! I truly hope to get the opportunity to continue this stimulating cooperation in the future, and hopefully with time, be able to give something in return. Second, I am also grateful to my co-supervisor Professor Enrico Zio for his important contributions throughout the writing process, especially the finalizing refinement of the papers. And last but not least, I am indebted to my family for the everlasting and loving support.

Stavanger, June 2012

Torbjørn Bjerga

Contents

Thesis background:

This thesis is founded on a working paper (submitted for publication) by Professor Terje Aven and Professor Enrico Zio, titled "Model Output Uncertainty in Risk Assessment", revised 22 December 2011. The paper discuss' several issues with respect to the prevailing views on, and treatment of, model uncertainty in risk assessment, and are concerned with cases where no experimental data exists. This motivates the introduction of a new framework for analyzing model (output) uncertainty where distinctions and clarifications are made with respect to meanings and concepts, and where they link the concept of model output uncertainty to the objectives of modeling and risk assessment. The framework also allows different approaches for describing these (epistemic) uncertainties, both probabilistic and non-probabilistic approaches.

In the paper they give three brief examples of how the framework may be applied. These examples are concerning a Poisson model for modeling undesired events, the consequence modeling of a release at an LNG (Liquefied Natural Gas) plant, and the groundwater flow modeling at a radioactive waste repository.

The objective of this thesis has been to demonstrate in more depth how the framework applies. In the first paper we are concerned with the LNG case, and the second paper looks into the Poisson case.

Summary

<u>The framework:</u>

In both papers we introduce the new framework for analyzing model (output) uncertainty in models used in risk assessment. The framework applies when no experimental data are available at the time of the risk assessment, and the main features can be summarized as follows (a more detailed description can be found in Paper I and II and the references within):

The following concepts and distinctions are given in the framework:

- The concepts and distinction between model error and model output uncertainty:

  > The difference between a true value of interest to be realized in the future, Z, and the model outcome (prediction) G(X) is called the model error, $\Delta G(X)=Z-G(X)$. Model output uncertainty is the epistemic uncertainty about the magnitude of the model error, $\Delta G(X)$.

- The concepts and distinction between structural model uncertainty and input quantity (parameter) uncertainty:

  > The concept model output uncertainty is divided into structural model uncertainty and input quantity (parameter) uncertainty. The structural model uncertainty is the model output uncertainty about the magnitude of the model error conditional on the true input quantity, $\Delta G(X_{True})$, while the input quantity uncertainty is uncertainty about the true value of the input quantity, X.

- The concept and distinctions regarding sources of uncertainty:

  > Sources of uncertainty are classified as belonging to either the input quantity uncertainty or the structural model uncertainty. Sources of input quantity uncertainty are sources that give uncertainty about the value of X. While sources of structural uncertainty are typically assumptions and approximations underpinning the model.

The framework also links the concept of model output uncertainty to the objectives of modeling and risk assessment and specifically model accreditation is given focus. Meaning that the models needs to have a certain level of quality for its intended use (the purpose) in the risk assessment and subsequent decision making process. In addition the framework is open for various tools to represent the epistemic uncertainties.

Uncertainty analysis within the framework:

In Paper I we investigate a model for predicting the number of fatalities, Z, in case of a flash (pool) fire scenario or an explosion scenario at an LNG plant; and a regulatory criterion in that concern must be met. We utilize three different approaches for representing the uncertainties; subjective probabilities, imprecision intervals and a qualitative importance score method. An elicitation process of uncertainties is performed, and uncertainties described via the chosen approaches. All three approaches lead us to a rejection of the initial model due to high structural model uncertainty and remodeling is required. An acceptable alternative model is advised.

In Paper II the value of interest, Z is the 95^{th} percentile of the true distribution of number of minor hydrocarbon releases at a commercial pilot facility/system handling hydrocarbons with new technology. The model we use is a homogeneous Poisson process with rate λ. A qualitative importance score and imprecision intervals are chosen as approaches for uncertainty representation. Then, several sources of structural model uncertainty are educed which could violate the Poisson assumptions, and the uncertainty about the magnitude of the model error due to these sources is described using the selected approaches. The result of the approaches is that remodeling is required, and potential alternatives like a non-homogeneous Poisson model is suggested.

Discussion and conclusions:

Paper II gives a discussion on the findings in the paper and is also implicit covering Paper I. The conduction of the uncertainty analysis in both papers follows these main steps.

1. Identify all concepts in the framework (Z,G,X)
2. Determine which approach should be used for assessing the uncertainties
3. Perform the uncertainty assessments
4. Make judgments about accreditation and possible remodeling

As for step 1 a key issue to determine whether the quantities of interest Z and X are probabilistic parameters of probability models as in Paper II or a physical quantity as in Paper I. For proper analysis of the model uncertainties, precision is required on this point as G is strongly dependent on this.

In step 2 it seems apparent for all the approaches used in both papers that the assignation/judgment of values, or scores, is critical, as being relative to the assignor, with the associated issue on arbitrariness in that respect. From the use of these various approaches it seems like the qualitative importance score, even though being crude and based on more or less precise definitions of scores, in many cases is a sound first approach, being swift and providing sufficient information to conclude on model accreditation or remodeling. Other cases may require other approaches or a mix of several approaches, both quantitative and qualitative.

The thesis conclusion is that based on the cases adopted in these papers, the framework has demonstrated to perform successfully for its intended use, and meaningful concepts and analyses can be defined and conducted.

Paper I – An application of a new framework for analyzing model (output) uncertainty in risk assessment.

To be published in the proceedings of PSAM'11/ESREL 2012, Jun 25-29 2012, Helsinki, Finland.

An application of a new framework for model (output) uncertainty analysis in risk assessment

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Abstract: The purpose of this paper is to demonstrate the applicability of a recently proposed framework for model (output) uncertainty analysis in a risk assessment context. The framework is based on a distinction between overall model output uncertainties (epistemic uncertainties about the differences between the true values of the output quantities and the values predicted by the model), structural model uncertainties and parameter (model input quantities) uncertainties. The framework also distinguishes clearly between model output uncertainty and sources of model output uncertainty, from incomplete/imprecise knowledge on the values of the parameters of the model, to model assumptions, simplifications and approximations introduced in the model. The application regards the modeling of the consequences from a scenario of hydrocarbon release in an LNG (Liquefied Natural Gas) plant located in an urban area. It is assumed that no experimental data are available at the time of the assessment. The application allows pointing at and discussing several issues of relevance for the successful implementation of the framework, related to inter alia the distinction between stochastic (aleatory) uncertainties and epistemic uncertainties, and the use of different types of probabilistic and non-probabilistic approaches for representing these uncertainties.

