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Abstract

This thesis tests the correlation between four commodities and eight stocks of companies consuming or producing the commodities. The commodities include oil, gas, salmon and aluminum, while the stocks are Statoil, Seadrill, DNO, Lufthansa, SAS, Norwegian, MHG and Norsk Hydro. Further it tests value-at-risk (VaR) estimation of market risk using EWMA and GARCH models to improve the historical model of volatility. This is done by modelling volatility of the data and using it to calculate VaR for the given sample period. All the results are backtested using Kupiec and Christoffersen tests to see what model produces the best results.

For the correlation, all stocks are affected by the volatility and follows the oil and gas prices. Also salmon is found to have an impact on all stocks, which is suspected to be related to the salmon price‘s correlation with the Norwegian economy, which in general follows the oil price. Aluminum has a more random affect on the stocks, affecting mostly airlines and oil service companies as they are more dependent on aluminum price for their equipment.

The VaR estimation results show that the EWMA performs best for the four commodities, although only at 5% significance level. The GARCH models, and in particular EGARCH has a slightly better result on the stock volatility. Also here only at 5% significance level, as all models fail to model the volatility at 1%. The extreme values are not modelled adequately, and in this thesis the VaR values during crisis are not acceptable, which is when it is needed most. A further improvement on the models using extreme value theory and generalised error distribution might elevate the accuracy.

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Abbreviations

ADF - Augmented Dickey Fuller

AIC - Akaike Information Criterion

AR - Autoregressive

ARCH - Autoregressive Conditional Variance

BIC - Bayessian Information Criterion

CAPM - Capital Asset Pricing Model

CML - Central Market Line

EGARCH - Exponential Generalised Autoregressive Conditional Heteroskedastic

EWMA - Exponentially Weighted Moving Average

GARCH - Generalised Autoregressive Conditional Heteroskedastic

GED - Generalised Error Distribution

JB - Jarque-Bera

L-B - Ljung-Box

LLF - Log Likelihood Function

MA - Moving Average

MLE - Maximum Likelihood Estimation

QMLE - Quasi-Maximum Likelihood Estimation

VaR - Value at Risk

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1 Introduction

In recent years, many different methods has been tested and used to deal with market risk. Variations in price for commodities affect both the producers and the consumers of the commodities, as well as their products. There will always be risk associated with a specific market. A producer in a given market takes risk by producing a certain product hoping someone will buy it. On the other hand the consumer takes risk through becoming dependent on the product, or through changes in costs of producing given product. Together they create a market risk that is dependent on supply and demand, and how those react to information and events occurring.

There are several studies on price volatility in commodity markets. Pindyck (2001, 2004) explains the volatility dynamics for commodities in general using oil and gas prices in particular. He finds relatively high volatility and argues that volatility affects the convinence yield of the commodity. A paper by Regnier (2007) show that price of crude oil is extremely volatile. This is also confirmed in Hamilton (2009) whom concludes that predicting the oil price is extremely difficult due to its fluctuations. Moreover, Narayan and Narayan (2007) study oil price volatility and find that shocks have permanent and assymetric effects on volatility. For the seafood industry, recent studies by Dahl and Oglend (2014) and Asche et al. (2015), show that volatility varies over time for seafood markets, and indicate high volatility for species such as salmon comparable to crude oil price volatility.

For stock markets, there are several noticable contributions. Schwert (1989, 1990) explained historical volatility leading to the stock market crash in 1989 and show how stock market volatility change over time. Fama and French (1989) showed that volatility is counter cyclical, a characterstic later studied by Hamilton and Lin (1996), whom relate stock market volatility to business cycles. Engle et al. (2013) follow up on Schwert's findings and show that macroeconomic fundamentals (i.e. business cycles) have a major impact on stock market volatility. In addition, Bekaert and Harvey (1997) use time-series analysis to estimate volatility in emerging stock markets, which typically have higher volatility. Recently, the VIX-index, a volatility index for US stock markets, has become frequently qouted in media due to economic uncertainty, and Bekaert and Hoerova (2014) extracts a variance premium from the stock market volatility.

Over the years risk management has become more and more important. One obvious example is the use of hedging, either through futures contracts or options. Also diversification has become an viable option in order to reduce risk on portfolio investments. The need to estimate risk has been increasingly necessary in order to have control of the financial risk. The Basel Committee on Banking Supervision (2013) has set the standard regulations, and each revision has had stronger requirements for the banks and financial institutions. As a result, more accurate measurements of financial risk has been developed in order to satisfy the regulations. One of these methods have been value-at-risk which aims to answer a single question; "How much can I possible lose?".

Since its introduction VaR has had many critics claiming that risk modelling does not work, criticising its viability to predict crisis such as the financial crisis. Danielsson (2002) and Taleb (2007) both claim that VaR does only work without crisis, and that it fails to cover extreme events such as the financial crisis. This has created an evolution of the VaR method from its simple historical application, through the use of normal VaR, Monte Carlo simulation, with or without a combination with student t-distribution, and delta-gamma approach. It has been obvious that some improvement have been necessary which leads us to the approach used in this thesis. By estimating the volatility using EWMA or different GARCH methods, one hopes to create a more real description of the uncertainty in the market. This would produce more accurate VaR calculation, and again give us more accurate ways to estimate risk. Different GARCH models using different recipes have been performed several times. Hung et al. (2008) use a fat-tailed GARCH approach to estimate VaR for energy commodities, and more recently Zhang and Zhang (2016) applies GARCHEVT for VaR on energy commodities. In addition, a study by Dahl (2017) applies VaR for risk management in seafood markets, and in particular considering the salmon markets.

The main objective of this thesis is to look at the main commodities of the Norwegian economy, and see how they relate to several stocks and the effect the commodities have on those stocks. Furthermore the thesis tests different models for calculating value-at risk in order to estimate market risk. These volatility models include, rolling analysis, EWMA and GARCH models, whose objective is to better the basic historical VaR calculations. Backtesting is performed see how the different models compare to each other, and which model best estimates the volatility of the chosen commodities and stocks (markets).

The thesis is set up as follows: Chapter 2 explain basic market theory and introduces our chosen companies and our expectations. Chapter 3 describes statistical theory including theory about our volatility models. In chapter 4 a descriptive analysis of the data is given, and the commodities and stocks are graphically presented and discussed related to our sample period. In chapter 5 value-at-risk is presented with methods for testing our models. Chapter 6 includes the results from the analysis and the models, before chapter 7 concludes the thesis.

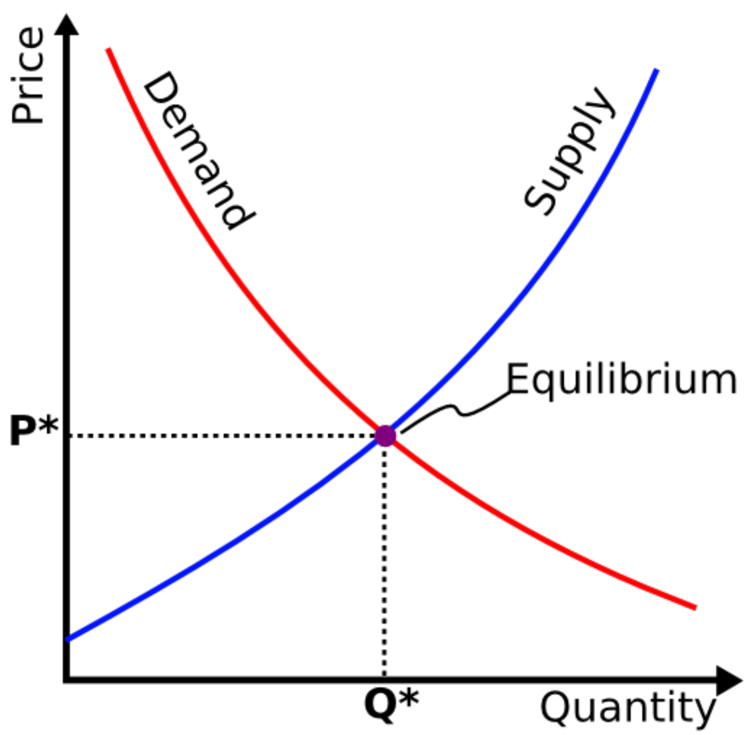
2 Market theory

First, this chapter provides theory on price setting in a market through its supply and demand equilibrium. Also presented are the companies and expectations related to how they affect each other. Lastly, the chapter will provide an overview of risk management strategies. As mentioned in the introduction the thesis focuses on four commodities; oil, gas, salmon and aluminum, and eight stocks consuming or producing these commodities.

2.1 Supply and demand

Before going into anything more, a brief explanation on price setting of commodities and why the prices vary over time will be given. For commodities this is basic supply and demand, and has both a long term price setting as well as a short term price setting. First, looking at the figure below we see a typical supply and demand graph where the supply and demand equilibrium represents the price of a given commodity.

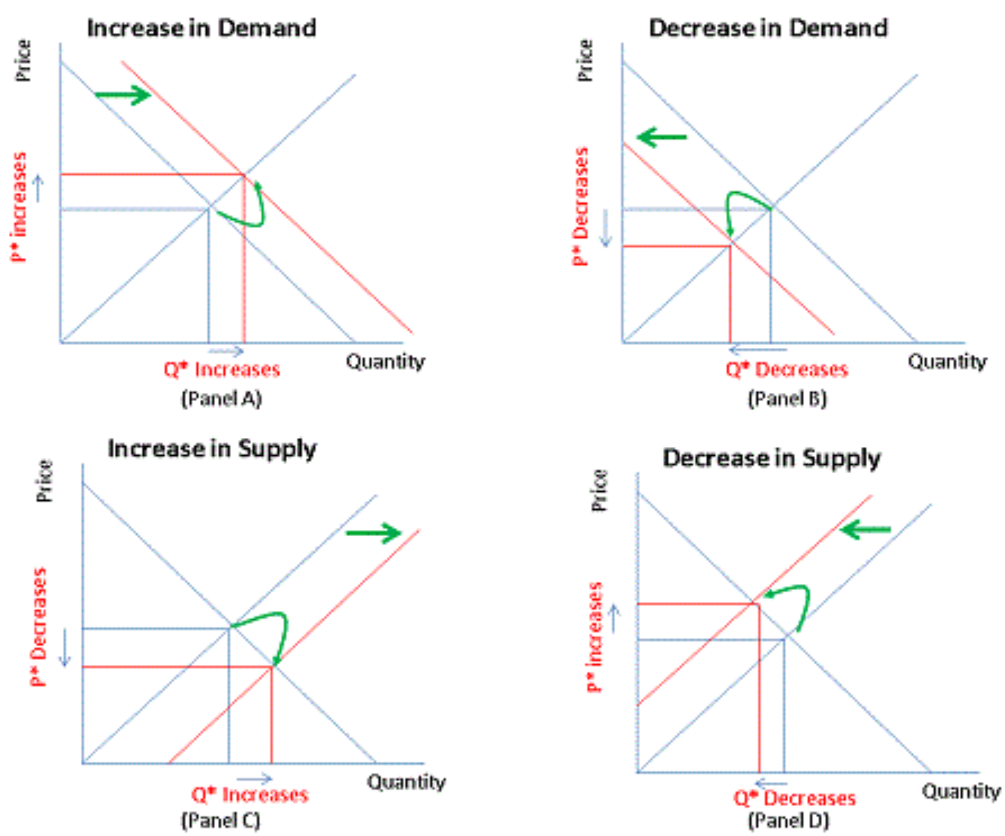
Figure 1: Supply and demand



(Stephansmithfx.com, 2017)

The basic supply and demand equilibrium may change as events occur and we usually split them into two. Long term trends in the market like increased need for oil during wars, and smaller corrections based on news like financial reports or natural catastrophes. These factors are often non-price related changes which results in shifts in supply or demand. In our case these factors are directly related to the stock price of our chosen companies, as they are dependent on the prices of the commodities. One example, which is described in chapter 4, is that in recent years the supply of oil increased, which led to a shift in supply and consequently the oil price dropped as the demand stayed the same. A simplified image of that is seen below at the bottom left.

Figure 2: Shifts in supply and demand



(Graduatetutor.com, 2017)

Another example might be a shift in demand of a commodity, and might be as mentioned a war that increases the demand for oil. This sets the new oil price which increases as long as the supply remains the same. These are all examples of new information that leads to changes in price due to changes in supply and demand. Another example can be short term correction leading to a higher oil price due to a new discovery, or as the price increases more exploration is done and supply increases. In the long run this will again lower the oil price.

Price volatility is the difference in price, usually day to day, of the commodity. A highly volatile good is usually due to supply and demand, for instance because it is not easily substituted with another good. Given an energy commodity, this means that the consumer are often stuck with the volatile energy price because substituting to another source of power is expensive and time consuming. Therefore a volatile energy commodity will be affected by different factors and the consumer will have to endure the highs and the lows. Compared to food and other commodities, which usually have substitutes which keeps the price relatively stable over time, it means the price will change all the time due to supply and demand (Eia.gov, 2017).

For the different markets we look at, which are described in the next section, they may react differently to new information. Short term a oil company might react more to news related to a American shale oil production, than an airline company. The important thing is our ability to model the volatility and use our knowledge and information in predicting the future.

2.2 Commodity and stocks

For the analysis of the markets, several companies have been selected to represent an industry to look at their correlation with the chosen commodities. A summary is found in the table below.

Table 1: Companies

Name	Industry	Market value
Statoil	Oil producer	481
DNO	Oil producer	8.25
Seadrill	Oilfield services	1.97
Lufthansa	Airline	83.5
Norwegian	Airline	8.16
SAS	Airline	5.38
MHG	Seafood	71.5
Norsk Hydro	Aluminum	93.9

Market value is given in billions NOK

As seen, Statoil, DNO and Seadrill represents the oil industry with different sizes and areas. Statoil being the larger one, DNO the smaller company and Seadrill representing the contracting and service companies. Norsk Hydro is a producer of aluminum, however until 2008 they were also an oil producer. Another industry with three companies is the airline industry, with Norwegian being low-cost, SAS a Scandinavian brand, and Lufthansa being one of the largest in the world. Lastly MHG represents the seafood industry as one of the largest in the world, and of importance to the Norwegian economy. The choosing of the companies for the thesis is mainly based on the effect of the oil and gas price, hence the amount of oil related companies. First and foremost it is of interest to see how the oil price affects the oil companies, and how other industries related to oil are affected. Therefore the airlines are chosen to see if there is a difference in the correlation with the oil price for oil companies and airline companies. From there we see if the seafood industry is affected by the volatility of the commodities through MHG. Since oil is such a powerful commodity in the Norwegian economy, it is interesting to see what effects it has on other industries Norway rely on. Continuing on that thought a former oil company, turned fully aluminum producer, is added to see if the commodities affects the aluminum production as well.

Even though the original thought was regarding the oil and gas volatility, it was decided to also look at how they all affect each other. Is a volatile salmon market related to the volatile oil market and vice versa. Before analysing the historical data and modelling the volatility, some expectations on the correlation are introduced next.

2.2.1 Expectations

Before analysing the data, some thoughts on what we expect. The oil price is without a doubt the "main attraction" if you will of this thesis and the economy. How volatile the oil price is affects many things, and in particular in Norway. Looking at our oil companies we expect the volatility in the oil price to affect the stock prices of these companies. Uncertainty in regards to an increase or decrease in the oil price will affect the companies. Statoil being a large, robust company it is also expected that they might have a advantage when dealing with a volatile oil price compared to the smaller DNO. Smaller companies tend to be more vulnerable when it comes to uncertainty and a volatile oil price is expected to showcase that. As a service company, Seadrill will most likely be heavily affected with a drop in oil price due to less work. When it comes to the airline companies, the affects are expected to be present as the prices for fuel will affect their revenue. High fuel prices is not good news for them, and low fuel prices will give them more profit. However we expect a volatile oil price, with a high degree of variation will be negative for both the oil and the airline companies. Norsk Hydro will definitely be affected by the oil price until 2008 in the same matter as Statoil, and after that we expect a smaller correlation with the oil price. Even so, we do expect some correlation due to the usage of aluminum for oil related work, and the affect oil has on the Norwegian economy in general. For seafood and MHG, the expectations are that a high oil price might produce some problems with exporting salmon, so it is interesting to see the correlation between the two.

For gas we assume that the oil and gas prices are highly correlated, in particular here in Europe. Therefore the oil companies are all expected to be correlated with gas and similar to oil. Same with the Norsk Hydro, and the airline companies even though they are not dependent on gas as of today.

Looking at salmon, MHG will definitely be affected by the volatile salmon price. On the other hand the oil companies, although not directly correlated, is believed to be in some what affected by the salmon price. This is because, as mentioned earlier, a high oil price might create a tough situation for salmon export, and therefore a high salmon price might mean a low oil price and therefore correlate with the oil companies. The same would then be true with Norsk Hydro at least up until 2008.

For aluminum, a correlation is most likely expected with the airlines except the obvious Norsk Hydro. For airlines, the aluminum price might have an affect especially at times of ordering new airplanes to their fleets. The same can be said for Seadrill whenever they buy or create oil related equipment like rigs etc that uses aluminum. MHG is not expected to be affected much, although they are both important parts of the Norwegian economy. A volatile aluminum price might be a representative for the economy in general and therefore affect MHG.

2.3 Risk management

Financial institutions are obliged to measure their risk of their assets in commodities or their investments in stocks, currency or other types. Over the years the Basel Committee on Banking Supervision (2013) has released a stricter control over the capital requirements. Combined with events like the financial crisis, the task of calculating risk has been a hot topic and an increased necessity to the financial institutions.

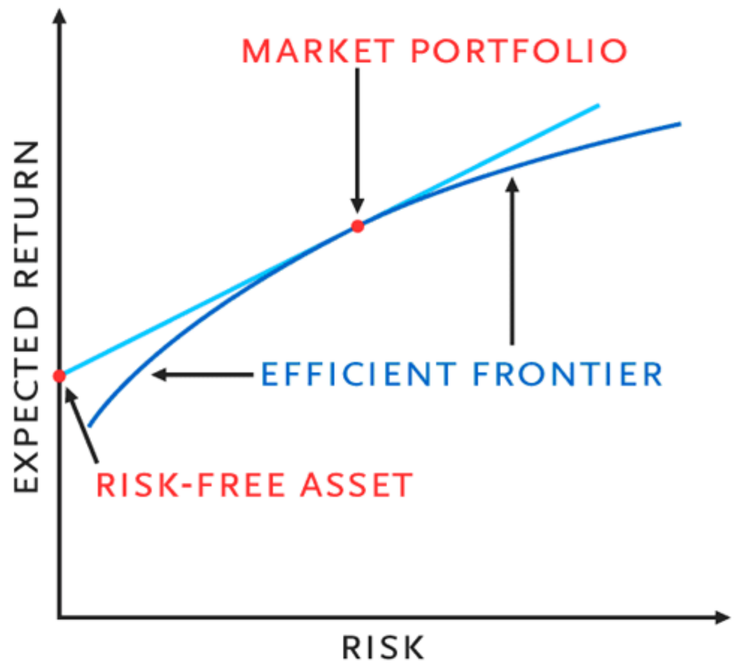
2.3.1 Portfolio theory

Markowitz (1952) introduced modern portfolio theory in order to address risk, and combined risk with return and utility to optimally allocate capital. He drew an efficient frontier given the different option, and the calculated minimum variance portfolio. All the portfolios on the efficient frontier line represents the optimal allocation of capital. Sharpe (1982) then followed with his theories on asset pricing with five market assumptions. These assumptions are:

1. Risk free interest rate, and no limits on borrowing/lending
2. Asset is fully described using mean, standard deviation and correlation
3. No limit to quantitiy sold of bought
4. Investors have same information
5. Investors are risk averse

These assumptions created the capital market line (CML), which goes tangentially on the efficient frontier line and creates the market portfolio. Combining these two we create the Capital Asset Pricing Model (CAPM), which tells us the relationship between risk of one asset compared to the risk in the market portfolio. Below is a figure to illustrate these theories.

Figure 3: Shifts in supply and demand



The CAPM formula is as follows

$$E_{ri} = r_f + (E_{rm} - r_f)\frac{\sigma_{i,M}}{\sigma_M^2} \tag{1}$$

This equation is identical to
Return on asset = risk free rate + (risk premium) market risk
As a result of financial crisis and importance of risk management, value-at-risk was invented to ease the way we calculate risk. This will be discussed further in chapter 5.

2.3.2 Hedging

Another way to manage risk is to hedge investments. A perfect hedge would be something that moves 100% in the opposite direction of our investment. This is not realistic so we often hedge in something that secures our investment if the market goes down. An example can be to hedge the price of oil by securing the downfall with airline stocks. If the price of oil goes down, the airlines make more profit and the stock will save us from the decline in our commodity investment.

Popular hedging methods are derivatives, which include options, future and forward contracts. An option is a contract that gives the buyer of that contract the right to either buy (call option) or sell (put option) at the price manifested in the contract (Staff, 2017).The contract also includes a expiration date. A forward contract is a type of hedge where the buyer and seller agrees upon a price and quantity of a commodity. In addition, they agree upon when the contract is to be executed, for instance buying x amount of sugar for price y six months from signing the contract. Compared to a forward contract, a future contract is similar but with a couple of differences. Futures contracts are traded on an exchange, and is settled daily instead of at the end of the contract (Staff, 2017). This makes its use mainly by price speculators.

All hedging does come at a cost, and even though it is always good to be secured one might miss out on a potential profit as a result. If your investment does well and you hedge it, you will make a smaller profit although you are secured against the downside. That is why hedging is easily described as being an insurance policy in case something goes wrong, which reduces the risk you take.

