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Studies in damage detection using flexibility method

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Abstract

Structural health monitoring (SHM) is a way of implementing a system that can monitor, and provide data to help engineers secure the integrity of different structures like civil infrastructure, bridges, offshore structures and aerospace structures. The main purpose of SHM is to detect damages in structures before they become so large that they threaten the integrity and the functionality of the structure. After natural disasters like hurricanes and tsunamis, there is always an uncertainty in the health and safety of structures that has been exposed in some way by these natural phenomena. SHM might in these cases be tool that can assist in determining the integrity of the structure. For aging and deterioration structures SHM can be used to help determine the remaining lifetime of the structure. The main subjects involved in SHM are detection of damage, its location and determination of the severity. SHM techniques might be based on changes in the structural dynamical characteristics (measured by experimental mechanical vibrations), impedance and non-destructive evaluation methods (NDE).

In this thesis, *the flexibility method*, which is a vibration-based damage detection method, is tested to determine the pros and cons of applying the method to a SHM system. The method is tested on a fixed-fixed beam structure. By using this method the identification of damage is determined from the change in frequency response data, obtained from frequency response functions (FRF). The location of the damage is determined by the flexibility diagrams.

A finite element model (FE-model) of the fixed-fixed beam structure is made in the numerical computing software MATLAB. The beam is modeled with two-noded finite beam elements, and the damage is detected and located by the flexibility method for different damage cases simulated in the FE-model. The damages are implemented by reducing the E-modulus of one or more of the two-noded finite beam elements. The dynamical characteristics of the beam structure are obtained from simulated mechanical vibrations, where different frequencies of vibration are tested. The accuracy of the flexibility method is tested for all damage scenarios. From this investigation it can be concluded that there are both advantages and disadvantages when using the method to detect damages. FRF can for instance only determine if there is damage present in a structure. The flexibility method is not able to determine the severity of the damage. The flexibility method is also less accurate when a structure is subjected to multiple damages that lie in close vicinity to each other. If the beam has two damages present, and the severity of one of the damages is greater than the other, then the damage with the highest severity is easier to detect compared to the other.

Preface

This master thesis represents the completion of a master degree in Structure and Materials with specialization in offshore structures at the University of Stavanger. The topic for the thesis was offered to the author in January 2012. The work for the thesis was carried out during the spring semester of 2012. The topic for the thesis was offered by Phd. Arora Vikas who, also acted as supervisor for the work on the thesis.

I am very grateful to Vikas Arora who, despite his busy schedule, agreed to be my supervisor at UiS and providing me with valuable support and comments.

I am also grateful to my parents Rita Irene, Asbjørn and my sister May Lise, for their love and support throughout my education.

“Give thanks to the LORD, for he is good; his love endures forever”

- 1 Chronicles 16:14

Notation and abbreviations

CM	Condition monitoring
dB	Decibel
DP	Damage prognosis
DOF	Degrees of freedom
FE	Finite element
FEA	Finite element analysis
FEM	Finite element method
FFT	Fast Fourier transformation
FRF	Frequency response function
Hz	Hertz
MDOF	Multiple degrees of freedom
NDE	Non-destructive evaluation
NDT	Non-destructive testing
UiS	University of Stavanger
SDOF	Single degree of freedom
SHM	Structural health monitoring
SPC	Statistical process control
VBSD	Vibration-based structural damage
VBSHM	Vibration-based structural health monitoring

Mathematical symbols and operators

f	Function
[]	Matrix

{}	Vector
.	Time differentiation; for example, $\dot{x} = dx/dt$, $\ddot{x} = d^2x/d^2t$

Latin symbols

A_e	Elemental cross-section
$[A]$	Example matrix
B	Width
$[B]$	Example matrix
c	Damping coefficient
C	initial value constant
$[D]$	Damping matrix
E	E-modulus, Young's modulus
$f(t)$	Force function, applied to a SDOF system
F	Flexibility
$F(t)$	Force function, applied to a MDOF system
F_0	Force amplitude
F_d	Flexibility for damaged structure
F_u	Flexibility for undamaged structure
H	Height
$H(\omega)$	Frequency response function
i	Complex number; $\sqrt{-1}$
I	Moment of inertia
$[I]$	Identity matrix
k	Stiffness
$[K]$	Stiffness matrix
$[K]_e$	Elemental stiffness matrix
L	Length
l_e	Elemental length
m	Mass
$[M]$	Mass matrix
$[M]_e$	Elemental mass matrix
n	Node, mode
t	Time
u_n	Vertical displacement at a node n
x	Displacement
$\{x\}$	Displacement vector
$\{\bar{x}\}$	Mode of vibration; eigenvector
\dot{x}	Velocity
\ddot{x}	Acceleration
$\{y\}$	Eigenvector
ΔF	Change in flexibility
w_n	Vertical displacement at node n
$\{F(t)\}$	Periodic force function vector

Greek symbols

α	Structural damping coefficient
β	Structural damping coefficient
Δ	Difference
δ	Maximum flexibility value
θ_n	Rotational degree of freedom at a node n
Λ	Eigenmatrix
λ_i	Eigenvalue
λ	Scalar
ζ	Damping ratio
Φ	Mode shape
ρ	Material density
ω	Circular frequency
ω_n	Natural frequency
ω_0	Natural frequency
ω_i	Vibration frequency
Ω	Phase angle

Basic definitions

Matrix $[A]$	A matrix, A , is a system of mn quantities, called <i>elements</i> , arranged in a rectangular array of m rows and n columns [1].
Matrix element	An element located in row i and column j of a matrix is denoted as a_{ij} .
Square matrix	If $m = n$, then A is a square matrix of order n [1].
$[A]^T$	Transpose of a matrix, $[A]^T$, is obtained by interchanging the rows and columns with each other.
Symmetric matrix	A matrix is symmetric if it is equal to its transpose, that is, if $a_{ij} = a_{ji}$ [1].
$[A]^{-1}$	The matrix $[A]$, is invertible if there exists an $n \times n$ matrix $[B]$ such that $[A][B] = [B][A] = [I]$. Where $[I]$ is the identity matrix.
Vector $\{V\}$	A vector $\{V\}$ is a quantity that is completely specified by a magnitude and a direction [1].

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1 Introduction

1.1 Structural Health Monitoring (SHM)

During the service life of a structure, the structure will be subjected to various factors that have the potential to cause damage to the structure. These factors might be corrosion, fatigue, extreme load scenarios and other natural hazards and load scenarios. If the structure gets damaged there will be an alteration in the dynamical characteristics of the structure, as well as a danger of local and global collapse of the structure. Structural damage will also increase the danger of the structure not being able to be functional for its intended lifetime. Structural collapse might result in loss of human life, environmental damage and high economical cost or loss.

Future intelligent structures demand high system performance, structural safety and integrity, and low maintenance cost. To meet the challenge, SHM has emerged as a reliable, efficient, and economical approach to monitor system performance, detect damages if they occur, assess/diagnose the structural health condition, and make corresponding maintenance decisions[2]. The SHM process involves the observation of a structure or mechanical system over time using periodically spaced measurements, the extraction of damage-sensitive features from these measurements and the statistical analysis of these features to determine the current state of system health. For long-term SHM, the output of this process is periodically updated information regarding the ability of the structure to continue to perform its intended function in light of the inevitable aging and damage accumulation resulting from the operational environments[3].

An ideal SHM system typically consists of two major components: a built-in network of sensors for collecting response measurements, and a data analysis algorithm/software for interpretation of the measurements in terms of the physical condition of the structures[2]. A structure installed with a SHM system can be considered as a full-scale experimental model and system. The loads and response of the structure are recorded directly, from which the performance of the structure is identified. Once the life-cycle performance, the total cost of the initial investment and maintenance of a structure have been collected, the life-cycle performance-based design can be conducted accordingly. Therefore, the SHM technology is the basis of the life-cycle performance-based design approach[4].

SHM has also been represented as a process of conventional inspection, inspection through combination of data acquisition and damage assessment; and more recently as an embodiment of an approach enabling a combination of non-destructive testing (NDT) and structural characterization to detect changes in structural response[5].

After an extreme event, such as an earthquake or unanticipated blast loading, SHM is used for rapid condition screening. This screening is intended to provide, in near real-time, reliable information about system performance during such extreme events and the subsequent integrity of the system. SHM is being developed for aerospace companies that want to have some kind of damage identification tool to monitor space shuttle control surfaces hidden by heat shields. Development of such a damage identification tool might have significant life-safety implications[3]. The danger of fatigue in aircrafts that has been used long passed its intended design-life time, is of major concern in the aerospace industry. An assessment is

needed of how the aircrafts functionality in airworthiness is affected by aging, and by the dangerous combination between fatigue and corrosion. Prevention of such unexpected occurrences could be improved through the installation of on-board SHM systems that could assess the structural integrity monitoring and detect incipient damage before a possible catastrophic failure occurs. Other areas of SHM application might be offshore structures, civil and mechanical structures, dams and bridges.

The developments of embedded distributed piezoelectric sensor arrays and embedded fibre optic sensors have made SHM more easily possible[6], and in present times the data values from the SHM system is obtained from these type of sensors installed in the monitored structure. As the technological development in the field of sensor, communication and signal processing equipment is evolving more and more, the reliability of SHM technology is also becoming more and more evident. The prices for the equipment necessary to conduct SHM of a structure is also decreasing, and thus making SHM more appealing for different private and governmental companies.

As there is a large amount of research material and literature on the subject and applications of SHM, it is impossible to cover every aspects of SHM in this thesis. For further reading on the subject references are made to [3, 5].

1.2 Structural damage detection

The possibility that structures will suffer some sort of damage during their service life is highly probable. The main purpose of SHM is to detect these damages before they become so severe that they threaten the integrity and functionality of the structure.

In the most general terms, damage can be defined as changes introduced into a system that adversely affects its current or future performance. Implicit in this definition is the concept that damage is not meaningful without a comparison between two different states of the system, one of which is assumed to represent the initial, and often undamaged, state. This thesis is focused on the study of SHM and damage identification in structural and mechanical systems. Therefore, the definition of damage will be limited to changes to the material and/or geometric properties of these systems, including changes to the boundary conditions and system connectivity, which adversely affect the current or future performance of these systems[3].

In terms of length-scales, all damage begins at the material level. Although not necessarily a universally accepted terminology, such damage is referred to as a defect or flaw and is present to some degree in all materials. Under appropriate loading scenarios, the defects or flaws grow and coalesce at various rates to cause component and then system-level damage. The term damage does not necessarily imply a total loss of system functionality, but rather that the system is no longer operating in its optimal manner. As the damage grows, it will reach a point where it affects the system operation to a point that is no longer acceptable to the user. This point is referred to as failure. In terms of time-scales, damage can accumulate incrementally over long periods of time such as that associated with fatigue or corrosion damage accumulation. On relatively shorter time-scales, damage can also result from scheduled discrete events such as aircraft landings and from unscheduled discrete events such as enemy fire on a military vehicle or natural phenomena hazards such as earthquakes[3].

The effects of damage on a structure can be classified as linear or nonlinear. A linear damage situation is defined as the case when the initially linear-elastic structure remains linear-elastic after damage. The changes in modal properties are a result of changes in the geometry and/or the material properties of the structure, but the structural response can still be modeled using linear equations of motion. Linear methods can be further classified as model-based and non-model based. Model-based methods assume that the monitored structure responds in some predetermined manner that can be accurately discretized by finite element analysis (FEA), such as the response described by Euler-Bernoulli beam theory. Nonlinear damage is defined as the case when the initially linear-elastic structure behaves in a nonlinear manner after the damage has been introduced. One example of nonlinear damage is the formation of a fatigue crack that subsequently opens and closes under the normal operating vibration environment. Other examples include loose connections that rattle and nonlinear material behavior such as that exhibited by polymers[7].

Damage identification is carried out in conjunction with five closely related disciplines that include SHM, condition monitoring (CM), non-destructive evaluation (NDE), statistical process control (SPC) and damage prognosis (DP). Typically, SHM is associated with online-global damage identification in structural systems such as aircraft and buildings. CM is analogous to SHM, but addresses damage identification in rotating and reciprocating machinery, such as those used in manufacturing and power generation. NDE is usually carried out off-line in a local manner after the damage has been located. There are exceptions to this rule, as NDE is also used as a monitoring tool for in situ structures such as pressure vessels and rails. NDE is therefore primarily used for damage characterization and as a severity check when there is a priori knowledge of the damage location. SPC is process-based rather than structure-based and uses a variety of sensors to monitor changes in a process, one cause of which can result from structural damage. Once damage has been detected, DP is used to predict the remaining useful life of a system[3].

A way of defining the presence and location of damage in a structure can be with vibration-based structural health monitoring (VBSHM). The fundamental idea for VBSHM is that the damage-induced changes in the physical properties (mass, damping, stiffness, etc.) will cause detectable changes in modal properties (natural frequencies, modal damping, mode shapes, etc.). For instance, reductions in stiffness resulting from the onset of cracks may change the natural frequencies and other modal parameters (as will be investigated in chap. 3). Therefore, it is intuitive that damage can be identified by analyzing the changes in vibration features of the structure[8]. By using VBSHM one has a global-damage detection technique, which is an alternative to the local-damage detection techniques like visual inspection and x-ray scanning. The main difference between global and local damage detection techniques is that you don't need to have prior information about the location of damage. This is of course a major advantage if you want to install a SHM system that can monitor the condition of the whole structure in a continuous or determined timeframe.

The vibration-response data can be analyzed and give an indication of damage. The presence and the subsequent determination of the location of damage can be determined through different application of frequency response (natural frequency shift), eigenvectors and other modal analysis like e.g. mode shape analysis. There has been published a lot of scientific papers covering different ways of how VBSHM can be used to determine presence, severity and location of damage of different types of structures.

One problem with VBSHM is that small cracks may not affect the eigenfrequencies or the first few eigenmodes of the structure[6], and because of this many researchers began to

develop damage detection methods based on fast Fourier transformation (FFT) including [9]. However, the drawback of FFT based methods is that FFT is unable to provide information about when at an instant of time/space a particular frequency band occurs. This drawback has been removed by the developed time-frequency analysis tool called wavelet[6].

According to Rytter (1993), damage detection technique can be classified as a five step process[3].

- (i) Existence: determine that there is damage located in the structure.
- (ii) Location: determine the location of the damage.
- (iii) Type: classify the type of damage.
- (iv) Extent: classify the severity of the damage.
- (v) Prognosis: estimate the remaining lifetime of the structure

Steps (i) and (ii) of the damage detection process is investigated in this thesis.

The methodology of VBSHM is investigated more in detail in chapter 2.

1.3 Finite Element Method/Analysis

The finite element method (FEM), also known as finite element analysis (FEA), is a numerical technique that is widely used in many fields of engineering. The range of application is mainly static-dynamic problems, heat transfer problems, fluid dynamic problems, electromechanics and geomechanics. The finite element method is a useful tool for finding approximate solutions of problems related to structural design, where the relevant problem can be idealized with a model in which geometry, material properties, loads and geometric boundary conditions are simplified in order to obtain a reasonable and logical solution to the problem. When performing a finite element analysis of a structural design problem, the given problem is usually evaluated by equations containing matrices and vectors. Reference is made to [10, 11] for more information of the general theory and applications of FEM/FEA.

FEM is a very useful to perform dynamic analysis of structures. The dynamic characteristics (i.e. modal properties) of a structural system (e.g. natural frequencies, frequency response) can be obtained from vibration and dynamical excitation analysis. As structural damage alters the dynamic characteristics of a given system, dynamic finite element analysis is very commonly used in damage detection methodologies.

In order to perform a structural FEA it is necessary to make an idealized finite element model of the physical problem. The model is then discretized by dividing the model into a mesh of finite elements[10]. In FEM theory there are 3 types of elements:

- 1) 1-D elements: bar, spring, pipe etc
- 2) 2-D elements: beam, membrane, plate, shells

3) 3-D elements: solids

In this thesis, the objective is to perform damage detection on a beam structure made up of two-noded finite beam elements. And thus, the finite element modeling will be conducted using 2-D elements. There are two main types of beam theories; one is known as Euler-Bernoulli beam theory, or simply known as elementary beam theory. In this theory, it is assumed that you can ignore the effects of transverse shear deformation. The other theory is that of Timoshenko beam theory where the transverse shear deformations are included. The Timoshenko beam theory is usually applied when vibrations of beams are studied and is the theory that is usually applied when vibration-based damage detection is performed on beams.

Fig. 1.3-1 shows a 2-D beam element with 2 nodes. At each node there are two degrees of freedom; θ_n represents the rotational degree of freedom at the relative node n . w_n represents the vertical displacement at the relative node n .



Figure.1.3-1 Degrees of freedom for a beam element[12].

For a 2-D beam element the elemental stiffness matrix is given as[10]

$$[K]_e = \frac{EI}{l_e^3} \begin{bmatrix} 12 & 6l_e & -12 & 6l_e \\ 6l_e & 4l_e^2 & -6l_e & 2l_e^2 \\ 12 & -6l_e & 12 & -6l_e \\ 6l_e & 2l_e^2 & -6l_e & 4l_e^2 \end{bmatrix} \quad (1)$$

where l_e is the length of the element, E is the E-modulus/Young's modulus and I is the moment of inertia of the element.

The elemental mass matrix for a 2-D beam element is given as[10]

$$[M]_e = \frac{\rho A_e l_e}{420} \begin{bmatrix} 156 & 22l_e & 54 & -13l_e \\ 22l_e & 4l_e^2 & 13l_e & -3l_e^2 \\ 54 & 13l_e & 156 & -22l_e \\ -13l_e & -3l_e^2 & -22l_e & 4l_e^2 \end{bmatrix} \quad (2)$$

where ρ is the material density, A_e is the cross-section of the element and l_e is the elemental length.

In order to obtain the global stiffness and mass matrices of a beam in an actual FE-model, the elemental stiffness and mass matrices are added together at the nodes that connect the elements together.

1.4 MATLAB®

MATLAB® is a high-level interactive programming language that is applicable for solving problems in various fields such as engineering and other scientific areas. The MATLAB® programming language is highly vectorized in that it handles data in vectors or matrices as a whole. This makes MATLAB® a very useful tool for FEA of structures. In this thesis, MATLAB® has been used to do a FEA of a beam structure.

1.5 Objectives

The objectives of this master thesis are as follows

- a) To study various SHM methods
- b) To make a FE-model of a fixed-fixed beam structure
- c) To investigate different damage scenarios with application of the flexibility method on a fixed-fixed beam structure

1.6 Organization of the work

The content of this master thesis is organized in four chapters.

Chapter 1 gives an introduction into the objectives of this thesis and general information about several important aspects such as use of software.

Chapter 2 presents how vibration-based structural health monitoring works. Various vibration-based damage detection methods are presented. Basic application and theory of dynamical finite element method and structural dynamics is presented. Single- and multi-degree of freedom systems are presented and explained. The theory and application of frequency response function analysis (FRF) is presented and application to damage identification is explained. The flexibility method is presented and application to SHM is explained. The effects of noise contamination in datasets are presented.

Chapter 3 presents the analysis part of this thesis. Here damages are implemented on a fixed-fixed beam structure and damage is detected and localized through application of FRFs and flexibility diagrams, respectively.

Chapter 4 presents conclusions and recommendations.

2 Vibration based structural health monitoring (VBSHM)

2.1 Introduction

In this thesis the concept of SHM is investigated through vibration data retrieved from simulation studies. The following chapter will give a description of how damage can be detected through various vibration-based damage detection techniques. The theory of the frequency response functions (FRF) is presented, as well as its application to damage detection. The application and theory of the flexibility method is presented as a method that can be used to localize the damage(s) through a non-parametric study of the structural state.

In this thesis, the flexibility method is used to perform damage detection studies on a fixed-fixed beam structure, where different damage scenarios are simulated and investigated.

2.2 Vibration-based damage detection methods

As mentioned in section 1.2, structural damage detections are usually determined as either local or global. Local damage techniques are better known as non-destructive testing techniques (NDT). These techniques are usually visual, liquid-penetrant, magnetic, ultrasonic, eddy current and x-ray testing methods. The NDT methods are a very good choice when you want to investigate small and regular structures, such as pressure vessels. However, for the large and complicated structures in invisible or closed environments, it is very difficult to detect damage using local damage detection method. The engineers have to make on-site structural damage detection. Therefore, local damage detection methodology can only be used to detect some special components of a structure. In order to detect damage throughout the whole structure, especially some large, complicated structures, the global damage detection methods based on vibration-based structural damage (VBSD) can be used[13].

Structures can be evaluated as a dynamic system with a specific stiffness, mass and damping. The presence of damage in a structure will alter its dynamic signature. This means that change in stiffness, mass and damping for a given system will give a dynamic signature that is different from the dynamic signature of the intact/healthy structure. The dynamic signature of a system is generally evaluated through modal parameters and the structural frequency-response function (FRF). Thus, the change of the structural modal parameters can be taken as the signal of early damage occurrence in the structural system. Recently (as of 2006), researches on vibration-based structural damage detection have become a hot area because it can solve this particular problem, i.e., to insure reliable operation of multitudinous important engineering structures by online and continuous damage detection using vibration-based methods[13].

When VBSD is used in SHM, the combined term is noted as VBSHM (vibration-based structural health monitoring). This methodology involves a combination of several disciplines, such as structural dynamics, artificial intelligence, signal processing and measure technology[13]. VBSHM is divided into two groups: traditional-and modern type. The traditional types involves the mechanical characteristics of a structure, i.e., eigenfrequencies, mode shapes, modal strain energy etc. However, this kind of method generally requires experimental modal analysis or transfer function measure, and this is not convenient for online detection of structures in service because these experimental measures often need multifarious instrument or manual operation. The modern-type refers to detection method for

structural damage based on online measured response signal of structures in service. Because realization of this method is simple and also feasible to build for continuous and automatic structural damage detection for structures in service, researches on this area have become the most important hot area for recent two decades. Among the modern-type methods for structural damage detection, the representative ones include wavelet analysis, Genetic algorithm (GA) and Artificial Neural Network (NN)[13]. In this thesis there will only be studies of the traditional types of VBSHM.

Generally, structural damage detection can be divided into five levels: (1) identification of damage existence in a structure; (2) localization of damage; (3) identification of the damage type; (4) quantification of damage severity; and (5) prediction of the remaining service life of the structure. The basic problems for structural damage detection are how to ascertain emergence, location and severity of structural damage using the given measured structural dynamic responses. In order to detect structural damage from structural dynamic response signals, the first problem is to select damage feature index to be constructed. The physical variable used to identify damage may be a global one, but the physical variable used to determine damage location is better to be a local one, and it should meet the following two requirements: (1) the variable must be sensitive to structural local-damage. (2) The variable must be monotone function of the location coordinate[13].

Generally, determination of structural damage location is equivalent to determining a region, where the structural stiffness and loading capacity decreases using a measurable quantity. The key factor of vibration based damage detection is to establish the calculation model and to estimate the vibration parameter to be measured. Especially, the selection and sensitivity of the structural damage feature index will affect the final results and accuracy of structural damage detection[13]. In this thesis, the damage location on a beam structure is predetermined by the author by selecting different finite beam elements and damage is implemented by reducing the elemental E-modulus, resulting in local stiffness decrease. The presence of damage is then identified through FRF data, where the vibration parameter is the excitation frequency. The damages are localized through the increase/change in flexibility (i.e. the reciprocal of stiffness) at the damaged elements.

2.3 Damage detection based on change in basic modal properties

The basic idea behind VBSHM is to monitor the dynamical characteristics of structures in order to detect the presence, location and severity of potential damages. The dynamical characteristics of a structure usually comprises the modal parameters, i.e., eigenfrequencies, mode shapes, modal damping.

Doebling, Farrar and Prime[7], categorized the VBSD methods as follows[14]:

- Methods based on frequency changes
 - 1) The forward problem
 - 2) The inverse problem
- Methods based on mode shape changes
- Methods based on mode shape curvature /strain mode shape changes
- Methods based on dynamically measured flexibility
 - 1) Comparison of flexibility changes
 - 2) Unity check methods

- 3) Stiffness error matrix methods
- 4) Effect of residual flexibility
- 5) Changes in measured stiffness matrix
- Methods based on updating structural model parameters
 - 1) Objective functions and constrains
 - 2) Optimal matrix update methods
 - 3) Sensitivity-based update methods
 - 4) Eigen-structure assigned methods
 - 5) Hybrid matrix update methods

All of these methods are thoroughly reviewed in [7]. The main disadvantage with all of these methods is that it is extremely difficult to determine the presence, location and severity of the structural damage[14]. Therefore, the indication of damage is determined from FRF, and the location of damage is determined from the flexibility diagrams. The damage severity is not investigated in this thesis.

2.4 Structural dynamics/Dynamic finite element

Loading on structures are generally static or dynamic. Static loads might be people standing inside an elevator or, the weight of a topside exerted to a jacket foundation on an offshore platform. Dynamical loads are loads that are usually time-dependent and they may be different impact loadings or just variable in nature. Examples of impact loads are blast-loads like explosions or, boat impacts on an offshore-located wind turbine structure. Variable loads might be wind loads or wave loads. The basic idea with dynamic loads is that if they hit the structure at a certain force magnitude, or at a certain frequency, it will make the structure vibrate/oscillate. This oscillation will occur about the initial-unloaded equilibrium position of the structure (as can be seen in Fig.2.4-1).

In the theory of mechanical vibrations of structures, it is significant to determine the structural properties like stiffness, mass and damping, but also to determine how many degrees of freedom (NDOF) the structure has. When you have decided on the boundary conditions, the type of finite element (bar, beam, shell etc) your structure should be modeled with, you can combine the theories of the finite element with the boundary conditions, structural properties and the NDOF of the structure to make a dynamic FE-model. When the FE-model is completed in a software-program like MATLAB, you can make simulated vibration analysis of the structure and from here obtain the dynamical characteristics of the structure.

