

Governments' and Terrorists' Defense and Attack in a T -Period Game

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We analyze how a government allocates its resources between attacking to downgrade a terrorist's resources and defending against a terrorist attack. Analogously, the terrorist allocates its resources between attacking a government's asset and defending its own resources. A two-stage game is considered where the government moves first and the terrorist moves second. We show that (a) when the terrorist's resources are low, the government attacks the terrorist's resources sufficiently to deter the terrorist from attacking and does not defend; (b) when the terrorist's resources are high, both the government and terrorist defend and attack. We analyze T periods of the two-stage game between two myopic players. First we assume no linkages between periods. We show that after an attack the government may enjoy incoming resources, which deter the terrorist for some periods. Between periods the terrorist's resources may change because of arithmetically and geometrically changing incoming funds. We allow the government's and the terrorist's resources to be determined randomly in each time period. We also allow the government's resources in one period to depend on the terrorist's attacks in earlier time periods for three dynamics, where the terrorist's resources are drawn from a normal distribution or change arithmetically or geometrically.

Key words: terrorism; terror capacity; threat; conflict; dynamics; contest success function; game theory; multiple-period game

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1. Introduction

Essential for terrorism assessment is the understanding of the terrorist's and government's objectives, resource capacities, and the role of time. Extending earlier research, which has typically assumed that the government defends and the terrorist attacks, this paper more realistically, and ambitiously, assumes that both players both defend and attack simultaneously. The government defends its assets and infrastructures, but may also attack the terrorist's resources. Consequently, the terrorist defends its resources in addition to attacking the government's assets. Both players can thus use their resources defensively and/or offensively. We furthermore consider how the government and terrorist may defend and attack through time. We analyze how the government may deter attacks (i.e., the terrorist does

not attack as a result of government deterrence), and how the terrorist responds to such deterrence. We model objectives as utilities, distinguish between unit costs of defense and attack, and allow different asset valuations for the government and the terrorist. The interplay of these factors causes a variety of different equilibrium strategies, which are analyzed.

We develop a model for how a government allocates resources between defending against a terrorist attack and attacking a terrorist's resources, and how a terrorist analogously allocates resources between attacking a government's asset and defending its own resources. We consider the government and terrorist as unitary players.

The government is usually more transparent than the terrorist. Most governments publish their defense budgets to the public (and therefore, to the terrorists).

We thus assume that the terrorist takes the government's defense information as given when choosing its attack strategy in each time period T . For each period, we analyze a two-stage game where the government moves in the first stage, and the terrorist moves in the second stage. Such a game is usually more descriptive than a simultaneous game where the players are unaware of each other's actions. For example, the U.S. homeland security defense budget and its Iraq and Afghanistan operations are well observed by the terrorist.¹

The two-stage game is played T times, referred to as periods (Zhuang et al. 2010). In §6 of this paper there are no linkages between the T periods. In §7, the government's resources depend on the terrorist's attacks in earlier time periods. The time between periods is assumed to be sufficiently longer than the time between stages so that each two-stage game can be solved with backward induction for each period. This means that the players are myopic and boundedly rational in the sense that they only consider one two-stage game in each period. Support for bounded rationality has been provided by Nobel Prize winner Herbert Simon (1955), and in an extensive subsequent literature; see, e.g., Lindblom (1959), Padgett (1980), Rubinstein (1998), and Gigerenzer and Selten (2001).

Clausewitz (1832) suggests that attack is the best defense. The principle is highly debated and does not always hold. This paper seeks to determine to what extent it is optimal to stay on the defensive and await the terrorist's attack and to what extent it is optimal to go on the offensive and actively decrease the terrorist's resources.

To facilitate analytical tractability of the attack versus defense balance for two players, accounting for the time factor, one asset is considered. The asset is interpreted broadly as something of value, which the government seeks to protect and the terrorist seeks to destroy or capture. Some terrorists have a broad

objective, such as inflicting damage on a country (e.g., the United States). In the face of such a terrorist, a government defends its entire country, which calls for a broad defense. The model also applies for collections of assets interpreted as a joint asset and assets defined more narrowly to the extent both the government and the terrorist can be perceived as allocating budgets for attack and defense by collections of assets or specific assets. One example of a collection of assets is the four targets of the 9/11 attack, i.e., the World Trade Center's north and south towers, the Pentagon, and the White House (which was not hit). Focusing on one asset means that we do not analyze how the government and terrorist substitutes resources across assets. For that research question, see Enders and Sandler (2004) and Hausken (2006), and see Bier et al. (2007) for when a government allocates defense to a collection of locations but a terrorist chooses a location to attack. Typically, the government's action against terrorism receives a lot of media and political attention, e.g., U.S. president Reagan's 1986 attack of Libyan president Gaddafi. Such attention can be expected to play a role in the government's decision on its level of attack and whether or not to attack. To avoid making the model too complex, and acknowledging that modeling media and political attention is challenging, this aspect has been left out of our model.

Section 2 presents a literature review. Section 3 develops the model. The government allocates its resources into defending its assets and attacking the terrorist's resources. The terrorist allocates the remaining part of its resources into attacking the government's assets and defending its resources. The probability of asset damage, utilities, decision variables, game structure, and equilibrium are specified. Section 4 analyzes the two-stage game and determines two cases for the solution; the terrorist is deterred when the government attacks but does not defend, or both players defend and attack. Section 5 illustrates the solution. Section 6 considers the T -period game with no linkages between periods. It is first illustrated how the two cases arise and which strategies are chosen in two subsequent time periods dependent on various sizes of the terrorist's resources. Thereafter, the impact of letting the government's resources recover after an attack is analyzed

¹ Observing a government's budget is not sufficient to know how the government allocates its resources into multiple attacks and defenses, though there may be instances where governments specify such allocations to some extent. Our simplifying assumption in this paper is one government, one terrorist, one asset, and one resource. Future research may focus on the players' resource allocations into multiple attacks and defenses.

(in Appendix B.2), which determines how the terrorist can be deterred. Finally, the terrorist's resources are allowed to increase arithmetically and geometrically, and the impact on deterring the terrorist from attacks is analyzed. Sections 7 and 8 consider the T -period game with linkages between periods. We model how the government's resources in one period change dependent on the changes of the terrorist's attacks in the two previous periods in §7, and vice versa in §8. Section 9 suggests how to validate the model and results. Section 10 concludes our findings. Appendix A provides technical solutions to the model, and Appendix B provides the T -period games where the government's resources are endogenously linked between periods.

2. Literature Review

To position the current paper within the stream of literature, we briefly outline earlier research. Earlier research has considered passive defense in the sense of defending against incoming attacks. Azaiez and Bier (2007) consider the optimal resource allocation for security in reliability systems. They determine closed-form results for moderately general systems, assuming that the cost of an attack against any given component increases linearly in the amount of defensive investment in that component. Bier et al. (2005) and Bier and Abhichandani (2002) assume that the government minimizes the success probability and expected damage of an attack. Bier et al. (2005) analyze the protection of series and parallel systems with components of different values. Bier and Abhichandani (2002) apply game theory to characterize optimal defensive strategies against intentional attacks. Levitin (2007) considers the optimal element separation and protection in a complex multi-state series-parallel system and suggests an algorithm for determining the expected damage caused by a strategic terrorist. Patterson and Apostolakis (2007) introduce importance measures for ranking the system elements in complex systems exposed to terrorist actions. Michaud and Apostolakis (2006) analyze such measures of damage caused by the terror and its impact on people, the environment, public image, etc. Dighe et al. (2009) consider secrecy in defensive allocations as a strategy for achieving more cost-effective terrorist deterrence. Zhuang and Bier (2007) consider

government resource allocation for countering terrorism and natural disasters. Levitin and Hausken (2008) consider a two-period model where the defender, moving first, distributes its resources between deploying redundant elements and protecting them from attacks.

Raczynski (2004) simulates the dynamic interactions between terror and antiterror groups. Feichtinger and Novak (2008) use differential game theory to study the intertemporal strategic interactions of Western governments and terror organizations. They illustrate long-run persistent oscillations. Berman and Gavious (2007) study a leader follower game, where the state provides counterterrorism support across multiple metropolitan areas to minimize losses, whereas the terrorist attacks one of the metropolitan areas to maximize his utility. Berrebi and Lakdawalla (2007) consider how terrorists sought targets in Israel between 1949 and 2004, responding to costs and benefits, and find that long periods without an attack signal lower risk for most localities, but higher risk for important areas. Barros et al. (2006) apply parametric and semiparametric hazard model specifications to study durations between Euskadi Ta Askatasuna's (a Spain-based terrorist group) terrorist attacks, which seem to increase in summer and decrease with respect to, e.g., deterrence and political variables. Udwardia et al. (2006) consider the dynamic behavior of terrorists, those susceptible to terrorist and pacifist propaganda, military/police intervention to reduce the terrorist population, and nonviolent, persuasive intervention to influence those susceptible to becoming pacifists. Hausken (2008) considers a terrorist that defends an asset that grows from the first to the second period. The terrorist seeks to eliminate the asset optimally across the two periods. Telesca and Lovallo (2006) find that a terror event is not independent from the time elapsed since the previous event, except for severe attacks, which approach a Poisson process. This latter finding suggests that attack and defense decisions are not unit periodic in nature, but that there are linkages through time. One objective of the current paper is to understand more thoroughly the nature of such linkages through time, affected by changes in resources, unit costs of defense and attack, etc.

Bakir (2008) develops a decision tree model for evaluating countermeasures to secure cargo at south-west U.S. border entries. Merrick and McLay (2010)

analyze whether screening cargo containers for smuggled nuclear threats is worthwhile. Barrett (2010) determines the cost effectiveness of on-site chlorine generation for chlorine truck attack prevention. Feng and Keller (2006) provide a multiple-objective decision analysis for terrorism protection, focusing on potassium iodide distribution in nuclear incidents. von Winterfeldt and O'Sullivan (2006) question whether we should protect commercial airplanes against surface-to-air missile attacks by terrorists.

For a recent survey of work that examines the strategic dynamics of governments versus terrorists, see Sandler and Siqueira (2009). They survey advances in game-theoretic analyses of terrorism, such as proactive versus defensive countermeasures, the impact of domestic politics, the interaction between political and militant factions within terrorist groups, and fixed budgets. Furthermore, Brown et al. (2006) consider defender-attacker-defender models. First the defender invests in protecting the infrastructure, subject to a budget constraint. Then, a resource-constrained attack is carried out. Finally, the defender operates the residual system as best possible. They exemplify with border control, the U.S. strategic petroleum reserve, and electric power grids. Trajtenberg (2006) studies a model with a nonstrategic terrorist, targets in a given country that choose defensive measures, and a government who chooses the proactive effort level.

Some research has focused on investment substitutions across time. First, Enders and Sandler (2004) suggest that a terrorist may compile and accumulate resources during times when the government's investments are high, awaiting times when the government may relax his efforts and choose lower investments. Second, Keohane and Zeckhauser (2003, pp. 201, 224) show that "the optimal control of terror stocks will rely on both ongoing abatement and periodic cleanup" of "a terrorist's 'stock of terror capacity.'" Enders and Sandler (2005) use time series to show that little has changed in the overall terrorism incidents before and after 9/11. Using 9/11 as a break date, they find that logistically complex hostage-taking events have fallen as a proportion of all events, whereas logistically simple, but deadly, bombings have increased as a proportion of deadly

incidents. Enders and Sandler (1993) apply data from 1968 to 1988 and find both substitutes and complements among the attack modes. Evaluating the effectiveness of six policies designed to thwart terrorism, they find that policies designed to reduce one type of attack may affect other attack modes.

Sandler and Siqueira (2006) model the differences between proactive and defensive policies with pseudo contest functions. They find that preemption is usually undersupplied. A country's deterrence decision involves both external benefits and costs as the terrorist threat is deflected, whereas its preemption decision typically gives external benefits when the threat is reduced for all potential targets. With damages limited to home interests, they find that a country would overdeter, whereas for globalized terror, a country would underdeter. Bandyopadhyay and Sandler (2009) consider in a two-stage game to study the interaction between preemption and defense. In the first stage, two countries decide their levels of preemption against a common threat. Preemption decreases damages at a diminishing rate. Preemption, as a public good, is subject to a free-rider problem. In the second stage, the countries decide their levels of defense against the threat adjusted by the first-stage preemption. An increase in one country's defense increases the probability of an attack against the other country. They find that high-cost defenders may rely on preemption, whereas too little preemption may give rise to subsequent excessive defense.

Cárceles-Poveda and Tauman (2011) study a two-stage game. In the first stage, an endogenously determined subset of countries choose their proactive effort levels, which downgrade through a functional form the resources available to the terrorist in the second stage. In the second stage, the terrorist allocates its remaining resources to attack the countries, while, at the same time, the countries choose their defensive measures.

