

# Risk, production and conflict when utilities are as if certain

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**Abstract:** The article analyses a production and conflict model of risk, supplementing the common rent seeking analysis. Agents differ in attitudes toward risk, productive efficiencies, fighting efficiencies and resources for production versus fighting. A fighting decisiveness parameter determines distribution of utilities. Skaperdas' (1991) analysis of conflict and risk attitudes is generalised from a symmetric to an asymmetric risk function, from two agents to many agents of two kinds, and the fraction of risk seekers is endogenised. Specific functional forms of the utility function and production function are used. The amount of fighting increases in the amount of risk aversion, contrary to received rent seeking theory, but consistently with much experience. Surprisingly, higher production costs or lower fighting costs for risk seekers cause higher utility for risk seekers, contrary to the received theory of higher utility to risk avoiders. We show how the first agent taking on risk benefits, given that the other agents remain risk averse, whereas risk seeking by all agents is the worst scenario.

**Keywords:** risk; production; fighting; conflict; cardinal utility.

## 1 Introduction

Risk is much analysed, but within conflict theory it has been largely ignored. In classical economics with focus on production and consumption, risk is well-understood (Machina and Rotschild, 1992). The same goes for financial economics, e.g., regarding principles of diversified portfolios. In economic psychology, an agent's departure from classical utility has received attention (Kahneman and Tversky, 1979). Applying evolutionary approaches, Dekel and Scotchmer (1999) consider risk-taking in winner-take-all games and Robson (1996) examines risk attitudes assuming lottery tickets. Hvide and Kristiansen (2003, p.74) let the strategic variable be the degree of risk taking rather than the amount of effort and 'show that the selection efficiency of a contest may be improved' by 'a small number of contestants, and by restricting contestant quality'. McGuire et al. (1991, p.329) depart from the fixed-probabilities case, focusing on probability-improving outlays. A risk avoider tends to insure and a risk seeker tends to gamble, though such that 'if the good outcome is less (more) likely than' a so-called 'switching probability', 'the expenditures represent gambling (insurance)'.

There has been some work on risk in the rent seeking literature, which has a substantial focus on rent dissipation. The received theory states that rent dissipation decreases in the number of risk averse agents. Modelling individual preferences with Taylor expansion, Hillman and Katz (1984) find that rent dissipation decreases with risk aversion for small rents. They consider numerical examples for large rents. Konrad and Schlesinger (1997) find that risk aversion reduces rent-augmenting expenditures and has an indeterminate impact on rent-seeking expenditures.

Analyses in the literature and in this paper consider both constant relative Arrow-Pratt risk aversion (CRRA) and constant absolute Arrow-Pratt risk aversion (CARA), introduced by Pratt (1964) and Arrow (1965). CRRA is defined such that  $-xu''(x) / u'(x)$  is constant, where  $u$  is the utility,  $x$  is the independent variable and ' means differentiation. CARA is defined such that  $-u''(x) / u'(x)$  is constant. Skaperdas and Gan (1995) find for CARA and a decisiveness parameter less than one<sup>1</sup> that the less risk averse agent injects higher effort and has a higher probability of winning the rent. Wärneryd (2002) analyses a rent seeking model assuming CRRA. He finds that although rents are perfectly dissipated, efficiency is greater than if all agents had been risk neutral since risk seekers specialise in rent seeking.

For CARA, Cornes and Hartley (2003) find that the proportion of the rent that is fully dissipated decreases from one for risk neutral agents towards zero for sufficiently risk averse agents, with more dissipation as the number of agents increases. If there are sufficiently many agents with less risk aversion than with more risk aversion, only the less risk averse will be active, and the more risk averse will not contest the rent.

Cornes and Hartley (2003, p.4) criticise what they call an anomaly, to wit, that the risk averse agents spend more on rent seeking than risk neutral agents (Konrad and Schlesinger, 1997). They intend to abstract from this and other anomalies by studying CARA. Skaperdas and Gan (1995) find that under limited liability (i.e., the loser's utility is independent of his effort since he goes bankrupt) the risk averse invest more in rent seeking, has a higher probability of winning the contest and earns a higher expected utility. Whether this is an anomaly or not requires further scrutiny. Skaperdas and Gan (1995) suggest the intuition that risk averse agents are more fearful of negative consequences, and put more effort into avoiding these.<sup>2</sup> This intuition is quite plausible and is also supported by this article. When competing for a rent, the fear for the risk averse of not obtaining may induce increased effort. More generally than rent seeking, the fear of looming disaster may cause the risk averse to take extraordinary precautions, while the risk seeking may hope that the disaster passes without ruin. Hence, not unexpectedly, McGuire et al. (1991, p.329) find that a risk avoider tends to insure, while a risk seeker tends to gamble.

There is a need to extend risk research beyond that of rent seeking. That a rent is fully dissipated means that the agents earn zero utilities. An exogenously given rent is realistic in some cases, but in other cases an endogenous rent is realistic. Rents are produced as well as competed for. Hausken (2005) compares rent seeking models and production and conflict models. That the rent is endogenised implies that it is never fully dissipated. The agents make a trade-off between production on one hand, and fighting for their production on the other hand. One such model has been offered by Skaperdas (1991). He considers a symmetric model and lets the two agents' fighting efforts determine their respective win probabilities.

This article fills a void in the literature by generalising Skaperdas' (1991) model in various senses. First, Skaperdas (1991, p.117) considers a symmetric function 'in order to isolate the effect of differential attitudes toward risk'. His main results are that 'being more risk averse than one's opponent is advantageous when conflict is inevitable', and 'as the two sides become more risk averse, the amount of resources expended on conflict increases' [Skaperdas, (1991), p.118]. More generally, which gives a richer set of results, this article lets agents differ with respect to productive efficiencies, fighting efficiencies, resources available for transformation (into production versus fighting) and attitudes toward risk. A fighting decisiveness parameter determines distribution of utilities.

Second, this article generalises too many agents of two kinds and endogenises the fraction of risk seekers. Given that one agent has a certain risk attitude, it is of interest to determine optimal behaviour for other agents with other risk attitudes. Risk seeking behaviour is sometimes beneficial, as illustrated by Hawkes (1990), but optimal behaviour depends on the parameter characteristics and equilibrium strategies of all agents. This article analyses the multiplicity of conditions under which risk seeking and risk avoiding behaviour is optimal.

Consequently, the objective of this article is quite ambitious. It extends beyond the common rent seeking approach, with focus on rent dissipation, to a production and conflict model which opens up for a rich analysis of a kind that has not been made earlier. The analysis generalises from symmetry to asymmetry, generalises from two to many agents of two kinds, and endogenises the fraction of risk seekers. The kind of heterogeneity is thus more general than Cornes and Hartley's (2003) heterogeneity in degree of risk aversion.

Generalisations usually come at a cost. Whereas, Skaperdas (1991) assumes symmetry and two agents, he uses a general utility and production function. In contrast, this paper uses specific functional forms of the utility function and production function. By using credible specific functional forms, we obtain analytical solutions, illustrated with simulations. In return for the sacrifice of generality, a successful specification shows that the minimal standard of internal consistency is satisfied. Particular functional forms are often illuminating. For example, Cobb-Douglas and CES production functions on one hand make special assumptions about the functional relations between inputs and outputs, but on the other hand have proved useful to enhance our understanding of production, growth and related phenomena. Using particular functional forms makes it possible to determine ranges of parameter values within which various equilibria can and cannot be obtained.

It is commonly argued that expected utility theory prevents interpersonal comparison of utilities for agents who have different risk attitudes and prevents comparing utilities

for the same agent under different risk attitudes.<sup>3</sup> One reason for this argument is that any increasing linear transformation of the utilities does not change the risk preferences. Another reason is that adding a constant to von Neumann Morgenstern (1944) utility makes an agent better off, but does not change the risk preferences. Marschak (1950, p.120) presents two variations of the well-known independence axiom (Postulates IVa and IVb) which is now considered the defining axiom for the derivation of the von Neumann-Morgenstern utility function, which is linear in probabilities and invariant to affine transformations. Marschak's (1950, p.125) Figure 4 illustrates the linearity in probabilities and his equation (35.1) (p.130) states the invariance to affine transformations. At the end of the paper, Marschak discusses the differences in the derivation from the original von Neumann-Morgenstern derivation. Similarly, Arrow (1965) and Pratt (1964) define measures of risk aversion for von Neumann-Morgenstern utilities, which are invariant to affine transformations of the utility functions.

Marschak (1950, p.120) introduced a new terminology by changing the denotation of 'risk attitude' to a concave 'as if certain' expected utility mapping. That is, if the as if certain utility function is linear, the agent is 'risk neutral'; if the as if certain utility function is concave, the agent is 'risk averse'; and if the as if certain utility function is convex, the agent is 'risk seeking'. Marschak's definition has subsequently been adopted by Arrow (1965) and Pratt (1964) in the so-called Arrow-Pratt measures of risk aversion, which are applied in this article. Marschak (1950), Arrow (1965), Pratt (1964) and von Neumann and Morgenstern (1944) consider both ordinal and cardinal utilities. The literature seems often confused when they consider which. For example, von Neumann and Morgenstern (1944) consider cardinal utility for events (e.g., sun shining) but not for money, while Marschak (1950) considers cardinal utility, e.g., for money.

Dodging interpersonal comparisons because of the arguments above is not a good idea. We all make such comparisons in our daily lives. Alternatives, such as introducing a separate reservation lottery for each risk attitude that involves the same reservation lottery, make the analysis in this article far more complicated and most likely unreadable.

