Universitetet
i Stavanger

Faculty of Science and Technology

MASTER'S THESIS

Study program/ Specialization: Offshore Technology/ Industrial Asset Management	Spring semester, 2016 Restricted access
Writer: Shyam Krishna Ramanan	……………………………………… (Writer's signature)
Faculty supervisor: Professor Jayantha Prasanna Liyanage External supervisor(s): Associate Professor Jawad Raza	
Thesis title: Developing a dynamic risk simulator for rotating equipment integrity management	
Credits (ECTS): 30	
Key words: Maintenance Strategies Maintenance Cost Optimal Maintenance Intervals Modeling Simulation	Pages: ………60………… + enclosure: …11…… Stavanger, 15th June 2016…… Date/year

Developing a Dynamic Risk Simulator for Rotating Equipment Integrity Management

Shyam Krishna Ramanan

15th June 2016

Department of
Mechanical and Structural Engineering and Materials Science
University of Stavanger

Advisors: Professor Jayantha Prasanna Liyanage, Associate Professor Jawad Raza

Preface

This thesis is submitted in partial fulfillment of the requirements for the degree of Master of Science (MSc) in Offshore Technology with the specialisation Industrial Asset Management at the University of Stavanger (UiS), Stavanger, Norway.

It has been completed in cooperation with Dr.Jawad Raza, Apply Sørco AS, Stavanger, and under the supervision of Professor Jayantha Prasanna Liyanage, University of Stavanger, during the period January to June 2016.

Abstract

The aim of this thesis is to understand the existing relevant practices, processes, Norwegian regulations for rotating equipment and develop a simulator to model and simulate the lifetime of single-unit repairable systems subjected to age-related failures towards qualitatively visualising the financial, safety and environmental risks associated with varying periodic preventive maintenance intervals for two maintenance strategies, the minimal-repair-periodic-overhaul strategy and the run-to-failure strategy. The simulator also provides quantitative estimates on the cost of maintenance, optimal maintenance interval and equipment availability for finite time horizon. The purpose of such a tool, the Dynamic Risk Simulator, is to serve as decision support to help the decision maker to value his/her options on proceeding with or delaying preventive maintenance.

The factors which affect selection of maintenance policies for equipment were identified and the equipment failure, repair and maintenance processes were mapped. This was used in the development of the underlying models and algorithm of the Dynamic Risk Simulator and the specifications for its input and output parameters were established. A proof-of-concept Dynamic Risk Simulator was built with a Graphical User Interface using the Microsoft Excel VBA language. The simulator has been partially validated with data provided by offshore operators through Apply Sørco AS. The limitation of the developed tool is that it cannot replace human judgement with regards to taking the final call on whether or not to postpone maintenance. The simulator provides quantitative and qualitative results and is reliant on the experience and insight of industry experts to take the most appropriate course of action.

Acknowledgments

I would like to thank Dr.Jawad Raza for giving me the opportunity to pursue this thesis and for taking time off his busy schedule to meet on a regular basis and for the knowledge sharing and feedback sessions he had arranged with principal and senior engineers from Apply Sørco AS.

I would like to express my gratitude and thank Professor Jayantha Prasanna Liyanage for his time, his guidance and insightful feedback which together with Dr.Jawad's guidance and feedback has helped shape the thesis. I have learnt much working under both supervisors and the journey has been intense, interesting and challenging.

I would like to thank Maurina Andriana of Apply Sørco AS and my fellow thesis student, André Vike, for the valuable support they both have provided.

I would like to acknowledge Professor Jan Terje Kvaløy for his course on probability and statistics, Professor Eirik Bjorheim Abrahamsen for his course on reliability analysis and Professor Gerhard Ersdal for his course on life extension of structures. Their course material and lectures have helped in developing this thesis.

I am indebted to my parents, relatives, cousins, family friends, friends and colleagues who have helped me in more ways than one on many an occasion and have knowingly or unknowingly left their imprints on this thesis and on my perception of life.

Shyam Krishna

Table of contents

List of Figures

List of Tables

Chapter 1

Introduction

For an offshore production facility comprising a large number of rotating equipment of many different types (pumps, compressors, motors and engines among others) of multiple OEMs where each complex equipment has multiple failure modes of differing criticality, condition monitoring could be resource intensive and cost ineffective. Preventive maintenance if done at too frequent an interval creates unnecessary downtime, underutilised spare parts, additional costs of spare parts holding and logistics for spare parts movement among other costs. If the interval is rather long, situations demanding corrective maintenance which is more expensive and leads to unplanned downtime could present themselves in-between preventive maintenance intervals. Hence, there exists a need to determine an optimal interval for preventive maintenance. Modelling failure and repair times with a reasonable level of accuracy hence becomes important towards estimating the expected number of failures in a given interval and using it to plan and manage the spare parts inventory and logistics and for scheduling preventive maintenance.

1.1 Problem description

The aim of this thesis is to understand, model and simulate the lifetime of single-unit repairable systems, with particular focus on rotating equipment - pumps, compressors etc. subjected to age-related failures and qualitatively visualise the financial, safety and environmental risks associated with varying periodic preventive maintenance intervals for two maintenance strategies, the minimal-repair-periodic-overhaul strategy and the run-to-failure strategy. The purpose of such a tool, the Dynamic Risk Simulator, is to serve as decision-support to help the decision maker to value his/her options on proceeding with or delaying the planned preventive maintenance.

1.2 Objectives

This thesis was proposed and completed in partnership with Apply Sørco AS with the objectives:

1. To understand existing relevant practices, processes, Norwegian regulations for rotating equipment.
2. To understand the basics of practical risk aspects in dynamic operational environment.
3. To map the process and identify required workflows and underlying processes.
4. To prepare specifications of dynamic risk visualization tool where required logics, input and output parameters are to be defined.
5. To develop and demonstrate a test module.

1.3 Methodology/Approach

The first two weeks were devoted to understanding the requirements of the thesis, in searching through relevant NORSOK standards and in identifying similar modelling and risk visualisation tools present in the market. The NORSOK Z-008 standard was looked into for its guidelines on maintenance of rotating equipment following which maintenance strategies and failure and repair models were studied. The problem scope during this period was refined to model age-related failure of repairable systems, in particular rotating equipment.

Over the next four and a half months, existing maintenance strategies in the industry were studied and probabilistic modelling methodologies were looked into. Simultaneously, the computer program was developed and improved upon through knowledge sharing and feedback sessions with the academic supervisors from UiS and engineers from Apply Sørco. These sessions helped with refining the problem scope and with understanding the thesis deliverables. The deliverables were finalised to be quantitative estimates on the costs for preventive and corrective maintenance and qualitative results on the safety and environment risks associated with varying preventive maintenance intervals. The Weibull distribution to model equipment failure times and the lognormal distribution to model equipment repair times were also finalised during this period. Two maintenance strategies were chosen to be adapted and implemented in the Dynamic Risk Simulator program. These were variants of the minimal-repair-periodic-overhaul maintenance strategy and the run-to-failure maintenance strategy with non-negligible repair times since these were two maintenance strategies which were often used in the industry for rotating equipment and hence would help with validating the model at a later stage. The tool was developed as a Microsoft Excel-based program with Graphical User Interface written in the Visual Basic for Applications (VBA) programing language. The simulation uses pseudo-random numbers generated based on the Mersenne Twister algorithm.

The final four weeks were spent in running preliminary validation of the program with the help of data provided from offshore operators through Apply Sørco and in compiling the thesis report.

1.4 Structure of the report

Chapter 2 of this report briefly covers a few categories of maintenance strategies which are practised in the industry and covers fundamental theoretical concepts and their governing equations which are of relevance to modelling lifetime of repairable equipment in this thesis, including modelling the failure and repair times and counting process which help with estimating the number of failures in a given time interval. Methods of estimating the Weibull α and β parameters, their underlying equations and goodness-of-fit tests for the Weibull and lognormal distributions are also included in this chapter.

Chapter 3 of this report covers the variants of the maintenance strategies developed and implemented in the Dynamic Risk Simulator program, its Graphical User Interface (GUI), the governing equations for arriving at preventive and corrective cost estimates, the theoretical optimal maintenance intervals and the limits and limitations of the computer program. The Mersenne Twister algorithm used for generation of pseudo-random numbers is also briefly discussed in this chapter.

Chapter 4 of this report includes the partial validation tests of the computer program.

Chapter 5 of this report summarises the report and provides direction on future work.

Appendix A includes Excel-VBA code fragments for some sections of the implemented maintenance strategies and includes sample calculations and Matlab code fragments for estimating the Weibull β parameter from data on equipment times to failure.

Appendix B includes the Poisson distribution table in relation to Chapter 2 equation 2.1 in page 12.

Chapter 2

Theory, General Concepts and Definitions

The main aim of this thesis is in simulating the lifetime of single-unit repairable systems and in qualitatively visualising the financial, safety and environmental risks associated with varying periodic preventive maintenance intervals for two particular maintenance strategies. Modelling the lifetime of the repairable equipment includes modelling the failure and repair times and the maintenance policy being practised. For this, there is a need to understand the underlying counting process, estimating the number of failures in a given time interval. In addition, the system restore mechanism and maintenance policy being practised - whether the system is restored to the state it was prior to failure or whether it is better than it was prior to failure among other possible system states could play a part in determining the number of failures which could occur at a later time interval. These and other related concepts are briefly discussed in this chapter.

2.1 Maintenance Strategies

In this section a few categories of maintenance strategies for single-unit systems are discussed which are of relevance to this thesis. It must be noted that there are many maintenance strategies which have been developed over the years to cater to a wide variety of constraints from budget to equipment availability to reducing down time due to preventive maintenance among other constraints. The maintenance policy most suited for a system depends not only on such constraints but also on other factors such as the load it is being subjected to, the environment in which it is present, its functionality as either a main unit or as a stand-by unit, whether run-to-failure is an option, whether compliance with governing industrial standards requires periodic inspection etc. For these reasons, the choice of maintenance policy for any given system is to be developed on a case-by-case basis following the generic strategies discussed in this section.

In this regard, NORSOK Standard Z-008 (2011) includes a general guideline on selection of maintenance strategies, included in figure 2.1

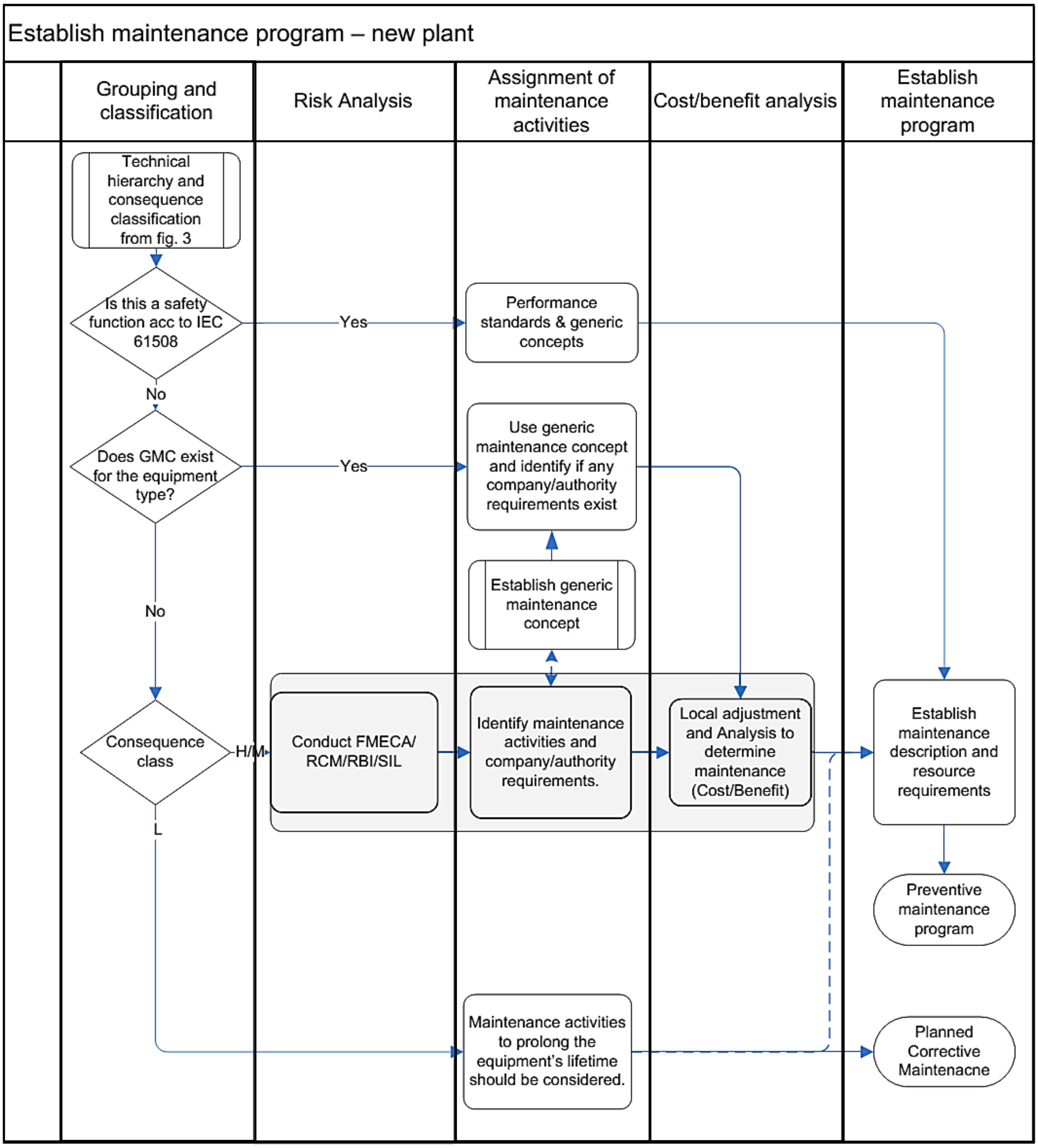

Figure 2.1: Process diagram for establishing maintenance program for new plants as included in (NORSOK Standard Z-008, 2011)

We can infer from figure 2.1 that the consequence classification of the equipment is a starting point to selection of a maintenance strategy and that for equipment classes which have a low consequence or impact on the entire system due to their failure, efforts towards extending the useful life of the equipment and planning for corrective maintenance could be a maintenance strategy. An example of this could be the run-to-failure strategy for motors. On the other hand, for equipment classes which have a medium to high consequence or impact on the entire system due to their failure, cost, resource constraints and compliance with company/regulatory standards are added factors which affect the selection of an optimal preventive maintenance policy.

Further to these, there are other factors which help in selecting an optimal maintenance policy for repairable systems. These factors are included in figure 2.2 below. The factors highlighted in figure 2.2 are those which have been considered in this thesis for modelling the minimal-repair-periodic-overhaul maintenance strategy and the run-to-failure maintenance strategy for rotating equipment:

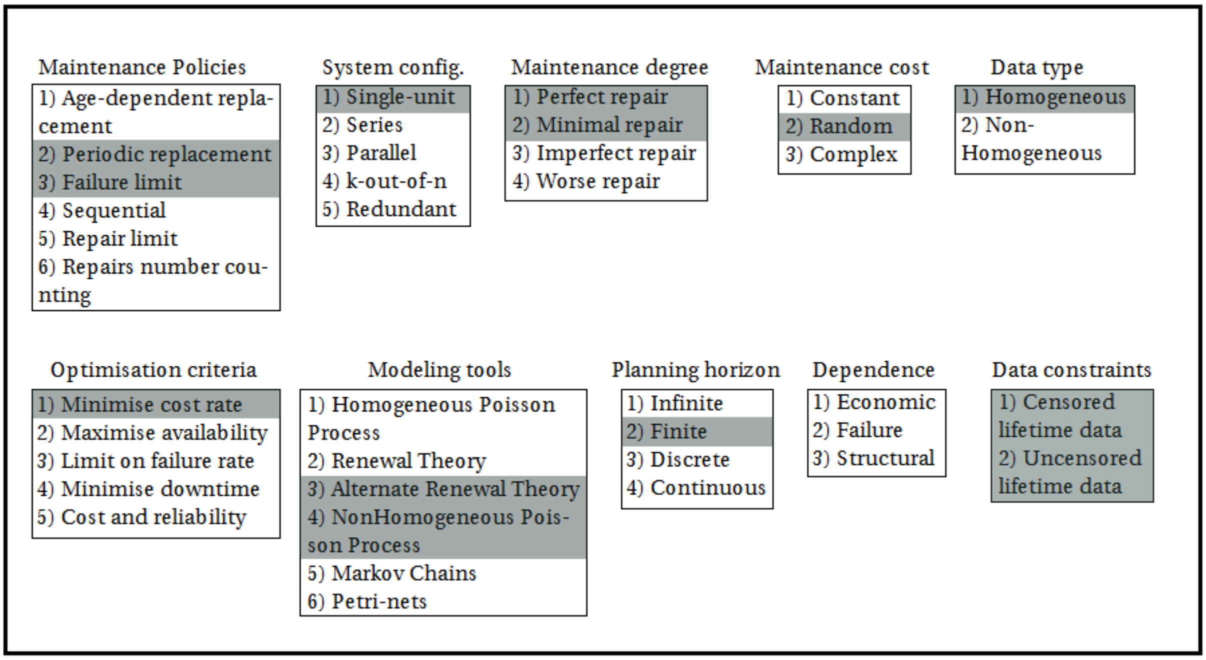

Figure 2.2: Factors which influence maintenance policies, adapted from (Wang, 2002) and modified. The factors highlighted are those which have been considered in this thesis for modelling the minimal-repair-periodic-overhaul maintenance strategy and the run-to-failure maintenance strategy for rotating equipment

Wang (2002) has grouped the most commonly practised maintenance policies for single-unit repairable systems into six categories of which the four which are of relevance to this thesis are included below.

Additional details on the policies discussed below and details on the remaining categories of maintenance policies can be found in Wang (2002). For further reading, the Handbook of Reliability Engineering by Hoang Pham (2003) has details on various combinations of maintenance strategies, for eg. minimal-repair with periodic perfect replacement, minimal-repair with periodic imperfect repair etc.

2.1.1 Periodic Preventive Maintenance policies

In a periodic preventive maintenance policy, equipment are replaced at periodic intervals which are multiples of a chosen time u_{const} or at failure whichever occurs earlier. A periodic preventive maintenance policy in which the system is repaired minimally upon failure and periodically replaced at intervals which are multiples of the constant u_{const} is termed as a minimal-repair-periodic-replacement maintenance policy (Barlow and Hunter, 1960). Under this policy, the minimal-repair portion which restores to a functional state in the earliest possible time is assumed to bring the system to the as-bad-as-old state while each periodic replacement restores the system to as-good-as-new state. This maintenance policy together with its governing equations for cost estimates and optimal interval for overhaul is further discussed in chapter 3 and is one of the two maintenance strategies implemented in the Dynamic Risk Simulator in this thesis.

2.1.2 Age-dependent Preventive Maintenance Policies

In the age replacement model, a component is replaced either at failure or at age u_{const}, whichever occurs earlier, where u_{const} is a constant (Barlow and Hunter, 1960).

If the time u is not a constant but varies, then the policy becomes one of a random age dependent maintenance policy applicable to systems for which a fixed age-based PM is not feasible/practical (Wang, 2002).

An age-dependent maintenance policy which combines the replacement at fixed intervals which are multiples of the constant u_{const} combined with a variable interval u_{var} after which the system is replaced at the first failure, whichever of the two times occurs earlier, where $u_{var} < u_{const}$ was mentioned by Tahara and Nishida (1975).

An age-dependent maintenance policy in which the system is replaced at fixed intervals u_{const} or after N failures whichever occurs earlier was mentioned by Nakagawa (1984).

Wang (2002) mentions that maintenance policies which are categorised as age-dependent policies have preventive maintenance at u and corrective maintenance at failure and both, preventive and corrective maintenance, can be either minimal (restoring system to as-bad-as-old), imperfect (between as-bad-as-old and as-good-as-new among other possible system states) or perfect (as-good-as-new).

2.1.3 Failure Limit Preventive Maintenance policies

In a failure limit preventive maintenance policy, equipment are replaced when the hazard rate (ROCOF) or other reliability metrics reach a predetermined threshold and failures in-between these times are either minimally or imperfectly repaired. One such maintenance policy is included in Lie and Chun (1986) wherein PM is performed on equipment when the Rate of Occurrence of Failure (ROCOF) reaches a predetermined threshold on the ROCOF and failures in-between are minimally repaired.