Keywords: Model Uncertainty, Risk Assessment, LNG Plant

1. INTRODUCTION

Model uncertainty in a risk assessment context has been studied by several authors, see e.g. Zio and Apostolakis (1996), Devooght (1998), Nilsen and Aven (2003), Helton et al. (2004), Droguett and Mosleh (2008) and Baraldi and Zio (2010). These references provide different perspectives on the concept of model uncertainty and different ways for analyzing it. The present paper refers to a framework introduced in a recent work by Aven and Zio (2011), and aims at testing its applicability by addressing a specific case, a risk assessment related to hydrocarbon releases in an LNG (Liquefied Natural Gas) plant in an urban area. Before we study the case, we give a short presentation of the framework.

2. THE FRAMEWORK AND ITS MAIN ATTRIBUTES

This Section gives a formal introduction of the framework set forth in Aven and Zio (2011), and presents its main attributes. These attributes are pertaining to two main schemes: the first relates to the model output uncertainty concept itself, breaking it down in manageable parts so as to gain insights and use appropriate analysis tools; the second relates to the objectives of modeling and risk assessment, and the links to model output uncertainty.

2.1. Model Output Uncertainty

Consider an event/system/process subject to a risk assessment, and assume that at the time of the assessment no experimental data is available. Let Z represent the true value found in an unfolding future, and let G(X) be the model prediction at the time of the assessment where X is the input parameters. Both X and Z may be vectors. Define:

Model error: The difference, $\Delta G(X)$, between the model predictions, G(X) and the true future value Z (i.e. $\Delta G(X) = Z - G(X)$), and:

Model output uncertainty: The uncertainty associated with the true value of the model error.

A closer look at this taxonomy reveals that the model output uncertainty is actually the *epistemic* uncertainty of the model error and hence it may in theory be assessed using a suitable tool for measuring this type of uncertainty, like subjective probabilities and interval probabilities.

The model output uncertainty is decomposed into two categories:

Structural model uncertainty: The conditional uncertainty associated to the model error $\Delta_G(X)$, given the true value X_{True} (i.e. $\Delta_G(X_{True})$).

Input quantity (parameter) uncertainty: The uncertainty associated with the true value of the input quantity X.

The structural model uncertainty is expressing the epistemic uncertainty under the assumption that the input parameters are known (the true values), and relates then to the model structure itself, typically associated with assumptions and suppositions, approximations and simplifications made in the model. Input quantity (parameter) uncertainty is on the other hand reflecting epistemic uncertainties relating to the model inputs X.

Sources of structural model uncertainty stem from actual "gaps" in knowledge which can take the form of poor understanding of phenomena that are known to occur in the system, as well as complete ignorance of other phenomena. This type of uncertainty can lead to "erroneous" assumptions regarding the model structure. Other sources of structural model uncertainty stems from approximations and simplifications introduced in order to translate the conceptual models into tractable mathematical expressions.

2.2. Objectives of Modeling and Risk Assessment in Relation to Model Output Uncertainty

The objectives of modeling and risk assessment in a model output uncertainty context, and set forth in the framework, are founded on four categories typical of industrial practice as presented in de Rocquigny et al. (2008):

- *Accredit*: To reach a required level of quality for the model by validation for its certified use.
- *Understand*: To understand the influence of uncertainties on the results of the analysis and rank their importance so as to guide additional efforts (measurements, research, etc.) for uncertainty mitigation.
- *Select*: To compare performances of alternative system designs, operation modes and maintenance policies for "optimal" choices.
- *Comply*: To demonstrate compliance of a system, process, procedure with regulatory criteria.

The framework proposed in Aven and Zio (2011) sets forth the links between these categories and the model output uncertainty analysis. It points at uncertainty analysis as a tool to accredit the model, so as to ensure a certain quality and possible certification. In the accreditation process, the understanding of the influence of uncertainties on the results of the analysis is of importance, to adequately guide the uncertainty reductions. If the model considered cannot be accredited, remodeling is required. When an accredited model is obtained, a risk analysis might be conducted to inform the decision makers on the selection and compliance in line with the objectives stated above.

The characteristic that no experimental data exist at the time of the assessment leads us away from classical statistical tools for validation and subsequent accreditation of the model. Instead validation transforms into utilizing expert/analyst argumentation based on established scientific theories and specific knowledge about the system, which the model assessed, intends to describe.

An important observation is that no restrictions pertain to utilizing a pure probability-based approach. The framework opens for both probabilistic and non-probabilistic approaches, and thereby injects flexibility into the uncertainty analysis, giving the opportunity to choose the approach that is judged to best represents/express the uncertainties, given the specific phenomena and surroundings examined.

In all instances a (accompanying) qualitative analysis is encouraged and necessary, since even if accredited, a model still has inherent limitations and weaknesses, and these should be presented as part of the total risk assessment.

3. CASE STUDY: UNCERTAINTY ANALYSIS CONCERNING CONSEQUENCE MODELING OF A HYDROCARBON RELEASE IN AN LNG PLANT.

The case is taken from a Quantitative Risk Assessment relating to an LNG (Liquefied Natural Gas) plant in an urban area, as described in Aven (2011), and assumes that the assessment is conducted prior to construction and that no experimental data are available. Also it is assumed that the purpose of the assessment is to demonstrate compliance with regulatory criteria for risk related to loss of lives. We consider the consequences of a potential hydrocarbon release modeled with the event tree of Figure 1. The quantity of interest is the number of fatalities, Z and we wish to assess the pertaining model output uncertainties in line with the framework introduced in Section 2, by a crude analysis of uncertainties including evaluation of the relation to the objectives of the modeling and risk assessment.

3.1. The Event Tree Model

The event tree model (Figure 1) looks into a potential release and considers four final consequences; pool fire, flash (pool) fire, explosion, or no effect. The final scenario outcome depends on the intermediate branching events; immediate ignition, A, delayed ignition, B, and a third stage branch event following a delayed ignition, leading to either flash (pool) fire, C, or explosion.

In the event tree model (Figure 1), X_0 = number of releases, which is approximately equal to 1 if a release occurs and 0 otherwise, ignoring the probability of two or more releases in the time period studied. Furthermore, $X_1 = I_{\{A\}}$, $X_2 = I_{\{B\}}$ and $X_3 = I_{\{C\}}$, where I is the indicator function which is equal to 1 if the argument is true and 0 otherwise. Together, the X_i's form a quadruplet for the input parameter vector, $X = (X_0, X_1, X_2, X_3)$.