To allow for interpersonal utility comparison, this paper considers cardinal utilities and considers expected utilities 'as if certain', in which case risk aversion is simply the concavity of the utility function. This means that once the origin and scale are fixed, so that each agent's minimum utility and maximum utility are fixed, and the form of the utility function is fixed, as in the Ramsey (1926) and von Neumann and Morgenstern (1944) standard gamble devices, utility in expected utility theory is not merely invariant up to affine transformations, but cardinal in whatever sense is wanted. This is also implicit in the consumer producer rent comparisons which require interpersonal cardinal utility. We assume CRRA and confirm with CARA in Appendix 2.

Considering utilities as if these are certain allows for substantial insight. Friedman and Savage (1948) stated that under expected utility, each outcome maps into a utility number evaluated 'as if certain'. That is, the outcome is mapped into the same number regardless of whether that outcome is sure, a bit risky or very risky. An agent may evaluate two risky outcomes of \$10 versus \$0 against a sure outcome of \$7. There is no difference in kind between receiving \$10, \$0 or \$7, since these are merely numbers. Under expected utility, probabilities play no role in how each of the outcomes \$10, \$0 and \$7 maps into a utility number. However, probabilities play an atemporal role in aggregating the mutually exclusive outcomes \$10 and \$0 into an expected utility. Hence,

under expected utility, risk attitude is a utility mapping that excludes all sources of satisfaction related to the agent's degree of knowledge ahead. For example, it excludes anticipating that a risky choice may involve the risk-based emotions and the problems of inter-temporal resource allocation under risk.

Section 2 presents a basic model for two agents. Section 3 generalises too many agents of two kinds where the fraction of risk seekers plays a role. Section 4 concludes.

## 2 A production and conflict model with risk for two agents

### 2.1 Assumptions of the model

Consider two agents 1 and 2. As formulated by Hirshleifer (1995, p.30) and Skaperdas and Syropoulos (1997, p.102), agent  $i$  has a resource  $R_i$  transformable into two kinds of efforts. The first is productive effort  $E_i$  designed to generate production from resources currently controlled. The second is fighting effort  $F_i$  designed to acquire the production of others or repel others as they attempt to do the same. With unit conversion costs  $a_i$  and  $b_i$  of transforming  $R_i$  into  $E_i$  and  $F_i$ , this gives  $R_i = a_i E_i + b_i F_i$ . Assume a production function where agent  $i$  produces  $(R_i - b_i F_i) / a_i$ , which means that production increases linearly with effort. Both agents produce  $\sum_{i=1}^2 (R_i - b_i F_i) / a_i$ . The production process is such that the joint production is readily available to be fought for by both agents, which mathematically means that it is placed in a common pool. The agents fight with each other with decisiveness  $m > 0$ . Agent  $i$  gets a ratio  $F_i^m / (F_1^m + F_2^m)$ , known as the contest success function (Tullock, 1980; Skaperdas, 1996) of the total production. Considering CRRA, agent  $i$ 's expected utility is:<sup>4</sup>

$$U_i = \frac{F_i^m}{F_1^m + F_2^m} \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right)^{1-\alpha_i} \leq 1, \quad -1 \leq \alpha_i \leq 1, \quad i = 1, 2 \quad (2.1)$$

where  $\alpha_i$  is the risk parameter.  $\alpha_i < 0$  means risk seeking,  $\alpha_i = 0$  means risk neutrality and  $\alpha_i > 0$  means risk avoidance.  $U_i(x, \alpha_i) = x^{1-\alpha_i} \leq 1$  ensures that equation (2.1) describes agent  $i$ 's attitude toward risk.<sup>5</sup> We consider the utility in equation (2.1) 'as if it is certain', which allows for interpersonal utility comparison.

Contest success functions can either be interpreted as the probability that production is kept by an agent, as in equation (2.1) where the ratio  $F_i^m / (F_1^m + F_2^m)$  expresses probability and is outside the utility index  $\left( \sum_{i=1}^2 (R_i - b_i F_i) / a_i \right)^{1-\alpha_i}$  or as what proportion of production is kept by an agent, as in equation (A12) in Appendix 3 where the ratio  $F_i^m / (F_1^m + F_2^m)$  is inside the utility index. These two interpretations are equivalent under risk neutrality  $\alpha_i = 0$ , but are generally different. This section and the generalisation to  $n$  agents of two kinds in the next section consider the first interpretation. Appendix 3 considers the second interpretation.

## 2.2 Analysis of the model

Calculating the two FOCs  $\partial U_1 / \partial F_1 = 0$  and  $\partial U_2 / \partial F_2 = 0$ , and solving with respect to  $F_1$  and  $F_2$  give the necessary conditions for optimality and an interior solution:

$$\begin{aligned}
 F_1 &= F_2 \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{1/(m+1)}, \\
 F_2 &= \frac{m(a_2 R_1 + a_1 R_2)}{a_2 b_1 (m+1 - \alpha_1) \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{1/(m+1)} + a_1 b_2 (m+1 - \alpha_2)}, \\
 0 &\leq F_i \leq R_i / b_i
 \end{aligned} \tag{2.2}$$

The second order condition for agent 1 is always satisfied:

$$\begin{aligned}
 \frac{\partial^2 U_1}{\partial F_1^2} &= - \frac{N_1 b_1 (1 - \alpha_1) \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{\frac{m-1}{m+1}}}{D_1 a_1^2 a_2 m \left( \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{\frac{m}{m+1}} + 1 \right)} < 0, \\
 N_1 &= \left( a_1 b_2 (m+1)(1 - \alpha_2) + a_2 b_1 (m - \alpha_1 + 1) \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{\frac{1}{m+1}} \right), \\
 D_1 &= \left( \frac{\left( \frac{R_1}{a_1} + \frac{R_2}{a_2} \right) \left( a_2 b_1 (1 - \alpha_1) \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{\frac{1}{m+1}} + a_1 b_2 (1 - \alpha_2) \right)}{a_1 b_2 (m - \alpha_2 + 1) + a_2 b_1 (m - \alpha_1 + 1) \left( \frac{a_1 b_2 (1 - \alpha_2)}{a_2 b_1 (1 - \alpha_1)} \right)^{\frac{1}{m+1}}} \right)^{\alpha_1 + 1}
 \end{aligned} \tag{2.3}$$

The second order condition for agent 2 is found by permuting the indices in equation (2.3). If agent  $i$  chooses zero fighting  $F_i = 0$ , he earns zero utility according to equation (2.1) since his entire production gets lost to the other agent, even if the other agent only fights minusculely. Hence, each agent chooses strictly positive fighting. Conversely, if both agents fight maximally,  $F_1 = R_1 / b_1$  and  $F_2 = R_2 / b_2$ , there is no production to fight for and both utilities are zero. Consequently, each agent faces a trade-off. In an interior equilibrium, both agents choose an intermediate amount of fighting. This also occurs when both agents are resource constrained with low  $R_1$  and  $R_2$ , which causes  $F_1$  and  $F_2$  to be correspondingly lower, as expressed in equation (2.2). Inserting equation (2.2) into the utility  $U_1$  in equation (2.1) gives:

$$U_1 = \frac{\left( \frac{\left( \frac{R_1}{a_1} + \frac{R_2}{a_2} \right) \left( (1-\alpha_2) + (1-\alpha_1) \frac{a_2 b_1}{a_1 b_2} \left( \frac{a_1 b_2 (1-\alpha_2)}{a_2 b_1 (1-\alpha_1)} \right)^{\frac{1}{1+m}} \right)}{\left( (1+m-\alpha_2) + (1+m-\alpha_1) \frac{a_2 b_1}{a_1 b_2} \left( \frac{a_1 b_2 (1-\alpha_2)}{a_2 b_1 (1-\alpha_1)} \right)^{\frac{1}{1+m}} \right)} \right)^{1-\alpha_1}}{1 + \left( \frac{a_2 b_1 (1-\alpha_1)}{a_1 b_2 (1-\alpha_2)} \right)^{\frac{m}{1+m}}} \quad (2.4)$$

where  $U_2$  is found by permuting the indices. An interior solution exists when  $0 \leq F_i \leq R_i / b_i$ . Otherwise, a corner solution exists, analysed later in this section.

Appendix 1 confirms the results in this section when applying the difference or logit formula, rather than the ratio formula in equation (2.1). Propositions 1–3 also hold for the difference or logit formula, with a slight rewording of Proposition 3 in Appendix 1. Appendix 2 finds results with CARA utilities which largely confirm the results with CRRA utilities and finds no results with CARA utilities which qualitatively contradict the results with CRRA utilities.

Two agents with equal risk attitudes  $\alpha_1 = \alpha_2$  and cost parameters  $a_1 / b_1 = a_2 / b_2$  fight equally hard,  $F_1 = F_2$ . The resources  $R_1$  and  $R_2$  have no influence on the ratio  $F_1 / F_2$ , since the joint production is placed in a common pool which both agents fight for.

*Proposition 1:* When two agents with the same risk attitude become more risk avoiding ( $\alpha_i$  increases), their expenditures  $F_1$  and  $F_2$  on fighting increase.

*Proof:* Inserting  $\alpha_1 = \alpha_2$  into equation (2.2) and differentiating gives  $\partial F_1 / \partial \alpha_1 > 0$  and  $\partial F_2 / \partial \alpha_2 > 0$ . Skaperdas (1991, p.118) finds the same result.

