Cointegration and Pairs Trading in Major Cryptocurrencies

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Preface

This master thesis has been written to fulfill the graduation requirements of the Master of Science in Business Administration at the University of Stavanger. The choice of topic is a result of my interest and passion for investments and the cryptocurrency market.

I would like to thank my thesis advisor, Peter Molnár, for his vital assistance and valuable contribution to my study. I would also thank Svein Olav Krakstad for his contribution with regards to economic and econometric knowledge, as well as valuable inputs with regards to the statistical software package Stata.

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Vegard Isaksen

Abstract

This paper applies cointegration tests to identify cryptocurrency pairs which can be used in pairs trading strategies. The aim of this research is twofold. First, I want to examine cointegration in a system of bitcoin, dashcoin, dogecoin and litecoin. In the second part, I create pairs trading strategies in order to determine whether excess return can be made, compared to a simple buy and hold approach. The results find evidence of cointegration between the cryptocurrencies and positive profitability using pairs trading. By creating a portfolio in which the funds are equally allocated to the strategies with an open position, excess return can be made.

Keywords: cryptocurrency, cointegration, pairs trading

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1. Introduction

The cryptocurrency market has since its origin grown tremendously and become popular for investors and researchers worldwide. The most known cryptocurrency, bitcoin, was created in 2008 and started trading at a price of $0.003 American dollars in 2010. The 17^{th} of December 2017, bitcoin peaked at a price of $19,783.06. The opportunity to make high returns in short time resulted in large investments from both speculators and professionals, which created a high demand and rapidly increasing prices. The cryptocurrency market is susceptible of being a bubble (Cheah and Fry, 2015) and is compared to the dot-com bubble of 1994 (Folkinshteyn, Lennon and Reilly, 2015).

Pairs trading is aiming to be a market-neutral statistical arbitrage strategy, developed in the early mid 1980s, which simultaneously matches a long position with a short position in a pair of highly correlated assets. The underlying premise in relative pricing is that assets, with similar characteristics, must be priced more or less the same (Vidyamurthy, 2004). Profits are generated when the spread between two correlated assets revert back to its historical mean after deviating from their equilibrium relationship. One of the most cited papers in the pairs trading literature is the work of Gatev (2006). The study back-tested pairs trading on U.S. equities between 1962 and 2002, yielding 11% annualized excess return. However, Do and Faff (2010) replicated the work of Gatev (2006) and found evidence of decreasing profits using the strategy, due to growing popularity and increased competition among arbitregeurs.

Acharya and Pedersen (2005) found evidence of pairs trading profitability being negatively correlated with market liquidity, while Do and Faff (2010) argued that the strategy performs stronger during longer periods of high volatility. The cryptomarket has since its origin been subject to a high degree of volatility. In addition, the liquidity is lower than other emerged currencies (Carrick, 2016), making cryptocurrencies a candidate for excess return using pairs trading.

The pricing mechanism of cryptocurrencies is studied in Sovbetov (2018), Cheah and Fry (2015) as well as Bouoiyour, Tiwari, Selmi & Olayeni (2016). Sovbetov (2018) found evidence of Google search frequency for “Bitcoin”, “BTC” or “Blockchain” being fairly correlated with bitcoin and altcoin prices, making it a significant explanatory factor of cryptocurrency prices. The work of Sovbetov (2018) is consistent with Aalborg, Molnár & Vries (2018) who found that the trading volume of bitcoin can be predicted from google search for “Bitcoin”. Cheah and Fry (2015) found empirical evidence which supported that the fundamental price of bitcoin is zero. However, Bouoiyour et al (2016) argues that cryptocurrencies may be driven by long-term fundamentals while short-term fluctuations are a result of news, media coverage, hacking, political regulations and other external factors.

Pairs trading relies on two highly correlated assets. In order to find suitable assets, three commonly methods are applied to pairs trading in the literature: the distance model, cointegration and stochastic spread. Gatev (2006) selected pairs and created trading signals according to the distance model but Do, Faff and Hamza (2006) argued that this approach provided little help in forecasting. Elliott, Hoek and Malcolm (2005) is the most referenced work with regards to pairs trading using the stochastic spread method. However, this thesis will take the basis of Vidyamurthy (2004) who developed the framework for pairs trading using cointegration.

Cointegration, introduced by Engle and Granger (1987), is a common method to determine if there is a cause-effect relationship between two time series variables (Vidyamurthy, 2004). In a multivariate case, the Johansen test is commonly used to determine cointegration in a system of more than two time series (Johansen, 1988). If two series are said to be cointegrated, it exists a long-run equilibrium between them, and a short-run deviation from equilibrium would be corrected for in a future period. However, from a financial theory perspective, two assets which are moving together in the long-run would be in violation of the efficient market hypothesis (Schleifer, 2000). In addition, decreased diversification effect is another implication of cointegration for investors who seek to minimize risks (Balarezo, 2010).

The examined cryptocurrencies in this paper are bitcoin, dogecoin, litecoin and dashcoin. These are selected based on their relatively longer existence compared to other cryptocurrencies[1]. After conducting both the Engle and Granger approach and the Johansen test, the vector error correction model (VECM) is executed in order to reveal the short- and long-term effects of one cryptocurrency on another. The pairs trading strategy is conducted in the last part of the thesis, where the spread is estimated by regression. In this thesis, there was evidence of three cointegration relationships between the examined cryptocurrencies, and positive profitability using the pairs trading strategy.

[1] Ripple is not considered due to its different characteristics

2. Overview of studied cryptocurrencies

A cryptocurrency is a digital asset with the purpose to serve as a medium of exchange, functioning in a similar matter as fiat currencies (Broek & Sharif, 2018). Cryptocurrencies are considered to be decentralized, meaning no central authority, government or corporation possess access to an individual's funds or private information. In other words, third parties such as banks or financial institutions are not needed. The decentralized control works through a distributed ledger technology, typically known as blockchain. According to the coinmarketcap website, more than 2000 cryptocurrencies exists, serving different purposes and functions. In this chapter, a brief overview of the blockchain technology, as well as the cryptocurrencies involved in this thesis, will be presented.

2.1 Blockchain

Blockchain is the decentralized, public digital ledger used to record transactions between two parties in a verifiable and permanent way. Blockchain is a chain of blocks that contains information which are linked using cryptography. Each block contains a cryptographic hash, containing information about the transaction, who is participating in the transaction and information of how the block distinguishes itself from other blocks. The blocks are secured and bound to each other using cryptographic principles. As an example, when one party initiates a transaction process by creating a block, this block is verified by thousands of computers distributed in the network. The verified block, which is unique with a unique history, is added to the chain. Therefore, falsifying a single record would imply falsifying the entire chain, which is virtually impossible.

There are no transaction costs associated with blockchain, but the so called "proof of work" system, used to validate transactions and create new blocks, is not free. Miners compete against each other to complete transaction, by solving advanced mathematical puzzles, and are rewarded in cryptocurrency. The proof of work system is also a mechanism used to slow down transactions, with the purpose of making it impossible to tamper with the blocks, due to the amount of time it would take to recalculate the proof of work for all blocks.

The blockchain is maintained by a peer-to-peer network, which is a collection of peers (commonly known as nodes) that are interconnected to each other. Accordingly, there is no need for a central point of storage as the information is constantly recorded and interchanged between all parties in the network. Each node verifies the block which is added to the blockchain, making it almost impossible for a tampered block to be accepted within the system.

As the existing blockchain is public, which everyone can join, developers can implement new technology with the intent to improve certain features, commonly known as a fork. Cryptocurrencies that are emerged from a fork or ICO (initial coin offering) is defined as altcoins.

2.2 Bitcoin

Bitcoin was the first ever cryptocurrency, created in 2008 by the unknown person/group named Satoshi Nakamoto. It serves as an open-source network where transactions and the issuing of bitcoins are carried out collectively through the network. Bitcoin uses peer-to-peer technology and is exchanged through the digital ledger blockchain. The verification and connection of blocks happens through mining. Mining occurs when computer power is used to solve advanced mathematical puzzles which yields a hash. The hash is the link between the new block and the old chain. Miners are competing in order to solve the puzzles and are compensated in bitcoins. There exists a total amount of 21 million bitcoins that can be mined, and today more than 16 million bitcoins are released.

Per May 2019, bitcoin accounts for more than 50% of the total market capitalization of cryptocurrencies with a market cap of more than $120 billion dollars, making it by far the largest cryptocurrency.