There are significant differences between our paper and Bandyopadhyay and Sandler's (2009) and Cárceles-Poveda and Tauman's (2011) papers. First, we assume that both the government and the terrorist are fully strategic when allocating their resources between defense and attack. The terrorist's resources are downgraded by two fully strategic players where

the government attacks and the terrorist defends its resources. In contrast, Bandyopadhyay and Sandler (2009) assume a nonstrategic threat and Cárceles-Poveda and Tauman (2011) assume that the resources available to the terrorist in the second stage are downgraded nonstrategically through a functional exponential form. The resources available to the terrorist in the second stage are applied in their entirety. The terrorist's strategic decision is how to allocate its downgraded resources across the countries. Second, we assume that the damage probability for the government's asset depends on the strategic decision by the government of how well to defend its asset, and the strategic decision by the terrorist of how well to attack the asset using its downgraded resources, accounting for a contest intensity. In contrast, Bandyopadhyay and Sandler (2009) assume that the terrorist's second-stage attack depends nonstrategically and functionally on the countries' first-stage preemption, and Cárceles-Poveda and Tauman (2011) assume that the damage inflicted on country i is determined by a functional form which is proportional to the resources allocated by the terrorist to country i , proportional to the political and/or economic power of country i , and inverse proportional to the defense of country i in the second stage. Third, we consider one unitary government, which means abstracting away the collective action problem of multiple governments. In contrast, Bandyopadhyay and Sandler (2009) and Cárceles-Poveda and Tauman (2011) account for the collective action problem with two and multiple players, respectively. Fourth, both Bandyopadhyay and Sandler (2009) and the present paper determine solutions where the government does not defend.

Our paper builds upon and extends earlier research. First, we enrich the one-period model by allowing both the government and terrorist to both defend and attack. The government defends itself and at the same time attacks the terrorist's resources. Analogously, the terrorist defends its resources, and, at the same time, uses its remaining resources to attack the government. Second, we repeat the one-period model T times to understand how long the terrorist can be deterred.

3. The Model

3.1. Motivation and Notation

The model in this paper seeks to answer the research question of how two players, a government and a terrorist, strike a balance between attack and defense over time. Game theory is chosen as the modeling methodology to account for the two players' strategic options. Important factors related to this research question are the players' resources, asset valuations, the contest intensity for asset damage, and unit costs of defense and attack. Throughout this paper we use the following notation.

Parameters:

- T number of time periods
- t time period, $t = 1, \dots, T$
- r_t government's resources in period t , $r_t \geq 0$
- R_t terrorist's resources in period t , $R_t \geq 0$
- g_t government's unit attack cost in period t , $g_t \geq 0$
- G_t terrorist's unit defense cost in period t , $G_t \geq 0$
- m_t contest intensity for asset damage, $m_t \geq 0$

Decision variables:

- d_t government's defense effort protecting the asset in period t , $d_t \geq 0$
- A_t terrorist's attack effort attacking the asset in period t , $A_t \geq 0$
- a_t government's attack effort attacking the terrorist's resources R_t in period t , $a_t \geq 0$
- D_t terrorist's defense effort protecting its resources in period t , $D_t \geq 0$

Functions:

- P_t probability of asset damage in period t , $1 \geq P_t \geq 0$
- Q_t proportion of terrorist resources remaining after the government's attack, $1 \geq Q_t \geq 0$
- u_t government's expected utility in period t
- U_t terrorist's expected utility in period t

3.2. Assumptions

In each time period t , $t = 1, 2, \dots, T$, the government has an available budget in terms of resources r_t . We first assume that r_t is exogenous, and in §7 we endogenize r_t . In each time period t , the government moves first by transforming the resources r_t to either defense d_t at unit cost 1 or attack a_t at unit cost g_t directed against the terrorist's resources. The resources r_t can be capital goods and/or labor. More

specifically, using Hirshleifer's (1995, p. 30), Skaperdas and Syropoulos's (1997, p. 102), and Hausken's (2005, p. 62) terminology, g_t is the unit conversion cost of transforming the resources r_t into a_t . The unit conversion cost of transforming r_t into d_t is 1. We thus get²

$$r_t = d_t + g_t a_t. \quad (1)$$

Equation (1) strikes a balance between defending an asset, and actively attacking and decreasing the terrorist's resources. The transformation into d_t and a_t can be considered as production processes where $1/g_t$ is the productive efficiency. Note that (1) implicitly requires that $a_t \in [0, r_t/g_t]$ and $d_t \in [0, r_t]$. Note also that allocating equal amounts of resources (e.g., a capital good such as money) into defense and attack ($r_t/2$ to each) generally does not mean that the defense effort d_t and attack effort a_t become equally large because the productive efficiencies of these two kinds of efforts may be different. For example, economies of scale, differences in competence and organizational structure, and different production processes, may cause 1 and $1/g_t$ to differ substantially.

An allocation of fixed and exogenously given resources into two kinds of efforts has been made earlier by Hirshleifer (1995) and Hausken (2005) in a one-period game. A feature of this paper is that r_t is first exogenously given in each time period (§§3–6), and thereafter endogenously (§7). When exogenous, neither the government nor the terrorist affects r_t over time, but r_t may change over time because of external factors. In §6.3, r_t is drawn from a random distribution, and in §7, r_t is endogenous determined by the terrorist's attacks in earlier time periods. Further endogenizing may be done in future research.

We consider the government and terrorist as unitary players, abstracting away the collective action problem within each of these players. Both governments and terrorists may, to some extent, have separate power fractions and decentralized decision making. For example, in the United States, terrorism defense is to some extent separated in a chain of command and funding channels from attack activities. However, moving toward the top of

the chain of command, which in the United States means Congress and the president, resource allocation inevitably occurs between defense and attack. Multiple terrorist threats generated by one or multiple terrorists are either perceived as independent, or, if they have commonalities, they can be grouped together as a large threat generated by a collective player, applying Simon's (1969) principle of "near decomposability," which means grouping together players with similar but not entirely aligned preferences. Future research may model the government and terrorist as nonunitary heterogeneous players.

The terrorist observes the government's choice of d_t and a_t in the first stage in each time period³ and allocates in the second stage its resources into defense D_t at unit cost G_t against the government's attack, and attack A_t at unit cost 1 against the asset controlled by the government. Although the players' decisions occur in two stages, the two contests, over the terrorist's resources and the asset, occur after the two stages, i.e., after the efforts d_t , a_t , D_t , and A_t have been chosen. This is illustrated in Figure 1.

We model the proportion of the remaining terrorist's resources, which is the part of the terrorist's resources that has not been destroyed by the government's attack, as a contest between the terrorist's defense of its resources and the government's attack. For this purpose we use the common ratio form (Tullock 1980, Skaperdas 1996) contest success function, i.e.,

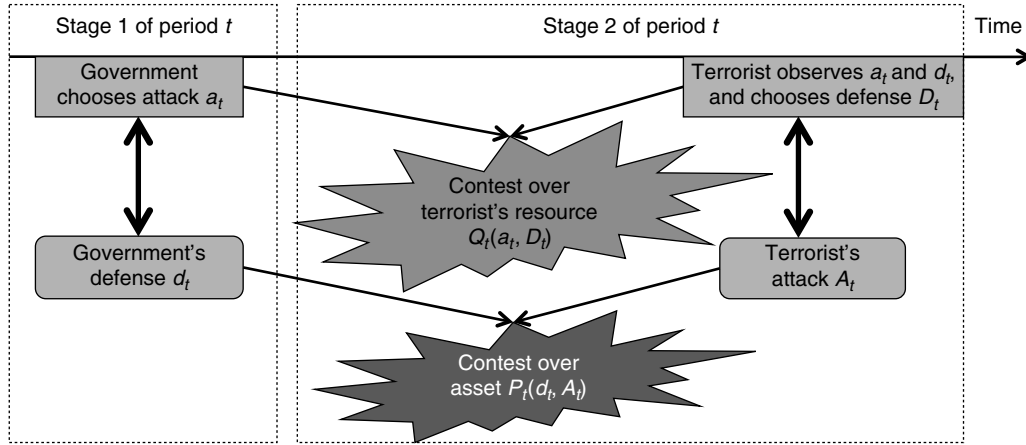
$$Q_t(a_t, D_t) = \frac{D_t}{D_t + a_t}, \quad (2)$$

where $\partial Q_t / \partial D_t > 0$ and $\partial Q_t / \partial a_t < 0$. Equation (2) expresses that the terrorist keeps a larger fraction of its resources when its defense D_t is large and the government's attack a_t is small. The terrorist's original resources in each period is R_t , but it decreases to $Q_t R_t$ because of the government's attack, where $Q_t R_t$ is the proportion of the remaining resources, and $(1 - Q_t) R_t$ is the proportion of the damaged terrorist's resources. The remaining resources $Q_t R_t$ are transformed into

² Equation (1) abstracts from a general unit defense cost x , obtained by replacing (1) with $x_t r_t = x_t d_t + x_t g_t a_t$.

³ For simplicity we assume that there is no secrecy or deception in government disclosure of attack and defense, in contrast to the studies by Zhuang et al. (2010) and Zhuang and Bier (2011).

Figure 1 Sequence of Moves of the Model



attack A_t and defense D_t . The terrorist's resource allocation equation can thus be expressed as⁴

$$Q_t R_t = \frac{D_t}{D_t + a_t} R_t = G_t D_t + A_t. \quad (3)$$

Analogously to (1), R_t can be a capital good and/or labor, and G_t is the unit conversion costs of transforming R_t into D_t . The unit conversion cost of transforming R_t into A_t is 1. Hence, $1/G_t$ is the productive efficiency. Equation (3) states that the terrorist's allocation into A_t and D_t at the same time determines $Q_t R_t$, which depends on D_t . This means that the terrorist possesses only $Q_t R_t$ when making its allocation in period t . The terrorist cannot allocate its proportion $(1 - Q_t)R_t$ into defense and attack in period t because that proportion gets eliminated by the government in period t . This immediate feedback is realistic because the terrorist needs to protect its entire resources. For example, when launching an attack, the terrorist needs to protect equipment and personnel involved in the attack. As an example, assume $g_t = G_t = 1$, $r_t = R_t = 2$, and that the government chooses $d_t = a_t = 1$, and the terrorist chooses $D_t = 1/2$. Hence, the terrorist has resources $Q_t R_t = (D_t / (D_t + a_t)) R_t = ((1/2) / (1/2 + 1)) 2 = 2/3$ available for defense and attack. Equation (3) implies $2/3 = 1/2 + A_t$, which gives $A_t = 1/6$.

⁴ Equation (3) abstracts from a general unit attack cost X_t obtained by replacing (3) with $X_t Q_t R_t = X_t G_t D_t + X_t A_t$.

For the probability of asset damage, we also consider the ratio form contest success function,

$$P_t(d_t, A_t) = \frac{A_t^{m_t}}{A_t^{m_t} + d_t^{m_t}}, \quad (4)$$

where $m_t \geq 0$ is a parameter for the contest intensity, $\partial P_t / \partial d_t < 0$, and $\partial P_t / \partial A_t > 0$. The model thus has five parameters, i.e., two unit costs g_t and G_t , two players' resources r_t and R_t , and the contest intensity for asset damage m_t .

When $m_t = 0$, the efforts d_t and A_t have no impact on the asset damage, which gives $P_t = 0.5$. When $0 < m_t < 1$, exerting more effort than one's opponent gives less advantage in terms of asset damage than the proportionality of the players' efforts specify. For example, when $m_t = 0.5$, high terrorist effort $A_t = 2$ and low government effort $d_t = 1$ give $P_t = 0.59 < 2/3$, which means that the terrorist gets a lower probability of asset damage than $2/3$ despite the higher effort. When $m_t = 1$, the efforts have a proportional impact on the damage. When $m_t > 1$, exerting more effort than one's opponent gives more advantage in terms of vulnerability than the proportionality of the agents' efforts specify. For example, $A_t = 2$, $d_t = 1$, $m_t = 2$ gives $P_t = 0.8 > 2/3$. Finally, $m_t = \infty$ gives a step function where $P_t = 1$ if and only if $A_t > d_t$. The parameter m is a characteristic of the contest, which can be illustrated by the history of warfare. Low levels of m occur for assets, which are defensible, predictable, and consisting of individual asset components, which are dispersed, i.e., physically distant or separated by barriers

of various kinds. Neither the government nor the terrorist can get a significant advantage. High levels of m occur for assets that are less predictable, easier to attack, and where the individual asset components are concentrated, i.e., close to each other or not separated by particular barriers. This may cause dictatorship by the strongest. Either the government or the terrorist may get the advantage.

3.3. Problem Formulation

The probability that the asset is not damaged is $1 - P_t(d_t, A_t)$, which the government maximizes. Analogously, the terrorist maximizes the probability of damage. The government's and terrorist's expected utilities in period t are

$$u_t(d_t, A_t) = 1 - P_t(d_t, A_t) = \frac{d_t^{m_t}}{A_t^{m_t} + d_t^{m_t}}, \quad (5)$$

$$U_t(d_t, A_t) = P_t(d_t, A_t) = \frac{A_t^{m_t}}{A_t^{m_t} + d_t^{m_t}}.$$

Inserting (1) and (3) into (5) gives

$$u_t(a_t, D_t) = \frac{(r_t - g_t a_t)^{m_t}}{\left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{m_t} + (r_t - g_t a_t)^{m_t}},$$

$$U_t(a_t, D_t) = \frac{\left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{m_t}}{\left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{m_t} + (r_t - g_t a_t)^{m_t}}. \quad (6)$$

The government's one decision variable is a_t , where d_t follows from (1). Analogously, the terrorist's one decision variable is D_t , where A_t follows from (3). We assume common knowledge and complete information so that both players know all the parameters and the game structure.

In each time period t we consider a two-stage game where the government moves first and the terrorist moves second. To determine the subgame perfect Nash equilibrium (see Mas-Collel et al. 1995, Chap. 9.B), we assume that the government chooses a_t in the first stage. The terrorist observes a_t and chooses D_t in the second stage.