2.1.4 Sequential Preventive Maintenance policies

The sequential preventive maintenance category of maintenance policies aim to mimic systems which experience increasing failure rates with age. Under this policy equipment are maintained at unequal intervals of time which become shorter with increasing age of the equipment to cater to the equipment's increasing failure rate with age (Barlow and Proschan, 1965). This category of maintenance policy could provide considerable cost savings over the periodic replacement maintenance policy in which the equipment is preventively maintained at fixed intervals of time irrespective of its age. An additional feature of this maintenance policy is that the time for the next maintenance is determined only at the end of the current maintenance and is not planned well in advance for the entire life of the equipment but on a case-by-case basis, at the end of an on-going maintenance the time for the subsequent maintenance is selected such that it minimises the maintenance cost over the remaining life of the equipment (Barlow and Proschan, 1965).

2.2 Modelling lifetime of repairable equipment

In this section the alternative renewal theory, Non-Homogeneous Poisson Process and imperfect repair models are included. These are counting processes used in estimating the number of failures in a given time interval under a set of assumptions. The Non-Homogeneous Poisson Process together with the minimal-repair-periodic-overhaul maintenance strategies is one of the two models which has been implemented in the Dynamic Risk Simulator computer program as part of this thesis. Within the Non-Homogeneous Poisson Process, the Weibull distribution has been used to model equipment failure times and the lognormal distribution has been used to model equipment repair times. In discussing imperfect repair models, the system restore mechanism will be briefly looked into - whether the system is restored to the state it was prior to failure or whether it is better than it was prior to failure among few other system-restore states which could play a part in determining the number of failures which could occur at a later time interval. Further information on the same can be found in Barlow and Proschan (1965), Pham and Wang (1996) and Rausand and Høyland (2004).

2.2.1 Counting Processes

Counting processes are stochastic process (non-deterministic) with values which are positive integers and which are increasing. Examples of counting process would be counting the number of heads when a coin is tossed a certain number of times (discrete events over a discrete variable, number of trials) and counting the number of times an equipment fails within a given time period (discrete events over a continuous variable, time). The Nelson-Aalen plot discussed later in section 2.3.2 is an example of a counting process since it is a plot of cumulative number of failed equipment against calendar time and can be used in understanding the age-reliability characteristics of equipment – whether fewer equipment fail with the passage of time (decreasing failure rate), whether equipment fail at a near constant rate with time (constant hazard rate) or whether more equipment fail with age (increasing failure rate). Counting process which have been used in this thesis are the Non-Homogeneous Poisson Process and the alternating renewal process. Other counting processes include the Homogeneous Poisson Process, renewal processes and imperfect repair processes among many others. Some of these counting processes will be briefly discussed in the sections below for completeness.

2.2.2 Modelling lifetime of repairable equipment - Perfect Repair Processes

2.2.2.1 Homogeneous Poisson Process

In a Homogeneous Poisson Process (HPP), the inter-occurrence times are considered to be independent and identical. This means that all inter-occurrence times have the same probability distribution with the same parameters (identical) and that the outcome and/or information about one inter-occurrence time does not have an influence over the probability distribution of other inter-occurrence times (independent). The HPP also exhibits the memoryless property, the number of events depends on the length of the interval and does not depend on the distance of the interval from the start of the process. Thus the hazard rate or the Rate of Occurrence of Failures (ROCOF) is independent of time and is a constant. An assumption of a system to exhibit the HPP is that the equipment does not experience ageing effects, i.e. replaced/renewed upon failure to a state which is as good as new with negligible repair times. For all of the above conditions to be met, the inter-occurrence times between equipment failures have to be exponentially distributed.

As an example, for a repairable equipment the identical, independent and memoryless conditions are met if:

1. The repair times to bring the equipment back to an operating state are negligible.

2. The equipment is restored to a condition which is as-good-as-new termed perfect repair (repair tasks involve replacement/renewal) and the equipment experiences the same operating loads and operating environment during the entire counting interval, hence the inter-occurrence times essentially have the same failure probability distribution with the same parameters and are considered ‘identical’, i.e. no ageing effects are experienced by the equipment.

3. The occurrence of previous events does not affect the occurrence of later events and the number of events depends only on the length of the chosen interval and not on the location of the interval.

Since the focus of this thesis is on repairable equipment which experience ageing effects and which require finite non-negligible repair times for major overhauls, the HPP was not found to be suitable to model equipment lifetime. Variants of the alternating renewal process and the Non-Homogeneous Poisson Process (NHPP) have been used to model equipment lifetime in this thesis. The HPP is by itself a special case of the renewal process, the NHPP and many other processes. The HPP is also a starting point for the other counting processes namely the renewal process, the alternating renewal process and the Non-Homogeneous Poisson Process and hence was included in this section for completeness.

2.2.2.2 Renewal Process

In the renewal process, the inter-occurrence times are assumed to be independent and identically distributed (i.i.d) with any lifetime distribution. The HPP becomes a special case of the renewal process where the inter-occurrence times are exponentially distributed. Similar to the HPP, the renewal process is applicable to equipment which do not experience ageing effects i.e. which are replaced/renewed upon failure to a state which is as good as new. Repair times are assumed negligible for the generic renewal process. However, there are many variants of the renewal processes such as the alternating renewal process and the delayed renewal process among others (Rausand and Høyland, 2004).

An interesting result on the probability of the number of failures being 'n' or more in a chosen time interval for renewal processes with inter-occurrence times which have an Increasing Failure Rate (IFR) distribution (for instance the Weibull distribution with $\beta > 1$) is included in Barlow and Proschan (1965), and has been included here:

$$P(N(t) \leq n) \leq 1 - \sum_{i=0}^{n-1} \frac{(t/\mu)^i}{i!} e^{-t/\mu} \quad \text{for } t < \mu \tag{2.1}$$

Barlow and Proschan (1965) mention that the above equation finds its relevance in being a conservative estimate of the probability of 'n' or number of failures which can occur for any IFR distribution for any length of time from the start, as long as the chosen length of time is less than the mean of the chosen IFR distribution. The right hand side of the above equation is the Poisson distribution, hence the poisson distribution provides the conservative estimate for planning the quantity of spare parts to be stocked for equipment whose failure can be described as a probability distribution with an Increasing Failure Rate (IFR).

The table for the Poisson distribution for $t < \mu$ computed for ratios of t/μ has been included in Appendix B for quick reference.

2.2.2.3 Alternating Renewal Process

An alternating renewal process is a process which alternates between the two system states – operational and failure, over time. It can be considered as a renewal process, restoring the system to as-good-as-new at each overhaul but with non-negligible repair times. It is of specific interest in this thesis since it accounts for repair times which are non-negligible.

The limiting equipment availability for a repairable equipment modelled using the alternating renewal process (for an *infinite* time horizon) is given by:

$$\lim_{t\to\infty} \text{Availability} = \frac{\text{MTBF}}{\text{MTBF} + \text{MTTR}} \tag{2.2}$$

A variant of the alternating renewal process with the times to failure modelled by the NHPP with the power law model is one of the two strategies implemented in the Dynamic Risk Simulator computer program. The concept of minimal-repair and NHPP is discussed next.

2.2.3 Modelling lifetime of repairable equipment - Minimal Repair Process

2.2.3.1 Non-Homogeneous Poisson Process

In the Non-Homogeneous Poisson Process (NHPP), the inter-occurrence times between events (or failures) are not independent and are not identical. This means that the inter-occurrence times all need not have the same probability distribution with the same parameters (not identical) and that the outcome and/or information about one inter-occurrence time has an influence over the probability distribution of other inter-occurrence times (not independent). The Rate of Occurrence of Failures (ROCOF) for a Non-Homogeneous Poisson Process is a function of time and not a constant unlike ROCOF of the Homogeneous Poisson Process. Under the main assumptions included below, an ageing equipment's inter-occurrence times could fit into a Non-Homogeneous Poisson Process model with an increasing ROCOF function (Rausand and Høyland, 2004):

1. Minimal repair, that is, the equipment is restored to a state which is as-bad-as-old or as bad as it was immediately prior to failure.

2. Negligible time to conduct minimal repairs.

3. Equipment will not experience any more than 1 failure at any instance in time.

4. The process has independent increments – the number of events (failures) in non-overlapping time intervals are independent and the number of failures in an earlier interval does not influence the number of failures in future (non-overlapping) intervals.

Rausand and Høyland (2004) also mention that the NHPP could be used to model a complex repairable equipment comprising a large number of parts since only a very small portion of the system is adjusted or replaced during minimal-repair and this would not have significant impact on the equipment's reliability after the minimal-repair, i.e. the equipment's reliability after minimal-repair is assumed essentially the same as it was immediately prior to the minimal-repair.

Further, the NHPP model is such that knowing the distribution for the time until the first failure for the equipment can help to determine the ROCOF for the entire lifetime (Rausand and Høyland, 2004)

In this thesis, the Weibull distribution has been used to model the time till the first failure for rotating equipment. Since the NHPP has been chosen to model the lifetime of the equipment from the time it is put in service till the next periodic overhaul with all failures in-between being minimally repaired, the ROCOF of the NHPP process follows the power law model of the NHPP wherein the hazard rate as a function of time is (which corresponds to ageing effects on equipment) as included in equation 2.3 below. The Weibull lifetime distribution which has been used in this thesis to model the times to failure of rotating equipment is discussed next.

$$\text{Rate Of Occurrence Of Failures: } \lambda(t) = \alpha\beta t^{\beta-1},\ \alpha > 0,\ \beta > 0,\ t \geq 0 \tag{2.3}$$

2.2.4 Modelling lifetime of repairable equipment - Lifetime Distributions

2.2.4.1 Modelling time until the first failure of the equipment

The Weibull distribution was described by a Swedish professor, Waloddi Weibull, in his paper titled "A Statistical Distribution Function of Wide Applicability" (Weibull, 1951). The distribution is an empirical distribution which is versatile and was used with examples in his

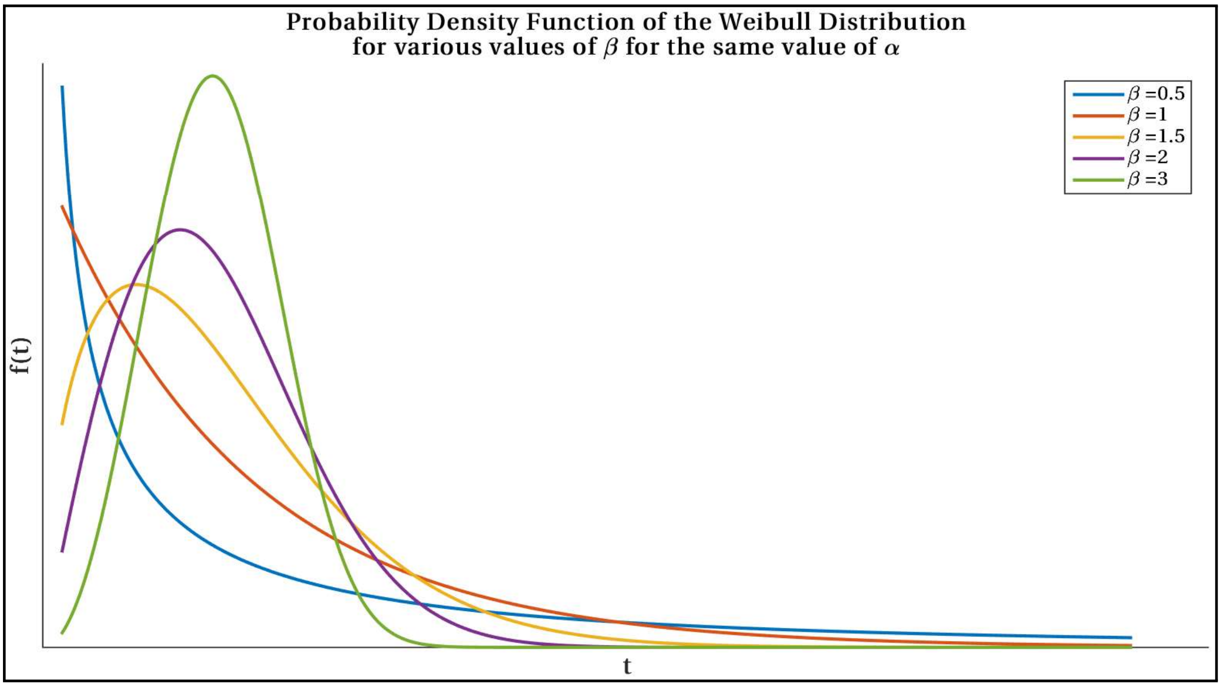

Figure 2.3: Probability Density Function of the Weibull Distribution for various values of β for the same α

paper to describe the yield strength and fatigue life of a few steels and the size distribution of fly ash among many other phenomenon.

The Weibull distribution used to describe the time until the first failure in the Dynamic Risk Simulator is of the form:

$$\begin{aligned}
\text{PDF of the Weibull distribution: } & f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} \\
\text{CDF of the Weibull distribution: } & F(t) = \int_0^t f(t)dt = 1 - e^{-\alpha t^{\beta}} \\
\text{Expectation: } & E(t) = \alpha^{-1/\beta}\Gamma(1+\frac{1}{\beta}) \\
\text{Variance: } & Var(t) = \alpha^{-2/\beta}\left[\Gamma(1+\frac{2}{\beta}) - \Gamma(1+\frac{1}{\beta})^2\right] \\
\text{Rate Of Occurrence Of Failures: } & \lambda(t) = \frac{f(t)}{R(t)} = \frac{f(t)}{(1-F(t))} = \frac{\alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}}{e^{-\alpha t^{\beta}}} = \alpha\beta t^{\beta-1}
\end{aligned} \tag{2.4}$$

The Weibull parameters α and β, could be estimated from a plot of the cumulative failure rate versus time and checked if the system being studied is with either an increasing, decreasing or a constant failure rate or rate of occurrence of failures. The cumulative failure can in turn be estimated from data on the equipment's time to failure using non-parametric estimators such as the Nelson estimator or the Kaplan-Meier estimator.

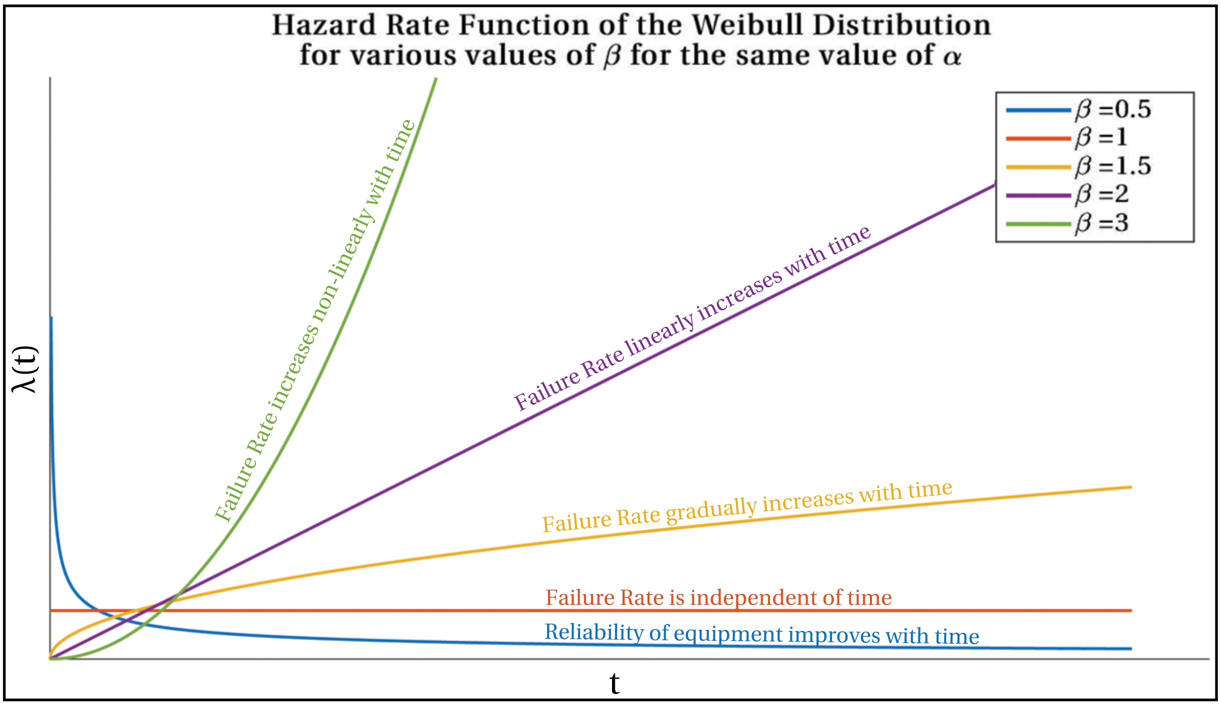

Figure 2.4: Hazard Rate Function of the Weibull Distribution for various values of β for the same α

$\beta < 1$ results in a Weibull distribution with a decreasing hazard rate and is usually used to represent the burn-in or infant mortality phase of equipment in the empirical bath-tub curve. $\beta < 1$ also represents certain types of electronic equipment whose reliability increases with time.

$\beta = 1$ is results in an exponential distribution, with failures which are independent of time (i.e. no ageing effects). $\beta = 1$ results in a constant hazard rate as shown below and generally represents the useful-life phase of equipment in the empirical bath-tub curve. A Weibull distribution with $\beta = 1$ becomes:

$$\begin{aligned}
&\text{PDF of the Weibull distribution: } f(t) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}} \\
&\text{For } \beta = 1\text{: } f(t) = \alpha .1.t^{0} e^{-\alpha t^{1}} \\
&\text{This is an exp. dist. of the form: } f(t) = \lambda e^{-\lambda t} \\
&\text{With the CDF: } F(t) = \int_0^t f(t)dt = 1 - e^{-\lambda t} \\
&\text{And Rate Of Occurrence Of Failures: } = \frac{f(t)}{R(t)} = \frac{f(t)}{(1-F(t))} = \frac{\lambda e^{-\lambda t}}{e^{-\lambda t}} = \lambda = constant
\end{aligned} \tag{2.5}$$

$\beta > 1$ represents ageing due to wear and corrosion among other ageing factors on equipment. If the equipment has either a decreasing or an increasing failure rate, then, it could be checked against a logarithmic plot of ln(Z(t)) vs ln(t) – which are the natural logarithm of the cumulative failure rate against the natural logarithm of the calendar time for linearity. The cumulative failure rate in turn could be estimated using the Nelson estimator. This is discussed in detail in section 2.2.4.3 on estimation of parameters from graphical plots.

2.2.4.2 Modelling equipment repair times

In this thesis, the lognormal distribution has been used to model *active repair times* which includes:

1. Troubleshooting activities to identify the component which needs to be repaired/replaced.
2. Disassembly of either the component alone or other additional components towards having access to remove the component at fault.
3. The repair/replacement task.
4. Re-assembly of the repaired/replacement component back to the main assembly including re-assembly of any additional components which were removed towards gaining accessibility to the repaired component.
5. Function tests conducted to verify system's state prior to reinstating it back in service.

In general, repair times in addition to the *active repair times* include time incurred in administrative tasks such as obtaining requisition for a work order to proceed with the repair, HSE approval and lock-downs/shut-downs, time incurred in waiting for replacement parts, tools and personnel etc.

The *active repair times* and the *additional down time* incurred are combined together to become the Mean Down Time or the Mean Time To Repair (MTTR). In this thesis the down time is assumed as a percent of the active repair time and the MTTR is used in calculations related to equipment availability.