As mentioned, when there is damage present in a structure the dynamical characteristics will be altered. Therefore it is important to have a FE-model of the structure in order to develop a damage detection algorithm that can be applied to the SHM system that will be applied to a structure in order to monitor its health condition. In subsections 2.4.1 to 2.4.3, it is explained how to obtain the modal properties of a structure.

2.4.1 Single-and Multi Degree Of Freedom systems

The minimum number of independent coordinates required to completely determine the position of all parts of a system at any instant of time defines the degree of freedom of the system [15]. Fig. 2.4-1 shows an illustration of a single degree of freedom system (SDOF). It is called a single-degree of freedom system because one coordinate (x) is sufficient to specify the position of the mass at any time[15].

Several mechanical and structural systems can be idealized as SDOF systems. In many practical systems, the mass is distributed, but for a simple analysis, it can be approximated by a single point mass m . Similarly, the elasticity of the system, which may be distributed throughout the system, can also be idealized by a single spring[15] with stiffness k . The damping in the system is represented as a dashpot with a viscous-damping constant c .

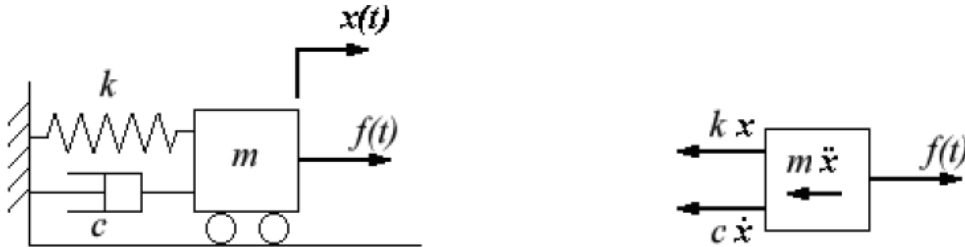


Figure.2.4-1 (Left) illustration of a SDOF system.(Right) free-body diagram[16].

In the left figure in Fig. 2.1, mass m is supported by frictionless rollers and can have translatory motion in the horizontal direction. The right figure of Fig. 2.1 is a free body diagram of the SDOF system represented in the left figure in Fig. 2.1. The free-body diagram gives an overview of the forces acting on the mass when the mass has been displaced in the horizontal x -direction (displaced from the equilibrium position). A summation of the forces in the free body diagram gives the following equation:

$$m\ddot{x} + c\dot{x} + kx = f(t) \quad (3)$$

Eq. (3) is a second order differential equation where $f(t)$ is the applied force, $c\dot{x}$ is the viscous damping force, kx is the force in the spring and $m\ddot{x}$ is the inertia force based on D'Alembert's principle. Eq. (3) is an equation that describes the dynamics of a forced vibration. The oscillation that arises in machines such as diesel engines is an example of forced vibration[15]. For free vibration studies $f(t)$ is set equal to zero. If there is no external force acting on a system that is left vibrating after an initial disturbance, the vibration is said to be a free vibration. Oscillation of a simple pendulum is an example of free vibration.

In many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes[15]. In the case of undamped vibration, Eq. (3) rewrites to

$$m\ddot{x} + kx = f(t) \quad (4)$$

For undamped free vibration, Eq. (3) rewrites to

$$m\ddot{x} + kx = 0 \quad (5)$$

If the force function $f(t)$ is an harmonic force such that $f(t) = F_0 \cos(\omega t)$, where F_0 is the static force and ω is the circular force frequency, the solution of the differential equation Eq. (3) becomes

$$x(t) = C_1 \cos(\omega_n t) + C_2 \sin(\omega_n t) + \frac{F_0}{k - m\omega^2} \cos(\omega t) \quad (6)$$

Where C_1 and C_2 are constants that depends on the initial conditions of the system, ω_n is the natural frequency of the system and is determined as $\omega_n = \sqrt{k/m}$.

For free undamped vibration the solution is

$$x(t) = x \sin(\omega t) \quad (7)$$

where x is the displacement amplitude.

When multiple degree of freedom systems (MDOF) are considered, Eq. (3) will be given as

$$[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (8)$$

where $[K]$, $[M]$, $[D]$, are $n \times n$ analytical stiffness, mass, and structural damping matrices, respectively, and $\{F(t)\}$ is the periodic loading vector. $\{\ddot{x}\}$, $\{\dot{x}\}$ and $\{x\}$ are vectors representing nodal accelerations, nodal velocities and nodal displacements, respectively. For a system which consists of n degrees of freedom, there will be n natural frequencies, and each of them will be associated with its own mode shape. As the number of DOF increases for a system, the more difficult it is to obtain a solution for Eq. (8). The development of the finite element method (FEM) and the different techniques associated with the method has enabled engineers to obtain solutions to very complex problems where the number of nodes might be several hundreds of thousands.

2.4.2 Eigenvalues and eigenvectors

Eigenvalues and eigenvectors are common subjects involved in structural dynamics[14]. If one considers two matrices $[A]$ and $[B]$, where both of them are $n \times n$ square matrices, and include them in the following equation

$$([A] - \lambda[B])\{y\} = \{0\} \quad (9)$$

Eq. (9) gives an example of an eigenvalue problem, and in these problems the interesting aspect is to obtain the values of a scalar λ such that the matrix equation has solutions other than the trivial solution $\{y\} = \{0\}$. The λ_i are called eigenvalues and corresponding to each λ_i is a $\{y\}_i$ called an eigenvector. Together, λ_i and its associated $\{y\}_i$ are called an eigenpair. Eq. (9) is called a generalized eigenproblem or simply an eigenproblem. If $[B]$ happens to be an identity matrix $[I]$, Eq. (9) is called a standard eigenproblem and the associated λ_i are called eigenvalues of $[A]$.

A common physical problem characterized by Eq. (9) is that of undamped free vibration for a structure, where $[A]$ is the structural stiffness matrix, $[B]$ is the structural mass matrix, $\sqrt{\lambda_i} = \omega_i$ is a vibration frequency, and $\{y\}_i$ is the associated mode of vibration[10].

2.4.3 Vibrational eigenvalue problem

The eigenvalue problem for undamped free vibration of a structural system is called a generalized eigenvalue problem. By assuming a steady-state condition, Eq. (3) rewrites to

$$x(t) = \{\bar{x}\} \sin(\omega t) \quad (10)$$

where $\{\bar{x}\}$ is the vector of nodal displacement amplitude and ω is the natural frequency of the structural system. Substituting Eq. (3) into Eq. (9) we get the following equation for the generalized eigenvalue problem

$$[[K] - \lambda[M]]\{\bar{x}\} = \{0\} \quad (11)$$

Where \bar{x} is the eigenvector that indicates the vibration mode, λ is the eigenvalue, and $\lambda = \omega^2$. $[K]$ and $[M]$ are symmetrical $n \times n$ stiffness and mass matrices, respectively. The matrix $[[K] - \lambda[M]]$, is called a dynamic stiffness matrix. A physical interpretation of vibration comes from writing Eq. (11) in the form $[K]\{\bar{x}\} = \lambda[M]\{\bar{x}\}$. Eq. (11) says that a vibration mode is a configuration in which elastic resistances are in balance with inertia loads[10].

By assuming that $\{\bar{x}\}$ only contains DOF that has nonzero values after all rigid-body modes and mechanisms (if any) are suppressed, $[K]$ will be a positive definite. If element mass matrices are consistent, or lumped with strictly positive diagonal coefficients, $[M]$ is also positive definite, then the number of nonzero ω_i is equal to the number of DOF in $\{\bar{x}\}$. Occasionally two or more ω_i are numerically equal. Then their associated vibration modes $\{\bar{x}\}_i$ are not unique, but mutually orthogonal modes for the repeated ω_i can be established[10].

2.5 Frequency response

A method for detecting damage and deterioration in structures is by analyzing how the structure will respond to different excitation frequencies. The presence of damage or deterioration in a structure will cause changes in the natural frequencies of the structure[17]. Therefore the frequency response function (FRF) of a structure can give an indication of damage or deterioration. In this thesis frequency response has been used as a damage indicator. This section will contain a description of the theoretical background of FRF analysis.

If assuming that a harmonic excitation is applied to the classical model of the SDOF system given in Fig 2.1, the solution of Eq.(3) is a steady state response and is expressed as

$$x(t) = F_0 H(\omega) \sin(\omega t - \Omega) \quad (12)$$

where Ω is the phase angle and is determined as $\Omega = \tan^{-1} \left(\frac{2\xi\omega/\omega_0}{1 - (\omega/\omega_0)^2} \right)$, ω is the excitation frequency, ω_0 is the eigenfrequency of the system, F_0 is the static force (excitation) and ξ is the damping ratio. $H(\omega)$ is the frequency response function (FRF) and is defined as

$$H(\omega) = \frac{1/k}{\sqrt{1 - \left(\frac{\omega}{\omega_0}\right)^2 + 2\xi \frac{\omega}{\omega_0}}} \quad (13)$$

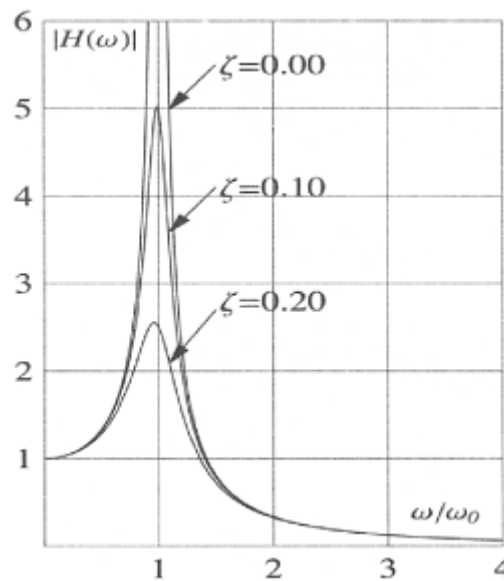


Figure.2.5-1 Response of a damped SDOF to harmonic excitation[18].

The FRF shows that in a small range of the ratio ω/ω_0 , when the frequency of the excitation approaches the natural frequency of the system, the response amplitude is much larger than the static response. This is called resonance. Furthermore, the amplitude of the steady-state response is linearly dependent on both F_0 and $H(\omega)$. By knowing $H(\omega)$ the response of a SDOF system to a harmonic excitation can be estimated[18]. Fig. 2.5-1 shows how the FRF varies with ω/ω_0 and ζ for a SDOF system. Notice that when $\zeta = 0$ and $\omega/\omega_0 \rightarrow 1$, the response $H(\omega) \rightarrow \infty$. This is the resonant motion and the structural response of the system gets indefinitely large. It has to be stated that the frequency response plotted in Fig. 2.5-1 is based on harmonic excitation. In reality forces are not solely harmonic, being frequently periodic or approximated closely by periodic forces[18].

Scientific damage detection methods based on modal parameter analysis has also been applied to detect structural damage and deterioration. Modal parameters can be easily and cheaply obtained from measured vibration responses[17]. However, since indirectly-measured modal data contain accumulative errors incurred in modal parameter extraction and provide much less information than FRF data, it is more reasonable and reliable to use directly-measured FRF data for structural damage detection. Another major advantage of using FRF data over using modal data in model updating comes from the fact that FRF data can provide much more information in a desired frequency range than modal data; this is because modal data are extracted mainly from a very limited number of FRF data around the resonance of the structure. In a broad sense, those methods using modal data are discrete versions of the methods using FRF data[19].

The equation of motion for a structure with multiple degrees of freedom (MDOF) can be expressed as

$$[M]\{\ddot{x}\} + [D]\{\dot{x}\} + [K]\{x\} = \{F(t)\} \quad (14)$$

where $[K]$, $[M]$ and $[D]$ are $n \times n$ analytical stiffness, mass, and structural damping matrices, respectively, and $\{F(t)\}$ is the periodic loading vector. From Eq. (3) we can determine the FRF as

$$H(\omega) = \frac{\{x\}}{F(t)} = \frac{1}{[K] + [D] - \omega^2[M]} \quad (15)$$

where $\{x\}$ is time dependent displacement vector. For Eq. (8), the structural damping matrix $[D]$ can be modeled as structural damping and is a quantity evaluated in the complex plane given as $[D] = \alpha[M] + \beta[K]$, where α and β are damping constants. Structural damping is independent of frequency; whereas viscous damping depends upon frequency.

For FRF evaluation of an undamped structure $[D]$ is assumed to be zero, and Eq. (15) rewrites

$$H(\omega) = \frac{\{x\}}{F(t)} = \frac{1}{[K] - \omega^2[M]} \quad (16)$$

Assuming that a structure has no damping is not realistic in nature. However, in many physical systems, the amount of damping is so small that it can be disregarded for most engineering purposes. However, consideration of damping becomes extremely important in analyzing vibratory systems near resonance[15]. In this thesis the damage detection procedure is conducted on an undamped fixed-fixed beam structure, so FRF will be calculated according to Eq. (16).

Fig. 2.5-2 shows the FRF plot for an intact undamped fixed-fixed beam structure. The peaks in the plots indicate how the resonant frequencies influence the frequency response of the structure. The response/receptance of the beam structure is measured in decibels (dB). As noticed properties such as stiffness and mass are involved in the FRF analysis and a change in any of these parameters will therefore alter the FRF signature of the structure. Therefore damage can be detected by using structural FRF analysis.

FRF analysis is a very useful tool to determine if there is damage present in a specific structure, however, it should be noted that significant frequency changes alone do not automatically imply the existence of damage since frequency shifts (exceeding 5%) due to changes in ambient conditions have been measured for both concrete and steel bridges within a single day[17], but a significant change in the frequency response pattern of a structure gives a strong indication of structural damage being present. Another drawback of detecting structural damage with FRF analysis is that changes in vibration data caused by small damages like surface or internal cracks, flaws, voids, thin spots, could be unobservable in presence of measurement noise[19]. The effect of noise is investigated further in section 2.7.

A major drawback of using FRF analysis in damage detection is that FRF analysis in general has little application in determining severity and localization of damage in a structure. However, there have been developed methods that enable FRF to be applicable in determining damage localization and severity. Reference is made to [17],[20] for more on this subject.

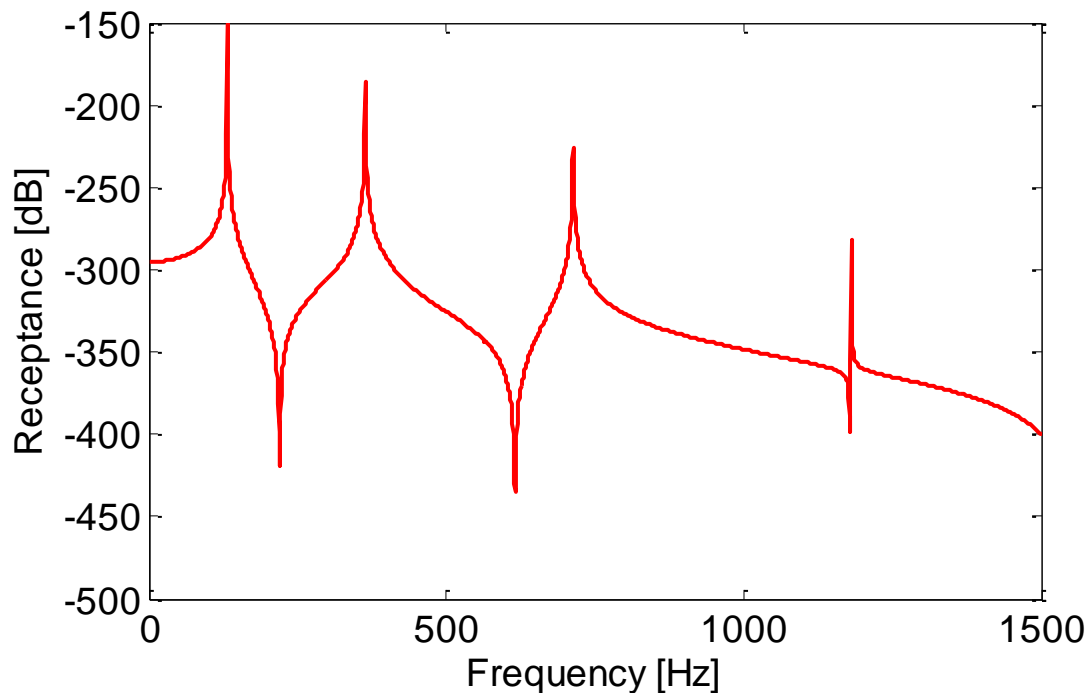


Figure.2.5-2 FRF for an undamaged fixed-fixed beam structure.

2.6 Flexibility

The flexibility method can be used to determine both presence and location of damage in a structure. The method is also known as matrix force method. When a structure becomes damaged the stiffness of the structure will decrease, and then the structure will become more flexible in the vicinity of the damage. In other words, the flexibility matrix of a structure can be regarded as the inverse of the structural stiffness matrix. The method is based on evaluation of the dynamic vibration characteristics of a structure. Typically, damage is detected using flexibility matrices by comparing the flexibility matrix synthesized using the modes of the damaged structure, to the flexibility matrix synthesized using the modes of the undamaged structure, or the flexibility matrix from a FEM. Because of the inverse relationship to the square of the modal frequencies, the measured flexibility matrix is most sensitive to changes in the lower-frequency modes of the structure [7]. The flexibility matrix can be easily and accurately estimated from a few of the lower frequency modes of vibration of the structure, which can be easily measured [21].

The flexibility method has been used in many different ways by scientists and researchers in order to detect damage in structures. Toksoy and Aktan [7] computed the measured flexibility of a bridge, and examined the cross-sectional deflection profiles with and without a baseline data set. They observed that anomalies in the deflection profile can indicate damage even without a baseline data set. Aktan et al. [7] proposed the use of measured flexibility as a “condition index” to indicate the relative integrity of a bridge. They applied this technique to two bridges and analyzed the accuracy of the flexibility measurements by comparing the measured flexibility to the static deflections induced by a set of truck-load tests.

The homogeneous dynamic equilibrium equation for a structure may be written as

$$[K][\Phi] = [M][\Phi][\Lambda] \quad (17)$$

Where $[K]$ and $[M]$ are respectively the stiffness and mass matrices, $[\Lambda] = \text{diag}(\omega_i^2)$ is the eigenmatrix and ω_i is the i th modal frequency. $[\Phi]$ is the $n \times n$ -dimensional mode shape matrix defined as $[\Phi] = [x_1 \ x_2 \ \dots \ x_n]$, with the individual vibration mode shapes x_1 to x_n , where n is the number of nodes[14]. $[K]$ and its inverse; the flexibility matrix $[F]$, may be expressed as

$$[K] = [M][\Phi][\Lambda][\Phi]^T[M] = \sum_{i=1}^n [M]\omega_i^2 \phi_i \phi_i^T [M] \quad (18)$$

$$[F] = [K]^{-1} = [\Phi][\Lambda]^{-1}[\Phi]^T = \sum_{i=1}^n \frac{1}{\omega_i^2} \phi_i \phi_i^T \quad (19)$$

where the mode-shape vectors have been mass-normalized such that $[\Phi]^T[M][\Phi] = [I]$, where $[I]$ is an $n \times n$ identity matrix. It can be seen from Eq. (18) that the modal contribution to the stiffness matrix increases as frequency increases. However, in most experimental surveys, only a few low-frequency modes can be measured. Besides, the mass matrix needs to be determined. These problems may constitute an obstacle to apply experimental stiffness for damage localization. Conversely, Eq. (19) shows that the modal contribution to the flexibility matrix decreases as frequency increases and generally the flexibility rapidly converges to a good approximation with a few low-frequency modes[22].

If the modal parameters are estimated from two sets of data: one from the initial (undamaged) reference structure denoted by u , and another from the damaged structure denoted by d , the corresponding flexibility matrices F_u and F_d may be constructed in a dimension of measured DOF. A simple damage localization method was proposed by Pandey and Biswas [21], which consists of calculating the flexibility change matrix and then observing the maximum value of each column[22].

The change in flexibility is defined as[14]

$$[\Delta F] = [F_d] - [F_u] \quad (20)$$

Damage in a structure will result in higher flexibility values for the elements near the damage point. A flexibility change vector is defined as[14]

$$\{\Delta F_v\} = \{F_{jj}^d\} - \{F_{jj}^u\} \quad (21)$$

The flexibility vectors for the damage and intact structure are[14]

$$\{F_{jj}^d\} = \{f_{11}^d \ \dots \ f_{jj}^d \ \dots \ f_{nn}^d\}^T \quad (22)$$

$$\{F_{jj}^u\} = \{f_{11}^u \ \dots \ f_{jj}^u \ \dots \ f_{nn}^u\}^T \quad (23)$$

When the structure has multiple damages present, a value δ_j is defined. For each column of $[\Delta F]$ the absolute maximum value of the element in the j th column is [14]

$$\delta_j = \max|\Delta F|, j = 1, 2, 3, \dots, n \quad (24)$$

The column of $[\Delta F]$ corresponding to the largest δ_j is the indicative of the j th DOF with damage. It has been verified that flexibility vectors δ_j and ΔF_v are equivalent [14]. In this thesis the flexibility method will only be used to investigate the lateral DOF for damage localization. The rotational DOF is not considered in the study.

A flexibility diagram is presented in Fig. 2.6-1. In the diagram the horizontal axis represents the element number in an FE-model. The vertical axis represents the flexibility value. The blue line represents the flexibility curve. This curve will have its maximum value $\max[\Delta F]$ at the element that is damaged. As can be seen from the figure $\max[\Delta F]$ is located at element 6, which indicates that damage is present in this element.

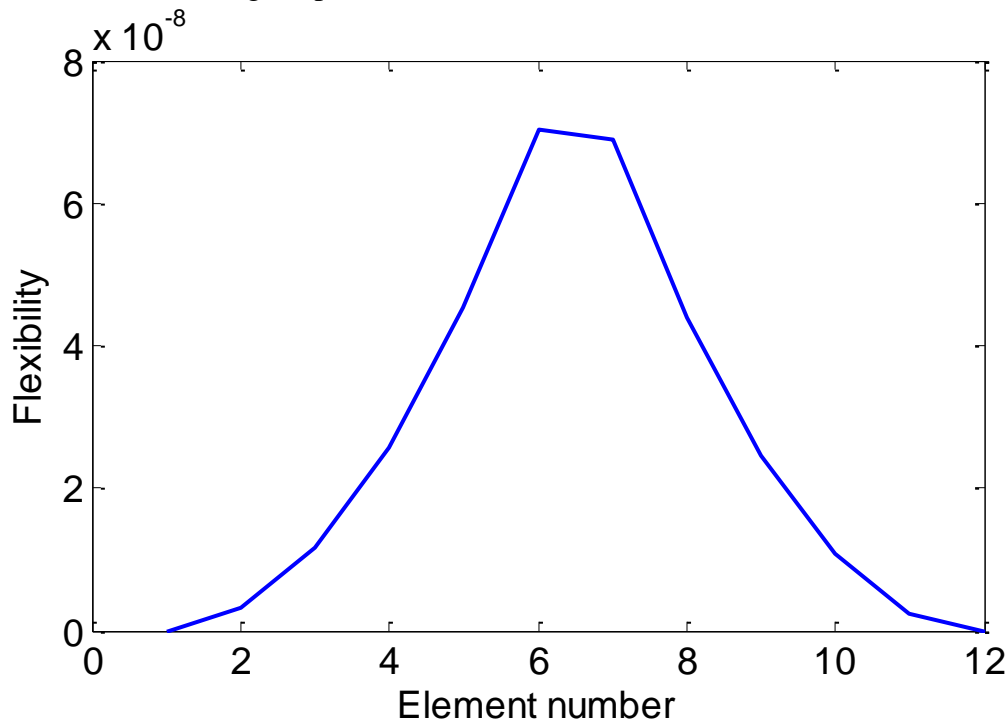


Figure.2.6-1 Flexibility diagram

2.7 Noise contamination in datasets

In experimental modal testing, measurement noise contamination is inevitable. Any signal corrupted by random noise is, by its nature, unpredictable [11]. To evaluate the robustness of a damage detection method, it is essential to investigate its noise immunity performance [8]. Since noise generally contains mostly high-frequency data, it can only significantly affect the high-frequency information [23]. This means that a FRF analysis might give a false-positive indication of damage when a structure is subjected to high load frequencies. When the measured dataset is contaminated by noise the information from tiny damages in a structure might be covered by the noise and this is of course highly unwanted. Simulated noise tests is a widely used method for determination of noise sensitivity of a given damage detection technique. The noise can be generated by the *noise generator* function in MATLAB, and then

be added to a simulated dataset as a random noise signal. After the noise has been added to the dataset you can apply Gaussian filter to locate the noise signal, and then filter it out of the dataset. In this thesis, the effects of noise on the flexibility results are investigated by contaminating datasets with 1-10% random noise. Reference is made to [8, 11, 24] for more info on the different aspects of noise contamination in datasets.

3 Damage detection analysis

3.1 Fixed-fixed beam structure

The different damage detection methods available are not all as effective in all parts of the damage detection process. Some of the methods are highly effective when the damage is detected from frequency response data, while others are not. Especially when it comes to defining the location and the severity of the damage it becomes clear that not all methods are accurate or applicable. Also, if there exists more than one damage in a structure the effectiveness of the methods varies.

In this thesis, the flexibility method is studied on a fixed-fixed beam structure in order to determine the advantages and disadvantages of using the method as a damage detection tool.

Theory and application for FRF and the flexibility method is presented in sections. 2.5 and 2.6, respectively.