2.3 Litecoin

Litecoin was announced in 2011 by Charlie Lee and is the 5th largest cryptocurrency with regards to market capitalization, with a market cap of more than $5 billion dollars. Litecoin serves many of the same features as bitcoin. However, litecoin manage to confirm transactions faster and can handle a higher volume of transactions. In order to create a new block, litecoin uses approximately 2.5 minutes, in contrast to bitcoin which uses approximately 10 minutes. Litecoin uses blockchain in order to verify transactions, and there is a maximum of 84 million litecoins that can be mined.

2.4 Dogecoin

Dogecoin was introduced in 2013 and has reached a market capitalization of more than $307 million dollars, being the 29th largest cryptocurrency per May 2019. Dogecoin can be compared to bitcoin as it enables peer-to-peer transactions across a decentralized network. However, dogecoin started much as a "joke" with the purpose of bringing something fun into cryptocurrencies. Even though the development team produces minimal updates, the dogecoin community stays active and loyal due to its feature of being both faster and cheaper than bitcoin.

2.5 Dashcoin

Dashcoin is a fork of bitcoin, introduced in 2014, and is the thirteenth biggest cryptocurrency with a market cap of more than $1 billion dollars. The purpose of dashcoin was to allow for fast and anonymous transactions which could overcome shortfalls in bitcoin through blockchain via so called masternodes. There is a maximum number of 18 million dashcoins which can exist, and average mining time is 2.5 minutes.

Bitcoin is based on an open source which can be found online. Accordingly, most of the created altcoins are created on the basis of bitcoin with the purpose of fixing certain issues or improve specific features related to bitcoin. Given the joint characteristics, it is natural to think that their prices may be connected. Due to the relatively high volatility and current regulatory situation, cryptocurrencies contribution to an asset portfolio is questionable. However, being a new developed asset class, diversification potential can be exploited as the characteristics of cryptocurrencies may potentially be less affected by shocks in the traditional market, making it a potential hedge option (Kurka, 2019). Chuen, Guo and Wang (2017) found that incorporating the cryptocurrency index CRIX, with a portfolio consisting of traditional assets, would improve the performance.

The intent of this paper is to examine whether pairs trading can obtain excess return in the cryptocurrency market, compared to a simple buy and hold strategy, based on a cointegration approach. The disposition of the thesis will be as follows: Chapter 3 presents the data used in the paper. The methodology is described in chapter 4 while chapter 5 presents the findings and a discussing of the results. Chapter 6 concludes.

3. Data

The data used are daily time series of bitcoin, dogecoin, dashcoin and litecoin. The data spans from 14th February 2014 until 25th January 2019, consisting of 1806 daily observations. According to the coinmarketcap website, the selected cryptocurrencies accounts for more than 60% of the total cryptocurrency market cap, per 9th of April 2019. The prices are collected from the cryptocurrency website coingecko and are expressed in American dollars. All variables are transformed by natural logarithm. Table 1 presents the correlation matrix between bitcoin, dashcoin, dogecoin and litecoin.

Table 1 – Correlation matrix

Crypto	Bitcoin	Dashcoin	Dogecoin	Litecoin
Bitcoin	1	-	-	-
Dashcoin	0.39	1	-	-
Dogecoin	0.53	0.29	1	-
Litecoin	0.61	0.34	0.49	1

Table 1 presents the correlation between the returns of bitcoin, dashcoin, dogecoin and litecoin, at daily intervals. Time period is from 14th of February 2014 until 25th of January 2019.

Cryptocurrencies are traded on digital exchanges and do not follow traditional opening and closing hours. Exchanges trading cryptocurrencies are open all day, 7 days a week, 365 days a year. The opportunity to trade at any time creates a wider window of trading possibilities.

As can be seen in the figures below, the price of bitcoin, dashcoin, dogecoin and litecoin increased rapidly until the end of 2017. At this time, the cryptocurrency market crashed and more than 50% of the market value vanished. The graphical representation of the cryptocurrencies may give an indication of their statistical properties (Bjørnland & Thorsrud, 2014). The figures present the price evolution, transformed by natural logarithm, of each cryptocurrency with the corresponding return.

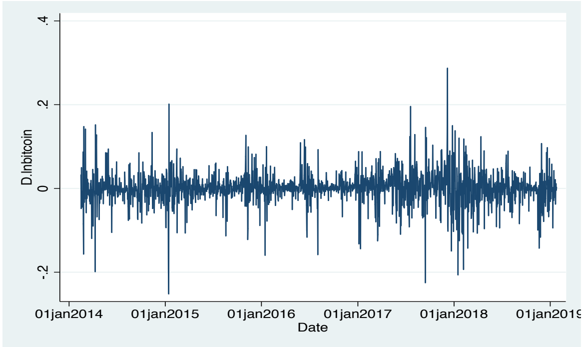
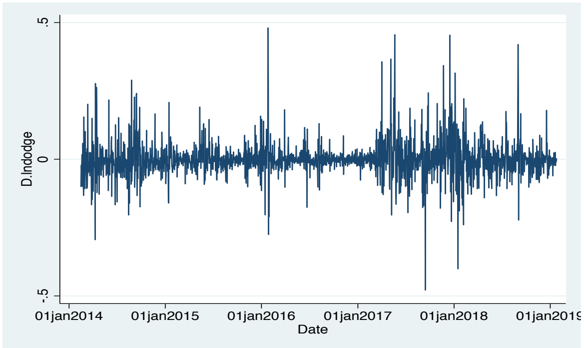
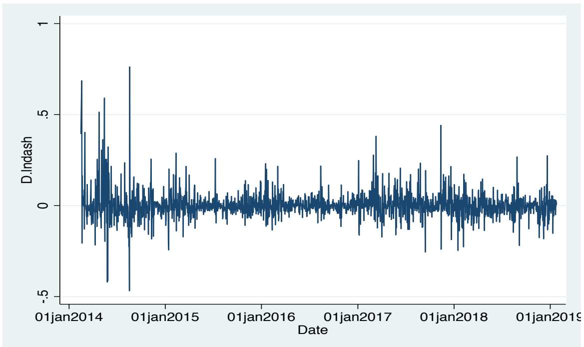


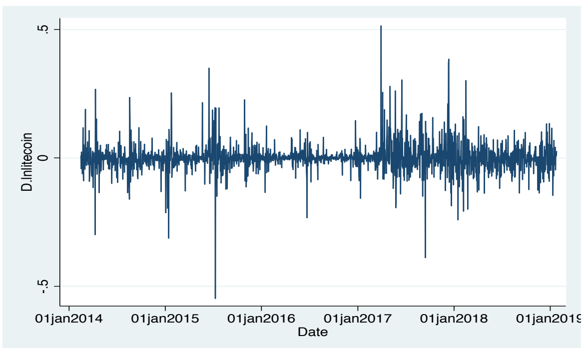
Figure 1 a) Bitcoin price *b) Return bitcoin*



c) Dogecoin price *d) Return dogecoin*



e) Dashcoin price *f) Return dashcoin*



g) Litecoin price *h) Return litecoin*

The visual inspection indicates non-stationarity in the original series, while the return seems to oscillate around a mean of zero, indicating stationarity. Table 2 presents descriptive statistics for the return of bitcoin, litecoin, dashcoin and dogecoin over the sample period.

Table 2 – Descriptive statistics of returns

Crypto	Number of observations	Mean	Std. Dev	Min	Max
Bitcoin	1806	0.0009	0.0396	-0.2518	0.2871
Litecoin	1806	0.0004	0.0583	-0.5472	0.5144
Dashcoin	1806	0.0032	0.0774	-0.4676	0.7619
Dogecoin	1806	0.0001	0.0669	-0.4781	0.4802

Table 2 Descriptive statistics for daily log returns from 14th February 2014 until 24th January 2019.

The logarithmic return is calculated according to equation (1)

$$r_t = \ln\left(\frac{P_t}{P_{t-1}}\right) \qquad (1)$$

Where r_t is the log return, P_t is the cryptocurrency price at day t and P_{t-1} is the cryptocurrency price from the previous day.

4. Methodology

A cointegrated relationship exists when asset prices are tied together in the long term, implying that short-run deviation from equilibrium will be corrected for in the long-run. Such a situation occurs when a linear combination of non-stationary variables is stationary (Enders 2010).

The most referenced methods used in the cointegration literature are the two-step Engle and Granger (1987) approach and the Johansen cointegration test (Johansen, 1988). Both methods will be used in this thesis. The Johansen test is appropriate when analyzing multivariate time series while the Engle and Granger is a bivariate test used on the relationship between two variables. Although both models share common characteristics, the Johansen test seeks the linear combinations which is most stationary. The Engle and Granger approach, on the other hand, is based on ordinary least squares (OLS) and locates the linear combination with minimum variance (Leung & Nguyen 2018).

