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Abstract

Steel have many design and functional advantages as a material in the construction process and in structures in use. However, deterioration of steel e.g. due to corrosion is one of the drawbacks for steel structures. Understanding the behaviour of steel structures under corrosion and assessment of remaining load carrying capacity of existing steel structures because of this possible corrosion become an important topic. In line with this challenge, this thesis is aiming to provide analytical formulas for calculating remaining capacity of structural steel members namely column, beam and a beam-column member

In this thesis a theoretical background for structural steel members and their behaviour is presented. In addition, analytical methods of strength calculations and design of structural steel members based on EC3 is discussed in brief. The cause and effect of corrosion on steel structures and the methods of corrosion prevention in design and service phases is also described. Some previous studies on similar topics are also evaluated and summarized.

The main parameter used in this thesis for formulating the equations is the eccentricity (shift) of the elastic and plastic neutral axes caused by the unsymmetrical material loss due to corrosion. Hence a corrosion damage model of an I-section is adapted for analysis purposes, where an eccentricity is introduced about the major axis. As result, a generalized and a specific geometrical property of the corroded model are presented as a function of the magnitude of the eccentricities and other parameters. Formulas for calculating the remaining capacities are also provide based on the section properties.

Typical I-sections in this thesis are selected for demonstrating the reduction in load carrying capacities and illustrations of effective capacity versus percent (%) area loss is provided. The results show that a significant capacity reduction occur for the member with axial compressive force. For 10% area loss the results of the analytical evaluations indicate a capacity reduction of about 40% & 50% reduction of critical elastic buckling load and axial plastic compressive capacity respectively about the major axis when the eccentricity due to unsymmetrical corrosion was introduced. For the member with a pure bending moment the reduction was about 22% & 18% of the elastic and plastic moment capacity. This indicates that the eccentricity due to corrosion have a significant effect, especially for members with axial forces.

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Symbols

A_i	-	area of the i^{th} component of the cross-section
z_i	-	moment arm of the i^{th} area from a reference point to the centroid of the i^{th} area
A_{uf}, A_{lf}, A_w	-	area of the upper flange, lower flange and web
t_{u_f}, t_{l_f}, s	-	thickness of the upper flange, lower flange and web
b_{uf}, b_{lf}	-	Width of the upper flange & lower flange
h	-	total height of section
d	-	moment arm from the centroid of the area to the reference axis
z_1, z_2, z_3	-	area moment arm of A_{l_f}, A_w & A_{u_f} respectively
I_{uf}, I_{bf}, I_w	-	second moment of area of the upper flange, lower flange and web about their respective centroids
σ_z	-	bending stress at the Z distance from the elastic neutral axis
M_y	-	bending moment about Y-axis
A_C , A_t	-	areas under compression and tension stress respectively
T	-	Torsion
φ	-	angle of twist
G	-	Shear modulus
I_t	-	Saint-Venant torsional constant
E	-	young's modulus of elasticity
I_w	-	Warping constant
m_x	-	torsional moment distributed along the beam
ψ_f	-	differential ratio of the second moments of area of the beam flanges about the minor axis (z-axis)
h_s	-	distance between the shear centre of the upper and lower flanges
N_y	-	squash load
f_y	-	yield strength of steel
A_0	-	gross cross-sectional area
k_σ	-	buckling coefficient
ν	-	poison`s ratio
b	-	Plate width
t	-	Plate thickness
L	-	Length of member
i	-	Radius of gyration
N_{Ed}	-	design value of axial compression force
$N_{c,Rd}$	-	design resistance of the cross-section
A_{eff}	-	effective area of cross-section
γ_{M0}	-	partial safety factor
$N_{b,Rd}$	-	design buckling resistance
x	-	reduction factor for the relevant buckling mode
N_{Cr}	-	elastic critical buckling load
$w_{e_l,y}$	-	elastic section modulus
M_P	-	Plastic moment
N_P	-	Plastic axial force
w_{p_l}	-	plastic section modulus
l_{d_i}	-	length of the i^{th} deteriorated area
t_{d_i}	-	thickness of the i^{th} deteriorated area
$\delta_{e_l y}$	-	Eccentricity of elastic neutral axis

$\delta_{p_l y}$	-	Eccentricity of plastic neutral axis
I_{effz}	-	Effective second moment of area of deteriorated section
I_{0z} & I_{0y}	-	second moment of area of the initial section along Z & Y
i_{eff}	-	Effective radius of gyration
w_{pleff}	-	Effective plastic section modulus
w_{eleff}	-	Effective elastic section modulus
A_d	-	Area of deterioration
δM	-	is an infinitesimal moment due to shift of PNA
N_{PlD}	-	full axial plastic capacity of the deteriorated section
N_{creff}	-	Effective elastic buckling load
M_{Pleff}	-	Effective plastic moment of deteriorated section
M_{eleff}	-	Effective elastic moment of deteriorated section
M_{pleffN}	-	Effective plastic moment due to axial force interaction of deteriorated section

Chapter 1

Introduction

1.1 Background and motivation

Structures in general and especially steel structures experience a decrease in performance due to deterioration during their service life. The main causes of deterioration for steel structures is corrosion, and material loss is the main effect of corrosion of steel structures. As a result, structural members lose their load carrying capacity due to the change in section properties and expose to additional loads due to possible eccentricities introduced by section loss. Therefore, a need for estimating the remaining strength of steel structures and a tool that provides a quick analysis is on demand.

Several research projects have been conducted to address the issue of deterioration of steel structures and some of these are presented in chapter 5. These studies typically select a specific corrosion pattern based on a data gathered from site inspections and hence calculate case specific load capacities. The results of such studies contribute for estimating the remaining capacity of similar corrosion patterns which may exist in some other structures.

As a structural engineer, I am always excited to study the behaviour of structures in general and steel structures in particular. My motivation to work in this area of study is to contribute to the above-mentioned demand and as well as to gain more knowledge and an in-depth understanding of steel structures. I believe such contribution will help to reduce the economic, social and environmental loses that comes with the deterioration of steel structures.

1.2 Objective

The main objective of this master thesis is to provide analytical formulas for estimating the remaining strength of corroded I-section structural steel members in relation to EC3 design of steel structures. The structural steel members include compression, beam and a beam-column member.

1.3 Scope of work

The scope of this thesis project includes:

- A theory part discussing about:
    - Structural steel members and their behaviour
    - Capacity of structural steel members and design methods based on EC3.
    - Deterioration mechanisms of steel structures
    - Review of previous research studies on remaining capacity of deteriorated steel structures
- Providing a generalized formula for section property of corroded I-section
- Developing a corrosion model for I-section (for analysis purposes)
- Providing formulas for estimating the remaining capacity of corroded I-section based on the corrosion model. Such formulas include:

  - Effective plastic axial compressive force capacity of a compression member
  - Effective elastic eccentric buckling load of compression member
  - Effective elastic and plastic moment capacity of beam member
  - Effective plastic moment capacity of a beam-column member

- Finally, a discussion and conclusion of the results obtained and suggestion for future works is provided

1.4 Limitations

This thesis is a beginning of a large research project aiming at developing generalized formulas for corroded I-section structural steel members. In order to obtain such generalized formulas, several corrosion models representing different corrosion patterns must be analysed, and the analytical formulas need to be verified by computer based FEA and full-scale laboratory tests. In this master thesis the following limitations have been used:

- To generalize and simplify the problem a rather simple corrosion pattern has been used.
- This thesis has studied the capacity of axially loaded beam with and without moment. Torsional buckling/lateral buckling has not been included in these evaluations.
- In order to force eccentricities of the neutral axes, a non-uniform corrosion could have been used. However, the corrosion pattern adapted for this thesis allows only asymmetry of the section about the main axis of the I-beam. A corrosion pattern that is unsymmetrical about the weak axis would introduce torsional buckling (lateral buckling), which is not studied in this thesis.
- At point of loads local contact and buckling issues are excluded

Chapter 2

Behaviour of steel structures

2.1 Introduction

A structure refers to a system of connected structural elements, which carry actions and resist forces, and transfer it to the foundation or supports. The actions and forces the structure carries are originated from the structures self-weight (permanent-actions), from the daily activities (variable-actions), and from the forces of nature acting on the structure (environmental actions), such as wind, snow, earth-quake. Structures may be built from several different materials, such as steel, concrete, wood, aluminium, stone, plastic, or a combination of them.

Steel structures combines several unique features that make them an ideal solution for many applications. In addition, material properties of steel allow much greater freedom at the design phase, thereby helping in achieving a greater flexibility and quality. Specially the high strength to weight ratio of steel, maximizes the useable area of a structure and minimizes the self-weight.

2.2 Types of structural steel members & their behaviour

Steel structures are composed of different types of structural members. Structural steel members can be classified as tension or compression members, beams, beam-columns, torsion members or plates (Figure 2.1), according to the technique by which they transmit the forces in the structure.

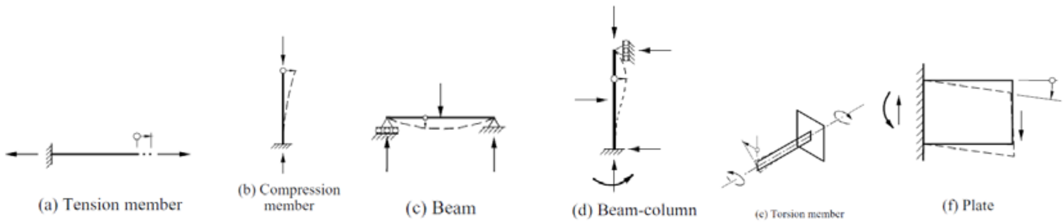


Figure 2.1: Load transmission by structural steel members[1]

Structural steel members transmit different types of forces such as axial, transverse force, moment and torque as shown in Figure 2.1 and their response to these forces are described by the load-deformation characteristics shown in Figure 2.2. Generally, a member may behave in a linear or non-linear fashion. The non-linearity behaviour of a member can further be classified as material-nonlinearity (curve 2), geometrical non-linearity (curve 3), or a combination of both material and geometrical non-linearity (curve 5) [1].

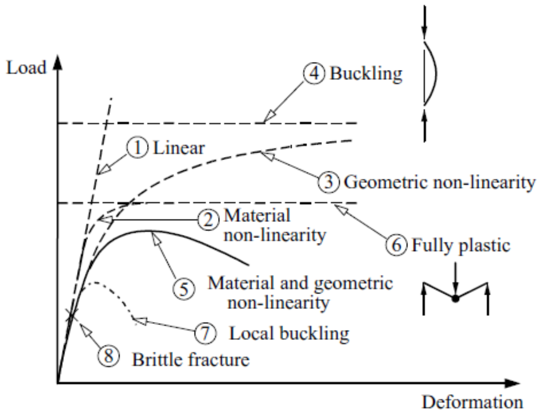


Figure 2.2: Structural steel member behaviour [1]

Curve 1 in figure 2.2, shows a linear response of a members until the material reaches the yield stress f_y . In this range the magnitude of the deformation depends on the material mechanical properties elastic moduli E and G. As the magnitude of the load increases, the member starts to behave in a non-linear way as it is seen in curve 2 of figure 2.2. and it reaches the condition of full plasticity representing by curve 6 in figure 2.2. On this stage the behaviour of the member depends on the yield stressf_y [1].

The above described behaviour of structural steel members is based on the material properties, but the member may also behave differently based on their geometry which is referred to as the geometrical non-linearity. In this case the forces acting on the member such as the bending moments and torques may be affected by the deformations of the member. In this condition the material non-linearity, may cause the deformations to become very large as shown in curve 3 figure 2.2 when the situation of elastic buckling is reached as in curve 4 figure 2.2. Besides the member may also fail suddenly in a brittle manner because of the presence of local buckling in the thin plate element of the member which is also represented by curve 7 in figure 2.2, or because of material fracture or cracks as shown in curve 8 of figure 2.2 [1].

The response of an individual member depends on the forces acting on it. For that reason, members which are not susceptible to member buckling such as tension members, laterally supported beams and torsion members exhibit linear behaviour until their material non-linearity becomes significant and then they approach the fully plastic state. Yet, members which are prone to compression forces such us compression members and laterally unsupported beams demonstrate geometric non-linearity as they approach their critical buckling loads. Members which transmit both transverse and axial load, beam columns, show both material and geometrical non-linearity.

2.3 Compression members

Compression members are axially loaded structural elements like tension members. The capacity of a steel member under axial compressive force depends on the cross-section resistance or on the possibility of member instability. All in all, the design of a compression member is governed by the instability phenomena as steel members are generally of medium to high slenderness[2]. In general, the behaviour (mode of failure) of compression members is characterized as:

- Failure due to yielding of cross section
- Local buckling of thin-plate compression elements (flange & web) or

- Overall flexural buckling of member

A compression member fails by yielding of cross section when the member is very stocky .Such compression members behave in an elastic way until the material begins to behave plastically at the squash load $N_y = A_0 \times f_y$ (plastic axial force), which is the maximum axial compressive load the member can carry based on its cross sectional area (A_0) and its material yield strength (f_y). The cross-sectional resistance of the member can be decreased by the local buckling capacity of the thin-plate elements, which depends on the plate's length to thickness ratio and yield strength capacity [1].

However, the resistance of a compression members is normally governed by their slenderness value and, their capacity is decreased significantly by the increase in slenderness. Hence the member can fail due to overall flexural buckling. The decrease in member resistance is due to the applied compressive load N which causes lateral bending in a member with initial curvature as shown in figure 2.4a. This lateral deflection of a compression member increase with the increase of the applied load, as shown in figure 2.4b [1].

Theoretically considering a perfectly straight elastic member, no bending occurs until the applied load reaches the critical elastic buckling load N_{cr}. At this load, the member starts to deflect laterally, as shown in figure 2.4b, and these deflections raise until the member fails. This failure mode of compression members is called flexural buckling. As a result, the critical elastic buckling load N_{cr} uses as a measure of the slenderness, while the squash load N_y provides an indication of its resistance to yielding[1].

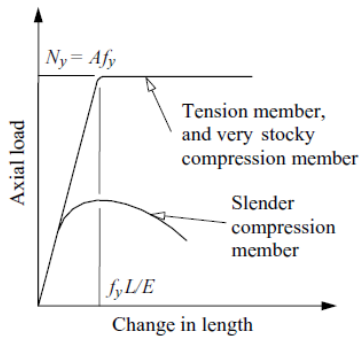


Figure 2.3: Resistance of axially loaded members [1]

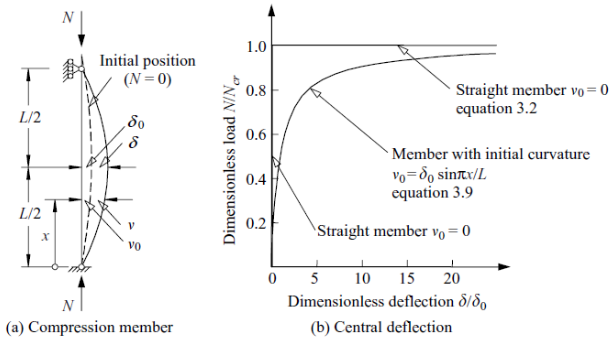


Figure 2.4: Elastic behaviour of a compression member [1]

2.4 Beam members

A beam is a member designed to resist a bending moment and shear force throughout its length. The performance of a steel beam member in bending depends on the cross-section resistance or lateral-torsional buckling. Over-all, like the compression members the lateral-torsional buckling is the governing mode of failure for steel members composed of I or H sections bent about the major axis [2]. The response of a beam member to the actions applied can be specified as:

- Yielding of compression flange and/or fracture of tension flange
- Shear yielding of web
- Bearing failure /crashing of web
- Local buckling of compression flange or local buckling of web
- And over-all flexural lateral torsional buckling

The first four modes of failure of a beam member depends on the geometrical properties of the beam in plane of loading and on its yield strength f_y. Usually I or H sections and rectangular hallow sections are chosen for beams due to high major axis bending resistance and bending stiffness of the cross-sections. The ultimate strength of a determinate beam is achieved when the cross-section under the maximum moment is fully yielded and a plastic hinge is formed. This ultimate moment capacity M_p is higher than the yield moment M_y as shown in figure 2.5.[1] .In other case if shear is the dominant force, as in case of deep beams of short span, the governing strength becomes the shear force which causes the web to be in full plastic state[1].

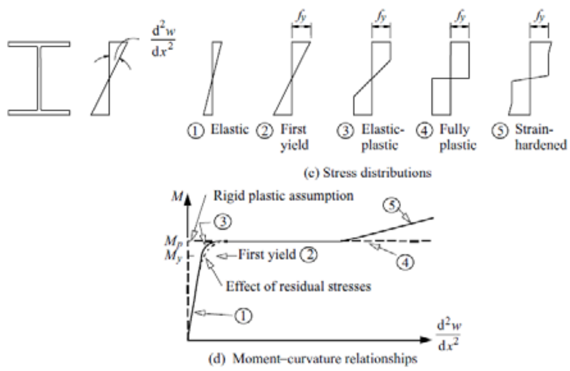


Figure 2.5: Moment-curvature relationships for steel beams [1]

The above in-plane behaviour of a beam member, was assumed that the beam deflects only in the plane of loading, which is vertical. However, if the beam does not have sufficient lateral stiffeners to ensure that the beam deflects only vertically, then it may buckle out of the plane of loading as shown in Figure 2.6. This significant instability behaviour of a beam member is called *lateral-torsional buckling*, which is the typical instability phenomenon in standard cross-sectional shapes, such as I or H bent about the major axis. Lateral torsional buckling is the failure mode of beam members in which the compressed part of the member`s cross section (the compressed flange in the case of I or H sections) deforms laterally [2]. The phenomenon of lateral torsional buckling is characterized by the elastic buckling moment M_{cr} , similar to that of compression members, which is determined by N_{cr} described in section 2.3.

Figure 2.6: Lateral buckling of a cantilever beam [1]

Whenever one of the following situations occurs in a beam, lateral-torsional buckling cannot develop, and the beam can be analysed based on the cross-section resistance only.

- The cross section of the beam is bent about its minor z axis.
- The beam is laterally restrained by means of secondary steel members or any other method
- The cross section of the beam has high torsional stiffness and similar flexural stiffness about both principal axes of bending as, for example, closed hollow sections.

Chapter 3

Capacity of structural steel members and design methods based on EC3

3.1 Introduction

Capacity(strength) is defined as the ability of a structure or structural member to resist the effect of the external forces acting upon it. The capacity of structural members or in general structures is determined by analysing the stresses, strains and displacements develops with in the structural members[3].In simple ways stress is defined as the force per unit area and strain is defined as the elongation /compression of the structural member along its length per unit length. Stresses and strain can be classified further as normal, bending, shear, bearing etc based on the external forces acting up on the structural members. The amount of stresses, strains and deflections the member tolerate depends on the member's geometrical properties and mechanical (material) properties.

Structures shall be designed to be safe during their service life. For that reason, design codes and regulations are introduced to account some safety margins on the capacity of the structural members and actions applied on structures. The safety factors of the capacity account the inherent material imperfections and manufacturing defects and the safety factors of the actions are introduced to account the uncertainty that comes with estimation of the magnitude of the actions. In addition, design codes provide a step by step design procedures and guidelines to help designers to deliver a safe and durable structure. There are several design codes in different parts of the world and one of the commonly used is Eurocode.

In this chapter the theoretical background of capacity calculation of structural steel members and design methods based on EC3 will be discussed.

3.2 Geometrical property

Structural members design requires the data of material strength (allowable stresses), critical shear and moment values and data about the cross-section properties. The shape and proportion of elements of cross-section of a structural member is critical in keeping the stresses developed with in the allowable limits and controlling the amount of deflection that results from the applied forces. Below are some of the basic and important geometrical properties of a member and the analytical formulas used to calculate their values.

3.2.1 Coordinate system

The coordinate system used in this thesis is following the method used in EC3, which is a Cartesian coordinate system in which the x-axis lies along the beam, the positive z-direction is vertical pointing upwards and the Y-direction is horizontal, perpendicular to the x -axis. For a vertically loaded beam the major axis is considered the Y-axis and the z-axis is its manor axis.

3.2.2 Neutral axis

A neutral axis is an axis in the cross-section of a beam member, where there are no longitudinal stresses or strains. For symmetric and isotropic member, the neutral axis coincides with the geometric centroid of the section. For structural steel members, as their mechanical properties can be used as in elastic and/or plastic states, both the elastic and plastic neutral axes of the cross-section are important.

In general, the location of Elastic Neutral Axis (*ENA*) and Plastic Neutral Axis (*PNA*) can be found respectively as:

$$y_{el} = \frac{\sum A_i z_i}{\sum A_i} \qquad (3.1)$$

Where: A_i – is the area of the i^{th} component of the cross-section
z_i - is the moment arm of the i^{th} area from a reference point to the centroid of the i^{th} area

And the Plastic neutral axis (*PNA*) can be found by dividing the cross-sectional area in to equal parts of compression and tension zones.

$$A_C = A_t \qquad (3.2)$$

ENA and *PNA* of a doubly symmetric I-section is given by respectively:

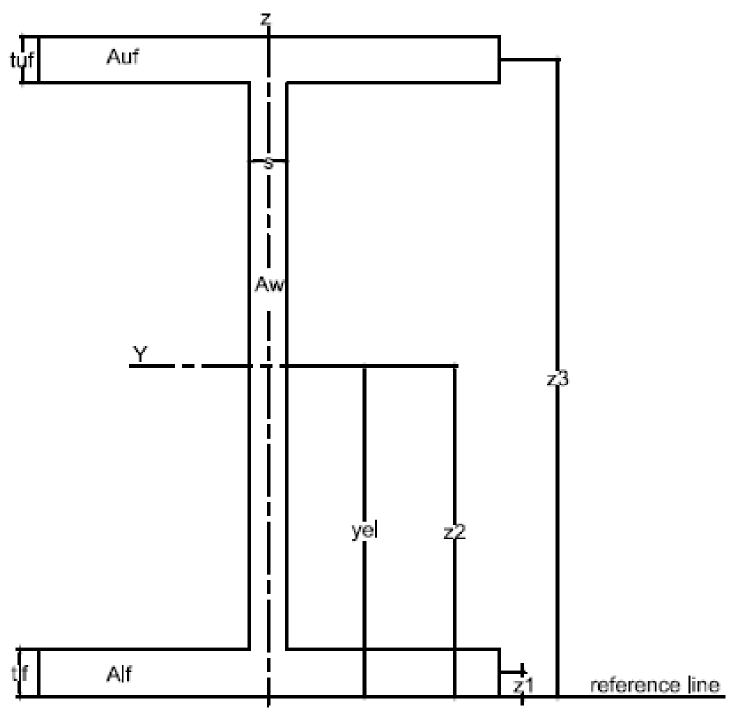


Figure 3.1: Elastic neutral axis(ENA)

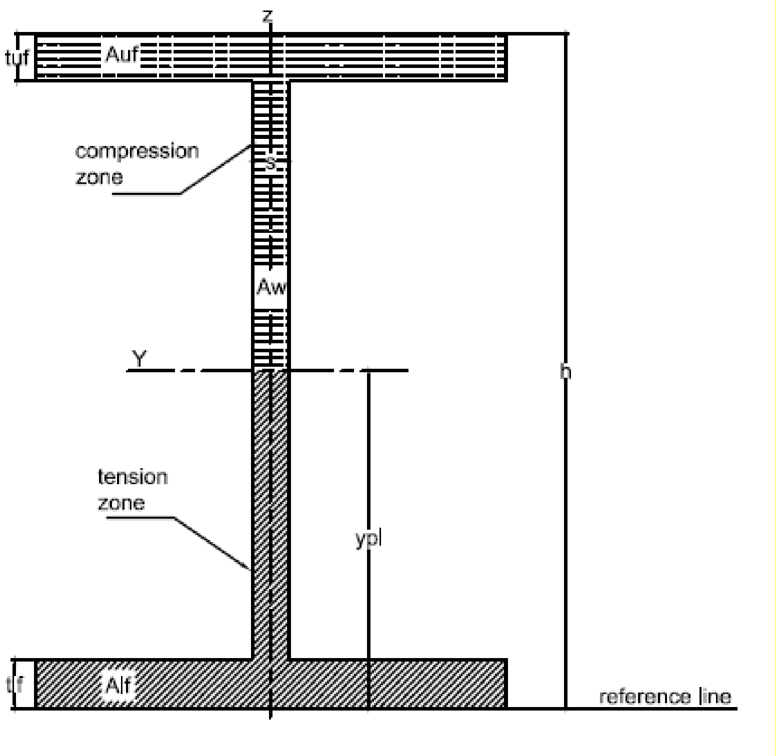


Figure 3.2: Plastic neutral axis(PNA)

$$y_{el} = \frac{A_{l_f} z_{1+} A_w z_{2+} A_{u_f} z_3}{A_{u_f} + A_w + A_{b_f}} = \frac{h}{2} \tag{3.3}$$

$$A_{l_f} + \left(y_{pl} - t_{l_f}\right) s = A_{u_f} + \left(h - t_{uf} - y_{pl}\right) s \tag{3.4}$$

$$y_{pl} = \frac{h}{2} \tag{3.5}$$

Where:

A_{u_f}, A_w, A_{l_f} – is the area of the upper flange, lower flange and web.

t_{u_f}, t_{l_f} , s - is the thickness of the upper flange, lower flange and web
z_1, z_2, z_3 – are the area moment arm of A_{l_f}, A_w & A_{u_f} respectively.
h – is the total height of the I-section

3.2.3 Moment of inertia (I)

The moment of inertia which is also known as the second moment of area is a geometrical property of a cross-section which reflects how its points are distributed with respect to a given axis, in this case the neutral axis. In structural analysis, the second moment of area of a beam cross-section is used in the calculations of the beams deflection and the calculations of a stress caused by a moment. In general, the second moment of area of an arbitrary cross-section about any axis can be found using the Parallel Axis Theorem as:

$$I_{xx} = I_C + Ad^2 \tag{3.6}$$

Where: d – is the moment arm from the centroid of the area to the reference axis
A – is area of the cross-section
I_C - Is the second moment of area about its centroid

For a doubly symmetric I-section, the second moment of area parallel to Y- axis about the elastic neutral axes is given by:

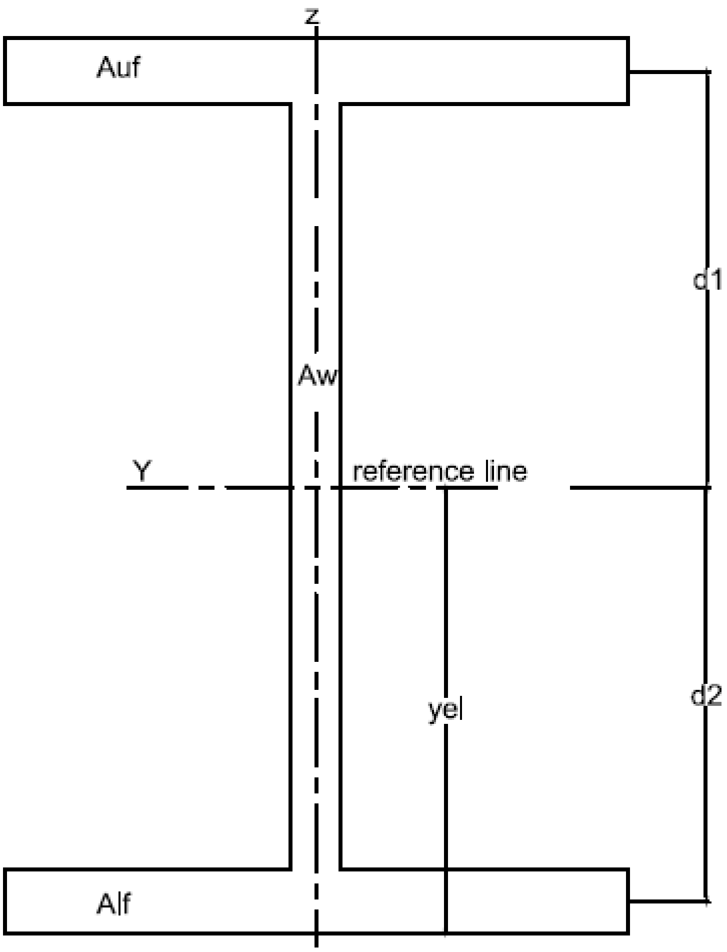


Figure 3.3: Second moment of area of an I-section

$$I_y = I_{uf} + A_{uf}d_1^2 + I_w + I_{lf} + A_{lf}d_2^2 \quad (3.7)$$

Where: I_{uf}, I_{bf}, I_w – is the second moment of area of the upper flange, lower flange and web about their respective centroids.
A_{uf}, A_{lf}, A_w – is the area of the upper flange, lower flange and web.
d_1, d_2 – are the distances between the centroid of the upper flange & lower flange and ENA of the I section.

