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USE OF PROBABILITY MANAGEMENT IN E&P PORTFOLIO ANALYSIS

Thesis submitted in partial fulfillment of
the requirements for the degree of
master of science in petroleum reservoir technology

by
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University of Stavanger

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Abstract

USE OF PROBABILITY MANAGEMENT IN E&P PORTFOLIO ANALYSIS

Jugal Chetankumar Bodawala

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Capital investment decisions are a critical decision that every organization must take in a careful manner to optimize its resources.In the industry driven by uncertainty such as upstream petroleum industry, it becomes vital to consider uncertainties in a proper way while making capital investment decisions.Even if one consider cyclical nature of petroleum industry, the historical financial performance of the industry as a whole has been discouraging.One of the key reason behind this can be attributed to use of average or single value in spreadsheet models used for economic evaluation of a project.

Interpretation of *Modern Portfolio Theory* has long been established as a valuable tool while evaluating investment opportunities in the context of petroleum projects-portfolio.We use a field of information management called *Probability Management* to build a project level and portfolio level model.*Probability management*, which uses an array of pre-generated random trials as an uncertain variable, provides a standardized way to communicate and model uncertainties across the organization without the need for any special program.

The model developed in this work used two kinds of price model to demonstrate an importance of including inter-asset dependencies in the form of stochastic oil and gas price model to show the usefulness of approach developed in this study compare to current practice.

The interactive model, developed in this study, is easily customizable and shareable to a broad audience.This study provides a portfolio and project level model with multiple attributes that decision maker can use for making capital investment decisions.We welcome further research for the use of *Probability Management* in another area of the petroleum industry.

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“Tell me and I forget
Teach me and I
remember.Involve
me and I learn.”

-Benjamin Franklin.

Acknowledgements

“ The only certainty is that nothing is certain.

-Pliny the Elder.

This study is an accumulation of not only my effort but the effort of several others to whom I must express gratitude.

My journey to the University of Stavanger was anything but certain.I must thank the University of Stavanger and this beautiful country for giving me an opportunity to develop as a human being.

My advisor Dr. Reidar Brumer Bratvold is always an inspiration.Our discussion on various matters has always motivated me.I am honored in working alongside him, and I look forward to the future.

I also wish to express my gratitude to all the teacher who has helped me to reach where I am today.It was not possible without you all.

My friends here and in India have always defined me as a person.I am more than proud to call them my friends.

One thing that I learned in the past two years is the importance of collaboration. I must thank many people especially, people associated with *Probability Management* for their support and their great initiative.

At last, My parents and family is a foundation to all in my life.No words can begin to justify my appreciation for their support and love to me.

Chapter 1

Introduction:

Capital investment decisions are a critical decision that every organization must take in a careful manner to optimize its resources.In the industry driven by uncertainty such as upstream petroleum industry, it becomes vital to consider uncertainties in a proper way while making capital investment decisions.Even if one consider cyclical nature of petroleum industry, the historical financial performance of the industry as a whole has been discouraging.One of the key reason behind this can be attributed to use of average or single value in spreadsheet models used for economic evaluation of a project.

In this research, we look into an emerging field of information management called, *Probability Management* as a way to communicate and model uncertain variable across the organization.We use portfolio analysis of petroleum projects-portfolio based on *Modern Portfolio Theory*, to demonstrate working and usefulness of *Probability Management*.

The primary objectives of this thesis are to:

- Introduce field of *Probability Management*.
- Develop project level and portfolio level model that can be useful for portfolio analysis of petroleum projects-portfolio.
- Demonstrate the effect of ignoring stochastic nature of oil and gas price models on the portfolio analysis of petroleum projects-portfolio.

Following the introduction, We have organized this thesis in the following manner:

1. Chapter-2 (Portfolio Theory):

   Chapter 2 introduces the *Modern Portfolio Theory* and further key development in an investment analysis.We then provide a brief review of *Mean-Variance Optimization*.We conclude this chapter by giving an account of the use of portfolio analysis in the petroleum industry.

2. Chapter-3 (Probability Management: Cure for Flaw of Averages):

   Chapter 3 introduces the field of *Probability Management* and discuss its key concepts.We also provide an overview of *SIPmath*^{TM} *Modeler Tools* used in this research.

3. Chapter-4 (Project Model):

   Chapter 4 gives an account of how the project level model utilized in this work have been developed using principles of *Probability Management* and *SIPmath^{TM} Modeler Tools*.

4. Chapter-5 (Portfolio Model):

   Chapter 5 discuss the development of spreadsheet models for portfolio analysis based on a project model developed in Chapter 4.

5. Chapter-6 (Case Study):

   Chapter 6 provides a case study to demonstrate the usefulness of model developed in this work in a real-life problem using a hypothetical scenario.

6. Chapter-7 (Summary and Further Research):

   Chapter 7 provides an overview of this work and general idea regarding further research directions.

Chapter 2

Portfolio Theory:

"My ventures are not in one bottom trusted,

Nor to one place; nor is my whole estate

Upon the fortune of this present year;

Therefore, my merchandise makes me not sad."

-Antonio

Act I, Scene 1,

The Merchant of Venice.

Portfolio Theory has its root in financial economics. First, we present a brief overview of Portfolio Theory before 1950 and discuss *Modern Portfolio Theory* and subsequent development in detail.We describe the mathematical concept behind *Mean-Variance Optimization*(MVO) at length.In the end, we present key differences between financial portfolio analysis and petroleum portfolio analysis and list key research done in the area of exploration & production portfolio analysis.

2.1 Financial Portfolio Management:

In one word, the result of portfolio theory is *Diversification*.Harry M. Markowitz,(H. M. Markowitz, 1999) explained diversification is not a new concept that was introduced by him, even Shakespeare knew about the diversification (See quote on this page).What he presented in his seminal paper *Portfolio Selection* (H. Markowitz, 1952) for the first time was the systematic thinking on how each asset affects return and the risk of the entire portfolio.

The Theory of Investment Value(Williams, 1938) is considered one of the most famous works in finance.He introduced discounted cash flow based valuation of the financial asset, especially *Dividend Discount Model*[1].The goal of the investor at that time was to find a stock at a good price and own it.As Markowitz himself explained in his Nobel

[1]Dividend Discount Model(DDM) is a procedure to value financial asset using the predicted dividends and to discount it back to present value.

Lecture(H. M. Markowitz, 1990), the inspiration behind what now known as *Modern Portfolio Theory*, came to Markowitz while reading The theory of investment by John Burr Williams during working towards his doctoral degree due to lack of a framework to consider the effect of an investment on the risk and reward of the entire portfolio of assets.

In 1952, Harry Markowitz (H. Markowitz, 1952) wrote *"Portfolio Selection."* In this work, he lay the foundation of *Modern Portfolio Theory*.He stated that any rational investor has its level of risk for which he is comfortable.Also, every investor has two objectives,

1. Maximize the return of the portfolio.
2. Minimize the risk of the portfolio.

He graphically showed, how it is possible to arrive at a combination of security which for a given risk, maximize the return and for a given return, minimize the risk.The collection of all these points will produce what is now known as *efficient frontier*.

Two fundamental assumption and limitations in his works are,

1. Variance is a correct measure of portfolio risk.
2. Joint elliptical distribution such as normal distribution can represent return distribution of individual asset and portfolio.

In 1959, Harry Markowitz published his book *Portfolio selection: efficient diversification of investments*(H. Markowitz, 1959), in which he presented a mathematical framework for *Mean-Variance Optimization* based on quadratic programming technique to construct an efficient frontier.He also identified semi-variance as a better measure of risk, but the computational limitation at that time forced him to make a trade, and he used variance as a measure of risk.

Willian Sharpe,(Sharpe, 1963) simplified the Markowitz *Mean-Variance Optimization* model by assuming that individual covariance between all securities are zero.It is known as a diagonal model of *Mean-Variance Optimization*.

Based on works of Harry Markowitz and William Sharpe, there has been considerable research done; we list few of the key works below:

1. Capital Asset Pricing Model(Lintner, 1965; Mossin, 1966; Sharpe, 1964) extends the work of Markowitz and deal with the economic equilibrium assuming all investors optimize in the manner which Markowitz proposes.It also includes for the first time simple but a significant relationship between risk and return of an asset.
2. Intertemporal Capital Asset Pricing Model(ICAPM)(Merton, 1973)extends the CAPM from the single period to the multiperiod economy.

3. Arbitrage Pricing Theory(APT)(Ross, 1976) includes multiple risk factors.CAPM can be considered as a particular case of APT with the single risk factor.
4. Black-Litterman Model(Black & Litterman, 1992) overcome one significant practical limitation of *Modern Portfolio Theory* associated with an adequate approximation of expected return of an asset.

All these research has given birth to the investment analysis industry, where the goal is to manage risk and return in a manner which is in alignment with investor's risk appetite and moreover, his targeted return.

2.2 Mean-Variance Optimization:

Mean-Variance Optimization, which is a quadratic programming model, is first introduced by Harry Markowitz(H. Markowitz, 1959).In this section, we will first compare uncertainty and risk.We then discuss mathematical concepts behind *Mean-Variance Optimization*.

In everyday life, risk and uncertainty is often used interchangeably, but based on (Bratvold & Begg, 2010; Sam Savage & Zweidler, 2006a), we define risk and uncertainty as:

Uncertainty: Uncertainty represents the state of our knowledge.We do not have any control over it.We express uncertainty regarding possible events and their associated probabilities.Usually, uncertainty expressed prior in terms of the probability distribution.

Risk: Risk is an undesirable consequence of uncertainties. It is a subjective to the person, as he or she determines what is the undesirable consequences.Definition of adverse event and its associated probability will determine the risk.

In layman's term, financial portfolio is a collection of asset which is held together in particular proportion to achieve a target portfolio return.There are two types of risk in the context of the portfolio:

systematic risk: Systematic Risk or non-diversifiable risk are risks which affect an entire market such as interest rate or inflation rate.

unsystematic risk: Unsystematic risk or diversifiable risk are risks which are related to particular company or a particular sector only such as the price of gold or production of coal.

Mean-Variance Optimization is a technique that helps to diversify a portfolio and reduce unsystematic risk[2] (See Figure 2.1 on the next page adopted from (Walls, 2004)). We consider having a total of 'n' opportunities available to invest in exchange traded

[2]Theoratically it is possible to totally eliminate unsystematic risk.

instruments such as stocks and bonds.Based on the historical data of return we can calculate mean and variance of the distribution of return for each asset as follows.

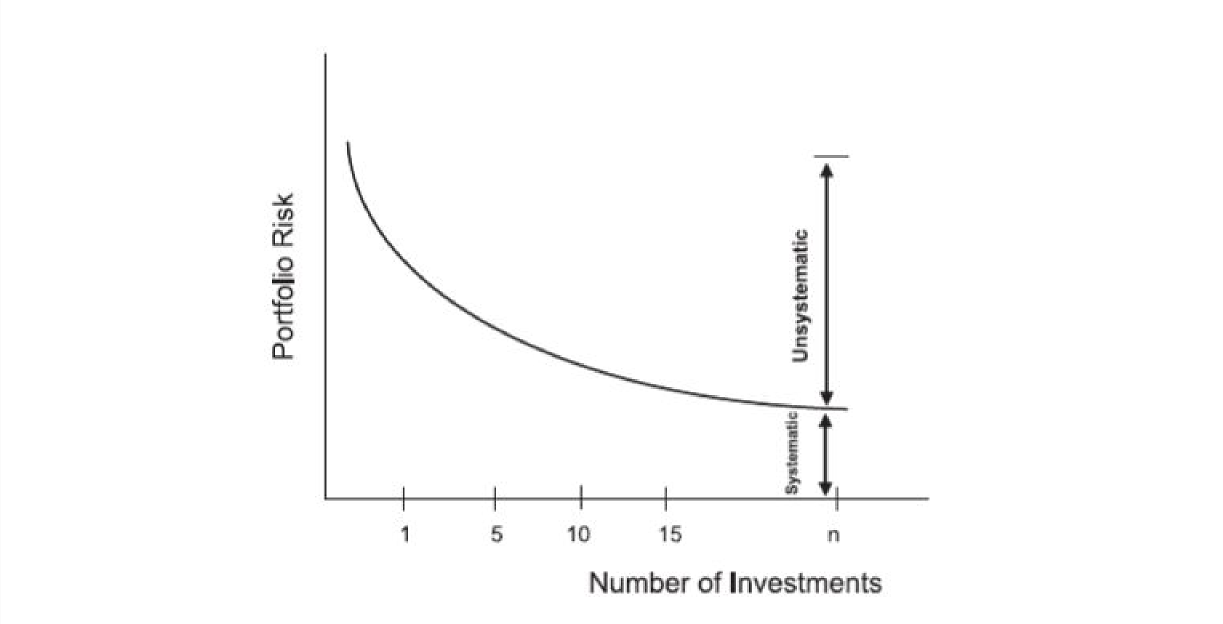


Figure 2.1: Portfolio Risk v/s Number of Investment

$$Mean(\mu_i) = \sum_{t=1,i=1}^{J,n} \frac{1}{J}(r_{it}) \tag{2.1}$$

$$Variance({\sigma_i}^2) = \sum_{t=1,i=1}^{J,n} \frac{1}{J}(r_{it} - \mu_i)^2 \tag{2.2}$$

Diversification is useful to reduce risk because returns of different assets do not move in the same direction.The result is the variance of the portfolio is less than the average variance of the assets that constitute the portfolio.Covariance is useful as an absolute measure of a linear relationship between the return of two assets over time(Walls, 2004).Covariance between assets i and j are defined as follows:

$$Covariance(COV_{i,j}) = \sum_{t=1}^{P} \sum_{t=1}^{Q} \frac{1}{PQ}[(r_{it} - \mu_i)(r_{jt} - \mu_j)] \tag{2.3}$$

Where,

$$Covariance(COV_{i,j}) = Variance(\sigma_i{}^2), if\ i = j.$$
$$Covariance > 0,\ return\ of\ asset\ i\ and\ j\ move\ in\ same\ direction.$$
$$Covariance = 0,\ no\ consistent\ relation\ between\ return\ of\ asset\ i\ and\ j.$$
$$Covariance < 0,\ return\ of\ asset\ i\ and\ j\ move\ in\ opposite\ direction.$$

(Pearson) Correlation Coefficient is the normalized version of covariance which shows the strength of linear relationship between returns of 2 assets i and j.

$$Correlation\ Coefficient(\rho_{i,j}) = \frac{COV_{i,j}}{\sigma_i \sigma_j} \tag{2.4}$$

where,

$$-1 \leq \rho_{i,j} \leq 1$$

For $\rho_{i,j} = +1.0$ or $\rho_{i,j} = -1.0$,if we know return of one asset, we can predict return of another asset with complete certainity.

The objective of *Mean-Variance optimization* is to solve for participation vector (X):

$$X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{pmatrix} \tag{2.5}$$

The mean and variance of portfolio is given by:

$$Portfolio\ Mean[E(r_p)] = \sum_{i=1}^{n} (x_i \mu_i) \tag{2.6}$$

$$Portfolio\ Variance(\sigma_p{}^2) = X^T S X \tag{2.7}$$

Where,

$$X^T = \begin{pmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{pmatrix} \tag{2.8}$$

$$S = \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} & \cdots & \sigma_{1n} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} & \cdots & \sigma_{2n} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} & \cdots & \sigma_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{n1} & \sigma_{n2} & \sigma_{n3} & \cdots & \sigma_{nn} \end{pmatrix} \tag{2.9}$$

Here,

$$\sigma_{ij} = \sigma_i \sigma_j \rho_{ij}. \tag{2.10}$$

Figure 2.2 shows the effect of diversification(Adopted from (Walls, 2004)).Shaded cells demonstrate the contribution of variance(unsystematic risk) by each asset and unshaded cells shows the contribution of covariance(systematic risk) by each pair of assets to the portfolio risk.If the return is less than perfectly positively correlated, it is possible to reduce portfolio risk with increase in a number of assets in the portfolio and investment amount unchanged.As figure 2.2 indicate, with assets $\rightarrow$ n, unsystematic risk in total portfolio risk $\rightarrow$ 0,which is the expected result of diversification.

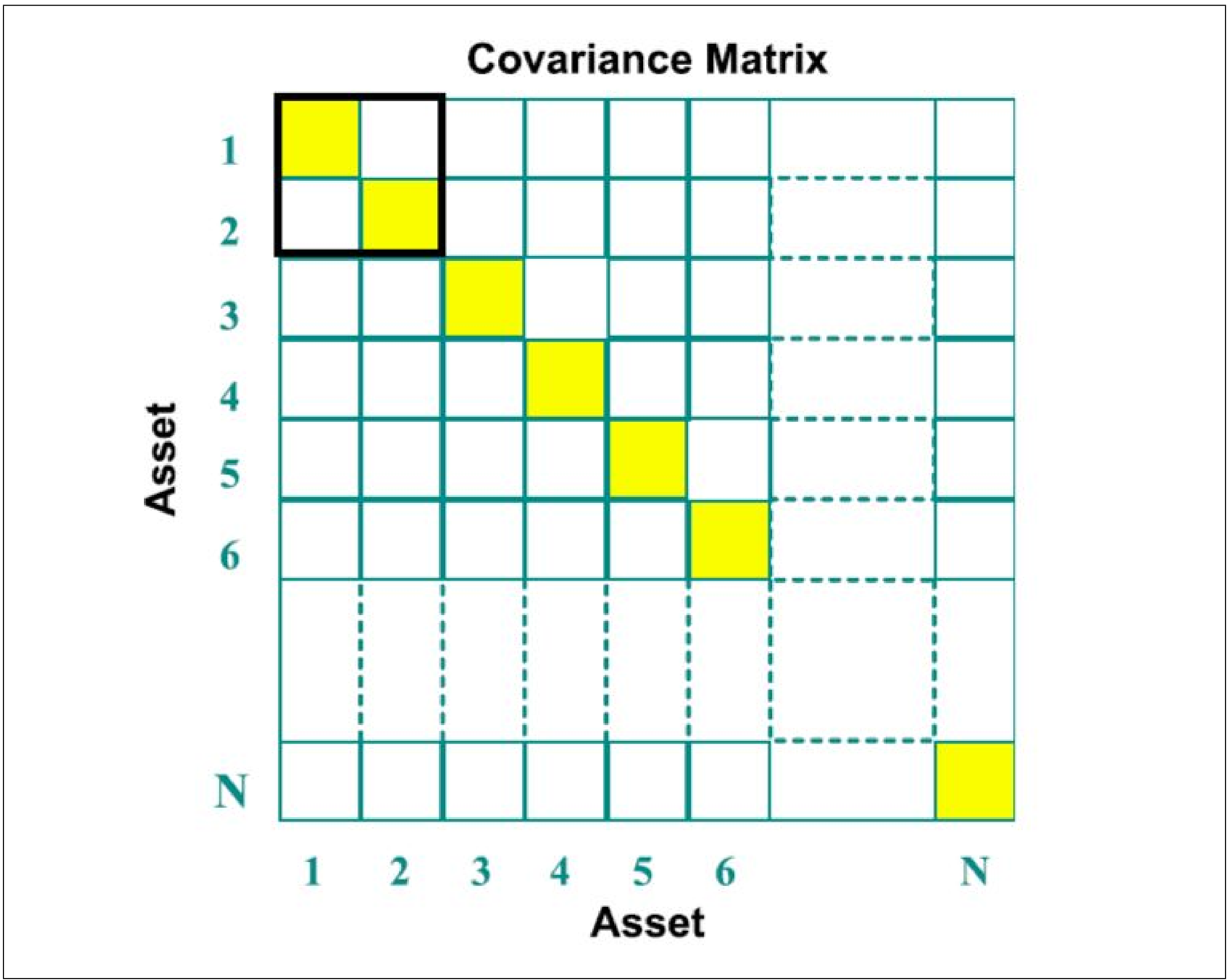


Figure 2.2: Effect of Diversification

Thus, *Mean-Variance Optimization* when introduced by Harry Markowitz represented significant theoretical advance.

