Hausken, K. (2007) The impact of the future in games with multiple equilibria. *Economics Letters* , 96(2), pp. 183-188

Link to official URL: http://dx.doi.org/10.1016/j.econlet.2006.12.027
(Access to content may be restricted)

The impact of the future in games with multiple equilibria

Kjell Hausken

Abstract

The article shows that in a game with multiple equilibria, where one player estimates that there is at least a minuscule probability that the other player acquiesces, then conflict is inevitable if both players value the future sufficiently highly.

Keywords: conflict; repeated game; discounting

It is commonly believed that in repeated games where the threat of punishment is credible and immediate and future cooperation is collectively desirable, the more the players value their future interactions (the greater is the discounted benefit of future cooperation) the more likely cooperation will be the equilibrium. In these games cooperation is immediate and e.g. a trigger strategy can be designed to continue that cooperation into the future. This works nicely when there is an agreed upon outcome that is best for everyone (e.g. prisoner's dilemma), where cooperation backed up by credible equilibrium threats elicit cooperation.[1] The threat of conflict thus enforces cooperative relations. The approach in this article is different. "Cooperation" is not immediate. One player is trying to get his preferred equilibrium at the

[1] The Folk Theorem (Fudenberg and Maskin, 1986:533) states that "any individually rational payoff vector of a one-shot game of complete information can arise in a perfect equilibrium of the infinitely repeated game if players are sufficiently patient." Observe also the tit-for-tat strategy analyzed by Axelrod (1984).

Table 1
Two-person two strategy game with two equilibria

	I	II
I	a_1,a_2	t_1,t_2
II	d_1,d_2	b_1,b_2

expense of the other. There is fundamental disagreement as to what is the best equilibrium. In addition, a player, by challenging the other player, is trying to induce him to give in to choosing the other equilibrium.

Consider the game in Table 1 where $a_1 \geq b_1 \geq t_1$, $b_2 \geq a_2 \geq t_2$, $a_1 \geq d_1$, $b_2 > d_2$ or $a_1 > d_1$, $b_2 \geq d_2$.[2] The two pure strategy equilibria are (a_1,a_2) and (b_1,b_2). Row player 1 prefers (a_1,a_2), and column player 2 prefers (b_1,b_2).[3]

Consider a "trigger strategy" (Osborne and Rubinstein, 1994:143–154) for player 2 where the players move simultaneously in each period and a challenge lasts only two periods. In period 0 player 1 gets to know that he is challenged, and in period 1 player 2 gets to know whether player 1 resists or acquiesces: Assume play of (I,I) in Table 1 in period −1 giving (a_1,a_2) and that player 1 also chooses I in period 0. Challenge player 1 in periods 0 and 1 if player 1 chooses strategy I in periods −1 and 0. This gives (t_1,t_2) i n period 0. If player 1 resists in period 1 giving (t_1,t_2), then revert to strategy I giving (a_1,a_2). If player 1 acquiesces in period 1, then continue with strategy II giving (b_1,b_2) until player 1 reverts back to strategy I.[4]

[2] Table 1 encompasses games 64–69 in Rapoport and Guyer's (1966:213) ordinal taxonomy. The most well-known of these are the Battle of the Sexes (game 68), Chicken (game 66), and game 69 with several names such as "Let George do it", "Apology", "Hero", "Sacrificed leader". The games 64, 65, 67 are hybrid asymmetric games.

[3] In the finitely and infinitely repeated versions of the game in Table 1 the two Nash equilibria are subgame perfect. In the infinitely repeated game the following two strategies constitute a subgame perfect equilibrium with payoff (a_1,a_2) in each period: Player 1: Choose strategy I when challenged, unless strategy 2 was chosen in the past, then always choose strategy II. Player 2: Choose strategy I unless player 1 failed to choose strategy I in the past, then always choose strategy II. The justification for the subgame perfect equilibrium with payoff (b_1,b_2) in each period is analogous. For these two subgame perfect equilibria one player acquires a reputation for recalcitrance, the other for acquiescence. One problem with these two equilibria is that the reputation is never tested. Table 1 is equivalent to the probably most well-known example of entry deterrence, viz. the chain store game on normal form, when $a_1 = d_1 \geq b_1 \geq t_1$, $b_2 \geq a_2 = d_2 \geq t_2$, where player 1 is the incumbent (fight = strategy I, acquiesce = strategy II) and player 2 the entrant (stay out = strategy I, enter = strategy II). Both games have the same two Nash equilibria, but the chain store game in its finitely repeated version has only one unique subgame perfect equilibrium (proved by backward induction); the entrant enters and the incumbent does not fight. Kreps and Wilson (1982) and Milgrom and Roberts (1982) were first to formalize reputation effects, where a small amount of incomplete information can be sufficient to overcome Selten's (1978) chain store paradox. As Kreps and Wilson (1982:255) point out, the second equilibrium (the entrant stays out and the incumbent chooses the strategy "fight if entry") i s "imperfect" and "not so plausible as the first. It depends on an expectation by the entrant of the <incumbent's> behavior that, faced with the fait accompli of entry, would be irrational behavior for the <incumbent>." For text book treatments see e.g. Fudenberg and Tirole (1991:369–374), Osborne and Rubinstein (1994:105–106,239–243), Rasmusen (1989:85–118, 2001:110,129), Wilson (1985:31–33).