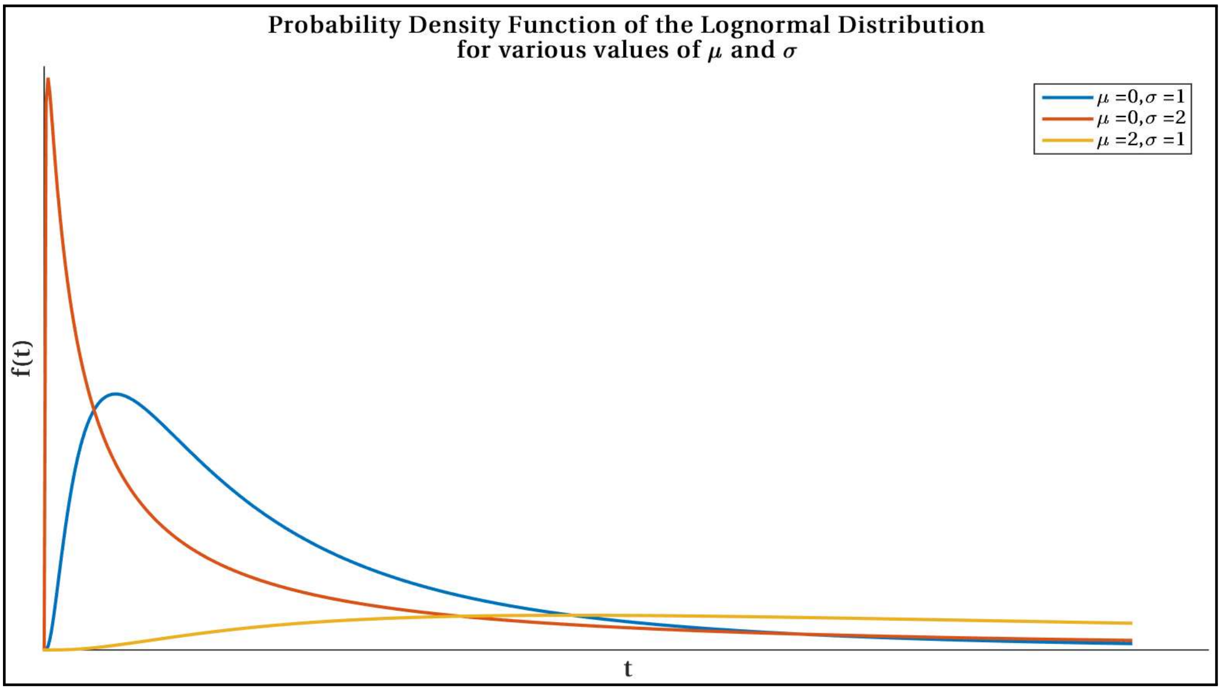

Figure 2.5: Probability Density Function of the Lognormal Distribution for various values of μ and σ

Mathematically, if X ~ Lognormal(μ_{LN}, σ^2_{LN}), then ln(X) ~ Normal(μ_N, σ^2_N). The lognormal distribution used in the Dynamic Risk Simulator is of the form:

$$\begin{aligned} \text{PDF: } f(t) &= \frac{1}{\sqrt{2\pi}\sigma t} e^{-\frac{1}{2}\frac{(ln(t)-\mu)^2}{\sigma^2}} \quad , t > 0 \\ \text{Expectation: } E(t) &= e^{\mu+\frac{\sigma^2}{2}} \\ \text{Variance: } Var(t) &= e^{2(\mu+\sigma^2)} - e^{2\mu+\sigma^2} \end{aligned} \tag{2.6}$$

Parameter estimation and goodness-of-fit tests

Parameter estimation: The graphical plot and Maximum Likelihood Estimation (MLE) methods help in estimating the value of parameters which cannot be directly observed/inferred from the experiments. For instance, in observing times to failure for an equipment which is known to have failure times which are exponential or Weibull distributed, the observable parameters are the times to failure while the unobservable parameters are the values of the exponential distribution's λ and the Weibull α and β parameters which are in turn estimated from the observed data.

Goodness-of-fit tests: Goodness-of-fit tests can be considered to be a subset of hypothesis tests and serve to ascertain the type of distribution (if any) to which observed data belong.

Two methods of estimating parameters, from graphical plots and using the MLE methods have been used in this thesis. These concepts are discussed next.

2.2.4.3 Estimating parameters - from graphical plots

This method involves estimating the Weibull parameters α and β graphically from a plot of cumulative failure rate ($\hat{Z}(t)$) against their calendar times (t) plotted in a log-log plot. The cumulative failure rate is in turn estimated using estimators such as the Nelson-Aalen estimator or the Kaplan-Meier estimator depending on the type of data available – whether uncensored lifetime data or whether one of the few types of censored lifetime data. In general, the Nelson-Aalen estimator and the Kaplan-Meier estimator are non-parametric estimators – which are independent of the underlying distribution. However, with the aid of the following equations, it is possible to obtain initial estimates of the Weibull distribution parameters:

Nelson-Aalen method for estimating the cumulative failure rate Z(t):

First step is to sort the calendar times to failure: $T_1 \leq T_2 \leq \ldots \leq T_j \leq \ldots \leq T_n$

Under the assumption that there is at most 1 failure in the interval $[t, t+\triangle t]$ we have:

No. of failures in $[t, t+\triangle t]$: $\approx$ (no. of equipment which have not failed at time T_j) z(t)dt

$$\approx (n-j+1)\int_t^{t+\triangle t} z(u)du \text{ with } \int_t^{t+\triangle t} \hat{z}(u)du = \begin{cases} \frac{1}{n-j+1}, & \text{if 1 failure in } [t, t+\triangle t]. \\ 0, & \text{otherwise.} \end{cases}$$

$\int_t^{t+\triangle t} \hat{z}(u)du = \hat{Z}(t)$ which is the Nelson-estimator for cumulative failure rate

Estimating Weibull parameters from the plot of ln(Z(t) vs ln(t):

Rate Of Occurrence Of Failures: $z(t) = \dfrac{f(t)}{R(t)} = \dfrac{f(t)}{(1-F(t))} = \dfrac{\alpha\beta t^{\beta-1}e^{-\alpha t^\beta}}{e^{-\alpha t^\beta}} = \alpha\beta t^{\beta-1}$

Integrating both sides of the equation: $\int z(t)dt = \int \alpha\beta t^{\beta-1}dt \implies Z(t) = \alpha t^\beta$ (2.7)

Taking natural logarithm on both sides of the equation:$ln(Z(t)) = ln(\alpha) + \beta ln(t)$

Replacing $Z(t)$ with the Nelson Estimator $\hat{Z}(t)$, we have:$ln(\hat{Z}(t)) = ln(\alpha) + \beta ln(t)$

Which is a linear equation of the form y = mx + c,

Weibull parameter β is the slope of the plot of ln($\hat{Z}(t)$) vs ln(t)

Weibull parameter α is the intercept of the plot of ln($\hat{Z}(t)$) vs ln(t)

Since the Nelson-Aalen estimator is an approximation of the cumulative failure rate, the estimated values of the Weibull parameters α and β are initial estimates and warrant other methods of estimation such as the Maximum Likelihood Estimation towards confirmation of the estimates obtained.

2.2.4.4 Estimating parameters - using the Maximum Likelihood method

The Maximum Likelihood Estimation (MLE) method involves setting up a likelihood function which corresponds to the probability of obtaining the observed values which are assumed to belong to a probability distribution known apriori. If the observed data are independent and identically distributed, then the MLE method can be used to estimate the Weibull parameters α and β.

The Maximum Likelihood Estimates $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ obtained by this method in turn have confidence intervals eg. a 95% confidence interval which means that if many such intervals are constructed, then 95% of them will contain the true parameter value $\hat{\alpha}_{MLE}$ (Kvaløy, 2014). It is incorrect to infer that there is a 95% probability that the confidence interval contains the actual value of α. Furthermore, if the distribution which is chosen to setup the likelihood function is incorrectly chosen due to epistemic uncertainty, then the MLE method could yield misleading estimates. Hence, the MLE method in conjunction with goodness-of-fit tests and/or hypothesis tests (the hypothesis being the validity of the chosen distribution) is required for a more robust evaluation of data.

For the two parameter Weibull distribution of the form used in the Dynamic Risk Simulator program (refer to equation 2.4), the likelihood function and the MLEs with their corresponding confidence intervals are thus obtained:

$$\text{PDF: } f(t) \equiv f(t = T) = \alpha\beta t^{\beta-1} e^{-\alpha t^{\beta}}$$

$$\begin{aligned}
\text{Likelihood function: } L(\alpha,\beta; t_1, t_2 \dots t_n) &= P(t_1 = T_1 \cap t_2 = T_2 \cap \dots \cap t_n = T_n) \\
&= f(T_1, T_2, \dots, T_n) \\
&= f(T_1;\alpha,\beta).f(T_2;\alpha,\beta)\dots f(T_n;\alpha,\beta) \\
&= \left(\alpha\beta {T_1}^{\beta-1} e^{-\alpha {T_1}^{\beta}}\right).\left(\alpha\beta {T_2}^{\beta-1} e^{-\alpha {T_2}^{\beta}}\right)\dots\left(\alpha\beta {T_n}^{\beta-1} e^{-\alpha {T_n}^{\beta}}\right) \\
&= \alpha^n.\beta^n.\left(\prod_{i=1}^{n} {T_i}^{\beta-1}\right).\left(e^{-\alpha\sum_{i=1}^{n} {T_i}^{\beta}}\right)
\end{aligned} \tag{2.8}$$

Taking the natural logarithm of eq 2.8,

$$ln(L(\alpha,\beta;t_1,t_2\dots t_n)) = (nln(\alpha)) + \big(nln(\beta)\big) + \left((\beta-1)\sum_{i=1}^{n} ln(T_i)\right) - \left(\alpha\sum_{i=1}^{n} {T_i}^{\beta}\right) \tag{2.9}$$

Partially differentiating eq 2.9, w.r.t α: $\frac{\partial}{\partial\alpha} ln(L(\alpha,\beta;t_1,t_2\dots t_n)) = \frac{n}{\alpha} + 0 + 0 - \sum_{i=1}^{n} {T_i}^{\beta}$

Setting this equal to zero, we obtain: $\boxed{\hat{\alpha}_{MLE} = \frac{n}{\sum_{i=1}^{n} {T_i}^{\beta}}}$ (2.10)

For the above estimate of $\hat{\alpha}_{MLE}$ the confidence intervals are constructed using the χ^2 distribution as derived below:

If T ~ Weibull(α,β) then, $Y = 2\alpha T^{\beta} \sim \chi^2$with 2 degrees of freedom

Let $T_1, T_2 \dots T_n \sim$ Weibull(α,β) and, $Y_1 = 2\alpha {T_1}^{\beta} \sim \chi^2_2, Y_2 = 2\alpha {T_2}^{\beta} \sim \chi^2_2 \dots Y_n = 2\alpha {T_n}^{\beta} \sim \chi^2_2$

$$\implies \sum_{i=1}^{n} 2\alpha {T_i}^{\beta} \sim \sum_{i=1}^{n} \chi^2_2$$

$$\implies \sum_{i=1}^{n} 2\alpha {T_i}^{\beta} \sim \chi^2_{2n}$$

$$\implies 2\alpha \sum_{i=1}^{n} {T_i}^{\beta} \sim \chi^2_{2n}$$

Substituting $\sum_{i=1}^{n} {T_i}^{\beta} = \frac{n}{\hat{\alpha}_{MLE}}$, we have: $\frac{2n\alpha}{\hat{\alpha}_{MLE}} \sim \chi^2_{2n}$

The (1-ω) confidence intervals are derived thus:

$$P\left(\chi^2_{1-\frac{\omega}{2},2n} \leq \frac{2n\alpha}{\hat{\alpha}_{MLE}} \leq \chi^2_{\frac{\omega}{2},2n}\right) = 1-\omega$$

$$\implies P\left(\frac{\hat{\alpha}_{MLE}}{2n}\chi^2_{1-\frac{\omega}{2},2n} \leq \alpha \leq \frac{\hat{\alpha}_{MLE}}{2n}\chi^2_{\frac{\omega}{2},2n}\right) = 1-\omega$$

The (1-ω) confidence intervals for α are: $\left[\frac{\hat{\alpha}_{MLE}}{2n}\chi^2_{1-\frac{\omega}{2},2n}, \frac{\hat{\alpha}_{MLE}}{2n}\chi^2_{\frac{\omega}{2},2n}\right]$ (2.11)

Next, substituting the value of $\hat{\alpha}_{MLE}$ in eq 2.9, log-likelihood function $ln(L(\alpha,\beta;t_1,t_2\dots t_n))$:

$$
\begin{aligned}
ln(L(\hat{\alpha}_{MLE},\beta;t_1,t_2\dots t_n)) &= (nln(\hat{\alpha}_{MLE})) + \big(nln(\beta)\big) + \left((\beta-1)\sum_{i=1}^{n} ln(T_i)\right) - \left(\hat{\alpha}_{MLE}\sum_{i=1}^{n} {T_i}^{\beta}\right)\\
ln(L(\beta;t_1,t_2\dots t_n)) &= \left(nln(\frac{n}{\sum_{i=1}^{n} {T_i}^{\beta}})\right) + \big(nln(\beta)\big) + \left((\beta-1)\sum_{i=1}^{n} ln(T_i)\right) - \left(\frac{n\ \cancel{\sum_{i=1}^{n} {T_i}^{\beta}}}{\cancel{\sum_{i=1}^{n} {T_i}^{\beta}}}\right)\\
&= \left(nln(n) - nln(\sum_{i=1}^{n} {T_i}^{\beta})\right) + \big(nln(\beta)\big) + \left((\beta-1)\sum_{i=1}^{n} ln(T_i)\right) - (n)
\end{aligned}
$$

Partially differentiating the above expression $ln(L(\beta;t_1,t_2\dots t_n))$ w.r.t β, we have:

$$
\begin{aligned}
\frac{\partial}{\partial\beta} ln(L(\beta;t_1,t_2\dots t_n)) &= 0 - \left(n\frac{\sum_{i=1}^{n} {T_i}^{\beta}.ln(T_i)}{\sum_{i=1}^{n} {T_i}^{\beta}}\right) + \left(\frac{n}{\beta}\right) + \left(\sum_{i=1}^{n} ln(T_i)\right) + 0 - 0\\
&= \boxed{-\left(n\frac{\sum_{i=1}^{n} {T_i}^{\beta}.ln(T_i)}{\sum_{i=1}^{n} {T_i}^{\beta}}\right) + \left(\frac{n}{\beta}\right) + \left(\sum_{i=1}^{n} ln(T_i)\right)}
\end{aligned}
\tag{2.12}
$$

Setting the above equal to zero, we find that unlike the trivial solution we had obtained for $\hat{\alpha}_{MLE}$ in equation 2.10, we cannot obtain a trivial solution for $\hat{\beta}_{MLE}$. Hence we will have to iteratively solve equation 2.12 for $\hat{\beta}_{MLE}$.

Correspondingly the confidence intervals for $\hat{\beta}_{MLE}$ are also non-trivial unlike the confidence intervals obtained earlier for $\hat{\alpha}_{MLE}$ in equation 2.11 and hence was obtained using the in-built functions in Matlab. The Matlab code for estimating $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ for a sample data is included in Appendix A.1. The estimates for $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ are included in Table 2.1. The sample data used are the failure times corresponding to 90 critical failures of a specific compressor at a Norwegian Process plant monitored between 1968-1989 as published in Table 7.1 of the book System Reliability Theory by Marvin Rausand and Arnljot Høyland (2004).

It must be noted that the equation for the Probability Density Function (PDF) of the two-parameter Weibull distribution in-built in Matlab is of a different form from the two-parameter Weibull distribution used in this thesis.

Table 2.1: Estimates for $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ computed using Excel and Matlab for the failure times of a specific compressor at a Norwegian Process plant monitored during the period 1968 till 1989 included here as a sample calculation; data on the compressor's failure times are included in Table 7.1 of the book System Reliability Theory by Marvin Rausand and Arnljot Høyland (2004) Matlab code for the same is included in Appendix A.1

		Excel*	Matlab	
			Unadjusted output	Output arithmetically adjusted for parametric form of the equation
Weibull Alpha	Estimate using MLE	0.00004	2888.70	0.00004
	0.95 χ^2 Confidence Intervals	[0.00003, 0.00005]	[2440.7, 3419]	[0.00003, 0.00005]
WeibullBeta	Estimate using MLE	1.27	1.27	NA
	0.95 χ^2 Confidence Interval	-	[1.07, 1.51]	NA

*Using the values $\chi^2_{180,0.95} = 212.304$, $\chi^2_{180,0.05} = 149.969$

The PDF of the Weibull distribution in-built in Matlab is of the form:

$$
\begin{aligned}
\text{Probability Density Function: } f(t) &= \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1} e^{-(\frac{t}{\eta})^{\beta}} \\
\text{Cumulative Distribution Function: } F(t) &= \int_0^t f(t)dt = 1 - e^{-(\frac{t}{\eta})^{\beta}} \\
\text{Expectation: } E(t) &= \eta\, \Gamma(1+\frac{1}{\beta}) \\
\text{Variance: } Var(t) &= \eta^2 \left[\Gamma(1+\frac{2}{\beta}) - \Gamma(1+\frac{1}{\beta})^2\right] \\
\text{Rate Of Occurrence Of Failures: } \lambda(t) &= \frac{f(t)}{R(t)} = \frac{f(t)}{(1-F(t))} \\
&= \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{\beta-1}
\end{aligned}
\tag{2.13}
$$

This is the same as the substituting $\eta = \alpha^{-1/\beta}$ in the two-parameter Weibull distribution used in this thesis, included in equation 2.4. Correspondingly the likelihood function and the $\hat{\alpha}_{MLE}$ also requires the rearranging of η and α in the expression $\eta = \alpha^{-1/\beta}$ before the results could be made use of. However, the $\hat{\beta}_{MLE}$ is of the same form of the two-parameter Weibull distribution used in this thesis and the output from Matlab can be used without further modification. This has been verified using the numerical what-if solver in Microsoft Excel against the $\hat{\beta}_{MLE}$ result from Matlab.

The MLE method for estimating the parameters of the Weibull distribution has limitations when $\beta \approx 1$. This is due to the asymptotic and hence discontinuous nature of the Weibull probability density function at $\beta = 1$ since the distribution becomes an exponential distribution at $\beta = 1$. For $\beta < 1$ and approaching 1, $f(0) \to \infty$ while for $\beta > 1$ and approaching 1, $f(0) \to 0$.

2.2.4.5 Goodness-of-fit and Hypothesis testing

Goodness-of-fit tests such as the χ^2 test and the Kolmogorov-Smirnov tests are required to confirm the validity of the assumed distribution and to check for Type I and Type II errors. A Type I error involves verifying whether the chosen distribution adequately describes the failure pattern (true positive) or whether the data fits into the distribution by accident (false positive). A Type II error involves verifying whether the distributions which were rejected since the data did not have a good fit against them were known and documented to adequately describe the failure pattern (false negative) or whether certain distributions are known with certainty to not describe the data (true negative). For the Weibull Process (Power Law Model of NHPP), one other goodness-of-fit test based on the Total Time on Test, known as a TTT plot is included in System Reliability Theory by Marvin Rausand and Arnljot Høyland (2004).

The χ^2 test and the Kolmogorov-Smirnov tests have not been included here since they are beyond the scope of this thesis which is on simulating the lifetime of single-unit repairable systems and in qualitatively visualising the financial, safety and environmental risks associated with varying periodic preventive maintenance intervals for particular maintenance strategies. However, the MLE and graphical methods of parameter estimation have been used in this thesis for Weibull distributed times to failure of the ageing equipment.

Other goodness-of-fit tests and methods of estimating parameters:

Besides the two methods of parameter estimation discussed above and which are used in this thesis, there exist many other graphical and numerical methods of parameter estimation for the Weibull distribution. These include graphical methods such as QQ-plots (Quantile-Quantile plots), TTT-plots (Total-Time-on-Test plots), using the Weibull plotting paper and numerical methods such as the method of least squares, method of moments and using Bayesian prior distributions among others. These and other goodness-of-fit approaches are included in detail in The Weibull Distribution: A Handbook by Horst Rinne (2009).

2.2.5 Modelling lifetime of repairable equipment - Imperfect Repair Processes

In the section on perfect repair models, the Homogeneous Poisson Process and renewal theory for repairable systems were included based on the assumption that the system is restored to a state which is as-good-as-new while the Non-Homogeneous Poisson Process is based on the assumption of minimal-repair, which is restoring the system to a state which is the state it was in prior to failure or, the as-bad-as-old state.

When a system is imperfectly repaired however, it is restored to a state which usually lies between the as-bad-as-old and the as-good-as-new system states. In total, there are five ranges of system states a system takes on when repaired which include: worse-than-old, as-bad-as-old, better-than-old-worse-than-new, as-good-as-new and better-than new states. The reasoning behind these 5 possible system states are (Brown and Proschan, 1983; Nakagawa and Yasui, 1987):

1. Due to repairing the wrong part.
2. Due to partial repair of the part which is at fault.
3. Due to damaging adjacent part/parts while repairing part/parts at fault.
4. Incorrectly assessing the state of the system being inspected.
5. Performing maintenance when it is not required which results in introducing problems to the then properly functioning system.
6. Human errors which could further damage the system.
7. Replacement of defective part/parts with defective part/parts.