2.3 Portfolio Analysis in E&P Industry:

Markowitz's work on portfolio theory provided a sound basis for its application in E&P industry.However, there is a significant disparity between portfolio analysis for investment in instruments of the financial market and E&P projects as follows(Jr. & Savage, 1999; Bratvold, Begg, & Campbell, 2003):

- Stock portfolios depend only on uncertain returns. E&P projects face both private uncertainties involving the discovery and production of oil at a given site, and market uncertainties involving such as commodity price and tax structure. Furthermore, uncertainties in stock returns usually follow a normal distribution while E&P uncertainties are highly skewed and stress rare events.
- Risk in stock portfolios is measured in terms of volatility, the degree to which the portfolio swings in value. In application to E&P portfolios, there is debate as to whether or not only the downside risk should be tracked.
- The stock market is considered to be efficient in that there are no barriers to each item being priced and traded at its actual value, as determined by a large number buyers and sellers with minimal transaction costs. The market for E&P projects is inefficient.
- E&P projects pay out over long time periods.Stocks can be bought or sold at will.
- A stock portfolio contains a small fraction of the outstanding shares of any one company, and there is no restriction on the precision of that fraction. An E&P portfolio usually consists of projects where the company takes a significant portion, and arbitrary fractional investments are not available (the portfolio is lumpy).
- The return of stock portfolios is usually measured as an annualized average percentage return that is independent of the budget of the investor. The performance of E&P capital projects is usually measured directly in terms of *Net Present Value* and is constrained by the capital available, creating budgetary effects.

Below, few of the key papers in the area of E&P Portfolio Management has been summarized:

- Hightower and David (Hightower & David, 1991) introduced the concept of portfolio optimization as an application in E&P industry for the first time.The discussion in their paper covers a broad range of topics that have been further studied as an application of portfolio theory in E&P industry.
- Ball and Savage (Jr. & Savage, 1999) discussed the use of portfolio optimization and difference between the financial portfolio and petroleum portfolio and its implication on the procedures.They introduced what is referred in their paper as *E&P Portfolio Optimization Model*.
- Mcvean(McVean, 2000) demonstrated the effect of different measures of portfolio risk on the efficient frontier.He showed that efficient portfolio according to one risk

measure may not be efficient according to another risk measure.

- Tyler and McVean(Tyler & McVean, 2001) gave insight into the effect of project risking method on the portfolio optimization using Stochastic and discrete evaluation of projects.They observed that inclusion of oil price makes the difference significant.
- Howell and Tyler(III & Tyler, 2001) give an example of how portfolio process can be used to develop corporate strategies based on goals defined instead of the roll-up process in which we derive the goal based on portfolio achieved.
- Fichter (Fichter, 2000) introduced thr use of a genetic algorithm to solve for E&P portfolio optimization problem.
- Allan(Allan, 2003) presented a case for reducing the impact of price uncertainty in E&P industry using portfolio management practices instead of financial hedging instruments.
- Campbell, Bratvold, and Begg (Campbell, Bratvold, & Begg, 2003) looked into the reasons behind underperformance in portfolio optimization case, especially looking into simplification caused by ignoring intra-asset,inter-asset dependencies and optimizing against expected value of the portfolio, ignoring stochastic nature.
- A couple of papers(Walls, 2004; Bratvold et al., 2003), looked into combining decision analysis, especially the use of utility theory into portfolio optimization problem.
- Costa Limma, Gaspar Ravagnani and Schizor(G.A.Costa Lima, Gaspar Ravagnani, & D.J.Schizor, 2012) presented a simplified way to measure the correlation between return of projects using Monte Carlo Simulation.
- Schuyler and Nieman(Schuyler & Nieman, 2008) look into the area of biases which can cause major underperformance in the portfolio performance.

We refer interested reader to various other papers (Orman & Duggan, 1999; DuBois, 2001; Merritt & de San Miguel, 2000; Willigers, weis, & Majou, 2011) which is an excellent read to one who wishes to study this area in depth.The study of this literature also reveals the need for an approach which is easy to implement and communicate at all level of portfolio analysis so real decision maker can interact with it to understand the nuance behind portfolio analysis problem in hand.In the next chapter, we look into details of one such promising approach called, *Probability Management* used in this work.

Chapter 3

Probability Management: Cure for Flaw of Averages

Vectors of scenarios or realization of probability distribution have ben used in stochastic optimization at least since 1991(Dembo, 1991).Savage, Scholtes, and Zweidleri formally introduced the discipline of *Probability Management* (Sam Savage & Zweidler, 2006a, 2006b) in 2006.

We start this chapter with discussion about *Flaw of Averages* and its implications on portfolio analysis.We then give an overview of *Probability Management*.We finish this chapter by describing *SIPmath^{TM} Modeler Tools* used in this work.

3.1 Flaw of Averages:

> "Plans based on average assumptions are wrong on average."
>
> -Sam L. Savage

Organizations often uses single average or base-case numbers to represent business parameters and metrics, due to lack of consistent approach to model and communicate uncertainities, which leads to class of systematic errors knows as the *flaw of averages*.In simple terms, the *flaw of averages* arises because the average of the result of non-linear problem computed based on uncertain distribution is not equal to the result calculated based on the average of the uncertain parameters.

Sam Savage in his book(Savage, 2009), present following seven consequences of *Flaw of Averages*, which arises due to use of the average or single number.

1. It leads to unrealistic and non-existent scenarios.
2. It fails to capture the possibility of delay in completing a task.
3. It fails to look for an opportunity to diversify, resulting in increased risk.
4. It fails to consider interdependence between investment opportunities.

5. The average profit is less than the profit associated with the average demand.
6. Average value is greater than the value associated with average price
7. The cost related to average demand is zero, but the average cost is positive.

This list not exhaustive, but it captures the need of finding the solution for *Flaw of Averages*.

3.2 Probability Management:

This section is based mainly on two-part article(Sam Savage & Zweidler, 2006a, 2006b),in which Savage, Scholtes, and Zweidler first discussed *flaw of averages*, and then in the second part, they presented the idea of *Probability Management* to cure the flaw of averages.*Probability Management*, focuses on estimating, maintaining and communicating the distributions of the random variables driving business.

Using the electrical transmission system, they explained the three underpinnings of *Probability Management* as follows:

1. Interactive Simulation: Interactive simulation plays a role of the light bulb.It provides an experiential understanding of uncertainty and risk.Present Technologies able to run simulations nearly instantaneously each time parameter of a business model is changed.
2. Stochastic Libraries: Stochastic libraries are analogous to the electric power grid.It contain certified probability distribution for use in simulations throughout an organization.
3. Certification Authority: Certification authority is similar to the local power authority, as it makes sure that right balance between complexity and practicality exist while developing and certifying organization's stochastic library.

3.2.1 Key Concepts:

We now discuss few of the key terms associated with *Probability Management*.

- Coherent Modeling:

  Coherent Modeling is an approach developed by Savage, Scholtes and Zweidler for *Probability Management*.The fundamental of coherent modeling lies in stochastic library structure, which consists of *Stochastic Information Packet(SIP)* and *Stochastic Library Unit with Relationship Preserved(SLURP)*.Basically,SIP and SLURP are a collection of pre-generated random trials.

  The benefits of coherent modeling are:

  1. Statistical dependence is modeled consistently across entire organizations.

2. Probabilistic models may be rolled up between levels of an organization.
3. Probabilistic result may be audited at a later date.

- Stochastic Information Packet(SIP):

  Stochastic Information Packet(SIP) is a data structure formalized for a new area of information management called *Probability Management*.It represents distribution in terms of an array of values and metadata.All the values in an array array the possible realization of an uncertain variable.Currently, each element in an array has a probability of 1/N where N is the total number of element in the array.

  The advantage of SIPs are:

  1. Actionable: The output for one application can become the input for a downstream simulation.
  2. Additive: SIp of a sum of the variable is equal to the sum of the SIP.
  3. Auditable: Input and output distributions are treated as data with provenance supporting an audit trail.
  4. Agnostic: SIPs comprise a simple data structure, which may be supported across many platforms.

- Stochastic Library Unit with Relationship Preserved(SLURP):

  SLURP is a coherent collection of SIPs that preserve stastistical relationships between uncertainty.Two or more SIPs are said to be coherent if the values of their corresponding samples are in some way interdependent and that relationship is preserved in the SIPs.Therefore, SLURPs coherence is maintained by permuting all of its constituent SIPs with same permutation index.SLURP make it possible that SIP of a sum of variable is the sum of the SIP.

- SIPmath:

  SIPmath is calculating with uncertain variables the way ordinary maths calculates with single values.It is calculating uncertainty with SIPs.Compare to Monte Carlo Simulation, SIPmath extracts the data part and put it in a SIP.So, in SIPmath generation of data and its use is of a different concern.Figure 3.1 on the following page shows the fundamental of SIPmath, which is

$$For\ all\ i; Output(i) = Model[Input(S(i))] \tag{3.1}$$

3.3 SIPmath™ Modeler Tools:

We discuss *SIPmath™ Modeler Tools* using a small example to highlight relevant features to this work.

Figure 3.2 on the next page shows the toolbar for current version (V.3.2.6) of SIPmath™ Modeler:

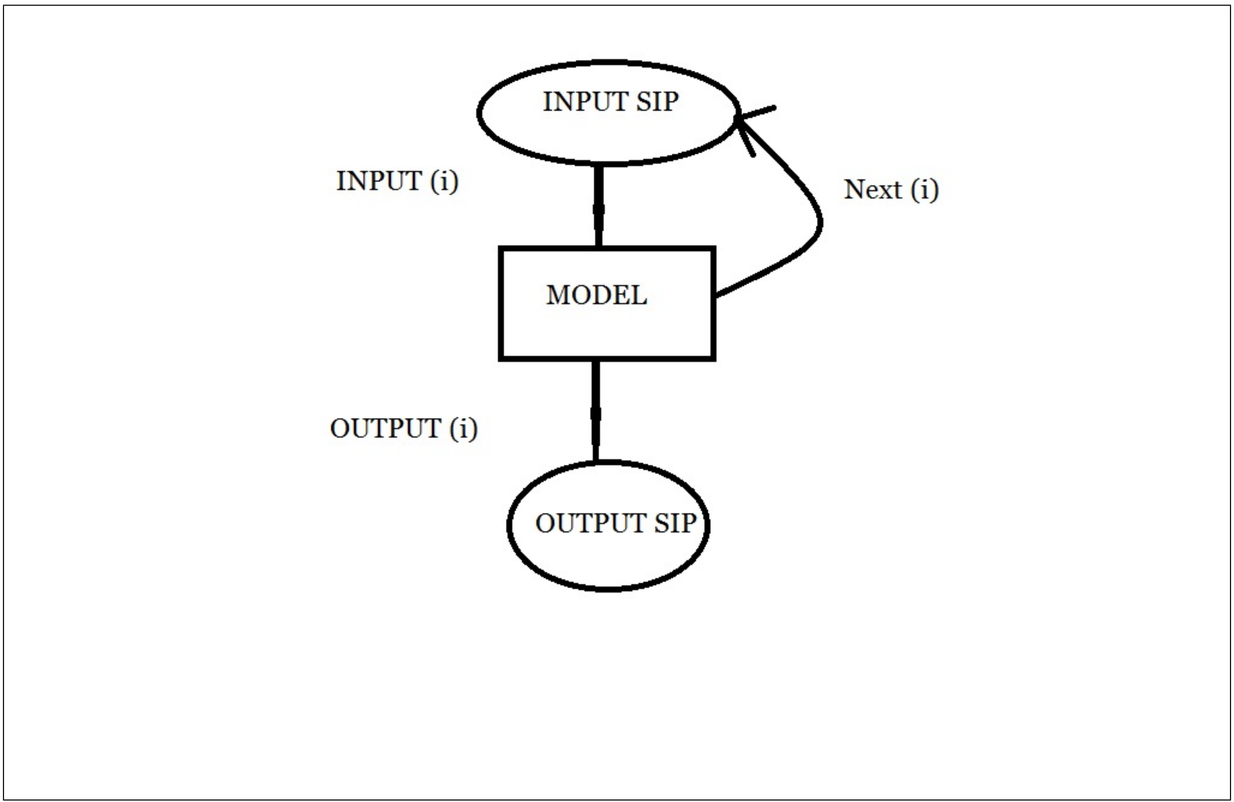


Figure 3.1: SIPmath

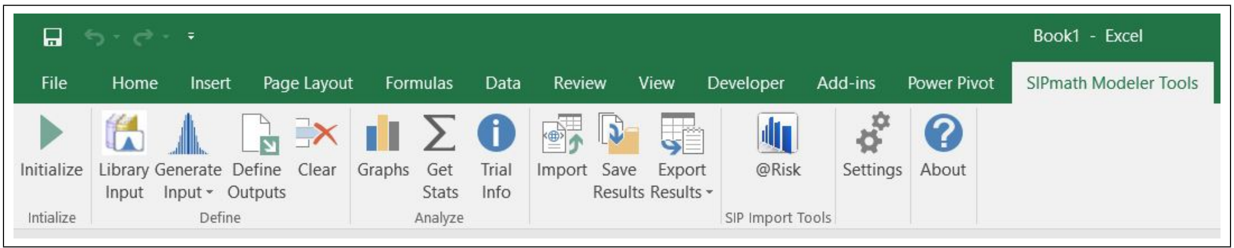


Figure 3.2: Toolbar for SIPmath^{TM} Modeler Tools

- Initialize:

  Figure 3.3 on the following page shows the dialogue box to initialize the model using *SIPmath^{TM} Modeler Tools*.In current workbook and External Workbook, both uses the pre-generated SIP library to initialize the model.In Generate mode[1] model is initialized using random number generation within the model itself.

  Number of Trials specify the number of iteration that we wish to have in each of the variables in our model.Variable ID in Hubbard Decision Research Random Number Generator (see section 4.1 on page 21) is useful to identify each variable.A number of bins are used to specify for histograms of variable output, with a maximum number of bins possible is 100 and the default value for a number of bins is 10.

  Once, we initialize the model, *SIPmath^{TM} Modeler Tools* will automatically add two sheet to the model, PMTable; where data table will be generated and SIPmath Chart Data; which generates and stores the data for graphs.

- Generate Input:

[1]Only generate mode is used in this work.

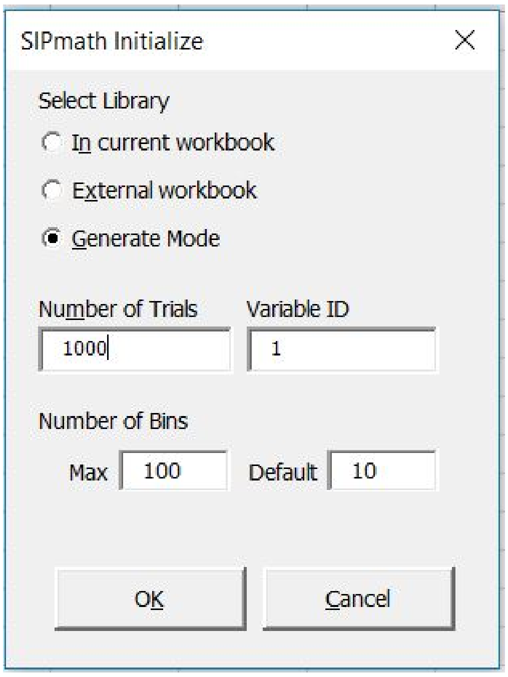


Figure 3.3: Dialogue Box for Initialize Button

Figure 3.4 on the following page shows all the distribution including correlated normal and uniform distribution using Cholesky factorization that can be produced using *SIPmath^{TM} Modeler Tools*.

Using various distribution type, we have generated a small example of 4 input variables A;B;C;D and output variable X related by Equation 3.2 and shown in Figure 3.5 on page 17.

$$X = \frac{(A - B) \times C}{D} \tag{3.2}$$

- Define Output:

  We can generate an output of SIP using this, figure 3.6 on page 17 shows the dialogue box.Figure 3.7 on page 18 shows the sparkline[2] generated using define output.Figures 3.8 on page 18 and 3.9 on page 19 shows corresponding PMTable and SIPmath Chart Data Sheet.PMTable sheet stores the data using data table and array function of excel.

- Graphs:

  Figure 3.10 on page 19 shows the dialogue box for Graphs and figure 3.7 on page 18 shows the histograms, and cumulative distribution function for distribution of output variable X.SIPmath Chart Datasheet automatically generate and store all the data necessary to plot the graphs.

- Library Input:

  Figure 3.11 on page 20 shows the dialogue box for library input, which is useful to import SIPs from one excel workbook to another excel workbook.

[2] A sparkline is a tiny chart in a worksheet cell that provides a visual representation of data.

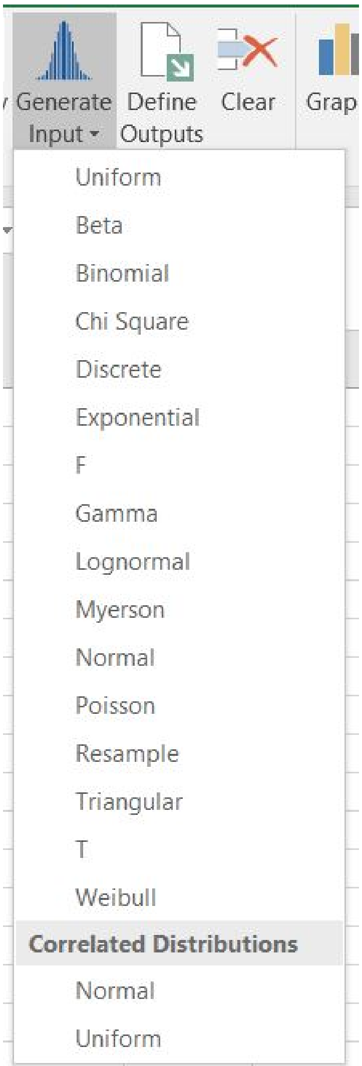


Figure 3.4: Dialogue Box for Generate Input

- Clear:

  Use clear to delete the output SIP.

- Get Stat:

  Get Stats is utilized for a calculation involving output cell(s). For example, figure 3.12 on page 20 shows the average and standard deviation of output distribution X (it shows error) and figure 3.13 on page 20 shows the average and standard deviation(correct) of output distribution X after using get stat button.

- Trial Info:

  It is used to move between individual trials from all trials.

- Import:

  SIPmath^{TM} Modeler Tools allow the import of file either in eXtensible Markup Language(XML) or Comma-Separated Values(CSV) format. It allows files from various kind of software to convert into SIP format.

	Variable	Distribution Name	Distribution	Random Number	Type
Input	A	d_A	0.8594685	0.376740337	Normal
	B	d_B	0.8143702	0.424722919	Log-Normal
	C	d_C	0.0099086		Uniform
	D	d_D	0.9155655	0.419130072	Triangular
Output	X	d_X	0.0004881		

Figure 3.5: SIPmath Example-1(a)-Distribution

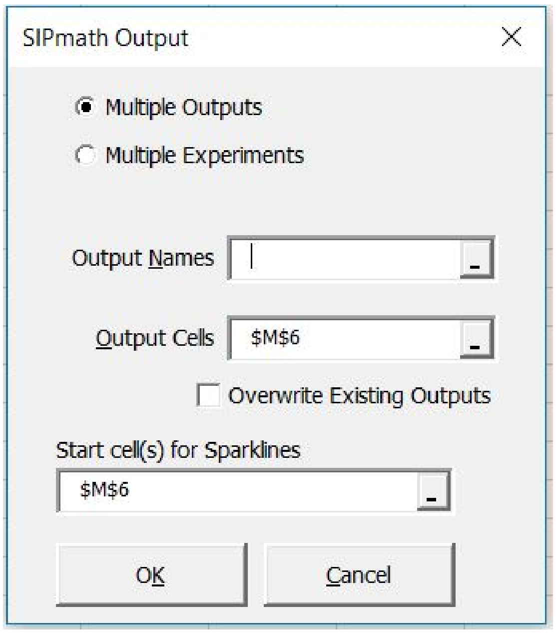


Figure 3.6: Dialogue Box for SIPmath Output

In addition, *SIPmath*^{TM} *Modeler Tools* also allows direct conversion of @Risk(http://www.palisade.com/risk/, 2017) or Crystal ball(http://www.oracle.com/us/products/applications/crystalball/overview/index.html, 2017) model into SIP library format.