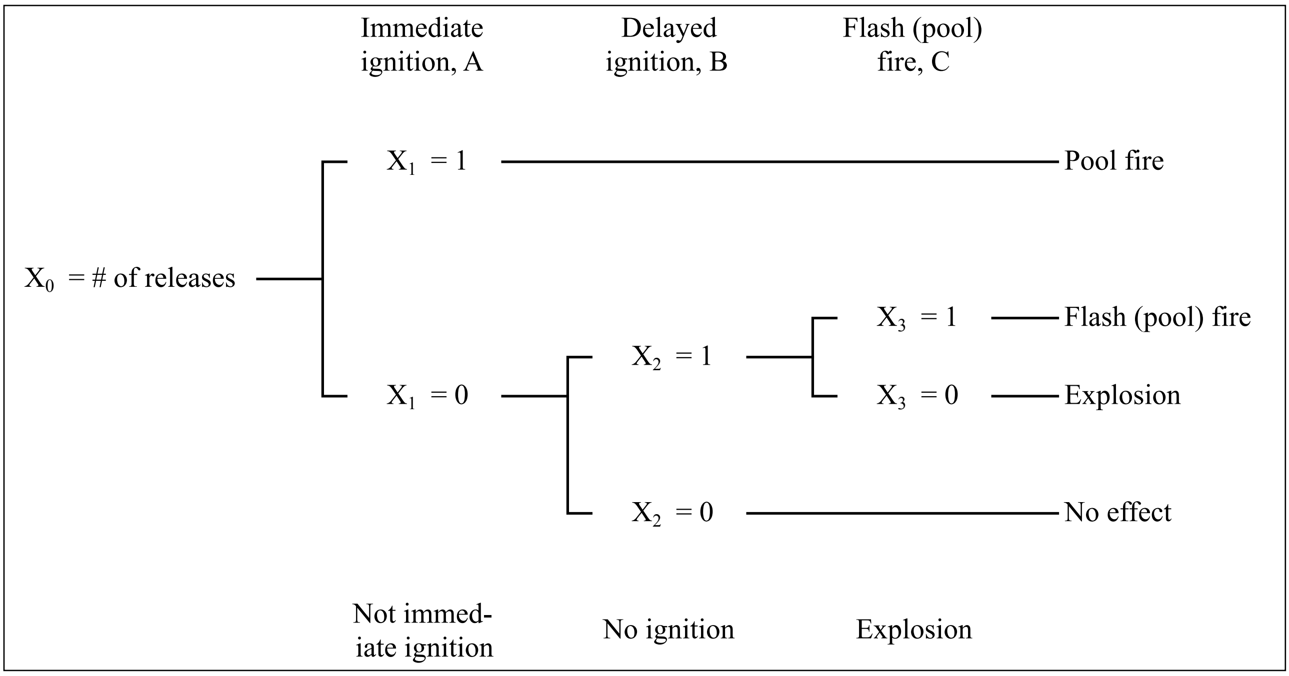

Figure 1: Event Tree Model for Hydrocarbon Release.

From the event tree we see that there are four paths, or scenarios, arising from the event tree. These are presented in Table 1, along with the numbers of people exposed and the associated fatalities. The fraction of fatalities is 0.1 for scenarios 2 and 3, causing 5 and 10 fatalities respectively, and 0 otherwise.

Table 1: Scenarios.

Scenario	Path	People exposed	Fraction of fatalities	Number of fatalities
1	release – A – pool fire	50	0.0	0
2	release – not A – B - flash (pool) fire	50	0.1	5
3	release – not A – B – explosion	100	0.1	10
4	release – not A – not B – no effect	50	0.0	0

Let Z be the true number of fatalities due to release events of the type here considered. To assess Z we introduce the model G, which according to the assumptions made above (Table 1) can be written:

$$G(X) = 5\, X_0\, (1 - X_1)\, X_2\, X_3 + 10\, X_0\, (1 - X_1)\, X_2\, (1 - X_3) \tag{1}$$

as the number of fatalities equals 5 if $X_0 (1 - X_1) X_2 X_3 = 1$ (scenario 2 occurs), and as the number of fatalities equals 10 if $X_0 (1 - X_1) X_2 (1 - X_3) = 1$ (scenario 3 occurs).

Following the terminology introduced in Section 2, the *model error* $\Delta_G(X)$ is defined as the difference between the true number of fatalities Z, and the model output G(X), i.e. $\Delta_G(X) = Z - G(X)$, and the *model output uncertainty* is the (epistemic) uncertainty associated with the model error $\Delta_G(X)$. Following the framework, we decompose the model output uncertainty into the *structural model uncertainty* which relates to $\Delta_G(X)$ if X is given the true value, and the *input quantity (parameter) uncertainty* which pertains to X and its true value.

3.2. Analyzing Uncertainties

There are many sources of the model output uncertainty. In this paper we focus on three examples:

1) The numbers 5 and 10 representing the number of fatalities following the scenarios 2 and 3, respectively.
2) That X_0 is approximated to be 0 or 1 release.
3) That X_3 is considered to be either a flash (pool) fire or an explosion in case of a delayed ignition.

Here source 1) relates to the structural model uncertainty, whereas the sources 2) and 3) relate to the input quantity (parameter) uncertainties. In the following, we will focus on source 1). There are also other sources of model output uncertainties, but they are assumed negligible and not further analyzed here.

For *uncertainty source 1)*, epistemic structural model uncertainty relates to the fixed values 5 and 10 representing the number of fatalities for scenarios 2 and 3. These numbers are based on some strong, simplifying assumptions. The numbers used can be considered the expected numbers of fatalities in these scenarios based on some crude analysis of the number of people exposed and the loss fraction in case the events occur. Clearly the actual number could deviate considerably from these computed expected values. The model output uncertainty can be reduced with more detailed modeling of the phenomena, reflecting for example the distributions of people being exposed and considering the number of fatalities as random quantities and not fixed numbers.

To conclude on the acceptance of the model, we will discuss the magnitude of the error $\Delta_G(X)$ given the current knowledge K. In the following, we will do this by using three different approaches:

i) Subjective probabilities: if we assign a probability of 0.1 say, it means that the assigner has the same uncertainty or degree of belief for this event to occur as drawing a specific ball out of an urn containing 10 balls (Lindley 2000).

ii) Imprecise probabilities: assigning an imprecision interval, e.g. [0.1, 0.2], means that the assigner states that his/her degree of belief is greater than or equal to the urn chance of 0.10 (the degree of belief of drawing a specific ball out of an urn containing 10) and less than or equal to the urn chance of 0.20. The analyst is not willing to make any further judgments.

iii) Using a qualitative scheme giving scores on the importance of the assumptions made, reflecting both the degree of sensitivity and the uncertainty (see e.g Flage and Aven 2009, Selvik and Aven 2011).

