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ABSTRACT

As the operational window is getting narrower, pressure control is becoming more important. Drilling from a semi-submersible platform can in this context offer new challenges as the top of the drill string will follow the semi-submersible platform's heave response to ocean waves during a connection. The heave movement can travel down the drill string and create pressure fluctuations around the drill bit.

The movement of the drill string has been numerical simulated with a numeric program created in Matlab. The movement of the semi-submersible platform was simulated using a combination of two sinus functions.

The results from the simulations show that the drill bit velocity and amplitude is generally increasing with heave amplitude, and generally decreasing with increased heave period and deviation angle. For the drill strings in vertical wells, the amplitudes are increasing with drill string length. However, as the deviation angle is increasing, more of the energy in the drill strings is lost due to contact friction, leading to a non-linear behavior and less distinctive oscillation patterns. The drill bit amplitude in deviation wells is then small up to a certain heave amplitude, where the drill bit amplitude increases rapidly.

The simulations show that some of the drill strings start to resonate at a certain heave period, but the resonance is terminated by contact friction if the deviation angle is larger than approximately 7 degrees. Simulations also showed that the surge and swab pressures during normal weather conditions and drill string oscillations are up to approximately 5 bar. The pressure calculations were done with a relatively large flow area between the BHA and wellbore wall, but preliminary calculations have shown that the pressure fluctuations can be drastically increased if the flow area between the BHA and the wellbore wall is decreased. However, the calculations performed during the simulations and pressure estimations are probably conservative.

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1. INTRODUCTION

Drilling on the Norwegian Continental Shelf is today associated with high cost and increasing complexity. The easiest accessible petroleum resources have up to now been produced in such a way that drilling into produced reservoirs to reach remaining pockets of oil is getting difficult, sometimes nearly impossible. In addition, the search for new recourses is forcing the industry into new areas with deeper waters and higher pressures and temperatures. This offers new challenges.

The pressure window between pore pressure and fracture pressure has traditionally been quite large. This difference is called the operational window, and is observed to shrink, for instance in high pressure wells. A narrower operational window leaves less room for pressure fluctuations during drilling operations.

When drilling from a floating platform the heave compensator is reducing the vertical drill string movements induced by waves. However, when doing a connection the drill string is disconnected from the heave compensator and wedged to the drill floor, the top of the string will then follow the rig movements. If these movements are transmitted down to the drill bit it may cause rapid changes in the bottom hole pressure.

The purpose of this thesis is primarily to calculate if the heave movements will travel down the string and reach the drill bit, and to what extent this movement will induce pressure fluctuations below the drill bit.

2. THEORY

The theory presented in chapters 2.1.2 and 2.1.3 are based on the presentations of professor Jonas Odland in the subject *Offshore Field Development* at the University of Stavanger, fall 2011. In addition some theory is collected from e-mail correspondence with the same professor.

The theory, equations and most of the figures in chapter 2.7 are based on the compendium *Dynamic Loading of Equipment* written in 1996 by Professor Erik Skaugen at the University of Stavanger.

2.1 Drilling in the North Sea

The average water depth in the North Sea is 94 meters, but varies between 25 meters in the south to 725 meters in the Norwegian Trench [1]. The North Sea has some of the harshest weather conditions in the world. The wind speed in the North Sea can exceed 50 m/s and the significant wave height (the mean of the highest third of the waves in a time-series) is 15 meters. The peak period (the period with highest wave energy) is between 15 and 17,5 seconds [2].

Drilling to find oil and gas in these conditions requires special designed equipment. Offshore wells are much more expensive to drill compared to land based wells because of high rig rates and limited space and weight issues. When drilling in water depths greater than around 120 meters it requires normally that operations are carried out from a floating vessel, as fixed structures are not practical.

The floating vessel can be a drill ship or a semi-submersible platform. For exploration wells drill ships can be used, but semi-submersible platforms are more commonly used. Drilling from semi-submersible platforms will be the basis for this thesis.

2.1.1 Semi-submersible platforms

Semi-submersible platforms (semi-subs) are the most common type of offshore drilling rigs. The floating structures obtain buoyancy from ballasted, watertight pontoons located below the ocean surface. The topside of a semi-sub are located high above the sea level, supported by structural columns connecting the pontoons to the top side. Semi-subs are quite easy to install on location since they can be ballasted up and down by altering the amount of sea water flooding in buoyancy tanks, usually located in the pontoons, for stability.

Figure 2.1: A semisubmersible platform with eight columns and two submerged pontoons. The drilling derrick is placed approximately in the middle of the platform [3].

The submerged location of the pontoons provides counteracting forces to the vertical motions on the vessel created by the waves as illustrated in Figure 2.3. A semi-sub can be designed for a fixed location or as a mobile drilling unit. A mobile semi-sub will in principle have only two pontoons for decreased drag during towing, while a semi-sub designed for a fixed location have four pontoons providing increased stability. Low relative contact area at the water line and wave action gives the design high operational stability in rough seas.

Semi-submersible platforms are capable of drilling in water depths up to 3000 meters. The water depth and weather conditions decide what kind of mooring system will be used. The rig movement characteristics will vary with the mooring system, water-line area and submerged volumes. The water-line area and submerged volumes are especially important factors for the movement characteristics in relation to drilling. The number of columns and pontoon are of less importance for the global behavior.

2.1.2 Movement of a semi-submersible platform

Semi-submersible platforms are generally designed to minimize the response to the environment, where each type has its own movement characteristics. A semi-subs movement characteristic can be decided from hydrodynamic calculations, which can be verified to some extent by model experiments.

The movement characteristics of a semi-sub can be considered to have six degrees of freedom, see Figure 2.2. This means that it can move in six different ways which can be divided into two main types of motion; translation and rotation.

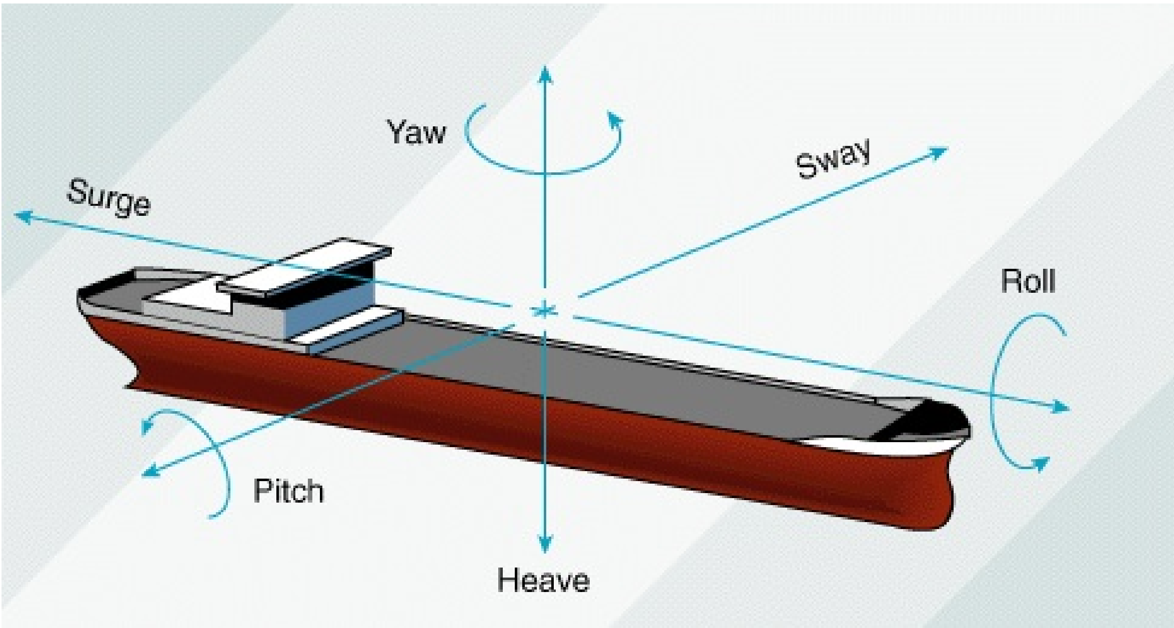

Figure 2.2: The different movements of a floating vessel [4].

Translation is movement that changes the position of the semi-sub. This is a linear type of displacement where the displacement values are common for each point of the vessel. The linear displacement motions can be divided into three different classes:

- Surge (forward/astern motion)
- Sway (starboard/port motion)
- Heave (up/down motion)

Rotation is movement of the vessel around an axis, which means the displacement values of each point of the vessel varies. Rotation is an angular displacement, and can be divided into three different classes:

- Roll (rotation about surge axis)
- Pitch (rotation about sway axis)
- Yaw (rotation about heave axis)

2.1.2.1 Vertical movement

Most semi-submersible drilling units are designed in such a way that the drilling derrick is placed in the center of the platform. Pitch and roll motions are both rotation in the vertical plane, which means that they will have minimum impact on drilling as long as the derrick is placed in the center of the platform. The motion of most importance in relation to drilling is the heave motion. Rotation about the heave axis will not affect the drill string at all.

The natural heave period is the period where the vessel will resonate with the waves. For a semi-sub the natural heave period is decided by the relation between water-line area and total volume submerged. To ensure as little resonance as possible, the natural heave period for a semi-sub in the North Sea is normally designed to be a bit longer than 20 seconds.

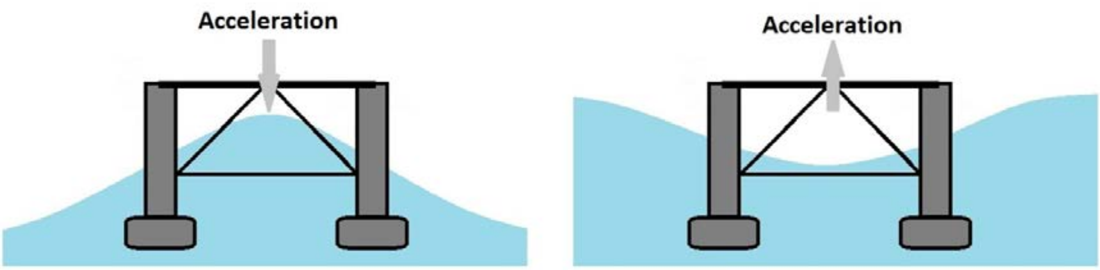

Figure 2.3: A twin pontoon semi-sub on a crest centered wave (left) and a trough centered wave (right). The platform is designed to counteract the passing waves, and is moving opposite to the wave movements.

The heave cancellation period is the period where the hydrodynamic forces on the pontoons are practically equal to the forces on the columns, and the heave response therefore tends to zero. The heave cancellation period for a semi-sub is dependent on water-line area and pontoon volume. For a semi-sub in the North Sea the heave cancellation period is designed to be somewhat shorter than 20 seconds.

2.1.2.2 Horizontal movement

The movements in the horizontal plane (surge, sway, yaw) consists of three components; fast-varying, slow-varying and quasi-static movements.

The fast-varying movement follows the waves in the same manner as heave, pitch and roll. This kind of movement is hard to mitigate.

The slow-varying movement is decided by the natural period which is dependent of the stiffness of the mooring system. This is where the water depth comes in. In shallow water the system is stiff and the natural period is short. In deep water the system is softer and the natural period is longer (maybe a couple of minutes).

The quasi-static movement is a shift of the equilibrium position due to wind and currents.

2.1.3 The semisubmersible platform mooring system

The type of mooring system has no or little effect on the vertical movement of a semi-sub (heave, pitch and roll). This means that the water depth is of less importance to the vertical movement. On the other hand, the type of mooring system and water depth are of larger importance when it comes to the horizontal movement. A semi-sub will experience a horizontal displacement changing the angle of the drill string to the seabed. This will however not be a problem due to the flexible qualities of the drill string. Deeper water gives a larger horizontal displacement, but is balanced by a longer span giving a gentler angle to the sea bottom.

The catenary mooring system is the most common type of mooring system used for a semi submersible platform in shallow water. Catenary refers to the shape of a free hanging line under the influence of gravity. The system provides restoring forces through the suspended weight of the mooring lines where its change in configuration is arising from the vessel motion. This means that external loading on the platform applied from the surrounding environment makes the vessel trying to lift the mooring lines. The catenary system requires that the mooring lines are terminated at the seabed horizontally, thus only applying horizontal loads on the anchor points. The lines must therefore be relatively long compared to the water depth. Traditional anchors which are designed for horizontal loads are used for the catenary systems.

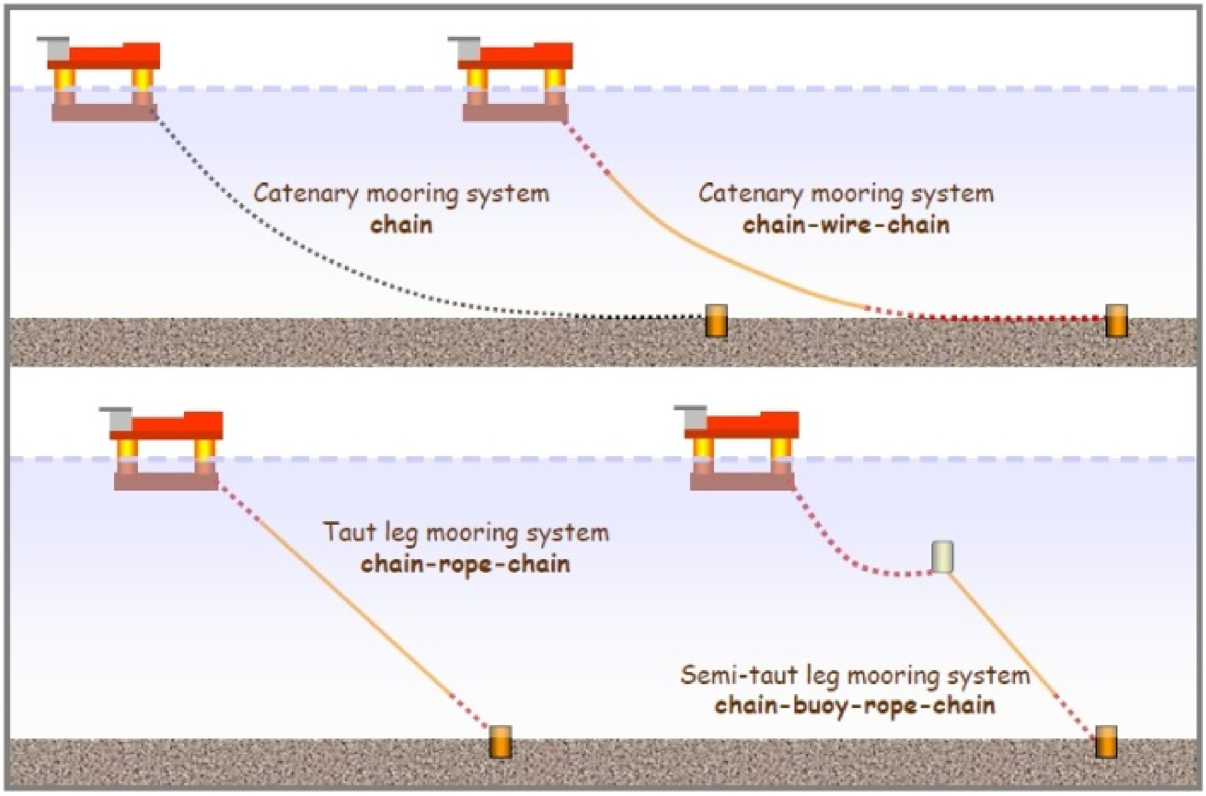

Figure 2.4: The different mooring systems used on semi-subs [3].

With the increase of the water depth the weight and the length of the mooring lines starts to increase rapidly. In deepwater the weight of the mooring lines becomes excessive and the mooring lines tend to hang directly down from the rig. The excessive weight diminishes the working payload of the platform. To overcome this problem, synthetic ropes are used in the taut leg mooring system.

The taut leg system is a much more cost effective system in deepwater. The system relies on the axial elastic stretching of the mooring lines rather than geometry changes. The lines terminate at an angle between 30 and 45 degrees at the seabed, which means that the anchor point are loaded by horizontal and vertical forces. A traditional anchor is therefore not suitable for taut leg systems, and suction anchors have to be applied instead. The restoring forces are determined by the stiffness and elasticity of the mooring lines.

A semi-taut system is a combination of the taut mooring system and the catenary system, and is better suited for deepwater application than catenary system.

2.1.4 Drilling from a semisubmersible platform

When drilling in water depths greater than around 120 meters a semi-sub must be employed, as fixed structures are not practical. Semi-subs can also be used for pre-drilling. Pre-drilling is a way of reducing the time before production of a new discovery can begin. In conventional offshore development, wells cannot be drilled until a platform has been constructed and installed. This means a delay of several years before production begins. Such delays can be considerably reduced by pre-drilling some of the wells using a semi-sub. Pre-drilling involves drilling and casing the wells to a convenient depth, normally through the shallow water flow zone or other potential hazards. Pre-drilling may also be suspended just above the production zone, while some wells may be drilled to total depth and completed [3].

When drilling from a semi-sub, the rig moves according to the movement characteristics of the rigs response to waves. To be able to keep a more constant weight on bit (WOB), the use of a heave compensator in the top drive is necessary. A heave compensator will reduce the vertical movement of the drill string, maintaining a more constant WOB. A constant WOB reduces torque variations at the bit, and will also improve the drilling rate of penetrations (ROP).

2.1.4.1 Making a connection

To make a connection is adding a length of drill pipe or a stand to the drill string in order to continue drilling. When the drill bit has drilled down to where the top drive is close to the drill floor, the drill string between the two must be lengthened by adding a stand (usually three joints) to the drill string. Once the rig crew is ready, the driller stops the rotary, picks up off bottom to expose a threaded connection and turns off the pumps. The crew sets the slips to grip the drill string temporarily and unscrews the top drive. The top of the drill string is now moving according to the rig movements. This can be critical, as the rig movements may travel down to the drill bit, possibly creating pressure fluctuations at the bottom of the well. The top drive is then screwed into the additional stand of pipe, the pipe is picked up and screwed into the top of the temporarily hanging drill string. The driller then picks up the entire drill string to remove the slips, and carefully lowers the drill string while starting the pumps and rotary system. The drilling resumes when the bit touches bottom. A skilled rig crew can make a connection in a minute or two [5].

2.1.4.2 Fluctuations of the bottom hole pressure during a connection

The bottom hole pressure in a well without circulation is decided by the fluid column above (no backpressure is applied). A surge effect can be created at the bottom of the well if the drill string moves down rapidly. This rapid movement can cause increased bottom hole pressure (BHP) due to the friction between the moving drill string and the stationary drilling fluid. The increase in the BHP due to a surge effect is referred to as surge pressure. The outcome of a increased BHP can be a fracture in the formation and loss of drilling fluid. In the worst case, a loss of drilling fluid can induce a kick from a formation further up the well.

The opposite of a surge effect is a swab effect. A swab effect can be created when the drill bit moves up rapidly, decreasing the BHP. The decrease in BHP due to a swab effect is referred to as swab pressure. The outcome of a decreased BHP can be a kick induced at the bottom of the well.

Traditionally the surge and swab effects have not been a problem during connections because of the large pressure margin between the pore pressure and the fracture pressure. However, as reservoirs are being depleted, they are getting more and more difficult to drill due to smaller pressure margins. The tendency observed is also that drilling in deeper water means a smaller margin between the fraction pressure and pore pressure (see section 2.5).

The surge/swab pressure below the drill bit can be calculated as the as the drill string moves up and down. The pressure fluctuations arise when fluid are forced to flow through an area with less flow area. This happens mainly at the drill bit and outside the lower part of the drill string which is called the bottom hole assembly (BHA) (see section 2.2.2).

The pressure drop over the drill bit can be calculated by using the nozzle equation from Drilling data handbook [6]:

$$\Delta P\ [Pa] = \frac{1}{2}\ \frac{v^2\ \rho_m}{{C_d}^2} \tag{2.1.1}$$

where v is the velocity, ρ_m is the density of mud, and ${C_d}^2$ is an efficiency factor for conservation of energy.

C_d is set to 0,95 and the velocity can be found using the equation below:

$$v = \ v_{bit}\ \frac{A_{BHA}}{A_{junk}} \tag{2.1.2}$$

where v_{bit} is the velocity of the drill bit, A_{BHA} is the cross section area of the bottom hole assembly, and A_{junk} is the total cross section area of junk areas in the drill bit. This assumes that the area A_{junk} is smaller than the annulus cross section around the BHA. If not, the latter should be used, replacing A_{junk} in equation (2.1.2).

The pressure drop due to friction along the outside of the BHA can be calculated using the equation below [7]:

$$\Delta P_F [Bar] = \frac{{\rho_R}^{0,8} \mu^{0,2} Q^{1,8}}{70696\ \ (D+d)^{1,8}\ (D-d)^3} L \tag{2.1.3}$$

where ρ_R is the relative density of mud, μ is the viscosity of mud [cP], Q is the flow rate [liter/minute], D is the wellbore diameter [inches], d is the diameter of the BHA [inches], and L is the length of the BHA [m].

As you can see not all of the units used in equation (2.1.3) are a part of the SI-system. The factor 70696 is adjusted in such a way that the result of the calculation comes out in bars.

The compressibility of the mud is not taken into account in the equations above, but the effect of the compressibility can easily be calculated as the mud is in a closed system.

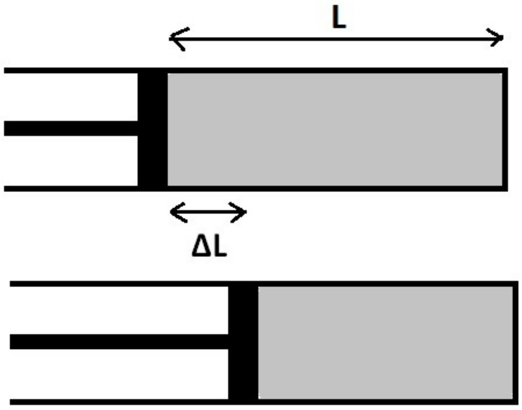

Figure 2.5: A cylinder of length L filled with mud, compressed by a piston travelling a distance ΔL.

Figure 2.5 shows a cylinder filled with mud having a density of ρ_m and the speed of sound in the mud is c. The volume inside the cylinder is proportional to the length of the cylinder, and the compressibility C_v of mud is equal to the inverse of c^2 multiplied with ρ_m. The pressure inside the mud as the piston has traveled the distance ΔL can be found by the equation below:

$$\Delta P = \frac{\Delta V}{V}\frac{1}{c} = \frac{\Delta L}{L}\frac{1}{c} = \frac{\Delta L}{L}c^2\ \rho_m \tag{2.1.4}$$

The situation in Figure 2.5 can be transferred to a well with a drill string inside. The pressure increase due to the movement of the drill string (without the compressibility of mud) is the sum of equation (2.1.1) and (2.1.3). The mud has then been compressed the distance ΔL:

$$\Delta L = \frac{\Delta P\ L}{c^2\ \rho_m} \tag{2.1.5}$$

This compression of the mud would have caused a slightly smaller pressure increase. If the cylinder is 10m long, the speed of sound in the mud is 1000m/s, the mud density is 1200kg/m^3, and the calculated pressure increase due to the string movement is 10 bar, the mud has been compressed the distance ΔL:

$$\Delta L = \frac{10{*}10^5 Pa * 10m}{(1000\,\frac{m}{s})^2 * 1200\,\frac{kg}{m^3}} = 0{,}0083m$$

This means that the mud have been compressed a distance of approximately 0,8 cm which is a small number compared to the expected drill string movement which is in the scale of meters. This shows that for all practical purposes the mud will here behave as an incompressible liquid. Any movement of the bit will generate a pressure that is sufficient to generate a mud flow that is equal to the mud displaced by the movement of the bit, as has been assumed in the equations 2.1.2 and 2.1.3.

2.2 The drill string

The drill string has two primary purposes; act as a passage for the drill fluid making it possible to circulate the well, and transmit torque from the top drive in addition to generating and transmitting weight on bit. The drill string consists of drill pipes, drill collars, drill bit and other equipment.

The drill string is custom built for each well and target. The length and weight will obviously change with the depth, but also the stiffness, diameter, strength of pipes, tools, drill bit, and other equipment will continuously be changed during the drilling of a well. The total length of a drill string can vary, but there are limitations when it comes to the TVD (True Vertical Depth). The heaviest loads are taken at the top of the drill string which must be able to stand the weight of the drill string below. A drill pipe of steel quality s135 are able to withstand a load equal to 135 000 psi. A 5" drill pipe can then bear a drill string having a TVD of more than 12000m. However, the string must also be able to take some extra loads due to friction, stuck pipe etc. A depth of 8000-9000m TVD is therefore looked at as the maximum depth of a wellbore.

2.2.1 Drill pipes

Most of the drill string consists of drill pipes. Drill pipes transmit rotation, vertical movement and drill fluid to the drill bit. There are many types and sizes, where the size denotes the pipe's outside diameter. The most used drill pipes in the North Sea have an outer diameter of 5" or 5 1/2", with a length of about 10 meters [8]. Typical nominal weight of a 5 1/2" drill pipe is 24,70 lb/ft (~37 kg/m), where the weight indicates the wall thickness. The actual weight of a drill pipe assemblies includes the weight of the tool joints [9].

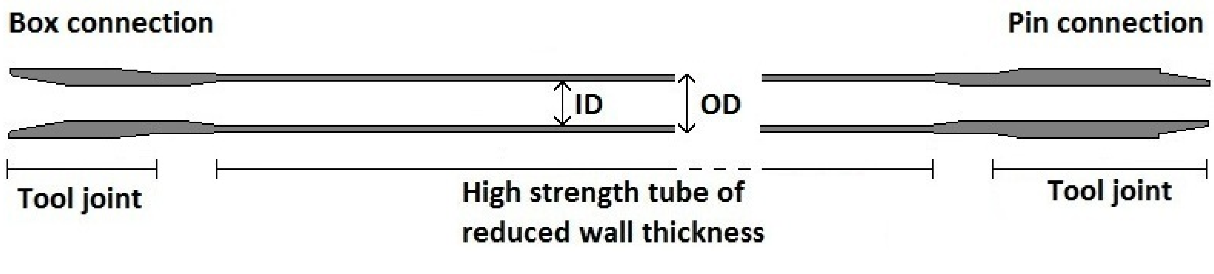

Figure 2.6: Cross section of a drill pipe seen from the side. Drill pipes come in different configurations, but a typical drill pipe cross section may look like this.

Both the inner and outer diameter of a drill pipe varies with the steel diameter variations. The inner diameter is smaller at the tool joints (3-4"), and the outer diameter is larger at the tool joints (7-7,5"). The reason for having thicker steel at the tool joints is the need for a low stress area where pipe tongs are used to grip the pipe. This means that relatively small cuts caused by the pipe tongs do not significantly impair the strength or life of the joint of drill pipe. Since the tool joints are having the largest outer diameter of the drill pipe, this is also where the main wear occurs. To extend the life time of the tool joints, the steel in this part have usually been heat treated to a greater hardness than the steel of the tube body [10]. To increase wear resistance even more some joints may have a surface layer of hard metal.

The tube section has a smaller outer diameter compared to the tool joints (5,5") making it less exposed to wear. However, since the tube section has a smaller steel cross section than the rest of the drill pipe, this is where most of the elongation and compression of the drill string occurs. Inner diameter of the tube section is typically 4,67". For standard drilling operations the drill pipe is always in tension, as any compression gives buckling that increases the risk of pipe failure.

2.2.2 Bottom hole assembly

The bottom hole assembly (BHA) is the lower part of the drill string that hangs below the drill pipes. Typical length of the BHA is 100-200 meters [7]. A simple example of a BHA consists of a drill bit, drill collars, heavy-weight drill pipes, and stabilizers. In addition, components as downhole motor, rotary steering system (RSS), measurement while drilling (MWD) and logging while drilling (LWD) can be installed. In general, the complexity of the drill string is increasing when deeper sections are drilled [8]. The BHA is sufficiently heavy to give the required WOB with a good safety margin, thus ensuring that the drill pipe section will not experience compression.

2.2.2.1 Drill collars

The drill collars are heavy thick-walled pipes made to withstand compression without significant bending, enabling the required WOB. The drill bit must be pressed towards the formation with a force depending on type of bit and type of formation. The drill collars are simpler compared to the drill pipes since there is no changes in inner or outer diameter, forming slick surfaces. This means that wear and friction are constant along the outside of the drill collars. Some drill collars can however have a spiral shaped surface to help the transportation of drill cuttings upwards in addition to reduce the differential pressure sticking.

The outer diameter of drill collars used are usually larger than the outer diameter of the drill pipes further up the string, but the inner diameter of the drill collars are nevertheless often smaller. Typical inner diameter of a 6,5" drill collar is 2,25" and the weight of the same pipe can be 100 lb/ft (~149 kg/m) [11].

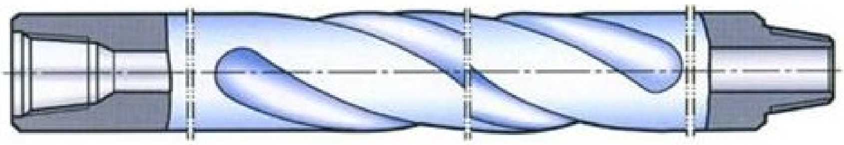

Figure 2.7: Spiral drill collar with constant inner and outer diameter [12].

Heavy weight drill pipes (HWDP) are often installed between the heavy rigid drill collars and the more flexible drill pipes to make a gradual transition between the two. This can often be an advantage if the drill string is exposed to vibrations because it reduces the stress concentration significantly.

2.2.2.2 Downhole motor

A downhole motor is most often a mud motor that can be placed in the BHA to provide additional power to the bit while drilling. The mud motor can draw energy from the flowing fluid, transferring it to torque power to the drill bit.

2.2.2.3 Stabilizers

Stabilizers are required in the BHA to keep the drill collars centralized in the hole. There are different types of stabilizers, but they all are used to maintain the BHA in the well centered in order to prevent unwanted deflection and vibrations. The dimensions of the stabilizers are usually the same as for the drill collars, the difference is the welded blades on the stabilizers, where the diameter is equal to the well diameter or slightly less.

Figure 2.8: Welded blade stabilizer [13].

2.2.2.4 Rotary Steering Systems (RSS)

Rotary steering systems are systems placed in the BHA enabling steering of the drill bit in the desired direction by the use of commands transmitted from the surface. The unit is placed directly above the bit and consists of a steering unit, a power unit and stabilizers among other things. The total length of a RSS can be around 10 meters [14].

2.2.2.5 Measurements While Drilling (MWD)

MWD tools are placed in the BHA near the bit to measure the exact direction of the tool, the vibration level, and the pressure and temperature in the well among other things. The measurements are recorded by a computer in the tool, and can be compressed and transmitted to the surface continuously while the hole is being drilled. The MWD tools are either installed inside a thick-walled drill collar or they are built directly into the collars at a factory prior to arriving on the drilling location [15]. A mud pulse tool will often be installed to transmit the signals from the MWD tools to surface.

2.2.2.6 Logging While Drilling (LWD)

LWD tools are logging tools that can be placed in the BHA, working with the MWD system to transmit the measurements to the surface. The logging tools can give information about the formation, borehole and formation fluid. The logging tools typically have a total length of 8-15 meters.

2.2.2.7 Jar

A jar can be placed in the BHA to better free stuck downhole equipment. The jar uses a principle where kinetic energy are stored and suddenly released allowing the jar to strike up or down. An accelerator is often used with the jar to increase the efficiency of the jar. The jar can be either mechanical or hydraulic, having a latched length of 20-25 meters [16].

2.2.2.8 Float valve

Inside the drill string there might be installed a float valve, allowing fluid to be pumped into the well, but preventing fluid flowing back into the drill string.

2.2.2.9 Drill bit

There are mainly two types of drill bits that are used to a large extent in the North Sea: tricone bit and polycrystalline diamond compact (PDC) bit. One can roughly say that tricone bit are used in the upper well sections, while PDC bit are used in the lower sections. The different drill bits operate in different ways, but all drill bits produce drill cuttings which must be transported to the surface. Junk slot areas in the bit allow the drill cuttings to pass, along with the drill mud. The junk slot area must be large enough to let the drill mud and cuttings escape without creating a large pressure build up below the drill bit, and at the same time be small enough to ensure effective cleaning of the drill bit. The junk slot area is different from bit to bit, but a 9 1/2" PDC bit can i.e. have a total junk slot area of 12,554 in^2, which corresponds to about 18% of the total area.

2.3 Well trajectory

An offshore drilling rig will have a limited number of well slots. The well slots are close to each other, which mean that there will always be a danger of one well intersecting with another. To avoid a conflict, every well trajectory is planned in detail with respect to where the kick-off point is located. The wells in the middle will have the deepest kick-off points, while the wells located further out will have shallower kick-off points.

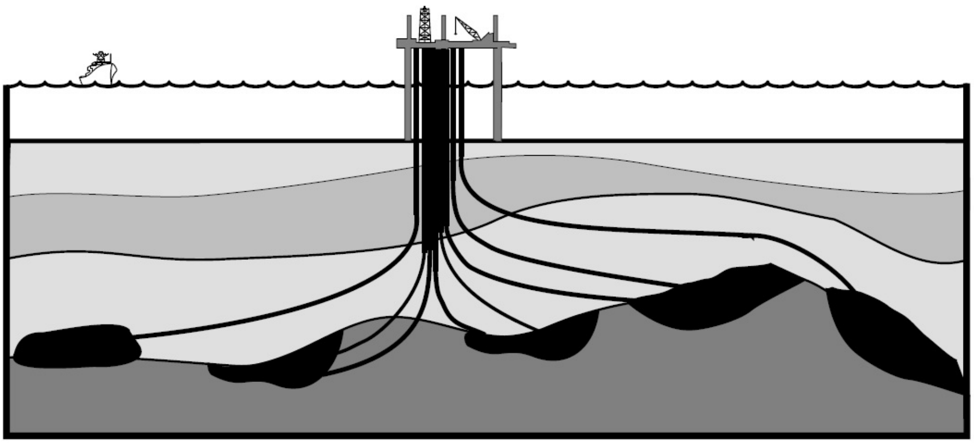

Figure 2.9: Multiple wells drilled from one offshore location having different kickoff points [17].

The trajectory of an offshore well will depend on a number of factors; hole pattern, casing program, mud program, required horizontal displacement and maximum tolerable inclination. Build-up rates are usually in the range 1,5°/100 ft MD to 4,0°/100 ft MD for normal directional wells [17]. It is however important to avoid unnecessary high dogleg severities which may lead to problems during completion and production.

The use of deviated and horizontal wells has made it possible to reach reservoirs several kilometers away from the drilling location. The introduction of steerable systems has resulted in wells that are planned and drilled with complex paths involving 3-dimensional turns. This is particularly true in the case of slot recoveries, where old wells are sidetracked and drilled to new targets. These complex well paths are harder to drill, and therefore most directional wells are still planned using traditional patterns. There are several basic types of wells, and some of the most common are listed below [17].

2.3.1 Build and hold

This type of well has a shallow kick-off point, followed by a build-up section and a tangent section. The build and hold are used to reach deep targets with a large horizontal displacement, and moderately deep targets with moderate horizontal displacement where intermediate casing in the well is not required [17].

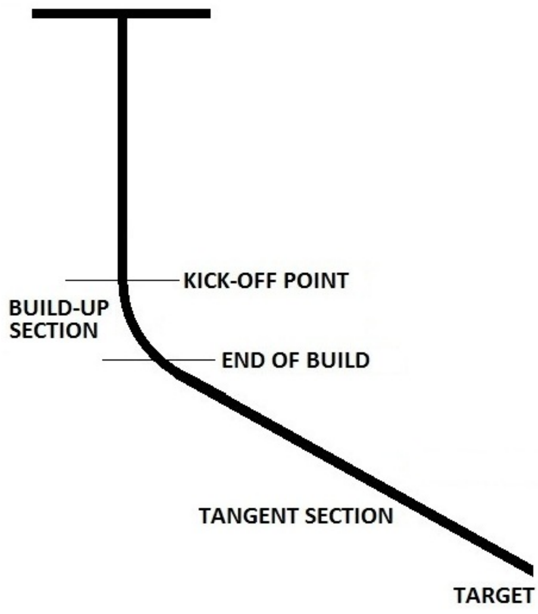

Figure 2.10: Build and hold.

2.3.2 Build, hold and drop

The build, hold and drop type of well have a shallow kick-off point, followed by a build-up section, then a tangent section, ending with a drop-off section. This type of well can have several variations:

- Build, hold & drop back to vertical
- Build, hold drop & hold (Figure 2.11)
- Build, hold & continuous drop through reservoir

Applications can be multiple pay zones, reduced angle in reservoir, lease or target limitations, well spacing requirements, and deep wells with small horizontal displacements.

