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Studying Value-at-Risk in Equity Markets

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Abstract

Value-at-Risk, in financial risk management, is a central method for estimating and controlling risk exposure for financial institutions. Challenges with current VaR methods is that its imprecise, especially under times of financial turmoil where precise estimations are most crucial and current methods suffer from too optimistic VaR estimations. Addressing these challenges and improving current methods is what serves as motivation for this paper. There are several methods for estimating VaR, and two of the most used methods is historical simulation and Monte Carlo simulation. And a study on how macroeconomic factors can improve these two methods is conducted and investigated for different confidence levels.

In this thesis, we investigate and develop methods for how historical VaR and Monte Carlo simulation can be improved by implementing macroeconomic variables, such as VIX, brent price, gold price, and US treasury note. The data analysis is performed on the American stock market index S&P 500 and the Norwegian Oslo Stock Exchange, and a selection of sectors for these two markets, with a span of two decades. With the intent to test how precise the VaR methods estimates are across well diversified and more specialized portfolios. An investigation on how well the VaR methods perform in financial stressing times was done by sectioning the time series into periods, to differentiate between where a market shock has occurred and when the market is in equilibrium.

The new versions of historical VaR and Monte Carlo simulation are compared to a benchmark model, historical VaR with 250 days rolling window, to see the advantages of implementing macroeconomic factors to these highly used methods. The conclusion of this thesis yield interesting results regarding how macroeconomic factors affect VaR estimation and give a contribution to and validate research and studies previously done on VaR. We find that implementing VIX, and the other macros studied, to current VaR methods can improve the estimates, especially concerning optimistic VaR estimates during financial turmoil.

Preface

This thesis marks the end of a five-year study in Industrial Economy at the University of Stavanger. This master program is a continuation of a bachelor's degree in science with two years of economics, project management and risk management as a field of study, resulting in a Master of Science degree.

Relevant skills obtained throughout our specialization within finance, investment and risk together with interest in financial markets and macroeconomics made it easy to accept this master thesis scope, proposed by Ph.D. Roy Endré Dahl.

The programming language Python used in Microsoft Visual Studio was selected as the programming platform for the analysis and generation of data. To write and construct the layout of this thesis Overleaf, an online LaTex editor, was selected as the platform. All of the data in this thesis was obtained from Thomson Reuters Eikon.

We are very thankful to our instructor Roy Endré Dahl at the University of Stavanger for providing help and insights to Value-at-Risk when needed.

Stavanger, 14 June 2019

Bjørnar Epland and Tor Erik P. Pettersen

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1 Introduction

The ability to measure the uncertainty of a portfolios profit and loss is a fundamental part of financial risk management. Mainly, downside risk is of great importance in regards to regulations, set by external parties, and for financial institutions ability to meet obligations and absorb unexpected losses. The gravity of handling deviations from target or expected values has become greater with stricter regulations being set. Risk management is especially crucial under times of crisis, where reliable and precise risk measures are vital. Guidelines and regulations, like the Basel Accords, set by Basel Committee on Bank Supervision, have risen as a result of severe disturbances in the global economy to enhance financial stability. With the financial crisis and collapse of Lehman Brothers in 2008 being the catalyst of the latest revision of the Basel Accords, referred to as Basel III. The accords set safeguards and requirements regarding minimum capital, liquidity, leverage ratio, measurement of risk, et cetera, that banks and other financial institutions are recommended to follow[23].

Value-at-Risk (VaR) is in financial risk management, a fundamental and widely used quantity of interest. The risk measure denotes the amount by which a portfolio of financial assets might fall the most, with a given probability and time horizon. This thesis limits its time horizon to one day and performs calculations with probabilities connected to the confidence levels 95% and 99%. There are numerous methods of calculating VaR, but within the scope of this thesis, the methods considered most relevant is *historical simulation, normal linear VaR* and *Monte Carlo simulation.*

It is well known that under times of crisis, e.g., the financial crisis of 2008 or the Dot-com bubble in the late 1990s, performing VaR calculations can be troublesome [13][35]. A tendency of achieving a too optimistic VaR under financial turmoil is a concern that will be addressed and a primary motivation for improving VaR accuracy. Adapting and modifying the methods mentioned above, by introducing macroeconomic factors, e.g., CBOE Volatility Index and gold price, is carried out and investigated. The fact that the most simple method for estimating VaR, historical VaR, is the most popular, suggests that there is room for improvement and further development. The second most used method is Monte Carlo simulation[31]. The two latter methods will serve as the foundation for further research and modification of the models in this thesis. Despite being a widely used risk measure, there is also controversy tied to VaR, as it cannot guarantee diversification and consequently is not a coherent risk measure. The issue of VaR violating the property of sub-additivity in some cases discourage the diversification benefits of expanding a portfolio, thus contradicts modern portfolio theory[3][22]. For the sake of simplicity, and within the scope of this thesis, VaR is considered as an adequate risk measure, and the shortcomings have been made aware of, but will not be addressed and discussed in depth within this thesis.

All calculations will be performed on the American large-cap stock market index S&P 500 and the Norwegian Oslo Stock Exchange index, to test VaR on well-diversified portfolios. A selection of sectors from the S&P 500 and Oslo Stock Exchange is also investigated to test VaR on more specialized portfolios. The relationships between sectors and indices are also analyzed based on the VaR results. Here a goal is to map which index and sectors carry the most risk and to uncover the behavior and intercorrelation of the sectors. To compare the American market versus the Norwegian market is of interest, due to the vast difference in volume and diversification. With Oslo Stock Exchange only containing approximately 1% of the S&P 500's total volume and Equinor being a quarter of the total stock volume, versus Microsoft only being about three percent of S&P 500's volume. S&P 500 is also used as a benchmark for how a well-diversified market index behaves.

The purpose of this thesis is to investigate how the implementation of macroeconomic factors affects VaR models accuracy, and use the results to investigate behavioral differences between the selected markets and sectors based on the VaR calculations.

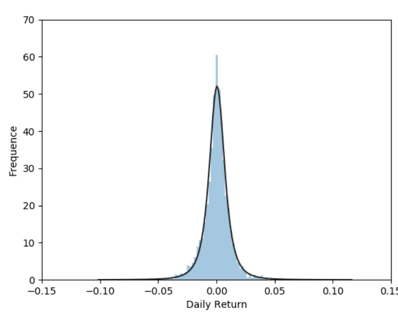
To find answers to the main objectives is done by organizing the thesis as follows: In the following section 2, characteristics and basic behavior of financial markets is presented. Followed up by section 3, where the theoretical background and the different methods used in this thesis is presented. In section 4 the data that will serve as foundation for the analysis, in section 5, will be presented and explained. And lastly in section 6 and 7 the results from the analysis will be evaluated and discussed, before concluded. In addition the appendix will supply the reader with additional figures and tables, together with complete results.

2 Characteristics of Financial Markets

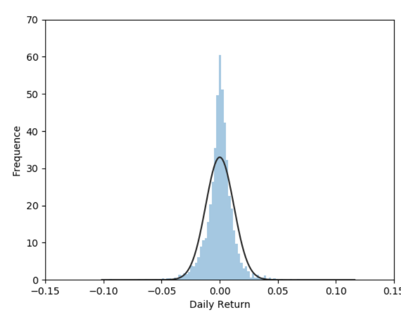
2.1 Stylized Facts

In the context of financial time series data, and specifically daily data series, there are empirical observations and inferences drawn from these observations that have held so long that it has been given the status of facts[14]. These stylized facts apply to the majority of daily series of risk factor changes, e.g., commodity prices and indexes. Additionally, in some cases, these stylized facts also continue to hold for other time series like intraday or weekly time series.

Return series appear to vary over time. One may argue that the market is psychological driven, i.e., fear and greed, and the resulting action by the investors due to world events, thus, that it is the reason for varying returns. For instance, consider the volatility in December 2018. It was the first time that the S&P 500 index had ended with red numbers after having green numbers throughout the three first quarters. 2018 was a record-setting year for stocks, from an all-time high on the 20th September 2018 to the worst December since 1931. Brexit, trade-war, a slowdown in the Chinese economy and fear of recession in the US led the way for the worst year since the financial crisis in 2008. In this period, one can also observe the concept of volatility clustering. Volatility clustering is also an example of stylized fact; extreme returns are generally followed by several other extreme returns, yet, not necessarily with the same sign.



(a) SPX with Student-T fit



(b) SPX with normal fit

Figure 1: SPX with different distribution fits

From figure 1a above, one can see the fact that return series are leptokurtic or fat-tailed. This means that the distribution is narrower around the mean, but has longer and fatter tails with respect to the normal distribution shown in figure 1b. This is an important fact due to simplification to the Gaussian curve yield too optimistic results regarding VaR.

Stylized facts summary for a single time series financial return [14]:

- Return series are not independent and identically distributed (iid), although they show little serial correlation.
- Series of absolute or squared returns show profound serial correlation.
- Conditional expected returns are close to zero. As per figure 1a.
- Volatility appears to vary over time
- Extreme return appears in clusters
- Return series are leptokurtic or fat-tailed.

Stylized facts for multivariate time series [14]:

- Multivariate return series show little evidence of cross-correlation, except for contemporaneous returns.
- Multivariate series of absolute returns show great evidence of cross-correlation.
- Correlations between series vary over time.
- If there is an extreme return in one series, it often coincides with extreme returns with several other series.

2.2 Statistics

Statistics' basic goal is to organize and summarize data, therefore, the mean and standard deviation are introduced as summary measures of the location and variability of a distribution. However, these measures do not say anything about the shape alone, and in order to better describe the tails and peak of a distribution, skewness and kurtosis are introduced. The mean, standard deviation, skewness, and kurtosis are respectively given by:

$$\mu = E[X] \tag{1}$$

$$\sigma = \sqrt{E[X - \mu^2]} \tag{2}$$

$$\tau = \frac{E[(X - \mu)^3]}{\sigma^3} \tag{3}$$

$$\kappa = \frac{E[(X - \mu)^4]}{\sigma^4} \tag{4}$$

The third equation yields the skewness for the distribution. If the majority of the data is at the left and the right tail is longer, with reference to the Gaussian curve, the distribution is right-skewed or positively skewed, i.e., $\tau > 0$. If the majority of the data is toward the right and the left tail is longer, the distribution is left-skewed or negatively skewed.

By definition, the Gaussian Curve has a kurtosis of three. A kurtosis higher than this number means that the distribution has excess kurtosis implying that the curve has a higher peak and fatter tails than a normal distribution and the series are known as leptokurtic. If the kurtosis is less than three the series is platykurtic.

2.3 Risk in Financial Markets

Risk is a term that is widely used within many areas, and therefore, it is difficult to pin down a precise definition of what risk is. It is commonly related to uncertainty, exposure, or volatility. Risk is often said to be the possibility of enduring an undesirable outcome, even though it is rather a vague term and root for ambiguity, which means that different people interpret it differently, so reducing it to a single number will always cause losing a substantial amount of information.

From an economic viewpoint, undesirable outcomes are universally in the form of monetary losses. The Basel Committee [23] recognizes that financial risk should be divided into three main areas:

- *Market risks* are related to changes in market prices. The value of a firm's financial position changes regularly due to changes in the value of its underlying components. The changes in underlying components could be due to fluctuations in exchange rates between two currencies (i.e., currency risk), adjusted interest rate, changes in stock and bond prices (i.e., equity risk), or the changes in the price of a commodity (i.e., commodity risk).
- *Credit risk* are related to the hazard of a trading partner not being able to fulfill his end of a contract and monetary losses caused by this, e.g., a borrower who is unable to pay his debt and defaults on it.
- *Operational risks* are related to monetary losses caused by failures or lack of quality in internal processes, systems, or people and losses caused by unexpected external events.

2.4 Commodities and macroeconomic variables

In the aftermath of the 2008 financial crisis, several articles are debating the financialization of commodity markets, i.e., an increased correlation between commodity markets and stock markets due to the role of financial investors in commodity markets [30]. Creti et al. [10] investigated the links between 25 commodities and stock markets over the period from January 2001 to November 2011. They found that the correlations between commodity and stock returns evolve through time, being highly volatile, particularly after the 2008 financial crisis. The highest correlations were observed during financial turmoil, showing increased links between stock and commodity markets. This is also supported by Mensi et Al [27] and Basher et Al. [4].

2.4.1 VIX

The CBOE Volatility Index, known by its ticker symbol VIX, is a measure of the stock market's expected volatility implied by the S&P 500 index options. It is commonly referred to as the fear index and is a real-time market index that represents the markets' expectation of a 30-day forward-looking volatility.

There are two ways of measuring volatility. First is based on performing statistical calculations on the historical prices over a specific time period. The resulting figure, standard deviation, is called "realized volatility." To predict future volatility for the next day, month, or year, it is common to use the calculated standard deviation and expect that the same pattern will follow. The second method measures volatility derived from option prices. Options are derivative instruments where the price depends upon the probability of a stock's current price moving to the strike price. The price of a call option will depend upon the market perceived probability of the stock price moving to above the strike price within the expiry date. Since the volatility factor represents the possibility of such price movements happening within the expiry date, various option pricing methods include volatility as an integrated input parameter. Therefore, such volatility, as implied by or inferred from market price, is called forward-looking "implied volatility."

The CBOE estimates VIX, or the expected volatility by aggregating the weighted prices of SPX puts and calls over a wide range of strike prices. Using SPX options with more than 23 days and less than 37 days to expiration ensures that the VIX index will always reflect an interpolation of two points along the S&P 500 volatility term structure [15]. Since the VIX is the expected annual standard deviation for the SPX, one can calculate the expected range of return the next 30 days. E.g., a random number drawn from a student-t distribution with three degrees of freedom has 95% chance of being within 2.353 standard deviations from 0. Since VIX is an annualized standard deviation, one must scale each of the coefficients by the square root of twelve to convert them to monthly volatilities (e.g., $2.353/\sqrt{12} = 0.6793$) [37].

$$Expected\,range\,at\,95\% = 0,6793 * VIX \tag{5}$$

2.4.2 Brent Oil

Brent blend is one of the main benchmarks for oil quality, and it stands for two-thirds of international traded raw oil in the world. In Hamilton's [19] article from 2011, he pointed out that a rise in oil prices had preceded ten out of eleven postwar recessions. There has been an endless attempt to prove the oil price's impact on equity prices. Mohan Nandha and Robert Faff [29] finds that rising oil prices harm all sectors except mining and oil and gas industries. Wang et al. [36] differentiated between oil-importing and oil-exporting countries and found that there is a significant difference on how an oil shock impact the stock market depending on whether the country is a net importer or net exporter in the world oil market. Furthermore, the relative contribution of the oil shocks depends on the level of importance of oil to national economy, the net position in the oil market and the driver behind the oil price changes. They also found that positive aggregate and precautionary demand shocks are shown to cause a higher degree of co-movement among the stock markets in oil-exporting countries. Bjørnland[8] found that Norway, as an oil exporting country, benefited from higher oil prices for prolonged periods, but other exporting countries like Canada and the UK tend to behave more in line with oil importing countries. Cunado and Perez de Gracia [11], on the contrary, found that the response of the European stock returns to an oil price shock may vary significantly on the underlying causes of the oil price change. This result suggests that there is a negative and significant impact of oil price changes on most European stock market returns and that oil supply shocks mostly drive it.

2.4.3 Gold

A safe-haven is an asset that is uncorrelated (weak safe-haven) or negatively correlated (strong safe-haven) with another asset or portfolio in financial turmoil or times of market stress. Hence, a safe-haven asset shields investors during a crisis, unlike a hedge which shields investors in normal times. Thus, a safe-haven asset is expected to keep or even increase its value during market turbulence, when most asset prices decline.

Gold is commonly believed to be a safe-haven in times of political or financial uncertainty, considering it is not at risk of becoming worthless like assets bearing credit risk. According to Baur and Lucy [6], gold is a hedge on stocks on average and safe-haven in extreme stock market conditions. The authors also found that gold can be utilized as safe-haven only for a limited time, approximately 15 trading days, suggesting that investors should buy gold on days of extreme negative returns and sell it when shareholders regain confidence, and the volatility is lower. Another study, from Baur and McDermott[7], using 30 years of data, confirms that gold is both a safe-haven asset and a hedge for major European and US equity markets, but not for Australia, Canada, Japan or major emerging markets such as the BRIC countries. Investors suffering losses in emerging markets tend to turn to developed markets rather than to gold.

2.4.4 US Treasury Note 10 Year

Robert Connoly, Chris Stivers and Licheng Sun[9], found that bond returns tend to be high relative to stock returns during days when implied volatility increases substantially and during days when stock turnover is unexpectedly high. Vice versa, bond returns tend to be low relative to stock returns during days when implied volatility decreases and during days when stock turnover is unexpectedly low. Their findings suggest that stock market uncertainty has important cross-market pricing influences and that stock-bond diversification benefits increase with stock market uncertainty. Jeff Flemming, Chris Kirby and Barbra Ostdiek[16], found that the linkage between stock, bond and money markets are strong. Also, they found that linkages have become stronger since the 1987 market crash. Baur and Lucey[5] believed that correlations between bond market and stock market would decrease due to external intervention events with the reason that the investors might move to safer allocations of assets. This effect is referred to as flight to quality. While, Hsiang-Hsi Liu, Teng-Kun Wang and Weny Li[25] investigated the dynamic interrelationships among US Stock, treasure bond cash and futures market with the VEC copula GJR-GARCH-skewed-t model. They found that regarding the dependence structure between the US stock and US treasury bond futures, the weight of Clayton Copula is the largest, indicating that there exists left tail dependence structure in these two markets. Implying that under negative shocks or within a bearish market, the degree of correlation, interdependence or co-movement among the markets increases.

3 Theoretical Background and Methods

The theory and practice of assets valuation over time is the main aspect of financial time series [34]. Recorded economic behavior can be quantified and represented by time series, such as price indexes, gross national product, production[18]. The first step in this study is to see how prices behave, and then use the knowledge of price behavior to help improve decision making. A thorough understanding of time series and how to manage the data can be profitable and a valuable tool in portfolio and risk management. Graphs of financial assets plotted over time can often reveal an intuitive interpretation of trends in prices[33]. Figure 2 below show a graphical representation of the time series, both price history and returns, for the S&P 500 and Oslo Stock Exchange index.

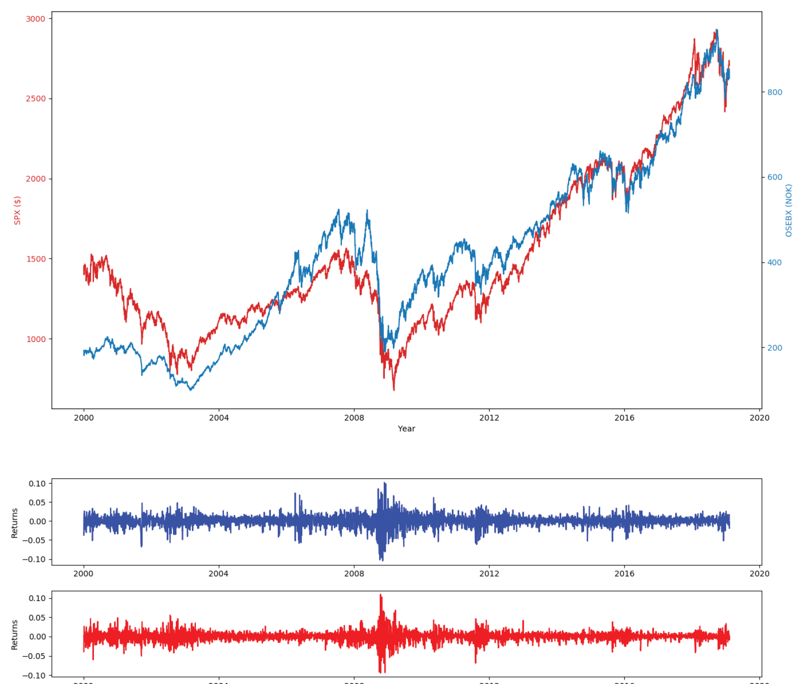


Figure 2: Time series for SPX and OSEBX. Price history and returns.

Common for studying financial time series is evaluating the returns, instead of the price, of assets. The return of an asset is scale free and easier to handle due to its statistical properties[34]. In addition there is uncertainty linked to economical principles and volatility of returns, where the usage of statistical and analytic tools play an important part in financial time series analysis. The monitoring of frequent price behavior and attempts to understand the probable price development is of particular interest when performing value at risk estimations. To process and handle these data, statistical and econometric tools such as regression and parameter estimation is performed, elaborated in the sections below. Lastly, methods for estimating VaR is presented as a conclusion of this section.

3.1 Ordinary least squares regression

Ordinary least squares regression, or OLS-regression, is a common and widely used method for fitting linear statistic models, i.e., choosing parameters to describe the relationship between one dependent variable and one or several independent variables. The parameters are chosen by the principle of least squares, where minimizing the sum of squares between the observed dependent variable given by the data set and the prediction of the dependent variable given by the regression line. Contrary to the use of correlation analysis, which only uncovers if there is a correlation between variables, regression also uncovers to which degree variables act in accordance with one another. This type of multivariate linear regression model can be expressed by several models, e.g., a level-level model shown in equation 6

$$Y_i = \beta_0 + \beta_{i1} X_{i1} + \beta_{i2} * X_{i2} + ... + \beta_n * X_{in} + \epsilon_i \tag{6}$$

or by a log-log model shown in equation 7

$$log(Y_i) = \beta_0 + \beta_{i1} log(X_{i1}) + \beta_{i2} * log(X_{i2}) + ... + \beta_n * log(X_{in}) + \epsilon_i. \tag{7}$$

Here Y_i is the value of the dependent variable in case i, β_0 is the regression constant, X_{in} is the n^{th} independent variable in case i, β_n is the independent variables regression weight and ϵ_i is the error for case i [20].

OLS-regression is based upon four assumptions [38]:

1. The mean value of Y_i is assumed to be a linear function of the independent variables X_{in}. For each value of X_{in} there is a probability distribution of Y_i.
2. OlS assumes homoskedasticity, i.e., that the standard deviation of each of the probability distributions is the same for all value of X_{in}.
3. The values of the dependent variable Y are independent of one another.
4. The probability distribution of ϵ_i is normally distributed, with a mean of zero and constant standard deviation.

3.2 Maximum likelihood estimation

Maximum likelihood estimation, MLE, is an intuitive procedure in statistics that uses observations from a data set, to make inferences and estimates about the parameters of a statistical model [32] and its distribution. The principle of MLE states that the desired probability distribution is the one that makes the observed data "most likely." Given the observed data and model of interest, one seek to find the probability density function, PDF, that is most likely to have produced the data. To solve this the likelihood function is defined, by reversing the roles of the data vector $\mathbf{y}$ and parameter vector $\mathbf{w}$ in the PDF denoted $f(\mathbf{y}|\mathbf{w})$, i.e. $L(\mathbf{w}|\mathbf{y}) = f(\mathbf{y}|\mathbf{w})$. Thus the likelihood function $L(\mathbf{w}|\mathbf{y})$ represents the likelihood of the parameter $\mathbf{w}$ given the observed data $\mathbf{y}$. Note that the likelihood function is a function given a particular set of given data. In general, for a model containing k parameters, the likelihood function takes the shape of a k-dimensional surface sitting above a k-dimensional plane spanned by the parameter vector $(w_1, w_2, ..., w_k)$.

After obtaining data and determining a likelihood function of a model given the data sample, statistical inferences about the distribution of the population can be done. One must seek the value of the parameter vector that maximizes the likelihood function. This parameter vector is called the MLE-estimate $\mathbf{w}_{MLE}$. According to the MLE principle, this is the population that is most likely to have generated the observed data [28].

3.3 Value-at-Risk models

Value-at-Risk is a risk measure that denotes the amount by which a portfolio of financial assets might fall the most, with a given probability and time horizon. The probability is expressed either by a significance level α or confidence level $1-\alpha$, and the time horizon, denoted h, is traditionally measured in trading days over which the VaR is measured. The assumption that the portfolio is left unmanaged throughout the time horizon, h, is also made.[2] The VaR of the portfolio at the confidence level α is given by the smallest number l such that the probability that the loss L exceeds l is no larger than $1-\alpha$. In probabilistic terms, VaR is thus simply a quantile of the loss distribution. VaR is probably the most widely used risk measure in financial institutions and has become a standard tool for calculating risk. [26]

$$VaR_\alpha = inf\{l \in \mathbb{R} : P(L > l) \leq 1 - \alpha\} \tag{8}$$

VaR has several attractive attributes such as[2]:

- Simplicity, it corresponds to an amount that could be lost with a given probability.

- It measures the risk of the risk factors as well as the risk factor sensitivities.
- It is comparable across markets and different exposures.
- It can be applied to all types of activities and all types of risk.
- It can be measured at any level. From individual trade or portfolio, up to enterprise-wide covering all the risks in a firm as a whole.
- When aggregated or disaggregated, it takes dependencies between constituents assets or portfolios into account.

However, there are also some discussions regarding VaR:

- With the largest discussion being the occurrence of a violation of subadditivity, making VaR a not coherent risk measure. According to modern portfolio theory, diversification should ensure that a portfolio of two assets should have a lower risk, compared to the sum of the two individual asset's risk. However, this is not always true when calculating the VaR, and can result in a negative diversification bonus. [17]
- Model risk. VaR estimate is subject to estimation error and the problem of model risk. Where model risk is tied to misspecified risk-management models or because some of the underlying assumptions are not met in practice.
- VaR also neglects any problems related to market liquidity. Here regarding the investor's ability to buy or sell large amounts of the security in a short time without affecting its price very much. [26]

There are several methods to calculate VaR, and in the following sections, the methods *Historical simulation, Normal linear VaR*, and *Monte Carlo simulation* will be elaborated.

3.3.1 Historical simulation

The historical VaR model is a fairly simple and frequently used method, and preferred by the majority of banks[1], for estimating VaR. Using empirical quantile estimation to estimate VaR, whereby theoretical quantiles of the loss distributions are estimated by sample quantiles of the data, is a common approach [26].

To estimate VaR for day $k + 1$, can be done by ordering the n data of the data sample and finding the $n * \alpha$ highest observed loss, where α is the chosen significance level. This observation will serve as the VaR estimate at a $1 - \alpha$ confidence level.

The main advantages are that historical VaR is easy to implement and does not have to assume the parametric form of the distribution of the risk factor returns. Errors regarding assumptions between dependencies of the risk factors are captured by this method, and simplifications often made are not necessary[1]. Drawbacks to historical VaR is that it is vulnerable to sample quantity and sample relevance. The model is also backward-looking and relies on history repeating itself[26].

3.3.2 Normal linear VaR

A simple parametric approach to estimate VaR is using the normal linear VaR model. The model assumes that the returns on the portfolio follow an independent normal distribution, i.e., the distribution of the returns X is i.i.d. and

$$X \sim N(\mu, \sigma^2). \tag{9}$$

By assuming that the joint distribution of the risk factor returns is multivariate normal, the covariance matrix of risk factor returns is all that is required to capture the dependency between the risk factor returns.

Defining x_α as the α quantile of the distribution X, i.e., $P(X < x_\alpha) = \alpha$. By applying the standard normal transformation we get

$$P(X < x_\alpha) = P(\frac{X - \mu}{\sigma} < \frac{x_\alpha - \mu}{\sigma}) = P(Z < \frac{x_\alpha - \mu}{\sigma}) = \alpha, \tag{10}$$

where Z is a standard normal variable, i.e., $Z \sim N(0, 1)$. Thus

$$\frac{x_\alpha - \mu}{\sigma} = \Phi^{-1}(\alpha), \tag{11}$$

where $\Phi^{-1}(\alpha)$ is the standard normal α quantile value. With $x_\alpha = -VaR_\alpha$ and by the symmetry of the normal distribution function $\Phi^{-1}(\alpha) = -\Phi^{-1}(1-\alpha)$ the latter equalities enables the possibility to express the $100\alpha\%$ parametric linear VaR, with time horizon h, as [1]

$$VaR_{h,\alpha} = \Phi^{-1}(1 - \alpha)\sigma_h - \mu_h. \tag{12}$$

The normal linear model is a simple and convenient model, hence its popularity. To obtain VaR values at higher significance levels e.g., 5% the method should suffice to correctly approximate distribution in the tails. However, financial assets often have distributions that exhibit fatter tails than the normal distribution, shown in figure 1a and figure 1b, resulting in too optimistic VaR estimates. Especially at low significance levels the normal linear model is vulnerable to estimation errors[21].

3.3.3 Monte Carlo simulation

The Monte Carlo method is a generic name for any approach to risk measurement that involves the simulation of an explicit parametric model for risk factor changes[26]. A favorable trait that the Monte Carlo simulation possesses is that it is extremely adaptable and various assumptions about the multivariate distribution of risk factor returns can be implemented into the model. It can be applied with any assumed distribution for risk factor returns[1].

The first step of the method is the choice of the model, and the calibration of this model to historical risk factor change data $\mathbf{X}_{t-n+1}, ..., \mathbf{X}_t$. The model should be from which one can promptly simulate since in the second stage it is generated m independent realizations of risk factor changes for the next time period, which can be denoted: $\tilde{\mathbf{X}}_{t+1}^{(1)}, ..., \tilde{\mathbf{X}}_{t+1}^{(m)}$. The number of replications, m, can be chosen freely, within reasonable constraints with respect to computation time. In general, m can be chosen to be much larger than n, to improve the accuracy in empirical VaR estimates. By applying a loss operator to these simulated vectors it can be obtained simulated realizations $\{\tilde{L}_{t+1}^{(1)} = l_{[t]}(\tilde{\mathbf{X}}_{t+1}^{(i)}) : i = 1, ..., m\}$from the loss distribution. These simulated loss data are used to estimate risk measures, often done by empirical quantile and shortfall estimation.

A vulnerability the method has is that the results obtained are only as good as the model that is used and that it can be time-consuming. Monte Carlo method does not solve the problem of finding a multivariate model for $\mathbf{X}_{t+1}$[26].

4 Data Presentation

The data set consists of 20 years of daily data, gathered from the Thomson Reuters Eikon databse, for the indices and for ten sectors on each stock exchange. The real estate sector was dropped due to incomplete data for both indices. The data ranges from 03. January 2000 to 08. February 2019 for the S&P 500, while the data from Oslo Stock Exchange ranges from 03. January 2000 to 11. February 2019. In order to keep within the time constraint, a selection of sectors is chosen for further investigations.

In this section, first, a presentation of the sector classification and structure will be done. Secondly, the indices are introduced, and the market capitalization for the sectors are shown. Lastly, a justification of the chosen sectors will be presented along with an overview of the selection.

4.1 Sector Presentation

Both, S&P 500 and Oslo Stock Exchange, opposed to Thomson Reuters Eikon, follows the GICS-classification. It is build up by various industries which form eleven sectors.

