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Author: Sølve Sætre Sem	
Programme coordinator: Dimitrios Pavlou Supervisors: Dimitrios Pavlou, Jørgen Apeland (external)	
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Optimization of Multirotor UAV Endurance

Sølve Sætre Sem

Supervisor: Dimitrios Pavlou
Associate Supervisor: Jørgen Apeland

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University of Stavanger
Norway

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Abstract

Advancements in the technology of electric motors, batteries and control modules have facilitated a steep increase in the interest of small battery-powered multirotor aircraft. Unmanned areal vehicles (UAV) of this type are mechanically simple, robust, have high maneuverability and a compact size due to their propulsion system. A critical issue in the design process is the UAV's limited flight time, or endurance, due to inefficiencies and a relatively low endurance to weight ratio. This thesis gives an overview of the factors that influence endurance, defines analytical models for the estimation of endurance and analyses the impact of propeller diameter, configuration and battery size for a 25 kg maximum take-off weight UAV. The endurance is found to be sensitive to all three factors. A case analysis is performed to compare a set of design options to the existing "Staaker BG-200" UAV produced by Nordic Unmanned AS, and improvement recommendations are presented. The maximum estimated endurance increase found is 61%, resulting in an endurance of 96 minutes and 36 seconds. This design is estimated to have a 91% increase in UAV length and a 52% decrease in payload capacity. A more practical design was recommended, with 23% increased endurance and $1892 decreased propulsion system cost compared to the Staaker BG-200 UAV.

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CHAPTER 1

Introduction

Advancements in the technology of electric motors, batteries and control modules have facilitated a steep increase in the interest of small battery-powered multirotor aircraft capable of hovering and vertical take-off and landing. The interest spans both recreation and commercial applications such as aerial photography, maintenance inspection, 3D reconstruction, search and rescue and disaster prevention [1]. Unmanned areal vehicles (UAV) of this type are mechanically simple, robust, have high maneuverability and a compact size due to their propulsion system. However, the associated drawbacks include efficiency losses compared to singe-rotor aircraft, and a low ratio between flight time and weight typical of battery powered electrical systems. This makes the flight time, henceforth addressed as endurance, a critical issue in the design process.

Nordic Unmanned AS is a company that uses unmanned areal systems to deliver services and solutions. The company was the driver behind the present thesis as well as two parallel theses. In one of these theses two Bachelor students conducted an experimental study on the performance of varying propeller sizes and the efficiency loss associated with coaxial rotors. Coaxial rotors have pairs of two rotors aligned on one axis, like in Figure 1.1. The other thesis studied the same by using numerical simulation. The present thesis will study endurance more generally, although rotor configuration is set as a primary interest. The basis for these three theses is the company's existing UAV called Staaker BG-200, which is a rotary wing coaxial 8-rotor aircraft with a maximum take-off weight (MTOW) of 25 kilograms.

FIGURE 1.1: Staaker BG-200

The main problem of this thesis is defined as: *How can the endurance of multirotor UAVs be optimized?* To pursue the answer to this question three scientific goals have been identified:

- Determine the factors that influence endurance
- Quantify the critical factors
- Suggest improvements to the Staaker BG-200 UAV and estimate the associated endurance improvement

First, an overview of the relevant factors will be given in chapter 2. Next, chapter 3 will focus on a selection of these factors and present analytical models. these models will then be used in chapter 4 for quantitative analysis. Based on these results, a selection of designs will be compared to the Staaker BG-200 in a case analysis. Finally, a recommendation to Nordic Unmanned will be presented along with suggestions for future work.

CHAPTER 2

Overview

In literature, such as References [2]–[5], endurance t is typically estimated as the ratio of the energy capacity E and the power consumed P.

$$t = \frac{E}{P} \tag{2.1}$$

There is a large number of factors that affect the energy capacity and the power consumption of multirotor UAVs. This thesis will attempt to organize the factors using the following three categories, and each category will be addressed in the present chapter:

(1) Propulsion system
(2) Electronics
(3) Airframe

2.1 Propulsion System

The components included in the propulsion system of a battery powered multirotor UAV is here defined as the propellers, motors, electronic speed controllers (ESC) and the battery/batteries, similar to the definition used in Reference [6]. These components are directly involved in the process of converting stored energy into aerodynamic lift. Together they account for a minimum of 46% of the weight of the Staaker BG-200 reference UAV. Understandably, optimization of the electrical propulsion system is critical to the vehicle's endurance.

The electronic speed controller will not be studied in detail in this thesis, but a brief explanation follows. The ESC is connected between the power supply and the motor. It controls the motor speed by manipulating the current or the voltage going to the motor based on signals from the control module. For brushless motors the ESC must convert the DC output of the battery to three-phase AC. Off-the-shelf ESCs can be purchased as a singe central unit to control all motors, or separate units intended to control one motor each. The ESC has to accommodate the necessary current strength and should be designed to minimize weight and maximise conversion efficiency.

2.1.1 Battery

Perhaps the most obvious component to consider for endurance is the battery. Increasing the battery size increases the energy capacity but also the mass of the propulsion system. This has diminishing returns for a given rotor setup due to the increased power consumption needed to lift the UAV. Therefore, the chosen battery should have a high specific energy. Specific energy is the amount of energy stored per mass, and may be measured in watt-hours per kilogram. Importantly, the battery must also be able to deliver a sufficient current to meet the power demand. Lastly the battery must be rechargeable. As an example, lithium-ion batteries have a high specific energy but a relatively low maximum current delivery [7]. Lithium-polymer (Li-Po) batteries on the other hand have seen wide adoption by small UAVs because they have both high specific energy and high maximum current delivery.

2.1.2 Propeller

For fixed wing applications, as well as for multirotor UAVs intended for high speed operations, the propellers are designed to have peak performance when flying at higher speeds. The propeller of a multirotor with hover and low speed applications should be specifically designed for hover efficiency. There is an inherit trade off between hover efficiency and cruising efficiency in the design of a propeller. This is in part due to the fact that the angle of attack of the propeller blade changes as the incoming air speed relative to the UAV changes. A propeller designed for high hover efficiency should have a relatively low pitch to diameter ratio, such that the angle of attack is optimal when the relative air speed is close to zero. Pitch controls the angle between the chord line and the plane of rotation, and should not be confused with angle of attack between the chord line and the wind direction relative to the rotating blade.

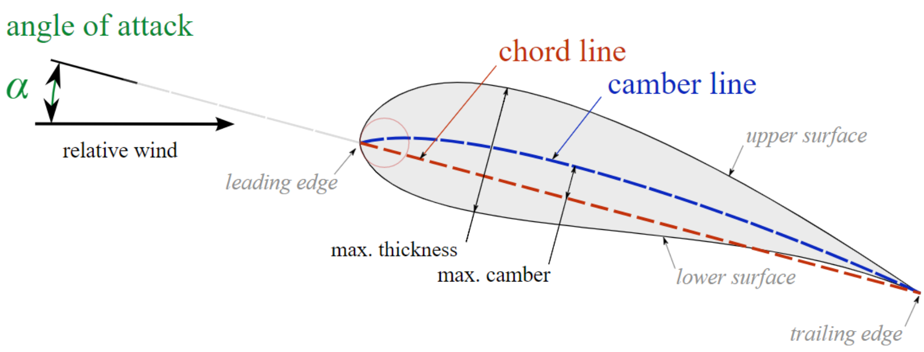

FIGURE 2.1: Airfoil Cross Section [8]

Winslow et. al. [9] did an experimental study on a micro quadrotor UAV to optimize endurance. The study found that to optimize endurance the propeller needed high solidity, a long chord length, camber of 4-6% of the chord, low pitch and two blades. Solitidy is tied to how much of the rotor disk area is covered by the blades when still. They found that the thickness of the propeller blades should be as thin as structurally feasible, and noted that the motors operate most efficiently at high speeds. They also noted that as the pitch decreased the thrust per power increased, but the thrust per voltage decreased. This means that when designing a propeller one may decrease the pitch to obtain a higher efficiency, but for a given voltage supply the maximum thrust is sacrificed.

Another study was done by Gur and Rosen [10] to optimize the propulsion system of a fixed-wing UAV using a multidisciplinary design optimization (MDO). While the application of a rotor on a fixed-wing aircraft is different to that of a rotorcraft, the results are still interesting as the goal of this study was also to maximize endurance (termed "loiter-time" in the study). Consistent with Reference [9], the optimized propeller design had a very small blade thickness profile. Interestingly, the optimized design also had a very short chord length and therefore a very small solidity, contrary to Reference [9]. The authors first considered the optimal thickness and chord to be structurally impractical, so they introduced structural constraints. Even with these constraints, the resulting thickness and chord were still considered impractical in terms of manufacturing.