2.3.3 Diversification

Divided into two types, risk is either undiversifiable or diversifiable. Undiversifiable risk is not possible to eliminate and is associated with market risk. This type of risk is exchange rates, inflation, interest rates or even wars and political new events. Diversifiable risk is the type of risk you can eliminate using diversification as the risk is related to a company or an industry (Staff, 2017). A major technique in regards to risk management is diversification of the investments. It is a good methods of reducing risk and is based on diversification of the portfolio. By that we mean to not to put all our eggs in one basket, and invest in different areas. By investing in different areas, you protect yourself against big events or news that would affect your stocks, because these events have opposite impact on your diverse portfolio. To give an example based on our chosen stocks, we could protect ourselves against a low oil price if we were an oil company, by including a airline company in our portfolio that makes more money when the fuel prices are low. Another way is to create future contracts with the airline companies to protect against a declining oil price, as the price would be set. However, risk will always be present in some degree.

3 Statistical theory

In this section we will cover some basic theory regarding volatility and different methods of modelling volatility based on historical data. The thesis focuses on three methods, which are later used in value-at-risk estimation of market risk. All estimations will be backtested in order to test the validity of the models.

3.1 Statistical Calculations

All the data for the commodities and stocks were collected and systemised in Excel. Ordered from oldest to newest, the daily returns were calculated using the natural logarithm.

$$\ln \frac{p_t}{p_{t-1}} \tag{2}$$

This led to the calculations of descriptive analysis presented in chapter 4 which included mean, standard deviation, skewness and kurtosis. The sample mean, also known as the expected value is calculated using the following formula.

$$\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \tag{3}$$

This is later used in calculations of variance (σ^2). Variance describes the distance between the observations and the mean of the sample. This is done through equation (4)

$$S^2 = \frac{\sum(X - \bar{X})^2}{n-1} \tag{4}$$

Standard deviation is the square root of variance in formula (4) Skewness is a measure of asymmetry compared to the normal distribution and shows if the curve is to the left or right of the normal distribution. This is done using equation (5)

$$Skewness = \frac{1}{n} \frac{\sum_{i=1}^{n}(X_i - \bar{X})^3}{s^3} \tag{5}$$

Kurtosis describes if the distribution of the data has a higher or lower peak than the normal distribution. A higher peak results is fatter tails, and a lower peak results in a more even distribution with even fatter tails. This statistical property is calculated by

$$K = \frac{1}{n} \frac{\sum_{i=1}^{n}(X_i - \bar{X})^4}{s^4} \tag{6}$$

To see if the data follows a normal distribution we test the data using a Jarque-Bera test introduced by Jarque and Bera (1980). This is done using the following formula.

$$JB = \frac{n-k+1}{6}\left(S^2 + \frac{1}{4}(C-3)^2\right) \tag{7}$$

The Jarque-Bera value follows a chi-squared (χ^2) distribution with 2 degrees of freedom. The null hypothesis is that the distribution is normal.

$$H_0 : S = 0, K = 3 \tag{8}$$

Where S is skewness and K i kurtosis, testing against the expected values of a normal distribution to have zero skewness and zero excess kurtosis.

Finally an Augmented Dickey Fuller (ADF) test, created by Dickey and Fuller (1979), is performed on our data to test for stationarity. This is done using the following formula

$$\Delta y_t = \psi y_{t-1} + \sum_{i=1}^{p} \alpha_1 \Delta y_{t-i} + u_t \tag{9}$$

The difference between the standard test and the augmented one is determining the optimal number of lags. In this thesis, the optimal number is calculated by a computer and differs slightly given the different data.

3.1.1 Correlation

To see how the different stocks correlate to the commodities, a window of n=250 days is chosen and moved throughout the sample. This means that we check for correlation during the 250 first days of one stock with the 250 first days of a commodity. The observation window is then moved one day ahead and looks at the next 250 days, giving us observations of 1:250, 2:251, 3:252 etc. This was done for all stocks with oil first, the results are graphically presented in chapter 6. The process was repeated with the stocks and gas.

For the correlation with salmon, all the data for the stock prices was changed into weekly returns. This was done by filtering out all the first and the last values for each week. The formula for daily returns was used as seen in equation (2), where p_t was the last value of each week, and p_{t-1} was the first value of each week.

Lastly for the correlation with aluminum it follows the same procedure as with salmon. Now the first and the last value for each month was filtered out, and used in equation (2), where p_t is the last the of each month and p_{t-1} is the first of each month.

3.2 Historical Volatility

The basic model of volatility is estimation based on historical data. It is one of the simplest methods as it only calculates the variance, or standard deviation, over a historical period and uses this in future volatility forecasts. Today, it is more common to use more sophisticated models, however the basic historical volatility model is still used for reference when comparing result with more complex models.

3.2.1 Rolling Analysis

To check the stability of a time series model over time, rolling analysis is a common choice. Discussed in Zivot (2006), rolling analysis assumes constant parameters over time, however this might not be reasonable because of changing parameters in real life. To test for the consistency of the model we estimate the parameter over a rolling window of a fixed size throughout the sample. The estimates over the rolling windows will not change if there are constant parameters and will differ if the parameters are non-constant. A common usage for the rolling analysis is to backtest the statistical models on our historical data to see if the data is stable and able to predict models accurately. To do this the data is split into an estimation sample and a prediction sample. Using the estimation sample to create the model and h-step ahead predictions are created from the prediction sample. By rolling the estimations sample ahead at a given increment, we then repeat the estimation and prediction exercise until no more h-step predictions are possible. Prediction error forms as the predictions are observed h-step ahead, and summarising those prediction errors the statistical model can be evaluated (Zivot, 2006).

3.2.2 Moving Average

A common rolling analysis is using moving average models, also known as the heart of financial time series analysis (Zivot, 2006). It is simply a linear combination of white noise processes, where $\mathrm{y_t}$ depends on current and previous values of the disturbance term. A lag operator is introduced to show the usage of previous values. A moving average process is one with a constant mean, variance and autocovariances, and may be non-zero until lag q, and always zero after. The properties of a moving average process, as presented in Brooks (2008), of a MA(q) are listed below.

$$E(y_t) = \mu \tag{10}$$

$$var(y_t) = \epsilon_0 = (1 + \theta_1^2 + \theta_2^2 + ... + \theta_q^2)\sigma^2 \tag{11}$$

covariances ϵ_s

$$= (\theta_s + \theta_{s+1}\theta_1 + \theta_{s+2}\theta_2 + ... + \theta_q\theta_{q-s})\sigma^2 \tag{12}$$

for s = 1, 2, ..., q
0 for s > q

3.2.3 Exponentially Weighted Moving Average

The rolling descriptive statistics discussed are based on equally weighted moving averages of the observed time series. This is useful to uncover unstable periods but may also give wrong predictions in the short-term. To avoid effects on extreme observations the rolling window can be weighted differently. Essentially, EWMA is a simple extension of the historical average volatility measure, which allows more recent observations to have a stronger impact on the forecast of volatility than older data point. As discussed in Brooks (2008), the latest observation carries the largest weight in EWMA, and the weights decline exponentially over time. Compared to the basic historical model EWMA will account for a stronger impact of recent news. Also as the event has passed, the effect of the news will decline exponentially over time. With the basic model the news will affect the volatility model as long as the news are in the sample and potentially show high volatility even when the market has settle down. Furthermore it can cause extreme change as the event falls out of the sample. Equation 13 shows the common way of expressing a EWMA model.

$$\sigma_t^2 = (1 - \lambda)\sum_{j=0}^{\infty} \lambda^j (r_{\text{t-j}} - \bar{r})^2 \tag{13}$$

Where ${\sigma_\text{t}}^2$ is the estimate of variance over period t, and also a forecast of the volatility for all periods. $\bar{r}$ is the average return estimated, and λ is the decay factor, which commonly has the value of 0.94. For daily data it is normal for the average return to be close to zero which simplifies the equation further.

Important to remember is that when the infinite sum is replaced with a finite sum of observable data, the weights from the given expression will now sum to less than one (Brooks, 2008 p. 384-385). In the case of small samples, this could make a large difference to the computed EWMA. Second, most time-series models such as GARCH, will have forecasts that tends towards the unconditional variance as the series increases. This is good because volatility is mean reverting, which is not accounted for in EWMA (Brooks, 2008 p. 384-385).

3.3 Autoregressive Model

The autoregressive model has its current value of y depending only on its previous value plus an error term. A model of order p, written as AR(p), is expressed as

$$y_t = \mu + \phi_1 y_{\text{t-1}} + \phi_2 y_{\text{t-2}} + ... + \phi_p y_{\text{t-p}} + u_t \tag{14}$$

Stationarity is desirable when estimating AR models and for several reasons. Firstly stationarity influences the behaviour of the time series. For instance stationary series will gradually be less impacted by shocks after they have occurred, while non-stationary series will feel the affect indefinitely. Another reason is that non stationary series might lead to spurious regressions, which means that two variables that are not related might show a high R^2 value if the variables are trending over time. Lastly non stationary regression models fails the standard assumptions of assymptotic analysis, which means that the F- and t- statistics will not follow their expected F- and t- distributions (Brooks, 2008 p 318-320).

3.3.1 ARMA

By combining the topics discussed, an autoregressive moving average model is obtained. This model combines the MA model with the AR model to form a ARMA(p,q) model. Now the model depends linearly on the previous values in addition to the combination of current and previous values of the white noise error term. By creating time series models we aim to model the underlying stochastic process. We then use this model to analyse the structure of the process to optimise our predictions. ARMA models is widely used for second-order stationary processes (Brooks, 2008 p. 223-224). It is also used to model time series dependency. With a ARMA model, we first model that the observations depend on its lagged observations. This can be shown by an AR(1) model.

$$x_t = \phi x_{\text{t-1}} + e_t \tag{15}$$

where $e_{\text{t}} \sim WN(0, \sigma^2)$. The error terms are assumed to be normally distributed with mean 0 and constant variance. Another part of the ARMA model is that we model the observations of a random variable at time t, and say that it is not only affected by the present shock, but also the shocks that already happened previous. This is expressed in a moving average

(MA) model were shocks affects the time after a negative or positive event has occurred.

$$x_t = e_t + \theta e_{\text{t-1}} \tag{16}$$

Combining these two models were have a general ARMA(p,q) model.

$$x_t = \phi_1 x_{\text{t-1}} + \phi_2 x_{\text{t-2}} + ... + \phi_p x_{\text{t-p}} + e_t + \theta e_{\text{t-1}} + ... + \theta_q e_{\text{t-q}} \tag{17}$$

This gives us the most basic tool in time series modelling (Brooks, 2008 p. 223-224).

3.3.2 ARCH

Autoregressive conditionally heteroskedastic (ARCH) models was first introduced by Engle (1982). It introduces the idea that financial time series are not homoskedastic, but heteroskedastic, meaning the variance is not constant. ARCH models want to explain how the variance changes, combined with describing what is known as volatility clustering, a term for large changes in prices following large changes, and small changes following small changes. Also volatility happens in bursts. Therefore, in order to described this common phenomenon in a model, the ARCH model was the first approach described.

3.3.3 GARCH

Generalised ARCH models, or GARCH was introduced by Bollerslev (1986), and allows the conditional variance to depend on its previous lags. In its simplest form expressed as:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{18}$$

Equation(18) is known as a GARCH(1,1) model. σ_t^2 is a one-period ahead estimate of the variance based on the historical data, and is what we call conditional variance. The GARCH(1,1) model can be expressed as a GARCH(p,q), where p is thee lags of conditional variance and q is the lags of the squared error.

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{19}$$

In general a GARCH(1,1) model is preferred to capture the volatility clustering throughout the data. A GARCH model is not estimated using OLS, because OLS will minimise the residual sum of squares, and focuses on the conditional mean and not the conditional variance. For GARCH model, the maximum likelihood is used for estimation, which uses the data to find the most likely value. For more detailed description, see Bollerslev (1986) or Hamilton (1994).

3.3.4 GARCH(1,1)

The rolling standard deviation model and the EWMA model for volatility has been sufficiently discussed. In this subsection some of the important steps for creating a GARCH(1,1) model are discussed.

Since we are dealing with the conditional variance, it must always be positive. A negative variance would not make sense and therefore ARCH models require a non-negative constraint. In order to estimate a GARCH model, it is necessary to test for ARCH effects to check if the GARCH model is appropriate for the data. In order to do so there are several steps, and a summary of those steps are found below and are taken from (Brooks, 2008 p 380).

1. Run any linear regression for instance

$$y_t = \beta_1 + \beta_2 x_{2t} + \beta_3 x3t + \beta_4 x4t + u_t \tag{20}$$

   saving the residuals $\hat{u}_t$

2. Calculate the squared residuals, and test ARCH(q) by running the regression

$$\hat{u}_t^2 = \gamma_0 + \gamma_1 \hat{u}_{t-1}^2 + \gamma_2 \hat{u}_{t-2}^2 + ... + \gamma_q \hat{u}_{t-q}^2 + v_t \tag{21}$$

   with v_t is the error term

3. We then test the number of observations multiplied with the R^2 from the regression, as a $\chi^2(q)$ distribution.

4. The null hypothesis is
   $H_0 : \gamma_1 = 0$ and $\gamma_2 = 0$ and $\gamma_3 = 0..$ and $\gamma_q = 0$
   alternative hypothesis is not equal to zero.

This test is also a test for autocorrelation in the squared residuals. In our data we have used a Ljung- Box (1978) test for autocorrelation and its formula is shown below.

$$Q* = T(T+2) \sum_{k=1}^{m} \frac{\hat{\tau}_k^2}{(T-k)} \sim \chi_m^2 \tag{22}$$

where $\hat{\tau}_k$ is sample autocorrelation at lag k, m is the number of lags tested with an n-size sample. This statistic uses the residuals of a fitted ARIMA model, and the degrees of freedom depends on the fitte model. For instance a ARIMA(1,1) model gives m-1-1 degrees of freedom. In our modelling we have used the Bayesian Information Criterion (BIC) introduced by Schwarz (1978) to fit this model. BIC adds a penalty term in order to prevent overfitting the model, and the lowest score is preferred. Since the returns are mean reverting, we expect the returns to be stationary and therefore fitting an ARIMA(p,d,q) model will actually be a ARMA(p,q) model.

When fitting a model one needs to check the data to select the most appropriate specification. After this one needs to estimate the parameters for the model, and the most common way to do so when it comes to ARMA modelling is maximum likelihood estimation (MLE). This means finding the parameters with the most likely values given our data. The steps from Brooks (2008) to estimate this is found below.

1. Specify the model with equations for mean and variance. Example is an AR(1)-GARCH(1,1) model.
$$y_t = \mu + \phi y_{t-1} + u_t, u_t \sim N(0, \sigma_t^2) \tag{23}$$
$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta\sigma_{t-1}^2 \tag{24}$$
2. Specifiying log-likelihood function (LLF)
$$L = -\frac{T}{2}log(2\pi) - \frac{1}{2}\sum_{t=1}^{T} log(\sigma_t^2) - \frac{1}{2}\sum_{t=1}^{T}(y_t - \mu - \phi y_{t-1})^2/\sigma_t^2 \tag{25}$$
3. Parameters that maximise the LLF and show their standard errors are estimated by the computer

These steps gives us the parameters that most likely have produced the data we have, and gives us our model.

3.3.5 EGARCH

An exponential version of the GARCH model was added by Nelson (1991). An example of the equation is found in (26).

$$ln(\sigma^2) = \omega + \beta ln({\sigma_{\text{t-1}}}^2) + \gamma\frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \alpha\left[\frac{|u_{t-1}|}{\sqrt{\sigma_{t-1}^2}} - \sqrt{\frac{2}{\pi}}\right] \tag{26}$$

Compared to GARCH, the EGARCH has several advantages as stated by Brooks (2008). The $\log(\sigma_t^2)$ is modelled, which will always be positive even if the parameters are negative. This mean we do not have to put constraints of non-negativity on the model and assymetries are allowed. This is because γ will follow the relationship between the volatility and the returns. Although, based on tests performed on who best describes the variance, simple GARCH and EGARCH are favourable according to Hamilton (1994), and that their basic (1,1) version are sufficient for estimations.

4 Data

The data used in the analysis is obtained from several places. All the stock data comes from Yahoo finance, and are historical data going back to January 3rd 2000, although several stocks have a shorter observation period. The historical oil and gas prices are taken from the US Energy Information Administration (EIA), and dates back to January 4th 2000. Salmon and aluminum prices have been obtained through the Norwegian statistical company SSB (Statistics Norway), and contains weekly salmon prices and monthly aluminum prices from the beginning of year 2000. The prices are in Norwegian Kroner (NOK), except oil and gas prices which are in Dollars. The natural logarithm is applied to the daily prices. A summary of the descriptive statistics of both the daily data and log-returns are summarised in Table 2-5.

4.1 Descriptive Statistics

This section summarises the data using common statistical properties. Table 2 describes the data for the commodities, table 3 describes the data for the stocks. Table 4 and 5 shows the statistical properties of the same data set with log returns. As you can see, the number of observations vary because oil and gas have daily prices, salmon has weekly prices and aluminum has monthly prices. All have positive skewness, which is used to describe asymmetry from normal distribution. In this case the data is positive meaning it is skewed to the right of the normal bell shaped curve. The kurtosis, often referred to as volatility of volatility, measures the weight of the distributions tail relative to the rest of the distribution. None of the price data have positive excess kurtosis, which is calculated by subtracting 3 from the kurtosis. A normally distributed data sett has a skewness of 0 and kurtosis of 3.

Table 2: Descriptive statistics of commodities; 3. January 2000 - 17. February 2017

Statistics	Daily prices			
	Oil	Gas	Salmon	Aluminum
Descriptive statistics				
Frequency	Daily	Daily	Weekly	Monthly
Observations	4329	4283	893	205
Mean	64.486	4.830	32.669	11626.178
Std. dev.	32.163	2.241	10.539	1696.870
Skewness	0.368	1.398	1.459	0.104
Kurtosis	-1.163	2.621	2.169	-0.570
Hypothesis test results				
J-B	804 (0.00)	62276(0.00)	487 (0.00)	3.29 (0.193)
ADF	-14.7 (0.00)	-17.1 (0.00)	-2.58 (0.00)	3.57 (0.0375)

Furthermore a Jarque-Bera test has been performed on the data sets to test for normality. All J-B values are high, with its p-value being close to 0. The null hypothesis of the test states that the data is normally distributed, and a rejection would mean the data is non-normally distributed. As we can see from table 2-5, we an reject the null hypothesis of normally distributed data sets for both the commodities and the stocks and the log-series at 1% significance level. Exception is the log series for aluminum, which we reject at 5% significance level.

To test for the stationarity of the data, Augmented Dickey Fuller (ADF) tests have been run to test data against a null hypothesis of a unit root. Stationarity is desired in autoregressive models, especially due to values not having a declining effect on the current value as time progresses. As seen in tables 2-5, all are data have small p- values, which rejects the null hypothesis of a unit root, and the sample data is indeed stationary. All data is given different lags in order to remove autocorrelation, which is the relationship between a variable‘s current value and its lagged value.

Table 3: Descriptive statistics of stocks; 3. January 2000 - 17. February 2017

Statistics	Daily prices							
	Statoil	Norsk Hydro	Norwegian	SAS	MHG	Seadrill	Lufthansa	DNO
Descriptive statistics								
Period start	03.01.2000	03.01.2000	18.12.2003	09.07.2001	10.10.2003	22.11.2005	03.01.2000	03.01.2000
Observations	4453	4453	3371	4059	3478	2888	4430	4452
Mean	91.816	22.420	140.177	85.849	53.578	128.929	12.497	7.400
Std. dev.	36.4	16.607	97.983	79.735	34.385	70.725	3.479	5.409
Skewness	0.305	0.124	0.644	0.970	1.211	0.244	0.645	0.813
Kurtosis	-1.081	-0.806	-0.742	0.252	0.962	-0.969	-0.156	0.596
Hypothesis test results								
J-B	1721 (0.00)	21683967 (0.00)	62.7 (0.00)	9050 (0.00)	25486 (0.00)	43988 (0.00)	948 (0.00)	107782 (0.00)
ADF	-17.6 (0.00)	-16.0 (0.00)	-13.3 (0.00)	-15.0 (0.00)	-13.2 (0.00)	-13.7 (0.00)	-16.1 (0.00)	-16.1 (0.00)

Looking at the distribution of the stock data in table 3, there are slight positive skewness in all the stocks, and the excess kurtosis will all be negative. This indicates fat tails and even distribution which is common for stocks as the prices of a stock is not normally distributed over time.

In this thesis we are interested in the daily returns, and therefore the log-series are more important to look at. Comparing to the previous tables we can see a much higher kurtosis for both the stocks and the commodities. Here we have positive excess kurtosis, also known as leptokurtic distribution which means a higher peaked distribution compared to a normal distribution and fat tails. The positive kurtosis is also a sign of a non-normal distribution which is strengthened with the rejected of the null hypothesis of the Jarque-Bera test of normality. We also note that the means are close to 0, which shows why we often simplify the equations by setting the mean = 0 for daily returns.