Starting with i) we aim at specifying a subjective probability distribution for $\Delta_G(X)$ which expresses the difference between the true number of fatalities Z and the model output G(X). First let us assume that the only source of model uncertainty stems from 1). Then, we need to compare G(X) defined in (1) with $G'(X) = X_4 X_0 (1 - X_1) X_2 X_3 + X_5 X_0 (1 - X_1) X_2 (1 - X_3)$, where X_4 and X_5 denote the numbers of fatalities in scenarios 2 (flash (pool) fire) and 3 (explosion), respectively: the error term becomes,

$$G'(X) - G(X) = (X_4 - 5)X_0 (1 - X_1) X_2 X_3 + (X_5 - 10) X_0 (1 - X_1) X_2 (1 - X_3). \tag{2}$$

The structural model uncertainty is the focus here, so we can condition on perfect knowledge of X, which in this case can be done by conditioning on the occurrence of either scenario 2 or 3. Given the occurrence of scenario 2, the structural uncertainty relates to the error $X_4 - 5$. The analyst group then assigns a probability distribution for $X_4 - 5$ reflecting this uncertainty. Figure 2 shows the assigned probability distribution. The analogous distribution for $X_5 - 10$ is shown in Figure 3.

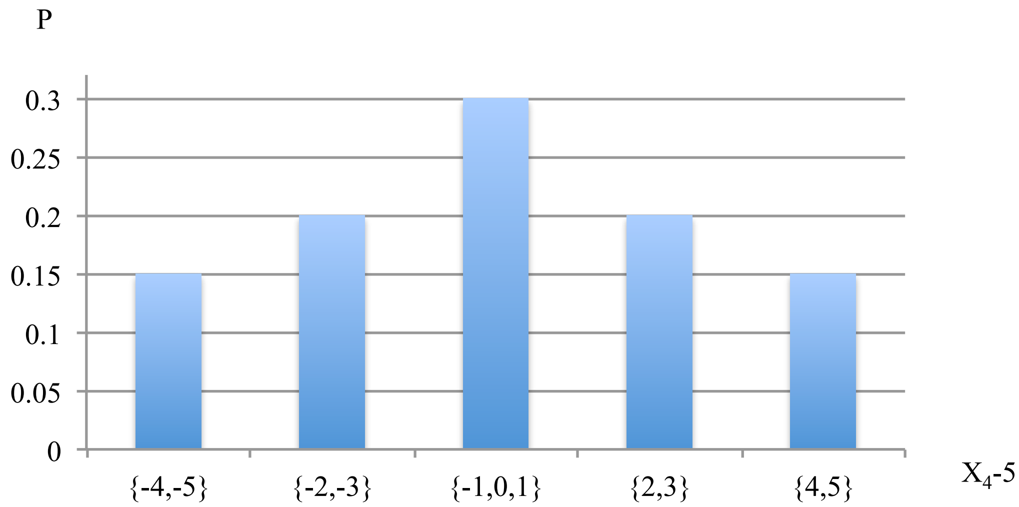

Figure 2: Assigned Probabilities for $X_4 - 5$

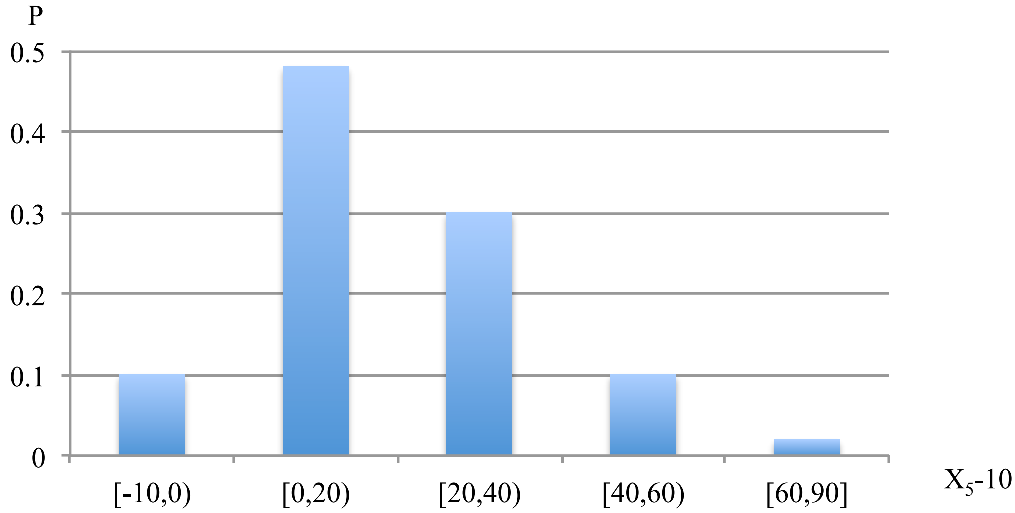

Figure 3: Assigned probabilities for $X_5 - 10$.

The estimate of 5 fatalities in case of a flash (pool) fire is rough and based on an estimated 50 people being exposed within the perimeters of the plant, and a fraction of fatalities set to 0.1. The analyst group review the models used and the assumptions made related to the number of fatalities, and excludes the possibility that more than 10 people can be killed due to this scenario. Within the range 0 - 10 fatalities, 5 is the best estimate but there are a number of factors that could lead to a different value in this range. A triangle-type distribution as shown in Figure 2 is used to reflect the analyst group uncertainty assessment. The uncertainties are larger in the explosion scenario 3, as Figure 3 shows. Here fatality numbers up to 100 are considered possible, although quite unlikely. For such extreme outcomes to occur, several barriers must fail in the system.

This rather crude assessment of the structural model uncertainty is then evaluated in view of the purpose of the risk assessment, which is to demonstrate compliance with regulatory criteria. The analyst group judges the uncertainty to be too large for model accreditation, and advise remodeling using G' in place of G. Adopting this new model the structural model uncertainties are considered small and the model is accredited.

Next considering ii), we seek to establish imprecision intervals on the uncertainty associated with the model error $\Delta_G(X)$, again conditional on X (scenario 2 or 3). Due to limited information and reluctance among the group members to assign exact probabilities, imprecise probability intervals are assigned to reflect the groups uncertainty assessment. The assigned intervals for $X_4 - 5$ and $X_5 - 10$ are presented in Figures 4 and 5, respectively.