2.1.3 Motor

The electric motors used for high endurance multirotor UAVs need to have high efficiency at typical operating conditions. The theory behind making the motor perform more efficiently is out of the scope of this thesis. In addition it is crucial that the motor is designed to be lightweight. Gur and Rosen [10] looked at 250 existing brushless motors from various web catalogues and tried to estimate a constant for the maximum rated power per mass, or power-to-weight ratio, of the motors. The variance was so large that the estimated value for the constant ranged from 110 to 800 W/kg, where individual motors were even more spread. For reference, the U8II KV100 sold by T-MOTOR and used by the UAV Staaker BG-200 has a maximum power-to-weight ratio of 5171 W/kg based on listed specifications [11]. Note that the maximum power for the motor is only rated for 180 second intervals. At some point reliability may be a concern when aiming to minimize motor weight, and other characteristics like the aforementioned efficiency must be taken into account by the designer. Still, the motor mass is an important factor for the endurance of a multirotor UAV.

Motor manufacturers often give a "KV" rating for each motor, called the speed constant. This characteristic is defined as the no-load RPM generated per volt supplied to the motor, and should not be confused with kilovolts. A motor with a higher KV rating can produce higher speeds, but will typically generate less torque. Similarly to the power-to-weight ratio estimate, Gur and Rosen [10] estimated the motor KV as a function of mass. The constant estimates ranged from 50 to 600 RPM kg/V. For reference, the UII8 KV100 motor has a value of 27.2 RPM kg/V. KV is notably modeled inversely proportional to mass. Because of the large spread, a sensitivity study was done for the design optimization. They found a high sensitivity for the power-to-weight ratio, but a negligible influence of the motor speed constant parameter on the optimal motor selection. For the latter the model predicted a variation of 4% in endurance, where the solution had a criteria of a minimum rate-of-climb of 2m/s.

2.1.4 Configuration

The configuration of a multirotor aircraft is the layout of the rotor components. The reference Staaker BG-200 UAV has a coaxial configuration with eight rotors. This means it has four pairs of rotors and four supporting arms (see Figure 1.1). Coaxial rotors have a significant efficiency loss due to aerodynamic interference, but can provide increased thrust or a smaller UAV size. Other common configurations are in-plane (non-coaxial) quad-, hexa- or octorotor. A coaxial UAV with three arms is possible and still has complete movement control over pitch/roll/yaw, but a more novel in-plane three-rotor design requires an additional mechanism to control yaw such as tilting one rotor. The configuration is closely tied to practical factors like the UAV size, availability of components and cost.

2.2 Electronics

The electronics category includes wiring and support components that are not considered part of the propulsion system. These components impact endurance through wiring losses and secondary power consumption. In addition, the control algorithm may significantly impact the dynamic power consumption associated with stabilizing and maneuvering the UAV. The present thesis assumes the effect of the electronics to be negligible in comparison to the large power consumption of the motors, but future studies should analyse the impact of these factors.

2.3 Airframe

The airframe category considers the weight, aerodynamics and solid dynamics of the UAV's airframe. High performance materials should be considered as mass is critical to any aircraft, and the benefits might outweigh the additional cost. Very small low-cost multirotor UAVs often use plastic as an inexpensive structural material with sufficient strength, toughness and stiffness coupled with low mass density. UAVs designed for higher performance and UAVs designed to carry small payloads often utilize more costly materials such as carbon fiber and aluminum. Carbon fiber is very light, provides excellent stiffness and good strength, but limited toughness which means it is prone to break under excessive load. It is also impractical to repair. Aluminum is a light metal, is repairable and provides very good toughness due to its ductility. It also exhibits good machinability and can benefit from economies of scale due to larger production volumes compared to more niche materials like carbon fiber. An economic alternative may be sandwiched materials that use thin layers of high performance materials like carbon fiber with a lightweight core material like foam or balsa wood.

Winslow et. al. [9] constructed an airframe for a micro-quadrotor out of sandwiched boards of carbon fiber and balsa wood. The airframe consisted of only 7.4% of the total weight. For comparison, similarly sized quadrotors available on the marked had airframes consisting of roughly 30-40% of the total mass, which leads to increases power consumption and decreased endurance. Robustness may be compromised when minimizing the mass of an airframe in terms of life span or damage resistance.

In terms of solid dynamics, stiffness and mass affects the resonance frequencies of the UAV. Care should be taken to avoid a large dynamic response to the excitation forces caused by the rotors. This response can in the best case lead to power losses. In the worst case it can lead to failure of the control algorithm if the harmonic excitation of the sensory components lead to a form of aliasing. The algorithm can misinterpret the signal from the vibrating sensors and attempt to stabilize the UAV based on this, leading to unstable behaviour and loss of control. The oscillation modes of the airframe of the reference UAV Staaker BG-200 was analyzed in a previous thesis written by Fischer [12]. Multiple modes were found to be within the range of the excitation frequencies of the rotors. The dynamic response was not included in the scope.

Another factor which impacts endurance is aerodynamics. Excluding the propeller aerodynamics, the UAV will experience drag due to the motion of air around it. For high-speed operations the designer may take extra care to manage the total drag on the airframe and the supporting components typically placed close to the center of the UAV. Regardless of the operating conditions, drag on the airframe arm should be taken into account when trying to maximize endurance. The arms are located directly in the high-speed airflow induced by the propellers, and will as such be particularly prone to drag. This drag causes a loss in the rotor thrust and therefore a reduced propulsion efficiency.

A study done by Theys et. al. [13] analyzed the relative efficiency impact from the aerodynamics of three arm designs. The first arm was a smooth 25 mm cylindrical tube. The second arm was produced by 3D-printing an aerodynamically shaped arm and nacelle. The study did not specify the dimensions of this arm, but from the figures the arm seems to have roughly the same frontal area facing the airflow as the first arm. The study also did not clarify if the 3D-printed arm had undergone any surface treatment. 3D-printed objects will typically have an uneven surface which may cause additional drag. The third arm was a smooth 10 mm square tube. The relative change in propulsion efficiency compared to the 25 mm tube was measured. For the 3D-printed arm an increase of 2-4$\%$ was measured at disk loading between 50-100 N/m^2. Disk loading (DL) is defined as the ratio between thrust force and the rotor disk area, and may be measured as N/m^2. For the 10 mm square tube an increase of 4-8$\%$ was measured in the same DL interval. For reference, the DL of the Staaker BG-200 UAV at maximum take-off weight is 77 N/m^2. The study, while limited in scope and perhaps robustness, suggests that significant gains in efficiency and by extension endurance can be achieved by engineering an airframe arm with optimized coefficient of drag and frontal area.

CHAPTER 3

Analytical Models

The analysis in chapter 4 will be limited to a few key factors. The influence of take-off weight on endurance is expected to be significant, and mass will therefore be analyzed. Rotor configuration will also be studied as the effect of changing the configuration is unclear, and is set as a primary interest by Nordic Unmanned. Parallel to the Bachelor thesis performing an experimental study and the Master thesis performing simulation of the Staaker BG-200 propellers, the propeller diameter will also be analysed in the present thesis. As a UAV with maximized endurance may involve impractical design features, this thesis will estimate the resulting size of the UAV.

3.1 Hover Power Model

Endurance will be estimated using equation 2.1. For this an estimate of the power consumed is needed, which will be adapted from Reference [14]. The UAV will be assumed to hover in position for the duration of the flight. Abdilla et. al. [14] compared the power consumption of a UAV in hover to the same UAV performing small harmonic motions in a vertical case and a horizontal case, and found very similar power consumption. This suggests that for UAV operations with slow movements, a hover assumption can be a good estimate for analysis.

Momentum theory gives steady state rotor hover power P_R [W] for a single rotor [15]:

$$P_R = \frac{T^{\frac{3}{2}}}{\eta_R r_P \sqrt{2\rho_a \pi}} \tag{3.1}$$

where, using SI units, T is the required thrust force, r_P is the propeller radius, ρ_a is the density of air and η_R is the rotor Figure of Merit (FM). FM was developed to study rotorcraft hover power efficiency during the first half of the 20th century. It is defined as the ratio between ideal hover power and actual hover power for a single rotor [16].

$$FM = \frac{Tv_i}{P_r} \tag{3.2}$$

where T is thrust, v_i is the induced air speed through the rotor disk and P_r is the actual power consumed by the rotor. This merit is useful for relative comparisons of performance between rotors, but not for absolute comparisons as was clarified by Leishman and Syal [17]. FM, although non-dimensional, is influenced by a number of factors that makes it biased when comparing rotors of different disk loading. Leishman and Syal [17] in their study presented alternative definitions of FM that can be used for coaxial rotors to account for inherit losses in the ideal power required, with the goal of judging the efficiency of a coaxial rotor more realistically. To measure the aerodynamic efficiency of comparable rotors is however out of the scope of this thesis, and therefore a single-rotor propulsion system efficiency factor η_{PS} will be used together with a rotor interaction efficiency factor η_{RI}. η_{PS} is the ratio between ideal hover power for an isolated rotor and the power consumed by the propulsion system for one rotor. This accounts for the propeller efficiency, the motor efficiency and the ESC efficiency. η_{RI} is the ratio between the sum of the power consumed by isolated rotors and the total power consumed by the rotors when positioned in the intended configuration. In other words, η_{RI} accounts for the efficiency loss of the non-isolated rotors due to aerodynamic interaction.