A common method in the cointegration literature is to take out the mean of the time series, following the seminal paper of Engle and Granger (1996) and MacKinnon (1996). This is done according to equation (2)

$$y_t = (X_t - \bar{X}) \tag{2}$$

By extracting the mean, the constant term is expected to be zero. The stationarity test and VECM estimation follows this approach.

4.1 Stationarity

In order to perform cointegration tests, it is necessary to determine stationarity and non-stationarity properties. Stationarity is a necessary assumption in modelling and analyzing most time series, as non-stationary data is unpredictable and must, in most cases, be transformed in order to become stationary. A stationary process is defined by Challis and Kitney (1991) as where the mean, variance and autocorrelation structure do not change over time.

The presence of non-stationary variables may cause spurious regression (Granger & Newbold, 1974). Spurious regression can provide misleading statistical evidence of a linear relationship between trending variables (Wooldridge, 2014). This may happen when two local trends are similar, but it may not be true that they move together. Typically, this involves a high degree of fit, R^2, and a significant t statistic (Brooks, 2008). This form of misspecification can occur from:

i) The omission of relevant variables
ii) The inclusion of irrelevant variables
iii) Autocorrelated residuals

In order to alleviate the problem, Granger and Newbold (1974) propose to either include a lagged dependent variable, take the first difference of the variables involved in the equation or to assume simple first-order autoregressive form for the residual of the equation.

4.1.1 Augmented Dickey-Fuller test

If a unit root is detected in a time series, it is said to be non-stationary. In order to test for unit roots in a time series y_t, the Augmented Dickey-Fuller test will be performed, while the DF-GLS test is included to complement the results. The Augmented Dickey-Fuller test is an extension of the Dickey-Fuller test which have been augmented with the lagged changes Δy_{t-p}. The inclusion of the lagged changes is intended to clean up any serial correlation in Δy_t. It follows equation (3)

$$\Delta y_t = \gamma y_{t-1} + \sum_{j=1}^{p} \beta_j \Delta y_{t-j} + e_t \qquad (3)$$

Where p is the number of lags and Δy_t denotes the value of a variable at time t ($\Delta y_t = y_t - y_{t-1}$). The hypothesis can be noted as:

H_0: $\gamma = 0$ (unit root)
H_1: $\gamma < 0$ (stationary data).

H_0 is rejected if $t_\theta < c$, where c is the critical value. If a unit root is detected, which implies that the time series are non-stationary, Granger and Newbold (1974) proposed differencing the series. If differencing the series *d* times yield stationary data, the original series is said to be integrated of order *d*, denoted $x_t \sim I(d)$.

The critical test statistic is obtained according to equation (4)

$$\text{test statistic, } t_\theta = \frac{\hat{\gamma}}{SE(\hat{\gamma})} \qquad (4)$$

Where $\hat{\gamma}$ is the estimated coefficient from equation 3. The test statistic does not follow a normal distribution, but rather a distribution developed by Dickey-Fuller, leading to non-asymptotic critical values. Appropriate critical values can be found in Dickey and Fuller (1979)

Campbell and Perron (1991) discussed the importance of correctly specifying deterministic components of the series. The critical values used to reject the null is sensitive to whether or not deterministic factors are included. As an example: not accounting for trend, if trend is present, may lead to spurious regression (Enders, 2010).

4.1.2 DF-GLS test

The Augmented Dickey-fuller test have been criticized for lack of power and size distortion as it may struggle to distinguish between $\gamma = 0.95$ and $\gamma = 1$ (Brooks, 2008). Accordingly, a modified Dickey-Fuller approach, DF-GLS, proposed by Elliott, Rothenberg and Stock (1996) will be used to complement the results. Studies has shown that this test has significantly greater power than the traditional augmented Dickey-Fuller (Becketti, 2013). The modified test performs a generalized least squares (GLS) transformation prior to the estimation of Dickey-Fuller. The problem of choosing appropriate deterministic factors is less critical in the DF-GLS method (Enders, 2010). Stock and Watson (2011) provides a detailed discussion of the DF-GLS approach. The DF-GLS test follows equation (5)

$$\Delta y_t^d = \gamma y_{t-1}^d + \sum_{j=1}^{p} \beta_i \Delta y_{t-j}^d + e_t \qquad (5)$$

The notation of *d* represents the GLS de-trending. The use of GLS de-trending gives a substantial gain in power and improves the ability to distinguish between the null and the alternative as it removes any linear trends in the series (Perron & Rodriguez, 2003).

The hypothesis tests are as follows:

H_0 = Series is a random walk
H_1 = Series is stationary

Appropriate critical values can be found in Cheung and Lai (1995). In addition, the results are affected by the maximum number of lags included. The maximum number of lags follows the work of Schwert (1989), and is calculated according to equation (6)

$$k_{max} = floor\left[12\left\{\frac{T+1}{100}\right\}^{0,25}\right] \qquad (6)$$

4.2 Lag length

It is crucial for both the stationarity test and cointegration analysis to select the appropriate number of lags. Cheung and Lai (1995) revealed that the lag order can significantly affect the critical values of the stationarity tests.

Studies are not consistent regarding the most correct way to determine the appropriate number of lags. However, the most widely used selection criteria are the Schwartz Bayesian Information Criterion (SBIC) and the Akaike Information Criterion, proposed by Schwartz (1978) and Akaike (1973), respectively. Lütkepohl (2005) argues that the SBIC provides consistent estimates of the true lag order, while the AIC overestimates the lag order with positive probability. Choosing SBIC over AIC is in accordance with Koehler and Murphree (1987) who argued that the AIC and SBIC often choose different number of lags and that AIC will overfit the data. Accordingly, the SBIC will be chosen if the two information criteria differ. The SBIC is computed according to equation (7)

$$SBIC = -2\left(\frac{LL}{T}\right) + \frac{2ln\{\ln(T)\}}{T} t_p \qquad (7)$$

where LL is the log likelihood, t_p is the total number of parameters in the model and T is the number of observations. The appropriate number of lags is chosen where SBIC is minimized.

4.3 The Johansen Test & Vector Error Correction Model (VECM)

The Johansen test belongs to the multivariate cointegration category. This involves testing for cointegration in a system of two or more variables. If there exist k variables, a maximum of k-1 cointegrating vectors can exist. The Johansen approach relies on a vector autoregressive (VAR) model (appendix). In summary, it is defined as a stochastic process used to identify the linear independencies between several time series (Wooldridge 2014). In a VAR model, all variables have an equation explaining its development based on its own, and other variables lagged values, as well as an error term. The Johansen test is used to obtain the number of ranks which would be included in the vector error correction model.

In order to use the Johansen test, the VAR must be transformed into a Vector Error Correction Model (VECM) by including error correction features to the VAR model (Brooks, 2008). A VECM is basically a VAR in first difference with a vector of cointegrating residuals. Any VAR(p) can after some algebraic manipulation be rewritten to a VECM of the form:

$$\Delta y_t = \mu + \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t \qquad (8)$$

Where

$$\Pi = \sum_{j=1}^{p} \Phi_j - I$$

And

$$\Gamma_j = -\sum_{j=i+1}^{p} \Phi_j$$

Where p-1 is the lags of the dependent variable, Γ is the coefficient matrix of every lagged variable and Π is the long-run coefficient matrix. The VECM in equation 8 contains information on both the long-run equilibrium and the short-run dynamics between the variables in y_t (Bjørnland and Thorsrud, 2014). Engle and Granger (1987) showed that Π has a rank r, $0 \leq r < K$, where r is the number of linearly independent cointegrating relationships among the elements of y_t.

The Π matrix is defined as the product of two matrices, α (short-term matrix) and β' (long-term matrix), of dimensions (g x r) and (r x g), respectively (Brooks, 2008). Π can be written in terms of:

$$\Pi = \alpha\beta' \qquad (9)$$

The β matrix gives the cointegrating vectors, while α (also known as the adjustment parameter) gives the amount of each cointegrating vector entering each equation of the VECM (Brooks, 2008). The interpretation would be that α measures the speed at which the variables adjust to the long-run equilibrium while β represents the long-run cointegrated relationship. If the Π matrix equals zero ($\Pi = 0$), the variables are not cointegrated.

The VECM is estimated by maximum likelihood estimation process, in contrast to Engle and Granger which used the ordinary least squares estimation. The Johansen method estimates the rank, r, of the matrix Π. The rank equals the number of characteristic roots (eigenvalues) which are significantly different from zero (Dwyer, 2015).