The disadvantages may be increased torque, risk of keyseating and logging problems due to inclination [17].

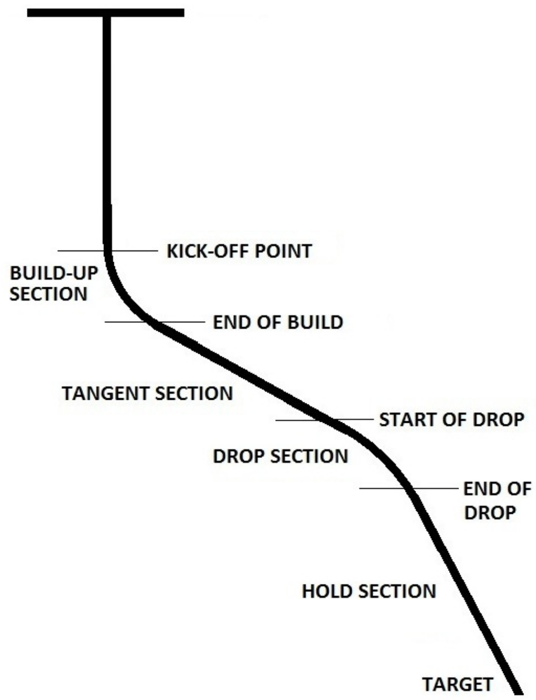

Figure 2.11: Build, hold and drop.

2.3.3 Deep Kick-Off and Build

This type of well have a deep kick-off point, a build-up section and may be followed by a short tangent section.

This configuration might be used for appraisal wells to access the extent of a newly discovered reservoir, repositioning of the bottom part of the hole, or salt dome drilling.

Since this type of well has a deep kick-off point, formations may be harder so that the initial deflection may be difficult to achieve. It may also be harder to achieve desired tool face orientation with downhole motor deflection assemblies because of more reactive torque [17].

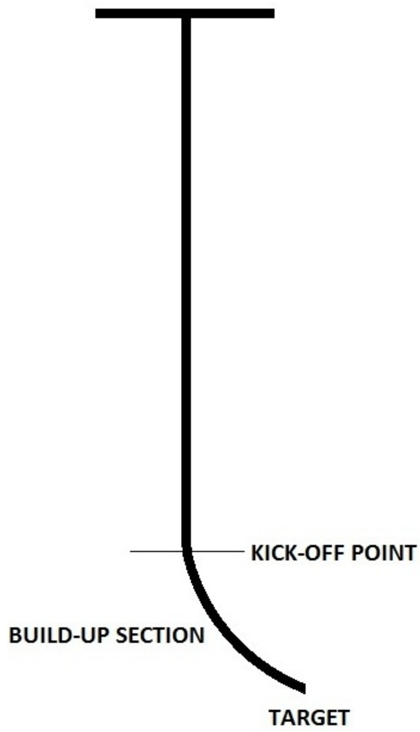

Figure 2.12: Deep kick-off and build.

2.3.4 Horizontal wells

Any of the well trajectories above can have a horizontal section, typical in the bottom part. In practice, any well where the bottom part is drilled inside and along a reservoir is called a horizontal well even though the reservoir, and therefore the well, is not horizontal. A horizontal well will have a large drainage area which is a major advantage, resulting in higher production rate. A horizontal section through a reservoir will also lead to less water and gas coning, ultimately increased oil recovery.

2.4 Drill string friction without rotation

The main object of this thesis is calculation of the drill string movement during a connection. As mentioned earlier, the top drive is not connected to the drill string during a connection, and rotation is therefore not possible. Friction while rotating is therefore not relevant in this setting.

When the drill string hangs in slips from the drill floor, the rig may move up and down due to waves. The movement of the rig will transfer to the top of the drill string and travel down the drill string. Since the drill string consists of mainly steel which is almost perfectly elastic, the downward movement will travel with the speed of sound in steel. The movement will however be dampened by friction which consists of mainly three components; friction between drill string and formation, friction between drill string and casing, and friction between drill string and drill fluid. The two first friction components will be referred to as contact friction, and the last component will be referred to as viscous friction.

As described in section 2.2, drill pipes are made up by a tool joint in each end with a tube section in the middle. The tool joints have a larger outer diameter than the tube section, and at the same time the tube section is rigid enough to resist bending when lying down. This makes it fairly reasonable to assume that only the tool boxes are in contact with the wellbore/casing wall, and not the tube section. When it comes to the BHA, most of the components are having a constant outer diameter, leaving the whole BHA-section in contact with the wellbore/casing wall.

In a perfectly vertical section, the drill string is barely touching the wellbore wall, which means that both fluid friction and contact friction are of importance. However, in a slightly deviated section the drill string is lying on the low side of the well, making the contact friction considerably larger than the viscous friction. A higher deviating well will result in a much higher contact friction, leaving the viscous friction even less significant [18].

2.4.1 Contact friction

Contact friction is in this context the friction between the drill string and formation or casing wall. The contact friction force mainly results from the drill string lying down at the borehole wall in deviated wells, and sometimes also axial force pressing the drill string towards the wellbore wall in bends and curvatures due to the well trajectory, see Figure 2.13 [7]. To be able to calculate the friction force along the whole drill string, one must consider the altering deviation angle, and then integrate the friction force along the whole well trajectory since the friction force is changing with the deviation angle. However, if one disregards the friction force that results from the extra axial force due to the curved sections, the total friction force is much easier to calculate [19].

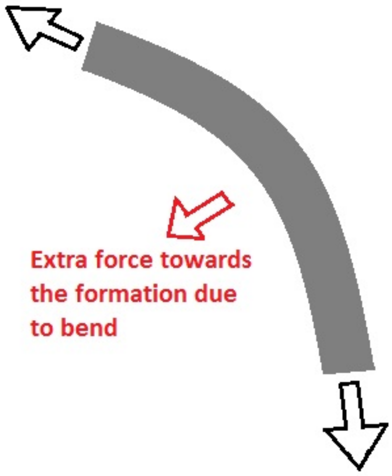

Figure 2.13: The drill string is forced towards the formation, creating more contact friction force in bends.

The friction force is proportional to the contact force, which is the force pressing the two bodies together. The contact force is always perpendicular to the contact point and is often called the normal force, N [18]. The normal force for a solid body lying on a slanting support is given by the equation below, where G is the gravitational force and α is the deviation angle:

$$N = G \sin \alpha \tag{2.4.1}$$

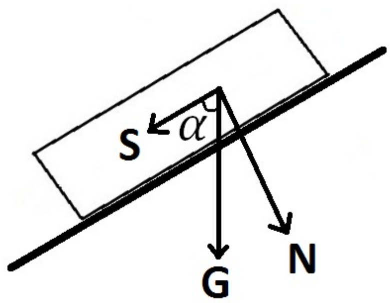

Figure 2.14: The normal force N is perpendicular to the contact point. N is here shown as the force from the support against the body lying on it.

The contact friction is normally highest when the relative speed between the two bodies is zero (static contact friction). When the speed is larger than zero, the contact friction is independent of the relative speed between the two bodies (kinetic contact friction). The friction force against one of the bodies is always directed in the opposite direction of the speed relative to the other body. In this way, the friction force against the other body will have the same value, but directed in the opposite direction. This fulfils Newton's third law [18].

The friction force is dependent upon the contacting surfaces of the two bodies. This dependence is given with a friction coefficient, μ.

Mathematically this relation is given by

$$\vec{F} = -\mu N \frac{\vec{v}}{|\vec{v}|} \tag{2.4.2}$$

where: $\vec{F}$ is the friction force with direction opposite to the velocity. $\vec{v}$ is the velocity of the body in question relative to the other.

A well that are being drilled will be cased off in the upper sections, having an open hole in the deepest section that are currently being drilled. The kinetic friction coefficient for a drill string inside a casing (steel against steel) will be approximately 0,15 - 0,2. For a drill string in contact with the formation in an open hole (steel against formation), the kinetic friction coefficient will be larger, approximately 0,25 - 0,3 [18]. The static friction coefficients are in this thesis set to be 20% higher than the kinetic friction coefficients for the same contacting surfaces.

2.4.2 Viscous friction

During a connection there will be no mud circulation and no rotation of the drill string as explained in section 2.1.4. The drill string can, however, move up and down with the rig movements, resulting in viscous friction against the drill string. The viscous friction towards the drill string is a function of many parameters such as fluid type, flow regime, relative speed, acceleration, roughness of surfaces, surface areas, and the position of the drill string in the wellbore.

Viscous friction is not always linear, even in the cases where there is laminar flow, as shown in Figure 2.16. It is to a large degree dependent upon acceleration and velocity. There is however no simple model for viscous friction, which makes modelling of the friction difficult. The total viscous friction during fluid circulation in a well can be estimated using the total fluid pressure drop. To find the total viscous friction against the drill string, the pressure drop can be divided between the drill string and wellbore surface according to the surface areas.

If one has a 14 cm OD drill string inside a 34 cm ID vertical wellbore, the distance from the drill string to the wellbore wall is 10 cm on every side. The speed of sound in drill fluid is approximately 1000 m/s. One can expect that the fluid velocity profile to propagate with the speed of sound in the drill fluid, reaching the wellbore wall in 1 ms. The velocity profile will probably use considerably more than 1 ms to get established as a static velocity profile, but since the drill string is oscillating with a period of around 15 seconds, the total effect of viscous friction will not be very far from a static viscous friction situation. It is therefore quite reasonable to assume that viscous friction can be specified by giving friction as function of string speed relative to the drill fluid.

To be able to calculate the viscous friction one can look at two parallel plates with fluid in between. If one of the plates moves with a constant speed relative to the other, a linear fluid velocity profile between the two plates will be obtained.

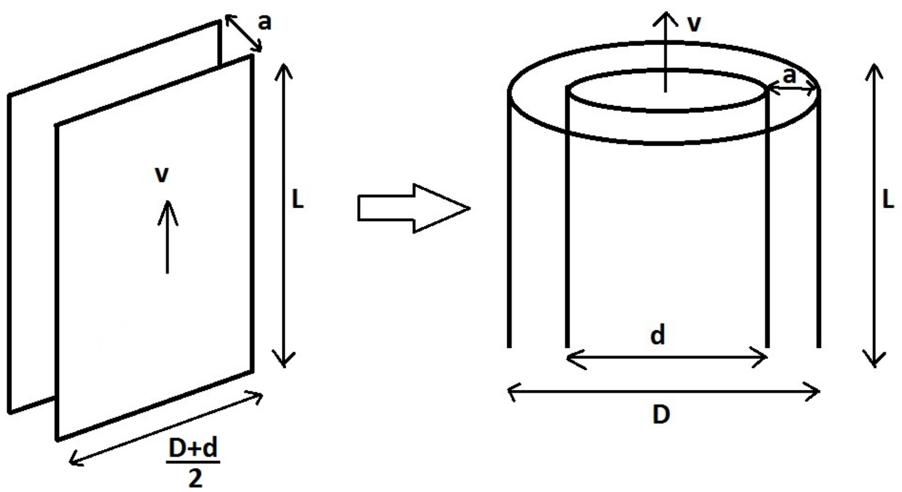

Figure 2.15: Left: Two parallel plates with fluid in between. One of the plates moves with a velocity v relative to the other. a is the distance between the plates. Right: A pipe inside another pipe with fluid in between also moves with a velocity v relative to the other. D is the diameter of the large pipe, d is the diameter of the smaller pipe, a is the distance between the two, and L is the length of the pipes.

The scenario with the two parallel plates can be transferred to the pipe inside a larger pipe which represents the drill string inside the wellbore. The difference is that the surface area of the larger pipe is larger compared to the surface area of the smaller pipe. The viscous friction force transferred in the fluid is however the same for both pipes and can be estimated by multiplying the shear stress τ with the contact area A:

$$F = \tau A \tag{2.4.3}$$

The shear stress is the product of the viscosity μ and shear velocity $\dot{\gamma}$:

$$\tau = \mu \dot{\gamma} \tag{2.4.4}$$

The shear velocity can be found by dividing the velocity v by the distance between the two pipes:

$$\dot{\gamma} = \frac{v}{a} = \frac{v}{\frac{D-d}{2}} = \frac{2v}{D-d} \tag{2.4.5}$$

Since the surface areas are not the same for both pipes, the fluid velocity must also be different, as the friction is the same. The fluid velocity near the larger pipe is lower compared to the velocity at the same distance from the small pipe. The result is a non-linear velocity profile. A more correct viscous friction can be estimated by using the surface area obtained by using the diameter lying between d and D:

$$A = \pi \frac{D+d}{2} L \tag{2.4.6}$$

The estimated viscous friction can then be found by the equation below:

$$F = \mu \pi L \frac{D+d}{D-d} v \tag{2.4.7}$$

The velocity profile in a well as in the example above will be different at different points along the drill string during a connection. The string may be oscillating where parts of the string move upwards while others move downwards. In the beginning of a movement the fluid velocity near the drill string will be very low, and the angle between the velocity profile and the drill string will be very small. The viscous friction is inversely proportional to the angle, and is thus very large in the beginning of the string movement. At the same time the velocity of the string is also very low (see Figure 2.16, image 1). When the velocity profile has propagated outwards, the angle of the velocity profile will be larger, which also applies for the velocity of the string (image 2). After a while the velocity profile is established and is approximately static (image 3).

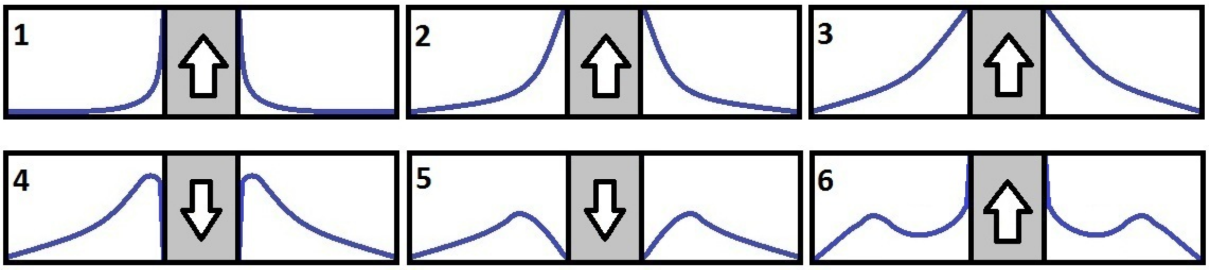

Figure 2.16: The velocity profile (blue line) of the fluid in the annulus outside a drill string when the string moves up and down (arrows). Image 1: The string starts moving upwards. Image 2: the string velocity is constant, but the velocity increases in the fluid outwards. Image 3: A approximate static velocity profile is established. Image 4: The string stops moving and then moves downwards. Image 5: The string is at rest and the fluid velocity starts decreasing outwards. Image 6: The string starts moving upward again, the velocity profile complexity is increasing.

During string oscillations some fluid velocity effects may arise and is addressed below:

As the string moves up and down it will act as a piston, displacing fluid when moving down, sucking fluid back when moving up. This piston effect will make the relative velocity between the string and fluid greater. The fluid velocity is also increasing with diminishing annulus, because of larger relative string volume to the wellbore volume. This effect is not included in the calculations.

As mentioned in section 2.2.2, there might be a float valve installed in the drill string. The valve can in principle refill the well with some fluid when the string is moving upwards, diminishing the suction effect. This is, however, neglected in the simulations, the string will be looked at as a closed string.

2.5 Formation and well pressures

During a drilling operation the objective is to reach the target using as little time and resources as possible with minimum formation damage to the reservoir. The formation types and their properties are constantly changing downwards, making it necessary to continuously monitor and adjust the bottom hole pressure (BHP). The BHP is usually altered by changing the density of the drilling fluid, but it is also possible to apply back pressure. If the BHP is too high the drilling fluid may fracture the formation, and if the BHP is too low, formation fluid may enter the well.

2.5.1 Pore pressure

The pore pressure is the fluid pressure inside the formation pores. In open porous formations where the pore fluid can flow freely to the surface, the pore pressure is the pressure exerted by a column of formation water from the formation's depth to sea level. When impermeable formations are compacted, the pore fluids cannot always escape and must then support the overlaying rock column, leading to anomalously high formation water pressures [20].

The pore pressure can be measured in a open hole section using different tools, but the pore pressure is also predicted in advance using data from adjacent wells and simulations.

If the well pressure in an open hole is lower than the pore pressure, fluid from the formation can start flowing into the well at this specific depth. This is called a kick, and has to be mitigated by increasing the well pressure.

2.5.2 Fracture pressure

The fracture pressure is the pressure where the formation will hydraulically fracture. The fracture induced in a vertical well will always be in the direction of maximum stress. Some formations have naturally occurring fractures, and the pressure needed to open these fractures is somewhat lower than the pressure needed to induce new fractures. This lower pressure is called the fracture propagation pressure.

The fracture pressure in a well is normally measured just after a new casing has been installed and cemented in the hole. The fracture pressure can be measured by increasing the well pressure until leak off occurs, which is called a leak off test (LOT). Sometimes it is only necessary to pressure up the well until a certain BHP is obtained. This is called a formation integrity test (FIT), and confirms that the formation at the shoe will be able to withstand a certain pressure.

If the well pressure during a drilling operation exceeds the fracture pressure, drill fluid in the well will be forced into the formation and hydraulically fracture it. The result of fracturing the formation can be huge losses of drill fluid to the formation until the pressure in the well is equalized to the formation pressure. This scenario can be critical, resulting in a damaged reservoir or worse, inducing a kick further up the well.

2.5.3 Operational window

As explained in the sections above, it is important to have a well pressure above the pore pressure and below the fracture pressure. The distance between these two pressures is called the operational window, and can be graphically described in a pore pressure plot as in the figure below.

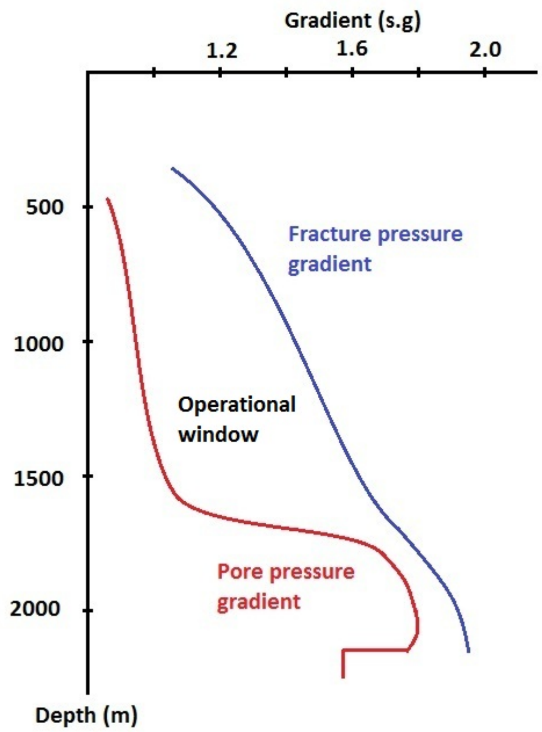

Figure 2.17: Pore pressure gradient plot. The red line is the pore pressure gradient, and the blue line is the fracture pressure gradient. The distance between the pore pressure gradient and the fracture pressure gradient is called the operational window.

The pressure in a well without circulation is the result of the pressure exerted by the mud column. However, if the mud is circulated, the friction pressure drop in the well along with the mud column results in a higher well pressure. This is called the equivalent mud weight (ECD), and is important to take into account when drilling. A high ECD results in a decreased operational window, and the risk of going outside the window increases.

2.5.4 Reservoir pressure

The reservoir pressure in an oil reservoir can be normal, high or low. To be able to determine if the reservoir pressure is normal, the pressure must be measured at the oil water contact (OWC). In an offshore oil reservoir the normal reservoir pressure is the pressure calculated using sea water gradient ρ_{sw} from mean sea level (MSL) to sea bottom, and mean formation water density ρ_p from sea bottom to the OWC (see Figure 2.18). The pressure inside the reservoir is higher above the OWC since oil is (in most cases) lighter than water. Normal pressure at the OWC can be calculated using following equation:

$$P_{OWC} = \rho_{sw}\, g\, h_{sw} + \rho_p\, g\, h_p \qquad (2.5.1)$$

where g is the gravity constant, h_{sw} is the sea water depth, and h_p is the formation water column from sea bottom to OWC.

If the measured pressure at the OWC is higher than the normal pressure, the reservoir pressure is high. If the measured pressure is lower than the normal pressure, the reservoir pressure is low.

To calculate the reservoir pressure at the gas oil contact (GOC) we can use the equation below:

$$P_{GOC} = \rho_{sw}\, g\, h_{sw} + \rho_p\, g\, h_p - \rho_o\, g\, h_o \qquad (2.5.2)$$

where ρ_o is the density of oil, and h_o is the height of the oil column.

The pressure at the top of the reservoir can be calculated using the equation below:

$$P_{Top} = \rho_{sw}\, g\, h_{sw} + \rho_p\, g\, h_p - \rho_o\, g\, h_o - \rho_g\, g\, h_g \qquad (2.5.3)$$

where ρ_g is the density of gas, and h_g is the height of the gas column.

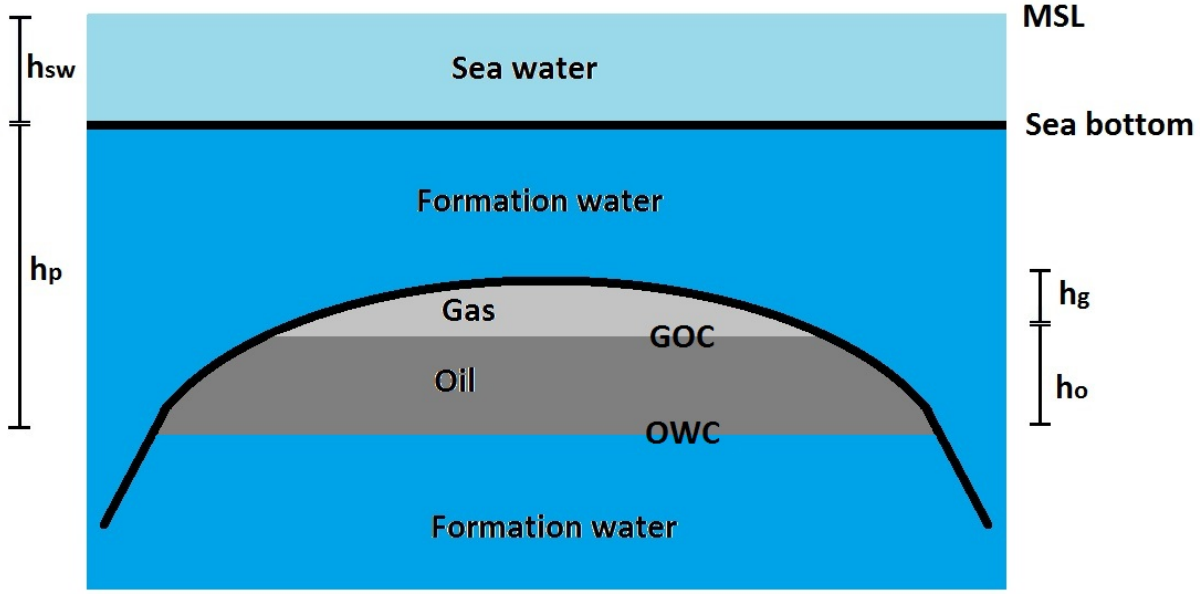

Figure 2.18: The pressure in a reservoir can easily be calculated if the depths and fluid densities are known.

2.5.4.1 Natural deviating reservoir pressures

The reservoirs with pressures naturally deviating from the normal pressure have probably been totally isolated from the surroundings while the reservoir depth has changed. The isolating rock that surrounds the reservoir must be completely impermeable as salt or shale. When an isolated reservoir is submerged or uplifted, the reservoir pressure stays almost constant while the surrounding pore pressure increases or decreases. This mechanism can create large pressure differences that can make drilling difficult.

2.5.4.2 Compartmentalized reservoirs

A reservoir that has been drained will usually have lower reservoir pressure than the normal pressure. Some reservoirs are compartmentalized, where there is no or very little communication between the different reservoir zones (see Figure 2.19). Before the production starts the pressures in the different zones are approximately the same. But since it is hard to predict the communication pattern in the reservoir, some zones will stay untouched, while others are drained. This can create large pressure differences where the drained zones have lower pore pressure than the virgin zones.

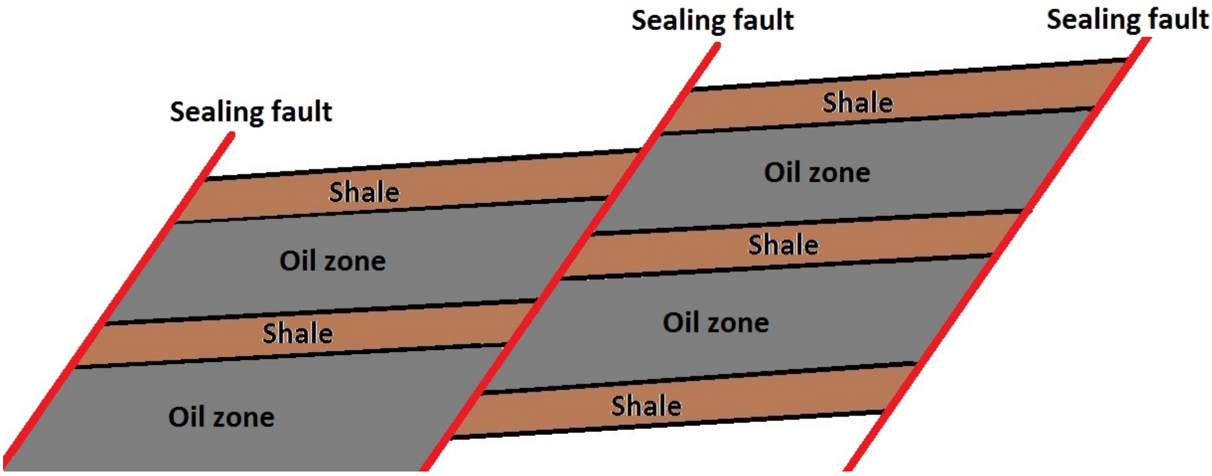

Figure 2.19: A compartmentalized oil reservoir with no pressure communication between the different zones.

The need for infill drilling will then be revealed. The use of 4D seismic (3D seismic shot at the same location at different times) can reveal the reservoir zones that have not been drained.

To reach the untouched zones it might be necessary to drill through a drained zone. The drained zone will have a lower pore pressure and fracture pressure than the untouched zone. If the pressure difference is too large, it can be seen as undrillable. The problem is that the low pore and fracture pressure will require a lower pressure in the well, while the high pore and fracture pressure requires a higher pressure in the well. If the high pore pressure is higher than the low fracture pressure, it can be impossible to complete the drilling. In some cases it can be done if a casing is set in the sealing zone between the reservoir zones.

2.5.4.3 Deep water reservoirs

Drilling in deep water can be very difficult due to a narrower operational window. The tendency in deep water is abruptly increasing pore pressures and weak fracture gradients. Pressure related drilling problems are the leading cause for abandoning deep water wells.

Problems as lost circulation and stuck pipe often suspend the drilling operation and force the operator to change the casing program or well path.

Overpressurized zones are more common in deep water wells, and may cause formation fluid inflow to the well.

2.6 Buoyancy

During a drilling operation the well is filled with mud which gives buoyancy to the drill string. The buoyancy forces acting on the drill string must be known in order to calculate the contact friction forces and the effective weight of the drill string.

Buoyancy is forces exerted by a fluid that opposes an object's weight. In a column of fluid, pressure increases with depth as a result of the overlying fluid. A submerged object in the fluid therefore experiences greater pressure at the bottom of the column than at the top. The pressure difference ΔP over a submerged object with a vertical distance h between the top and bottom is:

$$\Delta P = \rho_m \, g \, h \tag{2.6.1}$$

where ρ_m is the fluid density, and g is the gravity constant.

The difference in pressure results in a net force that accelerates the object upwards. There will however not be a pressure difference over vertical areas, which will not have a contribution to the buoyancy.

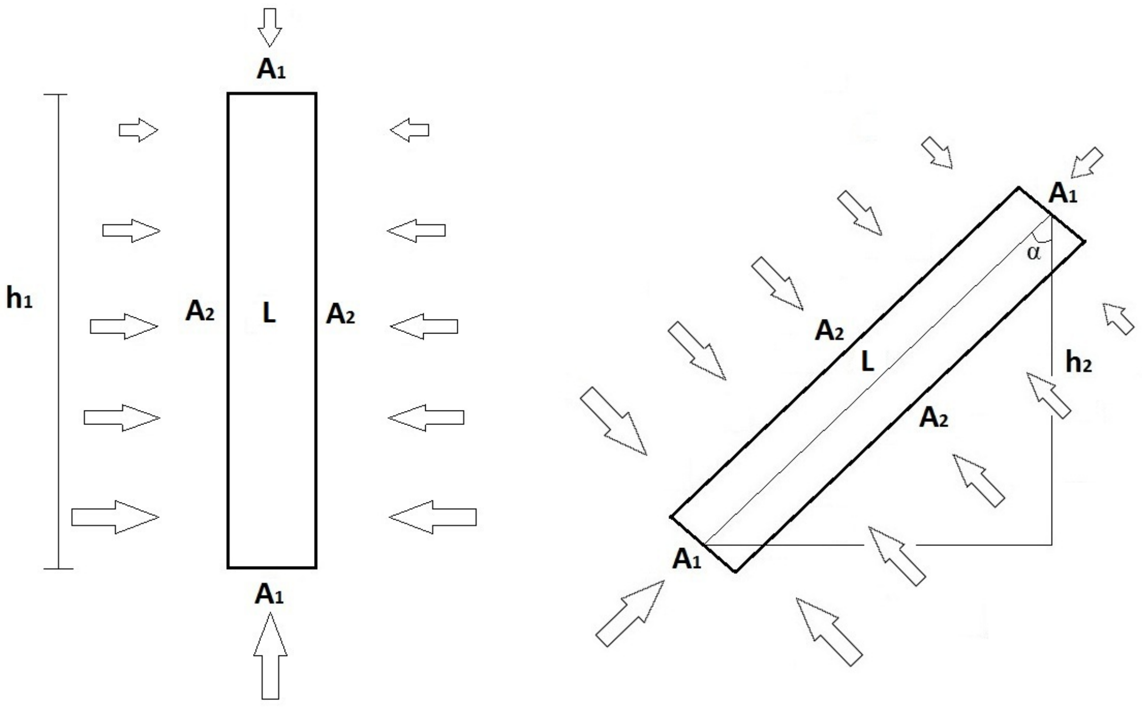

Figure 2.20: The pressure increases downwards in a fluid. The differential pressure over an submerged object accelerates the object upwards. This is called buoyancy. Pressure towards vertical surface areas gives no contribution to the buoyancy.

2.6.1 Buoyancy of a vertical submerged cylinder

A submerged cylinder with a vertical axis will experience a buoyancy force equal to the pressure difference over the length of the cylinder times the end surface area.

The area of the end surfaces is given by:

$$A_1 = \pi R^2 \tag{2.6.2}$$

The buoyancy of the cylinder becomes:

$$B = A_1 \, \Delta P = \pi R^2 \, h_1 \, \rho_m \, g \tag{2.6.3}$$

where h_1 is the height of the cylinder (see Figure 2.20).

2.6.2 Buoyancy of a horizontal submerged cylinder

A submerged cylinder lying horizontally will only experience a pressure difference over the side surface. The side surface is however curved, and the pressure difference is varying. This makes the calculation a bit more complex:

The following is derived by Professor Erik Skaugen. Let us look at a cylinder with length L and radius R. The difference in surface area A due to a small change in angle φ can be expressed as:

$$dA = R \, d\varphi \, L \tag{2.6.4}$$

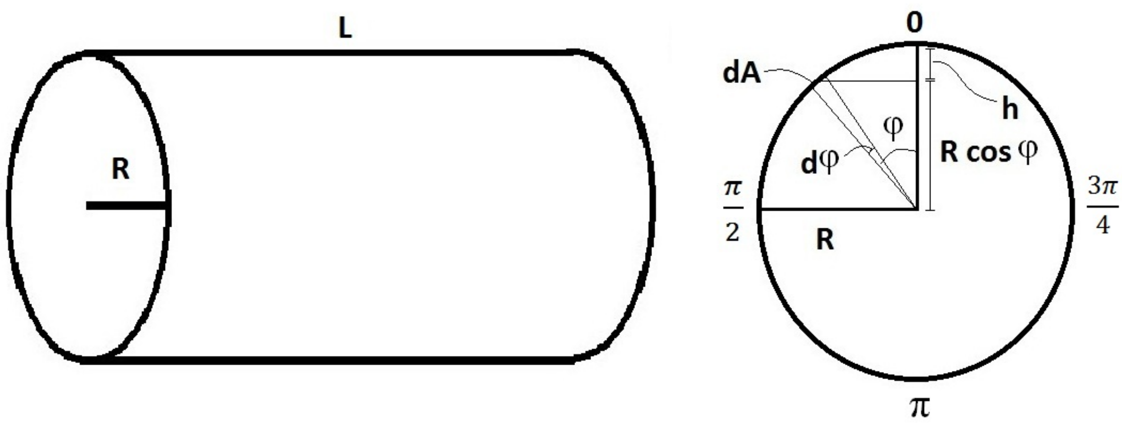

Figure 2.21: A cylinder with radius R and length L. A small change in the angle dφ will give a small change in surface area, dA.

According to equation (2.6.4) above, the pressure at dA is given by the column of fluid above. The height of the column is given by following equation (see Figure 2.21):

$$h = R - R\cos\varphi = (1 - \cos\varphi)R \tag{2.6.5}$$

The pressure at dA is:

$$P = \rho_m\, g\, h = \rho_m\, g\, (1 - \cos\varphi)R \tag{2.6.6}$$

The pressure is acting perpendicular to the surface, which means that the pressure have one vertical P_v and one horizontal component P_h.

The vertical component is the one contributing to the buoyancy (see Figure 2.22):

$$P_v = P\cos\varphi \tag{2.6.7}$$

$$P_v = \rho_m\, g\, (1 - \cos\varphi)R\cos\varphi \tag{2.6.8}$$

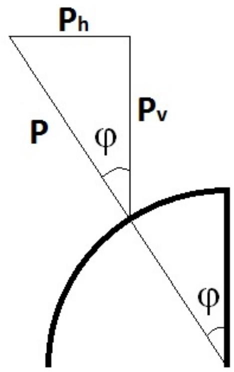

Figure 2.22: Pressure is acting perpendicular to the surface. P_h is the horizontal component, P_v is the vertical component.

In this setting, buoyancy is defined as positive, and pressure acting downwards is negative. The buoyancy acting on half the cylinder (from 0 - π, see Figure 2.21) is:

$$dB_{\frac{1}{2}} = -P_v\, dA \tag{2.6.9}$$

Inserting equation (2.6.4) and (2.6.8) into equation (2.6.9):

$$dB_{\frac{1}{2}} = -\rho_m\, g\, R^2\, L(1 - \cos\varphi)\cos\varphi\ d\varphi \tag{2.6.10}$$

Integrating around half the cylinder, and finds the buoyancy on half the cylinder:

$$B_{\frac{1}{2}} = \int_0^{\pi} dB_{\frac{1}{2}} = -\rho_m \, g \, R^2 \, L \int_0^{\pi} (\cos\varphi - \cos^2\varphi) \; d\varphi = \rho_m \, g \, R^2 \, L \, \frac{\pi}{2} \qquad (2.6.11)$$

The buoyancy for the whole cylinder is the buoyancy for half the cylinder given in equation (2.6.11) times two, since the cylinder is symmetrical:

$$B = \pi \, R^2 \, L \, \rho_m \, g \qquad (2.6.12)$$

This result is the same as equation (2.6.3). One should keep in mind that the buoyancy given in equation (2.6.12) is from the side of the cylinder. There is no pressure difference over the end pieces, and they do not contribute to the buoyancy.

2.6.3 Buoyancy of a submerged cylinder with an angle

A submerged cylinder that deviates with an angle α from vertical will experience an effective acceleration from both the bottom end piece and bottom side. The following is derived by Professor Erik Skaugen.