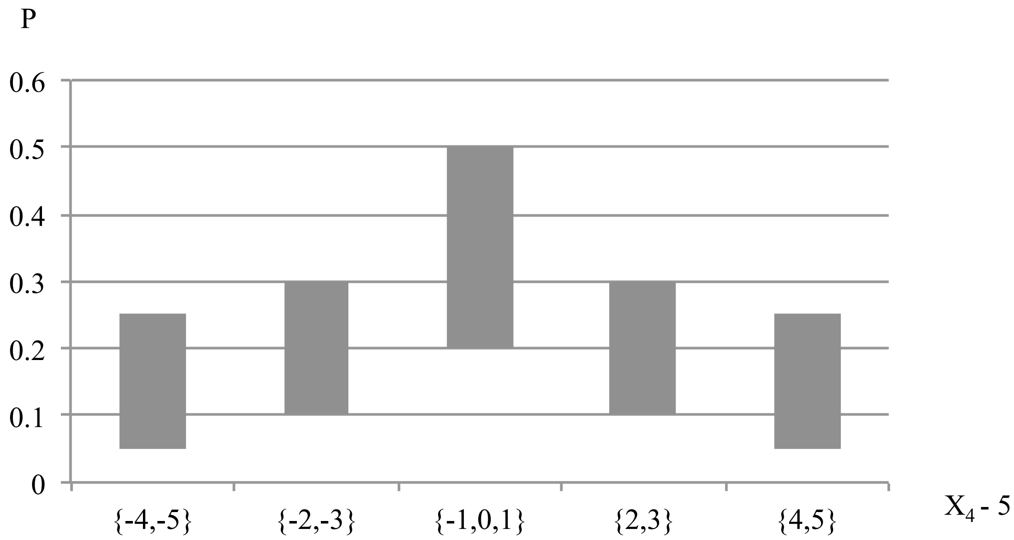

Figure 4: Assigned Imprecision Intervals for X_4 - 5.

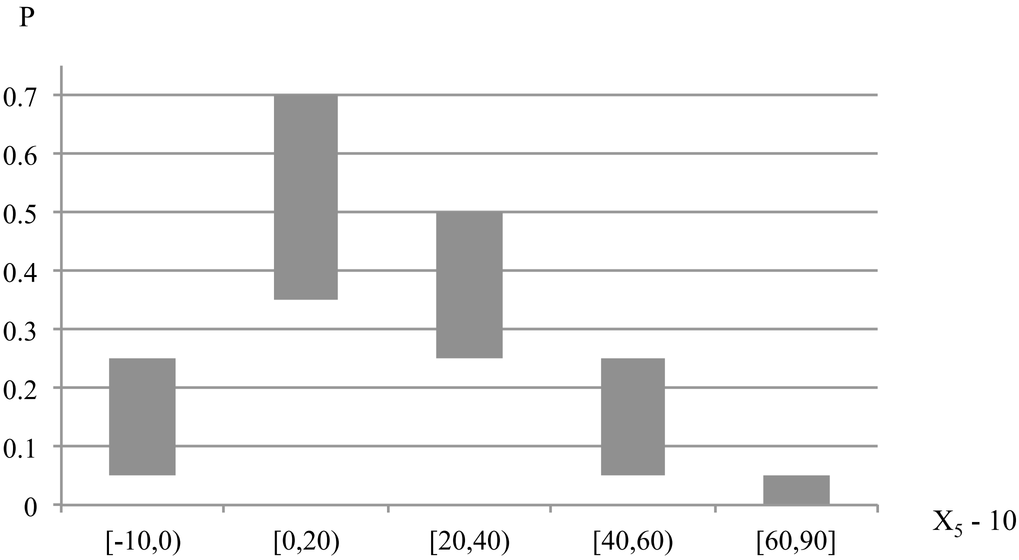

Figure 5: Assigned Imprecision Intervals for X_5 - 10.

The group concludes as in the case of i): the structural model uncertainty is too large for model accreditation. Remodeling is required, which leads to the adoption of G' as defined above.

The assumptions 2 and 3 are judged to give rise to small model output uncertainties. Low probabilities (less than 0.05) are assigned for these factors not to hold, and it is concluded that there is no need for remodeling. Models avoiding these assumptions would lead to considerably more complex models, but with little beneficial effect on the output model uncertainty.

Lastly, considering approach iii), we aim at giving a qualitative importance score of the assumptions made in the quantification. The first step is to perform a systematic identification of all the main assumptions that the assigned probabilities are based on. This task is carried out by the risk analysts, but to ensure that the identified list covers the key assumptions an independent review should be performed. Then, the importance of each assumption is measured by uncertainty and sensitivity analysis. A guideline for classifying the uncertainties and sensitivities in three categories (high, medium low) is shown in Appendix A. To obtain a high importance score, the probabilities assigned must be judged as sensitive to changes in the considered assumption, and the assumption must be subject to large uncertainties. The group identifies the following assumptions (in line with the above list), and gives the evaluations in Table 2:

1) The values 5 and 10 representing the number of fatalities following the scenarios 2 and 3 respectively.
2) That X_0 is either 0 or 1 release.
3) That X_3 is either a flash (pool) fire or an explosion in case of a delayed ignition.

Table 2: Importance Assessment.

Assump-tion	**Uncertainty**	**Sensitivity**	**Importance**
1	High	High	High
2	Low	Low	Low
3	Low	Medium	Low-Medium

From Table 2, we see that the uncertainties are judged high for assumption 1 as the values used represent strong simplifications, and low for the assumptions 2 and 3. The sensitivity is also judged high for assumption 1 as small changes in the number of fatalities for scenarios 2 and 3 would result in significant changes in the output probabilities. A change in the number of releases will also strongly affect the output probabilities, but as the likelihood is considered small for this event the sensitivity score is low. The sensitivity for assumption 3 is assigned a medium score reflecting that the number of fatalities and the output probabilities could be relatively strongly affected by a change in this assumption.

The practical conclusion is that remodeling is found necessary to reduce the criticality of assumption 1, whereas the assumptions 2 and 3 are judged acceptable.

4. CONCLUSION

We have applied a recently proposed framework for model (output) uncertainty to the consequence assessment from scenarios of hydrocarbon release in an LNG (Liquefied Natural Gas) plant located in an urban area. The assessment starts from the assumption that no experimental data are available. The purpose of the assessment is to demonstrate compliance with regulatory criteria for the risk associated with loss of lives. The framework is shown capable of supporting representations of the knowledge input into the assessment by alternative approaches of uncertainty modeling, such as subjective or imprecise probabilities, and qualitative schemes of value judgment. The present work is intended to be the first one of a number of applications of the proposed framework, aimed at verifying the suitability of its practical use in different problem and data/information contexts, and related uncertainty representations.

Appendix A. Uncertainty Assessment Score Interpretation.

Based on Flage and Aven (2009), see also Selvik and Aven (2011).

Aspect	Score	Interpretation
Uncertainty	High	One or more of the following conditions are met: - The assumptions made represent strong simplifications. - Data are not available, or are unreliable. - There is lack of agreement/consensus among experts. - The phenomena involved are not well understood; models are non-existent or known/believed to give poor predictions
	Medium	Conditions between those characterizing low and high uncertainty.
	Low	One or more of the following conditions are met: - The assumptions made are seen as very reasonable. - Much reliable data are available. - There is broad agreement/consensus among experts. - The phenomena involved are well understood; the models used are known to give predictions with the required accuracy.
Sensitivity	High	Relatively small changes in base case values needed to bring about altered conclusions.
	Medium	Relatively large changes in base case values needed to bring about altered conclusions.
	Low	Unrealistically large changes in base case values needed to bring about altered conclusions.
Importance	High/Medium/ Low	Average of the other two aspect scores.