DEFINITION 1. A strategy pair (a_t^S, D_t^S) is a *subgame perfect Nash equilibrium* if and only if

$$D_t^S = D_t(a_t^S) = \arg \max_{D_t \geq 0} U_t(a_t^S, D_t) \quad (7)$$

and

$$a_t^S = \arg \max_{a_t \geq 0} u_t(a_t, D_t(a_t)). \quad (8)$$

4. Solving the Two-Stage Game

Solving the game in period t with backward induction, Appendix A determines the subgame perfect Nash equilibrium solution in Table 1.

There are two cases of solutions classified by the terrorist's resources R_t : In Case 1, with an inactive terrorist and deterring government, when the terrorist's resources are low, the terrorist is deterred with $a_t > 0$; there is no terrorist activity, and the government keeps the whole asset. In Case 2, with an active terrorist and active government, when the terrorist's resources are high, both the government and terrorist defend and attack.

Table 1 also shows that at equilibrium, the terrorist chooses either $D_t = A_t = 0$ or $\{D_t > 0, A_t > 0\}$. Intuition for this can also be gathered from (3). The terrorist's defense effort D_t is positive if and only if its attack effort A_t is positive. This follows because the reason for the terrorist to defend is to ensure that resources are available to attack; and if the terrorist does not defend, then there are no resources available

Table 1 Solution to Subgame Perfect Nash Equilibrium for Period t

Cases	Case 1	Case 2
Conditions	$R_t < \frac{G_t r_t}{g_t}$	$R_t \geq \frac{G_t r_t}{g_t}$
Scenarios	Inactive terrorist and deterring government	Active terrorist and active government
a_t	$\frac{R_t}{G_t}$	$\frac{G_t r_t^2}{g_t^2 R_t}$
d_t	0	$r_t \left(1 - \frac{G_t r_t}{g_t R_t} \right)$
A_t	0	$R_t \left(1 - \frac{G_t r_t}{g_t R_t} \right)^2$
D_t	0	$\frac{r_t}{g_t} \left(1 - \frac{G_t r_t}{g_t R_t} \right)$
u_t	1	$\frac{1}{1 + (R_t/r_t - G_t/g_t)^{m_t}}$
U_t	0	$\frac{1}{1 + (R_t/r_t - G_t/g_t)^{-m_t}}$

to attack. The numerator for the terrorist's utility in (6) would be negative when $a_t \geq R_t/G_t - D_t$. Hence, when $a_t \geq R_t/G_t$, which means that the governments' attack is larger than (or equal to) the terrorist's resources divided by the terrorist's unit defense cost, the terrorist is guaranteed zero utility regardless of which strategy it chooses. If it chooses $D_t = 0$ after $a_t \geq R_t/G_t$, its resources are eliminated and it cannot attack. If it chooses $A_t = 0$ after $a_t \geq R_t/G_t$ in an attempt to defend its resources, (3) implies $D_t = 0$, which means that it cannot defend. Hence, when $a_t \geq R_t/G_t$, we assume that the terrorist's resources are eliminated and the terrorist chooses $D_t = A_t = 0$. As R_t decreases below $R_t = G_t r_t / g_t$, a smaller amount of government resources is needed to deter the terrorist. In particular, using (A1) in Appendix A, $a_t = R_t/G_t$ is the minimum attack needed to prevent the terrorist from attacking and is sufficient to deter the terrorist. In this case 1, using (1) when $d_t = 0$, we assume that the government uses $r_t^* = g_t R_t / G_t \leq r_t$, which deters the terrorist and saves $r_t - r_t^*$ of resources for the government.

For Case 1, intuitively, the government's attack that deters the terrorist increases in R_t and decreases in G_t , because a more resourceful or lower-cost terrorist needs a larger attack to be deterred. For Case 2, conversely, the government's attack decreases in R_t and increases in G_t because the government has lower capability to attack a more resourceful or lower-cost terrorist. Intuitively, a_t increases in r_t/g_t . The government's defense increases in R_t and decreases in G_t because the government according to (1) uses its resources to strike a balance between a_t and d_t . As the government's unit attack cost g_t increases, the government shifts its resources into increasing defense d_t , which is inverse U-shaped in r_t . When r_t is small, the government refrains from defense because of weakness. When r_t is large, the government refrains from defense because of strength, gradually shifting its resources into the attack which eventually, for large values of r_t , deters the terrorist. As the government becomes more resourceful, its defense initially increases strongly and concavely, $\partial d_t / \partial r_t > 0$ and $\partial^2 d_t / \partial r_t^2 < 0$, whereas its attack initially increases slowly and convexly, $\partial a_t / \partial r_t > 0$ and $\partial^2 a_t / \partial r_t^2 > 0$. With an intermediate amount of resources, the government's defense reaches a maximum where $\partial d_t / \partial r_t = 0$. With much resources the government shifts its

resources into the attack, which deters the government and makes less use for the defense, $\partial d_t / \partial r_t < 0$. Increasing terrorist resources R_t causes convexly increasing terrorist attack and concavely increasing terrorist defense. Both A_t and D_t decrease in G_t . Intuitively, the terrorist's attack decreases in the government's resources. Interestingly, the terrorist's defense D_t mirrors the government's defense d_t with an inverse U-shape as a function of the government's resources.

According to Table 1, the boundary conditions $D_t = R_t/G_t$ or $a_t = 0$ cannot arise at equilibrium. The terrorist will not choose $D_t = R_t/G_t$ because that leaves no resources for attack, which guarantees zero utility according to (5). Interestingly, $a_t = 0$ does not arise at equilibrium (see Appendix A). The intuition is that the government always prefers to proactively attack the terrorist's resources to degrade these, to a degree adjusted by R_t and G_t for Case 1 and R_t , G_t , r_t , and g_t for Case 2.

One should be careful when providing examples to illustrate the two cases because assessments are needed as to whether the examples fit the modeling assumptions. Different countries, organizations, and agencies have set up different lists of terrorist organizations according to various criteria.⁵ Case 1 suggests that governments may handle minor terrorist threats with proactive attacks against their modest resources. One example is the U.S. president Reagan's attack on Tripoli and Benghazi April 14, 1986, after which Libya disappeared from media attention as a sponsor of terrorist attacks. One example of a contradictory anecdote for Case 1 is when attacking a terrorist with scarce resources causes hatred (Glaeser 2005, Kress and Szechtman 2009⁶) to emerge within this terrorist, which draws resources so that this terrorist becomes a larger future threat. (This possibility is handled in §6.3

⁵ See <http://www.state.gov/s/ct/rls/other/des/123085.htm> for the United States July 7, 2009, list of 45 Designated Foreign Terrorist Organizations, and http://en.wikipedia.org/wiki/Terrorist_groups for the lists of various countries.

⁶ Kress and Szechtman (2009, p. 578) "model the dynamic relations among intelligence, collateral casualties in the population, attrition, recruitment to the insurgency, and reinforcement to the government force." They show that the government can contain the insurgency, but not eradicate it.

where the terrorist's resources are allowed to increase for a variety of reasons.) At the other extreme, Case 2 is illustrated by Al-Qaeda, which both attacks and defends, and faces governments, which attack and defend at home and abroad.⁷

Table 1 shows that the government always attacks the terrorist's resources, choosing $a_t > 0$, and does not defend in Case 1. This result is interesting, especially with the many minor terrorist threats around the world (assuming these are independent, or grouped together applying Simon's (1969) principle of "near decomposability"). It is really in a government's interest to eliminate these with active defense $a_t > 0$. We show that minor terrorists with $R_t \leq G_t r_t / g_t$ are fully deterred in Case 1. As a terrorist grows more resourceful, from Case 1 to Case 2 in Table 1, the government starts to use passive defense d_t as in Case 2. On the one hand, this decreases the terrorist's resources available for attack and protects the asset against the terrorist's attack furnished by the terrorist's resources, which have not been eliminated by a_t . As the terrorist's resources increase, from Case 1 to Case 2 in Table 1, the government suffers a more inferior position: when the terrorist's resources are low, the government applies a small amount of resources to destroy the terrorist's resources; otherwise, the government applies its entire resources striking a balance between active and passive defense.

In Case 1, the contest intensity plays no role. In Case 2, the government's and terrorist's strategic choices are equally unaffected by the contest intensity m , but their utilities are affected by m . Case 2

⁷Kaplan et al. (2010) consider how to confront entrenched insurgents. They develop one equilibrium with perfect government intelligence where "the insurgents concentrate their force in a single stronghold that the government either attacks or not depending upon the resulting casualty count" (p. 329). Under alternative assumptions they show how insurgents may "spread out" in a way that maximizes the number of soldiers required to win all battles. Taliban, operating mostly in Afghanistan and Pakistan, and various locally operating terrorists, are hybrids of Cases 1 and 2 as viewed by the United States. Attacking them locally, such as attacking Taliban in Afghanistan, contains and deters them from global attacks. Although intelligence is invested into terrorists operating more locally than Al-Qaeda, specific defenses against terrorists far away from their operating territories are less imperative.

implies

$$\begin{aligned} \frac{\partial u_t}{\partial m_t} > 0 &\Leftrightarrow r_t > \frac{R_t}{1 + G_t/g_t}, \\ \frac{\partial U_t}{\partial m_t} > 0 &\Leftrightarrow r_t < \frac{R_t}{1 + G_t/g_t}. \end{aligned} \quad (9)$$

Changes in the contest intensity always benefit one player and harm the other. The player advantaged with the most resources, or a low unit cost, benefits from increasing contest intensity. The terrorist's resource degradation in (3) implies that with equal resources $r_t = R_t$ and equal unit costs $G_t = g_t$, the government benefits from increasing contest intensity, whereas the terrorist does not.

THEOREM 1. (a) When $R_t < G_t r_t / g_t$, the terrorist is fully deterred with a government attack effort $a_t = R_t / G_t$, and the government does not defend. (b) When $R_t \geq G_t r_t / g_t$, both players attack and defend.

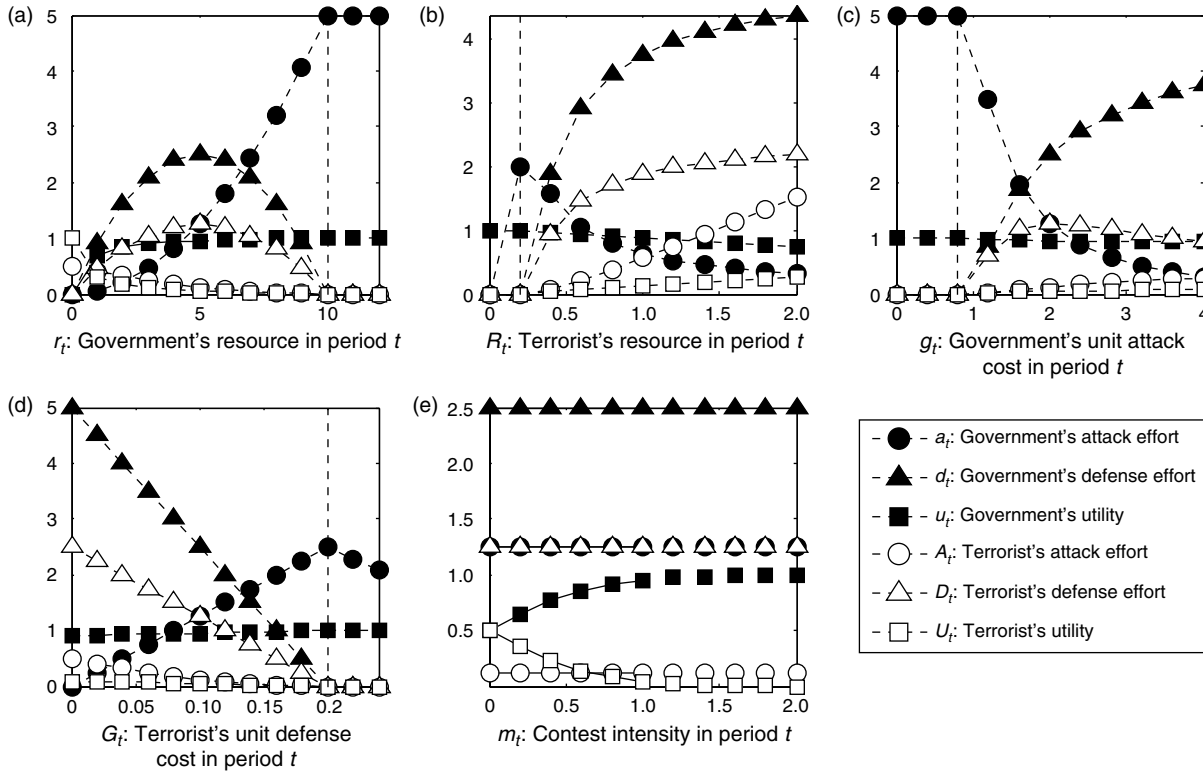
PROOF. The proof follows from Table 1. \square

5. Illustrating the Two-Stage Game

To determine plausible parameter values, we reason as follows. Both players may have a variety of production processes for their four kinds of efforts. An especially common and salient *ceteris paribus* starting point is to assume that all the four unit costs of effort are equal and set to 1; that is, $g_t = G_t = 1$. The most plausible value for the contest intensity is also $m_t = 1$, which means that the players' efforts have a proportional impact on the damage of the attack. A further benchmark is that the players are equally resourceful, $r_t = R_t$, though this latter assumption will be altered substantially as we proceed, through changing R_t . Using (1) and (3), where unit costs are 1, we choose $r_t = R_t > 1$ to get conveniently sized efforts. We analyzed the impact on the solution in §4 and Table 1 of varying $r_t = R_t$ upward and downward and found that $r_t = R_t = 10$ is a plausible benchmark, which illustrates important characteristics of the model.