[4] A generalization is the following "trigger strategy" for player 2 where a challenge may last $f+1$ periods, $f \geq 1$: Challenge player 1 in periods 0,1,2,…,f if player 1 chooses strategy I in periods −1,0,1,…,$f-1$. If player 1 resists in period f, then revert to strategy I giving (a_1,a_2). If player 1 acquiesces, continue with strategy II giving (b_1,b_2) until player 1 reverts back to strategy I. If this is before and including period f, continue to choose strategy II. If this is in period $f+1$ or thereafter, then choose strategy I.

In an infinitely repeated game with discount factors δ_1 and δ_2 for players 1 and 2, respectively, where player 1 resists, players 1 and 2 get

$$t_i + \delta_1 t_1 + \delta_1^2 a_1 + \delta_1^3 a_1 + \cdots + \delta_1^\infty a_1 = \frac{t_1 + (a_1 - t_1)\delta_1^2}{1-\delta_1} \tag{1}$$

$$t_2 + \delta_2 t_2 + \delta_2^2 a_2 + \delta_2^3 a_2 + \cdots + \delta_2^\infty a_2 = \frac{t_2 + (a_2 - t_2)\delta_2^2}{1-\delta_2} \tag{2}$$

respectively. If player 1 acquiesces, they get

$$t_1 + \delta_1 b_1 + \delta_1^2 b_1 + \delta_1^3 b_1 + \cdots + \delta_1^\infty b_1 = \frac{t_1 + (b_1 - t_1)\delta_1^2}{1-\delta_1} \tag{3}$$

$$t_2 + \delta_2 b_2 + \delta_2^2 b_2 + \delta_2^3 b_2 + \cdots + \delta_2^\infty b_2 = \frac{t_2 + (b_2 - t_2)\delta_2^2}{1-\delta_2}. \tag{4}$$

Player 1 resists in period 1 when his payoff in (1) is larger than that of (3), i.e.

$$\frac{t_1 + (a_1 - t_1)\delta_1^2}{1-\delta_1} > \frac{t_1 + (b_1 - t_1)\delta_1}{1-\delta_1} \Rightarrow \delta_1 > \frac{b_1 - t_1}{a_1 - t_1} = \delta_1^*. \tag{5}$$

According to (5) player 1 is more likely to resist the more important the future is. In order for player 2's challenge in period 0 to be part of a subgame perfect equilibrium, her expected payoff from challenging must be larger than the payoff $a_2/(1-\delta_2)$ of not challenging. For this calculation player 2 needs to make a conjecture of the probability q_1 that player 1 resists the challenge. Applying (2) and (4) player 2 challenges in period 0 when

$$\begin{aligned} &q_1 \frac{t_2 + (a_2 - t_2)\delta_2^2}{1-\delta_2} + (1-q_1)\frac{t_2 + (b_2 - t_2)\delta_2}{1-\delta_2} > \frac{a_2}{1-\delta_2} \\ &\Rightarrow \delta_2 > \frac{-(1-q_1)(b_2-t_2) + \sqrt{(1-q_1)^2(b_2-t_2)^2 + 4q_1(a_2-t_2)^2}}{2q_1(a_2-t_2)} = \delta_2^*. \end{aligned} \tag{6}$$

Eqs. (5) and (6) imply that if sufficiently low weight is placed on the future (δ_i is small, $i=1,2$), player 2 does not challenge, and neither does player 1 resist if there were a challenge, implying "peace" at (a_1,a_2). Satisfaction of (5) but not (6) implies that player 2 does not challenge, implying (a_1,a_2). Conversely, satisfaction of (6) but not (5) implies that player 2 challenges and player 1 acquiesces, implying "peace" at (b_1,b_2). Finally, if sufficiently high weight is placed on the future (δ_i is large), player 2 challenges and player 1 resists, implying conflict at (t_1,t_2).