Table 4: Descriptive statistics of log-returns for commodities; 3. January 2000 - 17. February 2017

Statistics	Log Returns			
	Oil	Gas	Salmon	Aluminum
Descriptive statistics				
Frequency	Daily	Daily	Weekly	Monthly
Observations	4328	4428	892	204
Mean	0.0002	0.0001	0.00095	0.00090
Std. dev.	0.023	0.045	0.0388	0.0462
Skewness	-0.132	0.649	0.125	-0.1003
Kurtosis	5.095	21.638	0.833	2.385
Hypothesis test results				
J-B	5262.3 (0.00)	94974 (0.00)	27.395 (0.00)	46.493 (0.00)
ADF	-15.233 (0.00)	-17.825 (0.00)	-9.280 (0.00)	-5.865 (0.00)
LB - Q(12)	8.0475 (0.781)	207.93 (0.00)	65.533 (0.00)	20.927 (0.0514)
LB - Qs(12)	506.50 (0.00)	1485.8 (0.00)	163.45 (0.00)	60.661 (0.00)

Table 5: Descriptive statistics of log-returns for stocks; 3. January 2000 - 17. February 2017

Statistics	Log Returns							
	Statoil	Norsk Hydro	Norwegian	SAS	MHG	Seadrill	Lufthansa	DNO
Descriptive statistics								
Period start	03.01.2000	03.01.2000	18.12.2003	09.07.2001	10.10.2003	22.11.2005	03.01.2000	03.01.2000
Observations	4452	4452	3370	4058	3477	2887	4429	4451
Mean	0.0003	0.0014	0.001	-0.001	0.0002	-0.0004	-0.0001	0.0003
Std. dev.	0.019	0.036	0.030	0.032	0.051	0.037	0.022	0.036
Skewness	-0.035	14.094	0.274	0.209	0.006	0.447	-0.105	-0.584
Kurtosis	6.046	343.735	3.383	10.304	16.264	22.102	5.257	27.079
Hypothesis test results								
J-B	6763 (0.00)	22015000 (0.00)	5548 (0.00)	24899 (0.00)	86812 (0.00)	43988 (0.00)	948(0.00)	107782 (0.00)
ADF	-17.6 (0.00)	-16.0 (0.00)	-14.6 (0.00)	-15.5 (0.00)	-14.3 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)
LB - Q(12)	40.3 (0.00)	10.3 (0.00)	39.5 (0.00)	30.4 (0.00)	579.9 (0.00)	1 (0.00)	1 (0.00)	1 (0.00)
LB - Qs(12)	878.7(0.00)	0.0000 (0.00)	652 (0.00)	440.9 (0.00)	58277.3 (0.00)	0.0000 (0.00)	0.0000 (0.00)	0.0000 (0.00)

We have also performed a Ljung-Box (1978) test on the log returns for autocorrelation. This has been done on the log returns denoted LB - Q, and on their squared residuals denoted LB - Qs. The test have been performed with 12 degrees of freedom, and we reject the null hypothesis of autocorrelation for both the stocks and the commodities.

4.2 Sample period

The sample period of this thesis is from the beginning of the century until today. During a period of 17 years, there will always be events that affect the prices, and certain events have a larger impact than others. As mentioned previously , these events give us new information and are used in order to set the price of the commodity. A war would give us the information that more oil will be demanded, and we would expect an increase in the price, which again affects our companies. Those periods of greater impact are important to address when doing analysis and creating models for the volatility of the different stocks and commodities.
One of the obvious events during our sample period is the financial crisis. Beginning in 2007 continuing into 2009, an American mortgage bubble led to a banking crisis which eventually led to the collapse of investment bank Lehman Brothers, one of the largest in the country. Even though a collapse of the world‘s financial system was avoided, a recession followed in the US and Europe (En.wikipedia.org, 2017). Hence, this period was a period with high volatility, which we will use to compare against the periods with low volatility. All the commodities and stocks used were affected by this period, which is shown graphically in the next two sections.

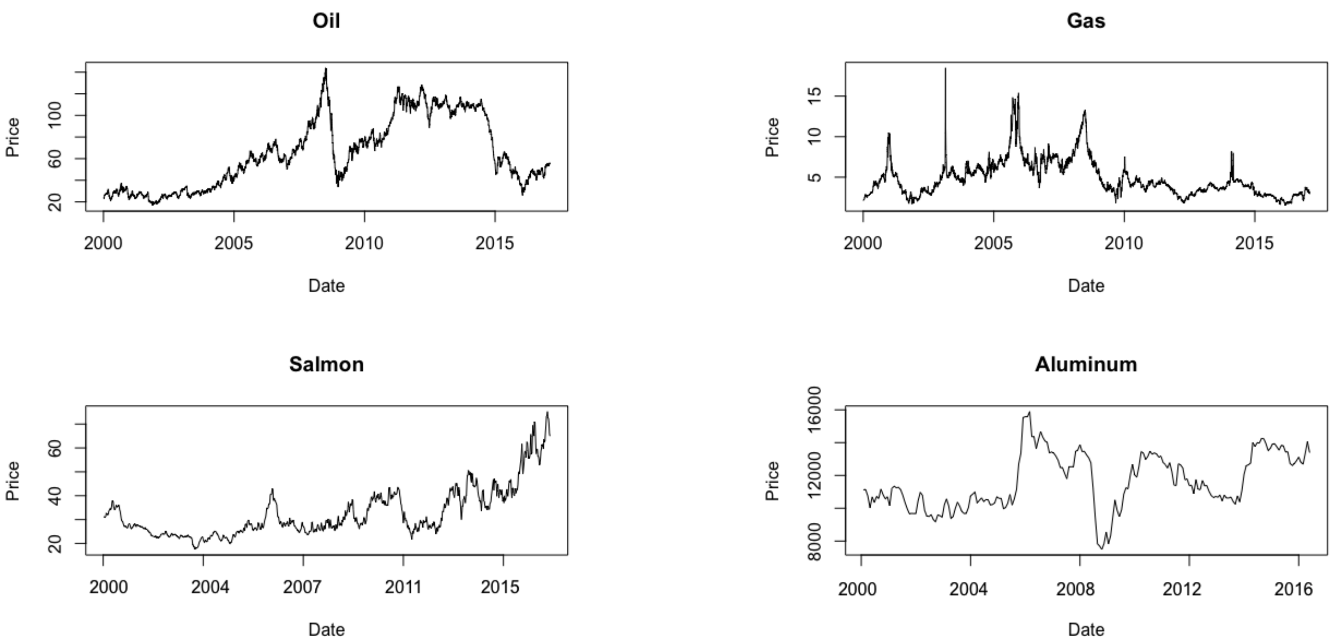
A more recent and interesting period, given our commodities, is the period after 2014. In 2014 the oil price started falling, and has remained low since. With oil-thirsty countries like China, Russia, India and Brazil having slower growth after 2010, the demand began to slow down. Combined with the American and Canadian success with shale oil, the demand for importing oil were drastically reduced. As the oil price went further down, Saudi-Arabia knew they could live with a lower oil price, as long as they kept their market share. Therefore they decided to not reduce their production in hope of squeezing the shale production out of business. Today the oil price has slowly recovered, but much slower and at a smaller scale than after previous blows to the price. As a result, the period during the last couple of years are also an example of the prices volatility. For the Norwegian markets this affects not only gas as well, but the whole economy which includes other commodities such as salmon and aluminum.

These two important periods are also an example on new information affecting the price setting of the commodities, and that the market is affected either by a larger trend like the decline in oil price, or a smaller correction due to a company having released positive numbers for their first quarter.

4.2.1 Commodities

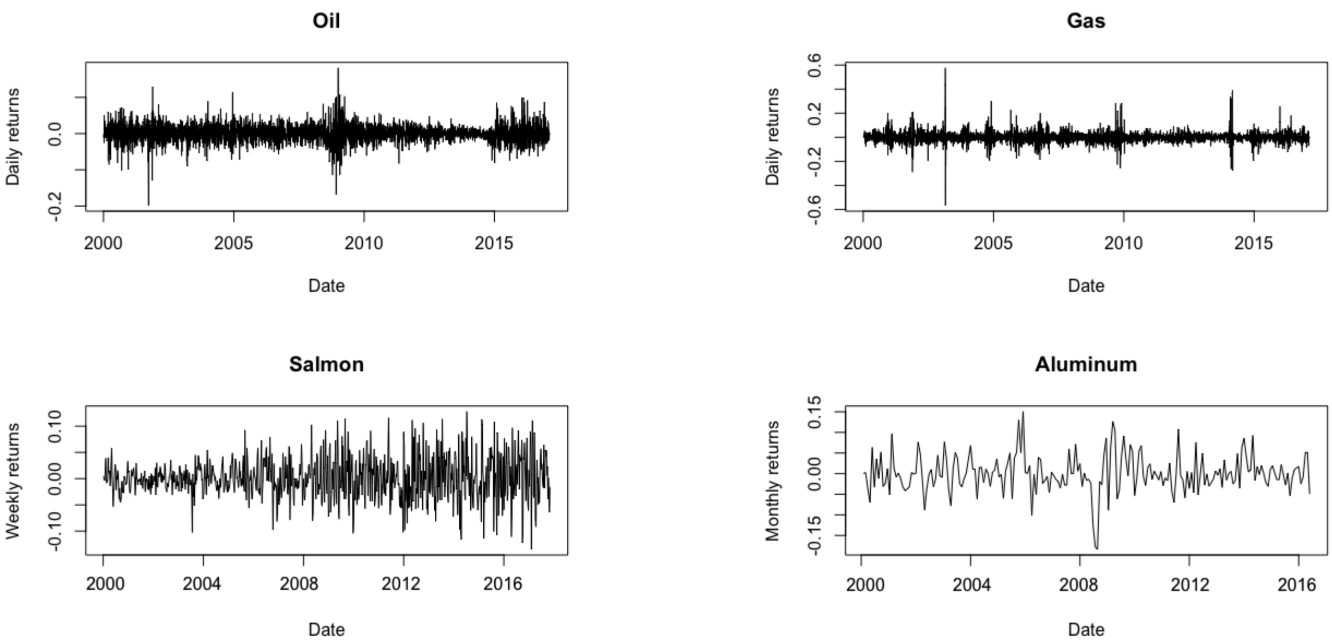
The data for our commodities will be graphically presented and compared to what we have discussed above about our sample period. Looking at figure 4 we see a graph of historical prices of the four commodities. As seen the oil price had an upward trend from the beginning of our sample up until our first major event, the financial crisis. The rapid fall of the oil price is a clear indication of a drastic downfall in the economy as well as uncertain times. The gas price has had volatile periods up until the financial crisis, and has maintained a low price ever since with less volatility. The salmon price has had volatile periods and was not as much affected by the financial crisis as the other. Also in recent years the salmon price has experienced a growth and led to the highest price during the period. The aluminum price has been fairly stable, although taking a massive hit in 2008, the price swings back and forth between a noticeable mean.

Figure 4: Historical price for commodities



In figure 5, we see a graph of the log return of the commodities prices over time. The daily returns look fairly stable, except for during the financial crisis and at the end of our sample period. The graph show evidence of volatility clustering, which means that large losses tend to follow large losses, and is what our models aim to address. During the financial crisis we also see clear evidence of leverage effect. A large price fall is followed by an increase in volatility, which we see in figure 5. The returns for gas is a quite identical one of the oil price, although at a smaller scale and has several more stable periods than the oil returns. Nonetheless there are some periods with quite high volatility burst for gas, which might be explained by non-oil related news even though oil and gas are correlated.

Figure 5: Log returns for commodities

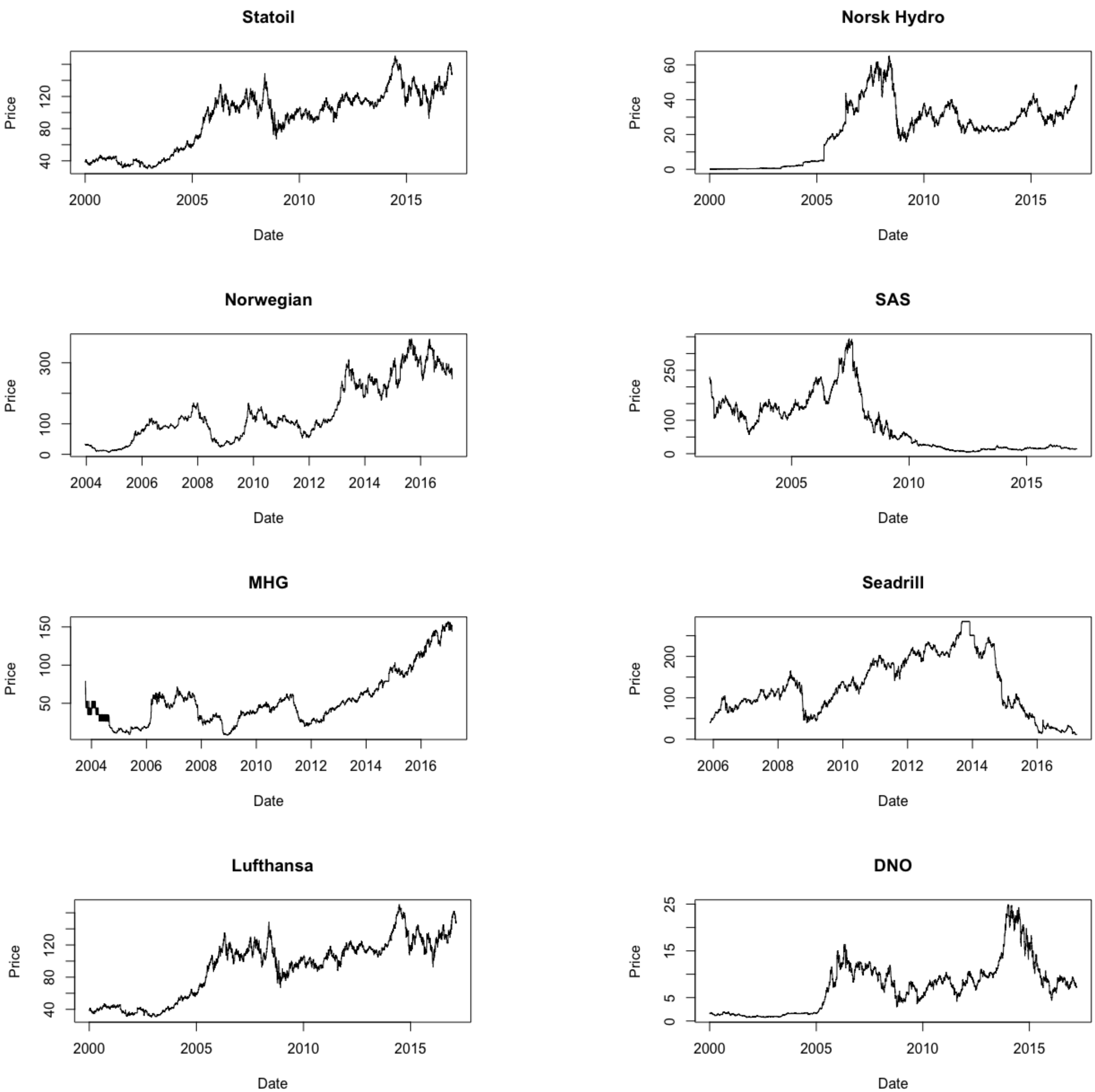


With the weekly and monthly returns of the salmon and aluminum prices the graph changes a bit from the previous two. Salmon shows a volatile but fairly stable beginning, before increasing rapidly and has a quite volatile weekly return for the second half of our sample. This begins during the financial crisis, and never stabilises after that. One reason behind this might be the uncertainty that the Norwegian seafood industry has had with China in recent years, combined with the financial uncertainty during the crisis and afterwards. For aluminum we see a relatively stable price given its monthly returns. The financial crisis is an obvious exception here, and is followed by some volatility in the price but also the least volatility during our sample period. The beginning of the millennium seems to have had a greater uncertainty for the aluminum price than recent years, which might have been due to wars or uncertainty in the airline business.

4.3 Stocks

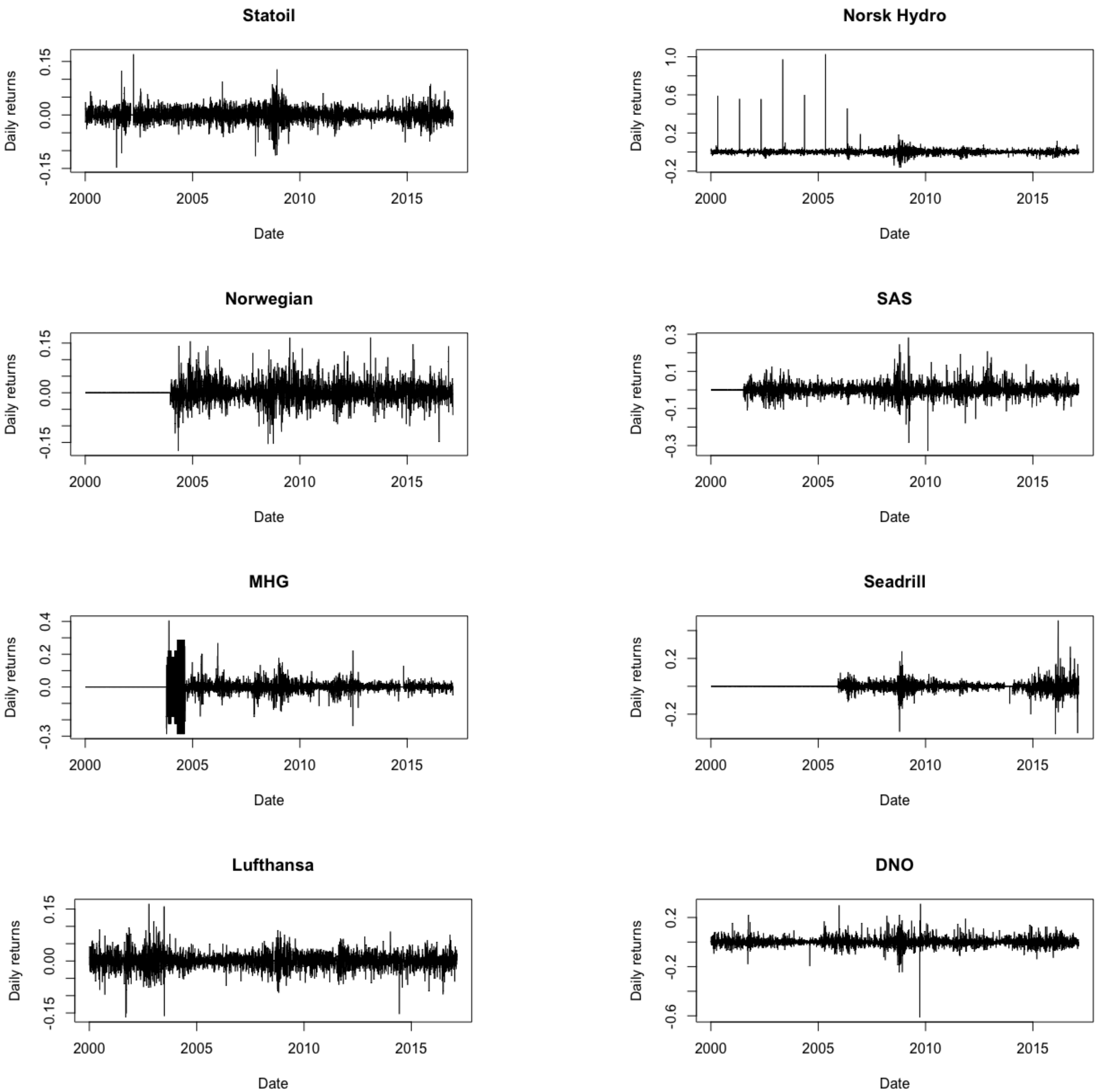
Next up is the stocks, and for comparison, figure 6 and 7 shows the historical stock prices and daily returns for all the stocks. For the oil companies we see the same pattern as the oil price, which is intuitive since they are bound by the price of oil. An increase, followed by a drop during the financial crisis is seen. In recent years the oil drop has affected Statoil, however we see a high stock price today. Comparing Statoil to the oil price, we see the graphs are quite similar which gives us a sense of expected correlation between the two. The only difference really is that the oil price is still quite low today, while Statoil‘s stock has risen up after not falling to much down after 2014. A reason for that is because of cutting costs and becoming more efficient with their projects. That means that they are as a company, regardless of the low oil price, managing to make profit and grow even though they have to do changes to achieve it. Looking at Seadrill and DNO, it seems they have not been able to recover as well as Statoil. Looking at the airlines both Norwegian and Lufthansa had growth up until the financial crisis, and continued growth after it passed. SAS has had many financial troubles in recent years, and therefore their stock has remained low since the crisis. MHG has a similar growth as the salmon price has reached its highest price ever at en of our sample period. Norsk Hydro has been fairly stable after the crisis and them leaving the oil industry.

Figure 6: Historical stock prices



Looking at the log returns, there might be evidence of even more volatility clustering for the Statoil graph, which implies that the stock was affected due to the high uncertainty in the oil price. The volatile oil price during the financial crisis and in recent years affected the stock price stability, but due to changes they managed to keep the stock at a reasonable level. The other oil companies show similar trends as Statoil, with Seadrill having a quite volatile stock in recent years. This indicate that our expectations about them being more vulnerable in this time due to fewer to no new projects. DNO also shows that during the financial crisis the stock witnesses quite the volatile burst, in which the size of the company might have effected their ability to handle the uncertain oil price.

Figure 7: Daily returns for stocks



Looking at the return for our airline companies, Lufthansa and SAS had a uncertain start. Especially Lufthansa who was affected by the aftermath of 9/11, and expansion of their fleet let to a volatile stock price. Most of the uncertainty related to airlines are related to their expansion of their fleet, and looking at Norwegian a lot of uncertainty has followed the stock throughout its existence. As a low cost airline they have always been dependent on profit from lower fuel prices, as well as not running into problems with their aircrafts. Norwegian has been in the news plenty of times over the years due to cancelations and delays. These have often been as a result of a new order of aircrafts which has created a volatile stock throughout its history. This can also be explained by them being a rapidly growing airline whom needed to expand often in order to keep up with their success.