This section illustrates the two-stage game with the baseline values $G_t = 0.1$, $R_t = 0.5$, $m_t = 1$, $g_t = 2$, and $r_t = 5$, which give Case 2 in Table 1. Figure 2 shows the four equilibrium choice variables a_t , d_t , A_t , and D_t and the two utilities u_t and U_t as the

Figure 2 Equilibrium Behaviors as Functions of $r_t, R_t, g_t, G_t,$ and m_t , with Baseline Values $G_t = 0.1, R_t = 0.5, m_t = 1, g_t = 2,$ and $r_t = 5$



parameter values $r_t, R_t, g_t, G_t,$ and m_t respectively change from the baseline value. The vertical dashed lines demarcate Case 1 from Case 2. For the baseline values, we assume that the terrorist is less resourceful than the government, which is often or usually realistic. The reason for assuming a low unit defense cost for the terrorist is that it might be cheaper for the terrorist to protect his resources (hiding in caves, on his home turf, etc.) than attacking the government (sending personnel and weapons to New York City, etc.).⁸

In the upper left panel, when $R_t < G_t r_t / g_t \Leftrightarrow 0.5 < 0.1 r_t / 2 \Leftrightarrow r_t > 10$ (Case 1), the terrorist withdraws. When $r_t \leq 10$ (Case 2), both players' defenses are inverse U-shaped in an interior solution. For high r_t , the government defends moderately (neither too much nor too little) out of strength, instead relying

on attack. For low r_t , the government defends moderately out of weakness, whereas the terrorist defends moderately out of strength, instead relying on attack. The government's and terrorist's attacks, and utilities, increase and decrease, respectively, in r_t .

In the upper middle panel, when $R_t < G_t r_t / g_t = (0.1 \times 5) / 2 = 0.25$ (Case 1), the terrorist withdraws. The government must increase its attack in $R_t, a_t = R_t / G_t$, to ensure the deterrence. When $R_t \geq 0.25$ (Case 2), the interior solution arises, in accordance with the right column in Table 1, both players' defenses and the terrorist's utility increase asymptotically toward constants as R_t reaches infinity; that is, $\lim_{R_t \rightarrow \infty} d_t = r_t = 5, \lim_{R_t \rightarrow \infty} D_t = r_t / g_t = 5 / 2 = 2.5, \lim_{R_t \rightarrow \infty} U_t = 1$, the terrorist's attack increases toward infinity, $\lim_{R_t \rightarrow \infty} A_t = +\infty$, and the government's attack and utility decrease toward zero, $\lim_{R_t \rightarrow \infty} a_t = 0, \lim_{R_t \rightarrow \infty} u_t = 0$.

In the upper right panel, $R_t < G_t r_t / g_t \Leftrightarrow g_t < G_t r_t / R_t = 0.1 \times 5 / 0.5 = 1$ (Case 1) deters the terrorist. As g_t increases above 1, the government allocates

⁸ Clausewitz (1832) argues in this regard for the superiority of defense over attack. The very low value $G_t = 0.1$ is needed because of the terrorist's resource degradation in (3). We analogously assume a larger unit attack cost for the government.

less resources to the more costly attack and more to defense. The terrorist allocates less resources to defense and more to attack when $g_t > 2$, as determined by

$$\begin{aligned} \frac{\partial D_t}{\partial g_t} &= \frac{\partial}{\partial g_t} \left[\frac{r_t}{g_t} \left(1 - \frac{G_t r_t}{g_t R_t} \right) \right] \\ &= \frac{r_t^2 G_t - r_t g_t R_t + G_t r_t^2}{g_t^3 R_t} < 0 \\ \Leftrightarrow g_t &> \frac{2r_t G_t}{R_t} = \frac{2 \times 5 \times 0.1}{0.5} = 2. \end{aligned} \quad (10)$$

In the lower left panel, a large unit defense cost $R_t < G_t r_t / g_t \Leftrightarrow G_t > R_t g_t / r_t = (0.5 \times 2) / 5 = 0.2$ for the terrorist (Case 1) deters the terrorist from defense and attack. As G_t increases above 0.1, less resources are needed by the government to deter. Hence, the government decreases its attack according to $a_t = R_t / G_t$ as G_t increases. As G_t decreases below 0.1, the terrorist is not deterred. Instead, the terrorist's attack, defense, and utility increase dramatically, whereas the government's attack, defense, and utility decrease dramatically.

Last, in the lower middle panel, where Case 2 applies throughout, we see a_t , d_t , A_t , and D_t are independent of m_t as shown in Table 1. However, the government's utility u_t increases in m_t , and the terrorist's utility U_t decreases in m_t , because the government is advantaged with $5 > 0.5 / (1 + 0.1/2)$ in (9).

6. The T-Period Game with No Linkages Between Periods

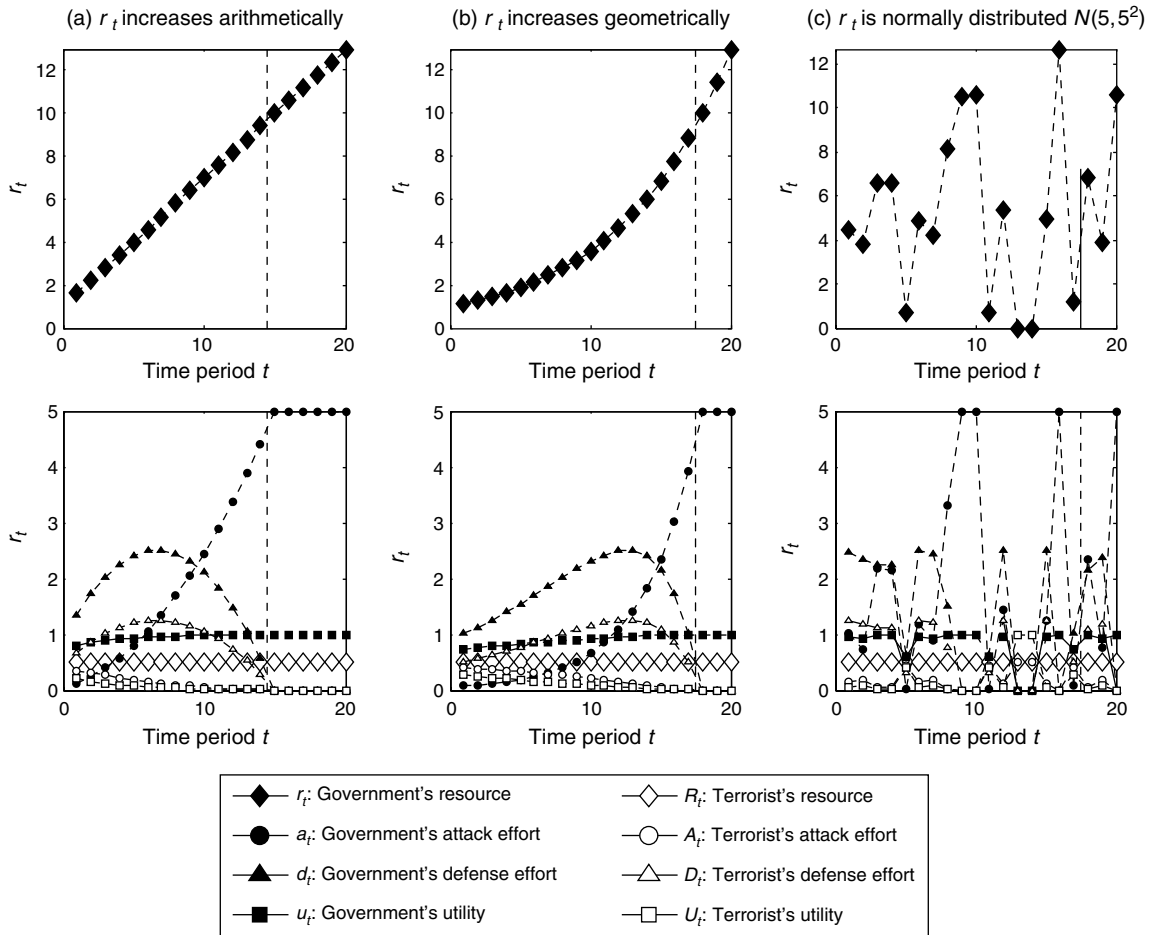
We assume that the time between periods is sufficiently longer than the time between stages so that each two-stage game can be solved with backward induction. This means that the players are myopic and boundedly rational, and only consider one two-stage game in each period. These assumptions are made for analytical tractability and because of the nature of real-world interactions. First, analyzing a T-period game with backward induction from period T, and simultaneously analyzing the embedded two-stage game with backward induction from Stage 2 in period t , $t = 1, \dots, T$, is an insurmountable task. Second, real-world players are indeed myopic and boundedly rational and do not look too far ahead

because of the plethora of eventualities and unforeseen contingencies that may arise. Survival in the present is important, and there is a tendency to discount events in the remote future unless they can be demonstrated to be important. Additionally, politically elected officials are usually elected for limited amounts of time and as a result there is not generally continuity between successive administrations. We thus assume that the government and the terrorist maximize u_t and U_t , respectively, in each period. This section considers the same baseline as in Figure 2, that is $G_t = 0.1$, $R_t = 0.5$, $g_t = 2$, $m_t = 1$, and $r_t = 5$.

6.1. Modeling the Government's Resources r_t

This section keeps the terrorist's resources at its baseline $R_t = 0.5$ and assumes that the government's resources r_t increase from a low level or decrease from a high level. An increase may occur after a political decision or after an earlier terrorist attack. A decrease may occur, e.g., when a substantial amount of time has elapsed after a terrorist attack. We investigate the following three functional forms of increment where r_t changes over 20 periods: arithmetic $r_t = 1 + 0.6t$ and geometric $r_t = 1.1368^t$, where r_t increases from 1 to 13, and stochastic r_t , which is randomly drawn from a normal distribution with mean 5 and standard deviation 5 (when negative, we let r_t be zero) for $t = 1, \dots, 20$. Figure 3 shows the equilibrium dynamics of these three functional forms. The functional forms when r_t decreases from 13 to 1 over 20 periods are the time-reversed version of the functional forms when r_t increases from 1 to 13 over 20 periods, shown by reading Figure 3(a) and 3(b), from right to left. With arithmetically increasing r_t , both players both defend and attack through period 14 (Case 2), where $r_t < 10$ and the government cannot deter. During periods 15–20 (Case 1), the terrorist is deterred by the government's attack. With geometrically increasing r_t , both players both defend and attack simultaneously through period 17 (Case 2), and then the terrorist is deterred starting from period 18 (Case 1). With geometrically increasing r_t , it takes much more time (three additional periods) than with arithmetically increasing r_t to cause $R_t < G_t r_t / g_t$, which is needed to deter the terrorist. Both the terrorist's and government's defense efforts are inverse U-shaped during the nondeterrence phase. Early on, the terrorist's defense is low or moderate because of strength. As the deterrence phase gets

Figure 3 Equilibrium Behaviors as a Function of Time Period When r_t Is Dynamic



approached, the terrorist's defense is low because of weakness.

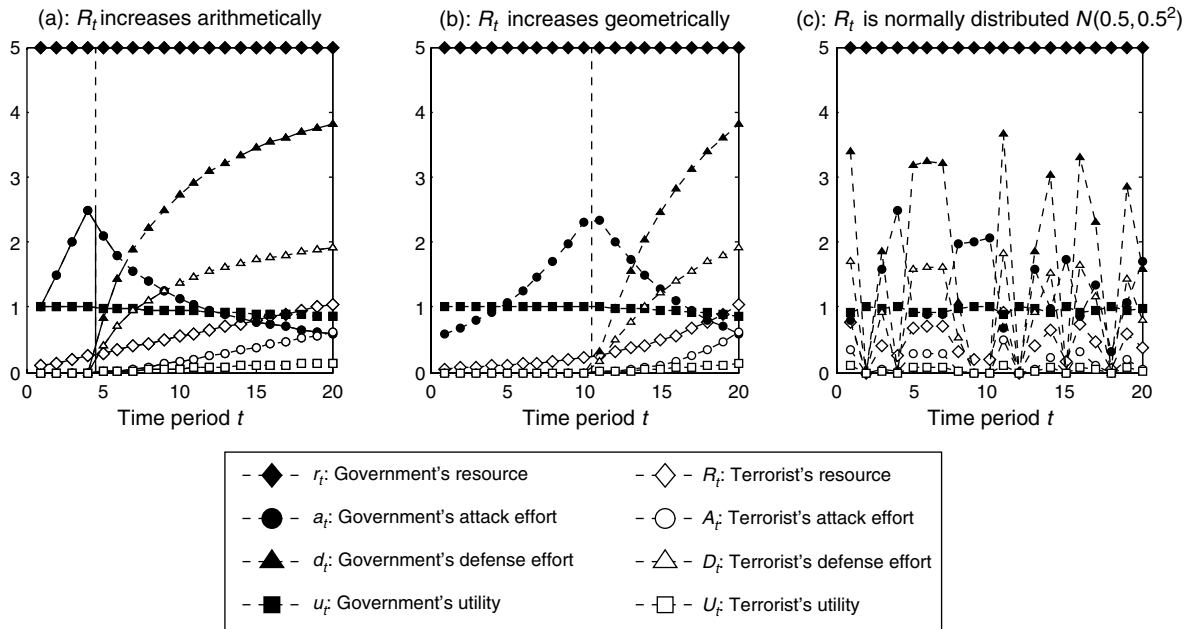
When drawing r_t from the normal distribution, Figure 3(c) shows that a_t and d_t fluctuate up and down when r_t fluctuates up and down, respectively. The low values of r_t in periods 5, 11, 13, and 14 cause low values of a_t , d_t , and u_t (Case 2). The large r_t values in periods $t = 9, 10, 16$, and 20 cause Case 1, which benefits the government with $u_t = 1$, $d_t = 0$, and $a_t = 5$, whereas the deterred terrorist suffers $U_t = 0$. Case 2 arises in the remaining periods. The low r_t values $r_t = 0$ in periods $t = 13$ and $t = 14$ cause the government to suffer $u_t = 0$ while the terrorist enjoys $U_t = 1$.