An FE-model of the beam and its boundary conditions are modeled in MATLAB. A damage detection algorithm has been established by codes written in MATLAB. This algorithm's functionality is to calculate the eigendata i.e., the dynamical characteristics of the beam. The location of damage is determined through changes in the flexibility of the beam. This can be done for any given condition. By analyzing the beam when it is in an undamaged state, you can obtain the dynamical characteristics of the intact/healthy beam. These characteristics can be used as a baseline for the health monitoring. When there is damage present in the beam the dynamical characteristics of the beam will be altered. This alteration can be detected by comparing the analyzed dynamical characteristics with the baseline values. If there is a difference in the values, then there is an indication of structural damage. In this chapter, the reliability of the change in flexibility method is checked through different damage studies. In subsection 3.1.1 the reliability of the change in flexibility method is checked when the E-modulus is progressively reduced at a point (element) in the beam. In subsections 3.1.2 and 3.1.3 the effect of noise contamination and multiple damage scenarios are investigated, respectively.

FE-Model and structural properties

The beam is analyzed as a steel-beam. The geometrical and material properties of the beam are presented in Table 3-1. The static system of the beam is presented in Fig. 3.1-1.

Table 3-1 Geometrical and material properties for beam structure

Geometrical and material properties	
Length: L	1 m
Area: b x h	0,025 x 0,025 m ²
E-modulus: E	2,07e11 N/m ²
Material density	7850 kg/m ³

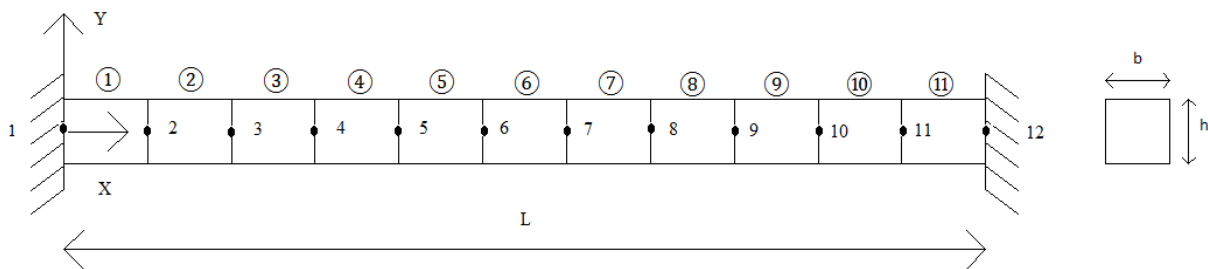


Figure.3.1-1 FE-model of the beam structure.

The FE-model of the beam has been modeled as an undamped fixed-fixed beam. In this model, the displacements and rotations are assumed to be zero at the fixed locations. The cross-section is rectangular, and massive. The mass of the beam is homogeneously distributed over the length of the beam. The beam is meshed with eleven finite 2D-beam elements, having the same geometry, E-modulus and mass content. There are twelve nodes in the model, having two DOFs, one rotational and one lateral. The damages are implemented by changing the E-modulus at one or more elements in the FE-model.

3.1.1 Change in E-modulus

Flexibility is defined as the inverse of structural stiffness. This means that if the structural stiffness is reduced, then the flexibility of the structure will increase. In this chapter, investigation is done to establish how the change in FRF diagrams can be used as a damage indicator, and how the change in flexibility of the beam can be used to determine the location of damage. The damages are introduced by reducing the local stiffness of the beam. This is done by progressive reduction of the E-modulus at element 6. The damaged cases are presented in Table 3-2.

Table 3-2 Overview of different damage cases

Damage case	Reduction of E-modulus [%]	Damage location at element nr
1	0	6
2	10	6
3	20	6
4	30	6
5	40	6
6	50	6
7	60	6
8	70	6
9	80	6
10	90	6
11	99	6

Damage indication: change in FRF data

As the structural stiffness is reduced, so will also the natural frequency of the structure. This can be explained from the expression for the natural frequency $\omega_n = \sqrt{(EI)/(l^3m)}$, where E is the E-modulus/Young's modulus of the material, I is the moment of inertia, l is the length of the beam and m is the structural mass. The expression for ω_n clearly indicates that if the E-modulus is reduced the natural frequency will also be reduced. The beam structure is excited by frequencies and the frequency response is determined from the damage detection

algorithm. It is made clear that it is not possible to obtain the location of the damage by just using FRF data and FRF plots. The values for the first five eigenmodes for the different damage cases are presented in Table 3-3. The percentage reduction of the five first modes for the damaged cases 2-11, relative to the undamaged case 1 is presented in Table 3-4. The frequency excitation range is set from 0 to 1500 Hz.

Table 3-3 Natural frequencies of the first five modes for damage cases 1 to 11

Mode nr	Damage cases										
	1	2	3	4	5	6	7	8	9	10	11
	Frequency [Hz]										
1	132.0	131.0	129.9	128.5	126.8	124.6	121.7	117.7	111.8	102.3	86.2
2	364.8	363.7	363.5	363.3	363.0	362.5	361.9	360.8	358.7	352.4	268.0
3	713.6	707.1	699.6	690.8	680.4	667.8	652.5	633.0	608.7	576.2	536.9
4	1181.1	1179.4	1177.5	1175.1	1171.8	1167.3	1160.5	1149.2	1126.8	1063.8	699.2
5	1767.9	1754.1	1738.6	1721.2	1701.6	1679.4	1654.0	1625.0	1591.5	1552.9	1513.1

Table 3-4 Reduction of the first five modes for the damaged cases 2 to 11 relative to case 1

Mode nr	Damage cases									
	2	3	4	5	6	7	8	9	10	11
	Reduction of Frequency [%]									
1	0.76	1.59	2.65	3.94	5.94	8.46	12.10	18.07	29.03	53.13
2	0.30	0.36	0.41	0.50	0.63	0.80	1.11	1.70	3.52	36.1
3	0.92	2.0	3.30	4.88	6.86	9.36	12.73	17.23	23.85	32.91
4	0.14	0.31	0.51	0.79	1.18	1.78	2.78	4.82	11.02	68.92
5	0.79	1.69	2.71	3.90	5.27	6.89	8.79	11.08	13.85	16.84

As can be seen in Table 3-4 the frequency reduction for the first five modes increases with the increase in the damage case number. This is coherent with expected results. This also shows that if the severity of damage increases, then the natural frequency of the structure values will decrease.

Figs. 3.1-2 to 3.1-11 illustrate how the structural FRF changes as the stiffness is progressively reduced. The difference in the FRF plots and the percentage increase in natural frequencies for the healthy-state versus the damaged-state clearly indicate that there is damaged present in the beam, and that the severity of the damage increases as the stiffness is progressively reduced. It is also noticeable that the FRF data can only determine that there is presence of damage in the beam structure. The severity and the location of the damage are impossible to determine from the FRF data and plots.

As Fig. 3.1-11 shows, the FRF for the damaged beam and the healthy beam is almost completely out of phase with each other. The reason for this is because the E-modulus at element 6 is reduced by 99 percent, and thus there is only 1 percentage of stiffness capacity left in this element. In reality there would be a big chance of imminent structural collapse for the beam in damage case 10. A reduction of 100 percent for the stiffness in element 6 has not been included in this study because in reality the beam has collapsed.

Overlay of FRF plots for the damaged cases and the undamaged case 1 is presented as follows

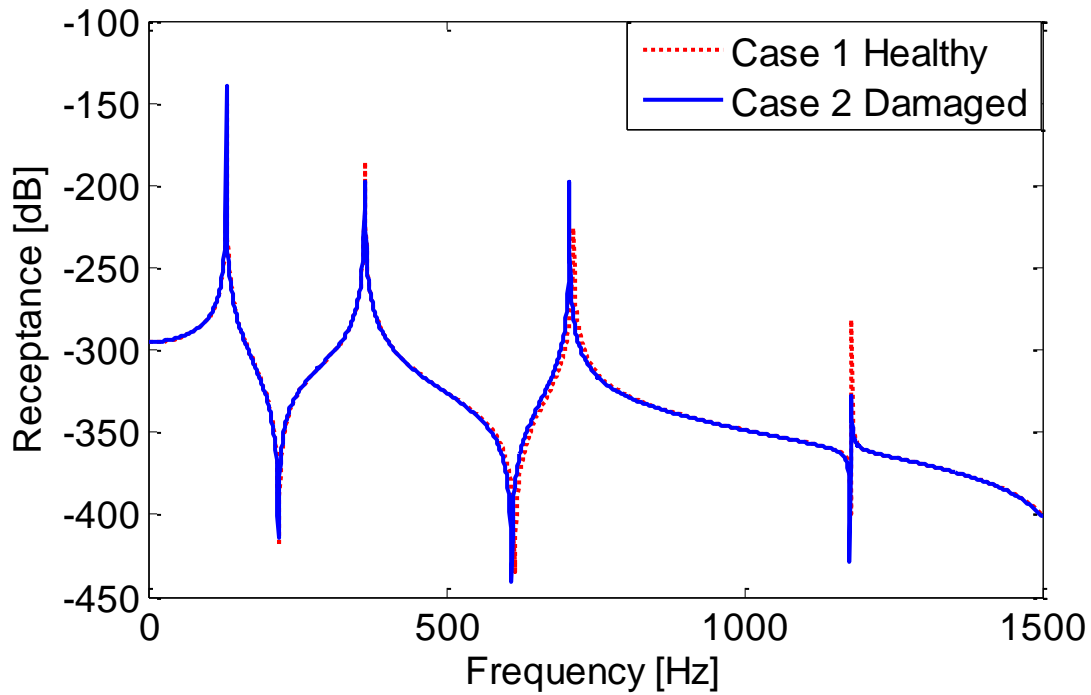


Figure.3.1-2 Overlay of FRF for damaged case 2 versus healthy case 1.

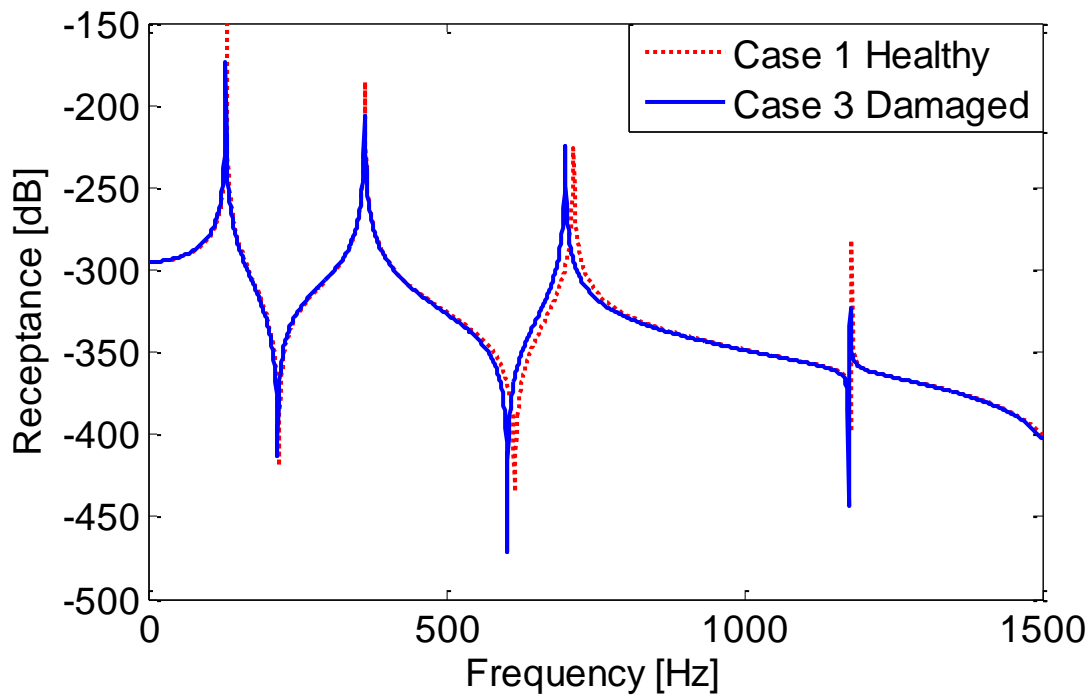


Figure.3.1-3 Overlay of FRF for damaged case 3 versus healthy case 1.

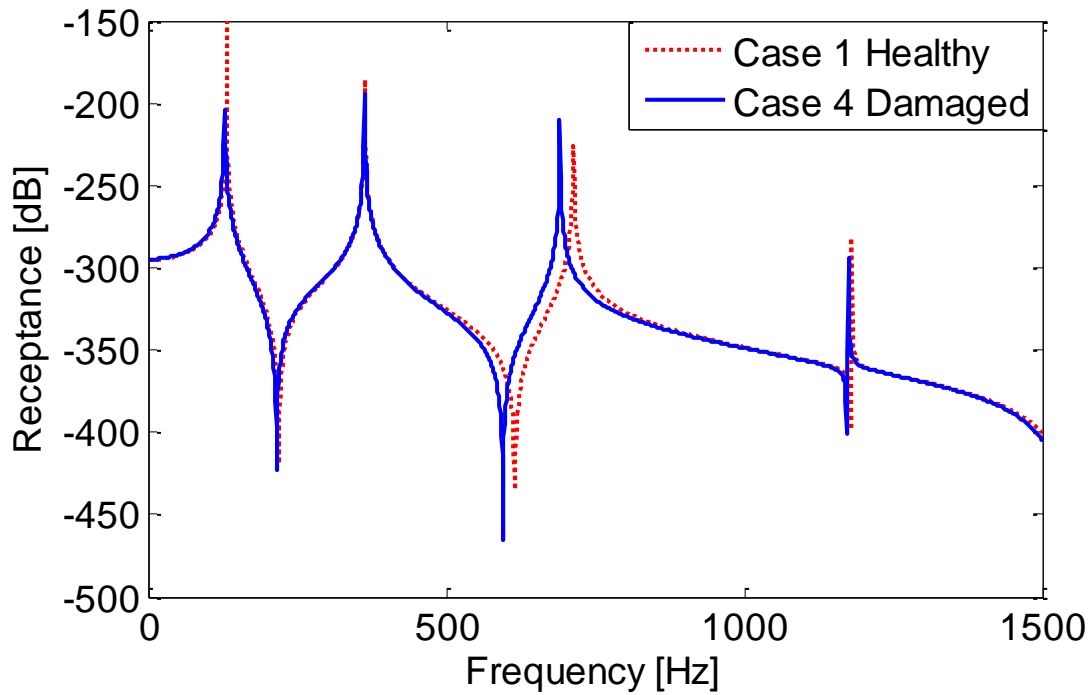


Figure.3.1-4 Overlay of FRF for damaged case 4 versus healthy case 1.

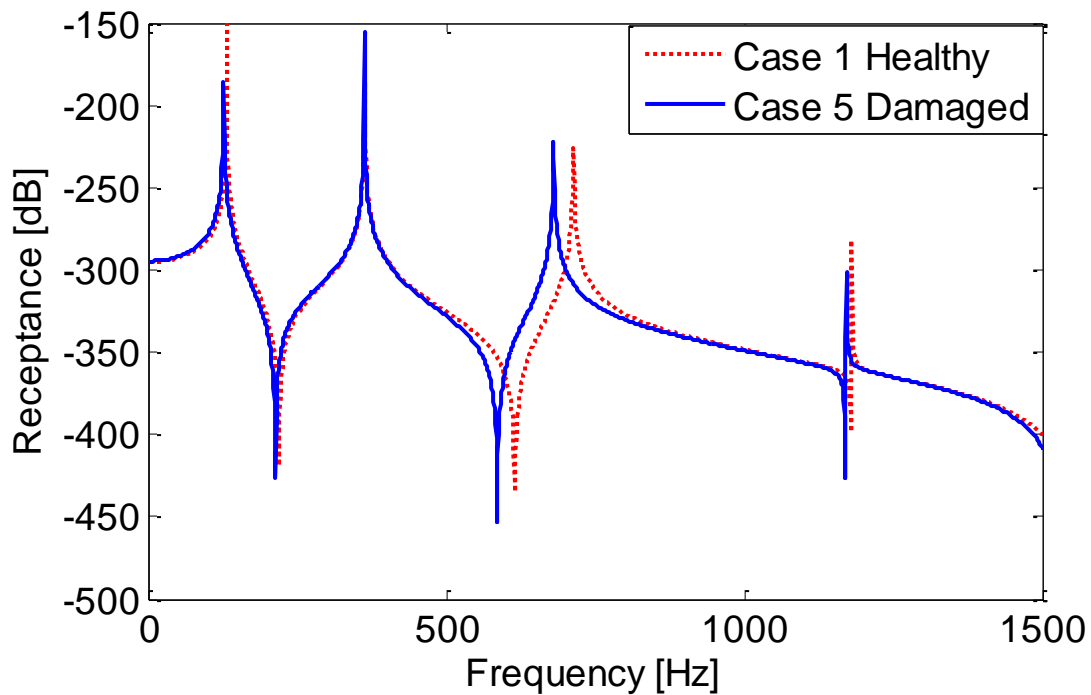


Figure.3.1-5 Overlay of FRF for damaged case 5 versus healthy case 1.

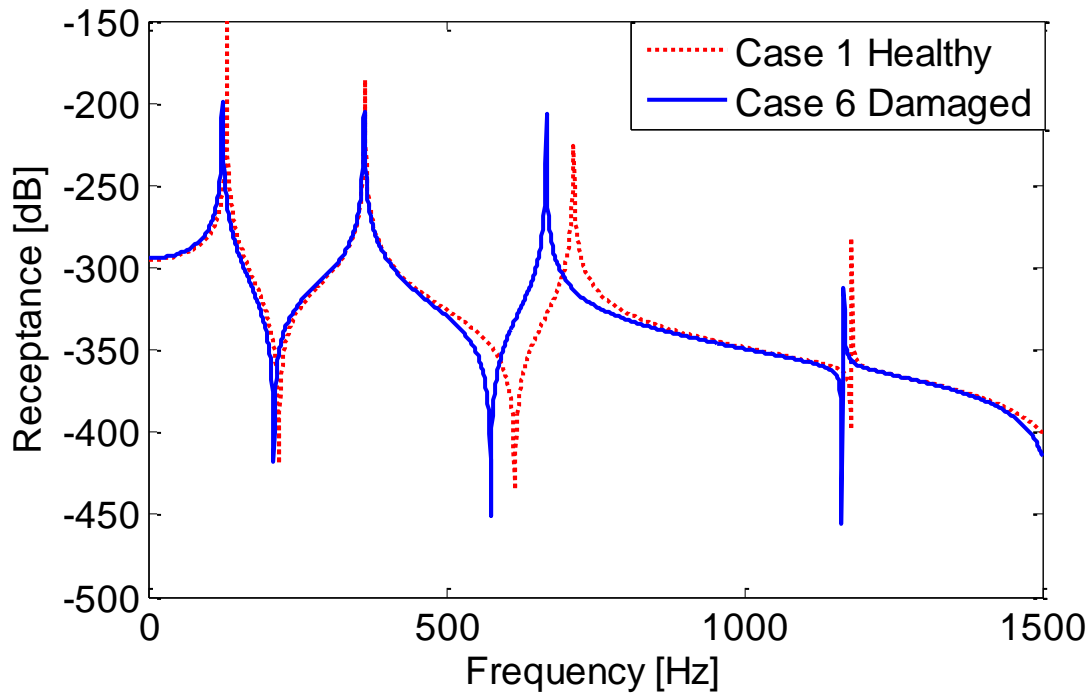


Fig.3.1-6 Overlay of FRF for damaged case 6 versus healthy case 1.

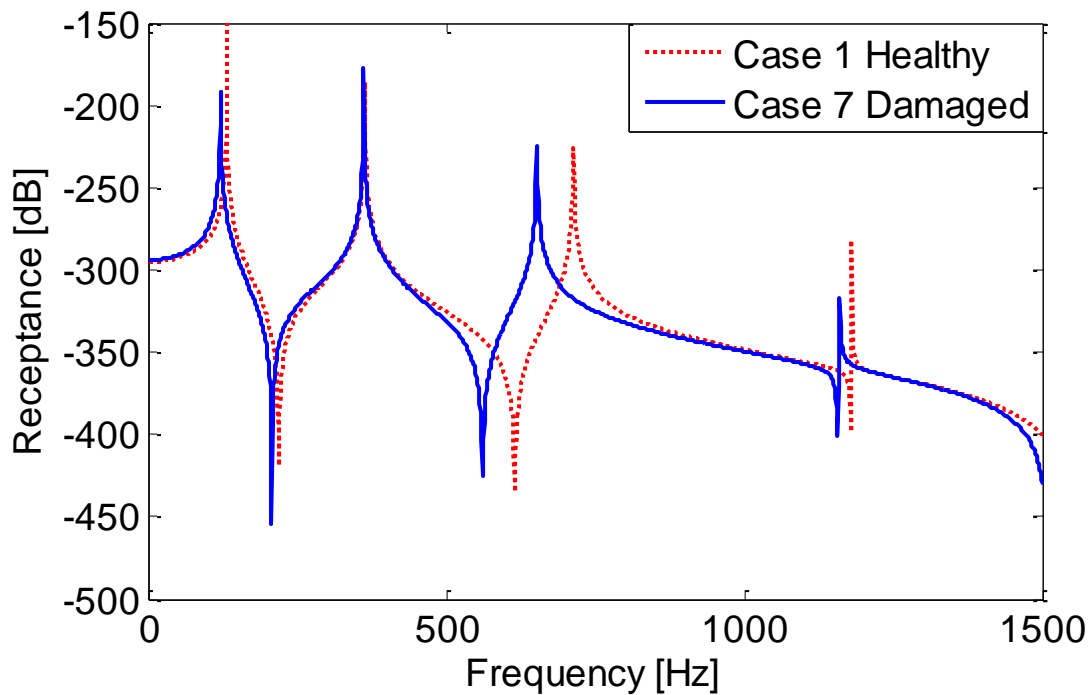


Figure. 1 Fig.3.1-7 Overlay of FRF for damaged case 7 versus healthy case 1.

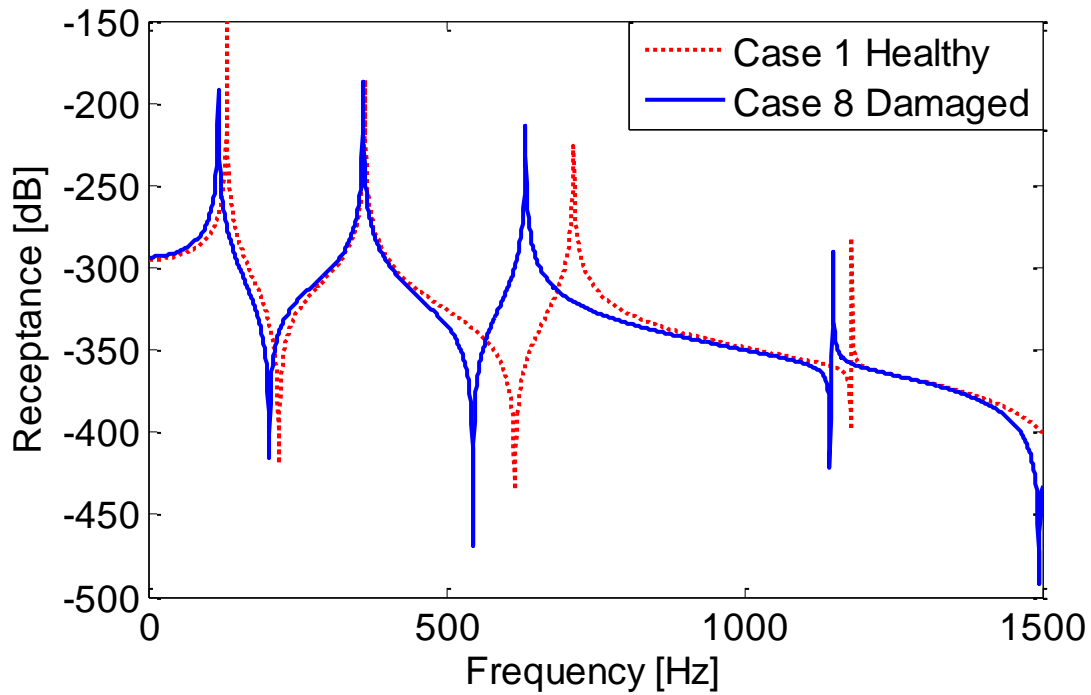


Figure. 2 Fig.3.1-8 Overlay of FRF for damaged case 8 versus healthy case 1.

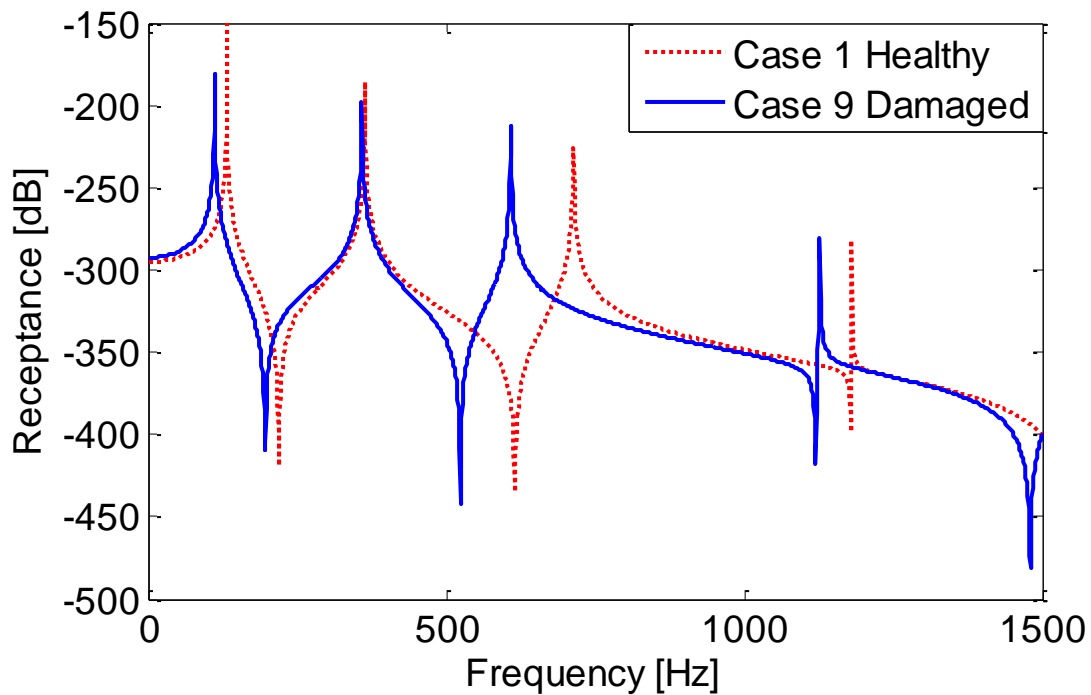


Figure. 3 Fig.3.1-9 Overlay of FRF for damaged case 9 versus healthy case 1.