We can look at a deviating cylinder in the same coordinate system as above. Since the cylinder is deviating from vertical with an angle α, the height of the pressure column for counter points are different compared to a cylinder lying horizontally (see Figure 2.23). The pressure is always higher at the lower side of the cylinder. The height difference is:

$$h = 2R \sin\alpha \qquad (2.6.13)$$

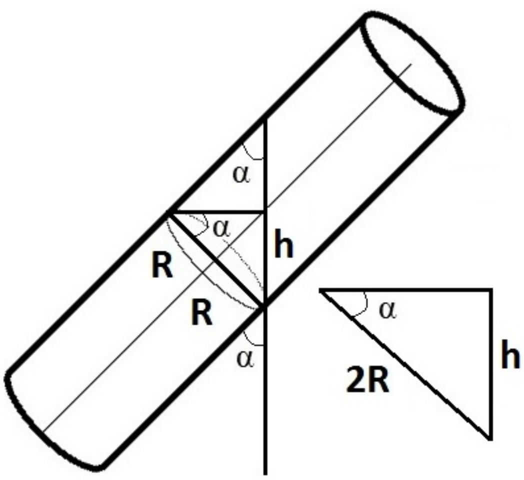

Figure 2.23: A cylinder submerged with a angle α deviating from vertical.

The vertical pressure component over the cylinder can be found by inserting equation (2.6.13) into equation (2.6.7):

$$P_v = \rho_m \, g \, 2R \sin\alpha \, \cos\varphi \tag{2.6.14}$$

The buoyancy over one half side (not including the end pieces) of the cylinder can be found by inserting equation (2.6.4) and (2.6.14) into equation (2.6.9):

$$dB_{\frac{1}{2}} = -\rho_m \, g \, L \, 2R^2 \, \sin\alpha \, \cos\varphi \, d\varphi \tag{2.6.15}$$

Integrating around half the cylinder, and finds the buoyancy over the half side of the cylinder:

$$B_{\frac{1}{2}} = \int_0^{\pi} dB_{\frac{1}{2}} = -\rho_m \, g \, R^2 \, L \, \sin\alpha \int_0^{\pi} (\cos\varphi) \, d\varphi = \pi \, R^2 \, L \, \rho_m \, g \, \sin\alpha \tag{2.6.16}$$

The buoyancy for the whole sides of the cylinder is the buoyancy for half the cylinder given in equation (2.6.16) times two, since the cylinder is symmetrical:

$$B_{sides} = \pi \, R^2 \, L \, \rho_m \, g \, \sin\alpha \tag{2.6.17}$$

We must also include the buoyancy acting on the end pieces. The average pressure towards the end pieces is equal to the pressure at the center of the cylinder. This pressure is given by the equation:

$$P = \rho_m \, g \, h = \rho_m \, g \, L \cos\alpha \tag{2.6.18}$$

The area of the end pieces are given by equation (2.6.2). The buoyancy acting at the end pieces are given by the equation:

$$B_{ends} = \pi \, R^2 \, L \, \rho_m \, g \, \cos\alpha \tag{2.6.19}$$

The total buoyancy is a combination of the buoyancy acting on the side pieces (equation (2.6.17)) and the buoyancy acting at the end pieces (equation (2.6.19)). The direction of the total buoyancy is vertical, as there are no rotational forces acting on a submerged object.

2.6.4 Archimedes' principle

To make the calculations as straightforward as possible, the use of Archimedes' principle can simplify them. The advantage of Archimedes' principle is that the orientation of an object is not considered.

Archimedes' principle states that the net upward buoyancy force B is equal to the magnitude of the weight of fluid displaced by the object:

$$B = \rho_m V_{disp} g \tag{2.6.20}$$

where V_{disp} is the displaced volume

The displaced volume of a cylinder with length L and radius R is:

$$V_{disp} = \pi R^2 L \tag{2.6.21}$$

Inserting equation (2.6.21) into equation (2.6.20), the buoyancy of the cylinder becomes:

$$B = \pi R^2 L \rho_m g \tag{2.6.22}$$

which is exactly the same as equation (2.6.3) and (2.6.12).

The displaced volume of i.e. a submerged drill pipe is not dependent of the orientation of the drill pipe, making Archimedes' principle practical in this thesis.

2.6.5 Apparent weight

The apparent weight of an object immersed in a fluid is given by the weight of the object in air subtracted the buoyancy:

$$W_{apparent} = W_{air} - B \tag{2.6.23}$$

The weight of a steel cylinder in air is:

$$W_{air} = \pi R^2 L \rho_s g \tag{2.6.24}$$

where ρ_s is density of steel

The apparent weight of the steel cylinder is equal to equation (2.6.24) subtracted equation (2.6.22):

$$W_{apparent} = \pi R^2 L \rho_s g - \pi R^2 L \rho_m g = \pi R^2 L g \left(1 - \frac{\rho_m}{\rho_s}\right) \tag{2.6.25}$$

2.7 Describing the drill string movement

In order to describe the drill string movement one can use a method by Professor Erik Skaugen at the University of Stavanger. The method is numerical and enables one to calculate the behavior of strings when exposed to changing forces. These forces can be movement of objects connected to the string, gravity and friction. Stress waves will be induced in the string by the different forces, and travel with the speed of sound in the material. The method provides equations for several situations, but has some limitations. If the forces acting on the string become so large that the string yields or break, the continued calculations will be incorrect. The numerical method is restricted to treat only one-dimensional waves, travelling along the string axis. These waves are pure stress waves which gives the local stress along the string, or torsion waves. Shear waves and waves that are induces across the string is however neglected [21].

2.7.1 String parameters

To be able to calculate the drill string movements it is important to establish a set of parameters and conditions:

<u>The physical parameters required for calculation are:</u>

- String length L
- String material cross section A
- Any two of the three string material parameters below. They are connected by the equation $c^2 = E/\rho$. If two of the parameters are given, the third can be calculated.
  - ρ, density
  - c, speed of sound in the material
  - E, the modulus of elasticity
- All external forces acting along the string and any connections must be specified as functions of time.
- Contact friction must be specified by the coefficients of friction.
- Liquid friction must be specified by giving friction as a function of string speed relative to liquid.

- The string axis deviation from the vertical direction must be given and may change along the string. This is used to calculate the normal force to find the contact friction, and to find the component of gravity acting along the axis.

The input numerical parameters required for calculation are:

- The time step length Δt (alternatively the space step length Δz)
- The displacement unit Δx. This unit is used as the basic unit in the numerical calculations and can be freely chosen. The displacement unit is often chosen to give integer values for hand calculations.

The numerical calculations give the actual movement of the segment mid points at all times that are whole multiples of the time step. The calculated movement is the actual movement of the string material at the segment mid point [21].

2.7.2 Division of a string into numerical segments

To be able to perform calculations on a string the whole string must be divided into a number of segments. Half segments can be used at the ends, but the rest of the string is always divided into whole segments. A string might have different sections with different diameters. Figure 2.24 is showing some possible divisions into numerical segments. The length of a segment is Δz, and the length of a half segment is $\Delta z/2$. It is worth to note that the "mid-point" of a half segment is actually at the end of the segment.

In a string with a total length L, having a total of N segments, the length of each segment Δz is L/N. A segment is defined to be inside the string if the mid point of the segment is a distance Δz or more away from the ends of the string.

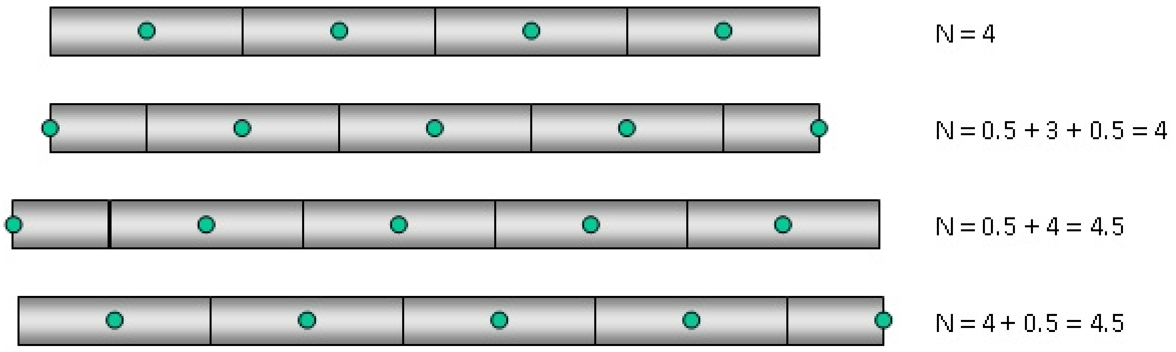

Figure 2.24: Some examples of how to divide strings correctly into numerical segments. The number of N segments is known, and the mid point of a segment is shown as a small circle [21].

The requirement for segment diameters is that each half of a whole segment must have a constant diameter. This means that a whole segment can have two different diameters, while a half segment can only have one diameter. Figure 2.25 shows the possible ways of representing a string of changing diameter.

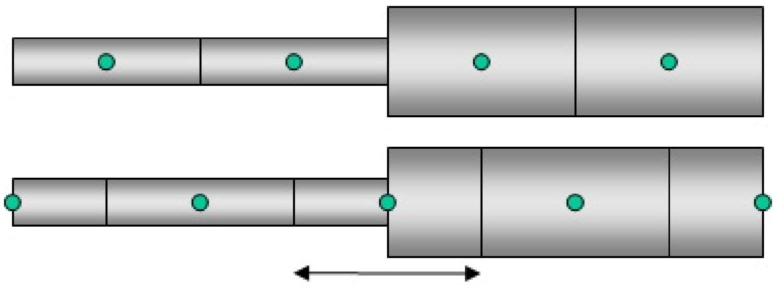

Figure 2.25: Two different ways of dividing a string with changing diameter into numerical segments. The lower case shows a whole segment having two different diameters. The number N of segments is 4 in both cases, but the number of segment mid points are 4 and 5 [21].

According to the rules only strings with abrupt changes of diameter can be modelled exactly. For real strings there may be more gradual changes of diameter. A possible approximation to this with numerical segments can be done, but requires that the distance between segment mid points is short compared to the transition zone.

The segment mid points should be numbered in the positive direction from 1 and upwards, where the positive direction can be chosen freely, but must be along the string axis. When gravity is included, positive direction is usually chosen to be from the top and downward along the string. The gravity component that acts along the string is then positive. Forces that act along the string in the positive direction are positive, and negative if they act in the negative direction. This convention is also used for velocities.

2.7.3 Displacements

In the beginning of a calculation, the string can be assumed to be completely relaxed, and the movement of each segment mid point is calculated as its displacement from its assumed initial position in the string. The calculation procedure and notations are as follows:

- The displacement of segment mid point number j at present time t is X_j.
- The displacement of segment number j at the time step before present time (time t - Δt) is XG_j.
- The new displacements XN_j at one time step into the future (time t + Δt) can be calculated from numerical equations as long as the displacements at present time X_j, and one time step before present time XG_j is known.
- When the new displacements are calculated the time t + Δt is taken as the present time t, and all XN_j becomes X_j, and all former X_j becomes XG_j.
- In order to start the first calculation, the initial conditions must be known. This means that in order to calculate all XN_j for the first time, a complete set of X_j and XG_j must be known. This is the initial condition or the boundary condition in time, and must be found from the actual condition of the string.

If no forces are acting on a string lying horizontally during the initial time, the string will remain relaxed, and all displacements X_j and XG_j are zero.

2.7.4 Boundary conditions in space

The initial conditions are left behind after two time steps of calculations. This means that they will have no influence on further calculations. However, there will usually be other conditions present that will keep having influence on the calculations. These are called boundary conditions in space, since they are usually found at a certain point along the string. Generally, all deviations from a freely moving, continuous string of constant cross section can be considered as a boundary condition. Some are mentioned here:

- The string is terminated with a free end. This is a end that can move freely, where no forces are acting on it. This is the case when a string ends in air, with nothing connected to the end.
- The string is terminated with a fixed end. This end has a fixed position, and movement of the end is not possible.
- Change of cross section of the string
- A piece of equipment clamped to the string and moving freely with the string.

As general forces as gravity and friction affect every part of the string, they are usually not called boundary conditions, but they can be considered as such. Fixed points and forced movement of any point in the string are given by an equation that is independent of the string movements. The equation for a fixed point will be a constant, while the equation for a forced moving point will be a function of time. For calculation purposes, the boundary condition should be applied on a segment min point or at a segment boundary. Often it will be smart to divide the string in such a way that fixed or forced points are placed at segment mid points.

For other boundary conditions as diameter changes and ends, special numerical equations for the segment mid points affected will be used.

2.7.5 The standard numerical equations

To calculate the new displacement (XN) of a segment mid point, the standard numerical equation is used. This is a equation used for a segment having a constant cross section. For segment number j, we have:

$$XN_j = X_{j-1} + X_{j+1} - XG_j \tag{2.7.1}$$

2.7.6 End boundary numerical equations

A free end of a string can be described by two different equations, depending on which of the two possible segment placements relative to the end is chosen. In most cases the recommended configuration is to end the string with a whole segment. The figure below shows a string ending with a whole segment, where the distance from the end to the mid point is $\Delta z/2$.

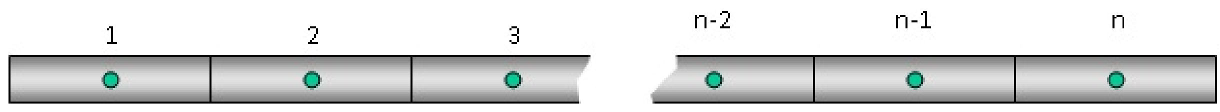

Figure 2.26: A string with free ends having whole segments at both ends. n is the number of segments in the string [21].

The numbering of the segments gives two possible equations for the free ends, depending if the free end is at the beginning or at the end of the string. The equation at the left is the equation for the first segment of the string, while the equation at the right is the equation for last segment of the string.

$$XN_1 = X_1 + X_2 - XG_1 \qquad XN_n = X_{n-1} + X_n - XG_n \qquad (2.7.2)$$

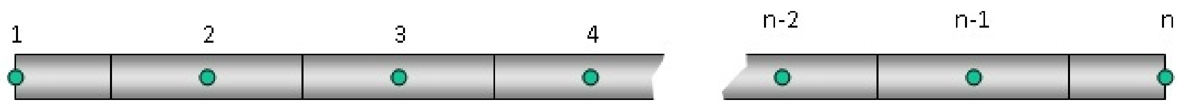

Figure 2.27: A string with free ends having half segments at both ends. n is the number of segment mid points, but the number of whole segments are n-1 [21].

Sometimes the string is ended with half ends. This gives segment mid points at the ends as shown above. The equations for the free ends then become:

$$XN_1 = 2X_2 - XG_1 \qquad XN_n = 2X_{n-1} - XG_n \qquad (2.7.3)$$

A string may end with a whole segment at one end, and a half segment at the other end. The equations will also be the same for changes in the cross section between the ends (if the change in cross section is at least a distance Δz away from the mid point of the end segment).

For fixed ends there are also two different numerical equations for segments at the string ends. The equations are also here depending on whether the string is ended with a half or a whole segment. If the string is ended with half segments at both ends, the fixed end equations are:

$XN_1 = X_{LE}$ $XN_n = X_{RE}$ (2.7.4)

where X_{LE} is the constant, fixed displacement of the left end, and X_{RE} is the constant, fixed displacement of the right end. This is the simplest type of fixed ends, and will never give any numerical problems. It is also the recommended type of fixed ends.

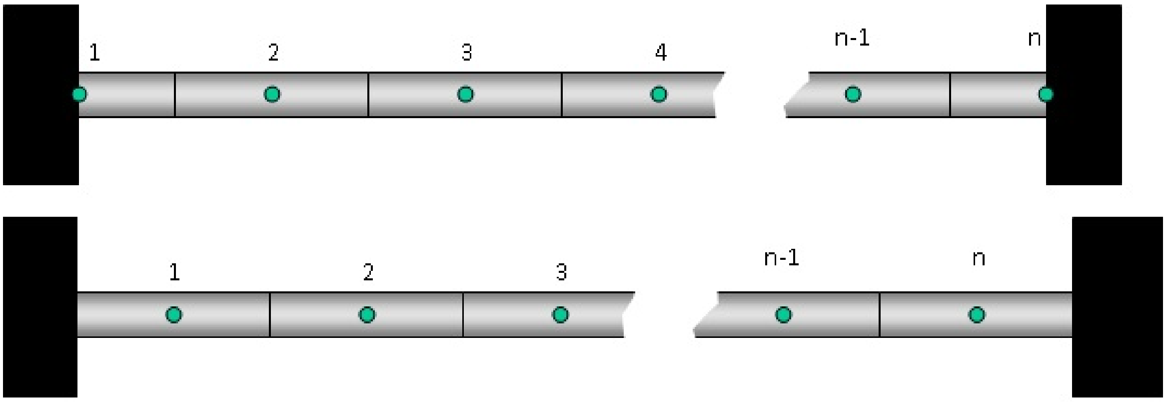

Figure 2.28: Two strings with fixed ends. The string above is ending with half segments at both ends, the lower with whole segments at both ends [21].

If the string is ended with whole segments at both ends, the fixed equations are:

$XN_1 = X_2 - X_1 - XG_1 + 2X_{LE}$ $XN_n = X_{n-1} - X_n - XG_n + 2X_{RE}$ (2.7.5)

For forced ends, the displacements will change with time and is then functions of time, $X_{LE} = X_{LE}(t)$ and $X_{RE} = X_{RE}(t)$. In some cases the end displacements may also be more complex.

2.7.7 Change of string cross section

A change in cross section is in this context a change in inner string diameter, outer string diameter or both. Any change of cross section is allowed at a segment mid point, or at a segment border. A change in cross section at a segment mid point is the simplest case and is shown in Figure 2.29 below. The equation for a change in cross section from A_1 to A_2 in the positive direction is:

$$XN_j = \frac{2A_1}{A_1 + A_2} X_{j-1} + \frac{2A_2}{A_1 + A_2} X_{j+1} - XG_j \qquad (2.7.6)$$

Neighboring segments will not be affected by the change of cross section. This is because their mid points are Δz away from the change, and the equation assumes that there is not another change of cross section closer than Δz to any segment mid point j.

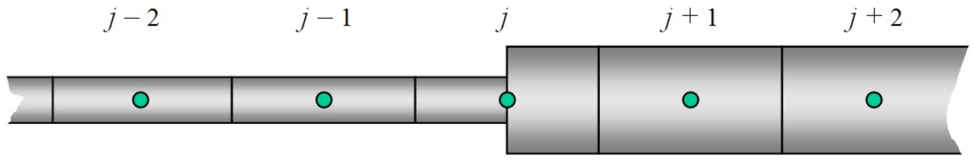

Figure 2.29: A change in the string cross section at the mid point of segment number j. The cross section is changed from A_1 to the left of this segment, to A_2 at the right of it [21].

If the change in cross section occurs at a segment border as shown in Figure 2.30 below, the two segment mid points that are closer than Δz are affected by it.

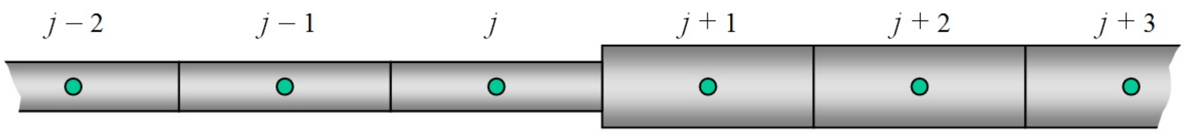

Figure 2.30: A change in the string cross section at the border between segment number j and j+1 [21].

The equations for the two segments are shown below:

$$XN_j = X_{j-1} + \frac{A_1 - A_2}{A_1 + A_2} X_j + \frac{2A_2}{A_1 + A_2} X_{j+1} - XG_j \tag{2.7.7}$$

$$XN_{j+1} = \frac{2A_1}{A_1 + A_2} X_j + \frac{A_2 - A_1}{A_1 + A_2} X_{j+1} + X_{j+2} - XG_{j+1} \tag{2.7.8}$$

2.7.8 Stress in the string

In the initial reference condition, the string is relaxed with no stress of the string material. However, when the string is affected by forces, the distance between the different segment mid points is altered, and stress is induced in the string. If the distance between two segment mid points is increased, the string is stretched and the stress induced is here defined to be positive. A decrease in the distance between two segment mid points means that the string is compressed, which is defined to be negative stress. The elongation between segment mid point number j and j+1 are given by the equation:

$$\Delta X_{j,j+1} = X_{j+1} - X_j \tag{2.7.9}$$

If $\Delta X_{j,\,j+1}$ is negative the string is compressed between segment mid point number j and j+1, and if $\Delta X_{j,\,j+1}$ is positive the string is stretched. The average stress $\sigma_{j,\,j+1}$ between these points can be found by the equation:

$$\sigma_{j,j+1} = E\frac{X_{j+1} - X_j}{\Delta z} \tag{2.7.10}$$

where E is the modulus of elasticity, with the unit N/m^2

Since the average stress is calculated between two points it is possible that the material yields, but we are not seeing it from our data. The calculations will be invalid from this point. This is however not a large problem, as the finite time steps limit the shortness of stress pulses we are able to put into the string. If there is a suspicion of existence of shorter pulses, one have always the possibility of reducing the time and space length to reveal them.

2.7.9 Velocity of the string material

Any change of the string state is spreading out along the string with a velocity equal to the speed of sound in the string. But the average material velocity v_j is the change of displacement of a segment mid point during a time step Δt at position j in the string, and can be estimatd by two different equations, equally valid:

$$v_j(t-\Delta t, t) = \frac{X_j - XG_j}{\Delta t} \qquad v_j(t, t+\Delta t) = \frac{XN_j - X_j}{\Delta t} \tag{2.7.11}$$

where t is the present time, t - Δt is the old time, and t + Δt is the new, future time.

However, these two equations are not good expressions for the velocity at the present time, as they are more suitable to find the velocity in the middle of X_j & XG_j, and XN_j & X_j.

The velocity of a segment mid point at any time t can be estimated by taking the average of the average velosities in the former and the future time interval Δt. This is more accurate and is shown matematically in [21]:

$$v_j(t) = \frac{v_j(t-\Delta t, t) + v_j(t+\Delta t, t)}{2} = \frac{X_j - XG_j}{2\Delta t} + \frac{XN_j - X_j}{2\Delta t} = \frac{XN_j - XG_j}{2\Delta t} \qquad (2.7.12)$$

This equation is quite accurate if the velocity changes smoothly, but may not be very useful when the velocity changes suddenly.

2.7.10 External forces

According to the physical ball-spring model, the expression for segment mid point force balance is:

$$ma = m\frac{XN_j - 2X_j + XG_j}{\Delta t^2} = \sum Internal\ \ forces + \sum External\ \ forces$$

(2.7.13)

where:

- m is the mass of a whole segment of length Δz, m = A Δz ρ
- A is the cross section of the string
- a is the acceleration of the segment
- Δt is the time step length, Δt = c/Δz
- c is the speed of sound in the string material, c = $\sqrt{E/\rho}$
- E is the modulus of elasticity of the string material
- ρ is the density of the string material

It is possible to calculate the internal forces and expressing them by known displacements in the equation above, resulting in the equation for the new position of a segment mid point including external forces:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + \frac{\Delta t^2}{m}\sum External\ \ forces \tag{2.7.14}$$

For a string of constant cross section hanging vertically in a well surrounded by air, the numerical equation for the segment mid points (except the end segments) are:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + \frac{\Delta t^2}{m} mg = X_{j-1} + X_{j+1} - XG_j + g\Delta t^2 \tag{2.7.15}$$

If the cross section changes from A_1 to A_2 at segment mid point number j, the equation is:

$$XN_j = \frac{2A_1}{A_1 + A_2} X_{j-1} + \frac{2A_2}{A_1 + A_2} X_{j+1} - XG_j + g\frac{2\Delta t^2}{m_1 + m_2} \tag{2.7.16}$$

where m_1 is the mass of the whole segment to the left of the cross section change, and m_2 is the mass of the whole segment to the right of the change.

The equation for a half end segment at the bottom end of the string is:

$$XN_1 = 2X_2 - XG_1 + \frac{2\Delta t^2}{m}\left(\frac{m}{2} g\right) = 2X_2 - XG_1 + g\Delta t^2 \tag{2.7.17}$$

2.7.11 Friction

Friction is a special case of external forces acting on the string, but can be treated in the same way as other external forces. Friction is usually acting on every segment in the string as gravity is, but can be more complex than gravity because it is always acting in the opposite direction of the segment velocity. It can therefore be a problem to determine when to use the plus sign or the negative sign in front of the friction term.

If the mid point displacement is positive (downward displacement), the friction should be negative and subtracted. If the mid point displacement is negative (upward displacement), the friction should be negative and added. The direction of the displacement of mid point j is determined by the difference between the current position and the new position, X_j - NX_j. Let us say that the displacement is positive, causing friction to be subtracted. If the friction term is larger than the difference in displacement, the mid point j would change direction due to the friction term. This is a problem since the friction is always acting in the oposite direction of the mid point direction. When the direction of the mid point is changed, so is the direction of friction, causing friction rather to be added. The mid point direction is once again changed due to the friction term, and the start of an endless loop is initiated. The scenario with a change of direction is illustrated in Figure 2.31. The problem can be solved with an assumption:

It can be assumed that a change of the displacement direction will include a brief stop of the movement. As the static contact friction is higher than the kinetic contact friction, a stop in the movement will also involve a higher contact friction. A solution to the problem is to either assume that the new position is equal to the current position, or that the mid point is equal to the position where the change of displacement direction occurred.

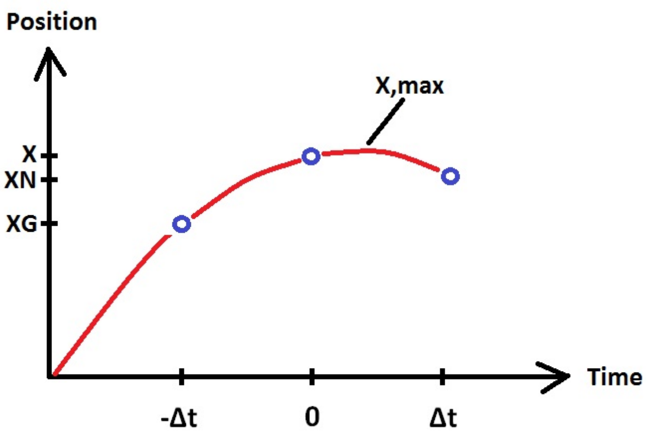

Figure 2.31: XG is the old position, X is the current position and XN is the new position. Δt is the time step, and downwards is defined as the positive direction. The red line represents the movement of a segment mid point, where the displacement direction changes between the current position and the next position due to friction forces. X,max is the turning point of the mid point where the movement stops before it changes direction.

If one assumes that the movement of the mid point can be described with a quadratic equation, the turning point of the curve is the position where the change of direction occurs. The turning point can easily be found, and has been derived by Professor Erik Skaugen:

The quadratic equation can be expressed using the following symbols:

$$y = at^2 + bt + c \qquad (2.7.18)$$

c is given at t = 0, and is equal to X:

$$y = at^2 + bt + X \tag{2.7.19}$$

The equation for XN is found at t = Δt:

$$XN = a\Delta t^2 + b\Delta t + X \tag{2.7.20}$$

The equation for XG is found at t = -Δt:

$$XG = a\Delta t^2 - b\Delta t + X \tag{2.7.21}$$

Finding the parameter a by adding XG to XN:

$$XN + XG = 2a\Delta t^2 + 2X \qquad \rightarrow \qquad a = \frac{XN+XG-2X}{2\Delta t^2} \tag{2.7.22}$$

Finding the parameter b by subtracting XG from XN:

$$XN - XG = 2b\Delta t \qquad \rightarrow \qquad b = \frac{XN-XG}{2\Delta t} \tag{2.7.23}$$

The top point of the curve is the derivate of equation (2.7.18):

$$\frac{dy}{dt} = 2at + b = 0 \tag{2.7.24}$$

This gives the time corresponding to the turning point:

$$t = -\frac{b}{2a} \tag{2.7.25}$$

By inserting equation (2.7.25) into equation (2.7.19), the position of the turning point is found:

$$y_{max} = a(-\frac{b}{2a})^2 + b\left(-\frac{b}{2a}\right) + X = \frac{b^2}{4a} - \frac{b^2}{2a} + X = X - \frac{b^2}{4a} \tag{2.7.26}$$

Inserting equation (2.7.22) and (2.7.23) into equation (2.7.26) gives us the position of the turning point:

$$y_{max} = X - \frac{(\frac{XN-XG}{2\Delta t})^2}{4(\frac{XN+XG-2X}{2\Delta t^2})} = X - \frac{(XN-XG)^2}{8(XN+XG-2X)} \tag{2.7.27}$$

By assigning either the current position or equation (2.7.27) as the new position, the calculations can continue. The mid point is now assumed to be stationary, and the static friction coefficient must be used in the calculation until it starts moving again.

2.7.11.1 Contact friction

Contact friction is discussed in section 2.4.1. For the strings considered in this thesis, there is only one dimension, along the string axis. The equation for the friction force due to contact friction in one dimension can be written as:

$$F = \mp \mu N \tag{2.7.28}$$

where μ is the coefficient of friction and N is the normal force. The minus sign means that if the velocity is positive, the friction force is negative, and vice versa.

We can use equation (2.7.14) (a string of constant cross section) and insert the equation above and get the equation for the new position when contact friction is included:

$$XN_j = X_{j-1} + X_{j+1} - XG_j \mp \frac{\Delta t^2}{m} \mu N \tag{2.7.29}$$

2.7.11.2 Viscous friction

The simplest case of viscous friction is linear friction where the friction force is proportional to the segment velocity, as discussed in section 2.4.2. The viscous friction force towards one numerical segment of the drill pipe with length Δz, drill pipe outer diameter OD_b, and inner diameter of the casing/wellbore D becomes:

$$F = \mu\, \pi\, \Delta z\, \frac{D+OD_b}{D-OD_b}\, v = \mu\, \pi\, \Delta z\, \frac{D+OD_b}{D-OD_b}\, \frac{XN_j - XG_j}{2\Delta t} \tag{2.7.30}$$

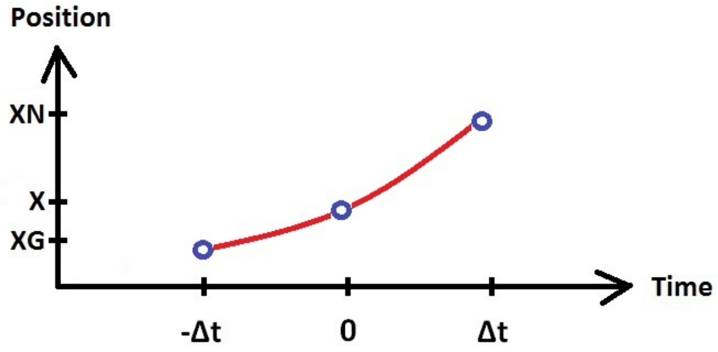

Figure 2.32: The average velocity from position XG to XN is (XN-XG)/2Δt.

Inserting the viscous friction force into equation (2.7.14):

$$XN_j = X_{j-1} + X_{j+1} - XG_j - \frac{\Delta t^2}{m}\,\mu\,\pi\,\Delta z\,\frac{D+OD_b}{D-OD_b}\,\frac{XN_j - XG_j}{2\Delta t} \tag{2.7.31}$$

For a drill pipe the segment mass m is the length of a segment Δz multiplied with the mass per length of drill pipe M_b, and we get:

$$XN_j = X_{j-1} + X_{j+1} - XG_j - \frac{\Delta t\,\mu\,\pi}{2\,M_b}\,\frac{D+OD_b}{D-OD_b}\,XN_j - XG_j \tag{2.7.32}$$

The equation can be rearranged and the constants can be collected into one constant C:

$$XN_j = \frac{1}{1+C}(X_{j-1} + X_{j+1}) - (1-C)XG_j \tag{2.7.33}$$

where $C = \frac{\Delta t\,\mu\,\pi}{2\,M_b}\,\frac{D+OD_b}{D-OD_b}$

We can rewrite equation (2.7.33) and get:

$$XN_j = (1-\varepsilon)X_{j-1} + (1-\varepsilon)X_{j+1} - (1-2\varepsilon)XG_j \tag{2.7.34}$$

where $\varepsilon = \frac{C}{1+C}$ and $C = \frac{\Delta t\,\mu\,\pi}{2\,M_b}\,\frac{D+OD_b}{D-OD_b}$

3. NUMERICAL SIMULATION

The numerical simulation of the string movements have been performed using Matlab. This is a programming environment for algorithm development, data analysis, visualization, and numerical computation [22].

The numerical simulation program has been developed from scratch by the author, but Professor Erik Skaugen has contributed with some of the programming and setup. It has been a goal to make a organized and time-efficient program, which is not too far from the final result. The reader must however bear in mind that the author has very little experience with programming prior to this thesis.

3.1 Input data

There are some input data that must be entered into the program before a simulation can begin; wellbore data, string data and fluid data. The program is built in such a way that most of the input data are entered in the upper part of the program, and the calculations are performed further down. There are however some exceptions from this in order to group the data and make the program more intuitive.

3.1.1 The wellbore

One of the first variables to decide before doing a simulation is the well path. The program is prepared to deal with different well trajectories, from simple vertical trajectories to quite complex ones. The most complex well trajectory the program can handle is a well with a vertical section at the top, then a build section, followed by a straight tangent section, after that a drop/build section, ended by a straight hold section. The following input data must be entered for the program to calculate the well trajectories:

- Length and deviation angle of straight sections
- Angular radius of build/drop sections

In order to calculate the contact friction forces, the friction coefficients and set depth of the last casing must be known. The program assumes the casing being suspended from surface having a constant ID and a constant friction coefficient between the drill string and the casing. The setting depth of the casing is adjusted to the nearest segment boundary. This means that the casing depth can be altered with ±5 meters by the program given a segment length of 10 meters. The new casing setting depth is however displayed in the command window. The program assumes an open hole below the casing setting depth.

3.1.2 The drill string

The drill string is divided into two main sections; drill pipes and drill collars. The length of the two sections must be entered, but these numbers can be changed by the program. The program will automatically adjust the section lengths according to the segment lengths, making sure of the string consisting of an integer number of segments. The adjustments performed by the program are small, and can vary up to ±5 meters, given a segment length of 10 meters. The usual setup is a 200 m drill collar section, and a drill pipe section of 1000 m-9000 m.

The program places the drill string inside the entered well trajectory and calculates the TVD, MD, horizontal displacement and deviation angle for every mid point in the string. It is not possible to run the program having a drill string that is longer than the wellbore. If this is attempted, an error message is displayed and the running of the program is terminated.

Other string data that must be entered is the material density, string diameters and Young's modulus of the string material. The cross section areas are then calculated, giving the basis for the string calculations.

3.1.3 Other data

Fluid density must be entered to include buoyancy, and start/stop simulation time must be specified for the program to start and stop the calculations at a desired time. The program then calculates the time step between every calculation based on the space step between segment mid points and speed of sound, and simulates the string movements one time step at a time. It is also possible to enter a specific time where the program starts sampling of the data. The simulation before the sampling start is then regarded as a "break in" of the string, which may be necessary in certain circumstances.

3.2 Assumptions

To be able to simulate the string movements effectively some assumptions have been made:

- Fluid:
  - The string is assumed to be closed at the end, not allowing any fluid flow in or out (this could affect the fluid velocity).
  - The changes in fluid velocity due to piston effect are neglected (when the string moves down, the string may displace some of the fluid, increasing the relative velocity between the fluid and the string. The opposite may happen when the string moves up).
  - It is assumed that the same mud is inside and outside the drill string.
  - The density of the fluid in the well is uniform and does not change with depth and temperature.
  - The fluid is incompressible

- Drill string calculations:
  - When a mid point moves up and down, a change of deviation angle are neglected. The angle used for every mid point is the same as in the beginning of the simulation.
  - The gravity is elongating the drill string slightly, possible giving a slight change in deviation angle of every mid point. This change in deviation angle is neglected.

- Drill string set-up:
  - The equipment in the BHA is assumed to have the same properties as drill collars (density, speed of sound, etc.). This may not be the case in a real set-up since equipment as MWD, LWD and RSS contains quite a lot of electronics. The equipment in the BHA is, however, in most cases only a fraction of the drill string as a total, and would most likely not have a large affect on the results.

- The drill string is assumed to be lifted far enough from the bottom of the well during a connection to ensure that the bit never touches the bottom, even with large drill string movements.
- The connections and the transition from drill pipes to BHA are assumed to be sudden with no upsets (a gradual transition from drill pipes to drill collars are not taken into account since the length of a segment is much larger than the length of a tool joint).
- The larger outer diameter of the drill string tool joints is neglected.

- Well paths:
  - The well trajectories are assumed to be two-dimensional, and not three-dimensional. A three-dimensional well trajectory would be more difficult to model, and is assumed to not have a large impact on the results.
  - The casings in the wellbore are suspended from surface which means that there are no liners in the well.
  - Open hole below the casing setting depth is assumed

- Friction:
  - The viscous friction is modeled as linear friction.
  - The drill string is assumed to not touch the wellbore wall in a vertical section, resulting in no contact friction.
  - The reduced or increased friction force in bends due to axial force pressing the string against the wellbore wall is neglected.
  - The extra friction force when the mud is squeezed past the drill bit and BHA when the drill string moves up and down is neglected.