6.2. Modeling the Terrorist's Resources R_t

This section keeps the government's resources at its baseline $r_t = 5$ and assumes that the terrorist's

resources R_t increases from a low level. First, the terrorist may be newly formed and may acquire increasing funding from various sources if its objective gains support. Second, the terrorist may be well established, may have depleted its resources in earlier attacks, and may, if earlier attacks were successful, more easily acquire further resources. For example, attacks such as 9/11-type attacks may generate sufficient momentum and willing investors among supporters of the attack to furnish a high R_t in subsequent years. Third, the terrorist may experience hatred (Glaeser 2005) arising from a variety of sources, which may recruit volunteers and funding, which may get directed at governments. We investigate the following three functional forms of increment where R_t changes over 20 periods: arithmetic

Figure 4 Equilibrium Behaviors as a Function of Time Period When R_t Is Dynamic



$R_t = 0.05 + 0.05t$ and geometric $R_t = 0.05 \times 1.1644^t$, where r_t increases from 0.05 to 1.05, and stochastic R_t , which is randomly drawn from a normal distribution with mean 0.5 and standard deviation 0.5 (when negative, we let R_t be zero) for $t = 1, \dots, 20$. Figure 4 shows the equilibrium dynamics of these three functional forms. With arithmetically increasing R_t , the terrorist is deterred by the government's attack through period 4, and the government does not defend (Case 1). From period 5, both players both defend and attack (Case 2). With geometrically increasing R_t , the terrorist is deterred through period 10 (Case 1), because it then takes much more time than with arithmetically increasing R_t to cause $R_t > G_t r_t / g_t$ needed to avoid deterrence. From periods 11 to 20 (Case 2), the terrorist's attack effort increases convexly. When R_t is drawn stochastically, A_t and D_t fluctuate up and down when R_t fluctuates up and down, respectively. Larger values of A_t and D_t cause lower government utility u_t .

6.3. Modeling Changes in Both r_t and R_t

This section assumes that R_t is randomly drawn from a normal distribution with mean 0.5 and standard deviation 0.5 (when negative, we let R_t be zero), and that r_t is randomly drawn from a normal distribution

with mean 5 and standard deviation 5 (when negative, we let r_t be zero) for $t = 1, \dots, 20$. Using the same baseline parameter values as above, Figure 5 shows the equilibrium behaviors as functions of time period. The terrorist is deterred (Case 1 in Table 1) earning zero utility in periods $t = 6, 7, 11, 13, 17$, and 20, where $R_t / r_t < G_t / g_t = 0.05$. Case 2 arises in the remaining periods. The low r_t values $r_t = 0$ in periods $t = 1, 8, 16$, and 19 cause the government to suffer $u_t = 0$ while the terrorist enjoys $U_t = 1$. Period 4 illustrates how a low r_t and high R_t cause low government utility and high terrorist utility.

7. The T-Period Game with the Terrorist's Resource Linkages Between Periods

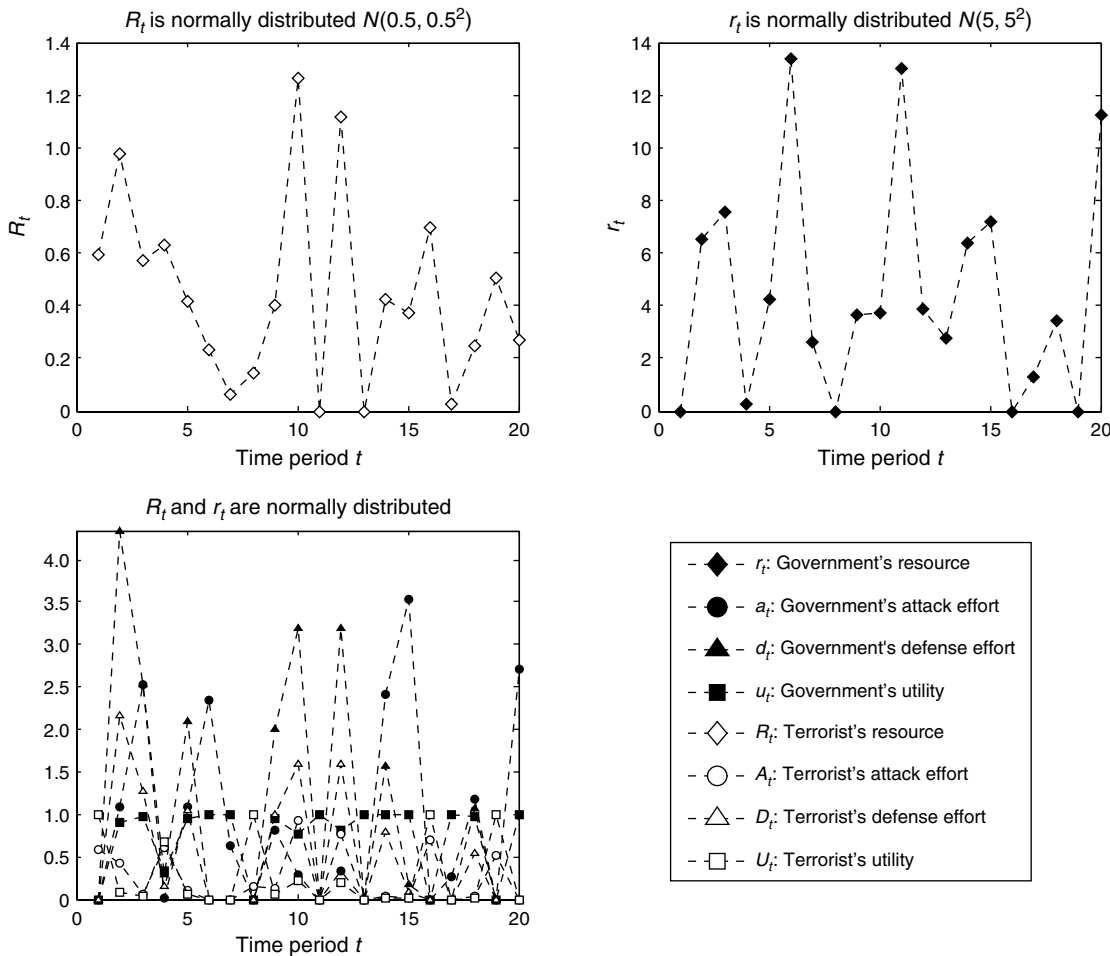
7.1. The Terrorist's Resources Depend on the Government's Attacks

To model the terrorist's resource linkage between periods, this section assumes, for all $t > 2$,

$$R_t = R_{t-1} e^{\Lambda(a_{t-1} - a_{t-2})} \implies R_t = R_2 e^{-\Lambda a_1} e^{\Lambda a_{t-1}}, \quad (11)$$

where Λ is a parameter and the implication (second part of (11)) follows straightforwardly. When

Figure 5 Equilibrium Behaviors as a Function of Time Period When Both r_t and R_t Are Normally Distributed

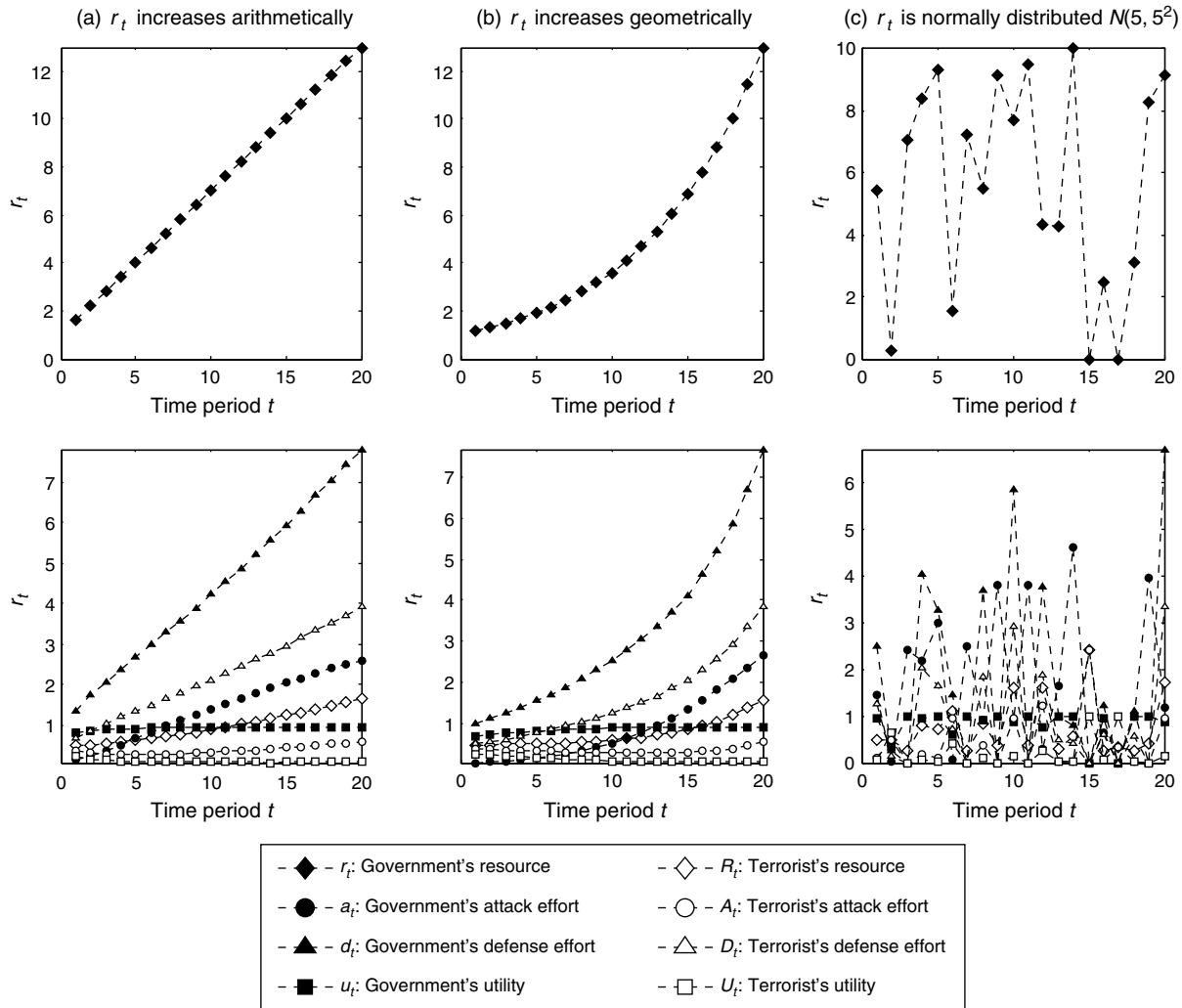


$\Lambda > 0$, (11) models how the terrorist's resources R_t in period t increase if the government launches an attack larger than in the previous period, decrease if the government launches an attack smaller than in the previous period, and remains unchanged if the government launches the same attacks in subsequent time periods. The reasoning is that the government's attack reinvigorates the terrorist because of various factors such as more easy access to funding, the emergence of hatred, etc. Conversely, when $\Lambda < 0$, (11) models how the terrorist's resources R_t in period t increase if $a_{t-1} < a_{t-2}$, decrease if $a_{t-1} > a_{t-2}$, and remains unchanged if $a_{t-1} = a_{t-2}$. The reasoning is that the government's attack successfully degrades the terrorist so that the terrorist's resources decrease.

Figure 6 shows the equilibrium behavior when $\Lambda = 0.5$, using the same parameter values and three dynamics for r_t used in §6.1. With arithmetically and geometrically increasing r_t in Figure 6(a) and 6(b), the government's utility increases, whereas the terrorist's utility decreases. Although the terrorist's utility is low, it is never deterred because $R_t \geq 0.05r_t$ (Case 2). When drawing r_t from the normal distribution, Case 1 occurs when $t = 3, 7, 9, 11, 19$, and Case 2 occurs otherwise.