- Save Result:

  This allows the user to generate SIP library to store several SIP in one place.

- Export Results:

  This allows SIP of model to converted into eXtensible Markup Language(XML), Comma-Separated Values(CSV) or JavaScript Object Notation(JSON) format.

With the help of *SIPmath*^{TM} *Modeler Tools*,we have developed project level and portfolio level model explained respectively in Chapter 4 and Chapter 5.

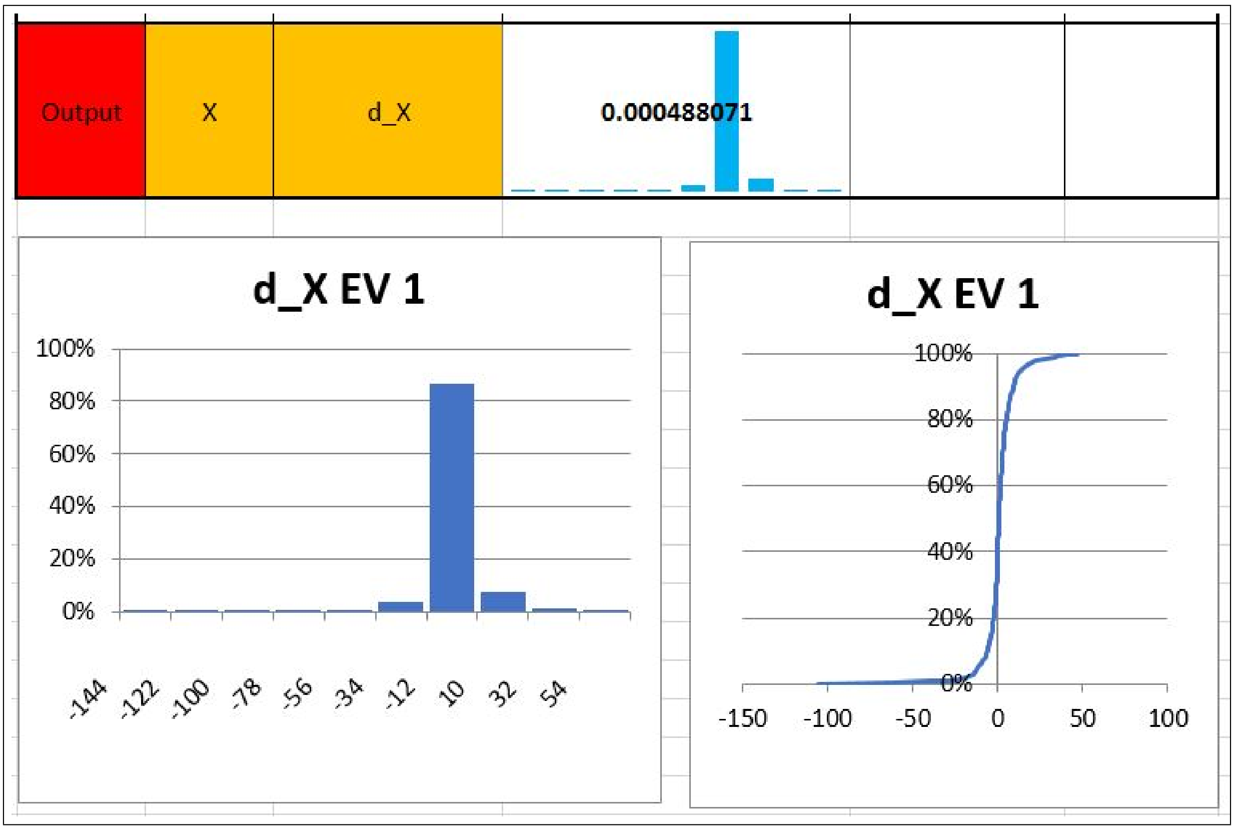
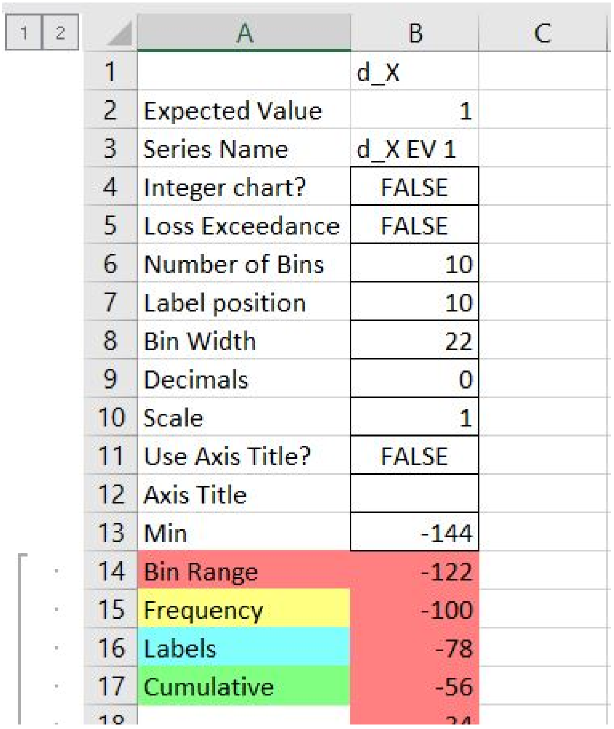


Figure 3.7: SIPmath Example-1(b)-Sparkline

Probability Management

1	Generated with the SIPmath™ M	
	Index	d_X
	Values	0.000488
	1	0.000488
	2	4.690563
	3	1.889338
	4	1.734898
	5	-10.9938
	6	1.416856
	7	10.15279
	8	2.690168
	9	-0.0222
	10	2.932085

Figure 3.8: SIPmath Example-1(c)-PM Table Data



	A	B	C
1		d_X	
2	Expected Value	1	
3	Series Name	d_X EV 1	
4	Integer chart?	FALSE	
5	Loss Exceedance	FALSE	
6	Number of Bins	10	
7	Label position	10	
8	Bin Width	22	
9	Decimals	0	
10	Scale	1	
11	Use Axis Title?	FALSE	
12	Axis Title		
13	Min	-144	
14	Bin Range	-122	
15	Frequency	-100	
16	Labels	-78	
17	Cumulative	-56	

Figure 3.9: SIPmath Example-1(d)-SIP Chart Data

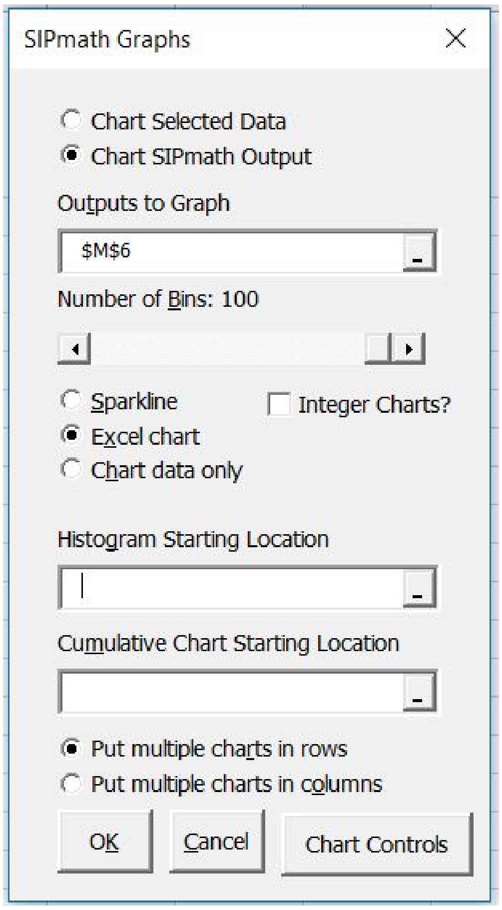


Figure 3.10: Dialogue Box for SIPmath Graphs

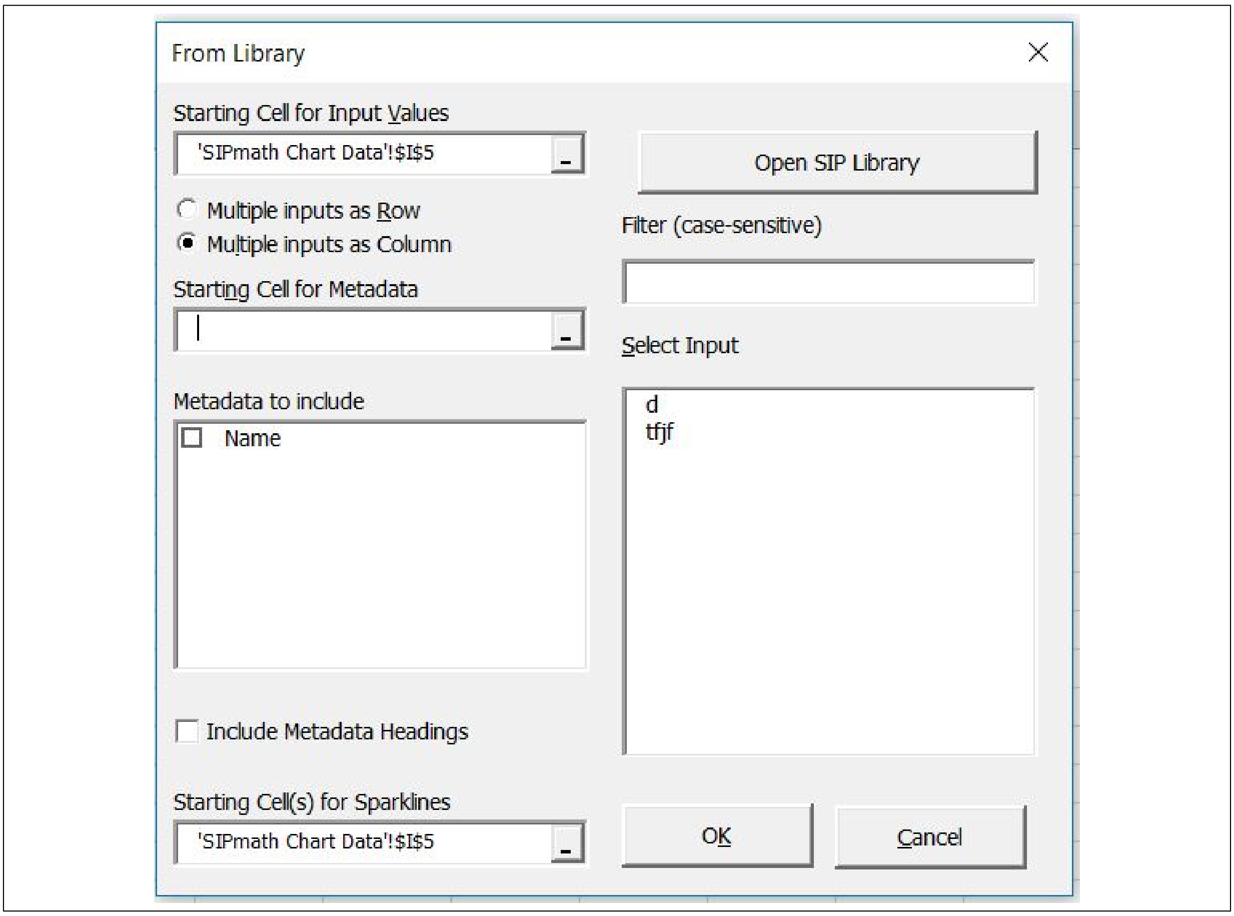


Figure 3.11: Dialogue Box for Library Input

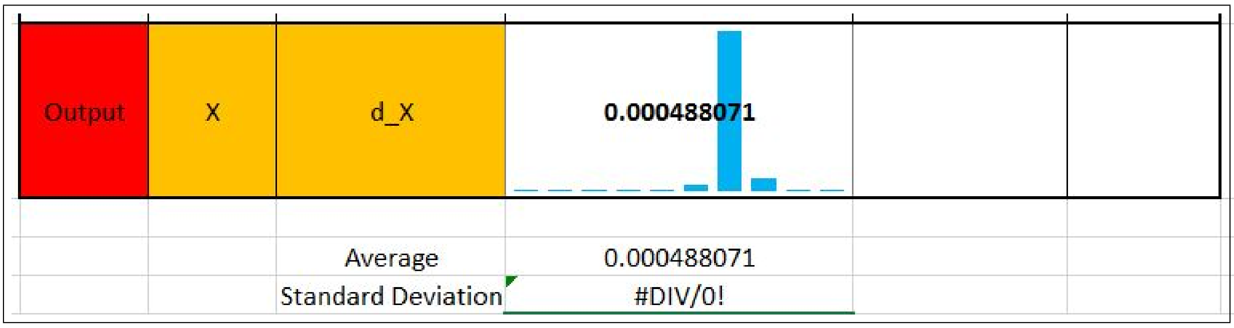


Figure 3.12: Before Using Get Stat Button

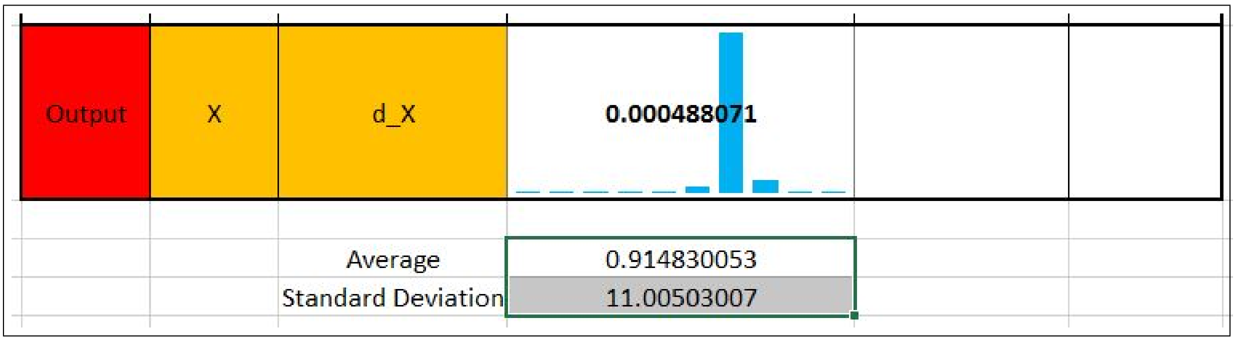


Figure 3.13: After Using Get Stat Button

Chapter 4

Project Model:

Distribution of return for a particular project is important to gauge its effect on the portfolio.In this work, we have used two kinds of price model for oil and gas to build two prototype models.We have developed these models using *The SIPmath^{TM} Modeler Tools for Excel v3.0*(http://probabilitymanagement.org/tools.html, 2017) based on concept explained in chapter 3. A total of 40 synthetic petroleum projects (10-new exploration, 10-new development,20-ongoing production) has been made using each of these two prototype models.

We start this chapter with a discussion on random number generator, especially *Hubbard Decision Research Random Number Generator*(http://probabilitymanagement.org/library/RARNG, 2017) used in this work.Two type of probability distributions are described which were used to generate Stochastic Information Packet(SIP) of input uncertainties using the SIPmath^{TM} Modeler Tools.Based on this two building blocks, We have divided each project model into seven modules.For each project,4000 trials have been run.We use pseudo algorithm format to inform about the thought process behind Excel formulas utilized in each module.

4.1 Random Number Generator:

> "Anyone who considers arithmetical methods of producing random digits is, of course, in a state of sin."
>
> -John von Neumann

As John von Neumann pointed out, it is not possible to generate truly random numbers using arithmetical methods.There are many well-tested pseudo random number generators available today, but *Probability Management* discipline in its current form has two new desirable conditions:

1. Prevent any two independent simulations from inadvertently using the same random number stream.
2. Allow intentionally dependent simulation to use the same stream of random numbers.

Pseudo random number generator with multiple seeds is needed to satisfy these two conditions, which is referred here as Random Access Random Number Generator(RARNG)(http://probabilitymanagement.org/library/RARNG, 2017).The current generation of pseudo random number generator does not have this property, as the generation of each random number depend on last random number generated instead of multiple input parameters.

Hubbard Decision Research(HDR) random number generator is the first step in this direction.It uses two type of IDS:

1. Trial ID(PM_Trials)-To identify particular trial.
2. Variable ID(Start Variable ID)-To define a variable within a simulation.

$$RandomNumber(RN_{HDR}) = X \bmod 2147483647 \tag{4.1}$$
$$Here, X = [(Y + 1000007) \times (Z + 1000013)] \bmod 2147483647 \tag{4.2}$$
$$Z = [P^2 + (P \times Y)] \bmod 99999989 \tag{4.3}$$
$$Y = [S^2 + (S \times P)] \bmod 99999989 \tag{4.4}$$
$$S = (Start\_Variable\_ID) + 1000000 \tag{4.5}$$
$$P = (PM\_Index) + 10000000 \tag{4.6}$$

Although in a primitive stage, it is useful to test *HDR random number generator* so it can further be improved and its use can be exemplified.We present the excel formula of *HDR random number generator* in the equation on this page.

4.2 Probability Distribution:

There is a maximum of 32 input uncertainties in this project, with some of the uncertainties becoming certain quantity at the various stage, e.g. exploration time uncertainty will become certain number after exploration ends in a particular project.To model this 32 input uncertainties, we have used two type of probability distribution from *SIPmath^{TM} Modeler Tools*:

- Discrete Distribution:

$$Realization = LOOKUP[(RN_{HDR}), (C_j), (V_k)] \tag{4.7}$$

$$Here, Cumulative\ Probability(C_j) = IF[\sum_{ALL}(P_j) \neq 1, NA(), \sum_{j=1}^{n}(P_j) - P_n] \tag{4.8}$$

$RN_{HDR} \rightarrow Random\ Number\ generated\ using\ HDR\ random\ number\ generator\ (See\ Equation\ 4.1)$

$V_k \rightarrow Values\ specifying\ possible\ outcomes\ of\ given\ discrete\ distribution$
$P_j \rightarrow Probability\ of\ Value\ (V_k)$