3.2.4 Elastic section modulus *(w_{el})*

Elastic section modulus of a structural element is a cross-sectional property which is used to relate the stress and moment at a specific distance from the neutral axis. The concept of the section modulus property is to optimize the bending resistance of a beam cross-sections by increasing the area at the extreme edge of a cross-section. For that reason, most beam cross-sections have an I-shape. As evident from the name, elastic section modulus is when stresses in the material are within the linear, elastic range, or that the strain in the extreme fibres do not exceed elastic limit. From the flexure formula:

$$\sigma_z = \frac{M_y z}{I_y} \tag{3.8}$$

$$\sigma_z = M_y w_{el} \tag{3.9}$$

Where: σ_z – is the bending stress at the Z distance from the elastic neutral axis
M_y – is the bending moment about Y-axis
I_y – is the second moment of area about Y-axis
z – is the distance from the *ENA*

From the above expression, elastic section modulus is then given by:

$$w_{el} = \frac{I_y}{z} \tag{3.10}$$

NOTE: For doubly symmetric I-sections $z = h/2$

3.2.5 Plastic section modulus *(w_{pl})*

Like the elastic section modulus, plastic section modulus is also a section property for structural steel members to estimate the plastic capacity of the section. Plastic section modulus is the summation of the first area moment of the areas under tension and compression about the plastic neutral axis.

$$W_{p_l} = A_C z_C + A_t z_t \tag{3.11}$$

Where: A_C , A_t – is the areas under compression and tension respectively
z_C , z_t – is the area moment arm of the compression and tension zones respectively

For doubly symmetric I-section, the plastic section modulus can be found by:

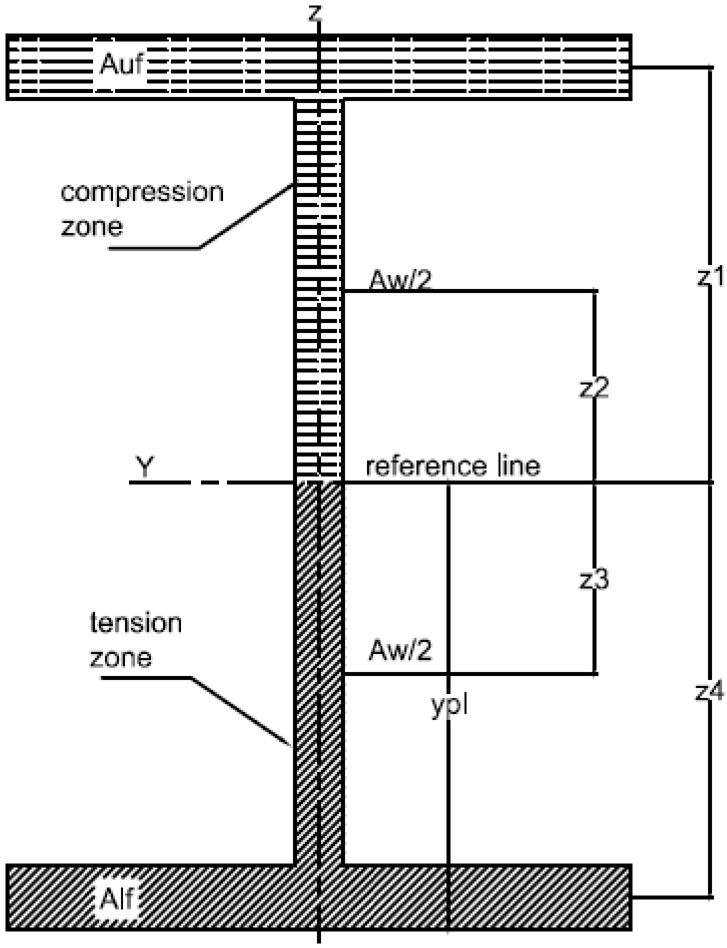


Figure 3.4: Plastic section modulus of I-section

$$W_{p_l} = A_{uf}z_1 + A_{lf}z_4 + \frac{A_\omega}{2}z_2 + \frac{A_\omega}{2}z_3 \qquad (3.12)$$

Where: A_{uf} , A_{lf} , A_ω – is the area of the upper flange, bottom flange and web.
z_1 , z_2 , z_3 , z_4 – is the respective moment arm of each area

3.2.6 Torsion constant (I_t)

Torsion constant is also one of the important geometrical properties of structural steel members**.** Mainly open cross-sections such us I-section, can easily be exposed to a torsion effect when they are exposed to eccentric loading. The basic equation for torsion according to Saint-Venant is as shown below[4].

$$\frac{d\varphi}{dx} = \frac{T}{GI_t} \qquad (3.13)$$

Where: T – Is Torsion
φ – angle of twist as shown in figure 3.5
G – Shear modulus
I_t – Saint-Venant torsional constant

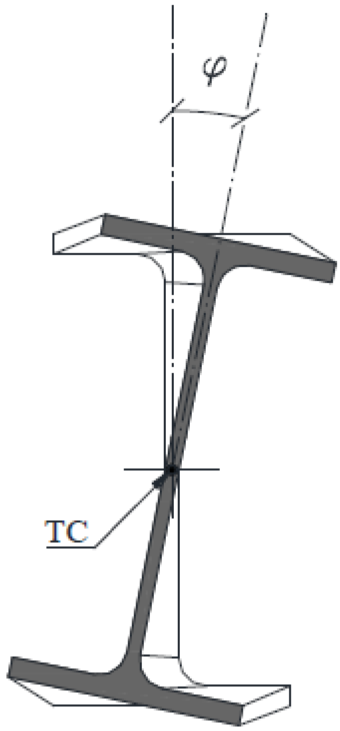


Figure 3.5: Twisting of I-shape with angle of twist (φ) [4]

The torsional constant, for cross-sections build up by rectangular parts can be approximated by:

$$I_t = \sum_{i=1}^{n} \frac{t_i h_i^3}{3} \tag{3.14}$$

Where: t_i – Thickness of cross-section part i
h_i – Height of cross-section part i
n – Number of cross-section parts

For members with and I-section, us we have considered in this thesis, the above expression, based on the above expression the torsional constant can be calculated by:

$$I_t = \frac{b_{uf}t_{uf}^3 + b_{lf}t_{lf}^3 + h_w t_w^3}{3} \tag{3.15}$$

Where: b_{uf}& t_{uf} – width & thickness of upper flange
b_{lf}& t_{lf} – width & thickness of lower flange
h_w& t_w – height & thickness of web

3.2.7 Warping constant

When a member is exposed to torsional effect, then there could be an effect of warping. The warping phenomenon is essentially a result of the top and bottom flange rotating about the torsion centre. As a result, for open section such as I-section, the strength against warping stress is essential. The above expression for torsion according to Saint-Venant is when warping effect is allowed to be free. But in real conditions warping can be restrained and the warping constant of the section must be calculated. For a member under torsion effect with a restrained warping the rotation of the member can be described by [4]:

$$EI_w \frac{d^4\varphi}{dx^4} - GI_t \frac{d^2\varphi}{dx^2} = m_x \tag{3.16}$$

Where: E – young's modulus of elasticity
I_w – Warping constant
m_x – torsional moment distributed along the beam.

The warping constant I_w can be calculated by:

$$I_w = \int_A \psi^2 \, dA \tag{3.17}$$

Where ψ is the warping function.

For I-section, just like the torsion constant, the warping constant can be approximated by:

$$I_w = \left(1 - \psi_f^2\right) I_z \left(\frac{h_s}{2}\right)^2 \tag{3.18}$$

$$\psi_f = \frac{I_{uf} - I_{lf}}{I_{uf} + I_{lf}} \tag{3.19}$$

Where: ψ_f – Differential ratio of the second moments of area of the beam flanges about the minor axis (z-axis)
I_z – Second moment of area about the minor axis
h_s – Distance between the shear centre of the upper and lower flanges
I_{uf} &I_{lf} – Second moment of area about the minor axis for the upper and lower flanges respectively

3.3 Mechanical properties

The significant mechanical properties of most structural steel are indicated in the tensile stress-strain diagram shown in figure 3.6. At the initial stage of the tensile loading, the steel has a linear stress-strain line whose slope is the young's modulus of elasticity E[1] .The steel behaves elastic while in this linear range and recovers perfectly to its original shape on unloading. The limit of the elastic state approximated by the yield stress f_y and the corresponding yield strain $\varepsilon_y = \frac{f_y}{E}$. Beyond this limit the steel behaves in a plastic way without any increase in stress till the strain is reached strain-hardening value ε_{st} . When the strain continues on increasing and exceeds strain-hardening ε_{st}, stress starts to increase above the yield value f_y , and this continues until the ultimate tensile stress f_u is reached. After this stage, a significant local reduction in the cross-section of the steel member occurs, and the load capacity decreases until tensile fracture takes place [1].

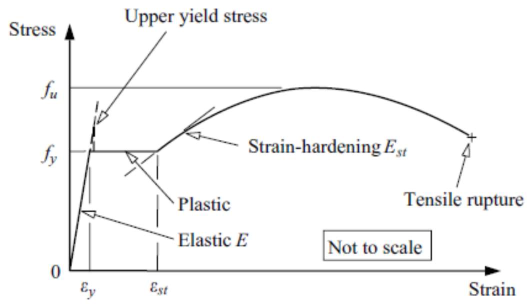


Figure 3.6: Idealised stress-strain relationship for structural steel [1]

As shown in figure 3.6 structural steel, unlike other construction materials, can be used up to its plastic limit (capacity) due to its ductile behaviour. However, the plastic capacity of structural steel is governed by the buckling behaviour of individual plates (web or flange) of the cross-section member.

This time Steel is one of the most used construction materials due to its outstanding mechanical properties related to behaviour under load or stress in tension, compression or shear. All structural steel is produced in several grades and according to different production process and chemical compositions, as specified in EN 10020(2000) [2].

Other important material properties of structural steel are[5]:

Modulus of elasticity E = 210000 $N\ /\ mm^2$
Shear modulus $G = \frac{E}{2(1+\nu)}$ ≈ 81000 $N\ /\ mm^2$
Poisson's ration in elastic stage $\nu = 0.3$

3.4 Capacity of compression members

As discussed in the section of behaviour of compression members, the modes of failure of the compression members are:

- Yielding of compressed area
- Local buckling of plates
- Flexural buckling of member

In this section the capacity (strength) of the compression members to these failure modes will be discusses.

3.4.1 Yielding capacity of compression members (stocky)

Stocky compression members can develop their full plastic capacity during axial compressive load, if the local buckling is not developed. The value of the maximum compressive force is given by [1]:

$$N_y = A_0 \times f_y \tag{3.20}$$

Where: N_y – is the squash load in KN
(maximum compressive force the member can carry)
f_y - is the yield strength of the material in N/mm^2
A_0 - is the gross cross-sectional area

3.4.2 Local buckling capacity of thin-plate components

The development of the above maximum compressive load can be prevented, if the thin- plate elements (flange and web) of the cross section starts to buckle locally. The local buckling of the plates mainly depends on the width-to-thickness ratio or plate slenderness and the material yield strength f_y .

The plastic and elastic buckling capacity (strength) of rectangular plate elements can be found as:

1. Elastic buckling capacity of Plate elements under compression

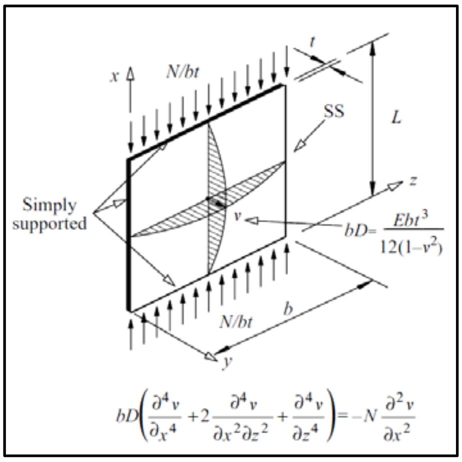


Figure 3.7: Plate buckling [1]

When a simply supported thin flat plate element of length L, width b, and thickness t as shown in figure 3.7 is under a uniform compressive force over both ends, then the elastic buckling load N_{Cr} at which the plate buckles corresponds to the buckling stress σ_{cr} is[1]:

$$\sigma_{cr} = \frac{N_{cr}}{b \times t} \tag{3.21}$$

Which is given by:

$$\sigma_{cr} = \frac{\pi^2 E}{12(1-\nu^2)} \frac{k_\sigma}{\left(\frac{b}{t}\right)^2} \tag{3.22}$$

Where: k_σ - is the buckling coefficient
E - Elastic modulus

ν - poison`s ratio
b - width of plate
t - thickness of plate

The elastic critical buckling stress σ_{cr} varies inversely proportional to the square of the plate slenderness or width-thickness ratio b/t ,in which the dimensionless buckling stress σ_{cr} / f_y can be expressed as:

$$\sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{b}{t}\sqrt{\frac{f_y}{E}\frac{12(1-\nu^2)}{\pi^2 k_\sigma}} \tag{3.23}$$

If the material ceases to be linear elastic at the yield stress f_y ,the above analysis is only valid for $\sqrt{(f_y / \sigma_{cr})} \geq 1$. This limit is equivalent to a width-thickness ratio b/t given by:

$$\frac{b}{t}\sqrt{\frac{f_y}{235}} = 56.8 \tag{3.24}$$

For plates which are free along one longitudinal edge and simply supported along the other three edges, for example the outstand flange of many structural steel elements, the above equation for elastic buckling stress can be use in which the buckling coefficient is approximated by[1]:

$$k_\sigma = 0.425 + \left(\frac{b}{L}\right)^2 \tag{3.25}$$

For the long-plate elements which are used as flange outstands in many structural steel members, the buckling coefficient is close to the minimum value of 0.425. In this case the elastic buckling stress (for a steel for which $E = 210000N / mm^2$ and $\nu = 0.3$) is equal to the yield stress f_y when[1]:

$$\frac{b}{t}\sqrt{\frac{f_y}{235}} = 18.5 \tag{3.26}$$

2. Inelastic buckling capacity of thick plates

For stocky steel plates where the calculated elastic buckling stress is above the limit of yield stress f_y ,modification must be made to the elastic analysis [1]. A simplified modification that can be applied to strain-hardened steel plates is to use the strain-hardening modulus E_{st} and $\sqrt{(E * E_{St})}$ instead of E. With these modifications, the strain-hardening buckling stress of a long simply supported plate is equal to the yield stress when:

$$\frac{b}{t}\sqrt{\frac{f_y}{235}} = 32.1 \tag{3.27}$$

For simply supported plates

$$\frac{b}{t}\sqrt{\frac{f_y}{235}} = 8.2 \quad (3.28)$$

For plates which are free along one longitudinal edge

3.4.3 Flexural buckling capacity of compression members

As discussed in section 2.3 the behaviour of axially loaded compression members, the strength of compression members is governed by the flexural buckling capacity which is characterized by the critical buckling load (N_{Cr}) in addition to the local buckling and yield capacity of the member. The critical buckling load N_{Cr} is a measure of slenderness of the compression members, and its value decreases as the length of the member increases.

The load N_{Cr} at which a straight compression member buckles laterally can be determined by finding a deflected position, and this position is given by [1].

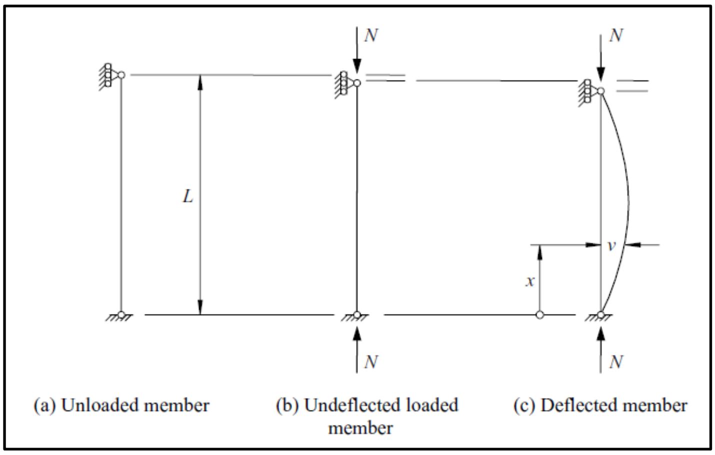


Figure 3.8: Lateral deflection (flexural buckling) of straight compression member [1]

$$V = \delta \sin \pi x / L \quad (3.29)$$

In which δ is the undetermined magnitude of the central deflection, and that the elastic buckling load is:

$$N_{Cr} = \pi^2 EI / L^2 \quad (3.30)$$

In which EI is the flexural rigidity of the compression member. The elastic buckling load N_{Cr} and the elastic buckling stress.

$$\sigma_{Cr} = \frac{N_{Cr}}{A_0} \quad (3.31)$$

can be expressed in terms of the geometrical slenderness ration $L/_i$ by :

$$N_{Cr} = \sigma_{Cr} A_0 = \frac{\Pi^2 E A_0}{\left(L/_i\right)^2} \quad (3.32)$$

In which $i = \sqrt{\frac{I}{A_0}}$ is the radius of gyration. From the above expression a dimensionless slenderness ration $\bar{\lambda}$ can be found from the ratio of the squash load $N_y = A_0 \times f_y$ to the critical elastic load N_{Cr}:

$$\bar{\lambda} = \sqrt{\frac{N_y}{N_{Cr}}} = \sqrt{\frac{f_y}{\sigma_{Cr}}} = \frac{L}{i}\sqrt{\frac{f_y}{\Pi^2 E}} \tag{3.33}$$

Where: L - is the critical length of the member
i - is the radius of gyration of the cross-section
f_y – is the yield strength of the material
E – modulus of elasticity of the material

3.5 Design of compression members based on EC3

3.5.1 Cross-section capacity (resistance)

For cross-section classes 1,2 and 3 the cross-section resistance is based on their plastic capacity, while in cross-section class 4 the plastic capacity should be calculated based on their effective cross-sectional area. According to clause 6.2.4(1), the cross-section resistance of axially compressed members is verified by the condition [5]:

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1.0 \tag{3.34}$$

Where: N_{Ed} – is the design value of the axial compression force
$N_{c,Rd}$ - is design resistance of the cross-section

Based on EC3 clause 6.2.4(2) the values of $N_{c,Rd}$ is given by [5]:

$$N_{C,Rd} = \frac{A_0 f_y}{\gamma_{M0}} \tag{3.35}$$

for cross-sections class 1,2 and 3;

$$N_{C,Rd} = \frac{A_{eff} f_y}{\gamma_{M0}} \tag{3.36}$$

for cross-section class 4.

Where: A_0 – is the gross area of the cross-section
A_{eff} – is the effective area of class 4 cross section
f_y - is the yield strength of the steel
γ_{M0} – is partial safety factor

3.5.2 Member capacity (resistance)

The resistance of compressed members is calculated based on the "European design buckling curves ", which are provided based on experimental and numerical research works. These buckling curves accounts the imperfections that are available in real compression members such as initial out-of-straightness, eccentricity of the loads, residual stresses [2]. The buckling resistance of a member under a design axial compression force N_{Ed} is verified by the following condition based on EC3 (clause 6.3.1.1(1)) [5]:

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1.0 \qquad (3.37)$$

Where: $N_{b,Rd}$ – is the design buckling resistance of the compression member.