Energy Sector (10GI) consists of companies that source, drill, extract and refine raw commodities, such as oil and gas. The energy sector hold two industries:

1. Energy equipment and services industry
2. Oil, gas and consumable fuels industry.

Materials Sector (15GI) consists of the businesses that logs, mines and manufacturers raw materials for other sectors. Its five industries are:

1. Chemicals industry
2. Construction materials industry
3. Containers and packaging industry
4. Metals and mining industry
5. Paper and forest products industry

Industrials Sector (20GI) range from railroads and airlines to military weapons and industrial conglomerates. The industrial sector comprises 14 industries:

1. Aerospace and defense industry
2. Air freight and logistics industry
3. Airlines industry
4. Building products industry
5. Commercial services and supplies industry
6. Construction and engineering industry
7. Electrical equipment industry
8. Industrial conglomerates industry
9. Machinery industry
10. Marine industry
11. Professional services industry
12. Road and rail industry
13. Trading companies and distributors industry
14. Transportation infrastructure industry

Consumer Discretionary Sector (25GI) contains businesses that have demand that falls and rises based on general economic conditions such as sporting goods and new cars. The 11 industries in the consumer discretionary sector are:

1. Automobile components industry
2. Automobiles industry
3. Distributors industry
4. Diversified consumer services industry
5. Hotels, restaurants and leisure industry
6. Household durables industry
7. Leisure products industry
8. Multiline retail industry

9. Specialty retail industry
10. Textile, apparel and luxury goods industry
11. Internet and direct marketing

Consumer Staples Sector (30GI) comprises of businesses that sell the necessities of life, ranging from milk and packaged food to laundry detergent and toothpaste. The six industries that comprise the sector is:

1. Beverages industry
2. Food and staples retailing industry
3. Food products industry
4. Household products industry
5. Personal Products industry
6. Tobacco industry

Health Care Sector (35GI) consists of drug companies, medical supply companies and other scientific-based operations that are concerned with improving and healing human life. There are six industries making up the sector:

1. Biotechnology industry
2. Health care equipment and supplies industry
3. Health care providers and services industry
4. Health care technology industry
5. Life sciences tools and services industry
6. Pharmaceuticals industry

Financials Sector (40GI) Banks, insurance companies, mortgage real estate investments trust and credit card issuers make up the financial sector. The seven industries are:

1. Banking industry
2. Capital markets industry

3. Consumer finance industry
4. Diversified financial services industry
5. Insurance industry
6. Mortgage real estate investment (REITs) industry
7. Thrifts and mortgage finance industry

Information Technology Sector (45GI) sector comprises of hardware, software, computer equipment and IT services operations. The information technology sector consists of six industries:

1. Communications equipment industry
2. Electronic equipment, instruments and components industry
3. IT services industry
4. Semiconductors and semiconductor equipment
5. Software industry
6. Technology hardware, storage and peripherals industry

Communication Services Sector (50GI) comprises of TV-streaming to high-speed internet companies. The communication services sector is made up of five industries:

1. Diversified telecommunication services
2. Wireless telecommunication services
3. Entertainment
4. Media
5. Interactive media and services

Real Estate Sector (55GI) includes all real estate investment trust (REITs) except for mortgage REITs, which is placed in the financials sector. The two industries that make up the sector is:

1. Equity real estate investment trusts
2. Real estate management and development

Utilities Sector (60GI) consists of the companies that let us turn on the power or make water come out of the tap and more. It is made up of five industries:

1. Electric utilities industry
2. Gas utilities industry
3. Independent power and renewable electricity producers industry
4. Multi-utilities industry
5. Water utilities industry

4.2 Indices Presentation

In the subsections below, the two indices and the selected sectors for the thesis are introduced, which is the energy, consumer staples, health care, financials and information technology sector.

4.2.1 SPX

The Standard & Poor 500 index (SPX) is a market-capitalization-weighted index of the 500 largest U.S. publicly traded companies and it is widely regarded as the best gauge of the large-cap U.S. equities [24]. Its market value is $24,57 trillion.

From table 1, one can see that information technology, with a market capitalization of 20%, is the largest contributor to the S&P 500 index. Hence, three out of the five most valuable companies, Apple, Microsoft and Alphabet, are from the IT sector. However, this is different on the Norwegian, OSEBX, where the most valuable sector is the energy sector with a 35% market capitalization. On the contrary, information technology is one of the least valuable with a market capitalization of 2%. Figure 3 shows an overview of the closing price over the data period for SPX and its sectors.

Index		Value		Market cap
SPX	$	24,57	Trillions	100 %
Energy (SPX10GI)	$	1,28	Trillions	5 %
Materials (SPX15GI)	$	0,645	Trillions	3 %
Industrials (SPX20GI)	$	2,40	Trillions	10 %
Consumer Discretionary (SPX25GI)	$	2,55	Trillions	10 %
Consumer Staples (SPX30GI)	$	1,94	Trillions	8 %
Health Care (SPX35GI)	$	3,55	Trillions	14 %
Financials (SPX40GI)	$	3,12	Trillions	13 %
Information Technology (SPX45GI)	$	5,01	Trillions	20 %
Communication services (SPX50GI)	$	2,58	Trillions	11 %
Utilities (SPX55GI)	$	0,76	Trillions	3 %
Real estate (SPX60GI)	$	0,70	Trillions	3 %

Table 1: S&P 500 overview (26.02.19)

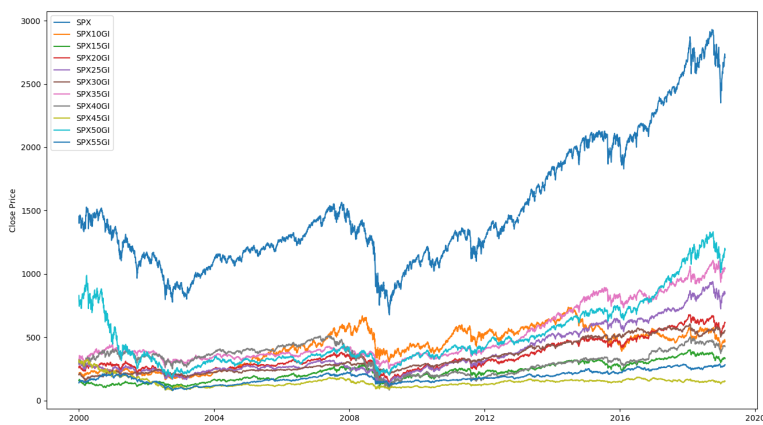


Figure 3: S&P 500 close price over the period

4.2.2 OSEBX

Oslo Stock Exchange Benchmark Index (OSEBX) contains a representative selection of all listed shares on the Oslo Stock Exchange. It is revised two times yearly, and the changes are implemented on the 1st of June and December. Its market value is $312,21 billion. In table 2, one can see that the energy, consumer staples and financials sector stands out with regards to market capitalization.

Figure 4 shows an overview of the closing price over the data period for OSEBX and its sectors.

Sector		Value		Market cap
OSEBX	$	312,21	Billions	100 %
Energy (OSE10GI)	$	109,93	Billions	35 %
Materials (OSE15GI)	$	22,91	Billions	7 %
Industrials (OSE20GI)	$	21,40	Billions	7 %
Consumer Discretionary (OSE25GI)	$	1,86	Billions	1 %
Consumer Staples (OSE30GI)	$	38,37	Billions	12 %
Health Care (OSE35GI)	$	1,047	Billions	0 %
Financials (OSE40GI)	$	56,86	Billions	18 %
Information Technology (OSE45GI)	$	5,58	Billions	2 %
Communication Services (OSE50GI)	$	38,37	Billions	12 %
Utilities (OSE55GI)	$	2,33	Billions	1 %
Real estate (OSE60GI)	$	13,95	Billions	4 %

Table 2: OSEBX overview (26.02.19)

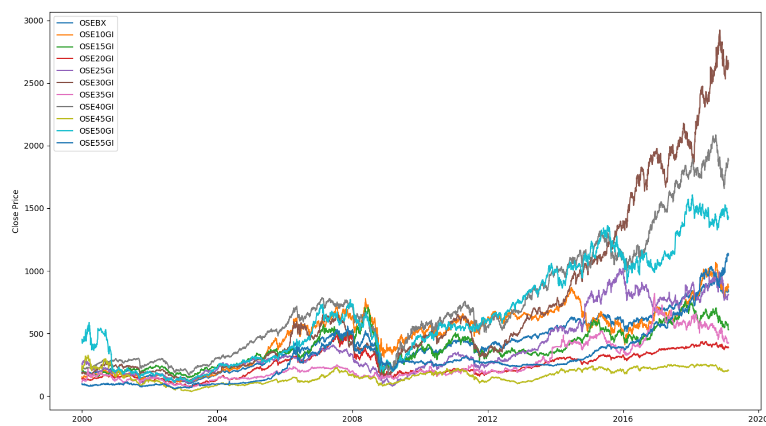


Figure 4: Oslo Stock Exchange close price over the period

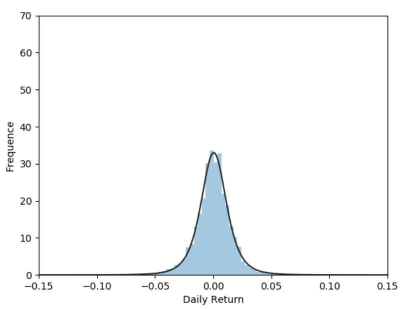
4.2.3 Energy Sector (10GI)

The energy sector on Oslo Stock Exchange consists of companies ranging from exploration to refining and marketing of oil products. The biggest companies are Equinor (previously Statoil) and Aker BP (previously Det Norske and British

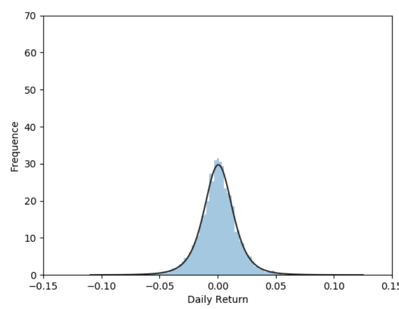
Petroleum), respectively worth $76 and $12 trillion. Thus, the results of the Energy sector, and even Oslo Stock Exchange, will be heavily influenced by Equinor's performance with a 24% of the value on Oslo Stock Exchange. As of February 26th, 2019, the total value of the sector is $110 billion, or about 35 % of the market.

In the U.S. one can find companies like National Oilwell Varco, Exxon Mobil Corp. and Baker Hughes, a GE Company. Common to them is their global presence. As of February 26th, 2019, the total value of the sector is $1,28 trillion, or about 5% of the S&P 500.

We chose to include this sector to further look into how the oil price influence the value of the energy sector for both an oil importing (USA) and an oil exporting (Norway) country. Also, the energy sector being 35% of the Norwegian market makes it almost impossible to neglect its influence on the index itself, and therefore, we must look into the sector.



(a) SPX10GI



(b) OSE10GI

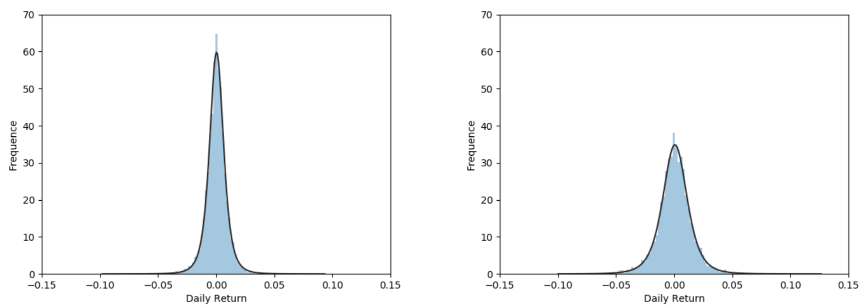
Figure 5: Distributions for Energy sectors

4.2.4 Consumer Staples (30GI)

This sector could be renamed to the salmon sector on the Oslo stock exchange. All, but two companies, Arcus and Orkla, are related to salmon production. MOWI (previously Marine Harvest) is the most valuable in the sector with a sector value of around 30%.

For the S&P 500 index, the most valuable company is Procter & Gamble Company with a value of $261 billion, or about 13,5% of the sector value. Other companies in the sector are; Coca-Cola Company, Walmart, Costco Wholesale Corporation and Sysco Corporation.

We chose to include this sector because of its lack of diversification and significant size on the Oslo Stock Exchange. While, on the other hand, the consumer staples sector is highly diversified in the S&P 500 index and therefore we can investigate the difference of our model on diversified and less diversified portfolios/sectors.



(a) SPX30GI

(b) OSE30GI

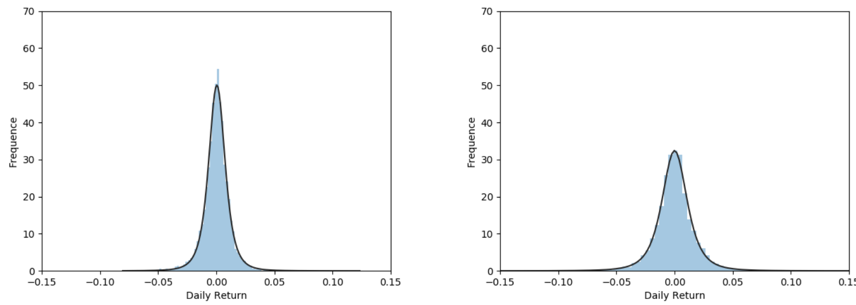
Figure 6: Distributions for Consumer Staples sectors

4.2.5 Health Care (35GI)

This is the least valuable sector on the Oslo Stock Exchange. Here, one can find companies like Nordic Nanovector, Medistim and BerGenBio. Also, these are the only companies worth over a billion within the sector.

The health care sector in the S&P 500 index is one of the largest. It consists of companies like CVS Health, Johnson& Johnson and Pfizer Inc.

Here, the difference between private- and public-driven health care shows its influence on the stock market. As the U.S. stock market with private health care, the health care sector has a market capitalization of 14%. Thus, compared to the Norwegian stock market, where the health care sector is nearly none existing with a market capitalization of less than 0,5%.



(a) SPX35GI (b) OSE35GI

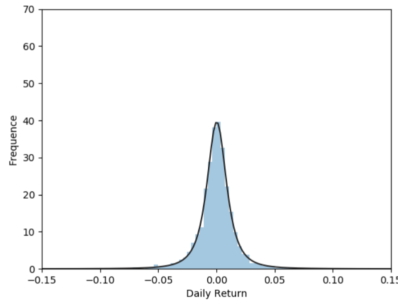
Figure 7: Distributions for Health Care sectors

4.2.6 Financials (40GI)

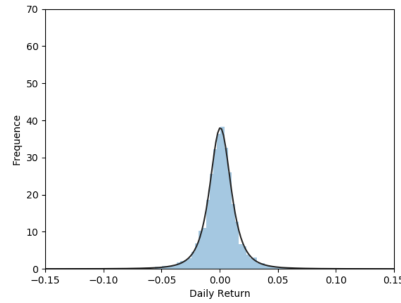
The financials sector is the second most valuable sector on the Oslo Stock Exchange, driven by DNB with around 50% of the sector value and ten percent of the market capitalization (Table 3). The financials sector on Oslo Stock Exchange comprises mostly of banks and insurance, and holding companies like Aker.

The financials sector is the third most valuable sector in the S&P 500 index. Here one finds investment banking, diversified banks, regional banks, insurance companies and multi-sector holdings, such as Berkshire Hathaway. It is also home for asset management like BlackRock.

The most apparent difference here is that the Norwegian government-owned DNB is the second most valuable company on the Oslo Stock Exchange and the most valuable within the financials sector. While, for S&P 500, the private American multinational conglomerate holding company led by Warren E. Buffet is the most valuable within the financials sector.



(a) SPX40GI



(b) OSE40GI

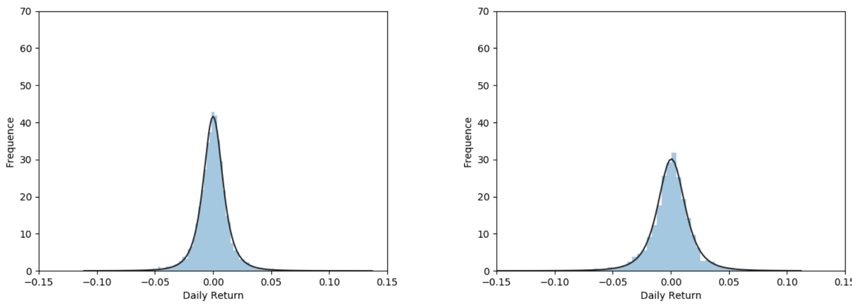
Figure 8: Distributions for Financials sectors

4.2.7 Information Technology (45GI)

With a value of $5,58 billion, IT is one of the least valuable sectors on Oslo Stock Exchange. The biggest companies in the sector specialize in building, maintaining and selling IT infrastructure. Smaller companies, like IDEX and Thin Film Electronics, specialize in biometric security and IoT.

The information technology sector is considered the most valuable sector in the S&P 500 index. Data has by many been introduced as "the new oil", in terms of value for a firm. Thus, the majority of the top five companies originates from the IT sector.

The IT sector on Oslo Stock Exchange and S&P 500 are entirely different. The IT sector on S&P 500 comprises of large tech companies like, Apple, Microsoft and Alphabet who stands for the entire production chain of their products. While the Norwegian companies on Oslo Stock Exchange typically specialize in IT infrastructure or one technology/product.



(a) SPX45GI

(b) OSE45GI

Figure 9: Distributions for Information Technology sectors

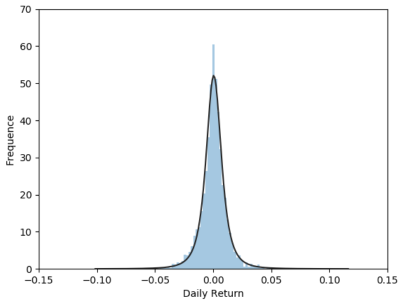
4.3 Summary

From the presentation of the sectors above and table 3, one can see that there is a clear discrepancy between the characteristics of the S&P 500 and Oslo Stock Exchange. While the smaller Norwegian Oslo Stock Exchange consists of few companies with a significant influence on the value and some smaller companies, the larger American market index S&P 500 is more diversified and less influenced by individual companies.

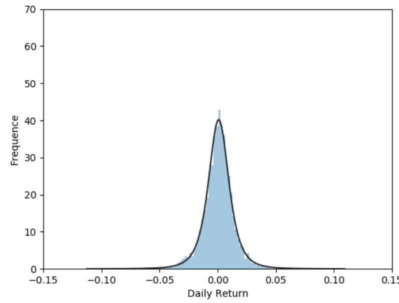
	S&P	Sector	Value (T$)	%	OSEBX	Sector	Value (T$)	%
1	Microsoft	IT	847	3%	Equinor	Energy	76	24%
2	Apple	IT	813	3%	DNB	Financial	30	10%
3	Amazon	CD	799	3%	Telenor	Com. Serv.	29	9%
4	Alphabet	IT	797	3%	Aker BP	Energy	13	4%
5	Berkshire Hathaway	Financial	491	2%	Yara	Materials	11	4%

Table 3: The five most valuable firms per 26.02.19

This is clear in figure 6a and figure 6b, where one can see a difference in the kurtosis. This is also shown in table 4, as the consumer staples' sector on Oslo Stock Exchange have a kurtosis of 3.74 and consists mostly of salmon companies, whereas the sector on S&P 500 is more diversified and have a kurtosis of 9.68. The distributions for the two indices are illustrated in figure 10, showing that SPX has a lower degree of freedom and standard deviation, and higher kurtosis.



(a) SPX



(b) OSEBX

Figure 10: Distributions for the indices

Sector	Degrees of Freedom		Mean		Standard Deviation		Skewness		Kurtosis		5% (1%) Quantile	
	S&P 500	OSE	S&P 500	OSE	S&P 500	OSE	S&P 500	OSE	S&P 500	OSE	S&P 500	OSE
Index	2.59	3.15	7.4E-5	9.0E-4	0.0070	0.0092	-0.21	-0.56	8.49	6.75	-0.019 (-0.035)	-0.023 (-0.043)
Energy	3.77	4.16	8.3E-7	6.3E-4	0.0113	0.0126	-0.31	-0.24	10.05	4.03	-0.025 (-0.044)	-0.027 (-0.048)
Consumer Staples	3.57	4.06	3.9E-4	7.1E-4	0.0062	0.0108	-0.17	-0.07	9.68	3.74	-0.014 (-0.026)	-0.023 (-0.041)
Health Care	3.22	3,00	5.4E-4	4.7E-5	0.007	0.011	-0.04	0.73	6.89	15.18	-0.017 (-0.034)	-0.026 (-0.050)
Financials	2.16	2.66	4.4E-5	2.7E-4	0.0090	0.0096	-0.11	-0.28	16.32	10.80	-0.026 (-0.053)	-0.023 (-0.047)
Information Technology	2.94	3.12	8.0E5	-1.5E-5	0.0088	0.0123	0.08	-1.37	7.13	19.73	-0.022 (-0.044)	-0.030) (-0.058)

Table 4: Descriptive statistics for the indices and sectors

5 Analysis and results

In common for all the Value-at-Risk models performed, four metrics were observed. This is to measure the performance and to make the results from the different models directly comparable and simple to evaluate against each other. The metrics observed were the following:

- The number of times the returns R on day n, exceeded the estimated VaR for day n, for the whole sample period
- The percentage of times $R < VaR$ for day n.
- The total distance from the returns R and the calculated VaR throughout the whole sample period. Measured from VaR to the return when $R < 0\%$ and from VaR up to 0% return when $R > 0\%$.
- The distance from the returns R and the calculated VaR, when $R < VaR$.

The calculations were done for both 99% and 95% VaR. Optimally when calculating 99% VaR one want the returns to exceed the VaR estimation as close to 1% as possible, for all the calculations. This is to ensure that the calculation performs at the level given by the confidence level and is a reliable metric. The magnitude of the distance between the returns R and VaR is a metric that optimally should be as low as possible so that the risk indicator VaR is as precise as possible. For days where $R > 0\%$ the distance is measured from VaR up to 0%, due to the fact, that if a portfolio manager had perfect information about the market on day n the Value-at-Risk on day n would be 0%. The portfolio manager would either keep the portfolio knowing $R > 0\%$ or reallocate to ensure a positive return. Thus, the Value-at-Risk is zero. The same traits are desired for the distance between R and VaR when $R < VaR$. The monitoring of this metric is to observe if VaR misses by a couple of percentage points or if $R << VaR$. Being aware of and considering this metric can reduce the severity of the consequences when $R < VaR$, by not being too far off in the VaR estimation. When optimizing the VaR calculations, a trade-off between distance from calculations to returns and reactiveness of VaR has to be evaluated and considered.

For all the VaR models different time periods are defined, to differentiate volatile periods from more stable financial times. This is to further investigate not only how the models' overall performance over 20 years, but also in more volatile periods when a precise Value-at-Risk estimate is critical. To see how the VaR models perform under less volatile periods are also of interest. When comparing the results, the time periods can reveal if the VaR models calculate pessimistic or optimistic estimates for the different periods or if the performance is stable throughout the whole data sample. The periods defined are:

- Period 1: 2002 - 2003
- Period 2: 2004 - 2006
- Period 3: 2007 - 2009
- Period 4: 2010 - 2013
- Period 5: 2014 - 2017
- Period 6: 2018 - 2019

The periods are chosen to capture moments like the financial crisis in 2008, period 3, and the oil price collapse in 2014, here period 5

5.1 Historical VaR

As a basis and due to its simple nature, historical VaR was calculated for the whole data period, the year 2000 - 2019, with the confidence levels 95% and 99%. This is a VaR calculation in one of its most simple forms, and is done to establish grounds to see how the VaR calculations will evolve throughout this thesis.

5.1.0.1 S&P 500

The historical VaR results for the S&P 500 index, SPX, was about 1.90% and 3.46% respectively, shown in table 5.

Confidence level	
95%	1.90 %
99%	3.46 %

Table 5: SPX: Historical VaR

5.1.0.2 OSEBX

The historical VaR results for the Oslo Stock Exchange index, OSEBX, was about 2.28% and 4.34% for the confidence levels 95% and 99% respectively, shown in table 6.

Confidence level	
95%	2.28 %
99%	4.34 %

Table 6: OSEBX: Historical VaR

5.2 Historical VaR with rolling window

The first step in the process and to serve as a benchmark model for the calculations performed throughout this thesis. Historical VaR with rolling window is computed due to its simplicity and popularity in the banking sector. The window size of 250 days will be the benchmark window size, which will be used as a basis for comparison between models later in the thesis. This model was performed to see how VaR calculations behaved and performed with different sample sizes. The different window sizes that were used in the computation was $\{22, 44, 250, 500, 750, 1000\}$ days.

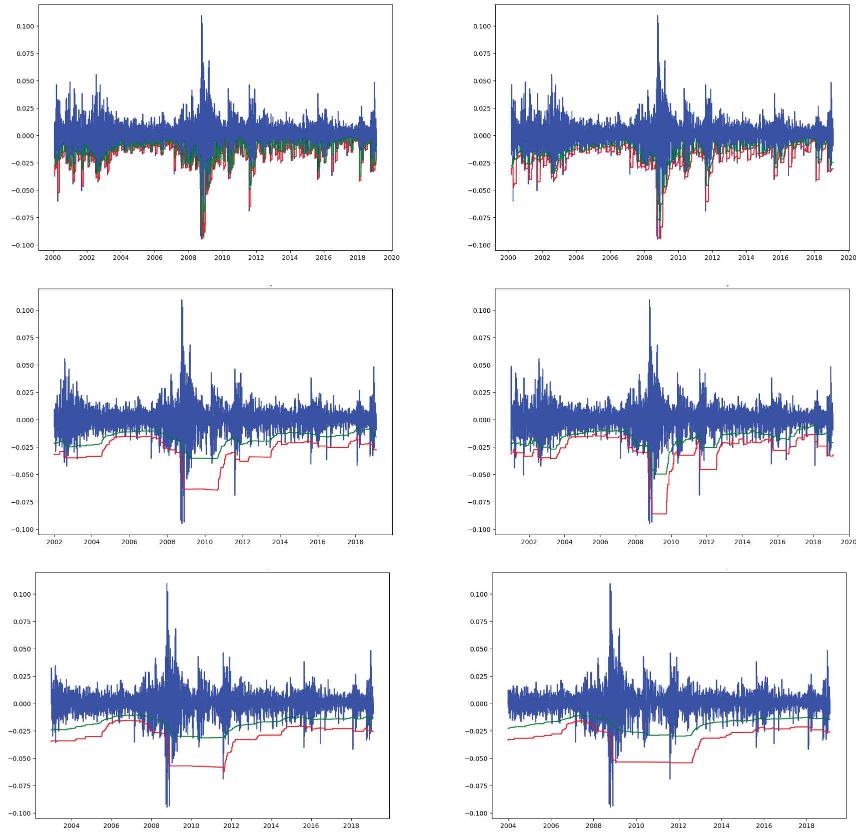


Figure 11: SPX: historical VaR with rolling window ranging from 22 - 1000 days, from top left to bottom right respectively. X-axis are dates and Y-axis are daily returns. Green line: 95% VaR. Red line: 99% VaR

For each iteration a plot was produced, where the daily returns were plotted against the calculated VaR throughout the data set. An observation from the plot is that the smaller the window is, the faster the VaR is to react to significant changes and market shocks. The larger windows struggle to adapt to changes, in terms of reacting after a shock has happened and also recognizing that the market is moving into a less volatile period, which can be seen from figure 11 above. From the plot, we also see that with smaller window sizes, e.g., 22 and 44 days, both 95% and 99% VaR follow the downside of the plotted returns relatively close. The bigger the window size, the number of times the daily returns R exceed VaR is reduced. A complete table with the results are presented in Appendix A.

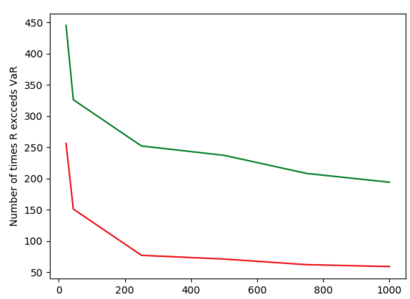
5.2.0.1 S&P 500

For 99% VaR it is observed that the percentage of times $R < VaR$ gets closer and closer to 1%, but stops just before it reaches the 1% mark for the S&P 500 index, seen in table 7 below. For the calculations with a 95% confidence level, VaR tends to calculate a too optimistic VaR estimation for the window sizes, meaning that the returns exceed VaR for more than 5% of the calculations.

Window size	22	44	250	500	750	1000
Number of times exceeding the 99% VaR:	256	151	77	71	62	59
Number of times exceeding the 95% VaR:	445	326	252	237	208	194
Percentage of times exceeding the 99% VaR:	5.31 %	3.15 %	1.68 %	1.63 %	1.51 %	1.53 %
Percentage of times exceeding the 95% VaR:	9.23 %	6.79 %	5.48 %	5.45 %	5.08 %	5.05 %
Sum of difference between historic 99% VaR and Observation:	77.07	91.3	116.19	116.78	117.19	115.67
Sum of difference between historic 95% VaR and Observation:	58.86	63.67	69.17	66.87	65.75	62.63
Sum of difference when R <99%VaR:	1.47	0.96	0.64	0.69	0.66	0.74
Sum of difference when R <95%VaR:	2.17	1.77	1.98	1.94	1.91	2.04

Table 7: SPX: historical rolling window output

The number of times the returns exceeded the VaR calculations was plotted as a dependent variable, with the number of days on the x-axis as the independent variable, seen on the left in figure 12. This was done to map the relationship between the two variables, which indicated itself to be exponential decreasing, i.e., $a * b^x$ where $0 < b < 1$, for both 95% and 99% VaR.



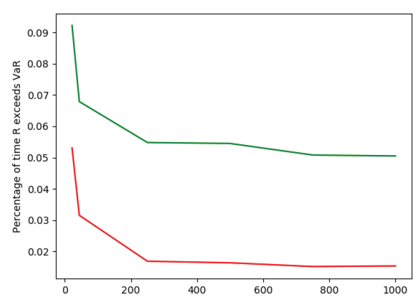


Figure 12: SPX: Number of times the returns exceeded VaR (left) and percentage of times the returns exceeded VaR (right). Green line: 95% VaR. Red line: 99% VaR

The relationship of the distance between the returns R and VaR was mapped, seen to the left in figure 13 below. This relationship seems to be inverse from the one between window size and number of times $R < VaR$, i.e. $a * b^x$ where $b > 1$, for 99% VaR. Whereas for 95%VaR the relationship of the distance between R and VaR has the shape of a function with diminishing returns. The performance regarding the total distance between the returns R and VaR reduces with increased window size. Regarding the distance between R and VaR, it seems to increase with increased window size up to a certain point where it flattens out, seen in figure 13 for the S&P 500 index, then slightly starts to decrease to some degree depending on which sector is observed, for the largest window sizes for 99% VaR. For 95% VaR the behavior of the number of times R exceed VaR is similar to 99% VaR, but the distance between R and VaR behaves a bit differently. When increasing the window size, the distance between R and VaR increases slightly before decreasing to an even lower level than for the smallest window size. This might come from the 95% VaR laying closer to R, due to a higher quantile, and being able to adapt and react quickly with small windows and with the large windows not being able to react, but on an average laying close to R. Also, for the values in the middle of the array of window sizes not being able to react or performing satisfying on average.

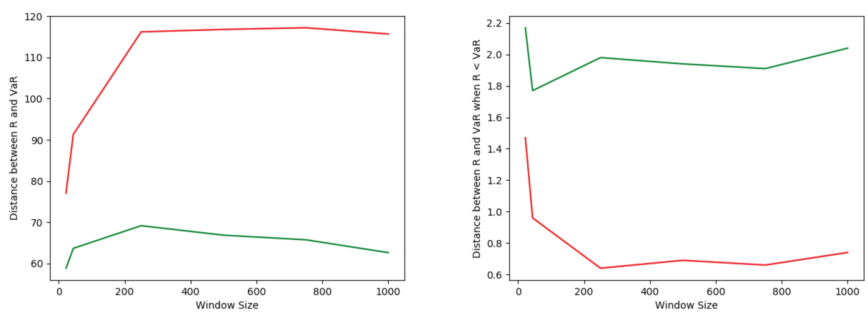


Figure 13: SPX: Sum of distance between returns and VaR (left). Sum of distance between returns and VaR, when $R < VaR$(right). Green line: 95% VaR. Red line: 99% VaR

The sum of the distance between R and VaR, when $R < VaR$, behaves differently for the two confidence levels. For 99% VaR the behavior is similar to the numbers of times and percentage of times the returns exceed VaR, a decreasing exponential function. The function has a steep slope in the start, before flattening somewhat out, depending on which sector is being studied. The behavior of the 95% VaR is a bit different. Generally, it lays higher than for the 99% VaR, and it also has a different characteristic. When simulating for the different sectors and indices, the behavior tends to be the same for the smaller window sizes, decreasing. However, for the larger window sizes, the behavior depends on which index and sector are studied, some decrease and others increase.