According to Bjørnland & Thorsrud (2014), there are three possible outcomes with regard to the number of ranks in the Π matrix:

1) Full rank (r = k): Every eigenvalue is different from zero and significant. This indicates that the original variables are stationary and cointegration is impossible.
2) Rank is zero (r = 0): There are no eigenvalues different from zero and thus no cointegration.
3) Reduced rank (0 < r < k): Cointegration exist in the system, with r cointegrating vectors.

In order to compute the number of ranks in the VECM, the Johansen test is used. The Johansen test defines the maximum eigenvalue and trace statistics, which are computed according to equations (10) and (11)

Maximum eigenvalue, $$\lambda_{max}(r, r+1) = -Tln(1 - \widehat{\lambda_{r+1}}) \quad (10)$$

Where:

H_0 = r cointegrating vectors

H_1 = r + 1 cointegrating vectors

Trace statistic, $$\lambda_{trace}(r) = -T \sum_{i=r+1}^{k} \ln(1 - \hat{\lambda}_i) \quad (11)$$

Where:

H_0 = r or less than r cointegrating vectors

H_1 = More than r cointegrating vectors

The $\widehat{\lambda_i}$ is the estimated eigenvalue, T is the number of observations and r is the number of cointegrating vectors. The critical values used in the model depends on the value k – r and the included deterministic features. This can be found in Johansen and Juselius (1990).

Five different cases can be used in estimation of both the Johansen test and VECM, with basis of equation 3 (Becketti, 2013):

1) Unrestricted trend: It is assumed that there are quadratic trends in the level of y_t and the cointegrating equations are trend stationary.
2) Restricted trend, $\tau = 0$: A linear trend, but not a quadratic trend, is included in y_t. The cointegrating equations are trend stationary.
3) Unrestricted constant, $\tau = \rho = 0$: The observable variables follows a linear trend, but the cointegrating equations are stationary around constant means.
4) Restricted constant, $\tau = \rho = \gamma = 0$: y_t includes no trend, but the cointegrating equations are stationary around a constant mean.
5) No trend, $\tau = \rho = \gamma = \mu = 0$: No non-zero means or trends.

4.4 Pairs trading strategy

Pairs trading strategy is based on the concept of relative pricing. If a cointegrated relationship is detected, and a long-run equilibrium exists, a temporary deviation from the equilibrium could be exploited by taking a long position in the relative cheap asset and short position in the relative expensive asset. Profits are generated when the deviation is corrected. Pairs trading strategy is a market neutral strategy where close to all risk is asset specific risk. Market neutrality holds as a long and short position of the same value is open and closed at the same time. Suitable pairs can be found using the Johansen test, and Engle and Granger test.

The bivariate Engle and Granger approach is used to estimate the beta in the cointegrated relationship between two assets, according to equation (12)

$$\ln(y_t) = \alpha + \beta \ln(z_t) + u_t \qquad (12)$$

Where the cointegration coefficient, β, is the expected percentage change in the price of y_t when the price of z_t change. When the cointegrated relationship has been established, the spread between asset y_t and z_t is defined according to equation (13)

$$S_t = \ln(y_t) - \alpha - \beta \ln(z_t) \tag{13}$$

The spread at time t, S_t, will be a stationary zero mean variable. In order to create a strategy, the spread must be monitored. If $S_t \neq 0$, a departure from the relationship in (12) is detected. If the spread exceed a predefined threshold, *d*, a position may be opened.

If:

1) $S_t < -d$: Asset z_t is overvalued relative to asset y_t. Then a long position in y_t and short position in z_t is created
2) $S_t > d$: Asset z_t is undervalued relative to asset y_t. Then a long position in z_t and short position in y_t is created.

In the pairs trading literatur, it is common to make a standardized value of the spread. The standardized value measures the distance to the long-term mean in terms of long-term standard deviation (Caldeira & Moura, 2013). This is done accordingly:

$$Standardized\ value = \frac{S_t - \bar{S_t}}{STD(S_t)} \tag{14}$$

Both positions are closed when the spread crosses a predefined threshold which is closer to the mean than the opening thresholds.

5. Results

5.1 Cointegration

5.1.1 Testing for unit roots

In order to determine whether the cryptocurrencies are cointegrated, a requirement is that they are intergrated of the same order. The examination of the figures in chapter 3 indicated integration of order 1, $I\ (1)$, in all cases. In order to determine this property, the Augmented Dickey-Fuller test is performed on the natural logarithm of each series. The DF-GLS test is included to complement the result of the Augmented Dickey-Fuller test.

The Schwarz information criteria is used in order to specify the number of lags. The null hypothesis, unit root in the times series, is rejected if the test statistic is lower than the critical values, $t_\theta < c$. There is no constant in the equation as the mean has been taken out of the time series. The results of the Augmented Dickey-Fuller test and DF-GLS test are presented in table 3

Table 3 – Stationarity test

	Augmented Dickey-Fuller		DF-GLS	
	Level	Return	Level	Return
Bitcoin	- 0.391	- 30.982***	0.032	- 18.022***
	(0.0007)	(0.0333)		
Dogecoin	- 0.985	- 28.630***	-0.851	- 10.744***
	(0.0011)	(0.0329)		
Dashcoin	- 1.863	- 30.013***	0.700	- 3.563***
	(0.0008)	(0.0329)		
Litecoin	- 0.696	- 31.607***	-0.739	-30.733***
	(0.0009)	(0.0331)		

*Table 3 The Augmented Dickey-Fuller test complemented with the DF-GLS test. Standard Error in parenthesis. Critical values for the DF-GLS test can be found in Elliott, Rothenberg and Stock (1996), while critical values for Augmented Dickey-Fuller can be found in MacKinnon (1996). Symbols ***, ** and * represents significance at 1%, 5% and 10% respectively.*

None of the test statistics are lower than the critical values of the series in original form. Hence, the null hypothesis cannot be rejected, indicating unit roots in the series and non-stationarity. The test statistics of the return are all signficantly lower than the critical values and the null hypothesis can be rejected. Accordingly, the return is stationary*,* implying that the series are integrated of order 1, *I(1)*. The DF-GLS test confirms the findings of the Augmented Dickey-Fuller test. The stationarity tests imply integration of the same order among all cryptocurrencies, making cointegration theoretically possible.

5.1.2 The Johansen test

The Johansen test is used in order to determine the number of cointegrating relationships in a system of several variables, which is used in the vector error correction model. The test is performed with no constant or trend ($\tau = \rho = \gamma = \mu = 0$).

Testing for cointegration in a multivariate case involves determining the rank of Π. The Johansen test consist of the trace and maximum eigenvalue tests where the null hypothesis, with regards to the trace statistic, is no more than r cointegration vectors, versus the alternative hypothesis of r > 0. If the null is rejected, it proceeds to r ≤ 1, versus r > 1. This continues until the null is accepted. The eigenvalue test performs a likelihood-ratio test where the null hypothesis is exactly r cointegrating vectors versus the alternative of r + 1 (Becketti, 2013). Table 4 presents the results from the Johansen test

Table 4 – Johansen cointegration test

	Cointegrating relationships (r)	Test statistics Trace	Max	Lags
	None r = 0	67.66 (39.89)	43.16 (23.80))	1
System	Up to 1 r = 1	24.50 (24.31)	14.10* (17.89)	1
	Up to 2 r = 2	10.39* (12.53)	9.15 (11.44)	1
	Up to 3 r = 3	1.24 (3.84)	1.24 (3.84)	1

Table 4 The trace and maximum eigenvalue test with 5% critical values in parentheses. The included variables are bitcoin, litecoin, dashcoin and dogecoin. Sample period is from 14th of February 2014 to 25th January 2019

The trace statistic accepts the null hypothesis of r ≤ 2, against the alternative of r > 2 cointegrating vectors. The maximum eigenvalue test accepts the null hypothesis of r =1 against r + 1 cointegrating vectors. However, as the trace statistic only marginally rejects three cointegrating vectors, and the bivariate Engle and Granger test (appendix) indicates cointegration among the pairs, it is assumed three cointegrating vectors when specifying the VECM.