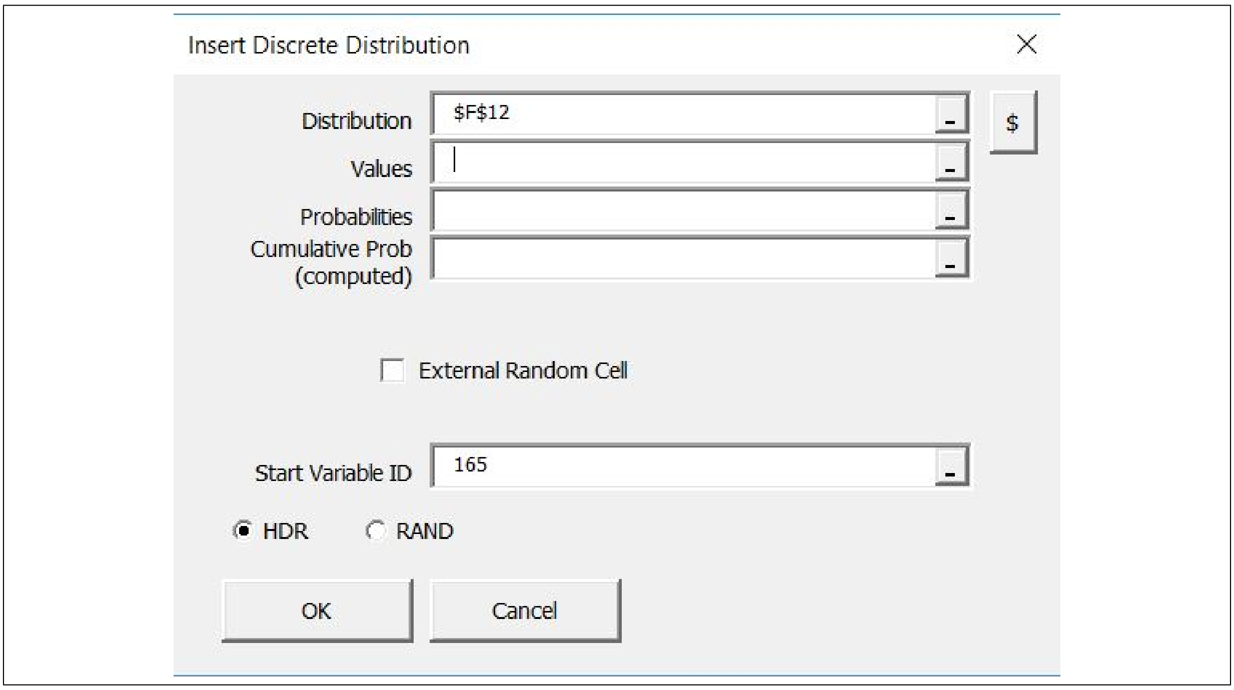


Figure 4.1: Input Window for Discrete Distribution

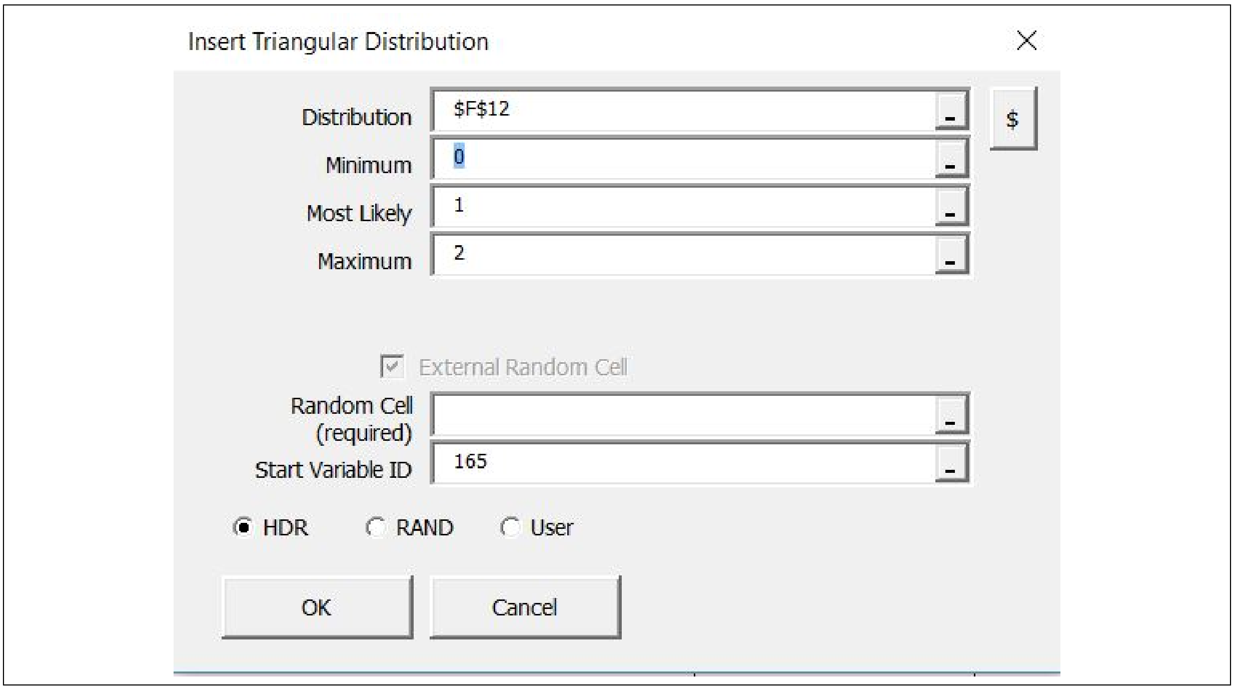


Figure 4.2: Input Window for Triangular Distribution

- Triangular Distribution: [See Algorithm 1 on page 29]

Figures 4.1 and 4.2 respectively shows the input window for discrete distribution and triangular distribution built using *SIPmath^{TM} Modeler Tools*.

4.3 Exploration Module:

Exploration module has four input uncertainties build as SIP as shown in Table 4.1.We also specify the current year(CY) and the starting year for exploration(SOE).

Uncertain Input Variable [Symbol for SIP]	**Distribution Type used for building SIP of Uncertain Input Variable**	**Unit/Possible Outcomes**
Chance of Success for Commercial Discovery [S(COS)]	Discrete	YES/NO
Exploration Time [S(ET)]	Discrete	Year
Average Exploration Cost per Year [S(AEC)]	Triangular	MM $/Year
Type of Hydrocarbon [S(THC)]	Discrete	OIL/GAS/BOTH

Table 4.1: Exploration Module

We use Algorithm 2 on page 29 and 3 on page 29 in this module to calculate for SIP of End of Exploration and SIP of Type of Hydrocarbon found based on the outcome of SIP of Chance of Success for Commercial Discovery.

This module has SIP of uncertain inputs only in projects that are on exploration stage, after that all parameters will become certain.At the end of this module, the presence of hydrocarbon and type of hydrocarbon present is main outputs, based on which other module described below will have their values for particular realization (PM_Trial).

4.4 Reserves Module:

Reserves Module has nine input uncertainties modeled as SIP as shown in the Table 4.2 on the following page.

We calculate four output variables in current section 4.4.

- Stock Tank Oil Originally in Place(STOOIP)[MMSTB]: [See Algorithm 4 on page 30]
- Stock Tank Gas Originally in Place(STGOIP)[BSCF]: [See Algorithm 5 on page 30]
- Initial Oil Reserves(OR_0)[MMSTB]: [See Algorithm 6 on page 30]
- Initial Gas Reserves(GR_0)[MMSTB]: [See Algorithm 7 on page 30]

Uncertain Input Variable [Symbol for SIP]	Distribution Type used for building SIP of Uncertain Input Variable	Unit
Area [S(A)]	Triangular	Acre
Gross Thickness of Oil Zone [S(T_O)]	Triangular	Feet
Gross Thickness of Gas Zone [S(T_G)]	Triangular	Feet
Net to Gross Ratio [S(NTG)]	Triangular	Fraction
Porosity [S(ϕ)]	Triangular	Fraction
Water Saturation [S(S_W)]	Triangular	Fraction
Oil Formation Volume Factor [S(B_O)]	Triangular	Rbbl/STB
Gas Formation Volume Factor [S(B_G)]	Triangular	RCF/SCF
Recovery Factor [S(RF)]	Triangular	Fraction

Table 4.2: Reserves Module

4.5 Facilities Module:

Table 4.3 on the following page shows the ten input uncertainties modeled as SIPs in this module.These input SIP, alongside with SIP of oil and gas reserves calculated in section 4.4 on the previous page is used to calculate production forecast in section 4.7.

4.6 Economic Module:

Table 4.4 on page 27 shows nine input uncertainties modeled as SIP in this module.We use straight-line depreciation method for six years of depreciation allowed.

$$Tax\ Rate(TTR) = 78\% = Normal\ Tax\ Rate(NTR) + Special\ Tax\ Rate(STR) \quad (4.9)$$

4.7 Production Data/Forecast Module:

This module calculates annual production forecast for oil and gas, figures 4.3 on the following page and 4.4 on page 27 shows example production forecast for a new

Uncertain Input Variable [Symbol for SIP]	**Distribution Type used for building SIP of Uncertain Input Variable**	**Unit**
Start of Production [S(SOP)]	Discrete	Year
Ramp Up Period [S(RUP)]	Discrete	Year
Field Potential [S(FP)]	Triangular	Fraction
Down Time [S(DT)]	Triangular	Fraction
Maximum Daily Rate-Oil [S(MDRO)]	Triangular	STB/Day
Maximum Daily Rate-Gas [S(MDRG)]	Triangular	MMSCF/Day
Facility Limit-Oil [S(FLO)]	Triangular	STB/Day
Facility Limit-Gas [S(FLG)]	Triangular	MMSCF/Day
Economic Rate-Oil [S(ERO)]	Triangular	STB/Day
Economic Rate-Gas [S(ERG)]	Triangular	MMSCF/Day

Table 4.3: Facilities Module

development project with the presence of both oil and gas.

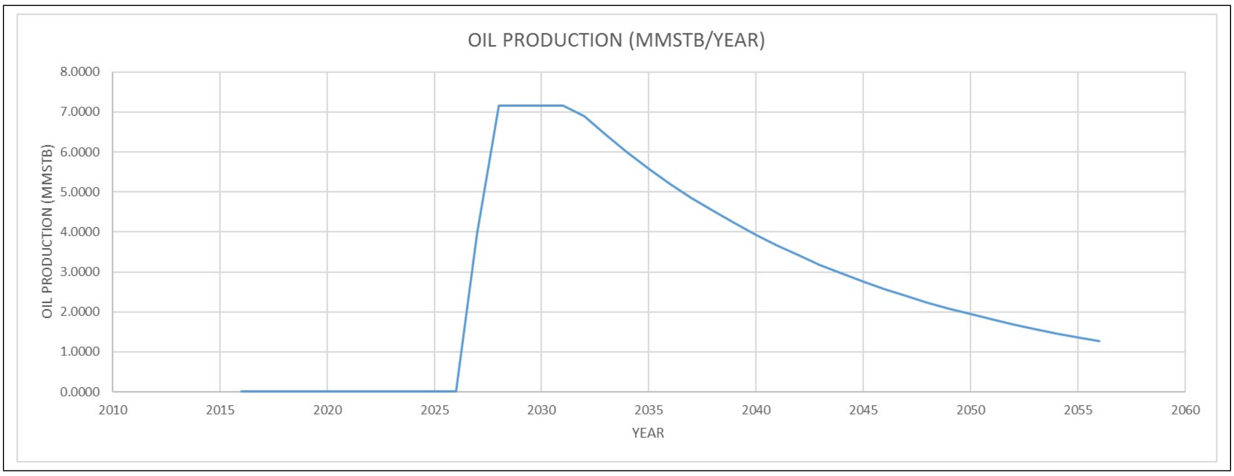


Figure 4.3: Example of Production Profile-Oil

Uncertain Input Variable [Symbol for SIP]	**Distribution Type used for building SIP of Uncertain Input Variable**	**Unit**
Initial Capital Expenditure [S(ICE)]	Triangular	MM $
Average Fixed Operating Expenditure per Year [S(FOE)]	Triangular	MM $/Year
Average Variable Operating Expenditure per Year-Oil [S(VOEO)]	Triangular	$/STB
Average Variable Operating Expenditure per Year-Gas [S(VOEG)]	Triangular	$/MMBTU
Normal Tax Rate [S(NTR)]	Triangular	Percentage
Risk Free Rate [S(RFR)]	Triangular	Percentage
Risk Premium Rate [S(RPR)]	Triangular	Percentage
Start Year for Gas Blowdown [S(SGP)]	Discrete	Year
Additional Expenditure for Gas Blowdown [S(AEX)]	Triangular	MM $

Table 4.4: Facilities Module

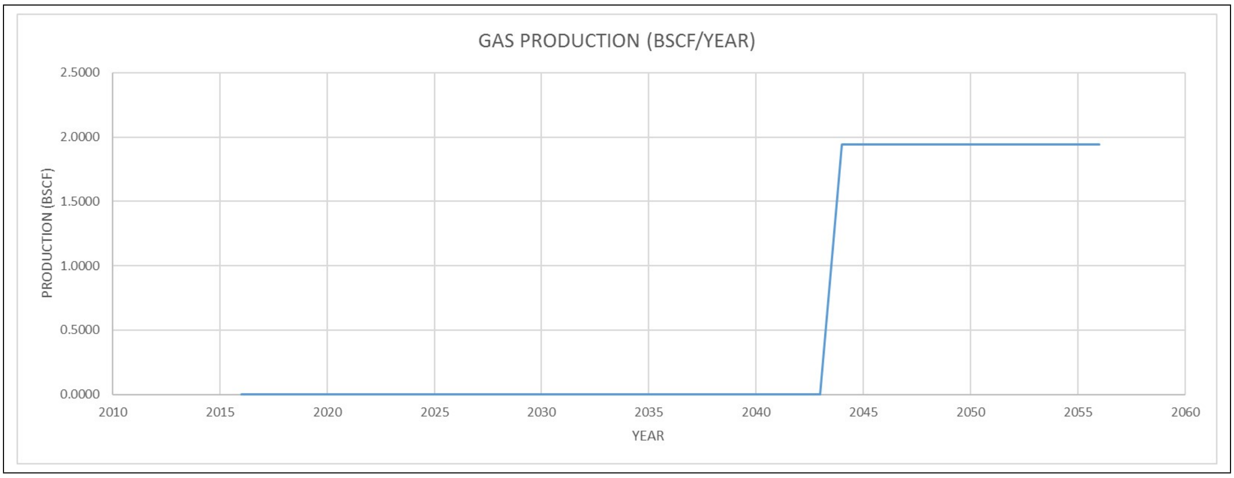


Figure 4.4: Example of Production Profile-Gas

There are 41 time steps(N) in our project model.We provide pseudo algorithms for calculating SIP for seven parameters in this module.

- Year: [See Algorithm 8 on page 31]
- Oil Reserves: [See Algorithm 9 on page 31]
- Daily Oil Production: [See Algorithm 10 on page 31]
- Annual Oil Production: [See Algorithm 11 on page 32]
- Gas Reserves: [See Algorithm 12 on page 32]
- Daily Gas Production: [See Algorithm 13 on page 33]
- Annual Gas Production [See Algorithm 14 on page 33]

4.8 Cash Flow Module:

In this module, the end objective is to calculate SIP of discounted net cash flow for 40 years, starting from 2017 to 2056.We start with obtaining the SIP of Oil Reserves(OR_t), Gas Reserves(GR_t), Annual Oil Production(AOP_t), Annual Gas Production(AGP_t) for a respective year using *VLOOKUP* function of *Microsoft Excel*.As mentioned on on page 21, Appendix A on page 64 explains two kinds of price model utilized in this work.We discuss step-by-step pseudo algorithms of all the components that ultimately leads to achieveing end objective of this module.

- Gross Revenue(RG): [See Algorithm 15 on page 34]
- Capital Expenditure(CEX): [See Algorithm 16 on page 34]
- Operating Expenditure(OEX): [See Algorithm 17 on page 35]
- Depreciation(DEP): [See Algorithm 18 on page 36]
- Normal Tax Base(NTB): [See Algorithm 19 on page 36]
- Normal Tax (NT): [See Algorithm 20 on page 37]
- UpLift (UL): [See Algorithm 21 on page 37]
- Special Tax Base(STB): [See Algorithm 22 on page 37]
- Special Tax (ST): [See Algorithm 23 on page 38]
- Total Tax (TT): [See Algorithm 24 on page 38]
- Net Cash Flow(NCF): [See Algorithm 25 on page 38]
- Discounted Net Cash Flow(DNCF): [See Algorithm 26 on page 39]

4.9 Result Module

We calculate SIP of Net Present Value(NPV)[Algorithm 27 on page 39] for a project and various statistical parameters such as average, variance and semivariance of that SIP using relevant *Microsoft Excel* formula.

We utilized output SIPs for net present value, and net cash flow, oil reserves, gas reserves, oil production and gas production from t=2017 to t=2026 in constructing portfolio model explained in next chapter on page 40.

4.10 Pseudo-Algorithms:

We present all the pseudo-algorithms used in this work in an orderly manner below:

Algorithm 1 Realization Triangular Distribution

procedure FOR CALCULATING REALIZATION OF TRIANGULAR DISTRIBUTION
  ▷ max-Maximum, ml-Most Likely
  ▷ min-Minimum, RN-Random Number
  for all PM_Trials **do**
    if $min > ml$ **then**
      $Realization = NA()$
    else if $ml > max$ **then**
      $Realization = NA()$
    else if $min = max$ **then**
      $Realization = ml$
    else if $RN < [\frac{(ml-min)}{(max-min)}]$ **then**
      $Realization = [min + \sqrt{\{RN \times (ml - min) \times (max - min)\}}]$
    else
      $Realization = [ml - \sqrt{\{(1 - RN) \times (ml - min) \times (max - min)\}}]$
    end if
  end for
end procedure

Algorithm 2 SIP of End of Exploration[S(EOE)]

procedure FOR CALCULATING SIP OF END OF EXPLORATION[S(EOE)]
  for all PM_Trials **do**
    $S(EOE) = [SOE + S(ET)]$
  end for
end procedure

Algorithm 3 SIP of Type of Hydrocarbon[S(THC)]

procedure FOR CALCULATING SIP OF TYPE OF HYDROCARBON[S(THC)]
  for all PM_Trials **do**
    if $S(COS) = NO$ **then**
      $S(THC) = NO\ HC$
    else
      $S(THC) = S(THC)$
    end if
  end for
end procedure

Algorithm 4 SIP of Stock Tank Oil Originally in Place[S(STOOIP)]

```
procedure FOR CALCULATING SIP OF STOCK TANK OIL ORIGINALLY IN PLACE[S(STOOIP)]
    for all PM_Trials do
        if [S(THC) = OIL] ∨ [S(THC) = BOTH] then
            S(STOOIP) = [(7758×S(A)×S(T_O)×S(NTG)×S(φ)×(1−S(S_W))) / (S(B_O)×1000000)]
        else
            S(STOOIP) = 0
        end if
    end for
end procedure
```

Algorithm 5 SIP of Stock Tank Gas Originally in Place[S(STGOIP)]

```
procedure FOR CALCULATING SIP OF STOCK TANK GAS ORIGINALLY IN PLACE[S(STGOIP)]
    for all PM_Trials do
        if [S(THC) = GAS] ∨ [S(THC) = BOTH] then
            S(STGOIP) = [(43560×S(A)×S(T_G)×S(NTG)×S(φ)×(1−S(S_W))) / (S(B_G)×1000000000)]
        else
            S(STGOIP) = 0
        end if
    end for
end procedure
```

Algorithm 6 SIP of Initial Oil Reserves[S(OR_0)]

```
procedure FOR CALCULATING SIP OF INITIAL OIL RESERVES[S(OR_0)]
    for all PM_Trials do
        if [S(THC) = OIL] ∨ [S(THC) = BOTH] then
            S(OR_0) = [S(RF) × S(STOOIP)]
        else
            S(OR_0) = 0
        end if
    end for
end procedure
```

Algorithm 7 SIP of Initial Gas Reserves[S(GR_0)]

```
procedure FOR CALCULATING SIP OF INITIAL GAS RESERVES[S(GR_0)]
    for all PM_Trials do
        if [S(THC) = GAS] ∨ [S(THC) = BOTH] then
            S(GR_0) = [S(RF) × S(STGOIP)]
        else
            S(GR_0) = 0
        end if
    end for
end procedure
```

Algorithm 8 SIP of Year[S(t)]

procedure FOR CALCULATING SIP OF YEAR [S(T)]
    for all PM_Trials **do**
        if $N = 1$ **then**
            $S(t) = [min(S(SOP) - 1, 2016)]$
        else
            $S(t) = [S(t)_{N-1} + 1]$
        end if
    end for
end procedure

Algorithm 9 SIP of Oil Reserves[$\mathrm{S(OR_t)}$]

procedure FOR CALCULATING SIP OF OIL RESERVES[$\mathrm{S(OR_T)}$]
    for all PM_Trials **do**
        for $t \leftarrow 2017, 2056$ **do**
            if $S(OR_0) = 0$ **then**
                $S(OR_t) = 0$
            else if $[S(OR_0) > 0] \wedge [S(t) < S(EOE)]$ **then**
                $S(OR_t) = 0$
            else if $[S(OR_0) > 0] \wedge [S(t) \geq S(EOE)] \wedge [S(t) \leq S(SOP)]$ **then**
                $S(OR_t) = S(OR_0)$
            else$[S(OR_0) > 0] \wedge [S(t) > S(SOP)] \wedge [S(OR_{t-1}) \geq S(AOP_{t-1})]$
                $S(OR_t) = [S(OR_{t-1}) - S(AOP_{t-1})]$
            end if
        end for
    end for
end procedure