Period	Window size	22	44	250	500	750	1000
1	Number of times exceeding the 99% VaR:	19	13	6	6	1	0
	Percentage of times exceeding the 99% VaR:	3.77 %	2.58 %	1.19 %	1.19 %	0.20 %	0.00 %
	Number of times exceeding the 95% VaR:	42	34	24	24	4	0
	Percentage of times exceeding the 95% VaR:	8.33 %	6.75 %	4.76 %	4.76 %	0.79 %	0.00 %
2	Number of times exceeding the 99% VaR:	40	25	9	6	4	0
	Percentage of times exceeding the 99% VaR:	5.28 %	3.30 %	1.19 %	0.79 %	0.51 %	0.00 %
	Number of times exceeding the 95% VaR:	66	49	32	16	11	5
	Percentage of times exceeding the 95% VaR:	8.71 %	6.94 %	4.22 %	2.11 %	1.45 %	0.66 %
3	Number of times exceeding the 99% VaR:	41	25	23	33	39	42
	Percentage of times exceeding the 99% VaR:	5.40 %	3.29 %	3.03 %	4.35 %	5.14 %	5.53 %
	Number of times exceeding the 95% VaR:	73	59	60	85	98	106
	Percentage of times exceeding the 95% VaR:	9.62 %	7.77 %	7.91 %	11.10 %	12.91 %	13.97 %
4	Number of times exceeding the 99% VaR:	56	35	12	5	1	1
	Percentage of times exceeding the 99%	5.39 %	3.37 %	1.15 %	0.48 %	0.10 %	0.10 %
	Number of times exceeding the 95% VaR:	100	65	43	29	19	16
	Percentage of times exceeding the 95% VaR:	9.62 %	6.26 %	4.14 %	2.79 %	1.83 %	1.54 %
5	Number of times exceeding the 99% VaR:	55	30	16	11	10	8
	Percentage of times exceeding the 99% VaR:	5.46 %	2.98 %	1.59 %	1.09 %	0.99 %	0.79 %
	Number of times exceeding the 95% VaR:	92	68	50	50	48	39
	Percentage of times exceeding the 95% VaR:	9.14 %	6.75 %	4.97 %	4.97 %	4.77 %	3.87 %
6	Number of times exceeding the 99% VaR:	18	11	8	10	7	8
	Percentage of times exceeding the 99% VaR	6.47 %	3.96 %	2.88 %	3.60 %	2.52 %	2.88 %
	Number of times exceeding the 95% VaR:	23	19	31	33	28	28
	Percentage of times exceeding the 95% VaR:	8.27 %	6.83 %	11.15 %	11.87 %	10.07 %	10.07 %

Table 8: SPX: Historical rolling window periods

From the results for the different time periods, shown in table 8, what the different window sizes have in common is that period 3 and 6 are the most challenging periods. For period 3, a window size of 250 days has the lowest percentage of times where the returns exceed VaR, for 99% confidence level, with a result of 3.03%. For 95% confidence level, the 44-day window gives the lowest percentage of times where $R < VaR$, with a result of 7.77%. For period 6 the 750-day window has the lowest percentage of times where the returns exceed VaR, for 99% confidence level, with a result of 2.52%. For the 95% confidence level, the 44-day window performs best again, with a result of 6.83%. For less volatile periods such as period 4, the model struggles with a too pessimistic VaR estimation, meaning a too low percentage of times where the returns exceed VaR, for both confidence levels when using the larger window sizes. For the window sizes 750 and 1000, note that data in period 1 and 2 partly or entirely fall away due to the size of the window in the first calculation.

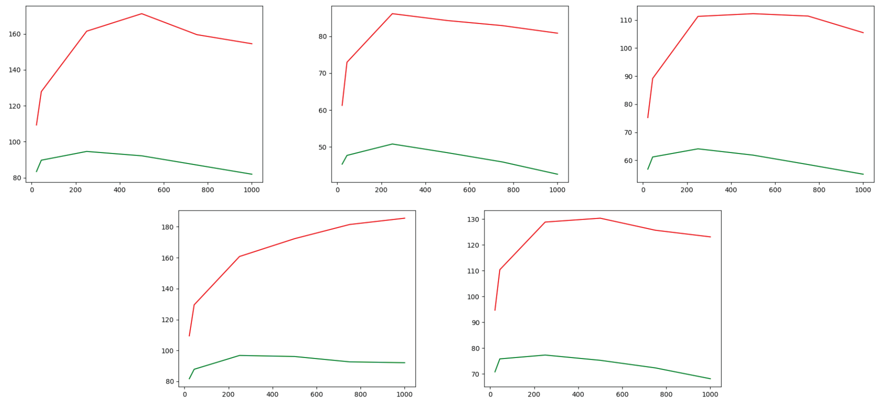


Figure 14: SPX sectors:X-axis: Window size. Y-axis:Sum of distance between returns and VaR. Green line: 95% VaR. Red line: 99% VaR

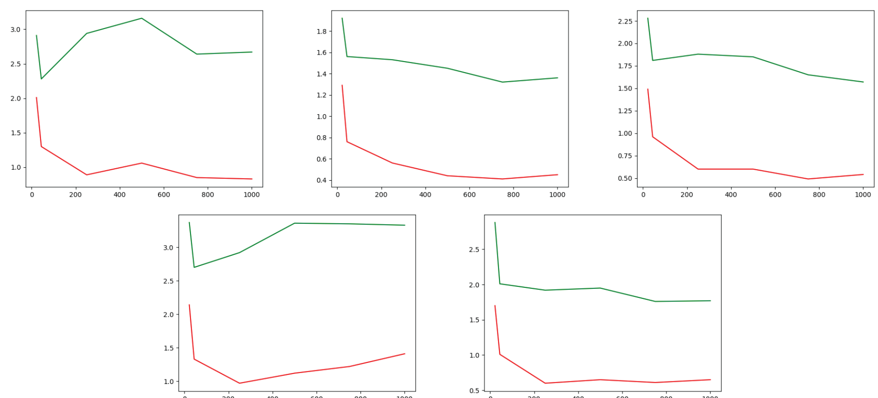


Figure 15: SPX sectors: X-axis: Window size. Y-axis: Sum of distance between returns and VaR, when $R < VaR$. Green line: 95% VaR. Red line: 99% VaR

From the results for the calculated 95% VaR for the sectors, one can see that the percentage of times the return exceed the calculated VaR decreases as the window size increases. However, at the 99% VaR one can see a significant difference, all of the sectors, except for the energy and information technology, reach the lowest percentage of times $R < VaR$ at the 750-day window and the percentage of times the return exceeds the calculated VaR increases or flattens towards 1000 days.

For the sum of the difference between returns and calculated 95% VaR, shown in figure 14, there are both different formations and magnitude. The energy and financials sectors stand out in magnitude, whereas the financials sector reaches a value of 185.56 for the 1000 days and energy sector reaches a value of 171.21 for the 500 days. The financials sector is also the only sector to increase from 22 to 1000 days; most of the other sectors follow the index and reaches their maximum with a window size between 250 and 500 days. Consumer Staples has the lowest distance between returns and calculated VaR, with a maximum value of 86.1 for the 250-day window size. It also has the smallest difference between the different window sizes. For the 99% VaR the maximum distance is reached for a window size of 250 days, and decreases for both smaller and larger window sizes. This is the case for all of the sectors, except financials. The distribution of the distances over the window sizes is similar for 95% VaR.

The sum of the distance between R and VaR, when $R < VaR$, shown in figure 15, behaves differently for the two confidence levels. Common for both confidence levels is a steep decrease from 22 to 44 days, but between 44 and 250 days, it increases or slightly decreases for the 95% VaR. On the other hand, for 99% VaR, the sum of the distance continues to decrease significantly from 44 to 250 days. In the window range from 500 to 1000 days, all but the financials sector flattens out. The sum of the distance for the financials sector increases linearly towards the 1000-day window. For the 95% VaR, consumer staples, health care, and information technology decrease slightly towards the 1000-day window while financials increases towards the 500-day window before stabilizing. Energy increase towards 500-day and decrease to 750-day before flattening out towards the 1000-day window.

For the different time periods the sectors, like for the index, period 3 and 6 are the most challenging periods to estimate VaR. For period 3 the 250-day window size performs the best for all the sectors, for 99% confidence level. For the 95% confidence level in period 3, the 44- and 250-day window sizes perform the best. For period 6, the window sizes 44, 250 and 750 tend to perform the best depending on which confidence level and sector is being observed. Period 4 with low volatility, calculations with large window sizes for the different sectors struggle with a too low percentage of times where the returns exceed VaR for both confidence levels. For the sector results, see appendix A.

5.2.0.2 Oslo Stock Exchange

For the largest window sizes for the index and sectors, the calculated VaR tends to lay just over or under the 1% mark for the larger window sizes, depending on which sector is observed. For the index the percentage of times $R < VaR$ goes down to 0.88% for a window size of 1000 days. For the calculations with a 95% confidence level, VaR tends to estimate a too pessimistic VaR, seen for the Oslo Stock Exchange index in table 9 below.

Historical VaR with rolling window	22	44	250	500	750	1000
Number of times exceeding the 99% VaR:	272	167	77	63	45	42
Number of times exceeding the 95% VaR:	469	372	262	246	211	196
Percentage of times exceeding the 99% VaR:	5.70 %	3.51 %	1.69 %	1.47 %	1.11 %	1.11 %
Percentage of times exceeding the 95% VaR:	9.82 %	7.82 %	5.76 %	5.72 %	5.21 %	5.16 %
Sum of difference between historic 99% VaR and Observation:	92.56	109.9	141.74	150.62	152.53	153.18
Sum of difference between historic 95% VaR and Observation:	70.26	76.74	82.89	80.27	76.74	72.87
Sum of difference when R <99%VaR:	1.82	1.18	0.78	0.82	0.65	0.68
Sum of difference when R <95%VaR:	2.85	2.31	2.44	2.9	2.82	2.82

Table 9: OSEBX: historical rolling window output

Both the number and percentage of times, the observation exceeded the calculated VaR was plotted against the window size. The index showed a similar relationship as the S&P 500 index for these two metrics, a decreasing exponential function shown in figure 16 below. The results are summarized in table 9 above.

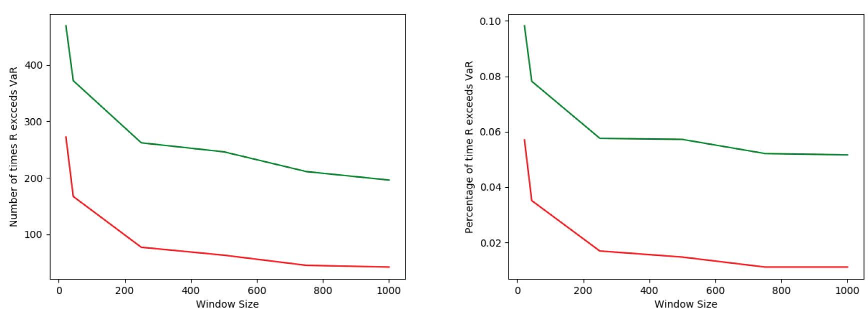


Figure 16: OSEBX: Number of times the returns exceeded VaR (left) and percentage of times the returns exceeded VaR (right). Green line: 95% VaR. Red line: 99% VaR

The relationship of the distance between the returns R and VaR is an increasing exponential function for 99% VaR, with an increase in total distance when increasing the window size. The 95% VaR has the shape of a function with diminishing returns. Here, first increasing, but then decreasing to an even lower level than for the smallest window size. Also, for the index this indicates that the 95% VaR moves away from the returns when increasing from 44 to 250 days, but from 250 to 1000 days the VaR calculations move closer to the returns again, shown to the left in figure 17.

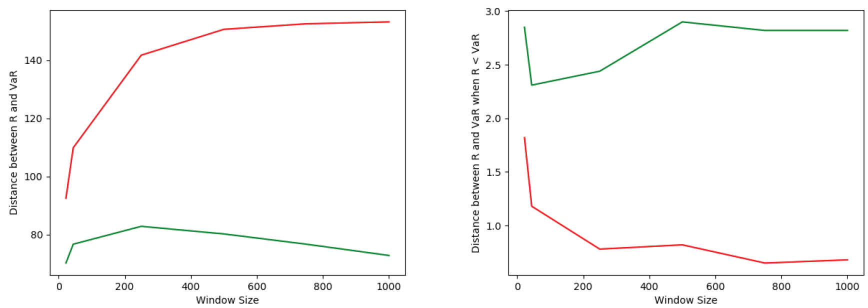


Figure 17: OSEBX: Sum of difference between returns and VaR (left). Sum of difference between returns and VaR, when $R < VaR$ (right). Green line: 95% VaR. Red line: 99% VaR

For the sum of the distance between returns and VaR when $R < VaR$, shown to the right in figure 17, the characteristics are different for the two confidence levels. With 99% VaR showing similar behavior as an exponentially decreasing function. Also, the 95% VaR is first decreasing, then increasing and lastly flattening out for the Oslo Stock Exchange index. Depending on which sector is being observed, the distance increases or decreases for the larger window sizes.

Period	Window size	22	44	250	500	750	1000
1	Number of times exceeding the 99% VaR:	27	14	3	5	1	0
	Percentage of times exceeding the 99% VaR:	5.41 %	2.81 %	0.60 %	1.00 %	0.20 %	0.00 %
	Number of times exceeding the 95% VaR:	49	36	24	22	4	0
	Percentage of times exceeding the 95% VaR:	9.82 %	7.21 %	4.81 %	4.41 %	0.80 %	0.00 %
2	Number of times exceeding the 99% VaR:	42	29	17	11	11	9
	Percentage of times exceeding the 99% VaR:	5.55 %	3.83 %	2.25 %	1.45 %	1.45 %	1.19 %
	Number of times exceeding the 95% VaR:	70	57	48	44	40	35
	Percentage of times exceeding the 95% VaR:	9.25 %	7.53 %	6.34 %	5.81 %	5.28 %	4.62 %
3	Number of times exceeding the 99% VaR:	45	30	20	15	20	22
	Percentage of times exceeding the 99% VaR:	5.98 %	3.98 %	2.66 %	199.00 %	2.66 %	2.92 %
	Number of times exceeding the 95% VaR:	79	67	51	67	77	88
	Percentage of times exceeding the 95% VaR:	10.49 %	8.90 %	6.77 %	8.90 %	10.23 %	11.69 %
4	Number of times exceeding the 99% VaR:	64	33	9	5	0	0
	Percentage of times exceeding the 99% VaR:	0.36 %	3.38 %	0.89 %	0.50 %	0.00 %	0.00 %
	Number of times exceeding the 95% VaR:	100	72	42	27	21	13
	Percentage of times exceeding the 95% VaR:	9.94 %	7.16 %	4.17 %	2.68 %	2.09 %	1.29 %
5	Number of times exceeding the 99% VaR:	52	31	12	20	12	9
	Percentage of times exceeding the 99% VaR:	5.17 %	3.08 %	1.19 %	1.19 %	1.19 %	0.90 %
	Number of times exceeding the 95% VaR:	100	80	59	62	58	49
	Percentage of times exceeding the 95% VaR:	9.95 %	7.96 %	5.87 %	6.17 %	5.77 %	4.88 %
6	Number of times exceeding the 99% VaR:	16	9	10	7	1	2
	Percentage of times exceeding the 99% VaR:	5.76 %	3.24 %	3.60 %	2.52 %	0.36 %	0.72 %
	Number of times exceeding the 95% VaR:	31	24	26	24	11	11
	Percentage of times exceeding the 95% VaR:	11.15 %	8.63 %	9.35 %	8.63 %	3.96 %	3.96 %

Table 10: OSEBX: Historial rolling window periods

From the different time periods, shown in table 10, period 3 and 6 are two of the periods where the VaR estimations struggle the most for both confidence levels. In period 3 a window size of 500 days gives the lowest percentage of times where the returns exceed VaR with the results of 1.99% and 8.90%, for 99% and 95% confidence level, respectively. For period 6, a rolling window of 1000 days gives the best results for both confidence levels with a result of 0.72% and 3.96%, for 99% and 95% confidence level, respectively. The larger window sizes struggle in period 4, with too pessimistic VaR estimates. Also for the Oslo Stock Exchange calculations, for the 750- and 1000-day window sizes, data from the periods 1 and 2 partly or entirely fall away due to the size of the window in the first calculation.

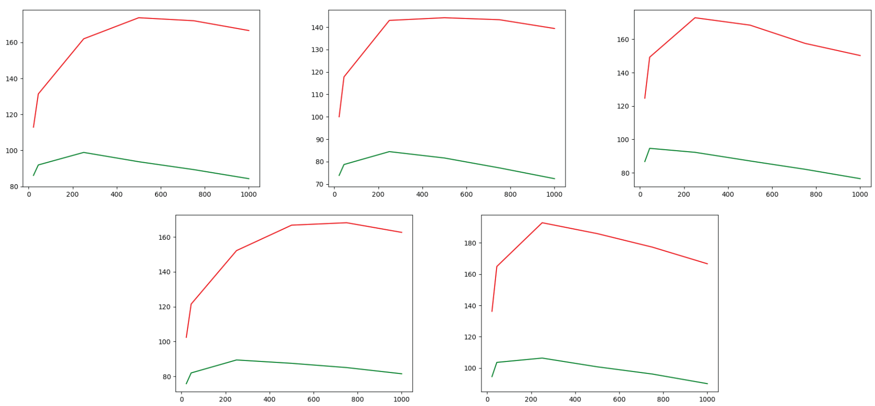


Figure 18: OSEBX sectors: X-axis: Window size. Y-axis: Sum of difference between returns and VaR. Green line: 95% VaR. Red line: 99% VaR

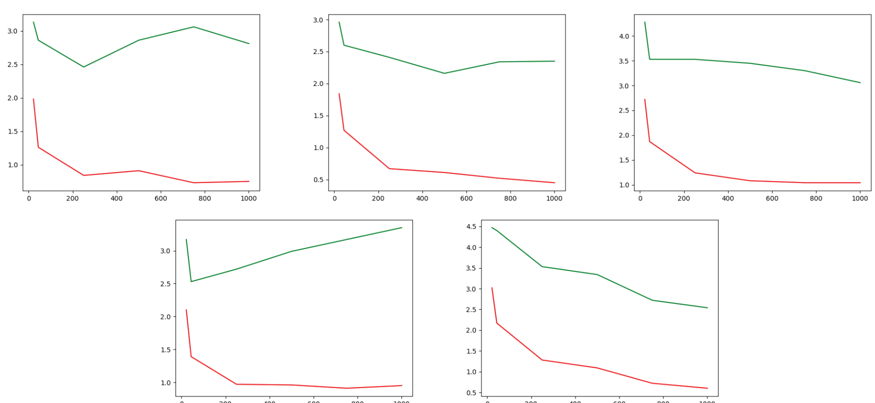


Figure 19: OSEBX sectors: X-axis: Window size. Y-axis: Sum of difference between returns and VaR, when $R < VaR$. Green line: 95% VaR. Red line: 99% VaR

The percentage of times R exceeds VaR for the Oslo Stock Exchange sectors are diminishing for both confidence levels and all sectors as the rolling window size increase from 22 to 1000 days. The only exception is the health care sector, where it increases from 750 to 1000 days. The energy and financials sectors reach the desired 1% mark at the 1000-day window. Consumer staples and health care approximately reach the desired 1% mark at 750-day and the information technology sector at 500-day. For the 95% VaR, all but information technology reach 5% at a window size of 500 days. Information technology reaches the percentage of times $R < VaR$ of 5% at the window size of 250 days.

The relationship of the distance between R and VaR, shown in figure 18, for the 95% confidence level increase towards a window size of 500 for energy and consumer staples, and 250 days for health care and information technology, while financials increases towards a window size of 750 days before decreasing to the window size of 1000 days. For 99%, the relationship of the distance between R and estimated VaR increases towards the window size of 250 days for all, except for the health care sector. It is also worth noting that the value for 99% VaR tends to return to approximately the same value for 1000 and 22 days, while the value for 95% tends to alter significantly from 22 to 1000 days.

For the sum of the distance between R and VaR when R exceeds the historical VaR, shown in figure 19, there is a clear relationship between the sectors in the window range from 22 to 44 days where the distance decreases. For the 95% from 44 days to 1000 days, the distance moves both up and down depending on which sector is being observed. For the 99% confidence level, all sectors have a decreasing exponential slope.

The different time periods for the Oslo Stock Exchange sectors share similar traits with the index. In period 3 and 6 the different window sizes struggle with a too optimistic VaR estimate for both confidence levels. For the energy and consumer staples period 2, especially for 95% VaR, also struggle with a too optimistic VaR. Large window sizes struggle in period 4, with a too pessimistic VaR for both confidence levels. The window sizes that give the best results are 250, 750, 500 and 1000, depending on which sector, period and confidence level is being observed. For the sector results, see appendix A.

5.3 Dynamic Historical VaR

From the results from historical VaR with a rolling window, it became interesting to investigate the effects of developing a model using a dynamic rolling window, a window that scales its size with respect to market uncertainty. To achieve this, the CBOE Volatility Index, VIX, was used as a scaling variable for the window size used to calculate the historical VaR. VIX is a forward-looking metric, and by implementing forward-looking variable into a backward-looking risk metric, can help make the estimate more precise and capable of performing under financial turmoil. The model is designed such that when the volatility index indicates an uncertain period it reduces the window size, thus making the most recent data the most relevant and enabling VaR to be more reactive to change. Also, when VIX indicates a less volatile period, the model expands the window and includes more data to calculate VaR to achieve as precise calculations as possible. The purpose of developing this model was to examine if the model could adopt the desired characteristics and qualities of the different window sizes into one model.

The model has a variable, avg, that enables the possibility to use the average of VIX for the last k days, instead of the daily trade close, as an independent variable. This is to prevent large and sudden changes to the daily VaR calculated, seen from the graphs in figure 20 and 21. The graphs show the behavior of the calculated VaR, for both 95% and 99% confidence level, with a $avg = 1$ and $avg = 15$, respectively. When using the average trade close of the last 15 days, the calculated VaR has less drastic and sudden changes, and the graph is smoother. VaR calculations with large fluctuations from day to day are undesirable regarding capital requirements tied to VaR reports and costs tied to portfolio management.

For complete results and graphs, see appendix B

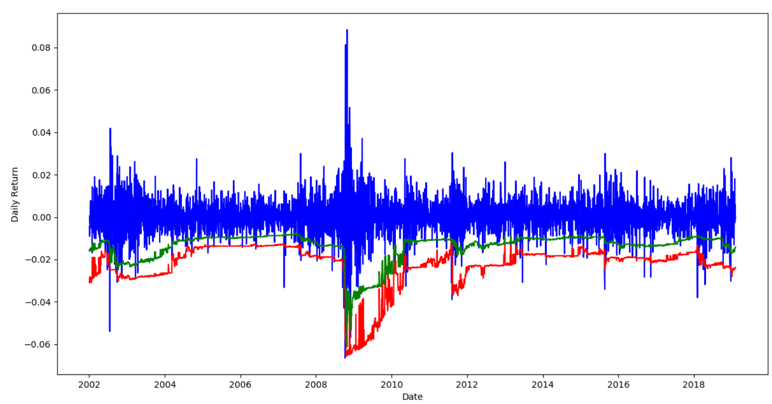


Figure 20: SPX: VaR with variable rolling window, 1-day average trading close and proportional change in VIX. Green line: 95% VaR. Red line: 99% VaR

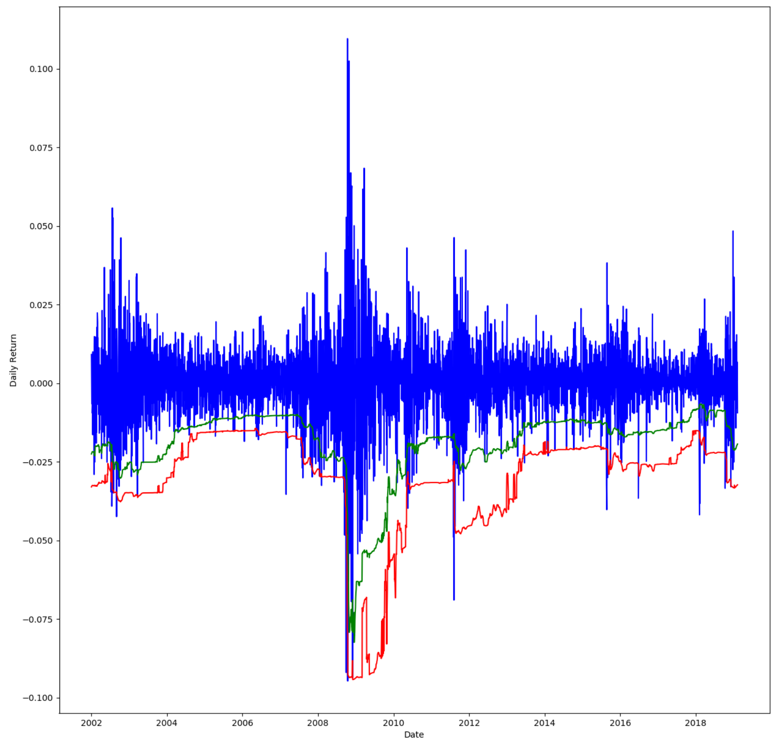


Figure 21: SPX: VaR with variable rolling window, 15-day average trading close and proportional change in VIX. Green line: 95% VaR. Red line: 99% VaR

5.3.1 DHV Model 1: Proportional Movement

The first version of this model, illustrated in the figures above, based itself on scaling the window size proportional to the VIX closing value. It used the range of VIX closing values $[9, 81]$ to scale the window size within the interval $[22, 500]$. The window size interval is based on the results from VaR with rolling window, excluding the window sizes of 750 and 1000 days due to less reactiveness to change and similar performance to the window size of 500 days. The relationship between the number of times $R < VaR$ and window size is exponential, uncovered from the results from historical VaR with a rolling window, and this model is set up after those findings. With the equation being:

$$Window\ size = 740 * 0.957^{VIX\ trade\ close} \tag{13}$$

5.3.1.1 S&P 500

DHV Model 1	
Number of times exceeding the 99% VaR:	59
Number of times exceeding the 95% VaR:	202
Percentage of times exceeding the 99% VaR:	1.38 %
Percentage of times exceeding the 95% VaR:	4.70 %
Sum of difference between historic 99% VaR and Observation:	117.32
Sum of difference between historic 95% VaR and Observation:	69.6
Sum of difference when R <99%VaR:	0.51
Sum of difference when R <95%VaR:	1.49

Table 11: SPX: Output for DHV Model 1

Table 11 shows the results from the proportional movement with $avg = 15$. Comparing these results with the results from different window sizes from calculating historical VaR with rolling window, the number of times $R < VaR$ is lower than any of the window sizes, for both confidence levels. For 99% and 95% VaR estimates, the results for SPX lay fairly close to where it is desired but is still considered imprecise with a percentage of 1.38% and 4.70%, respectively. The cumulative distance between the estimated VaR and R is 117.32 for 99% VaR, about the same as for the windows in the range $[250, 1000]$. For 95% VaR, this metric performed similarly to a window size of 250 days. The cumulative distance when $R < VaR$, performed better than any of the fixed window sizes for both 99% and 95% VaR, with the values 0.51 and 1.49, respectively.

		SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99%	1.47 %	1.38 %	1.47 %	1.52 %	1.56 %
	95%	5.54 %	5.03 %	4.75 %	4.84 %	4.93 %
Sum of diff between VaR and observation	99%	163,84	84,64	110,61	165,52	122,73
	95%	96.34	49.67	65.33	100.19	75.65
Sum of diff when R exceeds VaR	99%	0.78	0.43	0.5	0.83	0.53
	95%	2.45	1.32	1.38	2.04	1.6

Table 12: SPX sectors: Output for DHV Model 1 - Proportional movement

From table 12 one can see that for the 99% confidence level all the VaR estimations approximately lay within 0.5 percentage points or lower to the desired 1% mark. The information technology sector has the most optimistic VaR estimate, with 1.56%, while the consumer staples sector has the least optimistic, with a percent of 1.38%. The last three sectors have approximately 1.5% of times where the returns exceed VaR. For the 95% confidence level, the sector with the most pessimistic VaR estimate is health care, with 4.75%. Followed by financials, information technology, and consumer staples with respectively 4.84%, 4.93%, and 5.03%. Last is the energy sector, with the result of 5.54%.

For the cumulative distance between R and VaR, for both 95% and 99% confidence level, consumer staples stand out as its value is half of the two sectors with the highest values, energy and financials. Health care and information technology follow, after the consumer staples sector for both confidence levels.

For the cumulative distance between R and VaR, when $R < VaR$, the sector results are similar for the 99% confidence level. Consumer staples perform best, with almost half of the value compared to energy and financials. Health care and information technology perform approximately equal with 0.50 and 0.53, respectively. For the 95 % confidence level, consumer staples is lowest followed by; health care, information technology, financials, and energy.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	7	4	9	6	5	8
	Percentage of times exceeding the 99% VaR:	1.39 %	0.80 %	1.79 %	1.19 %	0.99 %	1.59 %
	Number of times exceeding the 95% VaR:	21	15	20	20	20	25
	Percentage of times exceeding the 95% VaR:	4.17 %	2.98 %	3.98 %	3.98 %	3.98 %	4.97 %
2	Number of times exceeding the 99% VaR:	6	12	6	5	7	9
	Percentage of times exceeding the 99% VaR:	0.80 %	1.60 %	0.80 %	0.67 %	0.93 %	1.20 %
	Number of times exceeding the 95% VaR:	23	59	26	25	25	19
	Percentage of times exceeding the 95% VaR:	3.07 %	7.88 %	3.47 %	3.34 %	3.34 %	2.54 %
3	Number of times exceeding the 99% VaR:	23	17	15	15	26	19
	Percentage of times exceeding the 99% VaR:	3.04 %	2.25 %	1.98 %	1.98 %	3.44 %	2.51 %
	Number of times exceeding the 95% VaR:	50	38	45	46	64	48
	Percentage of times exceeding the 95% VaR:	6.61 %	5.03 %	5.96 %	6.08 %	8.47 %	6.35 %
4	Number of times exceeding the 99% VaR:	6	8	10	9	6	11
	Percentage of times exceeding the 99% VaR:	0.60 %	0.80 %	0.99 %	0.89 %	0.60 %	1.09 %
	Number of times exceeding the 95% VaR:	35	37	44	43	28	46
	Percentage of times exceeding the 95% VaR:	3.48 %	3.68 %	4.37 %	4.27 %	2.78 %	4.57 %
5	Number of times exceeding the 99% VaR:	8	12	10	14	14	13
	Percentage of times exceeding the 99% VaR:	0.79 %	1.19 %	0.99 %	1.39 %	1.39 %	1.29 %
	Number of times exceeding the 95% VaR:	42	56	50	46	48	55
	Percentage of times exceeding the 95% VaR:	4.17 %	5.56 %	4.97 %	4.57 %	4.77 %	5.46 %
6	Number of times exceeding the 99% VaR:	9	9	9	12	7	7
	Percentage of times exceeding the 99% VaR:	3.25 %	3.25 %	2.89 %	4.33 %	2.53 %	2.53 %
	Number of times exceeding the 95% VaR:	31	35	29	26	23	21
	Percentage of times exceeding the 95% VaR:	11.19 %	12.64 %	10.47 %	9.39 %	8.30 %	7.58 %

Table 13: S&P 500: DHV Model 1 sectors

Table 13 shows the results of DHV Model 1 for the different time periods, for the index and sectors. Period 3 and 6 is when the percentage of times the returns exceed VaR is the highest for both confidence levels. With the highest result of 4.33%, for the health care sector, for 99% confidence interval in period 6. And 12.46%, for the energy sector, for the 95% confidence interval in period 6. In period 4, DHV Model 1 estimates a too pessimistic VaR, with lowest results for the index and financials sector of 0.60% and 2.78%, for 99% and 95% confidence level respectively.

5.3.1.2 Oslo Stock Exchange

Even though VIX is stock market expectations implied by S&P 500, it is also used to scale the window size for the calculations performed on Oslo Stock Exchange. Optimally a similar volatility index implied by Oslo Stock Exchange should have been used for calculations performed, but within the scope of this thesis, VIX is treated as a global macroeconomic variable containing information about the international status quo. Table 14 shows the results for the index with $avg = 15$. Comparing these results from the model with fixed window sizes, the number of times the returns exceed the 99% VaR is 67, which is comparable to a window size close to 250 days. The same argument applies to the percentage of times the returns exceed the estimated 99% VaR. 95% VaR performed similarly to a window size between 500 and 750 regarding the number and the percentage of times $R < VaR$.