*Proposition 2:* (a) With equal cost parameters  $a_1 / b_1 = a_2 / b_2$ , a risk seeker fights less than a risk avoider. (b) If  $\frac{a_1}{b_1} > \frac{a_2(1-\alpha_1)}{b_2(1-\alpha_2)}$ , which occurs when agent 1 has high unit cost

$a_1$  of production and low unit cost  $b_1$  of fighting, agent 1 fights more than agent 2. (c) Increasing decisiveness of fighting gives lower discrepancy in fighting between the risk seeker and the risk avoider.

*Proof:* (a) Agent 1 is more risk seeking than agent 2 when  $\alpha_1 < \alpha_2$  which implies  $(1 - \alpha_2) / (1 - \alpha_1) < 1$ . Hence,  $F_1 / F_2 < 1$  according to equation (2.2) when  $a_1 / b_1 = a_2 / b_2$ , (b) when the inequality is satisfied,  $F_1 / F_2 > 1$  in equation (2.2), (c) increasing  $m$  implies decreasing the exponent  $1 / (m + 1)$  in equation (2.2). Hence,  $\lim_{m \rightarrow \infty} F_1 / F_2 = 1$ . Q.E.D.

As an example, if  $\alpha_1 = -0.5$  and  $\alpha_2 = 0.5$ , the inequality in Proposition 2(b) simplifies to  $a_1 / b_1 > 3a_2 / b_2$ . In this case, the risk seeker must have a three times higher  $a_1 / b_1$  ratio in order to fight more than the risk avoider.

*Proposition 3:* A risk seeker becoming increasingly risk seeking earns a lower utility  $U_i$  and a risk avoider becoming increasingly risk avoiding earns a higher utility  $U_i$ .

*Proof:* The exponent  $(1 - \alpha_i)$  is positive and increases for a risk seeker becoming increasingly risk seeking. Aside from  $R_1 / a_1 + R_2 / a_2$ , the numerator in equation (2.4) is less than one due to the presence of  $m$  twice. Raising a number less than one to a positive number that increases gives a decreasing number. Furthermore, the denominator in equation (2.4) increases when  $\alpha_1$  increases. The opposite argument applies for the risk avoider. An advantage for the risk avoider is also found by Skaperdas (1991).<sup>6</sup>

Inserting  $R_1 = R_2 = R$ ,  $a_1 = a_2 = a$ ,  $b_1 = b_2 = b$ ,  $\alpha_1 = \alpha_2 = \alpha$  into equations (2.1) to (2.4) gives the symmetric solution:

$$F = \frac{mR}{b(m+1-\alpha)} \leq \frac{R}{b}, \quad U = 2^{-\alpha} \left( \frac{R(1-\alpha)}{a(m+1-\alpha)} \right)^{1-\alpha} = U(x, \alpha) = x^{1-\alpha} \quad (2.5)$$

$$\frac{U(x, \alpha) + U(x, -\alpha)}{2} - U(x, 0) = \frac{R(1-\alpha) \left( \frac{R(1-\alpha)}{a(m+1-\alpha)} \right)^{-\alpha} \left[ 2^\alpha \left( \frac{R(1-\alpha)}{a(m+1-\alpha)} \right)^\alpha - 1 \right]^2}{2^{1+\alpha} a(m+1-\alpha)} > 0 \quad (2.6)$$

Hence, the average utility of a risk seeker with risk parameter  $-\alpha$  and a risk avoider with risk parameter  $\alpha$  is always higher than or equal to the utility of a risk neutral agent with risk parameter 0, where the agents have otherwise equal parameters. Wärneryd (2002, p.344) similarly finds that when agents with differential risk attitudes ‘can either earn a certain income or enter a risky rent-seeking contest’, ‘all types’ will be present and ‘efficiency is greater than if everybody had been risk neutral, since risk lovers specialise in rent seeking’.

Let us consider the corner solutions. The solution in equation (2.2) cannot be negative. Zero fighting causes zero utility according to equation (2.1), so each agent will choose a strictly positive fighting effort. However, the upper fighting constraint  $R_i / b_i$  in equation (2.2) can be binding for an agent with a low resource and high unit conversion cost of fighting. Assume that the parameters in equation (2.2) are such that  $F_2 = R_2 / b_2$  is binding. (The analysis when  $F_1 = R_1 / b_1$  is binding is analogous.). Inserting into equation (2.1) and determining the FOC  $\partial U_1 / \partial F_1 = 0$  for agent 1 gives:

$$\left( \frac{R_2}{b_2} \right)^m (mR_1 - b_1(m+1-\alpha_1)F_1) - b_1(1-\alpha_1)F_1^{m+1}, \quad (2.7)$$

$$0 \leq F_1 \leq R_1 / b_1, \quad F_2 = R_2 / b_2$$

Solving the FOC when  $m = 1$  gives:

$$F_1 = \frac{\sqrt{b_1 R_2 (b_1 R_2 (2-\alpha_1)^2 + 4b_2 R_1 (1-\alpha_1))} - b_1 R_2 (2-\alpha_1)}{2b_1 b_2 (1-\alpha_1)}, \quad (2.8)$$

$$0 \leq F_1 \leq R_1 / b_1, \quad F_2 = R_2 / b_2$$

This expression is quite intractable so let us insert  $b_1 = b_2$  and  $R_1 = kR_2$ , where  $k > 0$  is a positive constant, which gives:

$$F_1 = \frac{R_2 \sqrt{(2-\alpha_1)^2 + 4k(1-\alpha_1)} - 2 + \alpha_1}{2b_2(1-\alpha_1)}, \quad 0 \leq F_1 \leq \frac{kR_2}{b_2},$$

$$\frac{(ka_2 + a_1)R_2}{a_2(2-\alpha_1) \sqrt{\frac{a_1(1-\alpha_2)}{a_2(1-\alpha_1)} + a_1(2-\alpha_2)}} \geq \frac{R_2}{b_2} = F_2 \quad (2.9)$$

where the inequality expresses the conditions on the parameters in equation (2.2) for agent 2 to be at his resource constraint for the corner solution. Increasing  $k$  on the left hand side, which means increasing the resource of agent 1, is a straightforward way of generating a corner solution. Differentiating  $F_1$  with respect to  $\alpha_1$  gives:

$$\frac{\partial F_1}{\partial \alpha_1} = \frac{R_2 \left( 2 - \alpha_1 + 2k(1-\alpha_1) - \sqrt{(2-\alpha_1)^2 + 4k(1-\alpha_1)} \right)}{2b_2 \sqrt{(2-\alpha_1)^2 + 4k(1-\alpha_1)} (1-\alpha_1)^2} > 0 \text{ when } k > 0 \quad (2.10)$$

and  $-1 \leq \alpha_1 \leq 1$

*Proposition 4:* Assuming  $m = 1$  and  $b_1 = b_2$ , if agent 2 is at his corner solution  $F_2 = R_2 / b_2$ , agent 1's expenditure  $F_1$  on fighting decreases when he becomes more risk seeking ( $\alpha_1$  decreases), and increases when he becomes more risk avoiding ( $\alpha_1$  increases), consistently with Proposition 1 for the interior solution.

*Proof:* Follows from the positive derivative  $\partial F_1 / \partial \alpha_1 > 0$  in equation (2.10).

An analogue of Proposition 2 does not apply for the corner solution. The reason is that one agent, say agent 2, can be at his resource constraint  $F_2 = R_2 / b_2$  determined by the parameters in equation (2.2) regardless of his risk attitude. Comparing the amount of fighting for two agents with different risk attitudes is thus less interesting for the corner solution.

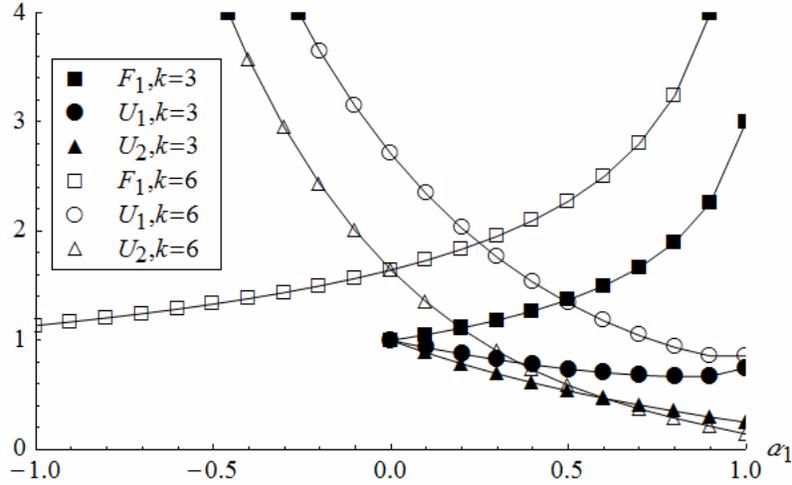
When  $a_1 = a_2$  and  $\alpha_1 = \alpha_2$ , the inequality in equation (2.9) simplifies to  $\alpha_1 \geq (3-k)/2$ . When  $k = 1, 2, 3, 4, 5$ , the inequality becomes  $\alpha_1 \geq 1, \alpha_1 \geq 1/2, \alpha_1 \geq 0, \alpha_1 \geq -1/2, \alpha_1 \geq -1$  respectively, and when  $k > 5$ , the inequality is always satisfied.

The utilities  $U_1$  and  $U_2$  are space consuming to set up for the corner solution, but are illustrated in Figure 1 for  $k = 3$  and  $k = 6$  as functions of  $\alpha_1$  when  $m = b_1 = b_2 = a_1 = a_2 = R_2 = 1, \alpha_2 = \alpha_1, R_1 = kR_2$ .