The design flexural buckling resistance of prismatic members is given by EC3 (clause 6.3.1.1(3)) [5]:

$$N_{b,Rd} = \frac{xA_0 f_y}{\gamma_{M1}} \qquad (3.38)$$

for cross-sections class 1,2 and 3;

$$N_{b,Rd} = \frac{xA_{eff} f_y}{\gamma_{M1}} \qquad (3.39)$$

for cross-section class 4;

Where: x - is the reduction factor for the relevant buckling mode
γ_{M1} – is a partial safety factor

The reduction factor x is given by EC3 (clause 6.3.1.2(1)) [5]:

$$x = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}} \quad \text{but} \quad x \leq 1.0 \qquad (3.40)$$

Where $\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$ and $\bar{\lambda}$ is the non-dimensional slenderness coefficient, given by [5]:

$$\bar{\lambda} = \sqrt{\frac{A_0 fy}{N_{Cr}}} = \frac{Lcr}{i}\frac{1}{\lambda_1} \qquad (3.41)$$

for cross-section class 1,2 and 3

$$\bar{\lambda} = \sqrt{\frac{A_{eff} fy}{N_{Cr}}} = \frac{Lcr}{i}\frac{\sqrt{\frac{A_{eff}}{A_0}}}{\lambda_1} \qquad (3.42)$$

for cross-section class 4

Where: N_{Cr} - is the elastic critical load for the relevant buckling mode
L_{Cr} - is the critical length of the corresponding buckling mode
i - is the radius of gyration of the cross-section
α - is the imperfection factor corresponding to the appropriate buckling curves which can be obtained from Table 6.1 & Table 6.2 EC3.
λ_1 - is given by $\lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93.9\varepsilon$, and $\varepsilon = \sqrt{\frac{235}{f_y}}$, f_y in $N\,/\,mm^2$

3.6 Capacity of Beam members

As discussed in *section 2.4* the behaviour of beam members, the modes of failure of beam members can be classified as:

- Yielding of compression flange and/or fracture of tension flange
- Shear yielding of web
- Bearing failure /crashing of web
- Local buckling of compression flange or local buckling of web
- And over-all flexural lateral torsional buckling

3.6.1 Elastic bending capacity

Case 1: Elastic bending capacity due to pure bending only

When a beam bends in the xz principal plane, plane cross-sections rotate as shown in Figure 3.9 , so that sections δx apart become inclined to each other at $-\left(\frac{d^2w}{dx^2}\right)\delta x$, where w is the deflection in the z principal direction , and δx is the length along the centroidal axis between the two cross-sections. The length between the two cross-sections at a distance z from the axis is greater than δx by $z\left(-\frac{d^2w}{dx^2}\right)\delta x$, so that the longitudinal strain is[1]:

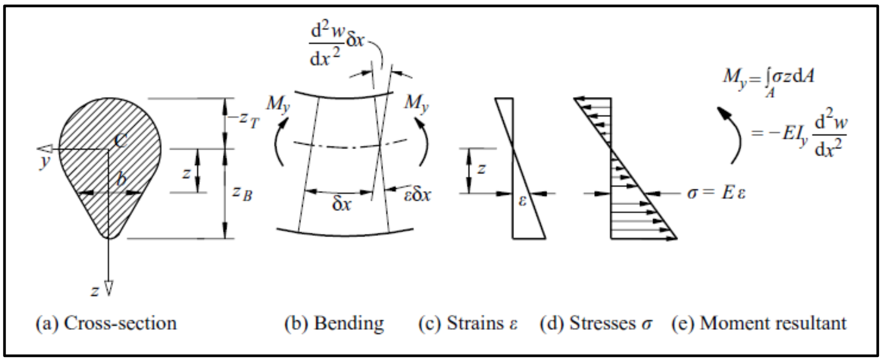


Figure 3.9: Elastic bending of beams [1]

$$\varepsilon = -z\,\frac{d^2w}{dx^2} \tag{3.43}$$

The corresponding tensile stress $\sigma = E\varepsilon$ is

$$\sigma = -Ez\,\frac{d^2w}{dx^2} \tag{3.44}$$

Which has the stress resultants

$$N = \int_A \sigma \, dA = 0 \tag{3.45}$$

Since $\int_A z \, dA = 0$ for centroidal axes, and $M_y = \int_A \sigma z \, dA$

Where:

$$M_y = -EI_y \frac{d^2w}{dx^2} \tag{3.46}$$

Substituting $\int_A z^2 \, dA = I_y$ into the above equation, the bending tensile stress σ is then obtained:

$$\sigma = \frac{M_y z}{I_y} \tag{3.47}$$

From this the elastic bending moment capacity of a given beam cross-section can be calculated by:

$$M_y = f_y w_{e_l,y} \tag{3.48}$$

Where: f_y – is the material yield strength
$w_{e_l,y}$ - is the elastic section modulus given by $\frac{I_y}{z}$

Case 2: Elastic capacity under combined loading (Bending and axial load)

Most often a structural member is subjected to different types of stresses that act simultaneously. In which, in this case is flexural and axial stresses due to bending and axial forces respectively. In such situations superposition method is used to determine the combined effect of these stress acting over the cross-section of the member.

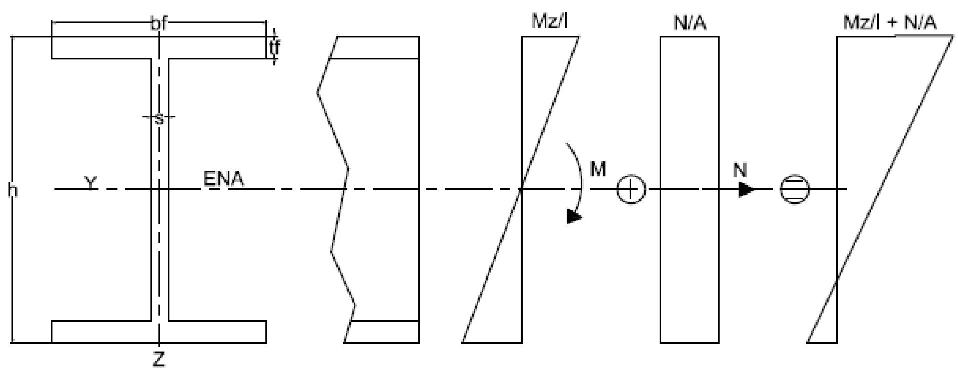


Figure 3.10: Combined loading in elastic state

And the maximum stress the member can resist can be found by:

$$\sigma = -\frac{N\sigma}{A} \pm \frac{Mz}{I} \tag{3.49}$$

Where: σ – is the maximum stress at the extreme fiber
N & M – are the external axial and moment forces respectively
A - is the gross area of the cross-section
I - is the moment of inertia of the section
z – is the distance from the ENA to the extreme fibre.

3.6.2 Plastic bending capacity

Case 1: Plastic bending capacity due to pure bending

While a beam is in a purely elastic state the curvature in any cross section of the beam is given by the ratio[6].

$$\frac{d^2w}{dx^2} = -\frac{M_y}{EI_y} \tag{3.50}$$

Where EI_y denotes the elastic cross-sectional stiffness. However, after yielding occurs in the area of maximum moment, the moment/curvature relationship ceases to be linear, and a plastic hinge is formed. Plastic hinge is defined as where all normal bending stresses (σ) along the total height of the cross-section attains the limit yield stress f_y as shown in Figure 3.11. At this point each fibre in the beam is yielding in either tension or compression. Corresponding bending moment is known as plastic moment capacity and is determined by the following equation[7].

$$M_p = \int_A f_y z\, dA = f_y w_{Ply} \tag{3.51}$$

Where w_{Ply} is the plastic section modulus, equal to the sum of the moments of the parts of the cross-section above and below plastic neutral axis with respect to the plastic neutral axis at the stage of full plasticisation of cross-section [7].

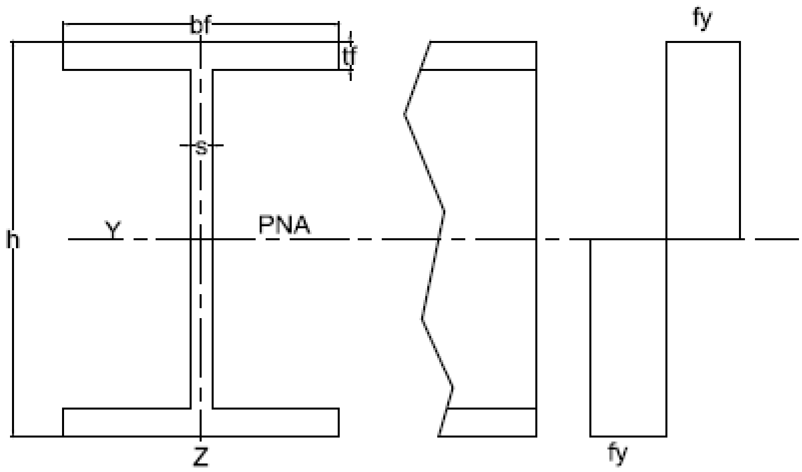


Figure 3.11: Pure bending plastic stress distribution

Case 2: Plastic capacity under combined loading (Bending and axial load)

For a fully plastic capacity of the cross-section during a combined loading of bending and axial load, the combination of the amount of moment and axial load is important. The reduced plastic moment capacity for an I-section under a combined loading can be determined as follows.

1. **When the *PNA* lies on the web**

The plastic stress distribution under combined moment and axial load on a symmetric I-section is as shown in Figure 3.12 when $N < A_w f_y$.

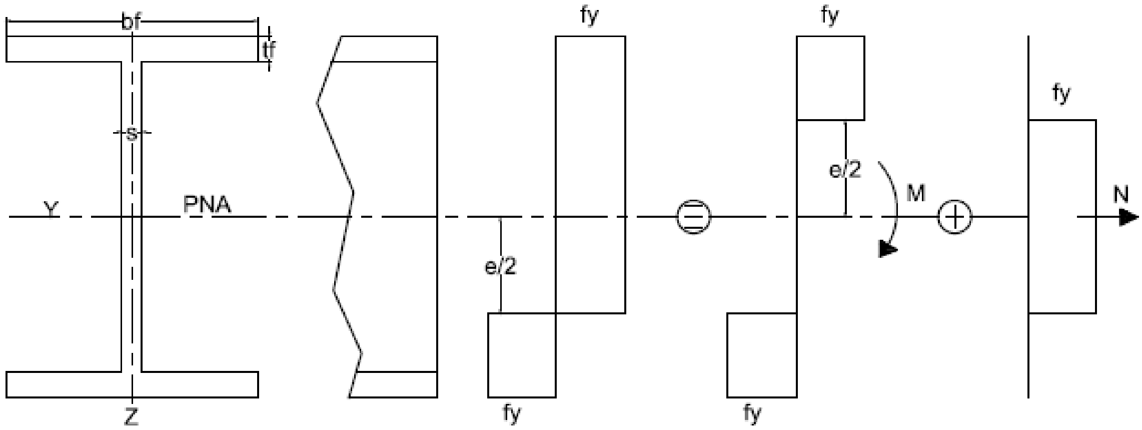


Figure 3.12 Combined loading of I-section. $N < A_w f_y$

In this case the axis of zero stress lies in the web at a distance $\frac{e}{2}$ from the centre axis of the section. The expression for the bending moment then will be [6].

$$M = M_P - \left(\frac{se^2}{4}\right) f_y \qquad (3.52)$$

The axial force is equal to

$$N = esf_y \qquad (3.53)$$

Combination of the above two equations gives:

$$M = M_P - \frac{N^2}{N_p^2}\left(\frac{A_0}{A_w}\right)\left(\frac{sh_w^2 f_y}{4}\right) \qquad (3.54)$$

Where N_P and M_P are the full plastic axial and moment capacities, A_0 and A_w are the gross area and web area respectively, and s and h_w are the thickness and height of web. Introducing the plastic section modulus of I-section and rearranging the above equation, the interaction formula become [6]*:*

$$\frac{M}{M_P} + \left(\frac{N}{N_P}\right)^2 \frac{1}{2\frac{A_W}{A_0} - \left(\frac{A_\omega}{A_0}\right)^2} = 1\,, \quad \frac{N}{N_P} \leq \frac{A_\omega}{A_0} \qquad (3.55)$$

NOTE: the above equation is valid for values of axial load N less than the web capacity $A_w f_y$ or for $\frac{N}{N_P} < \frac{A_w}{A_0}$.

2. When the *PNA* lies on the bottom flange

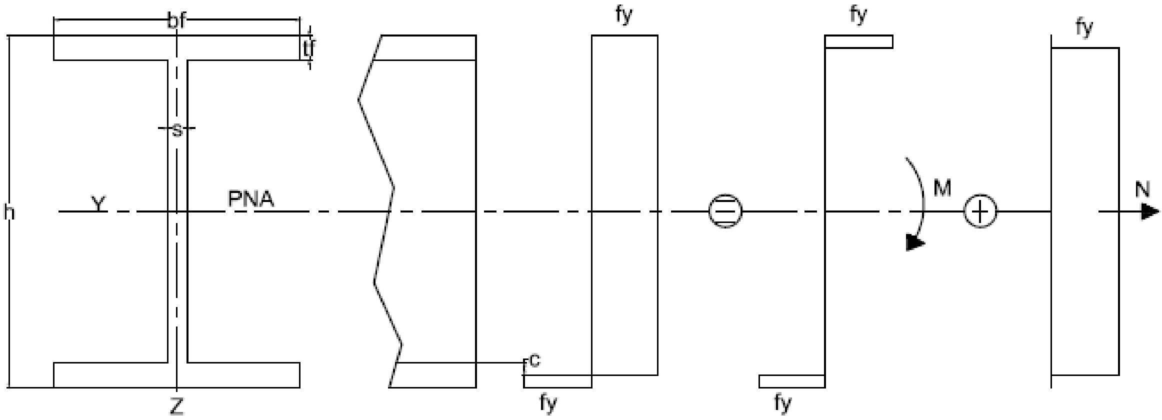


Figure 3.13: Combined loading of I-section. $N > A_w f_y$

For axial force N greater than the web capacity $A_w f_y$ the stress distribution will be as shown in Figure 3.13.

Denoting by c the distance from inner flange surface to axis of zero stress, the following expression

$$M = M_P - \left[b_f c(h_w + c) + \frac{A_w h_w}{4}\right] f_y \qquad (3.56)$$

$$N = \left(A_w + 2b_f c\right) f_y \qquad (3.57)$$

For thin-walled sections $C << h_w$. Combination of the above equations gives:

$$M = M_p - \frac{N h_w}{2} + \frac{A_w h_w}{4} f_y \qquad (3.58)$$

Introducing the plastic section modulus and $N_P = A_0 f_y$ the interaction formula becomes:

$$\frac{M}{M_p}\left(1-\frac{A_w}{A_0}\right)+\frac{N}{N_P}=1\,,\frac{N}{N_P}\ \geq \frac{A_w}{A_0} \qquad (3.59)$$

3.7 Design of beam members based on EC3

3.7.1 Cross-section capacity (resistance)

For a compact section (class 1 or 2), the bending resistance of the cross-section can be obtained from their plastic capacity, on the other hand for slender sections (class 3 or 4) which are susceptible to local buckling, the bending resistance calculated based on their elastic capacity.

In the absence of shear forces, the design value of the bending moment M_{Ed} at each cross-section should satisfy the condition below based on EC3(clause 6.2.5(1)) [5]:

$$\frac{M_{Ed}}{M_{c,Rd}} \leq 1.0 \qquad (3.60)$$

Where $M_{c,Rd}$ is the design resistance for bending, and the design bending resistance about a given principal axis of a cross-section is given by EC3 (clause 6.2.5(2)) [5]:

$$M_{c,Rd}=\frac{w_{p_l}f_y}{\gamma_{M0}} \quad \text{for cross-section class 1 and 2} \qquad (3.61)$$

$$M_{c,Rd}=\frac{w_{el,min}f_y}{\gamma_{M0}} \quad \text{for cross-section class 3} \qquad (3.62)$$

$$M_{c,Rd}=\frac{w_{eff,min}f_y}{\gamma_{M0}} \quad \text{for cross-section class 4} \qquad (3.63)$$

Where: w_{p_l} – is the plastic section modulus

$w_{el,min}$ - is the minimum elastic section modulus

$w_{eff,min}$ - is the minimum elastic section modulus of the reduced effective area

f_y - is the material yield strength

γ_{M0} – is the partial safety factor

Following to EC3(clause 6.2.9.1((1)) where an axial force is present, allowance should be made for its effect on the plastic moment resistance. For class 1 and 2 cross-sections, it can be verified by EC3 (clause 6.2.9.1((2)) [5]:

$$M_{Ed} \leq M_{N,Rd} \qquad (3.64)$$

For cross-sections where fastener holes are not to be accounted for, the following approximations may be used for standard rolled I or H sections and for welded I or H sections with equal flanges EC3 (clause 6.2.9.1((5))[5]:

$$M_{N,y,Rd} = M_{pl,y,Rd} * \frac{(1-n)}{(1-0.5a)} \text{ but } \quad M_{N,y,Rd} \leq M_{pl,y,Rd} \tag{3.65}$$

Where: $n = N_{Ed} \,/\, N_{p_l,Rd}$

$a = \left(A - 2bt_f\right) / A$ but $a \leq 0.5$

3.7.2 Member capacity (resistance)

A laterally unrestrained member subject to major axis bending should be verified against lateral-torsional buckling as follows (EC3 (clause 6.3.2.1((1)) [5]:

$$\frac{M_{Ed}}{M_{b,Rd}} \leq 1.0 \tag{3.66}$$

Where M_{Ed} is the design value of the moment and $M_{b,Rd}$ is the design buckling resistance moment given by EC3 (clause 6.3.2.1((3)) [5]:

$$M_{b,Rd} = x_{LT} w_y \frac{f_y}{\gamma_{M1}} \tag{3.67}$$

Where w_y is the appropriate section modulus given as follows:

- $w_y = w_{pl,y}$ for Class 1 or 2 cross-sections
- $w_y = w_{el,y}$ for Class 3 cross-sections
- $w_y = w_{eff,y}$ for Class 4 cross-sections

x_{LT} is the reduction factor for the lateral-torsional buckling.

In EC3-1-1 two methods for the calculation of the reduction coefficient x_{LT} are proposed, namely a general method that can be applied to any type of cross-sections (more conservative) and an alternative method that can be applied to rolled cross-sections or equivalent welded sections.

According to the general method (clause 6.3.2.2), the reduction factor x_{LT} is determined by the following equation [5]:

$$x_{LT} = \frac{1}{\phi_{LT} + (\phi_{LT}^2 - \bar{\lambda}_{LT}^2)\text{^}0.5} \qquad \text{But } \; x_{LT} \leq 1.0 \tag{3.68}$$

Chapter 4

Corrosion of Steel structures

4.1 Introduction

Structures in general undergoes different forms of change during their service life starting from the moment they are constructed. These changes may be grouped into 4 different types, namely Physical change, Technological change, change to knowledge and safety requirements and structural information change[8]. Deterioration of steel structures falls in to the category of physical change. Steel structures, as well as other structures, deteriorate over time. Deterioration of existing steel structures is the main problem and it specifically refers to any process involving degradation of metal structures or components caused by corrosion, mechanical wear and tear and fatigue. For steel structures one of the dominant deterioration factors is corrosion. In this thesis the only deterioration mechanism considered is corrosion, but several of the methods and formulas developed here may be applicable also for other types of deterioration.

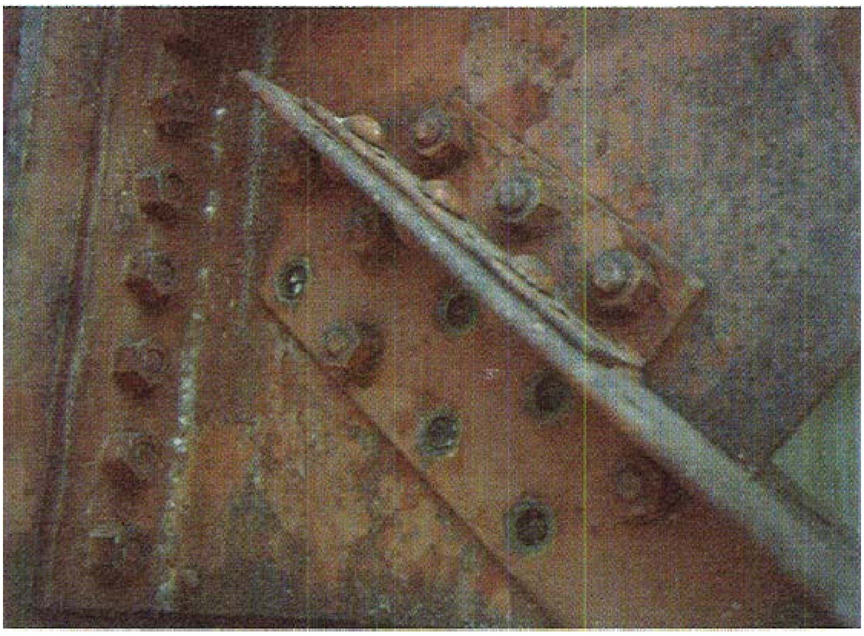


Figure 4.1: Corrosion of steel structures [9]

4.2 Corrosion

Corrosion is defined as the destruction or deterioration of a material because of a reaction with its environment[10]. There are many forms and classifications of corrosion, below are a list of corrosion forms based on their appearance.

- Uniform, or general attack
- Galvanic, or two-metal corrosion
- Crevice corrosion
- Pitting
- Intergranular corrosion

- Selective leaching, or parting
- Erosion corrosion
- Stress corrosion.

As it is seen from the list above most of the forms of corrosion are not easy to predict, and the damage is restricted to specific areas or parts of a structure. As a result, they tend to cause a sudden or an early failure of steel structures. Uniform attack, or general corrosion is the most commonly seen form of corrosion and it represents the greatest amount of the corrosion damage. Uniform attack normally is characterized by a chemical reaction of a metal with its environment that causes a section loss uniformly distributed over the entire exposed surface or over a large area. Consequently, the metal becomes progressively thinner and eventually fails [10].

Structures in general are built in different environments such as rural, urban, off-shore etc. As a result, the corrosivity of the environments and the corrosion damage rate is different from place to place. Corrosion rate is commonly quantified as the mass loss per unit time, or as the rate of the thickness loss. The rate of material loss due to uniform corrosion of steel in different environments has been studied and evaluated using a large amount of data and it can be estimated using an exponential function expressed below[11].

$$C = At^B \tag{4.1}$$

Where: C – represents the average corrosion penetration in μm
t - is number of years and
A & B – are random variable which depends on the environment where the structure is located.

4.3 Effects of corrosion on steel structures

Overall, corrosion affects the long-term performance and integrity of steel structures. The effects of corrosion differ with the type of structure, the location and extent of deterioration on a structure. As a result, steel structures can be affected by corrosion in many ways. Corrosion deteriorates the performance of steel structures with time. The results of this deterioration can range from progressive weakening of the structures to a sudden failure of the structures. The weakening of the structural members over a long time caused by section loss can lead to unpredicted change of behaviour of the structures and can affect the failure mode of the structure. Deterioration of steel have been the cause for many structural failures, for example corrosion has a great role in the collapse of both Point Pleasant (Silver) Bridge in 1967 and the Mianus River Bridge (Connecticut) in 1983[9].

The major effect of corrosion on steel structures is the loss of material from its exposed surfaces resulting a reduction of the section properties of the structural members and hence its load carrying capacity. There is also an increase in the level of uncertainty about the structural performance, due inherent randomness in the deterioration process [11]. Deterioration affects various structural parts differently due to their location and load carrying behaviour. Indeed, corrosion may reduce the ductility of steel member as well[12].

A reduction in the effective area of a member's cross-section will cause an overall increase in members stress for a given design load. If the loss of material is local, as in the case of pitting, small surface discontinuities or deep cut can cause stress increase which may increase the members vulnerability to fatigue [11]. A consequence of surface corrosion is the reduction in members cross-sectional properties, such as section modulus or the slenderness ratio, in which these properties are critical in a member's ability to resist bending moments or axial forces. Overall failure of a steel beam may result from web buckling, compression yielding, or a combination of both as discussed in the behaviour of beam members. Uniform corrosion can increase the width-thickness $b/_t$ ratio (plate slenderness) and result in localized buckling of the steel beam member. At higher levels of corrosion, the ultimate capacity of a steel member may fall below the service loads and in some cases the mode of failure due to loss of material can be changed since the class classification of the cross-section is changed. For example, a plastic section may become semi-compact section, and local buckling may prevent the development of full plastic capacity

Corrosion products (wastage) accumulated in steel structures such as bridges, may also affect structural performance [11]. These accumulated corrosion products can act as a drying agent by retaining moisture and so promote a further corrosion. In addition, a growing "pack rust" can apply a substantial amount of pressure on adjacent structural elements. Such pressure can cause a damage to connections by wedging apart connected elements [11].

In this thesis the focus is given to the shift (eccentricity) of neutral axes of the cross-section due to change of cross-sectional shape caused by material loss due to corrosion. The shift of neutral axes causes an additional external load and affect the behaviour of the structural members and it decrease the members buckling capacity. For laterally loaded member such as beam members, a shift of eccentricity will introduce torsion and for column or beam column members a torsion and eccentric buckling can be introduced. These additional external forces change the behaviour of the structural members and can cause unpredicted failure of the structures.

4.4 Prevention and mitigation techniques

Commonly recommended method of preventing corrosion is the selection of a proper metal or alloy for a given corrosive environment in which a structure is built. By introducing proper prevention and mitigation techniques based on the characteristics of the deterioration forms and the rate at which the damage occurs the service life of steel structures can be extended and protected from deterioration. Uniform attack of corrosion, for example, can be prevented or reduced by proper material choices, including coatings, inhibitors and Cathodic protection [10].

Surface treatment is the most common type of corrosion protection, were steel is usually painted or galvanized on its surface. Typical forms of surface protection provide a barrier layer between the metal and its environment, particularly water, which is the common electrolyte for corrosion. Sacrificial coatings such as aluminium, zinc or aluminium zinc alloy are also used to protect steel. Sacrificial coatings protect the steel by acting both as a barrier coating and a cathodic protection. Copper-bearing steel and high-strength, low-alloy steel (weathering steels) form an initial layer of rust when exposed to the environment which adheres to the metal surface to protect it from further exposure and corrosion. Another way of protecting structural failure due to corrosion damage is to repair the corrosion damage by restoration of the damaged member to its original cross-sectional area based on engineering evaluation of the inspection findings [9].

To prevent the effects of deterioration of steel structures due to corrosion, a durability-based design approach of steel structures is vital based on ultimate and serviceability limit states. In respect to that, most of the structural design codes provide only common protective principles. Eurocode (EN 1993-1-1) gives recommendations and requirements for a durable steel structures design, which can be practiced during design phase and/or during the life of the structures. For steel structures, the durability depends on the effects of corrosion, mechanical wear and fatigue, consequently, parts susceptible to corrosion, mechanical wear or fatigue should be designed such that inspection, maintenance and reconstruction can be carried out satisfactorily and access is available for in-service inspection and maintenance. For parts that cannot be inspected an appropriate corrosion allowance should be included, by introducing an additional thickness at the design stage .In addition, it recommends that the effects of deterioration of material, corrosion or fatigue where relevant should be taken into account by appropriate choice of material (use of weathering steel or stainless steel (EN 1993-1-4)) and details or by structural redundancy and by the choice of an appropriate corrosion protection system[5].