Algorithm 10 SIP of Daily Oil Production [$\mathrm{S(DOP_t)}$]

procedure FOR CALCULATING SIP OF DAILY OIL PRODUCTION[$\mathrm{S(DOP_T)}$]
    for all PM_Trials **do**
        for $t \leftarrow 2017, 2056$ **do**
            if $S(OR_0) = 0$ **then**
                $S(DOP_t) = 0$
            else if $S(t) < S(SOP)$ **then**
                $S(DOP_t) = 0$
            else if $[S(t) \geq S(SOP)] \wedge [S(t) \leq \{S(SOP) + S(RUP)\}]$ **then**
                $S(DOP_t) = \min[S(FLO), (S(MDRO) \times S(FP) \times F_{oil}), (S(MDRO) \times \frac{S(OR_t)}{S(OR_0)} \times F_{oil})]$ ▷ $F_{oil} = \frac{S(t)+1-S(SOP)}{S(RUP)}$
            else$[S(t) > [S(SOP) + S(RUP)]]$
                $S(DOP_t) = \min[S(FLO), (S(MDRO) \times S(FP)), (S(MDRO) \times \frac{S(OR_t)}{S(OR_0)})]$
            end if
        end for
    end for
end procedure

Algorithm 11 SIP of Annual Oil Production[S(AOP$_t$)]

```
procedure FOR CALCULATING SIP OF ANNUAL OIL PRODUCTION[S(AOP_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            if S(OR_0) = 0 then
                S(AOP_t) = 0
            else if S(OR_t) = 0 then
                S(AOP_t) = 0
            else if [S(OR_t) > 0] ∧ [S(DOP_t) ≥ S(ERO)] then
                S(AOP_t) = min[S(OR_t), S(DOP_t)×(1−S(DT))×365 / 1000000]
            else[S(DOP_t) < S(ERO)]
                S(AOP_t) = 0
            end if
        end for
    end for
end procedure
```

Algorithm 12 SIP of Gas Reserves[S(GR$_t$)]

```
procedure FOR CALCULATING SIP OF GAS RESERVES[S(GR_T)]
    for all PM_Trials do
        if S(GR_0) = 0 then
            S(GR_t) = 0
        else if [S(THC) = GAS] ∧ [S(t) < S(EOE)] then
            S(GR_t) = 0
        else if [S(THC) = GAS] ∧ [S(t) ≥ S(EOE)] ∧ [S(t) ≤ S(SOP)] then
            S(GR_t) = S(GR_0)
        else if [S(THC) = GAS] ∧ [S(t) > S(SOP)] ∧ [S(GR_{t−1}) ≥ S(AGP_{t−1})] then
            S(GR_t) = [S(GR_{t−1}) − S(AGP_{t−1})]
        else if [S(THC) = BOTH] ∧ [S(t) < S(EOE)] then
            S(GR_t) = 0
        else if [S(THC) = BOTH] ∧ [S(t) ≥ S(EOE)] ∧ [S(t) ≤ S(SGP)] then
            S(GR_t) = S(GR_0)
        else[S(THC) = BOTH] ∧ [S(t) > S(SGP)] ∧ [S(GR_{t−1}) ≥ S(AGP_{t−1})]
            S(GR_t) = [S(GR_{t−1}) − S(AGP_{t−1})]
        end if
    end for
end procedure
```

Algorithm 13 SIP of Daily Gas Production [S($\mathrm{DGP_t}$)]

procedure FOR CALCULATING SIP OF DAILY GAS PRODUCTION[S($\mathrm{DGP_T}$)]
  for all PM_Trials **do**
    for $t \leftarrow 2017, 2056$ **do**
      if $S(GR_0) = 0$ **then**
        $S(DGP_t) = 0$
      else if $[S(t) < S(SOP)] \wedge [S(THC) = GAS]$ **then**
        $S(DGP_t) = 0$
      else if $[S(t) \geq S(SOP)] \wedge [S(THC) = GAS] \wedge [S(t) \leq \{S(SOP) + S(RUP)\}]$ **then**
        $S(DGP_t) = \min[S(FLG), (S(MDRG) \times S(FP) \times F_{gas}), (S(MDRG) \times \frac{S(GR_t)}{S(GR_0)} \times F_{gas})]$ ▷ $F_{gas} = \frac{S(t)+1-S(SOP)}{S(RUP)}$
      else if $[S(t) > [S(SOP) + S(RUP)]] \wedge [S(THC) = GAS]$ **then**
        $S(DGP_t) = \min[S(FLG), (S(MDRG) \times S(FP)), (S(MDRG) \times \frac{S(GR_t)}{S(GR_0)})]$
      else if $[S(t) < S(SGP)] \wedge [S(THC) = BOTH]$ **then**
        $S(DGP_t) = 0$
      else if $[S(t) \geq S(SOP)] \wedge [S(THC) = BOTH] \wedge [S(t) \leq \{S(SOP) + S(RUP)\}]$ **then**
        $S(DGP_t) = \min[S(FLG), (S(MDRG) \times S(FP) \times F_{gas}), (S(MDRG) \times \frac{S(GR_t)}{S(GR_0)} \times F_{gas})]$ ▷ $F_{gas} = \frac{S(t)+1-S(SOP)}{S(RUP)}$
      else$[S(t) > [S(SOP) + S(RUP)]] \wedge [S(THC) = BOTH]$
        $S(DGP_t) = \min[S(FLG), (S(MDRG) \times S(FP)), (S(MDRG) \times \frac{S(GR_t)}{S(GR_0)})]$
      end if
    end for
  end for
end procedure

Algorithm 14 SIP of Annual Gas Production[S($\mathrm{AGP_t}$)]

procedure FOR CALCULATING SIP OF ANNUAL GAS PRODUCTION[S($\mathrm{AGP_T}$)]
  for all PM_Trials **do**
    if $S(GR_0) = 0$ **then**
      $S(AGP_t) = 0$
    else if $S(GR_t) = 0$ **then**
      $S(AGP_t) = 0$
    else$[S(GR_t) > 0] \vee [S(DGP_t) > S(EGR)]$
      $S(AGP_t) = \min[S(GR_t), \frac{S(DGP_t) \times (1-S(DT)) \times 365}{1000}]$
    end if
  end for
end procedure

Algorithm 15 SIP of Gross Revenue[S($\mathrm{RG_t}$)]

```
procedure FOR CALCULATING SIP OF GROSS REVENUE[S(RG_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            if S(COS) = NO then
                S(RG_t) = 0
            else if S(THC) = BOTH then
                S(RG_t) = [(S(AOP_t) × S(P_oil,t)) + (S(AGP_t)×S(P_gas,t) / 1000000)]
            else if S(THC) = OIL then
                S(RG_t) = [S(AOP_t) × S(P_oil,t)]
            elseS(THC) = GAS
                S(RG_t) = [S(AGP_t)×S(P_gas,t) / 1000000]
            end if
        end for
    end for
end procedure
```

$$S(RG_t) = [(S(AOP_t) \times S(P_{oil,t})) + (\frac{S(AGP_t)\times S(P_{gas,t})}{1000000})]$$

$$S(RG_t) = [\frac{S(AGP_t)\times S(P_{gas,t})}{1000000}]$$

Algorithm 16 SIP of Capital Expenditure[S($\mathrm{CEX_t}$)]

```
procedure FOR CALCULATING SIP OF CAPITAL EXPENDITURE[S(CEX_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            if [S(t) ≥ SOE] ∧ [S(t) < S(EOE)] then
                S(CEX_t) = S(AEC)
            else if S(COS) = NO then
                S(CEX_t) = 0
            else if [S(COS) = YES] ∧ [S(t) ≥ S(EOE)] ∧ [S(t) < S(SOP)] then
                S(CEX_t) = [S(ICE) / (S(SOP)−SOE−S(ET))]
            else if [S(COS) = BOTH] ∧ [S(t) = (S(SGP) − 1)] then
                S(CEX_t) = S(AEX)
            elseS(t) ≥ S(SOP)
                S(CEX_t) = 0
            end if
        end for
    end for
end procedure
```

$$S(CEX_t) = [\frac{S(ICE)}{(S(SOP)-SOE-S(ET))}]$$

Algorithm 17 SIP of Operating Expenditure [S(OEX_t)]

procedure FOR CALCULATING SIP OF OPERATING EXPENDITURE[S(OEX_T)]
    for all PM_Trials **do**
        for $t \leftarrow 2017, 2056$ **do**
            if $S(COS) = NO$ **then**
                $S(OEX_t) = 0$
            else if $S(t) < S(SOP)$ **then**
                $S(OEX_t) = 0$
            else if $[S(AOP_t) > 0] \wedge [S(THC) = OIL] \wedge [S(t) \geq S(SOP)]$ **then**
                $S(OEX_t) = [S(FOE) + (S(AOP_t) \times S(VOEO))]$
            else if $[S(AGP_t) > 0] \wedge [S(THC) = GAS] \wedge [S(t) \geq S(SOP)]$ **then**
                $S(OEX_t) = [S(FOE) + (\frac{S(AGP_t) \times S(VOEG)}{1000000})]$
            else if $[\{S(AOP_t) > 0 \vee S(AGP_t) > 0\}] \wedge [S(THC) = BOTH] \wedge [S(t) \geq S(SOP)] \wedge [S(t) < S(SGP)]$ **then**
                $S(OEX_t) = [S(FOE) + (S(AOP_t) \times S(VOEO))]$
            else if $[\{S(AOP_t) > 0 \vee S(AGP_t) > 0\}] \wedge [S(THC) = BOTH] \wedge [S(t) \geq S(SGP)]$ **then**
                $S(OEX_t) = [S(FOE) + (S(AOP_t) \times S(VOEO)) + (\frac{S(AGP_t) \times S(VOEG)}{1000000})]$
            else$[S(AOP_t) = 0] \wedge [S(AGP_t) = 0]$
                $S(OEX_t) = 0$
            end if
        end for
    end for
end procedure

Algorithm 18 SIP of Depreciation[S(DEP$_t$)]

procedure FOR CALCULATING SIP OF DEPRECIATION[S(DEP$_T$)]
  for all PM_Trials **do**
    for $t \leftarrow 2017, 2056$ **do**
      if $S(COS) = NO$ **then**
        $S(DEP_t) = 0$
      else if $[S(COS) = YES] \wedge [S(t) < S(SOP)]$ **then**
        $S(DEP_t) = 0$
      else if $[S(t) - S(SOP) \geq 0] \wedge [(S(t) - S(SOP)) < 6] \wedge [S(COS) = YES]$ **then**
        $S(DEP_t) = [\frac{S(ICE)+(S(ET)\times S(AEC))}{6}]$
      else if $[\{S(THC) = OIL \vee S(THC) = GAS\}] \wedge [S(t) \geq (S(SOP) + 6)]$ **then**
        $S(DEP_t) = 0$
      else if $[S(THC) = BOTH] \wedge [S(t) \geq S(SOP)] \wedge [S(t) < S(SGP)]$ **then**
        $S(DEP_t) = 0$
      else if $[S(THC) = BOTH] \wedge [S(t) < (S(SGP) + 6)] \wedge [S(t) \geq S(SGP)]$ **then**
        $S(DEP - t) = [\frac{S(AEC)}{6}]$
      else$[S(THC) = BOTH] \wedge [S(t) \geq (S(SGP) + 6)]$
        $S(DEP_t) = 0$
      end if
    end for
  end for
end procedure

Algorithm 19 SIP of Normal Tax Base[S(NTB$_t$)]

procedure FOR CALCULATING SIP OF NORMAL TAX BASE[S(NTB$_T$)]
  for all PM_Trials **do**
    for $t \leftarrow 2017, 2056$ **do**
      if $S(COS) = NO$ **then**
        $S(NTB_t) = 0$
      else
        $S(NTB_t) = [S(RG_t) - \sum\{S(CEX_t) - S(OEX_t) - S(DEP_t)\}]$
      end if
    end for
  end for
end procedure

Algorithm 20 SIP of Normal Tax [S(NT$_t$)]

procedure FOR CALCULATING SIP OF NORMAL TAX[S(NT$_T$)]
  for all PM_Trials **do**
    for $t \leftarrow 2017, 2056$ **do**
      if $S(COS) = NO$ **then**
        $S(NT_t) = 0$
      else if $[S(NTB_t) > 0] \wedge [S(STB_t) > 0]$ **then**
        $S(NT_t) = [S(NTB_t) \times S(NTR)]$
      else$[S(NTB_t) \leq 0] \vee [S(STB_t) \leq 0]$
        $S(NT_t) = 0$
      end if
    end for
  end for
end procedure

Algorithm 21 SIP of UpLift [S(UL$_t$)]

procedure FOR CALCULATING SIP OF UPLIFT[S(UL$_T$)]
  for all PM_Trials **do**
    for $t \leftarrow 2017, 2056$ **do**
      if $S(COS) = NO$ **then**
        $S(UL_t) = 0$
      else if $S(OEX_t) > 0$ **then**
        $S(UL_t) = [0.20 \times S(OEX_t)]$
      else$S(OEX_t) \leq 0$
        $S(UL_t) = 0$
      end if
    end for
  end for
end procedure

Algorithm 22 SIP of Special Tax Base[S(STB$_t$)]

procedure FOR CALCULATING SIP OF SPECIAL TAX BASE[S(STB$_T$)]
  for all PM_Trials **do**
    for $t \leftarrow 2017, 2056$ **do**
      if $S(COS) = NO$ **then**
        $S(STB_t) = 0$
      else
        $S(STB_t) = [S(NTB_t) - S(UL_t)]$
      end if
    end for
  end for
end procedure

Algorithm 23 SIP of Special Tax [$S(ST_t)$]

```
procedure FOR CALCULATING SIP OF SPECIAL TAX [S(ST_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            if S(COS) = NO then
                S(ST_t) = 0
            else if [S(NTB_t) > 0] ∧ [S(STB_t) > 0] then
                S(ST_t) = [S(STB_t) × S(STR)]
            else[S(NTB_t) ≤ 0] ∨ [S(STB_t) ≤ 0]
                S(ST_t) = 0
            end if
        end for
    end for
end procedure
```

Algorithm 24 SIP of Total Tax[$S(TT_t)$]

```
procedure FOR CALCULATING SIP OF TOTAL TAX[S(TT_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            if S(COS) = NO then
                S(TT_t) = 0
            else
                S(TT_t) = [S(NT_t) + S(ST_t)]
            end if
        end for
    end for
end procedure
```

Algorithm 25 SIP of Net Cash Flow[$S(NCF_t)$]

```
procedure FOR CALCULATING SIP OF NET CASH FLOW[S(NCF_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            if [S(COS) = NO] ∧ [S(CEX_t) ≥ 0] then
                S(NCF_t) = [−0.22 × S(CEX_t)]
            else if S(COS) = NO then
                S(NCF_t) = 0
            elseS(COS) = YES
                S(NCF_t) = [S(RG_t) − S(CEX_t) − S(OEX_t) − S(TT_t)]
            end if
        end for
    end for
end procedure
```

Algorithm 26 SIP of Discounted Net Cash Flow[S(DNCF$_t$)]

```
procedure FOR CALCULATING SIP OF DISCOUNTED NET CASH FLOW[S(DNCF_T)]
    for all PM_Trials do
        for t ← 2017, 2056 do
            S(DNCF_t) = [S(NCF_t) / (1+S(RFR)+S(RPR))^(N−0.5)]
        end for
    end for
end procedure
```

$$S(DNCF_t) = [\frac{S(NCF_t)}{(1+S(RFR)+S(RPR))^(N-0.5)}]$$

Algorithm 27 SIP of Net Present Value[S(NPV)]

```
procedure FOR CALCULATING SIP OF NET PRESENT VALUE[S(NPV)]
    for all PM_Trials do
        S(NPV) = [{∑_{t=2017}^{2056} S(NCF_t)} − IR]
    end for
end procedure
```

$$S(NPV) = [\{\sum_{t=2017}^{2056} S(NCF_t)\} - IR]$$

Chapter 5

Portfolio Model:

We now discuss the development of portfolio level model.We have developed 40 projects using each of the two base projects with different price model[See Chapter 4 on page 21 and Appendix A on page 64]. We have used *Define Output* and *Library Input* functions of *SIPmath^{TM} Modeler Tools* (Section 3.3 on page 15 and 3.3 on page 15) on SIP of following 51 parameters for each project.

- Net Present Value[S(NPV)].
- Net Cash Flow[S(NCF_t)], from t=2017 to t=2026.
- Annual Oil Production[S(AOP_t)], from t=2017 to t=2026.
- Annual Gas Production[S(AGP_t)], from t=2017 to t=2026.
- Oil Reserves[S(OR_t)], from t=2017 to t=2026.
- Gas Reserves[S(GR_t)], from t=2017 to t=2026.

We developed six *Microsoft Excel* spreadsheet model[1] Using *SIPmath^{TM} Modeler Tools*, one for portfolio level calculation and other five for probability calculation of achieving the target for given attribute for a given year.

As indicated in figure 5.1 on the next page, we calculate working interest (%) in portfolio model, which works as input in each of the five attribute model, where the probability of portfolio achieving the target for a given year is calculated by model.This probability is inserted back into portfolio model giving all the relevant details in one model.

[1]We divided the model into six part for simplicity and due to size consideration.

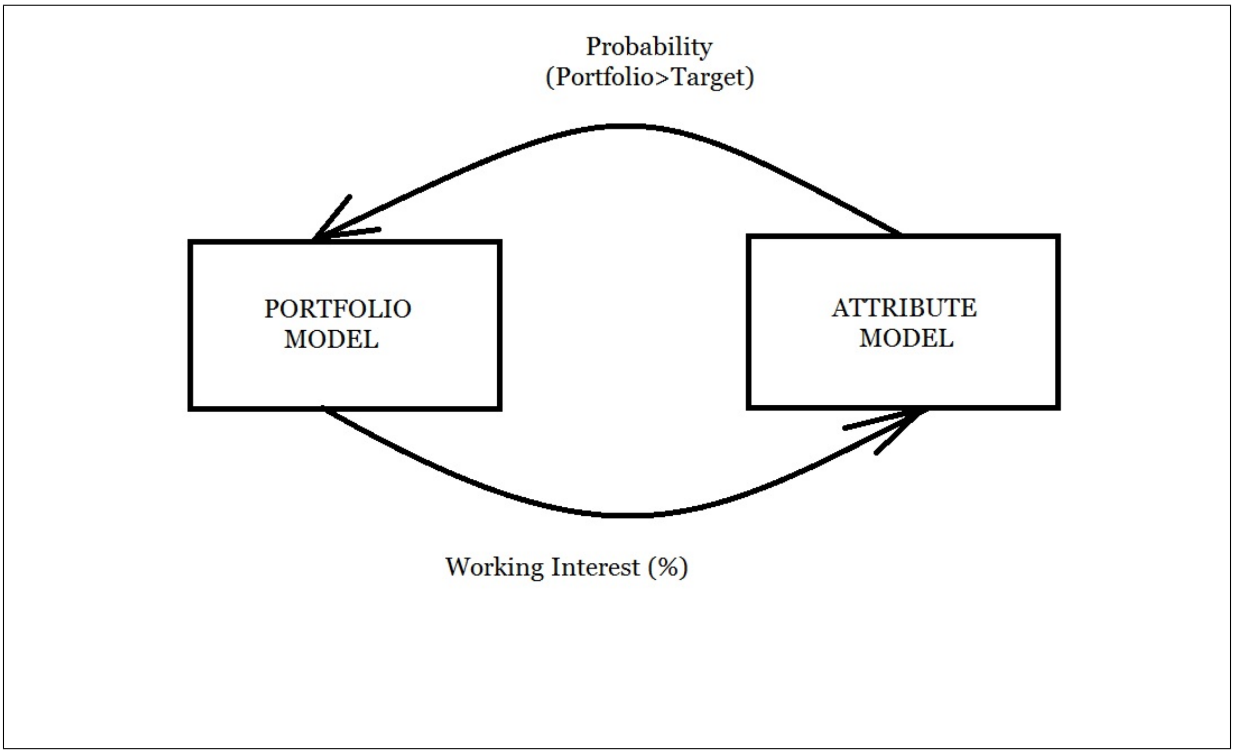


Figure 5.1: Portfolio Model

5.1 Portfolio Model:

Using Library Input function of SIPmath tools, we import SIP of Net Present Value for all 40 projects.SIP of portfolio net present value is calculated by equation 5.1.

$$S(NPV_{PORTFOLIO}) = \sum_{P=1}^{40}[WI_P \times S(NPV_P)] \tag{5.1}$$

Where,

$$\begin{aligned} S(NPV_{PORTFOLIO}) &: SIP\ of\ portfolio\ net\ present\ value \\ WI_P &: Working\ Interest\ (\%)\ in\ Project - P \\ S(NPV_P) &: SIP\ of\ net\ present\ value\ for\ Project - P \end{aligned}$$

SIP of portfolio net present value obtained using equation on the current page.We can use SIP of net present value to calculate various statistical parameters such as average, variance and semi-variance of that portfolio using simple Excel formulas.