Imperfect repair has not been modelled in this thesis. However, in view of the future recommended work on this thesis, a few imperfect repair models are included below which are of relevance.

2.2.5.1 Probabilistic models

In a probabilistic imperfect model described by Nakagawa (1979a, 1979b), the system is restored to the as-good-as-new state with a certain probability p and the system is restored to the as-bad-as-old state with the probability $q = 1 - p$. A modified version of the above model described by Block et al. (1985) is one in which p and q are functions of the one-unit system's age, i.e. $p(t)$ and $q(t)$ where t is the age of the item in use at the time of failure or the time since the last perfect repair.. Some of the probabilistic models assume negligible repair times which there are others which take into account non-negligible repair times in their models.

2.2.5.2 Failure rate reduction models

A failure rate reduction imperfect repair model described by Nakagawa and Yasui (1987) involves reducing the failure rate or ROCOF of the system by a fraction of its value at each PM. Operating the equipment increases the failure rate again which accounts for ageing and age-related failures of the system and there are other variants of this model.

2.2.5.3 Improvement factor models

The improvement factor model for imperfect repair described by Malik (1979) involves reducing the age of the system by a certain time at each repair. The degree of improvement was termed as the improvement factor by Malik (1979) and was to be assigned based on expert judgement (subjective probability). There are other variants of this model for both finite and infinite time horizons and for estimating the improvement factor using frequentist/deterministic probability.

2.3 Age-reliability characteristics of equipment

2.3.1 Age-reliability characteristics of equipment – Types of age-reliability curves

The age-reliability characteristics of simple components are in general described by the empirical bath-tub curve comprising of three phases – infant mortality, useful life (period of near constant hazard rate) and wear-out phases. The age-reliability characteristics of components can vary significantly depending on the loads the components/equipment are subjected to, the environment in which it is being put in service, the level of detail in commissioning and function checks before it is being put into service among others.

There can be appreciable difference in the age-reliability characteristics for the same components which are in service in an aircraft, in a marine vessel or in an offshore facility.

Based on a US-government commissioned study on aircraft components, Nowlan and Heap (1978) identified six age-reliability patterns into which a majority of the aircraft components could be grouped, these six age-reliability patterns are included in figure 2.6. The details and findings from the Nowlan and Heap studies and other similar studies conducted for marine vessels including submarines can be found in the United States Military Handbook on Reliability Centered Maintenance, Military Specification (MIL)-S9081-AB-GIB-010 (United States Naval Sea Systems Command, 2007).

Further, within the same operating environment, different components/equipment can have different age-reliability characteristics depending on their operating loads and functionality, for example, a centrifugal pump with the working fluid crude oil with dissolved gas from an oil well will have different age reliability characteristics, a different requirement for spare parts and a different maintenance plan compared to a centrifugal pump with the working fluid water with injected chemicals. Hence, there exists no single characteristic age-reliability curve for equipment and there exist many generic models of age-reliability curves describing the age reliability of components depending on its end application, industry and environment.

The six age-reliability patterns identified in the study by Nowlan and Heap (1978) can be described using the two-parameter Weibull distribution used in this thesis with appropriately chosen values for the Weibull β parameter. The versatility of the Weibull distribution in describing equipment with improving, with constant and with ageing reliability characteristics is one of the reasons for its selection to model equipment failure times in the Dynamic Risk Simulator.

2.3.2 Age-reliability characteristics of equipment – Constructing Nelson-Aalen plots

The Nelson-Aalen plot is a plot of cumulative number of failed equipment against calendar time and can be used in understanding the age-reliability characteristics of an equipment – whether fewer equipment fail with the passage of time (decreasing failure rate), whether equipment fail at a near constant rate with time (constant hazard rate) or whether more

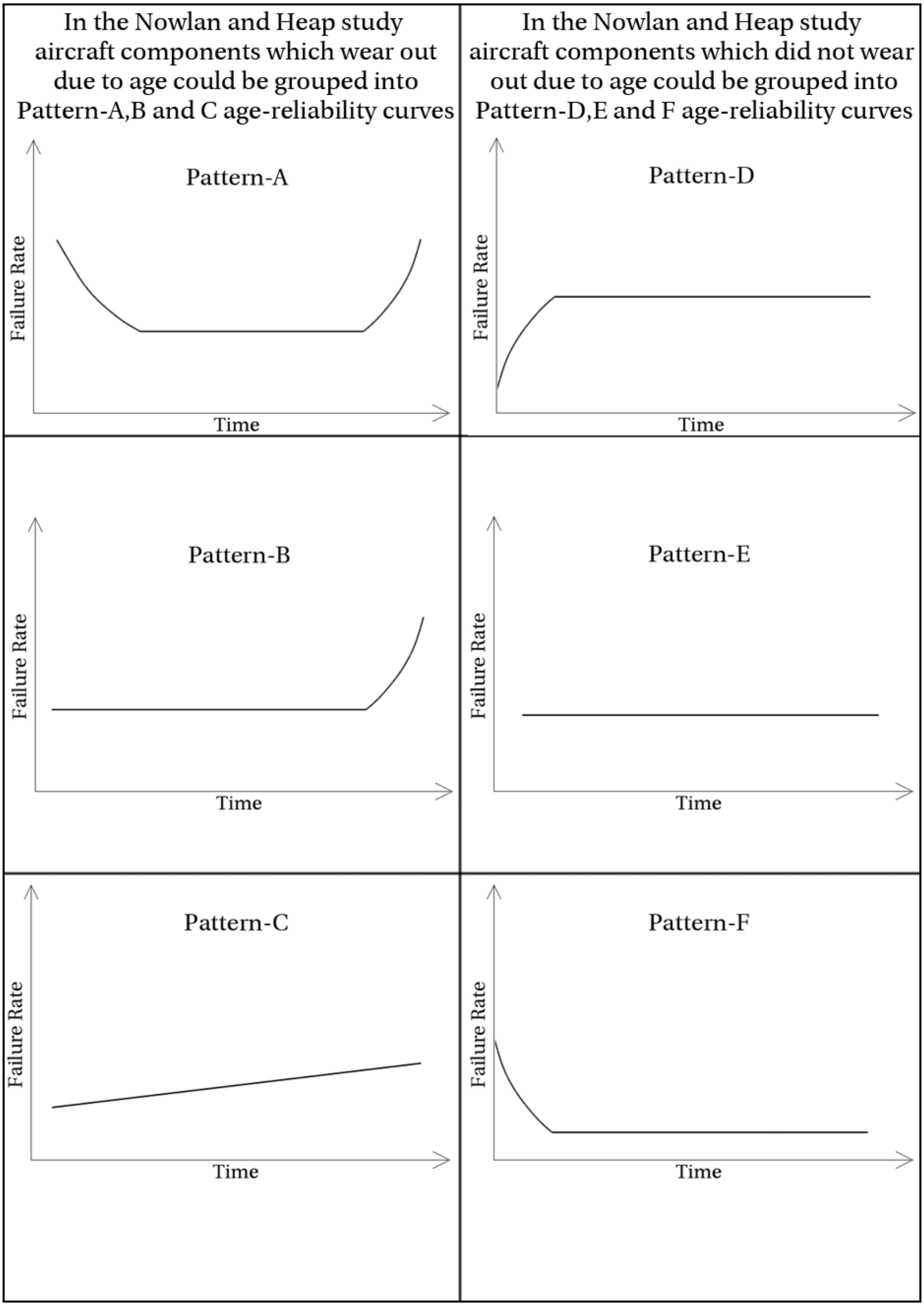

Figure 2.6: The six age-reliability patterns identified in the Nowlan and Heap study of aircraft components, adapted from (Nowlan and Heap, 1978)

equipment fail with age (increasing failure rate). The underlying equations for the same was included in equation 2.7.

When the data is obtained from many equipment of the same type belonging to multiple facilities, combining the different sets of data into a single plot would require verifying that all equipment being considered were subjected to nearly the same operating loads and nearly the same operating environment. For combining the different sets of data belonging to multiple sample-sets appropriate statistical estimators are to be used. Since the data made available for validating the model and computer program developed as part of this thesis were homogeneous, the details on combining data from multiple sample-sets is not being included here and can be found in Rausand and Høyland (2004).

2.3.3 Age-reliability characteristics of equipment – ISO standard 14224 and the OREDA handbook

The ISO standard 14224 (1999) provides guidelines and specifications for the collection of equipment failure and maintenance data for the purpose of reliability analysis, lifecycle costs analysis and optimising maintenance among others with specific focus on equipment used in the petroleum and natural gas industries including well-completion equipment, subsea equipment and process equipment. While the ISO standard 14224 provides specifications on which equipment data is to be collected, actual equipment data collected from offshore installations can be found in the OREDA handbook (2002) which has compiled data on various equipment, collected in phases starting from the year 1983.

The OREDA handbook (2002) also includes information on equipment failure modes, equipment calendar time and operation times, estimates for the failure rate of the equipment (under the assumption of a constant hazard rate which is independent of time which translates to an exponential time to failure), estimates for equipment active repair times, list of maintainable items among other data for various categories of static and rotating mechanical equipment, for both topside and subsea, which could be used as initial estimates where the assumptions of the OREDA database' data collection method are satistied. Special emphasis when using failure rate estimates from the OREDA handbook is to verify whether data samples are homogeneous or non-homogeneous. This is determined based on the standard deviation of the sample and has been explained in detail in the OREDA handbook (2002).

2.4 Summary

The NORSOK standard Z-008 guidelines on establishing maintenance policies based on consequence classification of equipment were briefly looked into. The factors which influence maintenance policies in general and the factors which are of relevance to this thesis were included in figure 2.2 and reproduced below in figure 2.7. Some of the factors which influence maintenance factors together with their underlying equations used in the development of the Dynamic Risk Simulator program were discussed in detail. The maintenance strategies implemented in the Dynamic Risk simulator and the equipment lifetime models which are variants of the fundamental concepts discussed in this chapter will be discussed in Chapter 3.

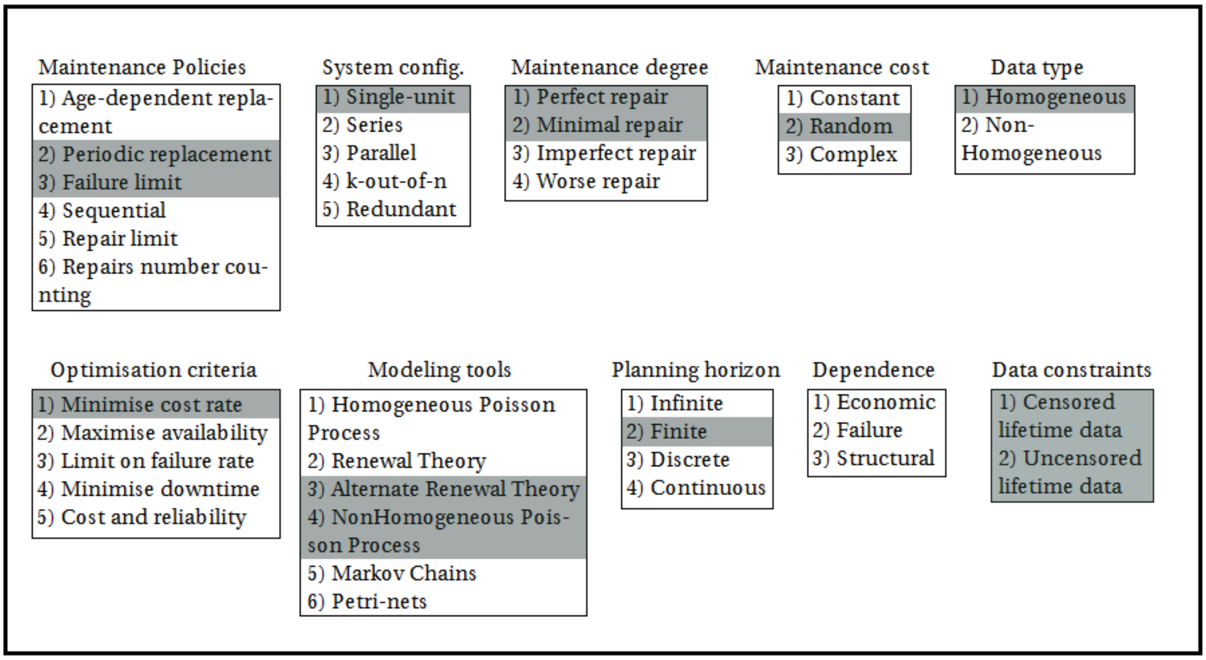

Figure 2.7: Factors which influence maintenance policies, adapted from (Wang, 2002) and modified. The factors highlighted are those which have been considered in this thesis for modelling the minimal-repair-periodic-overhaul maintenance strategy and the run-to-failure maintenance strategy for rotating equipment

Chapter 3

The Dynamic Risk Simulator

3.1 Introduction

The Dynamic Risk Simulator computer program developed as part of this thesis simulates maintenance strategies and visualises the financial, safety and environment risks qualitatively, to aid with decision making on whether postponing maintenance is within acceptable levels of risk. The program also provides quantitative estimates on the cost of preventive and corrective maintenance for variations in the equipment's time to failure and maintenance interval. The program is based on the lifetime model of a single-unit repairable equipment with the failure times modelled as a Non-Homogeneous Poisson Process with the power law model where the time to the first failure is Weibull distributed and the non-negligible repair times for the equipment are modelled as a lognormal distribution. Two maintenance strategies have been modelled in the equipment's lifetime - the minimal-repair-periodic-overhaul maintenance policy and the run-to-failure maintenance policy. These two maintenance strategies were chosen as a proof of concept and to check their applicability for rotating equipment.

The schematic for the Dynamic Risk Simulator is included in figure 3.1. The schematic begins with the process of user input validation, continues on with the modelling of equipment failure times as a Weibull distribution and equipment repair times as a lognormal distribution and then simulates the lifetime of the equipment for the chosen maintenance strategies and computes the qualitative and quantitative estimates for cost and equipment availability. Based on the schematic shown in figure 3.1, the Dynamic Risk Simulator computer program has been developed with a Graphical User Interface (GUI) in the Microsoft Excel Visual Basic

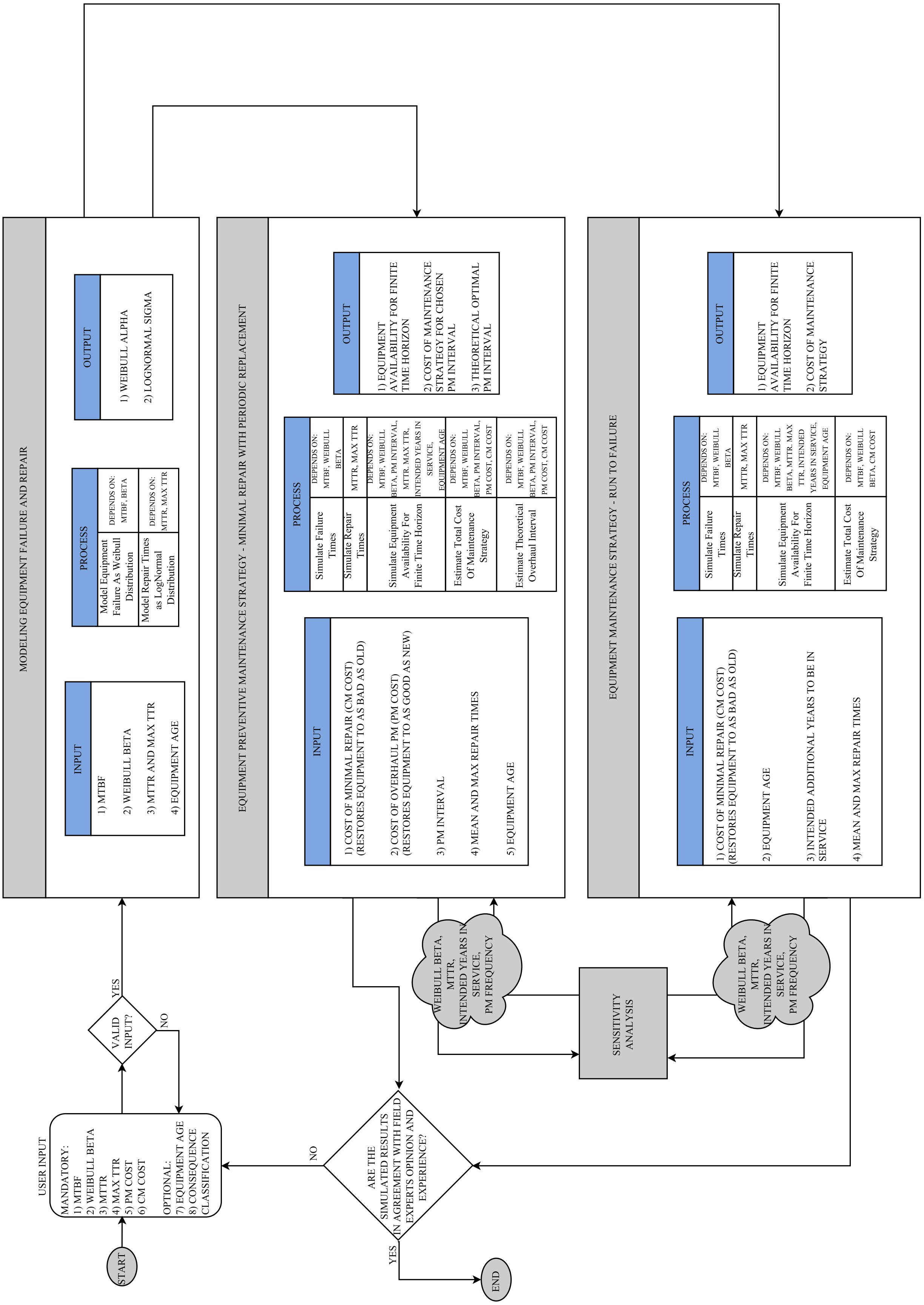

Figure 3.1: Schematic for the Dynamic Risk Simulator

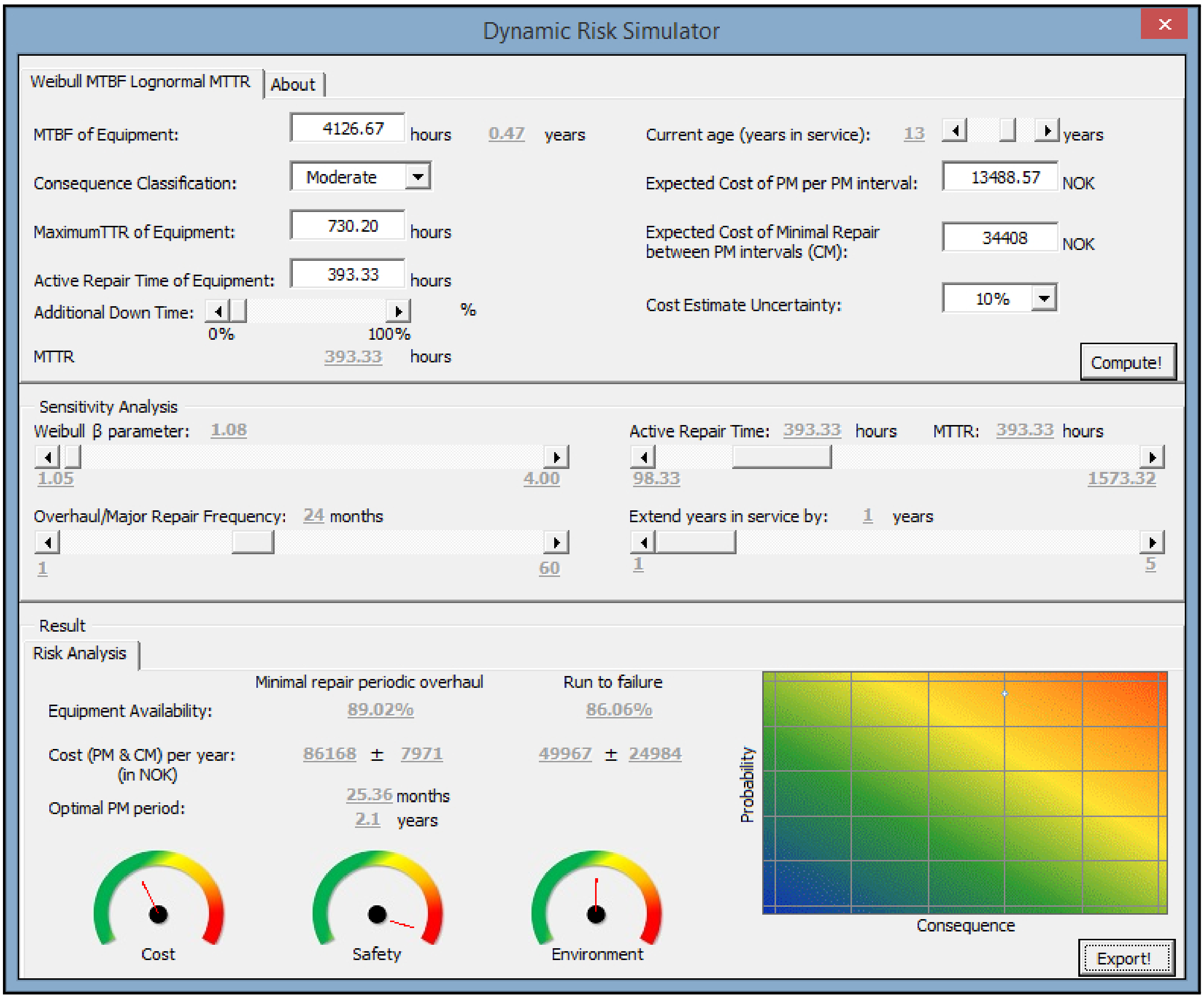

Figure 3.2: Screenshot of the Dynamic Risk Simulator program

for Applications language, Excel VBA in short. The Excel VBA language was chosen due to its ease of use as an introductory programming language which offers a relatively short learning curve for further development on the program. Development and compilation of the program are done from within the Visual Basic Developer environment in-built in Microsoft Excel. Excel's in-built functions and plugins are accessible from the Excel VBA developer's environment. Ease of deployment of programs developed in Excel VBA was another key decision criteria to choose Excel VBA as the language of development since the files can be shared as a native Excel macro-enabled file.