Figure 7 shows the equilibrium behavior when $\Lambda = -0.5$, using the same parameter values and three dynamics for r_t used in §6.1. In Figure 7(a), r_t increases arithmetically. Case 2 occurs when $t \leq 6$. The government's attack a_t increases, d_t is inverse U shaped, and the terrorist's utility decreases as the

Figure 6 Equilibrium Behaviors as Functions of Time Period When r_t Is Dynamic and the Terrorist's Resources Are Linked Between Periods for $\Lambda = 0.5$



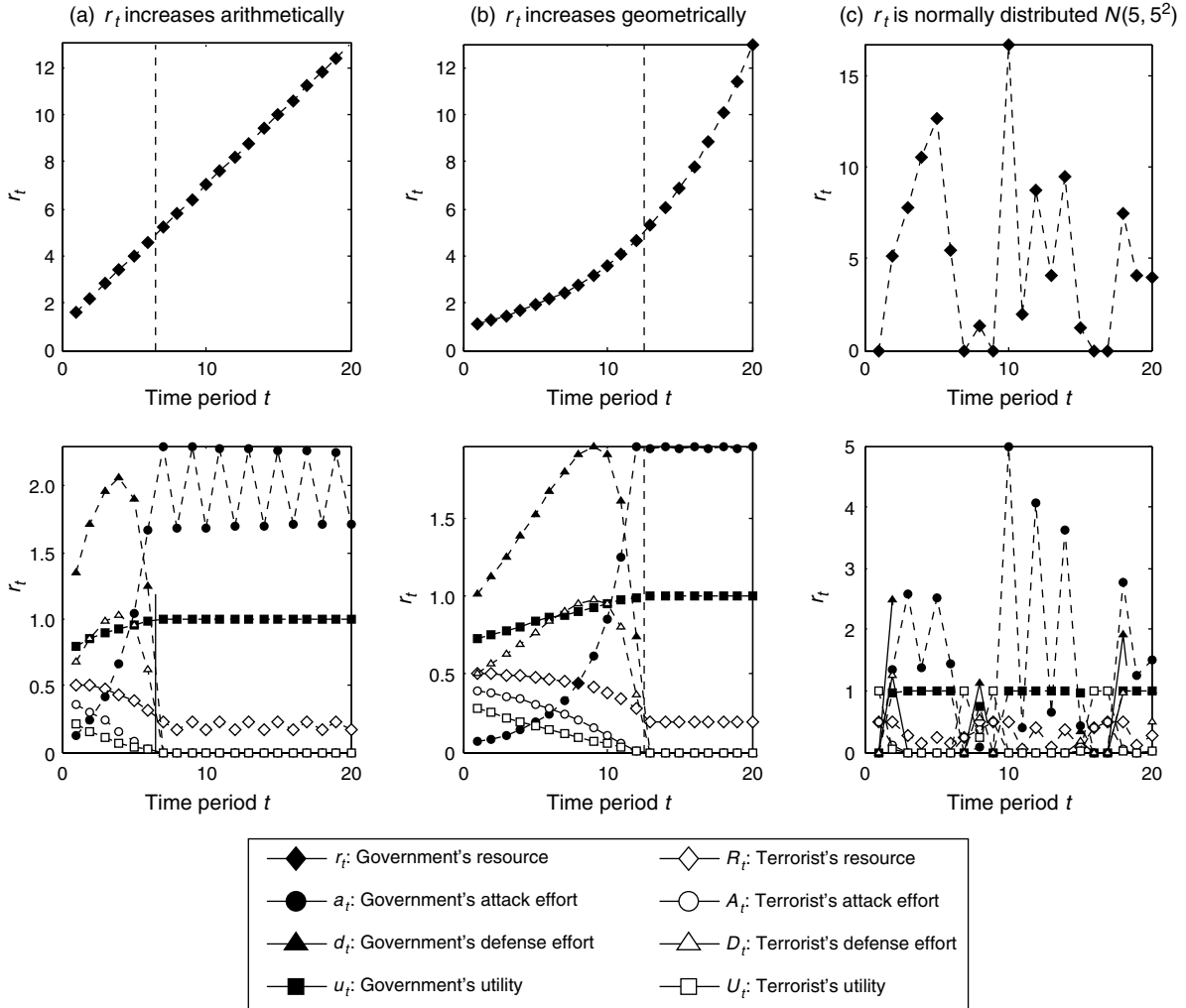
government becomes more resourceful. Case 1 occurs when $t \geq 7$ with oscillatory a_t and R_t according to (11). As a_t increases from $t = 6$ to $t = 7$ to deter the terrorist, R_t decreases from $t = 7$ to $t = 8$ according to (11). This allows the government to decrease a_t when $t = 8$, which is sufficient to deter the terrorist. This in turn causes R_t to increase when $t = 9$, causing oscillations. In Figure 7(b) r_t increases geometrically. Case 2 occurs when $t \leq 12$. Case 1 occurs when $t \geq 13$. Oscillations of a_t and R_t occur also here, but are much smaller. In Figure 7(c), r_t is drawn from the normal distribution. Case 1 occurs when

$t = 3, 4, 5, 6, 10, 11, 12, 13, 14$. Case 2 occurs otherwise. When $r_t = 0$ for $t = 1, 7, 9, 16, 17$, the terrorist enjoys maximal utility $U_t = 1$.

7.2. The Terrorist's Resources Depend on the Terrorist's Attacks

After a major attack by the terrorist, the terrorist may need time to recover. Its available resource in the next period may thus be lower than in the current period. Conversely, if the terrorist chooses no attack or a small attack in the current period, its available resource in the next period may be higher than in the current

Figure 7 Equilibrium Behaviors as a Function of Time Period When r_t Is Dynamic and the Terrorist's Resources Are Linked Between Periods for $\Lambda = -0.5$



period. We model that resource linkage as

$$R_t = R_{t-1}e^{-\Gamma(A_{t-1}-A_{t-2})} \implies R_t = R_2e^{\Gamma A_1}e^{-\Gamma A_{t-1}}, \quad (12)$$

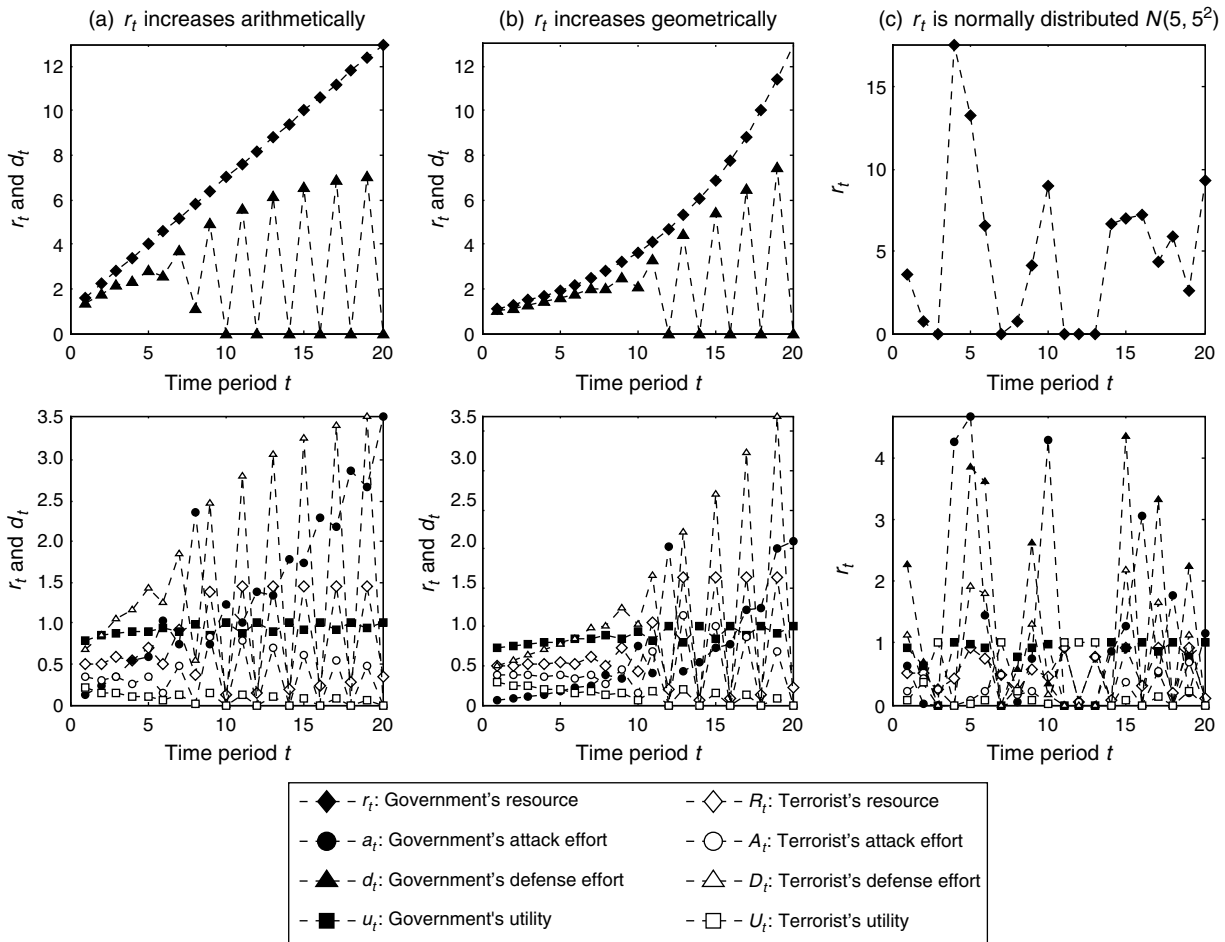
which accounts for the terrorist's production and funding technology, where $\Gamma > 0$ is a parameter.

The terrorist's resources R_t in period t decrease if the terrorist launches an attack larger than in the previous period, increase if the terrorist launches an attack smaller than in the previous period, and remain unchanged if the terrorist launches the same attack in subsequent time periods.

Setting $\Gamma = 3$, and using the same parameter values and three dynamics for r_t used in §6.1, Figure 8

shows the equilibrium behavior. When r_t is small, the terrorist's resources R_t and attack effort A_t fluctuate slightly around 0.5 and 0.3, respectively. When r_t becomes large, A_t decreases according to $A_t = R_t(1 - G_t r_t / (g_t R_t))^2$ in Table 1, which causes R_t to increase according to (12), which leads A_t to increase and thus causes a cycle, as observed in both Figure 8(a) and 8(b). We have Case 1 when $t = 10, 12, 14, 16, 18, 20$ in Figure 8(a), when $t = 12, 14, 16, 18, 20$ in Figure 8(b), and when $t = 4, 14, 16, 18, 20$ in Figure 8(c). We have Case 2 otherwise. Furthermore, in Figure 8(c) we have $r_t = 0$ for $t = 3, 7, 11, 12, 13$, which cause $U_t = 1$.

Figure 8 Equilibrium Behaviors as a Function of Time Period When r_t Is Dynamic and the Terrorist's Resources Are Linked Between Periods for $\Gamma = 3$



8. Suggestions for How to Validate the Model

This paper has developed a model with intuitive reasoning, has solved the model with game-theoretic tools, has presented two cases for the size of the terrorist's resources, has accounted for changes through time, and has illustrated the solution for various parameter values. Future research should support the model empirically and validate the results. Let us, somewhat ambitiously, sketch some directions for such research. Parameters should be estimated and tuned to match real-world cases. Cases that have occurred are a natural starting point. Proceeding through the parameters and variables in the nomenclature list, the government's resources, asset valuation, and unit costs of defense and attack are

determined from public records, interviews, and estimation techniques. The asset valuation can be estimated by letting people and elected officials rank the value of multiple assets against each other. The terrorist's resource, asset valuation, and unit costs of defense and attack could also be estimated, either in similar manners, or by applying covert techniques and espionage, exploring statements and interviewing defectors and sympathizers of potential terrorists, and applying expert judgments. Methods common in decision theory may also be used to estimate the parameters experimentally.

The contest intensity m_t for asset damage can be estimated by assessing how efforts by the government and terrorist have impact. If efforts have modest impact in an egalitarian contest, the intensity is low. If efforts have substantial impact (e.g., "winner

takes all”), the intensity is high. History can also be used to estimate m_i . An example of low intensity where no side easily gets a significant upper hand is the time prior to the emergence of cannons and modern fortifications in the 15th century. Another example is entrenchment combined with the machine gun, in multiple dispersed locations, in World War I (Hirshleifer 1995, pp. 32–33). “But in World War II the combination of airplanes, tanks, and mechanized infantry allowed the offense to concentrate firepower more rapidly than the defense, thus intensifying the effect of force superiority” (Hirshleifer 1995, p. 32).

Once a collection of real-world cases have been compiled with estimated values of parameters and variables through time, the next step is to analyze whether these cases comply with the solution predicted in this paper. First, we determine whether a low level of terrorist’s resources causes the government to attack them for deterrence purposes, whether intermediate terrorist’s resources cause the terrorist to attack and defend, and whether a high level of terrorist’s resources causes both players to defend and attack. Second, we estimate empirically how the parameters, especially the government’s resources, change after an attack and determine whether or for how long the terrorist gets deterred, and whether the deterrence period matches the deterrence period predicted in this paper. Third, we estimate empirically how the terrorist’s resources change after an attack, whether an arithmetic or geometric increase is descriptive, and whether the impact on the players’ strategies matches the solution predicted in this paper. This approach assumes that reactions of both players in the past were myopically optimal. This assumption may be an approximation, which we expect can be improved by compiling data from more cases.

The solutions are discussed with policy administrators. Once some agreement has been reached on parameter values, one may proceed with cases that may occur with varying degrees of likelihood and prescribe optimal policies for each case. Prior to the 9/11 attack, the notion of flying airplanes into buildings had been contemplated by various professionals, but had been assessed as too speculative for serious consideration. There is a need to proceed through both likely and unlikely scenarios, and

assess the optimal government response for each scenario. That requires some analysis of the players’ reactions after a hit by the terrorist, which has not been included in the model, and illustrates some challenges for future research.

9. Conclusion

This paper assumes that a government allocates resources between defending against a terrorist attack and attacking a terrorist’s resources. We also assume that a terrorist allocates resources between attacking a government’s asset and defending its own resources. The government builds the defense of its infrastructure over time. The terrorist takes this defense as given when choosing its attack strategy at each time period. In each period, we analyze a two-stage game where the government moves in the first stage, and the terrorist moves in the second stage.

