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Universitetet i Stavanger

FACULTY OF SCIENCE AND TECHNOLOGY

MASTER'S THESIS

Study programme/specialisation: Petroleum Engineering / Well Engineering	Spring semester, 2017 Open
Author: Camilo Andres Cardenas Medina	………………………………………… (signature of author)

Programme coordinator:

Supervisor(s): Dr. Reidar Bratvold

Title of master's thesis:

Offshore Exploratory Drilling Campaigns During Low Oil Price Period: Maximizing Value Creation from Flexibility

Credits: 30 ETCS

Keywords: Drilling campaigns Real Option Valuation Exploration campaigns Decision Analysis Project Valuation Uncertainty Flexibility	Number of pages: 77 + supplemental material/other: USB Stavanger, 15th June 2017

Title page for Master's Thesis
Faculty of Science and Technology

Acknowledgments

This thesis is the result of a research project of nine months, that it would have not been possible without the professor Reidar Bratvold. Under his supervision, I grew professionally and personally. His inspiring and motivating support, as well as his knowledge and expertise in this research field, were fundamental to complete this project.

I would like to thank Dr. Babak Jafarizadeh. Since the beginning of the project, he supported this research through insightful and inspiring discussions, that were essential to fulfill this thesis.

Finally, I would like to thank my family. My master program could not have been completed without their unconditional love and support. Also, to my beloved girlfriend Paula Vera, for her help and support during this thesis, but especially, for her words during tough times.

Abstract

During severe oil price downturns, many operating companies reduce or eliminate large investments with long time horizons such as exploratory drilling campaigns. This reduction in investments forces rig and drilling services providers to reduce their bids to be competitive. The result of this is lower initial investment in the oil and gas projects. In this research, a valuation approach is implemented to study the impact of this investment reduction on the decision-making process for executing exploratory drilling campaigns during low oil price periods. It is demonstrated that postponing exploration campaigns during low oil price periods does not necessary maximize value creation.

Value creation from investment in low price periods results from the combination of uncertainty and flexibility. The analysis of the value of flexibility (optionality) is usually referred to as Real Options Valuation (ROV). In this work, one of the most versatile approach for valuing options is applied: The Least-Square Monte Carlo Method (LSM). Two uncertainties were considered: oil price and drilling cost. Among the different oil price models, the two-factor stochastic price process developed by Schwartz and Smith (2000) was chosen because of its balance between realism and ease of communication to the managers. Drilling cost is modeled as a Geometric Brownian Motion process. By implementing a delayed correlation between the drilling cost and the oil price, the cost reduction observed in the market is accounted for.

In this research, it is shown how real option valuation can be used to determine the optimal time to start the exploratory drilling campaign. Furthermore, it is demonstrated that by including the correlation between the drilling cost and the oil price, the optimal time to execute the investment is during the year with the lowest expected oil price. The impact of this correlation is studied through the use of sensitivity analyses of the project value with respect to the correlation factor and the parameters in the stochastic price model. It is concluded that considering this correlation leads to more realistic project value estimations, resulting in portfolio decisions that maximize stakeholder value.

The key contribution of this thesis is the use of option valuating methods to demonstrate that value will be created by initiating the exploratory drilling campaigns during low oil price periods. The

real option model developed in this research is applicable to all types of exploration projects in the petroleum industry.

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1. Introduction

The price of crude oil, as other commodities, is governed by the supply–demand relationship in the markets. Low oil prices are signs of higher supply than demand, resulting from increased production levels or weakened demand (Geman, 2005). Current oil prices will consequently affect the investment policies of operating companies, forcing them to abandon expensive means of production. Among the companies' portfolio, large investments that involve high uncertainty such as exploratory drilling campaigns are avoided. Rig providers and services companies observe a substantial decrement in the operational activity, and they are forced to reduce the bidding cost to subsist in such competitive market. This in turn will have a ripple effect of reducing the exploration cost within the industry.

The exploration cost is a major expense for offshore projects. Therefore, a decline in the cost may have a major impact on the initial capital investment, positively impacting the overall value of the project. Although the correlation between oil price and drilling cost is clearly observed in the market, its effect on the project valuation has, to our knowledge, not been explicitly studied. In this research, this correlation is implemented to appraise its impact on the decision-making process for executing exploratory drilling campaigns during low oil price periods. The objective is to investigate if postponing exploration investments, as most companies do, is a value maximizing decision.

Prospects that involve high uncertainty are classified as "high-risk" in companies' portfolio. However, uncertainty also implies the possibility of having better than expected outcomes. Rejecting projects that involve significant downside risk could prevent capital lost, but at the same time, by not investing in uncertain projects the company removes the opportunity of investing in a prospect with a positive expected value. Ignoring project uncertainties do not lead to portfolio decisions that maximize the stakeholder value. As discussed in Begg et al. (2002) among others, the traditional deterministic Discount Cash Flow (DCF) method fails to reflect these uncertainties, and assumes that the investment is a now-or-never decision, which does not reflect the flexibility that managers have of making future decisions with the future knowledge from revealed uncertainties during the project lifetime. Value creation from investment in low price periods

results from the combination of uncertainty and flexibility. The analysis of the value of flexibility is usually referred to as Real Options Valuation (ROV).

ROV techniques have been applied before for studying the decision-making process in Oil and Gas (O&G) projects during oil downturns. For instance, Begg at al. (2004) implemented one of these methods to assess the abandonment decision during periods that oil price falls below the break-even value. They demonstrated that the return of investment can be increased when the uncertainties are included in the decision-making process. In this research, the most promising ROV method for solving real-world problems is implemented: The Least Squares Monte Carlo (LSM) approach developed by Longstaff and Schwartz (2001). This method is versatile and computationally efficient when multiple sources of uncertainty are considered.

Uncertainties changing over time are addressed in the evaluation of capital investment by using stochastic processes. These are implemented within the ROV method to model uncertain variables in the cash flow. The two uncertainties that typically have the largest impact on the Net Present Value (NPV) were considered in this study: the oil price and drilling cost. The two-factor stochastic price process developed by Schwartz and Smith (2000) was used to describe the behavior of oil prices because of its balance between realism and ease of communication. The drilling cost was modeled using a Geometric Brownian Motion (GBM) process, and it was assumed to be the main driver of the exploratory drilling campaign cost.

By implementing the LSM approach, the optimal time to start the exploratory drilling campaign in an offshore study case is evaluated. This method has been used in previous studies to evaluate optimal decisions in O&G projects: Thomas and Bratvold (2015) illustrated the implementation of this method to find the optimal blowdown decision, whereas Alkhatib and King (2011) used it to determine the optimal time to start surfactant flooding in Enhanced Oil Recovery (EOR) projects.

This research contributes to the literature of petroleum asset valuation in two aspects. First, it presents a ROV model for exploration projects that reflects the observed market correlation between the drilling cost and the oil price. Second, it implements the developed ROV model to demonstrate that value will be created by initiating the exploratory drilling campaigns during low oil price periods.

The first part of this dissertation illustrates the decision-making process in the exploration license. In the second section, the stochastic processes used for the oil price and the drilling cost, along with their correlation, are described. This is followed by the introduction of the ROV methods, including the LSM approach. Later the characteristics of the study case are specified, and the LSM implementation is defined. Finally, the findings, analysis, and conclusion are stated.

2. Offshore Exploration Projects Framework

As discussed in the introduction, uncertainty combined with flexibility may lead to value creation. The first step in the process of building a ROV model is identifying the variables that the decision-maker considers uncertain in the project, along with the flexibilities. In this chapter, decision trees are used to illustrate uncertainties inherent in the exploration projects, and the flexibilities the managers have along the exploration license.

Hydrocarbon resources are usually explored through investment vehicles called partnerships. In this arrangement, investors provide capital and a selected member, called the operator, operates and manages the projects. Exploration licenses are usually awarded on a fixed–term basis. The partnership formed by a group of companies has the option to drill the identified prospects until the contract maturity. If commercial hydrocarbons are discovered, the partnership may decide to extend the license. Otherwise, the license is returned to the authorities (Jafarizadeh and Bratvold, 2015).

Every milestone during the lifetime of the exploration project has associated different uncertainties. These projects require comprehensive strategic analysis because they include three types of uncertainties. First, the technical uncertainties such as reservoir properties. Second, the economic uncertainties that impact the value of the field, and finally, the strategic uncertainty related to the action of competitors in the near-area to be explored (Dias, 1997)[1]. Available information is never enough to remove these uncertainties, leading to the implementation of probabilistic models. A decision tree illustrating the main decisions and uncertainties for an exploration project is shown in the Figure 1. The decision to invest in exploration wells comprises the uncertainties of the existence and volume of hydrocarbons. Geological information is used to assess a probability of success, which may be different for every single well. As the main purpose of the exploratory wells is to reduce or reveal the subsurface uncertainties, they represent investments in new information. These wells will be tested for some months, and then, plugged and abandoned. The decision to drill an exploration well should be based on its expected value,

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[1] Dias (1997) illustrated that the operators have the option to postpone the execution of the exploratory campaign until results from other exploratory campaigns in the neighborhood are revealed. He implemented the game theory to argue that this may create an impact in the value of the exploration project.

calculated using the available information (Jafarizadeh and Bratvold, 2015). If economical feasible volume of hydrocarbons is discovered, the company should make the decision whether develop the field or sell the ownership of the license. Different development strategies are evaluated, considering the uncertainties in the production rates and the oil prices[2].
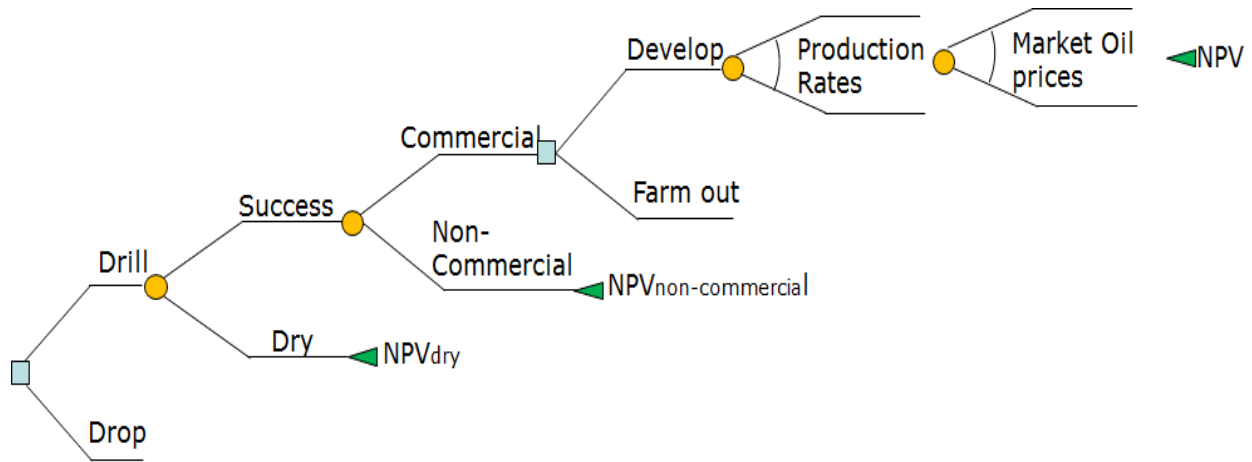
Figure 1 Decision tree representing the main decisions and uncertainties relevant for an exploration opportunity. Modified from Jafarizadeh and Bratvold (2015)

Along the time to maturity of the exploration license, the partnership has the flexibility to decide when to execute the exploratory drilling campaign. This is known as waiting option in the ROV context, and it is illustrated in the Figure 2. Every year, the partnership should decide on whether to start the drilling campaign, or to wait until the next year and observe the behavior of the oil price over that period. The same decision will be faced the following year if they choose to wait, but then, the uncertainty in the oil price of that year will be revealed, impacting the estimations of the Net Present Value (NPV).

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[2] Uncertainty in the oil price and production rates are represented as semi-circles in the Figure 1, indicating that they are modeled using continuous probability density functions.
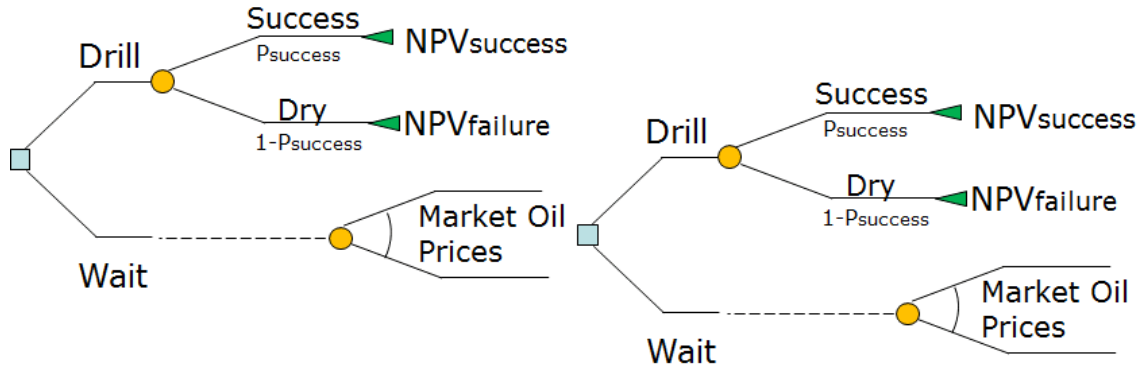
Figure 2 Exploration decision tree illustrating the waiting option

The NPV for each end–node is assessed by calculating cash flows from forward oil prices, production forecasts, tax rates, and costs. Figure 3 shows a typical cash flow diagram for development of a hydrocarbon discovery. The exploratory campaign represents the initial investment. After commerciality is determined, the company prepares the plan for development and operations, and delivers it to the government for approval[3]. The development expense includes the cost of facilities construction, drilling of production wells, and preparing downstream infrastructure. This expense depends on the size of the field, production strategy, number of production wells to be drilled, reservoir characteristics and distance to nearby fields, among others. After a period of development commonly called the lead time, first oil comes and positive cash flow starts to accumulate. Continuous operations require operational expenditure (OPEX), which consists of a fixed and a variable portion[4].

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[3] Regulations may change depending on the government. In the Norwegian Continental Shelf (NCS) the government shall approve the Plan for Development and Operations (PDO) before execution.
[4] Variable cost depends on the production rate and include processing and lifting cost, among others. Fixed OPEX's are independent of the production rate, and involves expenses such as tariffs or labor cost
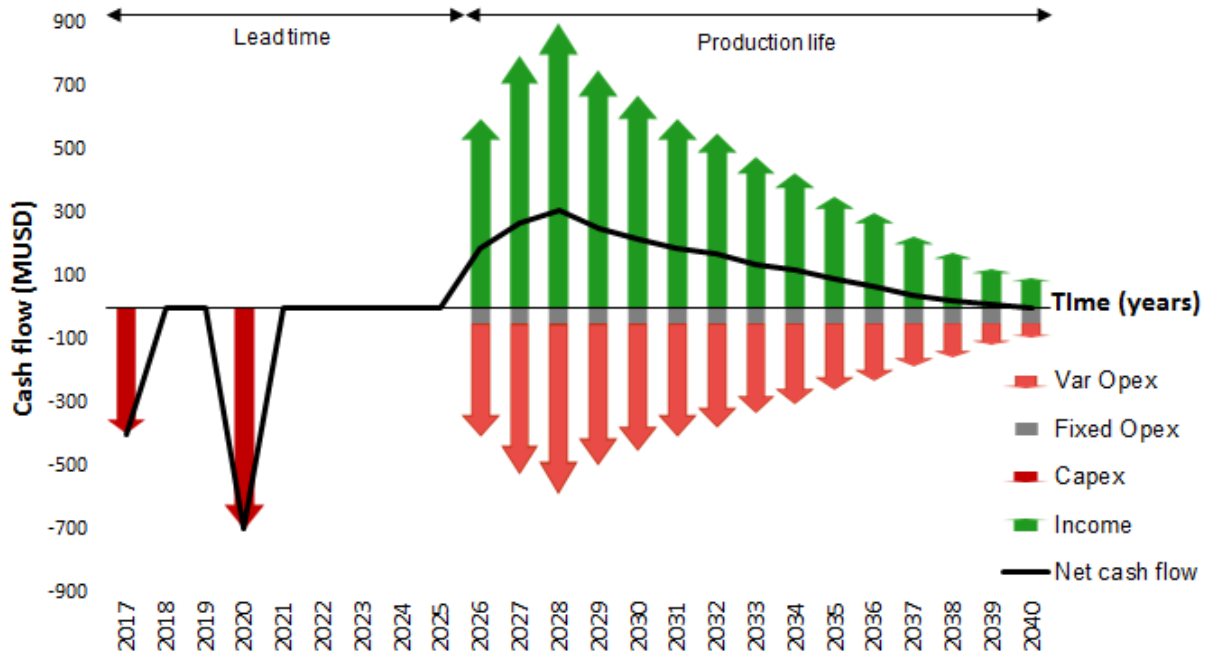
Figure 3 Typical cash flow diagram for an offshore exploration project

3. Stochastic processes

Probability distributions are used to quantify the lack of knowledge on a system variable. Its implementation entails the shift from deterministic calculations to a probabilistic form, that allows to address the uncertainty inherent in the outcomes. For time-dependent uncertain variables, a probability distribution must be assigned for every single time-step along the interval to be evaluated. Hence, stochastic processes are implemented, and they are used in the evaluation of capital investment to describe uncertain variables in the cash flow model including the oil price, the operational cost, drilling cost, and the capital expenditure.