3.3 Dividing the string into segments

To be able to simulate the drill string movements, the drill string is divided into segments. The length of every segment can be changed in the program, but is set to be the length of a drill pipe/drill collar which is 10 meters. The length of the drill string is then adapted to the segment lengths, so that the string is always divided into an integer number of segments.

The top and bottom segments are half segments, while the rest are whole segments. In order to keep track of the different segment mid points, they are counted from the top and downwards. N is the number of whole segments in the string (two half segments are counted as one whole segment), and N+1 are the total number of segment mid points.

The number of whole drill pipes in the string are NCB (the two half pipes at each end is counted as one whole segment). The mid point located at the transition between the drill pipes and drill collars are NCB+1.

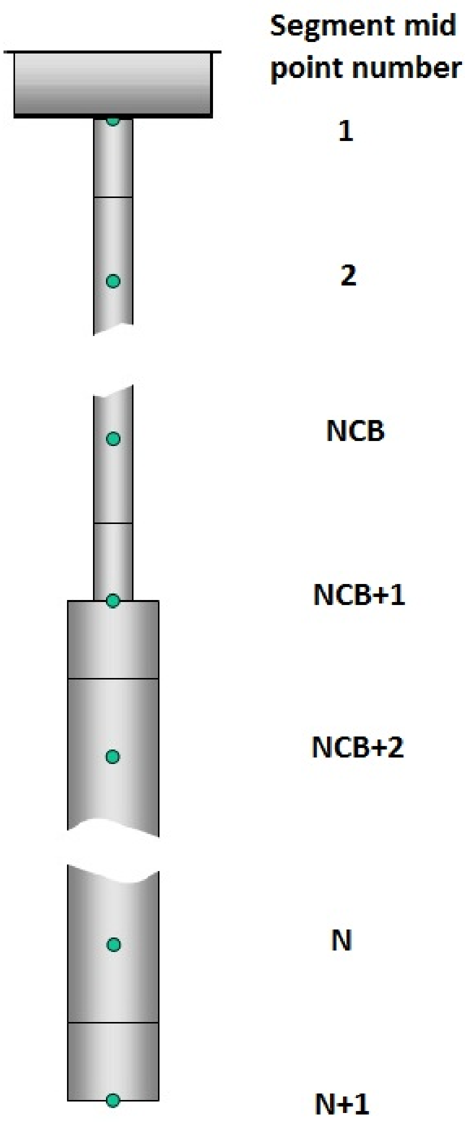

Figure 3.1: N is the number of whole pipes and segments in the string. NCB is the number of drill pipes in the string (the first drill pipe is counted as one drill pipe)

3.4 Forces acting on the string

There are three forces acting on the string; gravity, buoyancy and friction. All these forces are acting along the whole string, and can be calculated for every segment.

3.4.1 Gravity

Gravity is acting on every part of the string, and pulls the segments downwards with a force proportional to the mass of the segments. The gravitational force G_j of a segment number j with a mass m_j is given by the equation below:

$$G_j = m_j g \cos \alpha_j \tag{3.4.1}$$

where g is the gravity constant and α_j is the deviation angle.

The gravitational force is elongating the string, but also pulling the string towards the wellbore wall in a deviated well, giving contact friction. Equation (2.7.15) gives the equation for a string of constant cross section hanging in air, only affected by gravity. The gravity is given by the term g Δt^2, as the mass is conveniently eliminated from the equation.

$$XN_j = X_{j-1} + X_{j+1} - XG_j + g\Delta t^2 \cos \alpha_j \tag{3.4.2}$$

The mass of the segment is not including the mass of the tool joints. If one wish to include the mass of the tool joint in a string of constant cross section, the equation becomes:

$$XN_j = X_{j-1} + X_{j+1} - XG_j + \frac{\Delta t^2}{m_j}\left[m_j g - \Delta m_j g \left(1 - \frac{\rho_m}{\rho_s}\right)\right] \cos \alpha_j \tag{3.4.3}$$

where Δm_j is the mass of the tool joint.

The mass of the tool joints are however not included in the calculations.

3.4.2 Buoyancy

Buoyancy must be calculated for every segment mid point to be able to calculate the normal force along the string. The extra weight and volume of the tool joints are not taken into account when buoyancy is calculated. For convenience, it is assumed to have the same mud inside and outside the string when buoyancy is calculated.

Buoyancy is included in the calculations in two ways as the string is not always vertical. The buoyancy forces acting on the sides of the drill string is included in the normal force term by multiplying the normal force for every segment mid point with the buoyancy factor $(1-\frac{\rho_m}{\rho_s})$, where ρ_{m} is the density of the mud in the well and ρ_{s} is the density of the string material (density of steel). The equation for the normal force where buoyancy is included for segment number j is shown below:

$$N_j = m_j g \sin\alpha_j \ (1-\frac{\rho_m}{\rho_s}) \tag{3.4.4}$$

The buoyancy forces acting on the end pieces of the drill string is added to the segments it applies for, which is where there is change of cross section (transition between drill pipes and drill collars) and at the end of the string (drill bit). The buoyancy forces can easily be calculated by multiplying the area with the pressure given by the mud column. The complete equation for these two segment mid points is given in section 3.6.

3.4.3 Friction

The friction forces acting along the string is a combination of viscous and contact friction. By adding the contact friction term from equation (2.7.29) to equation (2.7.34), a combined equation for a segment of constant cross section is obtained below. The equation includes both viscous and contact friction:

$$XN_j = (1-\varepsilon)X_{j-1} + (1-\varepsilon)X_{j+1} - (1-2\varepsilon)XG_j \pm \frac{\Delta t^2}{m}\mu N_j \tag{3.4.5}$$

3.4.4 Combining the forces into one equation

By inserting the normal force from equation (3.4.4) into equation (3.4.5) we are able to obtain the final standard equation for a string of constant cross section including both viscous and contact friction, gravity and buoyancy:

$$XN_j = (1-\varepsilon)X_{j-1} + (1-\varepsilon)X_{j+1} - (1-2\varepsilon)XG_j + g\Delta t^2 \cos\alpha_j \pm g\Delta t^2\ \mu\ \sin\alpha_j\ (1-\frac{\rho_m}{\rho_s}) \tag{3.4.6}$$

3.5 Boundary conditions in time

There are several ways to initiate the drill string simulations. The forces acting on the string can either be gradually assigned to the string, or all forces can be assigned to the string from the start. If the simulations are started when the string is relaxed (no forces are acting on it), some initial waves may be induced in the string due to sudden introduction of gravity forces. The gravity force will elongate the drill string and induce waves, and it may take some time before these waves die out. The induced waves can therefore have an effect on the string movements for some time.

One solution may be to leave out the gravity that elongates the string from the calculations. Gravity is however included when the normal force is calculated, making the calculation of contact friction possible. Leaving out gravity will only affect the elongation of the string, and will also eliminate the initial waves induced by gravity. The advantage of leaving out gravity is that simulations can start immediately. The disadvantage is that the calculations of the string will not be completely accurate, as the position and angle of the segments will be slightly off.

Another scenario is to assign all forces acting on the string from the beginning, set the top of the string in motion, and then simulate the drill string movements for a long enough time allowing the initial waves to die out. When it is assumed that the initial waves have disappeared, the sampling can begin.

A last scenario is to start with a relaxed string hanging from a stationary drill floor (which can represent the top drive with heave compensator). Gravity is then assigned to the drill string, along with a linear friction. At this point the linear friction is adjusted to bring the drill string to rest as quickly as possible. Without linear friction is to stop the string from oscillating as gravity start acting, the string would keep on oscillating forever. The friction must therefore overcome the oscillations, and at the same time the friction must be small enough allowing the string to settle. When the string has settled, the simulations begin, where contact friction, buoyancy and the correct viscous friction are assigned to the drill string. The top of the drill string is set to move according to the chosen wave-function.

All the three scenarios mentioned above are tested, where the last one seems to be the better one and is therefore used in the simulations. The linear friction is set to gradually decrease in steps until it reaches the value of the correct viscous friction. Figure 3.2 shows a string being elongated by gravity, while Figure 3.3 is an example of a string being assigned all the other external forces after it has been elongated by gravity.

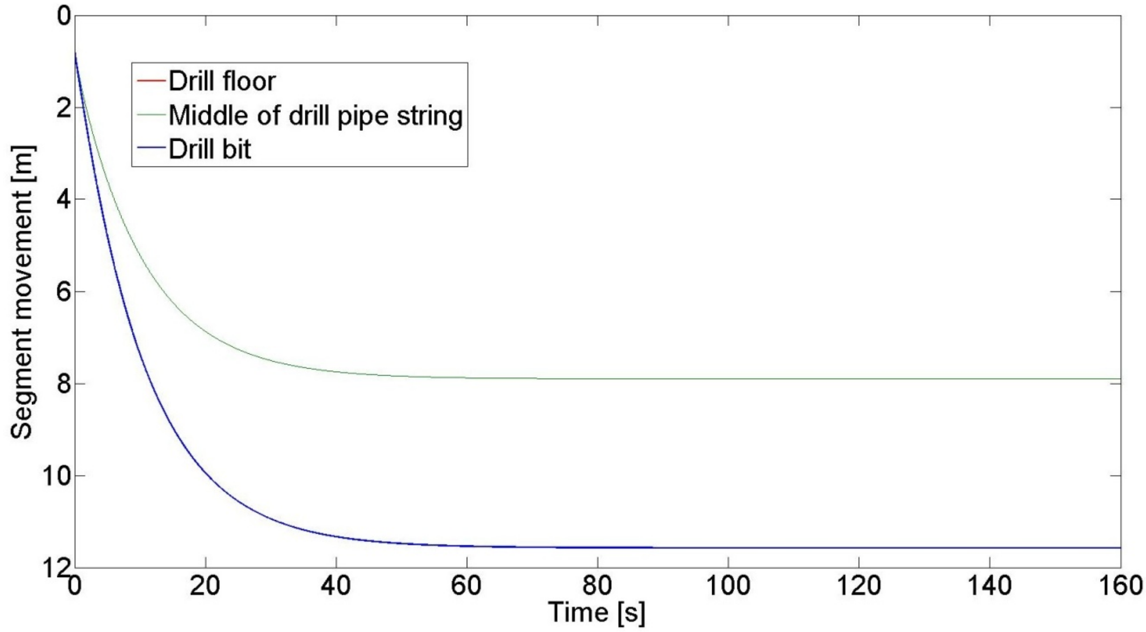

Figure 3.2: A 7200m drill string elongated by gravity. The drill bit is displaced approximately 12 meters due to gravity. Buoyancy is not included.

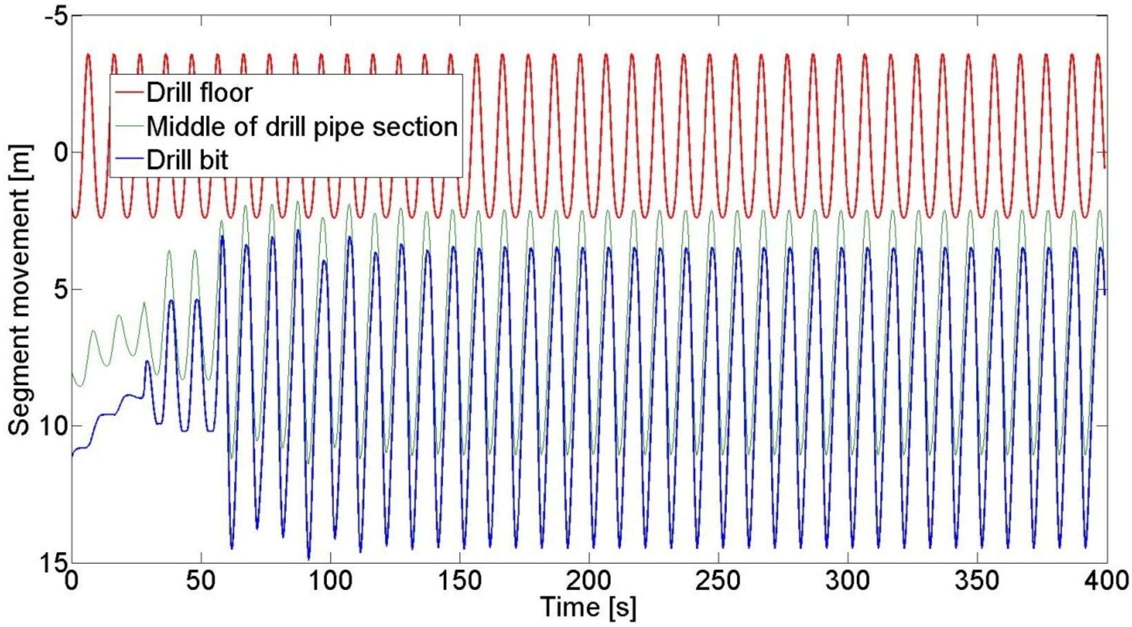

Figure 3.3: A 7200m drill string being assigned external forces as friction and buoyancy. The drill string starts climbing due to the introduction of buoyancy and the movement of the top segment. The viscous friction is decreased in steps, and the correct viscous friction is assigned at approximately 145 seconds. The initial waves due to the introduction of external forces disappear after approximately 80 seconds.

3.6 Boundary conditions in space

There are three different boundary conditions in space downward the string:

- The top segment is a forced end.
- There is a change of cross section at the transition between drill pipes and drill collars.
- The string is terminated with a free end.

Gravity and friction along the string affects every part of the string and is not considered a boundary condition in this context.

3.6.1 The top segment

The top of the string is a half segment where the mid point is located at the drill floor level. This location of the mid point is convenient as the movement of the drill floor is coupled to the drill string at this point, making calculations more exact. The top segment is modeled as a forced end, moving according to the drill floor motions, transmitting the movements downward.

As explained in section 2.1.4.1, this is also what is happening during a connection, and is quite realistic. However, when it comes to the movements of the drill floor, every semi-sub has its own movement characteristic, and it has unfortunately not been possible to obtain any movement characteristics data for a semi-sub. This is probably not critical for the simulations, as it is possible to base the rig movements on the shape of ocean waves. A semi-sub will probably move with a much longer time period compared to the time period of ocean waves, but as both the period and the amplitudes are changed throughout a simulation, the effect of these two variables can be revealed.

Ocean waves can be simulated by sinus functions. In real life, ocean waves have more pointed crests and rounded troughs compared to a single sinus function. The solution to successfully imitate the shape of ocean waves is adding a phase shifted sinus function to another sinus function, see equation below:

$$X(t) = A_1 \sin(\omega t) + A_2 \sin(2\omega t + \varphi) \qquad (3.6.1)$$

where A_1 and A_2 is amplitudes, ω is the angular frequency, t is the time, and φ is the phase shift.

The angular frequency is given by the equation below:

$$\omega = 2\pi/T \qquad (3.6.2)$$

where T is the time period.

The heave movement of the rig is here assumed to follow the asymmetric wave curve given by equation (3.6.1), where $A_2 = A_1/5$ and $\varphi = \pi/2$. The main point of this is that any asymmetric heave movement of the rig will probably have at least these two wave components. Possible resonances in the drill string due to the second wave component will then be seen. One graphical representation of equation (3.6.1) is shown in the figure below.

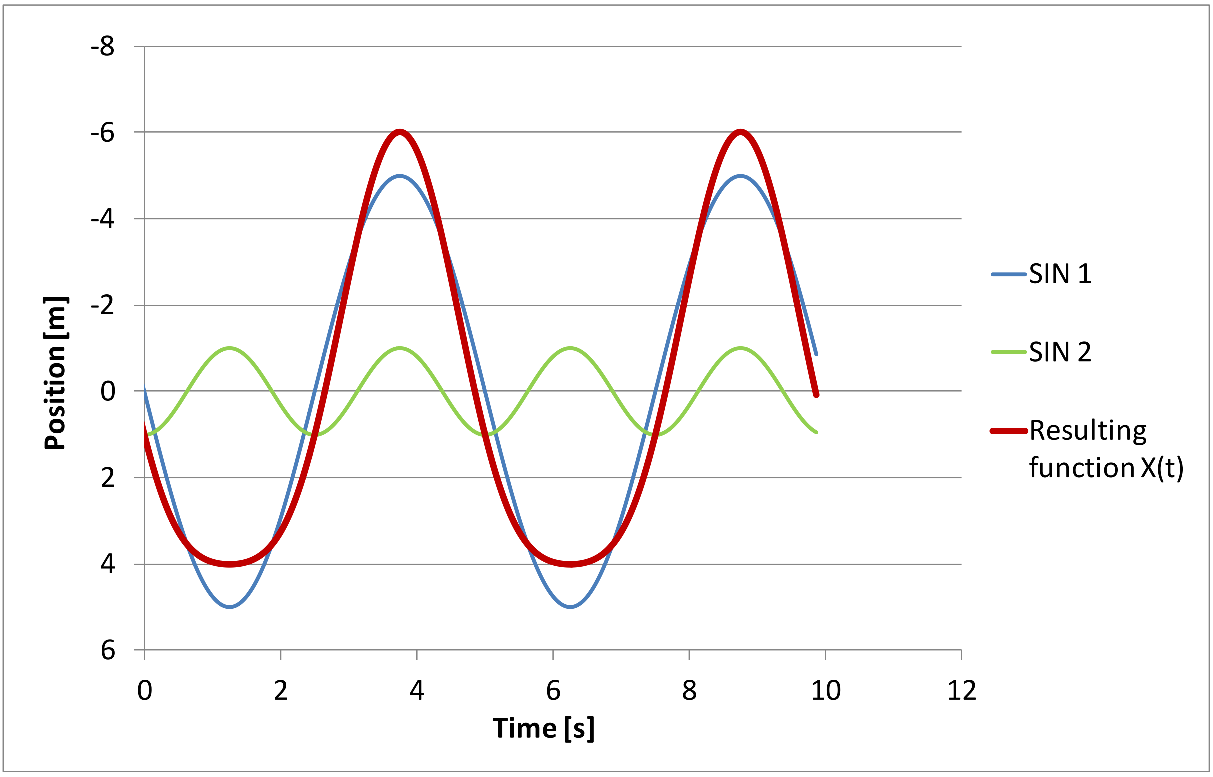

Figure 3.4: The red line shows the resulting function of two combined sinus functions imitating ocean waves. The ocean waves have a sharp crest and a round trough as achieved in the figure. The following values are used in equation (3.6.1) to get the resulting function: $A_1 = 5$, $A_2 = 1$, $T = 5$, $\varphi = \pi/2$.

This type of function is the base of the drill string simulations, making it easy to investigate how the top movements are influencing the drill string oscillations during a connection. The total vertical movement of the top segment is equal to $2A_1$, which is equal to 10 meters in this case. The relationship between A_1 and A_2 throughout the simulations have been set to $A_1 = 5A_2$, making sure that the shape of the waves stays the same. The results of the simulations will be presented in plots with $2A_1$ on the x-axis. It is therefore important to notice that the numbers on the x-axis represents the total wave amplitude from trough to crest.

3.6.2 Change of cross section

The segment at the transition between drill pipes and drill collars (segment mid point number NCB+1) is given a value shown by equation (2.7.6), where the change of cross section happens at the segment mid point. By including viscous friction, gravity and buoyancy, we obtain the final equation for the transition segment:

$$XN_{NCB+1} = (1-\varepsilon)\frac{2A_b}{A_b+A_v}X_{NCB} + (1-\varepsilon)\frac{2A_v}{A_b+A_v}X_{NCB+2} - (1-2\varepsilon)XG_j + g\Delta t^2\cos\alpha_j + KC \pm g\Delta t^2\,\mu\,\sin\alpha_j\,\left(1-\frac{\rho_m}{\rho_s}\right) \tag{3.6.3}$$

where A_b is the cross section of the drill pipe, and A_v is the cross section of the drill collar. KC is the negative buoyancy force due to the larger outer diameter of the drill collars compared to the drill pipes. KC is given below:

$$KC = 2\frac{M_v-M_b}{M_v+M_b}\frac{g\Delta t^2\,h_{NCB+1}\,\rho_m}{\Delta z\,\rho_s} \tag{3.6.4}$$

where M_v is the weight per meter of drill collars, M_b is the weight per meter of drill pipes, h_{NCB+1} is the true vertical depth of segment NCB+1, ρ_m is the density of mud, ρ_s is the density of steel, and Δz is the length of a segment.

3.6.3 The termination of the string

The string is terminated with a free end. The end segment is a half segment with the mid point at the very end of the drill string, ensuring that the calculation of movement at this numeric mid point gives the movement of the drill bit.

It is important to make sure of the end segment not hitting the bottom of the well. If this were to happen, the end segment is no longer regarded as a free end and the results from the calculations would be incorrect. If one would want to calculate the movements after an impact, the end segment should be regarded as a fixed end until it bounces off the bottom and becomes a free end again. This can be calculated, but is regarded as irrelevant in this setting.

The free end equation used in the program is equation (2.7.3), where the end is a half segment. All the forces as gravity, friction and buoyancy are also included in the equation.

The equation for the bottom end segment with mid point number N+1 with all external forces is:

$$XN_{N+1} = (1-\varepsilon)\,2X_N - (1-2\varepsilon)XG_{N+1} + g\Delta t^2 \cos\alpha_j - KB \pm \Delta t^2\,\mu\,g \sin\alpha_j\,\left(1-\frac{\rho_m}{\rho_s}\right) \tag{3.6.5}$$

where KB is the buoyancy force due to the pressure acting onto the end of the string. KB is given below:

$$KB = 2\frac{g\Delta t^2\, h_{N+1}\,\rho_m}{\Delta z\,\rho_s} \tag{3.6.6}$$

where h_{N+1} is the true vertical depth of the segment, ρ_m is the density of mud, ρ_s is the density of steel, and Δz is the length of a segment.

3.7 Determination of the parameters

Due to a large variety of parameters to be changed in a well, it is important to point out the ones that might have a relevant impact on the string movements. It is, however, not possible to take all parameters into account, but the most important is believed to be the drill floor movements (the top string movements) and the well path along the drill string length.

3.7.1 Drill floor movements

The amplitude of the top string movements is believed to heavily decide the movements of the drill bit. A large amplitude is expected to have a larger impact on the drill bit movements than a smaller amplitude.

The time period is expected to influence the drill strings in different ways. If the time period is the same as the natural frequency of the drill string, the string may start responding with a much larger displacement compared to the other string lengths.

3.7.2 Well path

It is required to use simple well paths in the simulations to be able to reveal some effects that could have been overlooked due to other effects in a more complicated well path. The simplest well path the simulations will be investigating is a straight vertical well with different depths. Different depths can reveal string resonance and the effect of viscous friction. To see effects due to contact friction, the well must have a deviated section. A KOP is then placed at 500m down the well, where the angle radius of the build section is set to 500m. The length of the well is then changed along with the deviation angle. It is worth to notice that the length of the build section will change with different deviation angles. A larger deviation angle gives a longer build section. The casing setting depth can also be changed to see if there are any significant effects of the changed contact friction.

To further complicate the well path, a build/drop section along with a hold section can be added. As long as the well is longer than the drill string, the drill string is automatically fitted into the well, regardless of the space beneath the drill bit. The space beneath the drill bit will not have any effects on the drill string simulations, but this space will be considered in section 5.

3.7.3 Table of parameters

The parameters used in the numerical simulations and pressure calculations are presented in the table below.

Parameter	**Value**	**Unit**
Drill string		
Drill pipe string length	1000 - 9000	m
BHA length	200	m
Total drill string length	1200 - 9200	m
Drill pipes		
Length of one drill pipe	10	m
Outer diameter	5	inches
Inner diameter	4	inches
Drill collars		
Length of one drill collar	10	m
Outer diameter	8	inches
Inner diameter	2,8125	inches
Steel properties		
Steel density	7850	kg/m^3
Young's modulus for steel	$210*10^9$	Pa
Yield strength	$9,3079*10^8$	Pa
Drill bit		
Outer diameter	12	inches
Junk area	0,01110	m^2
Casing		
Inner diameter	12	inches
Mud		
Mud density	1200	kg/m^3
Mud viscosity	5	cP
Friction coefficients		
Static steel-steel	0,20	
Dynamic steel-steel	0,24	
Static steel-rock	0,30	
Dynamic steel-rock	0,36	
Direct simulation data		
Numeric space step length, Δz	10	m
Start time	0	s
Start sample time	200	s
End simulation time	400	s
Heave parameters		
Heave amplitude (A1)	0,5 - 3	m
Heave period	10 - 20	s
Pressure calculations		
Nozzle factor, C_d	0,95	

Table 3.1: An overview of the parameters used in the Matlab program and pressure calculations. Some of the parameters as string length are changed throughout the simulations while others are constant.

3.8 Data sampling and presentation of the results

The main focus of this thesis is the movement of the bottom segment, but the global behaviour of the string is also of interest in order to be able to understand the overall string behaviour. It is therefore possible to plot the movement of selected segment positions throughout a simulation, making the string behaviour visual. Figure 3.3 is an example of a plot of the displacement of three different segment mid points.

Throughout a simulation the heave amplitudes and time period is continuously changed in steps. The movement of the drill bit segment is monitored and sampled throughout the simulation. The maximum amplitude and the maximum velocity of the drill bit is especially of interest.

3.8.1 Maximum drill bit velocity

The maximum velocity is relevant for the estimation of surge and swab effects. A large movement of the drill bit is not a threat in terms of pressure fluctuations as long as the velocity of the bit is relatively low. On the other hand, a high bit velocity is not a threat in terms of pressure fluctuations as long as the drill bit movement is small. The maximum values must therefore be seen in a larger setting.

The calculations are not always resulting in smooth lines in a micro-scale. The result is that the bit velocity within two time-steps (~0,004 s) can be very high, while the velocity in a more normal time perception (~0,5 s) is relatively low. The sampling of the drill bit velocity is therefore calculated as the average velocity over a time period of approximately 0,5 seconds. The maximum velocity is usually obtained around the point of equilibrium at which the drill bit is oscillating about.

3.8.2 Maximum drill bit movement

The maximum drill bit movement is relevant when seen in context with the drill bit speed. Both the minimum and the maximum value of the drill bit position are sampled throughout the simulation, where the maximum drill bit amplitude is the difference between the two. In a situation where the drill string oscillations are regular, the maximum drill bit amplitude will be a good quantification of the maximum drill bit movement. On the other hand, if the drill string oscillations are irregular, this might not be a good quantification of the maximum movement as the two values can be sampled within a large interval. It must always be seen in a larger context. The majority of the results from the simulations show regular movements. A regular movement can be seen in Figure 3.3. An irregular movement can be seen in the figure below.

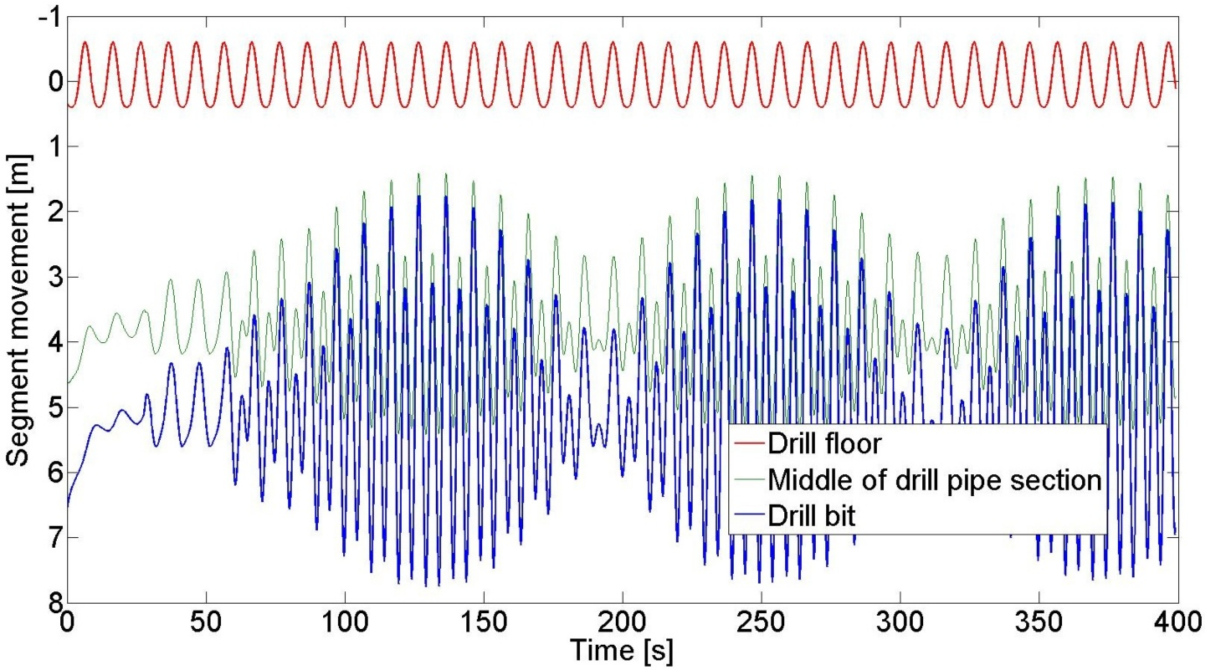

Figure 3.5: The drill string is oscillating with irregular movements on a small scale. On a large scale on the other hand, the movements can probably be said to be regular.

4. RESULTS

The simulations have been performed with nine different drill pipe string lengths (1000 m-9000 m), in both vertical and deviated wells. The BHA was given a constant length of 200 m regardless of total string length, and comes in addition to the drill pipe string length. In a deviated well, the KOP was set to 500 m below the top of the string, with a build radius of 500 m. The casing setting depth was set to 2/3 of the total drill string length.

Preliminary simulations showed that a time period of 5 seconds with heave amplitudes up to 5 m would shake the drill string violently causing the string to break regardless of string length. A time period of 5 seconds is too short to resemble the movements of a semi-sub, and the results from the 5 second time period are therefore left out. A heave amplitude of 5 m is also regarded as a too high amplitude to resemble a semi-sub, so the maximum heave amplitude was set to be 3 m. The total heave amplitude, which is the peak to peak movement, will then be 6 m.

The results are presented by evaluating the drill bit velocities and drill bit amplitudes for each heave period and well path. The x-axis in the maximum velocity plot represents the heave amplitude, which is measured from crest to trough, while the y-axis represents the maximum velocity measured at the given heave amplitude and time period. When it comes to the maximum drill bit amplitude plot, both the x-axis and the y-axis represents the maximum amplitude from crest to trough.

The lengths presented in the plots are the length of the drill pipe section, but the length of the BHA which is 200 m comes in addition to the drill pipe section. A drill string presented as 1000 m in the plots is therefore actually having a total length of 1200 m.

4.1 A vertical drill string

There is no contact friction in a vertical string, and the string is therefore expected to have larger amplitudes compared with the strings in deviated wells.

4.1.1 10 second time period

The drill bit amplitudes and velocities with a 10 second period are presented in the figures on the next page.

The maximum drill bit amplitudes in Figure 4.1 are quite regular in all cases, except for the 5000 m and the 9000 m drill strings. The 5000 m string has the largest amplitude, and seems to have a much more violent behaviour compared to the other strings. The 5000 m string broke at the top segment due to large load at a heave amplitude of 4 m. The maximum drill bit amplitude achieved before the 5000 m string broke was approximately 22 meters. The second largest amplitude was achieved by the drill bit at the 9000 m string just before it broke at a heave amplitude of 6 m. The maximum bit amplitude of the 9000 m string was 19,3 meters, but the behavior was not nearly as violent as the 5000 m string.

In general we can see that the drill bit at the end of a longer drill string was oscillating with larger amplitude as compared to the shorter drill strings. The distribution of the drill string amplitudes at a heave amplitude of 6m is within an interval of a little less than 16 meters.

When it comes to the drill bit velocities we can see that it is also behaving as a linear function of the input heave in most cases. The drill bit velocities follows the drill bit amplitudes to a large extent, but there are some exceptions. The 6000 m drill string achieved a higher velocity than both the 8000 m and the 9000 m string.

The bit at the 5000 m string achieved a maximum velocity of approximately 12,4 m/s before it broke. The second largest bit velocity was achieved by the bit at the 6000 m string, and was 7,2 m/s at a heave amplitude of 6 m.

For both the bit amplitude and velocity we can see a reduction in the slope just before the 5000 m and 9000 m strings broke.

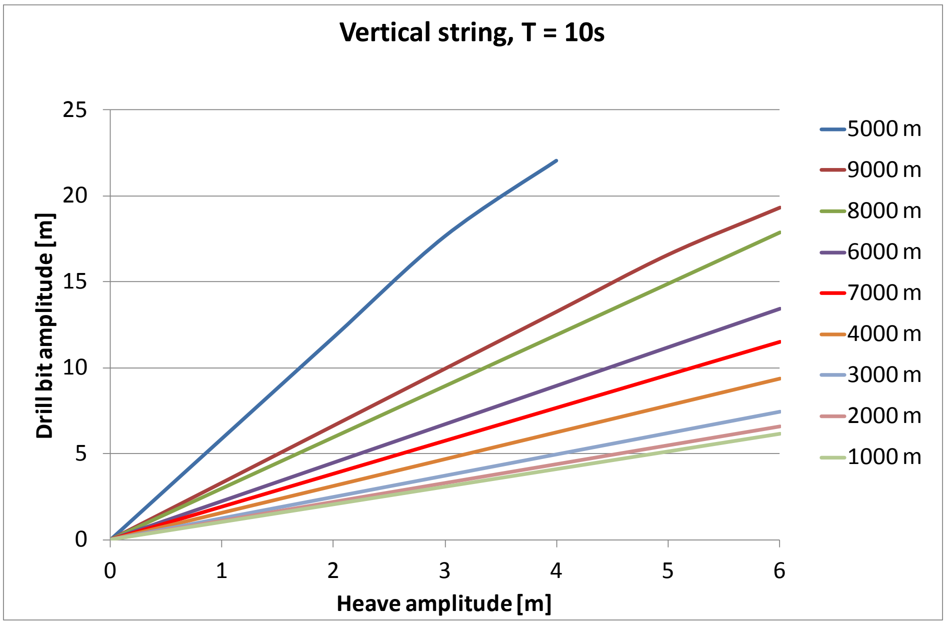

Figure 4.1: Drill bit amplitude vs. heave amplitude. A vertical drill string oscillating due to a heave-movement with a time period at 10 s at the top of the string. The 5000 m string broke at a heave amplitude of 4 m, while the 9000 m string broke at a heave amplitude of 6 m.

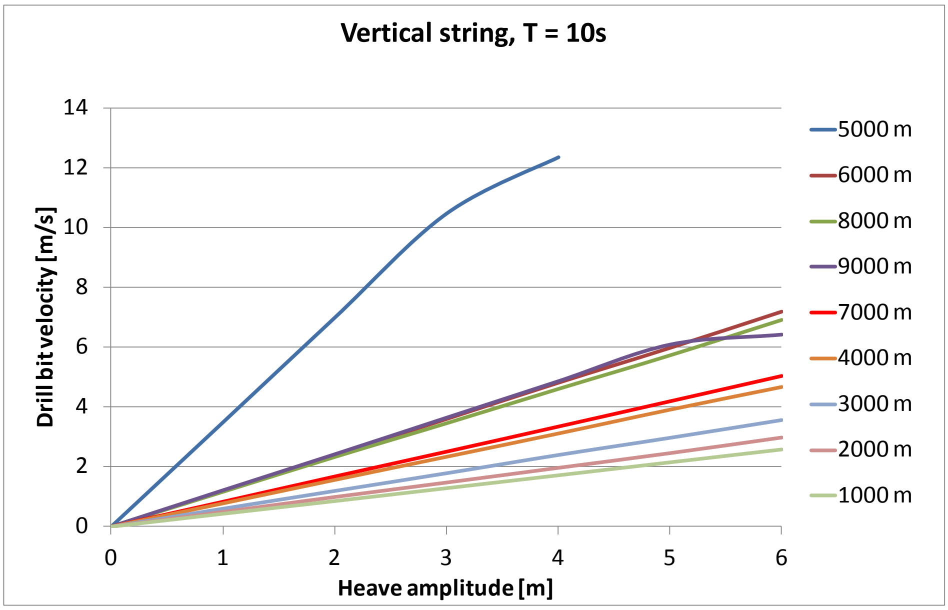

Figure 4.2: Drill bit velocity vs. heave amplitude. A vertical drill string oscillating due to a heave-movement with a time period at 10 s at the top of the string. The 5000 m string broke at a heave amplitude of 4 m, while the 9000 m string broke at a heave amplitude of 6 m.

4.1.2 15 second time period

The drill bit amplitudes and velocities achieved with a 15 second period are presented in the figures below.