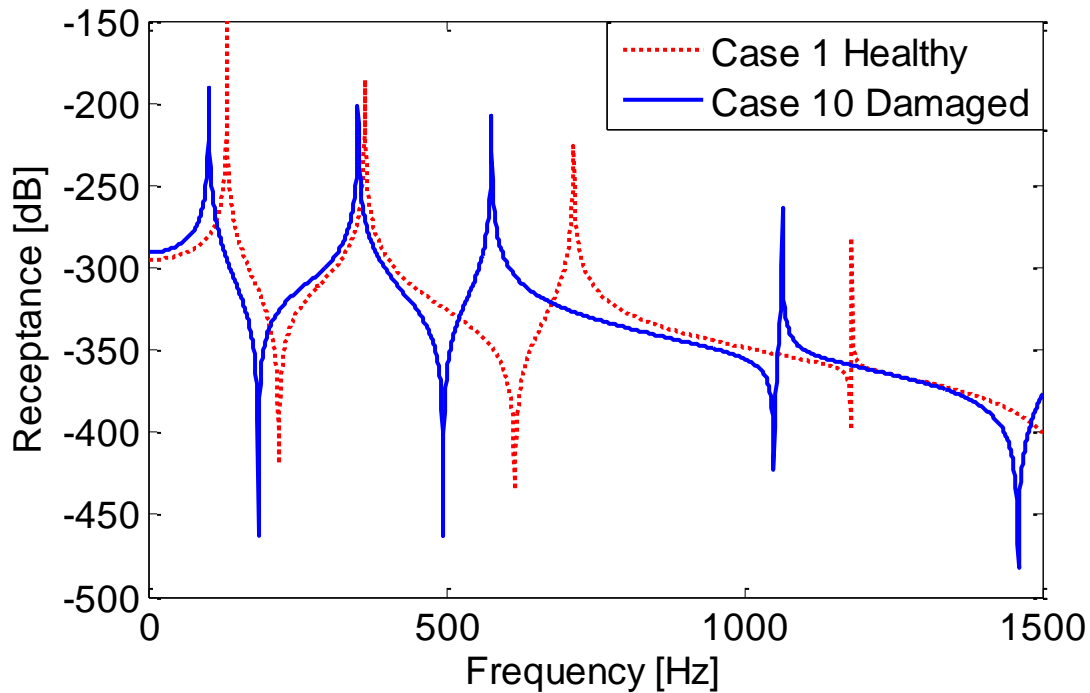


Figure.3.1-10 Overlay of FRF for damaged case 10 versus healthy case 1.

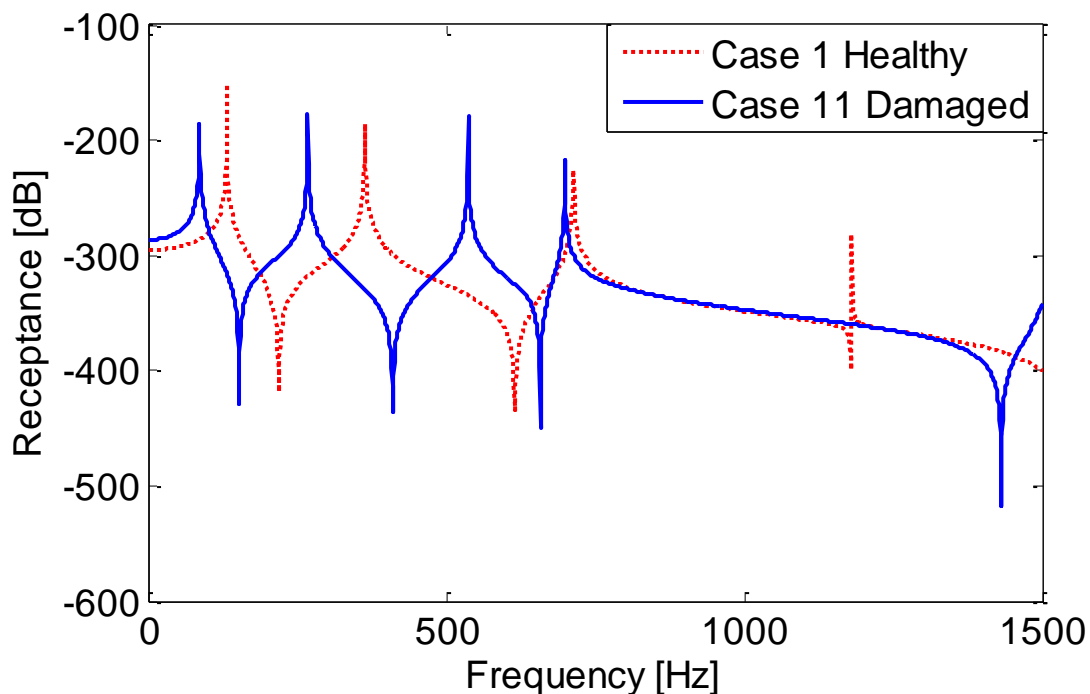


Figure.3.1-11 Overlay of FRF for damaged case 11 versus healthy case 1.

The difference in FRF data for the healthy beam case 1 and the damage cases 2 to 10, presented in Figs. 3.1-2 to 3.1-11, clearly indicates that there is damage present for every damage case. However, these plots do not present the location of damage in the beam. In order to solve this, the change in flexibility method is applied to every damage case.

Damage location: change in flexibility method

In order to determine the damage location, flexibility analysis is carried out for the different damage cases given in Table 3.1. The code written for the change in flexibility is based on the theory given in section 2.6. The maximum flexibility value F_{\max} for each damage case is presented in Table 3-5. The flexibility plots for each damage case are presented in Figs. 3.1-14 to 3.1-23.

It can be concluded that the change in flexibility method has successfully located the damage at element 6 for all damage cases.

Table 3-5 Change of maximum flexibility value for the damage cases 2 to 11

Damage case	Flexibility F	F_{\max}
2	F1	7.0409e-008
3	F2	1.5641e-007
4	F3	2.6394e-007
5	F4	4.0225e-007
6	F5	5.8675e-007
7	F6	8.4525e-007
8	F7	1.2335e-006
9	F8	1.8822e-006
10	F9	3.1894e-006
11	F10	6.8221e-006

The percentage reduction of E-modulus and the curve $F_{sim.Ered}(x)$ are presented together in Fig. 3.1-13. $F_{sim.Ered}(x)$ is the curve corresponding to the curve passing through the simulated maximum flexibility values for F1 to F10.

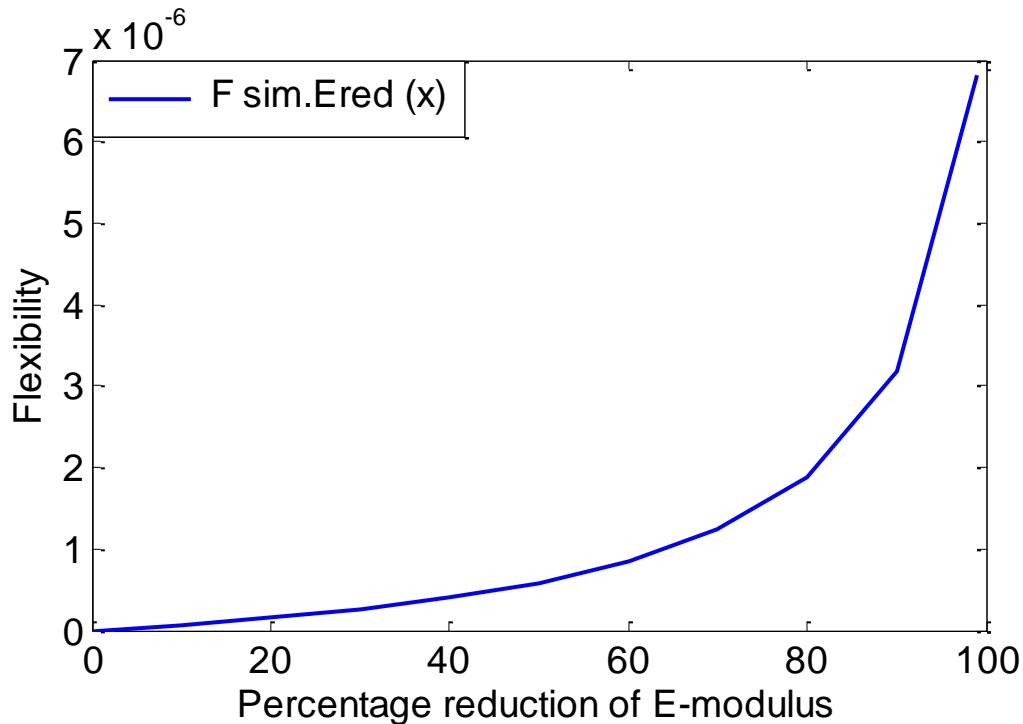


Figure.3.1-12 Percentage reductions of E-modulus and the corresponding maximum flexibility values.

The curve for $F_{sim.Ered}(x)$ shows how much the flexibility will increase as the E-modulus is reduced at element 6. $F_{sim.Ered}(x)$ isn't an analytical function, so in order to have an equation that fits the flexibility curve, i.e. that produces a curve passing near all the data points, a polynomial $F_{poly.Ered}(x)$ has been derived from the *Basic Fitting Interface* option found in MATLAB. The polynomial and its range are given as

$$F_{poly.Ered}(x) = a_1x^{10} + a_2x^9 + a_3x^8 + a_4x^7 + a_5x^6 + a_6x^5 + a_7x^4 + a_8x^3 + a_9x^2 + a_{10}x + a_{11}, x \in [0,99] \quad (25)$$

where x indicates the percentage-reduction of the E-modulus. p_1 to p_{11} are coefficients given in Table 3-6.

Table 3-6 Coefficients for polynomial $F_{poly.Ered}(x)$

Coefficients. p	Value
a1	3.2433e-023
a2	-1.4387e-020
a3	2.7508e-018
a4	-2.9615e-016
a5	1.9688e-014
a6	-8.3313e-013
a7	2.2289e-011
a8	-3.5924e-010
a9	3.1981e-009
a10	-4.6729e-009
a11	2.3932e-018

The polynomial $F_{poly.Ered}(x)$ and the curve obtained from simulation $F_{sim.Ered}(x)$ are presented together in Fig. 3.1-13. From viewing the figure it is clear that the polynomial, $F_{poly.Ered}(x)$, and the simulated curve, $F_{sim.Ered}(x)$, are highly compatible for E-modulus reduction of 0 to 70 percent. However, there are some slight deviations from 70 to 100 percent reduction. The *norm of residuals* obtained from MATLAB gives a good indication as to how accurate the polynomial is. The norm of residuals is calculated by MATLAB as follows

$$norm(d, 2) = \sum \left(\sqrt{abs(d)^2} \right) = \sqrt{\sum_{i=1}^n d_i^2} \quad (26)$$

Where d_i is the difference between the value of $F_{poly.Ered}(x_i)$, and the value of $F_{sim.Ered}(x_i)$, calculated at a given percentage reduction of the E-modulus x_i . The norm of residuals for the polynomial is obtained from MATLAB, and the value is $norm(d, 2) = 6.0688e-018$. This value is very small, and thus the polynomial can be used as a safe representation of the maximum flexibility value with respect to the reduction of E-modulus.

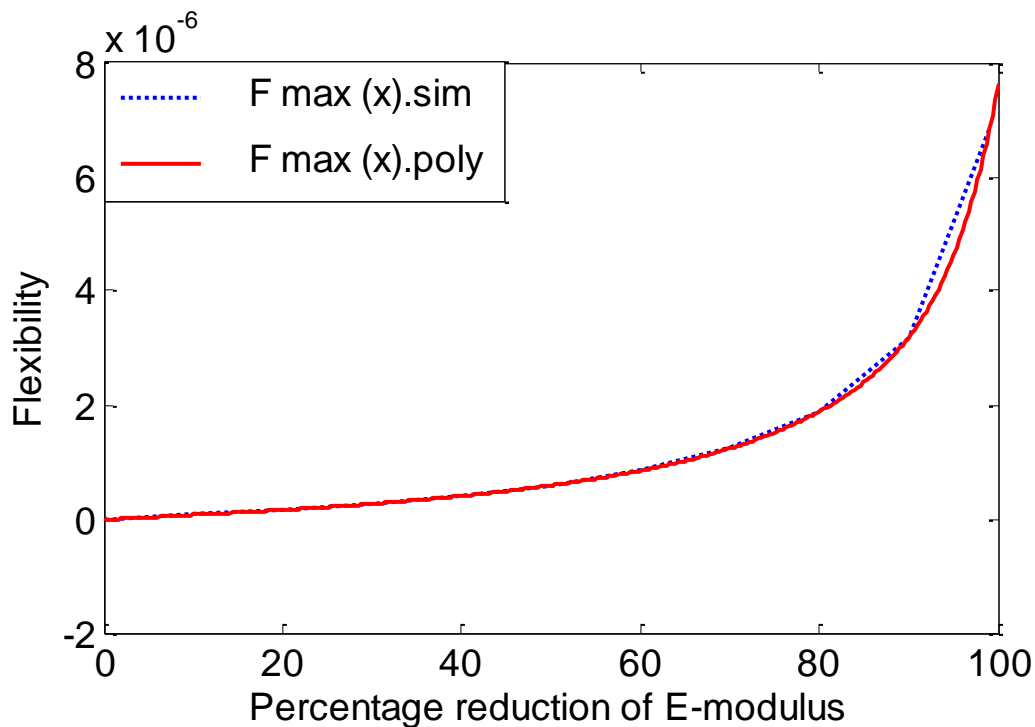


Figure.3.1-13 Overlay of polynomial function and simulated curve.

With the polynomial $F_{poly.Ered}(x)$, the flexibility can be estimated analytically if you know how much the E-modulus has been reduced at a location.

For the interval of 0 to 70 percent E-modulus reduction, the following expression and range can be used for $F_{poly.Ered}(x)$

$$F_{poly.Ered}(x) = b_1x^7 + b_2x^6 + b_3x^5 + b_4x^4 + b_5x^3 + b_6x^2 + b_7x + b_8, \quad x \in [0,70] \quad (27)$$

where coefficients b_1 to b_8 are given in Table 3-7.

Table 3-7 Coefficients for polynomial $F_{poly.Ered}(x)$

Coefficients. p	Value
b1	3.2433e-023
b2	-1.4387e-020
b3	2.7508e-018
b4	-2.9615e-016
b5	1.9688e-014
b6	-8.3313e-013
b7	2.2289e-011
b8	-3.5924e-010

The location of the damage in the damage cases is obtained from the respective flexibility plots given in Figs. 3.1-14 to 3.1-23. The peak in the flexibility curves indicates at which element number the damage is present. The vertical axis represents the flexibility values.

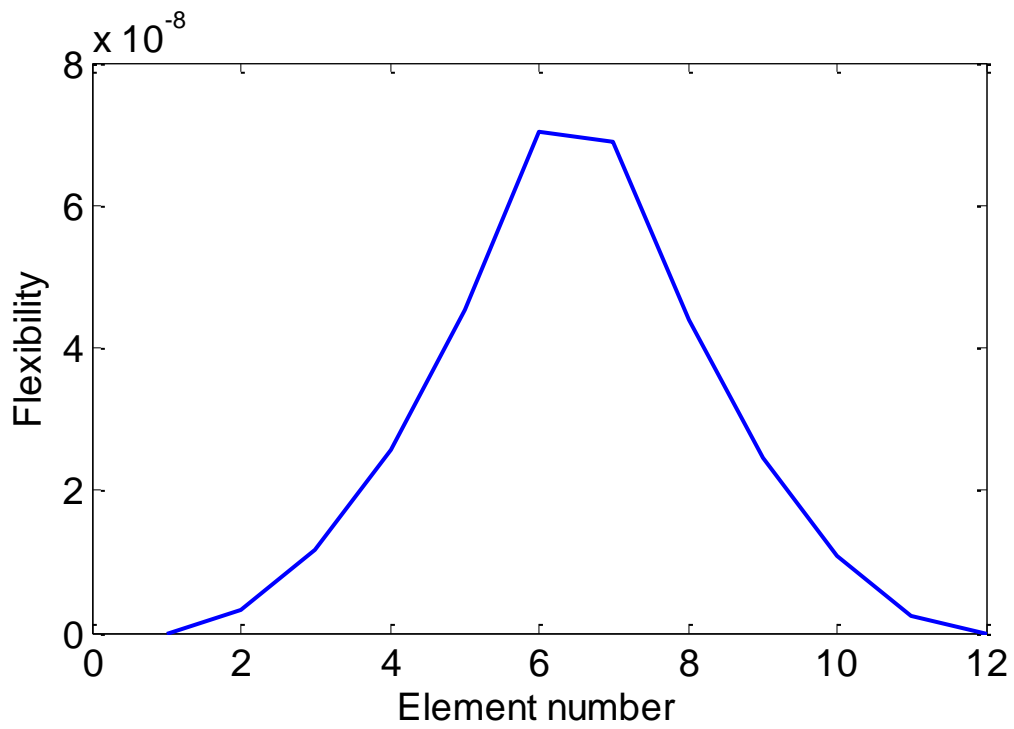


Figure.3.1-14 Flexibility for damage case 2. F1.

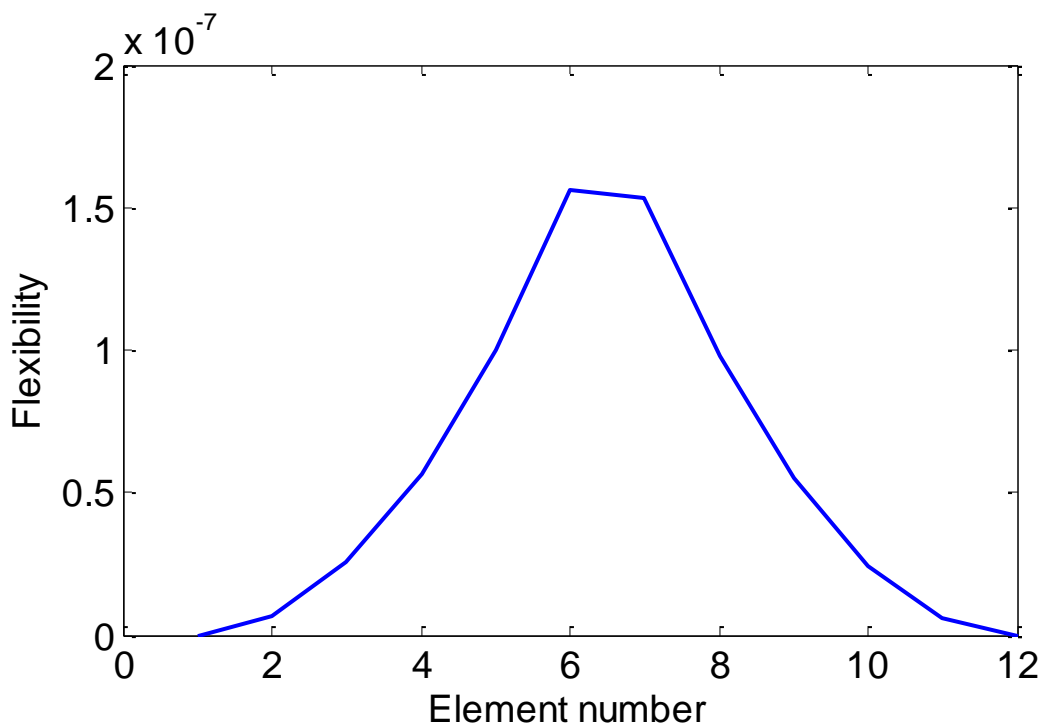


Figure.3.1-15 Flexibility for damage case 3. F2.

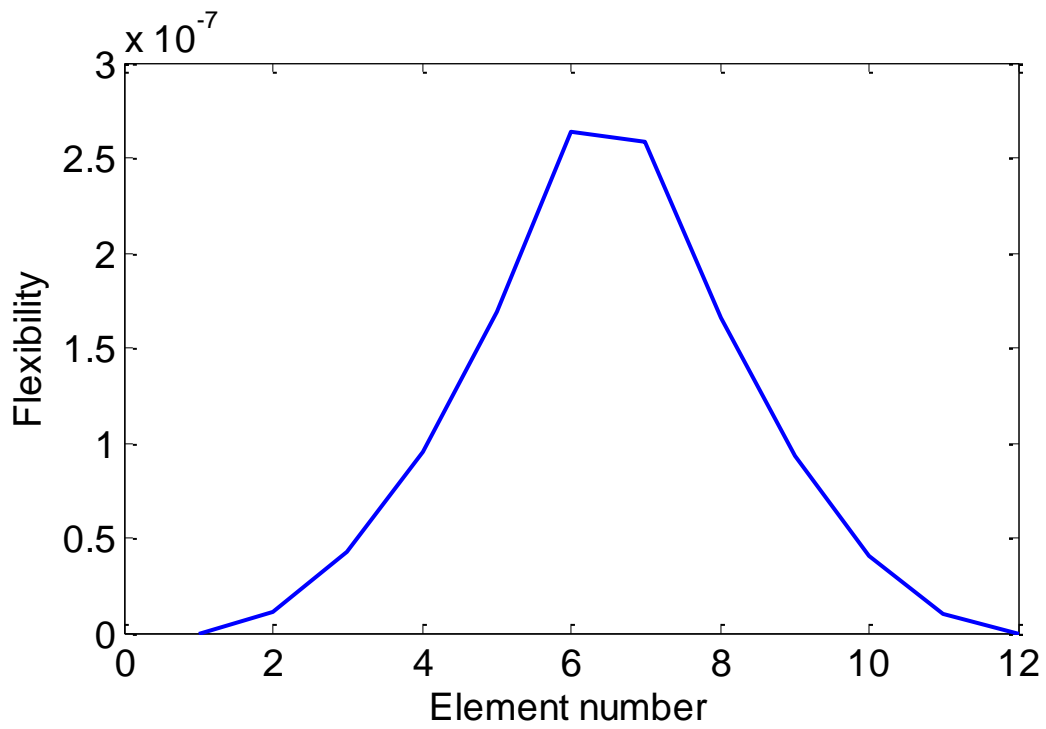


Figure.3.1-16 Flexibility for damage case 4. F3.

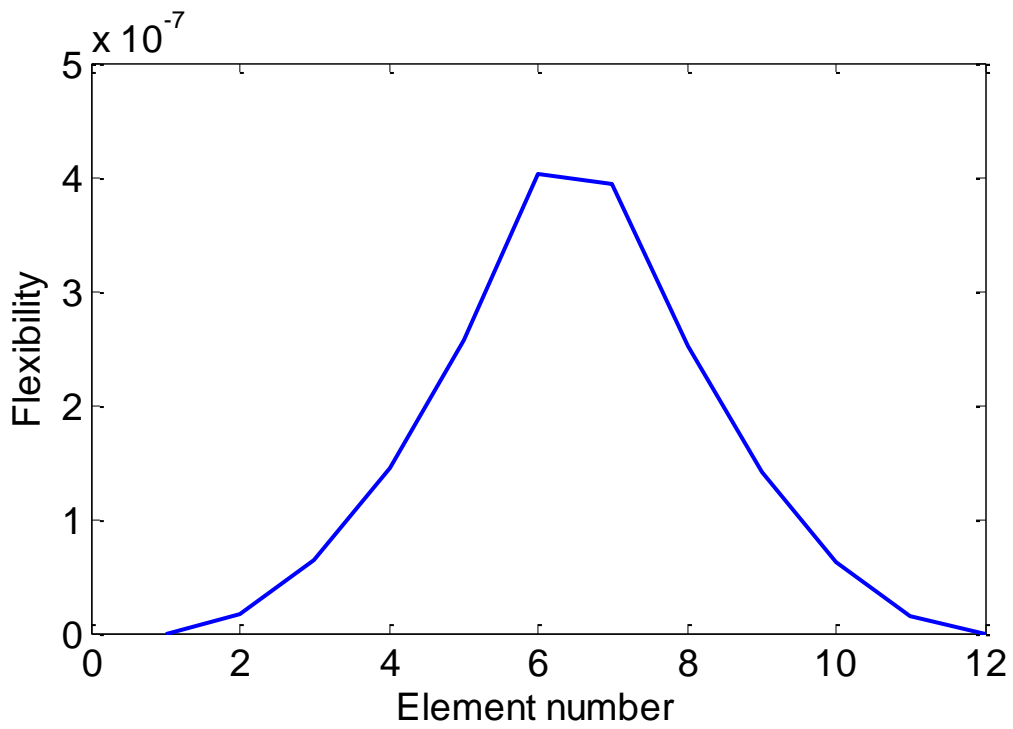


Figure.3.1-17 Flexibility for damage case 5. F4.