DHV Model 1	
Number of times exceeding the 99% VaR:	67
Number of times exceeding the 95% VaR:	224
Percentage of times exceeding the 99% VaR:	1.61 %
Percentage of times exceeding the 95% VaR:	5.39 %
Sum of difference between historic 99% VaR and Observation:	140.94
Sum of difference between historic 95% VaR and Observation:	83.42
Sum of difference when R <99%VaR:	0.66
Sum of difference when R <95%VaR:	2.07

Table 14: OSEBX: Output for DHV Model 1

The cumulative distance between VaR and returns performed close to the same as a fixed window size of 250 days for both 99% and 95% VaR, with a result of 140.94 and 83.42, respectively. The distance between VaR and R, when $R < VaR$, performed for 99% VaR, approximately as good as the best result from the previous VaR model, with a fixed window size of 750 days. For 95% VaR, this model performed better than any of the fixed window sizes in the previous model.

		OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99%	1.70 %	1.46 %	1.66 %	1.44 %	1.23 %
	95%	5.68%	5.47%	5.37%	4.94 %	4.99 %
Sum of diff between VaR and observation	99%	156.97	134.87	163.15	155.46	180.04
	95%	95.85	79.95	87.77	91.04	99.28
Sum of diff when R exceeds VaR	99%	0.74	0.69	1.06	0.68	1.12
	95%	2.25	2.22	2.86	2.1	3.15

Table 15: Oslo Stock Exchange Sectors: Output for DHV Model 1

The results for the sectors is presented in table 15. The sector that estimates the least optimistic 99% VaR is information technology, with a result of 1.23%. The sector with the most optimistic estimate is the energy sector, with a result of 1.70%. For the 95% confidence interval, both financials and information technology are relatively precise, with the results 4.94% and 4.99%, respectively. Thus, the rest of the sectors calculate too optimistic VaR estimates, with values 5.68%, 5.47%, and 5.37% for energy, consumer staples, and health care, respectively.

For the 99% confidence level, the cumulative distance between R and VaR ranges from 134.87 to 180.04, and for the 95% confidence level, the range is from 79.95 to 99.28. For both confidence levels, it is the consumer staples which performs best and the information technology that performs the worst with regards to the ability to follow the R.

For the cumulative distance between R and VaR, when $R < VaR$, at 99% confidence level, financials has the lowest value again with 0.68. It is followed by consumer staples, energy, health care, and information technology with 0.69, 0.74, 1.06, and 1.12, respectively. For the 95% confidence level, it follows the same pattern with the values; 2.10, 2.22, 2.25, 2.86, and 3.15, respectively.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	5	3	5	5	4	5
	Percentage of times exceeding the 99% VaR:	1.06 %	0.64 %	1.06 %	1.06 %	0.85 %	1.06 %
	Number of times exceeding the 95% VaR:	19	14	19	25	21	16
	Percentage of times exceeding the 95% VaR:	4.03 %	2.97 %	4.03 %	5.31 %	4.46 %	3.40 %
2	Number of times exceeding the 99% VaR:	13	15	15	12	12	7
	Percentage of times exceeding the 99% VaR:	1.76 %	2.03 %	2.03 %	1.63 %	1.63 %	0.95 %
	Number of times exceeding the 95% VaR:	45	56	54	36	37	33
	Percentage of times exceeding the 95% VaR:	6.10 %	7.59 %	7.32 %	4.88 %	5.01 %	4.47 %
3	Number of times exceeding the 99% VaR:	16	16	11	15	15	9
	Percentage of times exceeding the 99% VaR:	2.18 %	2.18 %	1.50 %	2.04 %	2.04 %	1.22 %
	Number of times exceeding the 95% VaR:	44	46	31	46	52	31
	Percentage of times exceeding the 95% VaR:	5.99 %	6.26 %	4.22 %	6.26 %	7.07 %	4.22 %
4	Number of times exceeding the 99% VaR:	6	9	10	11	8	9
	Percentage of times exceeding the 99% VaR:	0.61 %	0.92 %	1.02 %	1.12 %	0.82 %	0.92 %
	Number of times exceeding the 95% VaR:	35	37	46	41	28	54
	Percentage of times exceeding the 95% VaR:	3.58 %	3.78 %	4.70 %	4.19 %	2.86 %	5.52 %
5	Number of times exceeding the 99% VaR:	16	20	14	16	12	14
	Percentage of times exceeding the 99% VaR:	1.63 %	2.04 %	1.43 %	1.63 %	1.22 %	1.43 %
	Number of times exceeding the 95% VaR:	54	57	62	53	49	48
	Percentage of times exceeding the 95% VaR:	5.50 %	5.81 %	6.32 %	5.40 %	4.99 %	4.89 %
6	Number of times exceeding the 99% VaR:	10	8	6	9	8	7
	Percentage of times exceeding the 99% VaR:	3.73 %	2.99 %	2.24 %	3.36 %	2.99 %	2.26 %
	Number of times exceeding the 95% VaR:	25	25	16	23	17	26
	Percentage of times exceeding the 95% VaR:	9.33 %	9.33 %	5.97 %	8.58 %	6.34 %	9.70 %

Table 16: Oslo stock exchange: DHV Model 1 sectors

Table 16 shows that the most challenging periods are period 3 and 6. The most optimistic 99% VaR estimates are calculated in period 6 for Oslo Stock Exchange, where the returns exceed VaR 3.73% of the times. For 95% the highest percentage of times $R < VaR$ is 9.70% in period 6 for information technology. The lowest result is found in period 4 for both 99% VaR and 95%, where the model estimates a too pessimistic VaR estimate and the R exceed

VaR 0.61% of the times for 99% VaR for Oslo stock exchange, and 2.86% for 95% VaR for the financials sector.

5.3.2 DHV Model 2: Percentage Change

The second version of the model bases itself on the behavior of the daily percentage change of the VIX trade close, shown in figure 22 for the S&P 500 index. The purpose of this model is to see if taking into account the daily change in VIX could improve the historical VaR calculations. Scaling the window size within the same interval as DHV Model 1, $[22, 500]$ days. The assumption that the relationship between window size and percentage change in VIX is exponential is made. The change ranging within the interval $[0, 0.768]$, resulting in the equation:

$$Window\ size = 500 * 0.0171^{|\%VIX\ trade\ close|} \tag{14}$$

In equation 14 the absolute value of the change in VIX is the scaling parameter of the window size. The absolute value of the change is used to scale the window size, due to when a volatile market is present the equation only needs to capture that there are large fluctuations and do not need to differentiate between if the fluctuation is positive or negative.

5.3.2.1 S&P 500

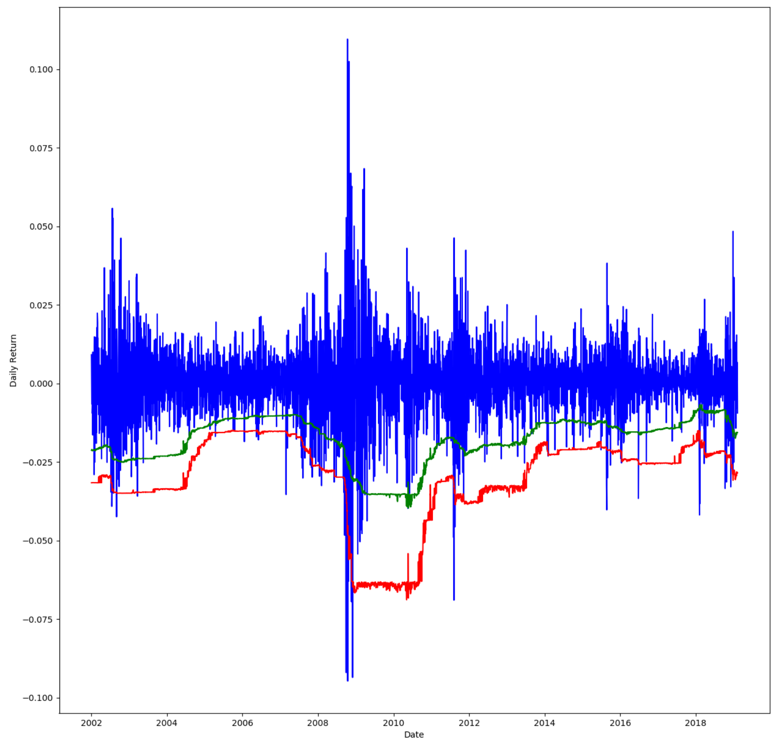


Figure 22: SPX: DHV Model 2. Green line: 95% VaR. Red line: 99% VaR

From table 17 it is shown that DHV Model 2 has a higher number of times $R < VaR$ for 99% VaR for the S&P 500 index, compared to the DHV Model 1. Thus, the percentage of times $R < VaR$ also being higher. The percentage of times $R < VaR$ is 1.63% for the 99% confidence level, meaning that the model generally estimates a too optimistic VaR. For 95% VaR the percentage of times where $R < VaR$ is also higher than DHV Model 1, but instead of laying under the five percent mark, the model is above with a result of 5.42%. Despite having higher measured metrics, the 99% VaR still lay close to the same window size, 250 days, when comparing to 5.2. The 95% VaR also performs somewhat similar to a window size between 250 and 500 days. The main difference between moving proportionally with the VIX trade close and moving with respect to the daily change is that the estimated VaR, for both 99% and 95% VaR, generally lays closer to the returns. The 99% VaR performing similar to a window size between 44 and 250 days and 95% VaR yielding results even with all of the window sizes tested in the historical VaR with rolling window. When $R < VaR$, the cumulative distance between R and VaR, 99% VaR performs the same as a window size of 750 days. 95% VaR performs better than the most of the window sizes, only a window size of 44 days gives better results, when considering the distance between the returns and VaR, when $R < VaR$.

DHV Model 2	
Number of times exceeding the 99% VaR:	70
Number of times exceeding the 95% VaR:	233
Percentage of times exceeding the 99% VaR:	1.63 %
Percentage of times exceeding the 95% VaR:	5.42 %
Sum of difference between historic 99% VaR and Observation:	114.78
Sum of difference between historic 95% VaR and Observation:	65.69
Sum of difference when R <99%VaR:	0.66
Sum of difference when R <95%VaR:	1.89

Table 17: SPX: Output for DHV Model 2

For DHV Model 2, shown in figure 18, the 99% VaR estimations are optimistic, with a range from the highest percentage of times where $R < VaR$ with a result of 1.70% for the financials sector, to the lowest result of 1.33% for consumer staples. For the 95% confidence level, the results are close to the five percent mark, with the best sector being health care with a result of 5.21%. The most optimistic estimations are observed for the financials sector, with 3.40% of times $R < VaR$ for the 99% confidence level, and the energy sector with a result of 5.86% of times $R < VaR$ for the 95% confidence level.

		SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99%	1.61 %	1.33 %	1.54 %	1.70%	1.58 %
	95%	5.86 %	5.63 %	5.21 %	5.72 %	5.38 %
Sum of diff between VaR and observation	99%	168.19	82.71	111.27	168.89	128.06
	95%	90.71	47.64	61.2	94.34	74.26
Sum of diff when R exceeds VaR	99%	1.03	0.43	0.59	1.1	0.63
	95%	3.1	1.44	1.79	3.32	1.9

Table 18: S&P 500 Sectors: Output for DHV Model 2

On the other hand, DHV Model 2 has beneficial traits considering the cumulative distance between R and VaR, especially for the 95% confidence level. The distance is low, notably for the consumer staples sector with a sum of 47.64. At the other end of the scale, financials and energy yield distances of 168.89 and 168.19 for the 99% confidence level.

For the sum of the distance between R and VaR when R exceeds VaR, the consumer staples perform best followed by health care, information technology, energy, and financials, both confidence levels.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	6	5	8	8	3	7
	Percentage of times exceeding the 99% VaR:	1.19 %	0.99 %	1.59 %	1.59 %	0.60 %	1.39 %
	Number of times exceeding the 95% VaR:	23	15	23	24	22	25
	Percentage of times exceeding the 95% VaR:	4.57 %	2.98 %	4.57 %	4.77 %	4.37 %	4.97 %
2	Number of times exceeding the 99% VaR:	7	12	4	4	5	6
	Percentage of times exceeding the 99% VaR:	0.93 %	1.60 %	0.53 %	0.53 %	0.67 %	0.80 %
	Number of times exceeding the 95% VaR:	17	52	24	19	19	17
	Percentage of times exceeding the 95% VaR:	2.27 %	6.94 %	3.20 %	2.53 %	2.54 %	2.27 %
3	Number of times exceeding the 99% VaR:	31	24	21	23	35	24
	Percentage of times exceeding the 99% VaR:	4.10 %	3.17 %	2.78 %	3.04 %	4.63 %	3.17 %
	Number of times exceeding the 95% VaR:	81	54	73	68	103	74
	Percentage of times exceeding the 95% VaR:	10.71 %	7.14 %	9.66 %	8.99 %	13.62 %	9.79 %
4	Number of times exceeding the 99% VaR:	5	7	5	5	6	8
	Percentage of times exceeding the 99% VaR:	0.50 %	0.69 %	0.50 %	0.50 %	0.60 %	0.80 %
	Number of times exceeding the 95% VaR:	31	28	40	39	26	38
	Percentage of times exceeding the 95% VaR:	3.08 %	2.78 %	3.98 %	3.88 %	2.58 %	3.78 %
5	Number of times exceeding the 99% VaR:	12	12	12	16	16	14
	Percentage of times exceeding the 99% VaR:	1.19 %	1.19 %	1.19 %	1.59 %	1.59 %	1.39 %
	Number of times exceeding the 95% VaR:	59	62	54	46	49	56
	Percentage of times exceeding the 95% VaR:	4.97 %	6.16 %	5.36 %	4.57 %	4.87 %	5.56 %
6	Number of times exceeding the 99% VaR:	8	9	9	10	8	8
	Percentage of times exceeding the 99% VaR:	2.89 %	3.25 %	3.25 %	3.61 %	2.89 %	2.89 %
	Number of times exceeding the 95% VaR:	33	38	29	29	26	22
	Percentage of times exceeding the 95% VaR:	11.01 %	13.72 %	10.47 %	10.47 %	9.39 %	7.94 %

Table 19: S&P 500: DHV Model 1 Sectors

Table 19 show the results for the S&P 500 index in the different time periods, and the most challenging periods to estimate VaR is period 3 and 6. Where the highest percentage of times the returns exceed the 99% VaR is 4.63% for the financials sector in period 3. The highest percentage of times where the returns exceed the 95% VaR is 13.72% in period 6, for the energy sector. For period 3 and 6, DHV Model 2 estimate a too optimistic VaR. In period 2 and 4 the VaR model obtains the lowest values. The lowest values for 99% VaR is found in period 4, with a result of 0.50% for the S&P 500 index, consumer staples and health care sector. The lowest value for 95% VaR is 2.27%, for the S&P 500 index in period 2. In these periods DHV Model 2 pessimistic VaR estimates are obtained.

5.3.2.2 Oslo Stock Exchange

Also for the Oslo Stock Exchange index, the two first metrics measured are higher for DHV Model 2 when considering 95% VaR, than for DHV Model 1, shown in table 20 below. However, the results for 99% VaR are lower, when considering the number and percentage of times $R < VaR$. With 99% VaR giving similar results as a fixed window size between 250 and 500 days. While 95% VaR gives similar results to a window size of 250. Comparing the results of how close the VaR estimations lays to the returns, the result for 99% is similar to a window size of 250 days and for 95% the performance is even with the majority of the window sizes. For the sum of distance when $R < VaR$, are higher for both 99% VaR and 95% VaR compared to DHV Model 1.

DHV Model 2	
Number of times exceeding the 99% VaR:	76
Number of times exceeding the 95% VaR:	253
Percentage of times exceeding the 99% VaR:	1.54 %
Percentage of times exceeding the 95% VaR:	5.80 %
Sum of difference between historic 99% VaR and Observation:	145.3
Sum of difference between historic 95% VaR and Observation:	78.22
Sum of difference when R <99%VaR:	0.79
Sum of difference when R <95%VaR:	2.83

Table 20: OSEBX: Output for DHV Model 2

99% VaR for the Oslo Stock Exchange sectors perform relatively close to the desired 1% mark. The most optimistic result being 1.66% for the energy sector and the closest value being 1.08% for information technology. The same two sectors have the extremities for the 95% VaR calculations, with the results 5.56% and 4.82%, respectively.

The information technology sector has the highest cumulative distance between the return and calculated VaR, while consumer staples has the lowest, for both confidence levels.

		OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99%	1.66 %	1.30 %	1.46 %	1.51 %	1.08 %
	95%	5.56 %	5.27 %	5.56 %	5.51 %	4.82 %
Sum of diff between VaR and observation	99%	166.51	139.94	165.03	161.93	181.65
	95%	91.08	79.41	84.79	84.39	98.17
Sum of diff when R exceeds VaR	99%	0.88	0.6	1.06	0.93	1.08
	95%	2.68	2.10	3.37	2.97	3.17

Table 21: Oslo Stock Exchange sectors: Output for DHV Model 2

For the cumulative distance when the return exceeds VaR, the sector performing closest to the returns is consumer staples for both confidence levels, with a result of 0.6 and 2.10 for the 99% and 95% confidence level, respectively. On the other end of the scale, information technology has the greatest distance for 99% confidence level, with a result of 1.08. Moreover, for the 95% confidence level health care has the highest result, with 3.37.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	7	3	3	3	5	2
	Percentage of times exceeding the 99% VaR:	1.49 %	0.64 %	0.64 %	0.64 %	1.06 %	0.42 %
	Number of times exceeding the 95% VaR:	22	9	21	22	32	15
	Percentage of times exceeding the 95% VaR:	4.67 %	1.91 %	4.46 %	4.67 %	6.79 %	2.97 %
2	Number of times exceeding the 99% VaR:	11	13	14	12	11	8
	Percentage of times exceeding the 99% VaR:	1.49 %	1.76 %	1.90 %	1.63 %	1.49 %	1.08 %
	Number of times exceeding the 95% VaR:	44	55	50	33	36	31
	Percentage of times exceeding the 95% VaR:	5.96 %	7.45 %	6.78 %	4.47 %	4.88 %	4.20 %
3	Number of times exceeding the 99% VaR:	14	17	12	15	21	8
	Percentage of times exceeding the 99% VaR:	1.90 %	2.31 %	1.63 %	2.04 %	2.86 %	1.09 %
	Number of times exceeding the 95% VaR:	63	49	36	56	72	38
	Percentage of times exceeding the 95% VaR:	8.57 %	6.67 %	4.90 %	7.76 %	9.80 %	5.17 %
4	Number of times exceeding the 99% VaR:	4	5	8	6	7	6
	Percentage of times exceeding the 99% VaR:	0.41 %	0.51 %	0.82 %	0.61 %	0.72 %	0.61 %
	Number of times exceeding the 95% VaR:	27	30	38	39	29	43
	Percentage of times exceeding the 95% VaR:	2.76 %	3.06 %	3.88 %	3.98 %	2.96 %	4.39 %
5	Number of times exceeding the 99% VaR:	19	23	12	15	12	12
	Percentage of times exceeding the 99% VaR:	1.19 %	2.34 %	1.22 %	1.53 %	1.22 %	1.22 %
	Number of times exceeding the 95% VaR:	58	65	59	57	46	48
	Percentage of times exceeding the 95% VaR:	2.76 %	6.63 %	6.01 %	5.81 %	5.69 %	4.89 %
6	Number of times exceeding the 99% VaR:	9	7	6	10	7	7
	Percentage of times exceeding the 99% VaR:	3.36 %	2.61 %	2.24 %	3.73 %	2.61 %	2.61 %
	Number of times exceeding the 95% VaR:	23	25	16	24	16	26
	Percentage of times exceeding the 95% VaR:	8.58 %	9.33 %	5.97 %	8.96 %	5.97 %	9.70 %

Table 22: Oslo Stock Exchange periods: DHV Model 2

Table 22 shows the results for the different time periods for the Oslo Stock Exchange's index and sectors. The DHV Model 2 struggles the most with too optimistic VaR in period 3 and 6. The highest value being 3.73% where $R < VaR$ in period 6 for the health care sector, for the 99% confidence level. The highest value obtained for the 95% confidence level is 9.80% in period 3, also from the health care sector. The time periods 1 and 4 gives the lowest percentage of times where the returns exceed VaR and are the periods where DHV Model 2 estimate a too pessimistic VaR. The lowest percentage for 99% VaR being 0.41% in period 4 from Oslo stock exchange index and for 95% VaR the lowest percentage is 1.91% in period 1 from the energy sector.

5.3.3 DHV Model 3: Proportional and Percentage Change

A combination of DHV Model 1 and DHV Model 2, shown in figure 23, was investigated to see how taking into accord both the proportional and percentage change in VIX affected VaR calculations. The idea behind combining the two models comes from a PD-controller, which is widely used in industrial control systems. That considering not only the change but also the magnitude of the change will lead to better VaR calculations. The model is set up such that for the daily calculations, the two models are equally weighted and then accumulated, expressed by equation:

$$Window\ size = \frac{740 * 0.957^{VIX\ trade\ close} + 500 * 0.0171^{|\%VIX\ trade\ close|}}{2} \tag{15}$$

and shown in table 23 below. Not surprisingly, most of the measured metrics lay somewhere in between the results from the two previous models. The 99% VaR still struggles with too optimistic VaR estimations and to perform close to the 1% mark, but the model performs quite precise when calculating 95% VaR.

5.3.3.1 S&P 500

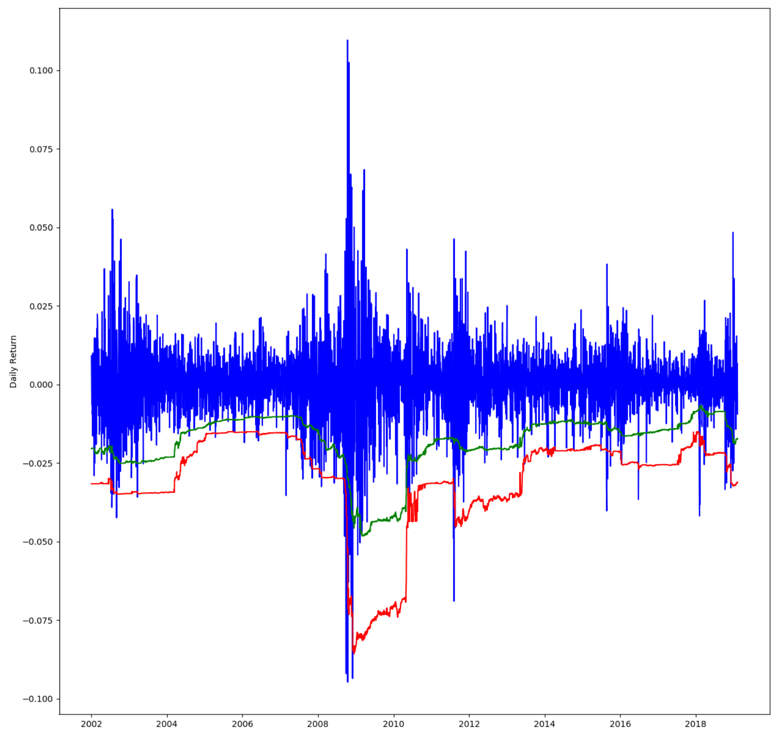


Figure 23: SPX: DHV Model 3. Green line: 95% VaR. Red line: 99% VaR

For the S&P 500 index, the percentage of times the returns exceed the VaR is 1.52% for 99% VaR and 5.42 % for 95% VaR. From table 23, we see that most of the results approximately lay in the middle of the results from the two previous models, DHV Model 1 and 2, for the index. Except for the sum of the distance from the returns and VaR, when $R < VaR$, for 95% VaR, which has a greater distance than both of the previous models.

DHV Model 3	
Number of times exceeding the 99% VaR:	67
Number of times exceeding the 95% VaR:	221
Percentage of times exceeding the 99% VaR:	1.52 %
Percentage of times exceeding the 95% VaR:	5.42 %
Sum of difference between historic 99% VaR and Observation:	116.09
Sum of difference between historic 95% VaR and Observation:	67.06
Sum of difference when R <99%VaR:	0.57
Sum of difference when R <95%VaR:	2.02

Table 23: SPX: Output for DHV Model 3

For DHV Model 3 the sectors returns exceed VaR with the following percentage; energy 1.47% and 5.72%, consumer staples 1.49% and 5.40%, health care 1.42% and 5.12%, financials 1.58% and 5.45%, and information technology 1.52% and 5.24%, for the 99% and 95% confidence level.

For the cumulative distance between returns and VaR, the results for the sectors also mostly give results in between the two previous models. The lowest distance is obtained for the consumer staples sector with 81.22 for the 99% confidence level and 47.26 for the 95% confidence level. On the other end, the highest distance obtained is from the energy sector with a value of 164.15 for the 99% confidence level. The highest value for the 95% confidence level is from the financials sector, with a result of 95.61.

		SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99%	1.47 %	1.49 %	1.42 %	1.58 %	1.52 %
	95%	5.72 %	5.40 %	5.12 %	5.45 %	5.24 %
Sum of diff between VaR and observation	99%	164.15	81.22	111.38	160.5	125.44
	95%	91.59	47.26	62.4	95.61	73.59
Sum of diff when R exceeds VaR	99%	0.9	0.44	0.51	0.97	0.58
	95%	2.91	1.34	1.68	2.69	1.79

Table 24: S$P 500 sectors: Output for DHV Model 3l change

For the cumulative distance between returns and calculated VaR when the return exceeds the calculated VaR, the lowest values still belong to the consumer staples sector. Consumer staples has a result of 0.44 for the 99% confidence level and 1.34 for the 95% confidence level. The highest value is 0.97 for the 99% confidence level, which is from the financials sector. The highest value for the 95% confidence level is from the energy sector, with a result of 2.91.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	6	4	10	9	6	7
	Percentage of times exceeding the 99% VaR:	1.19 %	0.80 %	1.99 %	1.79 %	1.19 %	1.39 %
	Number of times exceeding the 95% VaR:	24	13	25	24	23	26
	Percentage of times exceeding the 95% VaR:	4.77 %	2.58 %	4.97 %	5.77 %	4.57 %	5.17 %
2	Number of times exceeding the 99% VaR:	6	12	4	3	5	6
	Percentage of times exceeding the 99% VaR:	0.80 %	1.60 %	0.53 %	0.40 %	0.67 %	0.80 %
	Number of times exceeding the 95% VaR:	20	53	26	21	23	18
	Percentage of times exceeding the 95% VaR:	2.67 %	7.08 %	3.34 %	2.80 %	3.07 %	2.40 %
3	Number of times exceeding the 99% VaR:	26	21	18	18	30	20
	Percentage of times exceeding the 99% VaR:	0.44 %	2.78 %	0.99 %	2.38 %	3.97 %	2.65 %
	Number of times exceeding the 95% VaR:	69	48	59	57	84	62
	Percentage of times exceeding the 95% VaR:	9.13 %	6.35 %	7.80 %	7.54 %	11.11 %	8.20 %
4	Number of times exceeding the 99% VaR:	5	5	10	6	6	10
	Percentage of times exceeding the 99% VaR:	0.50 %	0.50 %	0.99 %	0.60 %	0.60 %	0.99 %
	Number of times exceeding the 95% VaR:	38	34	43	44	30	43
	Percentage of times exceeding the 95% VaR:	3.78 %	3.38 %	4.27 %	4.37 %	2.98 %	4.27 %
5	Number of times exceeding the 99% VaR:	13	11	11	16	13	14
	Percentage of times exceeding the 99% VaR:	1.29 %	1.09 %	1.09 %	1.59 %	1.29 %	1.39 %
	Number of times exceeding the 95% VaR:	48	61	51	49	48	56
	Percentage of times exceeding the 95% VaR:	4.77 %	6.06 %	5.06 %	4.87 %	4.77 %	5.56 %
6	Number of times exceeding the 99% VaR:	9	8	9	9	7	7
	Percentage of times exceeding the 99% VaR:	3.25 %	2.89 %	2.89 %	3.25 %	2.53 %	2.53 %
	Number of times exceeding the 95% VaR:	31	35	29	26	26	20
	Percentage of times exceeding the 95% VaR:	11.19 %	12.64 %	10.47 %	9.39 %	9.39 %	7.22 %

Table 25: S&P 500 periods: DHV Model 3

In table 25 the results from the different time periods are presented. Period 3 and 6 are the periods where calculating VaR for both confidence level are the most challenging and where VaR estimations are too optimistic, and consequently where the percentage of times the returns exceed VaR are the highest. The highest percentage of where $R < VaR$ for the 99% confidence level is 3.97% from the financials sector in period 3. For the 95% confidence level, the highest percent appears in period 6 for the S&P 500 index with a result of 11.19%. In period 2 and 4, DHV Model 3 estimates too pessimistic VaR estimates, and the lowest percentage of times $R < VaR$ appears. With the lowest value for 99% VaR being 0.40% and for 95% VaR being 2.40%, for the health care and information technology sector, respectively.

5.3.3.2 Oslo Stock Exchange

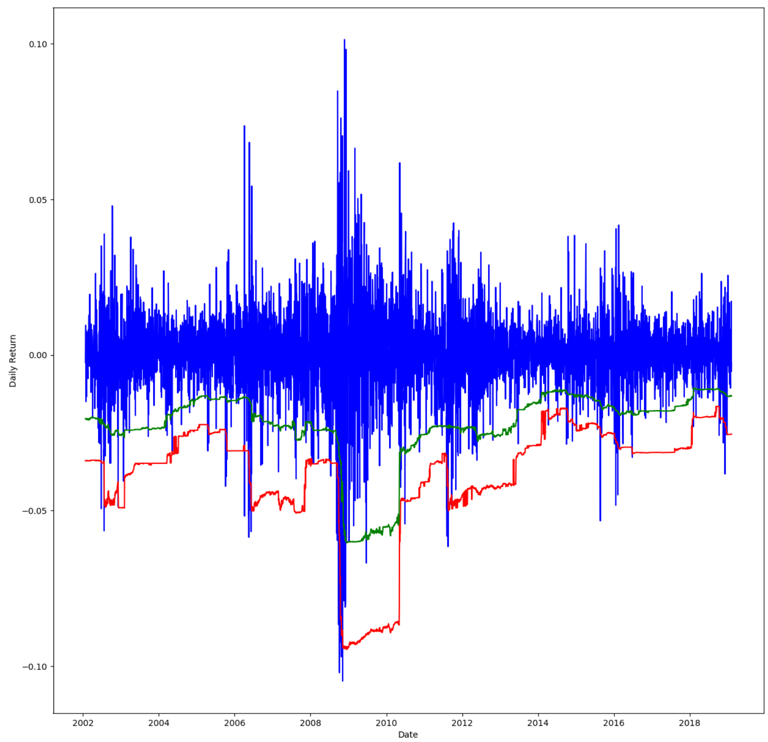


Figure 24: Oslo Stock Exchange: DHV Model 3. Green line: 95% VaR. Red line: 99% VaR

For DHV Model 3, both the 99% and 95% VaR performs fairly close to the 1% and 5% mark for the index. When combining DHV Model 1 and 2 the results, shown in the table 26 below, lay mostly in between the results from the latter models.

DHV Model 3	
Number of times exceeding the 99% VaR:	71
Number of times exceeding the 95% VaR:	232
Percentage of times exceeding the 99% VaR:	1.52 %
Percentage of times exceeding the 95% VaR:	5.68 %
Sum of difference between historic 99% VaR and Observation:	142.81
Sum of difference between historic 95% VaR and Observation:	80.15
Sum of difference when R <99%VaR:	0.70
Sum of difference when R <95%VaR:	2.54

Table 26: OSEBX: Output for DHV Model 3

For the Oslo Stock Exchange sectors, the percentage of times the returns exceed the 99% VaR range from 1.20% to 1.66%, for information technology and financials respectively. For 95% VaR, the results range from the information technology sector with 5.06% to 5.75% for the health care sector. The sector that performs best with regards to the VaR estimations laying close to the returns is consumer staples with a result of 136.56 and 79.36, for 99% and 95% confidence level, respectively. The sector that has the greatest distance between R and VaR is information technology, with the results 178.44 and 96.68 for 99% and 95% confidence level, respectively.

		OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99%	3.31 %	2.59 %	3.21 %	3.31 %	2.40 %
	95%	5.63 %	5.44 %	5.75 %	5.44%	5.06 %
Sum of diff between VaR and observation	99%	159.95	136.56	162.22	154.95	178.44
	95%	93.98	79.36	83.95	85.08	96.68
Sum of diff when R exceeds VaR	99%	0.80	0.60	1.12	0.82	1.18
	95%	2.36	2.29	3.46	2.68	3.24

Table 27: Oslo Stock Exchange Sectors: Output for DHV Model 3

The VaR calculations for consumer staples also lay closest to the returns, when $R < VaR$, for both confidence levels with the results 0.6 and 2.29 for 99% and 95% confidence level, respectively. The sector that has the greatest distance between VaR and R, when $R < VaR$, is information technology with a result of 1.18 for 99% confidence level. For 95% confidence level, the health care sector has the greatest distance between VaR and R when $R < VaR$, with a result of 3.46.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	4	2	3	4	4	6
	Percentage of times exceeding the 99% VaR:	0.85 %	0.42 %	0.64 %	0.85 %	0.85 %	1.27 %
	Number of times exceeding the 95% VaR:	22	11	22	26	29	18
	Percentage of times exceeding the 95% VaR:	4.67 %	2.34 %	4.67 %	5.52 %	6.16 %	3.82 %
2	Number of times exceeding the 99% VaR:	11	14	15	13	11	8
	Percentage of times exceeding the 99% VaR:	1.49 %	1.90 %	2.03 %	1.76 %	1.49 %	1.08 %
	Number of times exceeding the 95% VaR:	44	55	51	33	37	31
	Percentage of times exceeding the 95% VaR:	5.96 %	7.45 %	6.91 %	4.47 %	5.01 %	4.20 %
3	Number of times exceeding the 99% VaR:	15	17	10	16	23	9
	Percentage of times exceeding the 99% VaR:	2.04 %	2.31 %	1.36 %	2.18 %	3.13 %	1.22 %
	Number of times exceeding the 95% VaR:	53	48	35	52	61	34
	Percentage of times exceeding the 95% VaR:	7.21 %	6.53 %	4.76 %	7.07 %	8.30 %	4.63 %
4	Number of times exceeding the 99% VaR:	5	6	10	9	9	10
	Percentage of times exceeding the 99% VaR:	0.51 %	0.61 %	1.02 %	0.92 %	0.92 %	1.02 %
	Number of times exceeding the 95% VaR:	35	37	47	46	33	51
	Percentage of times exceeding the 95% VaR:	3.47 %	3.78 %	4.80 %	4.70 %	3.37 %	5.21 %
5	Number of times exceeding the 99% VaR:	17	22	14	16	12	13
	Percentage of times exceeding the 99% VaR:	1.73 %	2.24 %	1.12 %	1.63 %	1.22 %	1.33 %
	Number of times exceeding the 95% VaR:	57	60	59	56	47	50
	Percentage of times exceeding the 95% VaR:	5.81 %	6.12 %	6.01 %	5.71 %	4.79 %	5.10 %
6	Number of times exceeding the 99% VaR:	9	7	6	9	9	7
	Percentage of times exceeding the 99% VaR:	3.36 %	2.61 %	2.24 %	3.36 %	3.36 %	2.26 %
	Number of times exceeding the 95% VaR:	25	26	16	24	16	27
	Percentage of times exceeding the 95% VaR:	9.33 %	9.70 %	5.97 %	8.96 %	5.97 %	10.08 %

Table 28: Oslo Stock Exchange periods: DHV Model 3

From the results for the different time periods, shown in table 28, the periods where the models struggle the most with optimistic VaR estimates and percentage of times where the returns exceed VaR, is period 3 and 6. These periods are where the highest obtained values for when the returns exceed 99% VaR is 3.36%, in period 6 for the Oslo stock exchange index, health care and financials sector. The highest obtained value for when the returns exceed 95% VaR is 10.08%, in period 6 from the information technology sector. Period 1 and 4 are the periods where the model calculates the most pessimistic VaR estimations and obtain the lowest percentage of times $R < VaR$. With the lowest values being 0.42% and 2.34% from the energy sector, for 99% and 95% confidence interval respectively.

5.4 Monte Carlo simulation

5.4.1 MC Model 1: Single variable and constant regression coefficient

Obtaining interesting results with using VIX as a scaling factor for historical VaR calculations with rolling window, the same idea that a forward-looking factor could be implemented into a Monte Carlo simulation to improve parametric value at risk calculations was investigated. With the assumption that the returns follow a student t-distribution, the first step in developing this model a relationship between the distribution and VIX had to be established. From the student t-distribution, it is known that low degrees of freedom are linked to fat tails, thus in the context of VaR and VIX, linked to a high potential for loss and volatile periods. To uncover this relationship, a rolling window of 250 days was used across the data set to MLE-estimate the daily student t-parameters; degrees of freedom, standard deviation and mean. And then perform an OLS-regression using the degrees of freedom, denoted ν, as the dependent variable and VIX closing price as the independent variable, shown in figure 25 below. The regression model used to capture the percentage relationship is a log-log model, shown in equation 16.

$$log(\nu) = \beta_0 + \beta_1 * log(VIX) \tag{16}$$

In similarity to Dynamic Historical VaR, in order to obtain less sudden and rapid changes, an average of the VIX closing price, the last 15 days was used as an independent variable.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     nu   R-squared:                       0.063
Model:                            OLS   Adj. R-squared:                  0.063
Method:                 Least Squares   F-statistic:                     302.5
Date:                Sun, 19 May 2019   Prob (F-statistic):           1.27e-65
Time:                        14:04:11   Log-Likelihood:                -12231.
No. Observations:                4506   AIC:                         2.447e+04
Df Residuals:                    4504   BIC:                         2.448e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const         10.3005      0.435     23.667      0.000       9.447      11.154
VIX15avg      -2.5904      0.149    -17.392      0.000      -2.882      -2.298
==============================================================================
Omnibus:                     2065.860   Durbin-Watson:                   0.093
Prob(Omnibus):                  0.000   Jarque-Bera (JB):             7992.869
Skew:                           2.366   Prob(JB):                         0.00
Kurtosis:                       7.492   Cond. No.                         26.1
==============================================================================
```

Figure 25: SPX: OLS regression results

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     nu   R-squared:                       0.004
Model:                            OLS   Adj. R-squared:                  0.004
Method:                 Least Squares   F-statistic:                     18.72
Date:                Mon, 20 May 2019   Prob (F-statistic):           1.55e-05
Time:                        12:12:51   Log-Likelihood:                -10065.
No. Observations:                4421   AIC:                         2.013e+04
Df Residuals:                    4419   BIC:                         2.015e+04
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          1.0617      0.283      3.755      0.000       0.507       1.616
VIX15avg       0.4188      0.097      4.327      0.000       0.229       0.609
==============================================================================
Omnibus:                     4201.892   Durbin-Watson:                   0.041
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           117129.425
Skew:                           4.846   Prob(JB):                         0.00
Kurtosis:                      26.279   Cond. No.                         26.0
==============================================================================
```

Figure 26: OSEBX: OLS regression results

With the result of β_1 from the regressions, the formula for letting VIX regulate a dynamic degree of freedom was modeled to be

$$\nu_n = \nu_{n-1} + \nu_{n-1} * (\frac{VIX_n}{VIX_{n-1}} - 1) * \beta_1 \tag{17}$$

where n denotes the day the calculations are performed, for the VaR calculations for the next day $n+1$. This degree of freedom, decided by equation 17, is then used to generate a pseudorandom t-distributed variable. For each day n, m independent realizations of these pseudorandom t-distributed variables were generated and appended, and quantiles for 99% VaR and 95% VaR was extracted. The model is also restricted to the interval $[2, 30]$for the degrees of freedom. With the lower bound being set due to Pythons compatibility and being able to handle lower degrees of freedom with its built-in functions, and the higher bound being set to simplify calculations and with the assumptions that higher degrees of freedom than 30 is considered normal distributed. For the Monte Carlo simulation performed, one thousand of these variables were generated per day, i.e., $m = 1000$.

For complete results and graphs, see appendix C

5.4.1.1 S&P 500

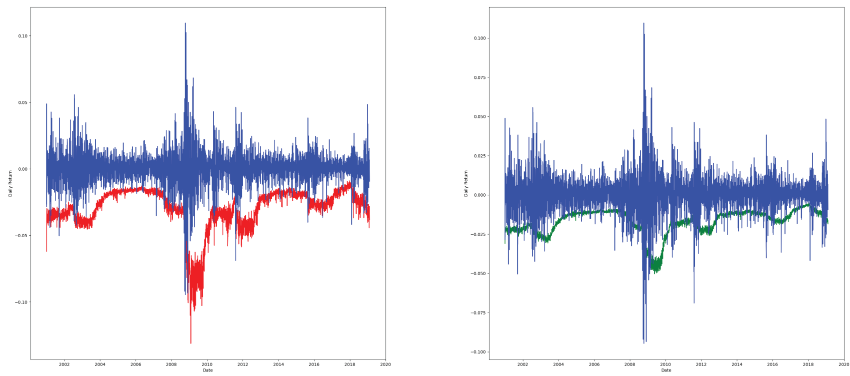


Figure 27: SPX: Graph from Monte Carlo model 1. Red line 99% VaR. Green line 95% VaR

Figure 27 above is a graphical representation of results for the S&P 500 index from the simulation and table 29 below shows a listing of the results. Through the whole period, the model performs close to the desired 1% mark, for 99% VaR, with a result of 1.62% for the index. The sum of distances between the calculated VaR and the returns is approximately 117 and the sum of distances between the calculated VaR and the returns when $R < VaR$ is about 0.62. For 95% VaR, for the index, the percentage of times the returns exceeds the calculated VaR shows a result of 6.02%. The sum of the distance between the VaR and observed value is approximately 64 and the sum of distances between the VaR and observed value when $R < VaR$ is 2.4.

		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99	1.62 %	1.75 %	1.55 %	1.95 %	1.55 %	1.62 %
	95	6.02 %	6.35 %	5.70 %	5.97 %	6.06 %	5.35 %
Sum of diff between VaR and observation	99	117.26	152.61	84.17	107.79	166.57	126.74
	95	64.2	89.33	49.01	61.02	91.86	73.87
Sum of diff when R exceeds VaR	99	0.62	0.99	0.52	0.67	0.85	0.68
	95	2.37	3.06	1.62	2.2	3.35	2.82

Table 29: S&P 500: Results from Monte Carlo model 1

For the different sectors, the 99% VaR range from 1.95% to 1.55% for the health care sector, and consumer staples and financials, respectively. The 95% VaR range from 6.06% to 5.35%, for finance and information technology, respectively. The highest distance between 99% VaR and returns is from the financials sector with a result of 166.57 and the lowest is 84.17 from consumer staples. For the

95% confidence level, the same sectors give the highest and lowest values, with 91.86 and 49.01, respectively. The sum of distance between VaR and R when $R < VaR$ range from 0.99 to 0.52 for the 99% confidence level and from 3.35 to 1.62 for the 95% confidence level.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	7	8	12	14	6	9
	Percentage of times exceeding the 99% VaR:	1.39 %	1.59 %	2.38 %	2.78 %	1.19 %	1.79 %
	Number of times exceeding the 95% VaR:	35	33	31	40	38	36
	Percentage of times exceeding the 95% VaR:	6.94 %	6.55 %	6.15 %	7.94 %	7.54 %	7.14 %
2	Number of times exceeding the 99% VaR:	6	10	6	7	8	6
	Percentage of times exceeding the 99% VaR:	0.81 %	1.35 %	0.81 %	0.94 %	1.08 %	0.81 %
	Number of times exceeding the 95% VaR:	30	53	31	27	30	20
	Percentage of times exceeding the 95% VaR:	4.04 %	7.14 %	4.18 %	3.64 %	4.04 %	2.70 %
3	Number of times exceeding the 99% VaR:	24	25	19	25	23	20
	Percentage of times exceeding the 99% VaR:	3.19 %	3.32 %	2.25 %	3.32 %	3.05 %	2.66 %
	Number of times exceeding the 95% VaR:	69	53	64	64	79	58
	Percentage of times exceeding the 95% VaR:	9.16 %	7.04 %	8.50 %	8.59 %	10.49 %	7.70 %
4	Number of times exceeding the 99% VaR:	10	8	14	16	9	14
	Percentage of times exceeding the 99% VaR:	1.03 %	0.82 %	1.44 %	1.64 %	0.92 %	1.44 %
	Number of times exceeding the 95% VaR:	50	46	52	57	48	47
	Percentage of times exceeding the 95% VaR:	5.13 %	4.72 %	5.34 %	5.85 %	4.93 %	4.83 %
5	Number of times exceeding the 99% VaR:	17	17	10	14	15	17
	Percentage of times exceeding the 99% VaR:	1.69 %	1.69 %	0.99 %	1.30 %	1.49 %	1.69 %
	Number of times exceeding the 95% VaR:	52	65	51	55	55	59
	Percentage of times exceeding the 95% VaR:	5.16 %	6.45 %	5.06 %	5.46 %	5.46 %	5.86 %
6	Number of times exceeding the 99% VaR:	9	11	9	12	9	7
	Percentage of times exceeding the 99% VaR:	3.25 %	3.97 %	3.25 %	4.33 %	3.25 %	2.53 %
	Number of times exceeding the 95% VaR:	35	36	28	26	23	21
	Percentage of times exceeding the 95% VaR:	12.64 %	13.00 %	10.11 %	9.39 %	8.30 %	7.58 %

Table 30: S&P 500: Results from Monte Carlo model 1 - periods

From the different periods, period 3 and 6 are the periods where MC Model1 estimates the most optimistic VaR, with a result of approximately 3.18% and 3.25% times of exceeding VaR respectively for 99% VaR for the index. For 95% VaR, the results were even higher, with a result of 9.16% and 12.64% for the respective periods for the index. For the sectors extremities like 4.33% for 99% VaR in period 6 for the health care sector and 13.00% for 95% VaR for the energy sector in period 6 are observed. The periods represent the time around the financial crisis and 2018's and the increase in volatility at the end of 2018. For the other periods the 99% VaR performs around the 1% mark for most of the calculations, but for 95% VaR there is a bit more spread in the results.

5.4.1.2 Oslo Stock Exchange

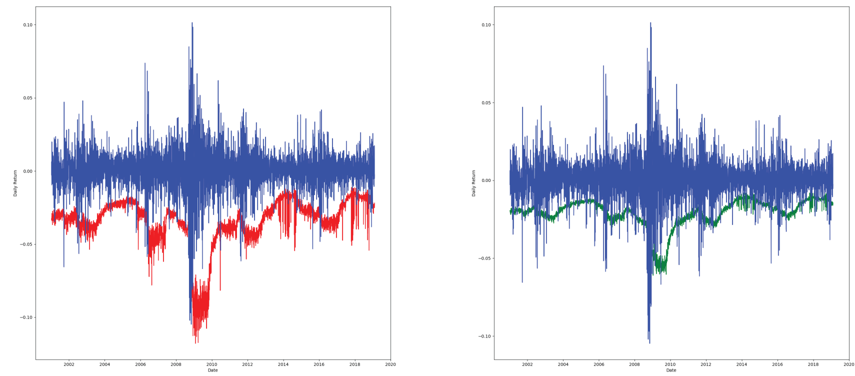


Figure 28: OSEBX: Graph from Monte Carlo model 1. Red line 99% VaR. Green line 95% VaR

From figure 28 and table 31 the results from the Monte Carlo simulations for the Oslo Stock Exchange is shown. For the index, the 99% VaR had a 1.81% times where the returns exceeded the estimated VaR. The sum of distance and distance when $R < VaR$ is approximately 133.68 and 0.95 respectively. The results for the 95% VaR calculations, the percentage of times the returns exceeded VaR was 5.90% for the index. The distances, both the total and for when $R < VaR$, are 77 and 2.85 respectively.

		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99	1.81 %	1.70 %	1.58 %	1.36 %	1.70 %	1.47 %
	95	5.90 %	5.70 %	5.70 %	5.27 %	5.63 %	5.45 %
Sum of diff between VaR and observation	99	132.68	158.96	142.63	178.78	152.10	178.80
	95	77.06	94.74	81.97	93.15	86.83	100.37
Sum of diff when R exceeds VaR	99	0.95	0.88	0.67	1.01	0.92	1.31
	95	2.85	2.89	2.59	3.17	3.03	3.51

Table 31: Oslo Stock Exchange: Results from Monte Carlo model 1

The sectors show similar results as the index, with 99% VaR ranging from 1.70% for the energy sector to 1.36% for the health care sector, 95% VaR range from 5.70% to 5.27% for the same sectors, respectively. The sum of distances between VaR and R vary from 178.80 to 142.63 for the 99% confidence level and from 100.37 to 81.97, for information technology and consumer staples respectively. 0.67 and 2.59 are the lowest results for the distance between VaR and R when $R < VaR$, for 99% and 95% confidence level, and the highest are 1.31 and 3.51, for respectively consumer staples and information technology.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	9	6	8	8	12	6
	Percentage of times exceeding the 99% VaR:	1.85 %	1.23 %	1.64 %	1.64 %	2.46 %	1.23 %
	Number of times exceeding the 95% VaR:	40	28	36	34	35	28
	Percentage of times exceeding the 95% VaR:	8.21 %	5.75 %	7.39 %	6.98 %	7.19 %	5.75 %
2	Number of times exceeding the 99% VaR:	17	17	12	8	13	11
	Percentage of times exceeding the 99% VaR:	2.30 %	2.30 %	1.62 %	1.08 %	1.76 %	1.49 %
	Number of times exceeding the 95% VaR:	42	41	41	31	32	40
	Percentage of times exceeding the 95% VaR:	5.69 %	5.56 %	5.56 %	4.20 %	4.34 %	5.42 %
3	Number of times exceeding the 99% VaR:	23	21	14	17	20	12
	Percentage of times exceeding the 99% VaR:	3.13 %	2.86 %	1.77 %	2.31 %	2.72 %	1.63 %
	Number of times exceeding the 95% VaR:	59	55	49	48	67	36
	Percentage of times exceeding the 95% VaR:	8.03 %	7.48 %	6.67 %	6.53 %	9.12 %	4.90 %
4	Number of times exceeding the 99% VaR:	8	8	16	13	9	14
	Percentage of times exceeding the 99% VaR:	0.82 %	0.82 %	1.64 %	1.33 %	0.92 %	1.43 %
	Number of times exceeding the 95% VaR:	44	43	54	46	40	61
	Percentage of times exceeding the 95% VaR:	4.50 %	4.40 %	5.52 %	4.70 %	4.09 %	6.24 %
5	Number of times exceeding the 99% VaR:	14	16	16	8	14	12
	Percentage of times exceeding the 99% VaR:	1.43 %	1.63 %	1.63 %	0.82 %	1.43 %	1.22 %
	Number of times exceeding the 95% VaR:	54	59	54	51	57	50
	Percentage of times exceeding the 95% VaR:	5.50 %	6.01 %	5.50 %	5.20 %	5.81 %	5.10 %
6	Number of times exceeding the 99% VaR:	9	7	5	6	7	10
	Percentage of times exceeding the 99% VaR:	3.36 %	2.61 %	1.87 %	2.24 %	2.61 %	3.73 %
	Number of times exceeding the 95% VaR:	22	26	18	23	18	26
	Percentage of times exceeding the 95% VaR:	8.82 %	9.70 %	6.72 %	8.58 %	6.72 %	9.70 %

Table 32: Oslo Stock Exchange: Results from Monte Carlo model 1 - periods

For the different periods, shown in table 32, period 3 and 6 are where the model estimate a too optimistic 99% VaR for the index, with 3.13% and 3.36% times exceeding the VaR, respectively. For 95% VaR the periods where MC Model 1 estimates the most optimistic VaR for the index are 1, 3 and 6 with 8.21%, 8.03% and 8.21% times exceeding VaR, respectively. The sectors perform similar to the index, with the same period 3 and 6 being the most challenging to calculate a precise 99% VaR and period 1, 3 and 6 being the most challenging for 95% VaR.

5.4.2 MC Model 2: Single variable and rolling regression coefficient

The second model where VIX is used to decide the degrees of freedom used in the VaR calculations is built up the same as the previous model, except for performing daily regressions over a rolling window of 250 days to continuously update the regression coefficient used to estimate the degrees of freedom. The parameters measured are the same as for MC Model 1. The idea behind continuously updating the regression coefficient used in equation 17 to scale the impact of VIX on the degrees of freedom, is to implement how the relationship between the two metrics changes and evolves.

5.4.2.1 S&P 500

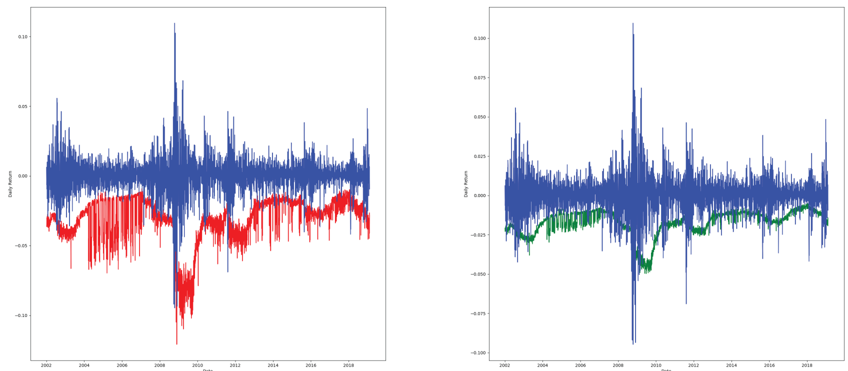


Figure 29: SPX: Results from Monte Carlo model 2. Red line 99% VaR. Green line 95% VaR

The results from this model shown in figure 29 above and table 33 below shows that daily updating the regression coefficient yields similar, but slightly different results for SPX. The graph in figure 29 containing a fair amount of noise, in the sense of rapid and substantial changes in the daily VaR. With a number of times exceeding 99% VaR being 1.55% for the index. The distance between the calculated VaR and returns and the distance when $R < VaR$ being 117.78 and 0.60 respectively. For 95% VaR number of times the index returns exceed the calculated VaR was approximately 5.90%. Moreover, the distances being 61.86 and 2.23, respectively.

		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99	1.55 %	1.41 %	1.60 %	1.81 %	1.48 %	1.62 %
	95	5.90 %	5.41 %	5.80 %	5.87 %	5.92 %	5.80 %
Sum of diff between VaR and observation	99	117.78	182.44	83.41	105.71	161.27	126.04
	95	61.86	93.30	46.50	57.75	86.94	70.16
Sum of diff when R exceeds VaR	99	0.60	0.94	0.50	0.60	0.87	0.71
	95	2.23	2.62	1.52	2.06	3.24	2.17

Table 33: S&P 500: Table from Monte Carlo model 2

The different sectors yield similar results as the index regarding 99% and 95% VaR. With the highest values being 1.81% from the health care sector and 5.92% from the financials sector, for 99% and 95% respectively, and the lowest values being 1.41% and 5.41% both from the energy sector. The cumulative distance between VaR and the returns range from 182.44 to 83.41 and from

93.30 to 46.50 for the 99% and 95% confidence level, respectively. The distance between R and VaR when $R < VaR$ also lay around the same values as the index, for the different sectors.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	4	2	8	14	2	9
	Percentage of times exceeding the 99% VaR:	0.80 %	0.40 %	1.59 %	2.79 %	0.40 %	1.79 %
	Number of times exceeding the 95% VaR:	24	14	26	29	25	33
	Percentage of times exceeding the 95% VaR:	4.78 %	2.79 %	5.18 %	5.78 %	4.98 %	6.57 %
2	Number of times exceeding the 99% VaR:	5	9	4	5	6	3
	Percentage of times exceeding the 99% VaR:	0.67 %	1.21 %	0.54 %	0.67 %	0.81 %	0.40 %
	Number of times exceeding the 95% VaR:	19	37	27	22	26	20
	Percentage of times exceeding the 95% VaR:	2.56 %	4.99 %	3.63 %	2.96 %	3.50 %	2.70 %
3	Number of times exceeding the 99% VaR:	24	18	24	23	24	24
	Percentage of times exceeding the 99% VaR:	3.19 %	2.39 %	3.19 %	3.05 %	3.19 %	319.00 %
	Number of times exceeding the 95% VaR:	73	41	64	63	81	58
	Percentage of times exceeding the 95% VaR:	9.69 %	5.44 %	8.50 %	8.37 %	10.76 %	7.70 %
4	Number of times exceeding the 99% VaR:	9	11	14	15	7	12
	Percentage of times exceeding the 99% VaR:	0.92 %	1.13 %	1.44 %	1.54 %	0.72 %	1.23 %
	Number of times exceeding the 95% VaR:	50	44	55	61	42	1
	Percentage of times exceeding the 95% VaR:	5.13 %	4.52 %	5.64 %	6.26 %	4.31 %	5.24 %
5	Number of times exceeding the 99% VaR:	15	11	10	13	15	13
	Percentage of times exceeding the 99% VaR:	1.49 %	1.09 %	0.99 %	1.29 %	1.49 %	1.29 %
	Number of times exceeding the 95% VaR:	52	58	57	52	55	63
	Percentage of times exceeding the 95% VaR:	5.16 %	5.76 %	5.67 %	5.16 %	5.46 %	6.26 %
6	Number of times exceeding the 99% VaR:	9	9	8	7	9	8
	Percentage of times exceeding the 99% VaR:	3.25 %	3.25 %	2.89 %	2.52 %	3.25 %	2.89 %
	Number of times exceeding the 95% VaR:	33	36	28	23	23	22
	Percentage of times exceeding the 95% VaR:	11.91 %	13.00 %	10.11 %	8.30 %	8.30 %	7.94 %

Table 34: S&P 500: Results from Monte Carlo model 2 - periods

The periods 3 and 6 are still the most challenging, shown in table 34. However, in comparison to model 1, period 1 and 2 has even fewer times where $R < VaR$ than desired, from MC Model 2 calculating too pessimistic VaR estimations, for the index. The same argument also applies to the majority of the sectors.

5.4.2.2 Oslo Stock Exchange

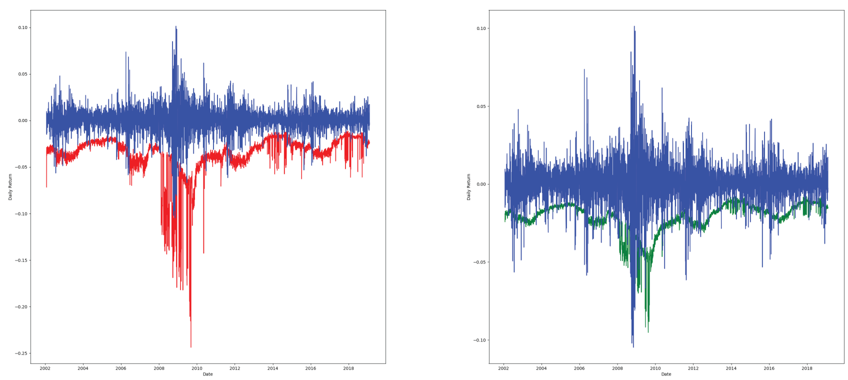


Figure 30: OSEBX: Results from Monte Carlo model 2. Red line 99% VaR. Green line 95% VaR

The results for Oslo Stock Exchange, shown in figure 30 for the index and table 35, are close to the same as from MC Model 1, with a constant regression coefficient, and also contain a fair amount of noise. With a result of 1.49% times exceeding 99%VaR for the index. The sum om distance between 99% VaR and the returns, both total and for when $R < VaR$, are 136.20 and 0.74 respectively. The percentage of times the returns exceed the index 95% VaR is 5.85%. The sum of distances between returns and calculated VaR, both total and for when $R < VaR$, are 74.80 and 2.41.

		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99	1.49 %	1.29 %	1.53 %	1.27 %	1.60 %	1.29 %
% exceeding the VaR	95	5.85 %	5.18 %	5.40 %	5.28 %	5.47 %	5.23 %
Sum of diff between VaR and observation	99	136.2	168.68	144.21	166.37	150.12	171.33
Sum of diff between VaR and observation	95	74.8	93.44	79.82	86.24	84.57	92.43
Sum of diff when R exceeds VaR	99	0.74	0.68	0.63	0.94	0.8	1.25
Sum of diff when R exceeds VaR	95	2.41	2.43	2.37	2.96	2.74	3.20

Table 35: Oslo Stock Exchange: Table from Monte Carlo model 2

The sector results are similar to the index for both 99% and 95% VaR, where all the sectors lay relatively close to the desired 1% and 5% mark. The cumulative distances between VaR and R range from 168.68 to 144.21 for 99% VaR and from 93.44 to 79.82 for 95% VaR. The sum of the distances when $R < VaR$ range from 1.25 to 0.63 and 3.20 to 2.37 for the 99% and 95% confidence level, respectively.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	5	1	5	6	4	4
	Percentage of times exceeding the 99% VaR:	1.06 %	0.21 %	1.06 %	1.28 %	0.85 %	0.85 %
	Number of times exceeding the 95% VaR:	27	16	23	26	20	20
	Percentage of times exceeding the 95% VaR:	5.74 %	3.40 %	4.89 %	5.53 %	4.26 %	4.26 %
2	Number of times exceeding the 99% VaR:	17	13	13	8	13	14
	Percentage of times exceeding the 99% VaR:	2.30 %	1.76 %	1.76 %	1.08 %	1.76 %	1.90 %
	Number of times exceeding the 95% VaR:	43	41	43	30	32	39
	Percentage of times exceeding the 95% VaR:	5.83 %	5.56 %	5.83 %	4.07 %	4.34 %	5.28 %
3	Number of times exceeding the 99% VaR:	12	14	9	17	18	10
	Percentage of times exceeding the 99% VaR:	1.63 %	1.90 %	1.22 %	2.31 %	2.45 %	1.36 %
	Number of times exceeding the 95% VaR:	55	43	35	48	65	36
	Percentage of times exceeding the 95% VaR:	7.48 %	5.85 %	12.00 %	6.53 %	8.84 %	4.90 %
4	Number of times exceeding the 99% VaR:	10	10	16	11	11	10
	Percentage of times exceeding the 99% VaR:	1.02 %	1.02 %	1.64 %	112.00 %	1.12 %	1.02 %
	Number of times exceeding the 95% VaR:	43	41	54	46	36	54
	Percentage of times exceeding the 95% VaR:	4.40 %	4.19 %	5.52 %	4.70 %	3.68 %	5.52 %
5	Number of times exceeding the 99% VaR:	10	13	19	7	13	10
	Percentage of times exceeding the 99% VaR:	1.02 %	1.32 %	1.94 %	0.71 %	1.33 %	1.02 %
	Number of times exceeding the 95% VaR:	54	58	53	46	58	44
	Percentage of times exceeding the 95% VaR:	5.50 %	5.91 %	5.40 %	4.69 %	5.91 %	4.49 %
6	Number of times exceeding the 99% VaR:	8	3	2	4	8	6
	Percentage of times exceeding the 99% VaR:	2.99 %	1.12 %	0.75 %	1.49 %	2.99 %	2.34 %
	Number of times exceeding the 95% VaR:	22	17	7	24	17	25
	Percentage of times exceeding the 95% VaR:	8.82 %	6.34 %	2.61 %	8.96 %	6.34 %	9.33 %

Table 36: Oslo Stock Exchange: Table from Monte Carlo model 2 - periods

The periods where MC Model 2 estimates too optimistic VaR estimations for 99% VaR for the index, are period 2 and 6 with the results 2.30% and 2.99%, respectively. For 95% VaR the periods where the model estimates too optimistic VaR estimations, for the index, are still the periods 1, 3 and 6. With the results 5.74%, 7.48% and 8.21%. The sectors struggle with VaR calculations for the same time periods as the index.