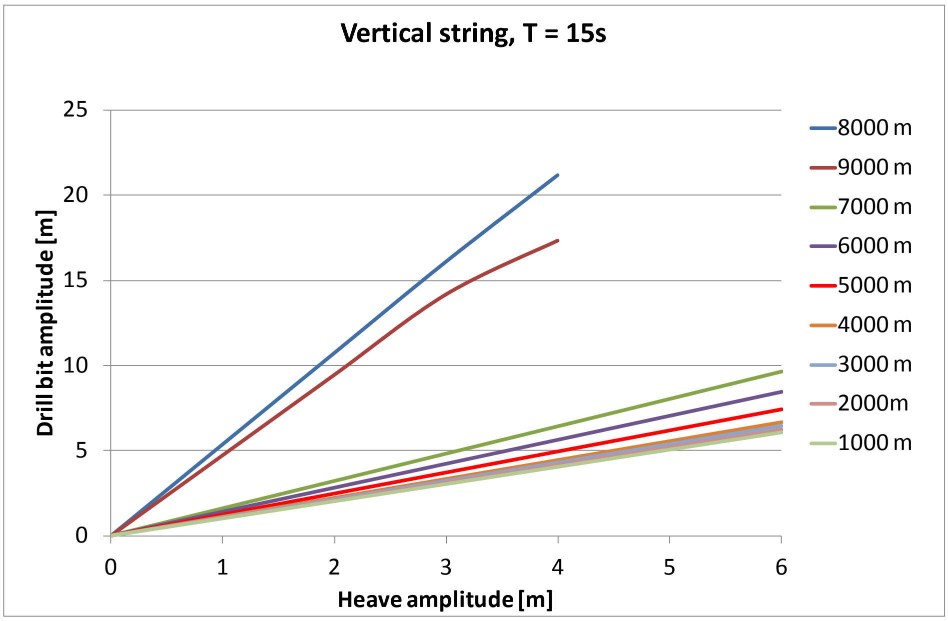

Figure 4.3: Drill bit amplitude vs. heave amplitude. A vertical drill string oscillating due to a heave-movement with a time period at 15 s at the top of the string. Both the 8000 m and the 9000 m string broke at the top at a heave amplitude of 4 m.

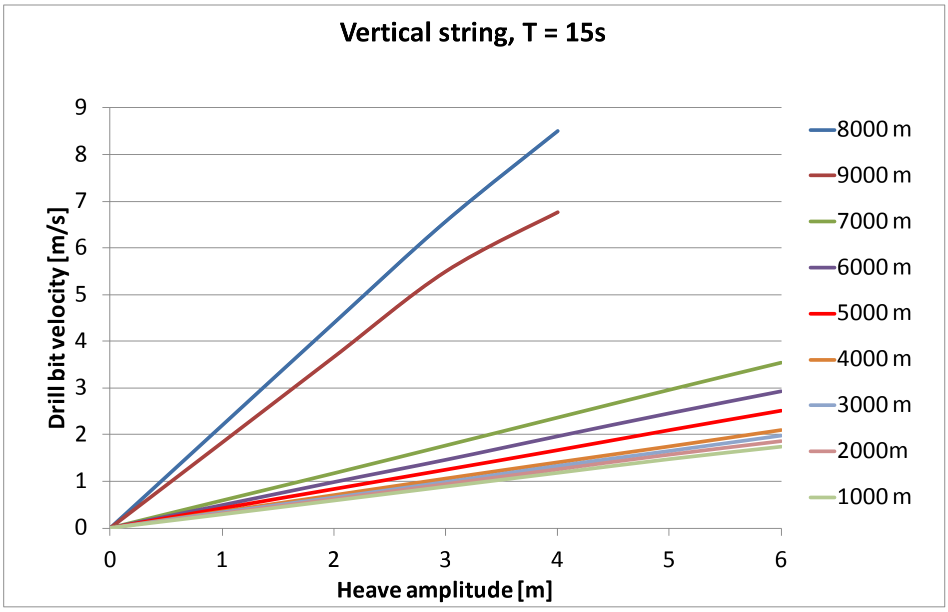

Figure 4.4: Drill bit velocity vs. heave amplitude. A vertical drill string oscillating due to a heave-movement with a time period at 15 s at the top of the string. Both the 8000 m and 9000 m string broke at heave amplitude of 4 m.

The drill bit amplitudes are also here acting linearly in most cases, and the amplitudes have generally diminished compared to T=10 s. There are however some exceptions. In

Figure 4.3 we can see that the 5000 m string is now acting linearly all the way, having a much smaller amplitude compared to T=10 s. We can also see that the 8000 m and 9000 m strings broke at a heave amplitude of 4 m.

The maximum amplitude for the 8000 m string was approximately 21,2 meters, while the maximum amplitude for the 9000 m string was approximately 17,3 meters. As for T=10 s, we can also here see a decline in the slope just before the strings breaks. For the other drill string lengths we can see that the amplitudes is roughly halved compared to the amplitudes at T=10 s, and the distributions of their bit amplitudes are also less, approximately 4 meters at a heave amplitude 6 m.

From Figure 4.4 we can see that the drill bit velocities corresponds perfectly to the drill bit amplitudes. All the strings behave linearly, except where the strings break as we have seen before. The largest maximum bit velocity is achieved for the 8000 m string just before it broke, and is approximately 8,5 m/s. The bit at the 9000 m string reached a speed of 6,8 m/s before it broke. The maximum bit velocity of the 7000 m string is 3,5 m/s.

4.1.3 20 second time period

The drill bit amplitudes and velocities achieved with a 20 second period are presented in the figures on the next page. The drill bit amplitudes and velocities are here acting linearly in all cases, and the amplitudes are quite similar compared to T=15 s. In Figure 4.5 we can see that the difference in amplitudes is more similar for short strings compared to the longer strings. The distance between the lines is especially increasing when the string reaches a length of more than 7000 m. We can also see that the amplitudes decrease with string length. The distribution of the drill string movements at heave amplitude equal to 6 m is a little less than 3 meters. The larges drill bit amplitude is achieved by the 9000 m string, where the bit has a maximum amplitude of 8,6 m.

In Figure 4.6 we can see that the drill bit velocity has generally decreased compared to the velocities at T=15s. The bit velocities correspond perfectly with the bit amplitudes where the string having the largest amplitudes, also achieves the highest velocities.

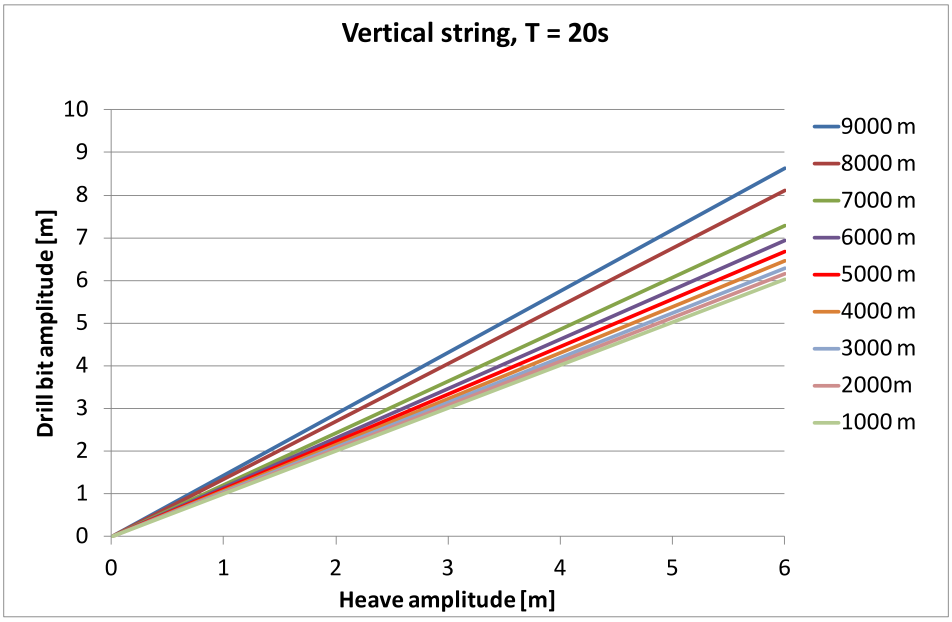

Figure 4.5: Drill bit amplitude vs. heave amplitude. A vertical drill string oscillating due to a heave-movement with a time period at 20 s at the top of the string. All strings behave linearly.

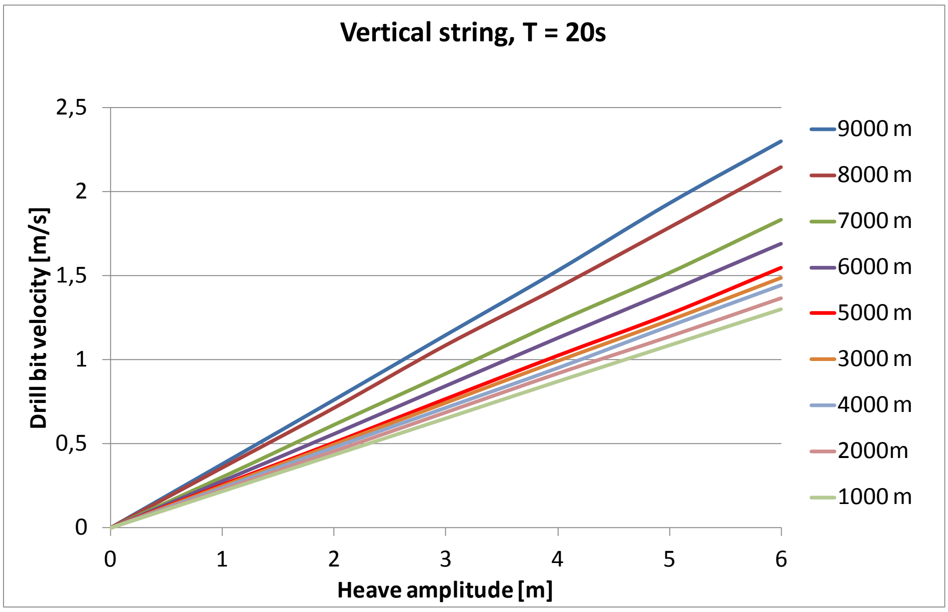

Figure 4.6: Drill bit velocity vs. heave amplitude. A vertical drill string oscillating due to a heave-movement with a time period at 20 s at the top of the string. All strings behave linearly.

4.2 A drill string deviating with 10 degrees below 500 m

As mentioned earlier, the KOP was 500 m, and the build radius was 500 m. This resulted in three different well sections: a 500 m vertical section, a build section of 87,3 m, and a 10 degrees deviated section of different lengths. A 1200 m string will have 42 % of the string in the vertical section, while a 9200 m string will have less than 6 % of the string in the vertical section.

4.2.1 10 second time period

From the figures on the next page we can see that the drill bit amplitudes and velocities have not changed significant compared to the vertical well at time period 10s. The largest difference is that none of the strings broke and some of the strings are not behaving linearly anymore. The 5000 m string amplitudes start off with a linear curve, but the gradient suddenly increases from a heave amplitude of 3m, and increases with a steady rate from there. The maximum drill bit amplitude for the 5000 m string was 21,6 m and occurred at a heave amplitude of 6 m. The drill bit at the 7000 m, 8000 m and 9000 m strings did not move at a heave amplitude of 1m, but they started to move at a heave amplitude of 2 m, where the movement increased steadily from there on. All strings shorter than 4000 m were acting linearly at all heave amplitudes.

From Figure 4.8 we can see that the drill bit velocity is corresponding quite well to the drill bit amplitudes. There is however some small bends on most of the drill bit velocity curves which means that the velocity is not acting linearly anymore. The drill bit velocities has generally decreased slightly, except from the bit in the 9000 m string which has increased by almost 1 m/s.

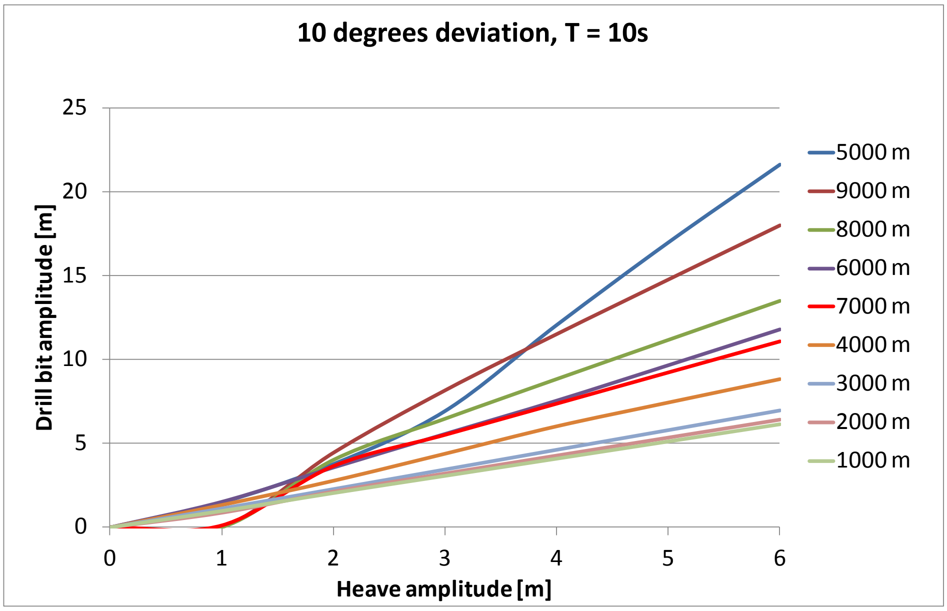

Figure 4.7: Drill bit amplitude vs. heave amplitude. A drill string with 10 degree deviation angle oscillating due to a heave movement with a time period at 10 s at the top of the string.

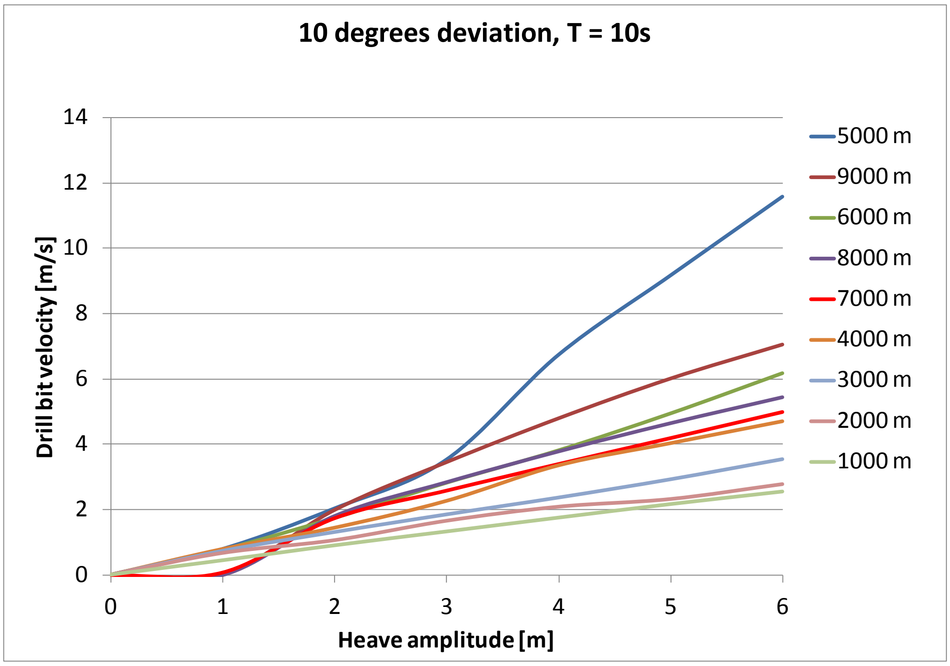

Figure 4.8: Drill bit velocity vs. heave amplitude. A drill string with 10 degree deviation angle oscillating due to a heave movement with a time period at 10 s at the top of the string.

4.2.2 15 second time period

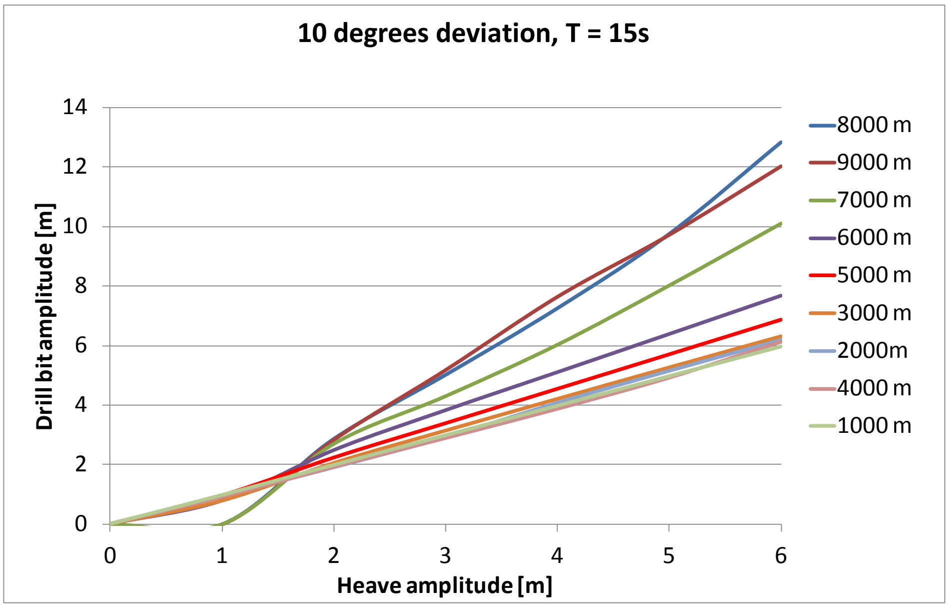

Figure 4.9: Drill bit amplitude vs. heave amplitude. A drill string with 10 degree deviation angle oscillating due to a heave movement with a time period at 15 s at the top of the string.

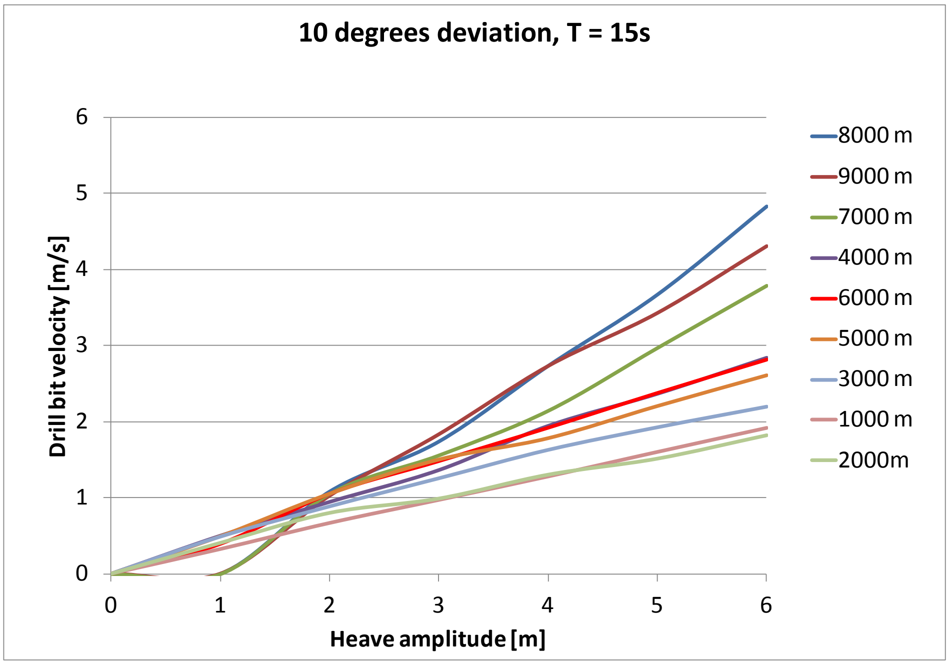

Figure 4.10: Drill bit velocity vs. heave amplitude. A drill string with 10 degree deviation angle oscillating due to a heave movement with a time period at 15 s at the top of the string.

In the figures above we see that all drill bit amplitudes have decreased compared to the drill bit amplitudes in a vertical well, and also compared to the strings having 10 degrees deviation with a 10 second heave period. The bit at the 7000 m, 8000 m and 9000 m strings are not moving at a heave amplitude of 1m, and the movement has decreased distinctly at a heave amplitude of 2 m compared to the movement at time period 10 s. The drill bits at the 5000 m string and shorter seems to behave linearly. The largest drill bit amplitude is achieved by the drill bit at the 8000 m string and is equal to 12,9 m at a heave amplitude of 6m.

When looking at the two figures above we can still see a correlation between the bit velocity and amplitude. As for the drill bit amplitudes, the highest drill bit velocity is achieved by the 8000 m string, and is 4,8 m/s at a heave amplitude of 6m.

4.2.3 20 second time period

In the figures on the next page we see that all drill bit amplitudes for the longest drill strings have generally decreased. The bit amplitudes of the 1000 m, 2000 m and 3000 m strings are still behaving linearly, while the bit at the 9000 m, 8000 m and 7000 m is still not moving at heave amplitude 1 m. A maximum bit amplitude of 8,1m is achieved by the 9000 m string at a heave amplitude of 6 m.

The drill bit velocities has almost halved for some of the bits compared to the 15 s time period. The correlation between bit velocity and amplitude is quite good, but we can see that the bit in the 6000 m string has the third lowest amplitude, but is maintaining the highest bit velocity at a heave amplitude of 6m compared to the shorter strings. A maximum bit velocity of 2,2 m/s is achieved by the 6000 m string at a heave amplitude of 6m.

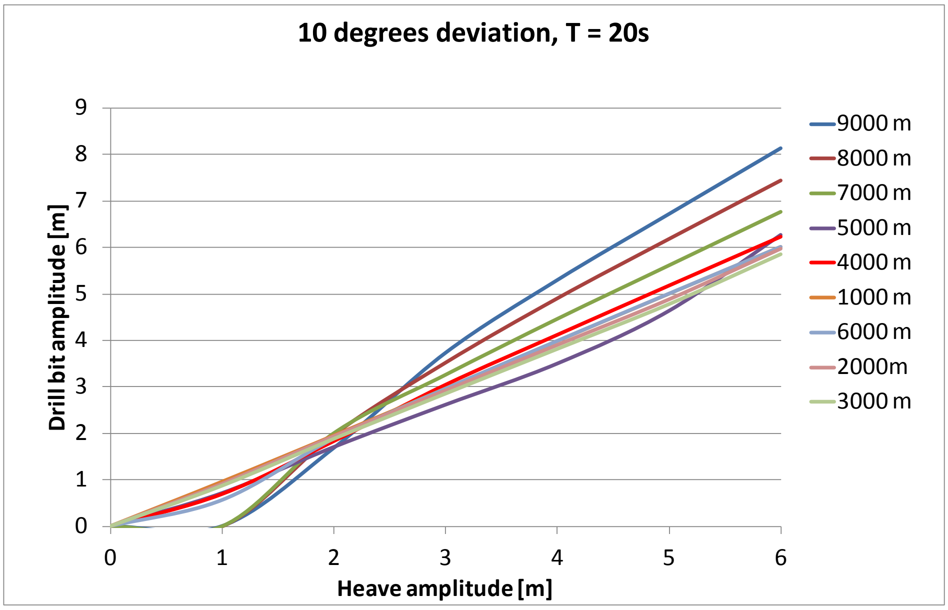

Figure 4.11: Drill bit amplitude vs. heave amplitude. A drill string with 10 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

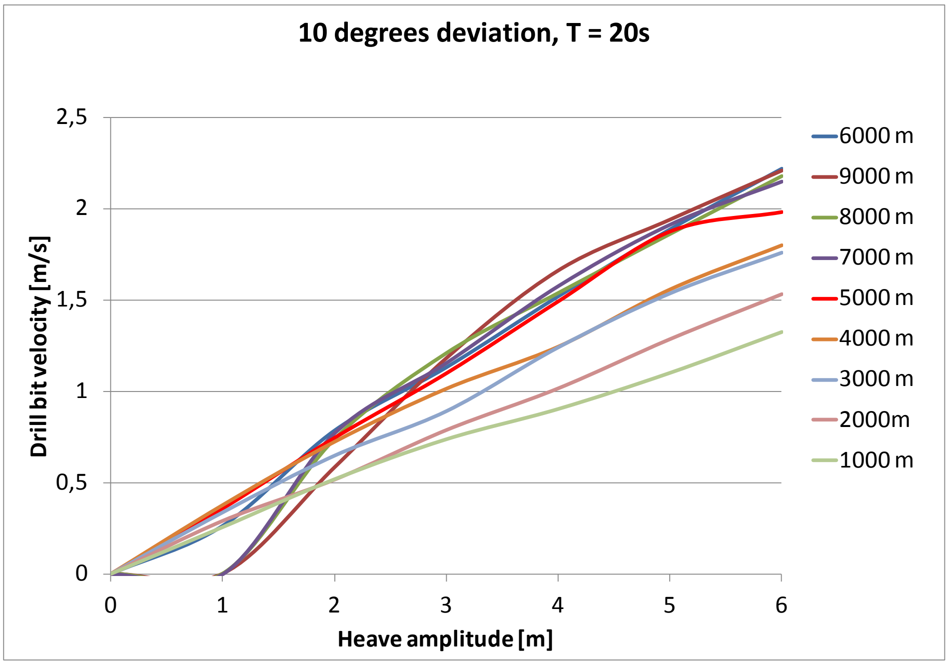

Figure 4.12: Drill bit velocity vs. heave amplitude. A drill string with 10 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

4.3 A drill string deviating with 20 degrees below 500 m

As mentioned earlier, the KOP was 500 m, and the build radius was 500 m. This resulted in three different well sections: a 500 m vertical section, a build section of 174,5 m, and a 20 degrees deviated section of different lengths.

4.3.1 10 second time period

In the figure on the next page we can see that it is now only the bit at the 1000 m string that responds with linear bit amplitude to the heave movement. The 2000 m and 3000 m strings are having a small dip around a heave amplitude of 1-2 m. We can also see that it is still the bits in the 9000 m and 5000 m strings that respond with the highest amplitudes at a heave amplitude of 3 m. A maximum bit amplitude of 16,4 m is achieved by the 9000 m string at a heave amplitude of 6 m. All the bits on the drill strings longer than 4000 m are stationary at a heave amplitude of 1 m, while all the bits at the drill strings longer than 6000 m are static at a heave amplitude of 2 m. We can generally see that the maximum amplitudes of the shorter are decreasing slightly at a heave amplitude of 6m compared to the same time period in a 10 degree deviated well, while the longer strings are decreasing a bit more.

The drill bit velocities in the figure on the next page are corresponding very well to the drill bit amplitudes. The 5000 m string has however a slightly higher velocity than the 9000 m string, even if the 9000 m amplitudes are higher than the 5000 m amplitudes. A maximum bit velocity of 7,0 m/s is achieved by the 5000 m string at a heave amplitude of 6 m.

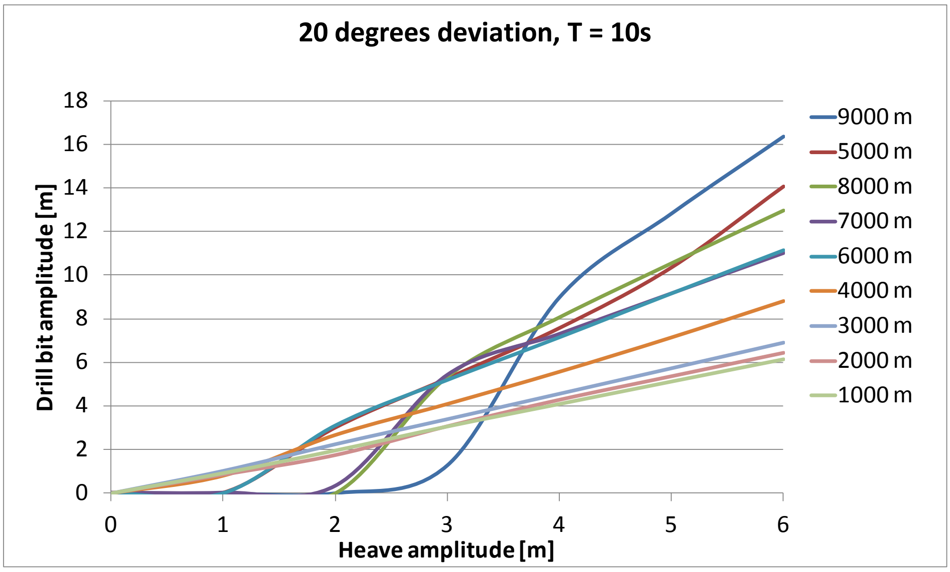

Figure 4.13: Drill bit amplitude vs. heave amplitude. A drill string with 20 degree deviation angle oscillating due to a heave movement with a time period at 10 s at the top of the string.

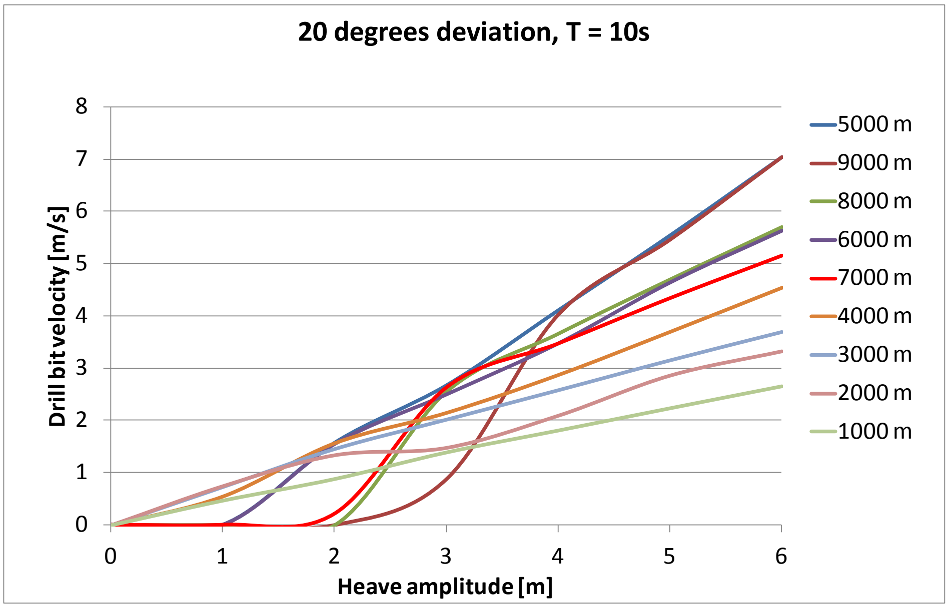

Figure 4.14: Drill bit velocity vs. heave amplitude. A drill string with 20 degree deviation angle oscillating due to a heave movement with a time period at 10 s at the top of the string.

4.3.2 15 second time period

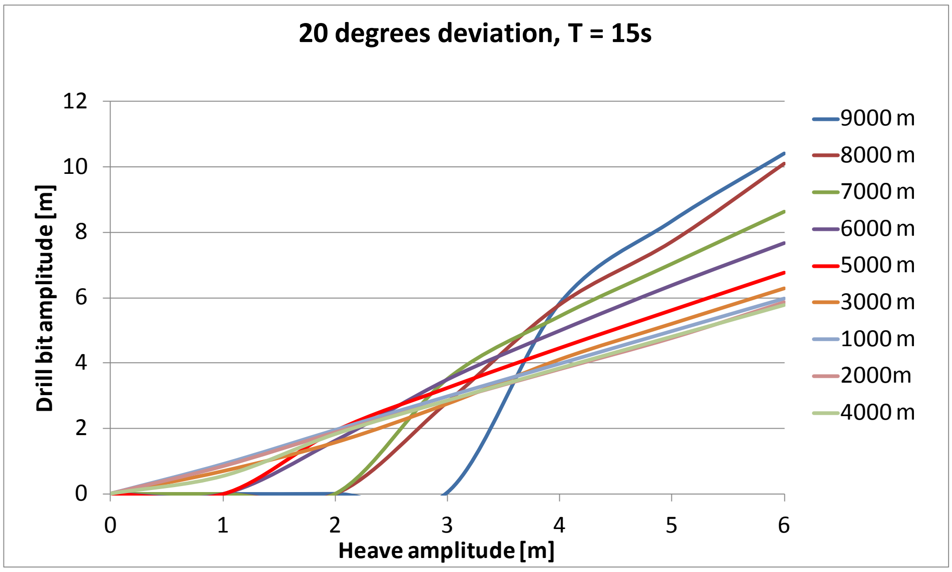

Figure 4.15: Drill bit amplitude vs. heave amplitude. A drill string with 20 degree deviation angle oscillating due to a heave movement with a time period at 15 s at the top of the string.

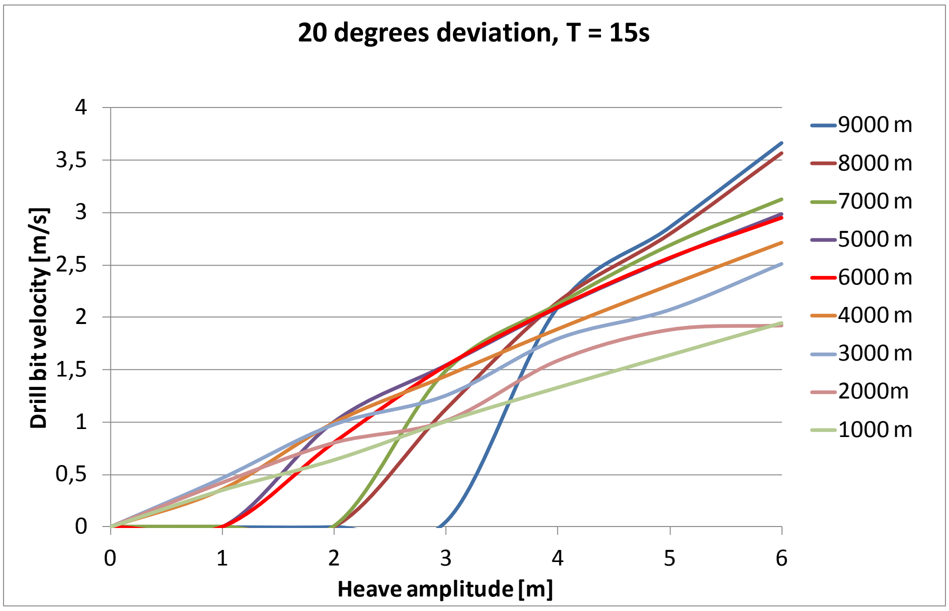

Figure 4.16: Drill bit velocity vs. heave amplitude. A drill string with 20 degree deviation angle oscillating due to a heave movement with a time period at 15 s at the top of the string.

From the figure above we can see that the drill bit amplitudes of the longer strings are getting smaller compared to the strings with a smaller deviation angle. It is now only the bits in the end of a string shorter than 5000 m that moves at all heave amplitudes. The bits at the longer strings are static up to a specific heave amplitude. The bit at the end of the 9000 m string is static until the heave amplitude reaches 3m.

The drill bit velocities in the figure above are corresponding very well to the drill bit amplitudes. The bit in the end of the 4000 m string is, however, having the smallest maximum bit amplitude, and at the same time the bit velocity of the same string is relatively high compared to the other strings.

4.3.3 20 second time period

The amplitudes in the figure at the next page is quite similar to the amplitudes at T=15 s, but smaller at a heave amplitude 6 m. The bit amplitude distribution for the different strings is also within a smaller interval at a heave amplitude of 6 m, where the bit at the 5000 m string has the lowest amplitude, and the bit at the 9000 m string has the highest amplitude.

There is a good correspondence between the amplitudes and velocities, and the distribution of the velocities is collected within an interval of 1 m/s.