Chapter 5

Previous studies on remaining capacity of corroded steel structures

5.1 Introduction

There have been several research works reported on remaining capacity of deteriorated steel structures in general. Most of the research works focuses particularly on a member of a structure such as a beam member, compression or a tension member. In addition, they study a specific capacity of the member, mostly the critical ones, which is the moment capacity, tension or compression capacity, and /or local or global buckling capacity. The shape of the members cross-section (I, H, Box girder etc) is also another differentiating factor among the studies done, because each cross section has a different section property and it makes it easy to deal with the change in section properties due to section loss. The main deterioration considered in most of the research papers is corrosion and some accounts for deterioration due to fatigue, creep and other factors. For the case of deterioration due to corrosion, uniform corrosion is the most common, and some papers are studying other type of localized corrosion types which occurs on a specific location of the member. Bridges and offshore structures in general was also the focus of the studies as they are the most susceptible structures to corrosion. In this section some of the research works related to this thesis project will be discussed.

These previous research studies are a base for this thesis project and motivated to analyse the remaining capacity of deteriorated steel structures based on the shift (eccentricity) of the neutral axes caused by the loss of material due to corrosion which is not been done in a previous works as far as my knowledge.

5.2 Simple assessment method of remaining moment capacity

A study conducted by Y. Sharifi and R. Rahgozar in 2010[13] aims to calculate the percentage remaining moment capacity of corroded I-beam sections using a simplified method. This method requires a thickness loss information provided by visual inspection and measurement. The corrosion type considered is a uniform corrosion and based on that two types of corrosion decay models were adopted for analysis purposes. The effects of corrosion in this assessment is basically based on section loss resulted on reduction of section properties such as cross-sectional area, moment of inertia, section modulus etc.

The analysis done in this study is based on the corrosion patterns which was specified by Kayser and Nowak where a severe corrosion occurs in the bottom one quarter of the web. Based on that two types of corrosion decay models were developed as shown in the figure 5.1 below.

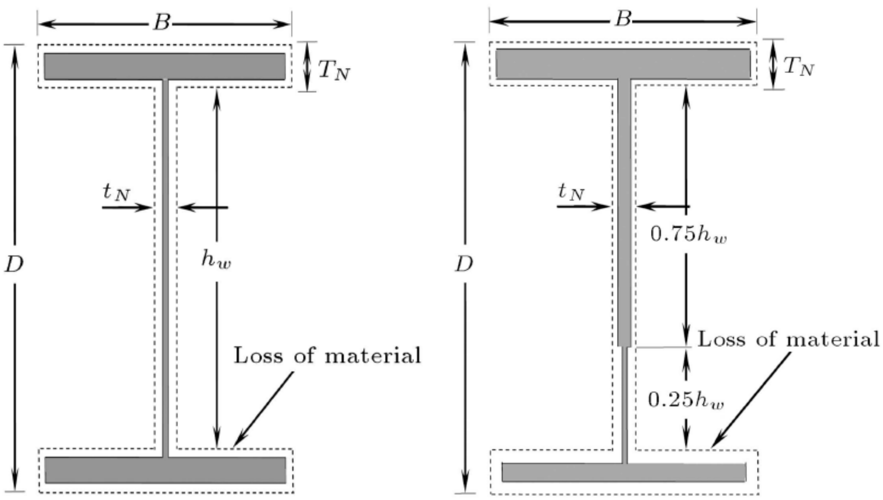


Figure 5.1: (a) Uniform thickness loss and (b) Varying thickness loss model [13]

Where: B – is the Flange width
D – is the depth of section
T_N – as-new thickness of flange
t_N - as-new thickness of web
h_w - height of web

For the uniform model both the flange and web is considered to loss material at the same rate ,while for the varying thickness loss model, the thickness loss in the bottom flange is approximately twice that of the loss in the top flange, and the thickness loss of the lower part in the web (0.25h_w) is nearly five times than the loss in the upper part of the web (0.75h_w). Based on the models the average thickness of flange and web of the corroded sections is calculated as follows.

NOTE: the results listed below is all studied and presented by Y. Sharifi and R. Rahgozar[13].

For the uniform thickness loss model:

Average thickness of corroded flanges:

$$T_C = T_N(1-\xi) \tag{5.1}$$

Thickness of the corroded web:

$$t_C = t_N(1-\xi) \tag{5.2}$$

Where: $\xi = \xi_F = \xi_w$ - % loss of flange and web thickness respectively given by

$$\xi = \%LFT/100 = \%LWT/100 \quad (5.3)$$

For the varying thickness loss model:

The loss of material is calculated as % of as-new section:

Top flanges $0.7\xi T_N$
Bottom flange $1.3\xi T_N$
Upper part of the web (0.75h_w) $0.25\xi t_N$
Lower part of the web (0.25h_w) $1.25\xi t_N$
Where in this case $\xi = \%LFT/100$

Therefore, based on the material loss the average flange and web thickness of the corroded section is calculated as follows:

Thickness of Top flange: $T_N(1 - 0.7\xi)$

Thickness of bottom flange: $T_N(1 - 1.3\xi)$

Average thickness of flanges: $T_C = T_N(1 - \xi)$

Similarly, the average thickness of the web is calculated based on the thickness loss of the upper and lower part of the web and is given by:

$$t_C = t_N(1 - 0.5\xi) \quad (5.4)$$

Where: $\xi = \xi_F = 2\xi_w$

These corroded dimensions are then used to calculate the relevant section properties of the corroded section. In this case to calculate the remaining moment capacity the plastic and elastic section modulus of the corroded section was calculated. For simplicity purposes the elastic section modulus is calculated from plastic section modulus divided by the shape-factor (SF) assumed a constant value of the shape factor for the new and corroded section.

The plastic section modulus for the corroded section is then calculated in relation to the plastic section modulus of the new section as:

For the uniform model:

$$z_{xc} \approx z_{xN} - \xi z_{xN} \approx z_{xN}(1 - \xi) \quad (5.5)$$

For the varying model:

$$z_{xc} \approx z_{xN} - \xi(z_{xN} - t_N \frac{h_w^2}{8}) \quad (5.6)$$

Where: z_{xc} & z_{xN} are the corroded and new plastic section modulus respectively.

The moment capacity of the corroded section is then calculated by:

$$M_{cC} = f_y Z_{xc} < 1.2 f_y Z_{xc} \quad (5.7)$$

For compact (plastic) sections

$$M_{cC} = f_y S_{xc} \quad (5.8)$$

For Semi-compact sections

$$S_{xc} = \frac{Z_{xc}}{SF} \quad (5.9)$$

Where S_{xc} is the elastic section modulus and SF is the shape factor for I-sections.

The percentage remaining moment capacity, $\%RMC$, is then determined as a function of the plastic section modulus of the corroded section (Z_{xc}) and the plastic section modulus of the new section (Z_{xN}) for both models using the equation below:

$$\%RMC = 100\left(\frac{Z_{xc}}{Z_{xN}}\right) \quad (5.10)$$

The study concluded that in addition to the section loss, some sections may change their class from compact to semi-compact due to corrosion while some sections may not. And this effect may prevent the development of full plastic capacity of corroded sections. The simple quantitative relationship between section loss and corresponding remaining capacity provided by this method helps on reaching a fast and reliable estimation of the remaining moment capacity and service life of the member.

5.3 Corrosion damage analysis considering lateral torsional buckling

In this recent study done by Secer and Uzun in 2017[14], investigates the effect of corrosion and lateral torsional buckling in the behaviour of steel structural frames using a non-linear structural analysis. In this study three types of steel structural frames were investigated under a period of 50 years of corrosion exposure. The time variant section loss was estimated for a given period of time based on the equation given below:

$$C = At^B \quad (5.11)$$

Where C is the average depth of corrosion (μm), t is the time of exposure in terms of years, A is the corrosion rate in the first year of exposure and B is the corrosion rate for representing the long-term decrease. A and B parameters depend on the environment in which the structure is located which can be found in relevant books for Rural, Urban and Marine areas. On this study the Urban area was used for the structures under investigation.

For evaluation of nonlinear analysis of steel frames, load increments were performed accounting the plastic interaction curve that represent full yielding of the cross-section, as a result, load parameters were determined using step by step procedure and these values were plotted at the end of each load step for each type of steel frames. In this study the cross-section plastic strength and lateral torsional buckling equations were used according to American National standard ANSI/AISC360-10 (specification for structural steel buildings). In that regard the cross sections plastic strength was defined by:

$$\frac{p}{p_y} + \frac{8M}{9Mp} = 1.0\ for\ \frac{p}{p_y} \geq 0.2 \quad (5.12)$$

$$\frac{p}{2p_y} + \frac{M}{Mp} = 1.0\ for\ \frac{p}{p_y} < 0.2 \quad (5.13)$$

Where: p is axial force, M is the bending moment, p_y is the squash load and M_y is the plastic moment capacity.

And the Lateral buckling equations based on the unbraced segment of the member (L_b) is given by the equations:

$$M_n = M_p = F_y Z_x \quad (5.14)$$

$$M_n = C_b(M_p - \left(M_p - 0.7F_y S_x\right)\left(\frac{L_b - L_p}{L_r - L_p}\right)) \leq M_p \quad (5.15)$$

$$M_n = F_{cr} S_x \leq M_p \quad (5.16)$$

Where : Z_x is the plastic section modulus about the strong axis , L_p & L_r are length limits, F_y is yield stress of the steel, S_x is the elastic section modulus, C_b is the lateral torsional buckling modification factor for non-uniform diagrams and F_{cr} is the critical yield point.

In this study the corrosion behaviour was investigated using a beam member, a single-story frame and a multi-story frame. In this thesis we will look only at the beam member to represent the result of the study as it is more related to the thesis.

Beam with fixed ends

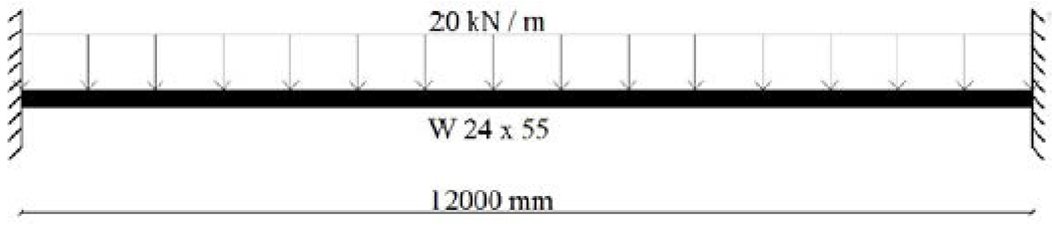
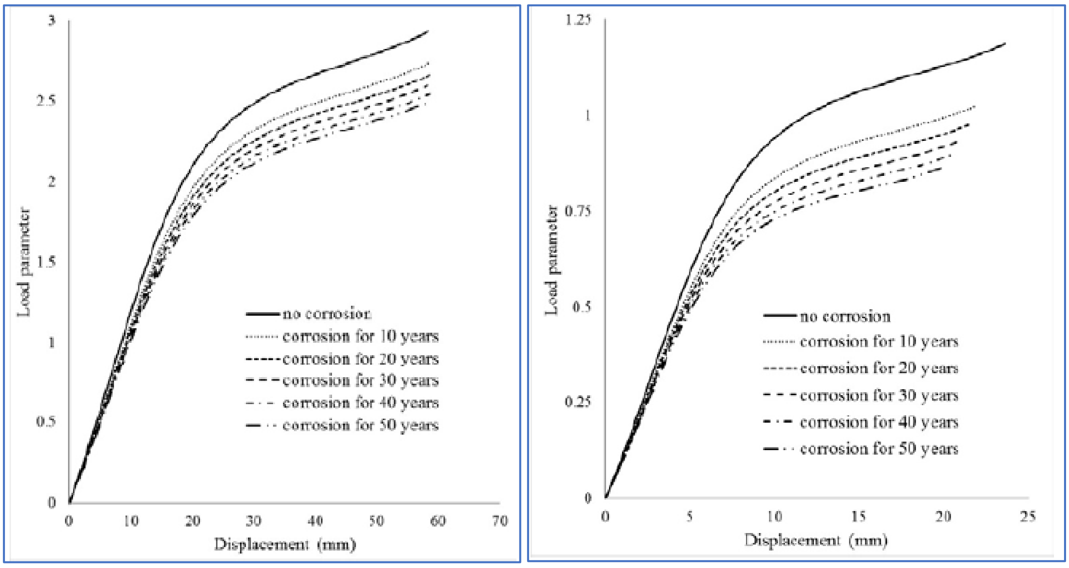


Figure 5.2: Beam member [14]

In this case a beam member was studied under a given load, support condition and beam dimensions as shown in figure 5.2 above and a structural behaviour was calculated accounting time-dependent corrosion for 50 years with and without lateral torsional buckling. For 10 years corrosion intervals the load increment parameter and midpoint vertical displacement relationships were plotted as shown in figure 5.3 below.



a b

Figure 5.3: Vertical point displacement (a) under corrosion only (b) under corrosion considering lateral torsional buckling [14]

As it can be seen from the graph (figure 5.3) the structural behaviour is highly influenced by corrosion particularly for the first 10 years and It was recorded that the load carrying capacity is decreased by 6.5% and 65.2% for cases without and with the consideration of lateral torsional buckling, which is clearly seen that accounting both corrosion and Lateral torsional buckling have a greater effect in the structural behaviour.

Based on the numerical study of the three steel frames the study then concludes that, the decrease on load carrying capacities and displacement capacities shows the influence of corrosion over the global structural behaviour. Consequently, this can lead to a decrease of performance to such an extent where the steel frames could not be able to fulfil their design service life.

5.4 Finite Element based analysis of deteriorated beams

Research conducted by John W. Van de Lindt and Theresa M. Ahlborn (2005)[15] was aimed to identify the common types of damage in steel beam ends and determine the critical damaged areas in a section and based on that to develop a simplified approach for calculating the reduced capacity of typical bridge beam sections.

A model for Finite Element Analysis was developed based on inspection reports of sixteen typical steel girder bridges. The Finite Element analysis was performed using a software called SDRC IDEAS and ABAQUS. In addition to that experimental work was performed to verify the validity of the Finite Element Analysis results. Damage parameters such as damage width, damage height and damage depth for both the bottom of the web and bottom flange were introduced for analysis purposes based on a sensitivity analysis.

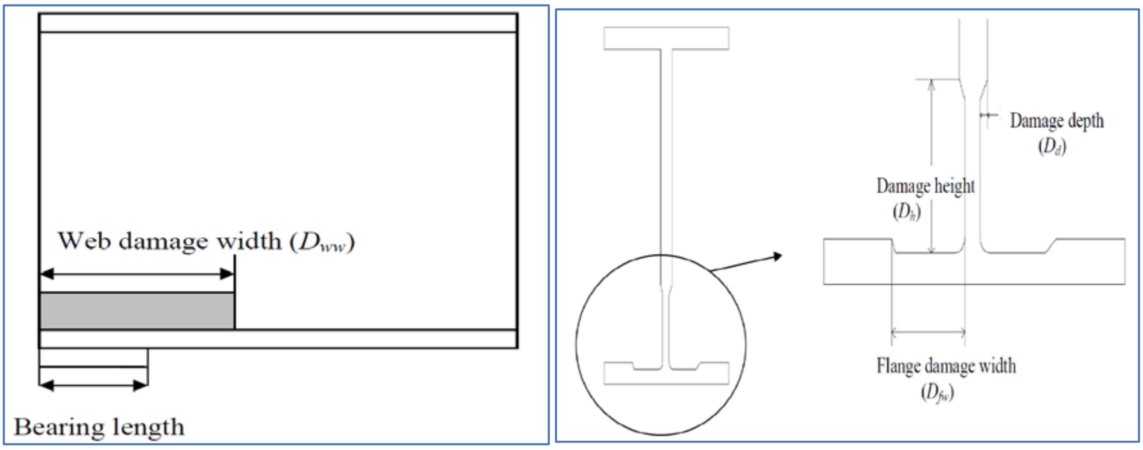


Figure 5.4: Damage parameters depth, width and height [15].

A numerical buckling/crushing analysis was then conducted to determine the remaining capacity and based on the results obtained a design charts were developed to assist the practicing structural engineer in evaluating damaged beams. The charts used the depth of damage and damage height ratio from the damage parameters to find a corresponding deterioration factor,ψ_d , for end beam corrosion damage. The damage height was determined by the ratio of damage height to unbraced web height. Finally, the reduced capacity can be found by multiplying the deterioration factor with the beams original capacity.

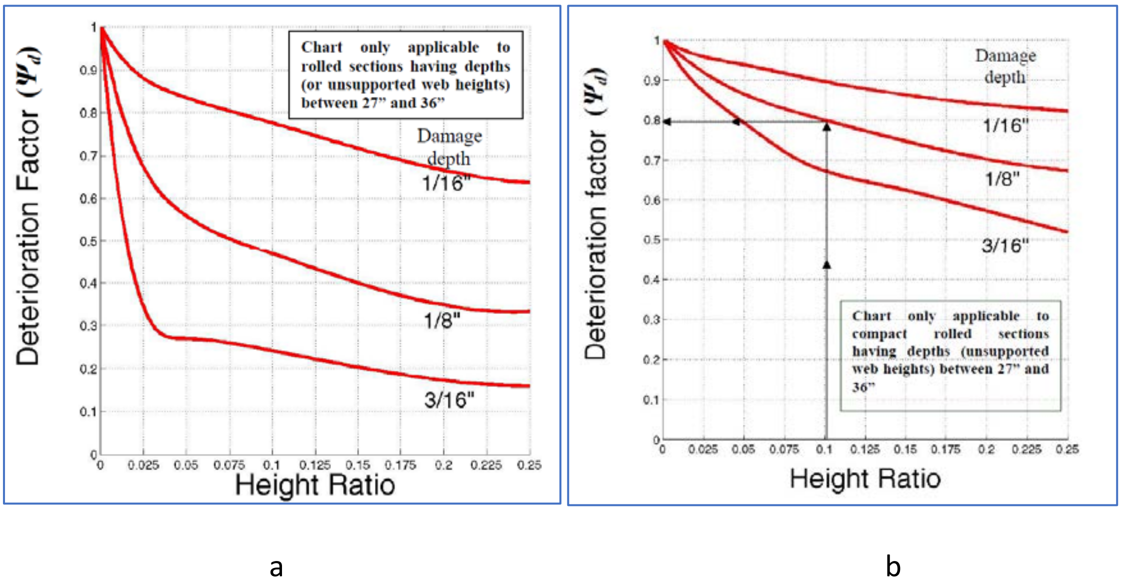


Figure 5.5: Design charts (a) for damage on one side of the web & (b) for damage on both sides of the web [15]

At the end of the study design charts were developed for various cases of damage based on FEA results. This guideline consists of factors used to estimate the reduced load capacity of the deteriorated steel beams from AISC capacity equations. The developed Deterioration Factor Method (DFM) is proved to be of the same accuracy as the traditional step by step calculation method which was previously in in practice.

Chapter 6

Geometrical properties of corroded I-section

6.1 Introduction

Even though some studies suggested that corrosion can have effect on the mechanical properties of steel, this thesis is mainly focuses on the effect of deterioration due to corrosion on section properties only. The main effect of corrosion is section loss of the member and consequently a reduction of the section properties such as the cross-sectional area, moment of inertia, elastic and plastic section modulus etc. The reduction of the section properties in return have effect on reducing the load carrying capacity of the member. Below is a list of methods on calculating the effective section properties for a doubly symmetric I-section relevant for calculation of the remaining capacity of structural steel members.

NOTE: The derivation of the section properties of deteriorated I-beams is based on the methods discussed on section 3.2 and some of the formulas are adopted from other research papers.

6.2 Effective cross-sectional area

The effective cross-sectional area $A_{e_{ff}}$, can be calculated considering the reduction of section loss due to general corrosion as follows.

$$A_{e_{ff}} = A_0 - \sum_{i=1}^{n} \left(t_{d_i} l_{d_i} \right) \tag{6.1}$$

Where: A_0 – is the gross area of the new section
l_{d_i} – is length of the i^{th} deteriorated area
t_{d_i} – is thickness of the i^{th} deteriorated area
n – is the total number of deteriorated parts

6.3 Elastic section properties

The following elastic section properties are a generalized formula for corroded I-sections, where the deterioration parts are located on different surface of the section as shown in Figure 6.1.

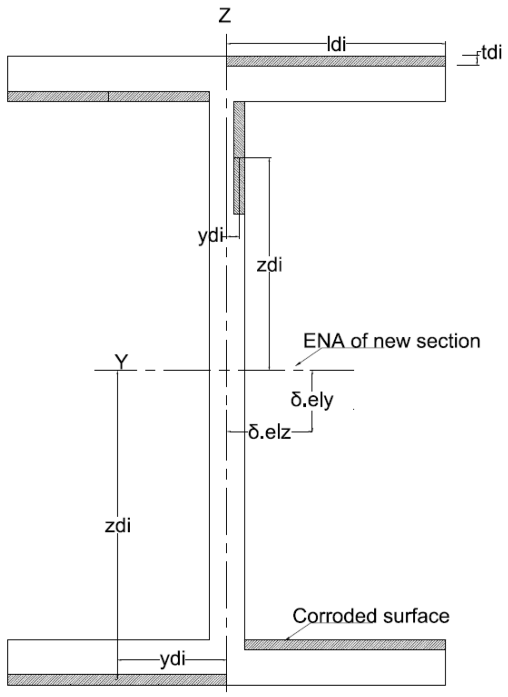


Figure 6.1: Deteriorated I-section

6.3.1 Eccentricity (shift) of Elastic Neutral Axis (ENA) (δ_{e_l})

For derivation see annex A-1

$$\delta_{e_l y} = \frac{\sum_{i=1}^{n}[A_{di} z_{di}]}{A_{e_{ff}}} \quad [16] \tag{6.2}$$

$$\delta_{e_l z} = \frac{\sum_{i=1}^{n}[A_{di} y_{di}]}{A_{e_{ff}}} \tag{6.3}$$

Where: $A_{e_{ff}}$ – is the effective area of the deteriorated section

A_{d_i} – is area of the ith deteriorated part given by $(t_{d_i} l_{d_i})$

y_{d_i} & z_{d_i} – is moment arm of A_{d_i} about the N.A of the original section along Y & Z axis respectively

n – is the total number of deteriorated parts

6.3.2 Effective second moment of area ($I_{e_{ff}}$)

For derivation see annex A-2

$$I_{effy} = I_{0y} + A_0 \delta_{ely}^2 - \sum_{i=1}^{n} \left[I_{dyi} + A_{di}\left(z_{di} + \delta_{e_l y}\right)^2 \right] \quad [16] \tag{6.4}$$

$$I_{effz} = I_{0z} + A_0 \delta_{elz}^2 - \sum_{i=1}^{n} \left[I_{dzi} + A_{di}\left(y_{di} + \delta_{e_l z}\right)^2 \right] \tag{6.5}$$

Where: I_{0z} & I_{0y} – is the second moment of area of the initial section along Z & Y axis respectively
A_0 – is the gross area of the original section
A_{d_i} – is area of the i^{th} deteriorated part
y_{d_i} & z_{d_i} – is moment arm of A_{d_i} about the N.A of the original section along Y & Z axis respectively
$\delta_{e_l y}$ & $\delta_{e_l z}$ - is eccentricity of the elastic N.A along Y & Z axis respectively

I_{dyi} & I_{dzi} - is the second moment of area of A_{d_i} about its own N.A parallel Y&Z axis respectively
n – is the total number of deteriorated parts

6.3.3 Effective elastic section modulus ($W_{ele_{ff}}$)

$$w_{eleffy} = \frac{I_{effy}}{Z_{max} + \delta_{ely}} \tag{6.6}$$

$$w_{eleffz} = \frac{I_{effz}}{y_{max}+\delta_{elz}} \tag{6.7}$$

Where: z_{max} & y_{max} – is the maximum distance from the ENA of the original section to the extreme fibre along Y&Z axes respectively

6.3.4 Effective radius of gyration ($i_{e_{ff}}$)

$$i_{effy} = \sqrt{\frac{I_{effy}}{A_{eff}}} \tag{6.8}$$

$$i_{effz} = \sqrt{\frac{I_{effz}}{A_{eff}}} \tag{6.9}$$

6.4 Plastic section properties

Formulation a generalized formula for plastic section properties is more complex, as the plastic neutral axis must divide the section in to equal areas of compression and tension zones. The best way is to consider a corrosion damage model and prepare a formula which can be used for similar cases. For that reason, below are some equations for the calculation of plastic section properties which are derived assuming the deterioration is on the upper or lower flanges only and the deteriorated section has an axis of symmetry about the z- axis. In addition, the plane of loading is considered perpendicular to the Y-axis, therefore the section properties suggested here are along the major axis only. For the parameters used in the equations reference should be made on figure 6.2 below.