The most commonly used stochastic process is the Geometric Brownian Motion (GBM). Initially, it was used in finance to model the stock price in the Black-Scholes model[5], but now it has been implemented in different areas. It assumes that at the time t, the uncertain variable has a log-normal probability distribution with a variance that increases with the time, and the expected value grows or declines exponentially with a constant drift. An example for the value of a developed field is shown in the Figure 4.
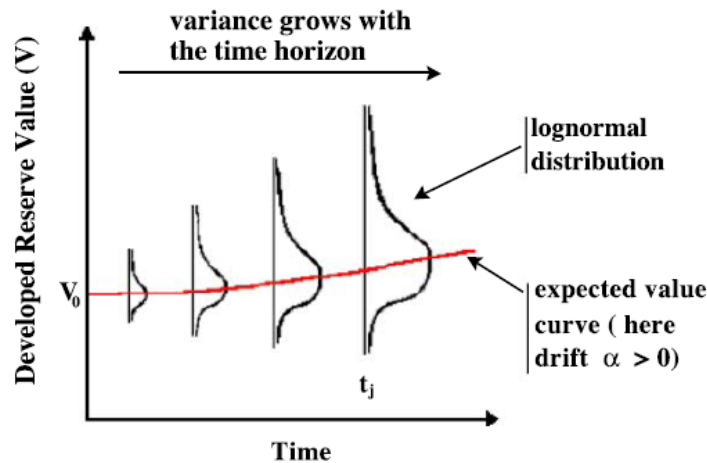
Figure 4 Example of GBM process for a developed field value. Taken from Dias (2004).

However, the GBM process fails to reflect the price behavior of some commodities in the market. In a liquid market, when the price of a commodity is above the long-term equilibrium, the

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[5] Introduced in 1973 by Fischer Black and Myron Scholes, the Black-Scholes model addressed the issue of estimating the value of European options. It is now implemented for stock and derivatives value estimations.

producers increase the investment in their production assets, raising the supply level. This production increment lowers the price to a long-term equilibrium. Similar effect is observed when the price is lower than the equilibrium level, driving down the supply level which leads to the increment in the commodity price. Thus, there is a mean-reverting force that is proportional to the difference between the spot price and the equilibrium level (Dias, 2004). The first mean-reverting model was introduced by Uhlenbeck and Ornstein (1930). It has been applied in several areas of study, and more recently for commodities pricing and petroleum valuation. As the GBM process, the uncertain variable has a lognormal distribution, but the difference is that the variance rises until a certain time, as shown in the Figure 5 for an oil price example. For this case, the variance grows until the time t_i and then remains constant. The expected value decreases from a value Po towards the equilibrium price.
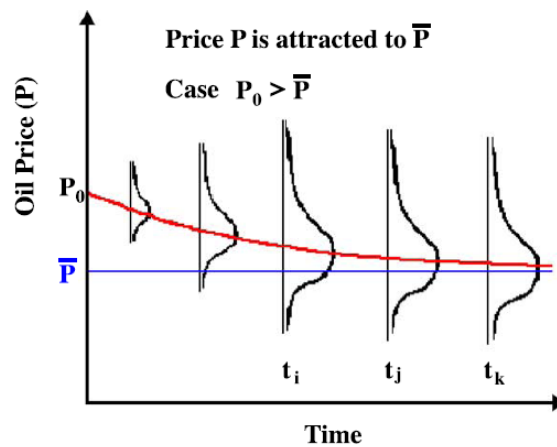
Figure 5 Example of a mean-reverting process for an oil price case. Taken from Dias (2004).

3.1. Oil price model

Both stochastic models previously discussed have advantages and disadvantages when it comes to their implementation for oil price modeling. The GBM process is simple to implement and use, but it fails to reflect the mean-reverting behavior observed in the market. The Uhlenbeck-Ornstein (OU) model address this issue, but it assumes that there is not uncertainty in the long-term equilibrium. Pindyck (1999) studied the historical data of the oil price for 127 years, and concluded that the oil price is a mean-reversion process, that reverts to a long-term equilibrium that itself is

a stochastic process. Schwartz (1997) compared the performance of four models for describing the oil price behavior: GBM, UO model, two-factor model, and three-factor model. He demonstrated that the two-factor model outperformed the one-factor models (GBM and UO models).

More recently, the short-term/long term (STLT) stochastic process developed by Schwartz and Smith (2000) has been the preferred approach in many implementations (Jafarizadeh and Bratvold, 2012, 2013, 2015; Ozorio et al., 2013; Hahn et al., 2014; Thomas and Bratvold, 2015, 2017). This STLT model has been chosen for this work because it provides consistency and relative ease of implementation. It states that the oil price follows a stochastic process that consists of two uncertain variables, a short-term factor and a long-term variable. The former works as a mean-reverting process to describe deviations from the equilibrium price (i.e. temporary supply disruptions), whereas the latter is defined as a GBM procedure which reflects the expectations of consumption of current reserves, the discovery of new reserves, or a technological change like the introduction of improved fracking methods. In this model, the log of spot oil price is the sum of the two uncertain elements:

$$S_t = exp\left(\chi_t + \xi_t\right)\tag{1}$$

where S_t is the spot oil price, χ_t is the short-term component, and ξ_t represents the long-term element. The short term is modeled as a mean-reverting process described in the risk-neutral version[6] as:

$$d\chi_t = \left(-\kappa\chi_t - \lambda_\chi\right)dt + \sigma_\chi dz_\chi^*\tag{2}$$

The short-term is a function of the volatility σ_χ, risk premium λ_χ, and the mean-reversion coefficient κ that represents the rate that the short-term deviations will vanish. The long-term factor (ξ_t) is modeled as a GBM process described in the risk-neutral version as:

$$d\xi_t = \left(\mu_\xi - \lambda_\xi\right)dt + \sigma_\xi dz_\xi^*\tag{3}$$

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[6] In traditional valuation, the discount factor applied accounts for risk and time. In the risk-neutral valuation introduced by Cox and Ross (1976), the stochastic processes of the uncertainties in the model are risk-adjusted, so the discount rate applied will only account for time value. This is done in the STLT model by subtracting the risk premium (λ).

The long-term is a function of the volatility σ_ξ, risk premium λ_ξ, and the drift μ_ξ which describes the rate that the long-term is expected to grow along the time, In the equation (2) and (3), dz_χ and dz_ξ are parameters that describe how the processes are incrementing along the time following a random process called the Browmian-motion. They are called increments of standard Brownian-motion process, and they are correlated as:

$$dz^*_\chi dz^*_\xi = \rho_{\chi\xi} dt \tag{4}$$

where $\rho_{\chi\xi}$ is the correlation coeeficient between the two factors. To simulate the short-term and long-term factors, it is required to discretize equations (2) and (3). Jafarizadeh and Bratvold (2012), and Davis (2012) proposed two different discretization methods. Although the methods differ in the formulation, they lead to the same simulated values. In this study, the discretization presented by Jafarizadeh and Bratvold (2012) is implemented. Hence, the discretize forms of the two factors are:

$$\xi_t = \xi_{t-\Delta t} + \left(\mu_\xi - \lambda_\xi \right) \Delta t + \sigma_\xi \varepsilon_\xi \sqrt{\Delta t} \tag{5}$$

$$\chi_t = \chi_{t-\Delta t} e^{-\kappa \Delta t} - \left(1 - e^{-\kappa \Delta t} \right) \frac{\lambda_\chi}{\kappa} \Delta t + \sigma_\chi \varepsilon_\chi \sqrt{\frac{1 - e^{-2\kappa \Delta t}}{2\kappa}} \tag{6}$$

where ε_χ and ε_ξ are standard normal random variables, correlated by $\rho_{\chi\xi}$. In other words, when implementing this discretization, ε_ξ and ε_χ are random numbers that are generated, and correlated between them. The model has a total of seven parameters, along with two initial conditions (χ_o, ξ_0), to be estimated.

3.1.1. Calibration

The parameters of the short-term and long-term equations are not directly observed in the market. To estimate them, a calibration method must be implemented that can deal with unobservable parameters. Three calibrations methods for the STLT price process have recently been studied: The Kalman Filter, Sequential optimization, and Implied Estimation. Thomas and Bratvold (2017) compared the performance of the calibration methods, and concluded that the operating company

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should select the method that reflect its point of view on the future oil prices. The Kalman filter or Sequential optimization methods is preferred if the company considers that the future oil price is better described by using historical and current futures data. On the other hand, the Implied Estimation should be chosen if the decision maker considers that the oil price model should portray the current market beliefs about the future oil price.

In this work, we assume that the decision maker is a public company[7].The overall value of the company is the financial market value of its assets. These values depend on uncertainties; hence, the stochastic models used in their valuation should reflect the market beliefs on the underlying uncertainties (Thomas and Bratvold, 2017). The Implied Estimation method was chosen in this investigation to calibrate the STLT model. Thus, the oil price uncertainty will embed the market beliefs regarding its future behavior, and an economical evaluation based on this, will lead to a value of the exploration prospect that is consistent with the current financial market concerns and expectations. This approach uses current market information about future price levels. In the market, crude is traded through spot contracts and future contracts[8]. Additional market information can be obtained from financial tools as the options on future contracts[9]. If an efficient market is assumed, this information reflects the perception of the participants in the market about the supply-demand relation in the future. Schwartz and Smith (2000) mentioned that the far-maturity future contracts can provide an insight about the long-term factor, and the spot and near-maturity future contracts provide information about the short-term factor. Using the STLT model, they derived the mathematical framework for valuing a future contract as follows:

$$ln\left(F_{T,0}\right) = e^{-\kappa T}\chi_0 + \xi_0 + A\left(T\right) \tag{7}$$

where A(T) is given by:

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[7] The shares of a public company are traded at a public exchange such as NYMEX or the Oslo Stock Exchange.
[8] Spot contracts are set for delivering the crude immediately. Future contracts are set for delivering the crude in a specific time, with pre-determined oil price.
[9] An option is a financial derivative whose value depends on an underlying variable, in this case, a future oil contract. It gives the buyer the right, but not the obligation, to buy or sell a predetermined asset.

$$A(T) = \left(\mu_\xi - \lambda_\xi\right)T - \left(1 - e^{-\kappa T}\right)\frac{\lambda_\chi}{\kappa} + \frac{1}{2}\left(\left(1 - e^{-2\kappa T}\right)\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2\left(1 - e^{-\kappa T}\right)\frac{\rho_{\chi\xi}\sigma_\chi\sigma_\xi}{\kappa}\right) \qquad (8)$$

The value of the future contract with time to maturity T in the time zero (t=0) is denoted as $F_{T,0}$.

Based on Schwartz and Smith' ideas (Schwartz and Smith, 2000), Jafarizadeh and Bratvold (2012) developed and implemented a method to calibrate the STLT model based on current spot contracts, future contracts and options on future contracts. This method is implemented using market data information observed on 19 October 2016. Details of the implementation are shown in Appendix 1. Results from the calibration are shown in the Table 1 and illustrated in the Figure 6.

Table 1 Parameters for the Two-factor price process

Parameter	Value
σ_ε	7%
μ_ε^*	0,96%
κ	1,16
σ_x	33,50%
$\rho_{\varepsilon\chi}$	0,34
λ_x	0
ξ_0	4,03
χ_0	-0,11
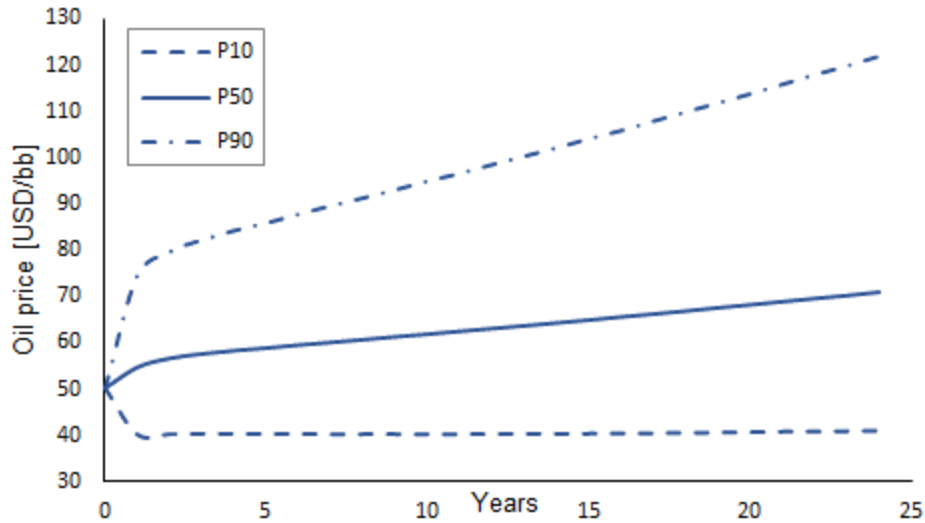
Figure 6 Oil price probabilistic model calibrated with data from 19 October 2016

3.2. Drilling Cost

The expenditures for drilling campaigns consist of the rig cost, drilling service fees, and man-hour expenses. These projects usually have low to medium capital requirements in a company's portfolio compared with the costs of major field developments. Yet perhaps because of uncertainties in finding hydrocarbons, such projects are often the first to undergo budget cuts during unfavorable economic conditions. During low price periods drilling campaigns are suspended, causing a decrease in demand for rigs and drilling services. This will in turn force the rig and service providers to reduce their rates. This correlation is clearly observed in the market. For instance, the demand of rigs with respect to the oil price is illustrated in Figure 7, where the international rig counts and the oil price from January 1995 to February 2017 is displayed.
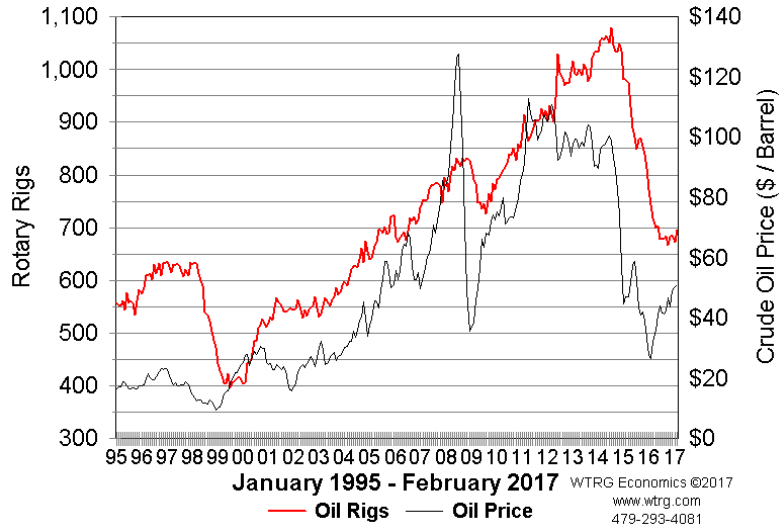
Figure 7 International Rig count. Taken from: http://www.wtrg.com/rotaryrigs.html

Willigers (2009) studied the relation between rig rates and oil price from 1995 to 2008. He determined correlation factors analyzing two types of rigs (Jack up and semi-submersible) in the Gulf of Mexico and the North Sea. The highest correlation factor was observed between the rig rates and the oil prices of the year before (close to 0.9), whereas the correlation factor without the time offset was less than 0.8. Results are shown in the Figure 8, where the rig rates are offset by one year with respect to the oil price.
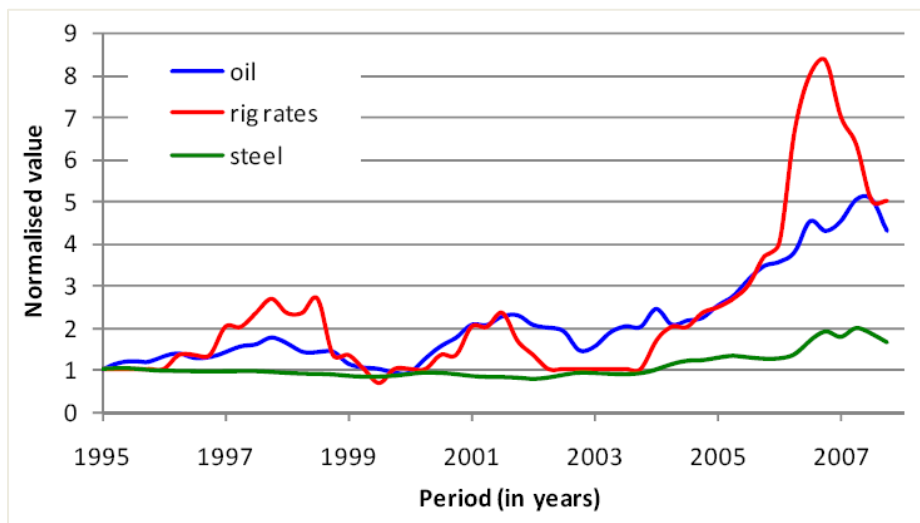
Figure 8 Historical development of oil price, rig rates and steel prices in the US market. Taken from Willigers (2009)

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In the risk–neutral valuation model, uncertainties can be categorized as market uncertainties (that can be hedged in the market, such as oil price) or private uncertainties (that cannot be hedged using market instruments, such as production levels). The market uncertainties are modeled using risk-adjusted probabilities and private uncertainties using assessed probabilities based on expert's beliefs or preferences (Smith and Nau, 1995). Drilling cost is a private uncertainty that also depends on oil market conditions, and its modelling includes the estimation of subjective probability[10] conditional on the oil price (Smith, 2005). In this research, the total cost of the drilling campaign is modeled using a GBM process, described by the following differential equation:

$$d\theta = \mu_\theta \theta dt + \sigma_\theta \theta dz_\theta \tag{9}$$

where θ represents the cost of the exploratory drilling campaign, μ_θ is the drift, σ_θ is the volatility, and dz_θ represents the Brownian increment. As discussed by Lima et al. (2005), the Equation 9 can be discretized as:

$$\theta_{t+1} = \theta_t e^{[(\mu_\theta - 0.5\sigma_\theta^2)\Delta t + \sigma_\theta \varepsilon_\theta \sqrt{\Delta t}]} \tag{10}$$

where Δt represents the time increment, and ε_θ is the standard normal random variable. The exploration cost is a private uncertainty that depend on the market. Smith (2005) stated that the stochastic process that describes this type of uncertainties should be assessed based on expert's opinion, and directly correlated with the market uncertainty (i.e. the oil price). Parameters for the cost of the exploratory drilling campaign in this research are shown in Table 2. The values of these parameters are determined by the local market, and the type of field. For instance, the supply-demand relationship of drilling services is not the same in a broad market such the North Sea, compare with a narrow market, i.e. the Caribbean Sea in Colombia. In addition, the calibration should account for the type of field to be explored, since the market conditions for land fields are different than for offshore prospects.