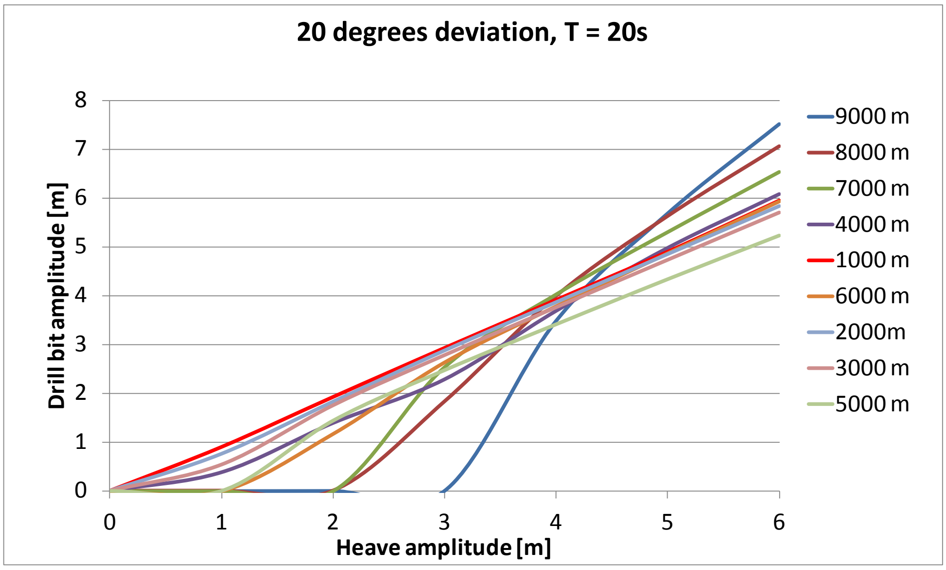

Figure 4.17: Drill bit amplitude vs. heave amplitude. A drill string with 20 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

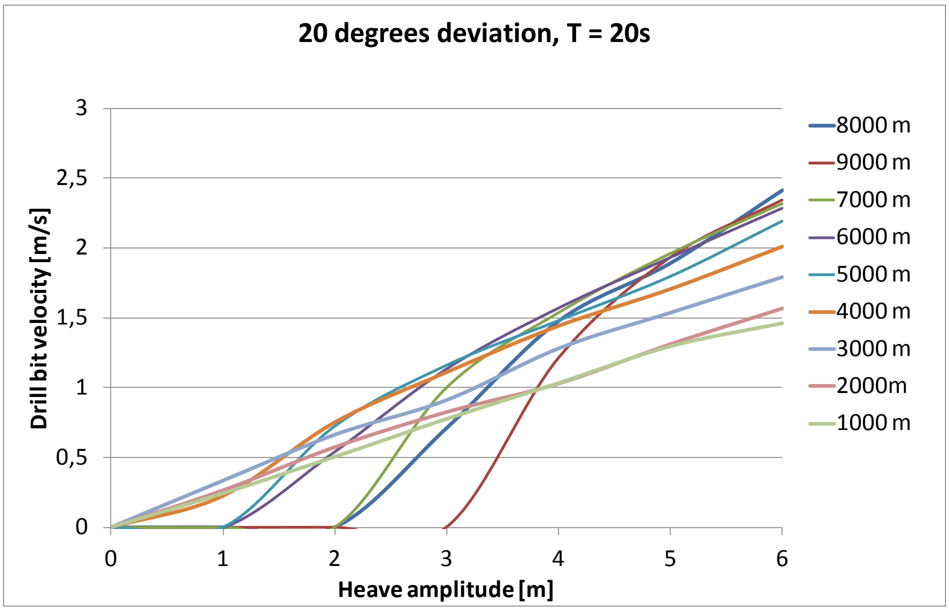

Figure 4.18: Drill bit velocity vs. heave amplitude. A drill string with 20 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

4.4 A drill string deviating with 40 degrees below 500 m

As mentioned earlier, the KOP was 500 m, and the build radius was 500 m. This resulted in three different well sections: a 500 m vertical section, a build section of 349 m, and a 40 degrees deviated section of different lengths.

4.4.1 10 second time period

In the figure in the next page we can see that the bits in the end of the longer strings are stationary until a certain heave amplitude is reached, where the bit amplitude suddenly increases. The bit in the end of the 9000 m string is now clearly the bit with the smallest amplitude of 4,0 m, while the bit in the end of the 7000 m string has the largest amplitude at a heave amplitude of 6 m, equal to 10,9 m.

The velocities correspond quite well to the drill bit amplitudes, where the bit at the 9000 m string has the lowest velocity. The distribution interval of both bit amplitudes and velocities are now increasing.

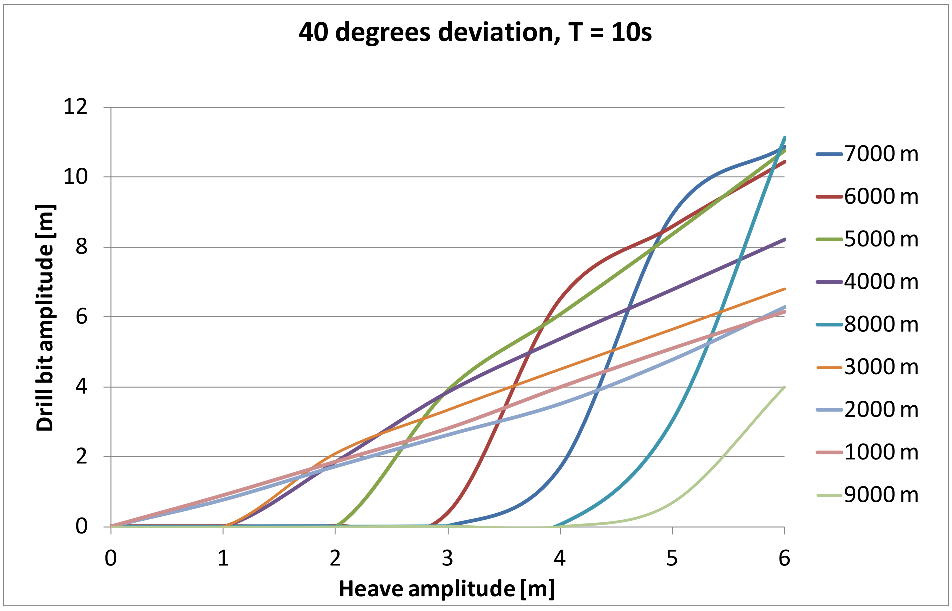

Figure 4.19: Drill bit amplitude vs. heave amplitude. A drill string with 40 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

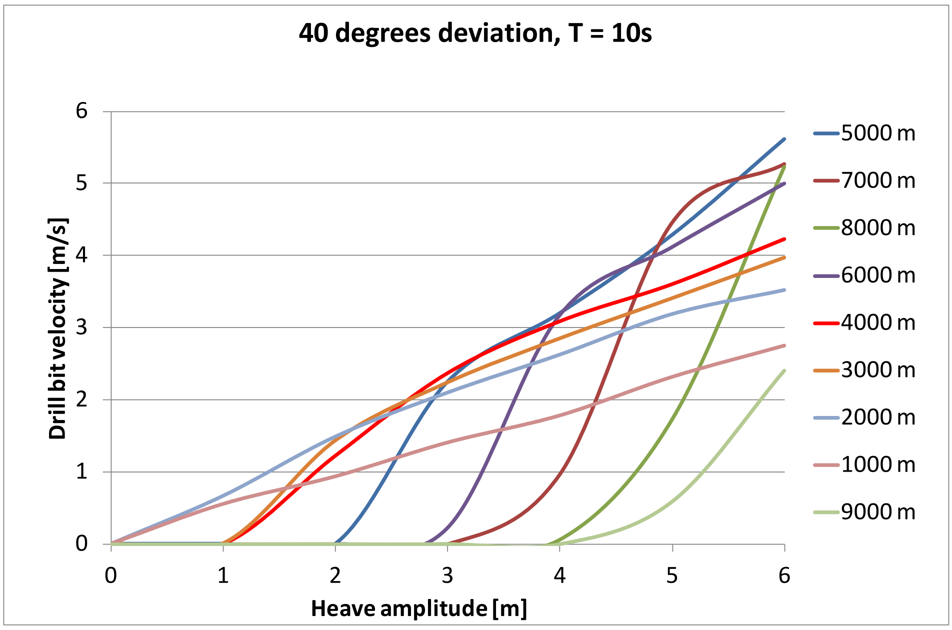

Figure 4.20: Drill bit velocity vs. heave amplitude. A drill string with 40 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

4.4.2 15 second time period

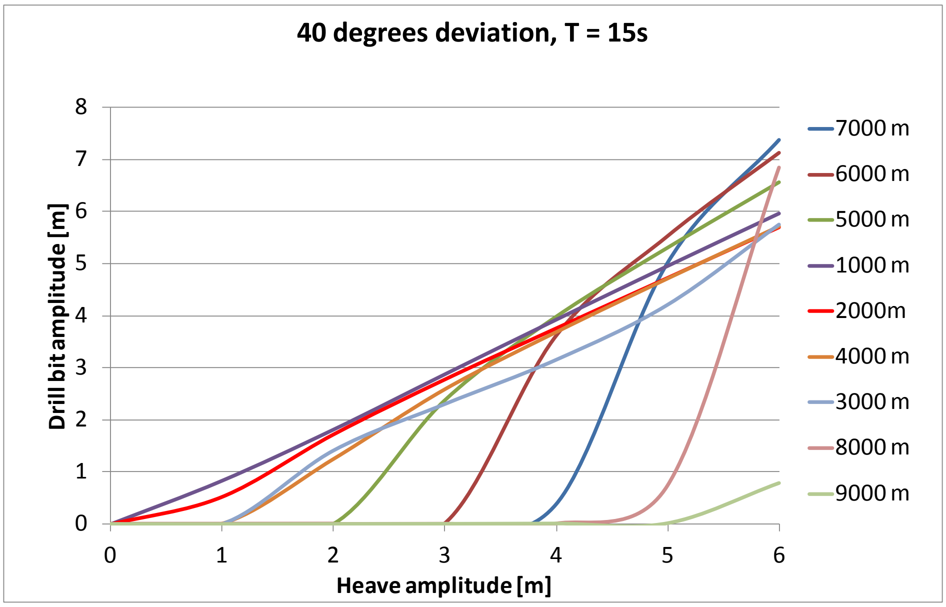

Figure 4.21: Drill bit amplitude vs. heave amplitude. A drill string with 40 degree deviation angle oscillating due to a heave movement with a time period at 15 s at the top of the string.

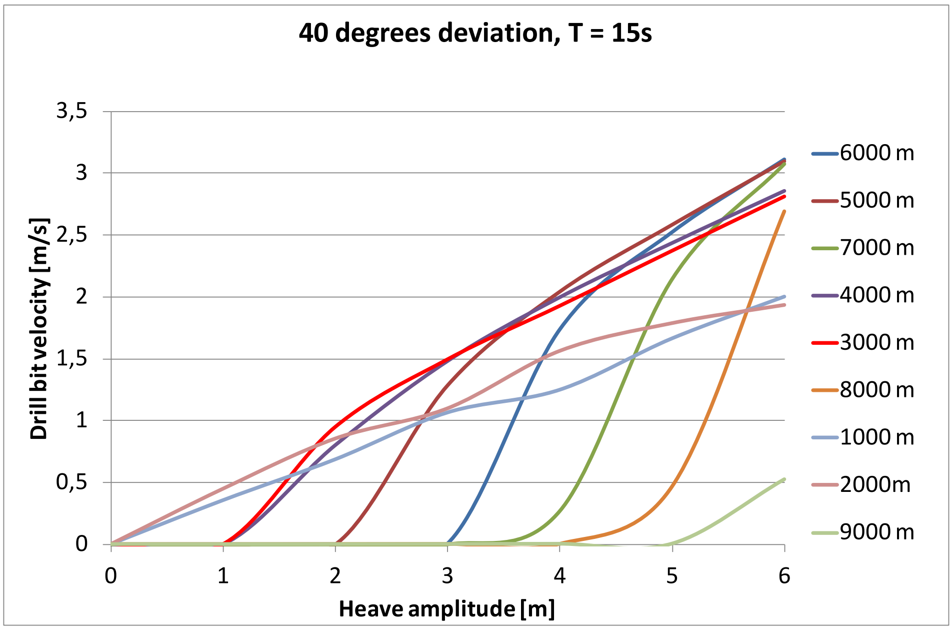

Figure 4.22: Drill bit velocity vs. heave amplitude. A drill string with 40 degree deviation angle oscillating due to a heave movement with a time period at 15 s at the top of the string.

The figure above shows that the bit at the 9000 m string is now static at all heave amplitudes up to 5 m, and the bit at the 8000 m string is moving first at a heave amplitude of 4 m. Both the bits at the 1000 m and 2000 m strings are responding to the top movement at all heave amplitudes, while the others are static until a certain non-zero heave amplitude.

Once again we can see that the velocities and amplitudes of the different strings are quite correlated, and the velocity distribution interval at a 6 m heave amplitude has increased to 2,6 m/s.

4.4.3 20 second time period

From the figures on the next page we can suspect a pattern emerging, where the bits at the longer strings are not having a movement respond at the smaller heave amplitudes. The drill bit amplitudes are also getting smaller with increased lengths.

The figures shows that the pattern mentioned for the drill bit amplitudes are not as clear as it is for the drill bit velocities, as some of the drill bits at the longer strings are having velocities larger than the shorter. The tendency is, however, that the drill bits at the longer drill strings has a lower velocity compared to the shorter strings.

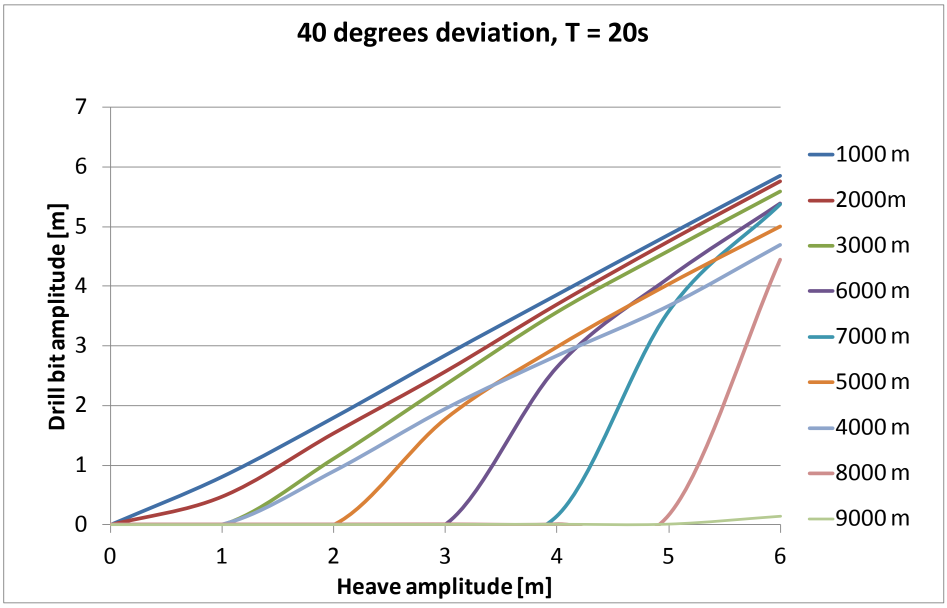

Figure 4.23: Drill bit amplitude vs. heave amplitude. A drill string with 40 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

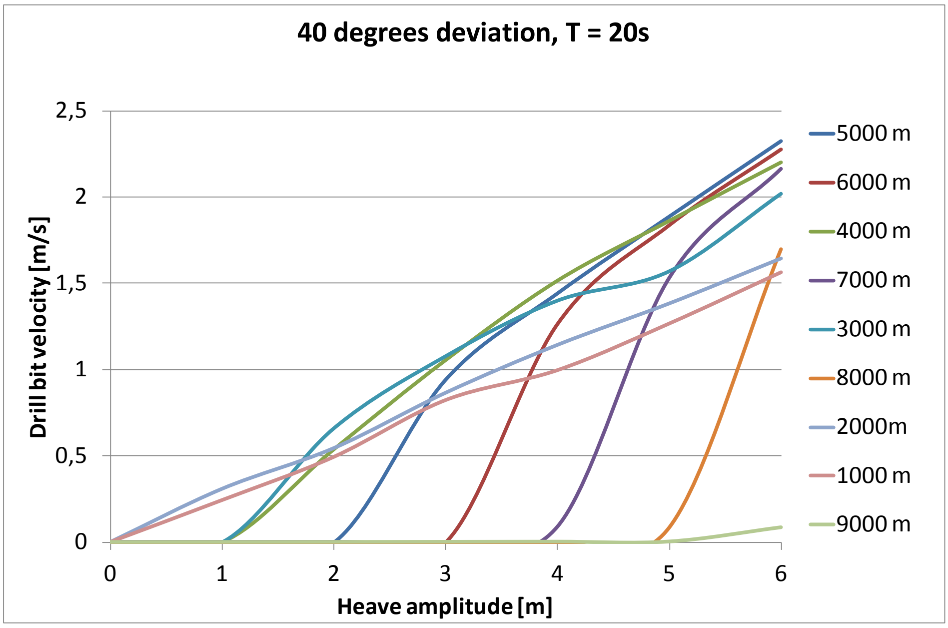

Figure 4.24: Drill bit velocity vs. heave amplitude. A drill string with 40 degree deviation angle oscillating due to a heave movement with a time period at 20 s at the top of the string.

5. DISCUSSION

When it comes to the calculations of the drill string movements, some aspects that may be important have been left out. One of them is the relative velocity between the drill string and the fluid. As the string moves up and down it will act as a piston, displacing fluid when moving down, sucking fluid back when moving up. This piston effect will make the relative velocity between the string and fluid greater. The result of this being included in the calculations would be a higher viscous friction which could dampen the movement, and at the same time a increase the pressure drop over the BHA. Another aspect that has been left out from the calculations is the dampening of the drill string movement due to the pressure drop over the BHA. It is not possible to forecast the outcomes from the simulations if these effects would have been included in the simulations, but it is reasonable to assume that the string velocity and amplitudes would have become smaller, and the calculations are in that sense conservative.

As mentioned earlier, the length of the BHA is set to a constant 200 m regardless of drill pipe string length. In reality the length of the BHA would vary with the type of well and length, but it turns out that the length of the BHA is not having a large impact on the string amplitude and velocity in a normal case. If the BHA length is decreased to 100 m, the bit amplitude and velocity are decreased slightly. The length of the BHA is, however, important when it comes to drill string resonance. Some of the vertical drill strings are experiencing resonance at a certain heave period. If the BHA length is changed, the string resonance is dampened or increased, depending on the natural frequency of the string. The natural frequency of a drill string is determined by the speed of sound in the string material combined with the string length, but also by the deflection of waves in the material due to a change in cross section. It is therefore not straight forward to determine the natural frequency of a drill string.

The surge and swab pressure calculations are the sum of the pressure drop over the drill bit and BHA. The total pressure drop depends heavily on the wellbore diameter, the junk area of the drill bit and the length and diameter of the BHA. To make the calculations consistent, the drill bit size is set to be the same as the wellbore diameter which is 12". The junk area is calculated to be 18% of the drill bit cross section area, while the diameter of the BHA is set to be 8". The BHA in a drill string is always having a smaller diameter than the wellbore, and if the wellbore diameter is decreased to 10" and the BHA diameter is decreased to 6", the pressure drop becomes slightly smaller. However, if the wellbore diameter is kept at 10" and the BHA diameter is increased to 8", the pressure drop is doubled. It is important to be aware of this, especially with the longer strings, where the wellbore diameter is often smaller than 12". One should then consider the length and diameter of the BHA, making sure that the pressure drop over the BHA stays small.

5.1 Vertical drill string

The results from the simulations on a vertical drill string shows that the drill bit amplitude and velocity are generally increasing with heave amplitude and drill string length, and are generally decreasing with increased heave period.

The bit amplitude and velocity increase linearly in response to a linear increase in heave amplitude. There is no contact friction along the strings in a vertical well, which means that the friction forces are more evenly distributed along the drill string. When the heave amplitude is increased linearly, so is the segment velocity, and thereby the viscous friction.

If we look at the drill bit amplitude of the 1000 m (actually 1200 m) drill string at a heave amplitude of 6 m and time period of 10 s, it has a maximum drill bit amplitude of approximately 6,2 m. This means that the drill bit is moving 3,1 m up and 3,1 m down from the point of equilibrium. An elongation and compression of 3,1 m corresponds to 0,26% of the total length of the drill string. This can be compared to the drill bit at the vertical 9000 m string, which had a maximum drill bit amplitude of approximately 19,3 m at the same heave amplitude and time period. This corresponds to approximately 9,6 m compression and elongation which is around 0,10% of the total string length. It then becomes clear that the longer strings have a smaller relative drill bit amplitude compared to the shorter strings, giving a smaller string material stress than for the 1000 m string. This result is quite reasonable as it is easier to elongate and compress a longer string the same distance as a short string. At the same time a longer string is experiencing more viscous friction which is damping the movement from the top segment, limiting the response.

As the heave period is increased, the bit amplitude and velocity is decreased. A longer heave period means less energy fed into the drill string in the same time interval, and a smaller response is the result. When the time period is increased from 10 s to 15 s, the relative decrease in amplitude is larger than the decrease in amplitude from 15 s to 20 s. This may also explain that the distribution interval of the amplitude and velocity is generally smaller as the time period is increased.

Some of the results from the simulations stands out, and must be further addressed. Some drill strings of certain lengths broke at the top due to excessive loads at the top segment. This happened at four occasions: the 5000 m and 9000 m string at a heave period of 10 s, and the 8000 m and 9000 m string at a heave period of 15 s. The reason why the 9000 m string broke at a heave period of 10 s was probably due to the sheer weight of the string in combination with the movement of the top segment. There is, however, a reason to believe that the three other strings broke due to another reason. The figure below shows the movement of the drill bit at the 5000 m string throughout one simulation with heave amplitude 0,5 m and time period 10 s.

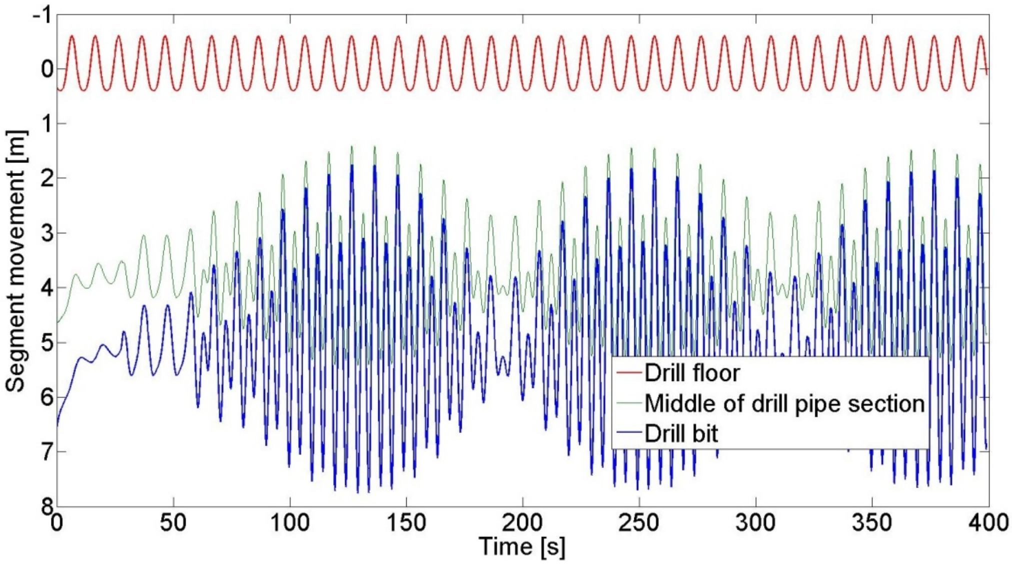

Figure 5.1: The blue line shows the drill bit movement at the 5000 m drill string at time period 10 s and heave amplitude 0,5 m.

The figure above shows typical signs of a string resonating. The string movement is increasing and decreasing in certain intervals, and becomes larger and larger as the heave amplitude is increased. The 5000 m drill string at a heave period of 10 s oscillated more and more violently as the heave amplitude was increased, until the top segment eventually broke due to excessive loads. The same pattern can be seen in the movement of the 8000 m and 9000 m strings at time period 15 s. A small decline in the slope for both the velocity and amplitude graphs is observed in Figure 4.1 to Figure 4.4, just before a string broke. This is because the simulation program stops the simulation at the moment a string breaks. The velocity and amplitude would therefore probably have continued increasing linearly if the string would have been able to withstand the loads.

The highest drill bit velocity was obtained by the bit at the 5000 m string just before the string broke at the top. The velocity was 12,4 m/s, and the bit amplitude was 22,0 m. The bit amplitude indicates a movement of approximately 11 m in both directions from the point of equilibrium. During a connection the drill bit is lifted a few meters from the bottom of the well, but normally not as much as 11 m. This means that the oscillating movement of the string due to resonance would have been reduced every time the bit had hit the bottom. However, if the drill string would have been lifted up more than 11 m from the bottom, the drill string could have started oscillating violently and eventually snapped, as seen in the simulations. The maximum surge/swab pressure the drill string could have caused before it broke was 43,4 bar according to equation 2.1.1 and 2.1.3. This is a very high pressure fluctuation and could certainly lead to both influx of formation fluid and loss of drill fluid. However, a 5000 m vertical well without any contact friction between the drill string and the wellbore wall is not very likely. It is impossible to keep the wellbore straight all the way, and it is very likely that the drill string would make contact with the wellbore wall. Some further investigation on the 5000 m string was conducted and a deviation angle of 7 degrees below 500 m was enough to stop the resonance completely. This shows that some contact friction due to an irregular wellbore could stop the resonance.

When it comes to the combination of a heave amplitude of 4 m and a heave period of 10 s in relation to the semi-sub movement, it is not very likely to occur. A heave amplitude of this magnitude involves the rig to i.e. move 1,6 m down, 2,4 m up, and back to zero again in 10 seconds. A semi-sub would probably not move this much in 10 seconds. If we however consider the lower and more probable heave amplitudes we can see that the drill bit at the 5000 m string moves with a velocity of 3,5 m/s at a heave amplitude of 1m. This corresponds to a surge/swab pressure of 4,3 bar. Pressure fluctuations of this magnitude can be enough to cause problems if the BHP in the well is already close to the pore or fracture pressure, but is more likely to not cause any serious problems. At a heave amplitude of 2 m, the bit obtains a maximum velocity of 7,0 m/s. This would give a surge/swab pressure of 15,3 bar, which is more likely to cause serious problems.

The most likely scenarios for vertical strings at a heave period of 10 s, however, involve the shorter strings as 1000 m, 2000 m and 3000 m. A shorter string would experience less contact friction, allowing the upper movement to travel down the string more easily. As mentioned earlier, a heave amplitude of 6 m at this time period is not very likely, but a heave amplitude of 2 m is. The bit amplitudes of all the three strings are roughly 2 m, and the velocities of all the three strings are approximately 1m/s. This would lead to a pressure increase/decrease of 0,4 bar, which is insignificant.

The maximum drill bit velocity at time period 15 s was the drill bit on the 8000 m string, and was measured to 8,5m/s just before the string broke at the top at a heave amplitude of 4 m. The maximum drill bit amplitude was 21,2 m, which is significant. It is also here expected that there would be some contact friction in the well making sure of reducing the bit amplitude along with the velocity, especially since this string is very long. Neither the 8000 m string or the 9000 m string is expected to be able to cause any problems, both because of the expected contact friction, but also since the lengths of the strings makes the whole scenario unlikely. Regardless of the likelihood of the scenarios, the surge/swab pressure would have been 21,7 bar, which is significant. If one disregards the results from the two strings that broke, the highest velocity measured at a heave period of 15 s are 3,5 m/s. This velocity was obtained at a heave amplitude of 6 m, and corresponds to a surge/swab pressure of 4,3 bar, which shows that there are no danger of harmful surge/swab effects due to heave at this time period.

The maximum drill bit velocity for the 20 s heave period was obtained at a heave amplitude of 6 m by the bit at the 9000 m string, and was 2,3 m/s which corresponds to a surge/swab pressure of 2,0 bar.

5.2 Deviated drill strings

As for the vertical strings, simulations of deviated strings also shows that the drill bit velocity and amplitude are generally increasing with heave amplitude, and are generally decreasing with increased heave period. The simulations also show that the bit velocity and amplitude are generally decreasing with increased deviation angle. However, as the deviation angle is increasing, more of the energy in the drill strings is lost due to contact friction, leading to a non-linear behavior and less distinctive patterns.

As the lower well section is getting more deviated, the amplitudes and velocities of the drill strings are declining. There are no definitive signs of resonance, even though the bit at the 5000 m drill string is having a large response to the heave movement. The maximum amplitude of the drill bit at the 5000 m string is 20,6 m and the maximum bit velocity is 11,6 m/s. This is obtained at a heave amplitude of 6m, and would give a surge/swab pressure of 38,4 bar. As before, a heave amplitude of this magnitude at a heave period of 10 s is not very likely. At a heave amplitude of 1m the drill bit amplitude is 1,5 m and the velocity is 0,8 m/s. This would give a surge/swab pressure of 0,3 bar, which is very small. The bit velocity at a heave amplitude of 2 m is 2,0 m/s, and would give a surge/swab pressure of 1,6 bar.

The maximum velocity for the 10 degree deviation and 15 s heave period is 4,8 m/s, and is obtained by the drill bit at the 8000 m drill string at a heave amplitude 6 m. This corresponds to a 7,7 bar surge/swab pressure which is significant. However, a more likely scenario for a semi-sub is in this case a maximum heave amplitude of 3 m. The maximum bit velocity at this heave amplitude is 1,8 m/s and would lead to a surge/swab pressure of 1,3 bar.

For the 20 s time period the maximum velocity was obtained at a heave amplitude of 6 m and is 2,2 m/s, which leads to a pressure increase/decrease of 1,8 bar.

As the well inclination reaches 20 degrees, the velocities are very similar to the ones obtained at a well deviation of 10 degrees. The bit at the 5000 m string (time period 10 s) still obtains the highest velocity at a heave amplitude of 6 m compared to the other strings. This indicates that the contact friction prevents string resonance, and it is also reducing the drill bit amplitudes.

As the wellbore inclination are further increased, none of the drill strings are showing any bit movements that are able to generate pressure fluctuations that are a threat to the drilling at heave amplitudes that is likely to occur during a drilling operation. The string behavior is getting more and more different from the linear behavior seen for small inclinations, probably due to the increased contact friction. In addition, for some of the longer strings the dynamic contact friction is overcoming the movements from the top of the drill string, stopping the bit movement altogether. As a consequence, the contact friction is in effect increased as the static contact friction is set to be 20% higher than the dynamic contact friction. The movement is also stopped further up the string, depending on inclination and heave amplitude. The 1000 m string is, however, not experiencing the same increase of contact friction, as it has only a shorter part of the string in the deviating well section. The 1000 m drill string is therefore able to maintain a behavior more consistent for changing deviations than the longer strings.

6. CONCLUSIONS

The results from the simulations show that the drill bit velocity and amplitude is generally increasing with heave amplitude, and generally decreasing with increased heave period and deviation angle. For the drill strings in vertical wells, the amplitudes are increasing with drill string length. However, as the deviation angle is increased, more of the energy in the drill strings is lost due to contact friction, leading to a non-linear behavior and less distinctive oscillation patterns. The drill bit amplitude is then small up to a certain heave amplitude, where the drill bit amplitude suddenly increases rapidly. A drill bit at a long deviating drill string may be completely stationary at small heave amplitudes, while the drill bit at a shorter string is moving with the heave amplitudes.

At a certain heave period, a vertical drill string can start resonating, leading to high loads mainly on the uppermost drill string segment, which may finally break due to excessive loads. This is, however, only observed in wells with deviations approximately less than 7 degrees, as the contact friction effectively prevent string resonance. In a real drilling operation it is therefore not believed that a drill string could start resonating due to the heave movement of a semi-sub, as the contact friction is likely to overcome the string resonance. Precautions must, however, be taken, as other combinations of the drill string sections in a shorter drill string can lead to drill string resonance, and thereby be much more likely to occur. A more thorough investigation of the natural frequency of the drill strings should be conducted to possibly predict when drill string resonance occurs, but this is outside the scope of this thesis.

As the pressure drop over the BHA and drill bit are not included in the calculations of the drill string movements, one can expect that the calculations are conservative. The drill bit amplitudes and velocities are therefore expected to be lower in a real drilling operation, but it is here not calculated whether the pressure fluctuations would be higher or lower. It is nevertheless important to be aware that if the flow area between the BHA and wellbore wall is decreased to less than a few inches, especially in a wellbore having a small inner diameter, high pressure fluctuations can easily be obtained for the bit amplitudes and velocities calculated here.

Based on a drill string a few inches smaller in diameter than the wellbore, the most likely scenarios shows that the drill string oscillations during normal weather conditions can cause surge and swab pressures up to approximately 5 bar. Pressure fluctuations of this magnitude have traditionally not been a problem, but as the operational window is decreased, it might turn out to be an issue.

Future work should focus on a better description of the semi-sub heave response to waves and a more comprehensive simulation program taking additional effects into account. The pressure drop over the BHA and drill bit should be included in the calculations, along with the extra weight of the tool joints. If the extra contact friction force due to bends in the well path is included in the program, the effects of additional contact friction due to one or more build/drop sections can be found.