For both strategies implemented in the program, the time until the first failure of the equipment follows a two-parameter Weibull distribution with a time-varying hazard rate which increases with the equipment's age as given by equation 2.3 and included below:

$$\text{Rate Of Occurrence Of Failures: } \lambda(t) = \alpha\beta t^{\beta-1}$$

Figure 3.3: Screenshot of the Dynamic Risk Simulator program showing the range of Weibull β values available for user control in the Sensitivity Analysis section of the simulator. $\beta \sim [1.05, 4.00]$.

where, the Weibull parameter α is the characteristic-life of the equipment or the time by which 63.2% of the equipment would have failed and the Weibull parameter β is the shape factor. Since this thesis focuses on modelling preventive maintenance strategies for ageing equipment, the Dynamic Risk Simulator program restricts values of the Weibull β to values greater than 1. Hence, Weibull distributions with $\beta < 1$ (decreasing failure rates) and $\beta = 1$ (constant failure rates) have not been included for user control in the computer program.

The least value of β for user control in the Dynamic Risk Simulator program has been set at 1.05 owing to the discontinuous nature of the Weibull distribution for Maximum Likelihood Estimation for $\beta \approx 1$ (discussed earlier in Chapter 2 section 2.2.4.4 on estimating parameters using the Maximum Likelihood method, refer to page 23 on the limitation of the Maximum Likelihood Estimation method for $\beta \approx 1$).

A further reasoning for not including user control for the Weibull parameter $\beta = 1$ is that the Weibull distribution becomes an exponential distribution when $\beta = 1$ (refer to equation 2.5, page 16). And, in introducing the exponential distribution to model failure, additional assumptions are introduced regarding the system's failure pattern. These include:

1. Failure has to be sudden and without warning or signs if the equipment's failure times are modelled as an exponential distribution.

2. Failure is time-independent for exponentially distributed failure times – older systems are equally likely to fail as newer systems – wear and tear due to age is not included in the exponential distribution model.

3. Exponential distribution is generally applicable to modelling failure of electrical components.

Since the failure is unpredictable and time-independent for the exponential distribution (any Weibull distribution with $\beta = 1$), an equipment which has been in operation for many years is as likely to fail as a new equipment; and an equipment is equally likely to fail in the next instant in time as well as at any chosen moment in the past or in the future. Due to this, theoretically, preventive maintenance is futile for equipment whose failure times can be modelled as an exponential distribution. In reality however, equipment failure times might not be truly exponential distributed or independent or identical and could have a $\beta < 1$ or a $\beta > 1$. They could be time-dependent, with varying operating loads and environments resulting in their reliability either improving or deteriorating with time and hence will require appropriate reliability improvement or preventive maintenance plans. This is the other reason for restricting the least value of Weibull parameter β for user control in the Dynamic Risk Simulator program to 1.05.

In the Dynamic Risk Simulator's user controls, the least value for the Weibull β parameter set as 1.05 was assigned arbitrarily. This lower bound can be lowered further to values closer to 1 in the source code if certain mechanical equipment's failure times yield estimates for Weibull β parameter lower than 1.05. In the two partial validation tests included in Chapter 4 and in the estimation of the $\hat{\beta}_{MLE}$ in the sample problem included in Chapter 2 Table 2.1 based on the compressor data, the values of β have been greater than 1.05 and hence the lower limit was left as is in the Dynamic Risk Simulator's user controls at the time of writing this thesis.

3.2 Input

The Dynamic Risk Simulator program requires the following inputs.

3.2.1 Mandatory Input 1

To model equipment failure as a Weibull distribution the Mean Time Between Failures (MTBF) and an estimate of the Weibull β parameter of the equipment is required.

Estimating the equipment MTBF (in hours)

Estimation method: Arithmetic mean of the times to failure.

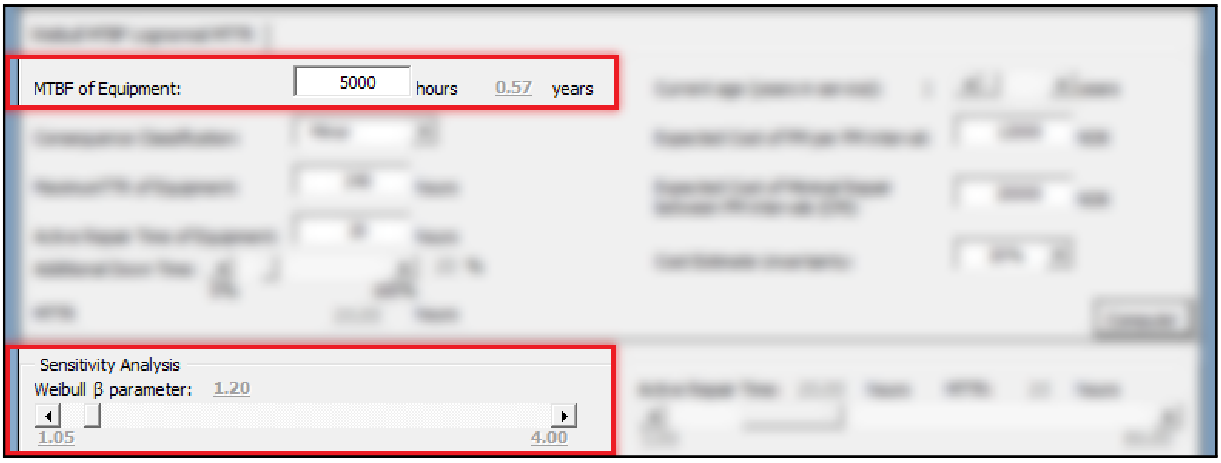

Figure 3.4: Screenshot of the Dynamic Risk Simulator program showing the MTBF and Weibull β input fields

Estimating the Weibull Beta parameter

Estimation method 1: Plotting the equipment's times to failure on a Weibull plot or on a log-log plot making use of the Nelson estimator (or other estimators such as the Kaplan-Meier estimator); If found to be linear, an estimate for β is the slope of the plot. The underlying equations for this method were included earlier in Chapter 2 in section 2.2.4.3 on estimating parameters from graphical plots (refer to page 19).

Estimation method 2: Obtaining $\hat{\beta}_{MLE}$ using the Maximum Likelihood Estimation method which involves solving equation 2.12 using the Matlab code included in Appendix A.1 or iteratively solving in Excel using the What-if analysis. The sample calculations for the same were included in Chapter 2 Table 2.1.

Alternative to methods 1 and 2: If data on the equipment's times to failure is unavailable and only the MTBF or MTBR (Mean Time Between Repairs) is available, an estimate of β, say $\hat{\beta}$, could be obtained using the user controls in the sensitivity analysis section of the Dynamic Risk Simulator to match expected output, either cost or optimal maintenance period. This $\hat{\beta}$ is then to be used to estimate the optimal overhaul period and costs per overhaul period on an equipment of the same OEM subjected to the same loads in the same environment and whose MTBF data were not used in the estimation of $\hat{\beta}$. This is to cross-verify the estimate for β. This alternative reverse-engineering estimation method is to be used as a last resort if methods 1 and 2 cannot be followed towards estimating the value of the Weibull β parameter.

Input Validation/Goodness-of-fit test: To check whether or not the Weibull distribution describes the equipment's times to failure, the goodness-of-fit tests mentioned in section 2.2.4.5, namely the TTT plots, χ^2 test and the Kolmogorov-Smirnov test should be conducted to test whether or not the Weibull distribution is an appropriate distribution within reasonable error limits to describe the given equipment's times to failure.

3.2.2 Mandatory Input 2

To model equipment repair times as a lognormal distribution, the mean *active repair time*, the mean down time or MTTR (*active repair time* plus *additional down time* which includes waiting time for labor, spare parts, work order creation etc.) and maximum time to repair the equipment is required.

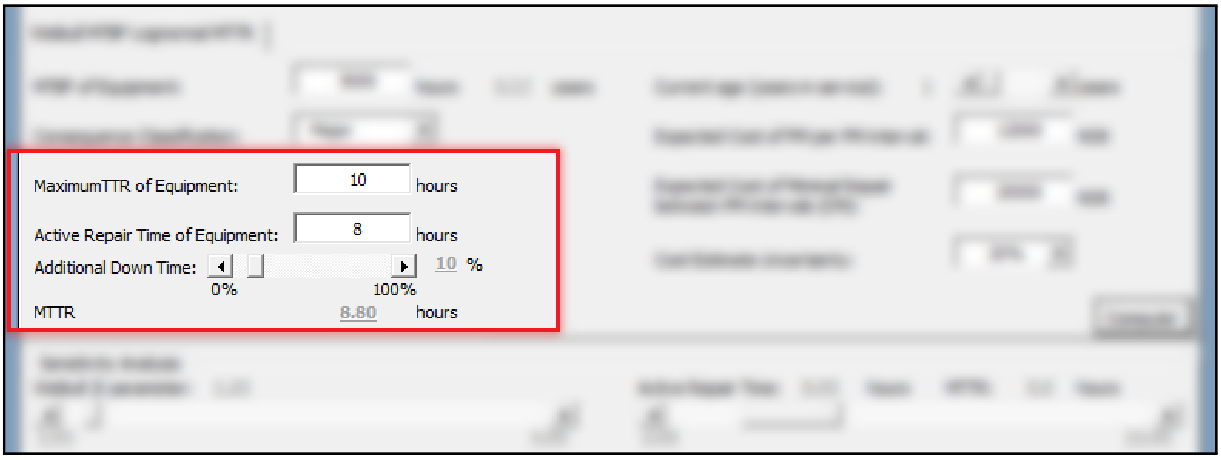

Figure 3.5: Screenshot of the Dynamic Risk Simulator program showing the Maximum Time To Repair, Active Repair Time, Additional Down Time input fields

Estimating the equipment MTTR (in hours) and the Maximum Time To Repair (in hours)

Estimation method for MTTR: Arithmetic mean of the down times of the system in a chosen period.

Estimation method for Maximum TTR: Maximum of the down times of the system in a chosen period.

Input Validation/Goodness-of-fit test: The χ^2 test and the Kolmogorov-Smirnov test mentioned in section 2.2.4.5 should be run on the data to check whether or not the lognormal distribution is an appropriate distribution to model the equipment's times to repair within reasonable error limits.

Limitation: To define a lognormal distribution, the expectation and standard deviation of the data is required. However, since repair times are subjective to the skill level of the person performing the repair, the latency and delay incurred in parts procurement and work order creation among others, obtaining a sizeable sample of repair times to estimate the standard deviation can be difficult. Repair times themselves could be assigned an expected value based on subjective probability of field experts. For these reasons, the lognormal distribution in the Dynamic Risk Simulator program is modelled using the expected Mean Time To Repair (MTTR) and the Maximum Time To Repair (Maximum TTR) under the following set of assumptions using the following equations:

$$\text{If the time to repair, T} \sim \text{Lognormal}(\mu_{LN}, \sigma^2_{LN}), \text{ then } \ln(\text{T}) \sim \text{Normal}(\mu_N, \sigma^2_N),$$
$$\text{where, MTTR} = \mu_{LN} = e^{\left(\mu_N + (\frac{\sigma^2_N}{2})\right)} \text{ and } \sigma^2_{LN} = e^{2\left(\mu_N+\sigma^2_N\right)} - e^{\left((2\mu_N)+\sigma^2_N\right)} \tag{3.1}$$
$$\implies \ln(\text{MTTR}) = \ln(\mu_{LN}) = \left(\mu_N + (\frac{\sigma^2_N}{2})\right)$$

$$\text{Since } \ln(\text{T}) \sim \text{N}(\mu_N, \sigma^2_N), \text{ we have: } Z_\alpha = \frac{ln(T_\alpha) - \mu_N}{\sigma_N} \implies ln(T_\alpha) = \mu_N + (Z_\alpha \sigma_N)$$
$$\implies T_\alpha = e^{\left(\mu_N + (Z_\alpha \sigma_N)\right)} \text{where, } Z_\alpha \text{is the } \alpha \text{ quantile of the Standard Normal distribution.}$$
$$\text{If we set } \alpha = 0.95, 0.99 \text{ which corresponds to 5\% or 1\% exceedance, then}$$
$$T_\alpha = \text{Maximum TTR } = e^{\left(\mu_N + (Z_\alpha \sigma_N)\right)} \implies \ln(\text{MaxTTR}) = \left(\mu_N + (Z_\alpha \sigma_N)\right) \tag{3.2}$$

Solving equations 3.1 and 3.2, with the constraints on times to repair and standard deviations not holding negative values, we can obtain the values of σ^2_N and σ^2_{LN} for a chosen α quantile of the Standard Normal distribution. The Dynamic Risk Simulator's default modelling of lognormal distribution assumes a 1% exceedance which corresponds to a Z_α value of 2.236 obtained from the Standard Normal distribution tables. The limitation of using this method to obtain estimates for σ^2_N and σ^2_{LN} is that it might not be possible to model all possible sets of MTTR and Maximum TTR inputs as a lognormal distribution. The limitation exists since equations 3.1 and 3.2 form a quadratic equation in σ^2_N,

$$\sigma^2_N - 2Z_\alpha + 2ln\left(\frac{\text{MaxTTR}}{\text{MTTR}}\right) = 0 \tag{3.3}$$

which could yield negative and/or imaginary roots for a given set of MTTR, MaxTTR and Z_α.

Table 3.1: Quantiles of the Standard Normal Distribution Z_α vs. Maximum allowable ratio of MTTR:MaxTTR in the Dynamic Risk Simulator due to the constraint imposed by eq 3.3

% Exceedance (α)	Standard Normal Distribution Quantile (Z_α)	Maximum allowable ratio of MTTR:MaxTTR in the DRS due to eqn. 3.3 constraints
5%	1.645	1:3
4%	1.751	1:4
3%	1.881	1:5
2%	2.054	1:8
1%	2.326	1:14

3.2.3 Mandatory Input 3

The third set of mandatory input are the expected cost of preventive repair (μ_{PM}) and the expected cost of minimal-repair in-between overhaul periods (corrective repair, μ_{CM}) for the equipment (both in NOK). Both costs have been modelled as normal distributions with their corresponding standard deviations σ_{PM} and σ_{CM}. The uncertainty in cost estimates can be selected from a drop-down menu and is captured as the standard deviation (σ_{PM} and σ_{CM} = x% of μ_{PM} and μ_{CM} respectively where x ~ [10%, 20%, 30%, 40%])

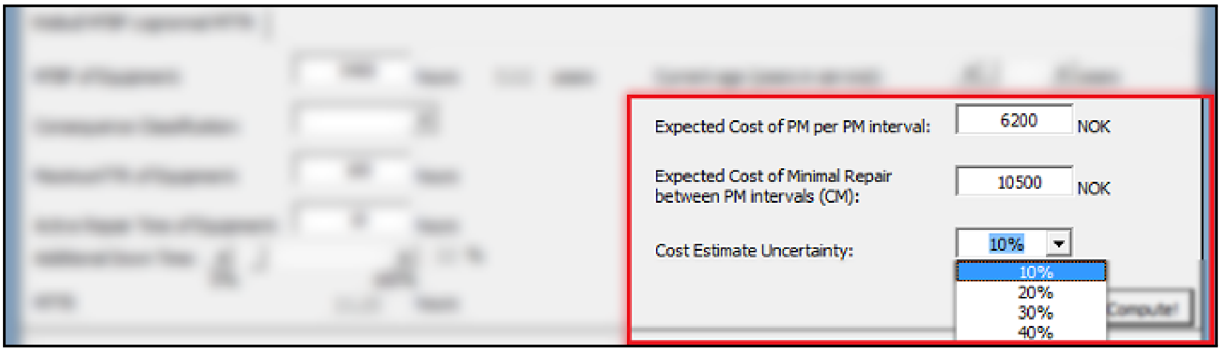

Figure 3.6: Screenshot of the Dynamic Risk Simulator program showing the maintenance cost input fields and the drop-down menu to select uncertainty in costs

3.2.4 Optional Inputs

1. The function/class of the equipment related to the consequence of the equipment's unavailability on the system can be selected as an optional input from the drop-down menu. If left unselected, the default consequence classification of moderate consequence is assigned.

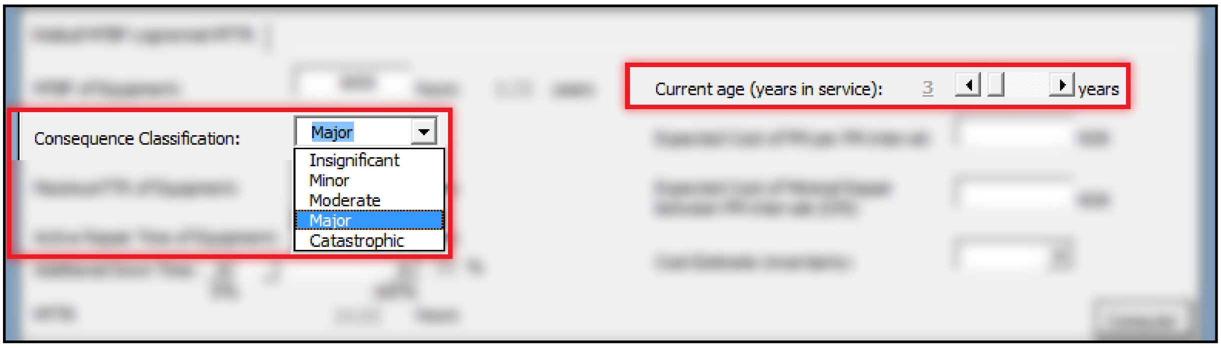

Figure 3.7: Screenshot of the Dynamic Risk Simulator program showing the drop-down menu to select consequence classification of equipment and input field for equipment age

2. The current age of the equipment can be input using the scrollbar. The equipment is assumed to be new by default and if the age is not input the current age of the equipment is set as zero.

3.3 Modelling the minimal-repair-periodic-overhaul strategy with non-negligible repair times

Among the two preventive maintenance strategies implemented, the first strategy is a minimal-repair-periodic-overhaul strategy. In this strategy the equipment will be renewed/replaced at fixed intervals in time to a state which is as-good-as-new and failures within the renewal period will be subjected to minimal-repair which will restore the equipment to a state which is as-bad-as-old or the state the equipment was in immediately prior to failure.