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Paper II – Application of a new framework for model (output) uncertainty analysis on a probabilistic model for risk assessment

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Enrico Zio, Politecnico di Milano/ Ecole Centrale Paris and Supelec

Application of a new framework for model (output) uncertainty analysis on a probabilistic model for risk assessment

Abstract: This paper aims at demonstrating the applicability of a recently developed framework for assessing model (output) uncertainties (epistemic uncertainties about the differences between the true values of the output quantities and the values predicted by the model) in models used in risk assessments where no experimental data are available at the time of the assessment. The framework is based on a distinction between model (output) uncertainty, structural model uncertainty and input quantity (parameter) uncertainty, and relates the model (output) uncertainty to the objectives of the modeling and the risk assessment. The application is related to the use of a Poisson model for representing the number of events occurring in specified intervals, and the use of both probabilistic and non-probabilistic approaches for representing epistemic uncertainties. We show how the selection of responses (remodeling, acceptance, or rejection of the model) is based on judgments of the different type of uncertainties.

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1. INTRODUCTION

Within risk assessment, models are commonly used to represent systems and provide predictions and estimates of relevant quantities. The quality of a risk assessment – its strength in providing decision support - relies strongly on the "goodness" of the models used.

The models are simplifications of the real systems, and their accuracy has to be balanced against their timely and efficient use. In this paper, the focus is on probability models describing the variation in quantities characterizing a huge (infinite) population of similar units, referred to as stochastic or aleatory uncertainty in the risk assessment literature. We exemplify the problem by considering the Poisson model for describing the variation in the occurrences of a specific event on the real axis. As the quantity of interest, we consider the number of events occurring in a specific period of time.

In this typical risk assessment setting, the issue of model uncertainty arises. This type of uncertainty has been studied by several authors, see e.g. Zio and Apostolakis (1996), Devooght (1998), Nilsen and Aven (2003), Helton et al. (2004), Droguett and Mosleh (2008), Baraldi and Zio (2010) and Aven and Zio (2011). The present paper is based on the framework introduced by Aven and Zio (2011). In Bjerga et al (2012) we presented an application of this framework within a risk assessment related to hydrocarbon releases in an LNG (Liquefied Natural Gas) plant in an urban area. The present paper extends the work by considering a probability model (the Poisson model). Through the example, we clarify the meaning of the various concepts of the model uncertainty framework and show how they can be described and measured using different approaches, including interval probabilities. Before we introduce the Poisson model and study its uncertainty, we give a short presentation of the framework.

2. MAIN FEATURES OF THE FRAMEWORK

This Section gives a formal introduction of the framework set forth in Aven and Zio (2011), and presents its main features from two main sides: the first relates to the model output uncertainty concept itself, breaking it down in manageable parts so as to gain insights and use appropriate analysis tools; the second relates to the objectives of modeling and risk assessment, and the links to model output uncertainty.

2.1. Model Output Uncertainty

Consider an event/system/process subject to a risk assessment, and assume that at the time of the assessment no experimental data is available. Let Z represent the true value found in an unfolding future, and let G(X) be the model prediction at the time of the assessment where X is the input parameters. Both X and Z may be vectors. Define:

> *Model error*: The difference, $\Delta G(X)$, between the model predictions, G(X) and the true future value Z (i.e. $\Delta G(X) = Z - G(X)$), and
>
> *Model output uncertainty*: The uncertainty associated with the true value of the model error.

A closer look at this taxonomy reveals that the model output uncertainty is actually the *epistemic* uncertainty of the model error and hence it may in theory be assessed using a suitable tool for measuring this type of uncertainty, like subjective probabilities and interval probabilities.

The model output uncertainty is decomposed into two categories:

> *Structural model uncertainty*: The conditional uncertainty associated to the model error $\Delta_G(X)$, given the true value X_{True} (i.e. $\Delta_G(X_{True})$).
>
> *Input quantity (parameter) uncertainty*: The uncertainty associated with the true value of the input quantity X.

The structural model uncertainty is expressing the epistemic uncertainty under the assumption that the input parameters are known (the true values), and relates then to the model structure itself, typically associated with assumptions and suppositions, approximations and simplifications made in the model. Input quantity (parameter) uncertainty is on the other hand reflecting epistemic uncertainties relating to the model inputs X.

Sources of structural model uncertainty stem from actual "gaps" in knowledge which can take the form of poor understanding of phenomena that are known to occur in the system, as well as complete ignorance of other phenomena. This type of uncertainty can lead to "erroneous" assumptions regarding the model structure. Other sources of structural model uncertainty stem from approximations and simplifications introduced in order to translate the conceptual models into tractable mathematical expressions.

2.2. Objectives of Modeling and Risk Assessment in Relation to Model Output Uncertainty

The objectives of modeling and risk assessment in a model output uncertainty context, and set forth in the framework, are founded on four categories typical of industrial practice as presented in de Rocquigny et al. (2008):

- *Accredit*: To reach a required level of quality for the model by validation for its certified use.
- *Understand*: To understand the influence of uncertainties on the results of the analysis and rank their importance so as to guide additional efforts (measurements, research, etc.) for uncertainty mitigation.
- *Select*: To compare performances of alternative system designs, operation modes and maintenance policies for "optimal" choices.
- *Comply*: To demonstrate compliance of a system, process, procedure with regulatory criteria.

The framework proposed in Aven and Zio (2011) sets forth the links between these categories and the model output uncertainty analysis. It points at uncertainty analysis as a tool to accredit the model, so as to ensure a certain quality and possible certification. In the accreditation process, the understanding of the influence of uncertainties on the results of the analysis is of importance, to adequately guide the uncertainty reductions. If the model considered cannot be accredited, remodeling is required. When an accredited model is obtained, a risk analysis might be conducted to inform the decision makers on the selection and compliance in line with the objectives stated above.

The characteristic that no experimental data exist at the time of the assessment leads us away from classical statistical tools for validation and subsequent accreditation of the model. Instead validation transforms into utilizing expert/analyst argumentation based on established scientific theories and specific knowledge about the system, which the model assessed, intends to describe.

An important observation is that no restrictions pertain to utilizing a pure probability-based approach. The framework opens for both probabilistic and non-probabilistic approaches, and thereby injects flexibility into the uncertainty analysis, giving the opportunity to choose the approach that is judged to best represent/express the uncertainties, given the specific phenomena and surroundings examined.

In all instances a (accompanying) qualitative analysis is encouraged and necessary, since even if accredited, a model still has inherent limitations and weaknesses, and these should be presented as part of the total risk assessment.