If the thrust of a multirotor is assumed equally distributed between the rotors, and the total thrust is substituted by $T = mg$ for the hover condition, the multirotor hover power can be derived as Equation 3.3 [14]:

$$P_{N_R} = \frac{m^{\frac{3}{2}} g^{\frac{3}{2}}}{\eta_{PS}\eta_{RI} r_P \sqrt{2N_R\rho_a\pi}} \tag{3.3}$$

Power consumption increases exponentially with mass, is inversely proportional to the radius of the propeller and decreases with increasing number of rotors. These factors are not independent however. Increasing the radius of the propeller increases its mass. A bigger propeller also occupies more space and will require longer and heavier arms. Increasing the number of rotors directly adds mass and requires more space with the aforementioned drawbacks. The optimal design cannot be determined by studying factors in isolation, and therefore this thesis will include a dynamic mass model when estimating endurance.

3.2 Mass Model

The mass of the UAV is modeled as he sum of a constant mass m_C and a variable mass m_V. Only the latter varies with the number of rotors, the size of the propellers and motors and the number of rotors per arm. It is assumed that the hub structure will be able to support the change in rotor mass and arm length.

$$m = m_C + m_V \tag{3.4}$$

The constant mass includes the mass of items such as the battery, the hub structure, the control units, the batteries and the payload. The variable mass includes the mass of the propellers, the motors, the ESCs, the arms and their structural connection components. The mass of the wiring is neglected. If the mass of each arm is m_{arm}, and the sum of the mass of the remaining variable components per rotor is m_{var} then the total variable mass is:

$$m_V = \begin{cases} N_R(m_{var} + m_{arm}), & \text{in-plane} \\ N_R(m_{var} + \frac{1}{2}m_{arm}), & \text{coaxial} \end{cases} \tag{3.5}$$

where a factor of $\frac{1}{2}$ is used in the coaxial case since the number of arms is half the number of rotors. The mass of each arm is modeled as:

$$m_{arm} = m_{mpl} L_{arm} \tag{3.6}$$

where m_{mpl} is the mass per length of the arm material and L_{arm} is the length of the arm. Larger battery powered multirotor UAVs such as the Staaker BG200 often use carbon fiber tubes for arms, in which case m_{mpl} is a constant. L_{arm} can be estimated as a function of the number of arms and the rotor size based on their geometric relationship by assuming a constant angle between adjacent rotors and equal length arms.

The distance between two horizontally adjacent rotors will in practice be larger than two times the propeller radius r_P to avoid collision between the propellers and to mitigate aerodynamic losses due to blade tip vertices. An effective radius $k_P r_P$ can be used to better reflect the distance between rotors, where the constant k_P is larger than one. A value of 1.05 was used in the analysis based on Reference [18]. The horizontal distance R from the center of the UAV to the rotor as a function of propeller size and configuration (see Figure 3.1) is defined as [19]:

$$R = \frac{k_P r_P}{\sin \frac{\pi}{N_A}} \tag{3.7}$$

where N_A is the number of arms. The ideal arm length then becomes $R - L_{0,hub}$ where $L_{0,hub}$ is the horizontal distance from the center of the hub to the root of the arm. $L_{0,hub}$ is assumed to be 0.171 m for the reference UAV Staaker BG-200. The propellers must not collide with other components connected to the hub such as a battery or a leg. For the propellers of the Staaker BG-200 to avoid colliding with the legs, a minimum distance $L_{0,arm}$ from the root of the arm to the propeller blade tip is assumed to be 0.158 m. With this consideration the arm length must be at least $r_P + L_{0,arm}$. Equation 3.8 uses the maximum of the two arm length models to ensure both the distance between propellers and the distance to protruding hub components is considered.

$$L_{arm} = max \begin{cases} R - L_{0,hub} \\ r_P + L_{0,arm} \end{cases} \tag{3.8}$$

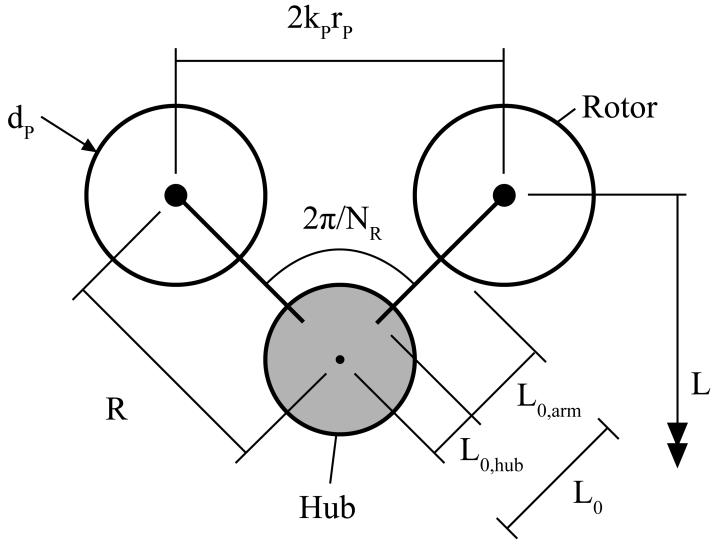

FIGURE 3.1: Multirotor Dimensions

3.3 UAV size

Similarly to Equation 3.8, the length of the UAV (along the roll axis) can be geometrically estimated. The length will be compared to the endurance of configuration options in Chapter 4 as a secondary design consideration. This length will be used instead of the diagonal length as it better represents the space occupied by the UAV for various configurations, especially in relation to storage containers. As with the arm length, the UAV length must consider both the distance between propellers and the distance from the propeller tip to protruding hub components. Respectfully, Equations 3.9 and 3.10 are used for these two considerations.

$$L_1 = \frac{2k_P r_P}{\tan \frac{\pi}{N_A}} \tag{3.9}$$

$$L_2 = 2(r_P + L_0)cos(\frac{\pi}{N_A}) \tag{3.10}$$

where L_0 is the minimum distance from the center of the UAV to the propeller blade tip necessary to avoid colliding with hub components. L_0 is then equal to the sum of $L_{0,hub}$ and $L_{0,arm}$. Finally the UAV length estimate is:

$$L = max\begin{cases} L_1 \\ L_2 \end{cases} \quad (3.11)$$

This equation assumes an even number of at least four rotors and will underestimate the length of configurations with three arms. A modification to Equation 3.11 will be used for the three arms case which takes into account the fact that one arm is in line with the length of the UAV:

$$L_3 = \frac{L}{2} + R \quad (3.12)$$

3.4 Battery Capacity Model

Abdilla et. al. [14] tested different battery models to determine a good estimate for the effective battery endurance for use in multirotor UAVs. They used a series of commercial off-the-shelf three-cell (3S) Li-Po batteries in the experiments, and compared four models:

1) Ideal battery capacity $t = \frac{C_{nom}}{I}$
2) 0.8 capacity offset $t = 0.8\frac{C_{nom}}{I}$
3) 0.9 capacity offset $t = 0.9\frac{C_{nom}}{I}$
4) Peukert equation $t = \frac{C_{nom}^{1.01} R_{nom}^{1-1.01}}{I^{1.01}}$

where C_{nom} is the nominal battery capacity, I is the circuit current, and 1.01 is the Peukert exponent which was the average value over the experiments conducted. The study highlighted that the Peukert equation is not suitable for Li-Po batteries as it is only valid for constant discharge rate and temperature in a limited working range. An offset of 0.9 was found to be the best fit. Based on the results in Reference [14], the following equation will be used for the energy capacity E:

$$E = k_B C_{nom} V_{nom} \quad (3.13)$$

where k_B is the battery capacity offset factor and C_{nom} and V_{nom} is the rated battery charge and volt respectively. Although a k_B value of 0.9 can be a good estimate, it can also be determined experimentally and is dependent on battery State of Health (SoH) and discharge rate. By substituting Equation 3.13 into the endurance equation (2.1) it is assumed that the UAV flies until the battery can no longer provide enough power to hover. It is advisable for normal operations to limit the depth of discharge (DOD) to less than 100%, for instance 80%. This will significantly improve the life-span of the battery.

CHAPTER 4

Analysis

Endurance was used as the primary characteristic for analysis in line with the scientific goals of this thesis. The specifications of the existing Staaker BG-200 UAV was used as the basis for the analysis. The energy capacity was estimated using Equation 3.13 with the reference case of 32 Ah nominal capacity and 44.4V nominal voltage. A MATLAB script was developed to execute the analysis, which can be found in Appendix A.