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[10] Subjective probabilities are estimated based on expert's opinion (not the information offered by the market) and they can explicit consider the effect of risk (Jafarizadeh and Bratvold, 2009).

Table 2 Parameters for the GBM process for the cost of the exploratory drilling campaign.

Parameter	Value
θ_0	280 Million
μ_θ	3%
σ_θ	20%

3.2.1. Correlation

As mentioned before, the exploration cost uncertainty is correlated with oil price. However, the oil price is composed of two uncertain variables that are themselves correlated: the short and long-term factors. The changes in the exploration cost are more affected by the variations in the short-term component in the spot price, than by the long-term equilibrium factor, because the exploratory drilling campaign can only be executed until the exploration license maturity[11], which is a relatively short time frame. For that reason, the stochastic process of the exploration cost (Eq. 10) will be correlated with the short-term factor (Eq. 6). This is implemented by correlating the normal random variables of the processes (ε) as described by Wiersema (2008):

$$\varepsilon_\theta = \varepsilon_\chi \rho_{\theta\chi} + \varepsilon \sqrt{1 - \rho_{\theta\chi}^2} \tag{11}$$

Same equation is used to correlate the short-term factor with the long-term factor in the STLT model. The correlation factor found by Willigers (2009) is used in this research ($\rho_{\theta\chi} = 0.89$). Nevertheless, this correlation factor was estimated for the North Sea and the Gulf of Mexico, and it may be different depending on the market location. Hence, a sensitivity analysis for the correlation factor is included in this research.

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[11] The duration of the exploration license is different for every country. In this research, the exploration license is assumed to be five years.

4. Real Option Valuation (ROV)

The dissatisfaction of corporate strategists and some academics with the traditional techniques of capital budgeting stimulated the search for new solutions as the ROV techniques (Trigeorgis, 1996). They realized traditional methods, as the Discount Cash Flow (DCF), fail to account for the flexibility that managers have for making decisions in the face of revealed uncertainties. The DCF method assumes the investment is a go/no-go decision, considering a passive strategic attitude from the manager, in the base of an expected cash flow. These assumptions differ from the corporate reality, where an uncertain cash flow is a function of underlying uncertainties. As new information arrives and the uncertainties start to be revealed, the managers have the flexibility to change their initial strategy, seeking for increasing the value of the project or mitigating possible losses. Managers often consider this operating flexibility as valuable as direct cash flow (Donaldson and Lorsch, 1983).

The term *Real Options* was introduced by Stewart C. Myers in 1977 (Myers, 1977), who suggested that valuation techniques for financial options can be applied to evaluate corporate projects. A call option[12] is a financial contract that offers the buyer the right, but not the obligation, to pursuit a stock by a pre-determined price (Exercise price). The time that the buyer can exercise the option depends on the type of option. American options can be exercised anytime until the option expires, whereas European options can only be exercised at the maturity time. It is a financial derivative used for hedging the risk that the underlying stock price falls. The strategy pattern observed in most of O&G projects, including the study case in this thesis, is similar to the American option pattern. When an operating company is awarded with an exploration license, it has the right, but not the obligation, to perform exploration activities in a specific area, until the license expires. The operating company has the flexibility to start the exploration campaign any year along the license. This flexibility is comparable to the flexibility of the financial option's buyer. The decision to exercise the financial option depends on the uncertain stock value, while the decision to start the exploration campaigns depends on the uncertain cash flow of the project.

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[12] Financial options are divided in call and put options. Call option gives the buyer the right to buy the stock, whereas the Put option gives to the holder the right to sell it.

The major breakthrough in the valuation of financial options was done by Black, Scholes and Merton in 1973 with the introduction of their Nobel-prized formula for valuation of European options. Early ROV methods were built based on that. However, two main issues arise in their implementation to the corporate field. First, the European option can only be exercised at the end of the maturity time, which is not the case for projects where strategic decisions are made anytime during the project life-time; and secondly, the formula assumes that the underlying stock price follows a GBM process. This is not applicable for O&G projects since their value depend on the oil price which is not described by a Brownian Motion. In this chapter, it is described how these issues were overcome, leading to the development of the ROV methods available nowadays. Moreover, the different numerical solutions will be shortly explained, to finally focus on the Least-Squares Monte Carlo method, which will be applied in this dissertation.

4.1. ROV Methods

Classic Approach

Early ROV models were developed based on valuation of the European Options using the Black-Scholes formula. This formula assumes that if two financial assets embed the same risk, and have the same cash-flow pattern, they should be traded with the same price in the market. Thus, the value of the financial option can be indirectly estimated if a portfolio of known traded assets with similar behavior is found. This assumption is appropriate for financial options, but it is improper for corporate assets. For valuing the real option, a replicate portfolio of financial assets that reflect the cash-flow behavior of the corporate project should be built. For most of the O&G cases, the return of the real option cannot be mimicked by a portfolio of traded assets. In addition, as mentioned before, the Black-Scholes formula assumes that the underlying asset price follows a GBM process, which does not apply for projects in the petroleum industry. Further attempts to overcome the issue of reliance on market information lead to the development of a "subjective approach". This relies on the same assumptions of the classic approach, but it replaced the market information for subjective information (experts' opinion). Nevertheless, this method was not well accepted since it combines assumptions of the replicating portfolio method, with subjective input that is not based on such portfolio (Borison, 2005, Jafarizadeh and Bratvold, 2009).

Marketed Asset Disclaimer Approach

Copeland and Antikarov (2001) proposed the Marketed Asset Disclaimer (MAD) method to overcome the challenges of the replicating portfolio approach. They stated that if the real option is traded in the market, the present value of future cash flows without considering flexibility is the best unbiased estimation of its market value. Subjective information is used initially to calculate the NPV, and a single probability distribution is calculated based on expert's opinion (Copeland and Antikarov, 2001). The resulted distribution is then implemented in a binomial lattice. Implementation of binomial lattices is expanded in the next section of this chapter.

The only market information used in this approach is the market stablished discount rate to calculate the NPV without flexibility. The rest of the information is based on experts' opinion. Jafarizadeh and Bratvold (2009) summarized the disadvantages and advantages of this method. The main drawbacks of this method are:

1. The extensive use of subjective information.
2. It ignores the fact some uncertainties depend on capital market information. Therefore, this method is suitable for projects involving uncertainties unrelated with the market.
3. The assumption that the project value follows a GBM process. As Jafarizadeh and Bratvold (2009) stated, there are not arguments to believe that the subjective evaluation in the MAD approach can lead to results that follow a Brownian motion.
4. Aggregating all market uncertainties into a single volatility makes it hard to do sensitivity analysis on individual uncertainties

They stated this method has two main advantages: as argued by Smith (2005), it is eminently applicable for some particular ROV situations (scaling options). Second, it closely mimics the well-known classical DCF approach, making it relatively easily accessible to practitioners familiar with the DCF approach

The integrated Approach

Projects in the O&G industry combine different type of uncertainties as illustrated previously in the Chapter 2. They can be categorized as: technical uncertainties (i.e. production levels);

uncertainties that can be hedge in the market (as the oil price); or private uncertainties that depend on the market (i.e. drilling cost). Such characterization of the uncertainties is the base for the Integrated Approach. It was elaborated by Smith and Nau (1995), Smith and McCardle (1999), Smith (2005), and Brandao et al. (2005b). It combines the decision analysis paradigms with option pricing methods.

Market uncertainties are assessed using probabilities derived from traded instruments. Thus, since these probabilities are calibrated using market instruments, the resulting parameters include the market's "view" of risk. These are called "risk-neutral" probabilities, and adding risk adjustment to the resulting probability distribution would be to "double-dip" in risk (Thomas, 2017). Private uncertainties are evaluated using experts' opinion, and they will also embed the associated risk. Uncertainties that fall between private and market uncertainties, are evaluated using subjective probabilities conditional on market conditions (Smith, 2005). As the probability distributions of the modeled uncertainties includes the relevant risks, the cash flow calculated based on them should be discounted using a rate that accounts for the time-value of money only, usually referred to as the "risk-free rate".

4.2. Numerical solutions for ROV

Prior to implementing a ROV method, two steps should be performed (Thomas, 2017):

1. Determining flexibilities in the project, meaning decisions, or options, that can be exercised when uncertainties are revealed

2. Identification and quantification of uncertainties in the model[13].

As discussed earlier, the Black-Scholes formula is an inadequate tool for modeling and valuing real world options. These limitations have led to research and improved implementation of numerical procedures for calculating the value of the options. In the following, the main approaches are discussed.

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[13] For the current research study, flexibilities were illustrated in Chapter 2, and the uncertainties were identified and quantified in Chapter 3.

Finite Difference Methods

The finite difference approach can be applied if the time development of the option value is described by a set of partial differential equations (Schulmerich, 2005). Numerically, by the use of finite difference methods, solving these equations provides the option value. In the finite difference methods, the differential equation is discretized in a grid. Initial and boundary conditions are determined. Trigeorgis (1996) defined the Finite Difference Method as more mechanical, requiring less intuition than lattice approaches, with the disadvantage that if the partial differential equations describing the value of the real option cannot be specified, this method becomes incompetent.

Decision Tree Approach

Decision trees are tools used to structure decision-making contexts. They can be employed to solve dynamic programming problems in the ROV methods. Hence, they are implemented to price sequential investment decision in which management decisions and the uncertainties are resolved at discrete points of time (Schulmerich, 2005). They provide advantages over the Finite Difference methods. First, they illustrate the uncertainties and decision nodes in the ROV problem, providing clarity and communication. Second, as Brandao et al. (2005a) argued, the decision tree approach is easier to inspect than more complex models, allowing a faster identification of issues in the model. Third, different authors have recognized that decision trees are more intuitive than the finite difference approach (Trigeorgis, 1996; Schulmerich, 2005; Brandao et al., 2005a; Bratvold and Begg, 2010).

Copeland and Antikarov (2001), and Brandao et al. (2005a) converged in the following steps to build a decision tree for a ROV problem. Start by calculating the NPV of the project without flexibilities. Second, evaluate the uncertainties in the model based experts' opinion, and combine them to calculate the variability of the NPV. Brandao et al. (2005b) proposed a method to incorporate the uncertainties into a single stochastic process for the expected value of the project. Finally, the distribution of the NPV is used to build a risk-neutral binomial tree (Brandao-Hahn-

Dyer approach), or a binomial lattice (MAD approach)[14]. The option price is calculated at each node of the tree, and the overall real option value is estimated using a roll-back procedure[15]. The previous steps account for market uncertainties. If private uncertainties are included, a chance node per every private uncertainty should be included. This constitutes one of the disadvantage of the decision tree approach. If the model includes several private uncertainties, the approach suffers dimensionality issues. The second drawback is the challenge to incorporate different market uncertainties in one single stochastic process, especially when those market uncertainties are described by high dimensional stochastic process (Smith, 2005; Brandao et al., 2005b).

Monte Carlo Simulation Approach

Monte Carlo (MC) simulation is widely used in the O&G industry for solving problems where the input values are uncertainties with probability distributions. It is based on performing the calculations N times, sampling different values from the probability functions used to represent the uncertain variables. In the ROV context, N trajectories for each uncertainty in the cash flow are simulated, and then, they are used to estimate a probability distribution for the value of the real option. The MC approach is appropriate for valuing options that are path-dependent or involve many underlying uncertainties (Willigers and Bratvold, 2009).

Boyle (1977) introduced the implementation of MC approach for valuing European options. However, this approach cannot be implemented for corporate options in which the optimal strategy is unknown. Therefore, more recent researches have been focused in the valuation of options that can be exercised anytime during the maturity time, where the MC approach allows for asset optimization (i.e. determining the optimal decision policy[16]) to be separated from the price-evolution model (Willigers et al., 2011). The optimal solution is calculated comparing the expected future value of the different alternatives at each decision point, conditioned in the revealed uncertainties up to that time. This recursive optimization approach, usually referred as dynamic

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[14] Decision trees (DT) differ from Binomial Lattices (BL) in that the decision nodes are illustrated in the DT whereas those are implicit in the BL. Furthermore, the BL assumes a Markov process which means that it does not matter how you reach a given node in the lattice, the resulting value will be the same (Bratvold and Begg, 2010).
[15] The rolling back is the common procedure to solve dynamic programing using a decision tress structure. It starts with the rightest part of the tree. Then it moves backwards, calculating the expected value and the optimal decision. (Bratvold and Begg, 2010)
[16] The term "Optimal policy" is used to define the decision that yields the highest value of the project.

programming (Dixit and Pindyck, 1994), uses the same principle than the "rolling back" procedure implemented in the decision tree approach. However, as Cortazar (2000) argued, the advantage of the MC method is the ability to handle the uncertainties, even when they involved complex stochastic models.

4.2.1. Least Squares Monte Carlo

Early MC approaches were developed for real options that are exercised in a pre-determined date (as the European Options). Later work was focused on options that are exercised anytime during the life-time of the project, which better reflects corporate reality. Introduced by Longstaff and Schwartz (2001), the Least-Squares Monte Carlo (LSM) has been the most used approach in the O&G industry during the last years (Smith, 2005; Willigers and Bratvold, 2009; Hem, at al. 2011; Willigers, et al, 2011; Alkhatib and King, 2011; Jafarizadeh and Bratvold 2012, 2013; Thomas and Bratvold, 2015). It starts by building a Monte Carlo simulation model that includes all relevant uncertainties in the model. Running the Monte Carlo model generates a range of outcomes for the possible values of the project without options. Then, the optimal policy is determined in every decision node by comparing the expected NPV if the option is exercised at the time t, with the expected value if the decision is to wait. The expected value of waiting, usually referred as the continuation value, is calculated by using a least-squares regression, where the dependent variable is the NPV if the option is exercised in the time $t+1$, and the independent variables are the underlying uncertainties revealed at time t. The LSM algorithm is illustrated using the research case in the chapter 6, LSM implementation.

Thomas (2017) summarized the five main advantages of this method:

1. It has been proved its accuracy in valuing options (Clement et al., 2002; Moreno and Navas, 2003; Stentoft 2004a, 2004b).
2. The decision structure can be represented using decision trees to improve communication and clarity.
3. High-dimensional stochastic process for modeling the underlying uncertainties can be easily combine into the model by simulating them individually (Brandao et al., 2005b; Smith, 2005).

4. Compared to the decision tree approach, it does not suffer from the curse of dimensionality, which means that it can incorporate several uncertainties and decision nodes (Willigers and Bratvold, 2009).

5. It provides different tools for asset optimization such as decision maps, or optimal-time histograms[17], that creates an additional insight of the decision-making process to managers.

Nevertheless, the method has some disadvantages compared with the decision tree approach. Brandao et al. (2005b) described the drawbacks of the LSM approach. First, it is more prone to include programming errors, but these can be minimized by using decision trees to state the problem, thus avoiding modeling errors; and secondly, the mathematical framework is more complicated, and it may seem as a "black box" to the managers. The advantages of this method were the reason to choose it for this work. Moreover, it will provide an additional value for estimating the optimal time to start the exploratory drilling campaign, by using the optimal-year histograms.

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[17] Charts where the frequency of the optimal time to exercise the option is displayed.

5. Study case

An offshore exploration license located in an unexplored area is awarded to an oil company. The expiration date is five years out. The current market conditions are unfavorable. Due to the relatively low oil price, oil companies are implementing cost-cutting policies and expensive investments are being postponed. The company must decide whether to start the exploration campaign now, or wait to see if the oil price improves. This was illustrated previously in Figure 2. The estimated lead time is 10 years. Based on seismic data, geologists have estimated the probability of success of 20% for the wildcat well. The company used available information to estimate the prospect properties shown in the Table 3. Besides, it decided to use a risk-free rate of 5%. The cash flow model was described in the Figure 3, and its calculation is detailed in the next chapter.

Table 3 Properties of the study case

Parameter	Value
Reserves	100 MSTB
Variable OPEX	15 USD/bbl
Fixed OPEX	10 MUSD/year
Production life	30 years
Development cost	500 MUSD

The managers have observed that rig providers and drilling services companies are willing to reduce their cost, to maintain a minimum level of operations during the current market conditions. Motivated by this significant reduction in the investment, the company wants to evaluate the optimal time to start the exploratory drilling campaign, considering that if the oil price increases, the investment would also increase, and the overall value of the project might be affected.

Even if the prospect has an attractive value, not all operators would be able to make the investment. The financial position of the company influences the investment policies in its portfolio. Large investments that involved high uncertainty as the exploration campaigns are classified as "high-risk" projects, and may not be considered in the portfolio of small companies, or companies with

a limited cash-reserve. In that case, the exploration campaign may make a larger dent in its overall viability and value, leading to a risk-averse attitude of the company.

In this work, we assume that the operator has access to good prospects and has a high overall value and large cash reserves (compared to the investment on the exploration campaign). This is the group of companies that can create value during the low-oil price, and they are usually involved in large merges and acquisitions, seeking opportunities to take advantage of the crisis to increase their assets. Therefore, an offshore drilling campaign represents a small percentage of their portfolio, and in the valuation context, their decision-makers can be assumed to be risk-neutral (Bratvold and Begg, 2010).