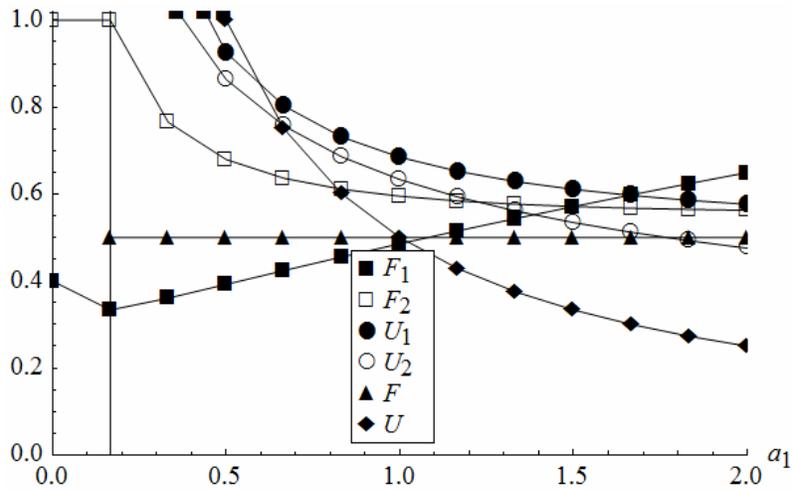
The plot for  $k = 3$  confines attention to  $\alpha_1 \geq 0$  where the corner solution exists. In accordance with Proposition 4,  $F_1$  increases in  $\alpha_1$ . In partial accordance with Proposition 3 for the interior solution,  $U_1$  and  $U_2$  decrease in  $\alpha_1$  except  $U_1$  when  $\alpha_1$  is close to one. The author has confirmed that the curves for  $F_1, U_1, U_2$  are similar for other values of  $k$ .

Proceeding to the general case of asymmetries, the risk avoider for most parameter combinations earns higher utility, but two noteworthy exceptions are when the risk seeker's cost  $a_1$  of production varies and the risk avoider's cost  $b_2$  of fighting varies. With parameters  $R_1 = R_2 = b_1 = b_2 = m = 1, \alpha_1 = -0.5, a_2 = \alpha_2 = 0.5$ , Figure 2 allows  $a_1$  to vary.

**Figure 1** Corner solution:  $F_1, U_1, U_2$  as functions of  $a_1$  when  $k = 3$  and  $k = 6, R_1 = kR_2$



**Figure 2** Variables as functions of  $a_1$



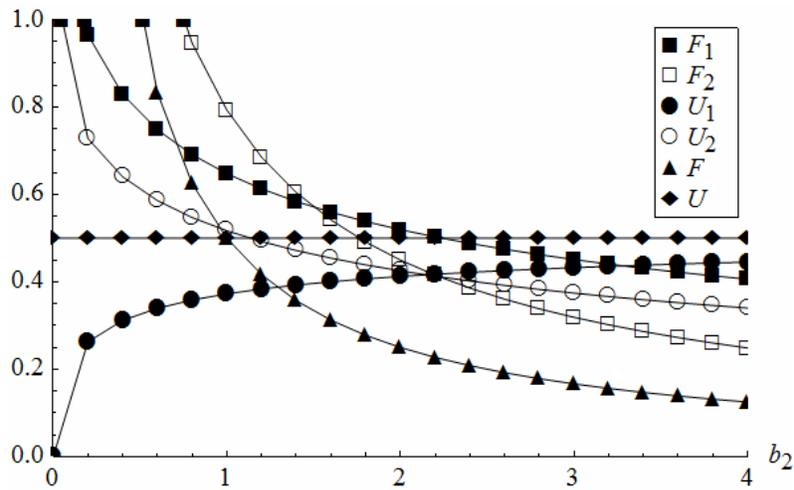
A corner solution with agent 2 at the resource constraint  $F_2 = R_2 / b_2 = 1$  occurs when  $a_1 < 0.17$ . A corner solution with agent 1 at the resource constraint  $F_1 = R_1 / b_1 = 1$  occurs when  $a_1 > 4.76$ . As the risk seeker 1's cost  $a_1$  of production increases above  $a_2 = 0.5$ , he searches for a more appropriate utilisation of his resources than transformation into production. Accordingly, he increases his fighting  $F_1$ , considerably, while the risk avoider reduces his fighting  $F_2$  moderately. The total production thus decreases, but the risk seeker's increased fighting  $F_1$  gives him a larger ratio of it. That is, the risk seeker's utility  $U_1$  decreases more slowly in  $a_1$  than the risk avoider's utility  $U_2$ . Although increasing  $a_1$  decreases both agents' utilities, risk seeker 1 compensates for his disadvantageous high  $a_1$  by fighting harder, which gives him a utility advantage relative

to that of the risk avoider 2. That is, a risk seeker 1 disadvantaged by a sufficiently high cost  $a_1$  of production earns a higher utility than a risk avoider with a low cost  $a_2$  of production. This result is quite surprising. If a risk seeker is concerned about increasing the relative difference between his utility and the utility of the risk avoider, he may do so by increasing his cost  $a_1$  of production, assuming that either he or someone else can alter system parameters. A higher cost  $a_1$  of production, however, causes lower utilities  $U_1$  and  $U_2$  for both agents. The average utility  $(U_1 + U_2) / 2$  decreases in  $a_1$ . Hence, increasing cost of production has a welfare reducing effect. For comparative purposes,  $F$  and  $U$  according to equation (2.5) are plotted for two risk neutral agents where  $\alpha = 0$ .

Figure 3 sets  $a_1 = a_2 = 1$ ,  $b_1 = 0.5$  and allows  $b_2$  to vary.

A corner solution with agent 2 at the resource constraint  $F_2 = R_2 / b_2 = 1 / b_2$  occurs when  $b_2 > 4.17$ . A corner solution with agent 1 at the resource constraint  $F_1 = R_1 / b_1 = 2$  does not occur. As the risk avoider 2's cost  $b_2$  of fighting increases, his fighting  $F_2$  decreases considerably and his utility  $U_2$  decreases moderately. Agent 1's need to fight diminishes and  $F_1$  decreases moderately. The net effect is that risk seeker 1's utility  $U_1$  increases moderately. Agent 1, thus, benefits from agent 2's misfortune. The utilities  $U_1$  and  $U_2$  cross each other at  $b_2 = 2.15$ . The risk avoider 2 must have a considerable cost disadvantage  $b_2$  of fighting for the risk seeker to earn a higher utility. As a comparison, equal parameters  $b_1 = b_2 = 0.5$  gives a distinct advantage to the risk avoider, as the conventional literature specifies. The average utility  $(U_1 + U_2) / 2$  decreases in  $b_2$ . Increasing cost of fighting also has a welfare reducing effect.

**Figure 3** Variables as functions of  $b_2$



The impact of altered risk attitudes is easily seen from Figures 2 and 3 using  $U_i(x, \alpha_i) = x^{1-\alpha_i} \leq 1$ . An increasingly risk seeking risk seeker applying a more negative  $\alpha_i$  earns a lower utility  $U_i$  and an increasingly risk averse risk avoider applying a more positive  $\alpha_i$  earns a higher utility  $U_i$ . This gives downward versus upward shifts of the utilities  $U_1$  and  $U_2$  in all figures in this article. Let us consider six similarities and differences between Figures 2 and 3. Four of these are intuitive and two are more unexpected. First, increasing the cost  $a_1$  of production gives a disadvantage to risk

seeker 1 and increasing the cost  $b_2$  of fighting gives a disadvantage to the risk avoider 2. Both results are intuitive since a change of a parameter in a disadvantageous direction causes a disadvantage to the agent associated with the parameter. Second, increasing  $a_1$  causes fighting  $F_1$  to increase, since agent 1 finds an alternative to costly production, which is fighting, and increasing  $b_2$  causes fighting  $F_2$  to decrease, since agent 2 prefers to avoid costly fighting. Third, increasing  $a_1$  causes the total production to decrease, while increasing  $b_2$  causes the total production to remain unchanged. Fourth, increasing  $a_1$  or  $b_2$  has the welfare reducing effect of reducing the average utility  $(U_1 + U_2) / 2$ .

The unexpected and more subtle results follow from interaction with the agent whose parameter remains unchanged. That is, changing  $a_1$  has impact on agent 2 and changing  $b_2$  has impact on agent 1. Hence, fifth, increasing  $a_1$  causes  $F_1$  to increase, but indirectly causes  $F_2$  to decrease. This gives a larger ratio of the production to agent 1, though both  $U_1$  and  $U_2$  decrease in  $a_1$  since the total production decreases in  $a_1$ . The manner in which agent 1 is disadvantaged by a large cost  $a_1$  of production gets more than compensated by a large equilibrium fighting effort  $F_1$ , giving agent 1 a larger share of the production than agent 2. Sixth, increasing  $b_2$  causes  $F_2$  to decrease, but indirectly causes  $F_1$  to decrease too, though more moderately. That  $F_1$  decreases is somewhat unexpected. The explanation is that risk seeker 1 ‘cashes in’ on facing a less fierce opponent. Since the risk avoider 2 fights so leniently (since the cost  $b_2$  is high), there is no need for agent 1 to fight fiercely and agent 1 decreases  $F_1$ . This causes risk seeker 1’s utility  $U_1$  to actually increase in  $b_2$ , while the risk avoider 2’s utility  $U_2$  more naturally decreases in  $b_2$ .  $U$  for a risk neutral agent decreases in Figure 2 since the total production decreases in  $a_1$ , while  $U$  is constant in Figure 3 where the total production does not depend on  $b_2$ .  $(U_1 + U_2) / 2 > U$  accordingly decreases in Figure 2 and increases in Figure 3 as specified by equation (2.6).