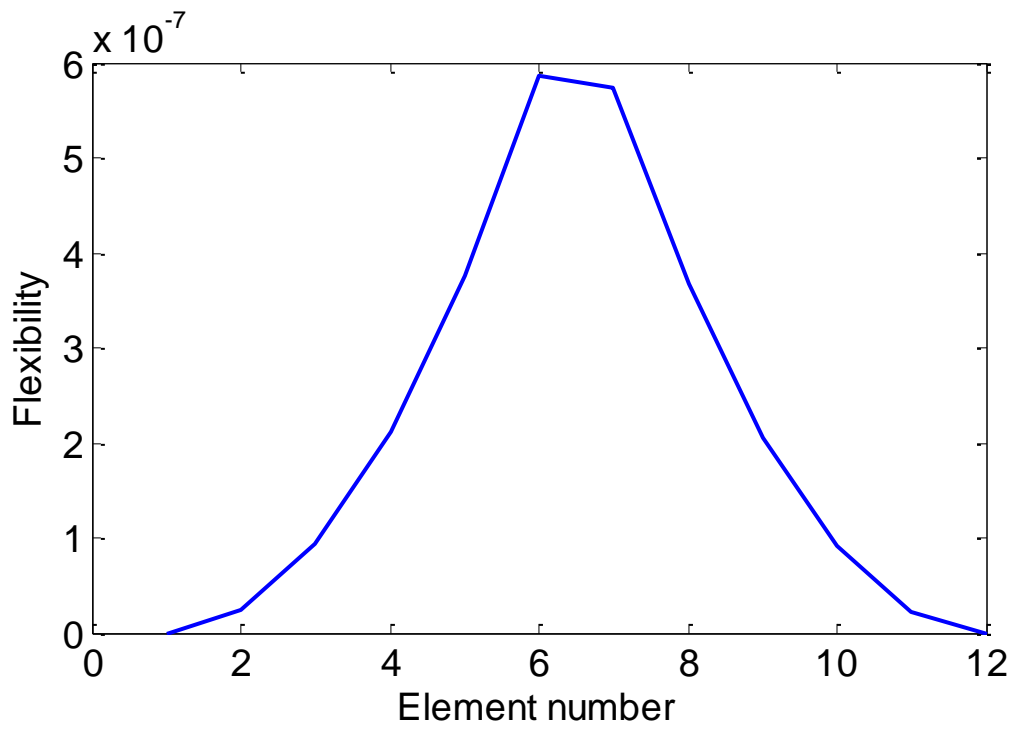


Figure.3.1-18 Flexibility for damage case 6. F5.

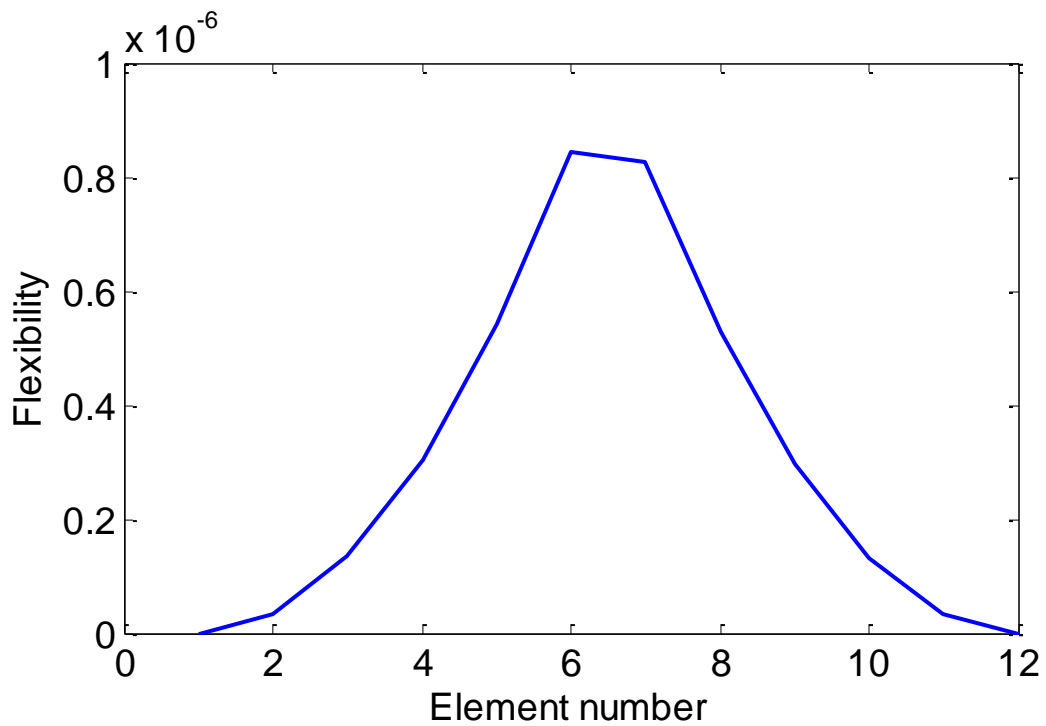


Figure.3.1-19 Flexibility for damage case 7. F6.

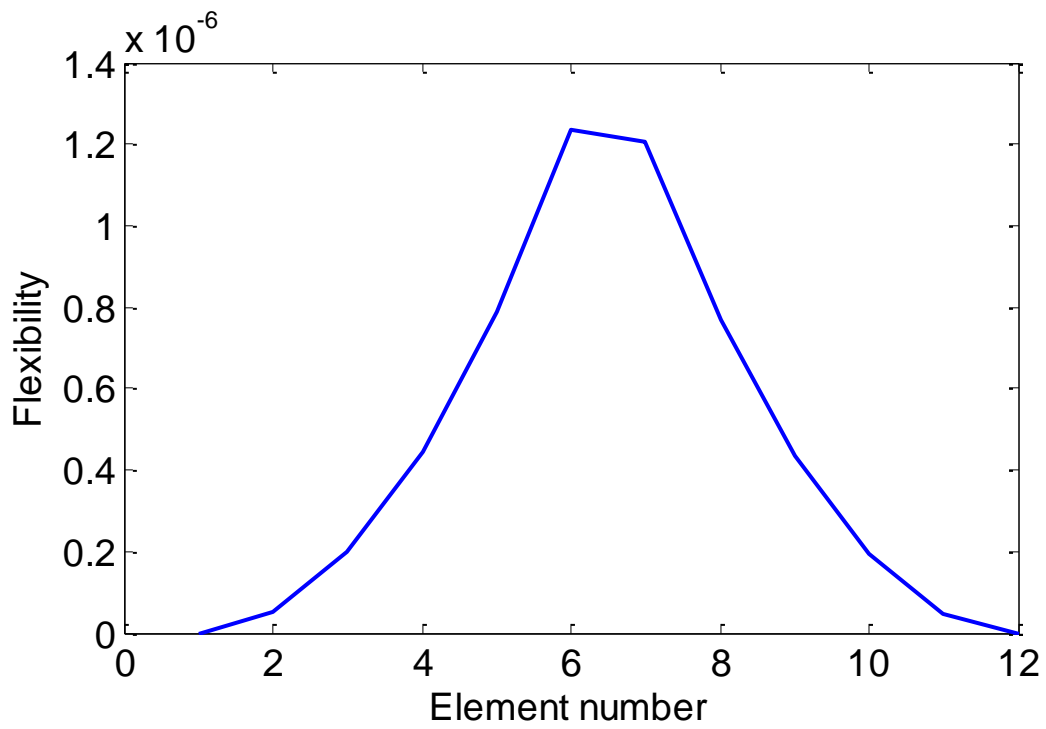


Figure.3.1-20 Flexibility for damage case 8. F7.

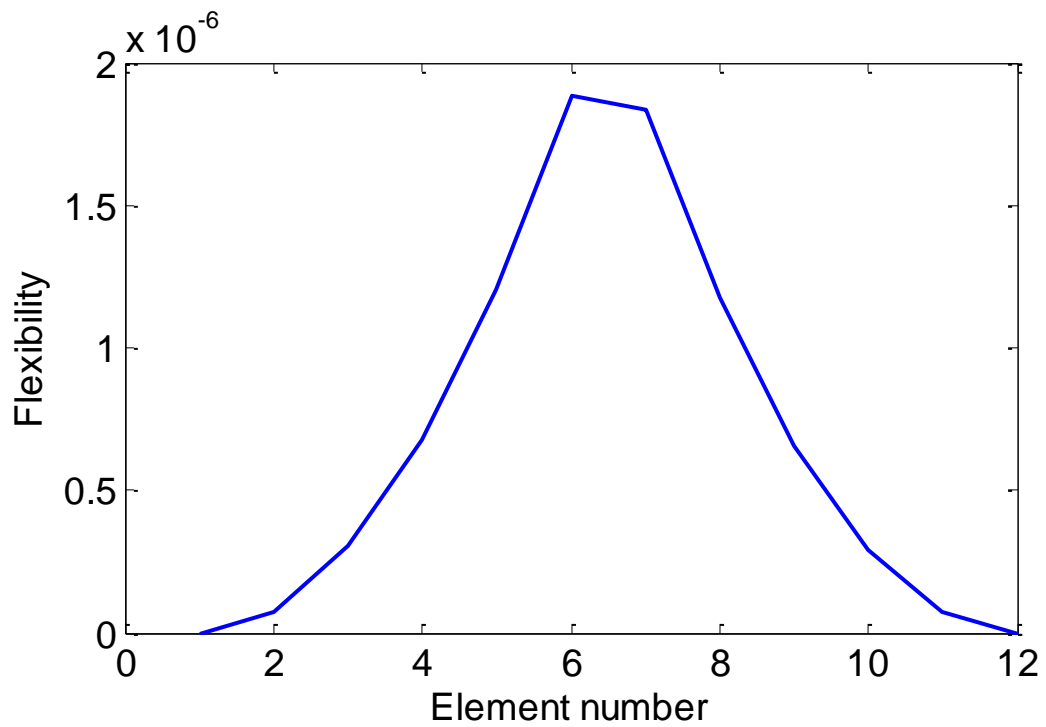


Figure.3.1-21 Flexibility for damage case 9. F8.

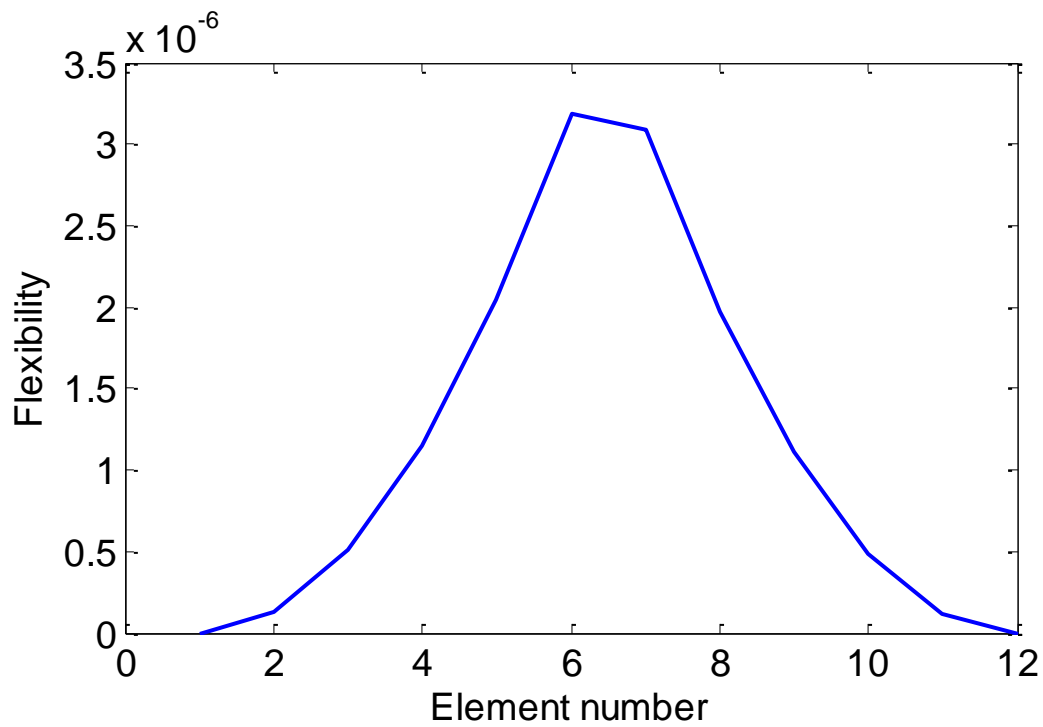


Figure.3.1-22 Flexibility for damage case 10. F9.

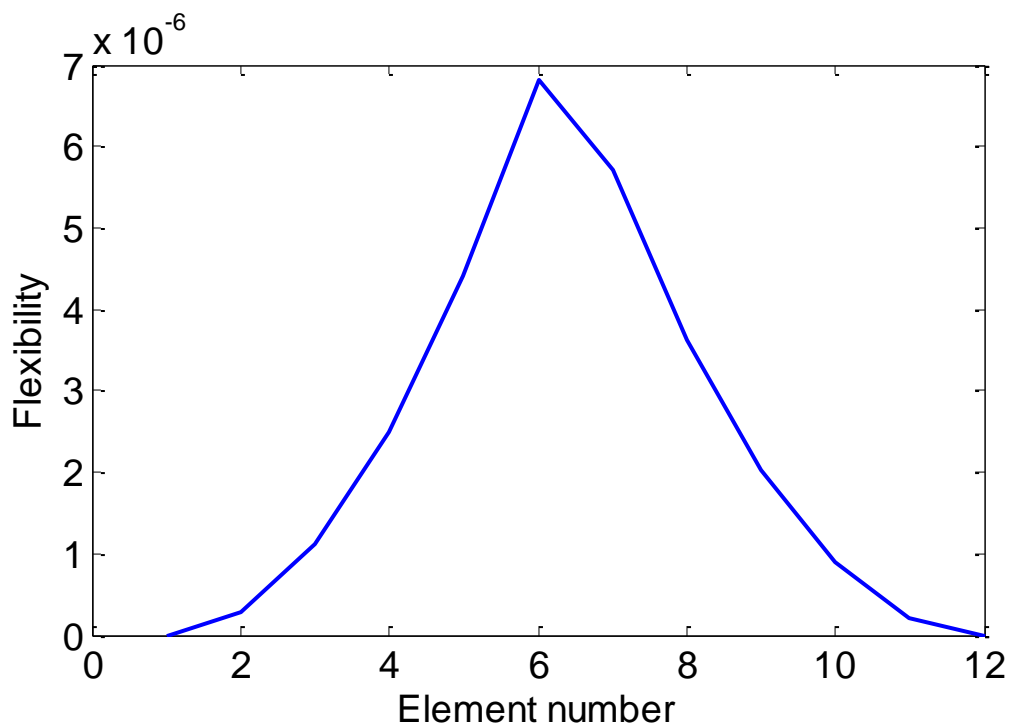


Figure.3.1-23 Flexibility for damage case 11. F10.

It can be seen from Figs. 3.1-14 to 3.1-23 that the change in flexibility method was able to correctly locate the damage at element 6, for all damage cases. The reduction of the E-modulus was from 0 to 99 percent. And thus, it seems safe to say that the flexibility method is

safe to use as a damage detection tool when analyzing damage cases where the structural stiffness is reduced to nearly 100 percent at one location (element) in the beam.

3.1.2 Noise contamination

As mentioned in section 2.7, there will always be some noise contamination in a recorded signal. The noise is usually random in nature, but can be generated into an analytic signal by using the *noise generator* function in MATLAB. In this chapter, the robustness of the flexibility method in terms of noise contamination has been investigated, and the results are presented.

Damage case 6 given in Table 3-2 has been contaminated with ten different combinations of random noise C1 to C10. The noise percentage is increased by 1 percent for C1, to 2 percent for C2 and so on up to 10 percent for C10. The flexibility values obtained from these combinations are then compared to the baseline case C0. C0 is the noise-free damaged state of damage case 6, which has been evaluated in subsection 3.1.1.

The different noise-damage combinations are presented in Table 3.8. The damage detection algorithm calculates the eigenfrequencies, eigenvalues, eigenvectors and maximum flexibility values for every combination. The flexibility plots and the percentage increase in flexibility are presented to illustrate how the flexibility changes for the different combinations. A maximum flexibility plot is presented in Fig. 3.1-24 where all of the maximum flexibility values for every combination have been plotted together to illustrate how the flexibility changes with the noise percentage input. The curve in Fig. 3.1-24 has been labeled $F_{sim.noise}(x)$.

Table 3-8 Overview of noise-damage combinations C0 to C10

Combination	Damage case	Noise input [%]	F_{max}	Increase F_{max} [%]
C0	6	0	5.8675e-007	0
C1	6	1	5.9935e-007	2.1
C2	6	2	6.0297e-007	2.69
C3	6	3	6.1903e-007	5.21
C4	6	4	6.3540e-007	7.66
C5	6	5	6.3912e-007	8.19
C6	6	6	6.0257e-007	2.63
C7	6	7	5.8925e-007	0.42
C8	6	8	6.0498e-007	3.01
C9	6	9	6.3566e-007	7.69
C10	6	10	6.1080e-007	3.94

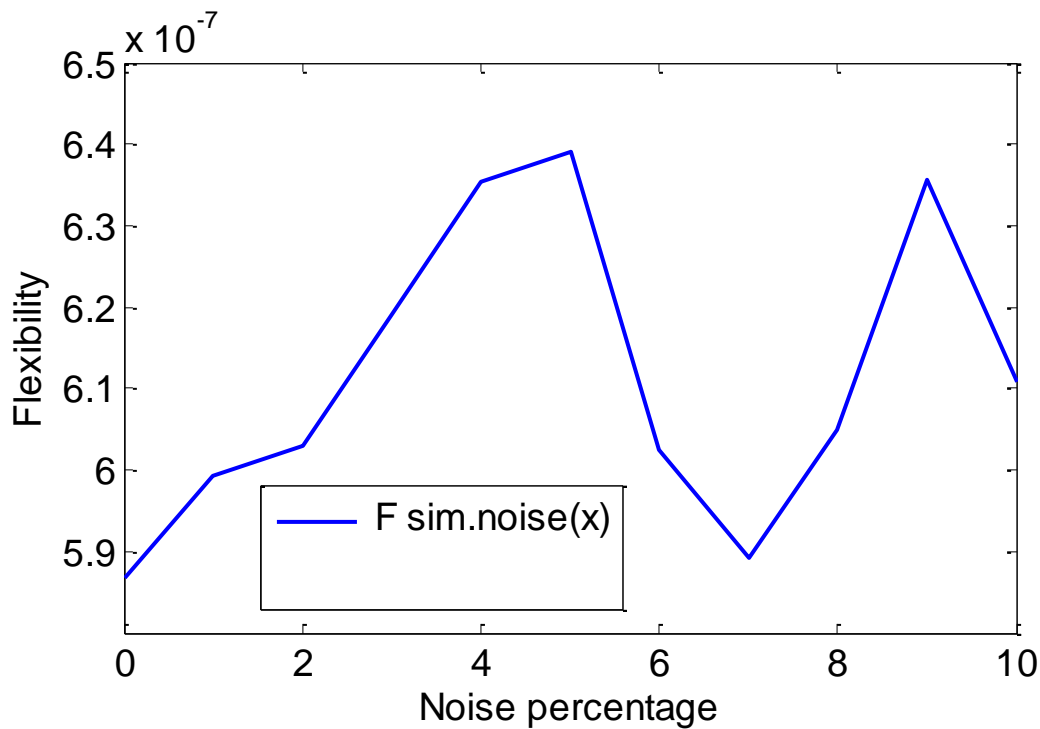


Figure.3.1-24 Change in flexibility from 1-10% noise input.

The curve for $F_{sim.noise}(x)$ indicates how the noise input percentage will influence the flexibility such that it will deviate from the baseline value equal to $5.8675e-007$. This deviation clearly shows that the change in flexibility method is sensitive to noise contamination. In order to have an analytical tool to control the noise influence, a best fit polynomial, $F_{poly.noise}(x)$ has been derived from the *Basic Fitting Interface* option found in MATLAB. The polynomial is given as

$$F_{poly.noise}(x) = c_1x^8 + c_2x^7 + c_3x^6 + c_4x^5 + c_5x^4 + c_6x^3 + c_7x^2 + c_8x + c_9, x \in [0,10] \quad (28)$$

where x indicates the percentage of noise input. c_1 to c_9 are coefficients given in table 3.9.

Table 3-9 Coefficients for polynomial $F_{poly.noise}(x)$

Coefficients. p	Value
c1	5.8675e-007
c2	-5.6895e-011
c3	8.6573e-010
c4	-6.5425e-009
c5	2.5518e-008
c6	-4.8503e-008
c7	3.7235e-008
c8	4.0602e-009
c9	5.8675e-007

The polynomial $F_{poly.noise}(x)$ and the curve obtained from simulation $F_{sim.noise}(x)$, are presented together in Fig. 3.1-25. From the figure it is clear that the curves don't match each other completely. However, the norm of residuals is equal to $6.8818e-009$. This is a low value and thus $F_{poly.noise}(x)$ can be used as a representation of how the flexibility will be altered when the data is contaminated with up to 10% noise. However, it should be stated that $F_{poly.noise}(x)$ is only valid of noise contamination of up to 10%. The signature plot of $F_{poly.noise}(x)$ might be different for percentage noise contamination larger than 10. Also, this means that the maximum flexibility value calculated for $x > 10$ might not be accurate. So the function and its range is

$$F_{poly.noise}(x) = c_1x^8 + c_2x^7 + c_3x^6 + c_4x^5 + c_5x^4 + c_6x^3 + c_7x^2 + c_8 + c_9, x \in [0,10] \quad (29)$$

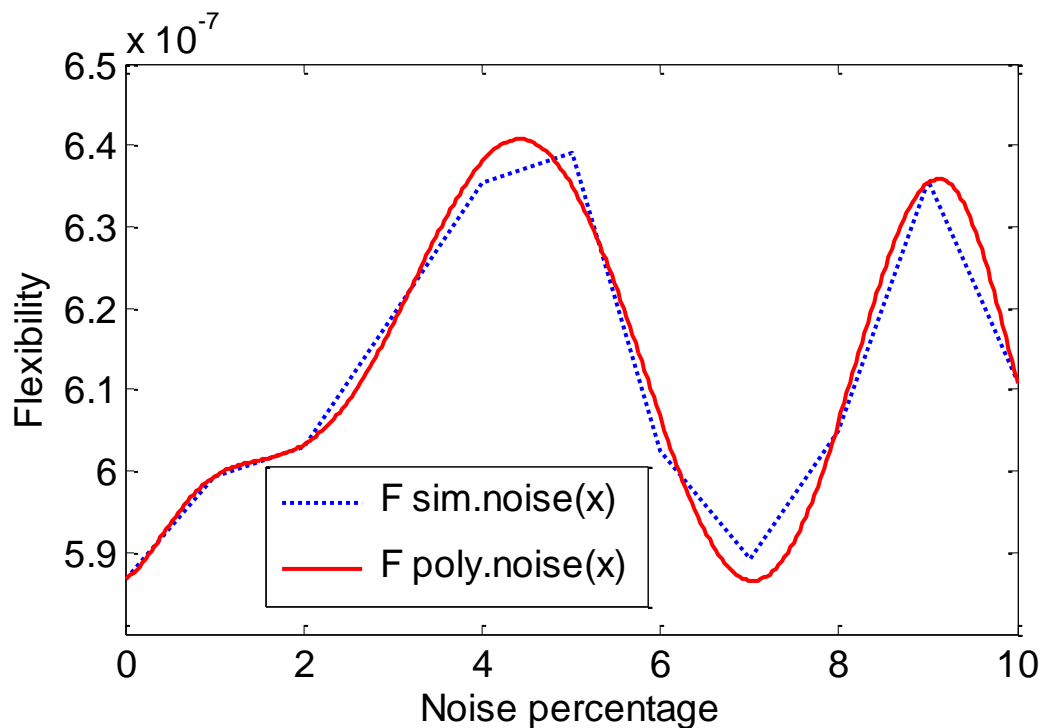


Figure.3.1-25 Overlay of polynomial function and simulated curve.

Flexibility plots for the different combinations in Table 3-8 are presented as follows

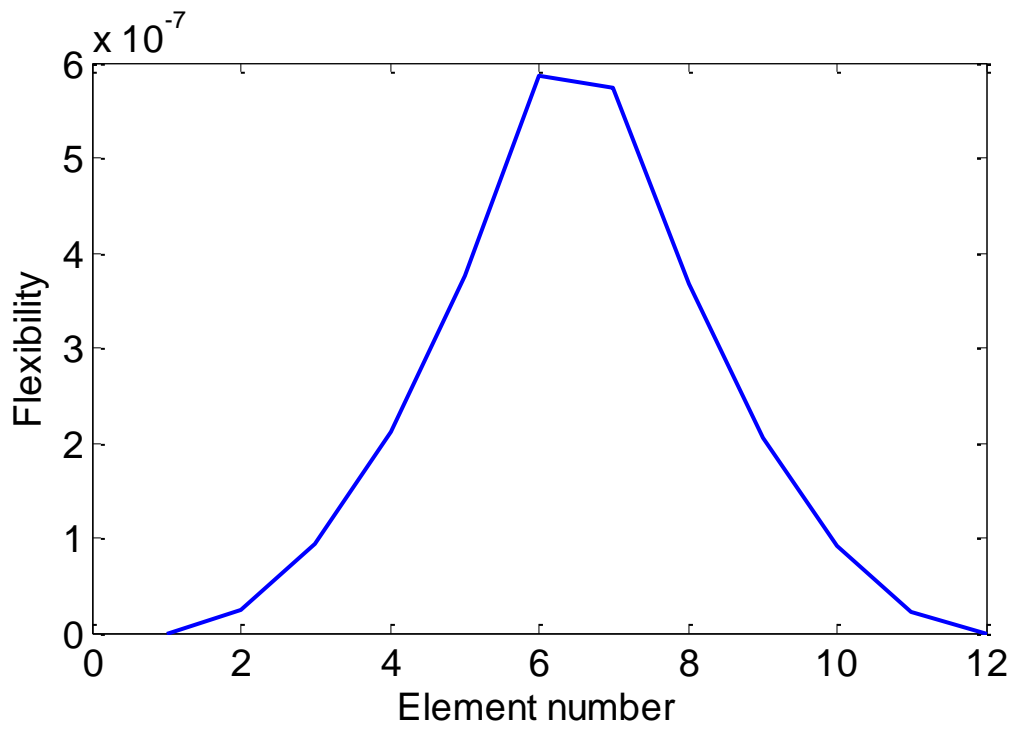


Figure.3.1-26 Flexibility for noise-damage combination C0.