6. LSM implementation

Every year, the company face the decision of whether to start the exploratory drilling campaign, or wait until next years to execute it. This decision should be made based on the expected value of the two alternatives. Therefore, calculating the total expected value of the project implies that the optimal decision policy over time has been evaluated using a dynamic programming solution. The step-by-step implementation of the LSM algorithm is described in this chapter. In this work, the algorithm has been implemented using MATLAB ®. The developed codes are illustrated in the Appendix 2.

1. Simulate the uncertainties

N paths for the uncertainties involved in the problem are simulated from time zero until the end of the exploration license (5 years). This is done by using the selected stochastic processes for the two uncertainties considered in the model: the oil price and the drilling cost. In this research, the Monte Carlo simulation is run with at least 100,000 samples (N≥100,000) to minimize the sampling error. The oil price uncertainty paths are simulated as a STLT process, using equations (1), (5) and (6), and can be represented for the five-year exploration license by:

$$
S = \begin{bmatrix}
S_{t=1}^{i=1} & S_{t=2}^{i=1} & S_{t=3}^{i=1} & S_{t=4}^{i=1} & S_{t=5}^{i=1} \\
S_{t=1}^{i=2} & S_{t=2}^{i=2} & S_{t=3}^{i=2} & S_{t=4}^{i=2} & S_{t=5}^{i=2} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
S_{t=1}^{i=N} & S_{t=2}^{i=N} & S_{t=3}^{i=N} & S_{t=4}^{i=N} & S_{t=5}^{i=N}
\end{bmatrix}
$$

where i represents each independent path of the MCS, and t the time in years. The second uncertain variable is the exploration cost, and includes the expenses from the exploratory drilling campaign, which are assumed to be completely driven by the drilling cost. This is simulated as a GBM process, using equations (10) and (11), and it can be represented by:

$$
D = \begin{bmatrix}
D_{t=1}^{i=1} & D_{t=2}^{i=1} & D_{t=3}^{i=1} & D_{t=4}^{i=1} & D_{t=5}^{i=1} \\
D_{t=1}^{i=2} & D_{t=2}^{i=2} & D_{t=3}^{i=2} & D_{t=4}^{i=2} & D_{t=5}^{i=2} \\
\vdots & \vdots & \vdots & \vdots & \vdots \\
D_{t=1}^{i=N} & D_{t=2}^{i=N} & D_{t=3}^{i=N} & D_{t=4}^{i=N} & D_{t=5}^{i=N}
\end{bmatrix}
$$

2. Estimate the cash flow and NPV

With the simulated paths, the NPV is estimated for every single element of the trajectories, thus, a matrix of discounted cash flows is built as:

$$NPV = \begin{bmatrix} NPV_{t=1}^{i=1} & NPV_{t=2}^{i=1} & NPV_{t=3}^{i=1} & NPV_{t=4}^{i=1} & NPV_{t=5}^{i=1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ NPV_{t=1}^{i=N} & NPV_{t=2}^{i=N} & NPV_{t=3}^{i=N} & NPV_{t=4}^{i=N} & NPV_{t=5}^{i=N} \end{bmatrix}$$

where the estimation of the element NPV_t^i uses the single values S_t^i and D_t^i.

The cash flow is estimated as:

$$CashFlow_t = ProductionRate_t * (OilPrice_t - VariabOpex_t) - FixedOpex_t - Capex_t \quad (12)$$

A production profile was estimated considering a peak profile, where the peak was assumed to occur after 3 years of production, and it is equivalent to the 15% of the total reserves. A logarithmic function was used for the incremental curve before the peak, whereas the decline curve after the peak was defined with an annual production rate set as the 10% of the remaining reserves.[18] The oil price curve for the lifetime of the project is calculated using equations (7) and (8)[19].The cost of the exploratory drilling campaign (D_t^i) is deducted in the year 1, whereas the development cost, that includes production drilling, facility construction and infrastructure development, is deducted in the year before productions starts (t=lead time-1). For simplicity, royalties and taxes are neglected in this research. All the cash flows are discounted using the risk-free rate to obtain NPV_t^i.

3. Calculate the expected value

In this study, the LSM algorithm presented by Jafarizadeh and Bratvold (2015) for exploration projects is implemented, where the optimal decisions are based on the expected value of drilling, considering the probability of success of the prospect. The expected value of drilling (E_d) is:

$$E_d = P_s * NPV_s^i + (1 - P_s) * NPV_f^i \quad (13)$$

---

[18] For more details about the production profile formulation please see the MATLAB® code "Pdecline" in the appendix 2.

[19] The initial values (χ_0, ξ_0) are the short term (χ) and long term (ξ) calculated in the previous step using equations (5) and (6). These are the only values changing in the forward curve (Equation (7) and (8)), and for this reason, every NPV_t^i estimation embeds a different oil price curve.

where P_s denotes the probability of success, NPV_s^i is the net present value discovering commercial reserves, and NPV_f^i is the net present value for dry hole. Equation (13) represents value from the upper branch of Figure 2. Therefore, a matrix of expected values that will be used in the following steps is developed, as illustrated:

$$Ed = \begin{bmatrix} Ed_{t=1}^{i=1} & Ed_{t=2}^{i=1} & Ed_{t=3}^{i=1} & Ed_{t=4}^{i=1} & Ed_{t=5}^{i=1} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ Ed_{t=1}^{i=N} & Ed_{t=2}^{i=N} & Ed_{t=3}^{i=N} & Ed_{t=4}^{i=N} & Ed_{t=5}^{i=N} \end{bmatrix}$$

4. Backward Induction

a) Decision at expiration time (t=T)

The backward induction is the dynamic programming to determine the optimal policy, and it is similar to the roll-back procedure used to evaluate decision trees (find the optimal decision). It starts at the rightmost part of the expected value matrix Ed, at the expiration date ($t = T = 5$). At that time, the company must decide whether to drill or let the exploration license expire. This decision is based on the expected value of drilling. If the expected value $Ed_{t=5}^i$ is zero or negative, the company should relinquish or let the license expire. On the other hand, if the expected value of drilling is positive the company should drill. The purpose of the backward induction is to determine the optimal decision in every path. Every path has its optimal policy and the optimal payoff associated with it. For instance, if the optimal decision in the path i is to start to drill at the year 3 of the exploration license, the optimal payoff would be expected value of drilling at this year (Ed_3^i). To estimate the optimal payoff of every path, the algorithm uses a vector of optimal payoffs (F), which is set initially with the values at the maturity time, and then updated during the following steps while implementing the backward induction. At the end, it will contain the optimal payoff of every MC path, that will depend on the optimal year to start to drill in that path. The initial values for the vector of optimal payoffs (F) is set with the expected values at the maturity time, as shown in the following statement:

$$F^i = \begin{cases} Ed_5^i, & Ed_5^i > 0 \\ 0, & Ed_5^i \leq 0 \end{cases}$$

b) Optimal decision for t<T

Then, the algorithm moves to year 4 ($t = T - 1$), where the optimal policy is determined for every MC trajectory by comparing the expected value of drilling at that year ($Ed^i_{t=4}$), with the expected value of continuation[20]. The challenge of calculating the expected continuation value is addressed by Longstaff and Schwartz (2001) using least-squares regression, where the continuation value is a function of the oil price and the exploration cost at time t:

$$C^i_t = \alpha_1 S^i_t + \alpha_2 D^i_t + \alpha_3(S^i_t)^2 + \alpha_4(D^i_t)^2 + \alpha_5(S^i_t)(D^i_t) \tag{13}$$

where C^i_t is the continuation value, S^i_t is the oil price, and D^i_t represents the exploration cost, for the i-th path in the year t. α_k (with k=1, …,5) are the regression coefficients, and they are calculated using a regression where the dependent variable is the expected value of drilling at time $t = t + 1$, and the independent variables are the oil price and exploration cost of the year t. For instance, for $t = 4$, the dependent variable is the expected value of drilling in year 5 ($Ed^i_{t=5}$)[21], and the independent variables are the exploration cost and the oil price of year 4 ($D^i_{t=4}, S^i_{t=4}$). The equation 13 is defined following the recommendation made by Jafarizadeh and Bratvold (2015). As mentioned by Longstaff and Schwartz (2001), this regression equation can contain fewer or more terms. When implemented in MATLAB, a matrix for the independent variables at time t is generated:

$$Indep.Variables = X = \begin{bmatrix} S^1_t & D^1_t & (S^1_t)^2 & (D^1_t)^2 & (S^1_t)(D^1_t) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ S^N_t & D^N_t & (S^N_t)^2 & (D^N_t)^2 & (S^N_t)(D^N_t) \end{bmatrix}$$

The values of the expected values of drilling in the year t+1 are assigned to a vector:

---

[20] Longstaff and Schwartz (2001) defined the expected value of continuation as the best unbiased estimation of the value of waiting and possibly exercise the option in the future (i.e. wait and possible start to drill in the next years), given the uncertainty observed today.

[21] One of the differences of the LSM algorithm with respect to other similar approaches, is that the regression is performed using only positive values of $Ed^i_{t=t+1}$ to improve the efficiency of the model.

$$Dependent\ Variable = Y = \begin{bmatrix} Ed_{t+1}^1 \\ \vdots \\ Ed_{t+1}^N \end{bmatrix}$$

Then, the multiple regression can be performed using a MATLAB® function. In this thesis, the MATLAB solver "\" is used to find the coefficients[22]. It is implemented setting $\alpha = X\backslash Y$, where X and Y are matrixes defined above, and α is the vector of regression coefficients:

$$\alpha = \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_5 \end{bmatrix}$$

After finding the regression coefficients, an equation to calculate the continuation value is built with the form of the equation (13). The continuation value is then calculated for every path in the year t:

$$C_t = \begin{bmatrix} C_t^1 \\ \vdots \\ C_t^N \end{bmatrix} = \begin{bmatrix} S_t^1 & D_t^1 & (S_t^1)^2 & (D_t^1)^2 & (S_t^1)(D_t^1) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ S_t^N & D_t^N & (S_t^N)^2 & (D_t^N)^2 & (S_t^N)(D_t^N) \end{bmatrix} x \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_5 \end{bmatrix}$$

Then, the optimal decision at the time t is estimated for every path. If the discounted continuation value is higher than the expected value to drill, the optimal decision is to wait, otherwise the decision should be to drill in this year. For the year 4 ($t = T - 1$), the optimal payoff vector is updated as:

$$F^i = \begin{cases} Ed_4^i, & Ed_4^i > C_4^i e^{-r} \\ F^i, & Ed_4^i \leq C_4^i e^{-r} \end{cases}$$

This means that, if the expected value of the year 4 (Ed_4^i) is higher than the discounted continuation value ($C_4^i e^{-r}$), the optimal decision is to drill, and the optimal payoff is the expected value of drilling in this year. If the expected value of drilling is less than the discounted continuation value, the optimal decision is to wait and drill in the year five, therefore the optimal payoff in this path

---

[22] The backslash \ solver gives m unknowns for n system of equations when $n=m$. If $n>m$ this function uses the linear least squares regression to estimate m.

would be the expect value of drilling in the year five Ed_5^i, which was storage in this vector in the previous step. This step is repeated moving backwards until it reaches $t = 1$.

5. Calculate the expected value of the project.

The values in the optimal payoff vector (F) are discounted to the year one considering the optimal exercise date of each path (F_{dis}). For instance, if the optimal decision in one of the paths was to start the exploration campaign in the year three, then the value saved in the optimal payoff vector (F) is the expected value of drilling in this year (Ed_3^i). By discounting this value to the year one using the risk-free rate (r), the expected present value is found. The expected value of the project is calculated as the average of the discounted optimal payoff vector:

$$E(Project\ value) = \frac{\sum_{i=1}^{N} F_{dis}^i}{N} \tag{14}$$

The vector values F_{dis} are also used to build the probability distribution curve of the overall expected value of the project. Furthermore, an optimal-time histogram is developed using the optimal time to start the exploratory campaign of each path, as illustrated in the next section.

7. Results and discussion

The impact of the reduction of the exploration cost in the decision-making process of executing drilling campaigns during low oil price periods is evaluated by including a delayed correlation between the exploration cost and the oil price in the LSM approach. This is presented and discussed in four sub-sections. First, by using optimal-time histograms, the impact of the correlation on the optimal year to start to drill is illustrated. Then, it is shown how this correlation affects the probability distribution of the expected value of the project, assessing the sensitivity of the project value with respect to the correlation factor. Third, a sensitivity analysis of the parameters for the stochastic process of the exploration cost is presented, evaluating it with respect to the project value and the correlation factor. Finally, decision maps are introduced for offshore exploratory drilling campaigns.

7.1. Optimal decision

Most often, cash flow models of upstream petroleum projects assume constant or expected drilling costs along with uncertain oil prices. Therefore, the first LSM implementation is performed considering the exploration cost as constant, taking the oil price as the only uncertainty in the model. The optimal-time histogram from this implementation is shown in the Figure 9. The expected oil price grows as shown previously in Figure 6. Thus, during the exploration license, the year one has the lowest expected oil price, whereas the year 5 has the highest. Considering the oil price as the only uncertainty means that it is assumed that the exploration cost will not increase when the oil price rises, which does not reflect the typical market behavior.
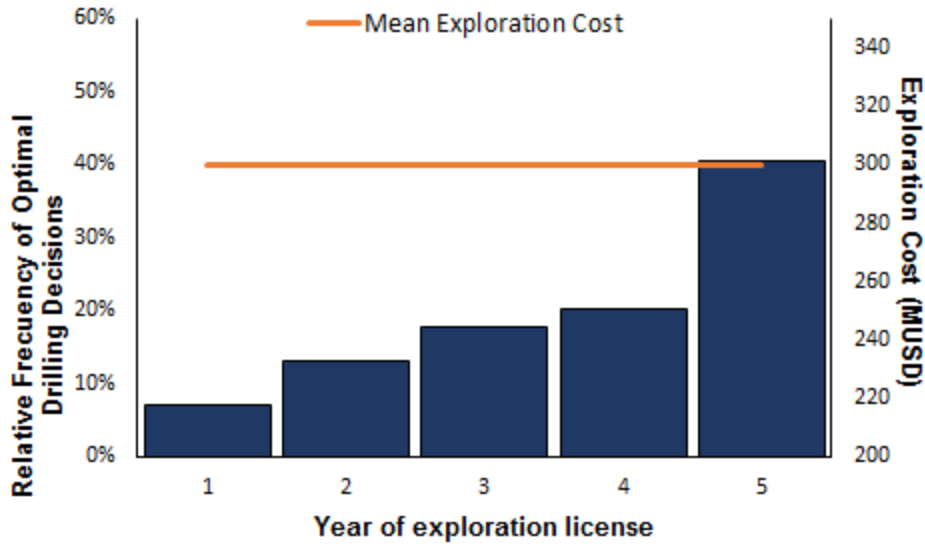
Figure 9 Optimal-time histogram for constant exploration cost.

The results show that, for many simulated paths, the most frequent optimal policy is to start the exploratory drilling campaign at the time of the highest expected oil price (year five). This illustrates the most common decision policy observed during low oil price periods: wait until the oil price increases to initiate the investment. These outcomes are a consequence of the positive slope in the long-term oil price. As the oil price is the only uncertainty, the production income (oil production rate times oil price) drives the cash flow and the value. Hence, the estimated NPVs in year five are higher because their production life will be in a time framework of higher expected oil prices, compared to the rest of the years of exploration license ($t < 5$). In other words, NPV will be highest in the final year of the license because oil prices are expected to continue to rise and costs are expected to be at their constant level.

Now, it is assumed that the managers have decided to account for the fact that the cost of the exploratory campaign changes with time and, thus, include it as an uncertainty in the model. A GBM process is implemented as described in chapter 3 of this work. First, it is included without a correlation with the oil price, with the objective of discriminating between the impact of adding the cost uncertainty and the impact of the cost correlation with prices. Results are illustrated in Figure 10, where the line represents the mean exploration cost uncertainty.
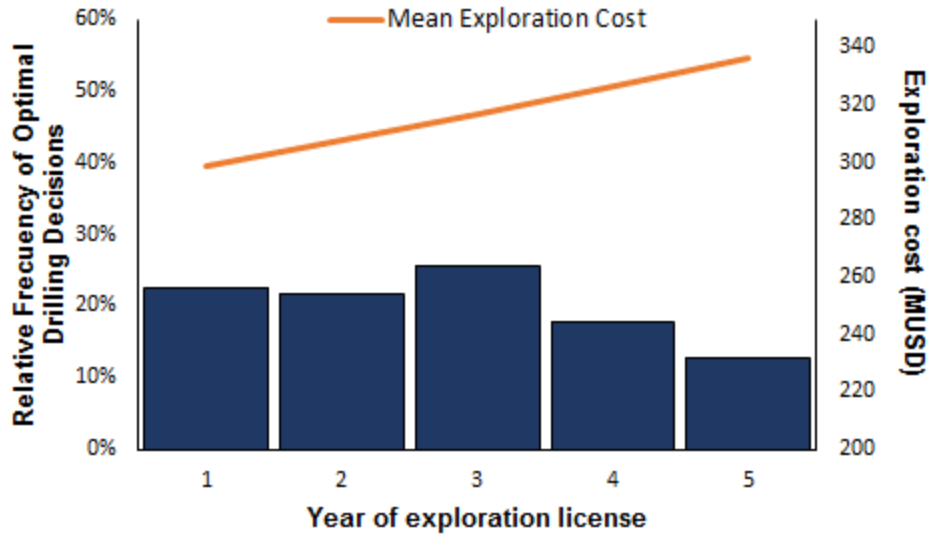
Figure 10 Optimal-time histogram for uncertain exploration cost with no correlation

Adding the non-correlated uncertainty impacts the optimal time to start the exploration campaign. Decision-makers may infer from Figure 10 that the value of the option is relatively independent of the time it is executed. Therefore, the optimal time to start the exploratory drilling campaign could be any year during the exploration license term. Even if the expected exploration cost grows with the oil price, this model does not reflect market reality, as it will be explained later.