3. CASE STUDY: A POISSON MODEL

The case study pertains to a risk assessment context, and the modeling of the occurrences of a type of undesirable event in the future for a specific activity. To represent such occurrences, a Poisson model is used. Let N(t) be the number of events occurring in the time interval [0,t]. It is assumed that the stochastic process N is a homogeneous Poisson process with occurrence rate λ. Hence N(t) has Poisson distribution with expected value λt, i.e. $P(N(t) = n \mid \lambda, t) = p(n| \lambda, t) = (\lambda t)^n e^{-\lambda t}/n!$, n = 0,1,2… We interpret λ as the expected number of events occurring per unit of time.

Furthermore let $p_0(n| t)$ be the "true" distribution of the number of events in [0,t], obtained by considering an infinite number of activities similar to the one considered. The average number of events occurring in [0,t] is defined as λ_0. The Poisson distribution $p(n| \lambda, t)$ is a model of this true distribution, and λ_0 is the true value of λ. The distributions p and p_0 represent the true variation in the number of events occurring in such intervals and the variation as modeled, respectively.
In the case study, the objective of the risk assessment is to verify that the 95th percentile, n_{95}, of $p_0(n|t_0)$ is in compliance with a regulatory threshold value n_M.

From Section 2, we identify n_{95} as the quantity of interest, Z, λ as the parameter X, and the model representing Z, $G(\lambda)$ as the 95th percentile of the Poisson distribution, which we refer to as $n_{95}(\lambda)$. The model error can thus be written $\Delta G(\lambda) = n_{95} - G(\lambda) = n_{95}- n_{95}(\lambda)$. The structural model uncertainty relates to uncertainty about the value of $\Delta G(\lambda_0) = n_{95} - G(\lambda_0) = n_{95}- n_{95}(\lambda_0)$, and the parameter uncertainty to the true value of λ, i.e. λ_0.

As a concrete example of this setting, we can consider potential releases from a commercial pilot facility/system handling hydrocarbons with new technology. This system is operating in a seasonal market following economic cycles and variations in demand. The system is in the planning phase, and compliance with regulatory regulations must be demonstrated prior to construction. Concerning the environmental risk and potential releases, the authorities acknowledge that releases could occur due to the novel technology and the limited operational experience. The authorities have specified an acceptance level of 5 releases during one month, to license construction and continuous operation; the system must be demonstrated capable of hold this true with 95% certainty.

To model the case, a homogeneous Poisson process is initially found to be representative for the number of releases. The parameter λ representing the average number of releases is estimated to be

1.75 per month. From this number the probability of having more than 5 releases is estimated to be 1% using the Poisson distribution, and it is concluded that requirement from the authorities is met.
But what about uncertainties and model uncertainties in particular? In the following we will address this issue by conducting an uncertainty analysis with different approaches and perspectives, all in line with the structure and terminology presented in Section 2.

3.1. Uncertainty Analysis

To evaluate and conclude on model accreditation, two different approaches i) and ii) for uncertainty assessments are used:

i) A *qualitative scheme* giving scores on the importance of the assumptions made, reflecting both the degree of sensitivity and the uncertainty (see e.g. Flage and Aven 2009, Selvik and Aven 2011). The basic features of the approach are outlined in Appendix A.
ii) *Imprecise probabilities*: assigning an imprecision interval, e.g. [0.1, 0.2], means that the assigner states that his/her degree of belief is greater than or equal to the urn chance of 0.10 (the degree of belief of drawing a specific ball out of an urn containing 10) and less than or equal to the urn chance of 0.20. The analyst is not willing to make any further judgments.

3.1.1. Structural Model Uncertainty

We first look into approach i). The analysis starts with evaluating the underlying assumptions for the Poisson process, and how they contribute to the structural model error $\Delta_G(\lambda_0)$.The homogeneous Poisson process can be defined as follows:

> A stochastic process N is a homogeneous Poisson process with rate λ if,
> E[N(t + h) - N(t) | history up to t]/h converges to λ, as h converges to zero, for all t.

From the definition we may identify two critical model conditions for the Poisson process (which we will refer to as the Poisson assumptions):

> 1. *Independence:* That the occurrence rate for the events at a specific point in time t is independent of the history up to that time.
> 2. *Stationarity*: The occurrence rate is a constant, λ.

The analyst group discusses the uncertainties related to these assumptions. Several sources of uncertainties in this regard are pointed to (formulated as assumptions):

a) There is no deterioration of the system over time.
b) A release will not cause long term learning effects.
c) Utilization of the system does not vary in time.
d) There is no extra stress imposed on the system during a release, increasing the chance of new releases close in time.

Following the procedure of approach i), a qualitative importance score is assigned for each of the assumptions justifying the model. The first step in the approach is a systematic identification of the assumptions, and is done by the risk analyst. An independent review should be performed to ensure that all key assumptions are identified. The sources of uncertainty a) through d) cover the list of assumptions in this case. Next, a classification of the uncertainties and sensitivities in three categories (high, medium and low) is performed following the guideline in Appendix A. To obtain a high importance score, the assumptions must be subject to large uncertainties, and the model conclusions must be sensitive to changes in the assumptions. The score evaluations are presented in Table 1, followed by brief explanations in the subsequent paragraphs.

Table 1. Importance Score for Assumptions a) through d).

Assumption	Uncertainty	Sensitivity	Importance
a)	Low	Medium	Low-Medium
b)	Medium	Medium	Medium
c)	High	High	High
d)	Low	Low	Low

a) Over time the system will experience deterioration (causing e.g. mechanical fatigue and rupture) which is increasing the chance of releases over time. On the other hand, maintenance plans are prepared to restrain the system degradation. Accordingly the analyst group rates the uncertainty as low. Relatively many releases due to deterioration, and over a short time interval, are considered necessary to affect the output, hence the sensitivity is judged medium.

b) In case of a release the system will likely experience causation focus, to be able to understand what happened and give input to how to prevent similar releases in the future. The occurrence rate thus depends to some extent on its history. The company has implemented a zero release vision, which indicates that the attention on improvements will be strong. Both the uncertainty and sensitivity are judged medium.

c) There are also concerns related to the utilization of the system, which is assumed responsive to shifting demand and economic cycles. For instance it is questioned whether the utilization via demand is correlated with volatile oil prices; i.e. high prices compels high utilization, hence a higher chance of releases, and vice versa. The group finds the assumption of fixed utilization as a strong simplification subject to large uncertainties, and the stationary assumption (2), in particular, is found glaringly sensitive to changes.

d) Lastly, the increased stress level is also a factor that may come to play. Due to extra stress imposed on the system during a release, a second release is more likely to occur close in time. However the group finds that the additional stress is minor and assigns low scores to this assumption.
Given the purpose of the risk assessment, the analyst group consider assumption a) and d) to be acceptable; the effect on the model error is judged small enough for accreditation. The importance score on assumptions b) and c) suggests that the homogeneous Poisson model is invalidated, and remodeling should be seriously considered.