One interesting and favorable point in using *SIPmath*^{TM} *Modeler Tools* is a comparison of variance and semivariance value using approach illustrated in this work and traditional method.Results were encouraging, as variance match perfectly and semi-variance approximation match well with the actual value of semivariance using the approach in this study.Below we present pseudo-algorithm for calculation of semivariance.

Algorithm 28 Semi-Variance(SVAR)

procedure FOR SEMI-VARIANCE CALCULATION
    for all PM_Trials **do**
        $SVAR_{PORTFOLIO} = [\frac{\sum\{min(S(NPV_{PORTFOLIO}) - CUTOFF, 0)^2\}}{N}]$
            ▷ $N : Total\ number\ of\ PM\_Trials$
            ▷ $S(NPV_{PORTFOLIO}) : SIP\ of\ portfolio\ net\ present\ value$
            ▷ $CUTOFF : Value\ specified\ to\ calculate\ semi - variance$
    end for
end procedure

5.2 Attribute Model for Calculating Probability:

Using Library Input function of *SIPmath*^{TM} *Modeler Tools*, we have made five attribute models to calculate the probability of portfolio achieving the target attribute.Equation 5.2 present calculation for SIP of portfolio attribute at time t, where t=2017 to t=2026.

$$S(ATT_{PORTFOLIO,t}) = \sum[WI_P \times S(ATT_{P,t})] \tag{5.2}$$

Where,
Attribute(ATT) can be net cash flow(NCF),oil reserves(OR),gas reserves(GR),annual oil production(AOP) or annual gas production(AGP).

Using equation 5.3, one can calculate the probability of portfolio attribute to be greater than or equal to the target level of that attribute at time t, where t=2017 to 2026.

$$P(Attribute_{PORTFOLIO} >= Attribute_{Target})_t = \{\frac{COUNTIF[S(ATT_{PORTFOLIO,t}) >= (ATT_{Target,t}), S(ATT_{PORTFOLIO,t}), NA()]}{COUNT[S(ATT_{PORTFOLIO,t})]}\} \tag{5.3}$$

In the next chapter, we present various analysis done by us using SIPmath model developed in this work.

Chapter 6

Case Study

We divide this chapter into two main section.In the first section, we will study efficient frontier according to various constraints and note our observations based on that.Second, we take a three synthetic portfolio.We observe their performance relative to the different efficient frontier and relative to the probability of achieving a target attribute for a given year.This analysis alone, can not give an answer but it opens up important questions and give us new insights into portfolio problem.

6.1 Efficient Frontier Generation:[1][2]

There are 4 main type of portfolio optimization problem:

1. Minimize risk of a portfolio.
2. Minimize risk of a portfolio for a specific target of various attributes.
3. Maximize return of a portfolio subject to target level of risk.
4. Maximize risk-adjusted return of a portfolio.

We focus on number 1 and 3 in this work.Portfolio Variance and Semivariance both have been taken as an indicator of portfolio risk.

Below are some common constraints[3],which we assume are true for entire work.

- Decision Variable:

$$0\% \leq WI_P{}^{4} \leq 50\%,\ P \in [1, 40] \tag{6.1}$$

$WI_P : Working\ Interest\ in\ Project\ P\ in\ Percentage(\%)$

[1]For efficient frontier part, we have used Evolver-Optimization Add-in for Microsoft Excel by Palisade Corporation for academic purpose only with serial number-7093608.

[2]For efficient frontier generation constrained mentioned in each case has been divided into nine equal part using 10 points including minimum and maximum.

[3]All constraints in this work have been assumed as hard constraints.

[4]Working Interest have been assumed to be changing in the step size of 5%.

- Total Budget:

$$Total\ Budget(B_{TOTAL}) = 2000\ MM\ \$ \tag{6.2}$$

- Portfolio Budget:

$$Portfolio\ Budget(B_{PORTFOLIO}) \leq 2000\ MM\ \$ \tag{6.3}$$

- Remaining Cash:

$$Remaining\ Cash\ (CASH_R) = B_{TOTAL} - B_{PORTFOLIO} \tag{6.4}$$

- Portfolio Total Expected Net Present Value ($\text{ENPV}_{\text{TOTAL}}$):

$$ENPV_{TOTAL} = [ENPV_{PORTFOLIO} + (0.05 \times CASH_R)] \tag{6.5}$$

- Number of Projects in Portfolio:

$$Number\ of\ Projects(N_{PROJECT}) \leq 30 \tag{6.6}$$

6.1.1 Constant Price Model

In this section,we present the details of various cases.

Case-1

$$Goal : Minimize\ Portfolio\ Variance\ (\sigma^2_{PORTFOLIO})$$
$$Subject\ to : ENPV_{PORTFOLIO} \geq [100, 800]$$
$$Number of Iteration = 25000$$

Table 6.1 show key statistics for this case.

Target Portfolio ENPV (MM \$)	Portfolio ENPV (MM \$)	Portfolio Variance (MM $\$^2$)	Portfolio Semi-Variance (MM $\$^2$)	Portfolio Budget (MM \$)	Remaining Cash (MM \$)	Total ENPV (MM \$)
800.00	800.08	12489.69	6016.54	1998.85	1.15	800.1375
722.22	722.56	8140.63	4006.10	1976.70	23.30	723.7250
644.44	645.39	6635.76	3249.67	1742.80	257.20	658.2500
566.67	567.87	4962.66	2430.96	1580.60	419.40	588.8400
488.89	489.54	4053.30	1949.73	1373.95	626.05	520.8425
411.11	411.29	3059.00	1491.20	1156.53	843.47	453.4635
333.33	335.55	1974.18	959.49	937.75	1062.25	388.6625
255.56	255.61	1256.23	602.83	716.08	1283.92	319.8060
177.78	196.17	788.27	382.49	536.38	1463.62	269.3510
100	100.28	267.11	130.06	274.93	1725.07	186.5335

Table 6.1: Constant Price Model-Minimize Portfolio Variance

Case-2

$$Goal : Minimize\ Portfolio\ Semi-Variance\ (S^2_{PORTFOLIO})$$
$$Subject\ to : ENPV_{PORTFOLIO} \geq [100, 800]$$
$$Number of Iteration = 25000$$

Table 6.2 show key statistics for this case.

Target Portfolio ENPV (MM \$)	Portfolio ENPV (MM \$)	Portfolio Variance (MM $\$^2$)	Portfolio Semi-Variance (MM $\$^2$)	Portfolio Budget (MM \$)	Remaining Cash (MM \$)	Total ENPV (MM \$)
800.00	800.09	11779.81	5711.72	1999.25	0.75	800.1269
722.22	723.58	8151.71	3995.97	1986.33	13.67	724.2620
644.44	645.31	6177.56	3004.77	1768.40	231.60	656.8883
566.67	567.69	5219.33	2558.53	1570.35	429.65	589.1733
488.89	488.94	4057.31	1953.60	1344.23	655.77	521.7288
411.11	411.13	2930.68	1448.27	1130.70	869.30	454.5960
333.33	336.21	2356.83	1166.36	943.50	1056.50	389.0368
255.56	255.67	1296.31	614.56	737.85	1262.15	318.7765
177.78	177.90	641.10	315.85	498.05	1501.95	252.9994
100	100.60	313.81	150.17	279.85	1720.15	186.6054

Table 6.2: Constant Price Model-Minimize Portfolio Semi-Variance

Case-3

$$Goal : Maximize\ Portfolio\ Expected\ NPV\ (ENPV_{PORTFOLIO})$$
$$Subject\ to : (\sigma^2_{PORTFOLIO}) \leq [300, 13000]$$
$$Number of Iteration = 25000$$

Table 6.3 on the following page show key statistics for this case.

Case-4

$$Goal : Maximize\ Portfolio\ Expected\ NPV\ (ENPV_{PORTFOLIO})$$
$$Subject\ to : (S^2_{PORTFOLIO}) \leq [200, 6000]$$
$$Number of Iteration = 25000$$

Table 6.4 on the next page show key statistics for this case.

6.1.2 Stochastic Price Model

In this section,we present the details of various cases.

Target Portfolio Variance (MM $\$^2$)	Portfolio ENPV (MM \$)	Portfolio Variance (MM $\$^2$)	Portfolio Semi-Variance (MM $\$^2$)	Portfolio Budget (MM \$)	Remaining Cash (MM \$)	Total ENPV (MM \$)
300.00	105.38	290.07	141.78	294.15	1705.85	190.6724
1711.11	224.82	1703.83	829.20	582.40	1417.60	295.7027
3122.22	393.89	3109.79	1505.18	1062.68	937.32	440.7552
4533.33	509.10	4500.98	2199.67	1418.10	581.90	538.1989
5944.44	576.23	5944.33	2889.42	1607.30	392.70	595.8652
7355.56	637.84	7313.11	3592.13	1759.50	240.50	649.8612
8766.67	706.05	8753.11	4259.09	1885.95	114.05	711.7484
10177.78	743.65	10004.85	4911.84	1999.55	0.45	743.6683
11588.89	766.19	11584.71	5645.84	1991.25	8.75	766.6312
13000	780.11	12958.09	6302.15	1992.1	7.90	780.5026

Table 6.3: Constant Price Model-Maximize Total ENPV (Constraint-Variance)

Target Portfolio Semi-Variance (MM $\$^2$)	Portfolio ENPV (MM \$)	Portfolio Variance (MM $\$^2$)	Portfolio Semi-Variance (MM $\$^2$)	Portfolio Budget (MM \$)	Remaining Cash (MM \$)	Total ENPV (MM \$)
200	132.42	413.65	199.99	355.55	1644.45	214.6407
844.44	292.28	1744.13	843.35	793.80	1206.20	352.5892
1488.89	383.98	3040.41	1480.97	1034.45	965.55	432.2512
2133.33	485.20	4383.99	2114.06	1305.15	694.85	519.9425
2777.78	559.73	5716.75	2774.75	1543.15	456.85	582.5702
3422.22	609.66	6919.44	3400.82	1689.45	310.55	625.1861
4066.67	684.00	8430.07	4062.14	1871.45	128.55	690.4268
4711.11	721.16	9569.79	4666.64	1946.70	53.30	723.8266
5355.56	770.05	10880.21	5286.56	1999.38	0.62	770.0794
6000	793.50	12332.33	5997.04	1991.13	8.87	793.9422

Table 6.4: Constant Price Model-Maximize Total ENPV(Constraint-Semi-Variance)

Case-5

$$Goal : Minimize\ Portfolio\ Variance\ (\sigma^2_{PORTFOLIO})$$
$$Subject\ to : ENPV_{PORTFOLIO} \geq [100, 800]$$
$$Number of Iteration = 25000$$

Table 6.5 on the following page show key statistics for this case.

Case-6

$$Goal : Minimize\ Portfolio\ Semi-Variance\ (S^2_{PORTFOLIO})$$
$$Subject\ to : ENPV_{PORTFOLIO} \geq [100, 800]$$
$$Number of Iteration = 25000$$

Table 6.6 on the next page show key statistics for this case.

Target Portfolio ENPV (MM \$)	Portfolio ENPV (MM \$)	Portfolio Variance (MM $\$^2$)	Portfolio Semi-Variance (MM $\$^2$)	Portfolio Budget (MM \$)	Remaining Cash (MM \$)	Total ENPV (MM \$)
800	800.06	281779.67	120480.89	2000.00	0	800.0618
722.22	722.32	197990.98	85537.73	1917.08	82.92	726.4642
644.44	644.55	141438.78	61285.91	1748.40	251.60	657.1261
566.67	567.58	106039.59	46065.58	1530.75	469.25	591.0451
488.89	488.90	72126.95	31434.34	1320.10	679.90	522.9002
411.11	411.20	48824.23	21265.49	1105.25	894.75	455.9406
333.33	334.22	30565.64	13370.34	912.30	1087.70	388.6046
255.56	255.65	19056.81	8259.67	698.90	1301.10	320.7090
177.78	177.83	10260.47	4460.17	494.95	1505.05	253.0817
100	100.52	3489.86	1491.01	273.55	1726.45	186.8417

Table 6.5: Stochastic Price Model-Minimize Variance

Target Portfolio ENPV (MM \$)	Portfolio ENPV (MM \$)	Portfolio Variance (MM $\$^2$)	Portfolio Semi-Variance (MM $\$^2$)	Portfolio Budget (MM \$)	Remaining Cash (MM \$)	Total ENPV (MM \$)
800	800.03	281100.57	120838.66	1998.35	1.65	800.1113
722.22	722.25	195414.84	84403.58	1897.50	102.50	727.3784
644.44	644.54	141072.09	61076.48	1754.68	245.32	656.8040
566.67	567.11	103367.46	44885.91	1521.60	478.40	591.0346
488.89	489.39	72680.12	31562.74	1311.45	688.55	523.8158
411.11	411.56	51902.76	22533.89	1115.05	884.95	455.8046
333.33	333.40	32875.40	14265.88	892.80	1107.20	388.7590
255.56	256.00	18557.52	8062.19	694.85	1305.15	321.2574
177.78	179.43	9114.48	3994.36	501.68	1498.32	254.3442
100	100.25	3089.84	1334.98	270.35	1729.65	186.7289

Table 6.6: Stochastic Price Model-Minimize Semi-Variance

Case-7

$$Goal: Maximize\ Portfolio\ Expected\ NPV\ (ENPV_{PORTFOLIO})$$
$$Subject\ to: (\sigma^2_{PORTFOLIO}) \leq [4000, 300000]$$
$$Number of Iteration = 25000$$

Table 6.7 on the following page show key statistics for this case.

Case-8

$$Goal: Maximize\ Portfolio\ Expected\ NPV\ (ENPV_{PORTFOLIO})$$
$$Subject\ to: (S^2_{PORTFOLIO}) \leq [1500, 125000]$$
$$Number of Iteration = 25000$$

Table 6.8 on the next page show key statistics for this case.

Target Portfolio Variance (MM ^{2})	Portfolio ENPV (MM $)	Portfolio Variance (MM ^{2})	Portfolio Semi-Variance (MM ^{2})	Portfolio Budget (MM $)	Remaining Cash (MM $)	Total ENPV (MM $)
4000.00	123.13	3943.20	1759.00	339.05	1660.95	206.1732
36888.89	347.98	36759.82	15965.01	957.00	1043.00	400.1269
69777.78	441.20	69362.14	29934.00	1218.00	782.00	480.3031
102666.67	512.35	101829.98	44308.64	1478.00	522.00	538.4475
135555.56	558.42	135521.79	58323.61	1457.88	542.12	585.5236
168444.44	654.15	167792.78	72734.84	1744.00	256.00	666.9461
201333.33	710.56	196730.60	85016.44	1867.25	137.25	717.1983
234222.22	736.51	233992.66	100894.36	1901.50	98.5	741.4353
267111.11	763.65	266618.32	115154.09	1974.35	25.65	764.9287
300000.00	786.97	299854.18	129341.12	1979.75	20.25	787.9827

Table 6.7: Stochastic Price Model-Maximize Total ENPV(Constraint-Variance)

Target Portfolio Semi-Variance (MM ^{2})	Portfolio ENPV (MM $)	Portfolio Variance (MM ^{2})	Portfolio Semi-Variance (MM ^{2})	Portfolio Budget (MM $)	Remaining Cash (MM $)	Total ENPV (MM $)
1500.00	105.03	3484.63	1498.04	272.90	1727.10	191.3885
15222.22	332.98	34912.24	15207.55	898.75	1101.25	388.0462
28944.44	411.84	67048.31	28857.04	1025.95	974.05	460.5409
42666.67	517.24	98711.59	42625.38	1418.95	581.05	546.2936
56388.89	611.47	130209.87	56369.71	1669.20	330.80	628.0140
70111.11	656.53	160207.38	69486.75	1835.00	165.00	664.7809
83833.33	711.11	192570.92	83708.30	1926.60	73.40	714.7770
97555.55	746.98	96957.00	96957.01	1999.20	0.80	747.0198
111277.78	765.12	253049.80	109773.55	1999.25	0.75	765.1614
125000.00	783.71	288745.32	124984.88	1994.00	6.00	784.0072

Table 6.8: Stochastic Price Model-Maximize Total ENPV(Constraint-Semi-Variance)

6.1.3 Efficient Frontier

Based on subsections 6.1.1 on page 44 and 6.1.2 on page 45, we plot figures shown in this section.

As Figure 6.1 on the following page shows, for a same expected net present value, stochastic price model has a large factor of volatility[5] compare to while one evaluate portfolio at fixed price model.It is advisable to add stochastic nature of oil and gas price, as it is a major contributor to inter-project correlation, to the portfolio analysis problem.

As Figure 6.2 on page 50 shows, for a same number of iteration, minimizing risk give better result compare to maximizing reward.This finding might not be universally true, but it is an interesting area to explore further.

[5]variance or semi-variance

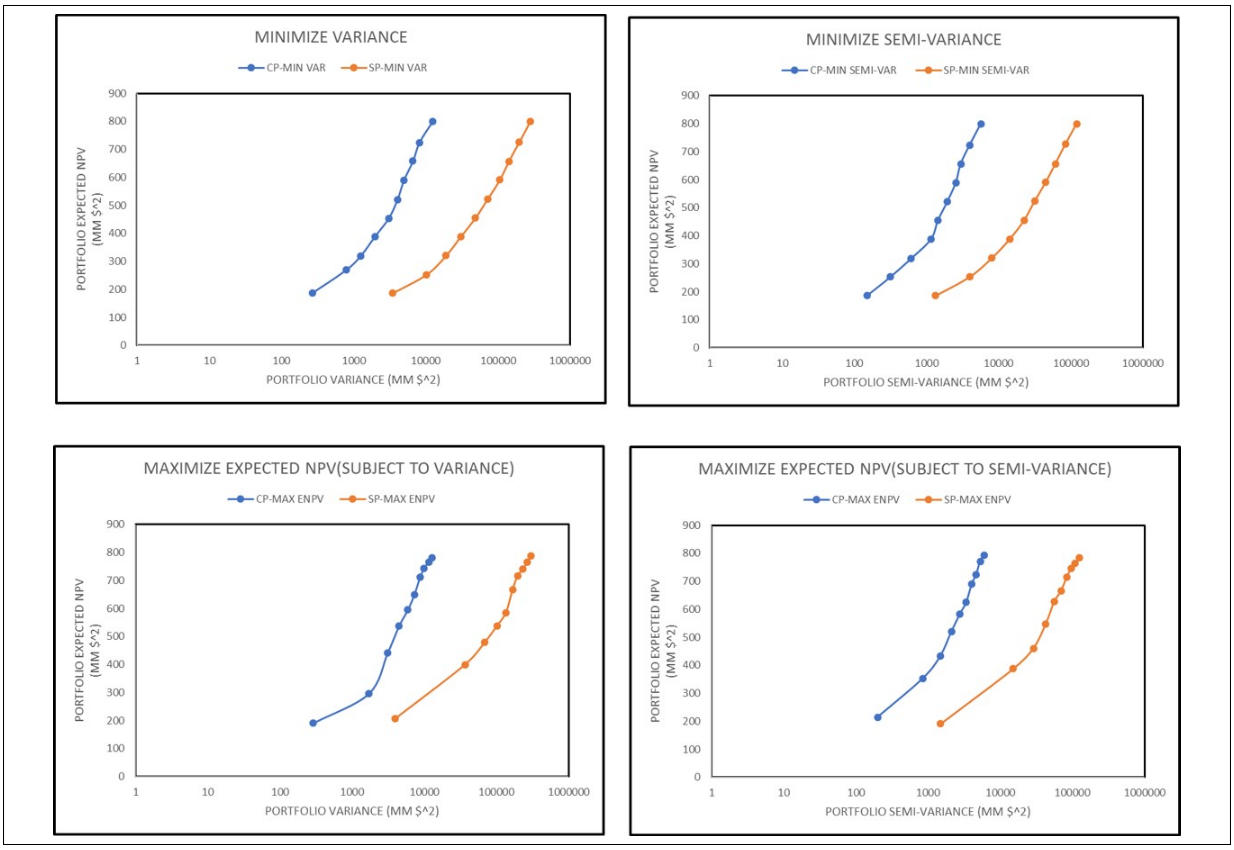


Figure 6.1: Effect of Price Model on Efficient Frontier

We repeat case-1 for >50000 trials and 5000 trials, to observe the effect of a number of trials as shown in figure 6.3 on page 51.As one can see, the difference in 25000 trials and 5000 trials is much larger, compared to >50000 trials to 25000 trials.As optimization problem,such in this work,is useful as a guiding principal[6],number of trials should be decided with care.