The idea of executing an exploration campaign during low oil price periods is investigated by including a correlation between the exploration cost and the oil price. This implies that for every MC trajectory, if the simulated oil price is low; there is a high probability that the exploration cost will be low as well. This mimics the observed market behavior. The correlation coefficient is set equal to the value found by Willigers (2009) ($\rho_{\theta\chi} = 0.89$) with a 1-year delay; i.e., such that the correlation between the short-term price in year t and the cost in year $t + 1$ is 0.89. The results are shown in Figure 11.
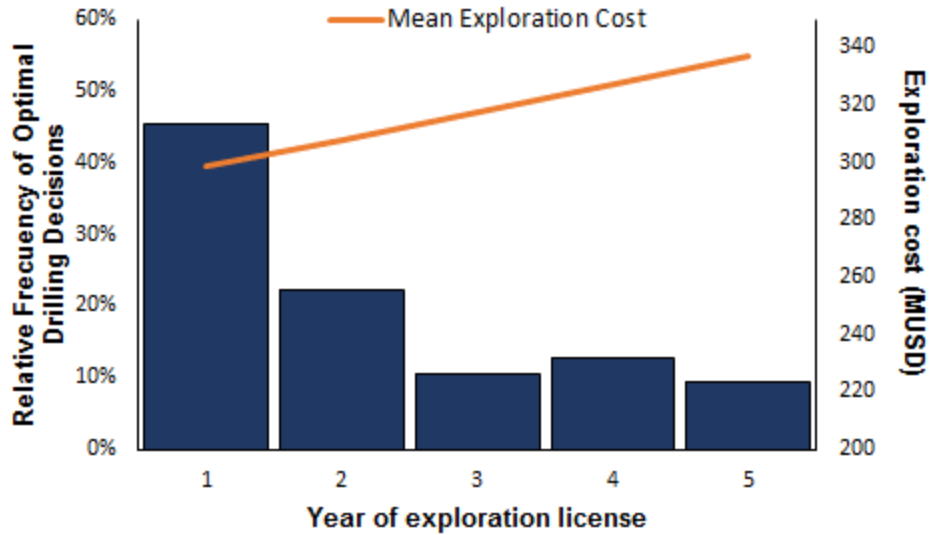
Figure 11 Optimal-time histogram for exploration cost with correlation factor equal 0.89

Unlike the common beliefs, as evidenced by actual decision making in the industry, the results indicate that the optimal time to start the exploratory drilling campaign is the year with the lowest expected oil price (year one). This is because the decrease of the investment in the exploration campaign increases the NPV's at that year, making them higher than the expected values of waiting. Another reason is that the cash flow is sensitive enough to the exploration cost, to be affected by a variance in their values. If the weight of the exploration expense component in the cash flow model is not significant, the optimal decision may not be affected. This could be the case for other type of projects, for instance onshore fields, where the cost of drilling the exploration wells is substantial lower than offshore projects.

Then, the full price-cost dependency is modeled by increasing the correlation coefficient to one ($\rho_{\theta\chi} = 1$). The results are shown in Figure 12. The optimal time to start drilling is the year one. However, the frequencies of optimal initiation in the final years decreased and now the decision to start to explore within the first two years is the optimal for 76% of the samples. When there was no dependence ($\rho_{\theta\chi} = 0$), shown in Figure 10, the frequencies in the last years were slightly lower than the early years. As the dependency was increased, these frequencies decrease more, whilst frequencies in the early years increased (Figure 11 and 12). Hence, as the dependency increases, the decision to drill when the expected oil price is low becomes more dominant. These results

indicate that executing the exploration campaign during the first year yields the highest expected project value.
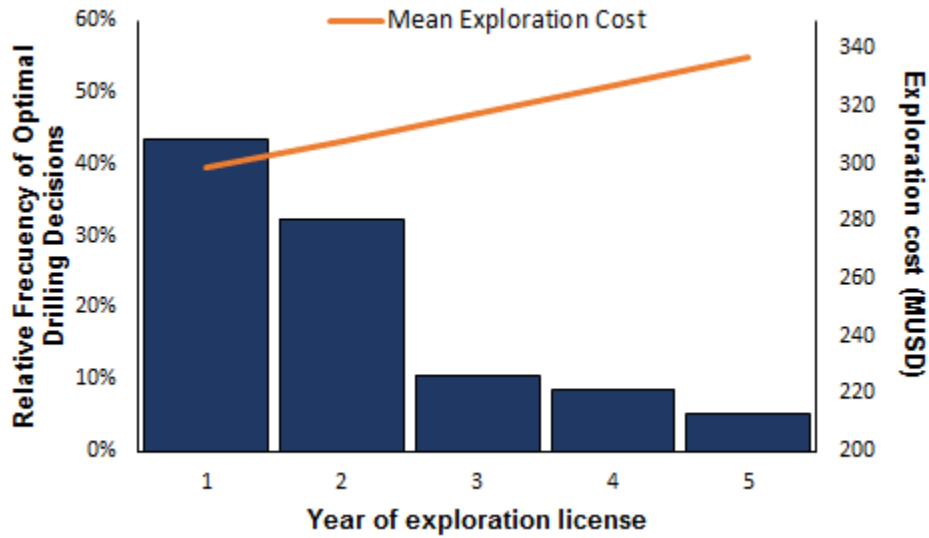

Figure 12 Optimal-time histogram for exploration cost with correlation factor equal 1

From the results displayed in Figures 10, 11 and 12, the impact of including the correlation can be inferred. In Figure 10, the exploration cost is included in the model using a stochastic process with a positive drift. It is observed that even if the positive slope reflects the increment in the drilling cost produced by the expected oil price, without the correlation, the model does not capture the observed cost-price dependency. Ignoring this dependence means that, in the simulated paths, low oil prices can be sampled with high exploration costs which does not reflect reality. The inclusion of the correlation leads to a model that is more realistic and the optimal-time histograms resulting from this model provide more useful and relevant insight to the decision-makers.

7.2. Expected project value

The expected value of the project is impacted by the correlation between the drilling cost and the oil price. Figure 13 compares the probability distribution for two of the cases: when the exploration cost is included as a constant, and when it is included as a correlated uncertainty with a correlation of 0.89. The expected project values are shown in the Table 4.

Table 4 Expected project values

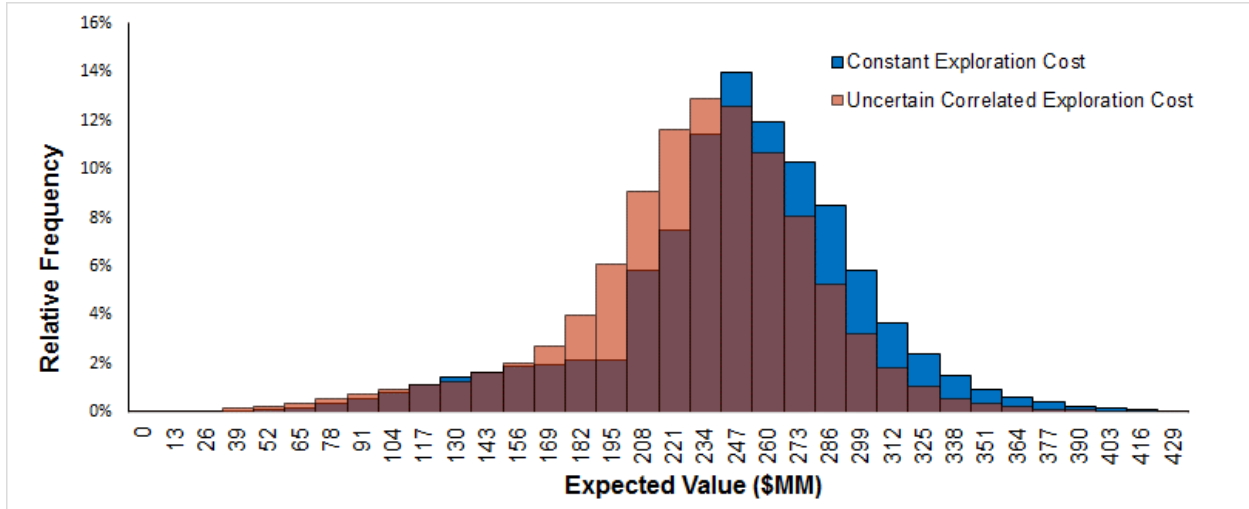|	Mean Expected Project Value (MUSD)
Constant Exploration Cost	253
Uncertain Correlated Exploration Cost	236

Figure 13 Probability distribution for the expected project value

Neglecting the cost-price dependency in the cash flow model leads to, on average, an overvaluation of the expected project value by 17 MUSD. Unbiased and consistent estimates of project values are key for making portfolio decisions that maximize capital efficiency and shareholder value. An over-estimation of expected value can incorrectly portray an exploration prospect as an attractive investment, or it can make the difference between relinquish, sell or pursue the exploration license. The dependency has a direct influence on the expected value of the project. As discussed in the chapter 3, the value of the correlation factor can vary with market locations. Therefore, a sensitivity analysis of the correlation coefficient on the expected project value was conducted with the results shown in Figure 14.
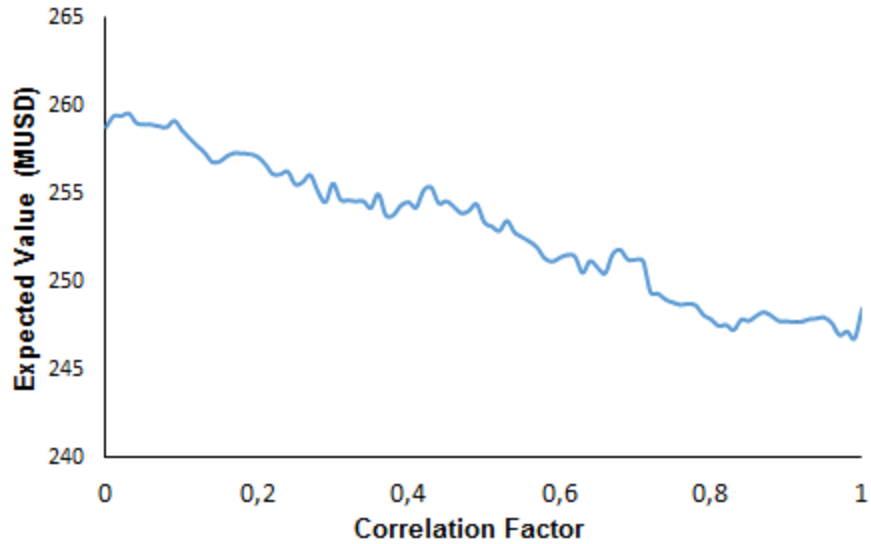
Figure 14 Sensitivity of the expected project value with respect to the correlation factor

As the cost-price dependency increases, the expected project value decreases. Increasing the correlation reduces the overall uncertainty in the project, and therefore, it lowers the value of the real option. In the LSM algorithm, the project value estimation depends on the optimal policy determined. As demonstrated in the previous section, the optimal policy changes as the correlation factor is increased. Therefore, the decrease in the project value is a consequence of the change on the optimal decision as the correlation factor increases.

In the MC simulation, if a low correlation coefficient is used, there will be some paths where the cash flow includes high oil prices along with low exploration cost, as well as the opposite case. These values affect the estimation of the decision strategy, and therefore, the project value. They are considered biased since they do not represent market reality. Increasing the correlation factor declines the number of these estimations in the model, leading to a more realistic project value estimation. Nevertheless, the most accurate expected project value is achieved when the correlation factor applicable for the area to be explored is used in the model.

7.3. Sensitivity of the exploration cost parameters

The exploration cost was included as an uncertainty in the cash flow model by means of a GBM process. As this is a private uncertainty that is correlated with the market, the parameters of its stochastic process are estimated using subject matter experts, and then, a correlation factor is

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chosen to describe its market dependency. However, these parameters depend on the type of field and the market location. The exploration cost does not grow at the same rate, nor does have the same volatility in all parts of world. Furthermore, the supply-demand relationship is not the same for offshore drilling services and for onshore projects. When implementing the methodology of this research on other type of projects, operators must observe and estimate these parameters for the relevant local market. This choice affects the value of the project, and therefore, its impact must be studied.

Sensitivity analyses of the expected project value with respect to the drift and the volatility of the exploration cost were performed. As it has been observed during this investigation that the inclusion of the cost-price dependency influences the project value and decisions, these analyses were performed for three different correlation coefficients ($\rho_{\theta\chi} = 0, 0.5, 1$).

Results from the sensitivity analysis for a zero correlation ($\rho_{\theta\chi} = 0$) are shown in Figure 15. Brandao et al. (2005a) stated that the volatility of a project value depends on the volatilities of the underlying uncertainties. Thus, if the volatility of one of its uncertainties increases, the project value volatility does as well. A higher project value volatility implies a higher value of the real option. This behavior is observed in Figure 15, where the project value increases when the exploration cost volatility grows. The drift has an opposite effect on the project value. Increasing the drift means that the values of the exploration cost will be higher over the time, thus decreasing the expected project value.
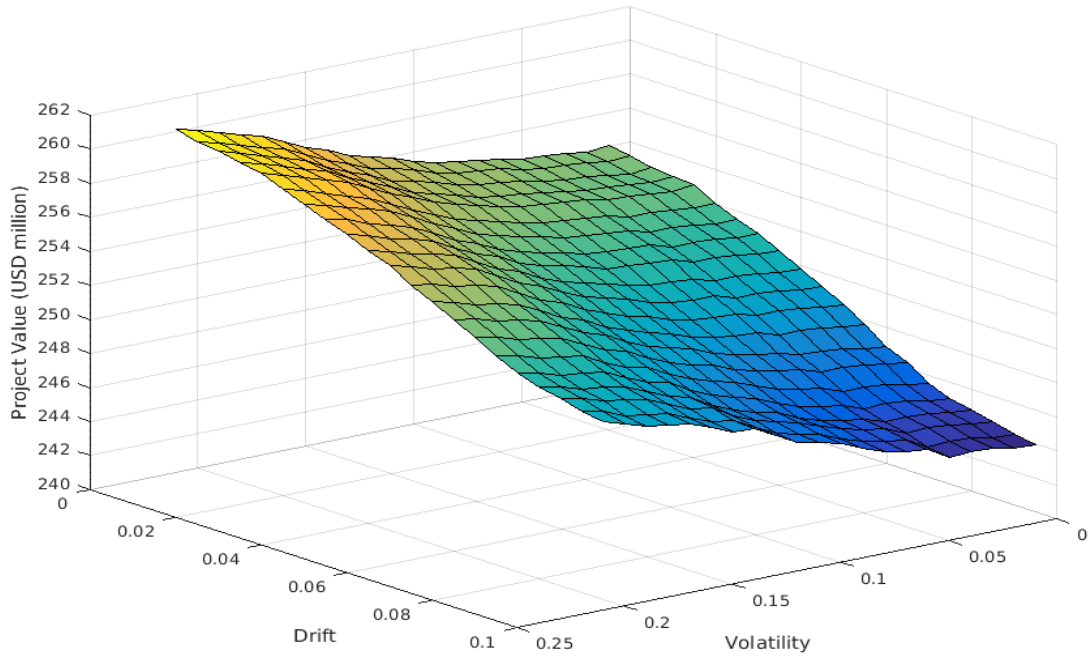
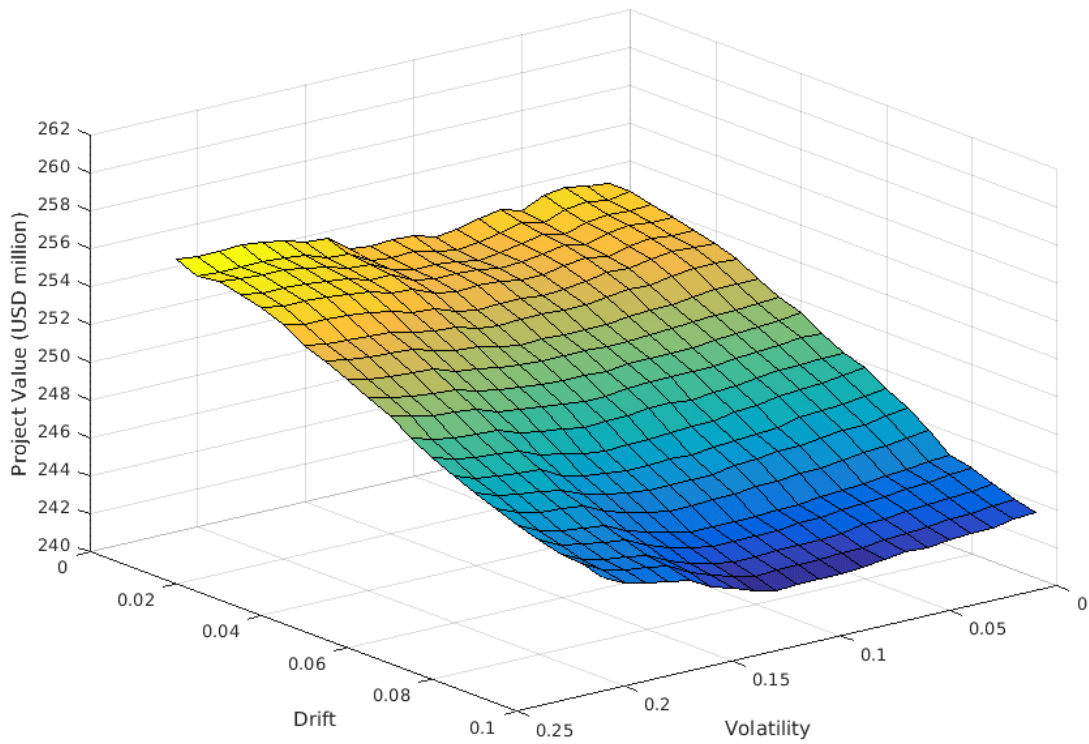**Figure 15 Sensitivity analysis of exploration cost parameters for $\rho_{\theta\chi} = 0$**