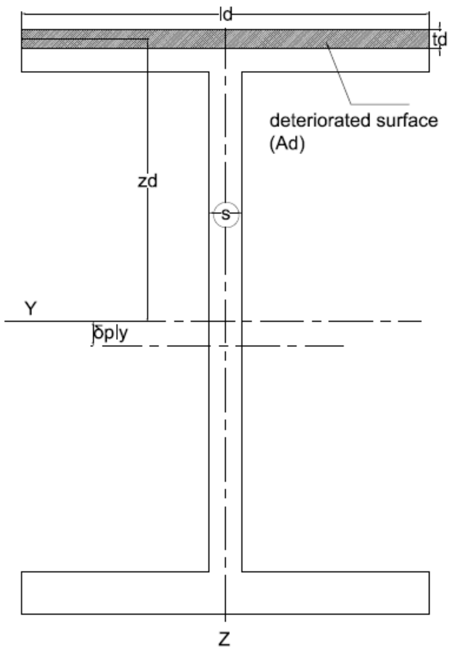


Figure 6.2: Deteriorated I-section model

6.4.1 Eccentricity (shift) of Plastic Neutral Axis (PNA) ($\boldsymbol{\delta_{pl}}$)

NOTE : For the equations below the deteriorated flange is considered to be the upper flange ,which is $\sum_{i=1}^{n_c}(l_{dc}t_{dc}) > \sum_{i=1}^{n_t}(l_{dt}t_{dt})$. Therefore, the PLA lies below the initial *PNA*.

For derivation see annex A-3

$$\delta_{ply} = \frac{1}{2s}\left[\sum_{i=1}^{n_c}(l_{dc}t_{dc}) - \sum_{i=1}^{n_t}(l_{dt}t_{dt})\right] \quad (6.10)$$

Note: The PNA lies on the web

6.4.2 Effective Plastic section modulus ($\boldsymbol{W_{ele_{ff}}}$)

For derivation see annex A-4

$$w_{pleffy(w)} = w_{ply} + s\delta_{ply}^2 - \left[\sum_{i=1}^{n_c} A_{dci}\left(z_{dci} + \delta_{ply}\right) + \sum_{i=1}^{n_t} A_{dti}\left(z_{dti} - \delta_{ply}\right)\right] \quad (6.11)$$

Note: The PNA lies on the web

Where: w_{ply} – is the plastic section modulus of the initial section about y-y axis
A_{d_i} – is area of the ith deteriorated part
z_{d_i} – is moment arm of A_{d_i} about the PNA of the initial section along y-y axis
δ_{pl_y} - is eccentricity of the plastic N.A along y-y axis
n_c/n_t – is the total number of deteriorated parts in the compression and tension zones respectively
s – is thickness of the web

Chapter 7

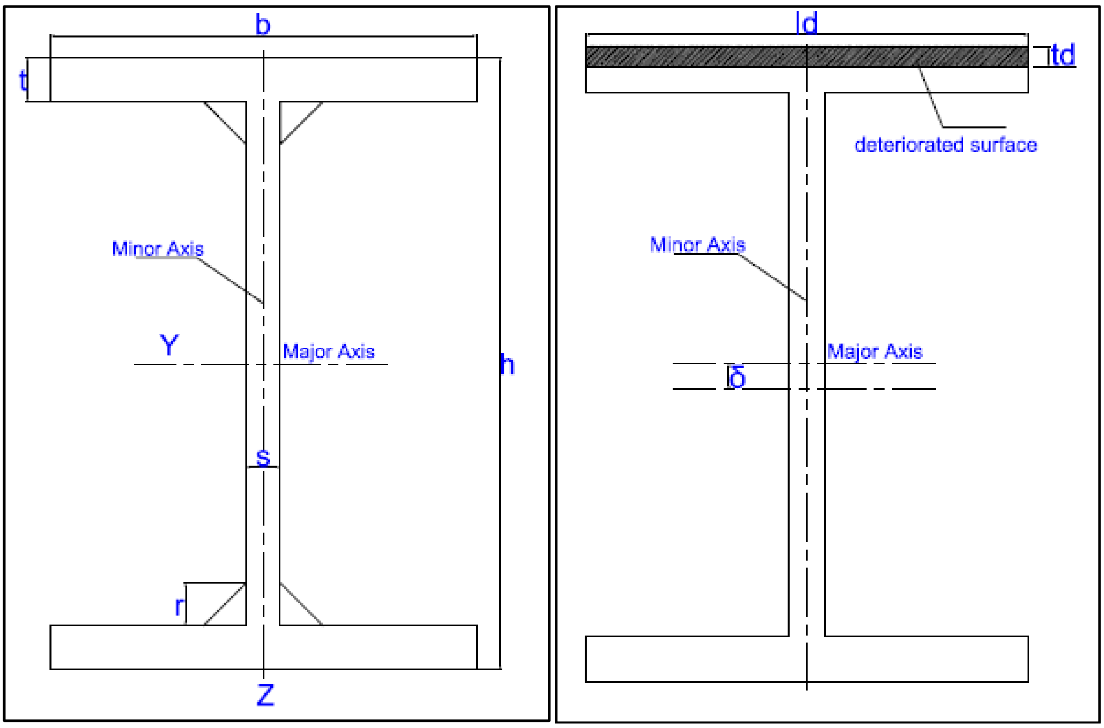
Remaining capacity of corroded I-section structural steel members

7.1 Introduction

Based on the observations of previous studies conducted on strength of deteriorated steel structures, the effective way to approach the issue is to assume a specified case of deteriorated member and develop a model of the deteriorated section. This can be done based on information gathered from site inspections of similar steel structures. For example, for steel bridges, especially in the US, several researches have been done based on data collected for many years. As a result, a common deterioration patterns were identified on different parts of the members and a simplified generalized equations and graphs were suggested based on % area loss and/or other parameters.

For this thesis an I-section steel member deteriorated on the upper flange is considered for analysis purposes and members such as compression, beam and a beam-column are investigated. Although the formulas developed here is based on this specific deterioration case, with some modifications it can be used for other deterioration cases too. The application for such deterioration models can be found in some bridge members and other steel structures, where only one outer face of the flange is exposed to corrosion.

7.2 Corroded I-section model



a b

Figure 7.1: (a) Initial I- Section and (b) Deteriorated I-Section model

As it can be seen in figure 7.1 (a), the initial I-beam section is a doubly symmetric I-section. Where t is thickness of flange, b is width of flange, h is total height of section, s is thickness of web and r is length of fillet. Similarly, the deteriorated section is defined as in figure 7.1 (b), where l_d and t_d are the length and thickness of deteriorated area and δ is the eccentricity of neutral axis due to deterioration.

7.3 Remaining capacity of corroded compression member

The behaviour and capacity of compression members is discussed in chapters 2 & 3. Based on that in this section the remaining strength of deteriorated compression members will be investigated. The compression member selected for this case is initially loaded with a concentric axial force as shown in figure 7.2 (a). In this case the member resists axial force and the section experience a normal stress due to the axial force throughout the section as shown in figure 7.3.

On the other hand, when the member is deteriorated on one side of the flange as shown in figure 7.1(b), there will be a moment due to the shift of the neutral axis (δ), as a result the member must resist both the axial force and the moment due to eccentricity. Consequently, the members capacity to carry axial force will be reduced due to the section loss and it will be exposed to additional external force of bending moment due to eccentricity. In this case the member can be analyses as a beam-column member to carry both the moment and the axial force. In addition to the axial compressive capacity the member local buckling capacity (flange & web) will be affected too.

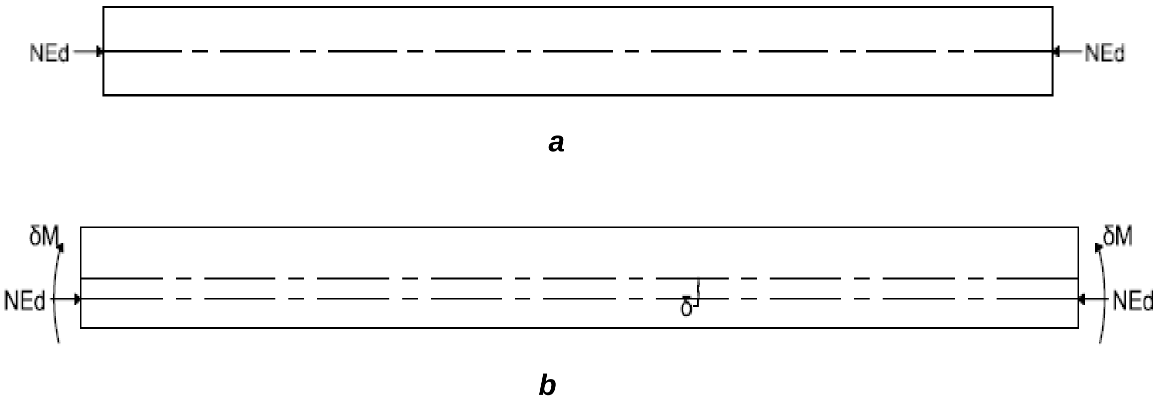


Figure 7.2: Load description of compression member (a) Initial & (b) deteriorated

7.3.1 Cross-section capacity

1. Effective axial compressive force capacity ($\mathbf{N_{pleff}}$)

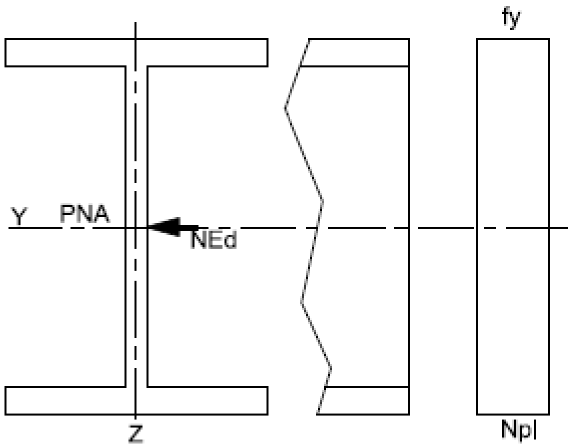


Figure 7.3: Normal plastic stress distribution of initial compression member

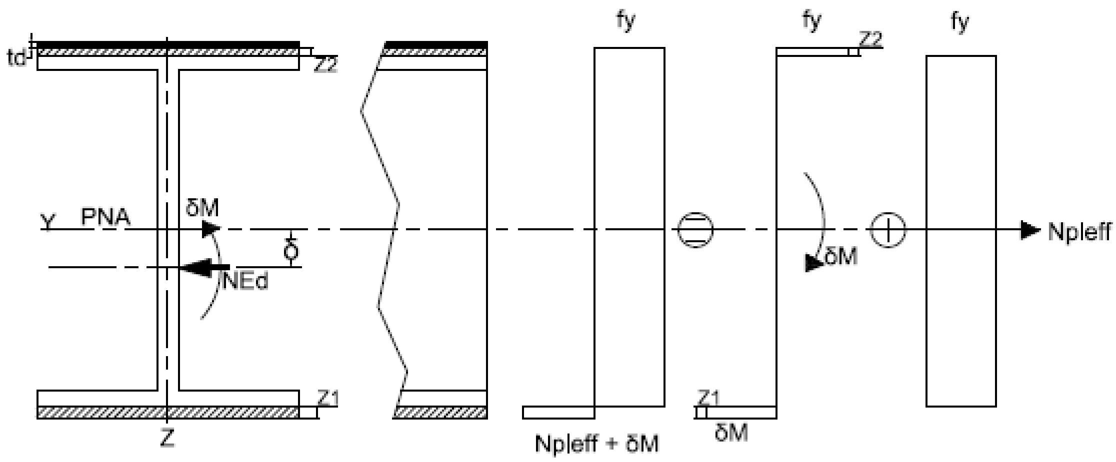


Figure 7.4: Normal plastic stress distribution of deteriorated compression member

Based on the stress distribution of the deteriorated section shown in figure 7.4, the effective plastic axial capacity can be estimated as:

For derivation see annex A-5

$$N_{pleff} = N_{PlD} - \left[\frac{\delta M h}{2\left(\frac{h}{2}-\delta_{Pl}\right)\left(\frac{h}{2}+\delta_{Pl}\right)}\right] \quad (7.1)$$

$$\delta M = N_{Ed}\delta_{Pl} \quad (7.2)$$

$$\delta_{Pl} = \frac{A_d}{2\,s} \quad (7.3)$$

$$A_d = l_d\, t_d \quad (7.4)$$

$$N_{PlD} = A_{eff} f_y \quad (7.5)$$

$$A_{eff} = A_0 - A_d \quad (7.6)$$

Where: N_{PlD}- is the full axial plastic capacity of the deteriorated section
δM – is an infinitesimal moment due to shift of PNA
δ_{Pl} – is eccentricity of PNA, assuming it lies within the web
A_d- is area of the deteriorated part
A_0- is the total area of the initial section

NOTE: In this model the moment (δM) is assumed to be resisted by the remaining flange area only, which means a moderate amount of section loss is considered. Therefore, for the above equation to be valid the maximum % section loss on the upper flange for a given I-section must be:

$$\left(\frac{A_d}{100}\right) < \frac{2hsA_f}{A_0(0.8A_0 + 2A_f)} \tag{7.7}$$

For demonstration purposes a standard I-section (HE-A 100) is selected for visualizing the effect of the section loss due to corrosion on strength capacities by plotting % area loss vs remaining capacity and based on equation (7.7) the maximum area loss for HE-A 100 is 12%. Other parameters which are considered for plotting the effects are listed below.

NOTE: The section considered here is an equivalent welded section, consequently the fillet area is not considered in any calculations.

- Yield strength of steel - $f_y = 275\, {}^{N}/_{mm^2}$
- Youngs modulus of elasticity - $E = 210000\, {}^{N}/_{mm^2}$

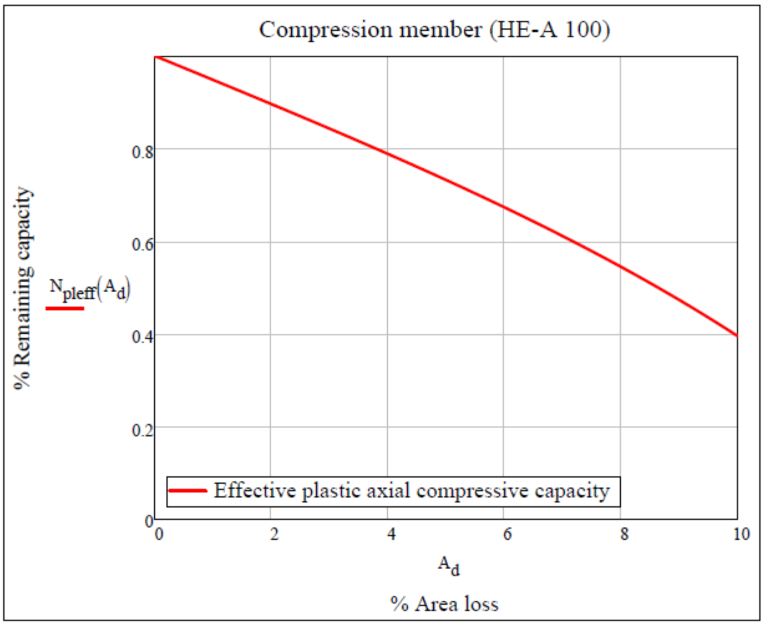


Figure 7.5: Effective plastic axial compressive capacity

2. Local buckling capacity

The local buckling capacity of thin plate elements of the I-section (flange and web) mainly depends on the plate slenderness value $\left(\frac{c}{t}\right)$. In this deteriorated model the dimensions of the web are not affected and the effective web height (C_w) assumed to remain in full compression stress. In case of the flange, the thickness is reduced by (t_d) ,therefore the buckling capacity of the flange will be reduced.

Plate slenderness value of web of the deteriorated section is equal to the value of the initial section.

$$\left(\frac{C_w}{s}\right)_D = \left(\frac{C_w}{s}\right)_I \quad (7.8)$$

$$C_w = h - 2t - 2r \quad (7.9)$$

Plate slenderness value of flange of the deteriorated section can be calculated by:

$$\left(\frac{C_f}{t}\right)_D = \frac{C_f}{t - t_d} \quad (7.10)$$

$$C_f = \frac{b - 2r - s}{2} \quad (7.11)$$

Similarly, the effect on flange class-classification can be seen from the graph below. Initial section HE-A 100 is class-1.

NOTE: The graph here is plotted for thickness loss as a % of total flange thickness.

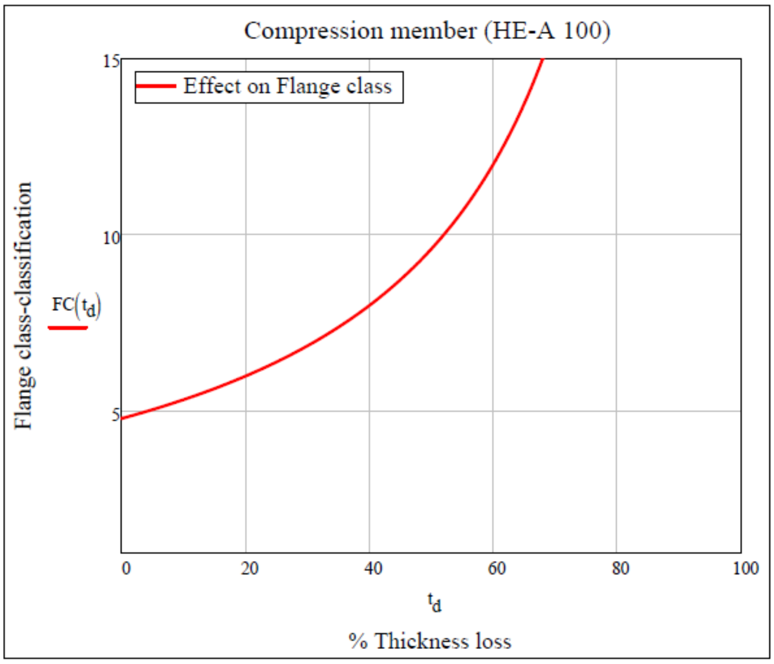


Figure 7.6: Effect on flange class-classification

7.3.2 Member Capacity

The capacity of a compression member is governed by the slenderness value, in return the slenderness value depends on the length, support condition and stiffness of the cross-section. Keeping all the other factors constant the critical elastic buckling load of the deteriorated section can be estimated by:

$$N_{creff} = \frac{\pi^2 E I_{eff}}{L^2} \tag{7.12}$$

Where E is modulus of elasticity and I_{eff} is given in section 6.3 for respective axis.

For initial sections the weakest axis to be considered for evaluation of the critical elastic buckling load is the z-axis. In case of the deteriorated section considered for this thesis, the evaluations must be conducted on both axes. The reason is about the major axis (y-axis) there is a shift of elastic neutral axis (eccentricity) as a result a moment is introduced in addition to the axial compressive load. Therefore, the member would deflect much more about the major axis y-axis due to the moment ∂M.

1. About the weak axis (z-axis) concentric buckling

In this axis there is no shift of neutral axis, therefore the critical elastic buckling load about z-axis is affected only due to reduction cross-sectional area and hence the second moment of area about z-axis. The reduction in critical elastic buckling load is plotted as shown in figure 7.7.

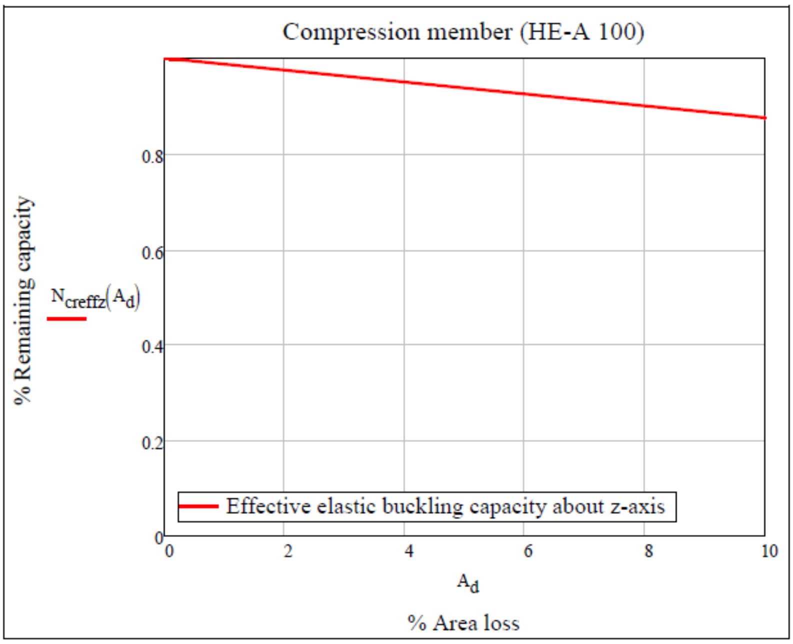


Figure 7.7: Effective elastic buckling capacity about z-axis

2. About the strong axis (y-axis) eccentric buckling

Analysis of eccentric buckling load of column is done based on the secant formula for buckling as shown below[17] .

$$\sigma_{max} = P\left[\frac{1}{A} + \frac{ec}{I}sec\left(\sqrt{\frac{P}{EI}}\frac{L}{2}\right)\right] \tag{7.13}$$

The above formula can be written as a function of P_{cr} :

$$\sigma_{max} = P\left[\frac{1}{A} + \frac{ec}{I}sec\left(\frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}}\right)\right] \tag{7.14}$$

$$P_{cr} = \frac{\pi^2 EI}{L^2} \tag{7.15}$$

Where:
σ_{max} – is the maximum stress
P_{cr} – is the critical elastic buckling load of concentric loading
e - is the eccentricity
c – is the maximum distance from the neutral axis
P – is the maximum elastic eccentric buckling load
A – is the cross-sectional
L – is the cross-sectional

For the deteriorated section the secant formula can be replaced by some parameters as shown below:

$$P_{cr} = N_{creffz} = \frac{\pi^2 EI_{effz}}{L^2} \tag{7.16}$$

$$e = \delta_{ely} = \frac{A_d * \left(\frac{h}{2} - \frac{t_d}{2}\right)}{A_{eff}} \tag{7.17}$$

$$c = \left(\frac{h}{2} - t_d + \delta_{ely}\right) \tag{7.18}$$

To find the critical eccentric buckling load P for the maximum stress f_y is a try and error process, therefore a graph is prepared for different deteriorated sections based on % area loss rearranging the secant formula in to a dimensionless equation given below:

$$\frac{\sigma_{max}A}{N_{cr}} = \left(\frac{N}{N_{cr}}\right)\left[1 + \left(\frac{ecA}{I}\right)sec\left(\frac{\pi}{2}\sqrt{\frac{N}{N_{cr}}}\right)\right] \tag{7.19}$$

$$\frac{ecA}{I} = \frac{\delta_{ely}\left(\frac{h}{2} - t_d + \delta_{ely}\right)A_{eff}}{I_{effy}} \tag{7.20}$$

$$\frac{\sigma_{max}A}{N_{cr}} = \frac{f_y A_{eff}}{N_{creffz}} \tag{7.21}$$

Based on the above dimensionless equation, a graph is plotted for 2.5%,5%,7.5%,10% and 12.5% area loss for the selected I-section (HE-A 100) as shown in figure 7.8 below.

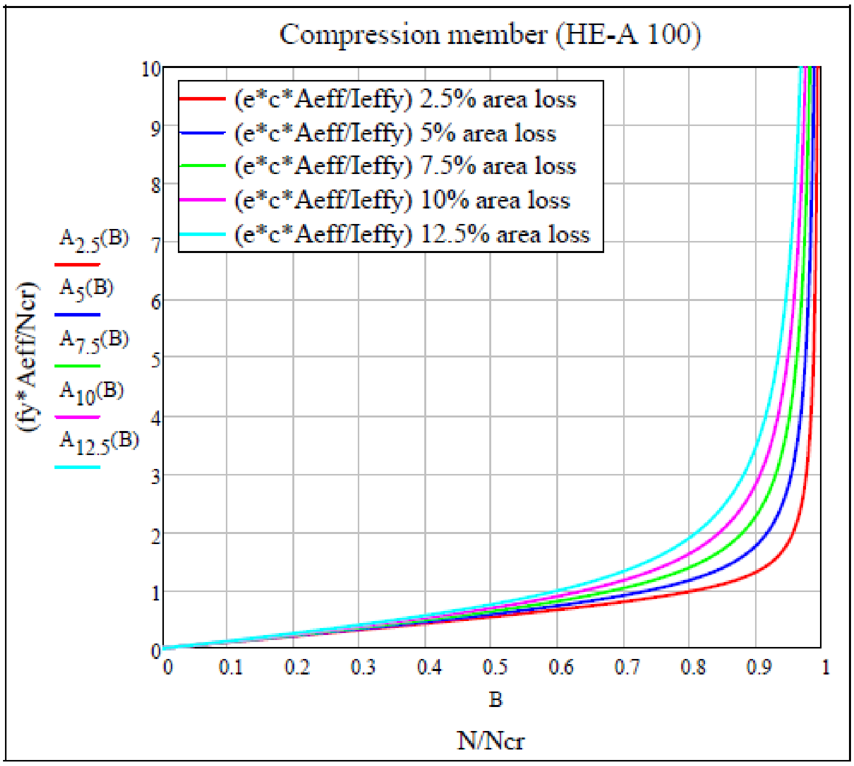


Figure 7.8: Non-dimensional eccentric buckling

For each $\frac{f_y A_{eff}}{N_{creffz}}$ value of corroded section the ratio of $\frac{N}{N_{cr}}$ is found from the above graph shown in figure 7.8 and the % reduction of the critical elastic buckling load with eccentric loading is plotted as shown in figure 7.9 below.