To this end a quantitative analysis according to approach ii) is adopted, using imprecision intervals. Figure 1 shows the results for the structural model error $\Delta_G(\lambda_0) = n_{95} - n_{95}(\lambda_0)$ due to b) and c). As we see from Figure 1 the group believes the error to be between -5 and 5.

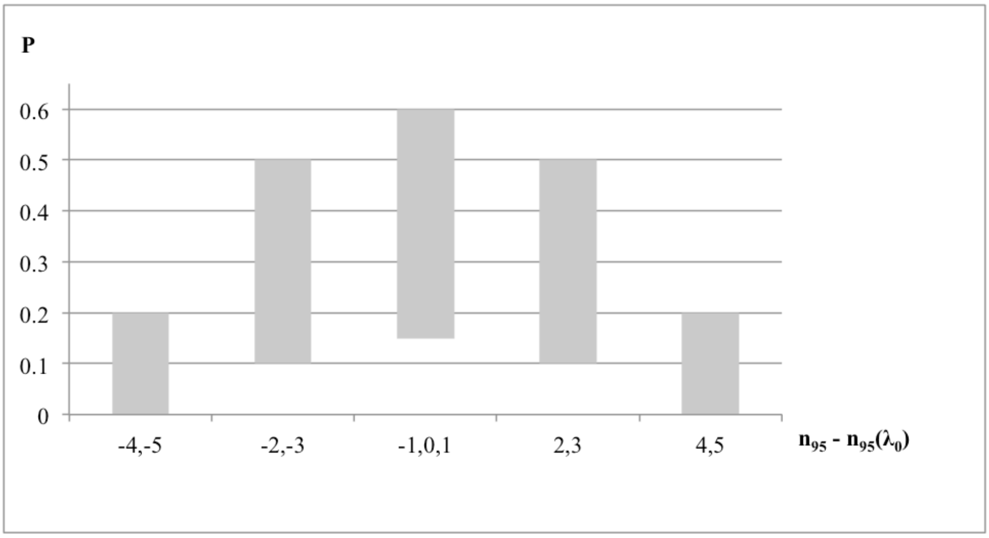

Figure 1. Imprecision Intervals for the Structural Model Error $\Delta_G(\lambda_0)$.

Based on this analysis, the analyst group concludes that the uncertainties are too high for accreditation - remodeling is advised. Demonstrating compliance with the regulatory threshold value of 5 releases per month cannot be justified when the model uncertainties are so large.

It is beyond the scope of the present paper to actually perform the remodeling, but some indications of the features addressed will be given. The uncertainties pertain mainly to the stationary assumption and to a less degree the independence condition. This suggests that a non-homogeneous Poisson process may be a candidate as a first test. Starting from this model we can repeat the above verification process and assess the model output uncertainties. If the model cannot be accredited, other models should be considered, including doubly stochastic Poisson process (also known as a Cox process or mixed Poisson process). If we also question the dependence assumptions, we need to open up for other types of counting processes (see Aven and Jensen 1999).

4. DISCUSSION

The motivation and use of the framework studied in the present paper is thoroughly discussed in Aven and Zio (2011). Of the many issues thereby raised, we emphasize one key point: model output uncertainty is not the same as the model error $\Delta_G(X)$: it is actually the epistemic uncertainty of it. To measure this uncertainty different tools can be used as illustrated above. The model output uncertainty and its measurement are considered in relation to the magnitude of the model error, as is clear from the analysis in Section 3. Hence what is a small model uncertainty output cannot be seen in isolation from the model error.

The present paper reports on the application of these concepts and the framework to a Poisson model. The analysis has shown that the framework provides meaningful definitions that allow performing uncertainty analyses of the model error and the structural model error. A critical step of the analyses is the approach used to assess the model uncertainties. In the above analysis we looked at interval probability assignments and a qualitative score method. The first approach is attractive because based on mathematically well-defined concepts, but is difficult to use in practice as it is typically based on assignments with little support. The arbitrariness in the assignment process is a problem for exact probability assignments, but is also present for the interval probabilities.

The qualitative approach is crude, based on more or less precise definitions of scores. Nonetheless, it can be a useful tool for quickly pointing at the most important sources of the model uncertainties and then in its turn, gives a basis for decision on whether the model should be accredited or not.

The analysis of the model uncertainties are based on the following four elements:

1. Identify all concepts in the framework (Z,G,X)
2. Determine which approach should be used for assessing the uncertainties
3. Perform the uncertainty assessments
4. Make judgments about accreditation and possible remodeling

A key issue to determine in relation to item 1 is whether the quantities of interest Z and X are probabilistic parameters of probability models (as in the example considered in Section 3) or some physical quantities, e.g. the number of events occurring or the number of fatalities as studied in Bjerga et al. (2012). For proper analysis of the model uncertainties, precision is required on this point as G is strongly dependent on this. For the choice of approach in item 2, it is clear that different situations call for different approaches. For a quick analysis, the qualitative approach may be preferred; in other situations both a qualitative and a quantitative approach may be used.

5. CONCLUSION

The aim of the present paper has been to demonstrate the applicability of a recently developed framework for assessing model (output) uncertainties in models used in risk assessments where no experimental data are available at the time of the assessment. The overall conclusion of the analysis is that the framework performs as intended in this case and that meaningful concepts and analyses can be defined/conducted. Several approaches for describing the model uncertainties are applicable, and in practice it could be most informative to use combinations of them, i.e. both qualitative and quantitative approaches.

Appendix A. Guidelines for providing scores in the qualitative approach for assessing model uncertainties

Based on Flage and Aven (2009), see also Selvik and Aven (2011).

Aspect	Score	Interpretation
Uncertainty	High	One or more of the following conditions are met: - The assumptions made represent strong simplifications. - Data are not available, or are unreliable. - There is lack of agreement/consensus among experts. - The phenomena involved are not well understood; models are non-existent or known/believed to give poor predictions
	Medium	Conditions between those characterizing low and high uncertainty.
	Low	One or more of the following conditions are met: - The assumptions made are seen as very reasonable. - Much reliable data are available. - There is broad agreement/consensus among experts. - The phenomena involved are well understood; the models used are known to give predictions with the required accuracy.
Sensitivity	High	Relatively small changes in base case values needed to bring about altered conclusions.
	Medium	Relatively large changes in base case values needed to bring about altered conclusions.
	Low	Unrealistically large changes in base case values needed to bring about altered conclusions.
Importance	High/Medium/ Low	Average of the other two aspect scores.

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