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NOMENCLATURE AND ABBREVIATIONS

Nomenclature

A	Area
a	Acceleration / Constant
B	Buoyancy force
b	Constant
C	Proportionality constant
c	Speed of sound in a material
E	Modulus of elasticity of a material
F	Force
g	Gravity constant
h	Height
j	Number
L	Length
m	Mass
N	Number of segments / Number of pipes / Normal force
NCB	Number of drill pipes
P	Pressure
R	Radius
T	Time period
t	Time
v	Velocity
X	Current displacement
XN	New displacement
XG	Old displacement
α	Deviation angle from vertical
ε	Constant
μ	Friction coefficient
ρ	Density of a material
σ	Stress
φ	Radial angle / Phase shift
ω	Angular frequency

Δt	The time step between calculations
Δx	Displacement unit
ΔZ	Distance between segment mid points
' (prime symbol)	Foot
'' (prime symbol)	Inch

Abbreviations

BHA	Bottom Hole Assembly
BHP	Bottom Hole Pressure
cm	Centimetre
ECD	Equivalent Circulating Density
FIT	Formation Integrity Test
ft	Foot
GOC	Gas Oil Contact
HWDP	Heavy Weight Drill Pipe
ID	Inner Diameter
kg	Kilogram
KOP	Kick Off Point
lb	Pound
LOT	Leak Off Test
LWD	Logging While Drilling
m	Metre
MD	Measured Depth
ms	Millisecond
MSL	Mean Sea Level
MWD	Measurement While Drilling
NCS	Norwegian Continental Shelf
OWC	Oil Water Contact
OD	Outer Diameter
ROP	Rate Of Penetration
RSS	Rotary Steerable System
s	Second
TVD	True Vertical Depth
WOB	Weight On Bit

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APPENDIX

A. The Matlab program text

The program text can be copied and pasted into Matlab. The program consists of mainly two separate sections: one section where the drill string simulations are carried out with only one heave period and amplitude, and one section where the drill string simulations are carried out with a chosen number of heave periods and amplitudes. Only one section can be used at a time, where the other must be deactivated by typing % in front of it.

```
% Simulerer bevegelsene i en streng som henger fra boredekk

%% INPUT DATA BORESTRENGEN
% Strenglengder
LB_inn =        2000;           % [m]       Total lengde borerørstreng
LV_inn =        200;            % [m]       Total lengde vektrørstreng
Lj =            10;             % [m]       Lengde av et borerør/vektrør

% Stålegenskaper
RHOs =          7850;           % [kg/m3]   Tetthet til stål
Es =            210*10^9;       % [Pa]      Youngs modulus til stål
c =(Es/RHOs)^0.5;               % [m/sec]   Lydhastighet i stål
Flytgrense = 930792234.579071;  % [Pa]      Flytgrense til stål

% Borerør
ODb =           5;              % [inches]  Ytre diameter borerør
IDb =           4;              % [inches]  Indre diameter borerør
RHOb =          RHOs;           % [kg/m3]   Tetthet til stål i borerør
Eb =            Es;             % [Pa]      Youngs modulus til borerør
Mtooljoint =    70.5;           % [kg]      Ekstra masse pga tool joint

% Vektrør
ODv =           8;              % [inches]  Ytre diameter vektrør
IDv =           2.8125;         % [inches]  Indre diameter vektrør
RHOv =          RHOs;           % [kg/m3]   Tetthet til stål i vektrør
Ev =            Es;             % [Pa]      Youngs modulus til vektrør

% Borekrone
Dslisse =       0.0244;         % [m]       Diameter på en slisse
Cd      =       0.95;           %           Dysefaktor

%% INPUT DATA BRØNNDATA
% Loddrett seksjon fra RKB til kick off:
L_rett_1 =          500;      % [m]       Lengde rett seksjon fra RKB til
kick-off

% Build-up seksjon:
Vinkel_1 =          10;         % [grader]  Vinkel på rett seksjon etter
kurve
Vinkel_radius_1 =   500;        % [m]       Radius på build-up kurve

% Tangent seksjon fra ende av build-up til bunn av brønn:
```

```
L_rett_2 =          12000;          % [m]       Lengde fra ende av kurve
til bunn av brønn eller til drop sekson

% Drop/build seksjon:
Vinkel_2 =          0;          % [grader]  Vinkel på rett seksjon etter
kurve
Vinkel_radius_2 =   0;          % [m]       Radius på drop kurve

% Hold seksjon:
L_rett_3 =          0;          % [m]       Lengde hold seksjon fra ende av
kurve til bunn av brønn

% Casing
Csg_set_depth_inn = LB_inn*2/3; % [m]       Settedybde MD casing
IDcsg = 12;                     % [inches]  Indre diameter casing


%% INPUT TID

tSTART =            200;        % Tiden sampling kan begynne
tEND =              200+tSTART; %50+tSTART; % [sek]     Tiden som skal
simuleres
tSLUTT =            tEND-1;     % Tiden sampling slutter
t=0;
%% DIVERSE INPUT DATA

% Boreslamsegenskaper
RHOm =                1200;     % [kg/m3]   Tetthet til boreslam
MYmud =               5;        % [cP]      Viskositet boreslam

% Vannegenskaper
RHOw =                1000;     % [kg/m3]   Tetthet til rent vann

%Konstanter
g =                   9.81;     % [m/sek^2] Gravitasjonskonstant
Inches_to_meters =    0.0254;   % [m/inch]  Omregningsfaktor

%Friksjonskoeffesienter
my =                  0;        % Friksjonskoeffesient uten avvik
my_staal_berg =       0.3;      % Friksjonskoeffesient stål/berg
my_staal_staal =      0.2;      % Friksjonskoeffesient stål/stål
my_stat_staal_berg =  my_staal_berg*1.2;    % Statisk friksjonskoeffesient
stål/berg
my_stat_staal_staal = my_staal_staal*1.2;   % Statisk friksjonskoeffesient
stål/stål

%% SMÅ BEREGNINGER

% Casing
Pipes_in_csg = convergent(Csg_set_depth_inn/Lj); %      Antall rør i casing
Csg_set_depth = Pipes_in_csg*Lj;             % [m]  Ny tilpasset settedybde
Acasing = pi/4*((IDcsg*Inches_to_meters)^2); % [m2]      Indre areal casing

% Borerør
Abo = pi/4*((ODb*Inches_to_meters)^2);  % [m2]      Ytre areal borerør
Abi = pi/4*((IDb*Inches_to_meters)^2);  % [m2]      Indre areal borerør
Ab = Abo-Abi;                           % [m2]      Stålareal borerør
```

```
Mb = Ab*RHOb;                          % [kg/m]    Masse pr lengde borerør
cb  = (Eb/RHOb)^0.5;                   % [m/sec]   Lydhastighet i borerør

% Vektrør
Avo = pi/4*((ODv*Inches_to_meters)^2);  % [m2]      Ytre areal borerør
Avi = pi/4*((IDv*Inches_to_meters)^2);  % [m2]      Indre areal borerør
Av = Avo-Avi;                          % [m2]      Stålareal borerør
Mv = Av*RHOb;                          % [kg/m]    Masse pr lengde vektrør
cv =(Ev/RHOv)^0.5;                     % [m/sec]   Lydhastighet i vektrør

% Ringrom BHA
Aring = Acasing-Avo;                   % [m2]      Strømningsareal utenfor
BHA

% Borekrone
Aslisser = pi/4*(6*Dslisse)^2;         % [m2]      Totalt areal slisser

% Relativ tetthet mud
RHOr = RHOm/RHOw;                      %           Relativ tetthet mud

% Viskositet mud i Pa s
MYmudPas=        MYmud/1000;    % [Pa s]    Viskositet boreslam

% Beregninger av antall koblinger:
NCB = convergent(LB_inn/Lj);    %   Antall borerør, avrundet til nærmeste
heltall
NCV = convergent(LV_inn/Lj);    %   Antall vektrør, avrundet til nærmeste
heltall
N = NCB+NCV;                  %   Totalt antall rør i strengen = antall hele
segmenter i strengen (har lagt to halve sammen)

% Definerer nye lengder ut ifra avrundet antall koblinger:
LB = NCB*Lj;                  % [m]       Ny lengde borerør
LV = NCV*Lj;                  % [m]       Ny lengde vektrør
L_streng = N*Lj;              % [m]       Ny totallengde

% Overgang borerør/vektrør
CF1 = 2*Ab/(Ab+Av);           %           Connection faktor 1
CF2 = 2*Av/(Ab+Av);           %           Connection faktor 2
NCO = NCB;                    %           Connection over overgang
NCU = NCB+2;                  %           Connection under overgang

% Tidssteg og midtpunkter
DELTAz = L_streng/N;          % [m]       Avstand mellom midtpunkter
DELTAt = DELTAz/c;            % [sek]     Lengde tidssteg
Z = convergent(tEND/DELTAt);%             Antall tidssteg

%% BRØNNBANE

% Build-up-kurve fra kick-off til tangentseksjon:
Vinkel_1_rad =  (2*pi/360)*Vinkel_1;        % [rad]     Vinkel på
tangentseksjon etter build-up-seksjon
L_bue_1 = Vinkel_1_rad*Vinkel_radius_1;     % [m]       Lengde build-up-
kurve
if L_bue_1 > 0
    Rad_per_m_1 =   Vinkel_1_rad/L_bue_1;   % [rad/m]   Vinkelstigning
build-up-kurve
else
```

```
    Rad_per_m_1 = 0;
end

% Drop-kurve fra tangentseksjon til holdseksjon:
Vinkel_2_rad = (2*pi/360)*Vinkel_2;              % [rad]     Vinkel på hold-
seksjon etter drop-seksjon
Vinkel_endring = Vinkel_1_rad-Vinkel_2_rad;      % [rad]     Endring i
vinkel fra tankentseksjon til holdseksjon
L_bue_2 = abs(Vinkel_endring)*Vinkel_radius_2;   % [m]       Lengde drop-
kurve
if L_bue_2 > 0
    Rad_per_m_2 =   Vinkel_endring/L_bue_2;      % [rad/m]   Vinkelstigning
build-up-kurve
else
    Rad_per_m_2 = 0;
end

% Brønnlengder:
% Measured Depth (MD)
MD_end_rett_1 =     L_rett_1;                    % [m]       MD fra toppen
til ende av loddrett seksjon
MD_end_buildup =    MD_end_rett_1 + L_bue_1;     % [m]       MD fra toppen
til ende av build-up seksjon
MD_end_tangent =    MD_end_buildup + L_rett_2;   % [m]       MD fra toppen
til ende av tangent seksjon
MD_end_drop =       MD_end_tangent + L_bue_2;    % [m]       MD fra toppen
til ende av drop seksjon
MD_end_hold =       MD_end_drop + L_rett_3;      % [m]       MD fra toppen
til ende av hold seksjon = total brønnlengde
L_bronn =           MD_end_hold;                 % [m]       Total MD
brønnlengde

% True Vertical Depth (TVD)
TVD_end_rett_1 =    L_rett_1;
% [m]       TVD til ende av loddrett seksjon
TVD_end_buildup =   TVD_end_rett_1 + (Vinkel_radius_1*sin(Vinkel_1_rad));
% [m]       TVD til ende av build-up seksjon
TVD_end_tangent =   TVD_end_buildup + (L_rett_2*cos(Vinkel_1_rad));
% [m]       TVD til ende av tangent seksjon
if Vinkel_1>=Vinkel_2   % Drop
    TVD_end_drop =  TVD_end_tangent + (Vinkel_radius_2*(sin(Vinkel_1_rad)-
sin(Vinkel_2_rad)));  % [m]       TVD til ende av drop seksjon
else                    % Build
    TVD_end_drop = TVD_end_tangent + (Vinkel_radius_2*(sin(Vinkel_2_rad)-
sin(Vinkel_1_rad)));  % [m]       TVD til ende av drop seksjon
end
TVD_end_hold =      TVD_end_drop + (L_rett_3*cos(Vinkel_2_rad));
% [m]       TVD til ende av hold seksjon = TVD brønn
TVD_bronn =         TVD_end_hold;
% [m]       TVD brønn

% Horisontal utstrekning
H_end_rett_1 =      0;
% [m]       Horisontal utstrekning til ende av loddrett seksjon
H_end_buildup =     H_end_rett_1 + (Vinkel_radius_1*(cos(0)-
(cos(Vinkel_1_rad)));           % [m]       Horisontal utstrekning til
ende av build-up seksjon
H_end_tangent =     H_end_buildup +
(L_rett_2*sin(Vinkel_1_rad));%((TVD_end_tangent-
```

```
TVD_end_buildup)*(tan(Vinkel_1_rad)));      % [m]       Horisontal
utstrekning til ende av tangent seksjon
if Vinkel_1>=Vinkel_2   % Drop
H_end_drop =        H_end_tangent + (Vinkel_radius_2*(cos(Vinkel_2_rad)-
cos(Vinkel_1_rad)));    % [m]       Horisontal utstrekning til ende av drop
seksjon
else                    % Build
    H_end_drop =    H_end_tangent + (Vinkel_radius_2*(cos(Vinkel_1_rad)-
cos(Vinkel_2_rad)));    % [m]       Horisontal utstrekning til ende av drop
seksjon
end
H_end_hold =        H_end_drop + (L_rett_3*sin(Vinkel_2_rad));
%((TVD_end_hold-TVD_end_drop)*(tan(Vinkel_2_rad)));              % [m]
Horisontal utstrekning til ende av hold seksjon = TVD brønn
H_bronn =           H_end_hold;
% [m]       Horisontal utstrekning brønn

%% KREFTER

% Gravitasjon
G = g*(DELTAt^2);   % [m]

% Oppdriftsfaktor
Oppdrift = 1-(RHOm/RHOs);   % Oppdriftsfaktor for stål

% Viskøs friksjonsfaktor
% Borerør:
epsB =
DELTAt*MYmudPas*pi*((IDcsg*Inches_to_meters)+(ODb*Inches_to_meters))/(2*Mb*
((IDcsg*Inches_to_meters)-(ODb*Inches_to_meters)));

% Overgang borerør/vektrør
epsOV =
DELTAt*MYmudPas*pi*((IDcsg*Inches_to_meters)+(((ODb+ODv)/2)*Inches_to_meter
s))/((Mv+Mb)*((IDcsg*Inches_to_meters)-(((ODb+ODv)/2)*Inches_to_meters)));

% Casing:
epsC =
DELTAt*MYmudPas*pi*((IDcsg*Inches_to_meters)+(ODv*Inches_to_meters))/(2*Mv*
((IDcsg*Inches_to_meters)-(ODv*Inches_to_meters)));

% Overgang vektrør/borekrone
epsOB =
DELTAt*MYmudPas*pi*((IDcsg*Inches_to_meters)+(ODv*Inches_to_meters))/(Mv*((
IDcsg*Inches_to_meters)-(ODv*Inches_to_meters)));

%% UTSKRIFTER I COMMAND WINDOW

% Skriver ut tilpassede strenglengder:
Streng = ['Ny tilpasset lengde borerør: ', num2str(LB), 'm, vektrør: ',
num2str(LV), 'm, total strenglengde: ' num2str(L_streng), 'm']

% Skriver ut brønnlengder:
Bronn = ['Brønnseksjoner: rett seksjon: ', num2str(L_rett_1), 'm, kurve 1:
', num2str(L_bue_1), 'm, tangent seksjon: ' num2str(L_rett_2), 'm, kurve 2:
' num2str(L_bue_2), 'm, hold seksjon: ' num2str(L_rett_3),'m, brønnlengde:
' num2str(L_bronn), 'm, ', 'Ny tilpasset casing settedybde: ',
num2str(Csg_set_depth), 'm ']
```

```
% Advarsel
if (L_bronn < L_streng);                            %           Skriver ut
advarsel dersom brønnen er kortere enn strengen
    kortere = L_streng - L_bronn;
    ADVARSEL = ['BRØNNEN ER ', num2str(kortere), ' METER KORTERE ENN
STRENGEN']
    break
end

%% Oppretter diverse tabeller:

% Tabeller med innsvingningsverdier
XG1 =           (N+1);  % Gammel verdi
X1 =            (N+1);  % Nåværende verdi
XN1 =           (N+1);  % Ny verdi

% Tabeller med simuleringsverdier
XG =            (N+1);  % Gammel verdi
X  =            (N+1);  % Nåværende verdi
XN =            (N+1);  % Ny verdi

% Brønnbane
V  =            (N+1);  % Vinkeltabell brønnbane
MD  =           (N+1);  % Tabell med MD for hvert midtpunkt
TVD  =          (N+1);  % Tabell med TVD for hvert midtpunkt
H  =            (N+1);  % Tabell med horisontal forflytning for hvert punkt

% Krefter
Gravitasjon  =  (N+1);  % Gravitasjonskrefter
MYG =           (N+1);  % Dynamisk friksjon
MYSG =          (N+1);  % Statisk friksjon
Eps =           (N+1);  % Viskøs friksjonskonstant
EpsA =          (N+1);
EpsB =          (N+1);

Bevegelse =     (N+1);  % Tabell for å lagre om i ro eller bevegelse. 1
betyr i ro fra XG til X, 0 betyr bevegelse

% Nuller ut tabellene
for j=1:N+1;
    XG1         (j:N+1,1) = 0;
    X1          (j:N+1,1) = 0;
    XN1         (j:N+1,1) = 0;

    XG          (j:N+1,1) = 0;
    X           (j:N+1,1) = 0;
    XN          (j:N+1,1) = 0;

    V           (j:N+1,1) = 0;
    MD          (j:N+1,1) = 0;
    TVD         (j:N+1,1) = 0;
    H           (j:N+1,1) = 0;

    Gravitasjon (j:N+1,1) = 0;
    MYG         (j:N+1,1) = 0;
    MYSG        (j:N+1,1) = 0;
    Eps         (j:N+1,1) = 0;
```

```
    EpsA        (j:N+1,1) = 0;
    EpsB        (j:N+1,1) = 0;

    Bevegelse   (j:N+1,1) = 0;
end

%% Tildelinger av verdier i tabellene
% Gir vinkel verdier til hvert punkt som korresponderer til hvert midtpunkt
for j=1:N+1;
    Dybde_streng = j * DELTAz;

    % Gir vinkel i loddrett seksjon
    if Dybde_streng <= MD_end_rett_1;
        V(j) = 0;
    end

    % Gir vinkel i build-up seksjon
    if  Dybde_streng > MD_end_rett_1 && Dybde_streng <= MD_end_buildup;
        V(j) = (Dybde_streng-MD_end_rett_1) * Rad_per_m_1;
    end

    % Gir vinkel i tangentseksjon
    if Dybde_streng > MD_end_buildup && Dybde_streng <= MD_end_tangent;
        V(j) = Vinkel_1_rad;
    end

    % Gir vinkel i drop seksjon
    if Dybde_streng > MD_end_tangent && Dybde_streng <= MD_end_drop;
        V(j) = Vinkel_1_rad - ((Dybde_streng-MD_end_tangent) *
Rad_per_m_2);
    end

      % Gir vinkel i hold seksjon
    if Dybde_streng > MD_end_drop && Dybde_streng <= MD_end_hold;
        V(j) = Vinkel_2_rad;
    end
end

% Finner TVD til hvert midtpunkt på strengen
for j=1:N+1;
    Dybde_streng = j * DELTAz;

    % Gir TVD i loddrett seksjon
    if Dybde_streng <= MD_end_rett_1;
        TVD(j) = Dybde_streng;
    end

    % Gir TVD i build-up seksjon
    if  Dybde_streng > MD_end_rett_1 && Dybde_streng <= MD_end_buildup;
        TVD(j) = TVD_end_rett_1 + (Vinkel_radius_1*sin(V(j)));
    end

    % Gir TVD i tangentseksjon
    if Dybde_streng > MD_end_buildup && Dybde_streng <= MD_end_tangent;
        TVD(j) = TVD_end_buildup + ((Dybde_streng-
MD_end_buildup)*cos(V(j)));
    end
```

```
        % Gir TVD i drop seksjon
    if Dybde_streng > MD_end_tangent && Dybde_streng <= MD_end_drop &&
Vinkel_1 >= Vinkel_2; %Drop
        TVD(j) = TVD_end_tangent + ((Vinkel_radius_2*sin(Vinkel_1_rad))-
(Vinkel_radius_2*sin(V(j))));
    end
    if Dybde_streng > MD_end_tangent && Dybde_streng <= MD_end_drop &&
Vinkel_1 < Vinkel_2;  %Build
        TVD(j) = TVD_end_tangent + ((Vinkel_radius_2*sin(V(j)))-
(Vinkel_radius_2*sin(Vinkel_1_rad)));
    end

      % Gir TVD i hold seksjon
    if Dybde_streng > MD_end_drop && Dybde_streng <= MD_end_hold;
        TVD(j) = TVD_end_drop + ((Dybde_streng-MD_end_drop)* cos(V(j)));
    end

end

% Finner MD til hvert midtpunkt på strengen
for j=1:N+1;
    Dybde_streng = j * DELTAz;

    MD(j) = Dybde_streng;
end

% Finner horisontal forflytning til hvert midtpunkt på strengen
for j=1:N+1;
    Dybde_streng = j * DELTAz;

    % Gir horisontal forflytning i loddrett seksjon
    if Dybde_streng <= MD_end_rett_1;
        H(j) = 0;
    end

    % Gir horisontal forflytning i build-up seksjon
    if  Dybde_streng > MD_end_rett_1 && Dybde_streng <= MD_end_buildup;
        H(j) = H_end_rett_1 + (Vinkel_radius_1*(1-cos(V(j))));
%(tan(V(j)/2)*(1-cos(V(j))));
    end

    % Gir horisontal forflytning i tangentseksjon
    if Dybde_streng > MD_end_buildup && Dybde_streng <= MD_end_tangent;
        H(j) = H_end_buildup + ((MD(j)-MD_end_buildup)*sin(V(j)));
%((TVD(j)-TVD_end_buildup)*(tan(V(j))));
    end

    % Gir horisontal forflytning i drop seksjon
    if Dybde_streng > MD_end_tangent && Dybde_streng <= MD_end_drop &&
Vinkel_1 >= Vinkel_2;
        H(j) = H_end_tangent + ((Vinkel_radius_2*cos(V(j)))-
(Vinkel_radius_2*cos(Vinkel_1_rad)));
    end
    if Dybde_streng > MD_end_tangent && Dybde_streng <= MD_end_drop &&
Vinkel_1 < Vinkel_2;
        H(j) = H_end_tangent + ((Vinkel_radius_2*(cos(Vinkel_1_rad)))-
(Vinkel_radius_2*cos(V(j))));
```

```
    end

      % Gir horisontal forflytning i hold seksjon
    if Dybde_streng > MD_end_drop && Dybde_streng <= MD_end_hold;
        H(j) = H_end_drop + ((TVD(j)-TVD_end_drop)*tan(V(j)));
    end

end

% Finner gravitasjon til hvert midtpunkt på strengen
for j=1:N+1;
Gravitasjon(j) = G*cos(V(j));
end

% Finner friksjon med oppdrift til hvert midtpunkt på strengen
for j=1:N+1;
    if j <= Pipes_in_csg;                               % Inni casing
        MYG(j) = my_staal_staal*G*sin(V(j))*Oppdrift;       % Beveger seg
        MYSG(j) = my_stat_staal_staal*G*sin(V(j))*Oppdrift; % Statisk

    else                                                % Utenfor
casing
        MYG(j) = my_staal_berg*G*sin(V(j));                 % Beveger seg
        MYSG(j) = my_stat_staal_berg*G*sin(V(j))*Oppdrift;  % Statisk
    end
end


%% Finner trykk og oppdrift på to punkter

% Overgang borerør/vektrør
Pc = RHOm*g*TVD(NCB+1);     % [Pa]      Trykk ved overgang borerør/vektrør
KC = 2*(Mv-Mb)*(DELTAt^2)*Pc/(DELTAz*RHOs*(Mb+Mv));% [m] Oppdrift overgang

% Borekrone
Pb = RHOm*g*TVD(N+1);       % [Pa]     Trykk ved borekrona
KB = 2*(DELTAt^2)*Pb /(DELTAz*RHOs);   % [m] Oppdrift borekrone

%Kstempel = (DELTAt^2)*(Mv^2)*RHOm/(DELTAz*(Aslisser^2)*(Cd^2)*(RHOs^3));
% [s2/m]    Stempelkraft borekrone uten hastighet

%BHAfriksjon =
(RHOr^0.8)*(MYmud^0.2)*LV*100000/(70696*((IDcsg+ODv)^1.8)*((IDcsg-ODv)^3));
% Konstant i friksjon langs BHA. Mangler Q. Omgjort til Pa/Q.
%BHAkonstant = BHAfriksjon*2*(DELTAt^2)/(DELTAz*RHOs);   % Klart til å
settes inn i formelen

%% FÅR STRENGEN TIL Å FALLE TIL RO

%% Henter ut verdier løkken der strengen faller til ro
t1 = 0;
tEND1 =1000;                                % [sek]     Max tid som simuleres

n_hopp1 =     1;                             % Vil hente ut verdier med avstand
mellom lik n_hopp
dtut1 =   n_hopp1 * DELTAt;                % Tidsintervall mellom verdiene som
blir hentet ut
```

```
PunkterUT1 = 3;                      % Antall punkter som skal lagres
XUT1 =    (tEND1/dtut1:PunkterUT1);   % Oppgir størrelsen på matrisen som
skrives ut. Størrelsen blir desimaltall... Problem?
tut1 =    1;                         % Variabel som sørger for at kun hver
n_hopp'ende verdi blir hentet ut. Når tiden er 1 vil den ikke plottes. Må
kanskje bruke tut=0?
k1 =      0;                         % Teller som holder styr på tiden
(Tiden = k*dtut)

% Oppretter matrise for å lagre noen posisjoner i kommende løkke:
M1 = (1:PunkterUT1);

% Velger hvilke segmenter som skal hentes ut:
M1(1) = 1;                      % Øvre segment
M1(2) = convergent((NCB+1)/2);   % Segmentet nærmest midten av borerørs-
delen.
M1(3) = N+1;                    % Det nederste segmentet

% Ønsker å plotte XUT1, men med sekunder i stedet for
tEND1/(DELTAt*n_hopp1). Lager da en matrise som holder rede på tiden:
XUTtid1 = (PunkterUT1);           % Oppgir størrelsen på matrisen som
skrives ut
for j=1:PunkterUT1;               % Nullstiller tabellen
    XUTtid1  (j:PunkterUT1,1) = 0;
end

%% Får strengen til å falle til ro med gravitasjon og lineær friksjon:
eps1 = 0.01;        % Lineær friksjonskonstant.
a1 = 1-eps1;       %
b1 = 1-(2*eps1);   %

Grense = 0.000000001;  % Verdi som avgjør om streng er i ro

while t1 < tEND1;     % Fortsetter til fastsatt tid

    X1(1) = 0;        % Det øverste punktet henger i ro

    for j=2:N;    % Tildeler punktene under toppen verdier
        XN1 (j)=a1*(X1(j-1)+X1(j+1))-(b1*XG1(j))+(G*cos(V(j)));
% Gir nye verdier basert på sinusformelen og a & b som demper bevegelsen
(lineær friksjon).
    end

    XN1 (NCB+1)= CF1*X1(NCO)+CF2*X1(NCU)-XG1(NCB+1)+(G*cos(V(NCB+1)));    %
Beregner verdi overgang borerør/vektrør
    XN1 (N+1)= (a1*2*X1(N))-(b1*XG1(N+1))+ (G*cos(V(N+1)));              %
Beregner verdi overgang vektrør/borekrone

    % Lagrer noen utvalgte verdier:
    if t1 > tut1;
        tut1 = tut1 + dtut1;
        k1 = k1+1;
        for m1 = 1:PunkterUT1;
            XUT1(k1,m1) = X1(M1(m1));
            XUTtid1(k1) = k1*dtut1;
        end
    end
```

```
    if t1>40 && abs(XN1(NCB+1)-XG1(NCB+1))<Grense &&
abs(XN1(convergent(NCB/2))-XG1(convergent(NCB/2)))<Grense && abs(XN1(N+1)-
XG1(N+1))<Grense;
        t1 = tEND1; % Avslutter løkken om strengen er i ro
    end

    for j=2:N+1;              % Gir de nye verdiene til de forrige
tabellene
        XG1 (j)= X1 (j);
        X1 (j)= XN1 (j);
    end

    t1=t1+DELTAt;             % Øker et tidssteg
end

%% Plotter bevegelsen slik at man kan avgjøre om i ro er tilfredsstillende
%
% % Trykket "File", "Generate Code"
% figure2 = figure;
%
% % Create axes
% axes1 = axes('Parent',figure2,'YDir','reverse','FontSize',30);
% box(axes1,'on');
% hold(axes1,'all');
%
% % Create multiple lines using matrix input to plot
% plot1 = plot(XUTtid1,XUT1,'Parent',axes1);
% set(plot1(1),'LineWidth',2,'Color',[1 0 0],'DisplayName','Drill floor');
% set(plot1(2),'DisplayName','Middle of drill pipe string');
% %set(plot1(3),'DisplayName','Transition drill pipes/drill collars');
% set(plot1(3),'LineWidth',2,'Color',[0 0 1],'DisplayName','Drill bit');
%
% % Create xlabel
% xlabel('Time [s]','FontSize',30);
%
% % Create ylabel
% ylabel('Segment movement [m]','FontSize',30);
%
% % Create legend
% legend1 = legend(axes1,'show');
% set(legend1,...
%     'Position',[0.275694444444444 0.767496342887659 0.136979166666667
0.0816044260027662]);

%% STRENGKALKULASJONER

%% Henter ut verdier fra kommende løkke

n_hopp =        1;                    % Vil hente ut verdier med avstand
mellom lik n_hopp
dtut =          n_hopp * DELTAt;        % Tidsintervall mellom verdiene som
blir hentet ut
PunkterUT =     3;                      % Antall punkter som skal lagres
XUT =           (tEND/dtut:PunkterUT);  % Oppgir størrelsen på matrisen som
skrives ut. Størrelsen blir desimaltall... Problem?
tut =           1;                      % Variabel som sørger for at kun
hver n_hopp'ende verdi blir hentet ut.
k2 =             0;                      % Teller som holder styr på tiden
(Tiden = k2*dtut)
```

```
% Matriser for å lagre noen verdier i kommende løkke:
M = (1:PunkterUT);                          % Oppretter matrise for å hente
ønskede posisjoner på strengen

% Velger hvilke segmenter som skal hentes ut:
M(1) = 1;                           % Øvre segment
M(2) = convergent((NCB+1)/2);   % Segmentet nærmest midten av borerørs-
delen.
M(3) = N+1;                         % Det nederste segmentet

% Lager en matrise som holder rede på tiden:
XUTtid = (PunkterUT);           % Oppgir størrelsen på matrisen som skrives
ut
for j=1:PunkterUT;              % Nullstiller tabellen
    XUTtid  (j:PunkterUT,1) = 0;
end


%% KALKULERER MED KUN EN PERIODE OG EN AMPLITUDE
%
% % Setter parametre
%
% % Hastighet
%
% DTFART = 350;
% XFART = 0;
% XHAST = 0;
%
% tHAST = 0;
% tMAX = 0;
% tMIN = 0;
%
% XMAX = -1000;   % Max utslag
% XMIN = 1000;    % Min utslag
% XGJS = 0;       % Gjennomsnitt
% XKVAD = 0;      % Kvadratet
% KFART = 0;
% XN1FART = 0;
% XHAST = 0;
% Teller = 0;
%
% A_MAXUT = 0;
% KVADUT = 0;
% A_HASTUT = 0;
%
% %% Verdier for bevegelse av boredekk
%
% % Parametre til sinuskurven:
% % Amplitude som endres
% A1 =  3;              % [m]   Amplitude 1
%
% % Periode som endres
% T =  10;              % [sek] Periode, en syklus
%
% A2 =    A1/5;         % [m]       Amplitude 2
% fi =      pi/2;           % [Rad]     Forskyvning
% omega = 2*pi/T;       % [Rad/sek] Vinkelfrekvensen [Angular frequency]
%
```

```
% %% Tildeler verdien til strengen slik at den starter i ro, men strekket
% for j=1:N+1;
%     XG(j) =     XG1(j);
%     X(j)=       X1(j);
%     XN(j) =     XN1 (j);
% end
%
% strengen_ryker = 1;
% XN1FART = 0;
%
% while t < tEND;   % Fortsetter til fastsatt tid
%
%     %% Væskefriksjon
%
%     if t<tSTART/7
%         for j=2:N+1;
%             Eps(j) = 0.01; % Bruker høy lineær friksjon for å drepe
initielle svingninger
%         end
%
%     elseif t<tSTART*2/7
%         for j=2:N+1;
%             Eps(j) = 0.001; % Bruker høy lineær friksjon for å drepe
initielle svingninger
%         end
%
%     elseif t<tSTART*3/7
%         for j=2:N+1;
%             Eps(j) = 0.0001; % Bruker høy lineær friksjon for å drepe
initielle svingninger
%         end
%
%     elseif t<tSTART*4/7
%         for j=2:N+1;
%             Eps(j) = 0.00001; % Bruker høy lineær friksjon for å drepe
initielle svingninger
%         end
%     elseif t<tSTART*5/7
%         for j=2:N+1;
%             Eps(j) = 0.000001; % Bruker høy lineær friksjon for å drepe
initielle svingninger
%         end
%     else
% % Finner virkelig viskøs friksjonskonstant til hvert midtpunkt på
strengen
%         for j=2:N+1;
%
%             if j>1 && j<NCB+1;      % Borerør
%                 Eps(j) = epsB;
%             end
%
%             if j== NCB+1;           % Overgang borerør/vektrør
%                 Eps(j) = epsOV;
%             end
%
%             if j>NCB+1 && j<N+1;    % Vektrør
%                 Eps(j) = epsC;
%             end
%
%             if j==N+1;              % Overgang vektrør/borekrone
```

```
%                   Eps(j) = epsOB;
%               end
%           end
%
%           % Her kan eps bli 0, mens du kjører inn den riktige?
%       end
%
%       for j=2:N+1;
%           EpsA(j) = 1-Eps(j);
%           EpsB(j) = 1-(2*Eps(j));
%       end
%
%
%       %% Øverste punkt
%
%       % Gir det øverste punktet en verdi ifølge bølgeligning
%       X(1) = A1*sin(omega*t) + A2*sin(2*omega*t+fi);
%
%
%       %% Alle punkter under toppen til overgang
%
%       for j=2:NCB;                          % Ønsker ikke å gi det øverste
punktet en verdi, men resten
%           XNY = (EpsA(j)*(X(j-1)+X(j+1)))-(EpsB(j)*XG(j))+Gravitasjon(j);
% Standardformelen+gravitasjon+væskefriksjon
%           %XNY = (X(j-1)+X(j+1))-XG(j)+Gravitasjon(j);      %
Standardformelen
%           DXN = XNY-X(j);                 % Avstand ny/gammel verdi
%
%           if Bevegelse(j) > 0.5;          % I RO
%               if abs(DXN) > MYSG(j);      % Dersom bevegelse er større enn
statisk friksjon
%                   Bevegelse(j) = 0;
%                   XN(j) = XNY - sign(DXN)*MYG(j);
%               else                        % Punktet er i ro
%                   XN(j) = X(j);
%               end
%
%           else                            % Bevegelse
%               if abs(DXN) > MYG(j);       % Bevegelsen er større enn
dynamisk friksjon
%                   XN(j) = XNY - sign(DXN)*MYG(j);
%               else
%                   XN(j) = X(j);
%                   Bevegelse(j) = 1;
%               end
%           end
%       end
%
%       % Forklaring Bevegelse(j):
%       % I ro fra XG(j) til X(j), Bevegelse(j)=1
%       % Beveger seg fra XG(j) til X(j), Bevegelse(j)=0
%
%       %% Overgang borerør/vektrør
%
%       XNY = EpsA(j)*(CF1*X(NCO)+CF2*X(NCU))-
(EpsB(j)*XG(NCB+1))+Gravitasjon(NCB+1)+KC;      % Overgang borerør/vektrør
%       %XNY = CF1*X(NCO)+CF2*X(NCU)-XG(NCB+1)+Gravitasjon(NCB+1)+KC;     %
Overgang borerør/vektrør
%
```

```
%     DXN = XNY-X(NCB+1);       % Avstand ny/gammel verdi
%
%     if Bevegelse(NCB+1) > 0.5;        % I RO
%         if abs(DXN) > MYSG(NCB+1);  % Dersom bevegelse er større enn
statisk friksjon
%             Bevegelse(NCB+1) = 0;
%             XN(NCB+1) = XNY - sign(DXN)*MYG(NCB+1);
%         else
%             XN(NCB+1) = X(NCB+1);
%         end
%
%     else                            % Bevegelse
%         if abs(DXN) > MYG(NCB+1);
%             XN(NCB+1) = XNY - sign(DXN)*MYG(NCB+1);
%         else
%             XN(NCB+1) = X(NCB+1);
%             Bevegelse(NCB+1) = 1;
%         end
%     end
%
%     %% Alle punkter under overgang til borekrone
%
%     for j=NCB+2:N;                            % Ønsker ikke å gi det
øverste punktet en verdi, men resten
%         XNY = (EpsA(j)*(X(j-1)+X(j+1)))-(EpsB(j)*XG(j))+Gravitasjon(j);
% Standardformelen legg til gravitasjon og friksjon
%         %XNY = (X(j-1)+X(j+1))-XG(j)+Gravitasjon(j);      %
Standardformelen
%
%         DXN = XNY-X(j);               % Avstand ny/gammel verdi
%
%         if Bevegelse(j) > 0.5;        % I RO
%             if abs(DXN) > MYSG(j);      % Dersom bevegelse er større enn
statisk friksjon
%                 Bevegelse(j) = 0;
%                 XN(j) = XNY - sign(DXN)*MYG(j);
%             else                        % Punktet er i ro
%                 XN(j) = X(j);
%             end
%
%         else                            % Bevegelse
%             if abs(DXN) > MYG(j);       % Bevegelsen er større enn
dynamisk friksjon
%                 XN(j) = XNY - sign(DXN)*MYG(j);
%             else
%                 XN(j) = X(j);
%                 Bevegelse(j) = 1;
%             end
%         end
%     end
%
%     %% Overgang vektrør/borekrone
%
%     % Finner stempelkraften
%     %Ks = Kstempel*(XN1FART^2);
%     %Kfriksjon = BHAkonstant*((XN1FART*Aring)^1.8);
%
%     XNY = (EpsA(j)*2*X(N))-(EpsB(j)*XG(N+1))+Gravitasjon(N+1)-KB;%
+(sign(X(N+1)-XG(N+1))*(Ks+Kfriksjon));
```

```
%     %XNY = 2*X(N)-XG(N+1)+Gravitasjon(N+1)-KB;        % overgang
vektrør/borekrone
%
%     DXN = XNY-X(N+1);         % Avstand ny/gammel verdi
%
%     if Bevegelse(N+1) > 0.5;          % I RO
%         if abs(DXN) > MYSG(N+1);  % Dersom bevegelse er større enn
statisk friksjon
%             Bevegelse(N+1) = 0;
%             XN(N+1) = XNY - sign(DXN)*MYG(N+1);
%         else
%             XN(N+1) = X(N+1);
%         end
%
%     else                            % Bevegelse
%         if abs(DXN) > MYG(N+1);
%             XN(N+1) = XNY - sign(DXN)*MYG(N+1);
%         else
%             XN(N+1) = X(N+1);
%             Bevegelse(N+1) = 1;
%         end
%     end
%
%     %% Lagrer noen utvalgte verdier:
%     if t > tut;
%         tut = tut + dtut;
%         k2 = k2+1;
%         for m = 1:PunkterUT;
%             XUT(k2,m) = X(M(m)); %(X(M(m))+XN(M(m)))/2;
%             XUTtid(k2) = k2*dtut;
%         end
%     end
%
%     %% Sjekker om strengen ryker øverst
%     if t>tSTART && t<tEND
%         if strengen_ryker > 0
%             if Es*abs((X(2)-X(1))/DELTAz)>Flytgrense;
%                 Brudd = ['STRENGEN RØK. Stresset i strengen er '
num2str((Es*abs((X(2)-X(1))/DELTAz))/100000), ' Bar. Tiden er ' num2str(t),
' sekunder. T er ', num2str(T), ' sekunder, A1 er ', num2str(A1), '
meter.']
%                 strengen_ryker = 0;
%                 t=tEND;
%             end
%         end
%     end
%
%     %% Hopper ut av while-løkken dersom strengen ryker
%     if strengen_ryker==0
%         break
%     end
%
%     %% Henter farten til borekronen
%     XN1FART=abs((XN(N+1)-X(N+1))/DELTAt);
%
%     %% Øker et tidssteg
%     t=t+DELTAt;
%
%
%
```

```
%     %% Henter ut maksverdier
%     if t>tSTART
%         if XN(N+1) > XMAX;
%             XMAX = XN(N+1);
%             tMAX = t-1;
%         end
%
%         if XN(N+1) < XMIN;
%             XMIN = XN(N+1);
%             tMIN = t-1;
%         end
%
%         if t < tSTART+1.5*DELTAt;
%             XFART = XN(N+1);
%         end
%
%         KFART = KFART+1;
%
%         if KFART > DTFART && t<tSLUTT;
%             if XHAST < abs((XN(N+1)-XFART)/(DTFART*DELTAt));
%                 XHAST = abs((XN(N+1)-XFART)/(DTFART*DELTAt));
%                 XFART = XN(N+1);
%                 KFART = 1;
%                 tHAST = t-1;
%             end
%         end
%     end
%
%     %% Gir de nye verdiene til de forrige tabellene
%     for j2=2:N+1;
%         XG (j2)= X (j2);
%         X (j2)= XN (j2);
%     end
%
% end
%
%  %% Henter ut verdier
%  A_MAXUT = XMAX-XMIN; % Største forskjell mellom min og maks
%  A_HASTUT = XHAST;    % Maks hastighet



%% KALKULERER MED FLERE PERIODER OG AMPLITUDER
% Setter parametre

% Amplitude:
AMP0 = 0.5;      % Startamplitude
DAMP = 0.5;      % Økning i amplitude
AMPMAKS = 3;     % Maks amplitude
AntallAmp = ((AMPMAKS-AMP0)/DAMP)+1;     % Antall amplituder

% Lager en matrise som holder rede på amplituden:
AmplitudeNr = (AntallAmp);  % Oppgir størrelsen på matrisen som skrives ut
for j=1:AntallAmp;          % Nullstiller tabellen
    AmplitudeNr  (j:AntallAmp,1) = 0;
end

% Periode
PTID0 = 10;      % Start bølgeperiode
```

```
DTID = 5;       % Økning i bølgeperiode
PTIDMAKS = 20;  % Maks bølgeperiode
AntallPer = ((PTIDMAKS-PTID0)/DTID)+1;     % Antall amplituder

% Lager en matrise som holder rede på perioden:
PeriodeNr = (AntallPer);    % Oppgir størrelsen på matrisen som skrives ut
for j=1:AntallPer;          % Nullstiller tabellen
    PeriodeNr  (j:AntallPer,1) = 0;
end

% Hastighet
kFART = 0;
DTFART = 350;
XFART = 0;
XHAST = 0;

kTID = convergent(((PTIDMAKS-PTID0)/DTID)+1);
jAMP = convergent(((AMPMAKS-AMP0)/DAMP)+1);

% Lager tabeller for å få utskrift av maksverdier
A_MAXUT =     (jAMP);
KVADUT =    (jAMP);
A_HASTUT =    (jAMP);


%% Nullstiller tabellene
for k5=1:kTID;
    for j5=1:jAMP;
        A_MAXUT(j5,k5) = 0;
        %KVADUT(j5,k5) = 0;
        A_HASTUT(j5,k5) = 0;
    end
end

for kk=1:kTID;
    PTD = PTID0+(kk-1)*DTID;

    PTD         % Skriver ut Perioden

    for jj=1:jAMP;
        AMP = AMP0+(jj-1)*DAMP;

        strengen_ryker = 1;

        AMP     % Skriver ut amplituden

        AmplitudeNr(jj) = AMP;
        PeriodeNr(kk) = PTD;

        %% Verdier for bevegelse av boredekk

        % Parametre til sinuskurven:

        % Amplitude som endres
        A1 =    AMP;                % [m]   Amplitude 1

        % Periode som endres
```

```
        T =     PTD;                 % [sek] Periode, en syklus

        A2 =    A1/5;            % [m]       Amplitude 2
        fi =    pi/2;            % [Rad]     Forskyvning
        omega = 2*pi/T;          % [Rad/sek] Vinkelfrekvensen [Angular
frequency]


        %% Nullstiller variabler
        t=0;

        for j1 = 2:(N+1);
            X(j1) = 0;
            XG(j1) = 0;
        end

        XMAX = -1000;   % Max utslag
        XMIN = 1000;    % Min utslag
        XGJS = 0;       % Gjennomsnitt
        XKVAD = 0;      % Kvadratet
        KFART = 0;
        XN1FART = 0;
        XHAST = 0;
        Teller = 0;

        % Tildeler verdien til strengen slik at den starter i ro, men
strekket
        for j7=2:N+1;
            XG(j7) =     XG1(j7);
            X(j7)=       X1(j7);
            XN(j7) =     XN1 (j7);
        end

        while t < tEND && strengen_ryker > 0    % Fortsetter til fastsatt
tid

%             XN1FART = 0;
            %% Væskefriksjon

            if t<tSTART/7
                for j=2:N+1;
                    Eps(j) = 0.01; % Bruker høy lineær friksjon for å drepe
initielle svingninger
                end

            elseif t<tSTART*2/7
                for j=2:N+1;
                    Eps(j) = 0.001; % Bruker høy lineær friksjon for å
drepe initielle svingninger
                end

            elseif t<tSTART*3/7
                for j=2:N+1;
                    Eps(j) = 0.0001; % Bruker høy lineær friksjon for å
drepe initielle svingninger
                end
```

```
            elseif t<tSTART*4/7
                for j=2:N+1;
                    Eps(j) = 0.00001; % Bruker høy lineær friksjon for å
drepe initielle svingninger
                end
            elseif t<tSTART*5/7
                for j=2:N+1;
                    Eps(j) = 0.000001; % Bruker høy lineær friksjon for å
drepe initielle svingninger
                end

            else
                % Finner virkelig viskøs friksjonskonstant til hvert
midtpunkt på strengen
                for j=2:N+1;

                    if j>1 && j<NCB+1;      % Borerør
                        Eps(j) = epsB;
                    end

                    if j== NCB+1;           % Overgang borerør/vektrør
                        Eps(j) = epsOV;
                    end

                    if j>NCB+1 && j<N+1;    % Vektrør
                        Eps(j) = epsC;
                    end

                    if j==N+1;              % Overgang vektrør/borekrone
                        Eps(j) = epsOB;
                    end
                end

                % Her kan eps bli 0, mens du kjører inn den riktige?
            end

            for j=2:N+1;
                EpsA(j) = 1-Eps(j);
                EpsB(j) = 1-(2*Eps(j));
            end

            %% Øverste punkt

            % Gir det øverste punktet en verdi ifølge bølgeligning
            X(1) = A1*sin(omega*t) + A2*sin(2*omega*t+fi);

            %% Alle punkter under toppen til overgang

            for j=2:NCB;                        % Ønsker ikke å gi det
øverste punktet en verdi, men resten
                XNY = (EpsA(j)*(X(j-1)+X(j+1)))-
(EpsB(j)*XG(j))+Gravitasjon(j);     % Standardformelen

                DXN = XNY-X(j);                 % Avstand ny/gammel verdi

                if Bevegelse(j) > 0.5;          % I RO
```

```
                    if abs(DXN) > MYSG(j);      % Dersom bevegelse er
større enn statisk friksjon
                        Bevegelse(j) = 0;
                        XN(j) = XNY - sign(DXN)*MYG(j);
                    else                        % Punktet er i ro
                        XN(j) = X(j);
                    end

                else                            % Bevegelse
                    if abs(DXN) > MYG(j);       % Bevegelsen er større enn
dynamisk friksjon
                        XN(j) = XNY - sign(DXN)*MYG(j);
                    else
                        XN(j) = X(j);
                        Bevegelse(j) = 1;
                    end
                end
            end

            % Forklaring Bevegelse(j):
            % I ro fra XG(j) til X(j), Bevegelse(j)=1
            % Beveger seg fra XG(j) til X(j), Bevegelse(j)=0

            %% Overgang borerør/vektrør

            XNY = (EpsA(j)*(CF1*X(NCO)+CF2*X(NCU)))-
(EpsB(j)*XG(NCB+1))+Gravitasjon(NCB+1)+KC;      % Overgang borerør/vektrør

            DXN = XNY-X(NCB+1);                 % Avstand ny/gammel verdi

            if Bevegelse(NCB+1) > 0.5;          % I RO
                if abs(DXN) > MYSG(NCB+1);      % Dersom bevegelse er større
enn statisk friksjon
                    Bevegelse(NCB+1) = 0;
                    XN(NCB+1) = XNY - sign(DXN)*MYG(NCB+1);
                else
                    XN(NCB+1) = X(NCB+1);
                end

            else                                % Bevegelse
                if abs(DXN) > MYG(NCB+1);
                    XN(NCB+1) = XNY - sign(DXN)*MYG(NCB+1);
                else
                    XN(NCB+1) = X(NCB+1);
                    Bevegelse(NCB+1) = 1;
                end
            end

            %% Alle punkter under overgang til borekrone

            for j=NCB+2:N;                      % Ønsker ikke å gi det øverste
punktet en verdi, men resten
                XNY = (EpsA(j)*(X(j-1)+X(j+1)))-
(EpsB(j)*XG(j))+Gravitasjon(j);     % Standardformelen

                DXN = XNY-X(j);                 % Avstand ny/gammel verdi

                if Bevegelse(j) > 0.5;          % I RO
```

```
                    if abs(DXN) > MYSG(j);  % Dersom bevegelse er større
enn statisk friksjon
                        Bevegelse(j) = 0;
                        XN(j) = XNY - sign(DXN)*MYG(j);
                    else                    % Punktet er i ro
                        XN(j) = X(j);
                    end

                else                        % Bevegelse
                    if abs(DXN) > MYG(j);   % Bevegelsen er større enn
dynamisk friksjon
                        XN(j) = XNY - sign(DXN)*MYG(j);
                    else
                        XN(j) = X(j);
                        Bevegelse(j) = 1;
                    end
                end
            end

            %% Overgang vektrør/borekrone

            % Finner stempelkraften
            %Ks = Kstempel*(XN1FART^2);
            %Kfriksjon = BHAkonstant*((XN1FART*Aring)^1.8);

            XNY = (EpsA(j)*2*X(N))-(EpsB(j)*XG(N+1))+Gravitasjon(N+1)-KB;
%+(sign(X(N+1)-XG(N+1))*(Ks+Kfriksjon));

            DXN = XNY-X(N+1);           % Avstand ny/gammel verdi

            if Bevegelse(N+1) > 0.5;    % I RO
                if abs(DXN) > MYSG(N+1);% Dersom bevegelse er større enn
statisk friksjon
                    Bevegelse(N+1) = 0;
                    XN(N+1) = XNY - sign(DXN)*MYG(N+1);
                else
                    XN(N+1) = X(N+1);
                end

            else                        % Bevegelse
                if abs(DXN) > MYG(N+1);
                    XN(N+1) = XNY - sign(DXN)*MYG(N+1);
                else
                    XN(N+1) = X(N+1);
                    Bevegelse(N+1) = 1;
                end
            end

            %% Lagrer noen utvalgte verdier:
            if t > tut;
                tut = tut + dtut;
                k2 = k2+1;
                for m = 1:PunkterUT;
                    XUT(k2,m) = X(M(m)); %(X(M(m))+XN(M(m)))/2;
                    XUTtid(k2) = k2*dtut;
                end
            end
```

```
            %% Sjekker om strengen ryker øverst
            if t>tSTART && t<tEND
                if strengen_ryker > 0
                    if Es*abs((X(2)-X(1))/DELTAz)>Flytgrense;
                        Brudd = ['STRENGEN RØK. Stresset i strengen er '
num2str((Es*abs((XN(2)-XN(1))/DELTAz))/100000), ' Bar. Tiden er '
num2str(t), ' sekunder. T er ', num2str(PTD), ' sekunder, A1 er ',
num2str(AMP), ' meter.']
                        strengen_ryker = 0;
                        t=tEND;
                    end
                end
            end

            %% Hopper ut av while-løkken dersom strengen ryker
            if strengen_ryker==0
                break
            end

            %% Henter ut hastigheten for hvert steg
            %XN1FART=abs((XN(N+1)-XFART)/(DTFART*DELTAt));

            %% Øker et tidssteg
            t=t+DELTAt;

            %% Henter ut maksverdier
            if t>tSTART && t<=tEND && strengen_ryker>0

                if XN(N+1) > XMAX;
                    XMAX = XN(N+1);
                end

                if XN(N+1) < XMIN;
                    XMIN = XN(N+1);
                end

                if t < tSTART+1.5*DELTAt;
                    XFART = XN(N+1);
                end

                KFART = KFART+1;

                if KFART > DTFART && t<tSLUTT;
                    if XHAST < abs((XN(N+1)-XFART)/(DTFART*DELTAt));
                        XHAST = abs((XN(N+1)-XFART)/(DTFART*DELTAt));
                        XFART = XN(N+1);
                        KFART = 1;
                    end
                end

                %XGJS = XGJS+XN(N+1);
                %XKVAD = XKVAD + (XN(N+1))^2;
                Teller = Teller+1;
            end

            %% Gir de nye verdiene til de forrige tabellene
            for j2=2:N+1;
                XG (j2)= X (j2);
```

```
                X (j2)= XN (j2);
            end

        end

            %% Henter ut verdier
            A_MAXUT(jj,kk) = XMAX-XMIN; % Største forskjell mellom min og
maks
            %KVADUT(jj,kk) = (XKVAD-((XGJS^2)/Teller)^0.5)/Teller;
            % KVADUT(jj,kk) = ((XKVAD/Teller)-(XGJS/Teller)^2)*2;
            A_HASTUT(jj,kk) = XHAST;    % Maks hastighet

        %% Hopper ut av for-løkken dersom strengen ryker
        if strengen_ryker==0
            break
        end

    end

end


%% PLOTTER BEVEGELSEN

figure1 = figure;

% Create axes
axes1 = axes('Parent',figure1,'YDir','reverse','FontSize',30);
box(axes1,'on');
hold(axes1,'all');

% Create multiple lines using matrix input to plot
plot1 = plot(XUTtid,XUT,'Parent',axes1);
set(plot1(1),'LineWidth',2,'Color',[1 0 0],'DisplayName','Drill floor');
set(plot1(2),'DisplayName','Middle of drill pipe section');
%set(plot1(3),'DisplayName','Transition drill pipes/drill collars');
set(plot1(3),'LineWidth',2,'Color',[0 0 1],'DisplayName','Drill bit');

% Create xlabel
xlabel('Time [s]','FontSize',30);

% Create ylabel
ylabel('Segment movement [m]','FontSize',30);

% Create legend
legend1 = legend(axes1,'show');
set(legend1,...
    'Position',[0.687934027777776 0.155463147866916 0.19375
0.112724757952974]);

%% PLOTTER BRØNNBANEN
%
% % Create figure
% figure2 = figure;
%
% % Create axes
% axes1 = axes('Parent',figure2,'YDir','reverse',...
```

```
%     'Position',[0.13 0.11 0.280416666666667 0.815],...
%     'FontSize',30);
% box(axes1,'on');
% hold(axes1,'all');
%
% % Create plot
% plot(H,TVD,'LineWidth',3,'DisplayName','TVD vs. H');
%
% % Create xlabel
% xlabel('Horizontal Reach [m]','FontSize',30);
%
% % Create ylabel
% ylabel('True Vertical Depth [m]','FontSize',30);


%% PLOTTER HASTIGHETEN
%
% % Create figure
% figure3 = figure;
%
% % Create axes
% axes1 = axes('Parent',figure3,'FontSize',30);
% box(axes1,'on');
% hold(axes1,'all');
%
% % Create multiple lines using matrix input to plot
% plot1 = plot(AmplitudeNr,A_HASTUT,'Parent',axes1,'LineWidth',2);
%
% set(plot1(1),'DisplayName','Time period = 10 seconds');
% set(plot1(2),'DisplayName','Time period = 15 seconds');
% set(plot1(3),'DisplayName','Time period = 20 seconds');
%
% % set(plot1(1),'DisplayName','Time period =  5 seconds');
% % set(plot1(2),'DisplayName','Time period = 10 seconds');
% % set(plot1(3),'DisplayName','Time period = 15 seconds');
% % set(plot1(4),'DisplayName','Time period = 20 seconds');
%
% % Create xlabel
% xlabel('Top segment amplitude [m]','FontSize',30);
%
% % Create ylabel
% ylabel('Velocity [m/s]','FontSize',30);
%
% % Create legend
% legend1 = legend(axes1,'show');
% set(legend1,...
%     'Position',[0.149913194444443 0.730992355222803 0.19375
0.148686030428769]);

%% PLOTTER MAKS UTSLAG
%
% % Create figure
% figure4 = figure;
%
% % Create axes
% axes1 = axes('Parent',figure4,'FontSize',30);
% box(axes1,'on');
% hold(axes1,'all');
%
```

```
% % Create multiple lines using matrix input to plot
% plot1 = plot(AmplitudeNr,A_MAXUT,'Parent',axes1,'LineWidth',2);
%
% set(plot1(1),'DisplayName','Time period = 10 seconds');
% set(plot1(2),'DisplayName','Time period = 15 seconds');
% set(plot1(3),'DisplayName','Time period = 20 seconds');
%
% % set(plot1(1),'DisplayName','Time period =  5 seconds');
% % set(plot1(2),'DisplayName','Time period = 10 seconds');
% % set(plot1(3),'DisplayName','Time period = 15 seconds');
% % set(plot1(4),'DisplayName','Time period = 20 seconds');
%
% % Create xlabel
% xlabel('Top segment amplitude [m]','FontSize',30);
%
% % Create ylabel
% ylabel('Max drill bit amplitude [m]','FontSize',30);
%
% % Create legend
% legend1 = legend(axes1,'show');
% set(legend1,...
%     'Position',[0.149913194444443 0.730992355222803 0.19375
0.148686030428769]);

%% PLOTTER DEN KVADRERTE
%
% % Create figure
% figure5 = figure;
%
% % Create axes
% axes1 = axes('Parent',figure5,'FontSize',30);
% box(axes1,'on');
% hold(axes1,'all');
%
% % Create multiple lines using matrix input to plot
% plot1 = plot(AmplitudeNr,KVADUT,'Parent',axes1,'LineWidth',2);
% set(plot1(1),'DisplayName','Time period = 10 seconds');
% set(plot1(2),'DisplayName','Time period = 15 seconds');
% set(plot1(3),'DisplayName','Time period = 20 seconds');

% % set(plot1(1),'DisplayName','Time period =  5 seconds');
% % set(plot1(2),'DisplayName','Time period = 10 seconds');
% % set(plot1(3),'DisplayName','Time period = 15 seconds');
% % set(plot1(4),'DisplayName','Time period = 20 seconds');
%
% % Create xlabel
% xlabel('Top segment amplitude [m]','FontSize',30);
%
% % Create ylabel
% ylabel('xxx','FontSize',30);
%
% % Create legend
% legend1 = legend(axes1,'show');
% set(legend1,...
%     'Position',[0.149913194444443 0.730992355222803 0.19375
0.148686030428769]);
```