Norsk Hydro has a similar volatility profile as the oil companies up until the financial crisis which also gave them an uncertain stock. Compared to the return graph for aluminum, they also have enjoyed a relatively stable stock price in recent years without too much volatility. The more volatility bursts happened early on, and is in coherence with the aluminum returns during that time. For MHG, they experience a lot of uncertainty during their beginning, and it lasted all the way into the financial crisis. This was followed by the Chinese boycott and the volatility did not drop until recent years, where MHG has enjoyed a more stable and growing environment for their products. This has happened even though the salmon price has been volatile and not easily predicted.

5 Method: Value-at-Risk (VaR)

In this chapter we present the method used for estimating market risk, and the different approaches for calculating value-at-risk.

5.1 Introduction to VaR

Value at risk, or VaR, is by Alexander (2008) described as a way to determine the maximum amount we expect to lose over a time-period. The two factors used in VaR are a significance level and a time horizon. When calculating VaR, we for instance end up with a number we are 95% sure will not be exceeded based on the historical data that we have. Looking at historical data, we can say with 95% certainty that our loss will not exceed the calculated amount the next day. There are several methods of calculating VaR, all relying on historical data, and are prone to history repeating itself. The most common methods are historical VaR, normal VaR and Monte Carlo VaR.

There are many concern with VaR and its use in calculating risk. It is based on history to repeat itself and calculating risk in special circumstances like the financial crisis can be hard to predict. The extreme events can be hard to map, and things like clustering effects have to be considered. When using a method, it is always important to do backtesting to confirm that the method is working. Here you compare the actual return (t+1) with VaR calculations up until a datapoint t. If you have a 95% VaR calculation then you expect the data to be violated 5% of the dataset.

5.2 VaR methods

Historical VaR is the most common and easiest. With a list of historical data, the daily returns are calculated and ordered by ranking. The 95% for instance, will give a number that shows the VaR, given the chosen significant level.

Normal VaR is also relatively easy to calculate and is also based on calculating the daily returns, but also standard deviation and correlation for the sample period. We then estimate the standard deviation for the returns, and calculate VaR based on the assumptions of normal, independent and identically distributed data. These are also some of the downside of this method as normal distribution often is an optimistic view. Also, a constant variance area assumed with no auto- or serial correlation.

Monte Carlo VaR is a more complex approach where random numbers are drawn. It can potentially map risk factors and assets more accurately which could make a more accurate prediction. The method uses expected return, standard deviation, kurtosis and skewness combined with identifying the underlying daily return distribution of the data. Next the correlation between the risk factors are estimated before random numbers are drawn from the distributions. Finally the VaR is calculated using historical data.

With several ways to improve the accuracy of the VaR calculations, there are two approaches that will be used in this thesis. By estimating the volatility with EWMA and GARCH, there will be a more accurate model of the volatility in the datasets, and with that a more accurate VaR calculations.

For EWMA VaR calculations, the EWMA variance is calculated with equation (13) from chapter 3, and this is then used the calculation of the daily VaR. Since VaR is just a combination of the calculated standard deviation (volatility), and our chosen confidence level, we use the volatility calculated with EWMA to calculated daily VaR. The volatility of the different stocks and commodities using EWMA is graphically presented in chapter 6. VaR is then, given our confidence level of either 95% or 99%, done by multiplying the daily volatility with the z-value corresponding to our confidence level, or using the historical approach. We then test the model by seeing how many times our calculated daily VaR is not captured by the volatility of the data using the EWMA method.

For GARCH and EGARCH VaR, we used the formulas in equation 18 and 26 to obtain the volatility of the data. Likewise as with the EWMA the daily VaR can be calculated in the same way, but switching out the EWMA volatility with GARCH. By either the normal VaR calculation or the historical, we estimate the daily VaR using GARCH and EGARCH volatility and compare them to the daily returns. For both models we have used a GARCH(1,1) and EGARCH(1,1) on the data combined with the fitted ARMA. This will be graphically presented in chapter 6.

Lastly, for these volatility models there are some expectations. Comparing our models to the basic of the historical volatility, we expect our more advanced models to improve the value-at-risk estimations. Firstly the EWMA model is expected to show a better estimation because of its assigning weights that exponentially decline over time. This increases the accuracy as shocks eventually stop impacting the prices of stocks and commodities. Our GARCH models are expected to estimate VaR even better as it also takes into account the mean reverting aspect of the prices. Backtesting will show which model performs best.

5.3 Backtesting

After the VaR calculations are successful, backtesting is a way of verifying if the models are good enough to to be used for the future. As mentioned , with a 99% VaR model we expect to have a 1% exceedance when we test the VaR calculations against our data. A backtest is therefore basically to see how many times this happens and compared it to the number of times we expect in a series.

Christoffersen (1998) came up with a independence test to measure if an exception is related to observing a exception on a given day. The test measures only dependencies with the next day, and is given by (as presented by Se.mathworks.com, 2017)

$$LR_{CCI} = -2log\left(\frac{(1-\pi)^{n00+n10}\pi^{n01+n11}}{(1-\pi_0)^{n00}\pi^{n01}(1-\pi_1)^{n10}\pi_1^{n11}}\right) \tag{27}$$

where the zeros represents no failure, and the ones represents failure, i.e 01 means no failure followed by failure. π represents the probability of failure, π_0 is the probability of failure given no failure in previous period, and π_1 is the probability of failure given failure in previous period. The Christoffersen forecast follows a χ^2 variable with one degree of freedom and is asymptotically distributed.

Another method of backtesting was introduced by Kupiec (1995), and is a variation of a binomial test which only compares actual number of exceptions to the expected number of exceptions. The Kupiec test uses the binomial distribution and combines it with the likelihood ratio to compare the probability of exceptions with the VaR confidence level. The Kupiec test is given by (as presented by Se.mathworks.com, 2017)

$$LR_{POF} = -2log\left(\frac{(1-p)^{N-x}p^x}{(1-\frac{x}{N})^{N-x}(\frac{x}{N})^x}\right) \tag{28}$$

where p is the expected exceedance percentage, N is the number of observations and x is the actual failures. This test follow the same distribution as the Christoffersen test. Kupiec also created a second test to see when the first rejection of the VaR model occurs. As with Kupiec‘s first test it is based on likelihood ratio, however it follows a geometric distribution and is given by (as presented by Se.mathworks.com, 2017)

$$LR_{TUFF} = -2log\left(\frac{p(1-p)^{n-1}}{(\frac{1}{n})(1-\frac{1}{n})^{n-1}}\right) \tag{29}$$

where n represents the number of days until the first rejection. Both the Kupiec and the Christoffersen test have been used on our models to see which model best captures the volatility of the stocks and commodities. The results are summarised and discussed in table 9.

6 Empirical Results

This chapter presents the results from the market correlation tests, and calculations of the VaR values based on the different volatility models. These calculations have been backtested and compared to see if they cover the volatility sufficiently with expected degree of violations.

6.1 Correlation

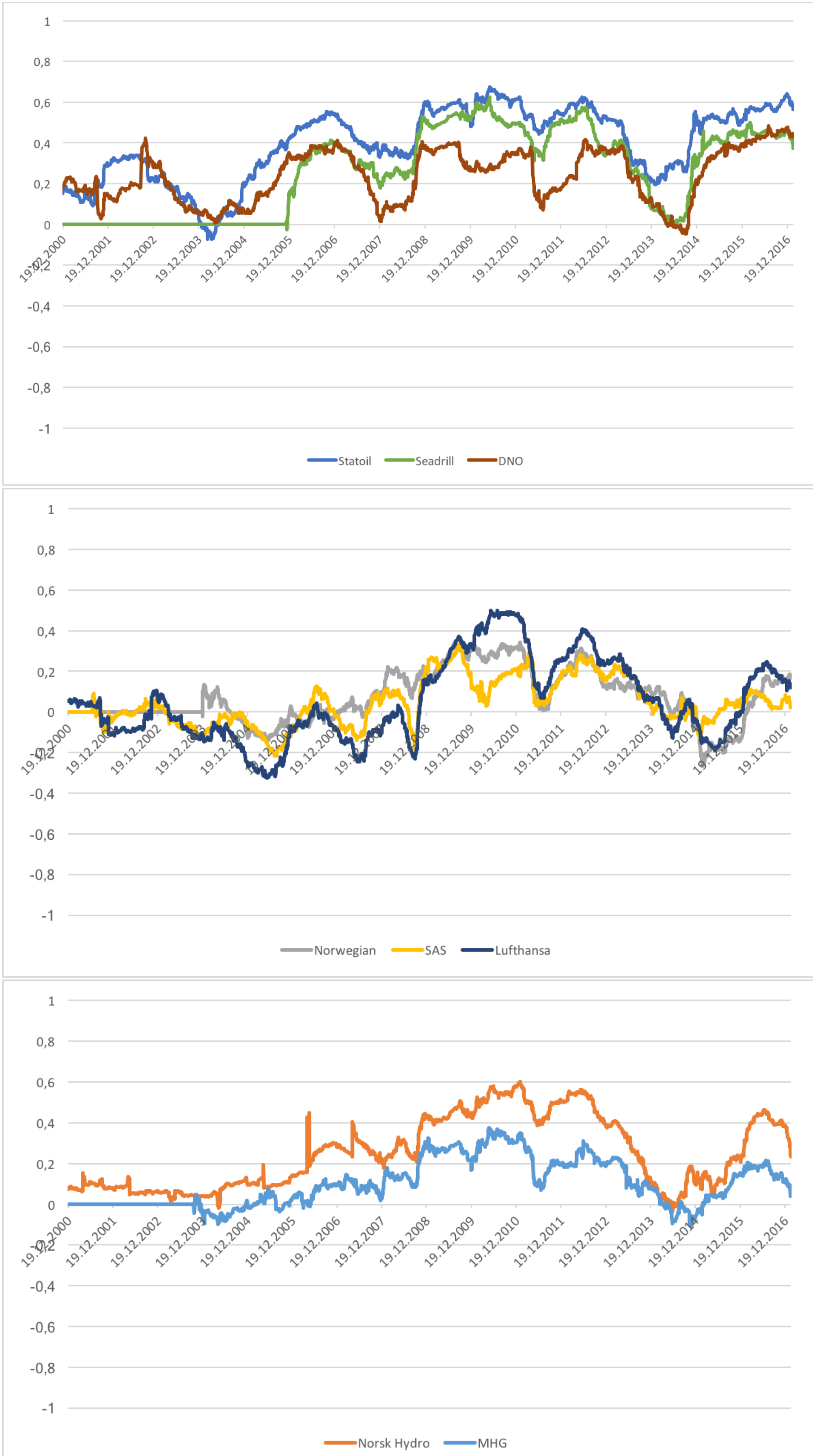
A test has been run to see the correlation between the stocks and the commodities chosen in this thesis. The results are shown below separated by the commodities the correlation is tested with.

6.1.1 Oil

First we have oil, and the correlation between the oil price and the stocks in this thesis. Figure (8) shows a graph of how the different stock correlates with oil. There is evidence of higher correlation with the oil price during periods of high volatility such as the financial crisis and during the oil price drop in 2014. Also we see that the oil companies are positively correlated with the oil price, with Statoil being the most affected. Also Norsk Hydro has a positive correlation for a long time until it drops down close to zero, which suggests a lower correlation after they moved on from oil and gas production.

The airlines have fairly little correlation which might be a little surprising given they use oil products daily for their operation. However, it suggests that airline companies use futures contracts to hedge against volatile oil prices to secure a stable income. MHG does not seem to be highly correlated during stable oil price periods, although it sees a spike in correlation during periods of volatile oil prices, again evident during the financial crisis. This supports our initial expectation that the oil price affects the whole economy, especially in Norway, and the salmon industry is no exception.

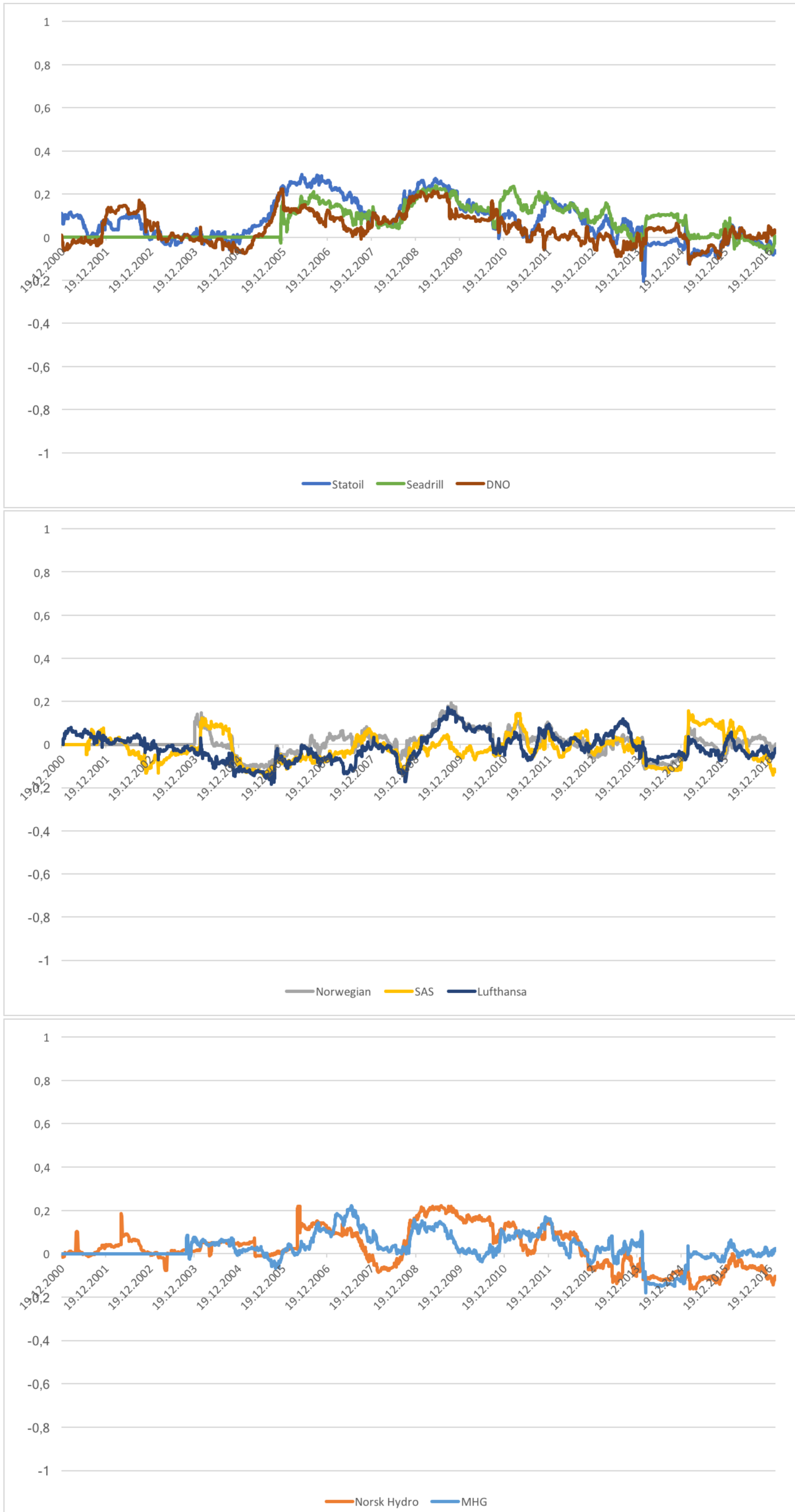
Figure 8: Correlation with oil price



6.1.2 Gas

Next up is the gas price and its correlation with the stocks. In figure (9) we see a similar correlation, although in a smaller scale. This supports our initial expectation that the oil and gas prices are correlated. Again the oil companies are correlated with the gas price, especially during volatile periods. The small correlation close to the end of our sample period compared to the oil, might be explained due to the rapid downfall of the oil price during that period. The oil price impact on the stocks were prominent, while the gas price did not suffer the same drastic drop and therefore not influencing the stocks much. The airlines does not show much correlation with the gas price. Even though the oil and gas price are correlated, the oil price affects them more than the gas price. Norsk Hydro has a slight positive correlation with the gas price prior to them exiting the oil industry. MHG has a correlation as expected, which follows the economy, in this case with the gas price.

Figure 9: Correlation with gas price

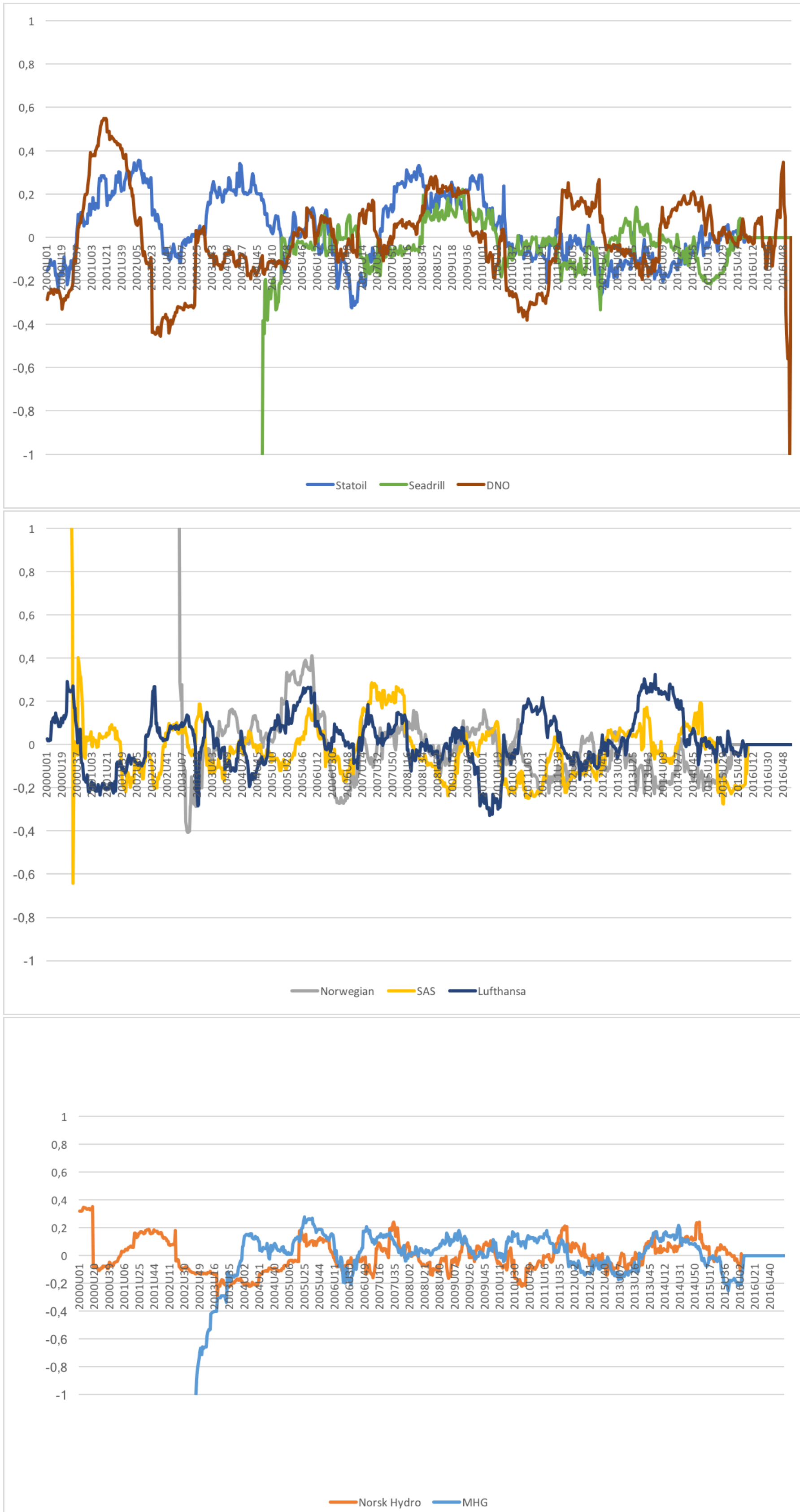


6.1.3 Salmon

To look at the correlation between the historical salmon price and the stocks, the stock prices was changed into weekly returns as described in chapter 3. The produced correlation graph is a bit messier than the previous two as seen in figure (10). More so than with oil and gas, many of the stocks experience negative correlation with the salmon price, which shows that they move in the opposite direction of the salmon price. This is seen in particular with DNO, the airlines and to some degree Seadrill. This might indicate that a declining salmon price, might also relate to a increasing oil price as we discussed in chapter 2. However, Statoil seems to be positively correlated together with the obvious MHG, indicating that the changes in salmon price is correlated with the oil price.

Also in several stocks, the graphs swing from positive to negative correlation which also might be an indication that the salmon price is somewhat random to the changes in the stock prices. This is also something that was expected at the beginning as most of the stocks are more related to oil and gas, even though salmon is a major part of Norwegian economy.