### 3 Generalising the model to $n$ agents of two kinds

#### 3.1 Assumptions of the model

First, this section generalises to  $n$  agents of two kinds, with  $n_R$  identical agents with risk attitude  $\alpha_R$  and  $n_S$  identical agents with risk attitude  $\alpha_S$ , where  $n_R + n_S = n$ . Without loss of generality, we assume  $-1 \leq \alpha_R \leq \alpha_S \leq 1$ , and hence, the  $n_R$  agents are more risk seeking than the  $n_S$  agents. Second, we generalise so that the fraction  $n_R / n$  of the first kind of agents plays a role. For expositional convenience, we refer to the  $n_R$  agents as risk seekers and the  $n_S$  agents as risk avoiding. Endogenising the fraction of risk seekers by accounting for the relative numbers of risk seekers and risk avoiders impacts the behaviour of both kinds of agents. It is often beneficial to be the first agent to take on risk, given that the other agents remain risk avoiding. We introduce boost factors to account for this. Assume that  $n$  risk seekers is the worst case scenario and that if all agents are risk avoiding, then it is beneficial to be the first risk seeker. Wärneryd (2002, p.347) similarly assumes in a rent seeking model that “the probability of getting a positive payoff from the risky activity declines in the total population proportion of agents who enter the activity”. Introducing a fighting benefit for risk seeker  $i$  is accomplished by multiplying his fighting effort  $F_{iR}$  with the scaling or boost parameter  $c_F$ . Similarly, introducing a production benefit for risk seeker  $i$  is accomplished by multiplying his production with the parameter  $c_P$ . Assume that  $c_F$  and  $c_P$  are proportional

to  $n$  and inverse proportional to  $n_R$ . Hence, a lone risk seeker  $n_R = 1$  boosts his fighting and production considerably when there are many agents. This is often witnessed in praxis where the benefit of being a lone risk seeker may be significant when surrounded by sufficiently many risk avoiders.  $U_{iR}$  is the utility of risk seeker  $i$ ,  $i = 1, \dots, n_R$ . Analogously,  $U_{iS}$  and  $F_{iS}$  are the utility and fighting effort of risk avoider  $i$ ,  $i = 1, \dots, n_S$ .  $a_R$  and  $a_S$  are unit costs of production, and  $b_R$  and  $b_S$  are unit costs of fighting. Generalising equation (2.1) by multiplying the two occurrences of  $F_{iR}$  with  $c_F$  and multiplying the production with  $c_P$ , gives:

$$U_{iR} = \frac{(F_{iR}c_F)^m}{\sum_{i=1}^{n_R}(F_{iR}c_F)^m + \sum_{i=1}^{n_S}F_{iS}^m} \left( \frac{\sum_{i=1}^{n_R}(R_R - b_R F_{iR})}{a_R} c_P + \frac{\sum_{i=1}^{n_S}(R_S - b_S F_{iS})}{a_S} \right)^{1-\alpha_R} \leq 1, \quad (3.1)$$

$$U_{iS} = \frac{F_{iS}^m}{\sum_{i=1}^{n_R}(F_{iR}c_F)^m + \sum_{i=1}^{n_S}F_{iS}^m} \left( \frac{\sum_{i=1}^{n_R}(R_R - b_R F_{iR})}{a_R} c_P + \frac{\sum_{i=1}^{n_S}(R_S - b_S F_{iS})}{a_S} \right)^{1-\alpha_S} \leq 1.$$

### 3.2 Analysis of the model

Calculating the two FOCs  $\partial U_{iR} / \partial F_{iR} = 0$  and  $\partial U_{iS} / \partial F_{iS} = 0$ , and solving with respect to  $F_{iR} = F_R$  and  $F_{iS} = F_S$ , where identical risk seekers behave equivalently in equilibrium and identical risk avoiders behave equivalently in equilibrium, gives the FOCs and interior solution:

$$\begin{aligned} & \left[ (n_R - 1)(F_R c_F)^m + n_S F_S^m \right] \left( \frac{n_R (R_R - b_R F_R) c_P}{a_R} + \frac{n_S (R_S - b_S F_S)}{a_S} \right) \\ &= \frac{b_R F_R c_P}{m a_R} \left[ n_R (F_R c_F)^m + n_S F_S^m \right] (1 - \alpha_R), \\ & \left[ n_R (F_R c_F)^m + (n_S - 1) F_S^m \right] \left( \frac{n_R (R_R - b_R F_R) c_P}{a_R} + \frac{n_S (R_S - b_S F_S)}{a_S} \right) \\ &= \frac{b_S F_S}{m a_S} \left[ n_R (F_R c_F)^m + n_S F_S^m \right] (1 - \alpha_S), \end{aligned} \quad (3.2)$$

Both FOCs apply when  $0 < n_R, n_S < n$ . When  $n_R = n$ , only the first FOC applies for  $n$  risk seekers. When  $n_R = 0$ , only the second FOC applies for  $n$  risk avoiders. Solving equation (3.2) gives:

$$\frac{F_R a_S b_R c_P}{F_S a_R b_S} = \frac{\left[ (n_R - 1)(F_R c_F)^m + n_S F_S^m \right] (1 - \alpha_S)}{\left[ n_R (F_R c_F)^m + (n_S - 1) F_S^m \right] (1 - \alpha_R)}, \quad F_R \leq \frac{R_R}{b_R}, \quad F_S \leq \frac{R_S}{b_S}. \quad (3.3)$$

For  $n$  equivalent agents with equal risk parameter  $\alpha$ , inserting  $R_R = R_S = R$ ,  $a_R = a_S = a$ ,  $b_R = b_S = b$ ,  $\alpha_R = \alpha_S = \alpha$ ,  $n_R = n$ ,  $n_S = 0$ ,  $c_F = c_P = 1$  into equations (3.1) to (3.3) gives the fighting effort and utility:

$$F = \frac{m(n-1)R}{b(m(n-1)+1-\alpha)} \leq \frac{R}{b} = \lim_{n \rightarrow \infty} F, \quad (3.4)$$

$$U = n^{-\alpha} \left( \frac{c_P R (1-\alpha)}{a(m(n-1)+1-\alpha)} \right)^{1-\alpha} = U(x, \alpha) = x^{1-\alpha},$$

for the interior solution which is equivalent to equation (2.5) when  $n = 2$ . With many agents, as  $n$  increases, fighting  $F$  asymptotically reaches the resource constraint  $R / b$ . This is consistent with rent dissipation increasing in the number of agents in the rent seeking literature. A similar logic applies for  $n_R$  risk seekers and  $n_S$  risk avoiders, where fighting  $F_R$  and  $F_S$  approach the resource constraints  $R_R / b_R$  and  $R_S / b_S$ .

*Proposition 5:* When  $n$  agents with the same risk attitude become more risk avoiding ( $\alpha$  increases), their expenditures  $F$  on fighting increase.

*Proof:* Differentiating equation (3.4) gives  $\partial F / \partial \alpha > 0$ .

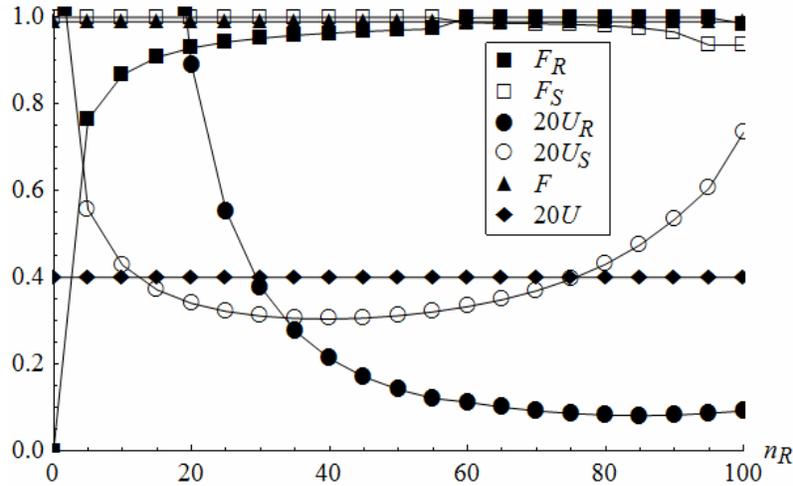
The solutions in this section are equivalent to the solutions in the previous section when  $n = 2$  and  $n_R = n_S = c_F = c_P = 1$ . We illustrate the solutions in this section with simulations. To increase realism in accordance with Wärneryd's (2002, p.347) observation discussed above and avoid the often unrealistic scenario that the agents' fighting quickly reaches the resource constraint, we hereafter assume boost parameters  $c_F = c_P = n / (5n_R)$ . The idea is that when there are few risk seekers, the advantage of being the first risk seekers can be substantial. With parameters as in Section 2,  $R_R = R_S = b_R = b_S = m = 1$ ,  $\alpha_R = -0.5$ ,  $a_R = a_S = \alpha_S = 0.5$ , and  $n = 100$  agents, Figure 4 illustrates as  $n_R$  varies.