Proposition 1. *1. If $\delta_1<\delta_1^*$ and $\delta_2<\delta_2^*$, then player 2 does not challenge, and player 1 does not resist, implying (a_1,a_2). 2. If $\delta_1>\delta_1^*$ and $\delta_2<\delta_2^*$, then player 2 does not challenge, implying (a_1,a_2). 3. If $\delta_1<\delta_1^*$ and $\delta_2>\delta_2^*$, then player 2 challenges, and player 1 acquiesces, implying (b_1,b_2). 4. If $\delta_1>\delta_1^*$ and $\delta_2>\delta_2^*$, then player 2 challenges, and player 1 resists, implying (t_1,t_2).*

Proof. Follows from (5) and (6). □

Proposition 2. *$\delta_2^* = (a_2 - t_2)/(b_2 - t_2)$ when $q_1 = 0$, and $\delta_2^* = 1$ when $q_1 = 1$.*

Proof. Follows from (6) applying L'Hopital's rule for the first equality. □

Proposition 3. *If $q_1 < 1$ and $a_2 < b_2$ and δ_i is sufficiently large, $i = 1,2$, the threat point (t_1,t_2) is guaranteed.*

Proof. Follows from Propositions 1 and 2. □

Proposition 3 states that if the future is sufficiently important, and player 2 estimates that player 1 is not 100% guaranteed to resist, then conflict is guaranteed. In other words, given that player 2 estimates at least a minuscule probability that player 1 acquiesces, sufficiently large emphasis on the future by both players makes conflict inevitable.

Fig. 1 illustrates the four areas in Proposition 1 assuming $(a_1,a_2) = (4,3)$, $(b_1,b_2) = (3,4)$, $(t_1,t_2) = (0,2)$. The horizontal axis is the probability q_1 estimated by player 2 that player 1 resists the challenge. The vertical axis is the discount factor δ_i which may be different for players 1 and 2. This means that one for each value of q_1 can choose one value δ_1 along the vertical axis for player 1's discount factor, and another value δ_2 along the vertical axis for player 2's discount factor, and read the optimal strategy and payoff for each player out of the diagram. For expositional convenience we focus on one value along the vertical axis, which is sufficient since all the four areas are present.

When $q_1 = 1$, which means that player 2 estimates that player 1 is guaranteed to resist, then player 2 is best off not challenging, regardless of her discount factor, as specified in Proposition 2. Conversely, when $q_1 = 0$, which means that player 2 estimates that player 1 is guaranteed not to resist, then player 2 may

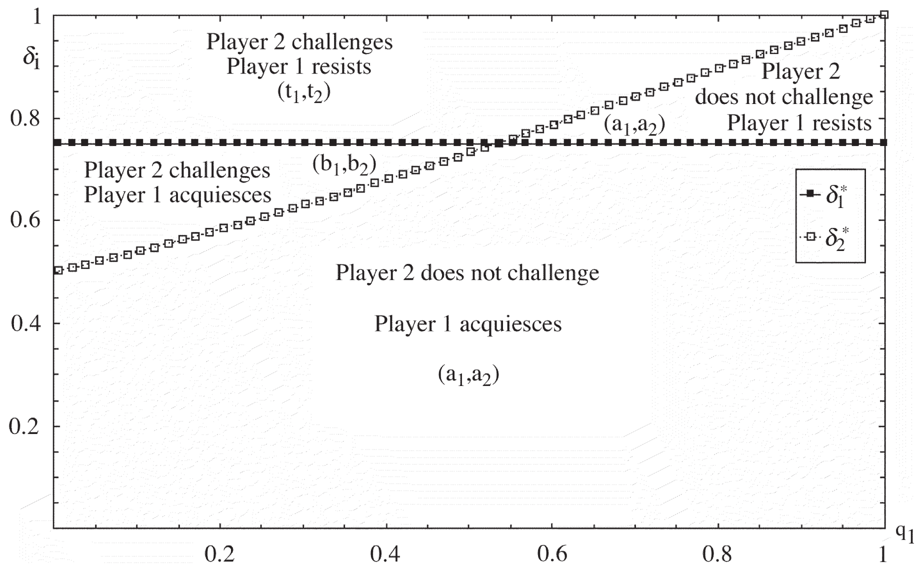

Fig. 1. The four areas in Proposition 1 dependent on the probability q_1 estimated by player 2 that player 1 resists the challenge, and the discount factor δ_i, $i = 1,2$ for both players. The payoffs within each area are shown in brackets.

challenge if the discount factor is large compared with the threat payoff t_2 that has to be endured in period 0 because of the challenge, see (4). Hence when $q_1=0$ and $a_2<b_2$, $\delta_2^*<1$ in accordance with Proposition 2. Fig. 1 shows how δ_2^* increases in q_1, while δ_1^* is constant in accordance with (5).