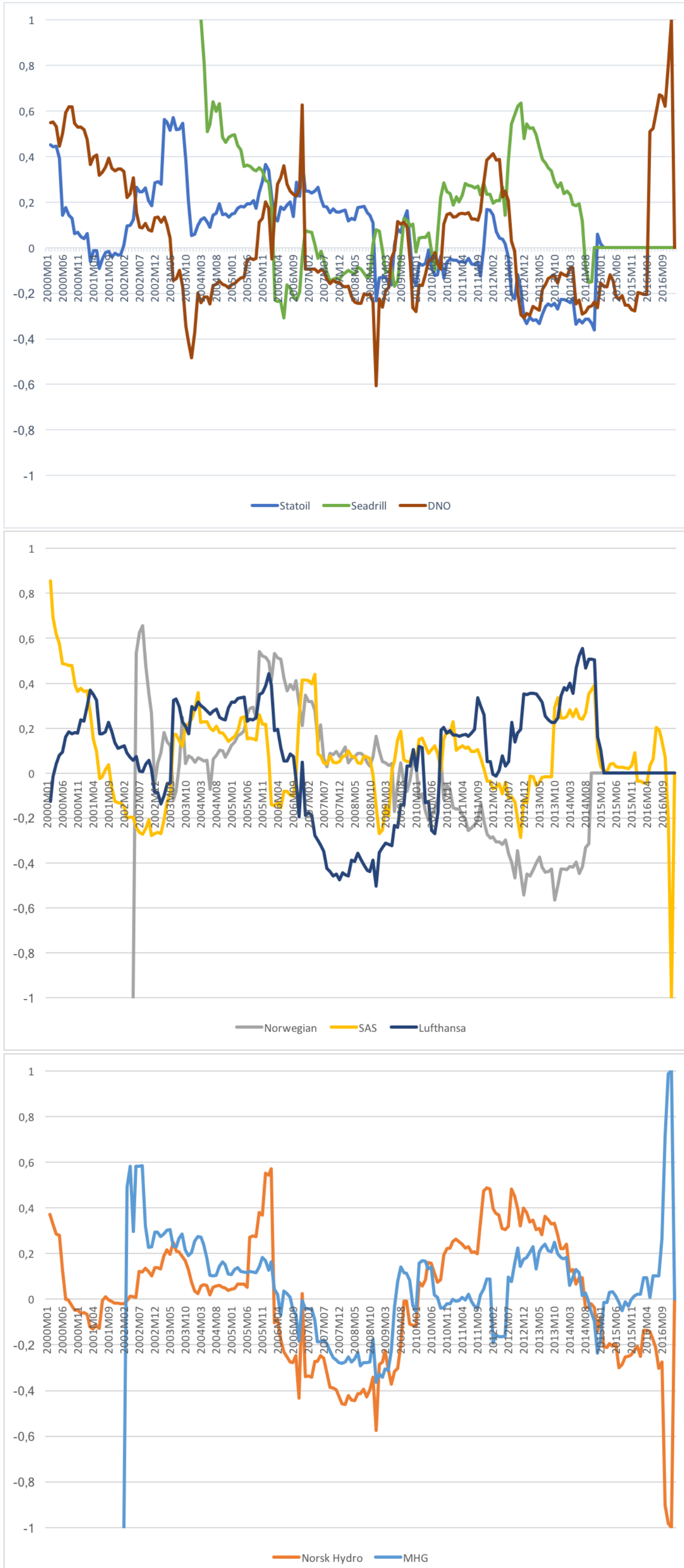
Figure 10: Correlation with salmon price



6.1.4 Aluminum

For the correlation with the aluminum price the historical stock data was changed into monthly returns as described in chapter 3. Looking at figure (11) the airlines experience times were it is correlated to the aluminum price, but also periods of negative correlation. This might be explained with airlines ordering new airplanes when the aluminum price is low, and their stock price increase as a result of a expansion of their fleet. During the financial crisis it is also interesting to see the negative correlation between Norsk Hydro and aluminum. This indicates that when the price of aluminum dropped in that period, Norsk Hydro experienced an opposite reaction which is something to look closer at as it contradicts the stock price at the time. It is also interesting to look at how Seadrill has a clear positive correlation with aluminum during a period of growth after the financial crisis, which might be related to their usage of aluminum in their services, and the increase in oil price during that time. Oil companies spent more money in those years creating opportunities for Seadrill to make money even with a higher aluminum price.

Figure 11: Correlation with aluminum price



6.2 Volatility models

The graphical results of our volatility models will be presented in this section along with the optimal parameters of the GARCH models.

6.2.1 Rolling Standard Deviation

The first model of volatility is a standard deviation model with a rolling window. This was chosen as the basis of the models for comparison with the two other methods. In this method the standard deviation was calculating by a similar approach as for the correlation. A rolling window of 250 days was used to calculate the moving standard deviation or the volatility for each stock and commodity. This means that day 1 through 250 of the returns were used to calculate the volatility on day 251. This was then moved ahead to day 2 through 251, and repeated throughout the data, and graphically presented in figure 12 and 13. The process was repeated for salmon using a moving window of 52 weeks to model the volatility of the weekly returns. For the volatility of aluminum the rolling window was 24 months for monthly returns.

As we see from in figure 12 and 13 the model captures the volatility during volatile times. Especially the financial crisis is evident in all stock and commodities. We also see a the model supporting that volatility happens in clusters and in bursts. Some of the stocks like Lufthansa and Statoil seems to have a fairly stable volatility during the sample period according to this model, although natural gas prices and DNO stocks in particular seems to be more volatile. DNO being a small oil producer this seems logical since they would be more affected by the changes in oil price, and the overall uncertainty in the economy. Compared to bigger companies like Statoil, their stock is more volatile due to the fact that they are more dependent on a solid market to survive. Seadrill on the other hand, also shows signs of high volatility in this model, especially during recent times where oil service companies have struggled with the low oil price. The volatility for salmon also shows that this model might not capture volatility in a reasonable way as the volatility increases through time, and does not revert back to a mean. The rolling window does seems to work better with the monthly aluminum prices as the volatility spikes are clear, the overall volatility reverts back to what seems to be a mean for the aluminum volatility.

Figure 12: Volatility using rolling standard deviation

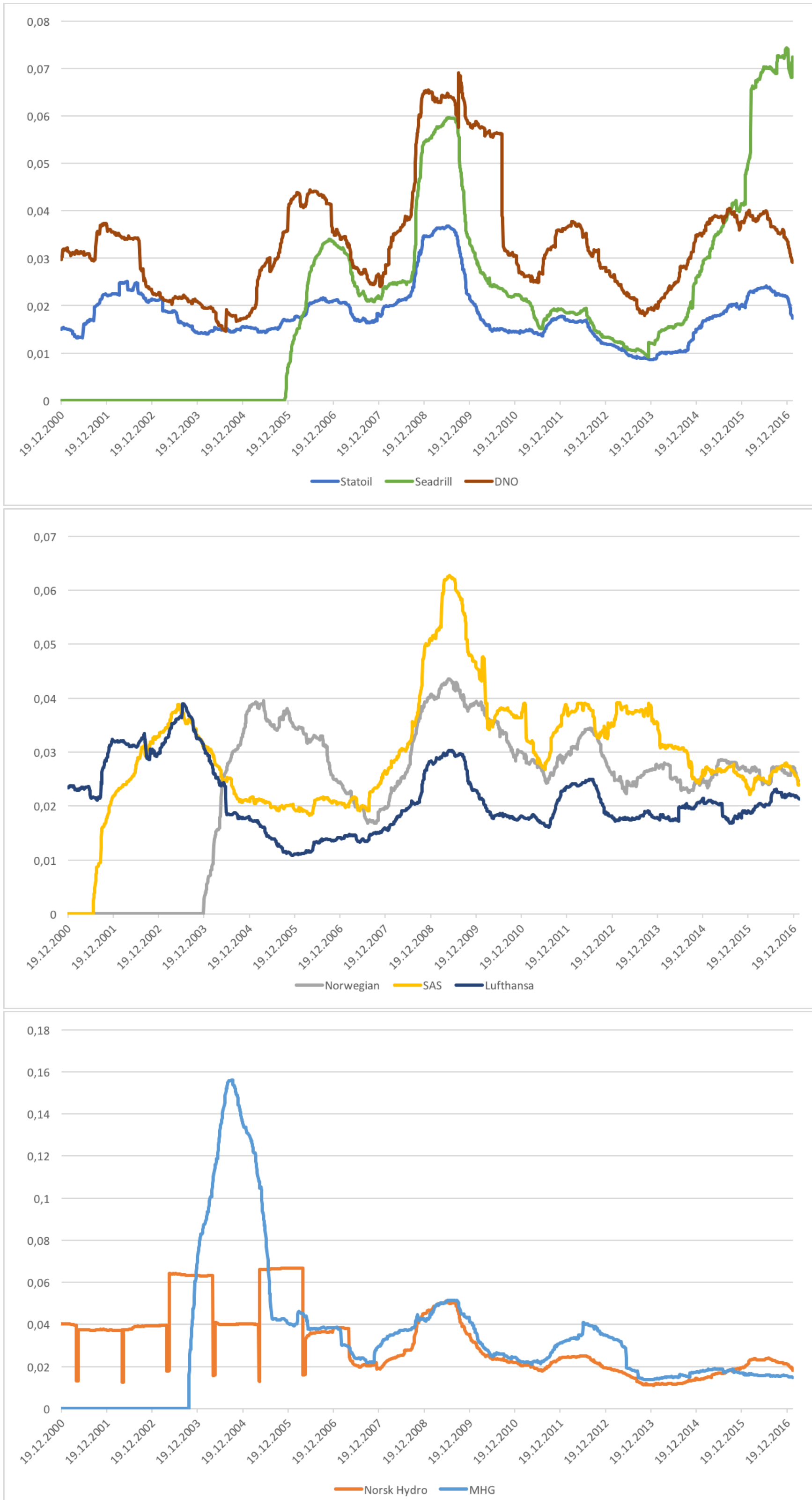
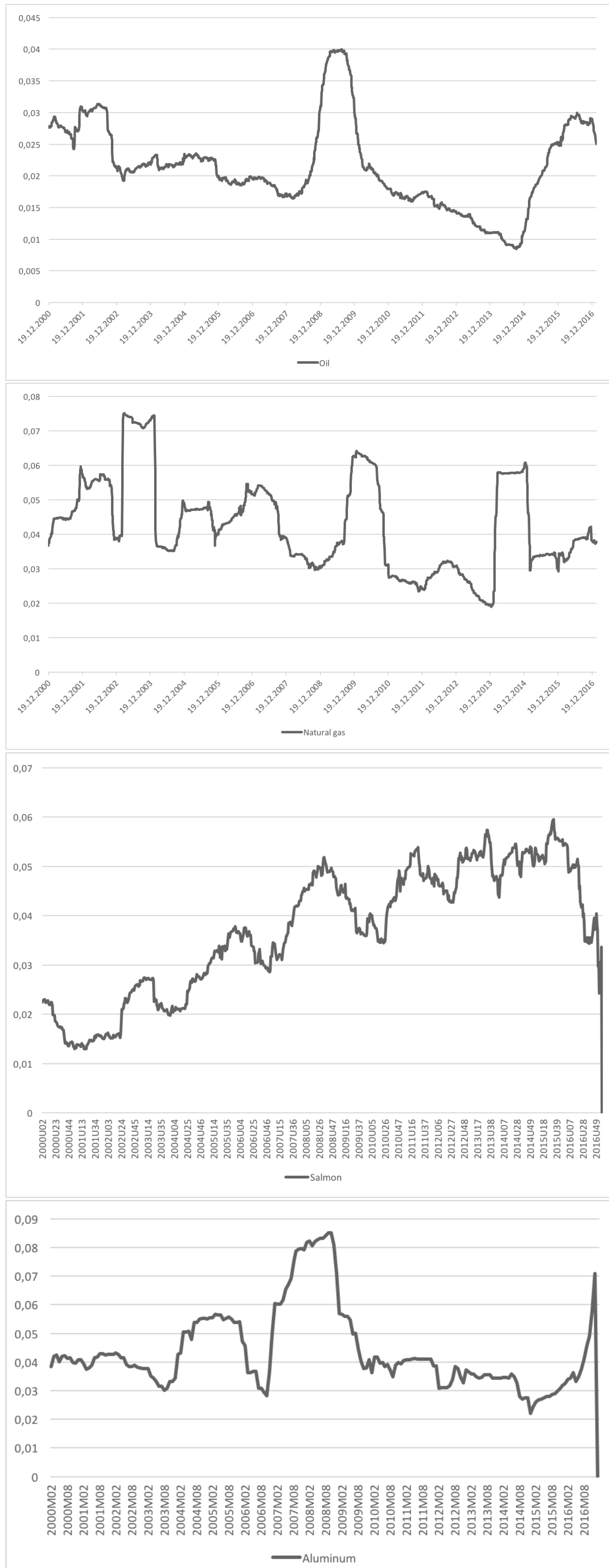


Figure 13: Volatility using rolling standard deviation

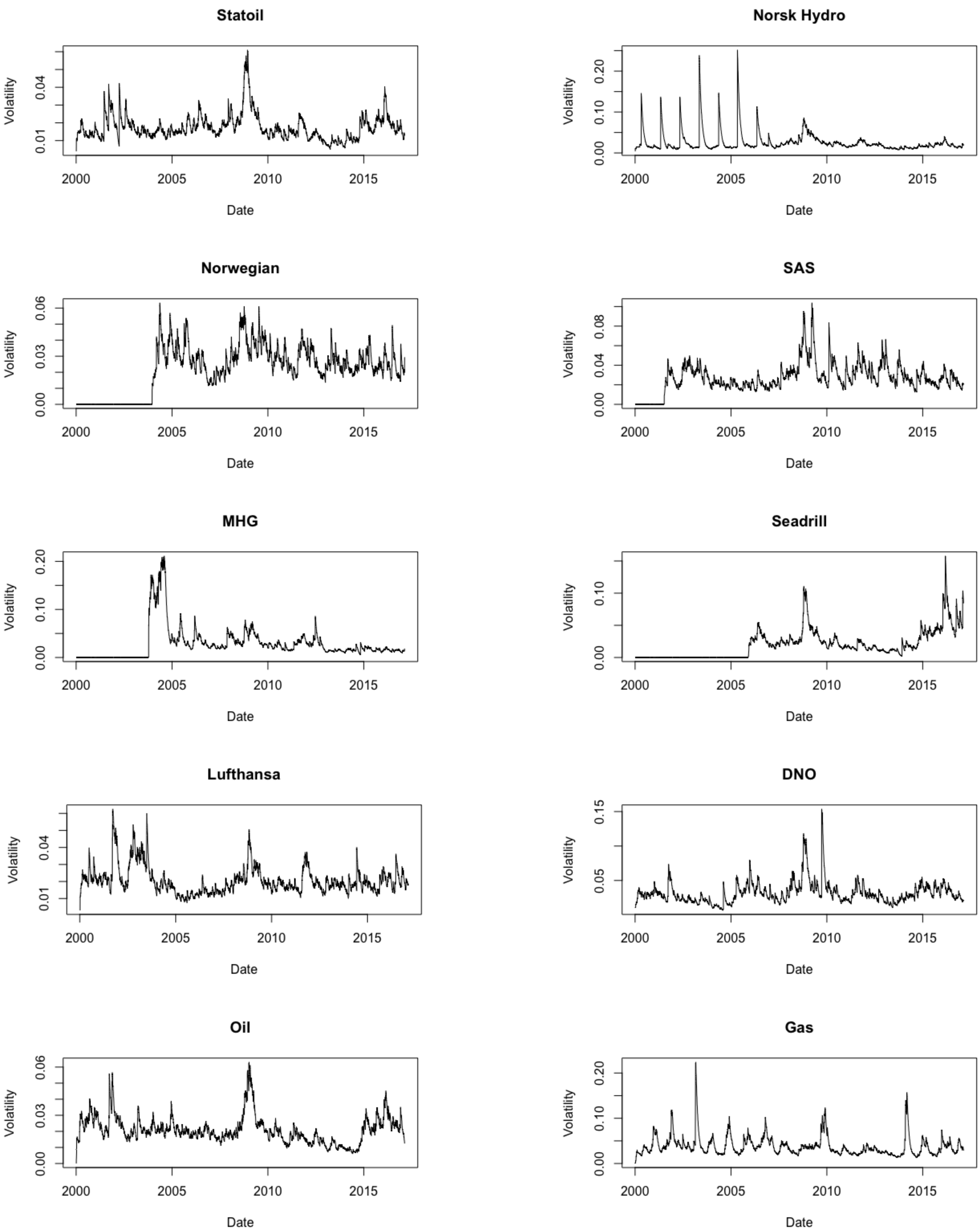


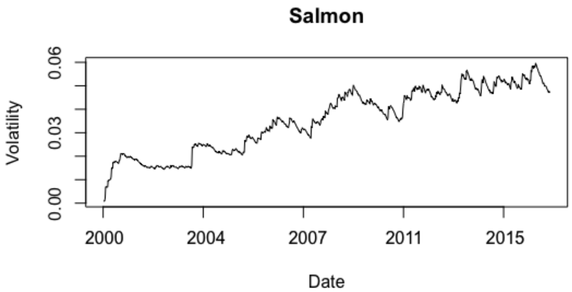
Model captures volatile periods

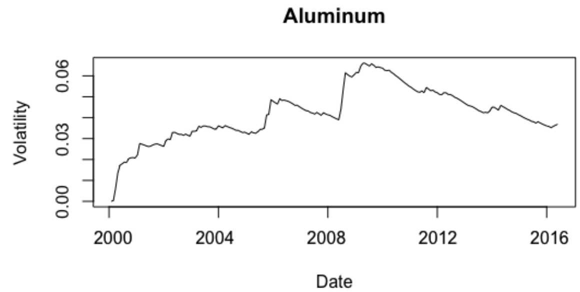
6.2.2 EWMA

For the EWMA volatility model, λ is set to 0.94 based on being the standard value for lambda. For monthly returns in the aluminum price, this has been changed to 0.97 based on recommended value. All the returns for the stocks and commodities have been used to create a moving window of the exponential weights, and calculated with equation (13) from chapter 3. The results are presented in the figure below, showing the volatility of the stocks and commodities using EWMA. As we can see the graph are smoother than with the rolling standard deviation, nonetheless we see similar trends as the previous model. As expected they all model the same spike in volatility during the financial crisis. In particular we see the same volatility spike in MHG at the start of their stock, and the high number of volatility spikes in both Norsk Hydro and gas during the first years of our sample. The model also captures the a more volatile stock for all the airlines, which indicated several factors affect the stability of the stock. One reason for this might because of volatility clustering and that in the model this does not fade quickly and affects the stock after an incident. Looking at the salmon and the aluminum price we almost see a identical graph, although not with as many ups and downs but more smoother increase in volatility. It is also not as mean reverting as the other, which also is not taken into account in EWMA as it is in GARCH.

Figure 14: Volatility using EWMA







6.2.3 GARCH & EGARCH

In addition to the EWMA model, we have fitted all the data into GARCH(1,1) and EGARCH(1,1) models and estimated the parameters using maximum likelihood. The results are shown in table 6 and 7. First we look at our commodities. For easy comparison the parameters can be compared to the EWMA model. The EWMA model is a non mean reverting GARCH model where the $\omega = 0$, and β_1 is the same as λ and fixed at 0.94. Both β and λ represents the decay factor in their respective models, and is a measure of how long volatility stays in the market after an event has occurred. This also makes the α_1 equal to 0.06 in the EWMA model, and this value is commonly referred to as the GARCH effect in GARCH models.

Looking at table 6 first we see that the β_1 values vary from 0.949 for oil to 0.564 for aluminum. This is an indication that the EWMA model might sufficiently cover the volatility for oil, meanwhile not that well for the volatility of aluminum, where we prefer the GARCH(1,1). On the other hand, we see fairly small ω values which again gives evidence that for the commodities EWMA and GARCH(1,1) will produced similar results. It is particularly interesting to look at the salmon parameters which are very similar to an EWMA model, and looking at figure 13 this is strengthened by the increasing volatility of that commodity.

Table 6: Estimation of univariate GARCH-type models for log-returns of commodities; 3. January 2000 - 17. February 2017

Statistics	Log Returns			
	Oil	Gas	Salmon	Aluminum
AR(1)-GARCH(1,1) model				
μ	0.000312 (0.244)	-0.000213 (0.614)	0.000193 (0.858)	-0.000237 (0.928)
ω	0.000001 (0.5316)	0.0000240 (0.00)	0.000006 (0.270)	0.000383 (0.0126)
α_1	0.0497 (0.00)	0.1287 (0.00)	0.0662 (0.00)	0.258 (0.004)
β_1	0.949(0.00)	0.870 (0.00)	0.932 (0.00)	0.564 (0.00)
BIC	-4.9548	-3.8175	-3.896	-3.394
AR(1)-EGARCH(1,1) model				
μ	-0.000171(0.00)	-0.00180 (0.459)	0.000026 (0.981)	-0.00152 (0.576)
ω	-0.0429(0.00)	-0.0973 (0.00)	0.0631 (0.012)	-1.0648 (0.0291)
α_1	-0.0429 (0.00)	0.00410 (0.641)	-0.06385 (0.828)	-0.0836 (0.244)
β_1	0.9939 (0.00)	0.9826 (0.00)	0.990 (0.00)	0.827 (0.00)
γ_1	0.101(0.00)	0.228 (0.00)	0.151 (0.00)	0.446 (0.00)
BIC	-4.9565	-3.823	-3.889	-3.3587

p-values are in parenthesis.

For the EGARCH(1,1) model the β_1 values are much higher, even close to 1. Since the parameters needs to add up to 1, this results in negative α_1 values. For the EGARCH model there is not any nonnegative constraint, and the γ_1 values show whether the volatility is affected by positive or negative news. A γ_1 values less than zero means negative news have most impact, and a γ_1 value above zero says positive news have greater impact. Looking at our commodities, positive news affect all of their volatility more than negative news. The ω values are much higher for the EGARCH(1,1) model and is a product of the long run variance of the model. With the GARCH(1,1) model being similar to EWMA for commodities, the EGARCH(1,1) model shows a significant long term variance for the data.

In table 7 the parameters for the stocks are presented. Similar trends as with the commodities. Some of the stocks like Statoil, SAS, Lufthansa and DNO show close to a EWMA model with β_1 values close to 0.94, and with a mean and long term variance close to zero. Other stocks like Norsk Hydro and Norwegian shows that the EWMA model might not cover the data, and that a GARCH or EGARCH model is more fitting.