Multiplication of utilities with 20 is for scaling purposes. With few risk seekers  $n_R$ , these engage in low degree of fighting, high degree of production and enjoy a high utility  $U_R$ , which decreases in  $n_R$  and crosses below  $U_S$  when  $n_R = 33$ . Risk avoiders produce nothing when  $n_R$  is low. Risk seekers produce nothing when  $n_R$  is high. The utility of risk seekers decreases in  $n_R$ , as for Wärneryd (2002), and the utility of risk avoiders is U shaped.  $n = 100$  risk neutral agents ( $\alpha = 0$ ) each invests fighting  $F = 0.99$  according to equation (3.4), with utility  $20U = 0.4$ . It is preferable to be a risk seeker if  $n_R \leq 29$  since the utility is above  $20U = 0.4$ . It is preferable to be risk neutral if  $29 < n_R < 76$  since  $20U = 0.4$  is the highest utility. It is preferable to be risk avoiding when  $n_R \geq 76$ . The 'beat the crowd' logic is that it is preferable to be among the few who are either risk seeking or risk avoiding.

Table 1 considers the utilities (left part) and fighting (right part) for one agent against 99 agents with the same parameter values. Agent 1 in the row can choose between risk attitude  $\alpha_S = 0.5$  and  $\alpha_R = -0.5$ , like choosing a profession, while agents 2–99 in the column also can choose between risk attitude  $\alpha_S = 0.5$  and  $\alpha_R = -0.5$ , where all these 99 agents are assumed to make the same choice. We, thus, get a  $2 \times 2$  matrix.

One hundred risk seekers earn utilities 0.01 and 100 risk avoiders earn utilities 0.0046. However, a lone risk seeker boosts his utility to 8.81.

**Figure 4** Variables as functions of  $n_R$  when  $n = 100$



**Table 1**  $R_R = R_S = b_R = b_S = m = 1$ ,  $\alpha_R = -0.5$ ,  $a_R = a_S = \alpha_S = 0.5$ , and  $n = 100$  agents,  $c_F = c_P = n / (5n_R)$

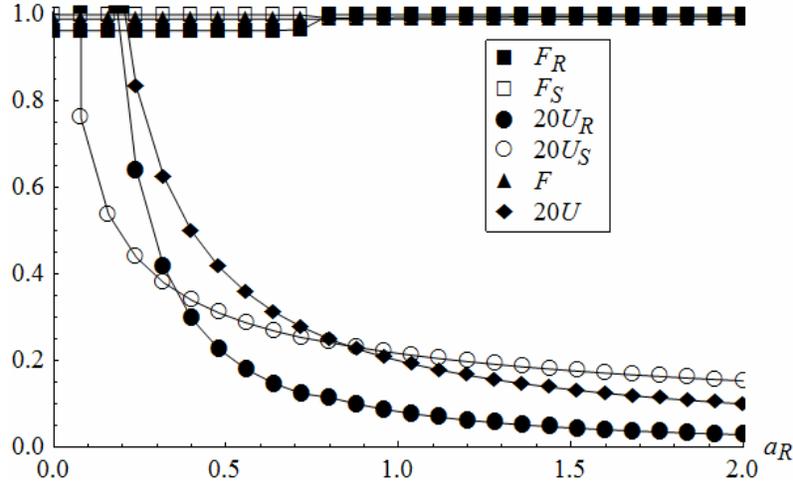
Utilities	Agents 2-100		Fighting	Agents 2-100			
	$\alpha_S = 0.5$	$\alpha_R = -0.5$		$\alpha_S = 0.5$	$\alpha_R = -0.5$		
Agent 1	$\alpha_S = 0.5$	0.01, 0.01	0.026, 0.0054	Agent 1	$\alpha_S = 0.5$	0.995, 0.995	1, 0.659
	$\alpha_R = -0.5$	8.81, 0.047	0.0046, 0.0046		$\alpha_R = -0.5$	0.382, 1	0.985, 0.985

Figure 5 assumes  $n_R = 40$  risk seekers and lets production cost  $a_R$  vary, with all other parameters as before.

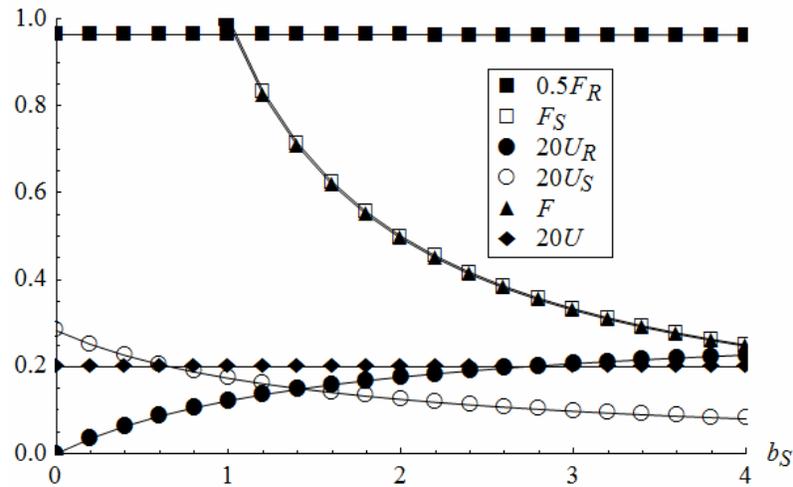
Risk seekers take care of all production when  $a_R$  is low, and conversely when  $a_R$  is high since production costs are too high. Both  $U_R$  and  $U_S$  decrease in  $a_R$ , but  $U_R$  decreases more due to the higher cost  $a_R$  of production for risk seekers. Contrary to Figure 2 where a risk seeker earns a higher utility than a risk avoider if he is disadvantaged by a higher cost  $a_1 > a_2$  of production, regardless how disadvantaged he is by  $a_1$ , in Figure 5, the risk seeker no longer earns a higher utility if he is too disadvantaged. The risk seekers' boost factors eventually become disadvantageous. That is, large boost factors for the risk seekers are not sufficient to compensate for large  $a_R$ , which causes low utility  $U_R$ . The utilities  $U_R$  and  $U_S$  cross at  $a_R = 0.35$ , and for higher  $a_R$ , the risk avoiders earn the highest utility. Decreasing  $a_R$  below  $a_R = 0.35$ , risk seekers earn higher utility, contrary to Figure 2.

Figure 6 sets  $a_R = a_S = b_R = 1$ ,  $n_R = 40$  and lets fighting cost  $b_S$  to vary for the risk avoiders.

**Figure 5** Variables as functions of  $a_R$  when  $n = 100, n_R = 40$



**Figure 6** Variables as functions of  $b_S$  when  $n = 100, n_R = 40$



Multiplication of  $F_R$  with 0.5 is for scaling purposes. The risk seekers' fighting  $F_R$  decreases negligibly, while the risk avoiders' fighting  $F_S$  decreases along its maximum border constraint  $F_S = R_S / b_S$ . The utilities  $U_R$  and  $U_S$  have a form similar to Figure 3, crossing at  $b_S = 1.42$ . The reason Figure 6 with  $b_S$  dependence is similar to Figure 3 with  $b_2$  dependence, while Figure 5 with  $a_R$  dependence is not similar to Figure 2 with  $a_1$  dependence, is the sharply decreasing fighting  $F_S$  for risk avoiders in Figure 6 which eventually becomes considerably smaller than  $F_R$ , contrary to Figure 5 where  $F_R$  and  $F_S$  are similar. As before, the impact of altered risk attitudes is seen using  $U_i(x, \alpha_i) = x^{1-\alpha_i} \leq 1$ . The utility  $U_R$  for an increasingly risk seeking risk seeker gets a downward shift (given that the number of risk seekers is fixed at  $n_R = 40$ ) and the utility

$U_S$  for an increasingly risk avoiding risk avoider gets an upward shift. This is in accordance with Proposition 3 for two agents where increasing risk aversion causes higher utility.

Wärneryd (2002, p.347) lets each agent choose whether or not to participate in a risky rent seeking contest, with threshold levels pertaining to an upper number of prizes. When production is endogenised, assume that agents choose among the attitudes risk seeking versus risk avoiding similarly to choosing among occupations and that each agent makes a choice that gives the highest utility. In equilibrium, the utilities are equal,  $U_R = U_S$  and the number  $n_R$  of risk seekers is adjusted accordingly, where  $0 \leq n_R \leq n$ .

As an alternative to a fixed fraction  $n_R / n$  of risk seekers, let us endogenise the fraction  $n_R / n$  by setting  $U_R = U_S$ . Utilities and parameters are then no longer given for each agent. Instead, similarly to choosing between occupations, agents choose freely between risk seeking and risk avoiding behaviour, taking on the utilities and parameters associated with either risk seeking or risk avoidance. In equilibrium,  $n_R / n$  gets endogenously adjusted so that all agents earn equal utilities. With the same parameter assumptions as in Figures 2–4, it is straightforward to show that as  $a_R$  increases above  $a_R = 0.35$ , to the disadvantage of the  $n_R = 40$  risk seekers,  $n_R$  decreases to fewer than  $n_R = 40$  risk seekers. The risk seekers suffer from their increased production cost and fewer agents choose to become risk seekers. Conversely, when  $a_R$  decreases below  $a_R = 0.35$ , there will be more than  $n_R = 40$  risk seekers. Similarly, increasing  $b_S$  above the crossing point  $b_S = 1.42$  gives a relative advantage to the risk seekers over the risk avoiders, causing  $n_R$  to increase to bring  $U_R$  down to  $U_S$ . The risk seekers benefit from the increased fighting cost of risk avoiders and more agents choose to become risk seekers. Conversely,  $n_R$  decreases as  $b_S$  decreases. Consequently, both unit costs of production and unit costs of fighting affect the relative numbers of risk seekers and risk avoiders.