It was desired to use the experimental data gathered from the parallel Bachelor thesis to calculate the single-rotor propulsion system efficiency η_{PS} for the propellers and the rotor interaction efficiency factor η_{RI} for the coaxial setup and use these in the analysis of this thesis. The goal was to use values averaged between 28 inch propellers and 32 inch propellers, and assume these values to be constant for all propeller and motor combinations. However, the η_{PS} calculated from the experiments underestimated the endurance by 27% compared to the reported maximum endurance of the existing reference UAV. The reason for the low experimental η_{PS} is unclear.

For the two 28 inch propellers at hover thrust η_{RI} was calculated to be 0.763. This value corresponds well with empirical data from previous experimental study [20]. The same factor was for two 32 inch propellers calculated to be 0.521. One reason for this decrease in efficiency is the spacing between the rotors which is fixed. With a fixed distance z between propellers the increased propeller diameter d_P leads to a decreased ratio $\frac{z}{d_P}$. A small $\frac{z}{d_P}$ (<0.15) ratio is tied to a decrease in efficiency as demonstrated in studies such as [20]. The η_{RI} factor was still not expected to decrease as much as it did. The Bachelor students discovered that a propeller was mounted incorrectly for many of the measurements, including the coaxial

32 inch test. The discovery was unfortunately made too late to rectify the mistake with new tests.

The reasonable η_{RI} calculated from the 28 inch propeller data was be used for further analysis instead of the averaged value initially desired. Note that the efficiency losses due to interaction between horizontally adjacent rotors is excluded as only one pair of rotors were used in the test. The effect horizontal interaction has been measured to be small when the propellers do not overlap in studies such as [20] and [13]. It may be noted that to the authors knowledge this effect has not been tested for adjacent pairs of coaxial rotors. The interaction may be more significant than what is observed with only two adjacent rotors. In place of the η_{PS} based directly on the experiments, a value of 0.793 was estimated by solving Equation 3.3 for η_{PS} using the reported maximum endurance of 60 minutes. It was assumed that $\frac{z}{d_P}$ is held constant, which means that the distance between the coaxial propellers must be increased as the propeller diameter increases.

4.1 Parameters

To demonstrate the effect of changing the mass of the UAV, Equations 3.3 and 2.1 were used with the specifications of the Staaker BG-200 UAV (26 inch coax 8-rotor). Figure 4.1 shows how endurance varies with take-off weight while holding all else constant. The endurance is estimated to be 33 minutes and 44 seconds at the MTOW of 25 kg assuming constant propulsion efficiency. This is 44% less than the reference endurance of 60 minutes. Figure 4.2 demonstrates the estimated relative change in endurance caused by a change in mass. Decreasing the mass of the reference UAV by one kilogram gives an estimated endurance increase of 5 minutes and 42 seconds.

A demonstration of how hover power depends on propeller size can be seen in Figure 4.3. The propeller diameter was varied from 20 to 40 inches holding all else constant. The mass was fixed at the MTOW of 25 kg, and Staaker BG-200 specifications were used otherwise. As the diameter doubled, the power consumption was halved for the same thrust.

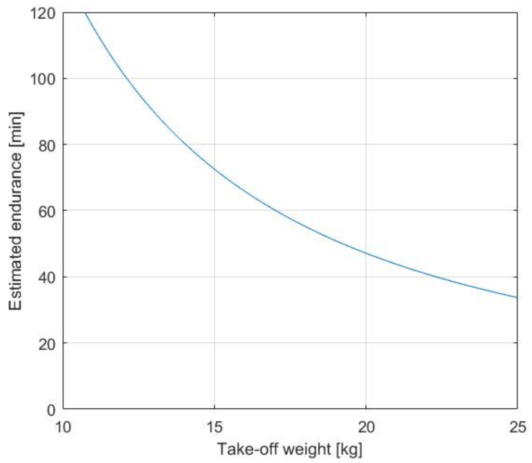

FIGURE 4.1: Mass Sensitivity

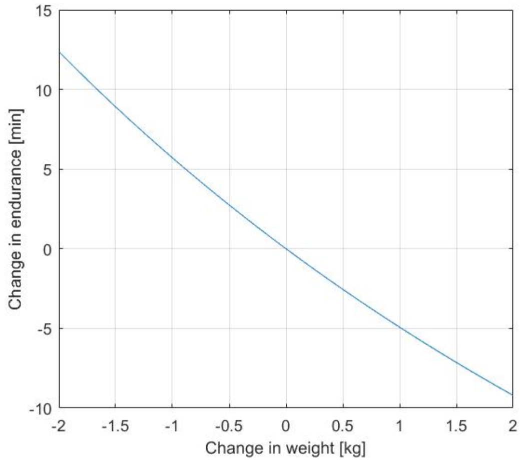

FIGURE 4.2: Relative Mass

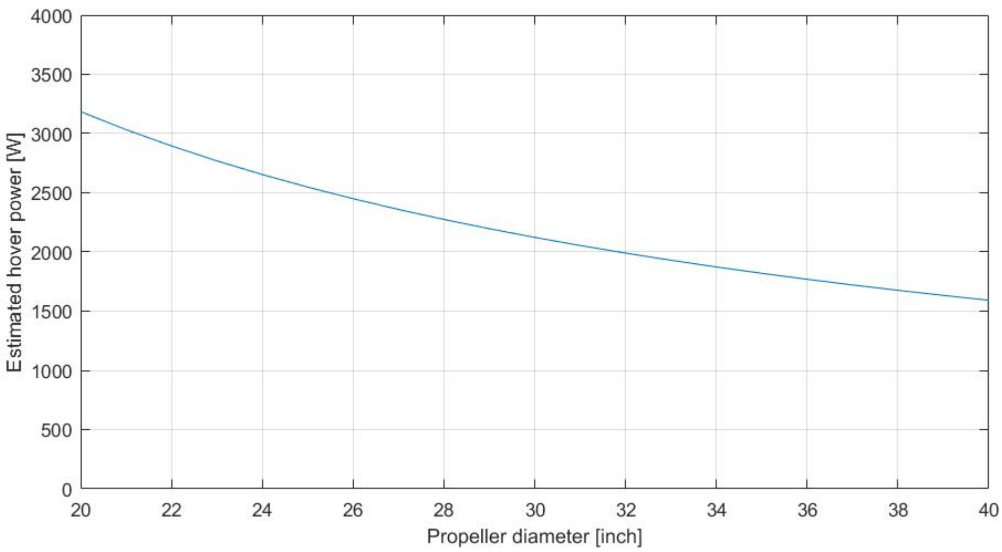

FIGURE 4.3: Effect of Propeller Size

4.2 Configuration Analysis

To compare rotor configurations, the dynamic mass model was used including Equations 3.4, 3.5, 3.6 and 3.8 with zero payload. The propeller-, motor- and ESC mass data was taken from the website of T-MOTOR [11], whom designs and sells UAV propulsion system components. The carbon fiber "glossy" propellers were used for analysis, which is the propellers used for the existing Staaker BG-200 UAV. This UAV has a thrust-to-weight ratio of 2.3 at MTOW based on the maximum rated thrust of the motor used. This thesis will use the criteria of 2.0 as the minimum thrust-to-weight ratio for analysis. Using a smaller ratio can allow for a smaller mass, but will negatively impact the dynamic maneuverability of the UAV. To achieve the set criteria the U8II KV100 motor found in the existing UAV was used for the 8-rotor configurations. For 6-rotor and 4-rotor configurations the U10II KV100 and the U12II KV120 motors were used, respectively. The two smallest propeller diameters did not meet the thrust criteria for the in-plane 4 rotor configuration and was omitted. The setup is presented in Table 4.1.

TABLE 4.1: Configuration Analysis Setup

Configuration	N_R	Motor	Propellers [in]
Coaxial	8	U8II KV100	20-40
	6	U10II KV100	20-40
In-plane	8	U8II KV100	20-40
	6	U10II KV100	20-40
	4	U12II KV120	24-40

Endurance was plotted against propeller size as seen in Figure 4.4. The reader should recall that the endurance estimate is for maximum flight time and assumes no limit to the depth of discharge. Generally, the endurance of the configurations increased with propeller diameter. When controlled for being coaxial or in-plane the endurance increased with number of rotors. The highest endurance estimate was achieved by the in-plane 8-rotor configuration at 82 minutes and 57 seconds. Compared to this, the in-plane 6-rotor configuration achieved an endurance estimate of only 0.7% lower. The lowest endurance estimate was achieved by the coaxial 6-rotor configuration at 39 minutes and 47 seconds. The difference between the worst and the best performing configurations was largely between 12 and 14 minutes for matching propeller sizes. The 36 inch propeller from T-MOTOR has a large mass relative to its size compared to the other propellers. This propeller had a smaller endurance gain per size increase compared to the other propellers due to it's mass. The effect is most apparent for the configurations with eight propellers.