5.4.3 MC Model 3: Multiple variables and constant regression coefficients

The third model in the Monte Carlo simulation, MC Model 3, was designed in the same way with the same restrictions as MC Model 1 and 2, but instead of one macroeconomic factor influencing the degrees of freedom used in the generation of pseudorandom student-t distributed variables, several factors are included. The new macroeconomic factors implemented into the model, in addition to VIX, is:

- Brent Price
- Gold Price
- Bond Price

As for the previous models, a need to uncover the relationship between the MLE-estimates of the parameters and the macros is necessary. A log-log OLS-regression was set up, shown in equation 18 below.

$$log(\nu) = \beta_0 + \beta_1 * log(VIX) + \beta_2 * log(Brent\, Price) + \beta_3 * log(Gold\, Price) + \beta_4 * log(Bond\, Price) \quad (18)$$

Also here the 15 day average of VIX was used to moderate rapid and sudden changes. For brent, gold and bond prices the daily price was used as variables in the regression.

OLS Regression Results

Dep. Variable:	nu	R-squared:	0.275
Model:	OLS	Adj. R-squared:	0.274
Method:	Least Squares	F-statistic:	387.4
Date:	Sun, 19 May 2019	Prob (F-statistic):	3.48e-283
Time:	14:32:39	Log-Likelihood:	-9623.4
No. Observations:	4086	AIC:	1.926e+04
Df Residuals:	4081	BIC:	1.929e+04
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-0.9084	7.976	-0.114	0.909	-16.545	14.728
VIX15avg	-1.7785	0.117	-15.167	0.000	-2.008	-1.549
Brent Price	0.7753	0.128	6.047	0.000	0.524	1.027
Gold Price	-3.2773	0.111	-29.580	0.000	-3.495	-3.060
Bond Price	5.9366	1.747	3.398	0.001	2.511	9.362

Omnibus:	2332.381	Durbin-Watson:	0.104
Prob(Omnibus):	0.000	Jarque-Bera (JB):	16548.145
Skew:	2.735	Prob(JB):	0.00
Kurtosis:	11.202	Cond. No.	1.98e+03

Figure 31: SPX: Regression from Monte Carlo model 3

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     nu   R-squared:                       0.144
Model:                            OLS   Adj. R-squared:                  0.143
Method:                 Least Squares   F-statistic:                     171.3
Date:                Mon, 20 May 2019   Prob (F-statistic):         7.81e-136
Time:                        20:28:36   Log-Likelihood:                -8960.5
No. Observations:                4086   AIC:                         1.793e+04
Df Residuals:                    4081   BIC:                         1.796e+04
Df Model:                           4
Covariance Type:            nonrobust
===============================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------
const          10.1146      6.781      1.492      0.136      -3.180      23.410
VIX15avg        0.6879      0.100      6.900      0.000       0.492       0.883
Brent Price     2.7846      0.109     25.545      0.000       2.571       2.998
Gold Price     -1.6246      0.094    -17.246      0.000      -1.809      -1.440
Bond Price     -2.2505      1.486     -1.515      0.130      -5.163       0.662
==============================================================================
Omnibus:                     3216.869   Durbin-Watson:                   0.023
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            55102.582
Skew:                           3.789   Prob(JB):                         0.00
Kurtosis:                      19.316   Cond. No.                     1.98e+03
==============================================================================
```

Figure 32: OSEBX: Regression from Monte Carlo model 3

From the results of the regressions the betas belonging to VIX, brent price, gold price and bond price are extracted, respectively $\beta_1, \beta_2, \beta_3$ and β_4, shown in figure 31 for SPX and figure 32 for OSEBX. These are then used in the new modified formula, equation 19 for calculating the degrees of freedom, which is going to be used in Pythons pseudorandom student-t variable generator.

$$\nu_n = \nu_{n-1} + \nu_{n-1} * ((\frac{VIX_n}{VIX_{n-1}} - 1) * \beta_1 + (\frac{Brent\ Price_n}{Brent\ Price_{n-1}} -)1 * \beta_2 + (\frac{Gold\ Price_n}{Gold\ Price_{n-1}} - 1) * \beta_3 + (\frac{Bond\ Price_n}{Bond\ Price_{n-1}} - 1) * \beta_4) \quad (19)$$

For each day n, $m = 1000$ independent realizations of these random variables are generated with the degree of freedom from equation 19.

5.4.3.1 S&P 500

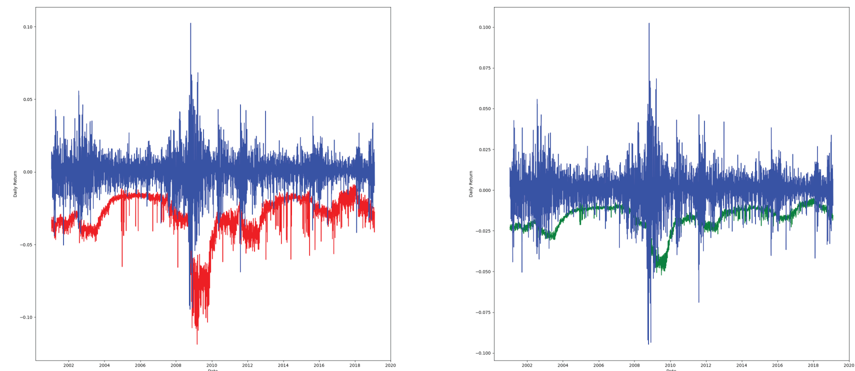


Figure 33: SPX: Graph from Monte Carlo model 3. Red line 99% VaR. Green line 95% VaR

Figure 33 above shows a graphical representation of 99% VaR to the left and 95% VaR to the right, for the S$P 500 index. Below, table 37 lists the results for the index and the sector. For 99% VaR, a result of 1.49% times where the return exceeds the estimated VaR was obtained for the index. The sum of the distance between VaR and returns and the sum of the distance between VaR and returns when $R < VaR$ are 107.85 and 0.55 respectively. The percentage of times the index returns exceed the 95% VaR is 6.19%. And the distances measured between VaR and returns, both total and for when $R < VaR$ are 58,48 and 2.24, respectively.

		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99	1.49 %	1.61 %	1.38 %	1.61 %	1.52 %	1.61 %
	95	6.19 %	6.39 %	5.56 %	5.70 %	5.54 %	5.47 %
Sum of diff between VaR and observation	99	107.86	148.78	82.88	104.44	162.60	124.43
	95	58.48	87.15	48.02	59.44	89.48	72.4
Sum of diff when R exceeds VaR	99	0.55	1.01	5.05	0.64	0.83	0.67
	95	2.24	2.95	1.58	2.09	3.29	2.27

Table 37: S&P 500: Table from Monte Carlo model 3

The sectors yield similar results as the index regarding calculating 99% and 95% VaR. The lowest values being 1.38% and 5.47% respectively. The cumulative distance between R and VaR range from 162.60 to 82.88 for the 99% confidence level and from 89.48 to 48.02 from the 95% confidence level. The distance when $R < VaR$ range from 5.05 to 0.64 and 3.29 to 1.58 for 99% and 95%, respectively.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	5	9	7	9	6	10
	Percentage of times exceeding the 99% VaR:	1.10 %	1.89 %	1.47 %	1.89 %	1.26 %	2.10 %
	Number of times exceeding the 95% VaR:	21	32	26	35	29	35
	Percentage of times exceeding the 95% VaR:	4.63 %	6.72 %	5.46 %	7.35 %	6.09 %	7.35 %
2	Number of times exceeding the 99% VaR:	5	11	5	5	5	5
	Percentage of times exceeding the 99% VaR:	0.69 %	1.53 %	0.69 %	0.69 %	0.69 %	0.69 %
	Number of times exceeding the 95% VaR:	28	53	29	23	25	19
	Percentage of times exceeding the 95% VaR:	3.89 %	7.36 %	4.03 %	3.19 %	3.47 %	2.64 %
3	Number of times exceeding the 99% VaR:	21	22	16	20	24	24
	Percentage of times exceeding the 99% VaR:	2.88 %	3.02 %	2.19 %	2.74 %	3.29 %	3.29 %
	Number of times exceeding the 95% VaR:	71	54	61	64	76	59
	Percentage of times exceeding the 95% VaR:	9.74 %	7.41 %	8.37 %	8.78 %	10.43 %	8.09 %
4	Number of times exceeding the 99% VaR:	7	8	11	12	7	11
	Percentage of times exceeding the 99% VaR:	0.74 %	0.85 %	1.16 %	1.27 %	0.74 %	1.16 %
	Number of times exceeding the 95% VaR:	50	46	51	54	41	48
	Percentage of times exceeding the 95% VaR:	5.29 %	4.87 %	5.40 %	5.71 %	4.34 %	5.08 %
5	Number of times exceeding the 99% VaR:	14	11	13	13	15	14
	Percentage of times exceeding the 99% VaR:	1.44 %	1.13 %	1.34 %	1.34 %	1.54 %	1.44 %
	Number of times exceeding the 95% VaR:	51	60	50	47	51	56
	Percentage of times exceeding the 95% VaR:	5.25 %	6.17 %	5.14 %	4.84 %	5.25 %	5.76 %
6	Number of times exceeding the 99% VaR:	9	9	8	11	9	6
	Percentage of times exceeding the 99% VaR:	3.40 %	3.40 %	3.02 %	4.15 %	3.40 %	2.26 %
	Number of times exceeding the 95% VaR:	32	32	24	24	18	20
	Percentage of times exceeding the 95% VaR:	12.08 %	12.08 %	9.06 %	9.06 %	6.79 %	7.55 %

Table 38: S&P 500: Table from Monte Carlo model 3 - periods

Period 3 and 6 are the periods where both 99% VaR and 95% VaR struggle the most, and the model estimates a too optimistic VaR. In period 3, the percentage of times the returns exceed the estimated VaR for the index are 2.88% and 9.74% respectively, and for period 6 the results are 3.40% and 12.08%. The highest values for the sectors being 4.15% for 99% VaR, from the health care sector, and 12.08% for 95% VaR, from the energy sector.

5.4.3.2 Oslo Stock Exchange

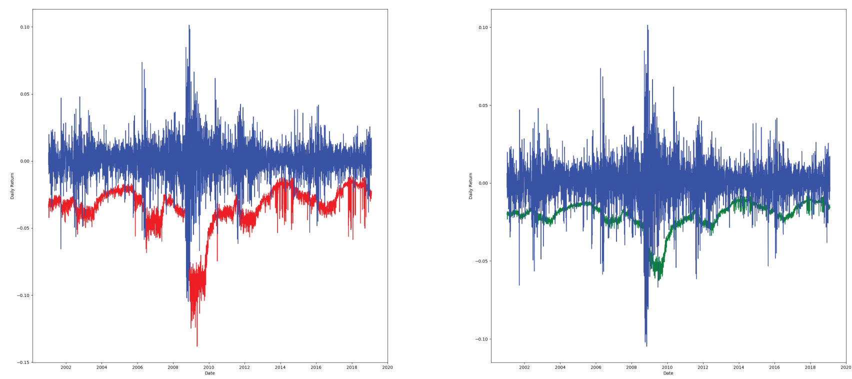


Figure 34: OSEBX: Graph from Monte Carlo model 3. Red line 99% VaR. Green line 95% VaR

For Oslo Stock Exchange the macroeconomic factor Bond Price is, not surprisingly, omitted from the VaR calculations. From the regression, shown in figure 32, it can be seen that US bond prices are not a significant factor for Oslo Stock Exchange, with a t-value equal to 0.13. In order to improve the regression and its coefficients, a new regression without Bond Price is conducted.

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                     nu   R-squared:                       0.143
Model:                            OLS   Adj. R-squared:                  0.143
Method:                 Least Squares   F-statistic:                     227.6
Date:                Mon, 20 May 2019   Prob (F-statistic):          1.49e-136
Time:                        20:44:02   Log-Likelihood:                -8961.7
No. Observations:                4086   AIC:                         1.793e+04
Df Residuals:                    4082   BIC:                         1.796e+04
Df Model:                           3
Covariance Type:            nonrobust
===============================================================================
                  coef    std err          t      P>|t|      [0.025      0.975]
-------------------------------------------------------------------------------
const          -0.1261      0.536     -0.235      0.814      -1.177       0.925
VIX15avg        0.6361      0.094      6.791      0.000       0.452       0.820
Brent Price     2.7938      0.109     25.664      0.000       2.580       3.007
Gold Price     -1.6259      0.094    -17.257      0.000      -1.811      -1.441
==============================================================================
Omnibus:                     3219.478   Durbin-Watson:                   0.023
Prob(Omnibus):                  0.000   Jarque-Bera (JB):            55255.227
Skew:                           3.793   Prob(JB):                         0.00
Kurtosis:                      19.340   Cond. No.                         136.
==============================================================================
```

Figure 35: OSEBX: Regression from Monte Carlo model 3 without Bond Price

From the new regression without US bond prices included, shown in figure 35, shows that the coefficients have changed its values slightly and now all macroeconomic factors included in the simulation are significant. The adjusted R-squared for both regressions are the same value, 0.143, which suggests that Bond Price does not add any value to the regression model for Oslo Stock Exchange.

The percentage of times the returns exceed 99% VaR is 1.78% for the index. The sum of distances between the R and the estimated VaR in total and for when $R < VaR$ for 99% VaR is 133.07 and 1.00, respectively. The percentage of times the returns exceed 95% VaR for the index is 5.74%. The distances are 76.58 and 2.88, for the total distance between returns and VaR and for the distance when $R < VaR$, respectively.

		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99	1.78 %	1.66	1.64	1.34 %	1.80 %	1.52 %
	95	5.74 %	5.56	5.51	5.29 %	5.67 %	5.40 %
Sum of diff between VaR and observation	99	133.07	157.26	142.17	177.47	153.56	176.55
	95	76.58	93.87	81.34	92.04	86.2	98.97
Sum of diff when R exceeds VaR	99	1.00	0.92	0.77	0.96	1.12	1.33
	95	2.88	2.97	2.68	3.06	3.23	3.45

Table 39: Oslo Stock Exchange: Table from Monte Carlo model 3

The VaR estimates calculated on the sectors yield a lower percentage of times where $R < VaR$ than the index for both confidence level, except for 99% VaR for the financials sector. The cumulative distance between R and VaR range from 177.47 to 142.17 and 98.97 to 81.34 for 99% and 95%, respectively. The cumulative distance when $R < VaR$ for the sectors perform around the same value as the index, for both confidence levels.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	11	8	10	9	12	8
	Percentage of times exceeding the 99% VaR:	2.31 %	1.68 %	2.10 %	1.89 %	2.52 %	1.68 %
	Number of times exceeding the 95% VaR:	36	28	30	37	43	29
	Percentage of times exceeding the 95% VaR:	7.56 %	5.88 %	6.30 %	7.77 %	9.03 %	6.91 %
2	Number of times exceeding the 99% VaR:	16	14	13	8	13	13
	Percentage of times exceeding the 99% VaR:	2.22 %	1.94 %	1.39 %	1.11 %	1.81 %	1.81 %
	Number of times exceeding the 95% VaR:	42	41	43	32	30	36
	Percentage of times exceeding the 95% VaR:	5.83 %	5.69 %	5.97 %	4.44 %	4.17 %	5.00 %
3	Number of times exceeding the 99% VaR:	22	23	13	18	19	11
	Percentage of times exceeding the 99% VaR:	3.02 %	3.15 %	1.78 %	2.47 %	2.61 %	1.51 %
	Number of times exceeding the 95% VaR:	56	48	45	48	63	37
	Percentage of times exceeding the 95% VaR:	7.68 %	6.58 %	6.17 %	6.58 %	8.64 %	5.08 %
4	Number of times exceeding the 99% VaR:	8	8	15	10	12	13
	Percentage of times exceeding the 99% VaR:	0.85 %	0.85 %	1.59 %	1.06 %	1.27 %	1.38 %
	Number of times exceeding the 95% VaR:	42	43	51	44	37	58
	Percentage of times exceeding the 95% VaR:	4.44 %	4.55 %	5.40 %	4.66 %	3.92 %	6.14 %
5	Number of times exceeding the 99% VaR:	11	13	18	7	14	12
	Percentage of times exceeding the 99% VaR:	1.13 %	1.34 %	1.85 %	0.72 %	1.44 %	1.23 %
	Number of times exceeding the 95% VaR:	52	55	54	45	56	47
	Percentage of times exceeding the 95% VaR:	5.34 %	5.66 %	5.56 %	4.63 %	5.76 %	4.84 %
6	Number of times exceeding the 99% VaR:	9	6	5	6	8	9
	Percentage of times exceeding the 99% VaR:	3.40 %	2.26 %	1.89 %	2.26 %	3.02 %	3.40 %
	Number of times exceeding the 95% VaR:	21	26	16	23	17	27
	Percentage of times exceeding the 95% VaR:	7.92 %	9.81 %	6.04 %	8.68 %	6.42 %	10.19 %

Table 40: Oslo Stock Exchange: Table from Monte Carlo model 3 - periods

The periods, shown in table 40, where this model struggles the most and estimates a too optimistic VaR, for the Oslo Stock Exchange, are period 3 and 6. For 99% VaR in period 3 the percentage of times the returns exceed the VaR estimation is 3.02% and for period 6 the result is 3.40%, for the index. For 95% VaR period 1, 3 and 6 are where the model struggles the most, with a percentage of times exceeding the index VaR of 7.56%, 7,68%, and 7,92%, respectively. The sectors give results high results in these periods as well, with the highest being 3.40% for the information technology for 99% VaR and 10.19% for 95% VaR for the same sector.

5.4.4 MC Model 4: Multiple variables and rolling regression coefficients

The fourth and last Monte Carlo model, MC Model 4, is a modification of MC Model 3, with the same assumptions and restrictions. The only difference is that MC Model 3 has constant regression coefficients; here, a new regression is performed each day, continuously updating the coefficients. The rolling window where the regression between the macroeconomic variables and the degrees of freedom has a size of 250 days. As for MC Model 2, continuously updating the regression coefficients is to implement how, in different periods, the relationship between the degrees of freedom and the macros change and evolve.

5.4.4.1 S&P 500

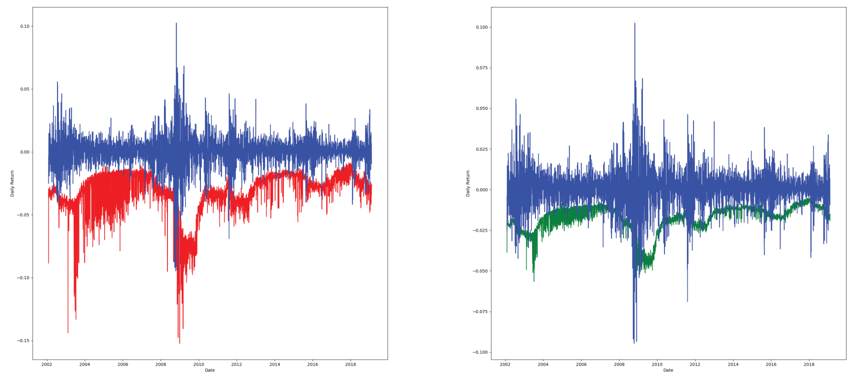


Figure 36: SPX: Graph from Monte Carlo model 4. Red line 99% VaR. Green line 95% VaR

Seen in the graphs from figure 36 a considerable amount of noise, in the form of large and rapid changes, appears in the VaR calculations. Especially between the years 2004 to 2006. For 99% VaR the percentage of times the returns exceed, shown in table 41, the estimated VaR is 1.47% for the index. The sum of the distance between the returns and VaR is 117.04 and the sum of the distance between returns and VaR when $R < VaR$ is 0.48, for 99% the 99% confidence level. For 95% VaR the percentage of times the index returns exceed VaR is 6.17%. The total distance between the returns and VaR calculations is 60.95, and the distance when $R < VaR$ is 2.18.

		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
% exceeding the VaR	99	1.47 %	1.47 %	1.64 %	1.69 %	1.52 %	1.59 %
	95	6.17 %	5.24 %	5.43 %	5.85 %	5.58 %	5.24 %
Sum of diff between VaR and observation	99	117.04	187.41	84.38	105.64	161.70	127.76
	95	60.95	93.25	46.00	56.87	85.92	69.58
Sum of diff when R exceeds VaR	99	0.48	0.86	0.49	0.62	0.75	0.63
	95	2.18	2.49	1.48	1.97	3.09	2.08

Table 41: S&P 500: Table from Monte Carlo model 4

Similar results are observed for the sectors, with the highest value for 99% VaR being 1.69% for consumer staples and the highest for 95% VaR being 5.85% for the same sector. The cumulative distance between R and VaR range from 187.41 to 84.38 and from 93.25 to 46.00 for the 99% and 95% confidence level, respectively. The distance when $R < VaR$ range from 0.86 to 0.49 and from 3.09 to 1.48 for the 99% and 95% confidence level, respectively.

Period		SPX	SPX10GI	SPX30GI	SPX35GI	SPX40GI	SPX45GI
1	Number of times exceeding the 99% VaR:	6	5	9	10	2	8
	Percentage of times exceeding the 99% VaR:	1.32 %	1.10 %	1.98 %	2.20 %	0.44 %	1.76 %
	Number of times exceeding the 95% VaR:	22	13	24	28	23	25
	Percentage of times exceeding the 95% VaR:	4.85 %	2.86 %	5.29 %	6.17 %	5.07 %	5.51 %
2	Number of times exceeding the 99% VaR:	6	10	7	6	6	5
	Percentage of times exceeding the 99% VaR:	0.83 %	1.39 %	0.98 %	0.83 %	0.83 %	0.69 %
	Number of times exceeding the 95% VaR:	26	36	22	24	20	14
	Percentage of times exceeding the 95% VaR:	3.61 %	5.00 %	3.06 %	3.33 %	2.78 %	1.94 %
3	Number of times exceeding the 99% VaR:	17	16	17	18	23	20
	Percentage of times exceeding the 99% VaR:	2.33 %	2.19 %	2.33 %	2.47 %	3.16 %	2.74 %
	Number of times exceeding the 95% VaR:	75	44	51	59	79	52
	Percentage of times exceeding the 95% VaR:	10.29 %	6.04 %	7.00 %	8.09 %	10.84 %	7.13 %
4	Number of times exceeding the 99% VaR:	8	9	14	14	8	13
	Percentage of times exceeding the 99% VaR:	0.85 %	0.95 %	1.48 %	1.38 %	0.85 %	1.38 %
	Number of times exceeding the 95% VaR:	51	40	50	55	42	45
	Percentage of times exceeding the 95% VaR:	5.40 %	4.23 %	5.29 %	5.82 %	4.44 %	4.76 %
5	Number of times exceeding the 99% VaR:	14	12	11	12	16	12
	Percentage of times exceeding the 99% VaR:	1.44 %	1.23 %	1.13 %	1.23 %	1.65 %	1.23 %
	Number of times exceeding the 95% VaR:	4	54	49	45	47	57
	Percentage of times exceeding the 95% VaR:	5.04 %	5.56 %	5.04 %	4.63 %	4.84 %	5.86 %
6	Number of times exceeding the 99% VaR:	9	8	9	10	7	7
	Percentage of times exceeding the 99% VaR:	3.40 %	3.02 %	3.40 %	3.77 %	2.64 %	2.64 %
	Number of times exceeding the 95% VaR:	29	27	26	28	17	21
	Percentage of times exceeding the 95% VaR:	10.94 %	10.19 %	9.81 %	10.57 %	6.42 %	7.92 %

Table 42: S&P 500: Table from Monte Carlo model 4 - periods

Period 3 and 6 are where the model struggles the most and VaR estimates are too optimistic, shown in table 42. The results for when the returns exceed 99% VaR are 2.33% and 3.40% respectively, and for 95% VaR the results are 10.29% and 10.94% for the Oslo stock exchange index. The sectors give similar results as the index in these periods.

5.4.4.2 Oslo Stock Exchange

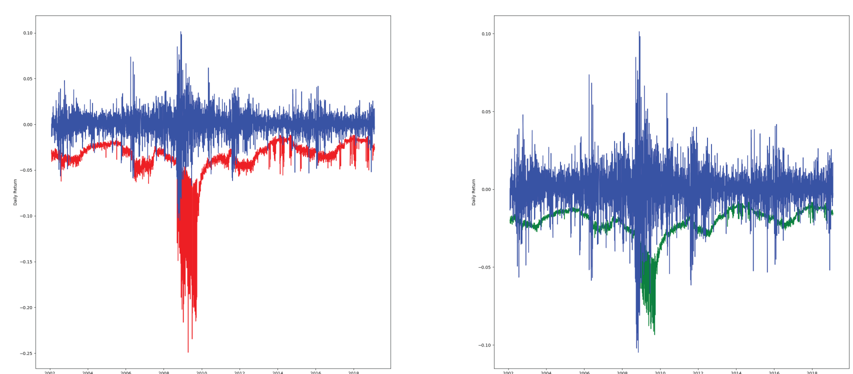


Figure 37: OSEBX: Graph from Monte Carlo model 4. Red line 99% VaR. Green line 95% VaR

A fair amount of noise can be observed from the graphs in figure 37 above. Especially around the financial crisis in 2008. The percentage of times the returns exceed the 99% VaR estimate is 1.71% for the Oslo Stock Exchange index. The total distance between R and VaR is 131.27 and the distance when $R < VaR$ is 0.93, for the 99% calculations. The percentage of times the returns exceed the 95% VaR for the index is 6.12%. The distances are 73.25 and 2.67, respectively.

		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
% exceeding the VaR	99	1.71 %	1.62 %	1.62 %	1.37 %	1.61 %	1.42 %
	95	6.12 %	5.43 %	5.65 %	5.17 %	5.56 %	5.46 %
Sum of diff between VaR and observation	99	131.27	166.13	142.36	163.22	151.52	166.33
	95	73.25	92.26	78.67	84.78	83.93	90.85
Sum of diff when R exceeds VaR	99	0.93	1.01	0.67	0.84	0.93	1.26
	95	2.67	2.73	2.49	2.88	2.98	3.27

Table 43: Oslo Stock Exchange: Table from Monte Carlo model 4

The sector results are all lower than the index results for both 99% and 95% VaR. The cumulative distance between R and VaR range from 166.33 to 142.36 for the 99% confidence level and from 92.26 to 78.67 for the 95% confidence level. The distance when $R < VaR$ range from 1.01 to 0.67 and from 3.37 to 2.49 for the 99% and 95% confidence level, respectively.

Period		OSEBX	OSE10GI	OSE30GI	OSE35GI	OSE40GI	OSE45GI
1	Number of times exceeding the 99% VaR:	4	2	6	3	4	6
	Percentage of times exceeding the 99% VaR:	0.88 %	0.44 %	1.32 %	0.66 %	0.88 %	1.32 %
	Number of times exceeding the 95% VaR:	25	15	20	25	20	17
	Percentage of times exceeding the 95% VaR:	5.51 %	3.30 %	4.41 %	5.51 %	4.41 %	3.74 %
2	Number of times exceeding the 99% VaR:	17	12	10	9	13	12
	Percentage of times exceeding the 99% VaR:	2.36 %	1.67 %	1.39 %	1.25 %	1.81 %	1.67 %
	Number of times exceeding the 95% VaR:	44	37	46	28	29	40
	Percentage of times exceeding the 95% VaR:	6.11 %	5.14 %	6.39 %	3.89 %	4.03 %	5.56 %
3	Number of times exceeding the 99% VaR:	21	23	13	17	20	11
	Percentage of times exceeding the 99% VaR:	2.88 %	3.16 %	1.78 %	2.33 %	2.74 %	1.51 %
	Number of times exceeding the 95% VaR:	59	50	44	47	66	36
	Percentage of times exceeding the 95% VaR:	8.09 %	6.59 %	6.04 %	6.45 %	9.05 %	4.94 %
4	Number of times exceeding the 99% VaR:	9	10	18	10	10	12
	Percentage of times exceeding the 99% VaR:	0.95 %	1.06 %	1.90 %	1.06 %	1.06 %	1.27 %
	Number of times exceeding the 95% VaR:	43	40	54	44	38	58
	Percentage of times exceeding the 95% VaR:	4.55 %	4.23 %	5.71 %	4.66 %	4.02 %	6.14 %
5	Number of times exceeding the 99% VaR:	10	13	16	9	13	9
	Percentage of times exceeding the 99% VaR:	1.03 %	1.34 %	1.64 %	0.93 %	1.34 %	0.93 %
	Number of times exceeding the 95% VaR:	57	55	57	44	56	44
	Percentage of times exceeding the 95% VaR:	5.86 %	5.66 %	5.89 %	4.53 %	5.76 %	4.53 %
6	Number of times exceeding the 99% VaR:	9	6	3	8	6	8
	Percentage of times exceeding the 99% VaR:	3.34 %	2.26 %	1.13 %	3.02 %	2.26 %	3.02 %
	Number of times exceeding the 95% VaR:	22	25	10	23	18	28
	Percentage of times exceeding the 95% VaR:	8.30 %	9.43 %	3.78 %	8.68 %	6.79 %	10.57 %

Table 44: Oslo Stock Exchange: Table from Monte Carlo model 4 - periods

The periods where the 99% VaR calculations struggle the most and estimate a too optimistic VaR, regarding the percentage of times where the returns exceed VaR, are 3 and 6. With the results 2.88% and 3.34%, respectively, for the index, and similar results for most of the sectors. Period 3 and 6 are also where the 95%

VaR estimates are the poorest, and too optimistic, with results for the index of 8.09% and 8.30%, respectively. The sectors also giving a high percentage of times the returns exceed the 95% VaR.

6 Discussion

In appendix B and C complete results for the models discussed can be found, where the results are color coded to make comparison between the models easier. In appendix D a table that compares the DHV model against the MC model can be found.

6.1 Evolution of the VaR model

6.1.1 Dynamic Historical VaR

In chapter 5.2 we started with inspecting and documenting the behavior of the different window sizes when calculating VaR with rolling window. By plotting and measuring the metrics defined in chapter 5, traits and characteristics from different window sizes were observed and taken into consideration. The key observations made, was that the VaR estimations with smaller window sizes followed the downside of the returns closely, but the returns exceeded the VaR calculations too many times. Moreover, VaR calculations for the larger window sizes did not follow the downside of the returns closely but performed better regarding the number of times the returns exceed VaR. The data obtained for this model, laid the grounds for the development of the first model using a macroeconomic variable as an input for calculating historical VaR. When comparing and discussing the results, comparison against the 250-day rolling window will be weighted and serve as a benchmark.

Comparing Dynamic Historical VaR to historical VaR with a fixed rolling window, it is observed that traits from the whole window size array {22,44,250,500, 750,1000} is implemented into one model. For DHV Model 1 and 2, results similar for window sizes 750 and 1000 is captured into the model, even though the scaling of the window size is limited to a maximum of 500 days. A dynamic window size enables the possibility to change the reactiveness of VaR based on the markets expected volatility, and when utilizing this model one does not have to compromise to the same degree on characteristics for the VaR calculation, as for when using a fixed window size.

6.1.1.1 S&P 500

For all three of the DHV models, the 99% VaR estimations perform closer to the 1% mark than the benchmark model. Except for the information technology sector for all three models, in addition to the energy sector for the DHV Model 2. The estimations for 95% VaR for all three models perform closer to the desired 5% mark than the benchmark model. All the 95% VaR estimations for the DHV Model 3 perform closer to the 5% mark than the benchmark of the 250-day historical rolling window. For DHV Model 2, all the 95% VaR estimations perform better than the benchmark, except for the financials and information technology sectors that perform the best and most precise, for DHV Model 1. Better results, meaning that the model scores better than the benchmark model across index and sectors, for all four metrics measured and for both confidence levels.

When comparing the three Dynamic Historical VaR models between themselves, DHV Model 1 performs best regarding the percentage of times R exceeds both 99% and 95% VaR for most of the S&P 500 calculations. This model contains all the best results for both confidence levels when considering the cumulative distance between R and VaR when $R < VaR$. Meaning that when the returns exceed VaR, this model estimates VaR the closest to the value of the return. Despite DHV model 1 having the majority of the best overall results, the model lacks in performance when it comes to the total distance between R and VaR, where the benchmark outperforms DHV Model 1 more times than not. The model that scales its window size with respect to the percentage change in VIX, DHV Model 2, performs the best out of the three models when it comes to the cumulative distance between R and VaR, for the 95% confidence interval. For the 99% confidence interval, the lowest cumulative distance between R and VaR depends on which model and sector are being observed.