## 4 Conclusions

The article fills a void in the literature. The literature on risk in rent seeking is supplemented with the analysis of a production and conflict model. One such has been offered by Skaperdas (1991). This article generalises in three ways. First, we generalise from a symmetric to an asymmetric risk function. Second, we generalise from two agents to many agents of two kinds. Third, we endogenise the fraction of risk seekers, and let this fraction be influential. Specific functional forms of the utility function and production function are used.

Section 2 equips each agent with a resource transformable into production and fighting. Agents differ with respect to resources, productive and fighting efficiencies and attitudes toward risk. A fighting decisiveness parameter determines distribution of utilities. Utilities are determined by a contest success function accounting for attitudes toward risk. The amount of fighting increases in the amount of risk aversion, contrary to received rent seeking theory where rent dissipation decreases with risk aversion (Hillman and Katz, 1984), but consistently with much experience. In the literature, the risk avoider typically earns the highest utility.

This article demonstrates two noteworthy exceptions. The first occurs when the risk seeker's cost of production increases, causing him to fight considerably harder than the risk avoider. The total production for the two agents' decreases, but the risk seeker enjoys a larger ratio of it, and surprisingly, earns a higher utility than the risk avoider. The

second exception occurs when the risk avoider's cost of fighting increases, which causes his fighting to decrease considerably. The risk seeker 'cashes in' on facing a less fierce opponent. The risk seeker's fighting decreases moderately and he enjoys the highest ratio of the increased production. These two results jointly benefit a risk seeker with low production capacity facing a risk avoider with low fighting capacity.

Section 3 presents a model with many agents of two kinds. It is beneficial to be the first risk seeker if all other agents are risk avoiders. The worst case scenario is that all agents are risk seekers. The utility to risk seekers decreases in the fraction of risk seekers, analogously to Wärneryd's (2002) analysis and the utility of risk avoiders is U shaped. The utility of risk seekers also decrease in the cost of production for risk seekers and eventually becomes lower than the utility of risk avoiders, contrary to Section 2. As the cost of fighting for risk avoiders increases, such fighting decreases compared with the fighting of risk seekers, causing results similar to those of Section 2. When agents can choose freely between risk seeking and risk avoiding behaviour, the fraction of risk seekers decreases in the cost of production for risk seekers and increases in the cost of fighting for risk avoiders.

## Acknowledgements

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## Notes

- 1 A decisiveness parameter  $m \leq 1$  less than gives disproportional advantage of injecting less effort than the other agent, while  $m > 1$  gives disproportional advantage of injecting more effort than the other agent.
- 2 Without limited liability, there are no clear results. When the prize is shared as a function of effort, Skaperdas and Gan (1995) find that the outcome is independent of the agents' risk aversion.
- 3 See Harsanyi (1987) regarding interpersonal utility comparison.
- 4 Equation (2.1) means placing the total production in a common pool for capture. The author has analysed the model where each agent defends his own production and appropriates the other agent's production. The results are qualitatively similar in many respects, though the FOCs are more complicated to discuss, with no analytical solutions.
- 5 Assuming  $U_i = U_i(x, \alpha_i) = x^{1-\alpha_i}$ , a risk avoider prefers the certain alternative over the uncertain alternative; i.e.,  $U_i(x, \alpha_i) > U_i(x, 0)$  when  $\alpha_i > 0$ . A risk seeker prefers the reverse; i.e.,  $U_i(x, \alpha_i) < U_i(x, 0)$  when  $\alpha_i < 0$ .
- 6 He assumes utilities  $\pi^1 = pU(C)$  and  $\pi^2 = (1-p)k(U(C))$ , where  $C(1-y_1, 1-y_2)$  is symmetric,  $y_i$  is quantity of arms,  $p$  is win probability and  $k(U(C))$  is increasing and strictly concave.

## Appendix 1

### *Difference or logit formula*

Applying the difference or logit formula rather than ratio formula in equation (2.1) gives:

$$U_i = \frac{e^{kF_i}}{e^{kF_1} + e^{kF_2}} \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right)^{1-\alpha_i} \leq 1, \quad -1 \leq \alpha_i \leq 1, \quad i=1,2 \quad (\text{A1})$$

where  $k > 0$  is a parameter. The two FOCs imply:

$$\frac{e^{kF_1}}{e^{kF_2}} = \frac{a_1 b_2 (1-\alpha_2)}{a_2 b_1 (1-\alpha_1)} \Leftrightarrow F_1 = F_2 + \text{Ln} \left( \frac{a_1 b_2 (1-\alpha_2)}{a_2 b_1 (1-\alpha_1)} \right) / k,$$

$$F_2 = \frac{a_2 (kR_1 - b_1 (1-\alpha_1)) + a_1 (kR_2 - b_2 (1-\alpha_2)) - a_2 b_1 \text{Ln} \left( \frac{a_1 b_2 (1-\alpha_2)}{a_2 b_1 (1-\alpha_1)} \right)}{k (a_2 b_1 + a_1 b_2)}, \quad (\text{A2})$$

$$0 \leq F_i \leq R_i / b_i,$$

$$U_1 = \frac{b_2 (1-\alpha_2)}{a_2 k} \left( \frac{a_2 b_1 (1-\alpha_1) + a_1 b_2 (1-\alpha_2)}{a_1 a_2 k} \right)^{-\alpha_1}, \quad (\text{A3})$$

$$U_2 = \frac{b_1 (1-\alpha_1)}{a_1 k} \left( \frac{a_2 b_1 (1-\alpha_1) + a_1 b_2 (1-\alpha_2)}{a_1 a_2 k} \right)^{-\alpha_2}$$

*Proposition A1:* A risk seeker becoming increasingly risk seeking earns a lower utility  $U_i$  and a risk avoider becoming increasingly risk avoiding earns a higher utility  $U_i$ .

*Proof:* The exponent  $-\alpha_i$  is positive and increases for a risk seeker becoming increasingly risk seeking. Raising a number less than one to a positive number that increases gives a decreasing number. The opposite argument applies for the risk avoider.

The author has confirmed that the simulations are qualitatively similar when using the difference or logit formula rather than the ratio formula.

## Appendix 2

### *Constant absolute risk aversion*

Considering CARA and applying the ratio formula, equation (2.1) becomes:

$$U_i = \frac{F_i^m}{F_1^m + F_2^m} \left( \frac{-1}{\alpha_i} \right) e^{-\alpha_i \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right)}, \quad -1 \leq \alpha_i \leq 1, \quad i=1,2 \quad (\text{A4})$$

where  $\alpha_i$  is the coefficient of absolute risk aversion. As  $\alpha_i$  increases, agent  $i$  becomes more risk averse. The two FOCs imply:

$$F_1 = F_2 \left( \frac{a_1 b_2 \alpha_2}{a_2 b_1 \alpha_1} \right)^{1/(m+1)}, \quad F_2 = \frac{m a_2}{b_2 (-\alpha_2)} \left( \frac{\left( \frac{a_1 b_2 \alpha_2}{a_2 b_1 \alpha_1} \right)^{m/(m+1)}}{1 + \left( \frac{a_1 b_2 \alpha_2}{a_2 b_1 \alpha_1} \right)^{m/(m+1)}} \right), \quad (\text{A5})$$

$$0 \leq F_i \leq R_i / b_i, \quad -1 \leq \alpha_i \leq 0$$

$F_2$  is positive when  $\alpha_2 < 0$  which means risk seeking and we must thus also have  $\alpha_1 < 0$  to ensure  $F_1$  positive. Hence, equations (A4) to (A5) are valid only when both agents are risk seeking or risk neutral.

*Proposition A2:* For two risk seeking agents, the results in equation (A5) for CARA are qualitatively similar to the results in equation (2.2) for CRRA.

*Proof:* Assuming CARA and that agent 2 is risk seeking ( $\alpha_2 < 0$ ), as agent 1 gets more risk seeking ( $\alpha_1 < 0$  gets more negative), the ratio  $a_1 b_2 \alpha_2 / a_2 b_1 \alpha_1$  in equation (A5) decreases, just as the ratio  $a_1 b_2 (1 - \alpha_2) / a_2 b_1 (1 - \alpha_1)$  in equation (2.2) decreases. Hence,  $F_1$  decreases in both cases, causing consistency between CARA and CRRA for two risk seeking agents.

Since the CARA in equation (A4) confirms the CRRA results only for risk seeking, let us add a constant  $C$  to specify the utility as:

$$U_i = \frac{F_i^m}{F_1^m + F_2^m} \left( C - \frac{1}{\alpha_i} \right) e^{-\alpha_i \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right)}, \quad -1 \leq \alpha_i \leq 1, \quad i = 1, 2 \quad (\text{A6})$$

The two FOCs become:

$$\begin{aligned} C \alpha_1 e^{\alpha_1 \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right)} - 1 &= \frac{\alpha_1 b_1 F_1 (F_1^m + F_2^m)}{m a_1 F_2^m}, \\ C \alpha_2 e^{\alpha_2 \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right)} - 1 &= \frac{\alpha_2 b_2 F_2 (F_1^m + F_2^m)}{m a_2 F_1^m} \end{aligned} \quad (\text{A7})$$

Equation (A7) is analytically solvable only when  $\alpha_1 = \alpha_2 = \alpha$ , with solution:

$$F_1 = F_2 \left( \frac{a_1 b_2}{a_2 b_1} \right)^{1/(m+1)} = F_2 q, \quad 0 \leq F_i \leq R_i / b_i, \quad -1 \leq \alpha \leq 1 \quad (\text{A8})$$

where

$$\begin{aligned} F_2 &= \frac{a_1}{\alpha} \left( \frac{a_2 \text{ProductLog}[z]}{a_1 b_2 + a_2 b_1 q} - \frac{m}{b_1 q (1 + q^m)} \right), \\ z &= \frac{C m (a_1 b_2 + a_2 b_1 q) \alpha}{a_2 b_1 q (1 + q^m)} e^{\frac{(a_1 b_2 + a_2 b_1 q) m}{a_2 b_1 q (1 + q^m)} + \left( \frac{R_1}{a_1} + \frac{R_2}{a_2} \right) \alpha}, \end{aligned} \quad (\text{A9})$$

where  $\text{ProductLog}[z]$  gives the principal solution for  $w$  in  $z = w e^w$ .