Figure 16 Sensitivity analysis of exploration cost parameters for $\rho_{\theta\chi} = 0.5$

The same surface is now generated using a correlation coefficient $\rho_{\theta\chi} = 0.5$ and the results are shown in Figure 16. The drift effect on the project value does not change when increasing the correlation factor. Hence, an increment in the drift has still negative repercussions on the expected project value. However, increasing the correlation factor modifies the behavior with respect to the volatility. As shown, the project value is relatively independent on changes in the exploration cost volatility for this correlation level.
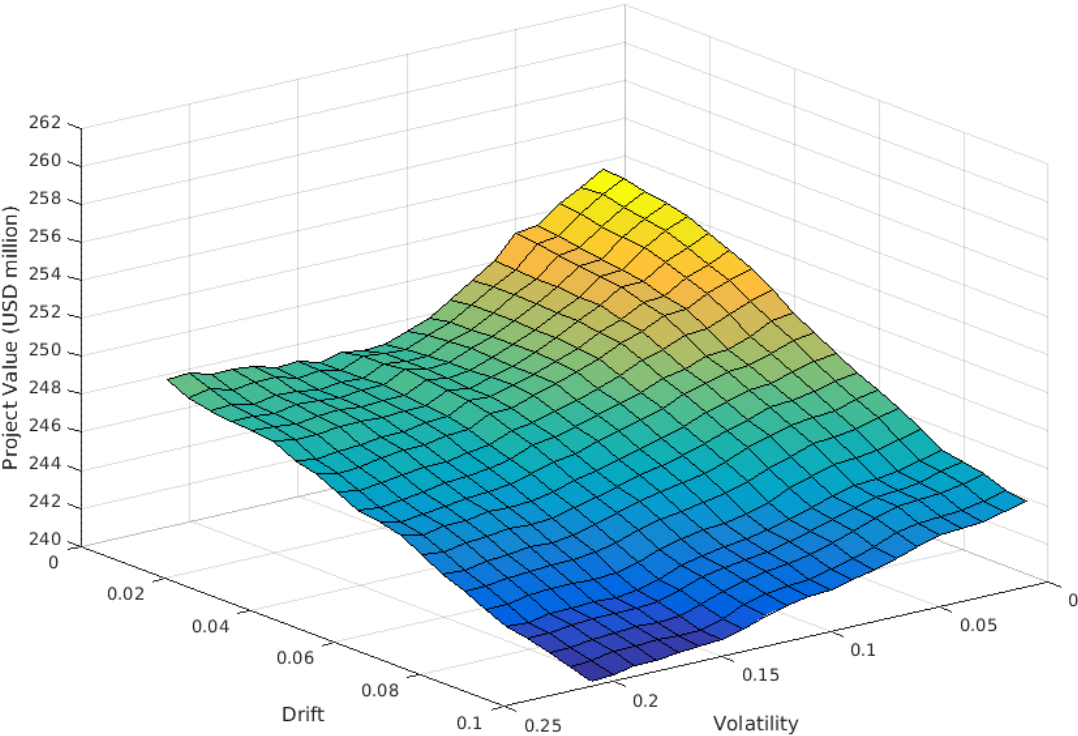
Figure 17 Sensitivity analysis of exploration cost parameters for $\rho_{\theta\chi} = 1$

Finally, a correlation coefficient of one ($\rho_{\theta\chi} = 1$) is used. The results are displayed in Figure 17. By analyzing the Figures 15, 16 and 17, it can be concluded that the drift impact in the project value is independent of the cost-price dependency. A maximum of 4% reduction in the project value was observed during the sensibility analyses performed. This percentage was constant as the correlation factor was increased.

The sensitivity of the project value with respect to the exploration cost volatility is dependent on the correlation factor. An increment in the volatility implies a positive effect on the project value

for a correlation factor of zero. As this correlation increases, the project value behavior changes. It could be concluded that the positive consequences on the project value when the volatility grows vanishes as the correlation coefficient increases from zero to 0.5, at which point the expected project value is independent of volatility changes in the range investigated. Beyond a correlation of 0.5, an increase in the volatility decreases the value of the project, and its most significant sensitivity is found when the correlation factor is equal to one.

For a better understanding of how sensitive the expected project value is on the volatility, consider the simulated data points in Table 5. Two estimations in a MC simulation with a cost volatility of 20% have been selected. They illustrate the scenarios where there is no dependency, and complete dependency. For the first case, the uncertainties are uncorrelated, therefore, the MC approach will have some simulated paths where the cash flow includes high oil price and low exploration cost. The oil price displayed is the initial value for the futures curve[23], thus, a high value entails high production incomes, leading to large estimations of NPV's. Having a low exploration cost increases these estimations. Still, if the volatility is increased in this case, it may find lower values of exploration cost associate with similar oil prices, which brings higher NPV's, explaining the behavior observed in the Figure 15.

Table 5 Example of data points for high exploration cost volatility

Correlation factor	Oil price (USD/BBL) $[S_t^i]$	Exploration Cost (MMUSD) $[D_t^i]$
$\rho_{\theta\chi} = 0$	98	116
$\rho_{\theta\chi} = 1$	96	502

When the correlation factor is equal to one, a high exploration cost is simulated when the oil price is high. Now, the NPV's will be lower because the investment decreases the value. This reflects the observed market reality, and as noted before, a calculated project value based on these estimations is considered a more realistic approach. In this case, if the volatility is increased, it will

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[23] Oil price curve used to be multiplied by the production rate to calculate the production income.

lead to lower estimated NPV's, and this will drive negative effects in the expected project value, as the behavior displayed in the Figure 17.

In the LSM algorithm, the project value depends directly on two factors: the optimal decision policy determined by the dynamic programming, and its vector of optimal payoff (as discussed in the section 5 of the chapter 6). In the MC simulation, for the case of uncorrelated uncertainties, there will be also biased NPV estimations resulted from low oil price and high exploration cost, which not represent the market reality. These estimations are lower compared to the rest since they entail lower production income and higher investment. They impact the estimation of the decision policy, but they may not be part of the vector of optimal payoff because this is composed by the highest NPV's of each path. That is the reason why they were not mentioned in the analysis above.

In addition, the effect of the correlation factor on the project value can be also analyzed in the figures above, since all the points on the surface decreases, except from the points on the line where the volatility is equal to zero, which remained constant.

7.4. Decision Maps

Decision maps is one of the possible decision support tools that can be extracted when the LSM approach is used to value options. The regression equations are used to create surfaces that provide insight to the decision-making process. To develop these maps, two linear equations are estimated: the expected value of drilling, and the continuation value. These regression equations will change for every year of the exploration license. Surfaces are then created from these equations as illustrated for year one in the Figure 18. A correlation factor $\rho_{\theta\chi} = 0.89$ was used.
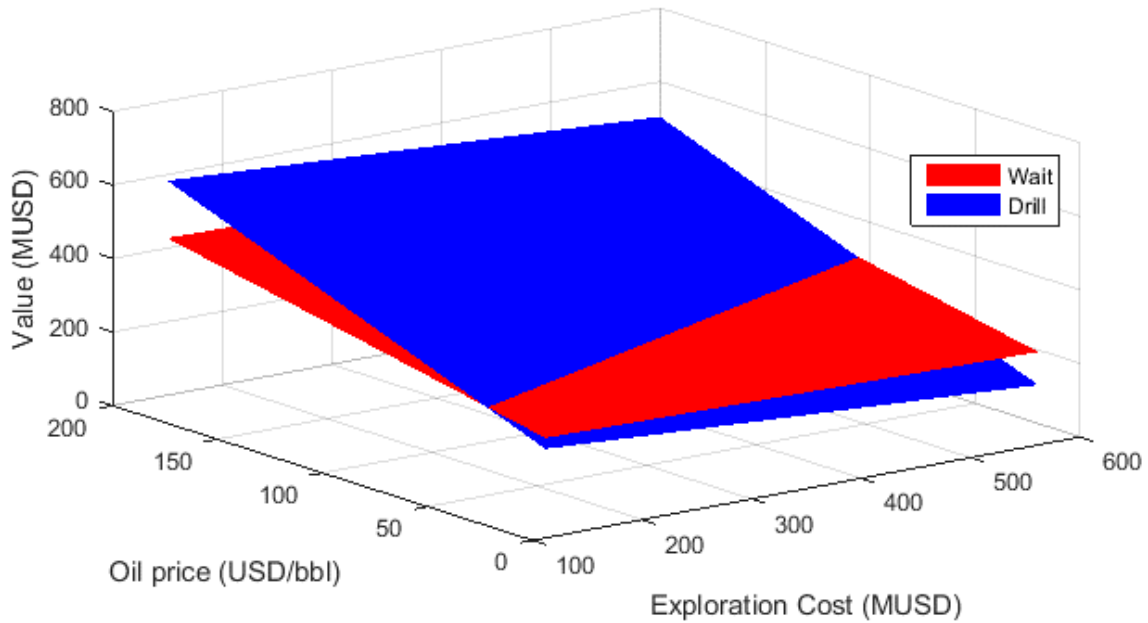
Figure 18 3D Decision Map for the year one

The exploration cost constitutes the initial investment in the project. Therefore, an increment in this value decreases the expected value of the project. This is illustrated in the Figure 18, where the expected value of drilling decreases as the exploration cost increases. Furthermore, the expected value of investing is a linear function of the oil price: as the oil price increases, the expected value of investing increases as well. For scenarios with low oil price and high exploration cost, the company should wait until the next year. The decision maps for the years 2, 3 and 4 using the same correlation are shown in the Figures 19, 20 and 21.

Figure 19 3D Decision Map for the year two
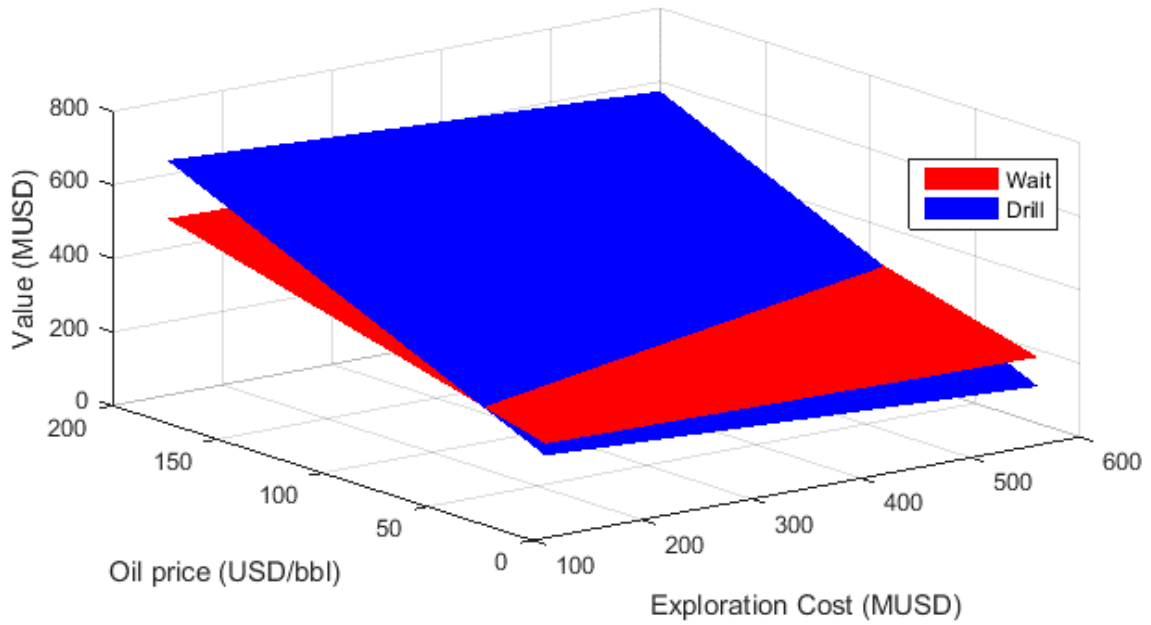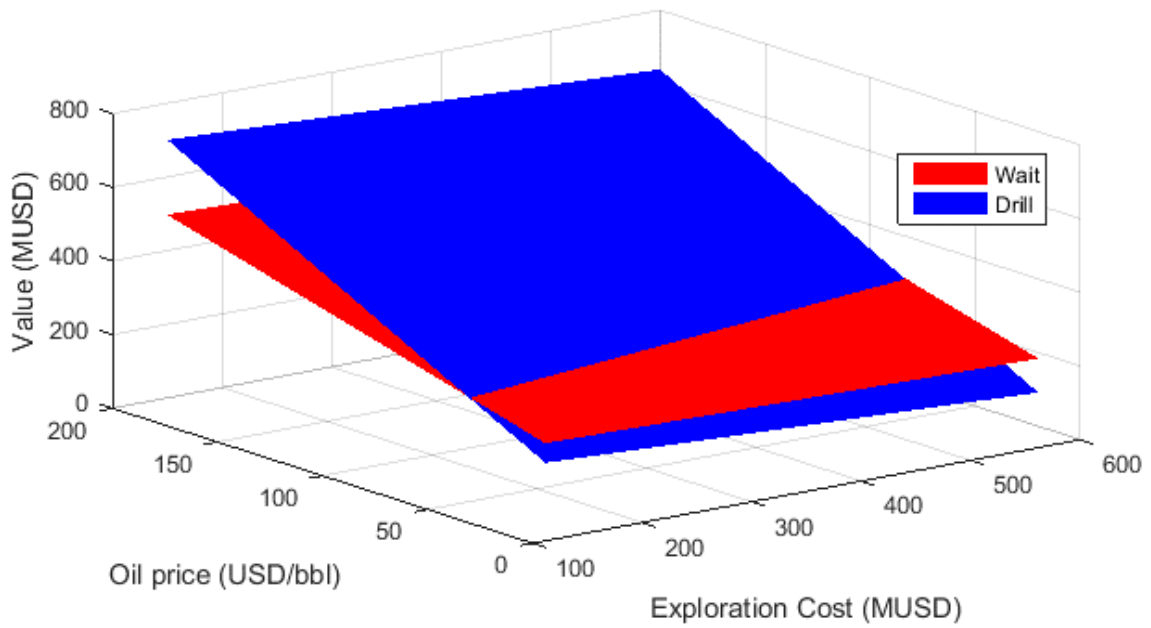

Figure 20 3D Decision Map for the year three
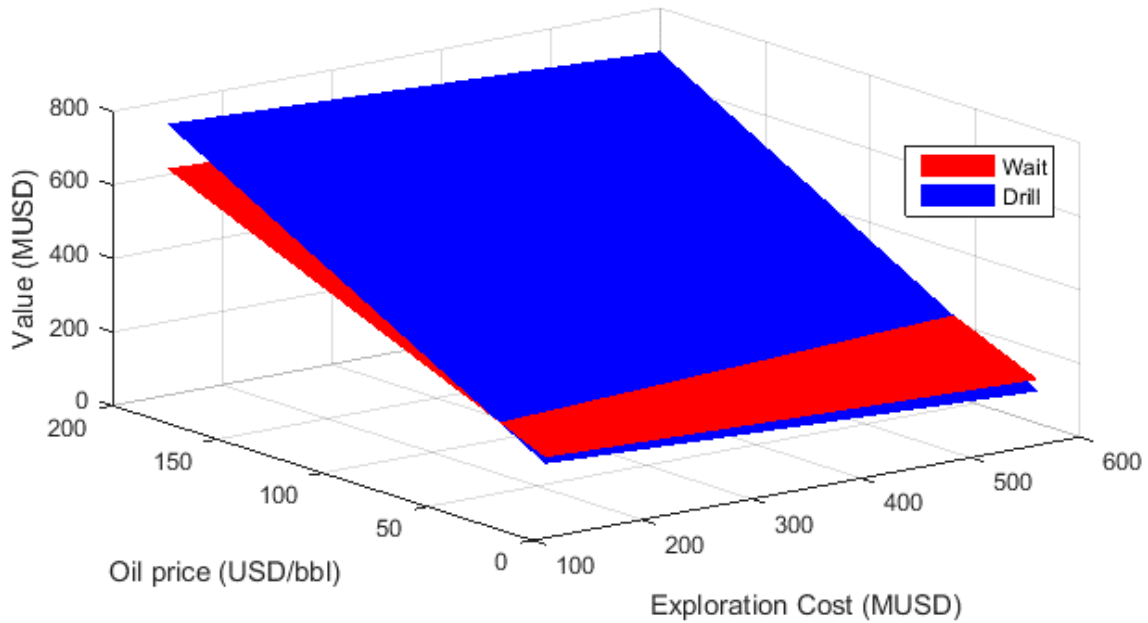
Figure 21 3D Decision Map for the year four