The last model, DHV Model 3, is an equally weighted combination of the two previously mentioned models. As explained in section 5.3 this model adopts the traits of the two latter models, resulting in better results than the benchmark for the majority of the metrics for the three models, when observing all four metrics for the index and sectors. Mostly having values in between the two latter models this model is based on, serving as a middle ground. DHV Model 3 also contains some of the best results when comparing the three models in between one another, depending on which metric and sector are being observed.

With all the three models calculating a more precise VaR for the data set when assessing the time periods, only DHV Model 1 perform better than the benchmark for more times than not for 99% VaR in the different periods. All three models perform especially well in period 1 and 5, for both confidence levels. When assessing the different time periods, the most interesting periods to observe are 3 and 6, where all models and the benchmark struggle with a high percentage of times the returns exceed the estimated VaR. These time periods represent the financial crisis and increased volatility at the end of 2018, respectively. Under these financial stressing times the only model out of the three Dynamic Historical VaR to perform better than the benchmark under the financial crisis is the DHV model 1 for both 99% and 95% VaR but still, suffer from too optimistic VaR estimates. Despite all three models performing more precise than the benchmark regarding the percentage of times the returns exceed VaR, for both confidence levels, when considering the whole data set, under financial stressing times the benchmark model performs more precise VaR estimations.

DHV Model 3 can be considered the more versatile model, out of the three. Depending on how important each of the four metrics observed is considered, this model can be weighted differently than 50/50 to emphasize the traits the portfolio manager considers the most important. For this model, an evaluation with regards to the weighting has to be done for the models' performance under financial stressing times.

6.1.1.2 Oslo Stock Exchange

The VaR models in chapter 5.3 yielded similar results for 99% and 95% VaR for Oslo stock exchange, as for S&P 500. Most of the sectors and the index outperforming the benchmark, for all three models, and performing VaR estimations closer to their respectively desired percentage, 1% and 5%. The only model to outperform the benchmark for all 99% VaR calculation is the DHV Model 2. However, DHV model 1 performs more precise than the benchmark model, with respect to the percentage of times the returns exceed 95% VaR, for Oslo stock exchange index and all the sectors. An interesting observation is that all three Dynamic Historical VaR models, the estimated VaR lay closer to the returns than for the benchmark for the majority of the calculations. The same argument applies for the distance between the returns R and VaR when $R < VaR$.

Comparing these three models between themselves, DHV Model 2 has the most precise results regarding the percentage of times the returns exceed 99% VaR. For the percentage of times the returns exceed 95% VaR DHV Model 1 has the most precise results out of the three. This is also the model that has the lowest value for most of the calculations with regards to the distance between the R and VaR when $R < VaR$, for both confidence levels. DHV Model 2 also performs better or as good as the latter model, for four observations, for this metric. Assessing the cumulative distance between the R and VaR, DHV Model 2 has the smallest distance for most of the Oslo Stock Exchange calculations, for the 95% confidence level. DHV Model 1 has the smallest distance for most of the 99% confidence level calculations. DHV Model 3 has two results for both confidence levels that outperform the two other models.

Assessing the different time periods, DHV Model 1 has VaR estimations, for both confidence levels, closest to the desired 1% and 5% mark the most often out of the three models. DHV Model 3 also outperform the benchmark model, more times than not, in the different time periods. On the other hand, DHV Model 2 is outperformed by the benchmark for the majority of the periods. Period 1 and 5 are the periods where all three MC Models perform more precise VaR calculations than the benchmark. When observing the performance under financial stressing times, period 3 and 6, DHV Model 1 calculates a more precise VaR than the benchmark for most of the calculations, for both confidence levels. The two other models also perform more precise VaR calculations, more often or as many times as the benchmark, in these periods for, both confidence levels. The exception being 99% VaR for period 6, where the three models still are outperformed more times by the benchmark. The results might indicate that in addition to having a more precise overall performance, the DHV Models can be more reliable in financial turmoil compared to historical VaR with fixed rolling window size.

Out of the three models, for Oslo stock exchange, the model which bases itself exclusively on proportional VIX movement performs better than the benchmark and the other models for most of the metrics measured, for both confidence levels. This model is also the most precise when assessing the different time periods and is the most reliable under financial turmoil. Nevertheless, DHV Model 3 can be considered the most versatile model. Moreover, the weighting between the proportional and percentage terms of the model can be adjusted to favor the proportional component, but yet incorporate the attractive attributes of the percentage component.

6.1.2 Monte Carlo simulation

From the results in chapter 5.3 we saw that implementing a macroeconomic variable to historical VaR was beneficial for the VaR calculations. In an attempt to perform even more precise and reliable VaR estimations, implementation of VIX, brent price, gold price and bond price, to a Monte Carlo simulation was performed as described in chapter 5.4. The objective being that a Monte Carlo simulation, with integrated OLS-regression and MLE-estimation of parameters, would capture the relationship between VaR and macroeconomic variables to a deeper extent. Also, to investigate if including more variables could improve the VaR estimations. Constructing a model that uses macroeconomic values to estimate the degrees of freedom for a student-t distributed variable, to be run in a Monte Carlo simulation, yielded interesting results.

Drawbacks for the Monte Carlo simulations is that with the data packs for Python used in this thesis, skewness could not be taken into account when generating pseudorandom variables.

6.1.2.1 S&P 500

Comparing the four Monte Carlo models with each other, we see from the results for S&P 500, presented in chapter 5.4, that running daily regression improves the results. The results also show that implementing more macroeconomic variables, in addition to VIX, improve the results.MC Model 4 that runs daily regression on VIX, brent price, gold price and bond price has the best overall results for the metrics measured. This suggests that for calculations done for S&P 500, including several relevant macroeconomic variables can help improve risk estimation. The results also imply that having a dynamic regression coefficient captures how the relationship between variables change over time, also improve calculations. MC Model 4 has the majority of the most precise results for the VaR estimations and the lowest distances measured, compared to the other Monte Carlo models.

Comparing MC model 4 with the benchmark, the Monte Carlo simulation obtains a more precise VaR estimation for three of the investigated sectors and index, for both confidence levels. With the lowest result for 99% VaR being 1.47% and 5.24% for 95% VaR for the energy sector. The highest values for 99% and 95% VaR being 1.69% and 6.17% for the health care sector and index, respectively. The cumulative distance between R and VaR, for the 99% confidence level, the consumer staples, health care, and information technology sector yield better results than the benchmark. With the lowest distance being 84.38 for consumer staples. The distance, for the 95% confidence level, is lower than the benchmark for all S&P 500 calculations, with the lowest value being 46.00 for consumer staples. These results indicate that on a regular basis, the VaR estimations lay closer to the returns. For the cumulative distance between R and VaR when $R < VaR$, for the 99% confidence level, MC model 4 outperform the benchmark for all calculations except two, health care and information technology. Also, for the 95% confidence level, the energy sector and consumer staples are the only sectors that give a lower distance than the benchmark. This indicates that when the returns exceed VaR, MC model 4 "misses" with less than the benchmark, for 99% confidence level.

With MC model 4 and 250-day rolling historical VaR giving similar results regarding VaR estimations for both confidence levels, comparing the performance in the different time periods is of interest. What we see from the period results is that MC model 4 and the benchmark model perform evenly, but the benchmark performs more precise VaR calculations more often than MC Model 4. However, more important, under financial stressing times, at period 3 and 6, the benchmark gives more precise VaR results. From the time periods, another observation that can be made is that MC model 4 perform better 99% VaR calculations in less volatile times, i.e., period 2 and 5. Meaning that MC Model 4 calculate VaR estimates that are too optimistic under financial turmoil, for both confidence levels, but in less volatile times the benchmark model estimates too pessimistic estimation for 99% VaR.

6.1.2.2 Oslo Stock Exchange

For Oslo Stock Exchange, the results for the different Monte Carlo models, shown in section 5.4, are different compared to the S&P 500. MC Model 2, the model that runs daily regression on VIX, performs best overall. It performs better than the benchmark and the other Monte Carlo models for most off the measured metrics.

MC Model 2 has the most precise VaR estimations for all Monte Carlo models and compared to the benchmark. Except for 99% VaR for the consumer staples sector and 95% VaR for the index. Ranging from health care with 1.27% to the finance sector with 1.6%, for the 99% VaR, and energy with 5.18 % to the index with 5.85%, for the 95% VaR. The cumulative distance between R and VaR, for the 99%, the index, health care, finance, and information technology sector yield better results than the benchmark, with the lowest distance being the index, with a value of 136.2. For the 95% confidence level, MC Model 2 outperforms the benchmark for all calculations with the index performing best, with a value of 74.8. An exception here is MC Model 4, which outperforms all of the other Monte Carlo models and the benchmark, for the distance measured for the 95% confidence level. These results indicate that the VaR estimated in MC Model 4 lay closer to the returns on a regular basis, for the 95% confidence level, and that a daily regression with the macros VIX, gold and brent price improves the VaR estimates. For the distance between R and VaR, when $R < VaR$, MC Model 2 performs best. It outperforms the benchmark for all calculation, except for the financial sector at the 95% confidence level. At the other end of the scale, we find MC Model 4 with being outperformed by the benchmark for six observations. This leads to the conclusion that MC Model 2 both follow the return closely on average and adjust better to financial turmoil than MC Model 4, which follows the return closely on average but does not adjust well to financial turmoil.

The results are also consistent with the findings when we decompose the timeline into periods representing calm and volatile periods. MC Model 2 performs on average best and beats the benchmark most times. The findings here also support that MC Model 2 performs best under financial turmoil as it outperforms the benchmark for the majority of VaR estimations in both periods 3 and 6, for the 99% confidence level. For period 4 and 5, both the MC Model 2 and MC model 4 perform equally well, with respect to the preciseness of VaR estimations for the 99% confidence level. This support the conclusion about the performance above, regarding calm financial times for both models. MC Model 2 outperforms the benchmark regarding the problems of estimating a too pessimistic VaR for calm financial times and estimating a too optimistic VaR under financial turmoil.

6.2 Dynamic Historical VaR versus Monte Carlo simulation

In addition to the results from the different models, other factors when calculating VaR needs to be considered. The computation time for the Monte Carlo simulation compared to the Dynamic Historical VaR is more tedious and is approximately 100 times longer. In general, a Monte Carlo simulation is highly characterized by and only as good as the input data. From the graphs produced for the two models, it is clear that there is significantly more noise, in the form of rapid and substantial changes in daily VaR calculations, for the Monte Carlo simulation. Having a daily VaR that varies to the degree that the Monte Carlo models produce, is unfavorable in practice due to the capital requirements, set by the Basel accords, which determines capital requirements based on the output of a financial institution's internal risk measurement system[12]. Besides, the costs related to re-balancing the portfolio will make it less favourable[39] to use a model that produces a substantial amount of noise for the VaR estimations.

6.2.1 S&P 500

When comparing DHV Model 3 to MC Model 4, presented in section 5.3 and 5.4, DHV Model 3 performs better for most of the calculations done when evaluated in total. Here only considering the calculations where the models perform a more precise VaR calculation than the benchmark and the other model. The historical model estimates a more precise VaR than MC model 4 for almost all S&P 500 calculations. DHV Model 3 has the smallest distance between R and VaR for the 99% confidence level but for the 95% confidence level the VaR calculations for MC Model 4 lay closer to the returns. DHV Model 3 performs the best regarding the sum of the distance between R and VaR when $R < VaR$, for both confidence levels.

When assessing the different time periods an interesting observation is that despite DHV Model 3 performs better in total, MC Model 4 performs more precise calculations for the different time periods, when evaluating both 99% and 95% VaR. This can indicate that the VaR estimates vary more for the historical model but that on average over 20 years, the VaR results seem more precise than the for MC model 4. The results suggest that the historical model tend to estimate a too pessimistic VaR in less volatile periods. So that in long the term this compensates for the optimistic VaR estimations calculated under financial turmoil. When assessing period 3 and 6, where VaR estimation is the most troublesome, MC Model 4 outperforms the historical model for both confidence levels in period 3 and for 95% VaR in period 6. The results indicate that MC model 4 is more reliable under financial turmoil and have less variation over time.

6.2.2 Oslo Stock Exchange

For the Oslo Stock Exchange, we compare the results from DHV Model 3 to MC Model 2, presented in section 5.3 and 5.4, respectively. In this section, we consider only the metrics that outperform the benchmark for the two models. MC Model 2 performs better for the majority of the calculations done, both on average for the whole data sample, and for the six time periods. DHV Model 3 and MC Model 2 performs relatively equally regarding calculating precise VaR estimates compared to the benchmark, and outperform the benchmark for the majority of times. Although, when we look at which of the models that perform closest to the desired values for the calculated metrics, MC Model 2 performs best in total, compared to DHV Model 3.

For the time periods, MC Model 2 estimates a more precise VaR than the benchmark and DHV Model 3. For both confidence levels, MC Model 2 is significantly more precise in period 5 and 6. Moreover, for the 99% confidence level, it outperforms the other model in period 1,3 and 4. With regard to the desired percentage of times the R exceed VaR, we observe that the MC Model 2 seems to perform better for the index, energy and consumer staples sector, while the DHV Model 3 performs better for the health care, financial and information technology sectors.

6.2.3 Comparison between Oslo Stock Exchange and S&P 500

From the data presented in subsection 4.3 and presented in table 4, the data clearly indicate that the S&P 500 index and sectors are more leptokurtic and have lower degrees of freedom than the Oslo Stock Exchange data. In addition, the S&P 500 data has a lower standard deviation over a two-decade horizon. What the models have in common, is that for the indexes, the cumulative distances between R and VaR is lower for S&P 500 compared to Oslo Stock Exchange. Which might indicate that S%P 500 is easier to estimate a more precise Value-at-Risk calculation for, than for Oslo Stock Exchange. Which again can be linked to S&P 500 being diversified to a much greater extent than Oslo Stock exchange.

From the DHV models, there is no unambiguous indication that either of the two markets is more challenging to calculate VaR for using the methods of this thesis, based on the results and discussion above and comparing to their respective benchmarks. However, from the Monte Carlo models, results for the different models indicate that there can be a distinction between the markets, in relation to risk calculation using the models from section 5.4. With MC model 4 being the better model for S&P 500 and MC model 2 being the better model for Oslo Stock Exchange. This implies that including brent and gold price, in addition to VIX, to estimate VaR using the models specified in section 5.4 does not strengthen the risk estimate for Oslo Stock Exchange. This does not exclude the possibility that these variables can improve VaR estimations for other models. The fact that 35% of the Oslo Stock Exchange value consists of shares from the energy sector, presented in table 3, and the fact that Norway is an oil exporting nation suggest that using brent price as an input for calculating VaR for the Norwegian market can be advantageous for other models. Considering the effect of gold price, on the degrees of freedom in the VaR estimations made on Oslo Stock Exchange, not having the effects as initially anticipated, together with brent price, can be linked to the Norwegian market being a niche market or in the same category as other emerging markets, presented in section 2.4.3. Consequently, investors suffering losses turn to developed markets rather than gold, when reallocating assets. The assumption that VIX is a global macro variable is supported by the results from the Oslo Stock Exchange calculations. Where the results from the Dynamic Historical VaR yield as good results for Oslo Stock Exchange, as the calculations performed on S&P 500. And for MC model 2, VIX works better to estimate VaR for the Norwegian market than the American market. Arguably the Oslo Stock Exchange calculations might have been improved if a similar volatility index based upon Oslo Stock Exchange market expectation was used to scale the window size of in the Dynamic Historical VaR models and as input in the Monte Carlo model.

6.2.3.1 The sectors

When comparing the sector results for both the DHV Models and MC Models, the percentage of times R exceed VaR for both confidence levels does not vary a lot from sector to sector for both markets. However, where the two markets differ regarding the behavior of sectors is when considering the cumulative distance between R and VaR and the distance when $R < VaR$. One thing that the models have in common is that the results for the consumer staples sector for S&P 500 stands out with low values for the distance between R and VaR in total and for when $R < VaR$. The same argument can be applied to the consumer staples sector for Oslo Stock Exchange but does not emerge from the results to the same degree as for S&P 500. The fact that the consumer staples sector consists of essential goods and products that are considered non-cyclical, meaning that they are always in demand regardless of the economy, can be considered as a contributor to why the VaR estimations lay closer the returns for this sector. The sectors that stand out in regards to high distances is energy and financials, which conforms with the fact that these are the two sectors with the highest standard deviation and kurtosis, for S&P 500. For Oslo Stock Exchange, the information technology sector tends to give the highest distances and is also the sector with the highest kurtosis and a high standard deviation.

The significant difference in diversification and volume between the two markets could conceivably be the reason why the sectors behave differently for the two markets. Where for Oslo Stock Exchange a few companies, or in some cases one company, make out a significant amount of the whole sector and consequently defining the behavior of the sector on the basis of its performance. The health care and information technology sectors are small sectors with a low daily volume, i.e., low liquidity. This can lead to an imbalance in trade orders and sudden drastic price movements, i.e., higher volatility.

7 Conclusion

In this thesis, a variety of Value-at-Risk models has been developed and tested for S&P 500 and Oslo Stock Exchange. In general, when developing and using methods for VaR challenges regarding the left tail is the primary concern. Especially in times of financial turmoil, as in its nature occurs suddenly and surprisingly. Moreover, for this purpose, we have introduced the concept of using macroeconomic variables as input data to the two most used Value-at-Risk models in the banking sector, Monte Carlo simulation and historical VaR. Based upon the results and discussion above, what is evident is that by implementing macroeconomic variables into different Value-at-Risk models can help to improve risk estimation for S&P 500 and Oslo Stock Exchange. Also, with Basel III still being in its implementation phase, further research and investigation can be advantageous for financial institutions in order to adapt to the new standards.

A study on how different window sizes for historical VaR affected VaR's ability to adapt to new information was conducted, without unambiguously obtaining information about how large a data window should be to perform optimally, especially under financial turmoil. Where in an attempt to solve this problem, the Dynamic Historical VaR Models emerged. The results, presented in section 5 and discussed in section 6, indicate that using VIX as an input to dynamically determine the window size for historical rolling VaR, improves the VaR estimations performed on both markets compared to their respective benchmark models. The Dynamic Historical VaR model that is considered the best and most reactive out of the three is DHV Model 3. This model also has the possibility to change the ratio between the proportional and percentage terms in equation 15, to further improve and optimize the risk estimates. This can be of interest and serve as motivation for further work.

For the Monte Carlo models, the model that is considered to perform best for S$P 500 is MC Model 4, and for Oslo Stock Exchange MC Model 2 is considered the best. This again, supports that VIX is a decent forward-looking macro variable for both the American and Norwegian market. However, the implementation of gold and brent price together does not strengthen VaR estimates for Oslo Stock Exchange, as it does for S&P 500 together with bond price. However, we only studied the effects of a few macroeconomic variables, and other macroeconomic variables that are considered relevant can be implemented or replace the variables used in this thesis, e.g., salmon price, non-farm payrolls, GDP, et cetera, to improve the models. For the Monte Carlo simulations, a model that takes skewness into account and performs a more advanced regression analysis, e.g., GARCH, can be further investigated to improve calculations.

Currently, the Dynamic Historical VaR Model 3 seemingly produces the best results over time, but further investigation shows that the model has more difficulties with pessimistic VaR estimation in calm financial times and optimistic VaR estimations under financial turmoil when compared to the Monte Carlo models. However, at the end of this thesis, this is the model that is the readiest to be used in practice and considered an improvement compared to the benchmark model. The Monte Carlo models have too much daily variation in the VaR calculated to be used in its current state. Further work addressing these challenges is required before the model is ready to use. Despite these challenges, the Monte Carlo model is considered to have the most potential for risk estimation, out of the models developed. The most intriguing results for the Monte Carlo simulations is found when studying the different time periods. This paper concludes that the Monte Carlo models, MC model 4 for S&P 500 and MC model 2 for Oslo Stock Exchange, produce more precise calculations for the different periods, and is able to adapt to financial distressing times and back to market equilibrium, to a greater extent than the other models investigated.

For the American and Norwegian market, we can conclude that the impact and relevance of the macroeconomic variables, used in this thesis, differ for the two markets. The markets also differ in how close the VaR estimations lay to the returns. Meaning that more precise calculations are easier to perform on S&P 500 with regards to target values. The results of this thesis also suggest that more risk is linked to the specific sectors investigated for both markets. And consequently, estimating risk using the methods of this thesis is adequately more challenging for these sectors. In the American market, more risk is linked to the energy and finance sector, while the consumer staples sector carries less risk. While for the Norwegian market, the different sectors do not differ to the same extent in terms of carrying risk. However, the information technology sector being more challenging to calculate a precise estimate risk for, while consumer staples being less challenging to calculate risk for.

The main result of this thesis can conclude that the analysis performed, emphasize and support the challenges tied to estimating VaR addressed by previously conducted research. With emphasize on testing how the VaR models perform for different time periods and across different types of portfolio, where the indexes represent diversified portfolios, and the sectors represent more specialized portfolios. Implementation of VIX, and the other macroeconomic variables used in this thesis can improve historical VaR with rolling window and VaR calculated by Monte Carlo simulation performed on S&P 500 and Oslo Stock Exchange. By taking into account these variables, VaR estimations lay closer to the desired 1% and 5% mark for the 99% and 95% confidence levels, respectively, compared to the benchmark model. In addition, the VaR estimate lay closer to the returns, which is favorable in regards to minimum capital requirements. However, in general, both of the models that include macroeconomic variables as an input yield too pessimistic and imprecise VaR estimates for the whole data sample, e.g., results as 1.47% for 99% VaR and 5.53% for 95% VaR, with different periods containing optimistic and pessimistic estimates. Although showing interesting and promising results, further work is needed in the form of sufficient back-testing and optimization, before we can conclude that financial institutions should use the models.

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Appendix A Historical VaR with rolling window

Appendix A contains the benchmark result, where the 250-day window is highlighted. For results in the different periods, see external Excel file.

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
1			SPX							SPX10GI					
2			22	44	**250**	500	750	1000		22	44	**250**	500	750	1000
3	% exceeding the VaR	99	5.31 %	3.15 %	**1.68 %**	1.63 %	1.51 %	1.53 %		5.20 %	3.42 %	**1.52 %**	1.59 %	1.34 %	1.40 %
4		95	9.23 %	6.79 %	**5.48 %**	5.45 %	5.08 %	5.05 %		9.08 %	6.89 %	**5.88 %**	5.89 %	5.23 %	5.41 %
5	Sum of diff between VaR and observation	99	70.07	91.3	**116.19**	116.78	117.19	115.67		109.31	127.87	**161.46**	171.21	159.55	154.48
6		95	58.86	63.67	**69.17**	66.87	65.75	62.63		83.39	89.72	**94.59**	92.16	87.05	81.82
7	Sum of diff when R exceeds VaR	99	1.47	0.96	**0.64**	0.69	0.66	0.74		2.01	1.3	**0.89**	1.06	0.85	0.83
8		95	2.17	1.77	**1.98**	1.94	1.91	2.04		2.91	2.28	**2.94**	3.16	2.64	2.67
9															
10			SPX30GI							SPX35GI					
11			22	44	**250**	500	750	1000		22	44	**250**	500	750	1000
12	% exceeding the VaR	99	5.29 %	3.02 %	**1.57 %**	1.33 %	1.29 %	1.43 %		5.22 %	3.06 %	**1.63 %**	1.59 %	1.34 %	1.43 %
13		95	9.16 %	7.25 %	**5.66 %**	5.59 %	5.08 %	5.33 %		9.00 %	6.83 %	**5.61 %**	5.32 %	4.76 %	4.79 %
14	Sum of diff between VaR and observation	99	61.25	72.94	**86.1**	84.23	82.86	80.81		75.25	89.18	**111.3**	112.26	111.41	105.49
15		95	45.33	47.7	**50.79**	48.43	45.92	42.6		56.88	61.22	**64.12**	61.88	58.53	55.08
16	Sum of diff when R exceeds VaR	99	1.29	0.76	**0.56**	0.44	0.41	0.45		1.49	0.96	**0.6**	0.6	0.49	0.54
17		95	1.92	1.56	**1.53**	1.45	1.32	1.36		2.28	1.81	**1.88**	1.85	1.65	1.57
18															
19			SPX40GI							SPX45GI					
20			22	44	**250**	500	750	1000		22	44	**250**	500	750	1000
21	% exceeding the VaR	99	5.18 %	3.17 %	**1.70 %**	1.70 %	1.66 %	1.82 %		4.89 %	3.02 %	**1.48 %**	1.57 %	1.51 %	1.53 %
22		95	9.12 %	7.04 %	**5.68 %**	5.64 %	5.57 %	5.31 %		9.02 %	6.69 %	**5.18 %**	5.36 %	5.03 %	4.97 %
23	Sum of diff between VaR and observation	99	109.47	129.51	**160.78**	172.3	181.47	185.56		94.69	110.35	**128.8**	130.3	125.64	123.06
24		95	81.71	87.83	**96.77**	90.09	92.64	92.08		70.79	75.82	**77.3**	75.27	72.33	68.13
25	Sum of diff when R exceeds VaR	99	2.14	1.33	**0.97**	1.12	1.22	1.41		1.7	1.01	**0.6**	0.65	0.61	0.65
26		95	3.37	2.7	**2.92**	3.36	3.35	3.33		2.88	2.01	**1.92**	1.95	1.76	1.77
27															
28			OSEBX							OSE10GI					
29			22	44	**250**	500	750	1000		22	44	**250**	500	750	1000
30	% exceeding the VaR	99	5.70 %	3.51 %	**1.69 %**	1.47 %	1.11 %	1.11 %		5.17 %	3.16 %	**1.67 %**	1.65 %	1.36 %	1.29 %
31		95	9.82 %	7.82 %	**5.76 %**	5.72 %	5.21 %	5.16 %		9.17 %	7.76 %	**5.76 %**	5.47 %	5.48 %	5.32 %
32	Sum of diff between VaR and observation	99	92.56	109.9	**141.74**	150.62	152.53	153.18		112.91	131.37	**161.94**	173.66	171.98	166.55
33		95	70.26	76.74	**82.89**	80.27	76.74	72.87		86.14	91.95	**98.9**	93.73	89.35	84.33
34	Sum of diff when R exceeds VaR	99	1.82	1.18	**0.78**	0.82	0.65	0.68		1.98	1.26	**0.84**	0.91	0.73	0.75
35		95	2.85	2.31	**2.44**	2.9	2.82	2.82		3.13	2.86	**2.46**	2.86	3.06	2.81
36															
37			OSE30GI							OSE35GI					
38			22	44	**250**	500	750	1000		22	44	**250**	500	750	1000
39	% exceeding the VaR	99	4.90 %	3.13 %	**1.43 %**	1.11 %	1.11 %	1.08 %		5.34 %	3.47 %	**1.67 %**	1.37 %	1.26 %	1.47 %
40		95	9.34 %	7.36 %	**5.78 %**	5.33 %	5.36 %	5.50 %		9.61 %	7.55 %	**5.74 %**	5.51 %	5.43 %	5.45 %
41	Sum of diff between VaR and observation	99	100.02	117.81	**143.02**	144.21	143.32	139.4		124.68	149.24	**172.89**	168.44	157.51	150.24
42		95	73.87	78.72	**84.48**	81.63	77.21	72.41		86.7	94.63	**92.25**	87.07	82.07	76.46
43	Sum of diff when R exceeds VaR	99	1.84	1.27	**0.67**	0.61	0.52	0.45		2.72	1.87	**1.25**	1.08	1.04	1.04
44		95	2.96	2.6	**2.41**	2.16	2.34	2.35		4.28	3.53	**3.53**	3.45	3.3	3.06
45															
46			OSE40GI							OSE45GI					
47			22	44	**250**	500	750	1000		22	44	**250**	500	750	1000
48	% exceeding the VaR	99	5.36 %	3.45 %	**1.63 %**	1.56 %	1.31 %	1.26 %		5.36 %	3.09 %	**1.34 %**	1.12 %	0.91 %	0.90 %
49		95	9.32 %	7.11 %	**5.74 %**	5.47 %	5.14 %	5.24 %		9.40 %	7.26 %	**5.23 %**	4.96 %	4.32 %	4.27 %
50	Sum of diff between VaR and observation	99	102.42	121.5	**152.2**	166.8	168.2	162.67		136.24	164.87	**192.82**	185.87	177.24	166.61
51		95	75.81	81.98	**89.42**	87.5	85.05	81.5		94.52	103.61	**106.36**	100.78	96.17	90.01
52	Sum of diff when R exceeds VaR	99	2.1	1.39	**0.97**	0.96	0.91	0.95		3.02	2.17	**1.28**	1.09	0.72	0.6
53		95	3.17	2.53	**2.72**	2.99	3.17	3.35		4.47	4.4	**3.53**	3.34	2.72	2.54

Appendix B Dynamic Historical VaR

Appendix B is structured in the following way:

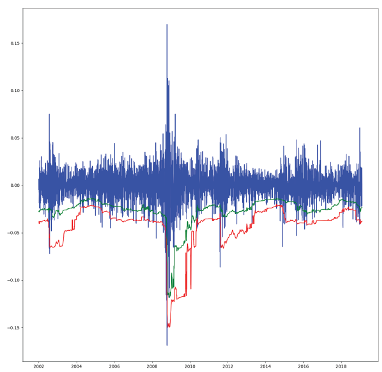
- The results are presented. The cells that are highlighted green are results considered better than the benchmark model. The cells that are highlighted dark green are results considered the best in total, between the DHV models and benchmark.
- The results from the different time periods are presented. The results for periods follow the same color coding.
- The graphs for 99% and 95% VaR are presented in the following order from top left to bottom right: energy (10GI), consumer staples (30GI), health care (35GI), financials (40GI), information technology (45GI).

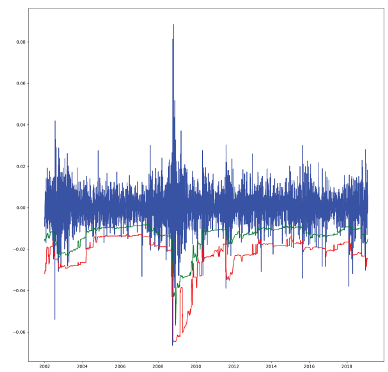
B.1 Results

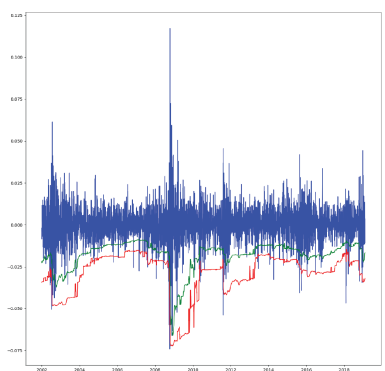
For results, see external Excel file.