Table 7: Estimation of variables for univariate GARCH-type models for log-returns of stocks; 3. January 2000 - 17. February 2017

Statistics	Log Returns							
	Statoil	Norsk Hydro	Norwegian	SAS	MHG	Seadrill	Lufthansa	DNO
AR(1)-GARCH(1,1) model								
μ	0.000420 (0.00)	0.00028 (0.00)	0 (1.00)	-0.000021 (0.0005)	0 (1)	0 (1)	0.000254 (0.368)	0.000744 (0.0820)
ω	0.000 (0.235)	0.000044 (0.00)	0 (1.00)	0 (0.00)	0 (1)	0 (1)	0 (0.00)	0 (0.00)
α_1	0.0812 (0.00)	0.405 (0.00)	0.4627 (0.00)	0.0663 (0.00)	0.158 (0.00)	0224 (0.00)	0.0519 (0.00)	0.0152 (0.00)
β_1	0.909 (0.00)	0.590 (0.00)	0.745 (0.00)	0.932 (0.00)	0.910 (0.00)	0.849 (0.00)	0.933 (0.00)	0.982 (0.00)
BIC	-5.2298	-4.6182	-10.316	-4.5563	-9.0178	-13.398	-4.9435	-4.0049
AR(1)-EGARCH(1,1) model								
μ	0.000184 (0.00)	0.000410 (0.00)	0 (1)	0 (0.426)	0 (1)	0 (1)	-0.000075 (0.787)	0.000474 (0.0233)
ω	-0.791 (0.00)	-0.729 (0.00)	-0.121 (0.00)	-0.208 (0.00)	-0.135 (0.00)	0.439 (0.00)	-0.0850 (0.00)	-0.0896 (0.00)
α_1	-0.0918 (0.00)	0.0547 (0.00)	0.359 (0.00)	-0.017 (0.207)	0.181 (0.00)	-1.11 (0.00)	-0.0384 (0.00)	-0.0384 (0.00)
β_1	0.900 (0.00)	0.900 (0.00)	0.986 (0.00)	0.966 (0.00)	0.985 (0.00)	0.921 (0.00)	0.988 (0.00)	0.985 (0.00)
γ_1	0.407 (0.00)	0.562 (0.00)	0.617 (0.00)	0.262 (0.00)	0.185 (0.00)	3.01 (0.00)	0.0840 (0.00)	0.0751 (0.00)
BIC	-5.2356	-4.4295	-11.481	-4.619	-6.5227	-9.3346	-4.9507	-4.0289

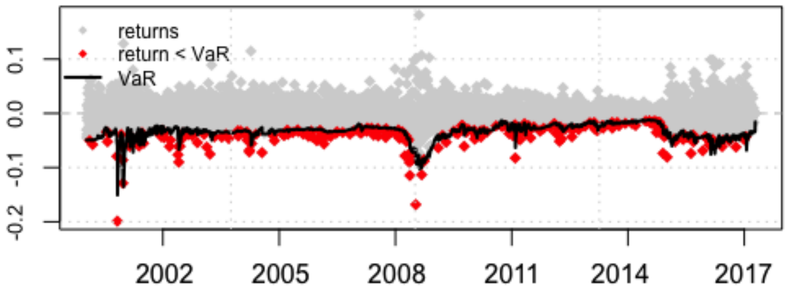
p- values are in parenthesis

The EGARCH(1,1) for the stocks also shows similar trends as with the commodities with a more long term variance, and with positive news affecting the volatility more than negative news. For all stocks and commodities, the BIC value is a measure of the relative quality of the model. Another thing to remember is that the EGARCH(1,1) model allows for testing of asymmetry which explain the often negative α_1 values. They represent the symmetry effect of the model, and since EGARCH allows for asymmetry this means the values can also be negative as we see in our parameters.

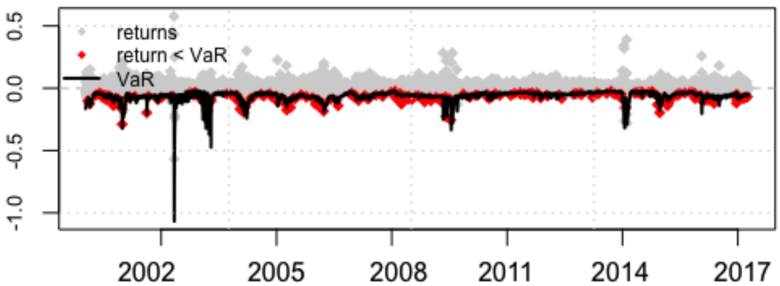
6.3 Value-at-risk

The VaR estimation has been done using EWMA, GARCH and EGARCH volatility, and their performance is discussed in the next section. The VaR will be discussed in this section in terms of the GARCH estimations, and are presented in figure 16 and 17. The VaR estimations using the other volatility models are graphically presented in the appendix.

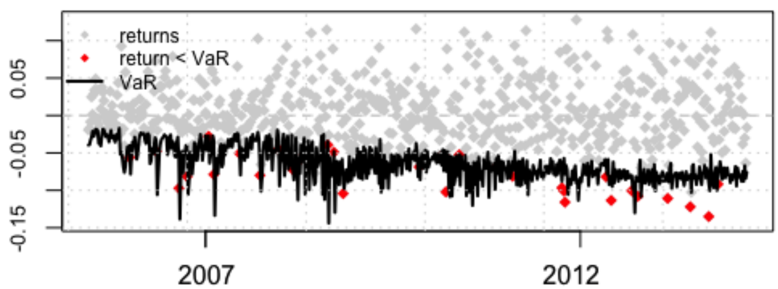
Figure 16: Daily VaR of commodities using GARCH(1,1)



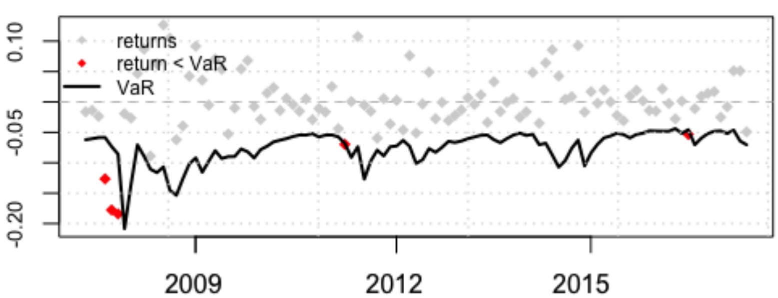
(a) Oil



(b) Gas



(c) Salmon

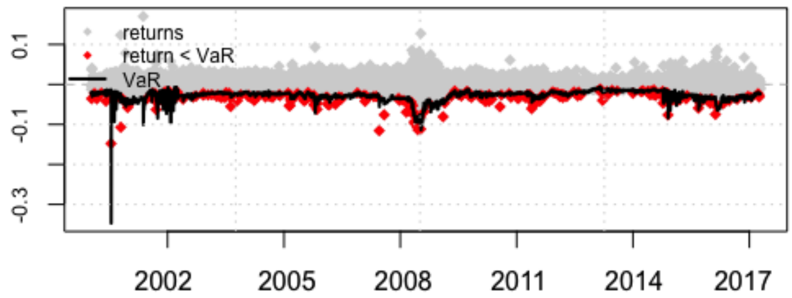


(d) Aluminum

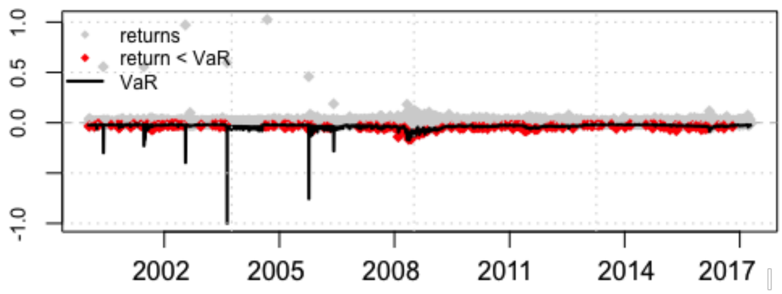
The figure shows daily VaR calculations using the GARCH(1,1) model plotted against the log returns. The confidence level is 95%

Figure 16 shows the downside 5% VaR in a black line plotting against the log returns with the red dots being the non captured volatility. With the exception of salmon, all the VaR calculation follows a fairly stable trend that follows the bottom part of the returns. Salmon has a more chaotic and volatile VaR. It is also clear, especially looking at he oil in the top left, that the 5% VaR covers the bottom part of the returns well. However the extreme events seems to be to extreme for the model to handle.

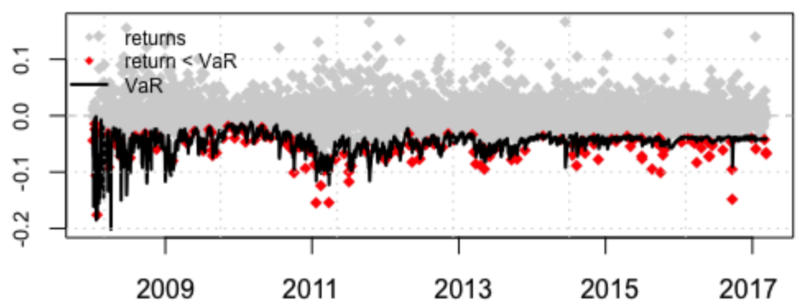
Figure 17: Daily VaR of stocks using GARCH(1,1)



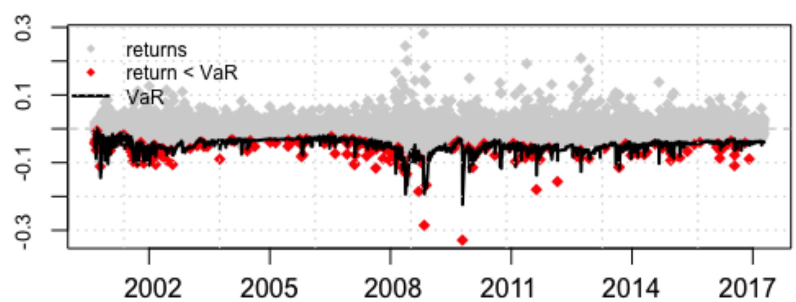
(a) Statoil



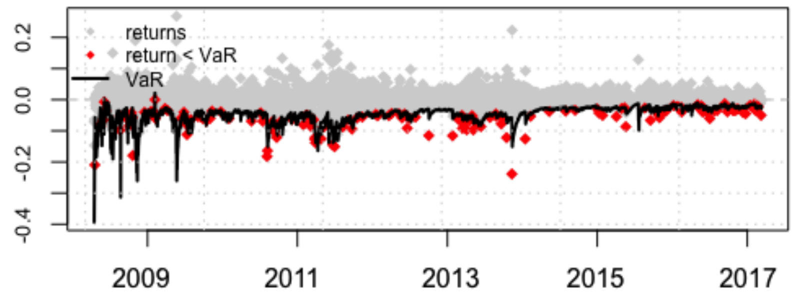
(b) Norsk Hydro



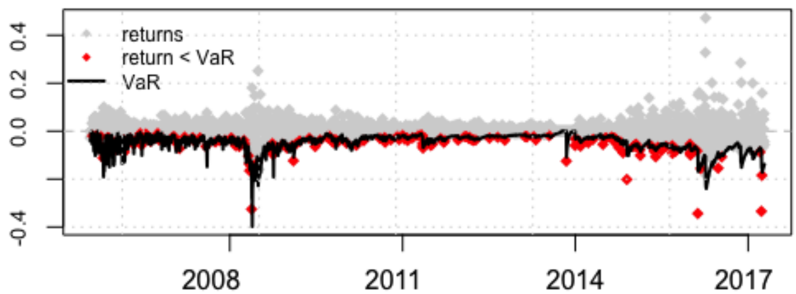
(c) Norwegian



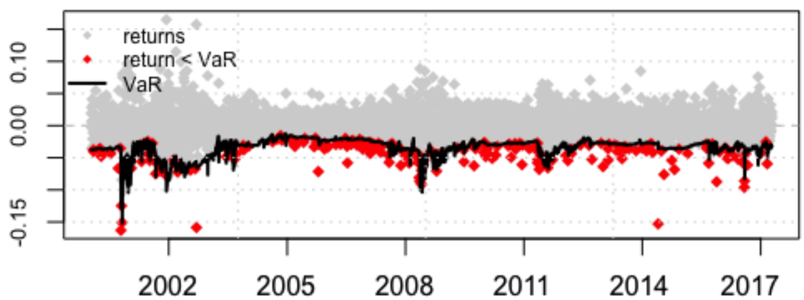
(d) SAS



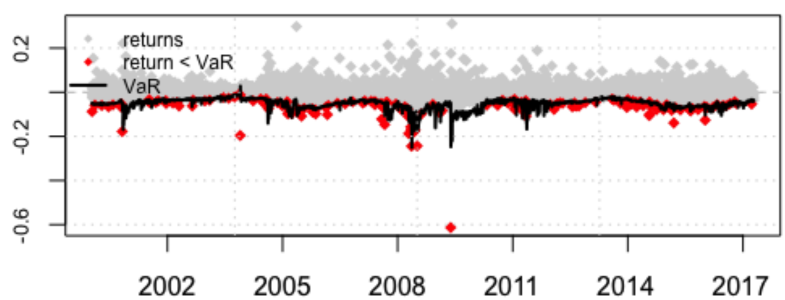
(a) MHG



(b) Seadrill



(c) Lufthansa



(d) DNO

The figure shows daily VaR calculations using the GARCH(1,1) model plotted against the log returns for the stocks. The confidence level is 95%

For the stocks, it is perhaps a little easier to compare since they have a more similar length with all daily returns. They are mostly steady VaR that again captures the bottom part of the volatility. As with the commodities we also see that the extreme values are far from the VaR line. This indicates that the model does sufficiently cover much of the volatility for both the stocks and commodities, however it struggles with capturing the volatility clustering and extreme volatility burst that happens during negative times. Also another observations might be that the affect of negative events die out too quickly after severe events like the financial crisis. This is evident in the many red dots after the financial crisis in for instance Lufthansa and Statoil. Some percentage of red dots are expected, and the next part covers if the amount is acceptable or not for all models.

6.3.1 Backtesting

To see which type of model best estimates volatility and thus is best fitted to be used in VaR estimation of market risk, both the Kupiec and the Christoffersen test is performed and the result are summarised in table 8 and 9. The table is split into three parts, EWMA, GARCH(1,1) and EGARCH(1,1). Each of those shows the result of the backtest done by the test explained in chapter 5, and is split into the two confidence levels used in the thesis. Each part finishes of with a decision to either reject the null hypothesis that the VaR estimations sufficiently covers the data, or fail to reject. As mentioned we expect the data to be violated 5% for the 95% confidence level and 1% for the 99% confidence level. The results are presented here, and discussed in chapter 7.
Based on the test run we first look at table 8 for the commodities. The EWMA model performs pretty accurately on our commodities, and sufficiently covers all of them at 5%, and also salmon and aluminum at 1% significance for the unconditional test. For the conditional test, the model for aluminum is rejected at any level, while oil, gas and salmon is accepted at 5%. Next up is the GARCH(1,1) model which does not seem to be an accepted model for the oil price at 5% significance, however for gas, salmon and aluminum the model is accepted. At 1% only salmon price is covered and accepted by both the Kupiec and the Christoffersen test.

Lastly the EGARCH(1,1) model is tested and shows promise in covering the gas, salmon and aluminum price at 5% significance with the Kupiec test. Also the EGARCH model fails to capture the volatility of the oil price, the same as the GARCH model. For the Christoffersen test only salmon is accepted at 5%, and surprisingly aluminum is accepted at 1% and not at 5%.

Table 8: Backtesting results of VaR from univariate GARCH-type models for log-returns of commodities; 3. January 2000 - 17. February 2017

Returns	Log Returns			
	Oil	Gas	Salmon	Aluminum
EWMA				
Kupiec's unconditional coverage test results				
$LR_{\alpha=0.05}$	0.042	0.877	0.6	1.193
EE/AE	210/213	210/197	39/34	5/53
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	9.175	20.043	0.622	2.436
EE/AE	42/63	42/74	8/10	1/3
Decision	Reject H_0	Reject H_0	Fail to reject H_0	Fail to reject H_0
Christoffersen's duration-based test results				
$LR_{\alpha=0.05}$	2.497	2.391	1.904	13.341
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Reject H_0
$LR_{\alpha=0.01}$	10.1	20.375	3.126	14.584
Decision	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0
GARCH(1,1)				
Kupiec's unconditional coverage test results				
$LR_{\alpha=0.05}$	5.486	1.088	0.116	0.005
EE/AE	210/244	210/225	32/34	5/5
Decision	Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	15.6	34.252	0.931	2.513
EE/AE	42/70	42/85	6/9	1/3
Decision	Reject H_0	Reject H_0	Fail to reject H_0	Fail to reject H_0
Christoffersen's duration-based test results				
$LR_{\alpha=0.05}$	6.57	4.963	0.139	6.418
Decision	Reject H_0	Fail to reject H_0	Fail to reject H_0	Reject H_0
$LR_{\alpha=0.01}$	16.1	36.571	3.5	14.582
Decision	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0
EGARCH(1,1)				
Kupiec's unconditional coverage test results				
$LR_{\alpha=0.05}$	9.841	3.488	0.481	0.292
EE/AE	210/255	210/237	32/36	5/4
Decision	Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	15.6	61.822	8.415	0.724
EE/AE	42/83	42/102	3/9	1/3
Decision	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0
Christoffersen's duration-based test results				
$LR_{\alpha=0.05}$	9.501	15.9	0.951	8.971
Decision	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0
$LR_{\alpha=0.01}$	32.39	70.168	9.27	6.439
Decision	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0

The null hypothesis states that the model sufficiently covers the sample period. EE is the expected exceedance, and AE is the actual exceedance. LR represents the likelihood ratio.

Next up is the stocks, and EWMA will lead the way. According to the Kupiec test, the EWMA model only covers Statoil and Seadrill at 5% significance level. Nonetheless the model seems to be sufficient for VaR estimation at 1% significance level for Statoil, Norsk Hydro, Norwegian, MHG and DNO. The same does the Christoffersen test, with the exception of Norwegian. This indicates that for the stocks used the EWMA does well with the volatility spikes, more than the average volatility model.

GARCH(1,1) model performs well at 5% significance level for our stocks, at least according to the Kupiec test, but not according to the Christoffersen test. Furthermore the model is thoroughly rejected at 1% significance level showing that for the chosen stocks the GARCH(1,1) does not cover the extreme values.

The EGARCH(1,1) performs in the same manner as the GARCH(1,1) and does cover volatility decently at the 5% significance level. Needless to say, also this models performs poorly against the extreme events and burst in volatility.

Table 9: Backtesting result of VaR for univariate GARCH-type models for log-returns of stocks; 3. January 2000 - 17. February 2017

Statistics	Log Returns							
	Statoil	Norsk Hydro	Norwegian	SAS	MHG	Seadrill	Lufthansa	DNO
EWMA								
Kupiec's unconditional coverage test results								
$LR_{\alpha=0.05}$	3.022	29.48	2.014	29.381	15.552	0.003	4.761	39.015
EE/AE	210/186	210/138	167/150	202/132	163/116	143/142	210/180	210/128
Decision	Fail to reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0	Reject H_0
$LR_{\alpha=0.01}$	1.112	0.003	1.192	7.463	0.364	16.94	11.778	0.401
EE/AE	42/49	42/42	33/40	40/59	32/36	28/53	42/66	42/38
Decision	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Reject H_0	Fail to reject H_0	Reject H_0	Reject H_0	Fail to Reject H_0
Christoffersen's duration-based test results								
$LR_{\alpha=0.05}$	16.614	31.823	16.983	35.477	24.209	7.677	9.355	1.248
Decision	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	3.351	0.592	11.994	17.552	1.112	17.782	12.508	1.248
Decision	Fail to reject H_0	Fail to reject H_0	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0	Reject H_0	Fail to reject H_0
GARCH(1,1)								
Kupiec's unconditional coverage test results								
$LR_{\alpha=0.05}$	7.911	0.018	0.016	1.656	2.326	5.235	0.173	6.529
EE/AE	210/251	210/212	167/166	202/185	163/144	143/170	210/216	210/175
Decision	Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Reject H_0	Fail to reject H_0	Reject H_0
$LR_{\alpha=0.01}$	44.833	21.204	13.73	26.242	16.358	48.963	18.909	2.226
EE/AE	42/92	42/75	33/57	40/77	32/58	28/73	0.022	0.036
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Fail to Reject H_0
Christoffersen's duration-based test results								
$LR_{\alpha=0.05}$	19.902	7.336	10.205	1.701	6.726	12.415	8.138	19.076
Decision	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0
$LR_{\alpha=0.01}$	52.868	21.498	19.458	27.539	18.953	49.577	19.282	2.4
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Fail to reject H_0
EGARCH(1,1)								
Kupiec's unconditional coverage test results								
$LR_{\alpha=0.05}$	7.175	1.56	1.576	3.55	5.563	4.8189	2.765	1.331
EE/AE	210/249	210/212	167/152	202/177	123/98	123/147	210/234	210/194
Decision	Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0	Reject H_0	Reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	37.159	31.44	17.116	21.249	13.859	79.324	41.695	12.708
EE/AE	42/87	42/83	33/60	40/73	26/45	25/80	42/96	42/67
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0
Christoffersen's duration-based test results								
$LR_{\alpha=0.05}$	13.982	3.316	6.478	4.008	9.186	5.392	4.685	5.12
Decision	Reject H_0	Fail to reject H_0	Reject H_0	Fail to reject H_0	Reject H_0	Fail to reject H_0	Fail to reject H_0	Fail to reject H_0
$LR_{\alpha=0.01}$	37.856	31.517	22.08	22.907	13.894	81.394	42.23	13.379
Decision	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0	Reject H_0

The null hypothesis states that the model sufficiently covers the sample period. EE is the expected exceedance, and AE is the actual exceedance. LR represents the likelihood ratio.