The payload capacity of the UAV is limited by the MTOW set by the regulations which the Staaker BG-200 falls under. Since the total weight cannot exceed 25 kg, the payload

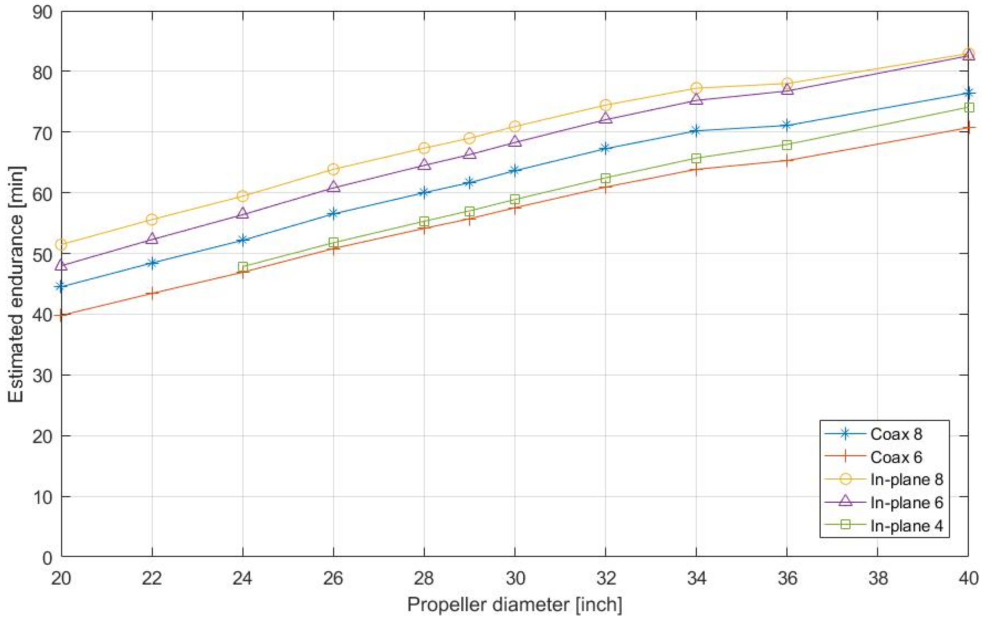

FIGURE 4.4: Configurations

capacity becomes the difference between the operating empty weight (OEW) and the MTOW. The estimated payload capacity in the analysis therefore was defined as 25 kg minus the estimated UAV mass. The results are presented in Figures 4.5 and 4.6. The in-plane 8-rotor configuration performed the worst in terms of payload capacity. While this and the in-plane 6-rotor configurations had very similar endurance estimates with 40 inch propellers, the former had 1.85 kg (31%) less estimated payload than the latter. This was attributed to the in-plane 8-rotor configuration having the highest number of arms, the longest arms and the maximum number of rotors since all of these properties increases mass. The mentioned large mass of the 36 inch propeller and to a lesser extent the 40 inch propeller from T-MOTOR had a significant impact on the difference in payload capacity between the configurations. This can be seen in Figure 4.5 as a deviation in performance for the first two data points starting from the left (40 and 36 inch respectfully) for each configuration. This phenomenon had again a larger effect on configurations with a higher number of rotors. For endurance of roughly 65 minutes and below, the coaxial 6-rotor configuration had the highest estimated payload capacity.

The in-plane 8-rotor and 6-rotor configurations showed the largest estimated UAV sizes as seen in Figure 4.7 (using Equation 3.11). Therefore, these two designs should perhaps be avoided if a small UAV size is desirable. The coaxial 8-rotor and the in-plane 4-rotor configurations have identical sizes (recall that the two smallest propellers were omitted for the latter configuration), but the latter delivers less estimated endurance for any given propeller diameter. It also provides less payload capacity for every propeller diameter except 40 inches, due to the larger and heavier motors. Therefore the in-plane 4-rotor configuration provides no performance benefits over the coaxial 8-rotor configuration with propellers below 40 inches based on the present estimations.