5.1.3 Fitting the VECM

The rank of Π is estimated to be three and the VECM can be specified. The VECM is used to determine the long- and short-run relationship between the series and is performed with no constant or trend ($\tau = \rho = \gamma = \mu = 0$). The estimated equations, with basis of equation 8, are as follows:

$$\Delta y_t = \alpha(\beta' y_{t-1}) + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t \qquad (15)$$

Where

$$\Delta dogecoin_t = \alpha_1 ECT_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta dogecoin_{t-i} + \sum_{j=1}^{p-1} \rho_j \Delta dashcoin_{t-j} + \sum_{k=1}^{p-1} \eta_k \Delta litecoin_{t-k} + \sum_{l=1}^{p-1} \varphi_l \Delta bitcoin_{t-l} + \epsilon_{1t}$$

$$\Delta dashcoin_t = \alpha_2 ECT_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta dogecoin_{t-i} + \sum_{j=1}^{p-1} \rho_j \Delta dashcoin_{t-j} + \sum_{k=1}^{p-1} \eta_k \Delta litecoin_{t-k} + \sum_{l=1}^{p-1} \varphi_l \Delta bitcoin_{t-l} + \epsilon_{2t}$$

$$\Delta litecoin_t = \alpha_3 ECT_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta dogecoin_{t-i} + \sum_{j=1}^{p-1} \rho_j \Delta dashcoin_{t-j} + \sum_{k=1}^{p-1} \eta_k \Delta litecoin_{t-k} + \sum_{l=1}^{p-1} \varphi_l \Delta bitcoin_{t-l} + \epsilon_{3t}$$

$$\Delta bitcoin_t = \alpha_4 ECT_{t-1} + \sum_{i=1}^{p-1} \delta_i \Delta dogecoin_{t-i} + \sum_{j=1}^{p-1} \rho_j \Delta dashcoin_{t-j} + \sum_{k=1}^{p-1} \eta_k \Delta litecoin_{t-k} + \sum_{l=1}^{p-1} \varphi_l \Delta bitcoin_{t-l} + \epsilon_{4t}$$

Where:

- Lag is 1
- ρ_j, φ_l, δ_i and η_k are short-run dynamic coefficients of the model's adjustment to long-run equilibrium
- α_i is the speed of adjustment coefficient
- ECT_{t-1} is the error correction term
- ϵ_{it} are the residuals

Due to $\Pi = \alpha\beta'$ is less than full rank, restrictions are needed in order to identify the elements of the two matrices (Becketti 2013). In the case of r cointegrating relationships, r^2 restrictions are necessary to estimate under the assumption of exact identification. The VECM will be performed both as an unrestricted and restricted model after testing for weak exogeneity. There are three cointegrating vectors and four variables in the system. Hence, the elements of Π, where α and β are (4 X 3), can be written as follows:

$$\Pi = \alpha\beta' = \begin{pmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} \\ \alpha_{12} & \alpha_{22} & \alpha_{32} \\ \alpha_{13} & \alpha_{23} & \alpha_{33} \\ \alpha_{14} & \alpha_{24} & \alpha_{34} \end{pmatrix} \begin{pmatrix} \beta_{11} & \beta_{12} & \beta_{13} & \beta_{14} \\ \beta_{21} & \beta_{22} & \beta_{23} & \beta_{24} \\ \beta_{31} & \beta_{32} & \beta_{33} & \beta_{34} \end{pmatrix}$$

The long-run coefficients are normalized in order for the relationships to be expressed with one of the variables being the dependent variable. This yields the following unrestricted α and β matrices with short-run coefficients:

Table 5 − β matrix

Variable	CV 1 β_{1i}	CV 2 β_{2i}	CV 3 β_{3i}
$Dogecoin_{t-1}$	1	0	0
$Dashcoin_{t-1}$	0	1	0
$Litecoin_{t-1}$	0	0	1
$Bitcoin_{t-1}$	-1.03***	-1.46***	-1.17***
	(0.087)	(0.089)	(0.105)

*Table 5 The unrestricted beta matrix shows the long-run relationship between dogecoin, dashcoin, litecoin and bitcoin. The following restrictions are imposed in order to estimate the beta matrix under the assumption of exact identification: $\beta_{11} = \beta_{22} = \beta_{33} = 1$ and $\beta_{21} = \beta_{31} = \beta_{12} = \beta_{32} = \beta_{13} = \beta_{23} = 0$. Standard Error in parentheses. CV 1, CV2 and CV3 are the cointegration vector 1, 2 and 3, respectively. Symbols ***, ** and * represents significance at 1%, 5% and 10%, respectively.*

Table 6 - α matrix and short-run coefficients

		Dependent Variables			
Variables	Coefficient	Dogecoin	Dashcoin	Litecoin	Bitcoin
Speed adjustment CV1	α_1	- 0.015*** (0.004)	- 0.001 (0.005)	0.001 (0.004)	- 0.003 (0.003)
Speed adjustment CV2	α_2	0.003 (0.003)	- 0.007** (0.003)	0.006*** (0.002)	0.005*** (0.002)
Speed adjustment CV3	α_3	0.004 (0.004)	- 0.001 (0.005)	- 0.006 (0.004)	0.004 (0.0027)
$\Delta dogecoin$	δ_i	0.0447 (0.0285)	-0.0115 (0.0330)	-0.0211 (0.025)	-0.011 (0.016)
$\Delta dashcoin$	ρ_j	0.0286 (0.0222)	0.0365 (0.025)	0.0207 (0.019)	0.011 (0.0132)
$\Delta litecoin$	η_k	0.0159 (0.0352)	0.0165 (0.041)	-0.010 (0.031)	-0.033 (0.0209)
$\Delta bitcoin$	φ_l	-0.1328** (0.0542)	-0.1477** (0.0627)	0.033 (0.047)	0.0222 (0.0322)

*Table 6 shows the unrestricted alpha matrix found by VECM and short-run coefficients. Standard Error in parenthesis. Symbols ***, ** and * represents significance at 1%, 5% and 10%, respectively. α_1, α_2 and α_3 represents the speed adjustment coefficient from cointegration vector 1, 2 and 3, respectively.*

A significant alpha indicates the speed of adjustment after deviating from the long-run equilibrium.

The next step is testing for over-identified restrictions. The first test involves testing for weak exogeneity, whether or not $\alpha = 0$ for the respective equations by using likelihood ratio (LR) test. If weak exogeneity is detected in a variable, it means that the variable does not adapt to deviations from equilibrium and that the stochastic process of the variable drives the system (Bjørnland & Thorsrud, 2014). Table 7 shows the result of the weak exogeneity test.

Table 7 – Weak exogeneity test

Test	Variable	Restrictions	LR test	p-value
1	Dogecoin	$\alpha_{11} = \alpha_{21} = \alpha_{31} = 0$	20.24	0.000
2	Dashcoin	$\alpha_{12} = \alpha_{22} = \alpha_{32} = 0$	5.14	0.161
3	Litecoin	$\alpha_{13} = \alpha_{23} = \alpha_{33} = 0$	13.63	0.003
4	Bitcoin	$\alpha_{14} = \alpha_{24} = \alpha_{34} = 0$	14.52	0.002

Table 7 Test 1-4 is used to check weak exogeneity for all variables. Restrictions imposed to beta matrix is under the assumption of exact identification: $\beta_{11} = \beta_{22} = \beta_{33} = 1$ and $\beta_{21} = \beta_{31} = \beta_{12} = \beta_{32} = \beta_{13} = \beta_{23} = 0$. LR = Likelihood ratio

The LR-tests in table 7 rejects the possibility of dogecoin, litecoin or bitcoin being weakly exogenous (p-values are below 5%). However, test 2 cannot reject the possibility that $\alpha_{12} = \alpha_{22} = \alpha_{32} = 0$. This result is surprising as the speed-adjustment coefficient in table 6 is significant. Intuitively, dashcoin is not the driver of the system. By re-doing the test with a sample period starting 6 months later, p-value is below 5%. Hence, dashcoin is assumed not to be weakly exogeneous in the restricted VECM.

Next, it tested whether $\beta_{14} = \beta_{24} = \beta_{34} = -1$. The LR test cannot reject that $\beta_{14} = \beta_{34} = -1$ (p-value > 0.1) but rejects that $\beta_{24} = -1$ (p-value = 0.004). Naturally, it is tested that $\beta_{14} = \beta_{34} = -1$, where β_{24} is freely estimated. The LR test cannot reject the null (p-value = 0.271), making β_{14} and β_{34} restricted to -1. Accordingly, the β vectors are equal [1, -1] for the pairs of dogecoin/bitcoin and litecoin/bitcoin.

Table 8 and 9 presents the VECM results with restrictions according to the overidentifying tests. The alpha values are the coefficients to be estimated. In addition, insignificant alphas are restricted to zero.