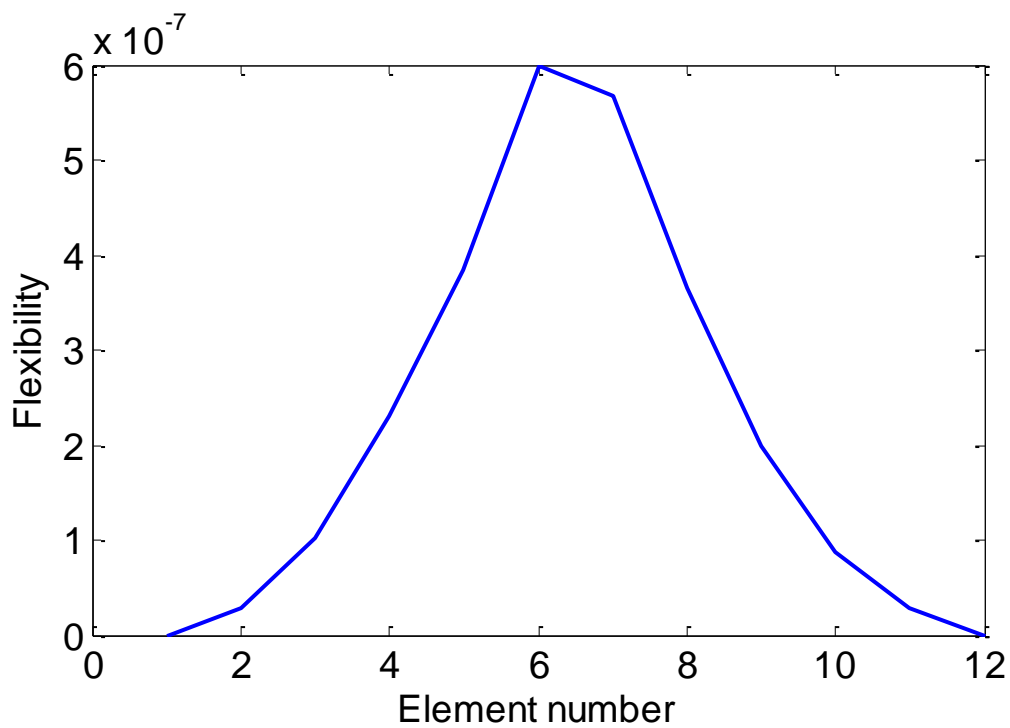


Figure.3.1-27 Flexibility for noise-damage combination C1.

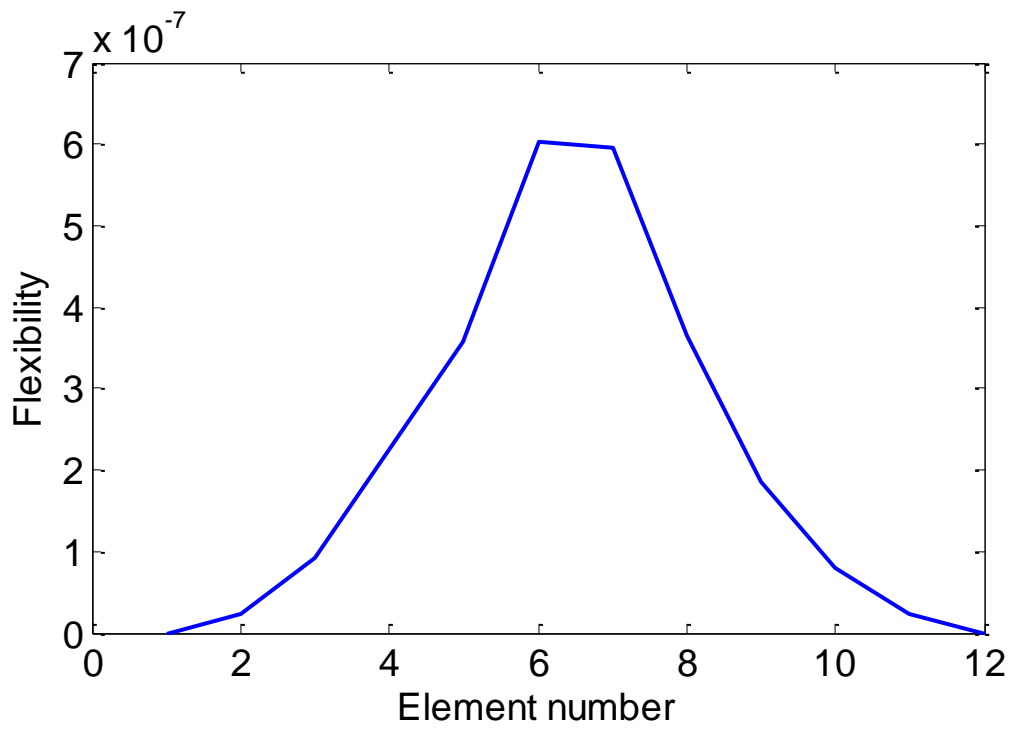


Figure.3.1-28 Flexibility for noise-damage combination C2.

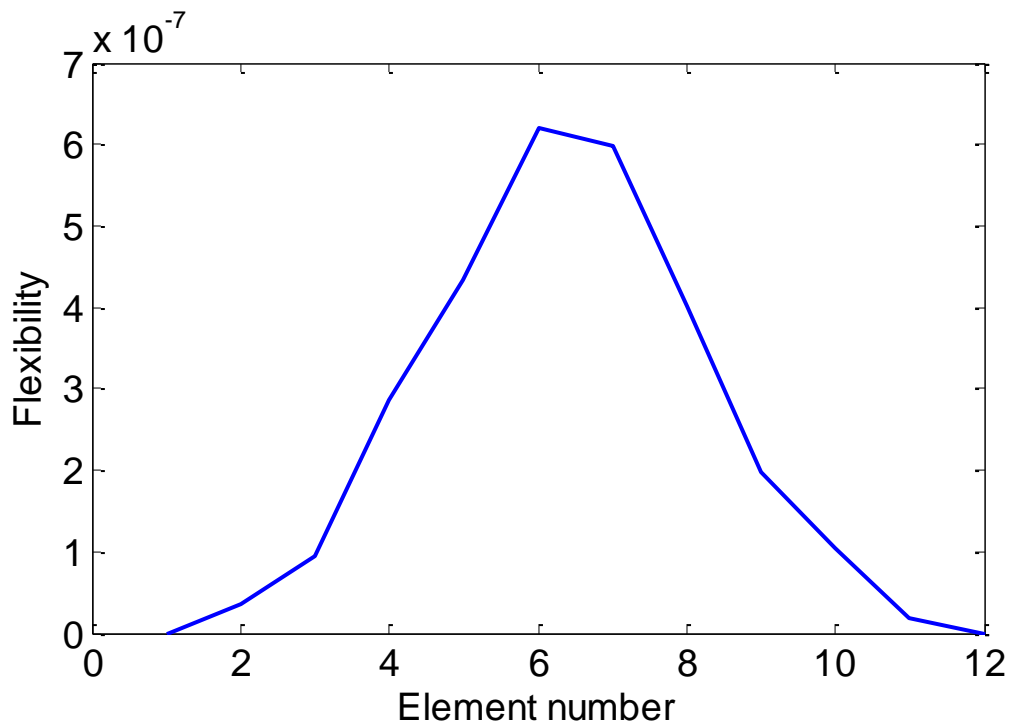


Figure.3.1-29 Flexibility for noise-damage combination C3.

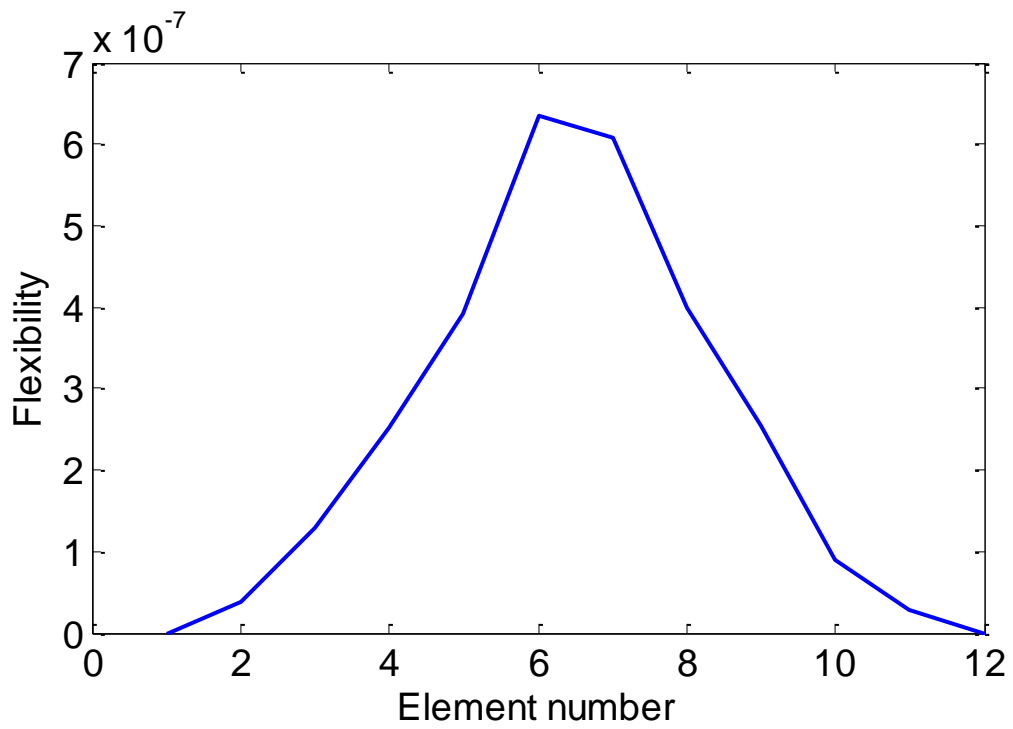


Figure.3.1-30 Flexibility for noise-damage combination C4.

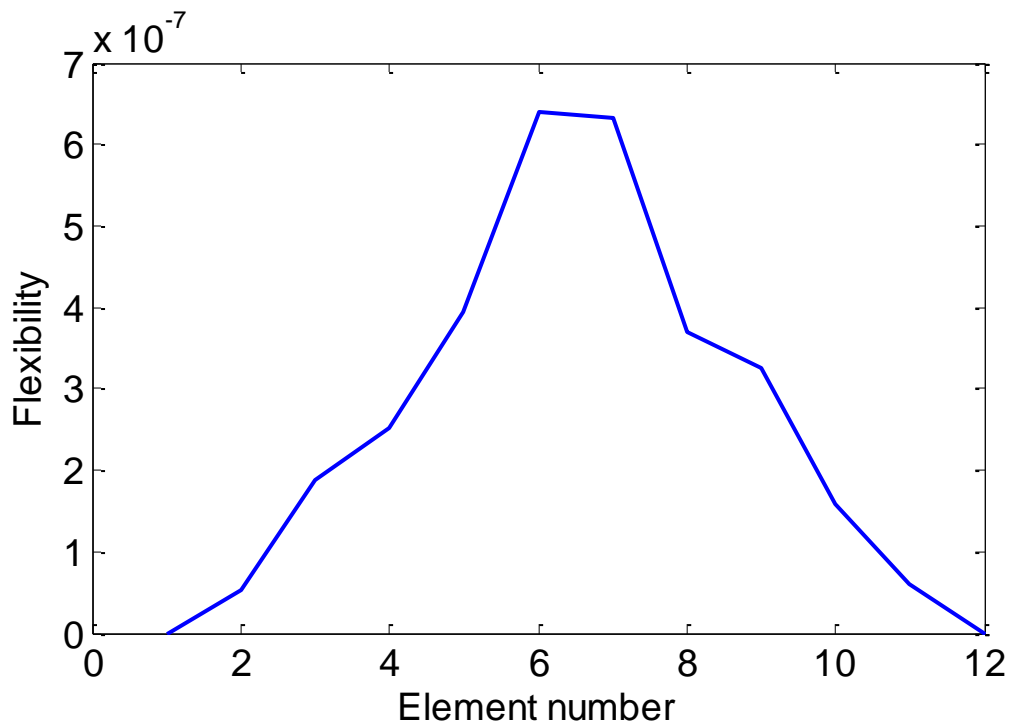


Figure.3.1-31 Flexibility for noise-damage combination C5.

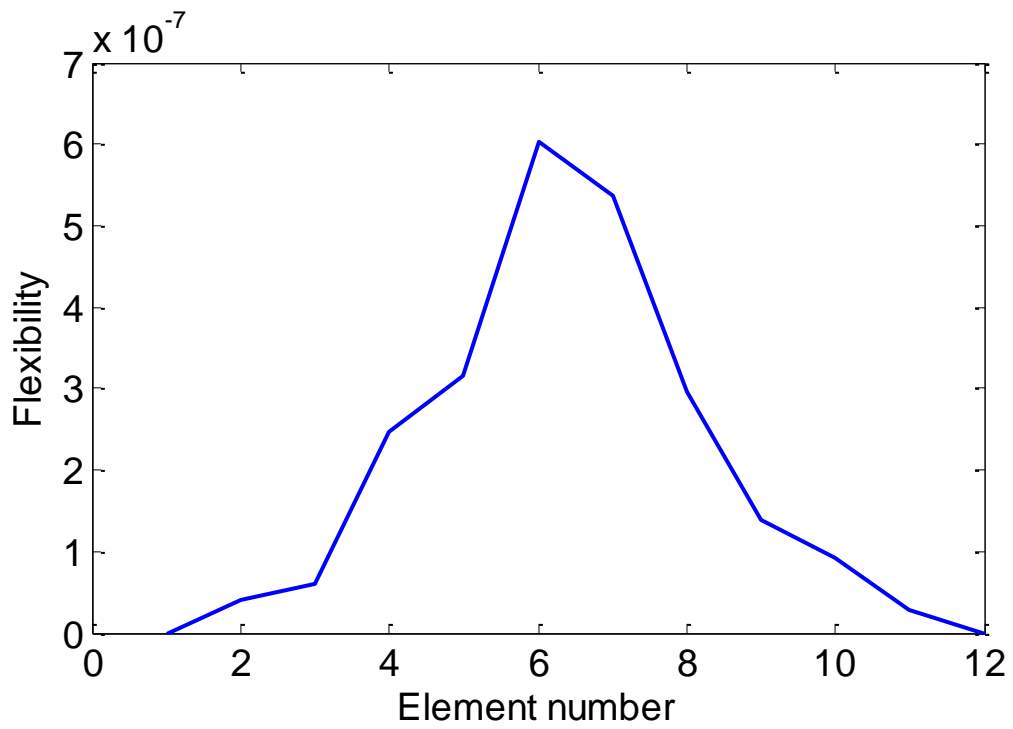


Figure.3.1-32 Flexibility for noise-damage combination C6.

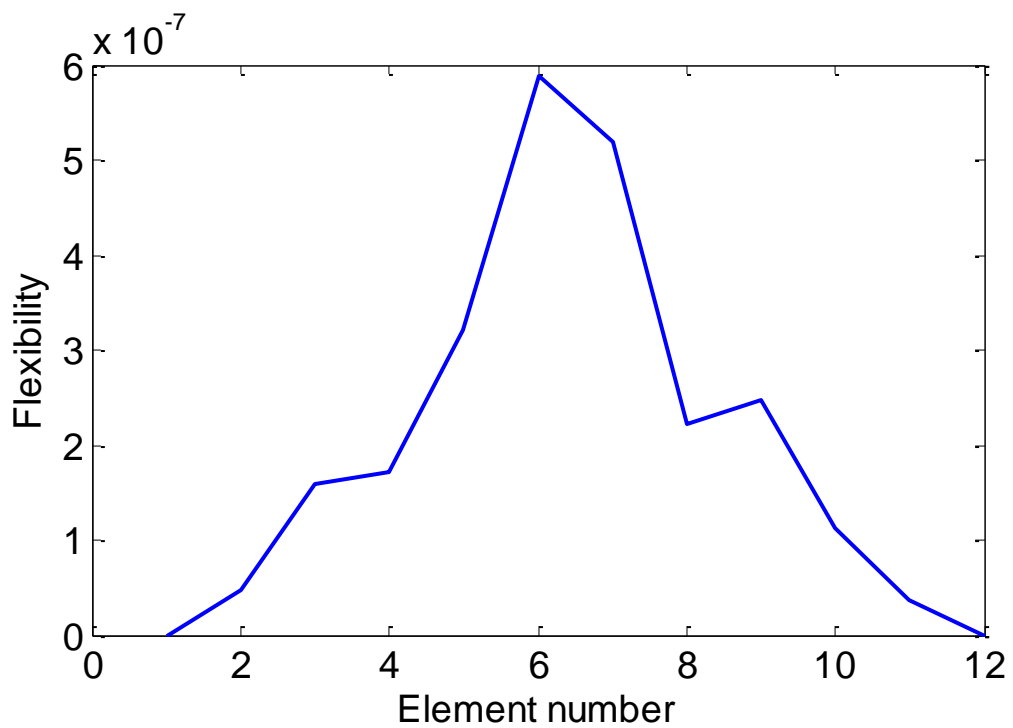


Figure.3.1-33 Flexibility for noise-damage combination C7.

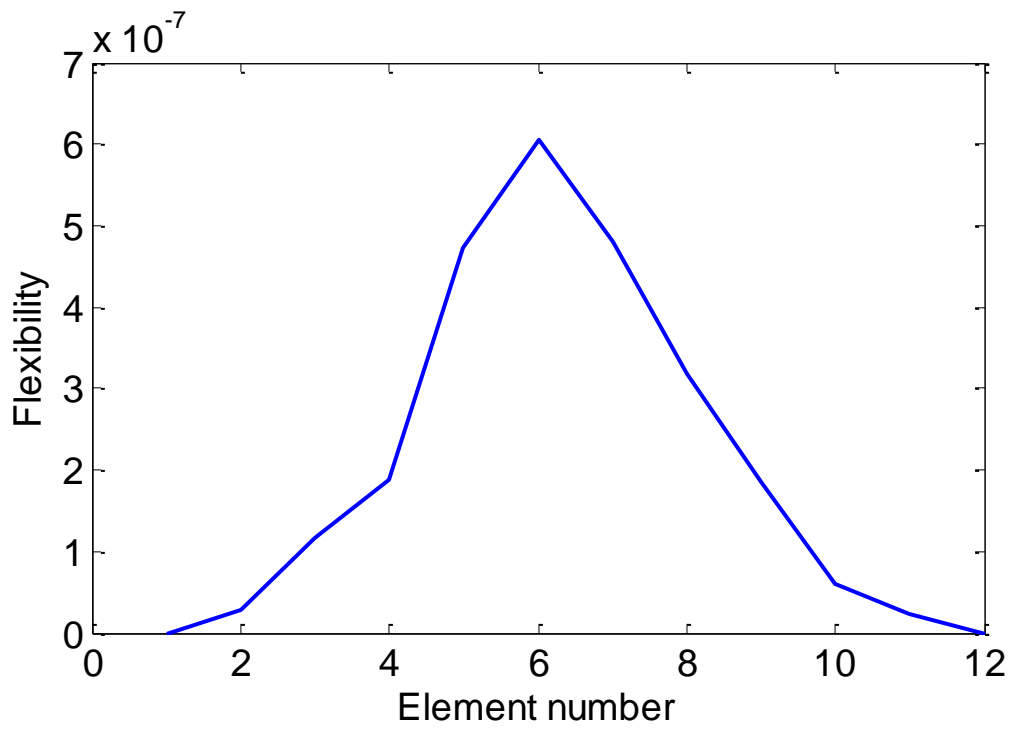


Figure.3.1-34 Flexibility for noise-damage combination C8.

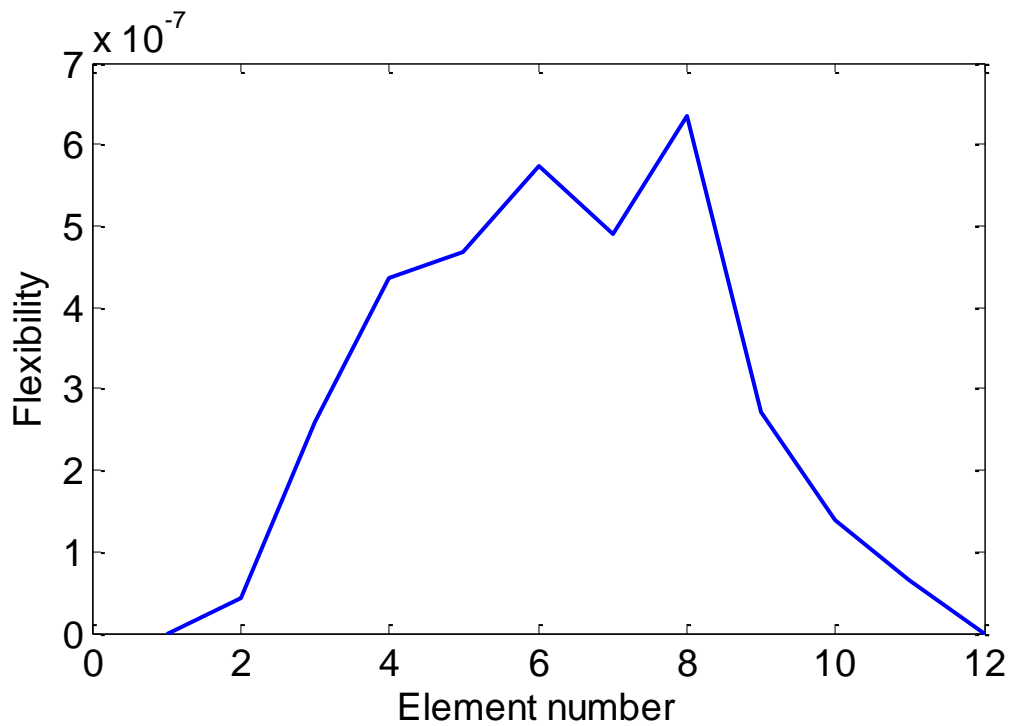


Figure.3.1-35 Flexibility for noise-damage combination C9.

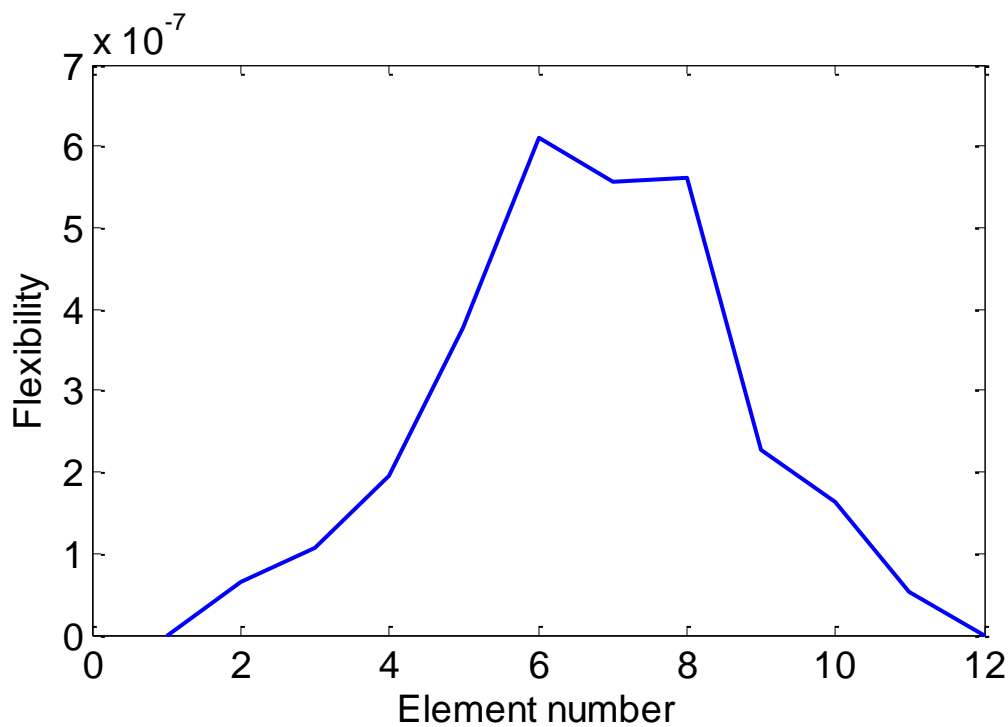


Figure.3.1-36 Flexibility for noise-damage combination C10.