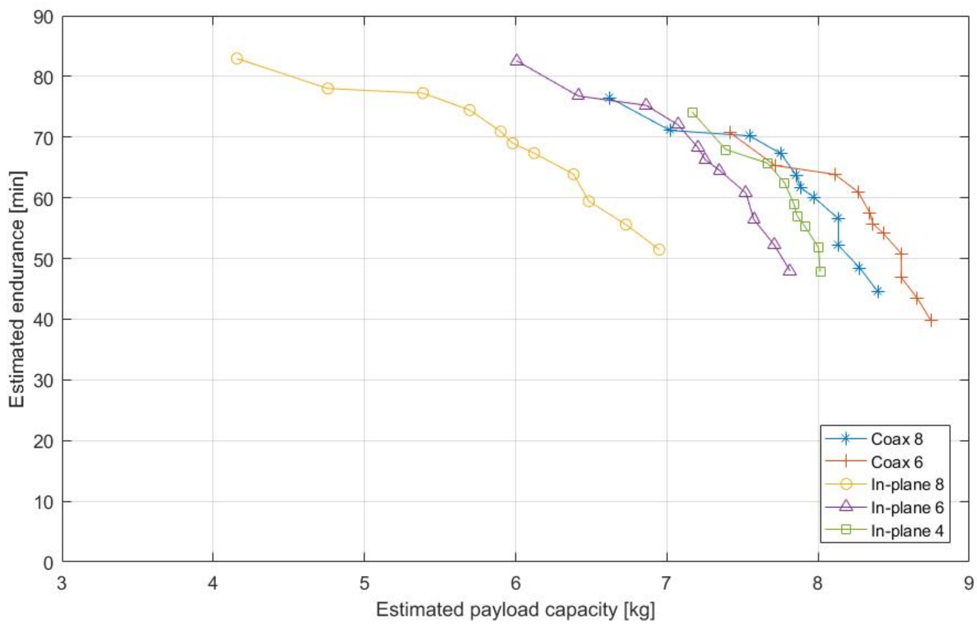

FIGURE 4.5: Configuration by Payload Capacity

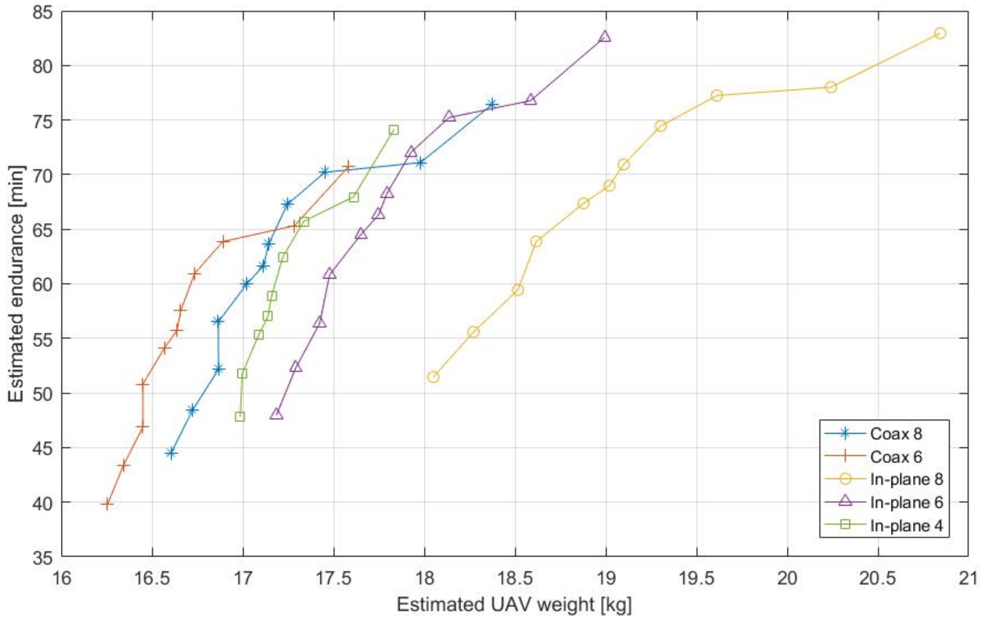

FIGURE 4.6: Configuration by mass

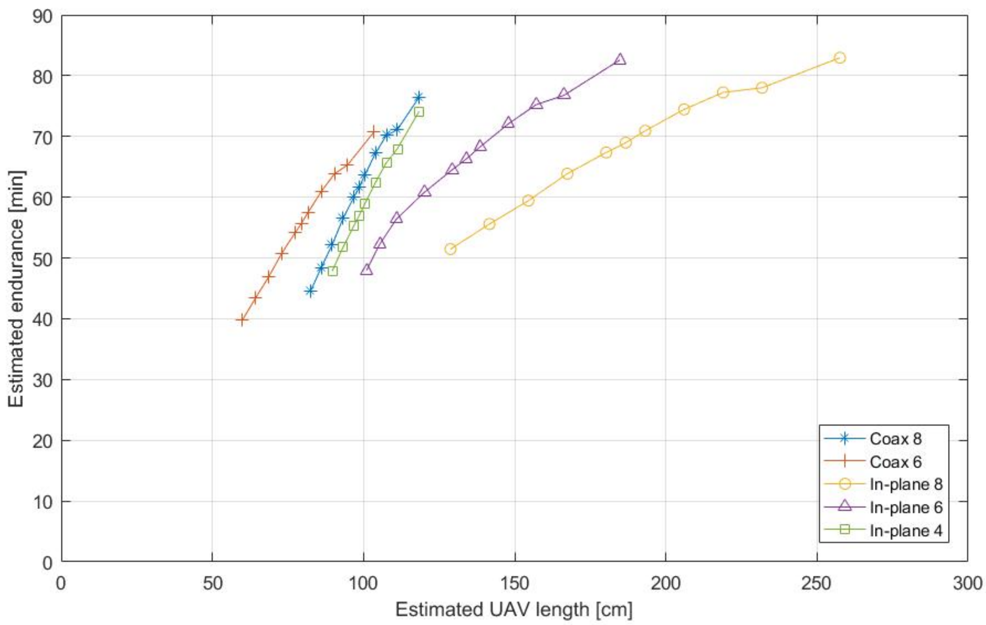

FIGURE 4.7: Configuration by Size

4.3 Case Analysis

Three alternative designs were compared to the reference Staaker BG-200 UAV based on the results of the configuration analysis. One design was selected for maximum endurance. The maximum endurance for each configuration was achieved with the largest propeller diameter at 40 inches (see Figure 4.4). As previously mentioned, the in-plane 8-rotor configuration achieved the overall highest estimated endurance. However, since the 40 inch in-plane 6-rotor design estimate was only 0.7% lower, and the design was significantly smaller and lighter, this configuration was instead chosen for the case analysis.

The next design was chosen to maximise payload, with the criteria that the endurance must be at least 80% of the reference UAV. The resulting configuration was coaxial with 6 rotors and 26 inch propellers. The last design was meant to be a balanced design that would increase endurance while still having good payload capacity and a manageable size. When appropriate criteria were applied, three designs with similar performance were found from three different configurations. Therefore a separate analysis was done for these three designs.

The results of the first comparison are presented in Table 4.2. A maximum estimated endurance of 82.6 minutes was found for the endurance optimized design, a large increase over the reference UAV. This design involved large compromises however. The length estimate of the UAV was almost double that of the Staaker BG-200, amd the payload capacity decreased by 2 kg. The payload optimized design only achieved a 7.2% increase in payload while sacrificing 15% endurance. Advantageously, the estimated length of this design was 27% smaller than the reference. While the length was here used to compare the size of the UAVs, one should not forget that the two dimensional size of the UAV changes with an exponent of two. The reason that a one dimensional estimate was used is because the geometry changes every time the number of arms changes, while the length can be modeled with a single formula for all configurations except the thee-armed ones.

To perform a simple analysis of the significance of the battery size, the original battery specifications were replaced by the next size available from seller MaxAmps. A battery

TABLE 4.2: Case Analysis - Original 32Ah Battery

	Endurance [min]		Capacity [kg]		Length [cm]	
Endurance optimized	82.6	(+38%)	6.01	(-25%)	185	(+91%)
Payload optimized	50.8	(-15%)	8.55	(+7.2%)	73.0	(-27%)
Staaker BG-200	60.0		7.98		96.8	

combination delivering a rated 44 Ah was used, increased from 32 Ah. The increased capacity and mass was used to run the same calculations as in Table 4.2. As seen in Table 4.3, endurance was increased and payload was decreased for all cases as expected. Interestingly, for these three cases the endurance gain increased with original endurance. In the order of payload optimized case, reference case and endurance optimized case, the endurance gain was 7.3 min (14%), 9.0 min (15%) and 14 min (17%). This effect might be connected to mass. While the absolute increase in mass was the same for all UAVs, the relative increase in total mass was not; it was ordered reversely to the observed increase in endurance gain. The endurance optimized design achieved a new maximum estimated endurance of 96.6 minutes, 61% more than the reference UAV. Even though this increase was large, the payload capacity was now less than half. UAV length was unaffected. The reference drone itself had a 15% increase in endurance but a 27% decrease in payload capacity with the larger battery. The 2156 gram mass difference between the battery options directly decreased the payload capacities by as much.

TABLE 4.3: Case Analysis - 44Ah Battery

	Endurance [min]		Capacity [kg]		Length [cm]	
Endurance optimized	96.6	(+61%)	3.85	(-52%)	185	(+91%)
Payload optimized	58.1	(-3.2%)	6.39	(-20%)	73.0	(-25%)
Staaker BG-200	69.0	(+15%)	5.82	(-27%)	96.8	

For the balanced design the following criteria were set. Endurance should be at least 10 minutes higher than the reference, payload capacity should be at least 7 kg and the length as a secondary consideration should not be too large. Three designs were chosen, and the setup is displayed in Table 4.4. Design A was coaxial, with 8 rotors and 34 inch propellers. Design B was coaxial with 6 rotors and 40 inch propellers, the largest option. To meet the thrust-to-weight ratio criteria of at least 2, this design used the heavier U10II KV100 motors. Design C was an in-plane 4-rotor configuration, also with 40 inch propellers. This design used the heaviest motors of this study, the U12II KV120. The original 16 Ah batteries were used.

Since the performance of the three designs were so similar, the sum of the off-the-shelf prices for the propulsion system for each design was found, excluding the battery. The prices were taken from the T-MOTORS web store 4th of June 2020. On the web store they recommend a larger ESC (see 2.1) for the U12II motor which is reasonable when only four motors are producing the necessary thrust. Therefore the difference in price between the ESC used for the other designs and the mentioned larger ESC was added to the summed price of design C. Note that the change in mass for the ESC was not considered. The results are presented in Table 4.5. Most interestingly, both design B and C amounted to lower costs compared to the components used in the reference UAV Staaker BG-200. The price of the design C components amounted to $3956, which was $1892 less than BG-200. The coupled endurance estimate was 14.1 minutes (23%) more than reference, with a decrease in payload capacity and an increased UAV length. The UAV sizes of all cases are compared in Figure 4.8.

TABLE 4.4: Case Analysis - Balanced Design Setup

	Configuration	d_P [in]	Motor	ESC [in]
Design A	Coaxial 8	34	U8II KV100	Alpha 60V HV
Design B	Coaxial 6	40	U10II KV100	Alpha 60V HV
Design C	In-plane 4	40	U12II KV120	Alpha 120V HV
Staaker BG-200	Coaxial 8	28	U8II KV100	Alpha 60V HV

TABLE 4.5: Case Analysis - Balanced Design Results

	Endurance [min]	Capacity [kg]	Length [cm]	Partial price
Design A	70.2	7.55	108	$6400
Design B	70.7	7.42	103	$5394
Design C	74.1	7.17	118	$3956
Staaker BG-200	60.0	7.98	96.8	$5848

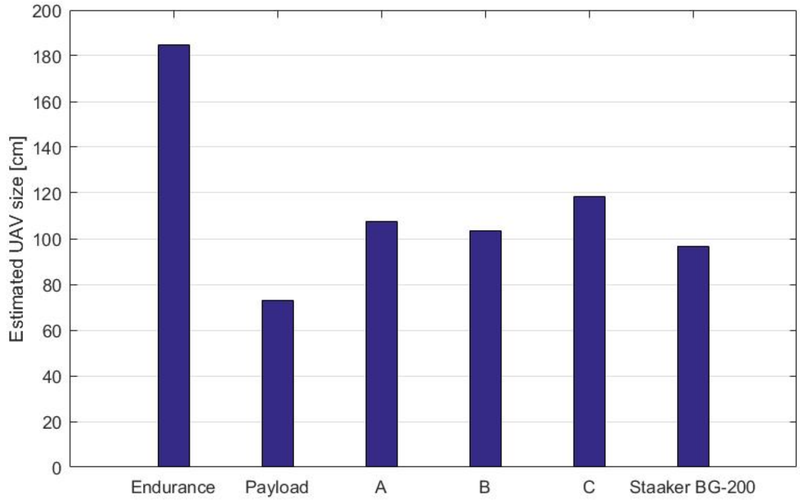

FIGURE 4.8: Size Comparison

CHAPTER 5

Discussion and Conclusion

5.1 Discussion

The configuration analysis showed that the size of the UAV is sensitive to both the configuration and the propeller size. It also showed that endurance is sensitive to the same factors. The configurations with coaxial rotors achieved lower endurance estimates compared to the in-plane rotor configurations when controlling for number of rotors and propeller diameter. This was expected due to the efficiency loss that occurs in the interaction between the upper and lower propellers for coaxial rotors. The mass of the arms used in the coaxial configurations is lower, because they are shorter in length and halved in quantity. This decrease in mass was not enough to make up for the efficiency loss.