NOTE: $A_{d1} = A_d$ area loss.

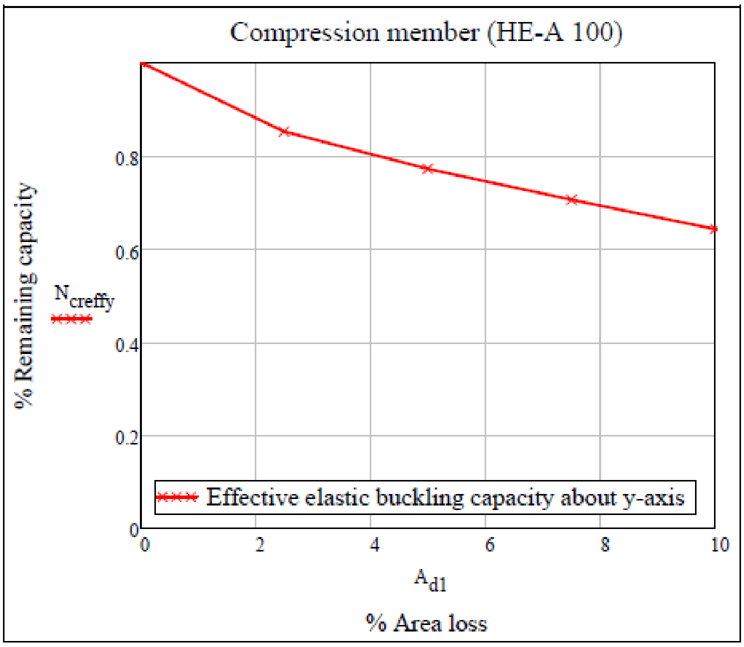


Figure 7.9: Effective elastic buckling capacity about y-axis

7.4 Remaining capacity of corroded beam member

In this case a beam member loaded with bending forces at both ends will be investigated. Deteriorated member bending capacity will be reduced, but the external moment will remain constant. Like the compression member discussed above the shift of the neutral axis will be considered in the web part only.

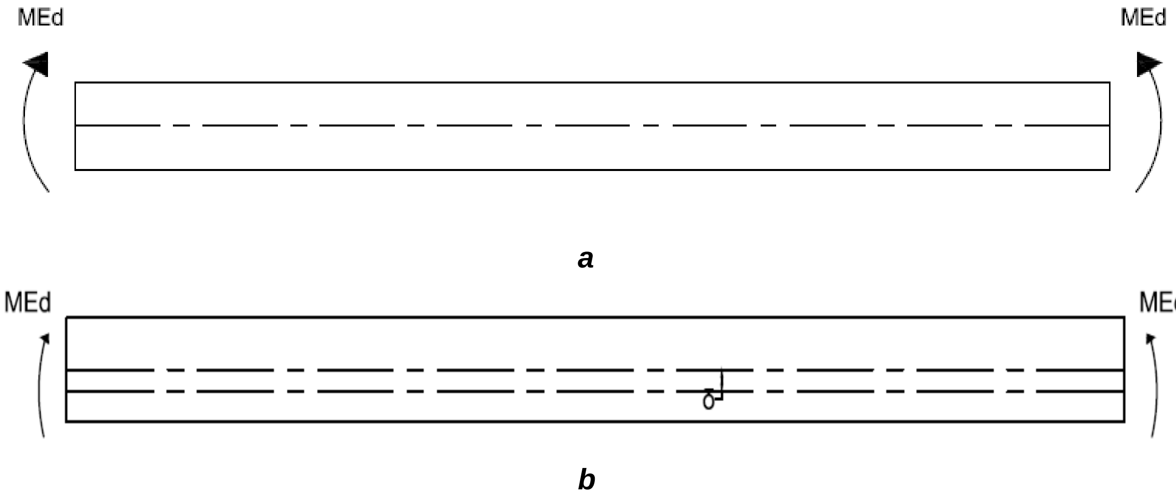


Figure 7.10: Load description of beam member (a) Initial & (b) deteriorated

7.4.1 Cross-section capacity

1. Effective plastic moment capacity (M_{pleff})

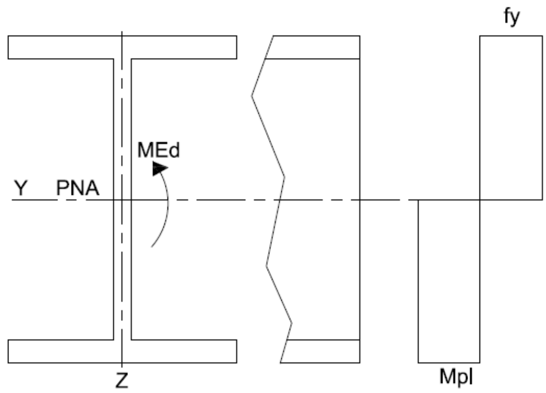


Figure 7.11: Plastic bending stress distribution of initial beam member

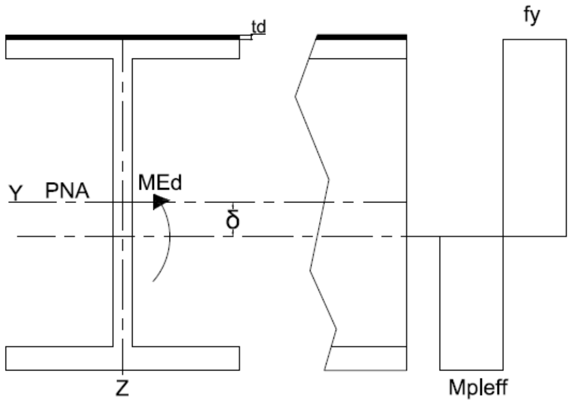


Figure 7.12: Plastic bending stress distribution of deteriorated beam member

Derived from the stress distribution as shown in figure 7.12, the effective plastic moment capacity can be found by:

$$M_{Pleff} = w_{Pleff} f_y \quad (7.22)$$

$$w_{pleff} = w_{pl} + s\,\delta_{pl}^2 - A_d\left(z_d + \delta_{pl}\right) \quad (7.23)$$

$$\delta_{Pl} = \frac{A_d}{2\,s} \quad (7.24)$$

$$A_d = l_d\,t_d \quad (7.25)$$

$$z_d = \frac{h}{2} - \frac{t_d}{2} \quad (7.26)$$

Where: w_{pl} - is the plastic section modulus of the initial I–section
w_{pleff} - is the effective plastic section modulus of the deteriorated I–section
s - is web thickness
z_d- is the moment arm of A_d about the PNA of the initial section
δ_{Pl} – is eccentricity of PNA, assuming it lies within the web
A_d- is area of the deteriorated part
f_y- is yield strength of steel

For demonstration purposes a universal beam member (IPE-240) is selected for plotting the reduction in moment capacity, and the results are shown in figure 7.13 below.

Note: the material properties used for plotting is the same as the compression member.

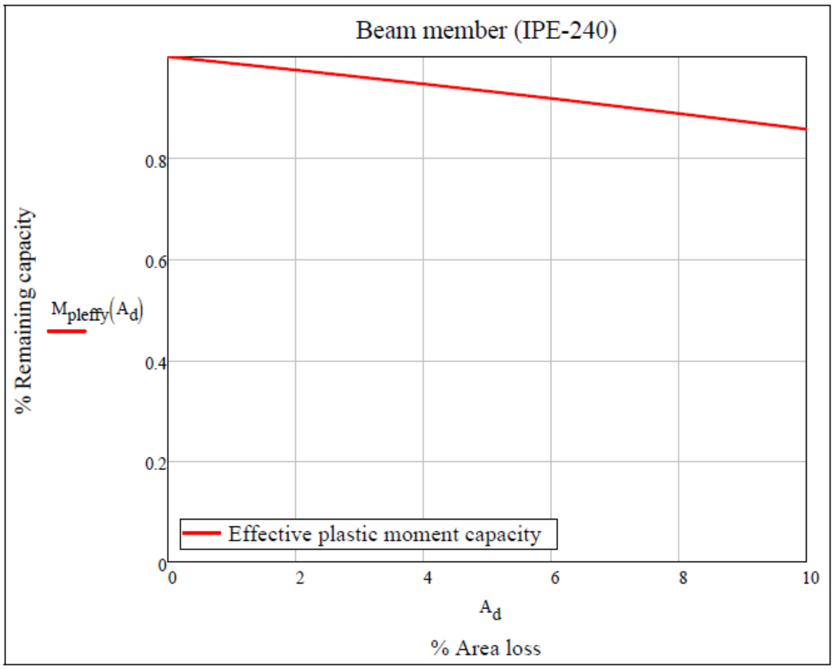


Figure 7.13: Effective plastic moment capacity

2. Effective elastic moment capacity (M_{eleff})

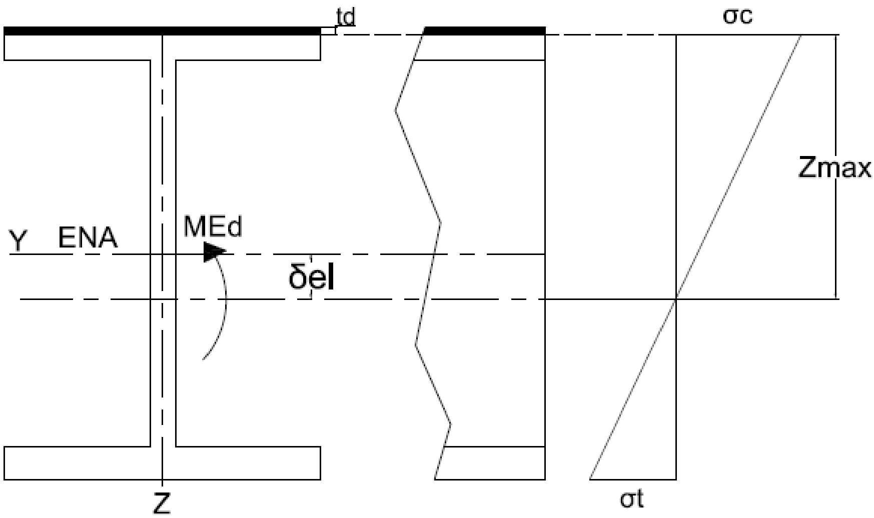


Figure 7.14: Elastic bending stress distribution of deteriorated beam member

The effective elastic moment capacity of the deteriorated section can be estimated by:

$$M_{eleff} = w_{eleff} f_y \tag{7.27}$$

Considering bending about the major axis (y-y)

$$w_{eleff} = w_{eleffy} = \frac{I_{effy}}{Z_{max}+\delta_{ely}} \tag{7.28}$$

$$I_{effy} = I_{0y} + A_0\delta_{ely}^2 - \left[I_{dy} + A_d\left(z_d + \delta_{ely}\right)^2\right] \quad (7.29)$$

$$I_{dy} = \frac{l_d t_d^3}{12} \quad (7.30)$$

$$\delta_{ely} = \frac{A_d Z_d}{A_{eff}} \quad (7.31)$$

$$A_d = l_d\, t_d \quad (7.32)$$

$$z_d = \frac{h}{2} - \frac{t_d}{2} \quad (7.33)$$

Where: w_{eleff} - is the effective elastic section modulus of the deteriorated section
I_{effy}- is the effective second moment of area of the deteriorated section about y-y axis
I_{0y}- is the second moment of area of the initial section about y-y axis
I_{dy}- is the second moment of area of the deteriorated part (A_d) about its neutral axis parallel to y-y axis
A_0 - is the total area of the initial section
z_d- is the moment arm of A_d about the ENA of the initial section
δ_{el} – is eccentricity of ENA, assuming it lies within the web
A_d- is area of the deteriorated part
f_y- is yield strength of steel

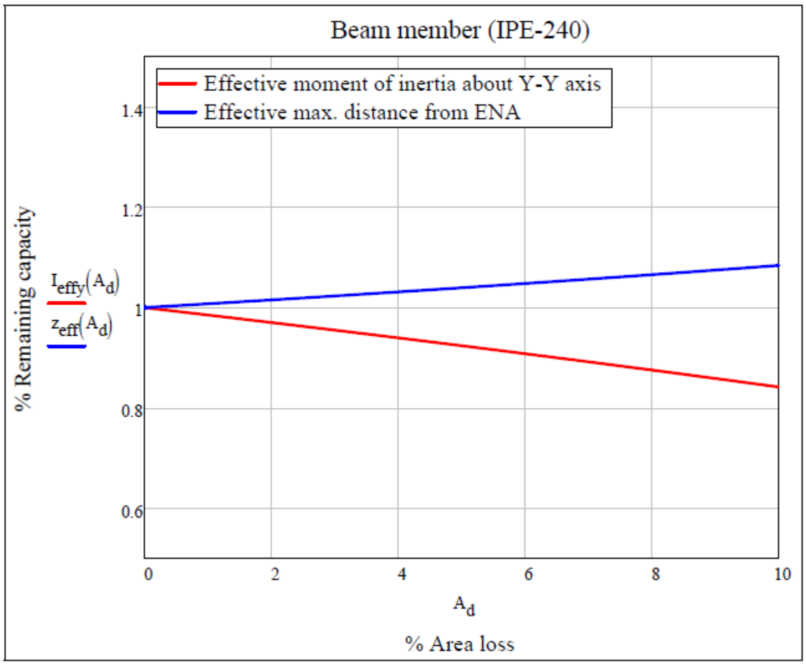


Figure 7.15: Effect on elastic section modulus

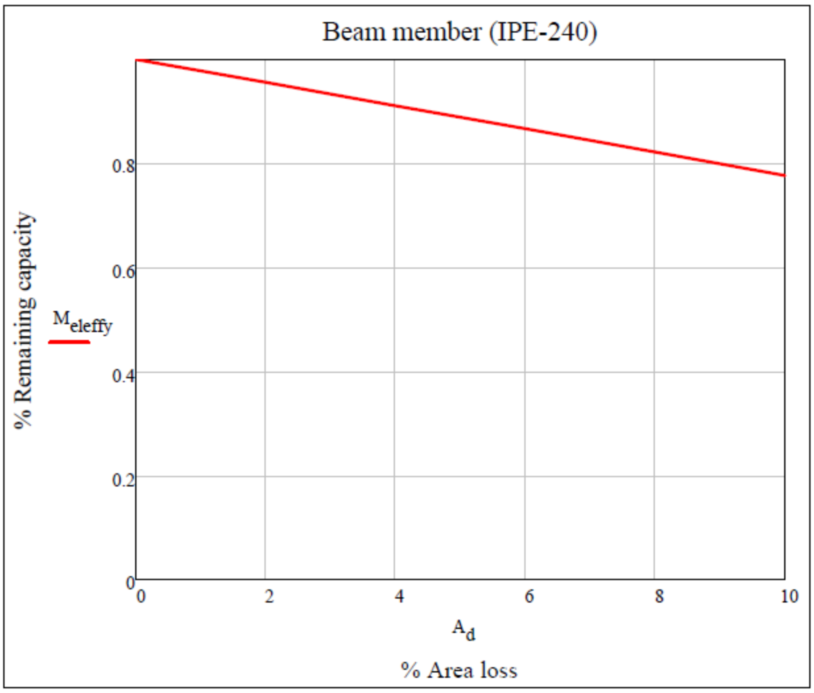


Figure 7.16: Effective elastic moment capacity

3. Local buckling capacity

Like the compression member discussed in section 7.3.1, the compression flange local buckling capacity can be found by:

$$\left(\frac{C_f}{t}\right)_D = \frac{C_f}{t - t_d} \tag{7.34}$$

$$C_f = \frac{b - 2r - s}{2} \tag{7.35}$$

For the case of web element there is no change in section properties as the deterioration is only in the upper flange. But the local buckling capacity is affected by the shift of the PNA and the compressive stress on the web element will be more than the tension stress. Therefore, to account the effect of the shift of PNA the value of α for determining the class of the cross-section in EC3 can be modified by:

$$\alpha_D = \frac{1}{2} + \frac{\delta_{pl}}{C_w} \tag{7.36}$$

$$\delta_{Pl} = \frac{A_d}{2\, s} \tag{7.37}$$

$$C_w = h - 2t - 2r \tag{7.38}$$

NOTE: this is valid only for class-section 1 & 2.

7.4.2 Member capacity

As it is discussed in section 7.3.2 the global buckling capacity of compression members is governed by the elastic critical buckling load N_{cr}. Similarly, the global buckling capacity of beam members against lateral-torsional buckling is governed by the critical bending moment M_{cr}. The equations available for calculation of M_{cr} involve many geometrical properties of the cross-section and they become more complicated when the section becomes unsymmetrical due to deterioration. For that reason, in this thesis a simplified method suggested in EC3, clause 6.3.2.4 (1) is used for checking the capacity of the deteriorated member against global buckling[5].

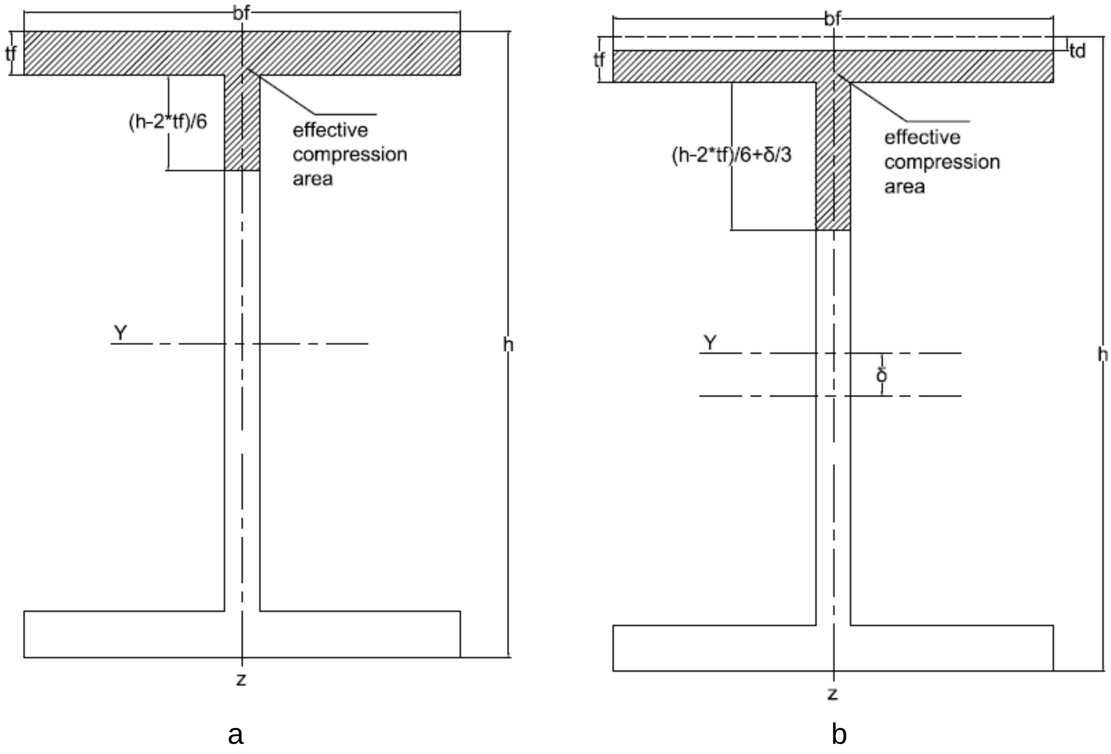


Figure 7.17: Effective compression area (a) initial section & (b) deteriorated section

Members with lateral restraint to the compression flange are not susceptible to lateral torsional buckling if the slenderness $\bar{\lambda}_f$ satisfies the condition below[5].

$$\bar{\lambda}_f = \frac{k_c L_c}{i_{fz}\lambda_1} \leq \bar{\lambda}_{co} \frac{M_{cRd}}{M_{yEd}} \tag{7.39}$$

Keeping all the other factors constant the value of M_{cRd} and i_{fz} for the deteriorated section can be calculated by:

$$M_{cRd} = M_{pleff} \text{ or } M_{cRd} = M_{eleff} \tag{7.40}$$

$$i_{fzeff} = \sqrt{\left(\frac{i_{fz}}{A_{fz}}\right) + \frac{\frac{1}{12}\left(\frac{\delta}{3}s^3 - t_d b_f^3\right)}{\left(\frac{\delta}{3}s - t_d b_f\right)}} \tag{7.41}$$

Where: $\frac{i_{fz}}{A_{fz}}$ - is the value of the new section
δ- is the eccentricity of the δ_{pl} or δ_{el} depending on the moment considered
s- is thickness of web
t_d- is thickness of deterioration
b_f – is width of flange

Substituting these values into equation 7.39 the deteriorated section`s capacity against lateral torsional buckling can be checked.

7.5 Remaining capacity of corroded beam-column member

As their name implies the beam-column members carries both a moment and axial loads and exhibits the behaviour of both columns and beams. In this case a beam-column member with a concentric axial compressive force and an end moment as shown in figure 7.18 will be investigated under the deterioration case specified above. As it is seen from the loading condition of the deteriorated member there will be an additional moment due to the shift of eccentricity, therefore the member must resist the initial design bending axial compressive forces and an additional moment due to the shift of the neutral axes. The strength of the member to resist moment and axial forces will be reduced due to deterioration and the reduction in local buckling capacity will contribute more to the reduction of the strength of the member.

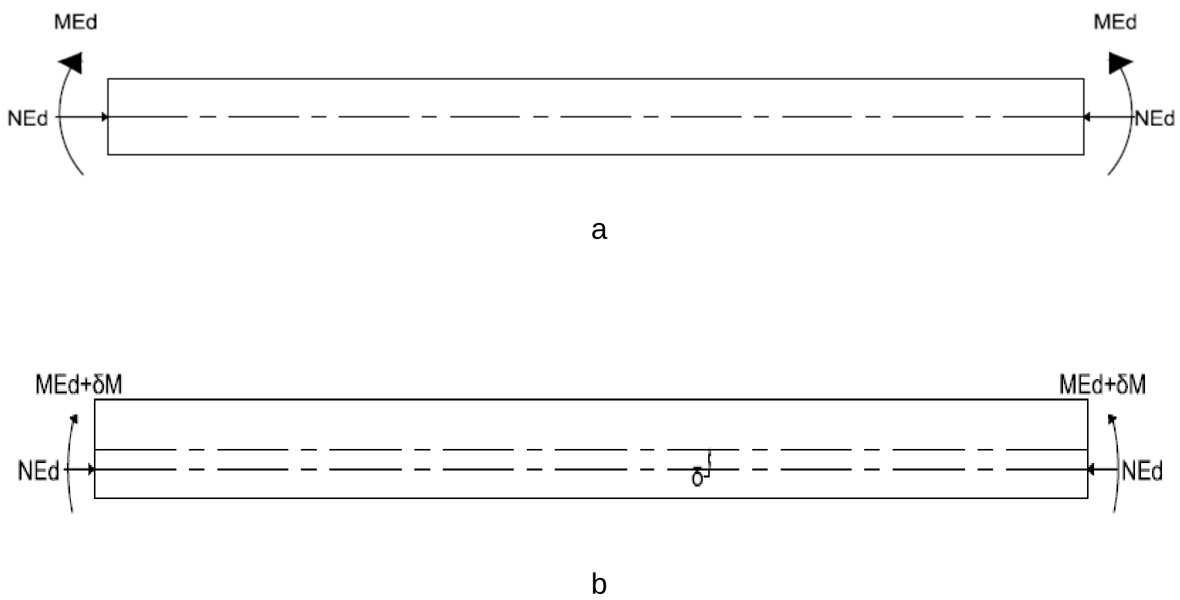
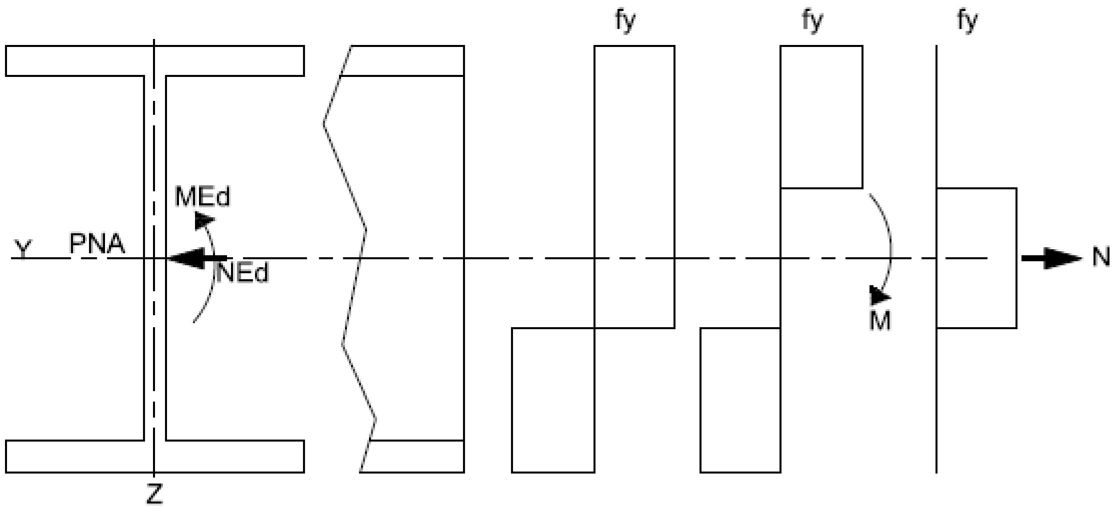


Figure 7.18: Load description of beam-column member (a) initial & (b) deteriorated

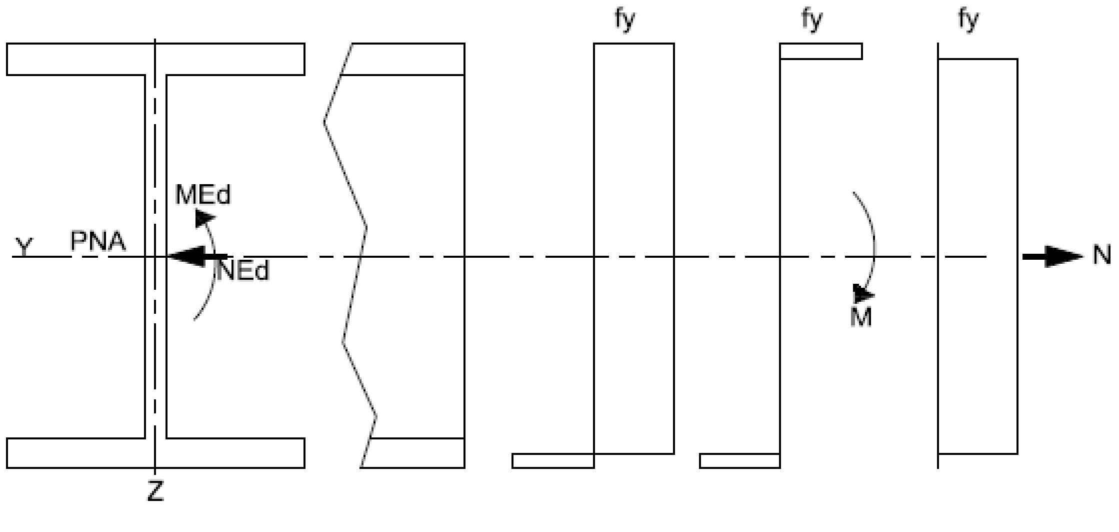
7.5.1 Cross-section capacity

1. Effective plastic moment capacity (M_{pleffN})

In this case focus will be given to the remaining plastic moment capacity due to the interaction with axial force (N_{Ed}) ,and it will be investigated both when the magnitude of the axial compressive force (N_{Ed}) is less than and greater than the capacity of the web ($A_w f_y$).The general stress distribution of both the initial and deteriorated members is as shown in figure 7.19 and 7.20 respectively ,and based on that the remaining effective plastic moment capacity can be estimated as:
For derivation see annex A-6.