As it is illustrated in Figs. 3.1-26 to 3.1-34 and Fig. 3.1-36, the change in flexibility method is capable of detecting the damage at element 6 with the presence of up to 10 % noise contamination. Generally, peaks in the plot indicate the location of damage. This is constant for combinations C1 to C8, and for combination C10. For these combinations, location of damage is correctly identified at element 6. However, the flexibility plots given in Figs. 3.1-31 to 3.1-36 have more than one peak. This is a problem because this indicates that there are more than one damage present in the beam. In combination C9 the damage is incorrectly indicated at element 8. The damage at element 6 can be noticed by the flexibility peak located over the element illustrated in Fig. 3.1-35. However it can be concluded that the flexibility method was inaccurate in locating the single damage at element 6 for combination C9. From the studies conducted in this chapter, it can be concluded that noise contamination in the analytical datasets, will have an effect on the accuracy of the flexibility method. For some noise input percentages the noise contamination might result in inaccurate damage localization by the flexibility method.

3.1.3 Multiple damages

A problem with many damage detection algorithms is that they are not completely accurate when you have structural damages that are in close vicinity to each other. If the vicinity is very small, then the results might indicate that there is only one damage present, instead of several. This problem might also arise when the flexibility method is applied to a VBSHM system. The flexibility diagrams might show a flexibility value peak (which indicates presence of damage) at a healthy element that lie in-between the damaged elements, and if you don't know the exact location of damage (which is seldom the case); the results might indicate a false damage location. Another problem is that the damage detection algorithms are often severity-of damage sensitive, meaning that the algorithm locates the damages that have

the highest severity more accurately than the damages with lower severity. In order to have a reliable VBSHM system, the flexibility method needs to be checked for these problems,

In this chapter, the flexibility method is tested for four case studies of multiple-damage. The four case studies are labeled A, B, C and D, respectively. These case studies are simulated in the FE-model for the fixed-fixed beam structure. In these case studies, there are two damages simulated at two different elements in the FE-model. The aim is to investigate how the vicinity of the damages will affect the accuracy of the flexibility method. Four different damage cases are investigated for every case-study. For every damage case the E-modulus at one damage location is reduced by 20 percent. This reduction is fixed for all damage cases. The damage at the other location is changed by reducing the E-modulus by 20, 40, 60 and 80 percent, respectively for the four damage cases. By analyzing this way you can determine the flexibility methods robustness for locating different damages that are in close vicinity to each other, as well as determining the methods accuracy in locating damages with severity differences. The different flexibility diagrams obtained from the analysis is presented, and the results are discussed.

From the investigation of the case studies A, B, C and D, it is quite clear that the flexibility method is highly inaccurate in locating the damages in a fixed-fixed beam structure when there is more than one damage present in the beam. The method is also severity of damage sensitive, meaning that in some cases the method manages to locate the damage that has the largest severity. Because of problems related to the amount of time given for the work on this thesis the author wasn't able to find out what types of modifications that can be applied to the flexibility method in order to make the method more applicable to multiple-damage detection. However, there is an indication that modifications can be done with the eigenvectors of the healthy state and the damaged state of the beam. The flexibility matrix, $[F] = \sum_{i=1}^n \frac{1}{\omega_i^2} \Phi_i \Phi_i^T$. This indicates that some modifications of the eigenvectors, which will result in modification of the mass-normalized mode shape vectors, Φ_i , might make the method more applicable to multiple damage detection.

Case studies A, B, C and D are presented in Tables 3.10 to 3.13, respectively.

Case study A:

Table 3-10 Overview of case study A

Damage locations at elements: 5,7	
Damage case	E-modulus reduction [%]
12	20,20
13	20,40
14	20,60
15	20,80

For case study A, there is only one healthy element separating the two damaged elements. The flexibility plots for the different damage cases are presented in Figs. 3.1-37 to 3.1-40, and the results are discussed. From the study of damage cases 12 to 15, it can be concluded that the flexibility method isn't accurate in locating two damaged elements when there is only one healthy element separating them. There is also a clear pattern that the flexibility method is

very sensitive to the difference in severity between the two damages. The damage with the highest severity is easier to locate in the flexibility diagrams.

Flexibility plots and discussions of the results are presented as follows

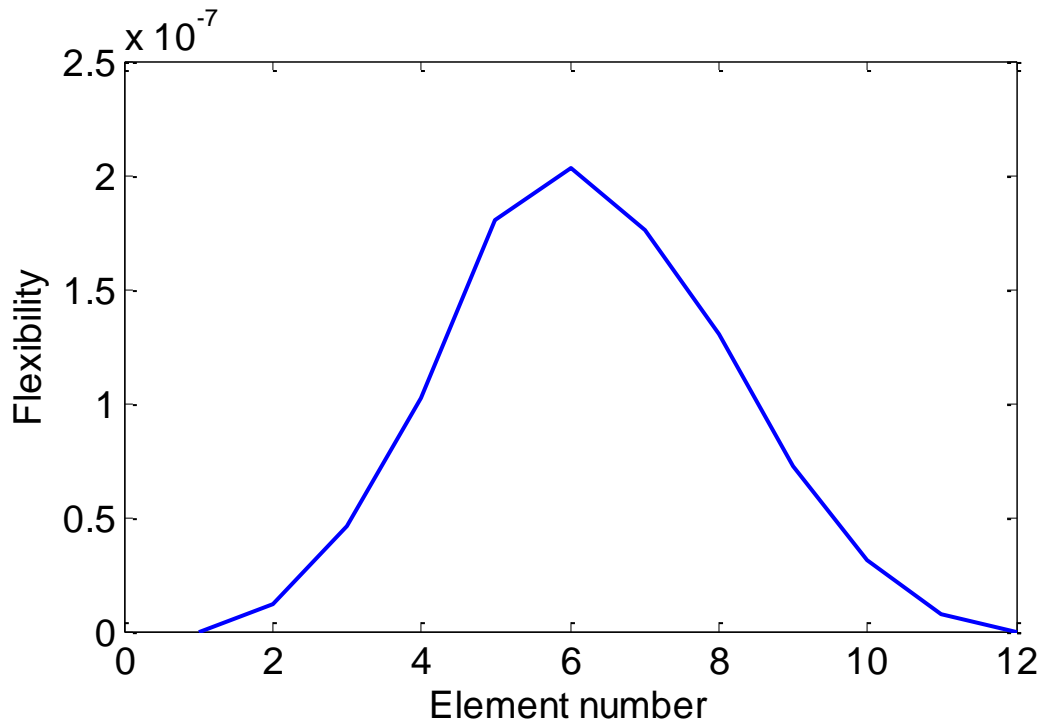


Figure.3.1-37 Flexibility for damage case 12.

It can be seen from the flexibility diagram in Fig 3.1-37 that there is a peak at element 6. This indicates that there is damage present in this element. This is not accurate. However, the damages are simulated at the two elements 5 and 7, and they are placed on both sides of element 6. Both damages are simulated with the same amount of stiffness reduction (20 %), so in this case the two damages are calculated as a single-damage case, where the single damage is located at element 6, instead of two separate damages, located at elements 5 and 7. From this it is clear that the method is not able to detect damages located at two separate elements, having the same amount of damage present and having only one healthy element separating them.

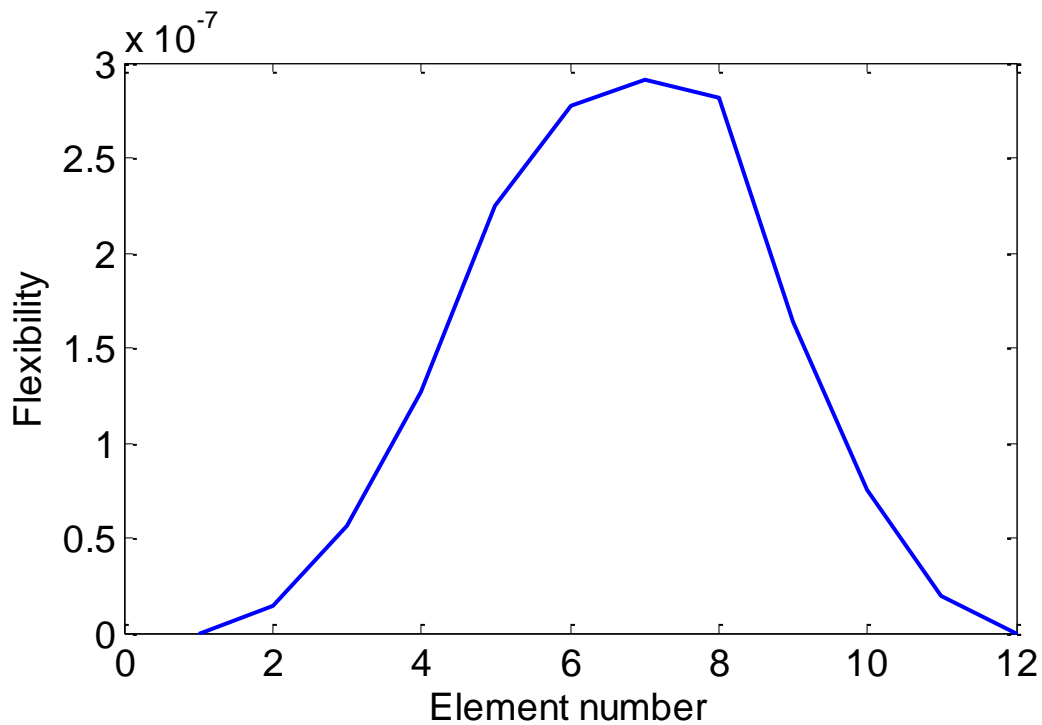


Figure.3.1-38 Flexibility for damage case 13.

In the flexibility diagram in Fig. 3.1-38, the peak of the flexibility curve is now at element 7. This is a positive localization of one of the damage location. However, there is no peak at element 5. There is however a slight deviation present in the flexibility curve at element 5, but this deviation is no more apparent than the ones that can be seen at elements 2 and 8. From this it is clear that the flexibility method is not able to detect damages located at two separate elements, where one element has a 20 percent reduction of E-modulus, and the other has a 40 percent reduction.

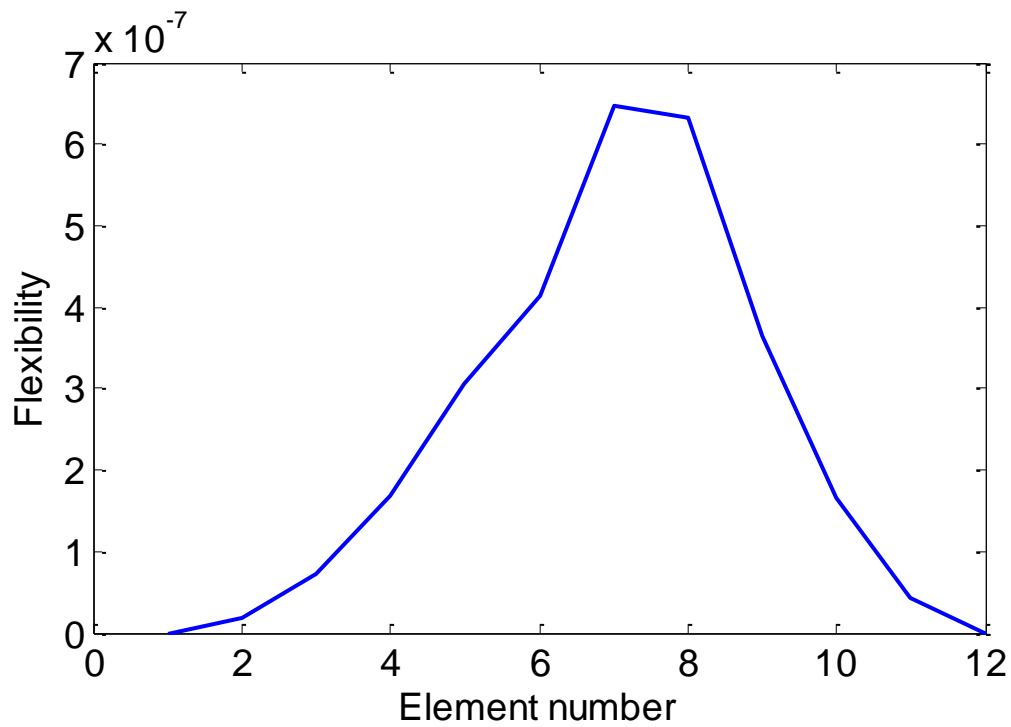


Figure.3.1-39 Flexibility for damage case 14.

From the flexibility diagram presented in Fig. 3.1-39 the flexibility method is able to accurately detect the damage located at element 7, but the damage at element 5 is not easily detectable. From this it is clear that the method is not able to detect both damages which have been placed separately at two elements, where one element has a 20 percent reduction of E-modulus and the other has a 60 percent reduction. This indicates that the method is sensitive to the specific severity of the damages.

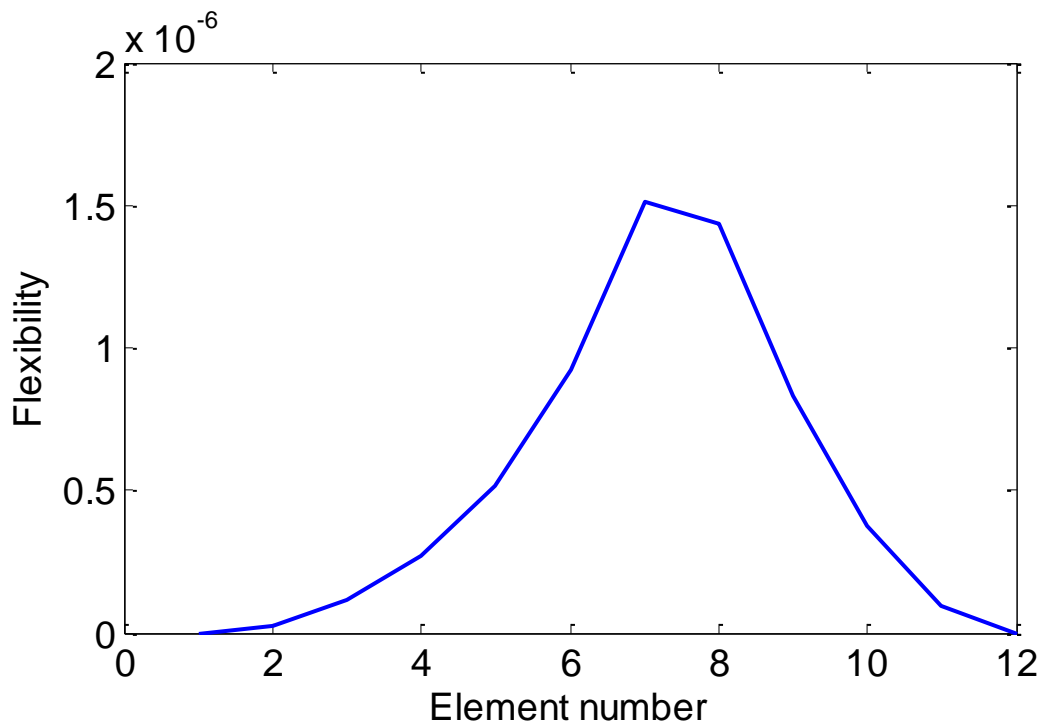


Figure.3.1-40 Flexibility for damage case 15.

From Fig, 3.1-40 it is clear that the flexibility method is able to detect the damage located at element 7, but the damage at element 5 is highly undetectable. From this it is clear that the flexibility method is not able to detect both damages which have been placed separately at two elements, where one element has a 20 percent reduction of E-modulus, and the other has 80 percent reduction. The damage located at element 7 is clearly visible, but this is probably because the severity of the damage there is four times the severity in element 5.

Case study B:

Table 3-11 Overview of case study B

Damage locations at elements: 4,8	
Damage case	E- modulus reduction [%]
16	20,20
17	20,40
18	20,60
19	20,80

For case study B, the vicinity of the damaged locations is increased by separating the damaged elements with three healthy elements. The flexibility plots for the damage cases are presented in Figs 3.1-41 to 3.1-44 and the results are discussed. From the study of damage cases 16 to 19, it can be concluded that the change in flexibility method has successfully identified, and located the damage at element 8 for all damage cases. In Figs 3.1-41 to 3.1-42 there is a change in the flexibility diagrams at element 4, however the left-hand flexibility peaks are over element 5 and this might give an indication of damage in this element, and this

is inaccurate. It can be concluded that the flexibility method isn't able to locate both of the damages in the damage cases 16 to 19

The flexibility diagrams for damage cases 16 to 19 are presented as follows and the results are discussed.

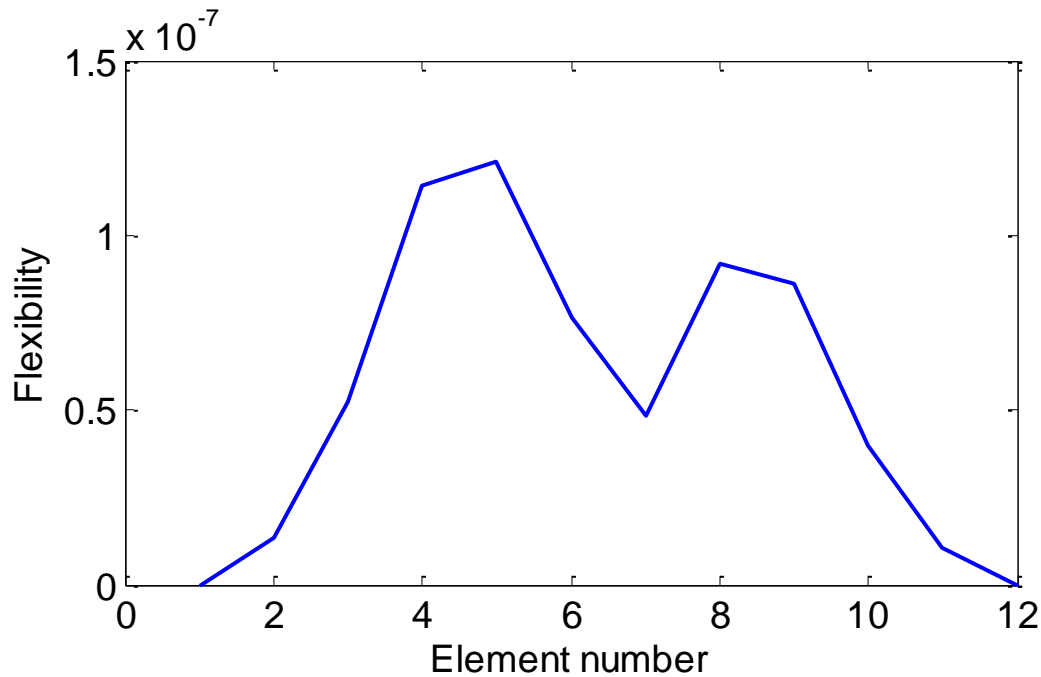


Figure.3.1-41 Flexibility for damage case 16.

In the flexibility diagram in Fig. 3.1-41 there are two flexibility peaks present and this gives an indication that there are two different damages present in the beam. The method has successfully indicated the damage in element 8, which can be clearly seen by the right-hand flexibility peak. The damage in element 4 has also been detected, as can be seen by the change in the flexibility curve at element 4, but the left-hand flexibility peak is at element 5, and this might give an indication of damage present in element 5, which is incorrect.

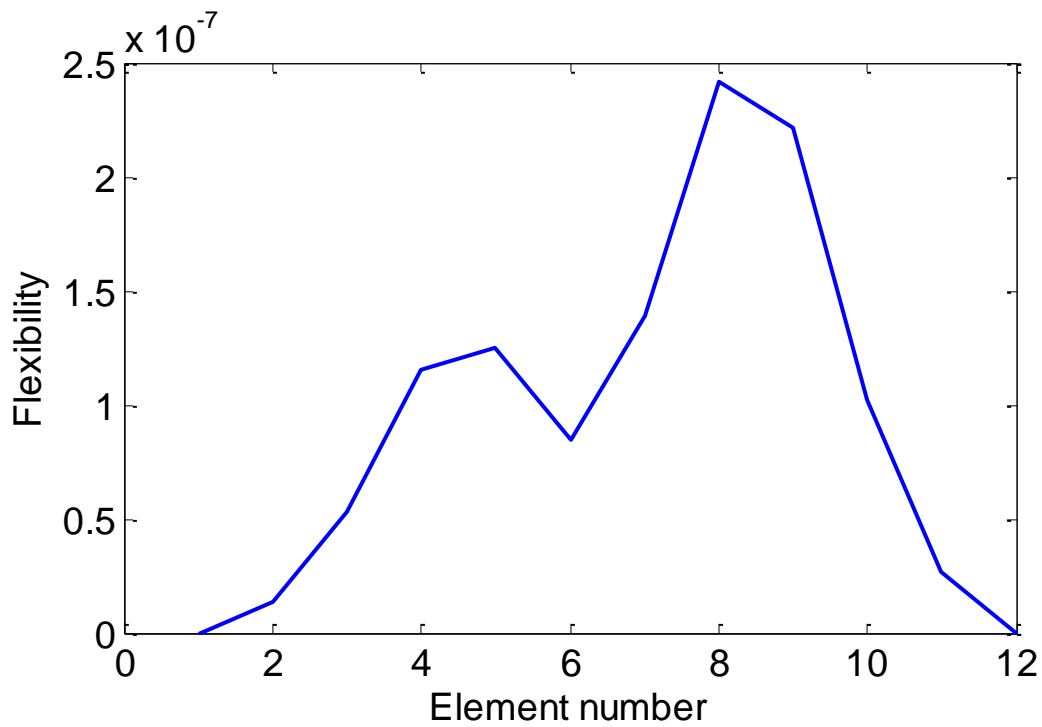


Figure.3.1-42 Flexibility for damage case 17.

In the flexibility diagram in Fig. 3.1-42, the damage located in element 8 has been successfully detected, as can be clearly seen by the right-hand peak. The damage in element 4 can also be noticed, but the left-hand peak is still at element 5, and this might give a false indication of damage present in element 5.

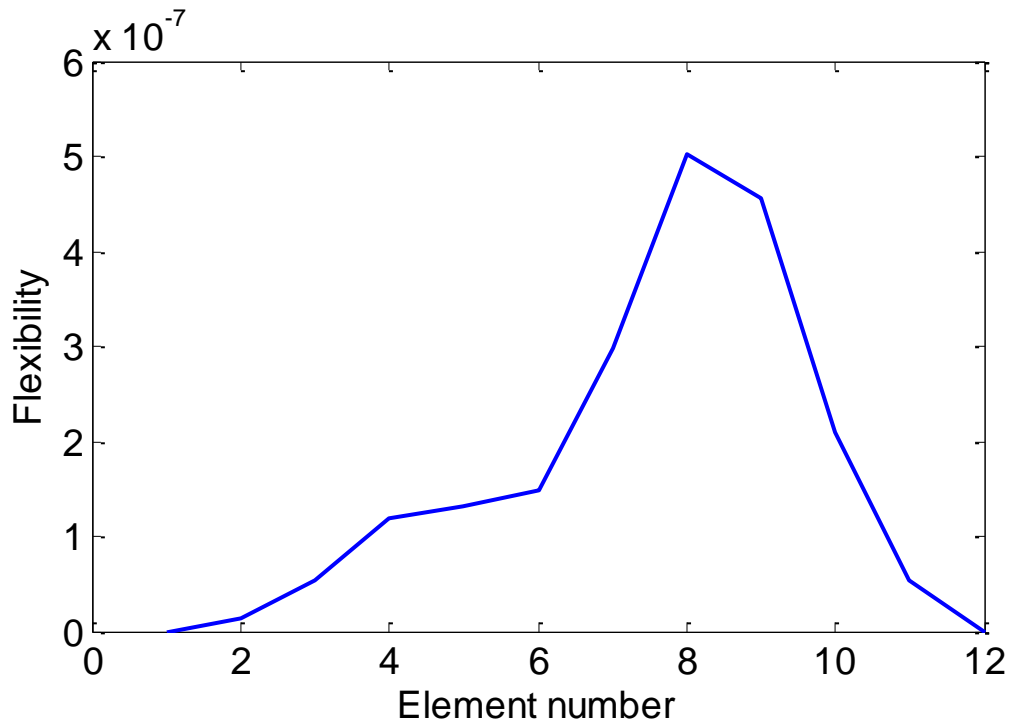


Figure.3.1-43 Flexibility for damage case 18.

From the flexibility diagram in Fig. 3.1-43 the damage located in element 8 can be clearly seen by the flexibility peak over the element number. The damage at element 4 has also been obtained. However, the change in the diagram at element 5 gives an indication that there is damage present at element 5, which is incorrect.

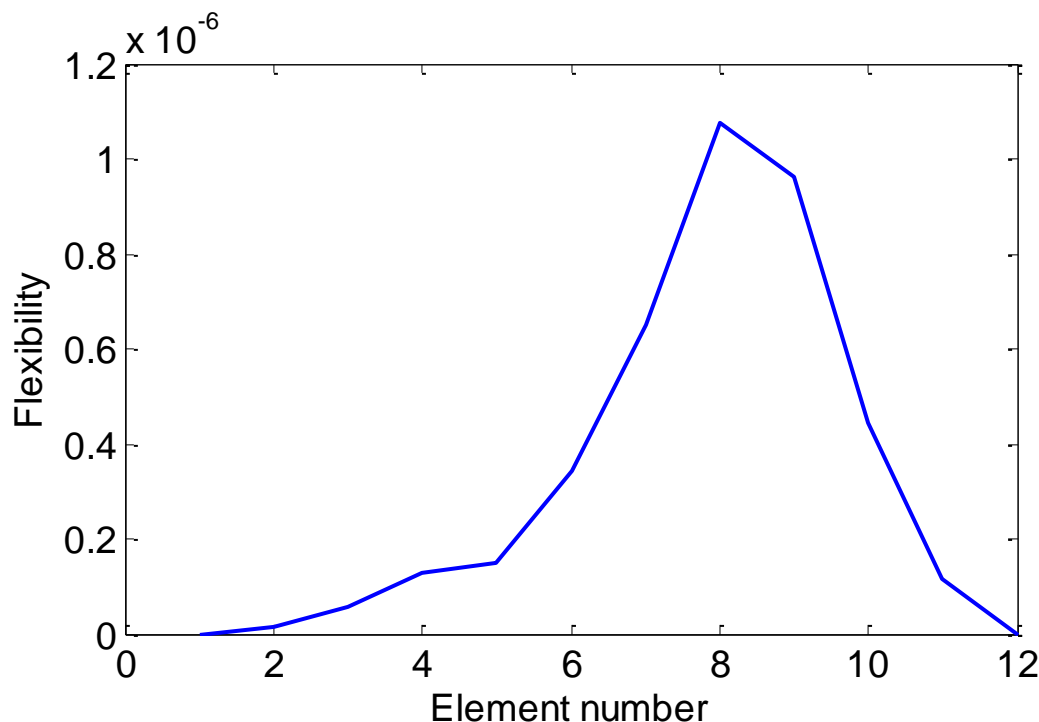


Figure.3.1-44 Flexibility for damage case 19.