At first the in-plane 4-rotor configuration was not considered advantageous because for each rotor diameter it displayed a lower endurance for the same UAV size as the coaxial 8-rotor configuration, while also generally having a smaller payload capacity. In the case analysis however, the in-plane 4-rotor proved to be superior in component pricing. Design C achieved better endurance for a significantly lower component price compared to the reference UAV Staaker BG-200, at the cost of size and payload capacity. A designer could choose smaller propellers for the same configuration to obtain the desired size and payload capacity. This would decrease the endurance, but would further decrease the component cost. The coaxial 6-rotor configuration cost less than the coaxial 8-rotor configuration as well. The in-plane 8-rotor configuration performed poorly in terms of size and payload capacity, while being largely matched in endurance by the in-plane 6-rotor configuration. The former had a lower power consumption per thrust due to the highest number of rotors, but the added mass associated with the rotors and the eight arms increased the necessary thrust to hover. This allowed the in-plane 6-rotor configuration to achieve a similar peak endurance with a smaller rotor disk area.

Except for the in-plane 8-rotor configuration, all other configurations displayed unique benefits. In-plane 6 achieved high endurance and in-plane 4 achieved low cost. Coaxial 6 achieved small size and low weight, and coaxial 8 at some limited points achieved the highest endurance for a given size and payload capacity. It can be argued that if the 36 inch and the 40 inch propellers did not deviate from the trend of the other propellers in terms of mass then the coaxial 8 might have been surpassed by the coaxial 6-rotor configuration. In Figure 4.5, excluding the two largest propellers, the coaxial 6-rotor configuration seems to maintain the gap in payload capacity to the coaxial 8-rotor data. Figure 4.7 suggests that the curves for the same two configurations may cross for some larger propeller size, although they might not. Therefore it is possible that the coaxial 6-rotor could achieve the best endurance for any given UAV size or payload capacity if the propellers were more carefully designed.

5.1.1 Improvement Suggestions

The recommendations for optimizing the endurance of the existing Staaker BG-200 UAV are as follows. Increasing the propeller size will generally improve the endurance at the cost of payload capacity, UAV size and price. A coaxial 6-rotor configuration should be considered as the estimations in this thesis suggest it generally has the highest payload capacity and the smallest UAV size for a given endurance target. The coaxial 6-rotor design in the case analysis also had a less costly motor and propeller combination than the Staaker BG-200. A 4-rotor configuration should also be considered if some performance can be sacrificed, as the cost of the propulsion system for this configuration is meaningfully lower than the coaxial 8-rotor configuration used today. An in-plane 4-rotor design with 40 inch propellers is proposed with an estimated endurance improvement of 14.1 minutes (23%) and a $1892 decrease in cost, with 0.81 kg (10%) decreased payload capacity and 14 cm (14%) increased UAV length.

5.1.2 Limitations

The present thesis has assumed that the motors will produce their rated thrust using any propeller size. However, larger propellers have higher moment of inertia and requires more torque to operate. It is possible that some of the motor-propeller combinations used in Chapter 4 are not appropriate. The parallel Bachelor thesis noticed in the experiments that the U8II KV100 motor produced more heat when pared with the largest propeller tested, which was 32 inches in diameter. Heat generation is synonymous with efficiency loss. This effect was observed at maximum throttle, and not at a normal hover/cruising throttle. The extent of this effect with further increased propeller size is unknown, and could have an impact on the performance of the designs.

Perhaps the most significant limitation of the present study is the assumption that the propulsion system efficiency stays constant for all propellers and motors used. Generally, this efficiency increases with RPM [2], [21], [22] while holding propeller diameter fixed. Ramasamy [23] found that FM increased with Reynolds number (Re), where Re increases with RPM and chord length. The study explained that this is expected because "viscous parasitic drag losses which do not contribute toward thrust generation are more prominent at low Re" [23]. This means that when Re is increased by increasing RPM, the ratio between useful and non-useful drag increases. When the total weight of the UAV is close to constant, the required thrust is close to constant and the pitch-to-diameter ratio is constant, increasing the propeller diameter will reduce the RPM necessary to hover.

While the reduction in ideal power consumption due to a larger rotor disk area has been captured by the model used in this thesis, the change in propeller and motor efficiency due to a reduction in RPM has not. The necessary RPM also changes with thrust, but the change in thrust has been limited throughout the analysis, where the largest mass variation can be found for the largest designs that have been considered impractical. The differences in efficiency between different propellers and motors have also not been captured. The experimental data from the Bachelor thesis showed very similar efficiencies around hover thrust when comparing the 28 inch and the 32 inch propellers, but the range of propeller diameters was much larger in the present study. In hindsight, the efficiency should not be assumed to be fixed when the change in RPM is unknown and a variety of components are used.

Ideally the range of components should be tested experimentally. To achieve accurate results without testing every propeller-motor combination the following should be determined: For each motor; the power efficiency as a function of torque and rotational speed - For each propeller the torque, rotational speed and the Figure of Merit as functions of thrust. These could be implemented in the MATLAB script used for this thesis or in a numerical solver. Component manufacturers have data sheets with varying levels of detail, but these are prone to be inaccurate and inflated, and they do not form a rigorous foundation for analysis. Other known limitations of the present analysis are not expected to have a large impact on the result, although they could affect the relation between closely matched designs. These limitations include the constant battery capacity model, excluding adjacent rotor interaction, fixed arm mass-per-length, inaccuracies in the data gathered from the manufacturer and assuming no secondary power consumption.

Undoubtedly, the problem of optimizing endurance of a multirotor UAV is complex with interdependent factors. These factors involve several fields of technology, and the designer must balance conflicting goals when searching for the most beneficial solution. The analysis done in the present thesis included multiple factors, and an attempt was done to account for multiple goals. A more holistic analysis can be done by using a multidisciplinary design optimization (MDO) method as done in Reference [10]. Here, a series of factors can be considered simultaneously, such as propeller blade design, structural integrity, electrical system design and propulsion system design. A solution can be solved numerically to give a design that balances the interdependent parameters and goals. Such a method requires deep as well as broad knowledge by the developer(s), but is ultimately very useful for this complex problem.

5.2 Conclusion

This thesis has studied how the endurance of multirotor UAVs can be optimized. The scientific goals were largely met. A broad overview was given for the identified factors that influence endurance. A set of critical factors were quantified and improvement suggestions to the Staaker BG-200 were made.

Endurance was found to increase with increasing propeller diameter, increasing number of propellers and decreasing mass. The highest endurance estimate achieved without replacing the battery was 82 minutes and 57 seconds, a 38% increase over Staaker BG-200. This in-plane 8-rotor configuration was relatively large and had a significantly decreased payload capacity. The coaxial 6-rotor configuration had the smallest size and the highest payload capacity of the configurations when controlled for propeller size. In the case analysis, the relative endurance gain from increasing the battery capacity from 32 AH to 44 Ah was between 14% to 17%. The payload capacity was lowered by the battery weight difference of roughly 2.2 kg. When propulsion system prices were considered, the in-plane 4-rotor configuration with 40 inch propellers amounted to $1892 less than the reference UAV while having an estimated endurance increase of 14.1 minutes (23%). Payload capacity was decreased by 0.81 kg (10%) and the UAV length increased by 14 cm (14%) for this design.

The recommendations for optimizing the endurance of the existing Staaker BG-200 UAV are to consider increasing propeller size, consider a coaxial 6-rotor configuration for performance or consider an in-plane 4-rotor configuration for its lower cost. In particular, the 4-rotor configuration with 40 inch propellers is proposed with the aforementioned benefits of increased endurance and decreased cost.

A significant limitation of the estimates done is the assumption that the ratio between ideal and actual power consumption stays constant when varying the propellers, motors and the thrust. This assumption may disproportionally benefit the larger propellers.

5.3 Future Work

A set of recommendations follows for future work on optimizing multirotor UAVs based on the experience and knowledge gathered in the present work. A dynamic model for the ratio of ideal to actual hover power should be included in future studies. Experimental data should be gathered for components considered to avoid inaccuracy and inflation of data reported by the manufacturers. To provide a holistic solution to this multifactored interdependent problem, a multidisciplinary design optimization method should be used. Such methods are intended to evaluate a series of parameters simultaneously, and is very well suited for this application. The interference between adjacent coaxial rotor pairs has to the authors knowledge not been studied. Experiments on the matter could reveal an ideal ratio between in-plane separation distance and propeller diameter for such systems. Limiting the UAV size at the cost of efficiency is perhaps the main advantage of a coaxial configuration. Therefore the option of using in-plane thee-bladed propellers in place of coaxial rotors could provide the same qualitative benefit, which warrants further quantitative analysis.

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Appendices

APPENDIX A

Master Script

Here follows the master script that executed all the calculations and plot generation for the analysis in this thesis. The script uses three functions, which are presented in Appendices B, C and D.