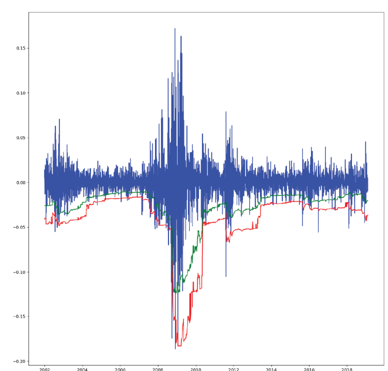
B.2 Graphs

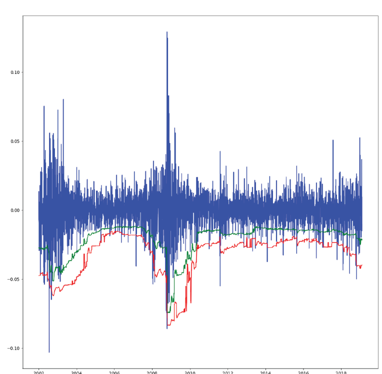
B.2.1 DHV Model 1 S&P 500



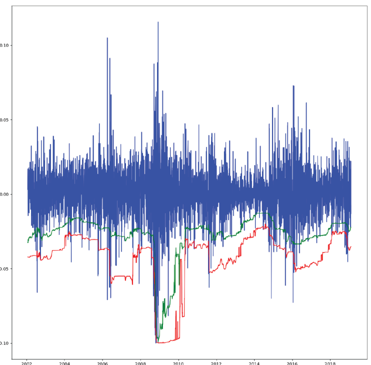


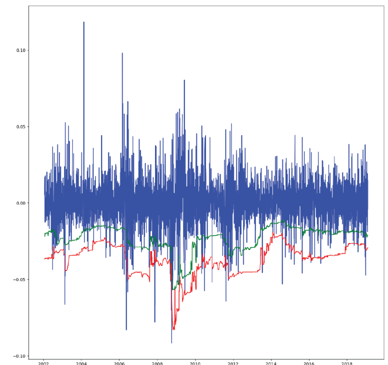


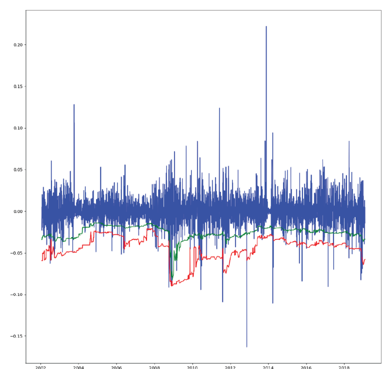


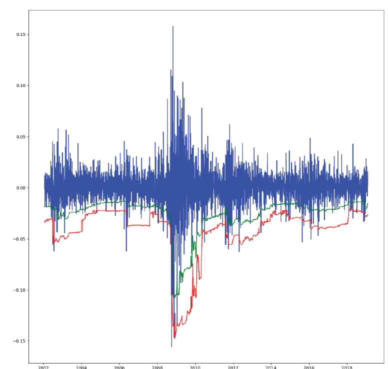


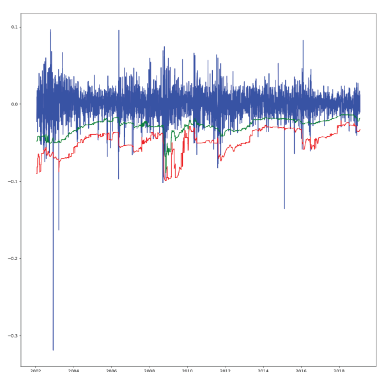
B.2.2 DHV Model 1 Oslo Stock Exchange



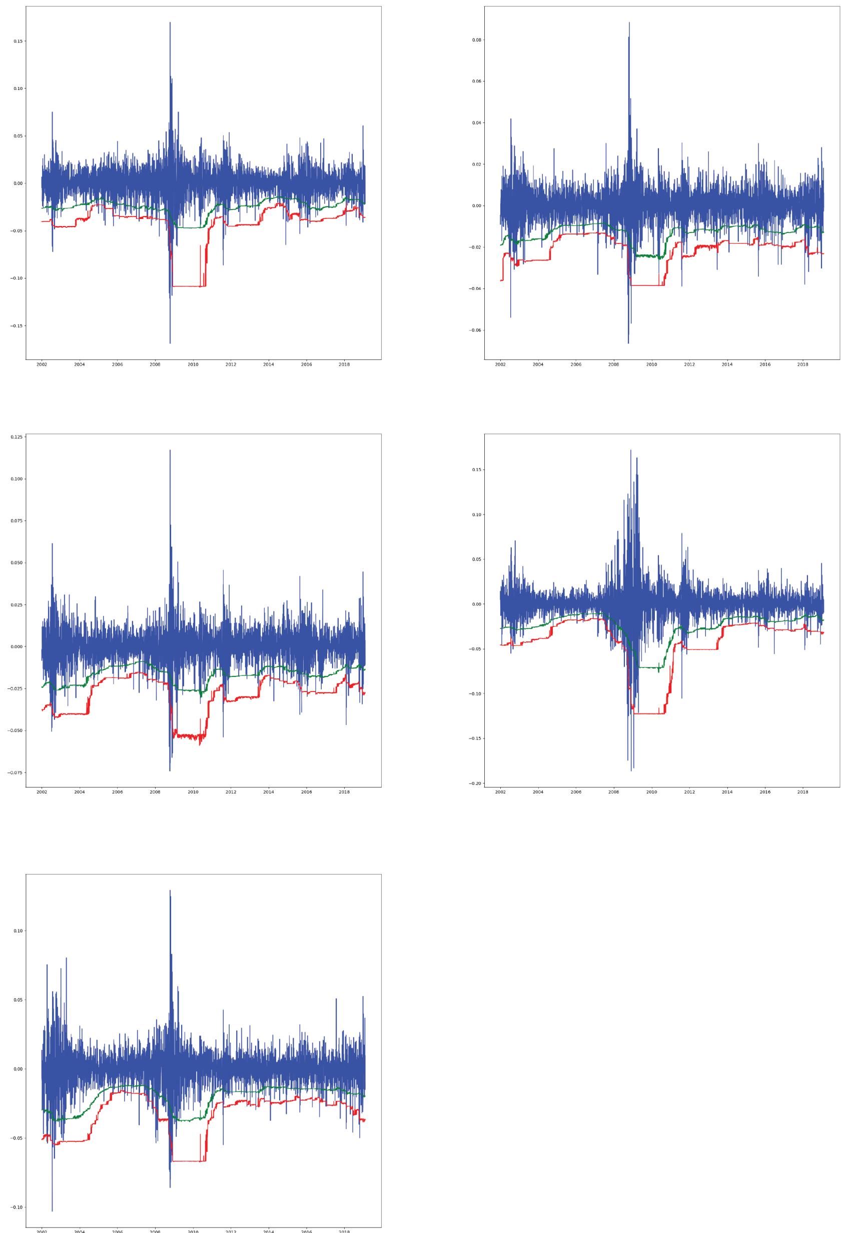




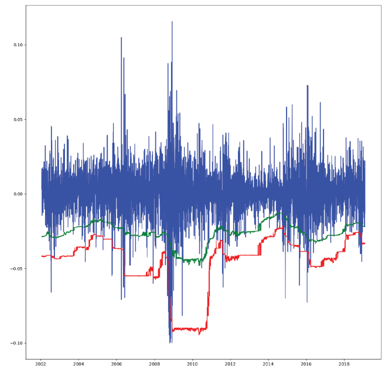


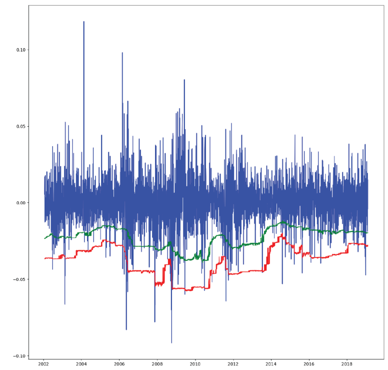


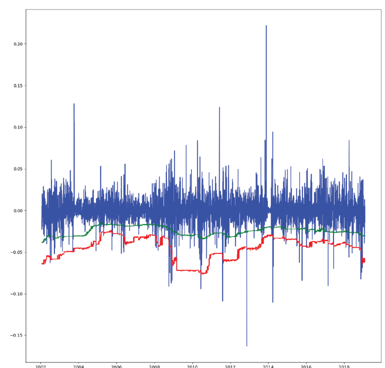
B.2.3 DHV Model 2 S&P 500

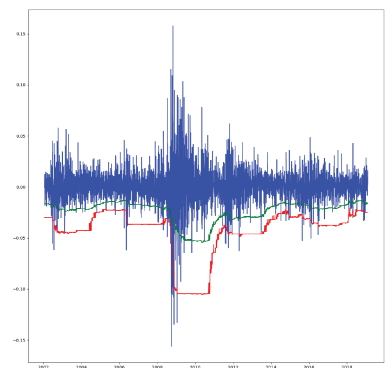


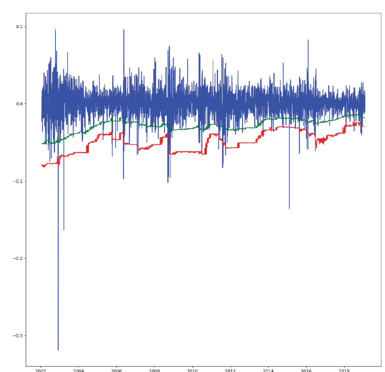
B.2.4 DHV Model 2 Oslo Stock Exchange



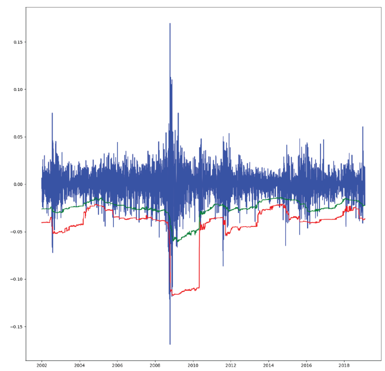


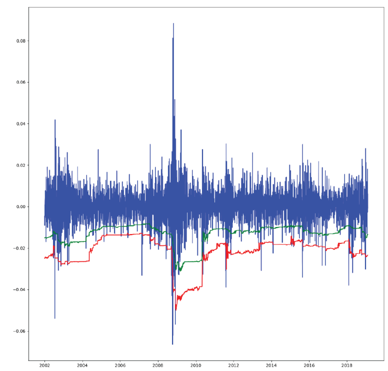


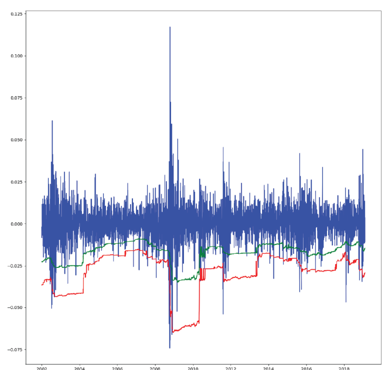


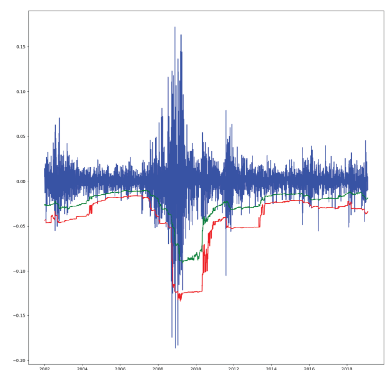


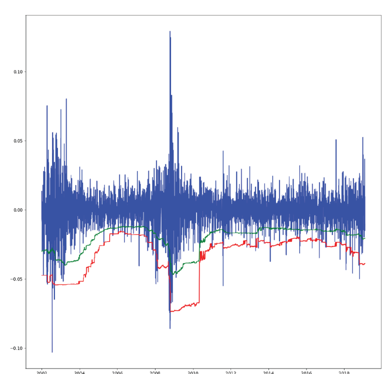
B.2.5 DHV Model 3 S&P 500



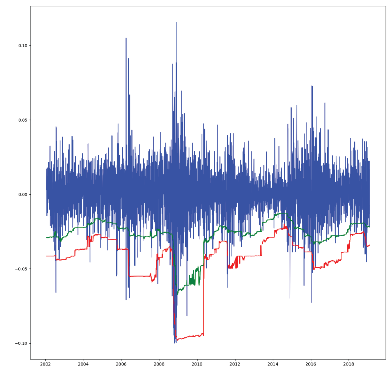


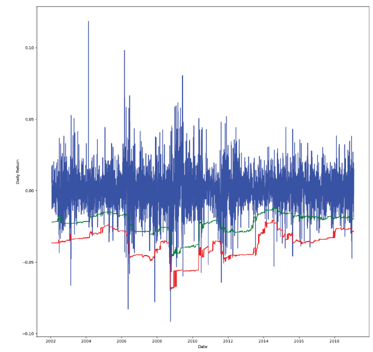


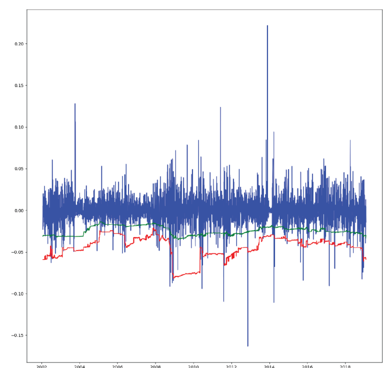


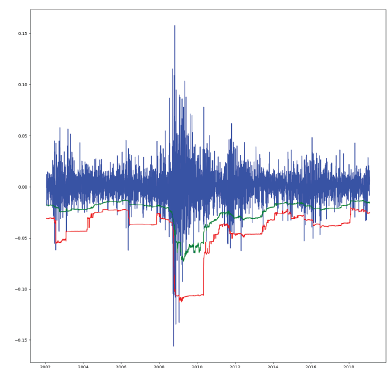


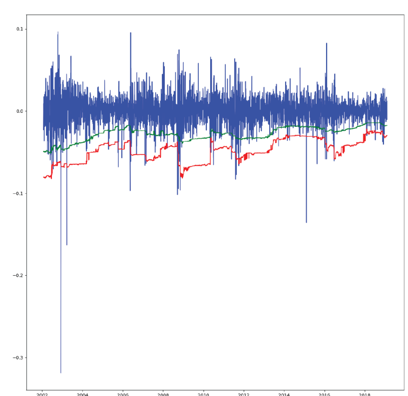
B.2.6 DHV Model 3 Oslo Stock Exchange











Appendix C Monte Carlo simulations

Appendix C is structured in the following way:

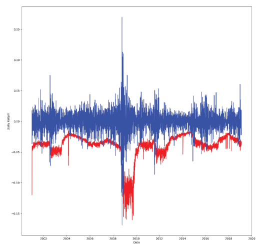
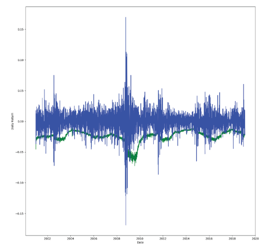
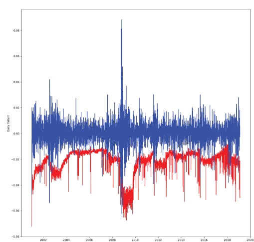
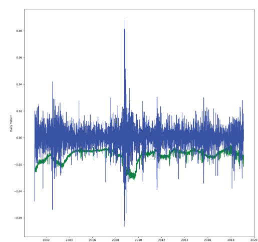
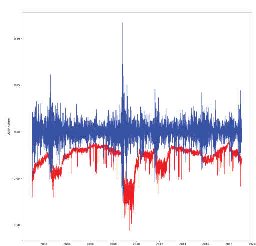
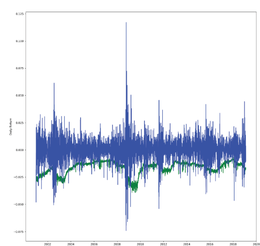
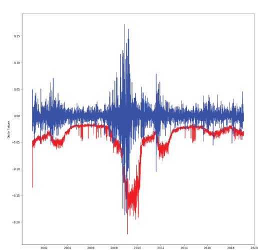
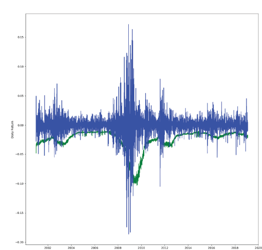
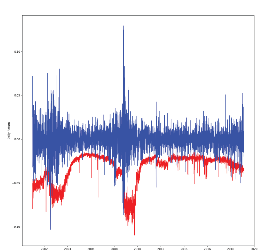
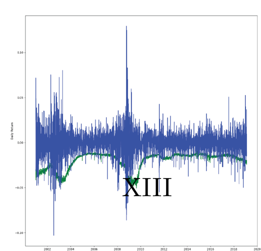
- The results are presented. The cells that are highlighted green are results considered better than the benchmark model. The cells that are highlighted dark green are results considered the best in total, between the MC models and benchmark.
- The results from the different time periods are presented. The results for periods follow the same color coding.
- The graphs for 99% and 95% VaR are presented in the following order, from top to bottom: energy (10GI), consumer staples (30GI), health care (35GI), financials (40GI), information technology (45GI).

C.1 Results

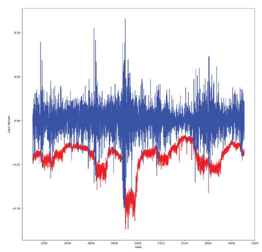
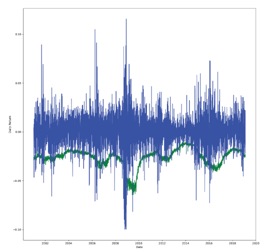
See external Excel file.

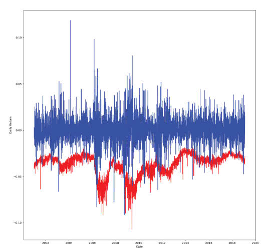
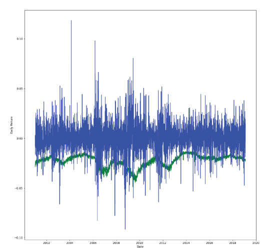
C.2 Graphs

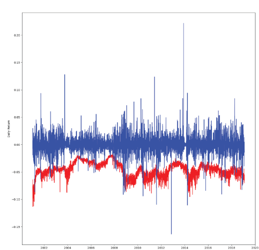
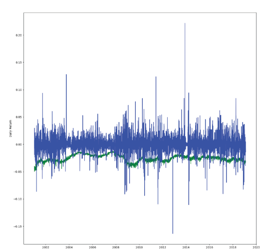
C.2.1 MC Model 1 S&P 500

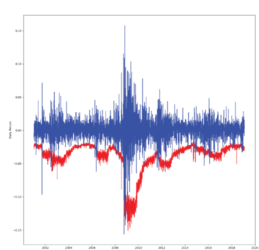
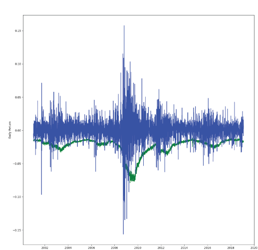











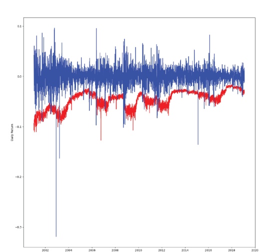
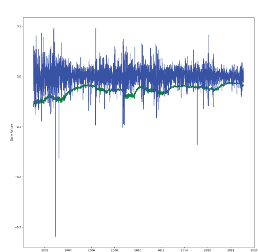
C.2.2 MC Model 1 Oslo Stock Exchange

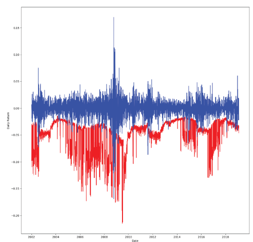



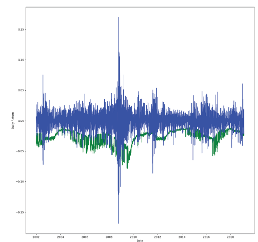



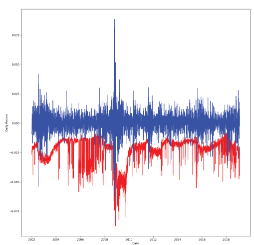



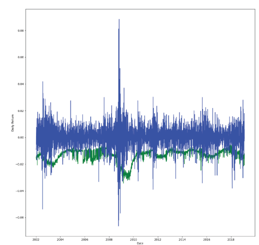



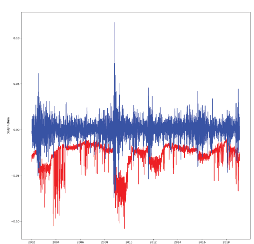
C.2.3 MC Model 2 S&P 500

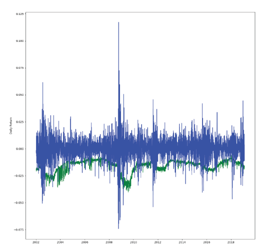


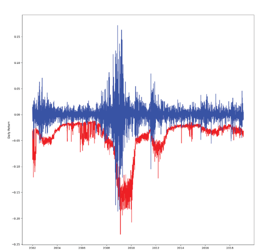


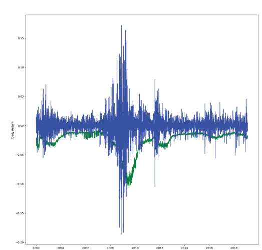


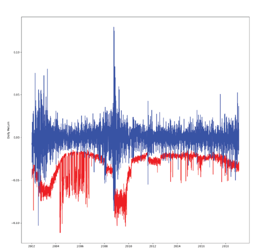


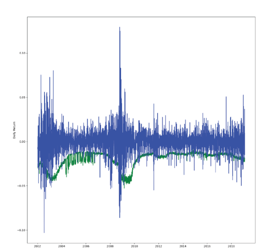




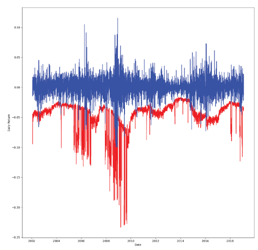


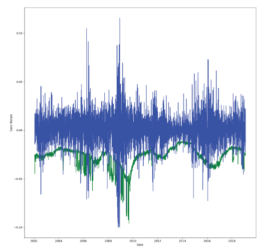


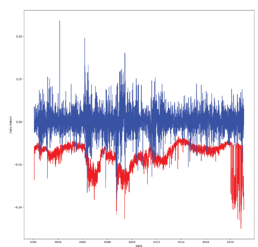


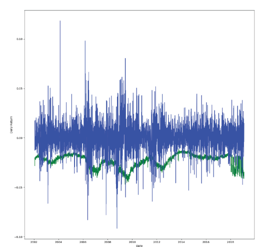


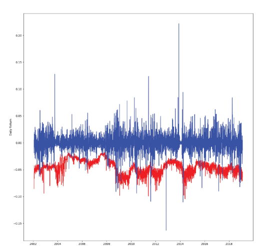
C.2.4 MC Model 2 Oslo Stock Exchange

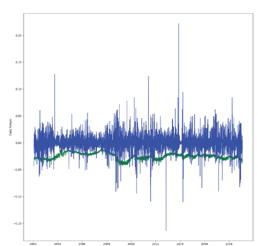


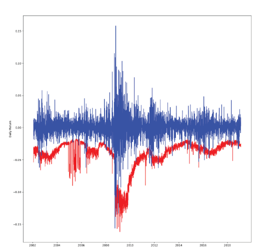


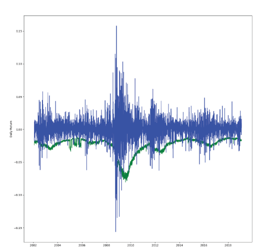


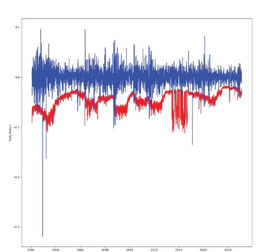


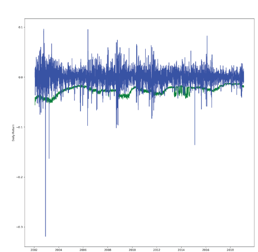




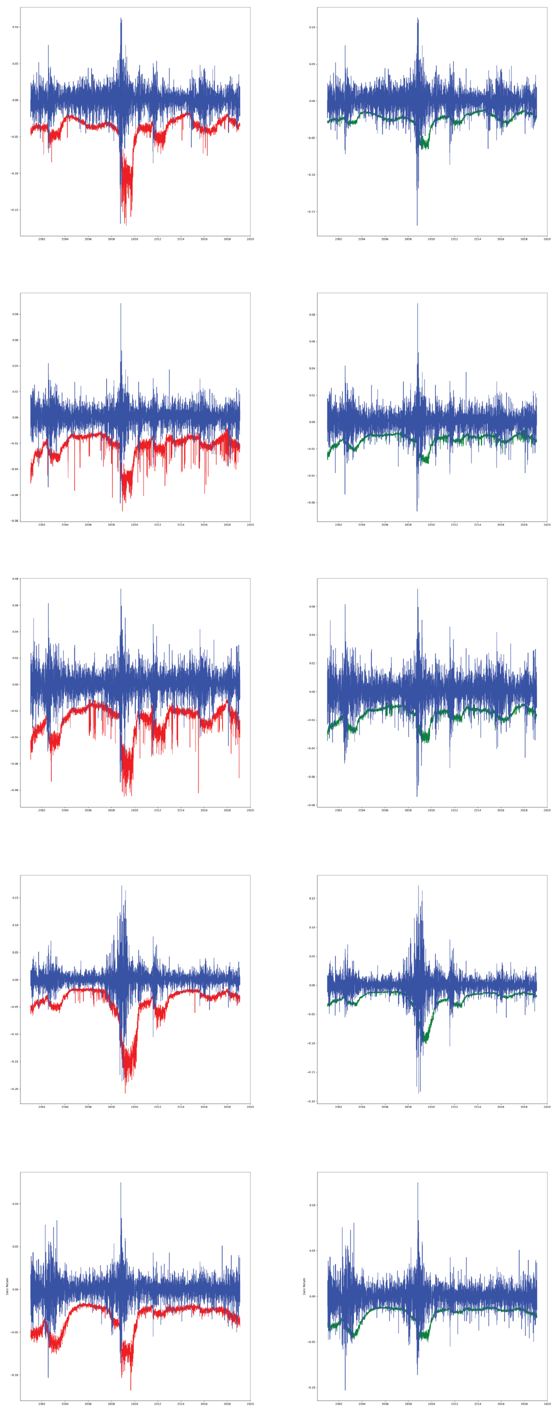




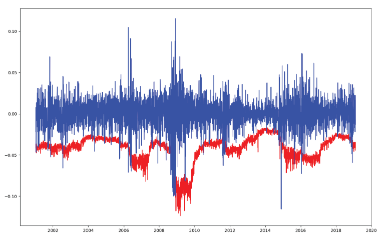
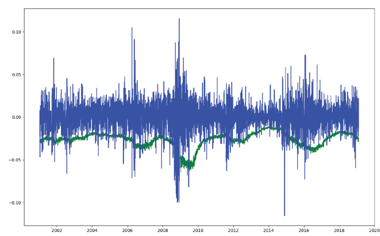
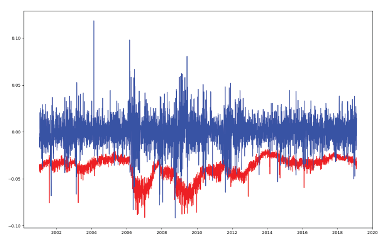
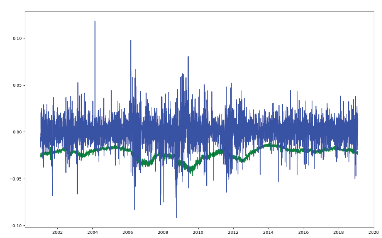
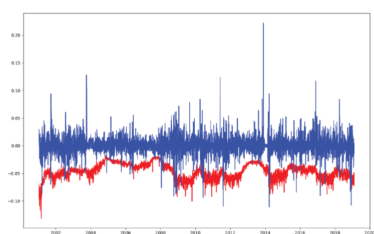
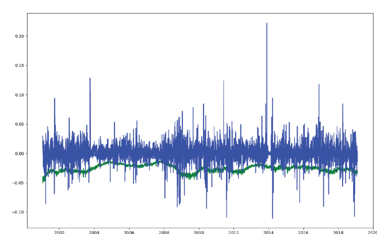
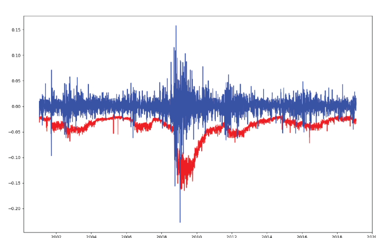
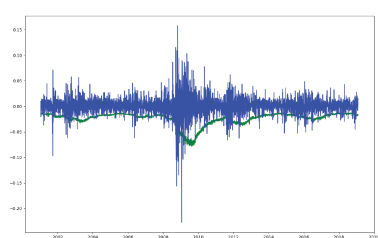
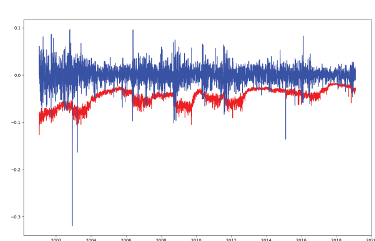




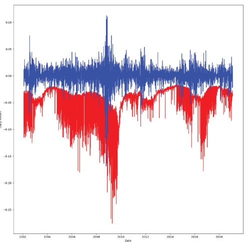
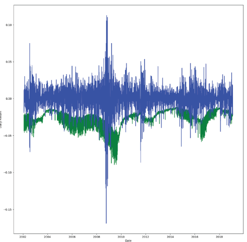
C.2.5 MC Model 3 S&P 500

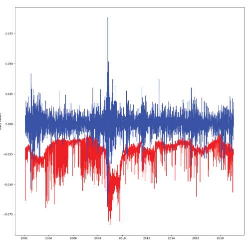
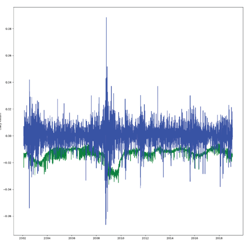


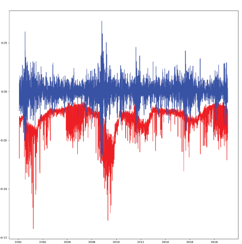
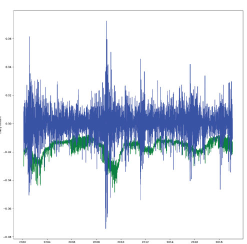
C.2.6 MC Model 3 Oslo Stock Exchange

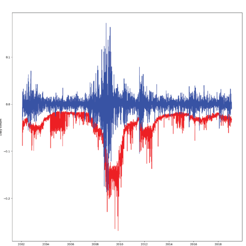
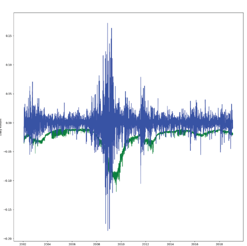










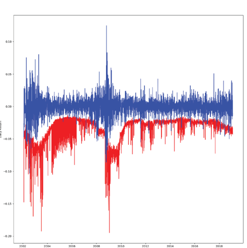
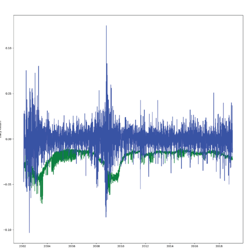

C.2.7 MC Model 4 S&P 500

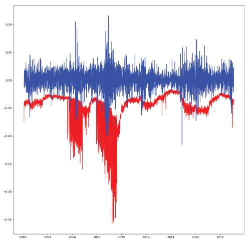
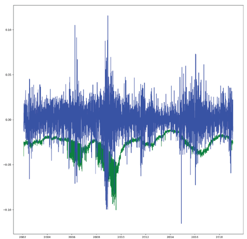
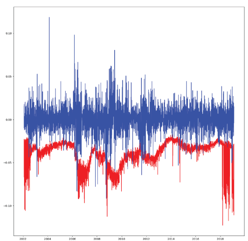
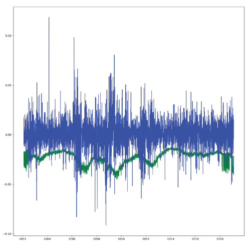
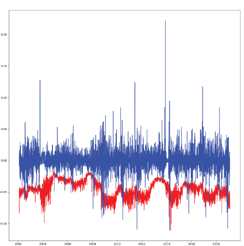
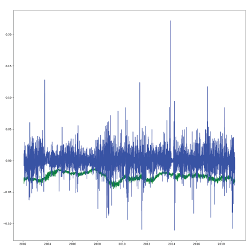
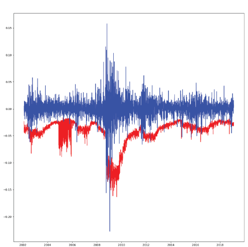
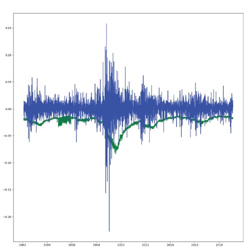
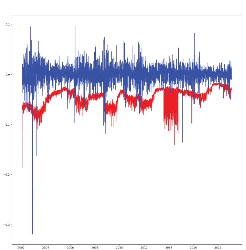
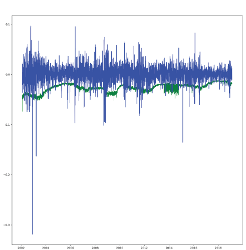



C.2.8 MC Model 4 Oslo Stock Exchange

Appendix D DHV versus MC

The results that are highlighted green are results considered better than the benchmark model. The cells that are highlighted dark green are results considered the best in total, between the DHV model, MC model and benchmark.

See external Excel file.