Figure 6.4 on page 51 shows the efficient frontier with a different definition for semi-variance[7].This shows the flexibility of model developed in this work.

6.2 3 Portfolio Choices:

In this section, we take an hypothetical example of three portfolio options and analysis them with the help of our model.Table 6.9 on page 52 show composition of each portfolio choice according to its working interest(%) in each project.

Table 6.10 on page 53 shows the key stastistics for three portfolio choices with both price model.Figure 6.5 on page 51 shows cumulative distribution functions for all cases.

[6]As E&P industry is inefficient and lumpy, it is challenging to achieve same working interest in project as obtained in efficient portfolio, nevertheless, this calculation gives useful insights regarding which project should we pursue with more interest.

[7]Cutoff for semivariance is taken as half of portfolio Expected Net Present Value.

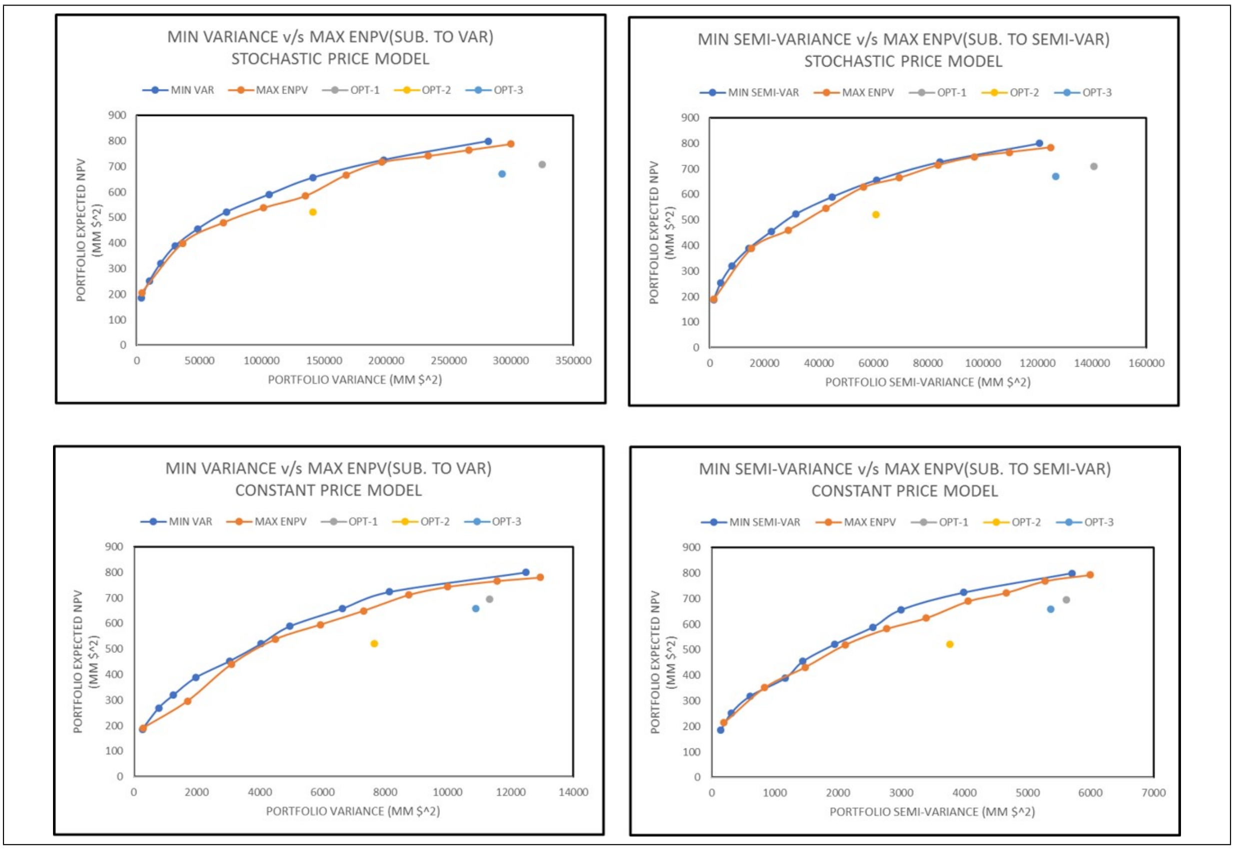


Figure 6.2: Effect of Type of Optimization on Efficient Frontier

Figure 6.6 on page 53 to 6.11 on page 56 shows the probability of achieving target attribute for a given choice of portfolio.

The choice of optimal portfolio out of three options provided here can not be made with using this model alone.What this model provide, is an interactive way for the decision maker to discuss alternatives and agree on optimal choice according to organization goals and strategy.

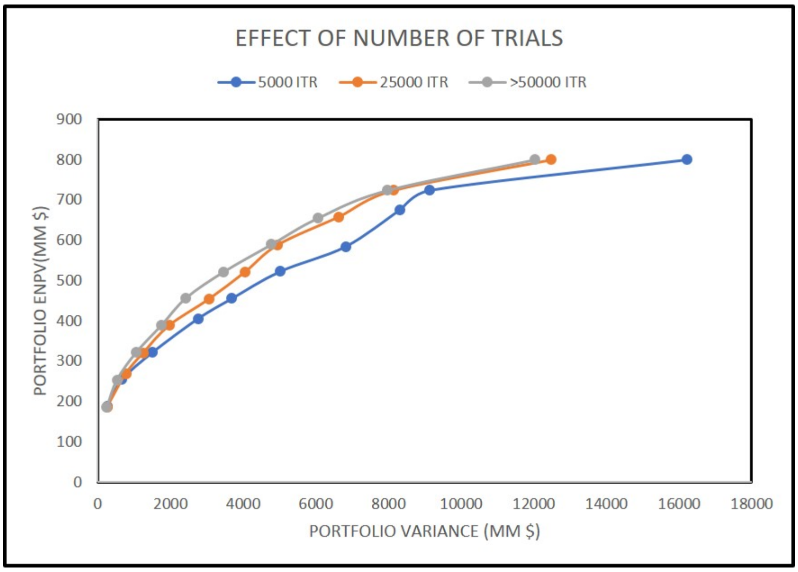


Figure 6.3: Effect of Number of Trials on Efficient Frontier

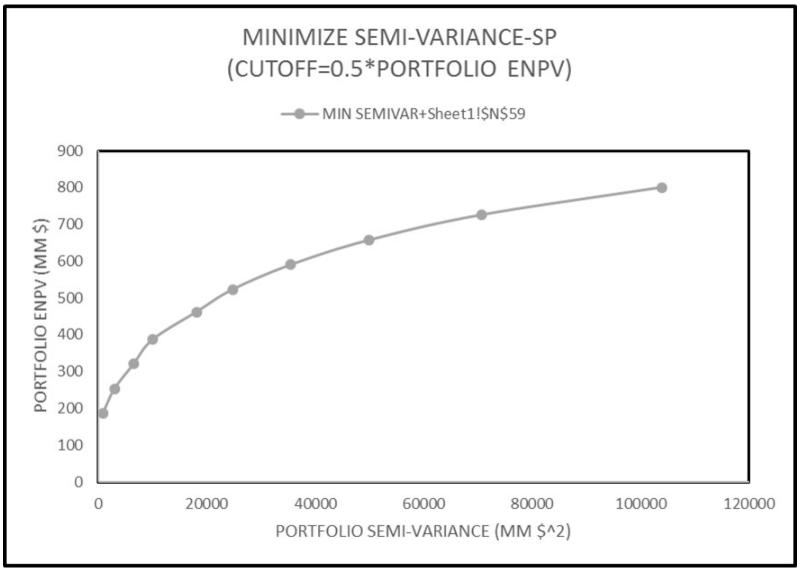


Figure 6.4: Efficient Frontier with Different Cut-Off for Semi-Variance

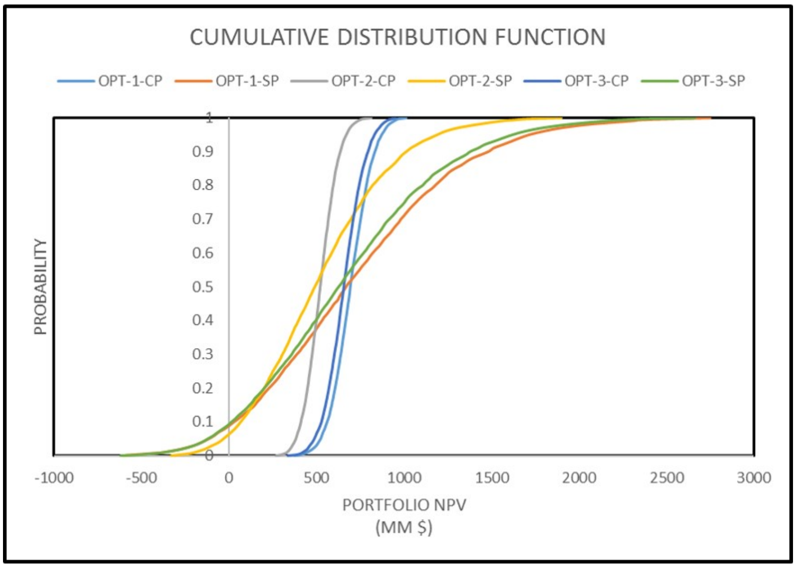


Figure 6.5: Cumulative Distribution Function

	PORTFOLIO OPT-1	PORTFOLIO OPT-2	PORTFOLIO OPT-3
Working Interest in Project (%)			
CP-1	0	10	0
CP-2	0	20	20
CP-3	0	25	0
CP-4	0	30	0
CP-5	0	15	15
CP-6	0	15	0
CP-7	0	20	0
CP-8	0	20	20
CP-9	0	40	0
CP-10	0	20	20
CP-11	25	0	25
CP-12	35	0	35
CP-13	30	0	30
CP-14	25	0	25
CP-15	25	0	25
CP-16	25	0	25
CP-17	25	0	25
CP-18	15	0	0
CP-19	20	0	20
CP-20	35	0	35
CP-21	40	40	40
CP-22	30	30	30
CP-23	25	25	25
CP-24	30	30	30
CP-25	45	45	45
CP-26	45	45	45
CP-27	30	30	30
CP-28	40	40	40
CP-29	35	35	35
CP-30	35	35	35
CP-31	15	15	0
CP-32	40	40	0
CP-33	35	35	0
CP-34	0	25	25
CP-35	0	35	35
CP-36	25	0	25
CP-37	35	0	35
CP-38	25	25	25
CP-39	30	30	30
CP-40	20	20	20

Table 6.9: Composition of Three Portfolio Option

	Opt-1 (Constant Price Model)	Opt-1 (Stochastic Price Model)	Opt-2 (Constant Price Model)	Opt-2 (Stochastic Price Model)	Opt-3 (Constant Price Model)	Opt-3 (Stochastic Price Model)
Total ENPV (MM $)	696.15	709.04	521.66	521.13	658.14	670.54
Portfolio Variance (MM $2̂)	11331.39	325439.00	7669.22	141546.68	10891.81	293093.23
Portfolio Semi-Variance (MM $2̂)	5620.79	140769.55	3772.62	61017.57	5374.60	126748.28
P10 (MM $)	561.22	-131.69	409.90	-36.73	528.11	-123.86
P50 (MM $)	693.93	657.80	520.00	489.09	656.88	626.94
P90 (MM $)	834.91	1476.00	635.43	1014.54	796.34	1381.52
Portfolio Budget (MM $)	1778.13	1778.13	1618.75	1618.75	1662.38	1662.38
Number of Projects (MM $)	28	28	28	28	30	30