The repair times are assumed to be *negligible* during the minimal-repair phase since the focus is on restoring the equipment to a functional state at the shortest possible time following failure.

Since the system is renewed/replaced to as-good-as-new at fixed periodic preventive maintenance intervals, the repair times for the renewal/replacement are *non-negligible* since it involves troubleshooting, identifying specific components to repair and includes waiting times associated with spare parts logistics and assembly-disassembly of the equipment.

The inter-occurrence times between failures are not independent since the system is restored to the as-bad-as-old state which results in a dependency on the times between equipment failures. This dependency represents the ageing of the equipment. The minimal-repair

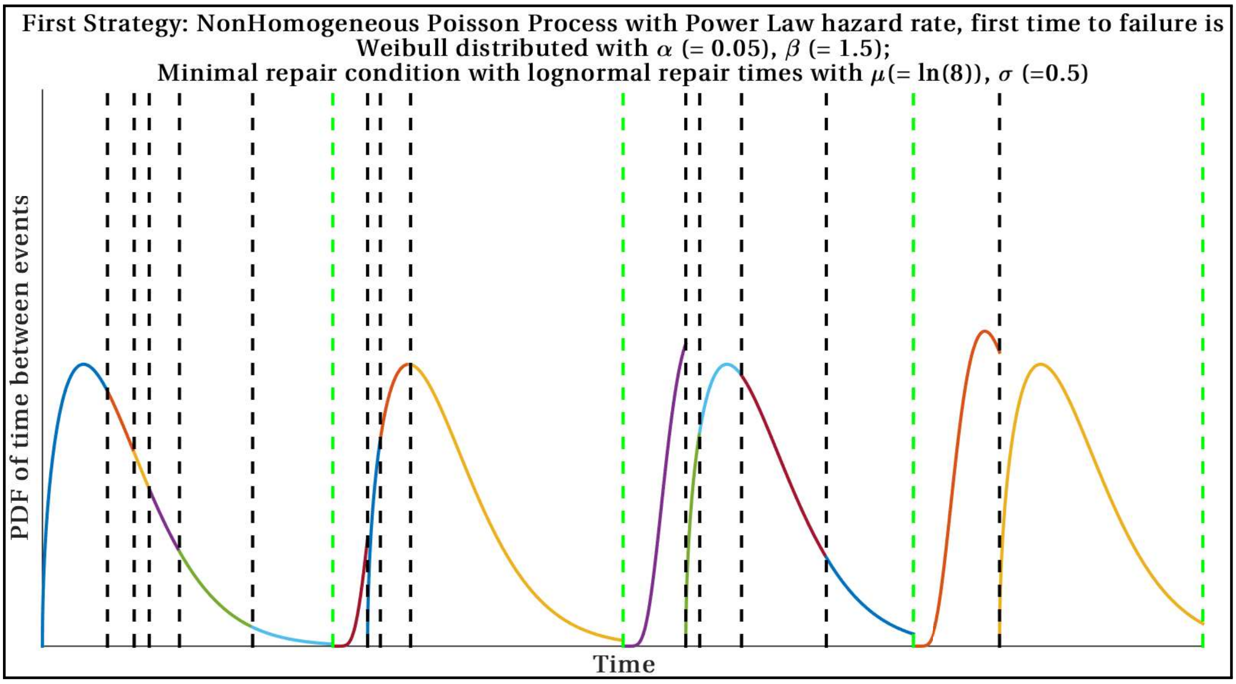

Figure 3.8: PDF of the times between failures and times to perform repairs of the first maintenance strategy proposed as part of this thesis; Asymptotes in green depict the scheduled periodic maintenance; Failures in-between scheduled periodic maintenance are repaired minimally restoring the system to as-bad-as it was prior to failure (NHPP).This is a variant of the minimal-repair-periodic-overhaul maintenance strategy with non-negligible repair times for periodic overhauls.

part of the strategy has been modelled using the NHPP with Power Law model (discussed in Chapter 2 section 2.2.3.1, refer to page 13). The periodic preventive maintenance part of the strategy which restores the system to its as-good-as-new state has been modelled using an alternating renewal process (discussed in Chapter 2 section 2.2.2.3, refer to page 13).

3.3.1 Expected Cost Per Maintenance Interval

The underlying equations for the theoretical expected cost per maintenance interval for the model discussed in the previous section is based on the equations for the expected cost per maintenance interval for the minimal-repair-periodic-replacement strategy included in Pham (2003):

For a NHPP with the power law model (Weibull first time to failure), we have:

$$\text{Rate Of Occurrence Of Failures (ROCOF): } \lambda(t) = \alpha\beta t^{\beta-1},\ \alpha > 0,\ \beta > 0,\ t \geq 0$$

$$\text{For a NHPP, the expected number of failures in [0,u] } = \int_0^u \lambda(t)dt$$

Failures in [0,u] are minimally repaired with expected cost k_m;

Expected cost of PM at periodic intervals u, 2u, 3u etc. is k_p per interval;

$$\text{Expected total cost of PM and CM per interval } u,\ E(K(u)) = \frac{\left(k_m \int_0^u \lambda(t)dt\right) + k_p}{u}$$
$$= k_m \alpha u^{\beta-1} + k_p u^{-1} \tag{3.4}$$

In equation 3.4 above, the expected total cost per PM interval is based on the expected costs of PM and CM, k_p and k_m respectively. Since the PM and CM costs have been assumed to be Normal distributed $X_p \sim N(\mu_p, \sigma_p^2)$ with $\mu_p = k_p$, $\sigma_p = x\%$ of k_p and $X_m \sim N(\mu_m, \sigma_m^2)$ with $\mu_m = k_m$, $\sigma_m = x\%$ of k_m, where x is either 10%, 20%, 30% or 40% and is user-input, the uncertainty in cost estimates are obtained thus:

If $X_i \sim N(\mu_i, \sigma_i^2)$, then $Y = \sum_i^n a_i X_i$ has E(Y) $= \sum_i^n a_i \mu_i$ and Var(Y) $= \sum_i^n a_i^2 \sigma_i^2$

From equation 3.4 above, we have, $Y = K(u) = X_m \alpha u^{\beta-1} + X_p u^{-1}$

with Normal dist. variables, $X_1 = X_m$ and $X_2 = X_p$ and co-efficients $a_1 = \alpha u^{\beta-1}$ and $a_2 = u^{-1}$

Thus, E(Y) $= E(K(u)) = k_m \alpha u^{\beta-1} + k_p u^{-1}$

$$\text{Var(Y)} = \left((\alpha u^{\beta-1})^2 * (\sigma_m^2)\right) + \left((u^{-1})^2 * (\sigma_p^2)\right) \tag{3.5}$$

The VBA code for the same is included in Appendix A.2.

3.3.2 Optimal Maintenance Interval

The optimal maintenance interval for the minimal-repair-periodic-replacement strategy is derived by differentiating and setting the expected cost per interval to zero (Abrahamsen, 2015), the VBA code for the same is included in Appendix A.2:

$$\text{Eq 3.4, Expected cost of PM and CM per interval } u,\ E(K(u)) = \frac{k_m \alpha u^{\beta-1} + k_p u^{-1}}{u}$$

Differentiating w.r.t u and setting it to zero, we have:

$$\begin{aligned} k_m \alpha(\beta-1) {u_{optimal}}^{\beta-2} - k_p {u_{optimal}}^{-2} &= 0 \\ \implies k_m \alpha(\beta-1) {u_{optimal}}^{\beta} &= k_p \\ \implies {u_{optimal}}^{\beta} &= \frac{k_p}{k_m \alpha(\beta-1)} \\ \implies \boxed{u_{optimal} = \left(\frac{k_p}{k_m \alpha(\beta-1)}\right)^{\frac{1}{\beta}}} \end{aligned} \tag{3.6}$$

3.3.3 Equipment Availability over finite time horizon

1. Method 1: One method of estimating equipment availability in a finite time horizon is by cumulatively adding MTBF and MTTR and fractions of each till the desired age is reached. This method is independent of the underlying probability distribution but can produce significant deviations from expected/observed equipment availability if the estimates for MTBF and MTTR have been obtained from few data points on the times to failure and the times to repair. The error could be even more pronounced if the data were to belong to a skewed distribution such as the lognormal distribution chosen to model repair times and the Weibull distribution chosen to model failure times in this thesis (for certain values of β).

2. Method 2: Another method of estimating equipment availability is through simulating the equipment's times to failure and times to repair through true-random/pseudo-random number generation over a large number of simulations and obtaining the mean availability. This method has been implemented in the Dynamic Risk Simulator program through pseudo-random numbers generated using the Mersenne Twister algorithm. The method of generating pseudo-random numbers which are distributed according to a specific probability distribution is discussed next.

Generating pseudo-random numbers which are Weibull distributed can be achieved using the method of Inverse Transform Sampling beginning with the CDF of the Weibull distribution, the VBA code for the same is included in Appendix A.3:

PDF of the Weibull distribution: $f(t) = \alpha\beta t^{\beta-1}e^{-\alpha t^{\beta}}$

CDF of the Weibull distribution: $F(t) = \int_0^t f(t)dt = 1 - e^{-\alpha t^{\beta}}$

By definition, all values of F(t) lie in [0,1]; let R be a random-number in [0,1]

$$\begin{aligned}
\text{Setting } R &= 1 - e^{-\alpha t^{\beta}} \text{ we have,} \\
e^{-\alpha t^{\beta}} &= 1 - R \\
\implies -\alpha t^{\beta} &= ln(1-R) \\
\implies t^{\beta} &= \frac{-1}{\alpha} ln(1-R) \\
\implies t &= \left[\frac{-1}{\alpha} ln(1-R)\right]^{\frac{1}{\beta}}
\end{aligned} \tag{3.7}$$

If R is sampled from the Uniform distribution U(0,1), then t~Weibull(α,β)

Generating pseudo-random numbers which are lognormal distributed can be achieved using the method of Inverse Transform Sampling beginning with the CDF of the lognormal distribution, the VBA code for the same is included in Appendix A.4:

If X ~ Lognormal(μ_{LN}, σ^2_{LN}), then ln(X) ~ Normal(μ_N, σ^2_N)

CDF of the Normal distribution: $N\left(\frac{ln(x) - \mu_{LN}}{\sigma_{LN}}\right)$

By definition, all values of F(t) lie in [0,1]; let R be a random-number in [0,1]

$$\begin{aligned}
\text{Setting } R &= N\left(\frac{ln(x) - \mu_{LN}}{\sigma_{LN}}\right) \text{ we have,} \\
\implies N^{-1}(R) &= \left(\frac{ln(x) - \mu_{LN}}{\sigma_{LN}}\right) \\
\implies ln(x) &= \mu_{LN} + \left(\sigma_{LN} * N^{-1}(R)\right) \\
\implies x &= e^{\mu_{LN} + \left(\sigma_{LN} * N^{-1}(R)\right)}
\end{aligned} \tag{3.8}$$

If R is sampled from the Uniform distribution U(0,1), then x~Lognormal(μ_{LN}, σ^2_{LN})

To test whether an algorithm generates pseudo-random numbers which are uniformly distributed in [0,1], there is a set of tests which pseudo-random number generating algorithms should pass. Some of these include the 'diehard' series of tests developed by George Marsaglia, the series of tests stipulated in NIST Statistical Test Suite included in NIST Special Publication 800-22 (Rukhin et. al, 2010), the TestU01 set of tests among others. Excel's in-built pseudo-random number generator was found to be insufficient for the purpose since its underlying algorithm was found to have been incorrectly implemented in the past (McCullough, 2008; Belsley and Kontoghiorghes, 2009) and also because it can be changed with software patch updates making it unreliable for further developments which are based on it. For these reasons, an algorithm known as the Mersenne Twister algorithm was used in the generation of pseudo-random numbers which are Uniform distributed in (0,1) in this thesis. These uniformly distributed pseudo-random numbers were in turn used in the generation of the Weibull and Lognormal distributed pseudo-random numbers using the transformations described in equations 3.7 and 3.8. The Mersenne Twister algorithm is discussed next.

3.3.4 Mersenne Twister algorithm

The Mersenne Twister algorithm for generation of pseudo-random numbers was developed by Japanese mathematicians, Professor Makoto Matsumoto and Professor Takuji Nishimura of Keio University, Hiroshima, in their paper titled "Mersenne Twister: A 623-dimensionally equidistributed uniform pseudorandom number generator" in 1997. Unlike early pseudo-random number generators such as John von Neumann's middle-square method which involves squaring the number and taking the middle digits and Lehmer's linear congruence method (Knuth, 1981) which involves multiplying by a constant, adding another constant and obtaining the last n-residual digits, the Mersenne Twister method of pseudo-random number generation does not involve arithmetic operations. It instead makes use of the boolean logic operator Xor and other operators. For all practical purposes and Monte Carlo simulations, the Mersenne Twister algorithm with a period of $2^{19937}-1$ has become a widely adopted algorithm with available libraries for most programming languages R, Matlab, Python, C++ and is also available in other software packages.

Based on the author's understanding, the algorithm involves the generation of a sequence of 623 random unsigned (positive) integers which are initially generated based on a starting seed used to initialise the program to a certain state. This sequence is cached and is repeated with the re-use of the starting seed. When a random number is required, it is obtained from this sequence by traversing through the sequence. Once the current sequence of 623 random numbers have all been traversed, the initial state is then transformed using a transformation function called a 'twist' function and used in the generation of another 623 random integers using a different function called a ’tempering’ function and the process of pseudo-random number generation is repeated.

In this thesis, the NtRand Mersenne Twister Excel add-in built by Numerical Technologies Incorporated, Japan, and made available for free on the internet under certain terms of use mentioned in their license agreement, was used for the generation of uniformly distributed pseudo-random numbers which were transformed into Weibull and lognormally distributed pseudo-random numbers using the transformations derived in equations 3.7 and 3.8. The library functions of NtRand’s Mersenne Twister add-in could be accessed from within the Excel VBA environment and was hence useful in this thesis. The NtRand Mersenne Twister Excel add-in was available for download at the time of writing this thesis from the webpage: www (dot) ntrand (dot) com (forward slash) download

The limitation of the Mersenne Twister algorithm lies with the ability to decode and predict all future sequence of pseudo-random numbers if a sufficient number of the pseudo-random generated numbers have been observed. However, for general and academic purposes of generating pseudo-random numbers for simulations in non-cryptographic applications, the Mersenne Twister algorithm still holds good. This is discussed further in the Chapter 5 section 5.3 on recommendations for future work on this thesis.

3.4 Modelling the run-to-failure strategy with non-negligible repair times

The run-to-failure maintenance strategy has been implemented as the second maintenance strategy as part of this thesis. In this strategy, no preventive maintenance is done on the equipment and the equipment is repaired when it fails. It is applicable to equipment for which equipment availability is critical and for which down time incurred due to period preventive maintenance activities are detrimental to its function. Due to this, with increase in equipment age more frequent failures are expected. For this reason, the NHPP with power law model (first time to failure is Weibull distributed) has been used to model the run-to-failure maintenance strategy in this thesis. The method of simulating availability over a finite time horizon is different between the run-to-failure and the minimal-repair-periodic-overhaul maintenance strategies and is discussed over the next few sections.

3.4.1 Expected Cost of Maintenance

The expected cost of maintenance is dependent on the MTBF, the current age of the equipment and the number of years it is planned to be in service. Due to equipment ageing, the equipment will experience greater number of failures the longer it is in service. The expected cost of maintenance is thus derived by the author:

For a NHPP with the power law model (Weibull first time to failure), we have:

Rate Of Occurrence Of Failures (ROCOF): $\lambda(t) = \alpha\beta t^{\beta-1},\ \alpha > 0,\ \beta > 0,\ t \geq 0$

For a NHPP, the expected number of failures in [CA, IY] $= \int_{CA}^{CA+IY} \lambda(t)dt$

Failures in [CA, IY] are repaired with expected cost k_m;

$$\text{Expected total cost of CM in [CA, IY]}, \quad = k_m \int_{CA}^{CA+IY} \lambda(t)dt = k_m\alpha(CA+IY)^{\beta} - k_m\alpha(CA)^{\beta} \tag{3.9}$$

where CA: Current Age, IY: Intended Years In Service

3.4.2 Equipment Availability over finite time horizon

One method of estimating equipment availability is through simulating the equipment's times to failure and times to repair through true-random/pseudo-random number generation over a large number of simulations and obtaining the mean availability. The sequence of steps involved in estimating availability are different from those involved in estimating availability for the minimal-repair-periodic-overhaul strategy.

A pseudo-random Weibull distributed Time To Failure (TTF) is first generated as described in equation 3.7. If this time to failure is less than the current age of the equipment plus the intended years the equipment is planned to be kept in service, then it means that the equipment has a chance of failure(s) in this period and hence would incur corrective repair(s) when it fails during this period. This would require the generation of a pseudo-random log-normally distributed repair time as described in equation 3.8 which will be added to the previously generated TTF. This process will continue cumulatively till the intended years in service is reached. If instead, the generated TTF is beyond the current age of the equipment plus the intended years the equipment is planned to be kept in service, then the equipment has a hundred percent availability over this period. Repeating this over a large number of cycles of pseudo-random number generated TTF, we can estimate the availability for the run-to-failure strategy.

As with the minimal-repair-periodic-overhaul strategy, this method has been implemented in the Dynamic Risk Simulator program using pseudo-random numbers generated by the Mersenne Twister algorithm.

3.5 Sensitivity Analysis

The Dynamic Risk Simulator has the option to perform sensitivity analysis to verify the impact of the change in equipment loads and accelerated ageing (through varying the Weibull β parameter), the change in active repair times and change in mean times to repair, the change in PM frequency (overhaul period for the minimal-repair-periodic-overhaul maintenance strategy) and the change in the intended number of years the equipment is planned to be kept in service, the latter is of importance to the run-to-failure strategy for ageing equipment.

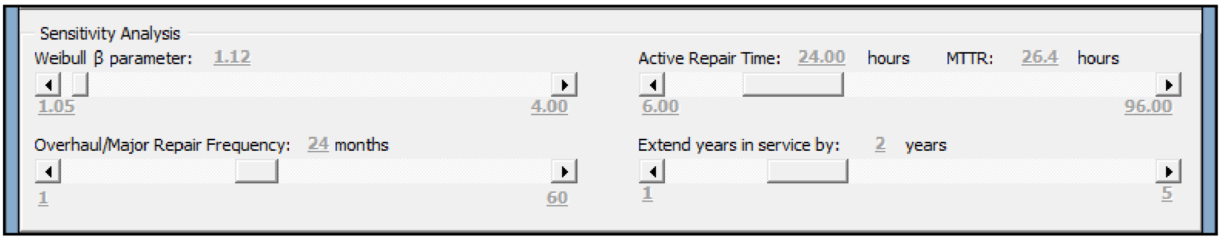

Figure 3.9: Screenshot of the Dynamic Risk Simulator program showing the sensitivity analysis section

3.6 Summary

The two maintenance strategies which were implemented in the Dynamic Risk Simulator program, namely the variants of the minimal-repair-periodic-overhaul and the run-to-failure maintenance strategies were discussed in detail in this chapter. The schematic diagram (figure 3.1), the inputs, data collection methods for the inputs, the underlying equations and code-excerpts for the models developed, outputs and the graphical user interface of the Dynamic Risk Simulator were also included in this chapter. The partial validation tests for the Dynamic Risk Simulator program using data provided by offshore operators will be discussed in Chapter 4.

Chapter 4

Validation Tests, Discussion and Conclusion

4.1 Partial validation test 1: Water injection booster pump

The lifetime data for a water injection booster pump on-board an offshore platform located in the Norwegian Sea was made available from an offshore operator via "AligniT", Apply Sørco's tool for Asset Integrity Solution (Table 4.1).

Assumptions: Apart from the data provided for validation included in Table 4.1, the following assumptions were made regarding the optional inputs for the Dynamic Risk Simulator.

1. Based on the earliest available data, the equipment is considered to have been in operation since 2003. Hence the current age of the equipment was assumed to be 13 years.

2. Information on the consequence class was not available and hence assumed to be the default consequence class of moderate. The uncertainty in cost estimates was assumed to be 10%.

3. Separate information on the active repair times and additional down times was not available. Hence the MTTR (which includes the active repair times and the additional down times) was assumed to be the data provided. This was input to the Dynamic Risk Simulator by setting the % of additional down time to 0% and setting the active repair time to the data provided.