Evidently, the constant  $C$  can be adjusted so that  $F_2$  is positive within the entire range  $-1 \leq \alpha \leq 1$ , which allows for risk seeking, risk aversion and risk neutrality. Hence, equations (A6) to (A9) are valid for all equivalent risk attitudes for the two agents.

*Proposition A3:* For two agents with equivalent risk attitudes  $\alpha_1 = \alpha_2 = \alpha$ , which may be risk seeking, risk aversion or risk neutrality, the fighting effort ratio  $F_1 / F_2 = (a_1 b_2 / a_2 b_1)^{1/(m+1)}$  in equation (A8) for CARA is equivalent to the ratio in equation (2.2) for CRRA.

*Proof:* Follows from comparing equation (A8) with equation (2.2).

As a further example of CARA, let us place the entire fighting ratio  $F_i^m / (F_1^m + F_2^m)$  into the exponent, which gives:

$$U_i = \left( \frac{-1}{\alpha_i} \right) e^{\frac{-\alpha_i F_i^m}{F_1^m + F_2^m} \left( \frac{R_1 - b_1 F_1 + R_2 - b_2 F_2}{a_1} \right)}, \quad -1 \leq \alpha_i \leq 1, \quad i = 1, 2 \quad (\text{A10})$$

The two FOCs imply:

$$F_1 = F_2 \left( \frac{a_1 b_2}{a_2 b_1} \right)^{1/(m+1)}, \quad F_2 = \frac{m(a_2 R_1 + a_1 R_2)}{\left[ a_2 b_1 \left( \frac{a_1 b_2}{a_2 b_1} \right)^{1/(m+1)} + a_1 b_2 \right] (m+1)},$$

$$0 \leq F_i \leq R_i / b_i,$$

$$U_1 = \left( \frac{-1}{\alpha_1} \right) e^{\frac{-\alpha_1 (a_2 R_1 + a_1 R_2)}{a_1 a_2 \left[ 1 + \left( \frac{a_2 b_1}{a_1 b_2} \right)^{m/(m+1)} \right] (m+1)}}, \quad (\text{A11})$$

$$U_2 = \left( \frac{-1}{\alpha_2} \right) e^{\frac{-\alpha_2 (a_2 R_1 + a_1 R_2)}{a_1 a_2 \left[ 1 + \left( \frac{a_1 b_2}{a_2 b_1} \right)^{m/(m+1)} \right] (m+1)}}$$

where risk attitude plays no role for fighting efforts, but does play a role for the utilities.

*Proposition A4:* Assuming the CARA utility in equation (A10), a risk seeker becoming increasingly risk seeking earns a lower utility  $U_i$  and a risk avoider becoming increasingly risk avoiding earns a higher utility  $U_i$ , which confirms the result for CRRA utility in Proposition 3.

*Proof:* Follows from equation (A11).

### Appendix 3

#### The ratio inside the utility index

When the contest success function is interpreted as what proportion of production is kept by an agent, agent  $i$ 's expected utility is:

$$U_i = \left[ \frac{F_i^m}{F_1^m + F_2^m} \left( \frac{R_1 - b_1 F_1}{a_1} + \frac{R_2 - b_2 F_2}{a_2} \right) \right]^{1-\alpha_i} \leq 1, \quad -1 \leq \alpha_i \leq 1, \quad i=1,2, \quad (\text{A12})$$

Calculating the FOCs  $\partial U_1 / \partial F_1 = 0$ ,  $\partial U_2 / \partial F_2 = 0$  and solving with respect to  $F_1$  and  $F_2$  gives:

$$\begin{aligned} F_1 &= F_2 \left( \frac{a_1 b_2}{a_2 b_1} \right)^{1/(m+1)}, \\ F_2 &= \frac{m(a_2 R_1 + a_1 R_2)}{(m+1) \left[ a_2 b_1 (a_1 b_2 / (a_2 b_1))^{1/(m+1)} + a_1 b_2 \right]}, \\ F_1 &\leq \frac{R_1}{b_1}, \quad F_2 \leq \frac{R_2}{b_2}. \end{aligned} \quad (\text{A13})$$

Inserting equation (A13) into equation (A12) gives:

$$U_1 = \left[ \frac{\frac{R_1}{a_1} + \frac{R_2}{a_2}}{(m+1) \left[ \frac{a_2 b_1}{a_1 b_2} (a_1 b_2 / (a_2 b_1))^{1/(m+1)} + 1 \right]} \right]^{1-\alpha_1} \quad (\text{A14})$$

Propositions 1–3 become:

*Proposition A5:*  $F_1$  and  $F_2$  do not depend on risk attitudes.

*Proof:* Follows from equation (A13).

*Proposition A6:* (a) With equal cost parameters  $a_1 / b_1 = a_2 / b_2$ , a risk seeker and risk avoider fight equally much,  $F_1 = F_2$ . (b) If  $a_1 / b_1 > a_2 / b_2$ , which occurs when agent 1 has high unit cost  $a_1$  of production and low unit cost  $b_1$  of fighting, agent 1 fights more than agent 2. (c) Increasing decisiveness of fighting gives lower discrepancy in fighting between the risk seeker and the risk avoider.

*Proof:* (a) Follows from equation (A13). (b) to (c) Equivalent to the proof of Proposition 2.

*Proposition A7:* A risk seeker becoming increasingly risk seeking earns a lower utility  $U_i$  and a risk avoider becoming increasingly risk avoiding earns a higher utility  $U_i$ .

*Proof:* Follows from the proof of Proposition 3.

Inserting  $R_1 = R_2 = R$ ,  $a_1 = a_2 = a$ ,  $b_1 = b_2 = b$ ,  $\alpha_1 = \alpha_2 = \alpha$  into equations (A12) to (A13) gives:

$$F = mR / ((m+1)b), \quad U = \left[ (R - mR / (m+1)) / a \right]^{1-\alpha} = U(x, \alpha) = x^{1-\alpha}, \quad (\text{A15})$$

$$\frac{U(x, \alpha) + U(x, -\alpha)}{2} - U(x, 0) = \frac{R(R / (a(m+1)))^{-\alpha} \left[ (R / (a(m+1)))^\alpha - 1 \right]^2}{2a(m+1)}. \quad (\text{A16})$$

Equivalent to the paragraph after equations (2.5) to (2.6), the average utility of a risk seeker with risk parameter  $-\alpha$  and a risk avoider with risk parameter  $\alpha$  is always higher than or equal to the utility of a risk neutral agent with risk parameter 0, where the agents have otherwise equal parameters.

Contrary to equation (2.2), in equation (A13), the agents' expenditures  $F_1$  and  $F_2$  on fighting are independent of the agents' risk attitudes  $\alpha_1$  and  $\alpha_2$ . When  $\alpha_1 = \alpha_2$ ,  $F_1 / F_2$  in equation (A13) and equation (2.2) are the same.  $\alpha_1 = \alpha_2 < 0$  causes  $F_2$  to be larger in equation (A13) than in equation (2.2), while  $\alpha_1 = \alpha_2 > 0$  causes  $F_2$  to be smaller in equation (A13) than in equation (2.2). This implies that Propositions A5 and A6(a) differ from Propositions 1 and 2(a), while Propositions A6(b) to (c) and A7 are equivalent to Propositions 2(b) to (c) and 3.

## Notation

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$R_i$	Agent $i$ 's resource
$E_i$	Agent $i$ 's productive effort
$F_i$	Agent $i$ 's fighting effort
$a_i$	Agent $i$ 's unit cost of production
$b_i$	Agent $i$ 's unit cost of fighting
$m$	Decisiveness of fighting
$\alpha_i$	Agent $i$ 's risk parameter
$U_i$	Agent $i$ 's expected utility
$n$	Number of agents, $n = n_R + n_S$
$n_R$	Number of risk seekers
$n_S$	Number of risk avoiders
$R_R$	Risk seeker's resource
$R_S$	Risk avoider's resource
$F_{iR}$	Risk seeker $i$ 's fighting effort
$F_{iS}$	Risk avoider $i$ 's fighting effort
$a_R$	Risk seeker's unit cost of production
$a_S$	Risk avoider's unit cost of production
$b_R$	Risk seeker's unit cost of fighting
$b_S$	Risk avoider's unit cost of fighting
$\alpha_R$	Risk seeker's risk parameter
$\alpha_S$	Risk avoider's risk parameter
$c_P$	Production boost parameter
$c_F$	Fighting boost parameter
$U_{iR}$	Risk seeker $i$ 's expected utility
$U_{iS}$	Risk avoider $i$ 's expected utility

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