Table 6.10: Portfolio Choices

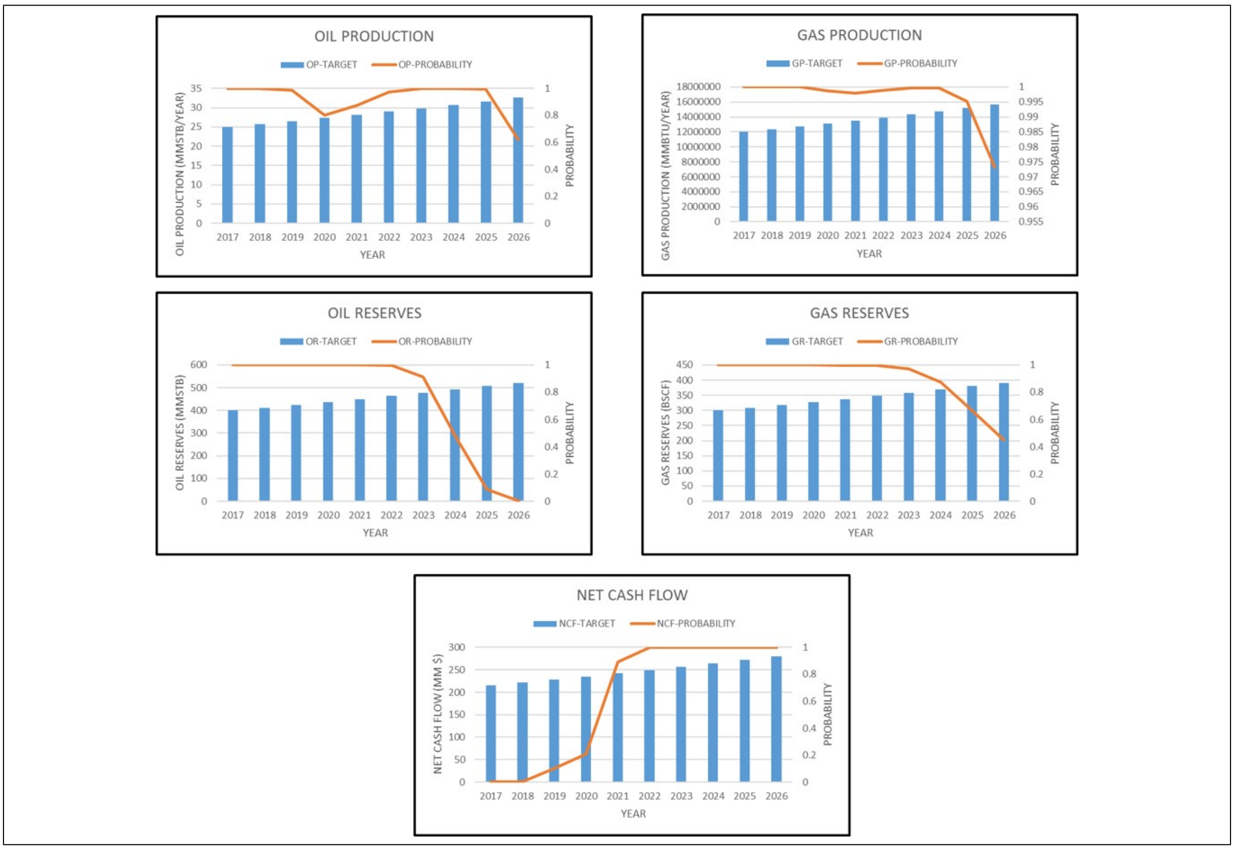


Figure 6.6: Choice-1 Constant Price

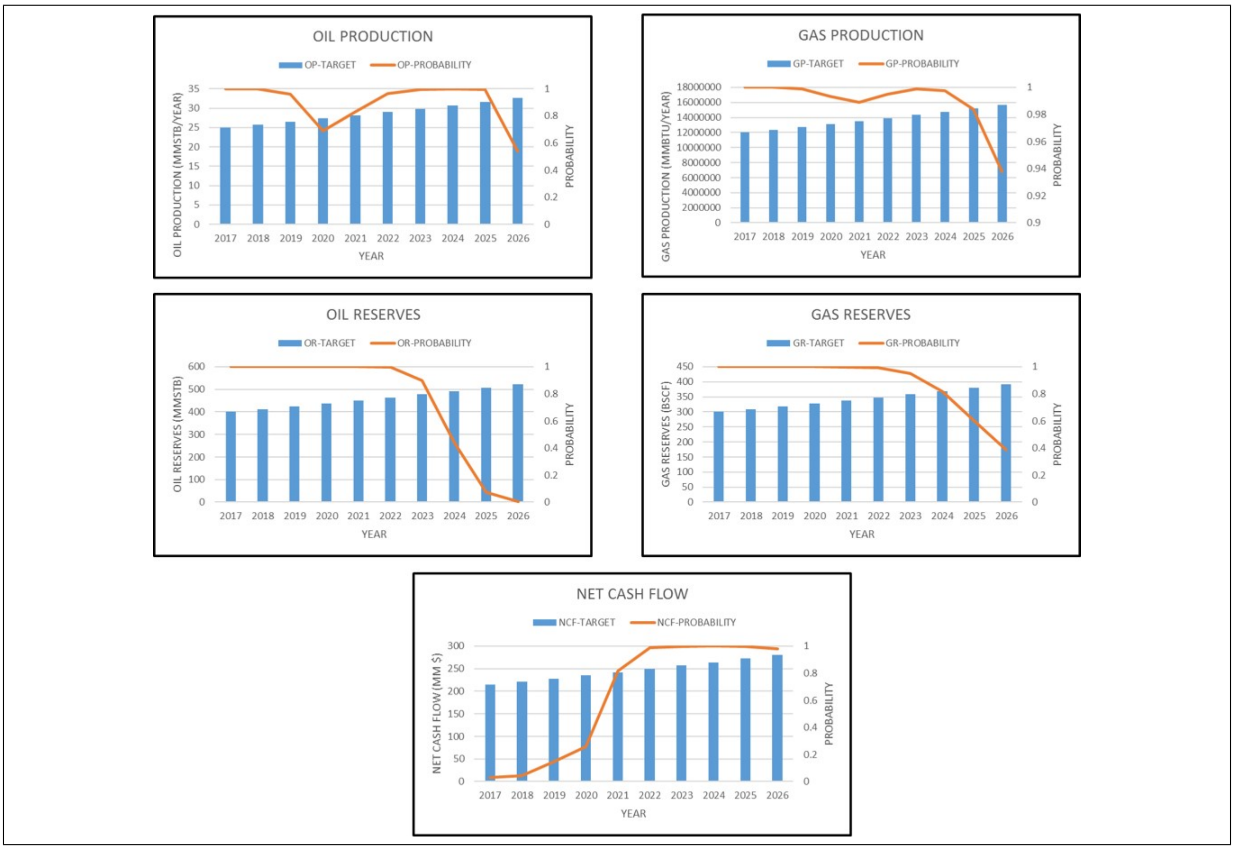


Figure 6.7: Choice-1 Stochastic Price

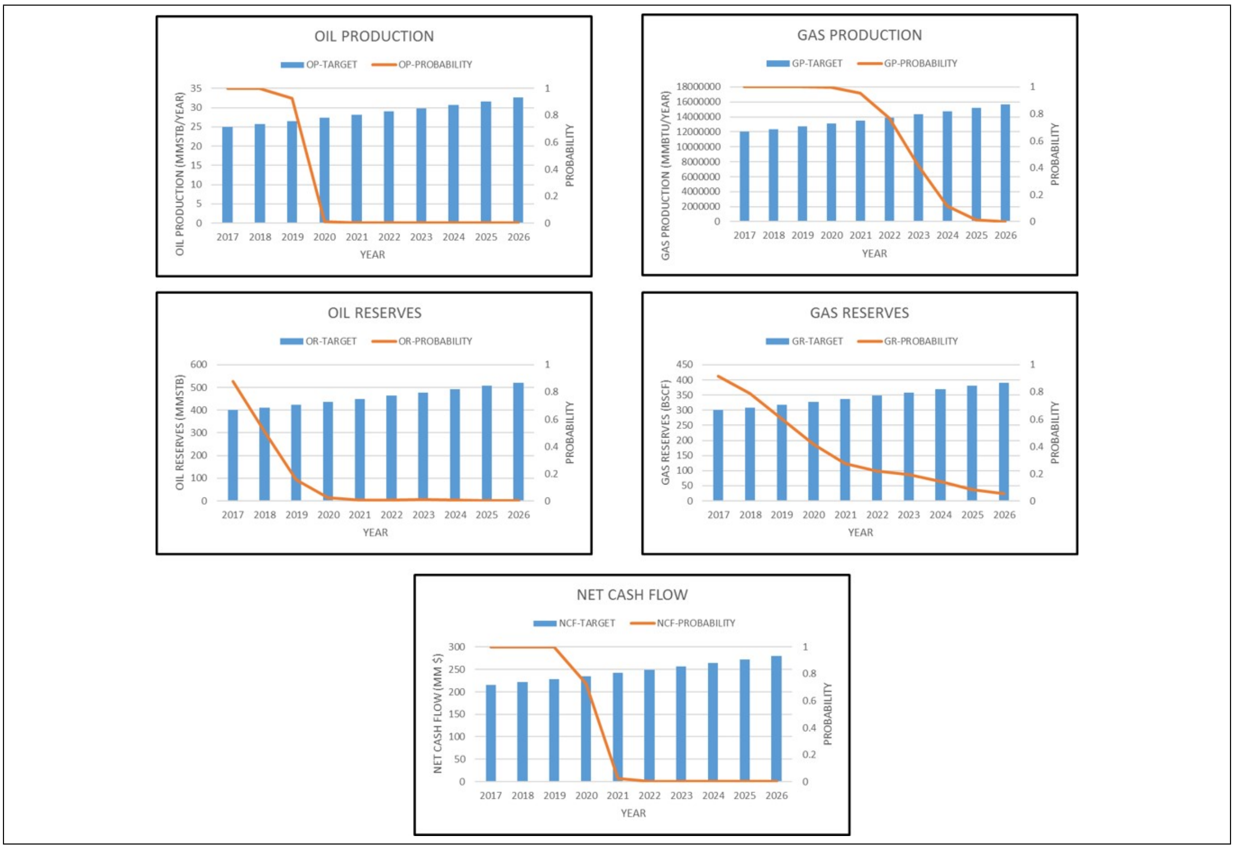


Figure 6.8: Choice-2 Constant Price

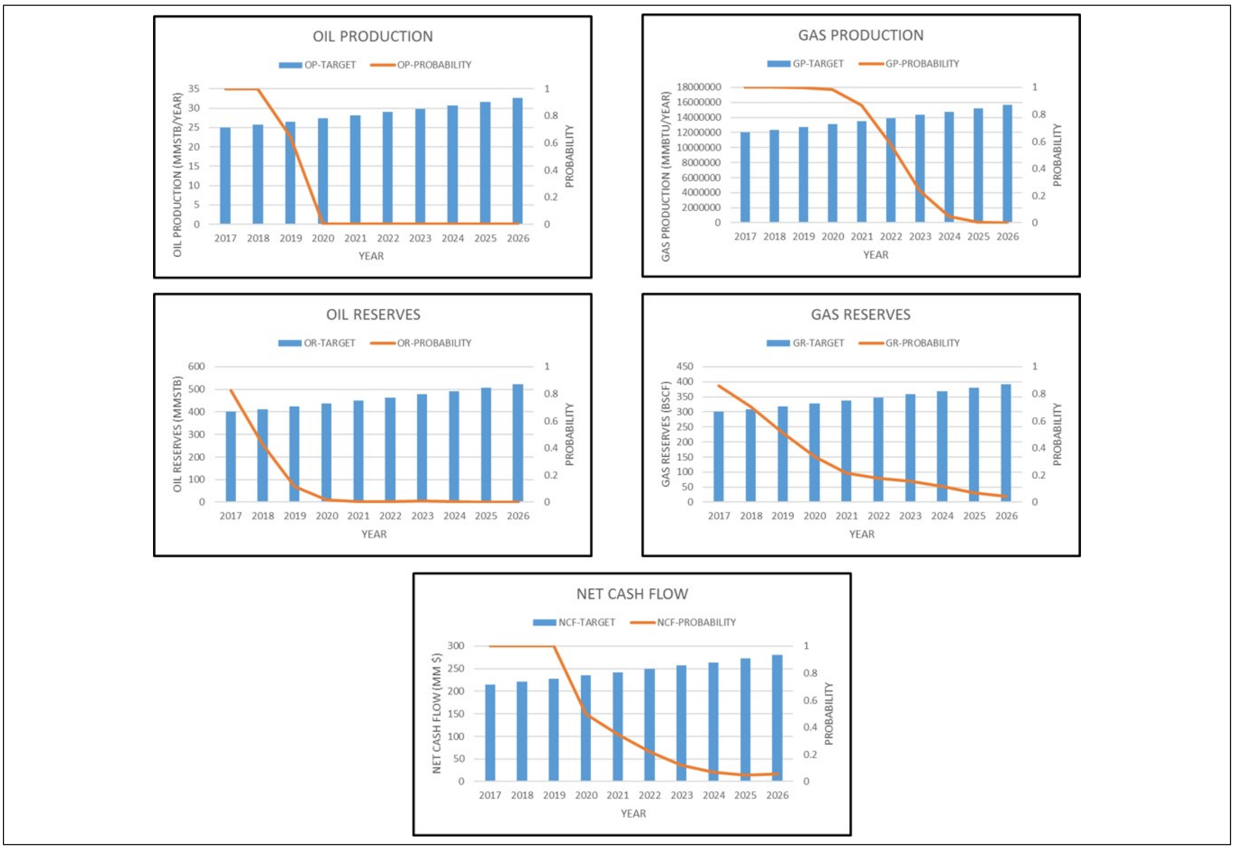


Figure 6.9: Choice-2 Stochastic Price

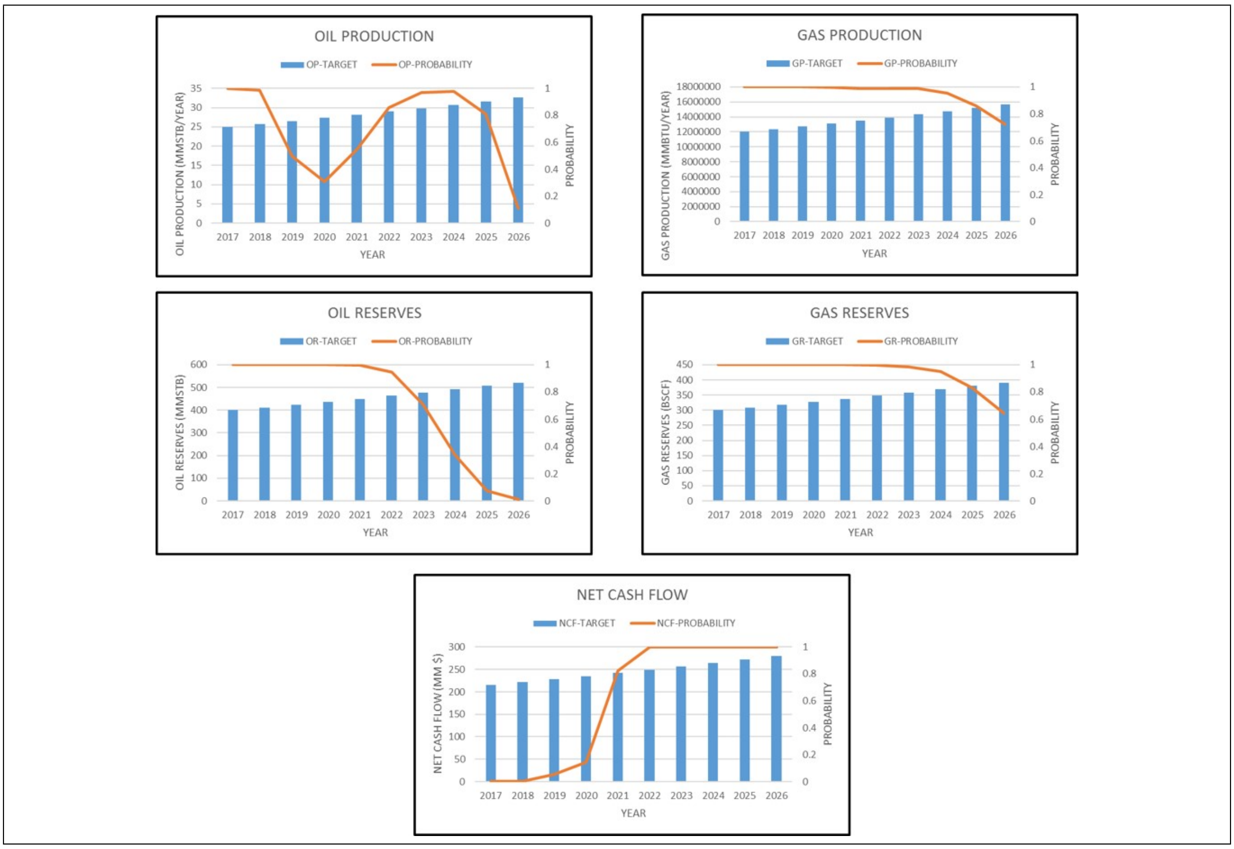


Figure 6.10: Choice-3 Constant Price

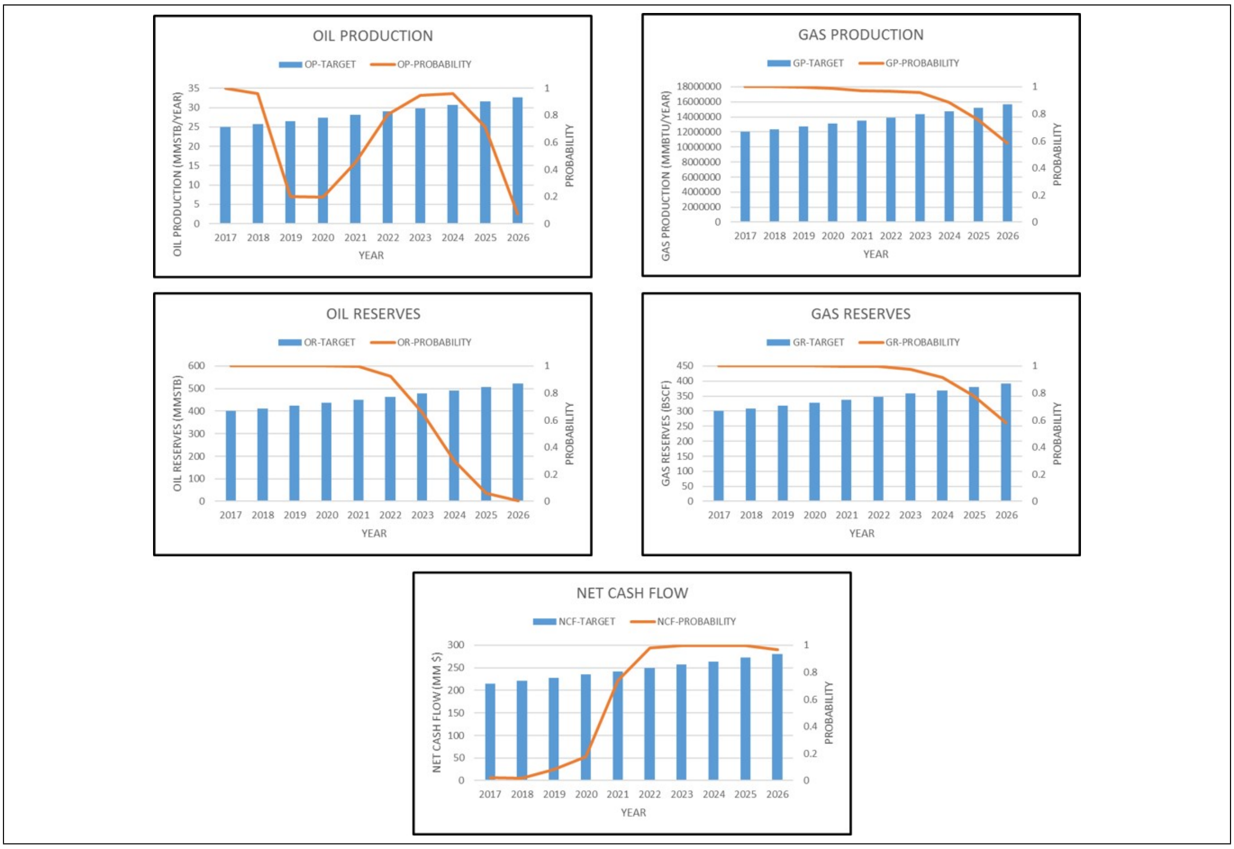


Figure 6.11: Choice-3 Stochastic Price

Chapter 7

Summary and Future Research:

Portfolio Analysis in an important tool for evaluating a capital investment opportunity in the petroleum industry.Current practice ignores inherent uncertainty in parameters while evaluating project and the portfolio.This occurs because either uncertainity is represented by a single number or uncertainty modeling becomes too complex that it requires experts to perform.In both scenarios, the actual decision maker is at a loss.

In this work, we have looked into an area of information management called, *Probability Management* to develop a complete asset-portfolio level model of petroleum projects.We tested this model with a series of analysis.The development of this model showed the usefulness of *Probability Management* in communication and modeling of uncertain variables.

We have specifically introduced the *Probability Management* and showed its use in petroleum portfolio analysis.The model developed help decision maker to interactively use it while discussing portfolio analysis problem.Also, the model does not require any special program to run and *SIPmath*^{TM} *Modeler Tools* was only needed to build the model, not to run it.Therefore it is easily shareable and customizable.At the same time, we have illustrated the need to included stochastic price model in portfolio analysis in this work.

We give following three research ideas to pursue to interested person:

- One can look into using SIPmath with other format or program to develop more complex but interactive portfolio analysis model.
- One can look into adding multi-period portfolio optimization problem using this approach.
- One can look into effect of random number and probability distribution type onto portfolio analysis.

We believe further development of *Probability Management* will be beneficial for decision maker in petroleum industry as shifting from calculation with single number to calculation with uncertainty distribution will remove *Flaw of Averages.*

Nomenclature

ϕ	Porosity
A	Area
AEC	Average Exploration Cost per Year
AEX	Additional Expenditure for Gas Blowdown
AGP	Annual Gas Production
AOP	Annual Oil Production
ATT	Attribute
BSCF	Billion Standard Cubic Feet
CEX	Capital Expenditure
COS	Chance of Success
CY	Current Year
DEP	Depreciation
DGP	Daily Gas Production
DNCF	Discounted Net Cash Flow
DOP	Daily Oil Production
DT	Down Time
ERG	Economic Rate-Gas
ERO	Economic Rate-Oil
ET	Exploration Time
FLG	Facility Limit-Gas
FLO	Facility Limit-Oil
FOE	Average Fixed Operating Expenditure per Year
FP	Field Potential
GR	Gas Reserves
HDR	Hubbard Decision Research
ICE	Initial Capital Expenditure
MDRG	Maximum Daily Rate-Gas

MDRO Maximum Daily Rate-Oil

MMSTB Million Stock Tank Barrel

MPT Modern Portfolio Theory

MVO Mean-Variance Optimization

N Time Step

NCF Net Cash Flow

NPV Net Present Value

NT Normal Tax

NTB Normal Tax Base

NTG Net To Gross

NTR Normal Tax Rate

OEX Operating Expenditure

OR Oil Reserves

PM Probability Management

RF Recovery Factor

RFR Risk Free Rate

RG Gross Revenue

RN Random Number

RPR Risk Premium Rate

RUP Ramp Up Period

SGP Start Year for Gas Blowdown

SIP Stochastic Information Packet

SLURP Stochastic Library Unit with Relationship Preserved

SOE Starting Year for Exploration

SOP Start of Production

ST Special Tax

STB Special Tax Base

STGOIP Stock Tank Gas Originally in Place

STOOIP Stock Tank Oil Originally in Place

STR Special Tax Rate

t Year

THC Type of Hydrocarbon

TT Total Tax

TTR Total Tax Rate

UL UpLift

VOEG Average Variable Operation Expenditure per Year-Gas

VOEO Average Variable Operating Expenditure per Year-Oil

WI Working Interest

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Appendix A

Two-Factor Price Model:

In this appendix, based on (Jafarizadeh & Bratvold, 2012; Thomas & Bratvold, 2015), we explain how we use two-factor price model and *SIPmath*^{TM} *Modeler Tools* to generate:

1. 4000 realizations of stochastic oil and gas price.
2. Mean oil and gas price of Stochastic oil and gas price derived above.

Two Factor Price Model			
Parameter	**Description**	**Parameter Value for Oil**	**Parameter Value for Gas**
κ	Mean-reversion coefficient	1.07	1.56
σ_χ	Volatality of the short-term factor	0.25	0.38
χ_0	Short-term increment of the log of the spot price	-0.34	-0.22
λ_χ	Risk-premium for short-term factor	0.08	0.02
μ_ξ	Risk-neutral drift rate for the long term factor	0.05	0.04
σ_ξ	Volatality of the long term factor	0.19	0.17
ξ_0	Long-term increment of the log of the spot price	4.2	1.17
λ_ξ	Risk premium for long-term factor	0.08	0.06
$\rho_{\chi\xi}$	Correlation coefficient between the random increments	0.34	0.75
Δt	Time-step	0.08333	0.08333

Table A.1: Two-Factor Price model

	Long Term Oil	**Short Term Oil**	**Long Term Gas**	**Short Term Gas**
Long Term Oil	1	0.34	0.63	0.48
Short Term Oil	0.34	1	0.36	0.64
Long Term Gas	0.63	0.36	1	0.75
Short Term Gas	0.48	0.64	0.75	1

Table A.2: Oil and Gas Price Correlation Matrix

Table A.1 on the previous page and Table A.2 shows the required data we used for two-factor price model.

Below are the steps for using *SIPmath Modeler tools* for implementing two-factor price model:

1. Using uniform distribution, we generate random number for long-term oil, short-term oil,long-term gas, and short-term gas.
2. Set mean for long-term oil, short-term oil, long-term gas, and short-term gas equal to 0.
3. Generate correlated normal distribution for all four using random number and mean in and a correlation matrix is shown in Table A.2.
4. Long-term factor(ξ_t):

   For time step(N) = 0,

   $$\xi_N = \xi_0 - (\frac{\lambda_\chi}{\kappa}) \tag{A.1}$$

   For time step(N) > 0,

   $$\xi_N = \xi_{N-1} + (\mu_\xi - \lambda_\xi)\Delta N + \sigma_\xi \varepsilon_\xi \sqrt{\Delta N} \tag{A.2}$$

   Where ε_ξ = Correlated normal random variable for long-term factor.
5. Short-term factor(χ_N):

   For time step(N) = 0,

   $$\chi_N = \chi_0 + (\frac{\lambda_\chi}{\kappa}) \tag{A.3}$$

   For time step(N) > 0,

   $$\chi_N = \chi_{N-1} e^{-\kappa \Delta N} - (1 - e^{-\kappa \Delta N})\frac{\lambda_\chi}{\kappa}\Delta N + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa \Delta N}}{2\kappa}} \tag{A.4}$$

   Where ε_χ = Correlated normal random variable for short-term factor.

6. Price (S_N):

$$S_N = e^{(\xi_N + \chi_N)} \tag{A.5}$$

7. Mean Price can be obtained using below formula:

$$Mean_{Price} = expe^{-\kappa N \Delta N} \chi_0 + \xi_0 - ((1 - e^{-\kappa N \Delta N}) \frac{\lambda_\chi}{\kappa}) + ((\mu_\xi - \lambda_\xi) N \Delta N) \tag{A.6}$$

Using this steps, we have obtained 4000 price realization.Figure A.2 shows one such realization and mean price path for oil and gas each.

Figure A.2: Two-Factor Price Model