There are two cases of solutions classified by the terrorist’s resources. First, when the terrorist’s resources are low, the government attacks the terrorist’s resources sufficiently to deter the terrorist from attacking and does not defend. This interesting result suggests that governments may handle the many minor terrorist threats around the world with proactive attacks against their modest resources, rather than designing a passive defense to passively await these to grow large and require more substantial resources. Second, when the terrorist’s resources are high, both the government and terrorist defend and attack.

Repeating the two-stage game, we first consider two periods and show how the two cases arise dependent on changes in the terrorist’s resources. Second, we allow the government to recover after an attack, caused by an increase in its resources driven by easier access to funding, with subsequent gradual decrease to its initial level. We show how effectively the government deters attacks during this period of higher resource availability, and when the terrorist is no longer deterred from further attacks. This transition to renewed terrorist attacks occurs when the terrorist’s resources are again larger than the government’s resources multiplied with the ratio of the terrorist’s unit defense cost and the government’s unit attack cost. The intuition is that when the terrorist is resourceful and enjoys a low unit defense cost, while the government is not resourceful and has a high

unit attack cost, then terrorist attacks resume. Third, we consider the impact of increasing the terrorist's resources arithmetically and geometrically from a low level, because the terrorist is either unestablished and gaining increased funding, or is established and gaining easier access to funding because of the success of earlier attacks. Fourth, we allow the government's and the terrorist's resources to be determined randomly in each time period. Fifth, we allow the government's resources in one period to depend on the terrorist's attacks in earlier time periods for the three events for which the terrorist's resources are drawn from a normal distribution or change arithmetically or geometrically.

The model in this paper is intended to be useful for legislators, the military, and government leaders. Based on some plausible assumptions, the strategic nature of the interaction between a government and a terrorist is captured, causing policy recommendations for when, and to what extent, the government should defend itself versus attacking the terrorist and how the terrorist responds by either being deterred, attacking, or defending its resources. Striking a balance between realism and parsimony, the model accounts for the players' resources, asset valuations, the contest intensity for asset damage, and the unit costs of defense and attack. Future research may account for alternative factors. We have also assumed boundedly rational players, where the time between periods is sufficiently longer than the time between stages so that each two-stage game can be solved with backward induction for each period. Future research may search for alternative ways of modeling the challenges of these interactions through time.

In this paper we provide two basic dynamics on the linkages for the terrorist's resources. Future research could investigate more scenarios such as a combination of the dynamics studied in §§7.1 and 7.2, or dynamics depending on the government's and/or terrorist's defense levels.

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conclusions or recommendations in this document are those of the authors and do not necessarily reflect views of the U.S. Department of Homeland Security. Author names are listed alphabetically by last name. Jun Zhuang is the corresponding author.

Appendix A. Solving the Model

We solve the game in period t with backward induction, starting with the second stage. For any given government's attack a_t , maximizing the terrorist's utility $U_t(a_t, D_t)$ specified in (6) gives the terrorist's best-response function $D_t(a_t) = \arg \max_{D_t \geq 0} U_t(a_t, D_t)$. In particular, the first derivative for $U_t(a_t, D_t)$ specified in (6) is

$$\begin{aligned} & \frac{\partial}{\partial D_t} U_t(a_t, D_t) \\ &= \frac{\left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{m_t} m_t \left[\frac{R_t}{D_t + a_t} - G_t - \left(\frac{D_t R_t}{D_t + a_t} \right)^2 \right]}{D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \left\{ \left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{m_t} + (r_t - g_t a_t)^{m_t} \right\}} \\ & \quad - \frac{\left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{2m_t} m_t \left(\frac{R_t}{D_t + a_t} - G_t - \frac{D_t R_t}{(D_t + a_t)^2} \right)}{\left\{ \left[D_t \left(\frac{R_t}{D_t + a_t} - G_t \right) \right]^{m_t} + (r_t - g_t a_t)^{m_t} \right\}^2 D_t \left(\frac{R_t}{D_t + a_t} - G_t \right)} \end{aligned} \quad (A1)$$

Equating (A1) with zero, we solve for D_t as a function of a_t , which gives $D_t = \sqrt{R_t a_t / G_t} - a_t$, which is positive if and only if $a_t \leq R_t / G_t$. If $a_t > R_t / G_t$, we have a boundary condition where $D_t = 0$. We thus express the terrorist's best-response function as follows:

$$\begin{aligned} D_t(a_t) &= \arg \max_{D_t \geq 0} U_t(a_t, D_t) \\ &= \begin{cases} 0 & \text{if } a_t \geq R_t / G_t, \\ \sqrt{R_t a_t / G_t} - a_t & \text{if } a_t \leq R_t / G_t. \end{cases} \end{aligned} \quad (A2)$$

Note, that both the terrorist and government's utility functions are concave in their respective decision variables, and as a result the second-order conditions are satisfied, and the first-order conditions ensure a maximum. From (A2) we see that the optimal terrorist's defense level D_t increases in the amount of available resources R_t and decreases in the terrorist's unit defense cost G_t (as long as $G_t < R_t / a_t$).

Inserting the terrorist's best response (A2) into (6) yields the government's first-stage utility:

$$u_t(a_t) = \begin{cases} 1 & \text{if } a_t \geq R_t / G_t, \\ \frac{(r_t - g_t a_t)^{m_t}}{[\sqrt{R_t} - \sqrt{G_t a_t}]^{2m_t} + (r_t - g_t a_t)^{m_t}} & \text{if } a_t \leq R_t / G_t. \end{cases} \quad (A3)$$

We first determine the interior solution. The government's first-order condition in the first stage implies

$$\frac{\partial u_t(a_t)}{\partial a_t} = 0 \implies a_t = \frac{G_t r_t^2}{g_t^2 R_t} \implies d_t = r_t - \frac{G_t r_t}{g_t R_t}, \quad (\text{A4})$$

which is inserted into (A2) to yield

$$\begin{aligned} D_t &= \frac{r_t}{g_t} \left(1 - \frac{G_t r_t}{g_t R_t} \right), \\ A_t &= \frac{(G_t r_t - g_t R_t)^2}{g_t^2 R_t} = R_t \left(1 - \frac{G_t r_t}{g_t R_t} \right)^2. \end{aligned} \quad (\text{A5})$$

From (A3), when $a_t \geq R_t/G_t$, it follows that the second-order derivative is $\partial^2 u_t(a_t)/\partial a_t^2 = 0$. When $a_t \leq R_t/G_t$ it does not seem possible to prove analytically that the government's second-order condition

$$\begin{aligned} \frac{\partial^2 u_t(a_t)}{\partial a_t^2} &= \frac{(r_t - g_t a_t)^{m_t} g_t^2 (m_t^2 - m_t)}{(r_t - g_t a_t)^2 [(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} + (r_t - g_t a_t)^{m_t}]} \\ &+ \frac{2(r_t - g_t a_t)^{m_t} m_t g_t}{(r_t - g_t a_t) [(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} + (r_t - g_t a_t)^{m_t}]^2} \\ &\cdot \left[-\frac{(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} m_t G_t}{\sqrt{G_t a_t} (\sqrt{R_t} - \sqrt{G_t a_t})} - \frac{(r_t - g_t a_t)^{m_t} m_t g_t}{r_t - g_t a_t} \right] \\ &+ \frac{2(r_t - g_t a_t)^{m_t} m_t g_t}{[(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} + (r_t - g_t a_t)^{m_t}]^3} \\ &\cdot \left[-\frac{(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} m_t G_t}{\sqrt{G_t a_t} (\sqrt{R_t} - \sqrt{G_t a_t})} - \frac{(r_t - g_t a_t)^{m_t} m_t g_t}{r_t - g_t a_t} \right] \\ &- \frac{(r_t - g_t a_t)^{m_t}}{[(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} + (r_t - g_t a_t)^{m_t}]^2} \\ &\cdot \left\{ \frac{(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} m_t^2 G_t}{a_t [\sqrt{R_t} - \sqrt{G_t a_t}]^2} + \frac{1}{2} \frac{(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} m_t G_t^2}{(G_t a_t)^{3/2} (\sqrt{R_t} - \sqrt{G_t a_t})} \right. \\ &\left. - \frac{1}{2} \frac{(\sqrt{R_t} - \sqrt{G_t a_t})^{2m_t} m_t G_t}{a_t (\sqrt{R_t} - \sqrt{G_t a_t})^2} + \frac{(r_t - g_t a_t)^{m_t} g_t^2 (m_t^2 - m_t)}{(r_t - g_t a_t)^2} \right\} \\ &\leq 0 \end{aligned}$$

is satisfied, but our numerical tests suggest that it is satisfied. Both players' choice variables are positive when

$$R_t \geq \frac{G_t r_t}{g_t}, \quad (\text{A6})$$

which means that an interior solution of positive government's attack and defense and positive terrorist's attack and defense is guaranteed when the terrorist is sufficiently resourceful. When (A6) is not satisfied, the government refrains from defending, $d_t = 0$, and focuses on attack, $a_t = r_t/g_t$, as determined by (1). Note here we have $a_t = r_t/g_t > R_t/G_t$, and therefore, using (A2), we have $D_t = 0$, and applying (3) we have $A_t = 0$.

We now consider the noninterior solutions (boundary conditions). The players have four decision variables, but d_t follows from (1) when a_t is given, and A_t follows from (3) when D_t is given. Hence, we focus on the noninterior solutions generated by a_t and D_t .

The variable a_t can take two extreme values, $a_t = 0$ and $a_t = R_t/G_t$. We first consider $a_t = 0$ chosen by the government in the first stage. Inserting $a_t = 0$ into (6) and (A2) to determine the terrorist's best response in the second stage gives $D_t = 0$. Using (1) gives $d_t = r_t$, using (3) gives $A_t = R_t$, and using (4) and (5) gives $P_t = U_t = [1 + (R_t/r_t)^{-m_t}]^{-1}$ and $u_t = [1 + (R_t/r_t)^{m_t}]^{-1}$. The government prefers the higher utilities in Table 1 for both Cases 1 and 2 and thus, will not choose $a_t = 0$ in equilibrium. Second, we consider $a_t = R_t/G_t$, which cannot exceed $a_t = r_t/g_t$, meaning that only Case 1 has to be considered. Inserting $a_t = R_t/G_t$ into (A2) gives $D_t = 0$. Using (1) gives $d_t = r_t - g_t R_t/G_t$, (3) gives $A_t = 0$, and (4) and (5) give $P_t = U_t = 0$ and $u_t = 1$, respectively, where the government is already guaranteed maximum utility 1 in Table 1. Third, we consider the event that the government chooses the interior solution for a_t and that the terrorist responds with the noninterior solution $D_t = 0$. The terrorist then gets 0, which is inferior to the utility from interior solution, which is 0 for Case 1 and $[1 + (R_t/r_t - G_t/g_t)^{-m_t}]^{-1}$ for Case 2. Fourth, we consider the event that the government chooses the interior solution for a_t and that the terrorist responds with the noninterior solution $D_t = 0$ as determined by solving (3) when $A_t = 0$. The terrorist then gets 0, which is inferior to the utility from interior solution, which is 0 for Case 1 and $[1 + (R_t/r_t - G_t/g_t)^{-m_t}]^{-1}$ for Case 2. Hence, the four noninterior solutions do not apply, and the equilibrium solution in Table 1 is exhaustive.

Appendix B. The T-Period Game with the Government's Resource Linkages Between Periods

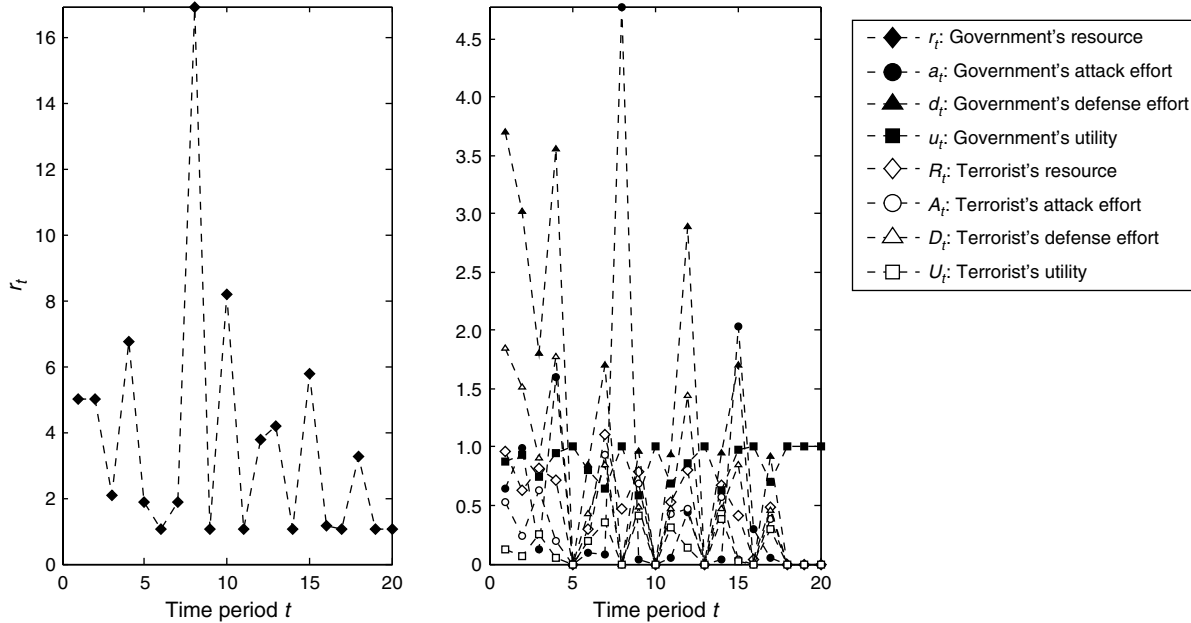
B.1. The Government's Resources Depend on the Terrorist's Attacks

To model the government's resource linkage between periods, this section assumes, for all $t > 2$,

$$r_t = r_{t-1} e^{\lambda(A_{t-1} - A_{t-2})} \implies r_t = r_2 e^{-\lambda A_1} e^{\lambda A_{t-1}}, \quad (\text{B1})$$

where $\lambda \geq 0$ is a parameter, and the implication (second part of (B1)) follows straightforwardly. Equation (B1) models how the government's resources r_t in period t increase if the terrorist launches an attack larger than in the previous period, decrease if the terrorist launches an attack smaller than in the previous period, and remain unchanged if the terrorist launches the same attacks in subsequent time periods. The resiliency of the U.S. economy through 9/11 and Hurricane Katrina suggests that λ should not be too large, where $\lambda = 0$ means $r_t = r_{t-1}$.