For a better understanding, decision maps are usually display in two dimensions, as shown in the Figure 22 for the first four years of exploration license. For the year 1, the figure provides an insight of the favorable conditions to execute the drilling campaign, besides the market conditions where the company should wait. As time progresses, the portion of the area that represents the decision to wait decreases. This is consistent with the expected oil price shown in the Figure 6. Executing the investment in late years of the exploration license implies that the production life would be in a time period of higher expected oil prices, compared with the previous years, increasing the area where the optimal decision is to invest.
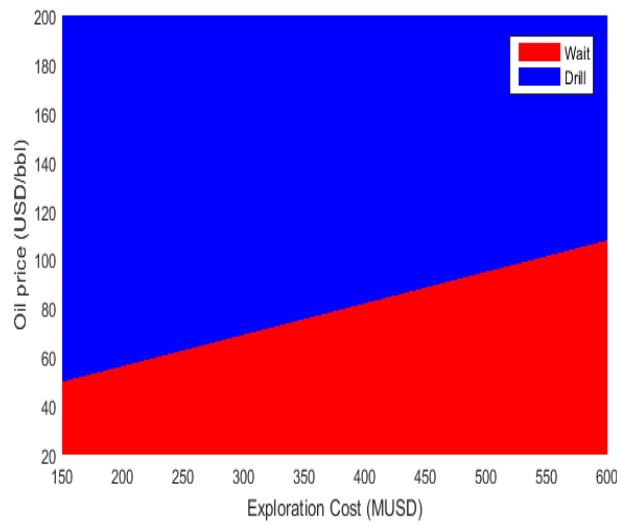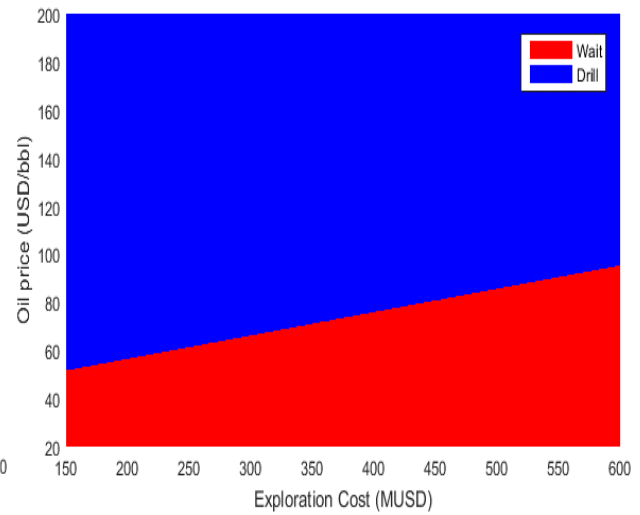
a) year 1   b) year 2
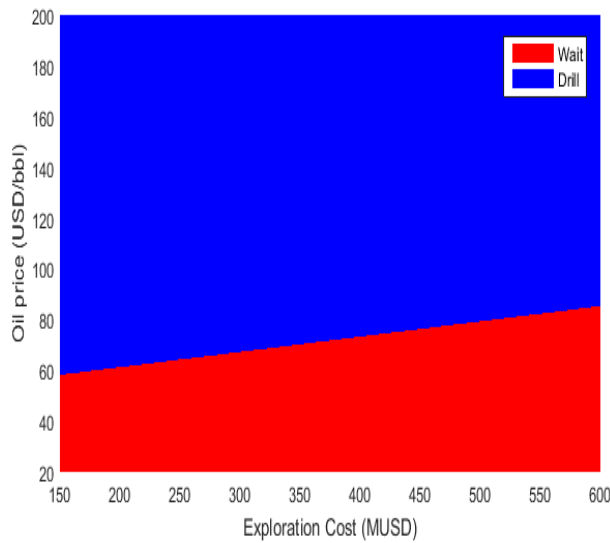
c) year 3   d) year 4

Figure 22 Decision maps for the four years

8. Conclusions

During low oil price periods, operating companies tend to avoid large investment. Oil downturns make companies more risk-averse than the high-price periods. The operator's common belief is that postponing the investment during this time will maximize the share-holder value. Yet, they do not seem to realize that the investment risk of some projects is lower in low-price than in high-price periods.
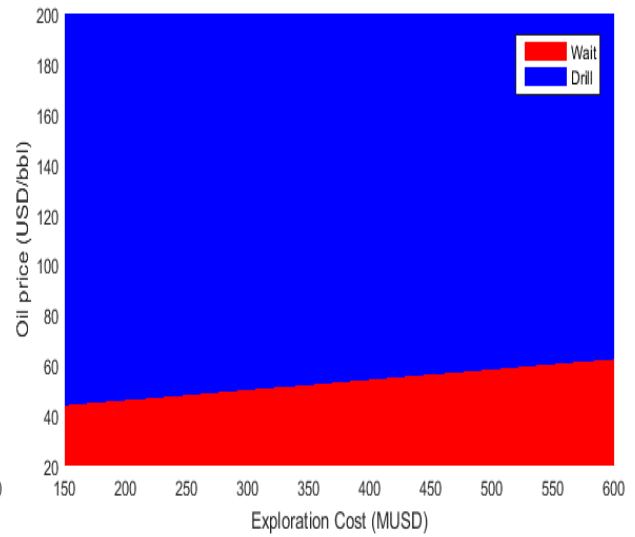
In this work, a ROV model has been implemented to demonstrate value can be created from investing in offshore exploratory drilling campaigns during low oil prices. Uncertainties in the oil price and the exploration cost were included in the model using stochastic processes. To reflect market reality, these uncertainties were correlated, and the impact of the correlation on the valuation and decision-making was studied. It was observed that the inclusion of the correlation contributes to a more realistic project value estimation, leading to portfolio decisions that create value.

There are different reasons for the common behavior of postponing investments during low oil price periods. First, the companies are concerned about their short-term cash flow and its impact in the share value. Second, the companies often fail to consider, and properly value, future flexibilities. In this work, optimal-time histograms were used to illustrate that this market belief is also a consequence of assuming that the oil price is the only uncertainty in the cash flow model. Including the uncertainty in the exploration cost impacted the optimal time to start the exploration campaign. However, if the correlation observed in the market between the drilling cost and the oil price is ignored, this may lead to suboptimal estimation of the optimal policy. By including the cost-price dependency, the decreasing effect of exploration cost during low oil price periods is accounted. For projects where the exploratory drilling campaign cost constitutes a substantial part of the cash flow, a reduction in this cost clearly impacts the optimal decision policy. This is the case of the offshore prospect studied in this thesis, where the optimal time to start the exploratory drilling campaign is the year with the lowest expected oil price, differing from the common operators' belief.

In this thesis, project valuations were performed to assess the impact of the cost-price dependency on the expected project value for a specific case. It was concluded that implementing the LSM method without this dependency will lead to over-valuation of the project. By including the cost-price correlation, the overall uncertainty of the project is reduced, which decreases the value of the real option. Moreover, the value of the project depends directly of the optimal policy. Therefore, it was demonstrated that the variation in the optimal decision policy as the correlation is increased impacts the expected value of the prospect.

In the MC simulation, if the cost-price correlation is neglected, cash flows estimations based on high oil prices and low oil exploration cost (as well as the opposite case) can be found. These estimations are considered biased since they do not represent the market reality, and their values affect the estimation of the optimal policy, and consequently, the project value. As the correlation factor is increased, these biased estimations cease, leading to more realistic project value estimations. Nevertheless, the correlation factor to be implemented should correspond to the local market, as this value depends on the location of the prospect.

Since the exploration cost is a private uncertainty that depends on the market, the parameters of the chosen stochastic models are estimated based on experts' opinion. When choosing a model and determining its parameters, the subject matter experts must account for the cost-price dependency characteristics in the relevant local market. In this work, a GBM stochastic process was used to quantify the decision-makers uncertainty in the exploration cost. A sensitivity analyses was conducted to analyze the impact of changes in the drift and volatility values. It was observed that, a growth in the drift decreases the expected value of the project, as it produces an increase in the exploration campaign investment. This behavior was found to be independent of the correlation. Previous studies have argued that increasing the volatility in one of the underlying uncertainties will raise the project volatility, and consequently, the value of the real option. However, in this work, it has been demonstrated the opposite behavior for correlated uncertainties. In studying the specific case in this thesis, if the cost-price correlation is large, increasing the exploration cost volatility decreases the project value. Nevertheless, the exploration cost is a negative component in the cash flow model, and further studies should be done to evaluate if this behavior is replicated by two positive uncertain components of the cash flow model that are correlated.

It is easy to generate decision maps when using the LSM approach. These maps can be used to assess future decisions conditional on the uncertainties of the model. In this thesis, they were used to illustrate the favorable market conditions for executing the exploratory drilling campaign during the different years of the exploration license.

Based on the LSM algorithm presented by Jafarizadeh and Bratvold (2015) for exploration projects, a series of MATLAB codes were developed that can be easy extended to other types of projects in the O&G industry. In this research, uncertainties such as operational cost or production were not included. However, as the LSM approach is relatively insensitive to the number of uncertainties in the problem, and the model can easily be extended to include them.

Finally, it has been illustrated that value can be created during low oil price periods by combining uncertainty and flexibility. Identifying value creation opportunities was possible by using ROV methods. Correlation between uncertainties in the cash flow are relatively easy to include in the LSM approach, and their implementation leads to a more realistic value estimate.

Although the study case in this work was realistic in terms of its cash flow, valuation, and decision making components, the conclusions reached in this thesis cannot be extended to all investment of this nature without a proper assessment. Nevertheless, the developed ROV model can be easy modified to be implemented in different type of exploration projects.

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Appendix 1. Calibration of the STLT oil price model

Jafarizadeh and Bratvold (2012) developed the method for calibrating the STLT price model based on current spot contracts, future contracts and options on future contracts. It is implemented in this thesis using data reported in the New York Mercantile Exchange on 19th of October 2016.

Step 1: Estimation of σ_ξ

Information about the long-term factor is embedded in options of future contracts with long maturity time. Schwartz and Smith (2000) derived the formula for estimating the value of European options in term of their STLT model parameters as:

$$C = e^{-rT}\left(F_{T,0}N(d) - KN(d - \sigma_\phi(T,t))\right) \tag{A-1}$$

$$P = e^{-rT}\left(KN(d - \sigma_\phi(T,t)) - F_{T,0}N(d)\right) \tag{A-2}$$

$$d = \frac{\ln\left(F/K\right)}{\sigma_\phi(t,\mathrm{T})} + \frac{1}{2}\sigma_\phi(t,\mathrm{T})$$

The value of a European call option (C) or put option (P) is based on today's (t=0) price of the underlying future contract $F_{T,0}$, the risk-free rate r, the maturity time of the contract T, the volatility $\sigma_\phi(T,t)$, and the strike price K. N(d) represents the cumulative probabilities for standard normal distribution. As the price of the option is observed in the market, equations A-1 or A-2 can applied to estimate the implied volatility of the option, using an inverse function. In this research, Microsoft Excel's Goal Seek function was used to find the option's volatility σ.

The volatility of the long-term factor (σ_ξ) is calculated based on the implied volatility of options that expire in 6 to 8 years. Observed marked data for a call option that expires on December 2024 is shown in Table A-1. Outcome of using the Goal seek function on equation A-1 gives an implied volatility $\sigma_\phi(T,t)$ equal to 19.46%.

Table A- 1 Observed market data for implied volatility calculation.

Term	Value
$F_{T,0}$	59.43 USD
K	45 USD
P	9.83 USD

The volatility of the long-term factor is calculated as the annualized implied volatility:

$$\sigma_\xi = \frac{\sigma_\phi(T,T)}{\sqrt{T}} = \frac{0.1946}{\sqrt{8.1667}} = 7\%$$

Step 2: Estimation of μ_ξ^* and κ

Figure A-1 shows the log of the future contract prices for different maturity dates. As argued by the Schwartz and Smith (2000), the slope of the line that represents the long-maturity futures is equal to $\mu_\xi^* + \frac{1}{2}\sigma_\xi^2$. The slope of the trend line shown in Figure A-1 is 0.012. Therefore, as the volatility of the long-term factor was calculated in the step before, the risk-neutral drift for the long-term factor is calculated as:

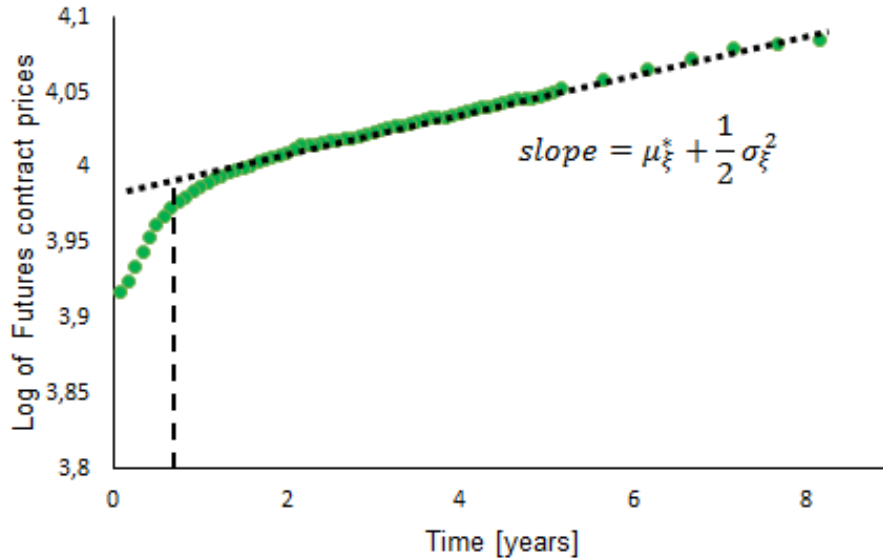$$\mu_\xi^* = 0.012 - \frac{1}{2}\sigma_\xi^2 = 0.012 - \frac{1}{2}0.07^2 = 0.96\%$$

Figure A- 1 Log of the Futures contract prices for different maturity dates

In addition, the mean-reversion coefficient κ is also estimated from the last figure. The half-life of the deviation is the length of time that the short-term deviations are expected to halve and is equal to ln (2)/ κ. The futures curve (Figure A-1) shows that the deviations from the equilibrium will halve at 0.6 years, thus, the mean-reversion coefficient is:

$$\kappa = \frac{\ln(2)}{0.6} = 1.16$$

Step 3: Estimation of σ_χ and $\rho_{\xi\chi}$

The annualized volatility of options with short maturity time ($T \approx 0$) is approximate as (Schwartz and Smith, 2000):

$$\sigma_\phi(T,T)/\sqrt{T} \cong \sqrt{e^{-2\kappa T}\sigma_\chi^2 + \sigma_\xi^2 T + 2e^{-\kappa T}\rho_{\xi\chi}\sigma_\chi\sigma_\xi} \qquad \text{(A-3)}$$

In the equation A-3, there are two unknown values: the short-term volatility (σ_χ) and the correlation coefficient ($\rho_{\xi\chi}$). Using equation A-3 on any two options with short maturity time (1,2,3 months), a system of two equations with two unknown values can be formed. The call option that expires on November 2016 (T=1/12), and the put option that expires on January 2017 (T=3/12) were used. The system of equations is then:

$$0.32727 \cong \sqrt{e^{-2*1.16*(1/12)}\sigma_\chi^2 + 0.07^2 * (1/12) + 2e^{-1.16*(1/12)} * 0.07\rho_{\xi\chi}\sigma_\chi} \quad \text{(A-4)}$$

$$0.273 \cong \sqrt{e^{-2*1.16*(3/12)}\sigma_\chi^2 + 0.07^2 * (3/12) + 2e^{-1.16*(3/12)} * 0.07\rho_{\xi\chi}\sigma_\chi} \quad \text{(A-5)}$$

Solving the system of equations, the values of the short-term volatility and the correlation coefficient found are:

$$\sigma_\chi = 0.335$$

$$\rho_{\xi\chi} = 0.34$$

Step 4: Estimation of ξ_0, χ_0, and λ_χ

Jafarizadeh and Bratvold (2012) argued that setting the risk premium for the short-term equal to 0 ($\lambda_\chi = 0$) will not affect the risk-neutral stochastic process, and the initial state variables can be estimated solving the following system of equations:

$$\chi_0 = \ln(S_0) - \xi_0 \tag{A-6}$$

$$\ln(F_{T,0}) = e^{-\kappa T}\chi_0 + \xi_0 + \mu_\xi^* T + 1/2\left[(1 - e^{-2\kappa T})\frac{\sigma_\chi^2}{2\kappa} + \sigma_\xi^2 T + 2(1 - e^{-\kappa T})\frac{\rho_{\xi\chi}\sigma_\chi\sigma_\xi}{\kappa}\right] \tag{A-7}$$

The input of the equations A-6 and A-7 are the spot price S_0 (50.29 USD/BBL on 19th of October 2016) and the price of a future contract $F_{T,0}$ with maturity time T. The future contract with expiration date on December 2024 (Observed price $F_{T,0}$=59.53 USD/BBL) was used. Solving the system of two equations with two unknow variables, the long-term factor of the spot price at time zero, and the deviation from the equilibrium at time zero are:

$$\xi_0 = 4.03$$

$$\chi_0 = -0.11$$

A summary of the parameters estimated is shown in the Table 1, in the chapter 3.1.1.

Appendix 2. MATLAB codes for exploration projects.

The LSM approach is implemented in this research using MATLAB® programming. In this appendix, the input variables, as well as the main codes are illustrated. The ROV model developed in this thesis is based on Jafarizadeh and Bratvold (2015) codes. The table A-2 describes the main MATLAB® scripts and functions in this model, which will be shown later in this section.

Table A- 2 Description of the main functions of the developed ROV model.

Function Name	Description
OptionsTime	The main script. It defines the input variables for the function "EXPWait", and plots its results.
EXPWait	Calculates the value of the exploration project by calling all the functions mentioned below, and implementing the LSM algorithm.
PriceSS	Generates a Ntrial x T matrix for each of the uncertainties (Long, short terms, and the exploration cost). The input arguments of this function are the parameters of the STLT process and the GBM process, along with the time of expiration of the license (T), and the number of simulation trials (Ntrial)
NPVGen	Creates a Ntrial x T matrix of NPV values. This function calculates the NPV values by calling the function "NPVFutures"
NPVFutures	Calculates the NPV of a project based on the cash flow model described previously. It calls the function "Futures" to calculate the oil price used in the cash flow estimation, and the function "Pdecline" to define the production profile.
Futures	Generates a vector of futures oil prices to be used in the cash flow estimation, by implementing the equation 7.
Pdecline	Defines an oil production profile for the project. The input of this function is the decline rate and the years of production.
OneTree	Calculates the expected value of the project based on the decision tree structure showed in the Figure 2.