Table 4.1: Data for water injection booster pump

MTBF		12112.33	hours
MTTR		163.68	hours
Max TTR		286.55	hours
PM	Sept 2007	1422.68	NOK
	Cost of Condition Monitoring per week for 23 pumps	4555.20	NOK
	Cost of Condition Monitoring per week per pump	198.05	NOK
	Cost of Condition Monitoring per PM interval	20597.43	NOK
	Total Cost of PM per PM interval	22020.11	NOK per 2 years
	PM Interval	2	years
CM	June 2003	80455.00	NOK
	Aug 2005	43551.00	NOK
	Average Cost of CM per PM interval	62003.00	NOK per 2 years

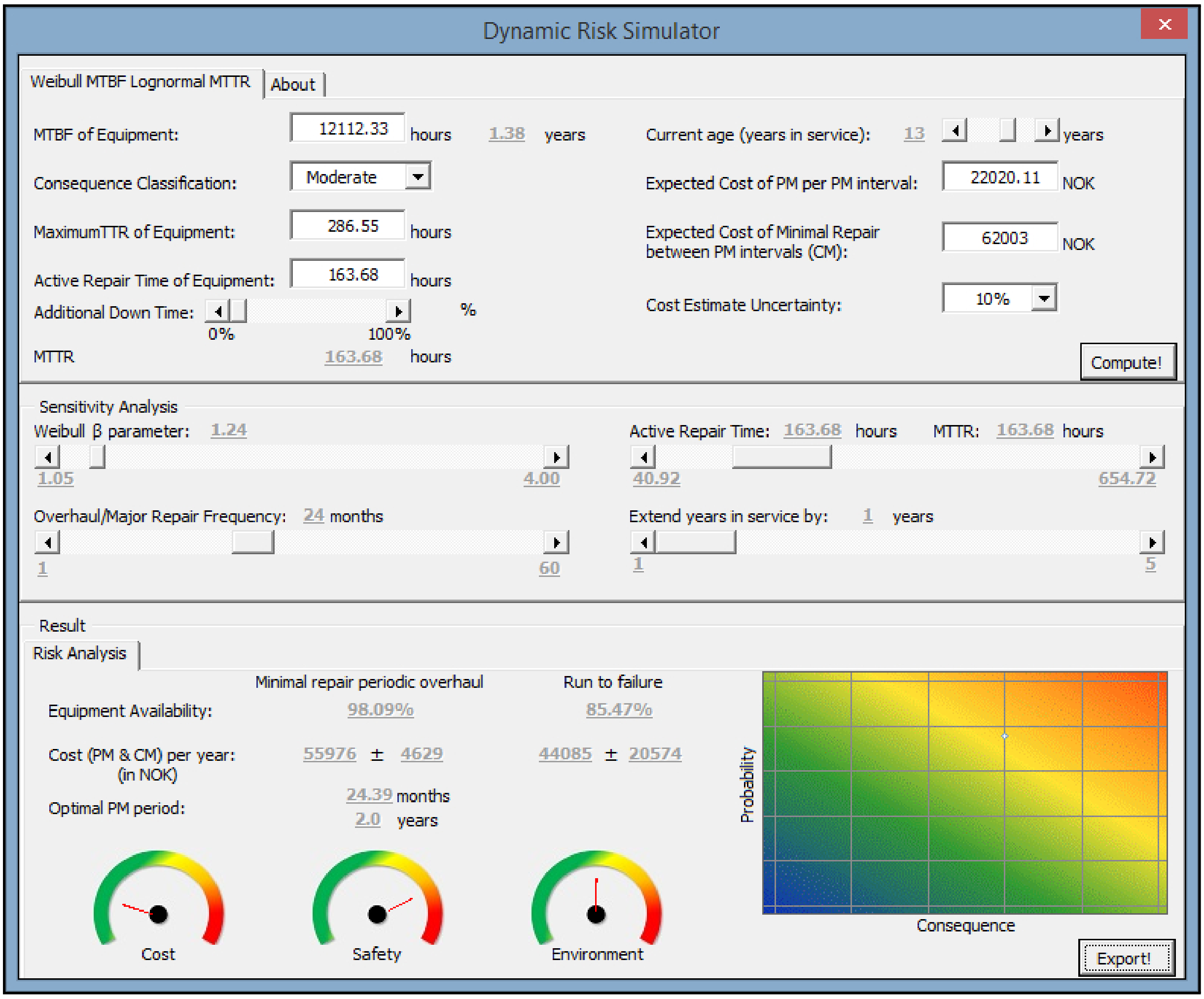

Figure 4.1: Screenshot of the Dynamic Risk Simulator with input data from the water injection booster pump

Data on the individual times to failures for a reasonable sample size which is required for goodness-of-fit testing against the Weibull distribution and for the estimation of the Weibull distribution parameters α and β was not available. Using the third estimation method described in page 36 by adjusting the Weibull β parameter in the sensitivity analysis section of the Dynamic Risk Simulator to obtain the optimal PM interval as 2 years, the Weibull β parameter for the pump was estimated to be 1.24.

The availability of the pump over a finite time horizon with the intended plan to keep the pump in service for one year was 98.09% for the minimal-repair-periodic-overhaul and 85.47% for the run-to-failure strategy.

The estimated maintenance cost for pump per year (10% uncertainty in input costs): 55976 ± 4629 NOK for the minimal-repair-periodic-overhaul strategy and this lies between the PM and CM costs data provided for the analysis and seems to be within expected values for an initial estimate. The estimated maintenance cost for pump per year (10% uncertainty in input costs) is 44085 ± 20574 NOK for the run-to-failure strategy which has a very large uncertainty associated with the costs. One plausible explanation could be the difference between the MTBF which is 1.38 years and the age of the equipment which is 13 years. Under the run-to-failure strategy, no preventive maintenance is assumed to have been done on the pump and the pump is repaired at failure, hence simulating the life of the pump over 13 years with its MTBF 1.38 years and a Weibull beta of 1.24 could result in fewer or more failures and hence lower or higher corrective repair costs resulting in large uncertainty in overall maintenance costs per year. It has been observed that for newer equipment (equipment with current age set as 0), the uncertainty in costs for the run-to-failure strategy is of similar magnitude to the uncertainty in costs associated with the minimal-repair-periodic-overhaul strategy based on the output from the Dynamic Risk Simulator. The uncertainty in costs for the run-to-failure strategy was observed to increase with equipment age while the uncertainty in costs for the minimal-repair-periodic-overhaul strategy continued to remain a magnitude lower than the cost estimates for the minimal-repair-periodic-overhaul strategy irrespective of equipment age.

4.2 Partial validation test 2: Motor driving the water injection booster pump

The lifetime data for the motor driving the water injection booster pump mentioned in the section above, on-board an offshore platform located in the Norwegian Sea was made available from an offshore operator via "AligniT", Apply Sørco's tool for Asset Integrity Solution (Table 4.2).

Assumptions: As with the pump data in the previous section, for the motor, apart from the data provided for validation included in Table 4.2, the following assumptions were made regarding the optional inputs for the Dynamic Risk Simulator.

1. Based on the earliest available data, the equipment is considered to have been in operation since 2003. Hence the current age of the equipment was assumed to be 13 years.

2. Information on the consequence class was not available and hence assumed to be the default consequence class of moderate. The uncertainty in cost estimates was assumed to be 10%.

3. Separate information on the active repair times and additional down times was not available. Hence the MTTR (which includes the active repair times and the additional down times) was assumed to be the data provided. This was input to the Dynamic Risk Simulator by setting the % of additional down time to 0% and setting the active repair time to the data provided.

As with the pump data in the previous section, data on the individual times to failures for a reasonable sample size which is required for goodness-of-fit testing against the Weibull distribution and for the estimation of the Weibull distribution parameters α and β was not available. Using the third estimation method described in page 36 by adjusting the Weibull β parameter in the sensitivity analysis section of the Dynamic Risk Simulator to obtain the optimal PM interval as 2 years, the Weibull β parameter for the motor was estimated to be 1.08.

The availability of the motor over a finite time horizon with the intended plan to keep the motor in service for one year was 89.02% for the minimal-repair-periodic-overhaul and 86.06% for the run-to-failure strategy.

Table 4.2: Data for motor driving water injection booster pump

MTBF		4126.67	hours
MTTR		393.33	hours
Max TTR		730.20	hours
PM	2013	11726.64	NOK
	2015	15250.50	NOK
	Average Cost of PM per PM interval	13488.57	NOK per 2 years
	PM Interval	2	years
CM	June 2003	5060.00	NOK
	Oct 2003	12144.00	NOK
	Total 2003	17204.00	NOK
	Estimated Cost of CM per PM interval	34408.00	NOK per 2 years

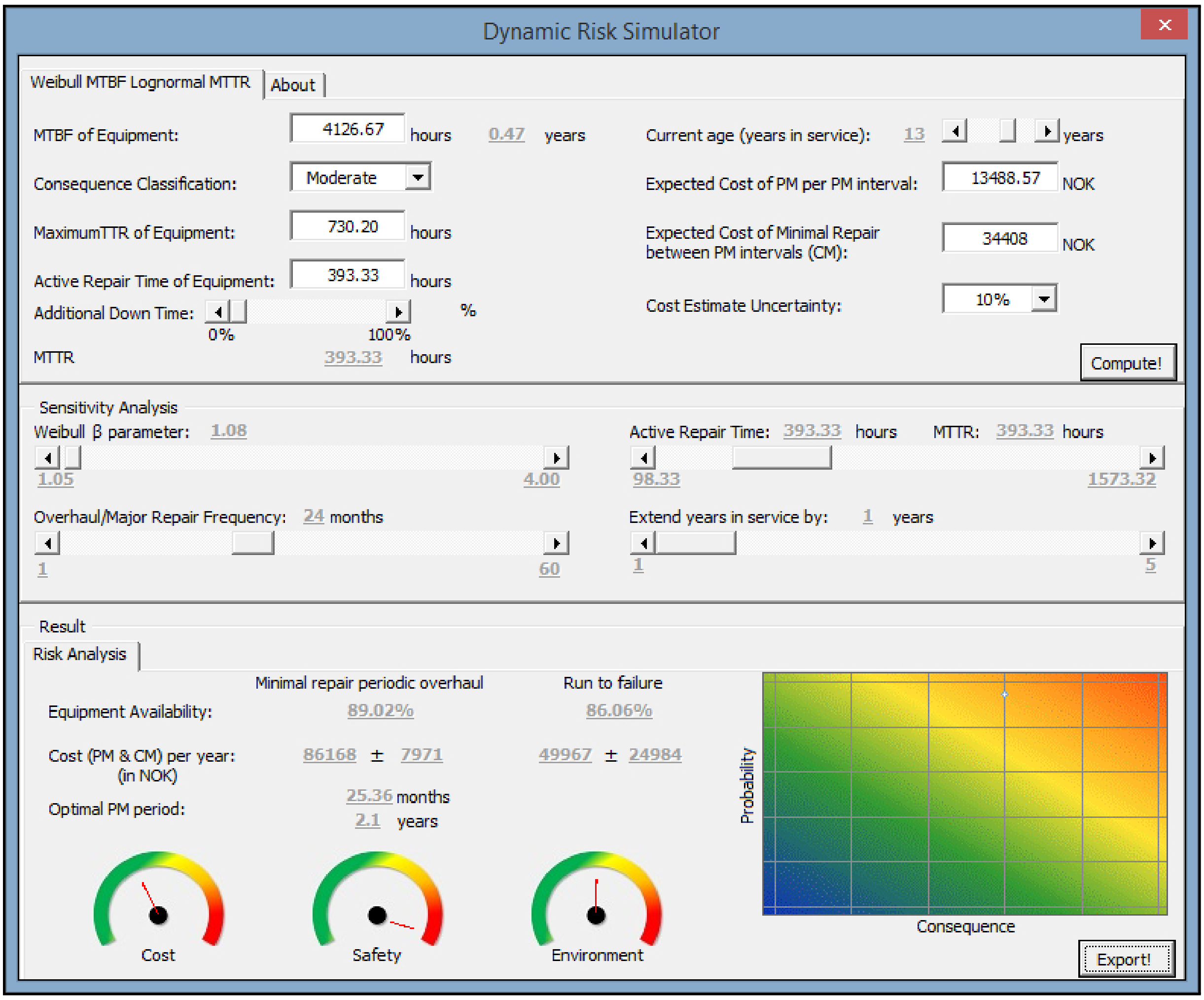

Figure 4.2: Screenshot of the Dynamic Risk Simulator with input data from the motor driving the water injection booster pump

The estimated maintenance cost for motor per year (10% uncertainty in input costs): 86168 ± 7971 NOK for the minimal-repair-periodic-overhaul strategy, which is considerably higher than the costs data provided. The estimated maintenance cost for the motor per year (10% uncertainty in input costs) is 49967 ± 24984 NOK for the run-to-failure strategy which has a very large uncertainty associated with the costs. As with the pump, one plausible explanation could be the difference between the MTBF which is 0.47 years and the age of the equipment which is 13 years. Under the run-to-failure strategy, no preventive maintenance is assumed to have been done on the motor and the motor is repaired at failure, hence simulating the life of the motor over 13 years with its MTBF 0.47 years and a Weibull beta of 1.08 could result in fewer or more failures and hence lower or higher corrective repair costs resulting in large uncertainty in overall maintenance costs per year. As was the case with the pump, it has been observed that for newer equipment (equipment with current age set as 0), the uncertainty in costs for the run-to-failure strategy is of similar magnitude to the uncertainty in costs associated with the minimal-repair-periodic-overhaul strategy based on the output from the Dynamic Risk Simulator. The uncertainty in costs for the run-to-failure strategy was observed to increase with equipment age while the uncertainty in costs for the minimal-repair-periodic-overhaul strategy continued to remain a magnitude lower than the cost estimates for the minimal-repair-periodic-overhaul strategy irrespective of equipment age.

4.3 Discussion and conclusion

Discussion

Both tests included in this Chapter were partial validation tests. To complete the validation and estimate the true optimal PM interval would require using the β value estimated above for each equipment to run simulations using the data from an equipment of the same type located in the same offshore platform. There was another water injection booster pump of the same manufacturer located at the same offshore platform. However, the MTBF, MTTR and Maximum TTR data on this pump were a magnitude higher than the limited times to failure data available for the pump included in this section. The reason for this as recorded in the system and informed to the author was that the second pump had to be sent onshore for a major overhaul, combined with a long lead time of spare parts procurement which resulted in the MTBF, MTTR and Maximum TTR data to be a magnitude higher than the data available for the pump included in table 4.1. The data on the second motor-pump pair were insufficient for further validation. Hence the validation tests remain partially completed.

For the water injection booster pump, a combination of condition monitoring and periodic preventive maintenance seems to have been the maintenance strategies followed. This could question the validity of whether or not the results from the minimal-repair-periodic-overhaul strategy modelled in the Dynamic Risk Simulator are applicable to this equipment.

For the motor driving the water injection booster pump, the discrepancy in the cost estimates from the Dynamic Risk Simulator and the data which was provided by the offshore operator for the motor equipment for the minimal-repair-periodic-overhaul strategy requires re-examining the assumptions made in the applicability of the maintenance strategy modelled in the Dynamic Risk Simulator versus the maintenance strategy followed on the equipment and in arriving at the results thereof.

For both equipment, based on availability, between the two maintenance strategies, the minimal-repair-periodic-overhaul strategy seems preferrable. While, for both equipment, based on the costs (including uncertainty), between the two maintenance strategies, the run-to-failure maintenance strategy seems preferrable.

Conclusion

Towards fulfilling the mandatory objectives included in the project proposal, refer Chapter 1 section 1.2, the development of the equipment lifetime models and the implementing of maintenance strategies in the computer program, five of the six months allotted for this thesis were dedicated. The final four weeks were spent in running preliminary validation of the program and in compiling this thesis report. Given the limited partial validation test results, further validation of the Dynamic Risk Simulator program is required with equipment data before it can be considered for integration into existing maintenance management systems.

Chapter 5

Summary, Limitations and Recommendations for Further Work

5.1 Summary

The objectives of this thesis as included in Chapter 1, refer 1.2, have been fulfilled beginning with understanding the NORSOK Standard Z-008 guidelines on establishing maintenance policies based on consequence classification of equipment. The factors which influence selection of maintenance policies in general and the factors which are of relevance to this thesis were identified to model and simulate single-unit repairable equipment which undergo ageing. The specifications of the Dynamic Risk Simulator tool including the inputs, underlying models/algorithms and outputs have been established. A proof-of-concept test simulator, the Dynamic Risk Simulator was built with a Graphical User Interface using the Excel VBA language and has been partially validated.

The computer program simulates maintenance strategies and visualises the financial, safety and environment risks qualitatively, to aid with decision making on whether postponing maintenance is within acceptable levels of risk. The program also provides quantitative estimates on the cost of preventive and corrective maintenance for variations in the equipment's time to failure and maintenance interval. The program is based on the lifetime model of a single-unit repairable equipment with the failure times modelled as a Non-Homogeneous Poisson Process with the power law model where the time to the first failure is Weibull distributed and the non-negligible repair times for the equipment are modelled as a lognormal distribution. Two maintenance strategies have been modelled in the equipment's lifetime

- the minimal-repair-periodic-overhaul maintenance policy and the run-to-failure maintenance policy. These two maintenance strategies were chosen as a proof of concept and to check their applicability for rotating equipment.

The purpose of the developed tool, the Dynamic Risk Simulator, is to serve as decision-support to help the decision maker to value his/her options on proceeding with or delaying the planned preventive maintenance. The main features of the Dynamic Risk Simulator program developed as part of this thesis include:

1. Modelling an equipment's times to failure as a Weibull distribution and the times to repair as a lognormal distribution.

2. Modelling the minimal-repair-periodic-overhaul and the run-to-failure maintenance strategies for single-unit systems and providing quantitative cost estimates and availability and qualitative visualisation of the safety aspect for these two maintenance strategies.

3. The uncertainty in available PM and CM cost estimates can be adjusted based on available estimates (±10%, ±20%, ±30%, ±40%).

4. The change in operating loads can be simulated by adjusting the Weibull β parameter ($\beta \sim [1.05, 4]$) and the impact of the change in loads on the change in financial, safety and environmental risks can be qualitatively visualised.

5. The equipment availability for a finite time horizon (for run-to-failure equipment – only age-related failure) can be simulated for new and aged equipment (new to 20 year old equipment forward simulation from 1 upto 5 years). Together with the equipment availability and cost estimates for the minimal-repair-periodic-overhaul strategy, this could help with the decision-making on whether to overhaul the equipment or to run the equipment to failure and their pros and cons from a financial and safety aspect.

6. To reduce equipment down time for a given operating load, managing and controlling MTTR is within the maintenance operator's domain while substantial improvements in MTBF is beyond the maintenance operator's control since MTBF is based on OEM design parameters. MTTR can be dynamically changed in the Dynamic Risk Simulator to understand its impact on equipment availability over a finite time horizon.

5.2 Limitations

1. The assigned time of six months for this project has helped with gaining much insight into the workings of the industry and the maintenance practises in place. Beginning with the literature survey to refining of the scope and development and partial validation of the computer program, there could be areas which were overlooked and which could have helped with modelling repairable systems to closely align with their behaviour in their service environment. The quantitative results of the simulation tool were developed to serve as a theoretical aid and to be used together with the subjective-probability based preventive maintenance interval, drawing on the experience of field experts.

2. A second limitation of the developed tool is that it cannot replace human judgement with regards to taking the final call on whether or not to postpone maintenance. The simulation tool provides quantitative and qualitative results and is reliant on the experience and insight of industry experts to take the most appropriate course of action.

3. The developed Dynamic Risk Simulator tool has been partially validated with limited data available from offshore operators. It requires further validation before it can be considered for integration into existing maintenance management systems.

4. The Dynamic Risk Simulator program with its graphical user interface (GUI) has been built as a proof of concept using Excel VBA making use of the Mersenne Twister algorithm for pseudo-random number generation. However, whether or not to scale up the developed program in the Excel VBA language or to switch over to another language will need to be looked into and decided from a software lifecycle management perspective. Further development of the simulator to include combinations of various maintenance strategies which are statistically and computationally intensive could be done in the Matlab environment with GUI.