In the flexibility diagram in Fig. 3.1-44 the damage located in element 8 can be clearly seen by the right-hand peak. The damage in element 4 has also been located. However there is also an indication that there is damage in element 5 and this is incorrect.

Case study C:

Table 3-12 Overview of case study C

Damage locations at elements: 3,9	
Damage case	E- modulus reduction [%]
20	20,20
21	20,40
22	20,60
23	20,80

For case study C, there are five healthy elements separating the two damaged elements. Studies are conducted to investigate if the flexibility method is able to locate the two damages when the damage severity is the same at both elements and when the damage severity is different.

The flexibility plots for the different damage cases are presented in Figs. 3.1-45 to 3.1-48 and the results are discussed. From the studies of the damage cases 20 to 23, it can be concluded

that the accuracy of the flexibility method is not sufficient when it comes to detecting both damages located at two elements having five healthy elements separating them. The damage at element 9 is successfully determined for all the damage cases, but this is because the severity is larger here for all damage cases instead of damage case 20, where the severity is equal for both elements.

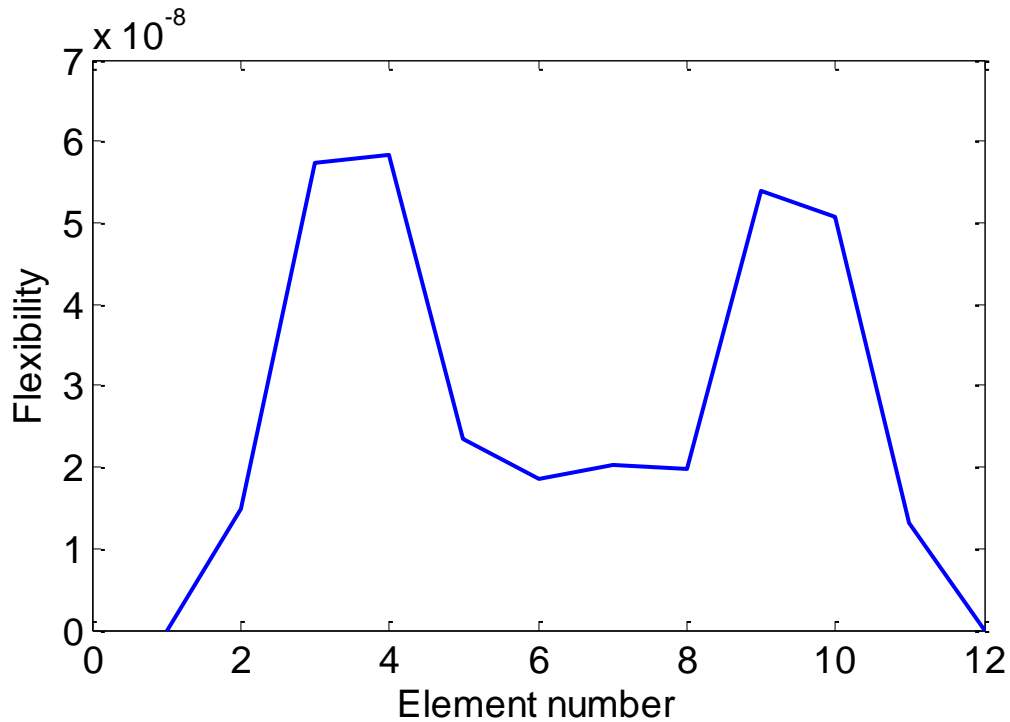


Figure.3.1-45 Flexibility for damage case 20.

From the flexibility diagram it can be seen that the damage located in element 9 has been successfully located. This can be seen by the right-hand peak. The damage in element 3 has also been detected but the left-hand peak is at element 4 and this might give a false damage location.

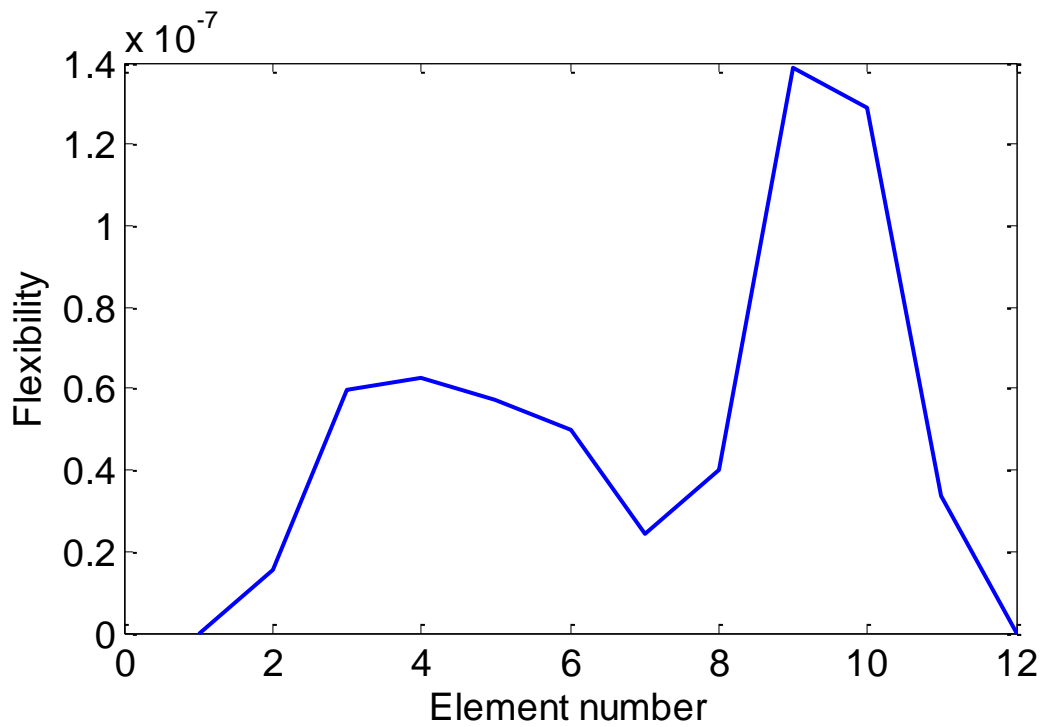


Figure.3.1-46 Flexibility for damage case 21.

From the flexibility diagram the damage located in element 9 can be clearly seen by the right-hand peak. The damage in element 4 has also been detected but the left-hand peak is at element 5 and this is inaccurate.

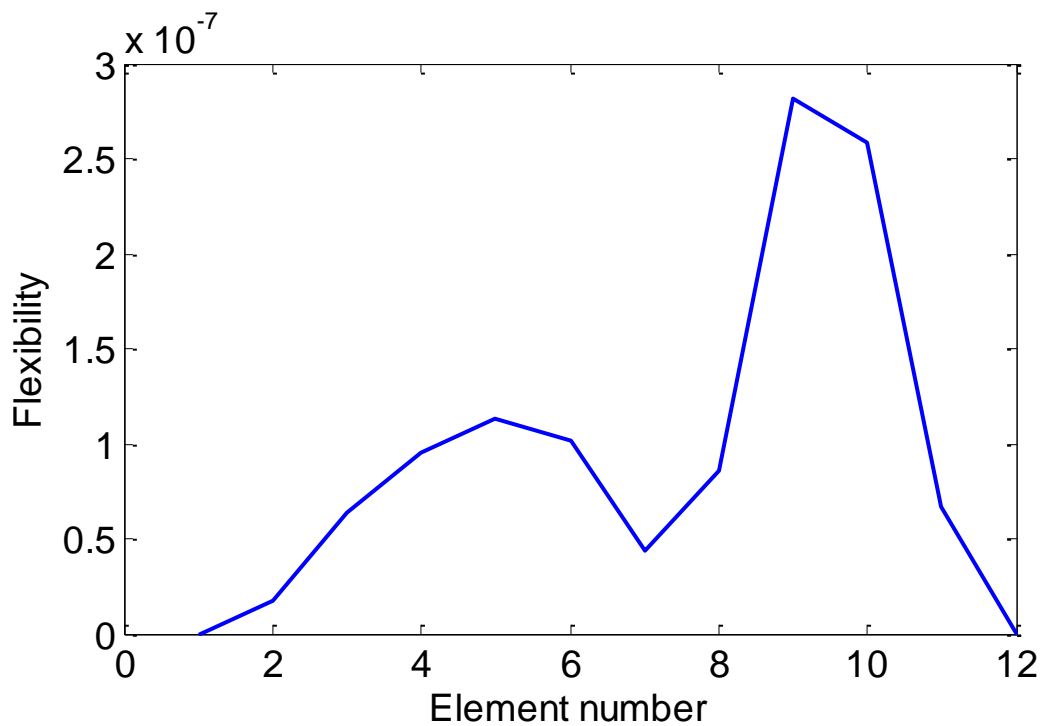


Figure.3.1-47 Flexibility for damage case 22.

From the flexibility diagram it can be seen that the damage located in element 9 has been successfully determined. This can be seen by the right-hand peak. The damage in element 3 has also been detected but the left-hand peak is at element 5 and this is inaccurate.

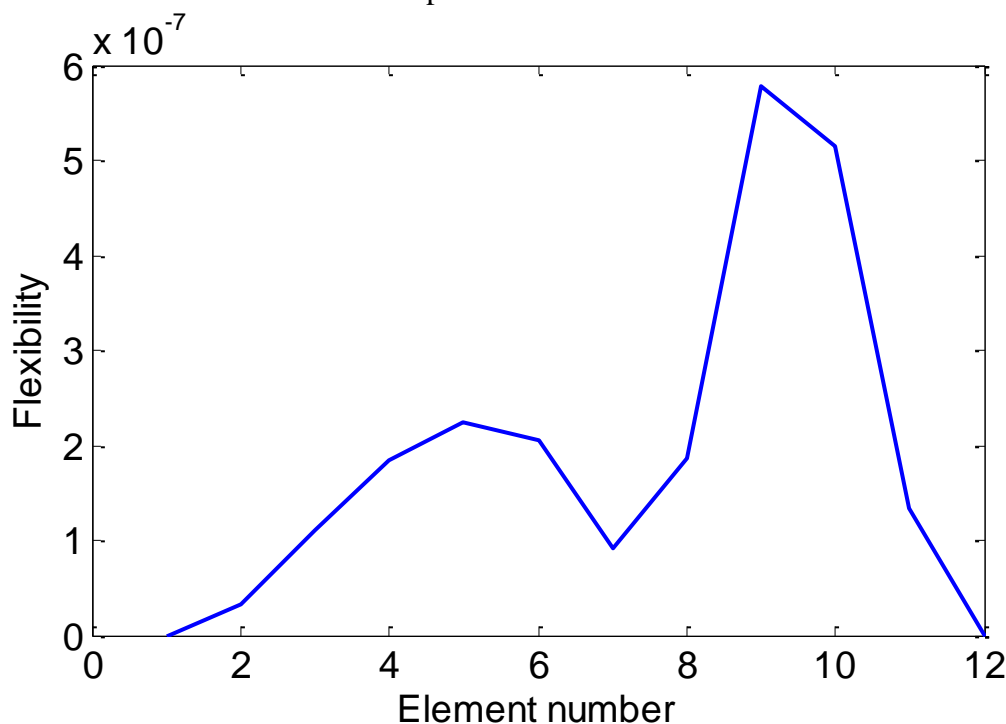


Figure.3.1-48 Flexibility for damage case 23.

From the flexibility diagram the damage located in element 9 can be clearly seen by the right-hand peak. The damage in element 3 has also been detected but the left-hand peak is at element 5 and this is inaccurate.

Case study D:

Table 3-13 Overview of case study D

Damage locations at elements: 2,10	
Damage case	E- modulus reduction [%]
24	20,20
25	20,40
26	20,60
27	20,80

In case study D, there are seven healthy elements separating the two damaged elements. Studies are conducted to investigate if the flexibility method is able to locate the damages at elements 2 and 10 when there is a same amount and a different amount of damage present at the two elements. In damage cases 25 to 27 there is an indication of damage in element 11 instead of element 10. The indication of damage at elements 2 and 10 is only detectable for damage case 1. The maximum flexibility peak is located at element 6 for damage cases 24 and 25, which can be seen in Figs 3.1-49 and 3.1-50, and a maximum flexibility peak is located at element 7 for damage cases 26 and 27, which can be seen in Figs. 3.1-51 and 3.1-52. As there is no damage simulated at neither element 6, nor element 7, it can be concluded that the

flexibility method was unable to locate both damages for the four damage cases simulated in this case study D.

The flexibility plots for the different damage cases are presented in Figs. 3.1-49 to 3.1-52.

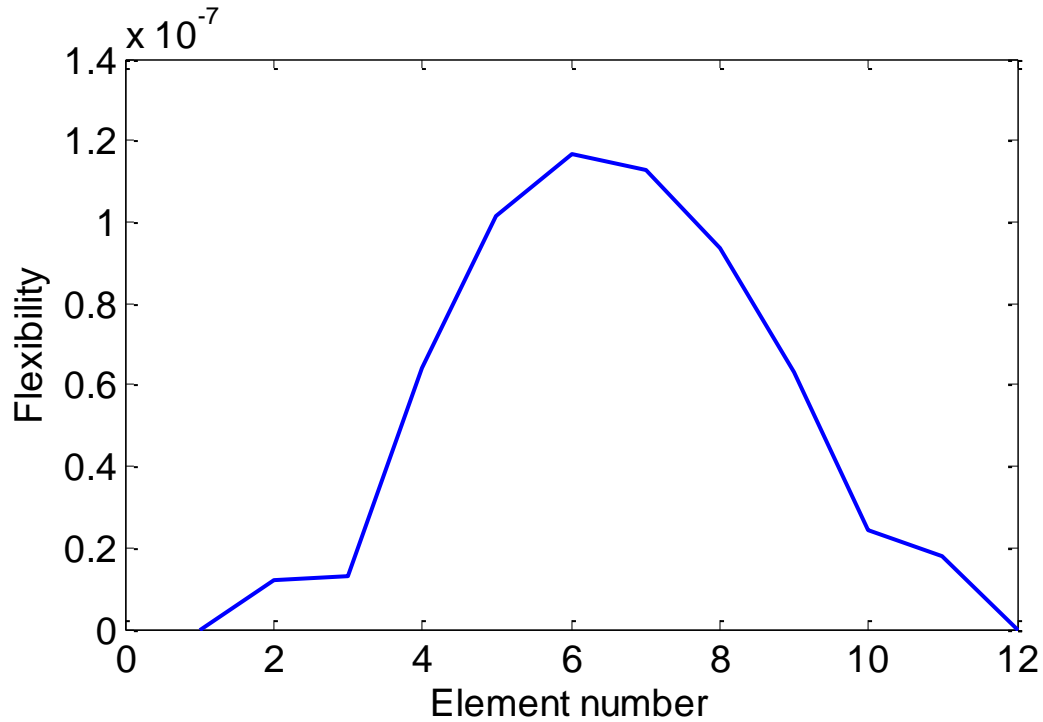


Figure.3.1-49 Flexibility for damage case 24.

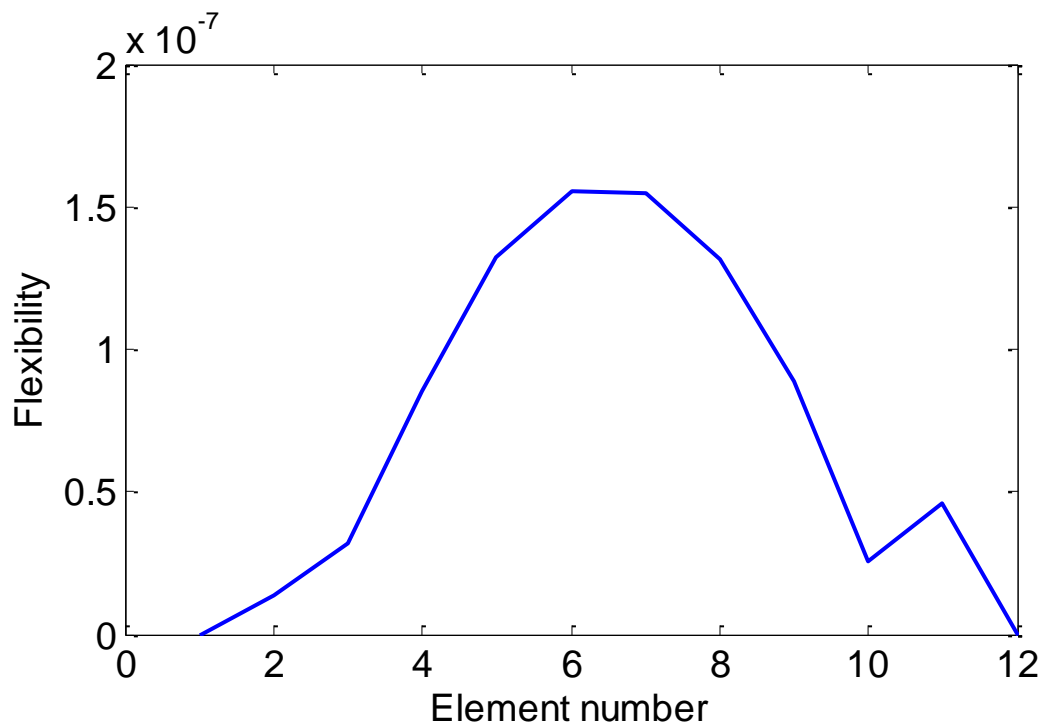


Fig.3.1-50 Flexibility for damage case 25.

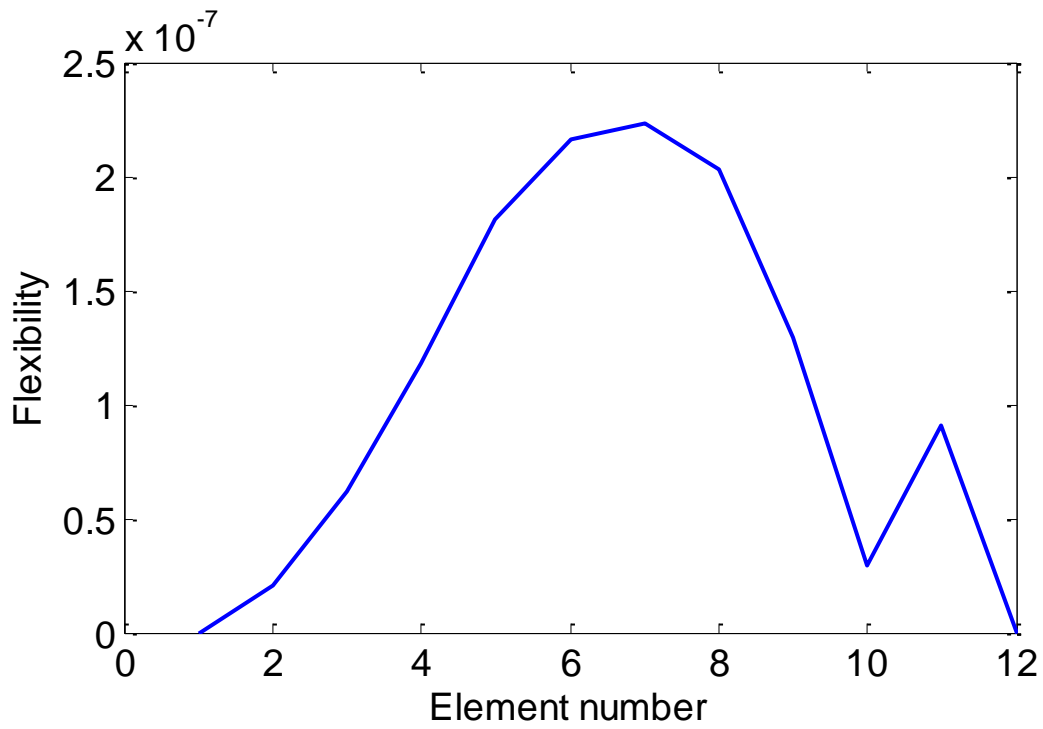


Figure.3.1-51 Flexibility for damage case 26.

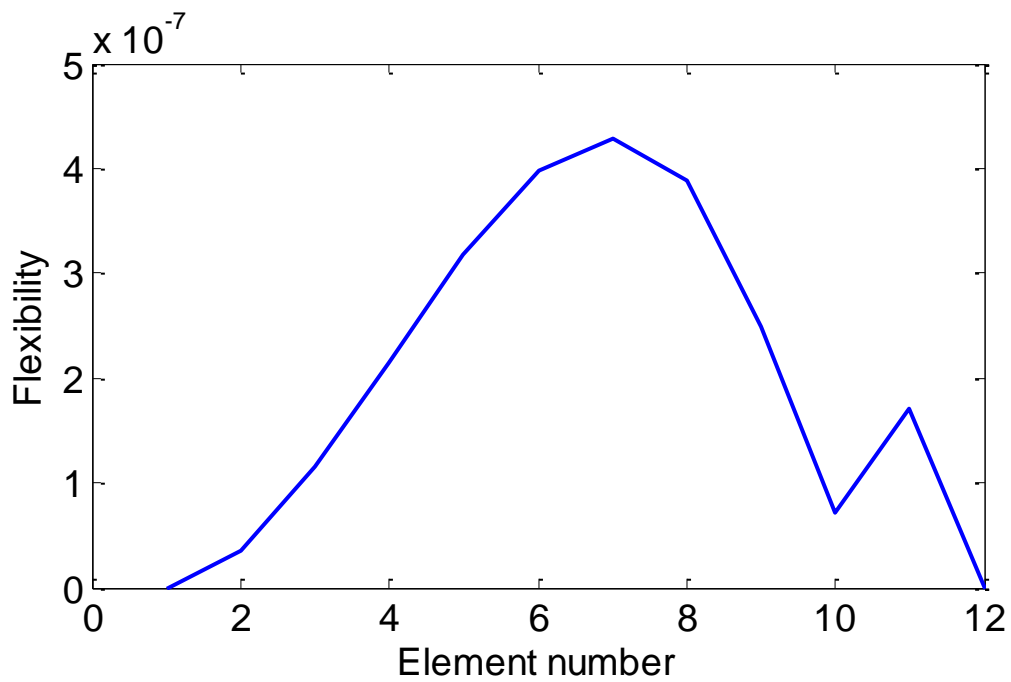


Figure.3.1-52 Flexibility for damage case 27.

4 Conclusions and recommendations

Conclusions

- Non-parametric methods such as frequency response of structure are useful tools for identifying the presence of structural damage. This is however very dependent on the complexity of the structure. Environmental conditions are also aspects that can alter the modal properties of a structure and should be considered when FRFs are used.
- The flexibility method is very sensitive to local damages. From the studies of the method performed in this thesis it is clear that single-damages simulated by reduction of E-modulus at one element in the FE-model can be located by the method.
- The flexibility method is also accurate in localizing single-damages when there is a presence of 1 to 10 percent noise contamination in measured datasets. However, the flexibility values obtained from noise contaminated damage cases will deviate from the flexibility values obtained from the noise-free damage cases.
- The flexibility method's accuracy is not sufficient when there are multiple damages located in the beam structure. The accuracy is highly dependent on the vicinity of the damages. The combination of vicinity and the difference in severity between damages will also influence the accuracy. Some modifications to the parameters in Eq. (19) should be investigated in order to make the flexibility method more applicable to multiple damage scenarios.

Recommendations

- 1) Study other vibration-based damage detection methods; implement them on the same beam structure with the same boundary conditions. Use the same excitation frequency-ranges used in this thesis and compare the results and outcome with the study done in this thesis.
- 2) For analysis of more complex structures it is recommended that parametric-damage detection methods are implemented for the establishment of an SHM system.
- 3) The flexibility method is highly sensitive to noise contamination in the measured dataset. When the flexibility method is used as a damage location tool in an SHM system, there should be some modification to the damage detection algorithm in order to filter out the noise signals to prevent errors in the flexibility values obtained from analysis.
- 4) The flexibility method is highly sensitive to the vicinity of different damages. If the vicinity is very small, the method might localize only one damage, instead of several. If a damage detection algorithm based on the flexibility method is to be implemented as a tool in the SHM system, it is highly recommended that the algorithms sensitivity to vicinity of different damages is investigated.
- 5) It is also probable that structures might sustain more than one damage inside a small timeframe. This can be experienced in structures after they have been subjected to different blast loads or to natural phenomena like storms or tsunamis. Therefore there should be some considerations into this when the flexibility method is used as a damage detection tool in an SHM system.
- 6) The author suggests that the flexibility method can be made more applicable to multiple damages by investigating if there can be made modifications to the mass-normalized mode shape vectors.

- 7) The FE-model of the beam structure was modeled by using eleven 2D-beam elements. It is a well known concept in FEM theory that the number of elements used in a FE-model will influence the results. Therefore it is noted that the results found in this thesis is a product of the FE-model used in the analysis. Investigating how the results will be by using different number of beam elements, other than the eleven elements used in this study is an interesting aspect that should be looked into.

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