OptionsensrhoC	Perform the sensitivity analysis of the project value with respect to the correlation factor.
SensitivityDriftandsigmaC	Perform the sensitivity analysis of the project value with respect to the drift and volatility of the stochastic process of the exploration cost
DecisionMap	Creates the decision maps for an exploration project

The input values that must be defined beforehand are described in the table A-3.

Table A- 3 List of input arguments of the MATLAB algorithm

Variable	Description
Capacity	The annual capacity of the production infrastructure. When the annual production is more than this value, the excess of oil will be produced in the next year
Chi0	Initial value of the short-term factor (χ_0)
DeclineRate	The annual decline rate of the production profile.
Develop	The overall development cost of the project. It includes production drilling, facility construction and infrastructure development cost
Explo0	The initial value of the exploration cost in the GBM process
Kappa	The mean-reverting coefficient in the STLT process (κ)
Lambda	The risk-premium for the short-term fact (λ_χ)
Lead	The estimated lead time of the project (time between the exploration campaign and the first production)
mu	The drift of the long-term factor in the STLT process (μ_ξ^*)
muC	The drift of the exploration cost in the GBM process (μ_θ)
NPVdry	The NPV in case the prospect is dry
Opex	Fixed operation cost per year of production. It includes expenses such as tariffs or labor cost
Pg	The probability of success on finding hydrocarbons in the prospect.

r	Risk-free ratio (Discount ratio)
rho	Correlation factor in the STLT process ($\rho_{\chi\xi}$)
rhoC	Correlation coefficient between the exploration cost and the short-term factor ($\rho_{\theta\chi}$)
sigmaC	Volatility of the exploration cost in the GBM process (σ_θ)
sigmaChi	Volatility of the short-term factor in the STLT process (σ_χ)
sigmaXi	Volatility of the long-term factor in the STLT process (σ_ξ)
T	Time of expiration of the exploration license
VariableOpex	Operational cost per barrel produced. It includes processing and lifting cost, among others.
Volume	Total recoverable reserve in the prospect [Barrels]
Xio	Initial value of the long-term factor (ξ_0)
Years	Expected years of production of the prospect.

The MATLAB® codes are presented starting from the main script, and thereafter, introducing the functions used in each simulation level.

OptionsTime

```
% The main script. It defines the input variables for the function EXPWait,
% and plots its results.
% written by Camilo Cardenas (15/11/2016)

clear
%load the input data
load defer.mat
%Set the number of Monte Carlo simulations
Ntrial=100000;
%Set the correlation factor for the exploration cost
rhoC=0.89;

% Call the function of EXPWait to estimate:
% Option=The expected value of the project
% ExerciseDate=The vector of the optimal exercise year for every simulation
% trial
% Price and Explo=The maxtrixes of the simulated uncertainties.
% k=The vector of optimal payoff for every simulation trial
% Coeff and Coeff are the regression coefficients for creating the decision maps.
[Option ExerciseDate Price Explo k Coeff
Coeff2]=EXPWait(Develop,Explo0,rhoC,muC,sigmaC,Xi0,mu,sigmaXi,Chi0,kappa,...
```

```matlab
            sigmaChi,lambda,rho,Opex,VariableOpex,Volume,...
            Capacity,DeclineRate,Years,Lead,NPVdry,Pg,r,T,Ntrial);
% Estimate the mean values of the uncertainty.
for i=1:T
    Exploo(i)=mean(Explo(:,i));
    X(i)=i;
end
Option
%Plot the optimal-time histograms
figure
subplot(2,1,1);
plot(X,Exploo)
xlabel('Years of exploration license')
ylabel('Capex')
subplot(2,1,2)
histogram(ExerciseDate)
xlabel('Years of exploration license')
ylabel('Frequency')
```

EXPWait

```matlab
function [Option ExerciseDate Price Explo k
Coeff,Coeff2]=EXPWait(Develop,Explo0,rhoC,muC,sigmaC,Xi0,mu,sigmaXi,Chi0,kappa,sigmaChi,lambda,rh
o,Opex,VariableOpex,Volume,Capacity,DeclineRate,Years,Lead,NPVdry,Pg,r,T,Ntrial)
% calculates the value of exploration project with the option to wait
% Written by Babak Jafarizadeh
% Modified by Camilo Cardenas (15/11/2016)

% Estimate the uncertainties.
[long,short,Explo]=PriceSS(Xi0,mu,sigmaXi,Chi0,kappa,sigmaChi,lambda,rho,T,T,Ntrial,Explo0,rhoC,m
uC,sigmaC);
Price=exp(long+short);
% Creates the time shift between the uncertainties
long(:,2)=[];
short(:,2)=[];
Price(:,2)=[];
long(:,1)=[];
short(:,1)=[];
Price(:,1)=[];
Explo(:,T+2)=[];
Explo(:,1)=[];
% Creates the matrix of NPV of the project without options.
NPVmat=NPVGen(Develop,long,mu,sigmaXi,short,kappa,sigmaChi,lambda,rho,r,Opex,VariableOpex,Volume,
Capacity,DeclineRate,Years,Explo,Lead,T,Ntrial);
% creates the discount vector
DiscountVec=exp(-r*(1:T));
% Set the values at t=T as the initial values for the vectors of optimal
% payoffs (ValueVec) and optimal exercise time (ExerciseDate)
NPVest=NPVmat(:,T);
ValueVec=OneTree(NPVest(:,1),NPVdry,Pg);
ExerciseDate=T*ones(Ntrial,1);
```

```
% Set the dimensions of the regression coefficients for the decision maps
Coeff=zeros(3,T-1);
Coeff2=zeros(3,T-1);
% Performs the backward induction
for step=T-1:-1:1
    NPVest=NPVmat(:,step);
    % Estimates the continuation value
    InMoney=find(OneTree(NPVest(:,1),NPVdry,Pg)>0);
    Regression=zeros(length(InMoney),5);
    Regression(:,1)=Price(InMoney,step);
    Regression(:,2)=Explo(InMoney,step);
    Regression(:,3)=Regression(:,1).^2;
    Regression(:,4)=Regression(:,2).^2;
    Regression(:,5)=Regression(:,1).*Regression(:,2);
    alpha=Regression\ValueVec(InMoney);
    continuation=Regression*alpha;

    % For the decision maps code, estimate linear coefficients of the
    % continuation values.
    Contdisc=continuation.*DiscountVec(ExerciseDate(InMoney)-step)';
    Regression2=zeros(length(InMoney),3);
    Regression2(:,1)=ones;
    Regression2(:,2)=Price(InMoney,step);
    Regression2(:,3)=Explo(InMoney,step);
    aalpha=Regression2\Contdisc;
    Coeff(:,step)=aalpha;

    % Estimates the optimal decision at step i
    Intrinsic=OneTree(NPVest(InMoney),NPVdry,Pg);
    index=find(Intrinsic>(continuation.*DiscountVec(ExerciseDate(InMoney)-step)'));
    ValueVec(index,1)=Intrinsic(index,1);
    ExerciseDate(index)=step;

    %For the decision maps code, estimates the linear regression coefficients of the drilling
option
    alpha2=Regression2\Intrinsic;
    Coeff2(:,step)=alpha2;
end
% Discounts the vector of optimal payoffs
k=ValueVec.*DiscountVec(ExerciseDate)';
% Calculate the value of the real option
Option=max(mean(k));
end
```

PriceSS

```
function [long short
Explo]=PriceSS(Xi0,mu,sigmaXi,Chi0,kappa,sigmaChi,lambda,rho,T,Nstep,Ntrial,Explo0,rhoC,muC,sigma
C)
% Generates price paths based on the Schwartz and Smith's (2000) two-factor price model
% Generate the Exploration cost as a GBM process
```

```
dt=T/Nstep;
short=zeros(Ntrial,Nstep+2);
long=zeros(Ntrial,Nstep+2);
Explo=zeros(Ntrial,Nstep+2);
% Set inital values
long(:,1)=Xi0;
short(:,1)=Chi0;
Explo(:,1)=Explo0;
% Generates the rest of the values
for i=2:Nstep+2
    epsilon=randn(Ntrial,1);
    long(:,i)=long(:,i-1)+mu*dt+sigmaXi*sqrt(dt).*epsilon;
    epsilonxi=rho.*epsilon+sqrt(1-rho^2).*randn(Ntrial,1);
        short(:,i)=short(:,i-1).*exp(-kappa*dt)-(1-exp(-kappa*dt))*lambda/kappa ...
        +sigmaChi*sqrt((1-exp(-2*kappa*dt))/(2*kappa)).*epsilonxi;
    % Include Exploration cost as GBM
    epsiloncapex=rhoC.*epsilonxi+sqrt(1-rhoC^2).*randn(Ntrial,1);
    Explo(:,i)=Explo(:,i-1).*exp((muC-0.5*(sigmaC^2))*dt+((sigmaC*sqrt(dt)).*epsiloncapex));
end
end
```

NPVGen

```
function
NPVmat=NPVGen(Develop,long,mu,sigmaXi,short,kappa,sigmaChi,lambda,rho,r,Opex,VariableOpex,Volume,
Capacity,DeclineRate,Years,Explo,Lead,T,Ntrial)
% Generates the NPV Matrix based on the cash flow model of the project
% written by Babak Jafarizadeh
% Modify by Camilo Cardenas (15/11/2016)

NPVmat=zeros(Ntrial,T);
for j=1:T
   for i=1:Ntrial
        NPVmat(i,j)=NPVFutures(Develop,long(i,j),mu,sigmaXi,short(i,j),kappa, ...
            sigmaChi,lambda,rho,r,Explo(i,j),Opex,VariableOpex,Volume, ...
            Capacity,DeclineRate,Years,Lead);
   end
end
end
```

NPVFutures

```
function
NPVF=NPVFutures(Develop,Xi0,mu,sigmaXi,Chi0,kappa,sigmaChi,lambda,rho,r,Explo,Opex,VariableOpex,V
olume,Capacity,DeclineRate,Years,Lead)
% Generates the NPV figure using the futures curve and production parameters
% written by Babak Jafarizadeh
% Modify by Camilo Cardenas (15/11/2016)

% Estimates the production profile.
```

```
production=Pdecline(Volume,Capacity,DeclineRate,Years);
ProjectProduction=[zeros(1,Lead) production];
% Set the discount vector
DiscountVec=[1 exp(-r*(1:Years+Lead-1))];
% Calculates the futures oil price
F=zeros(1,Years+Lead-1);
F=Futures(Xi0,mu,sigmaXi,Chi0,kappa,sigmaChi,lambda,rho,Years+Lead-1);
PriceVec=[exp(Xi0+Chi0) F];
% Calculate the gross revenue
Cashflow=zeros(1,Years+Lead);
Cashflow=ProjectProduction.*PriceVec;
% Subtract the variable opex
Cashflow(1,Lead+1:Years+Lead)=Cashflow(1,Lead+1:Years+Lead)- ...
    ProjectProduction(1,Lead+1:Years+Lead)*VariableOpex;
% Subtract the exploration cost
Cashflow(1,1)=Cashflow(1,1)-Explo;
% Subtract the development cost
Cashflow(1,Lead-1)=Cashflow(1,Lead-1)-Develop;
% Subtract the fixed Opex
Cashflow(1,Lead+1:Years+Lead)=Cashflow(1,Lead+1:Years+Lead)-Opex;
% Discount the cash flow to calculate the NPV
NPVF=sum(Cashflow(1,1:Years+Lead).*DiscountVec(1,1:Years+Lead));
end
```

Pdecline

```
function production=Pdecline(Volume,Capacity, DeclineRate, Years)
% returns the yearly production profile with a positive logarithm profile
% before the peak, and negative exponential profile after the peak
% written by Camilo Cardenas (15/11/2016)

% Defines the production peak
peaky=0.095*Years;
Peaky=round(peaky);
Peakp=0.15*Volume;
C1=(Peakp-1)/log(Peaky);
production=zeros(1,Years);
Remain=Volume;
%Estimate the production profile before the peak
for i=1:Peaky
P=(C1*log(i))+1;
production(1,i)=min(Capacity,P);
Remain=Remain-production(1,i)';
end
%Estimate the production profile after the peak
for k=Peaky+1:Years
P=DeclineRate*Remain;
production(1,k)=max(0,P);
Remain=Remain-production(1,k);
end
end
```

Futures

```
function F=Futures(Xi0,mu,sigmaXi,Chi0,kappa,sigmaChi,lambda,rho,T)
% returns the futures price using the parameters of the two-factor price process
F=zeros(size(T));
lnF=zeros(size(T));
lnF=exp(-kappa.*(1:T)).*Chi0+Xi0+mu.*(1:T)-(1-exp(-kappa.*(1:T)))*lambda/kappa+ ...
    0.5*((1-exp(-2*kappa.*(1:T)))*sigmaChi^2/(2*kappa)+sigmaXi^2.*(1:T)+ ...
    2*(1-exp(-kappa.*(1:T)))*rho*sigmaChi*sigmaXi/kappa);
F=exp(lnF);
end
```

OneTree

```
function OptimalAlt=OneTree(NPVoil,NPVdry,Pg)
% uses a one-step decision tree to find the alternative with the highest
% expected value
OptimalAlt=max(0,Pg*NPVoil+(1-Pg)*NPVdry);
end
```

OptionsensrhoC

```
% Perform the sensitivity analysis of the project value with respect to the
% correlation factor.
% written by Camilo Cardenas (15/02/2017)

clear
% load the data
load defer.mat
% Define the number of Monte Carlo simulations
Ntrial=1000000;
% Define the interval of correlation factor to be evaluated
rhoClist=0:0.01:1;
% Define the dimentions of the solution matrix
Optionlist=zeros(length(rhoClist),1);
%Perform the sensitivity analysis
for i=1:length(rhoClist)
    rhoC=rhoClist(i);
    [Option ExerciseDate Price Explo k Coeff
Coeff2]=EXPWait(Develop,Explo0,rhoC,muC,sigmaC,Xi0,mu,sigmaXi,Chi0,kappa,...
            sigmaChi,lambda,rho,Opex,VariableOpex,Volume,...
            Capacity,DeclineRate,Years,Lead,NPVdry,Pg,r,T,Ntrial);
        Optionlist(i)=Option;
end

%Plot the results
plot(rhoClist,Optionlist)
xlabel('Correlation factor')
ylabel('Expected project value')
```

SensitivityDriftandsigmaC

```
% Perform the sensitivity analysis of the project value with respect to the
% drift and volatility of the exploration cost stochastic process.
% written by Camilo Cardenas (15/02/2017)

clear
%Load the data
load defer.mat
% Define the number of Monte Carlo simulations
Ntrial=1000000;
% Define the interval of the variables for the sensitivity analysis
driftlist=0:0.01:0.1;
volalist=0.01:0.01:0.21;
% Define the dimention of the solution matrix
Optionlist=zeros(length(driftlist),length(volalist));
% Define the correlation factor to be evaluated
rhoC=1;
% Perform the sensitivity analysis
for i=1:length(driftlist)
    for j=1:length(volalist)
        sigmaC=volalist(j);
        muC=driftlist(i);
        [Option ExerciseDate Price Explo k Coeff
Coeff2]=EXPWait(Develop,Explo0,rhoC,muC,sigmaC,Xi0,mu,sigmaXi,Chi0,kappa,...
            sigmaChi,lambda,rho,Opex,VariableOpex,Volume,...
            Capacity,DeclineRate,Years,Lead,NPVdry,Pg,r,T,Ntrial);

        Optionlist(i,j)=Option;
    end
end
% Plot the results in a 3D graph
surf(volalist,driftlist,Optionlist);
xlabel('Volatility');
ylabel('Drift');
zlabel('Project Value (USD million)');
title('Sensitivity of Exploration Cost Parameters');
```

DecisionMap

```
% Creates the decision maps for an exploration project.
% written by Camilo Cardenas (15/02/2017)

clear
% load the data
load defer.mat
% Define the number of Monte Carlo simulations
Ntrial=1000000;
% Define the correlation factor applicable to your project
rhoC=0.89;
% Estimate the value of the coefficients of the regression.
```

```matlab
% Coeff= Linear regression coefficients matrix of the continuation value
% Coeff2= Linear regression coefficients matrix of the drilling decision
[Option ExerciseDate Price Explo k Coeff
Coeff2]=EXPWait(Develop,Explo0,rhoC,muC,sigmaC,Xi0,mu,sigmaXi,Chi0,kappa,...
            sigmaChi,lambda,rho,Opex,VariableOpex,Volume,...
            Capacity,DeclineRate,Years,Lead,NPVdry,Pg,r,T,Ntrial);


%Plot the decision maps 3D

%Specify the interval of the exploration cost and the price
Exppxx=150:5:600;
Pricex=20:5:200;
% Define the year to plot the decision map
w=1;
% Define the dimensions of the matrixes
Cont=zeros(length(Pricex),length(Exppxx));
NNPV=zeros(length(Pricex),length(Exppxx));
%Perform the calculations for the surfaces plots
for i=1:length(Pricex)
      for j=1:length(Exppxx)
          Cont(i,j)= Coeff(1,w)+(Coeff(2,w)*Pricex(i))+(Coeff(3,w)*Exppxx(j));
          NNPV(i,j)=Coeff2(1,w)+(Coeff2(2,w)*Pricex(i))+(Coeff2(3,w)*Exppxx(j));
      end
end

%Plot the decision map
figure
hSurface = surf(Exppxx,Pricex,Cont);
set(hSurface,'FaceColor',[1 0 0]);
hold on
hSurface = surf(Exppxx,Pricex,NNPV);
set(hSurface,'FaceColor',[0 0 1]);
ylabel('Oil price (USD/bbl)')
xlabel('Exploration Cost (MUSD)')
zlabel('Value (MUSD)')
legend('Wait','Drill'),
% Desactivate the following if 2D decision map is wanted
%view(2);
```