Table 8 − β matrix

Variable	CV 1 β_{i1}	CV 2 β_{i2}	CV 3 β_{i3}
$Dogecoin_{t-1}$	1	0	0
$Dashcoin_{t-1}$	0	1	0
$Litecoin_{t-1}$	0	0	1
$Bitcoin_{t-1}$	-1.00***	-1.46*** (0.089)	-1.00***

Table 8 This table presents the restricted beta matrix from VECM. The following restrictions are imposed: $\beta_{11} = \beta_{22} = \beta_{33} = 1$ *and* $\beta_{21} = \beta_{31} = \beta_{12} = \beta_{32} = \beta_{13} = \beta_{23} = 0$ *and* $\beta_{14} = \beta_{34} = -1$. *Symbols ***, ** and * represents significance at 1%, 5% and 10% respectively.*

Table 9 − α matrix and short-run coefficients

		Dependent Variables			
Variables	Coefficient	Dogecoin	Dashcoin	Litecoin	Bitcoin
Speed adjustment CV1	α_1	- 0.013*** (0.023)	0	0	0
Speed adjustment CV2	α_2	0	- 0.007** (0.003)	0.006*** (0.002)	0.004*** (0.001)
Speed adjustment CV3	α_3	0	0	0	0
$\Delta dogecoin$	δ_i	0.0444 (0.0285)	-0.0109 (0.0329)	-0.0182 (0.0249)	-0.0131 (0.0169)
$\Delta dashcoin$	ρ_j	0.0279 (0.0222)	0.0357 (0.0256)	0.0198 (0.0194)	0.0105 (0.0132)
$\Delta litecoin$	η_k	0.0197 (0.0351)	0.0164 (0.0406)	-0.0122 (0.0307)	-0.0311 (0.0209)
$\Delta bitcoin$	φ_l	-0.1328** (0.0542)	-0.1459** (0.0625)	0.0347 (0.0473)	0.0239 (0.0321)

Table 9 This table presents the results of the restricted VECM. Insignificant alphas are restricted to zero while the beta coefficients are restricted as follows: $\beta_{11} = \beta_{22} = \beta_{33} = 1$, $\beta_{21} = \beta_{31} = \beta_{12} = \beta_{32} = \beta_{13} = \beta_{23} = 0$ *and* $\beta_{14} = \beta_{34} = -1$. *CV1, CV2 and CV3 are the first, second and third cointegration vector, respectively. Symbols ***, ** and * represents significance at 1%, 5% and 10%*

As presented in table 8, the β vectors are significantly equal [1, -1] for the pairs of dogecoin/bitcoin and litecoin/bitcoin. After excluding insignificant alphas, table 9 shows the speed-adjustment coefficients which are all highly significant.

The results are in line with Nguyen & Leung (2018) who found long-term relationships between bitcoin, bitcoin cash, ethereum and litecoin. Broek et al (2018) found 31 cointegrated pairs within the cryptocurrency market after testing 952 potential pairs. Sovbetov (2018) found long-run equilibrium among a system consisting of bitcoin, ethereum, dash, litecoin and monero, using weekly observations. More specific comparable literature is hard to find as the cryptocurrency market is not heavily researched. As the long- and short-run relationship has been established, a pairs trading strategy can be implemented in order to potentially exploit arbitrage opportunities.

5.2 Pairs trading strategy

The pairs trading strategy is aiming not to be affected by market movements, as a long and short position, having the same value, is opened and closed simultaneously. However, due to historical limited possibilities, with regards to shorting cryptocurrencies, a second more practical investment strategy, which only allows long positions, will be implemented. In the remaining part of the thesis, these strategies will be denoted unrestricted and restricted strategy, respectively. The strategies will be performed on the pair of bitcoin/dogecoin, bitcoin/dashcoin and bitcoin/litecoin, due to the evidence of cointegration among the pairs. Only pairs involving bitcoin will be included as bitcoin is the largest cryptocurrency, with longest history, in which the other cryptocurrencies most intuitively are dependent on. Moreover, bitcoin is the most liquid, hence trading bitcoin is more easily implementable. The strategy follows the work of Vidyamurthy (2004) and Caldeira and Moura (2013).

The bivariate Engle and Granger approach is used to estimate the cointegration coefficient β and to establish the relationship:

$$\ln(Y) = \alpha + \beta \ln(z) \tag{16}$$

Where α is the constant. Due to the frequently changing structure of volatility and expected return of bitcoin (Molnár and Thies, 2018), performing regression based on historical returns, over the whole sample period, could give inaccurate results. Hence, the beta value in equation 16 is moving with one-year interval. This implies that each beta represents the relationship between the cryptocurrency pair for 365 days. The spread between cryptocurrency y and z, which is used to open or close a position, is calculated as follows:

$$S_t = \ln(Y) - \alpha - \beta\ln(z) \tag{17}$$

And is standardized according to

$$Spread_{standardized} = \frac{S_t - \bar{S_t}}{STD(S_t)} \tag{18}$$

The following figures present the standardized spread of each pair and their respective threshold levels. The widest thresholds, at 0.7 and -0.7, represents the opening signals, while the narrower thresholds, at 0.4 and -0.4, represents the closing signals.

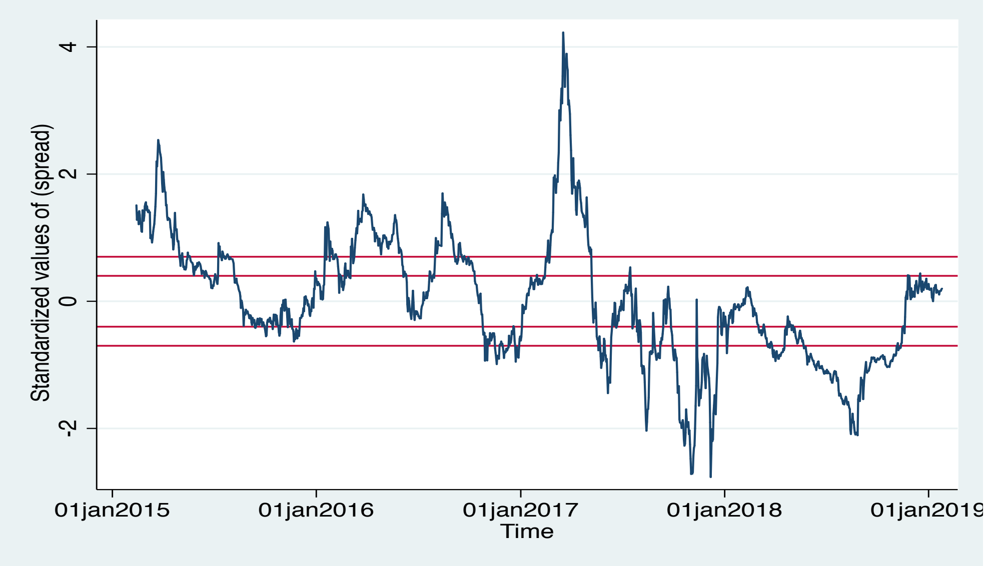


Figure 2 Standardized spread between dashcoin and bitcoin. Thresholds at -0.7, 0.7, -0.4 and 0.4

Figure 3 Standardized spread between bitcoin and dogecoin. Thresholds at -0.7, 0.7, -0.4 and 0.4

Figure 4 Standardized spread between litecoin and bitcoin. Thresholds at -0.7, 0.7, -0.4 and 0.4

In order to open a position, Caldeira and Moura (2013) suggest using a threshold level of 2/-2 and close the position at 1.5/-1.5. These specifications would, in this thesis, result in a low number of trades. Hence, the thresholds used are thereby lower. A simple buy and hold strategy is used as a benchmark while performance measures of the S&P 500 is included for illustrative purposes. In order to back-test the pairs trading strategy, average return per day, annualized return, annualized volatility and Sharpe Ratio is examined. The price of the crypto in which a long position has been opened is represented as X^l, while the price of the crypto in which a short position has been opened is represented as X^s. The statistics are computed as follows:

Return at day t	$\frac{ln\left[\frac{X_t^l}{X^l{}_{t-1}}\right] - \beta ln\left[\frac{X^s{}_t}{X^s{}_{t-1}}\right]}{1+\beta}$	(19)
Annualized return	$\bar{r} * 365$	(20)
Volatility, σ	$\sqrt{\frac{1}{N-1}\sum_{i=1}^{n}(z_i - \bar{z})}$	(21)
Annualized volatility	$\sigma * \sqrt{365}$	(22)
Sharpe Ratio	$\frac{r_t - r_f}{Std_t}$	(23)

Where r_f is the 12 months US treasury rate and r_t is annualized return.

The performance of the buy and hold strategy is calculated as a portfolio with equal weights assigned to the two respective cryptocurrencies. The portfolio return is calculated according to equation (24):

Portfolio return at day t, R_t	$\sum_{i=1}^{N} w_i R_i$	(24)

Annualized return, volatility and Sharpe ratio are calculated according to equations (20), (21) and (23).