B. Results from the simulations

B.1 Vertical well

1000 m				2000m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,42921	0,291	0,21764	1	0,49354	0,31068	0,22687
2	0,85153	0,58701	0,43436	2	0,99095	0,62015	0,45591
3	1,28007	0,88454	0,65144	3	1,47435	0,93996	0,68709
4	1,7088	1,17938	0,87054	4	1,96523	1,25897	0,91801
5	2,14069	1,47402	1,08529	5	2,4567	1,57236	1,13795
6	2,5688	1,74047	1,30003	6	2,97887	1,86206	1,36615
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	1,0298	1,01528	1,00509	1	1,10185	1,04061	1,02587
2	2,05958	2,03065	2,0102	2	2,20317	2,08149	2,05226
3	3,08935	3,04603	3,01531	3	3,30448	3,12238	3,07865
4	4,11913	4,0614	4,02042	4	4,40579	4,16327	4,10503
5	5,1489	5,07677	5,02553	5	5,5071	5,20415	5,13142
6	6,17867	6,09215	6,03063	6	6,60842	6,24504	6,15781

3000 m				4000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,5986	0,33373	0,23768	1	0,77977	0,35113	0,24885
2	1,19148	0,65126	0,4789	2	1,56506	0,70146	0,49653
3	1,78318	0,98299	0,71443	3	2,33885	1,05644	0,74491
4	2,39143	1,32774	0,95037	4	3,11768	1,40273	0,99326
5	2,96537	1,64847	1,1995	5	3,91568	1,74335	1,23539
6	3,55825	1,97726	1,44276	6	4,67607	2,09297	1,48865
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	1,23755	1,07916	1,04792	1	1,56237	1,11645	1,07422
2	2,47414	2,15851	2,09677	2	3,12305	2,2331	2,1501
3	3,71074	3,23786	3,14562	3	4,68374	3,34977	3,22598
4	4,94733	4,31721	4,19447	4	6,24443	4,46644	4,30186
5	6,18393	5,39656	5,24331	5	7,80512	5,5831	5,37775
6	7,42053	6,47591	6,29216	6	9,3658	6,69977	6,45363

5000 m				6000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	3,49005	0,42088	0,25618	1	1,1988	0,48814	0,27854
2	6,9928	0,83563	0,5079	2	2,38721	0,98094	0,55939
3	10,4734	1,24974	0,76826	3	3,5892	1,45775	0,8455
4	12,3569	1,66948	1,02551	4	4,79767	1,96063	1,13025
5	0	2,10008	1,27329	5	5,96094	2,45315	1,40944
6	0	2,51812	1,54929	6	7,17026	2,92446	1,69212
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	5,88721	1,23941	1,11267	1	2,24647	1,412	1,15258
2	11,774	2,47799	2,22762	2	4,48975	2,82203	2,30774
3	17,6608	3,71657	3,34258	3	6,73303	4,23206	3,46318
4	22,0602	4,95515	4,45753	4	8,97631	5,6421	4,61863
5	0	6,19374	5,57249	5	11,2196	7,05213	5,77408
6	0	7,43232	6,68744	6	13,4629	8,46216	6,92953

7000 m				8000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,83264	0,58962	0,30204	1	1,16164	2,19801	0,35799
2	1,67444	1,17403	0,6136	2	2,32163	4,39573	0,71324
3	2,50281	1,77021	0,91804	3	3,44497	6,57354	1,08638
4	3,33968	2,36954	1,22715	4	4,58903	8,50991	1,42801
5	4,18456	2,96565	1,51777	5	5,70991	0	1,78628
6	5,03085	3,54915	1,83433	6	6,8917	0	2,14581
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	1,91591	1,61029	1,21302	1	2,98692	5,37247	1,34943
2	3,83219	3,21973	2,42746	2	5,96804	10,7393	2,7018
3	5,74847	4,82917	3,64189	3	8,94916	16,106	4,05417
4	7,66475	6,43861	4,85633	4	11,9303	21,1892	5,40655
5	9,58104	8,04805	6,07076	5	14,9114	0	6,75893
6	11,4973	9,65749	7,2852	6	17,8925	0	8,11132

9000 m			
Max bit velocity			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	1,21237	1,83073	0,3798
2	2,41194	3,6688	0,76178
3	3,63021	5,50225	1,1477
4	4,84508	6,77091	1,53047
5	6,06633	0	1,93142
6	6,40039	0	2,30113
Max bit amplitude			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	3,31405	4,72645	1,4379
2	6,62848	9,45351	2,87674
3	9,9429	14,1806	4,31559
4	13,2573	17,3433	5,75443
5	16,5718	0	7,19327
6	19,3035	0	8,63212

B.2 10 degrees deviation

1000 m				2000m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,44348	0,32491	0,25539	1	0,67067	0,40535	0,28989
2	0,90169	0,66799	0,51717	2	1,06391	0,79886	0,51526
3	1,32676	0,97155	0,73586	3	1,66155	0,98681	0,78662
4	1,75173	1,27992	0,90137	4	2,09375	1,30042	1,01464
5	2,16659	1,60242	1,09904	5	2,33025	1,51114	1,28406
6	2,55218	1,91857	1,32255	6	2,78897	1,81776	1,53018
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,97835	0,97319	0,9588	1	0,87438	0,93598	0,90917
2	2,04924	1,98956	1,95433	2	2,1382	1,90165	1,93149
3	3,07515	2,9876	2,9675	3	3,21236	2,91186	2,91644
4	4,10083	3,98469	3,99134	4	4,28314	4,06987	3,89389
5	5,12682	4,98181	5,00957	5	5,35409	5,1451	4,87405
6	6,15326	5,98532	6,021	6	6,4253	6,18134	5,96364

3000 m				4000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,72934	0,48907	0,33682	1	0,78739	0,49961	0,37686
2	1,30477	0,88762	0,64739	2	1,4396	0,94381	0,72331
3	1,84346	1,25795	0,89101	3	2,26193	1,36253	1,01391
4	2,3589	1,63192	1,24044	4	3,34825	1,94322	1,24404
5	2,91605	1,92951	1,53626	5	4,02725	2,3652	1,55759
6	3,53174	2,20119	1,75955	6	4,69909	2,83836	1,79963
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	1,1227	0,78265	0,87162	1	1,3327	0,90793	0,69035
2	2,27773	2,05175	1,88088	2	2,77857	1,91747	1,82706
3	3,44694	3,14331	2,85382	3	4,38886	2,8846	3,03862
4	4,62006	4,21147	3,81542	4	6,0234	3,86613	4,11206
5	5,79016	5,27094	4,77991	5	7,43943	4,9136	5,17108
6	6,96268	6,32069	5,85566	6	8,83308	6,11665	6,22211

5000 m				6000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,78877	0,49289	0,35753	1	0,767	0,39195	0,26303
2	2,02983	1,05161	0,7443	2	1,74096	1,04914	0,78484
3	3,52467	1,50074	1,09746	3	2,82093	1,47778	1,13087
4	6,73251	1,78133	1,49089	4	3,8143	1,91533	1,51942
5	9,16396	2,20394	1,87364	5	4,94612	2,37067	1,88656
6	11,5792	2,60784	1,98003	6	6,1811	2,80854	2,21888
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	1,48651	0,95744	0,71267	1	1,52021	0,7866	0,5641
2	3,76742	2,22911	1,7037	2	3,56597	2,48463	1,88648
3	6,92809	3,38306	2,61479	3	5,56259	3,82861	2,96149
4	12,0512	4,53968	3,5005	4	7,54706	5,1088	3,99177
5	16,9673	5,69773	4,6364	5	9,6546	6,39409	5,01051
6	21,6068	6,86262	6,27195	6	11,7879	7,68546	6,01945

7000 m				8000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,08175	0,00034	0	1	0,00056	0,00063	0,00033
2	1,739	1,05359	0,77048	2	1,79987	1,0799	0,7379
3	2,58098	1,55898	1,15396	3	2,83367	1,73635	1,20869
4	3,39235	2,14391	1,57388	4	3,77279	2,72819	1,53808
5	4,18352	2,9691	1,90976	5	4,63396	3,66369	1,86151
6	4,98023	3,78639	2,14533	6	5,42792	4,82041	2,17906
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,11873	0,00132	0	1	0,00304	0,00231	0,00111
2	3,64605	2,7073	2,00213	2	4,0159	2,87649	1,97014
3	5,50394	4,31024	3,25876	3	6,46986	5,02801	3,51749
4	7,35554	6,03952	4,46143	4	8,83576	7,26733	4,89774
5	9,21067	8,03278	5,61324	5	11,1694	9,76572	6,17421
6	11,0564	10,1166	6,76543	6	13,4976	12,8467	7,43018

9000 m			
Max bit velocity			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	0,00107	0,0006	0,00051
2	1,99199	1,0258	0,58279
3	3,45767	1,833	1,18019
4	4,8009	2,72969	1,65968
5	6,02017	3,42674	1,93841
6	7,06367	4,30519	2,2069
Max bit amplitude			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	0,00535	0,00227	0,00155
2	4,42891	2,84057	1,68082
3	8,14914	5,18451	3,7274
4	11,5017	7,64157	5,29291
5	14,7557	9,71989	6,71492
6	17,9879	12,0219	8,13026

B.3 20 degrees deviation

1000 m				2000m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,46997	0,35014	0,24246	1	0,74552	0,42208	0,26435
2	0,88107	0,63841	0,50434	2	1,33564	0,80222	0,57425
3	1,3869	1,00588	0,77568	3	1,47613	1,01257	0,82546
4	1,80386	1,32809	1,03198	4	2,08917	1,58441	1,03012
5	2,2273	1,63733	1,29548	5	2,85393	1,87869	1,31245
6	2,64545	1,94332	1,46051	6	3,31688	1,91974	1,56866
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,92523	0,90437	0,89602	1	0,85427	0,83711	0,7569
2	1,96465	1,95008	1,9205	2	1,74929	1,87356	1,82519
3	3,06787	2,97862	2,92516	3	3,067	2,84383	2,87449
4	4,09895	3,97956	3,90879	4	4,27743	3,80342	3,8643
5	5,12858	4,978	4,89558	5	5,3538	4,75709	4,84912
6	6,15067	5,9754	5,94445	6	6,42471	5,84876	5,83382

3000 m				4000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,72985	0,46532	0,33343	1	0,54886	0,35892	0,2247
2	1,45294	0,97645	0,66152	2	1,56472	0,99572	0,75244
3	2,01292	1,25334	0,90967	3	2,14314	1,44154	1,11167
4	2,57316	1,79445	1,27827	4	2,86532	1,88973	1,44362
5	3,14234	2,07049	1,53434	5	3,68723	2,30861	1,70839
6	3,68236	2,50795	1,78925	6	4,52798	2,71394	2,01258
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	1,02722	0,68781	0,53609	1	0,80899	0,53867	0,37749
2	2,2466	1,56672	1,75234	2	2,67006	1,82125	1,38848
3	3,39337	2,7529	2,78243	3	4,08993	2,84784	2,28419
4	4,55688	4,11143	3,76392	4	5,56399	3,83648	3,68881
5	5,72368	5,20683	4,73801	5	7,13503	4,80352	4,98053
6	6,89607	6,28971	5,70912	6	8,79972	5,76993	6,0842

5000 m				6000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00017	0	0	1	0,00028	1,01E-05	0
2	1,5702	1,00214	0,72198	2	1,54593	0,80537	0,54164
3	2,66937	1,5416	1,15887	3	2,49482	1,53818	1,13389
4	4,1072	2,08844	1,47793	4	3,47088	2,09759	1,5684
5	5,54208	2,55904	1,79391	5	4,63562	2,56883	1,92985
6	7,04315	2,98165	2,18939	6	5,62596	2,95146	2,28189
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00101	0	0	1	0,00155	1,69E-05	0
2	2,99038	1,93172	1,43902	2	3,10189	1,62672	1,17119
3	5,25144	3,24675	2,47301	3	5,18918	3,49411	2,63505
4	7,5594	4,46279	3,41217	4	7,14349	4,98291	3,78308
5	10,3388	5,6217	4,33604	5	9,153	6,36907	4,86692
6	14,0589	6,76714	5,23085	6	11,1332	7,65779	5,92911

7000 m				8000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00024	0,00025	0	1	0,00023	0,00018	0
2	0,21457	0,00547	0	2	0,00107	0,00127	0,00045
3	2,62889	1,48838	0,99964	3	2,55244	1,12305	0,71242
4	3,46846	2,12619	1,53637	4	3,66127	2,14579	1,47828
5	4,33617	2,68869	1,96089	5	4,69882	2,79472	1,89369
6	5,14927	3,12831	2,31827	6	5,70163	3,56675	2,4159
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00139	0,00094	0	1	0,00127	0,00062	0
2	0,3187	0,00472	0	2	0,0059	0,0047	0,0016
3	5,41565	3,52052	2,53587	3	5,34382	2,81431	1,83031
4	7,29345	5,43673	4,02508	4	8,06521	5,79291	3,99135
5	9,15534	7,04222	5,2999	5	10,5415	7,7236	5,62725
6	11,0067	8,63687	6,53079	6	12,9589	10,109	7,05812

9000 m			
Max bit velocity			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	0,00031	0,0002	0,00012
2	0,00206	0,00126	0,001
3	0,87955	0,05151	0,00172
4	4,02175	2,09847	1,21108
5	5,45608	2,86243	1,93192
6	7,03394	3,66754	2,34377
Max bit amplitude			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	0,00163	0,0007	0,0004
2	0,0106	0,00468	0,00308
3	1,29081	0,05018	0,00541
4	8,98927	5,83552	3,47919
5	12,8347	8,35239	5,67548
6	16,3664	10,4222	7,50139

B.4 40 degrees deviation

1000 m				2000m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,55323	0,35513	0,24342	1	0,6659	0,44857	0,30867
2	0,93563	0,68536	0,4943	2	1,487	0,85462	0,54468
3	1,4055	1,06386	0,82396	3	2,09871	1,09895	0,86555
4	1,78047	1,2477	0,99748	4	2,62521	1,56403	1,14311
5	2,31501	1,66239	1,26806	5	3,18788	1,78843	1,38342
6	2,74917	2	1,56559	6	3,52153	1,93559	1,64452
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,8952	0,83336	0,80609	1	0,76349	0,52326	0,47349
2	1,85404	1,8137	1,79811	2	1,71593	1,71942	1,53608
3	2,80508	2,87505	2,84523	3	2,61717	2,77522	2,57124
4	3,98626	3,92384	3,86008	4	3,50344	3,75716	3,69716
5	5,09634	4,9539	4,86598	5	4,76876	4,72452	4,75472
6	6,14415	5,96172	5,85736	6	6,27072	5,69562	5,76324

3000 m				4000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,0042	3,90E-06	0	1	0	0	0
2	1,43418	0,94793	0,65993	2	1,22195	0,80103	0,53778
3	2,24184	1,49466	1,07751	3	2,36818	1,48047	1,05453
4	2,85374	1,92738	1,39917	4	3,08649	1,99598	1,51464
5	3,41214	2,37282	1,56916	5	3,60127	2,43419	1,86104
6	3,97683	2,81194	2,01803	6	4,22692	2,85533	2,20015
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00334	1,28E-05	0	1	0	0	0
2	2,09599	1,40852	1,11023	2	1,8502	1,23922	0,89171
3	3,33732	2,30223	2,34905	3	3,84533	2,58067	1,94127
4	4,5003	3,15315	3,56553	4	5,36669	3,68048	2,83281
5	5,64139	4,20962	4,59225	5	6,77461	4,7108	3,66935
6	6,79476	5,75036	5,58948	6	8,20757	5,71343	4,69488

5000 m				6000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00011	0	0	1	0	9,23E-05	0
2	0,00033	0	0	2	0,0006	0,00047	0
3	2,24599	1,28541	0,94245	3	0,22815	0,00518	0,00042
4	3,19656	2,04347	1,44271	4	3,17366	1,73836	1,2649
5	4,28029	2,58418	1,88785	5	4,11573	2,5253	1,83805
6	5,60929	3,09801	2,3252	6	4,99666	3,11449	2,27752
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00069	0	0	1	0	0,00039	0
2	0,00194	0	0	2	0,00327	0,0019	0
3	3,90546	2,36794	1,77014	3	0,38635	0,00393	0,00052
4	6,07889	3,97427	2,97674	4	6,50544	3,6096	2,6382
5	8,37159	5,29732	4,02818	5	8,58561	5,52834	4,13633
6	10,7633	6,55334	4,99539	6	10,4417	7,13033	5,38819

7000 m				8000 m			
Max bit velocity				Max bit velocity			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00011	8,85E-05	0	1	7,21E-05	2,68E-05	0
2	0,00054	0,00052	0	2	0,00056	0,00034	0,00024
3	0,00139	0,00122	0	3	0,00251	0,00141	0,00089
4	0,96807	0,27016	0,08774	4	0,06235	0,00205	0,00144
5	4,45481	2,14161	1,53009	5	1,72555	0,46631	0,08587
6	5,26893	3,07206	2,16317	6	5,22375	2,69354	1,69497
Max bit amplitude				Max bit amplitude			
2*A1	T = 10	T = 15	T = 20	2*A1	T = 10	T = 15	T = 20
0	0	0	0	0	0	0	0
1	0,00047	0,00026	0	1	0,00014	1,87E-05	0
2	0,00317	0,00205	0	2	0,0031	0,00155	0,00082
3	0,0076	0,00455	0	3	0,01343	0,00529	0,00288
4	1,68479	0,38281	0,14862	4	0,05783	0,00779	0,00459
5	8,93266	5,01559	3,57839	5	3,04502	0,73727	0,15135
6	10,8648	7,37635	5,37745	6	11,1433	6,84287	4,44214

9000 m			
Max bit velocity			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	6,69E-05	0	0,00016
2	0,00073	0,0004	0,00051
3	0,00374	0,00118	0,00119
4	0,0033	0,00276	0,00215
5	0,59005	0,00422	0,00297
6	2,40256	0,52347	0,08919
Max bit amplitude			
2*A1	T = 10	T = 15	T = 20
0	0	0	0
1	0,00018	0	0,00035
2	0,00381	0,00132	0,00158
3	0,02019	0,0043	0,00343
4	0,00835	0,01045	0,00676
5	0,69372	0,01544	0,00932
6	4,00362	0,7824	0,14204