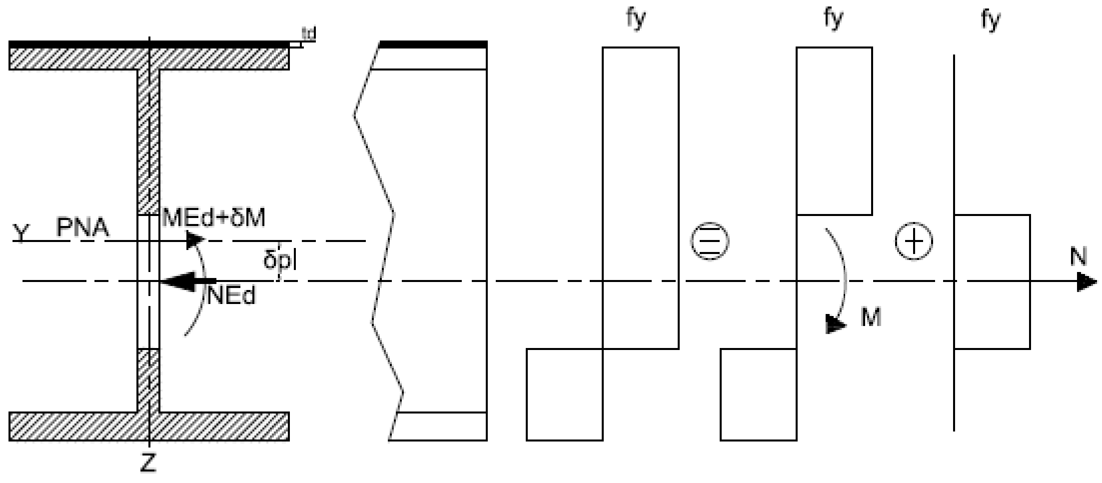


a

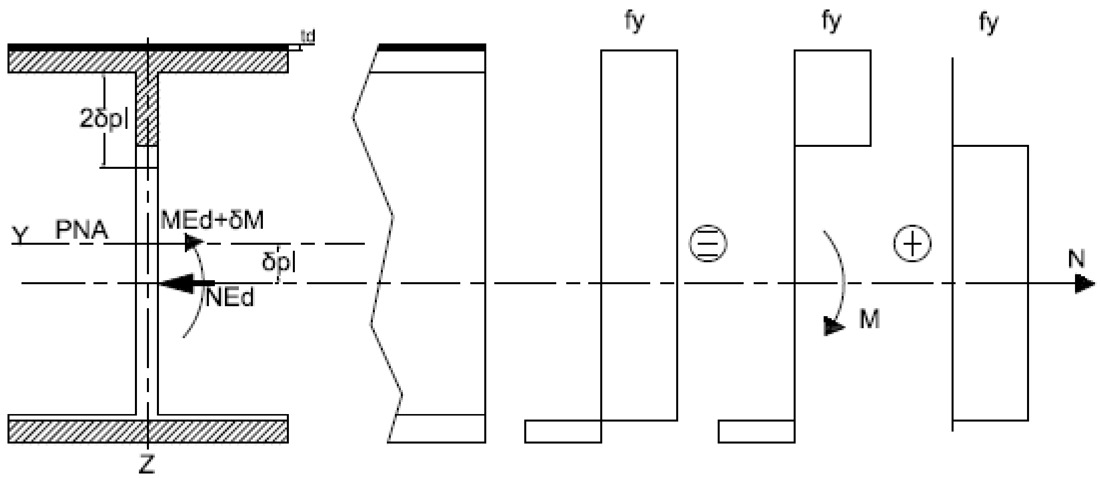


b

Figure 7.19: Plastic stress distribution of initial beam-column member (a) ($N_{Ed} < A_w f_y$) & (b) ($N_{Ed} > A_w f_y$)



a



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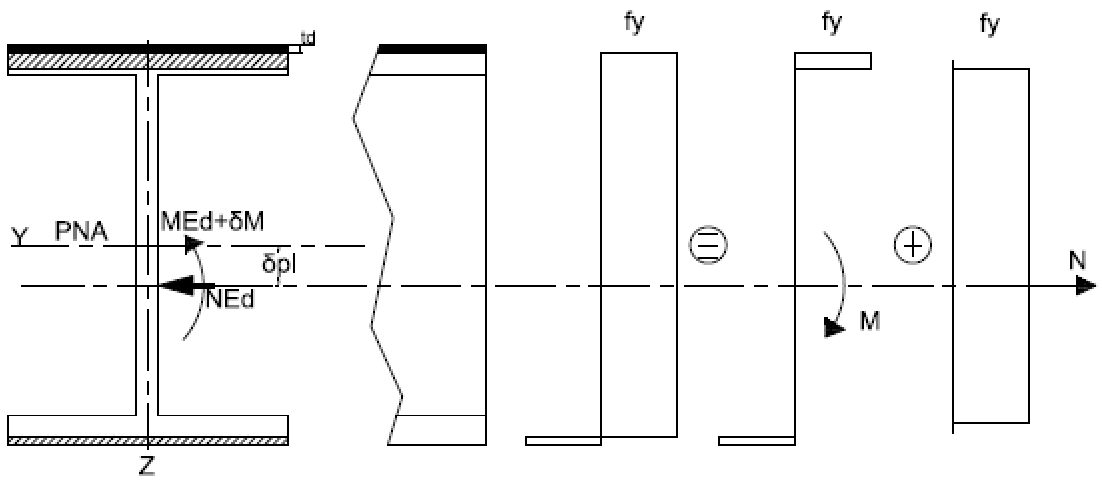

c

Figure 7.20: Plastic stress distribution of deteriorated beam-column member
(a) $N_{Ed} < (A_w - A_d)f_y$, (b) $(A_w - A_d)f_y < N_{Ed} < (A_w + A_d)f_y$ & (c) $N_{Ed} > (A_w + A_d)f_y$

Case I: $N_{Ed} < (A_w - A_d)f_y$

$$M_{pleffN} = M_{pleff} - \frac{N_{Ed}^2}{4Sf_y} \tag{7.42}$$

Case II: $(A_w - A_d)f_y < N_{Ed} < (A_w + A_d)f_y$

$$M_{pleffN} = M_{pleff} - \left(\frac{h_w}{2} - \delta_{pl}\right)^2 \left[\frac{N_{Ed}}{f_y\left(\frac{h_w}{2} - \delta_{pl}\right)} - s\right] f_y \tag{7.43}$$

Case III: $N_{Ed} > (A_w + A_d)f_y$

$$M_{pleffN} = M_{pleff} - \left[2(Cb)\left(\frac{h_w}{2} - \delta_{pl} + \frac{t_d}{2}\right) + 2(t_d b)\left(\frac{h_w}{2} - \delta_{pl} + \frac{t_d}{2}\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)^2 s + \left(s\,\delta_{pl} h_w\right)\right] f_y \tag{7.44}$$

$$C = \left[\frac{N_{Ed}}{f_y} - (t_d b) - (A_w)\right] \frac{\left(\frac{h_w}{2} + \delta_{pl}\right)}{(t_d + h_w)b} \tag{7.45}$$

Where: M_{pleff} - is the effective plastic moment capacity as defined in section 7.4.
s - is web thickness
δ_{Pl} – is eccentricity of PNA, assuming it lies within the web
A_d- is area of the deteriorated part
t_d- is thickness of the deteriorated part
A_w- is area of web
h_w- is height of web
f_y- is yield strength of steel

2. Local buckling capacity

The local buckling capacity of the compression flange can be estimated similar to the compression and beam members described above. While for the web element the value of α for determining the class of the cross-section according to EC3 can be found by[5]:

$$\left(\frac{C_w}{s}\right)_D = \left(\frac{C_w}{s}\right)_I \leq \frac{396\varepsilon}{13\alpha_D - 1} \quad \text{for class 1 cross-section} \tag{7.46}$$

$$\left(\frac{C_w}{s}\right)_D = \left(\frac{C_w}{s}\right)_I \leq \frac{456\varepsilon}{13\alpha_D - 1} \quad \text{for class 2 cross-section} \tag{7.47}$$

Where α_D for deteriorated sections can be given by:

$$\alpha_D = \frac{1}{2}\left[\frac{N_{Ed}}{\left(s\,c_w f_y\right)} + \frac{\delta_{pl}}{c_w} + 1\right] \tag{7.48}$$

$$C_w = h - 2b - 2r \tag{7.49}$$

Chapter 8

Design verification of corroded I-section structural steel members based on EC3

8.1 Introduction

The calculated remaining capacity of the different structural steel members based on the formulas developed in this thesis can be verified (checked) in relation to EC3 to see if the structural member is still in safe condition. As a result, in this chapter the steps and procedures on how the verification can be made will be discussed.

Based on Eurocodes generally, the design of a steel member, for the various combinations of external forces, is performed in two steps [2]:

- Cross section capacity (resistance) and
- Member capacity (resistance)

The capacity of cross-sections depends on their class classifications such as class 1,2,3 or 4 in accordance with clause 6.2.1(3) of EC3-1-1. Cross-section classes 1 and 2 reach their full plastic capacity, while class 3 cross sections only reach their elastic capacity. Class 4 cross sections are not able to reach their elastic capacity because of local buckling [5].

In addition to the cross-section capacity check, columns, beams and beam-columns members must be checked against overall member buckling capacity according to clauses 6.3 and 6.4 of EC3-1-1[2].

8.2 Verification of corroded compression member

8.2.1 Cross-section verification

1. **Local buckling capacity**

1.1 Compression flange

According to EC3: clause 5.5, Table 5.2, local buckling capacity of the outstand compression flange is verified by [5]:

$$\frac{c}{t} \leq 9\varepsilon \quad \text{for class 1} \tag{8.1}$$

$$\frac{c}{t} \leq 10\varepsilon \quad \text{for class 2} \tag{8.2}$$

Where $\varepsilon = \sqrt{\frac{235}{f_y}}$

For the corroded model used in this thesis, the ratio $\frac{c}{t}$ can be found by:

$$\frac{C}{t} = \frac{c_f}{t - t_d} \tag{8.3}$$

1.2 Compression web

In this case the web remains in compression state, therefore the class of the web will not be changed from its initial condition.

1.3 Axial compressive force

Based on EC3 clause 6.2.4 (1), the design value of the compression force N_{Ed} at each cross-section should satisfy the condition [5]:

$$\frac{N_{Ed}}{N_{cRd}} \leq 1.0 \tag{8.4}$$

Where N_{cRd} is the compressive resistance of the section. For the corroded section assumed in this thesis, N_{cRd} is given by:

$$N_{cRd} = \frac{N_{pleff}}{\gamma_{M0}} \tag{8.5}$$

for class 1,2 or 3

Where N_{pleff} is defined in section 7.3 and γ_{M0} is a partial safety factor.

2. Member capacity

A compression member should be verified against buckling as per EC3, clause 6.3.1.1 (1) by:

$$\frac{N_{Ed}}{N_{bRd}} \leq 1.0 \tag{8.6}$$

Where N_{bRd} is the design buckling resistance of the compression member given by EC3 clause 6.3.1.1 (3) [5]:

$$N_{bRd} = \frac{x A_0 f_y}{\gamma_{M1}} \tag{8.7}$$

for class 1,2 & 3

Where x is a reduction factor given by:

$$x = \frac{1}{\phi + \sqrt{\phi^2 - \lambda_b^2}} \quad (8.8)$$

$$\phi = 0.5\left[1 + \alpha(\lambda_b - 0.2) + \lambda_b^2\right] \quad (8.9)$$

$$\lambda_b = \sqrt{\frac{A_0 f_y}{N_{Cr}}} = \frac{L_{Cr}}{i}\frac{1}{\lambda_1} \quad (8.10)$$

For the corroded section considered in this thesis the value of N_{bRd} for class 1,2 & 3 can be estimated by:

$$N_{bRdeff} = \frac{x_{eff} A_{eff} f_y}{\gamma_{M1}} \quad (8.11)$$

$$x_{eff} = \frac{1}{\phi_{eff} + \sqrt{{\phi_{eff}}^2 - \lambda_{beff}^2}} \quad (8.12)$$

$$\phi_{eff} = 0.5\left[1 + \alpha\left(\lambda_{beff} - 0.2\right) + \lambda_{beff}^2\right] \quad (8.13)$$

The values of A_{eff} & λ_{beff} for the corroded section is given by:

$$A_{eff} = A_0 - A_d \quad (8.14)$$

$$\lambda_{beff} = \sqrt{\frac{N_{Pleff}}{N_{creff}}} \quad (8.15)$$

$$N_{Pleff} = A_{eff} f_y \quad (8.16)$$

$$N_{creff} = \frac{\pi^2 A_{eff} E}{\lambda_{eff}} \quad (8.17)$$

Note: N_{creff} can be taken the minimum of N_{creffy} or N_{creffz}.

8.3 Verification of corroded beam member

8.3.1 Cross-section capacity

1. Local buckling capacity

1.1 Compression flange

The compression flange remains under compression stress and can be verified similar to the compression members discussed in section 8.2(1).

1.2 Web under bending

According to EC3 clause 5.5, Table 5.2 parts subjected to bending and compression the web part is verified by [5]:

$$\frac{C}{t} \leq \frac{396\varepsilon}{13\alpha - 1} \quad (8.18)$$

for class 1 cross-section

$$\frac{C}{t} \leq \frac{456\varepsilon}{13\alpha - 1} \quad (8.19)$$

for class 2 cross-section

Note: The above equations are for $\alpha > 0.5$.

For the deteriorated model considered in this thesis, the value of α can be found by:

$$\alpha = \frac{1}{2} + \frac{\delta_{pl}}{C_w} \quad (8.20)$$

Where δ_{Pl} and C_w are given by:

$$\delta_{Pl} = \frac{A_d}{2\,s} \quad (8.21)$$

$$C_w = h - 2t - 2r \quad (8.22)$$

2. Moment capacity verification

The design value of the bending moment M_{Ed} at each cross-section should satisfy EC3 clause 6.2.5 (1) [5].

$$\frac{M_{Ed}}{M_{cRd}} \leq 1.0 \quad (8.23)$$

Where M_{cRd} is the bending resistance of the section. For the corroded model considered in this thesis M_{cRd} is calculated by:

$$M_{cRd} = M_{pleff} = \frac{w_{pleff} f_y}{\gamma_{M0}} \quad (8.24)$$

For class 1 or 2 cross -sections

$$M_{cRd} = M_{eleff} = \frac{w_{eleff} f_y}{\gamma_{M0}} \quad (8.25)$$

For class 3 cross -sections

Where M_{pleff} & M_{eleff} are the effective plastic and elastic moment capacity of the deteriorated section respectively as described in section 7.4.

8.4 Verification of corroded beam-column member

8.4.1 Cross-section capacity

1. Local buckling capacity

1.1 Compression flange

The compression flange remains under compression stress and can be verified similar to compression members discussed in section 8.2(1).

1.2 Web under bending and compression

According to EC3 clause 5.5, Table 5.2 parts subjected to bending and compression the web part is verified by [5]:

$$\frac{C}{t} \leq \frac{396\varepsilon}{13\alpha-1} \quad (8.26)$$

for class 1 cross-section

$$\frac{C}{t} \leq \frac{456\varepsilon}{13\alpha-1} \quad (8.27)$$

for class 2 cross-section

The above equations are for $\alpha > 0.5$.

For the deteriorated model considered in this thesis, the value of α can be found by:

$$\alpha = \frac{1}{2}\left[\frac{N_{Ed}}{\left(s\, c_w f_y\right)} + \frac{\delta_{pl}}{c_w} + 1\right] \quad (8.28)$$

Where δ_{Pl} and C_w are given by:

$$\delta_{Pl} = \frac{A_d}{2\,s} \quad (8.29)$$

$$C_w = h - 2t - 2r \quad (8.30)$$

2. Plastic moment capacity verification

For class 1 and 2 the following criterion should be satisfied based on EC3 clause 6.2.9.1(2) [5]

$$\frac{M_{Ed}}{MN_{Rd}} \leq 1.0 \quad (8.31)$$

For the deteriorated section considered in this thesis the above verification equation should be modified to:

$$\frac{M_{Ed}+\delta M}{MN_{Rd}} \leq 1.0 \qquad (8.32)$$

Where $\delta M = N_{Ed}\delta_{Pl}$ and the value of MN_{Rd} can be found based on the magnitude of N_{Ed} by:

$$MN_{Rd} = \frac{M_{pleffN}}{\gamma_{M0}} \qquad (8.33)$$

Where M_{pleffN} is as given in section 7.5.

Chapter 9

Discussion and conclusion

9.1 Discussion

9.1.1 limit on eccentricity (shift) of neutral axis

The focus of this thesis is to provide formulas for calculating the remaining capacity of corroded I-section structural members in relation to the possible eccentricity (shift) of the neutral axes introduced by the material loss due to corrosion. Based on that equations are formulated, but the eccentricities δ_{el} & δ_{pl} are restricted with in the web part only. Therefore, to apply any of the above formulas first the eccentricity should be checked based on the following steps.

- First relate the corrosion damage to the corrosion model in this thesis (corrosion on lower flange can be used) with some modifications.
- Then calculate:
  - ✓ $A_d = t_d b$ - corroded area
  - ✓ $z_d = \frac{1}{2}(h - t_d)$ – distance from the respective neutral axis to the centroid of the corroded area.
  - ✓ $A_{eff} = A_0 - A_d$ – Effective area
  - ✓ $\delta_{el} = \frac{A_d z_d}{A_{eff}}$ and $\delta_{pl} = \frac{A_d}{2S}$ – Eccentricity of the plastic or elastic neutral axis
- Then check if: δ_{el} & $\delta_{pl} < \frac{h_w}{2}$ – check if the distance is within the web height

9.1.2 Maximum area loss for calculating (N_{pleff})

For calculating the effective axial plastic compressive force as described in section 7.3.1(1), in addition to the requirement specified in section 9.1.1 the maximum area loss need to be satisfied by equation (7.7).

9.1.3 Effect on compression members

It is clearly seen that, the eccentricities of the neutral axes introduced due to corrosion have a large effect on the reduction of the load carrying capacity of the compression member. As it is shown in figure 7.5, the plastic axial compressive capacity (N_{pleff}) is reduced by approximately 60% of its initial capacity for an area loss of 10% of the total cross-sectional area. This is because the plastic capacity of the section must be shared by the additional external force of moment (δM) due to loss of material by corrosion. Flange local buckling capacity is also affected, even though the effect is much less than the other capacities.

Another significant effect of the introduced eccentricity is the reduction in elastic buckling load of the compression member. For the initial section the critical elastic buckling load is governed by the weak axis (z-axis), but as we can see in figure 9.1 below, the elastic buckling load about the strong axis is the governing criterion for the corroded member. The reduction in elastic buckling load about y-axis and z-axis is approximately 40% and 16% respectively for 10% area reduction. The reason for this large reduction in capacity about the strong axis (y-axis) is because of the eccentricity the buckling load must be analysed as an eccentric buckling load about the strong axis (y-axis).

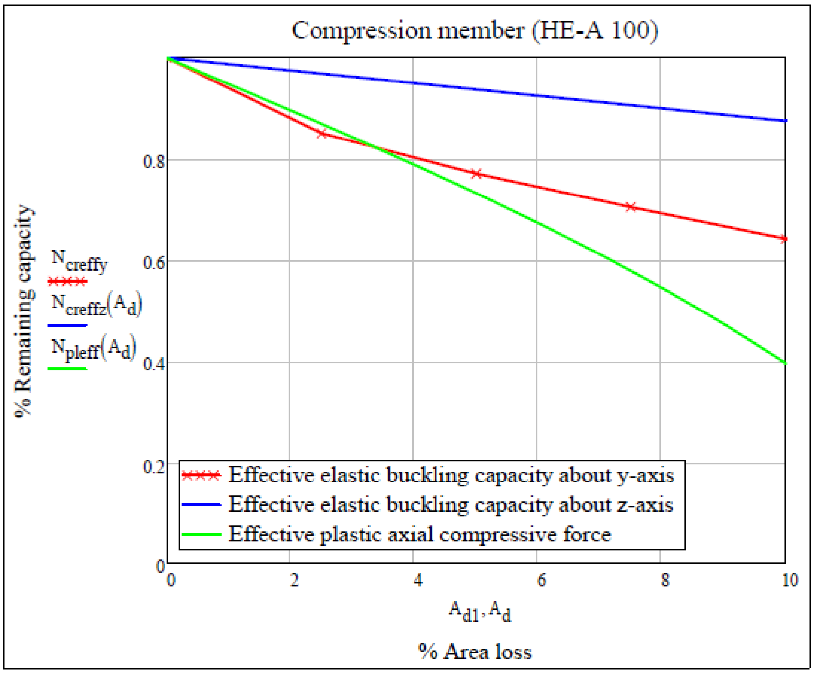

Figure 9.1: Elastic buckling capacity about y & z axis and effective axial plastic compressive force of the corroded section

9.1.4 Effect on beam members

For beam and beam-column members the effect on bending moment capacity is mainly analysed and a suggestion is given for local and global buckling capacity estimation. For the beam member the elastic and plastic capacity reduction is almost similar with approximately 22% and 18% moment capacity reduction respectively for a 10% area loss. Flange local buckling capacity have the same effect for all members as the flange element remains under compression stress for all members. But local buckling capacity of the web will be affected due to the shift (eccentricity) of neutral axis with in the web. This shift of neutral axis will create a more compression zone within the effective web height. Beam-column members are a more complicated one as they combine both axial force and moment, and the effect on their capacities reduction will be more due to additional moment due to the eccentricity.