5.2.1 The unrestricted strategy

In the unrestricted strategy, there are no restrictions attached to shorting. In order to create trading signals, and determine when to open and close a position, some predefined investment rules must be specified. As the spread has been calculated as $S_t = ln\ (y_t) - \alpha - \beta_1\ ln(z_t)$ and standardized according to equation (18), the trading signal is based on the distance to the long-term mean in units of standard deviation. When the standardized spread crosses a threshold of 0.7 from above or -0.7 from below, the cryptocurrencies in the pair are mispriced in terms of their relative value and a position is opened. If the standardized spread is above 0.7 it is overvalued and should be sold short, meaning shorting $\frac{1}{\beta}$ units of crypto y and buying 1 unit of crypto z. On the other hand, if the standardized spread is below -0.7, it is undervalued and the portfolio should be bought, meaning buying 1 unit of crypto y and shorting β units of crypto z. The position is closed when the standardized spread rises above -0.4 or declines below 0.4. The strategy can be summarized as follows:

Buy (long) to open position	if	$z_t < -0.7$
Sell (short) to open position	if	$z_t > 0.7$
Close short position	if	$z_t < 0.4$
Close long position	if	$z_t > -0.4$

5.2.2 The restricted strategy

In the restricted strategy, shorting is not allowed. From the spread $S_t = ln\ (y_t) - \alpha - \beta_1\ ln(z_t)$ would a buy signal imply going long in crypto *y,* while a sell signal would imply going long in crypto z. Investment signals and trading rules are similar to the unrestricted model. Hence, a buy signal when the standardized spread crosses -0.7 threshold and a sell signal when the spread crosses 0.7 threshold. The restricted strategy can be summarized as follows:

Buy signal, long in crypto y	if	$z_t < -0.7$
Close long position	if	$z_t > -0.4$
Sell signal, long in crypto z	if	$z_t > 0.7$
Close long position	if	$z_t < 0.4$

The performance statistics are calculated in the same way as the unrestricted strategy, except for return. Return is calculated according to equations (25) and (26)

$$\text{If buy position: } \ln\left(\frac{P(y)_t}{P(y)_{t-1}}\right) \quad (25)$$

$$\text{If sell position: } \ln\left(\frac{P(z)_t}{P(z)_{t-1}}\right) \quad (26)$$

5.2.3 Performance measures of pairs trading

In both strategies, the portfolio is calculated as a weighted average of the pairs in which a position is open. As an example, if only the strategy of the pair bitcoin/dashcoin is open, 100% is allocated to the pair of bitcoin/dashcoin. On the other hand, if all three strategies are open, 1/3 are allocated to the respective portfolios. A simple buy and hold approach, with daily returns over the full period, is used as a benchmark. The benchmark portfolio is weighted 1/3 to each pair over the whole period. Table 10 presents the performance of the pairs trading strategies

Table 10 – Performance statistics of pairs trading strategies

			Daily		Annualized			Total
Strategy	Pair	Winner/ losers	Avg. return	Vol.	Avg. Return	Vol.	S. R	Return
Buy and Hold								
	Bitcoin/Dogecoin		0.19%	4.6%	68.0%	87.7%	0.75	269%
	Bitcoin/Dashcoin		0.20%	4.2%	74.8%	80.5%	0.90	296%
	Bitcoin/Litecoin		0.19%	4.3%	70.8%	82.9%	0.83	280%
	Portfolio		0.20%	3.9%	71.2%	75.4%	0.91	281%
Unrestricted								
	Bitcoin/Dogecoin	7/3	0.04%	1.5%	15.3%	28.8%	0.45	60%
	Bitcoin/Dashcoin	9/7	0.48%	13.2%	175.1%	252.6%	0.68	691%
	Bitcoin/Litecoin	7/5	0.06%	2.2%	23.5%	41.6%	0.51	92%
	Portfolio		0.59%	13.5%	213.9%	257.7%	0.82	844%
Restricted								
	Bitcoin/Dogecoin	6/4	0.13%	4.2%	47.5%	80.2%	0.56	187.6%
	Bitcoin/Dashcoin	10/6	0.21%	5.4%	76.1%	103.7%	0.71	300.5%
	Bitcoin/Litecoin	8/4	0.18%	6.3%	66.9%	119.9%	0.54	264.2%
	Portfolio		0.52%	9.3%	190.5%	177.5%	1.06	752.2%

Table 10 The performance of the buy and hold, restricted and unrestricted strategy. Trading period is between 14.02.2015 - 25.01.2019. Vol and S.R indicates volatility and Sharpe Ratio.

It can be seen from table 10 that only the pair of bitcoin/dashcoin would outperform a simple buy and hold strategy, in both cases, with regards to return. However, the Sharpe Ratio is lower which indicates a poorer trade-off between risk and return. Due to the low amount of trades over the time period, the portfolio return is close to the sum of the return of the three pairs. Hence, an equally weighted portfolio of the pairs, in which a position is open, would outperform the market portfolio in both the unrestricted and restricted case. The S&P 500 obtained an annualized return of 6.2% and Sharpe ratio of 0.27 for the same period (appendix).

The statistics in table 10 is calculated per day, including the days when no position is open. Accordingly, table 11 presents the daily performance of both strategies, only for the days when a position is open.

Table 11 - Performance when position is open

Strategy	Pair	Winners / losers	Average days in position	Daily Avg. Return	Daily Vol.	Annualized Avg. Return	Annualized Vol.	Max/min
Unrestricted								
	Bitcoin/Dogecoin	7/3	56.7	0.44%	12.1%	162.3%	231.3%	217.5% / -147.0%
	Bitcoin/Dashcoin	9/7	52.5	0.43%	12.2%	155.7%	151.0%	110.6% / -183.3%
	Bitcoin/Litecoin	7/5	52.2	0.13%	5.5%	47.4%	104.6%	66.2% / -29.9%
Restricted								
	Bitcoin/Dogecoin	6/4	56.7	0.25%	3.53%	65.0%	67.4%	34.3% / -47.8%
	Bitcoin/Dashcoin	10/6	52.5	0.23%	4.25%	60.7%	62.0%	26.8% / -24.0%
	Bitcoin/Litecoin	8/4	52.2	0.31%	3.05%	113.3%	58.2%	36.7% / -19.4%

Table 11 Performance of the pairs trading strategy on daily basis, when position is open.

Table 11 shows the average daily return in the days when a position is open. This illustrates that the average daily return is higher when a position is open, in both strategies, compared to the whole period. In addition, the unrestricted strategy seems to yield a higher average return, as well as having a higher volatility, than the restricted strategy.

The selection of upper and lower threshold levels, used to trigger a buy/sell signal, is a matter of preference and risk aversion for the individual investor. Accordingly, a sensitivity analysis will be performed on both strategies in order to measure how sensitive returns and volatility are to the predefined investment rules. Table 12 summarizes the performance of the unrestricted and restricted strategy at different threshold levels. The selected threshold levels are 1/-1, 1.5/-1.5 and 2/-2, while the position is closed at 0.4/-0.4 in all cases.

Table 12 – Sensitivity Analysis

	Bitcoin/Dogecoin			Bitcoin/Dashcoin			Bitcoin/Litecoin		
Thresholds	A. R.	Vol.	S.R	A. R.	Vol.	S.R	A. R.	Vol.	S.R
Unrestricted									
1/-1	22.3%	32.4%	0.61	60.4%	115.9%	0.50	5.2%	8.6%	0.33
1.5/-1.5	16.6%	28.1%	0.51	20.5%	16.7%	1.08	6.4%	8.2%	0.49
2/-2	30.9%	41.9%	0.68	31.2%	29.7%	0.97	8.7%	9.90%	0.65
Restricted									
1/-1	48.2%	70.8%	0.65	71.0%	89.7%	0.76	62.0%	121.0%	0.49
1.5/-1.5	40.3%	71.9%	0.53	67.6%	86.8%	0.75	94.0%	112.2%	0.82
2/-2	35.3%	73.7%	0.45	56.2%	61.3%	0.88	77.5%	98.4%	0.76
Buy & Hold									
	68.0%	87.7%	0.75	74.8%	80.5%	0.90	70.8%	82.9%	0.83

Table 12 Sensitivity analysis for the unrestricted and restricted strategy. A.R, VOL. and S.R indicates annualized return, annualized volatility and Sharpe Ratio, respectively

It can be seen from table 12 that pairs trading does not guarantee a higher return than a simple buy and hold approach, for the individual pairs. The restricted strategy outperforms the unrestricted strategy in terms of return and Sharpe Ratio. It can be noted that this thesis assumes no transaction costs, which would affect the active trading strategies in a negative direction.