7 Conclusion

The thesis set out to find out how important Norwegian commodities and stock producing or consuming those commodities affected each other. In addition, the thesis wanted to test different approaches to model volatility of the commodity and stock prices, and test which performs best at estimating market risk.

Looking at how the four commodities affects the eight stocks in the thesis, the results are as expected. Correlation with the oil price is the most noticeable especially when it comes to the stocks following its direction. This means that oil companies as well as airlines and seafood companies are affected by the oil price. As a vital part of the Norwegian economy this comes as no surprise, even though the oil companies are naturally more correlated with the price than the others. One potential explanation is hedging and forward contracts for the airlines, and MHG being affected by the overall look of the economy which follows the oil price. The similar commodity of natural gas follows the same patterns as the oil price, although at a smaller scale. This is according to our expectations since the oil and gas prices are correlated, and gas being the one with less power over the stocks. For salmon, MHG is naturally positively correlated with it, while the other show somewhat random changes. The oil companies appears to follow a similar trend as MHG, indicating that salmon follows the economy which is highly effected by the oil price in Norway. The airlines show some similar trends as the others, which again supports the theory that salmon is affected by the changes in oil price, and therefore these stocks all follow the salmon price in that way. For aluminum we have a switching correlation for the airlines suggesting there stock depends on the aluminum price at the time of new orders. Same with the oil companies when it comes to equipment, and then especially for Seadrill as we see a positive correlation with the aluminum price after the financial crisis. This because increased activity in the oil industry increases their profit. Norsk Hydro has been more positively correlated after they stopped with oil and focused only on aluminum.

With the application of EWMA, GARCH and EGARCH we have estimated daily VaR to see if the volatility models sufficiently covers the log returns with the expected exceptions. For this a GARCH(1,1) and EGARCH(1,1) have been used on the data combined with a different ARMA models fitting the data. The results was graphically presented in chapter 6, and the parameters presented in tables. A summary of the results are found in the table below.

Table 10: Summary of findings

Model	Commodities	Stocks
EWMA		
Kupiec 5%	4/4	2/8
Kupiec 1%	2/4	5/8
Christoffersen 5%	3/4	2/8
Christoffersen 1%	1/4	4/8
GARCH		
Kupiec 5%	3/4	5/8
Kupiec 1%	2/4	1/8
Christoffersen 5%	2/4	1/8
Christoffersen 1%	1/4	1/8
EGARCH		
Kupiec 5%	3/4	5/8
Kupiec 1%	1/4	0/8
Christoffersen 5%	1/4	5/8
Christoffersen 1%	1/4	0/8

Table summarises how many of the commodities and stocks the different model covers against the total number of commodities and stocks.

With regards to our stocks and commodities all the models used in this thesis shows varying results. Based on backtesting using Kupiec and Christoffersen test, EWMA is the preferred volatility model in VaR estimation with superior coverage on the commodities. Although both GARCH and EGARCH models perform well, they both fail to adequately model the oil volatility. For our stocks, EWMA does not perform as well as with the commodities, even though it outperforms the other at 1% significance. Nonetheless the GARCH and EGARCH model is preferred as volatility models for VaR estimation at 5% significance level. Of the two GARCH models, they both do similarly well, and it is difficult to separate the two. EGARCH does perform better on the Christoffersen test and might therefore be a slightly better model for volatility in this case. Overall none of the models does well enough to cover the volatility at 1% significance and thus fails to cover the extreme events during our sample period. To conclude this means our models can mostly be used to a 95% certainty which gives a 5% chance of a greater loss.

The thesis has tried to capture the volatility of eight stocks and four commodities in order to estimate market risk. In general, EWMA and basic GARCH(1,1) model significantly increases the precision of the estimation, and it has been the case for our data at 5% significance level. The evidence from the results does however indicate that there is room for improvement. Some of our data might have been better off with a different order GARCH model. Also our models have used the simplicity of the normal distribution, and other options such as a generalised error distribution might have given better results. This has been the case in Zhang & Zhang (2016). They also implemented the use of extreme value theory in order to cover the extreme values our models failed to capture. By using this theory on the residuals negative side, a better model of the extreme negative events would possibly been covered. This would potentially bettered our models so that they also covered the volatility at 1% significance. Overall the basic GARCH(1,1) does as expected cover the volatility in a decent way, and rectifies its standing as an easy and relatively effective volatility model.

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A Code

Presented here is the R code used to create the volatility models and calculations of VaR, as well as backtesting. The code has Statoil as an example.

A.1 EWMA

This is the code used for EWMA plots.

```
#EWMA
ewma.func <- function(rets, lambda) {
  sig.p <- 0
  sig.s <- vapply(rets, function(r) sig.p <<- sig.p*lambda + (r^2)*(1 - lambda), 0)
  return(sqrt(sig.s))
}
lambda <- 0.94
rets <- 0.02*rnorm(100)
system.time( replicate(1000, ewma.func(rets, lambda)) )
ewma_fit <- ewma.func(Data$Statoil,lambda)
#Plot for EWMA
plot(ewma_fit, type="line", xlab="Date", ylab="Volatility", main="Statoil")
```

This is the code used for EWMA VaR calculations and backtesting

```
# EWMA med iGARCH
spec2 <- ugarchspec (variance.model = list(model = "iGARCH", garchOrder =
                            c(1,1)), mean.model=list(armaOrder=c(0,0),
                            include.mean=FALSE),distribution.model = "norm",
                             fixed.pars = list(alpha1=0.06))

fit2 <- ugarchfit(spec=spec2, data=Data$Statoil,
out.sample=0, solver="solnp",solver.control=list(trace=0))
show(fit2)

filt2= ugarchfilter(spec=spec2, data=Data$Statoil)
```

```
show(filt2)

roll2 <- ugarchroll(spec=spec2, data=Data$Statoil, n.start =250,
   refit.every=1, refit.window = "moving",
   solver="solnp", solver.control=ctrl, calculate.VaR=TRUE, VaR.alpha=c(0.01,0.05),
   keep.coef=TRUE,fit.control = list(scale=1))

roll2 = resume(roll2, solver="gosolnp")
report(roll2, type="VaR", VaR.alpha=0.01, conf.level=0.99)
report(roll2, type="VaR", VaR.alpha=0.05, conf.level=0.95)
plot(roll2, which = 4)
plot(roll2, VaR.alpha=0.05, which = 4)
plot(fit2, which =2)
```

A.2 GARCH & EGARCH

This is for GARCH and EGARCH modelling with VaR and backtesting

```
# install packages tseries, zoo, forecast, FinTS, rugarch

#Log plot
plot(Data$Statoil, type = "line", xlab = "Date", ylab = "Daily returns",
     main = "Statoil")

fit1 <- auto.arima(Data$Statoil, trace=TRUE, test="adf", ic="bic")
fit1
Box.test(Data$Statoil, lag=12, type="Ljung-Box")
Box.test(fit1$residuals^2, lag=12, type= "Ljung-Box")
adf.test(Data$Statoil, alternative = "stationary")
jarque.bera.test(Data$Statoil)

#GARCH(1,1)
res_garch11_spec <- ugarchspec(variance.model = list(model="sGARCH",
garchOrder = c(1,1)),
mean.model= list(armaOrder = c(1,1)))
```

```
res_garch11_fit <- ugarchfit(spec=res_garch11_spec,
data =Data$Statoil, out.sample = 0,
solver="hybrid", distribution.model ="norm")
res_garch11_fit

#VaR with GARCH(1,1)
ctrl = list(tol=1e-7, delta=1e-9)
res_garch11_roll <- ugarchroll(res_garch11_spec,Data$Statoil,n.start=250,
refit.every = 1,
refit.window= "moving", solver= "hybrid", calculate.VaR =TRUE,
VaR.alpha=c(0.01,0.05), keep.coef=TRUE, solver.control =ctrl,
fit.control = list(scale=1))
res_garch11_roll = resume(res_garch11_roll, solver="gosolnp")
#Backtesting
report(res_garch11_roll, type ="VaR", VaR.alpha=0.01, conf.level=0.99)
report(res_garch11_roll, type ="VaR", VaR.alpha=0.05, conf.level=0.95)
#Plot VaR
plot(res_garch11_fit, which=2)
plot(res_garch11_roll, VaR.alpha=0.05, which=4)
plot(res_garch11_roll, which=4)
#EGARCH(1,1)
res_garch11_spec2 <- ugarchspec(variance.model = list(model="eGARCH",
garchOrder = c(1,1)),
mean.model= list(armaOrder = c(1,1)))
res_garch11_fit2 <- ugarchfit(spec=res_garch11_spec2,
data =Data$Statoil,
out.sample = 0, solver="hybrid", distribution.model ="norm")
res_garch11_fit2
#VaR with EGARCH(1,1)
res_garch11_roll2 <- ugarchroll(res_garch11_spec2,Data$Statoil,n.start=2000,
refit.every = 1,
refit.window= "moving", solver= "hybrid", calculate.VaR =TRUE,
VaR.alpha=c(0.01,0.05), keep.coef =TRUE, solver.control = ctrl,
fit.control = list(scale=1))
```

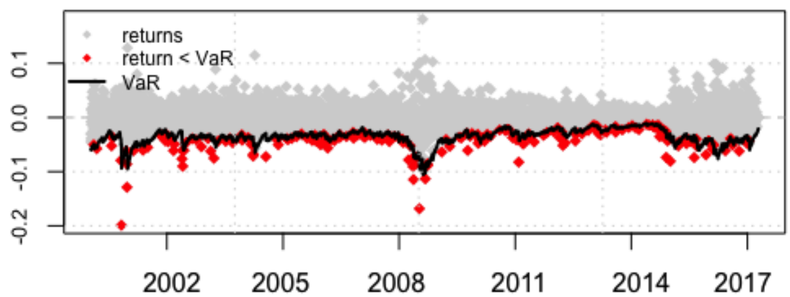
```
res_garch11_roll2 = resume(res_garch11_roll2, solver="gosolnp")
#Backtesting
report(res_garch11_roll2, type ="VaR", VaR.alpha=0.01, conf.level=0.99)
report(res_garch11_roll2, type ="VaR", VaR.alpha=0.05, conf.level=0.95)
# Plot VaR
plot(res_garch11_fit2, which=2)
plot(res_garch11_roll2, VaR.alpha= 0.05, which=4)
plot(res_garch11_roll2, which = 4)
```

B Graphs

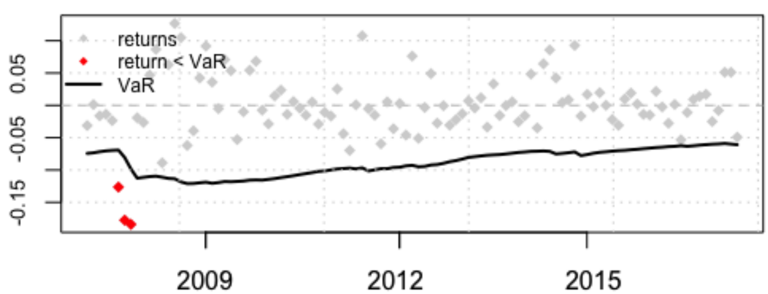
Presented here are graphical presentation of VaR calculations identical to those presented in figure 16 and 17 in chapter 6.

B.1 EWMA

Figure 19: Daily VaR of commodities using EWMA



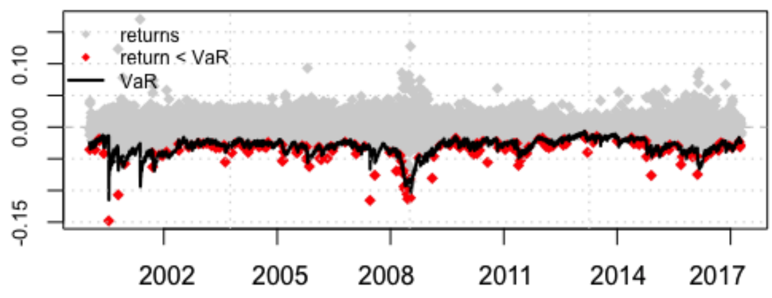
(a) Oil



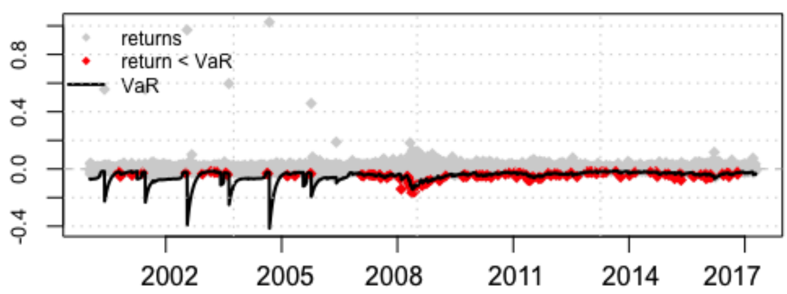
(b) Aluminum

The figure shows daily VaR calculations using the EWMA model plotted against the log returns. The confidence level is 95%

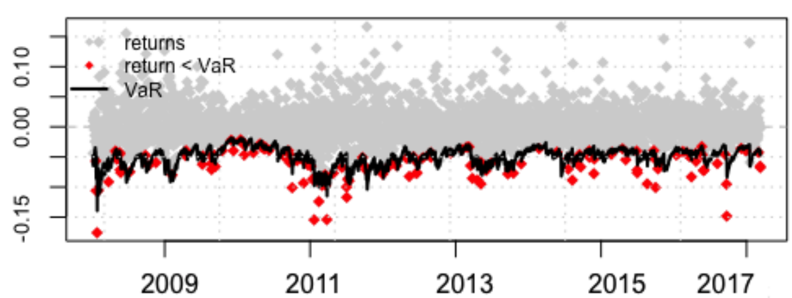
Figure 20: Daily VaR of stocks using EWMA



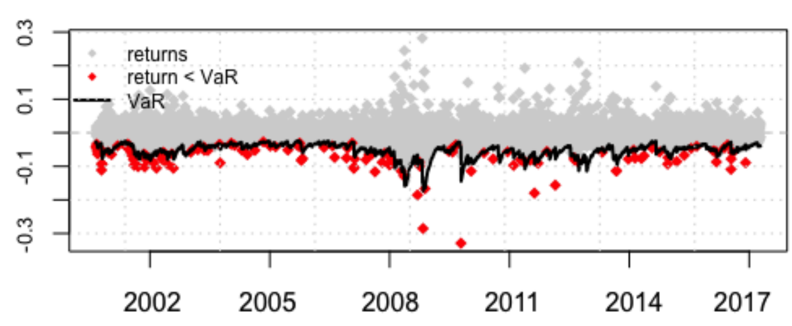
(a) Statoil



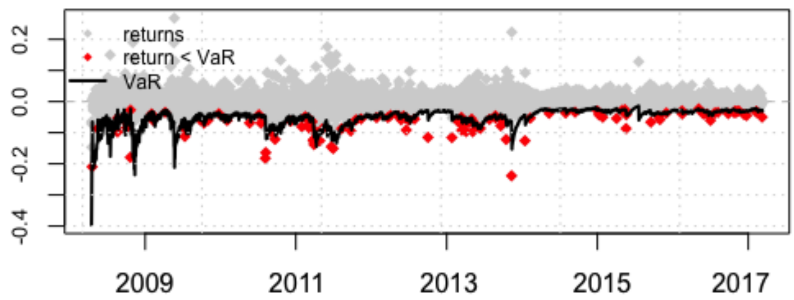
(b) Norsk Hydro



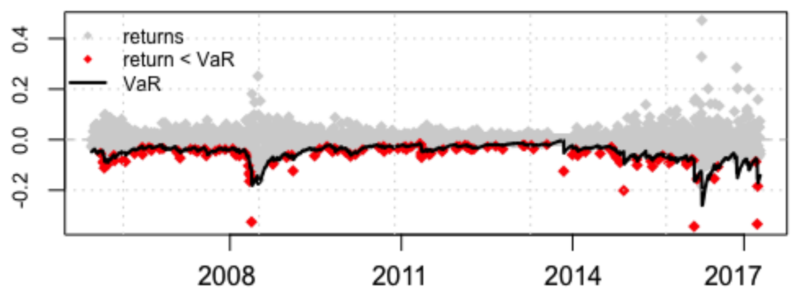
(c) Norwegian



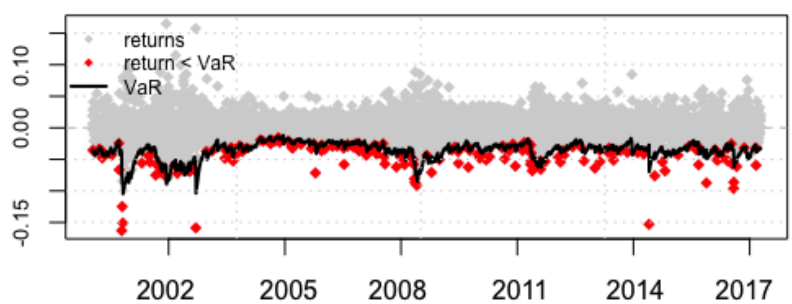
(d) SAS



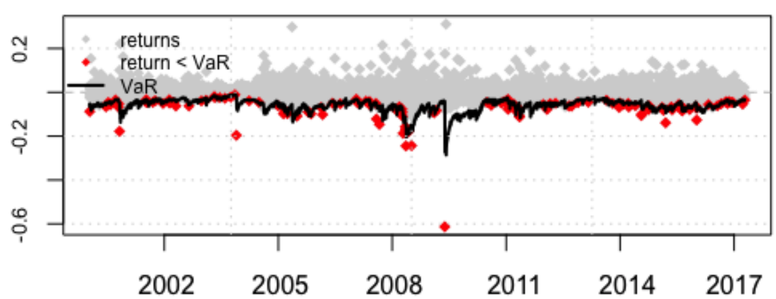
(a) MHG



(b) Seadrill



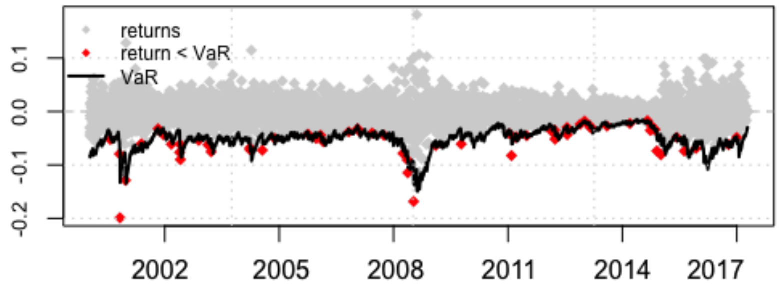
(c) Lufthansa



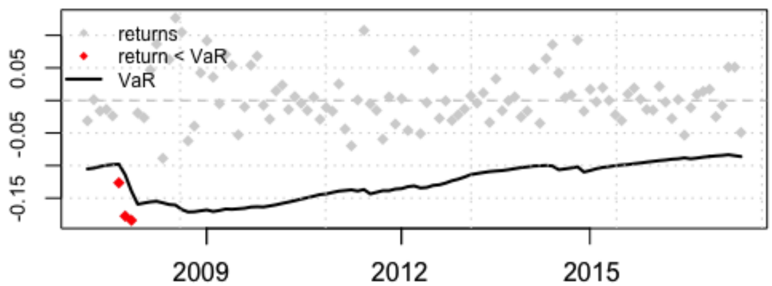
(d) DNO

The figure shows daily VaR calculations using the EWMA model plotted against the log returns for the stocks. The confidence level is 95%

Figure 22: Daily VaR of commodities using EWMA



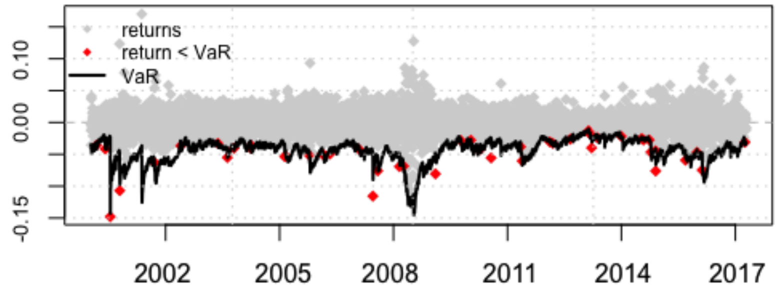
(a) Oil



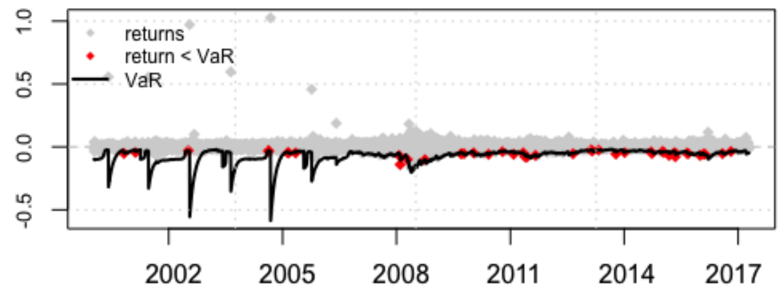
(b) Aluminum

The figure shows daily VaR calculations using the EWMA model plotted against the log returns. The confidence level is 99%