Figure B.1 Equilibrium Behaviors as a Function of Time Period When R_t Is Randomly Drawn from a Normal Distribution and the Government's Resources Are Linked Between Periods for $\lambda = 3$



When R_t increases arithmetically or geometrically, the terrorist stays inactive ($t < 5$ in Figure 4(a) and $t < 11$ in Figure 4(b)), and thus the government's resources stay constant according to (B1). When R_t increases above 0.25, the terrorist starts attacking, which increases the government's resources.

We use the same parameter values and three dynamics for R_t used in §6.2. Figure B.1 shows the equilibrium behavior when $\lambda = 3$ and R_t is randomly drawn, as in §6.2. The large R_t drawn in period 7 causes a large A_t in period 7, which, according to (B1), causes a large r_t and a_t in period 8, which deters the terrorist. Case 1 arises when $t = 5, 8, 10, 13, 16, 18, 19$, or 20, and Case 2 otherwise. Equation (B1) states that if no attack occurs in period $t - 1$, that is, $A_{t-1} = 0$, which corresponds to Case 1, the government's resources in the subsequent period are $r_t = r_{t-1}e^{-\lambda A_{t-1}} = 5e^{-3(0.5)} = 1.1$, shown in periods $t = 6, 9, 11, 14, 17, 19$, and 20 in Figure B.1.

Figure B.2 shows the equilibrium behavior when $\lambda = 9$ and R_t increases arithmetically and geometrically as in §6.2. When R_t increases arithmetically, starting in period $t = 5$, R_t becomes sufficiently large to allow the terrorist to attack ($A_t > 0$), which leads the government's resources to grow from 5.0 starting in period $t = 6$. As R_t increases, A_t increases, causing r_t to increase. A large r_t in a given period causes a low A_t in the same period, causing r_{t+1} to decrease and thereafter to fluctuate up and down in a cycle of increasing and unbounded r_t . More specifically, the r_t values for the periods 5–20 are 5.0, 5.4, 5.9, 6.4, 7.0, 7.5, 8.3, 8.3, 10.9, 6.8, 37.2, 5.0, 226.2, 5.0, 518.8, and 5.0,

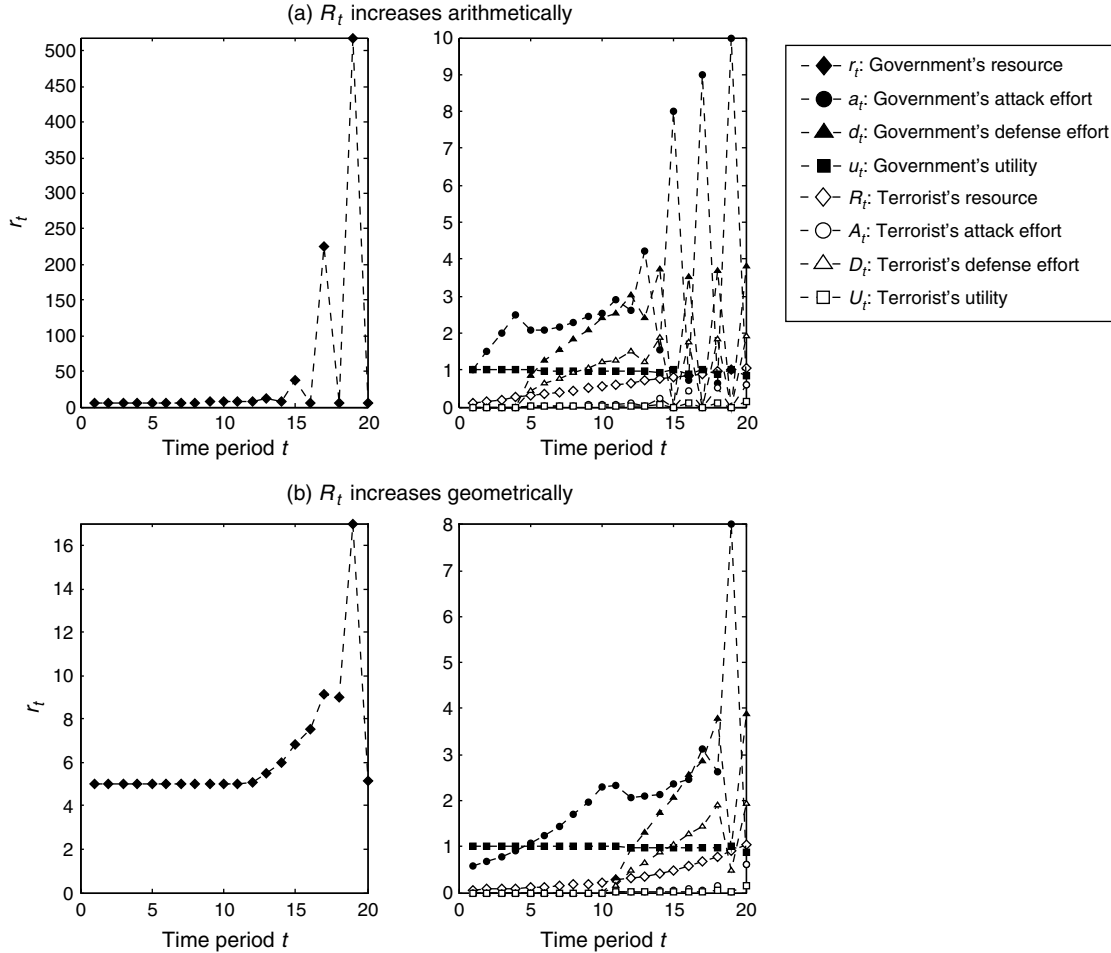
respectively. When R_t increases geometrically, starting in period $t = 11$, R_t becomes sufficiently large to allow the terrorist to attack, which leads the government's resources to grow from 5.0 starting in period $t = 12$. The r_t values increase to 9.1 in period 17, decrease to 9.0, increase to 17, and finally decrease to 5.1. For both the arithmetically and geometrically increasing R_t , the large $\lambda = 9$ implies that the increases of r_t are substantial, which causes A_t to decrease according to Table 1. This in turn causes r_t in (B1) to decrease. This explains how r_t moves up and down in this fully deterministic scenario. This result stands in contrast to Figure 3 in §6.1, where r_t is exogenously specified to increase, which causes A_t to decrease, but §6.1 models no feedback from this decrease of A_t to the subsequent decrease of r_t .

We tested for large values of the time periods $T = 200$ and $T = 2,000$ and did not find a similar cycle for r_t when R_t increases geometrically. This is because when r_t increases geometrically, R_t grows so fast that the government has no chance to recover. Furthermore, we have found no additional qualitative insights by studying linkage with a longer memory. In particular, we tested the case $r_t = r_{t-1}e^{\lambda(A_{t-1} - 0.5A_{t-2} - 0.5A_{t-3})}$ for $t = 20, 200$, and 2000 and did not observe cycles when R_t increases geometrically.

B.2. Modeling the Government's Resource r_t Bouncing Back After an Attack

Assume that $R_s \geq G_s r_s / g_s$ so that an attack occurs in period s . Assume for simplicity that all other parameters remain fixed for all $t \geq s$ while the government's resource r_t

Figure B.2 Equilibrium Behaviors as a Function of Time Period When R_t Increases Arithmetically and Geometrically and the Government's Resources Are Linked Between Periods for $\lambda = 9$



changes. This is because after an attack in period s the government more easily acquires funding from various sources. With a significantly higher r_t , the condition $R_t \geq G_t r_t / g_t$ is no longer satisfied, which deters the terrorist. Assume that r_t follows the form for $t \geq s$:

$$r_t = \begin{cases} r_s & \text{if } t = s, \\ r_s + [r_{\max} - r_s]e^{-\Phi(t-s-1)} & \text{if } s < t \leq t^*, \end{cases} \quad (\text{B2})$$

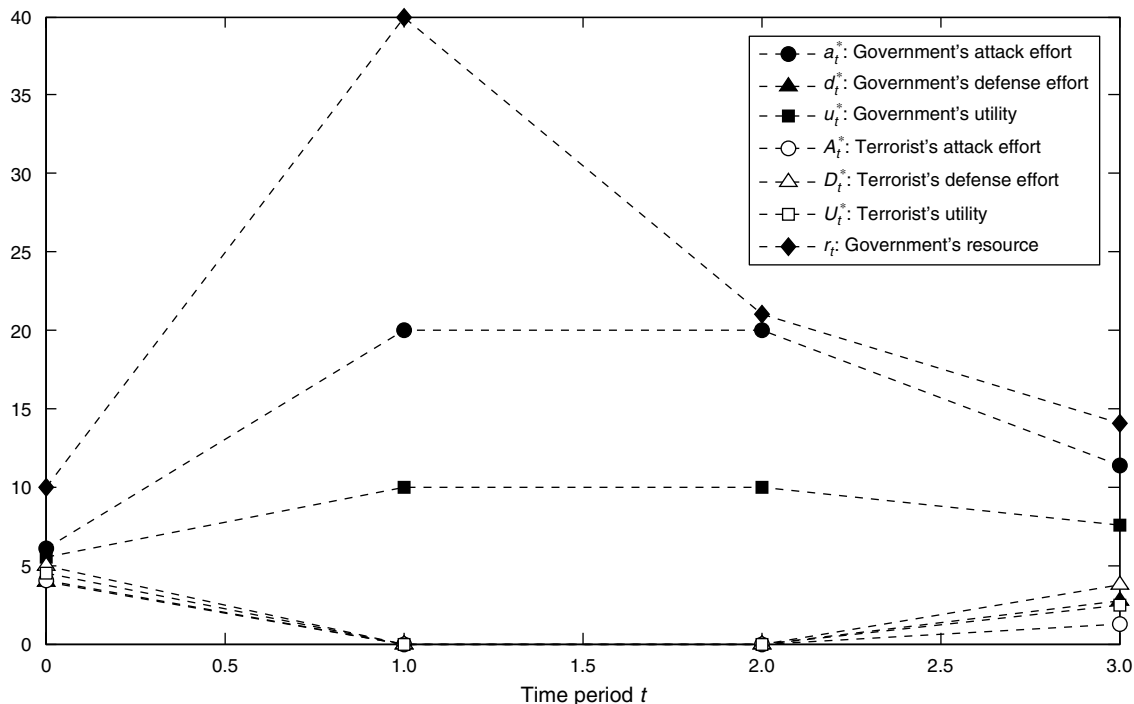
where r_{\max} is the maximum value that r_t acquires in period $s + 1$ after a terrorist attack A_s in period s , and $\Phi > 0$ regulates how quickly r_t bounces concavely back and decreases to its original level r_s because of defense reinforcement. Equation (B2) states that $r_t = r_s$ in period s , that $r_t = r_{\max}$ in period $s + 1$, and thereafter decreases. As discussed below (1), r_t continues to be exogenously given in each time period. Modeling the political processes after a terrorist attack is challenging and left for future research. Because

$\lim_{t \rightarrow \infty} r_t = r_s$ and $\partial r_t / \partial t < 0$, a second attack eventually occurs in period t^* determined by the smallest t such that $R_t \geq G_t r_t / g_t$ (see Table 1), which implies

$$t^* = \left\lceil s + 1 + \frac{1}{\Phi} \ln \left(\frac{r_{\max} - r_s}{R_t g_t / G_t - r_s} \right) \right\rceil, \quad (\text{B3})$$

where $\lceil z \rceil$ is the least integer that is not less than z . When $F = 0$, $r_t = r_{\max}(A_s)$ for all $t > s$, and the terrorist is always deterred. When $F = \infty$, $r_t = r_s$ for all $t > s$, and the attacker attacks in each subsequent period. Using the same parameter values as in §5, aside from r_t , Figure B.3 shows the six equilibrium values u_t , U_t , a_t , d_t , A_t , D_t , and r_t as functions of time t when $s = 0$, $r_{\max} = 40$, and $F = 1$. The terrorist is deterred in periods 1 and 2 by the substantial government's attack $a_t = 20$ because $R_t < G_t r_t / g_t$ is satisfied, causing Case 1. In period $t^* = 3$, r_t drops to 14 according to (B2). Therefore, $R_t < G_t r_t / g_t$ is no longer satisfied, and we get Case 2, in which the terrorist resumes activities.

Figure B.3 Equilibrium Behavior as a Function of Time Period When r_t Is Dynamic



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