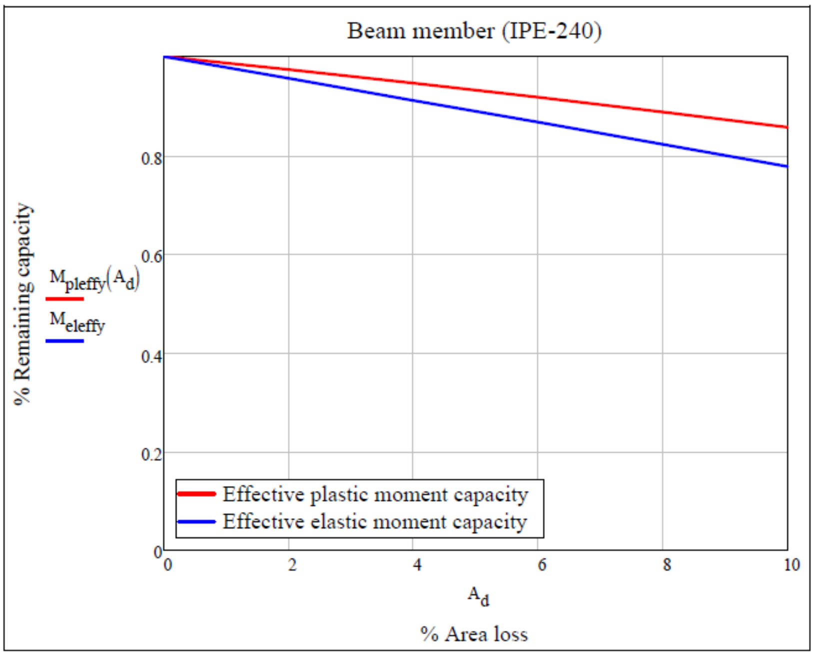

Figure 9.2: Effective elastic & plastic moment capacity

9.2 Conclusion

Analysis of load carrying capacities of corroded structural steel members based on the eccentricity (shift) of the neutral axis caused by material loss due to corrosion is proven to be effective in this thesis. The results show that the shift (eccentricity) of the neutral axes not only change the section properties, but they introduce additional external moments and hence decreases the cross-section and members resistance by great amount.

In addition to calculating the remaining capacity of a member for a given area loss, the results show too that we can predict further reduction of the capacity in the future by plotting % area loss vs remaining capacity as we have seen in chapter 7. This is very important because corrosion is time dependent and the damage of corrosion increase with time.

9.3 Recommendations and future works

The formulas recommended in this thesis project are based on analytical analysis only and they are applicable to the corrosion model established in this thesis. In addition, assumption is made for shift or eccentricity of the plastic and elastic neutral axis with in the web part. Therefore, the following recommendations could be given for any related future works:

- To develop more corrosion models based on data available from site inspections.
- Verify the formulas by conducting laboratory experiments and Finite Element analysis.
- Consider shift (eccentricity) that lies within the flange of the cross-section.
- Improved local and global buckling analysis for beam and beam-column member
- Analysis of remaining capacity for doubly unsymmetrical corrosion damage models with eccentricity aslo along the weak axis (z-axis). This will introduce torsional effect and decrease shear capacity of the member.

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Appendices

Appendix A: Derivation of some basic formulas developed for this thesis

A-1: Derivation of eccentricity of elastic neutral axis (δ_{ely})

For the derivation of this formula reference should be made to figure 6.1.

$$\delta_{ely} = \frac{\sum_{i=1}^{n} (A_i \cdot z_i) - \sum_{i=1}^{n_d} (A_{di} \cdot z_{di})}{\sum_{i=1}^{n} (A_i) - \sum_{i=1}^{n_d} (A_{di})}$$

$$\delta_{ely} = \frac{\sum_{i=1}^{n} \int_{z1}^{z2} b \cdot z \, dz - \sum_{i=1}^{n_d} \int_{zd1}^{zd2} l_d \cdot z_d \, dzd}{\sum_{i=1}^{n} \int_{z1}^{z2} b \, dz - \sum_{i=1}^{n_d} \int_{zd1}^{zd2} l_d \, dzd}$$

NOTE : $\sum_{i=1}^{n} (A_i \cdot z_i)$ *of the new section is equal to zero, (which is the location of the elastic N.A)* due to symmetry along the y-axis the area moments will cancel each other.

Therefore δ_{ely} can be written as:

$$\boxed{\delta_{ely} = \frac{\left[\sum_{i=1}^{n_d} (A_{di} \cdot z_{di})\right]}{A_{eff}}}$$

NOTE: The area moment arm z_i is taken $-ve$ if the i^{th} corroded area is located on the $-ve$ arm of the z-axis and $+ve$ if it is on the $+ve$ arm.

NOTE: Negative value of δ_{ely} means the neutral axis of the corroded section lies below the neutral axis of the initial section.

A-2: Derivation of effective second moment of area (I_{effy})

For the notations used in this derivation refer to figure 6.1.

$$I_{effy} = I_{uf} + A_{uf} \cdot d_{uf}^2 + I_{lf} + A_{lf} \cdot d_{lf}^2 + I_w + A_w \cdot d_w^2 - \left(I_d + A_d \cdot d_d^2\right)$$

$$I_{effy} = I_{uf} + A_{uf}\cdot\left(d_{ufn} + \delta_y\right)^2 + I_{lf} + A_{lf}\cdot\left(d_{lfn} - \delta_y\right)^2 + I_w + A_w\cdot\delta_y^{\,2} - \left(I_d + A_d\cdot d_d^{\,2}\right)$$

$$I_{effy} = I_{uf} + \left(A_{uf}\cdot d_{ufn}^{\,2} + 2\cdot A_{uf}\cdot d_{ufn}\cdot\delta_y + A_{uf}\cdot\delta_y^{\,2}\right) + I_{lf} + \left(A_{lf}\cdot d_{lfn}^{\,2} - 2\cdot A_{lf}\cdot d_{lfn}\cdot\delta_y + A_{lf}\cdot\delta_y^{\,2}\right) + I_w + \left(A_w\cdot\delta_y^{\,2}\right) - \left(I_d + A_d\cdot d_d^{\,2}\right)$$

$$I_{0y} = I_{uf} + A_{uf}\cdot d_{ufn}^{\,2} + I_{lf} + A_{lf}\cdot d_{lfn}^{\,2} + I_w$$ (moment of inertia of the initial section)

$$I_{effy} = I_{0y} + 2\cdot A_{uf}\cdot d_{ufn}\cdot\delta_y + A_{uf}\cdot\delta_y^{\,2} - 2\cdot A_{lf}\cdot d_{lfn}\cdot\delta_y + A_{lf}\cdot\delta_y^{\,2} + A_w\cdot\delta_y^{\,2} - \left(I_d + A_d\cdot d_d^{\,2}\right)$$

$$I_{effy} = I_{0y} + A_{uf}\cdot\delta_y^{\,2} + A_{lf}\cdot\delta_y^{\,2} + A_w\cdot\delta_y^{\,2} - \left(I_d + A_d\cdot d_d^{\,2}\right)$$

$A_0 = A_{uf} + A_{lf} + A_w$ (total area of the initial section)

$$I_{effy} = I_{0y} - I_d + A_0\cdot\delta_y^{\,2} - A_d\cdot d_d^{\,2}$$

$$d_d = \frac{h}{2} - \frac{h_d}{2} + \delta_y$$

$$z_d = \left(\frac{h}{2} - \frac{h_d}{2}\right)$$

$$d_d = z_d + \delta_y$$

Therefore, the general formula for the effective second moment of area of the deteriorated section about the new elastic neutral axis can be given by:

$$\boxed{I_{effy} = I_{0y} + A_0\cdot\delta_{ely}^{\,2} - \sum_{i=1}^{n}\left[I_{dyi} + t_{d1}\cdot l_{d1}\cdot\left(z_i + \delta_{ely}\right)^2\right]}$$

A-3: Derivation of eccentricity of plastic neutral axis (δ_{ply})

Based on the corrosion model shown in figure 6.2.

$$A_{lf} + \left(y_{pld} - t_{lf}\right)\cdot S - \sum_{i=1}^{n_t}\left(l_{dt}\cdot h_{dt}\right) = \left(h - y_{pld} - t_{uf}\right)\cdot S + A_{uf} - \sum_{i=1}^{n_c}\left(l_{dc}\cdot h_{dc}\right)$$

where y_{pld} is the plastic neutral axis of the corroded section about y-axis.

where: n_c and n_t are the number of deteriorated areas on the compression and tension parts respectively.

$A_{lf} = A_{uf}$ (doubly symmetric I-section)

$$y_{pld}\cdot S - t_{lf}\cdot S - \sum_{i=1}^{n_t}\left(l_{dt}\cdot h_{dt}\right) = h\cdot S - y_{pld}\cdot S - t_{uf}\cdot S - \sum_{i=1}^{n_c}\left(l_{dc}\cdot h_{dc}\right)$$

$S \cdot t_{lf} = S \cdot t_{uf}$ (doubly symmetric I-section)

$$y_{pld} \cdot S - \sum_{i=1}^{n_t} \left(l_{dt} \cdot h_{dt}\right) = h \cdot S - y_{pld} \cdot S - \sum_{i=1}^{n_c} \left(l_{dc} \cdot h_{dc}\right)$$

$$y_{pld} = \frac{1}{2S}\left[\sum_{i=1}^{n_t} \left(l_{dt} \cdot h_{dt}\right) - \sum_{i=1}^{n_c} \left(l_{dc} \cdot h_{dc}\right)\right] + \frac{1}{2S}(h \cdot S)$$

$$y_{pld} = \frac{1}{2S}\left[\sum_{i=1}^{n_t} \left(l_{dt} \cdot h_{dt}\right) - \sum_{i=1}^{n_c} \left(l_{dc} \cdot h_{dc}\right)\right] + \frac{h}{2}$$

$y_{pln} = \frac{h}{2}$ (Plastic neutral axis of the initial section about y-axis)

.

Therefore eccentricity of plastic neutral axis can be given by:

$$\delta_{pl} = y_{pld} - y_{pln}$$

$$\boxed{\delta_{ply} = \frac{1}{2S}\left[\sum_{i=1}^{n_t} \left(l_{dt} \cdot h_{dt}\right) - \sum_{i=1}^{n_c} \left(l_{dc} \cdot h_{dc}\right)\right]}$$

A-4: Derivation of plastic section modulus (w_{pleffy})

Based on the deterioration model shown in figure 6.2.

$$w_{pleffy} = \left[A_{uf}\left(z_{uf} + \delta_{pl}\right) + \left(\frac{h_w}{2} + \delta_{pl}\right)^2 \cdot \frac{S}{2} - A_{di}\left(z_{id} + \delta_{pl}\right) + A_{lf}\left(z_{lf} - \delta_{pl}\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)^2 \cdot \frac{S}{2} - A_{di}\left(z_{id} - \delta_{pl}\right)\right]$$

$$\left(\frac{h_w}{2} + \delta_{pl}\right)^2 = \frac{h_w^2}{4} + h_w \cdot \delta_{pl} + \delta_{pl}^2$$

$$\left(\frac{h_w}{2} - \delta_{pl}\right)^2 = \frac{h_w^2}{4} - h_w \cdot \delta_{pl} + \delta_{pl}^2$$

$w_{ply} = A_{uf} \cdot z_{uf} + A_{lf} \cdot z_{lf} + \frac{S \cdot h_w^2}{4}$ (plastic section modulus of the initial section)

$$w_{pleffy} = w_{ply} + \left[A_{uf} \cdot \delta_{pl} + \left(\frac{\delta_{pl}^2 \cdot S}{2}\right) - A_{di}\left(z_{id} + \delta_{pl}\right) - A_{lf} \cdot \delta_{pl} + \frac{\delta_{pl}^2 \cdot S}{2} - A_{di}\left(z_{id} - \delta_{pl}\right)\right]$$

$A_{uf} \cdot \delta_{pl} = A_{lf} \cdot \delta_{pl}$ (equal flange area of the initial section)

$$w_{pleffy} = w_{ply} + \delta_{pl}^2 \cdot S + \left(-A_{di}\left(z_{id} + \delta_{pl}\right) - A_{di}\left(z_{id} - \delta_{pl}\right)\right)$$

The plastic section modulus of the corroded section can be written as:

$$\boxed{w_{pleffy} = w_{ply} + \delta_{pl}^2 \cdot S - \left(\sum_{i=1}^{n_c} A_{di}\left(z_{id} + \delta_{pl}\right) + \sum_{i=1}^{n_t} A_{di}\left(z_{id} - \delta_{pl}\right)\right)}$$

A-5: Derivation of Effective plastic axial compressive force (N_{pleff})

The derivation of this formula is based on the plastic stress distribution as shown in figure 7.4.

$$N_{pleff} = N_{plD} - Z_1 \cdot b_f \cdot f_y - Z_2 \cdot b_f \cdot f_y = N_{plD} - b_f \cdot f_y\left(Z_1 + Z_2\right)$$

$$\left(Z_1 \cdot b_f\right) \cdot \left(\frac{h}{2} - \delta_{pl} - \frac{Z_1}{2}\right) = \left(Z_2 \cdot b_f\right) \cdot \left(\frac{h}{2} + \delta_{pl} - t_d - \frac{Z_2}{2}\right)$$

$$\frac{Z_2}{2} = \frac{Z_1}{2} = t_d << \frac{h}{2}$$ (For thin wall plate sections)

$$\left(Z_1 \cdot b_f\right) \cdot \left(\frac{h}{2} - \delta_{pl}\right) = \left(Z_2 \cdot b_f\right) \cdot \left(\frac{h}{2} + \delta_{pl}\right)$$

$$Z_2 = Z_1 \cdot \frac{\left(\frac{h}{2} - \delta_{pl}\right)}{\left(\frac{h}{2} + \delta_{pl}\right)}$$

$$N_{pleff} = N_{plD} - b_f \cdot f_y\left(Z_1 + Z_2\right) = N_{plD} - b_f \cdot f_y\left[Z_1 + \left[Z_1 \cdot \frac{\left(\frac{h}{2} - \delta_{pl}\right)}{\left(\frac{h}{2} + \delta_{pl}\right)}\right]\right]$$

Rearranging the above equation, it will give as:

$$N_{pleff} = N_{plD} - \frac{b_f \cdot f_y \cdot Z_1 \cdot h}{\left(\frac{h}{2} + \delta_{pl}\right)}$$

To find Z1

$$\left[\left(Z_1 \cdot b_f\right) \cdot \left(\frac{h}{2} - \delta_{pl}\right) + \left(Z_2 \cdot b_f\right) \cdot \left(\frac{h}{2} + \delta_{pl}\right)\right] \cdot f_y = \delta M$$

$$\left[\left(Z_1 \cdot b_f\right) \cdot \left(\frac{h}{2} - \delta_{pl}\right) + \left[\left[Z_1 \cdot \frac{\left(\frac{h}{2} - \delta_{pl}\right)}{\left(\frac{h}{2} + \delta_{pl}\right)}\right] \cdot b_f\right] \cdot \left(\frac{h}{2} + \delta_{pl}\right)\right] \cdot f_y = \delta M$$

$$2\cdot\left[\left[\left(Z_1\cdot b_f\right)\cdot\left(\frac{h}{2}-\delta_{pl}\right)\right]\right]f_y = \delta M$$

$$Z_1 = \frac{\delta M}{2f_y\cdot b_f\cdot\left(\frac{h}{2}-\delta_{pl}\right)}$$

Then substituting for Z1

$$N_{pleff} = N_{plD} - \frac{b_f\cdot f_y\cdot Z_1\cdot h}{\left(\frac{h}{2}+\delta_{pl}\right)}$$

$$N_{pleff} = N_{plD} - \frac{b_f\cdot f_y\cdot h}{\left(\frac{h}{2}+\delta_{pl}\right)}\cdot\left[\frac{\delta M}{2f_y\cdot b_f\cdot\left(\frac{h}{2}-\delta_{pl}\right)}\right]$$

$$\boxed{N_{pleff} = N_{plD} - \left[\frac{\delta M\cdot h}{2\left(\frac{h}{2}-\delta_{pl}\right)\cdot\left(\frac{h}{2}+\delta_{pl}\right)}\right]}$$

A-6: Derivation of effective plastic moment due to N (M_{pleffN})

Case 1 $N_{Ed} < \left(A_w - A_d\right)\cdot f_y$

Based on stress block diagram of figure 7.20a.

$$M_{pleffN} = M_{pleff} - \frac{S\cdot e^2\cdot f_y}{4}$$

$$N_{Ed} = N_{pleffM} = e\cdot S\cdot f_y$$

$$e = \frac{N_{Ed}}{S\cdot f_y}$$

$$M_{pleffN} = M_{pleff} - \frac{S}{4}\left(\frac{N_{Ed}}{S\cdot f_y}\right)^2\cdot f_y$$

$$\boxed{M_{pleffN} = M_{pleff} - \frac{N_{Ed}^{\;2}}{4\cdot S\cdot f_y}}$$

Case 2 $\left(A_w - A_d\right)\cdot f_y \le N_{Ed} < \left(A_w + A_d\right)\cdot f_y$

Based on stress block diagram of figure 7.20b.

$$M_{pleffN} = M_{pleff} - \left[\left(Z_1 \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + \frac{Z_1}{2}\right) + 2\left(\frac{h_w}{2} - \delta_{pl}\right)\cdot S \cdot \frac{\left(\frac{h_w}{2} - \delta_{pl}\right)}{2} + \left(Z_2 \cdot S\right)\left(\frac{h_w}{2} - \delta_{pl} + \frac{Z_2}{2}\right)\right]\cdot f_y$$

$$N_{Ed} = N_{pleffM} = \left[\left(Z_1 \cdot b_f\right) + 2\left(\frac{h_w}{2} - \delta_{pl}\right)\cdot S + \left(Z_2 \cdot S\right)\right]\cdot f_y$$

$\frac{Z_1}{2} = \frac{Z_2}{2} << \frac{h_w}{2}$ (for thin wall plates)

$$\frac{Z_1}{Z_2} = \frac{S}{b_f}$$

$$M_{pleffN} = M_{pleff} - \left(\frac{h_w}{2} - \delta_{pl}\right)\left[2\left(Z_1 \cdot b_f\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot S\right]\cdot f_y$$

$$N_{Ed} = \left[2\left(Z_1 \cdot b_f\right) + 2\left(\frac{h_w}{2} - \delta_{pl}\right)\cdot S\right]\cdot f_y$$

Z1 can be found from N_{pleffM}

$$Z_1 = \frac{N_{Ed}}{2 \cdot b_f\, f_y} - \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot \frac{S}{b_f}$$

Substituting Z1 in M_{pleffN}

$$M_{pleffN} = M_{pleff} - \left(\frac{h_w}{2} - \delta_{pl}\right)\left[2\left(C_1 \cdot b_f\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot t_w\right]\cdot f_y$$

$$M_{pleffN} = M_{pleff} - \left(\frac{h_w}{2} - \delta_{pl}\right)\left[2\left[\left[\frac{N_{Ed}}{2 \cdot b_f\, f_y} - \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot \frac{S}{b_f}\right]\cdot b_f\right] + \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot S\right]\cdot f_y$$

$$M_{pleffN} = M_{pleff} - \left(\frac{h_w}{2} - \delta_{pl}\right)\left[\left[\frac{N_{Ed}}{f_y} - \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot 2S + \left(\frac{h_w}{2} - \delta_{pl}\right)\cdot S\right]\right]\cdot f_y$$

$$\boxed{M_{pleffN} = M_{pleff} - \left(\frac{h_w}{2} - \delta_{pl}\right)^2\left[\left(\frac{N_{Ed}}{f_y\left(\frac{h_w}{2} - \delta_{pl}\right)} - S\right)\cdot f_y\right]}$$

Case 3 $N_{Ed} > \left(A_w + A_d\right)\cdot f_y$

Based on stress block diagram of figure 7.20c

$$M_{pleffN} = M_{pleff} - \left[\left(Z_3 \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + t_d + \frac{Z_3}{2}\right) + \left(t_d \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + \frac{t_d}{2}\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)^2 \cdot S + \cdots \right.$$

$$\left. \left(2 \cdot \delta_{pl} \cdot S\right) \cdot \left(\frac{h_w}{2} - \delta_{pl} + \frac{2\delta_{pl}}{2}\right) + \left(Z_4 \cdot b_f\right)\left(\frac{h_w}{2} + \delta_{pl} + \frac{Z_4}{2}\right) \right] \cdot f_y$$

simplifying the above equation

$$M_{pleffN} = M_{pleff} - \left[\left[\left(Z_3 \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + t_d\right) + \left(t_d \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + \frac{t_d}{2}\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)^2 \cdot S + \cdots \right. \right.$$

$$\left. \left. \left(\delta_{pl} \cdot S\right) \cdot \left(h_w\right) + \left(Z_4 \cdot b_f\right)\left(\frac{h_w}{2} + \delta_{pl}\right) \right] \right] \cdot f_y$$

$$N_{Ed} = N_{pleffM} = \left[\left(Z_3 \cdot b_f\right) + \left(t_d \cdot b_f\right) + 2\left(\frac{h_w}{2} - \delta_{pl}\right) \cdot S + \left(2 \cdot \delta_{pl} \cdot S\right) + \left(Z_4 \cdot b_f\right)\right] \cdot f_y$$

$$N_{Ed} = N_{pleffM} = \left[\left(Z_3 \cdot b_f\right) + \left(t_d \cdot b_f\right) + \left(A_w\right) + \left(Z_4 \cdot b_f\right)\right] \cdot f_y$$

$$\frac{Z_3}{Z_4} = \frac{\left(\frac{h_w}{2} + \delta_{pl}\right)}{\left(\frac{h_w}{2} - \delta_{pl} + t_d\right)}$$

$$Z_4 = Z_3 \cdot \frac{\left(\frac{h_w}{2} - \delta_{pl} + t_d\right)}{\left(\frac{h_w}{2} + \delta_{pl}\right)}$$

substituting Z.4 in M_{pleffN} and N_{Ed}

$$M_{pleffN} = M_{pleff} - \left[\left[2\left(Z_3 \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + t_d\right) + \left(t_d \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + \frac{t_d}{2}\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)^2 \cdot S + \left(\delta_{pl} \cdot S\right) \cdot \left(h_w\right)\right]\right] \cdot f_y$$

$$N_{Ed} = N_{pleffM} = \left[\left(Z_3 \cdot b_f\right) + \left(t_d \cdot b_f\right) + \left(A_w\right) + \left[Z_3 \cdot \frac{b_f\left(\frac{h_w}{2} - \delta_{pl} + t_d\right)}{\left(\frac{h_w}{2} + \delta_{pl}\right)}\right]\right] \cdot f_y$$

$$N_{Ed} = N_{pleffM} = \left[\left(Z_3 \cdot b_f\right)\left(\frac{h_w + t_d}{\frac{h_w}{2} + \delta_{pl}}\right) + \left(t_d \cdot b_f\right) + \left(A_w\right)\right] \cdot f_y$$

Z3 can be found from N_{Ed}

$$Z_3 = \left[\frac{N_{Ed}}{f_y} - \left(t_d \cdot b_f\right) - \left(A_w\right)\right] \cdot \frac{\left(\frac{h_w}{2} + \delta_{pl}\right)}{\left(h_w + t_d\right) b_f}$$

Therefore M_{pleffN} can be calculated by :

$$M_{pleffN} = M_{pleff} - \left[\left[2\left(C \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + t_d\right) + \left(t_d \cdot b_f\right)\left(\frac{h_w}{2} - \delta_{pl} + \frac{t_d}{2}\right) + \left(\frac{h_w}{2} - \delta_{pl}\right)^2 \cdot S + \left(\delta_{pl} \cdot S\right) \cdot \left(h_w\right)\right]\right] \cdot f_y$$

where C is

$$C = \left[\frac{N_{Ed}}{f_y} - \left(t_d \cdot b_f\right) - \left(A_w\right)\right] \cdot \frac{\left(\frac{h_w}{2} + \delta_{pl}\right)}{\left(h_w + t_d\right) b_f}$$

Appendix B: Equations for plotting remaining capacity vs percentage area loss

B-1: Remaining effective plastic axial compressive force vs area loss

$$N_{pleff}\left(A_d\right) = \left(1 - \frac{A_d}{100}\right) - \left[\frac{\left(\frac{A_0}{2 \cdot S} \cdot h\right) \cdot \left(\frac{A_d}{100}\right)}{2\left[\frac{h}{2} - \left(\frac{A_0}{2 \cdot S}\right) \cdot \left(\frac{A_d}{100}\right)\right] \cdot \left[\frac{h}{2} + \left(\frac{A_0}{2 \cdot S}\right) \cdot \left(\frac{A_d}{100}\right)\right]}\right]$$

B-2: Remaining critical elastic axial compressive force vs area loss (about z-axis)

$$N_{creff}\left(A_d\right) = 1 - \left[\frac{b_f^{\,2}\cdot\left[\left(\frac{A_d}{100}\right)\cdot A_0\right]}{12\cdot I_{0z}}\right]$$

B-3: Remaining effective plastic moment capacity vs area loss

$$M_{pleffy}\left(A_d\right) = 1 + \frac{\left[\left(\frac{A_0^{\,2}}{2\cdot b_f}\right)\left(\frac{A_d}{100}\right)^2 - \left(\frac{A_0\cdot h}{2}\right)\left(\frac{A_d}{100}\right) - \left(\frac{A_0^{\,2}}{4S}\right)\left(\frac{A_d}{100}\right)^2\right]}{w_{ply}}$$