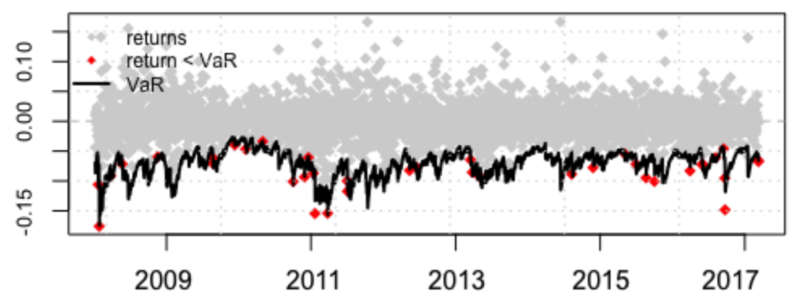
Figure 23: Daily VaR of stocks using EWMA



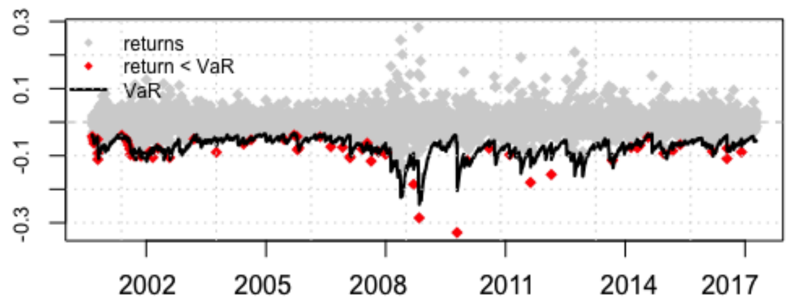
(a) Statoil



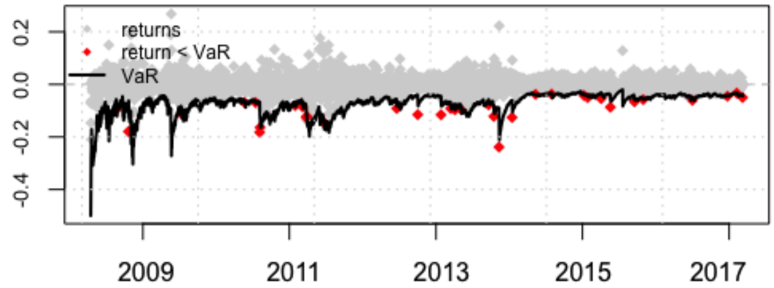
(b) Norsk Hydro



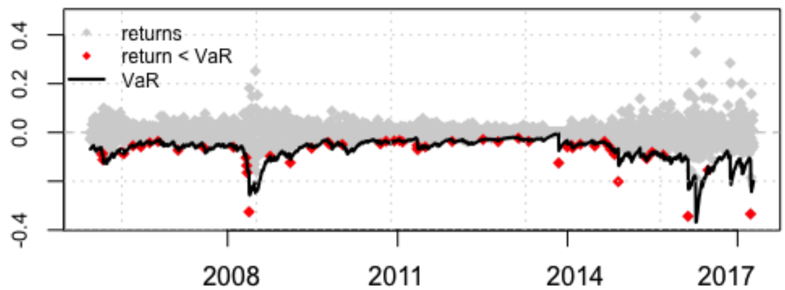
(c) Norwegian



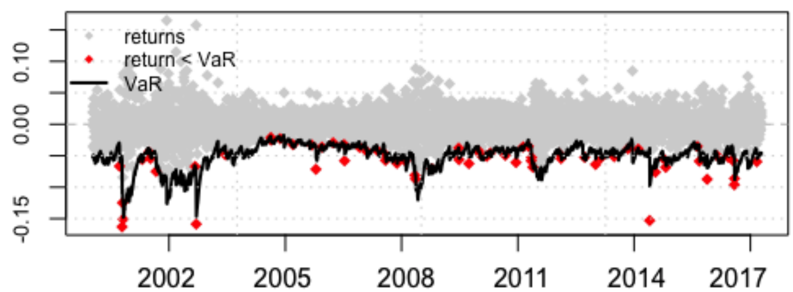
(d) SAS



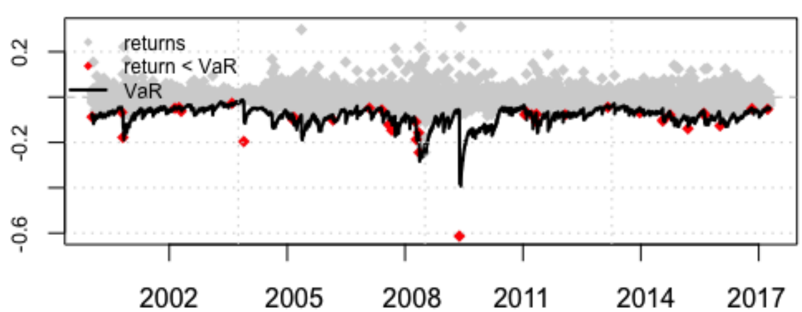
(a) MHG



(b) Seadrill



(c) Lufthansa



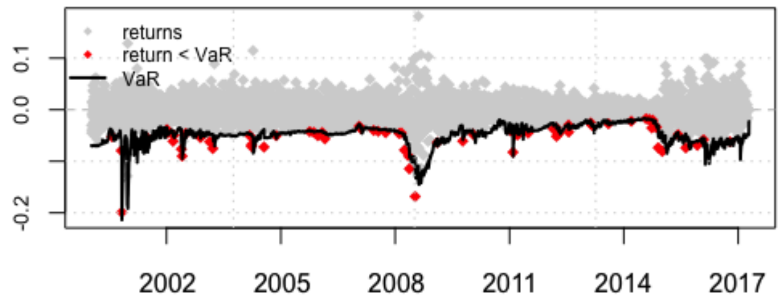
(d) DNO

The figure shows daily VaR calculations using the EWMA model plotted against the log returns for the stocks. The confidence level is 99%

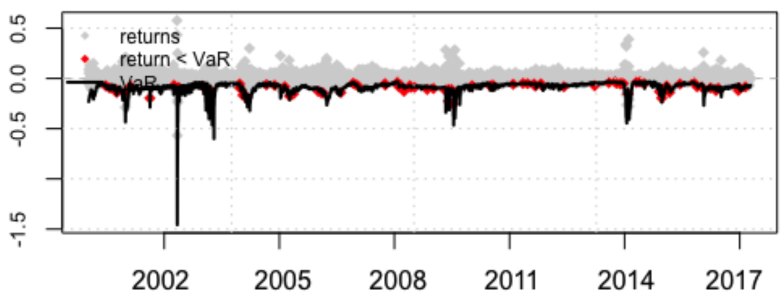
B.2 GARCH

Here are GARCH VaR graphs with 99% confidence level.

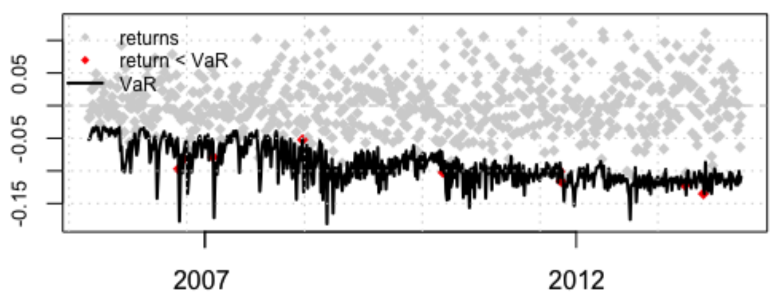
Figure 25: Daily VaR of commodities using GARCH(1,1)



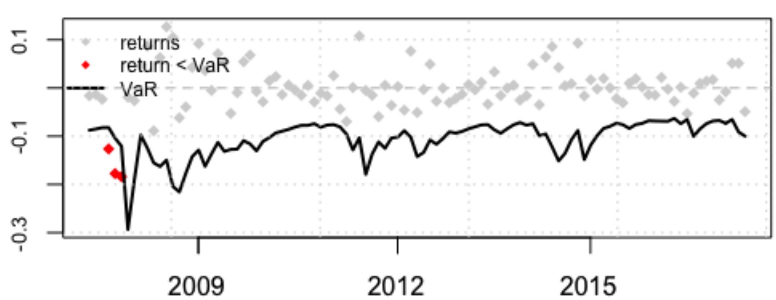
(a) Oil



(b) Gas



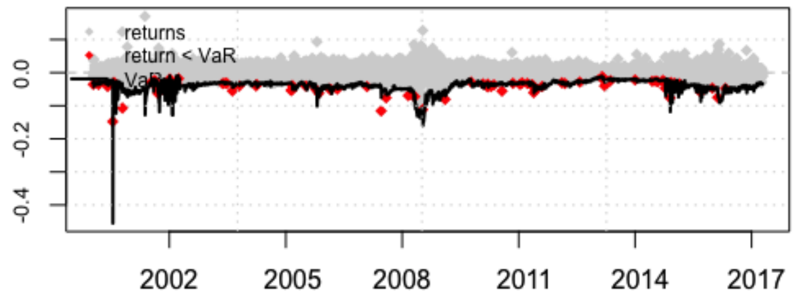
(c) Salmon



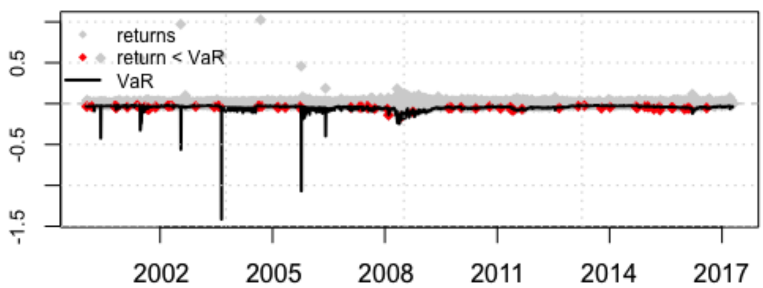
(d) Aluminum

The figure shows daily VaR calculations using the GARCH(1,1) model plotted against the log returns. The confidence level is 99%

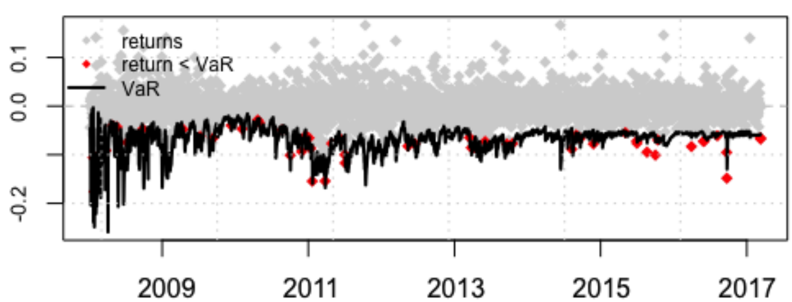
Figure 26: Daily VaR of stocks using GARCH(1,1)



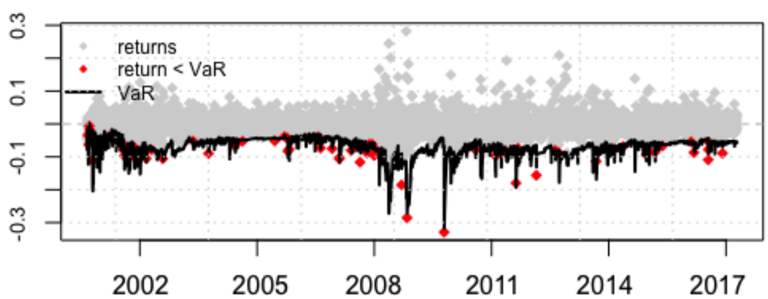
(a) Statoil



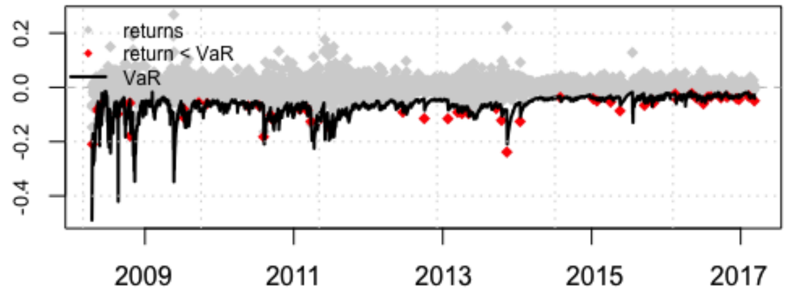
(b) Norsk Hydro



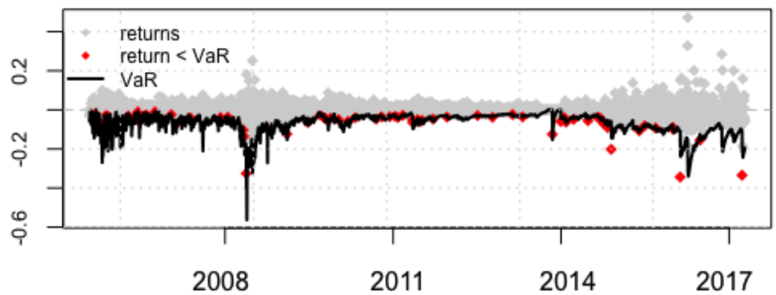
(c) Norwegian



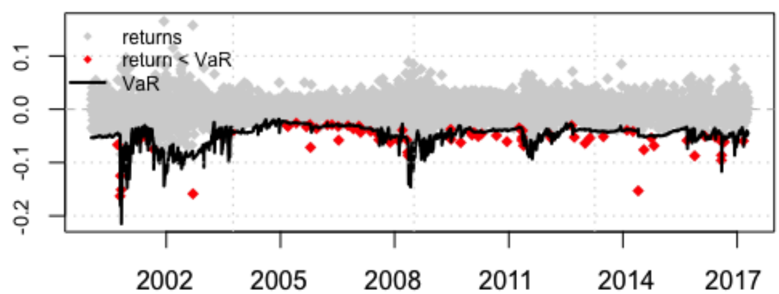
(d) SAS



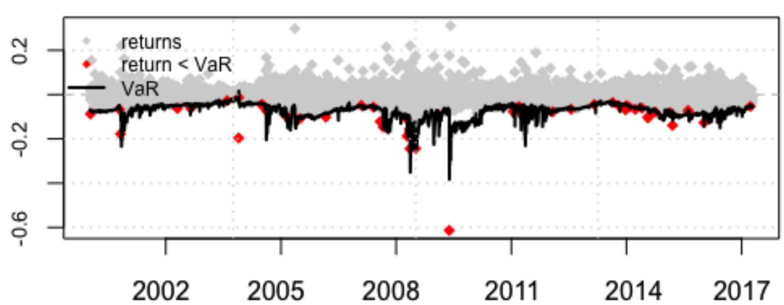
(a) MHG



(b) Seadrill



(c) Lufthansa



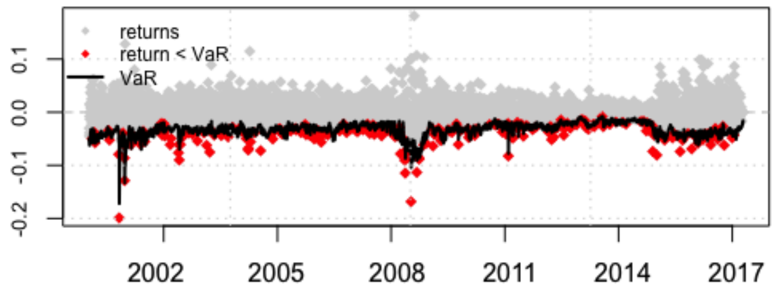
(d) DNO

The figure shows daily VaR calculations using the GARCH(1,1) model plotted against the log returns for the stocks. The confidence level is 99%

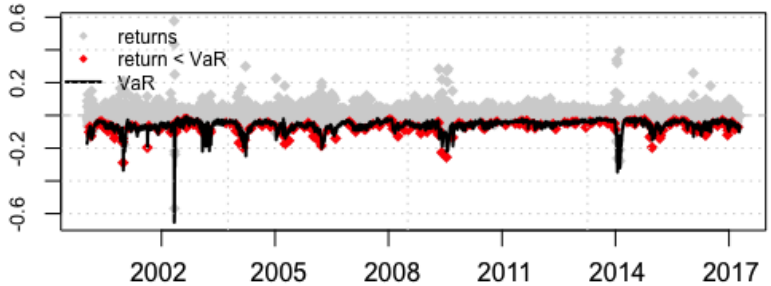
B.3 EGARCH

Presented here are EGARCH VaR of commodities for 95% and 99% confidence level.

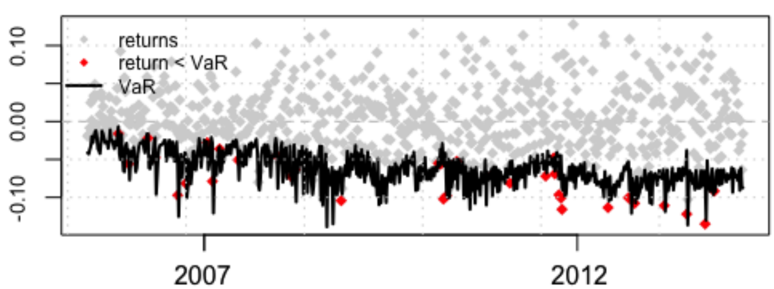
Figure 28: Daily VaR of commodities using EGARCH(1,1)



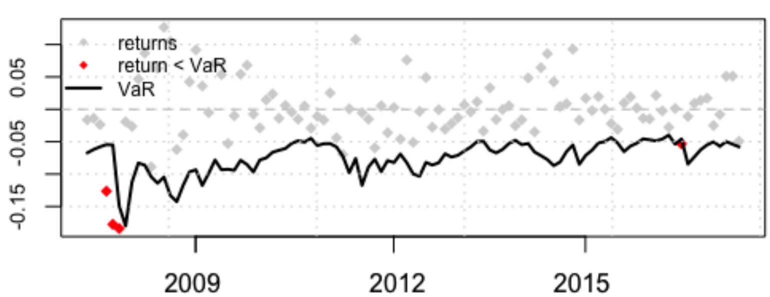
(a) Oil



(b) Gas



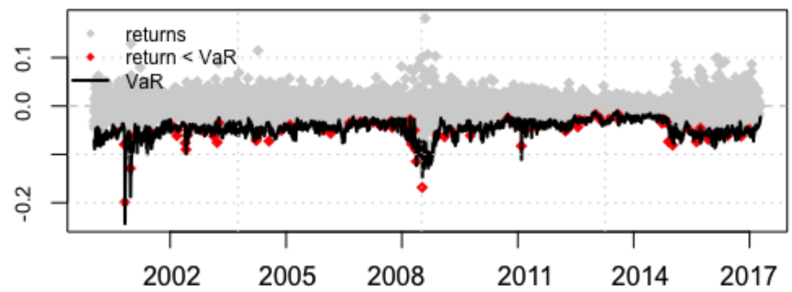
(c) Salmon



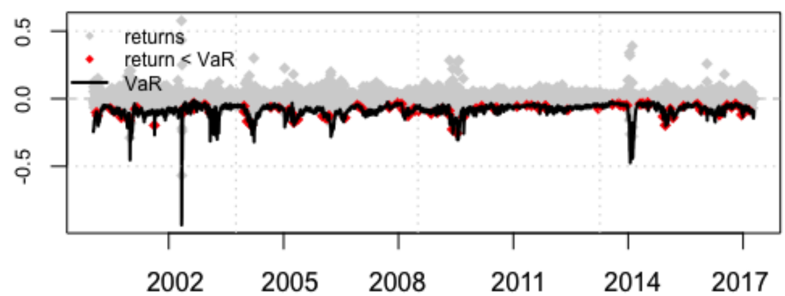
(d) Aluminum

The figure shows daily VaR calculations using the EGARCH(1,1) model plotted against the log returns. The confidence level is 95%

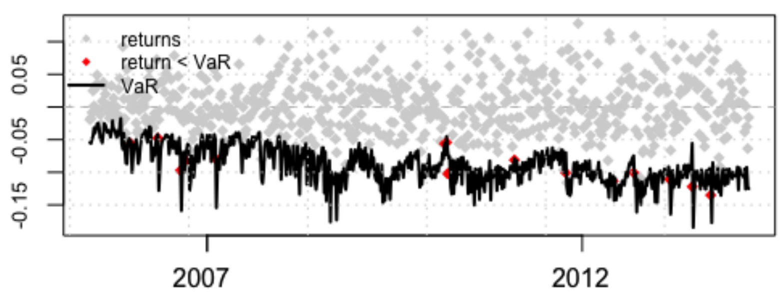
Figure 29: Daily VaR of commodities using GARCH(1,1)



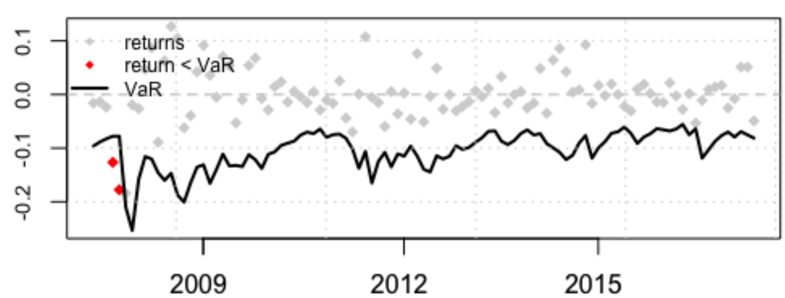
(a) Oil



(b) Gas



(c) Salmon



(d) Aluminum

The figure shows daily VaR calculations using the GARCH(1,1) model plotted against the log returns. The confidence level is 99%