Nguyen and Leung (2018) found evidence of wider threshold levels yield higher profits in the period December 2017 until June 2018, in a sample of bitcoin, bitcoin cash, ethereum and litecoin. This thesis does not find consistent results that support this statement. In addition, Nguyen and Leung found the threshold level 1.5/-1.5 to be optimal. Given the results in table 12, it is hard to determine the optimal threshold level. Broek et al (2018) concluded that the efficient market hypothesis does not hold for the cryptocurrency market, as the paper found positive profitability from arbitrage opportunities by using pairs trading, following the work of Gatev et al (1999). In addition, Broek et al (2018) concluded that pairs trading strategy can successfully be applied to the cryptocurrency market and generate profits. This research concludes that positive profitability can be obtained by using pairs trading, and that a portfolio which is equally weighted in the pairs, in which a position is open, outperform the market portfolio. The individual pairs, on the other hand, does not outperform a buy and hold strategy. A final observation is that return and volatility are highly sensitive to the selected threshold levels.

6. Conclusion

In this research, the statistical arbitrage strategy, known as pairs trading, have been conducted on selected cryptocurrencies. The purpose of this study was to examine whether the strategy could obtain excess return, compared to a simple buy and hold approach. The strategy is based on cointegration. Cointegration tests are applied to bitcoin, dashcoin, dogecoin and litecoin, which are selected based on their relative long existence, in order to identify whether they share a long-run equilibrium relationship. After conducting both the Johansen test, and Engle and Granger test, cointegration was found between the variables. Hence, three cointegration relationships were included when specifying the vector error correction model. Subsequently, the standardized spread between each pair was calculated, and used as signals in order to open or close a position. Only pairs involving bitcoin were included, as bitcoin is the largest cryptocurrencies in which the other currencies most intuitively are dependent on. An unrestricted and restricted strategy was created, where the unrestricted allowed shorting, while the restricted only allowed long positions. A simple buy and hold approach, involving an equally weighted portfolio of the pairs, was used as a benchmark.

The proposed strategies in this thesis did not consistently perform better than their benchmark. The pair of bitcoin/dashcoin outperformed the buy and hold approach with regards to return but yielded a lower Sharpe Ratio. However, the average daily return of each pair was higher in the period in which a position was open, than for the whole period. In addition, holding an equally weighted portfolio in the pairs in which a position is open, outperformed the market portfolio with regards to return in both cases and Sharpe Ratio in the restricted case. The sensitivity analysis indicated that the performance of the respective strategies is sensitive to the predefined threshold levels. No evidence of a superior threshold level was found, which is inconsistent with the findings of Nguyen and Leung (2018).

This research could be expanded by including transaction costs. In addition, as bitcoin frequently changes in terms of volatility and expected return (Thies and Molnàr, 2018), the estimations, which includes historical returns, could be divided into shorter periods. Due to evidence of stronger performance under high volatility (Do and Faff, 2010), the strategy could be tested before, after and during the crash of 2017/2018. To modify the strategy, a stop-loss or profit exit points could be implemented, although, Nguyen and Leung (2018) found the pairs trading strategy, without stop-loss, to yield higher return.

7. References

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8. Appendix

8.1 Vector Autoregressive model (VAR) model

A brief explanation of the VAR model is included as the Vector Error Correction Model (VECM) is an extension of VAR. VAR is proposed by Sims (1980) and is an extension of the univariate autoregressive model (AR) by including more than one evolving variable. The model is used to explain the dynamic behavior of time series. A VAR fits a multivariate time series regression of each dependent variable on both lags of its own, and all other dependent variables. The model contains K variables and a VAR with p lags are often denoted VAR(p). The equations for a simple case of VAR(1), with two variables are as follows

$$y_{1,t} = \eta_1 + \delta_1 y_{1t-1} + \alpha_1 y_{2t-1} + e_{1,t}$$
$$z_{2,t} = \eta_2 + \delta_2 y_{1t-1} + \alpha_2 y_{2t-1} + e_{2,t}$$

Where e_t is an error term with expected value of zero, E(e_t) = 0. It is not necessary to specify endogenous or exogenous variables in a VAR model, which makes it a flexible and easy model to work with.

8.2 Engle and Granger

Engle and Granger (1987) formalized the cointegration vector approach. The Engle and Granger approach belongs to the bivariate category of cointegration. Bivariate approach indicates a pairwise analysis with one endogenous and one exogenous variable. If a cointegrated relationship is found, an error-correction model can be specified.

Mathematically would cointegration of order d, b, denoted $x_t \sim CI(d, b)$, if (i) all components of x_t are $I(d)$; (ii) there exist a vector $\alpha(\neq 0)$ so that $(z_t = \alpha' x_t \sim i(d-b), b > 0)$. The vector β is called the cointegration vector (Engle and Granger 1987).

The Engle and Granger method consist of four steps (Enders 2010):

1) Pretest the variables for their order of integration. If both series are integrated of the same order, cointegration is theoretically possible. On the other hand, if they are not integrated of the same order, cointegration is not possible.

2) If the series are integrated of the same order, estimate the long-run equilibrium using ordinary least squares of the form

$$y_t = \beta_0 + \beta_1 z_t + e_t$$

In order to determine whether or not the variables are cointegrated, the residuals are stored and tested if they are stationary using the Augmented Dickey-Fuller test, where the following equation is tested:

$$\Delta\widehat{e_t} = a_1\widehat{e_{t-1}} + \sum_{i=1}^{n} a_{i+1}\,\Delta\widehat{e_{t-\iota}} + e_t$$

where:

$H_0\colon a_1 = 0,\ \widehat{u_t} \sim I(1)$ - Nonstationary residuals and no cointegration

$H_1\colon a_1 < 0,\ \widehat{u_t} \sim I(0)$ - Stationary residuals and cointegration

If the test statistic indicates rejecting the null, there exist a cointegration relationship between the variables. MacKinnon (1990) provides a valid table of critical values used to test the residuals for stationarity.

3) If cointegration is detected, estimate the error correction model. In an error correction model, the short-term dynamics of the variables are influenced by the deviation from equilibrium. In this case, the residuals from the equilibrium regression are used to estimate the error correction model of the form:

$$\Delta y_t = \alpha_y\widehat{e_{t-1}} + \sum_{i=1}^{k}\delta_{11}^{i}\,\Delta y_{t-i} + \sum_{i=1}^{k}\delta_{12}^{i}\,\Delta x_{t-i} + \varepsilon_{yt}$$

$$\Delta x_t = \alpha_x\widehat{e_{t-1}} + \sum_{i=1}^{k}\delta_{21}^{i}\,\Delta y_{t-i} + \sum_{i=1}^{k}\delta_{22}^{i}\,\Delta y_{t-i} + \varepsilon_{xt}$$

Where Δy is the dependent variable being tested, ε_{yt} are white noise disturbance, i is number of lags and δ_{11}, δ_{21}, δ_{12} and δ_{22}, are all short-run parameters. The speed adjustment parameter is expressed by α_y, while $\alpha_y\widehat{e_{t-1}}$ being the error correction term.

4) Determine whether or not the error-correction estimated is appropriate.

Table 13 presents the results of the Engle and Granger test. The Bayasian Information criteria is used to determine the number of lags. The sample period is 14th of February 2014 until 25th of January 2019. Trend is excluded in each test.

Table 13 – Engle and Granger cointegration test

Crypto	Bitcoin	Dogecoin	Litecoin	Dashcoin
Bitcoin	-	- 3.689**	- 2.776	- 4.977***
Dogecoin	- 3.574**	-	- 3.612**	- 5.136***
Litecoin	- 2.706	- 3.682**	-	- 4.717***
Dashcoin	- 4.569***	- 4.615***	- 4.178***	-

*Table 13 The Engle and Granger test. Symbols ***, ** and * represents significance at 1%, 5% and 10%, respectively. Critical values can be found in MacKinnon (1990).*

8.3 Performance of the S&P 500

The S&P 500 index accounts for more than 80% of the stock market in the United States and is considered to be a good representation of the US stock market. Accordingly, performance of the index is included for illustrative purposes, with regards to the pairs trading strategy. The price of S&P 500 is gathered from Yahoo Finance. Table 15 presents the performance over the period 13th February 2015 until 25th January 2019. The statistics are calculated according to equation (20), (22) and (23) but with 252 trading days.

Table 15 - Performance of the S&P 500

Statistics	Performance
Average return	0.02%
Annualized return	6.2%
Annualized standard deviation	13.73%
Sharpe ratio	0.278

Table 15 The performance of S&P 500 over the period 13th February 2015 – 25th January 2019.