5. Beyond the simulator's results, there could be epistemic uncertainty associated with modelling equipment failure and repair times, with the level of loads on the system, unintended faults and damage introduced into the system during maintenance among other such factors in the industry which will lead to deviations from expected financial and optimal maintenance interval results estimated by the Dynamic Risk Simulator.

5.3 Recommendations for further work

1. The Dynamic Risk Simulator has been developed to be as versatile as possible to model equipment failure and repair times of single-unit aging systems through the use of the Weibull and lognormal distributions. Future work could include modelling equipment failure and repair times as the Gamma distribution, the Normal distribution and other distributions depending on the failure and repair characteristics of the system being optimised.

2. In this thesis the goodness-of-fit tests described in Chapter 2 section 2.2.4.5 which were relevant to test the validity of the assumed Weibull failure times were computed using Excel. However, the goodness-of-fit tests described in Chapter 2 section 2.2.4.5 and other goodness-of-fit tests could be included as a feature in the Dynamic Risk Simulator to estimate the modelling error and help the user decide which probability distribution (Weibull, Gamma, Normal, Lognormal etc.) has the least error and best describes their equipment failure and repair characteristics and is a reasonable/realistic distribution for the particular type of equipment eg. Weibull distributions with $\beta > 1$ for equipment which experience ageing etc.

3. The minimal-repair-periodic-overhaul and the run-to-failure maintenance strategies have been implemented for single-unit systems in the Dynamic Risk Simulator computer program developed as part of this thesis. As discussed in Chapter 2 section 2.1, there exist many other maintenance strategies which have been developed over the years and various combinations of individual maintenance strategies which cater to a wide variety of constraints from budget to avoiding unnecessary maintenance to reducing down time due to preventive maintenance among other constraints which are being practised in many industries and which are available for the reliability engineer to choose from to address the problem at hand. As mentioned earlier, the maintenance policy most suited for a system depends not only on the above constraints but also on other factors such as the load it is being subjected to, the environment in which it is present, its functionality as either a main unit or as a stand-by unit, whether run-to-failure is also an option, whether compliance with governing industrial standards requires an inspection etc. The Handbook of Reliability Engineering by Hoang Pham

(2003) has details on various combinations of maintenance strategies, for eg. minimal-repair with periodic perfect replacement, minimal-repair with periodic imperfect repair among others.

In this regard, future efforts on the Dynamic Risk Simulator program should be towards customisation of the simulator to emulate the maintenance strategies of the system being optimised. Of particular significance would be the modelling of imperfect repair process and substituting the minimal-repair-periodic-overhaul strategy which is based on restoring the system to as-good-as-new state following repair with a minimal-repair-imperfect-repair strategy wherein the repair associated with the imperfect repair process is assumed to have five possible outcomes, namely: worse-than old, as-bad-as-old, better-than old-but-not-as-good-as-new, as-good-as-new and better-than-new.

4. The limitation of pseudo-random number generating algorithms such as the Mersenne Twister algorithm lies with the ability to decode and predict all future sequence of pseudo-random numbers if a sufficient number of the pseudo-random generated numbers have been observed. The better alternative would be the generation and use of true-random numbers using data measured from atmospheric noise, ambient temperature, relative humidity, CPU fan speed among others which would require interfacing the Dynamic Risk Simulator program developed in this thesis with temperature sensors or thermocouples and data acquisition systems. The National Instruments (NI) DAQ product line of data acquisition systems together with NI LabView could be employed for this purpose and the generated output could be exported to a suitable file format to be read by Excel for generating true-random numbers to be used in conjunction with the Dynamic Risk Simulator.

5. The Dynamic Risk Simulator computer program developed as part of this thesis is for single-unit systems. A final recommendation for future work would be the extension of the Dynamic Risk Simulator to multi-unit systems. Multiple instances of the same equipment within the same facility which would provide homogeneous data and whose availability, maintenance scheduling and demand for spare parts could all be simulated to provide financial and spare parts inventory holding estimates.

Appendix - A

A.1

Matlab code for computing the estimates for $\hat{\alpha}_{MLE}$ and $\hat{\beta}_{MLE}$ and their respective confidence intervals for the failure times of a specific compressor at a Norwegian Process plant monitored during the period 1968 till 1989 included as a sample calculation; data on the compressor's failure times are included in Table 7.1 of the book System Reliability Theory by Marvin Rausand and Arnljot Høyland (2004).

```
% Code snippet to obtain Weibull alpha and beta parameters for the ...
    failure times data corresponding to critical failures of a ...
    specific compressor at a Norwegian Process plant monitored during ...
    the period 1968 till 1989 as included in Table 7.1 of the book ...
    System Reliability Theory by Marvin Rausand and Arnljot Hoyland ...
    (2004).
temp1 = [1, 4, 4.5, 92, 252, 277, 277.5, 284.5, 374, 440, 444, 475,  ...
    536, 568, 744, 884, 904, 1017.5, 1288, 1337, 1338, 1351, 1393, ...
    1412, 1413, 1414, 1546, 1546.5,  1575, 1576, 1666, 1752, 1884, ...
    1884.2,  1884.4,  1884.6,  1884.8,  1887, 1894, 1907, 1939, 1998, ...
    2178, 2179, 2188.5, 2195.5, 2826, 2847, 2914, 3156, 3156.5, 3159, ...
    3211, 3268, 3276, 3277, 3321, 3566.5,  3573, 3594, 3640, 3663, ...
    3740, 3806, 3806.5, 3809, 3886, 3886.5, 3892, 3962, 4004, 4187, ...
    4191, 4719, 4843, 4942, 4946, 5084, 5084.5, 5355, 5503, 5545, ...
    5545.2, 5545.5, 5671, 5939, 6077, 6206, 6206.5, 6305];

% Function to obtain MLE estimates for Weibull parameters alpha and beta
weibull_parameters_temp1_mle = mle(temp1,'distribution','Weibull');

% Alternate function to obtain MLE estimates for Weibull parameters
% alpha and beta and their respective confidence intervals
[weibull_parameters_temp1_wblfit, CI_temp1_wblfit] = wblfit(temp1);
```

A.2

Excel VBA code for computing the expected cost per maintenance interval for the variant of the minimal-repair periodic-replacement model and the optimal maintenance interval for the minimal-repair periodic-replacement strategy.

```
Function PMStrategy1CostPerInterval(MinimalCorrectiveRepairCost As Double, ...
    PreventiveRepairCost As Double, OverhaulFrequency As Double) As Double
'Units of below CostPerInterval is in NOK/hour since WeibullAlpha has units ...
    hours^(-WeibullBeta) and Overhaul Frequency has units hours^(WeibullBeta-1); Hence ...
    convert it to cost per interval by multiplying with hours in that PM interval.

Select Case DynamicRiskSimulator.CostUncertaintyComboBox.ListIndex
    Case 0
        CostUncertainty = 0.1
    Case 1
        CostUncertainty = 0.2
    Case 2
        CostUncertainty = 0.3
    Case 3
        CostUncertainty = 0.4
End Select

CostUncertaintyStdDev = CostUncertainty * Application.WorksheetFunction.Power( ...
    (Application.WorksheetFunction.Power( (MinimalCorrectiveRepairCost * ...
    Module1.WeibullAlpha * Application.WorksheetFunction.Power(OverhaulFrequency, ...
    (Module1.WeibullBeta - 1) ) ), 2) + ...
    Application.WorksheetFunction.Power((PreventiveRepairCost / OverhaulFrequency), 2)), 0.5)

L3PMTheoreticalOptimalFrequency = ...
    ((Application.WorksheetFunction.Power(PreventiveRepairCost / ...
    (MinimalCorrectiveRepairCost * Module1.WeibullAlpha * (Module1.WeibullBeta - 1)), (1 ...
    / Module1.WeibullBeta)))) / (24 * 365 / 12) 'To convert it to months
PMStrategy1CostPerInterval = (MinimalCorrectiveRepairCost * Module1.WeibullAlpha * ...
    Application.WorksheetFunction.Power(OverhaulFrequency, (Module1.WeibullBeta - 1))) + ...
    (PreventiveRepairCost / OverhaulFrequency)

End Function
```

A.3

Excel VBA code for generating pseudo-random numbers which are Weibull distributed using the method of Inverse Transform Sampling and the Mersenne Twister algorithm function from the Mersenne Twister Excel Add-in by NtRand.

```
Sub WeibullSim(WAlpha As Double, WBeta As Double)
' Function to generate random numbers which are Weibull distributed
' with a given set of Weibull alpha and Weibull beta parameters.

' PDF: f(t) = alpha*beta*(t^(beta-1))*exp(-alpha*(t^beta))
' CDF: F(t) = 1 - exp(-alpha*(t^beta))
```

```
' We know F(t)~[0,1]
' Let the value F(t) takes be U
' => U = 1 - exp(-alpha*(t^beta))
' => e^(-alpha*(t^beta)) = 1 - U
' => (-alpha*(t^beta)) = ln(1-U)
' => (t^beta) = -1/alpha * ln(1-U)
' => t = [-1/alpha * ln(1-U)]^(1/beta), where U~[0,1]
' Therefore, generating a random number U between 0 and 1 and
' using the values for WeibullAlpha (WAlpha) and Weibull Beta
' WBeta, a random time t can be generated which fits with the
' Weibull distribution

' Application.Run("NtRand",n) generates 'n' random numbers
' between 0 and 1 belonging to the Uniform distribution

Dim Seed_1 As Integer
Dim Seed_2 As Integer
Dim Row As Long

' Generate random value between 1 and 21997 and 1 and 28541, which are both random
' 5 digit primes chosen arbitrarily as an upper limit for the two seeds to be used
' for pseudorandom number generation.
Seed_1 = Int((21997 * Rnd) + 1)
Seed_2 = Int((28541 * Rnd) + 1)

' Function to generate random numbers between 0 and 1 belonging to the Uniform distribution
' First argument is number of numbers to be generated
' Second argument is: 0 - Mersenne Twister 2002 algorithm, 1 - Mersenne Twister 1998 ...
    algorithm, 2 - Other algorithm
' Third argument is 1st seed and Fourth argument is 2nd seed
' WeibullDist is of datatype Variant declared outside this Sub
WeibullDist = Application.Run("NtRand", NumberOfSimulations, 0, Seed_1, Seed_2)
'WeibullDist(1, 1) = Application.WorksheetFunction.Power(((-1 / WAlpha) * ...
    Application.WorksheetFunction.Ln(1 - WeibullDist(1, 1))), (1 / WBeta))
'Range("R1").value = WeibullDist(1, 1)

For Row = 1 To UBound(WeibullDist, 1)
    WeibullDist(Row, 1) = Application.WorksheetFunction.Power(((-1 / WAlpha) * ...
        Application.WorksheetFunction.Ln(1 - WeibullDist(Row, 1))), (1 / WBeta))
Next Row
'Range("R1:R100000").value = WeibullDist

End Sub
```

A.4

Excel VBA code for generating pseudo-random numbers which are lognormal distributed using the method of Inverse Transform Sampling and the Mersenne Twister algorithm function from the Mersenne Twister Excel Add-in by NtRand.

```
Sub LogNormalSim(muN As Double, sigmaN As Double)
' Function to generate random numbers which are LogNormal distributed
' with a given set of muN mu Normal and sN sigma Normal parameters.

' If x~lognormal(muLN,sigmaLN^2), then,
' ln(x)~Normal(muN,sigmaN^2)

' P(x<=X) = P(ln(x)<=ln(X))
' P(x<=X) = P((ln(x)-muN) <= (ln(X)-muN))
' => P(x<=X) = P(((ln(x)-muN)/sigmaN) <= ((ln(X)-muN)/sigmaN))
' => F(x) = P(x<=X) = Normal((ln(x)-muN)/sigmaN)

' We know F(x)~[0,1]
' Let the value F(x) takes be U
' => U = Normal((ln(x)-muN)/sigmaN)
' => (ln(x)-muN)/sigmaN = Normal^-1(U)
' => (ln(x)-muN) = InverseNormal(U) * sigmaN
' => ln(x) = muN + (InverseNormal(U) * sigmaN)
' => x = exp(muN + (InverseNormal(U) * sigmaN))
' Therefore, generating a random number U between 0 and 1 and
' using the values for Normal mu muN and Normal sigma
' sigmaN, a random time x can be generated which fits with the
' LogNormal distribution

' Application.Run("NtRand",n) generates 'n' random numbers
' between 0 and 1 belonging to the Uniform distribution

Dim Seed_1 As Integer
Dim Seed_2 As Integer
Dim Row As Long

' Generate random value between 1 and 31231 and 1 and 27823, which are both random
' 5 digit primes chosen arbitrarily as an upper limit for the two seeds to be used
' for pseudorandom number generation.
Seed_1 = Int((31231 * Rnd) + 1)
Seed_2 = Int((27823 * Rnd) + 1)

' Generate random numbers between 0 and 1 belonging to the Uniform distribution
' First argument is number of numbers to be generated
' Second argument is: 0 - Mersenne Twister 2002 algorithm, 1 - Mersenne Twister 1998 ...
    algorithm, 2 - Other algorithm
' Third argument is 1st seed and Fourth argument is 2nd seed
' LogNormDist is of datatype Variant declared outside this Sub
LogNormDist = Application.Run("NtRand", NumberOfSimulations, 0, Seed_1, Seed_2)
```

```

For Row = 1 To UBound(LogNormDist, 1)
    LogNormDist(Row, 1) = Exp(muN + ...
        (Application.WorksheetFunction.NormSInv(LogNormDist(Row, 1)) * sigmaN))
Next Row
'Range("W1:W100000").value = LogNormDist
End Sub
```

Appendix - B

		For a Renewal process whose interoccurrence times are described by any distribution with an increasing failure rate with mean μ, for (t < μ), P(n-th event (eg.failure) occurs in [0,t]) is less than:									
		1	2	3	4	5	6	7	8	9	10
t/μ	0.05	0.0488	0.0012	0	0	0	0	0	0	0	0
	0.1	0.0952	0.0047	0.0002	0	0	0	0	0	0	0
	0.15	0.1393	0.0102	0.0005	0	0	0	0	0	0	0
	0.2	0.1813	0.0175	0.0011	0.0001	0	0	0	0	0	0
	0.25	0.2212	0.0265	0.0022	0.0001	0	0	0	0	0	0
	0.3	0.2592	0.0369	0.0036	0.0003	0	0	0	0	0	0
	0.35	0.2953	0.0487	0.0055	0.0005	0	0	0	0	0	0
	0.4	0.3297	0.0616	0.0079	0.0008	0.0001	0	0	0	0	0
	0.45	0.3624	0.0754	0.0109	0.0012	0.0001	0	0	0	0	0
	0.5	0.3935	0.0902	0.0144	0.0018	0.0002	0	0	0	0	0
	0.55	0.4231	0.1057	0.0185	0.0025	0.0003	0	0	0	0	0
	0.6	0.4512	0.1219	0.0231	0.0034	0.0004	0	0	0	0	0
	0.65	0.4780	0.1386	0.0283	0.0044	0.0006	0.0001	0	0	0	0
	0.7	0.5034	0.1558	0.0341	0.0058	0.0008	0.0001	0	0	0	0
	0.75	0.5276	0.1734	0.0405	0.0073	0.0011	0.0001	0	0	0	0
	0.8	0.5507	0.1912	0.0474	0.0091	0.0014	0.0002	0	0	0	0
	0.85	0.5726	0.2093	0.0549	0.0111	0.0018	0.0003	0	0	0	0
	0.9	0.5934	0.2275	0.0629	0.0135	0.0023	0.0003	0	0	0	0
	0.95	0.6133	0.2459	0.0713	0.0161	0.0029	0.0005	0.0001	0	0	0

Note: The above tabulated values of the Poisson distribution and their application to serve as an upperbound for the likelihood that the n-th event occurs before a certain time t for a Renewal process whose interoccurrence times are described by any distribution with an increasing failure rate are based on the equations and proof of Barlow and Proschan (1965). The values were computed and tabulated to serve as a quick reference guide in this thesis.

Bibliography

Abrahamsen, E.B. (2015). *Optimal Maintenance intervals*, lecture notes distributed in RIS510 Reliability Analysis at the University of Stavanger, Stavanger on 22nd March 2015.

Barlow, R. and Hunter, L. (1960). Optimum Preventive Maintenance Policies. *Operations Research*, 8(1), pp.90-100.

Barlow, R. and Proschan, F. (1965). *Mathematical theory of reliability*. New York: Wiley, pp.48-49.

Belsley, D. and Kontoghiorghes, E. (2009). *Handbook of computational econometrics*. Chichester, U.K.: Wiley, p.73.

Block, H., Borges, W. and Savits, T. (1985). Age-Dependent Minimal Repair. *Journal of Applied Probability*, 22(2), p.370.

Brown, M. and Proschan, F. (1983). Imperfect Repair. *Journal of Applied Probability*, 20(4), p.851.

International Organization for Standardization, (1999). *ISO Standard 14224:1999(E) Petroleum and natural gas industries - Collection and exchange of reliability and maintenance data for equipment.* International Organization for Standardization. Geneva, Switzerland.

Knuth, D. (1981). *The Art of Computer Programming, Volume 2: Seminumerical Algorithms*. 2nd ed. Reading, Mass: Addison-Wesley, pp.9-10.

Kvaløy, J.T. (2014). *Confidence intervals*, lecture notes distributed in STA500 Introduction to Probability and Statistics at the University of Stavanger, Stavanger on 19th September 2014.

Lie, C. and Chun, Y. (1986). An Algorithm for Preventive Maintenance Policy. *IEEE Transactions on Reliability*, 35(1), pp.71-75.

Malik, M.A.K. (1979). Reliable Preventive Maintenance Scheduling. *AIIE Transactions*, 11(3), pp.221-228.

McCullough, B. (2008). Microsoft Excel's 'Not The Wichmann–Hill' random number generators. *Computational Statistics & Data Analysis*, 52(10), pp.4587-4593.

Nakagawa, T. (1979a). Optimum Policies When Preventive Maintenance is Imperfect. *IEEE Transactions on Reliability*, R-28(4), pp.331-332.

Nakagawa, T. (1979b). Imperfect Preventive-Maintenance. *IEEE Transactions on Reliability*, R-28(5), pp.402-402.

Nakagawa, T. (1984). Optimal policy of continuous and discrete replacement with minimal repair at failure. *Naval Research Logistics Quarterly*, 31(4), pp.543-550.

Nakagawa, T. and Yasui, K. (1987). Optimum Policies for a System with Imperfect Maintenance. *IEEE Transactions on Reliability*, R-36(5), pp.631-633.

Nowlan, S. and Heap, H. (1978). *Report Number AD-A066579.* Reliability-Centered Maintenance. United States Department of Defense.

OREDA (2002). OREDA – Offshore Reliability Data Handbook. 4th ed. OREDA Participants, SINTEF Industrial Management, Det Norske Veritas. Høvik, Norway.

Pham, H. and Wang, H. (1996). Imperfect maintenance. *European Journal of Operational Research*, 94(3), pp.425-438.

Pham, H. (2003). *Handbook of reliability engineering*. London: Springer, p.353.

Rausand, M. and Høyland, A. (2004). *System reliability theory: Models, statistical methods and applications*. Hoboken, NJ: Wiley-Interscience.

Rinne, H. (2009). *The Weibull distribution: A handbook*. Boca Raton: CRC Press.

Rukhin, A., Soto, J., Nechvatal, J., Smid, M., Barker, E., Leigh, S., Levenson, M., Vangel, M., Banks, D., Heckert, A., Dray, A. and Vo, S. (2010). *National Institute of Standards and Technology (NIST) Special Publication 800-22 Revision 1a* A statistical test suite for random and pseudorandom number generators for cryptographic applications. United States Department of Commerce.

Standards Norway (2011). *NORSOK STANDARD Z-008* Risk based maintenance and consequence classification. Norway: Standards Norway.

Tahara, A. and Nishida, T. (1975). Optimal replacement policy for minimal repair model. *Journal of Operations Research Society of Japan*, 18(3-4), pp.113-124.

United States Naval Sea Systems Command (2007). *Military Specification (MIL)-S9081-AB-GIB-010* Reliability-Centered Maintenance (RCM) Handbook. United States Naval Sea Systems Command.

Wang, H. (2002). A survey of maintenance policies of deteriorating systems. *European Journal of Operational Research*, 139(3), pp.469-489.

Weibull, W. (1951). A Statistical Distribution Function of Wide Applicability. *Journal of Applied Mechanics,* 18(3), pp.293-297.