Below the curve δ_2^* in Fig. 1, the behavior of player 1 is irrelevant since player 2 does not challenge, which always causes (a_1,a_2). However, above the curve δ_2^*, the threat point (t_1,t_2) follows if player 1 resists, while (b_1,b_2) follows if player 1 acquiesces. When the parameters in (5) are adjusted so that δ_1^* decreases, Fig. 1 reduces to three areas making (b_1,b_2) impossible. For example, increasing t_1 from $t_1=0$ to $t_1=2$ causes $\delta_1^*=1/2$ and three areas.

The development above assumes that player 2 challenges the equilibrium (a_1,a_2). The development is analogous when player 1 challenges the equilibrium (b_1,b_2). In that case player 1 estimates a probability q_2 that player 2 resists the challenge.

Conflict at the threat point (t_1,t_2) raises the question of whether the off-the-equilibrium-path conjectures the players make of q_2 and q_1, and of each others' trigger strategies, when located in (I,II) (which is not an equilibrium) in period 1 receiving (t_1,t_2) are incompatible. Eq. (5) assumes that player 1 knows player 2's trigger strategy of acquiescing when player 1 resists. Knowing this conjecture, and assuming that (5) is satisfied, player 2 may rationally conjecture that player 1 is certain to resist, that is estimate $q_1=1$, which when inserted into (6) gives $\delta_2>1$. Hence player 2 does not challenge if player 1 is deemed certain to resist. This again ensures "peace" at (a_1,a_2). But deeming player 1 certain to resist is problematic. It means that player 1 commits in advance to the sequence of actions he will take. If sufficiently challenged it is not rational for player 1 to keep such a commitment. Neither are the remedies suggested by Schelling (1960) for ensuring that one's commitment is trustworthy present in this complete information game,[5] and neither are there any salient focal points. The analysis suggests on one hand that sufficiently high weight on the future increases the likelihood of conflict. But on the other hand, this presupposes that incompatible and sufficiently low off-the-equilibrium-path conjectures are admissible. The players are forced to play the game and cannot bargain themselves out of the game. Although the game has complete information, the players may well choose to resolve their situation by resorting to incompatible conjectures.

References

Axelrod, R., 1984. The Evolution of Cooperation. Basic Books, New York.

Fudenberg, D.M., Maskin, E., 1986. The folk theorem in repeated games with discounting or with incomplete information. Econometrica 54, 533–554.

Fudenberg, D.M., Tirole, J., 1991. Game Theory. MIT Press, Cambridge.

Kreps, D.M., Wilson, R., 1982. Reputation and imperfect information. Journal of Economic Theory 27, 253–279.

Milgrom, P., Roberts, J., 1982. Predation, reputation and entry deterrence. Journal of Economic Theory 27, 280–312.

Osborne, M.J., Rubinstein, A., 1994. A Course in Game Theory. MIT Press, Cambridge.

Rapoport, A., Guyer, M., 1966. A taxonomy of 2×2 games. General Systems 11, 203–214.

Rasmusen, E., 1989. Games and Information. Basil Blackwell, Inc., Cambridge.

Rasmusen, E., 2001. Games and Information. Basil Blackwell, Inc., Cambridge.

Schelling, T., 1960. The Strategy of Conflict. Harvard University Press, Cambridge, MA.

Selten, R., 1978. The chain store paradox. Theory and Decision 9, 127–159.

Wilson, R., 1985. Reputations in games and markets. In: Roth, A.E. (Ed.), Game— Theoretic Models of Bargaining. Cambridge University Press, Cambridge, pp. 27–62.

[5] In a sense, commitment involves choosing an action, and then "burning one's bridges" (Schelling, 1960), thereby ensuring some degree of irreversibility. Commitment implies that one has placed restrictions on oneself. If a player is able to commit in advance to a certain strategy, his behavior will be perfectly predictable, which implies that his reputation is irrelevant since neither his nor the other players' strategies depend on this reputation. Ensuring a role for reputation is done by introducing some degree of uncertainty regarding how the player will behave in the future.