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- Analysis of Multirotor UAV Endurance -

Sølve Sætre Sem, University of Stavanger, 15.06.2020

```
% Reset
clear
close ALL
```

Setting Constants

```
g = 0.001;  % grams per kg, for convenience

% Existing Staaker BG-200 specifications
VBatt = 44.4;               % Nominal battery voltage rating
CBatt = 32;                 % Nominal battery charge [Ah]
    E = VBatt * CBatt;   % Resulting nominal battery capacity

% Factor assumptions
kB = 0.9;        % Battery capacity offset constant
etaPS = 0.793;  % Propulsion system efficiency, excluding rotor
 interaction
                % Reverse engineered from benchmark t=1hr and
                % experimental nRI = 0.763
```

Calcucation of Endurance for Base Case

Staaker BG-200

```
% Configuration
NR      = 8;        % Number of rotors
dP      = 28;       % Propeller diameter in inches
isCoax  = 1;        % 1 if UAV has coaxial rotors, 0 if otherwise

m = mass(NR, dP, isCoax);
P = PMulti(NR, dP, m, etaPS, isCoax);
t = kB * E ./P;

L = DroneSize(NR,dP,isCoax);

minutes = t*60;
disp('Estimated endurance of base case in minutes is:');
disp([num2str(round(minutes)),' minutes']); disp(' ');
```

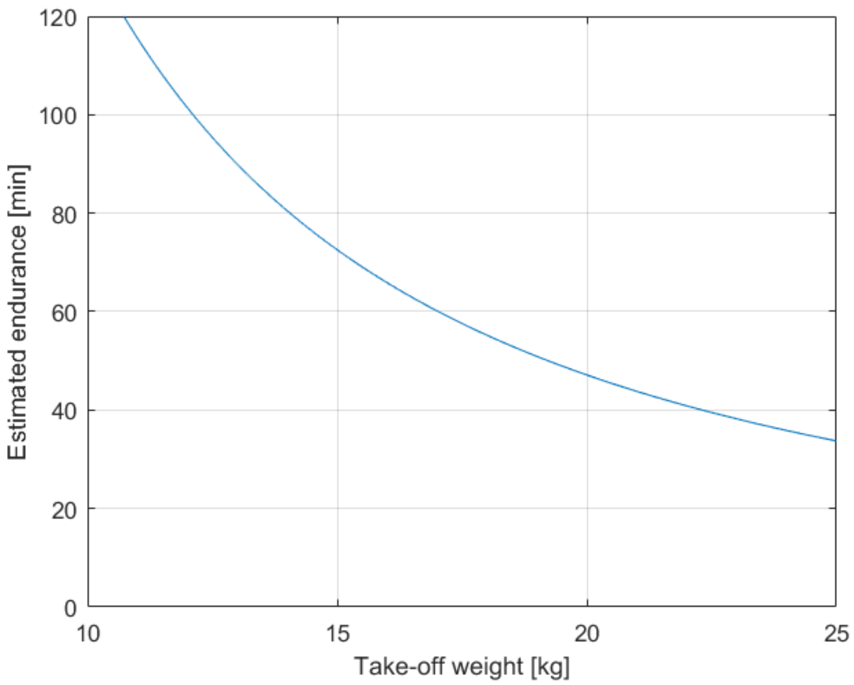
Estimated endurance of base case in minutes is:
60 minutes

Effect of Mass on Endurance

Absolute

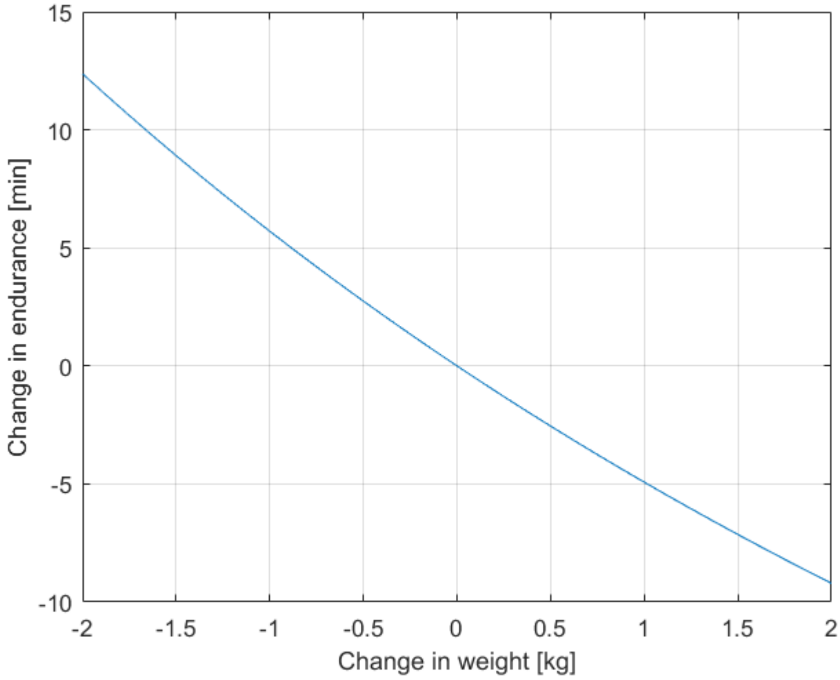
```
m2_Delta = 125*g;
m2 = 10:m2_Delta:25; % Varying mass from 12kg to MTOW 25kg in steps of
 125g

P2 = PMulti(NR, dP, m2, etaPS, isCoax);
t2 = kB * E ./P2;
minutes2 = t2.*60;

figure(1);
plot(m2, minutes2);
xlim([m2(1),m2(end)])
ylim([0,120]);
xlabel('Take-off weight [kg]');
ylabel('Estimated endurance [min]');
grid on

disp(['Estimated endurance at MTOW is ', num2str(minutes2(end))]);
 disp(' ');
```

Estimated endurance at MTOW is 33.7268



Relative

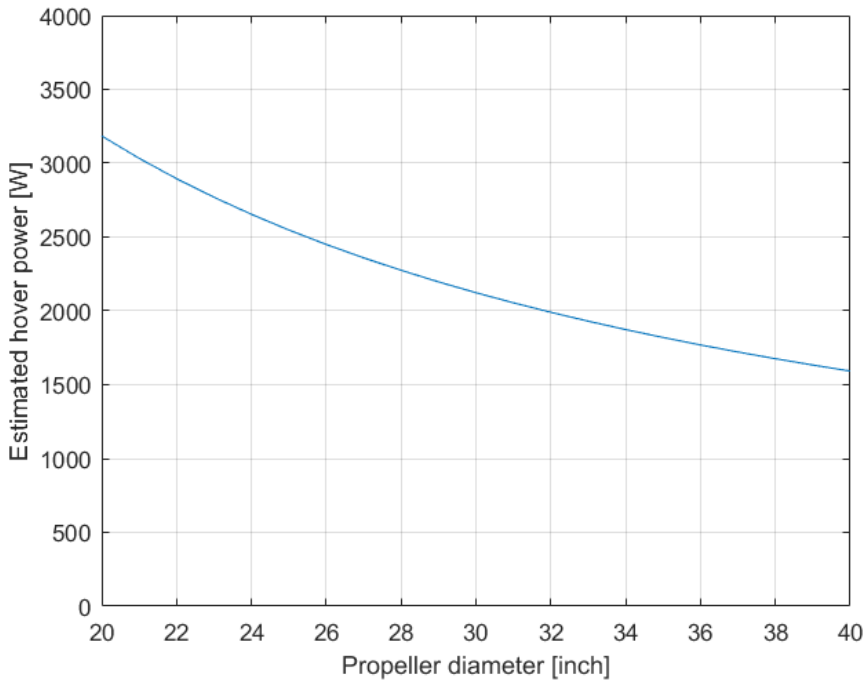
```
m3_Delta = m2_Delta;
m3_Steps = -2:m3_Delta:2;
m3 = m + m3_Steps; % Varying reference mass by -2 to +2kg in steps of
 125g

P3 = PMulti(NR, dP, m3, etaPS, isCoax);
t3 = kB * E ./P3;
t3_Delta = t3 - t;

figure(2);
plot(m3_Steps, t3_Delta.*60);
xlabel('Change in weight [kg]');
ylabel('Change in endurance [min]');
grid on
```



Effect of Propeller Diameter on Power

```
m4 = 25;          % Maximum Take-Off Weight (MTOW)
                  % Excluding effects of variable mass which depending
 on dP

NR4 = 8;          % Base case from BG-200
isCoax4 = true; %

dP4 = 20:1:40;   % Varying propeller diameter from 20 to 20*2 inches


P4 = PMulti(NR4, dP4, m4, etaPS, isCoax4); % Associated hover power

%P_MTOW = PMulti(NR4, 20, m4, FM, isCoax4); % Power for reference- 20"
 prop
%P4_Normalized = P4/P_MTOW;      % Hover power compared to base case
%dP4_Normalized = dP4/20;

% Plotting theoretical hover power vs propeller diameter
figure(4);
%plot(dP4_Normalized, P4_Normalized);
plot(dP4,P4);
ylim([0,4000]);
xlabel('Propeller diameter [inch]');
```

```
ylabel('Estimated hover power [W]');
grid on
```

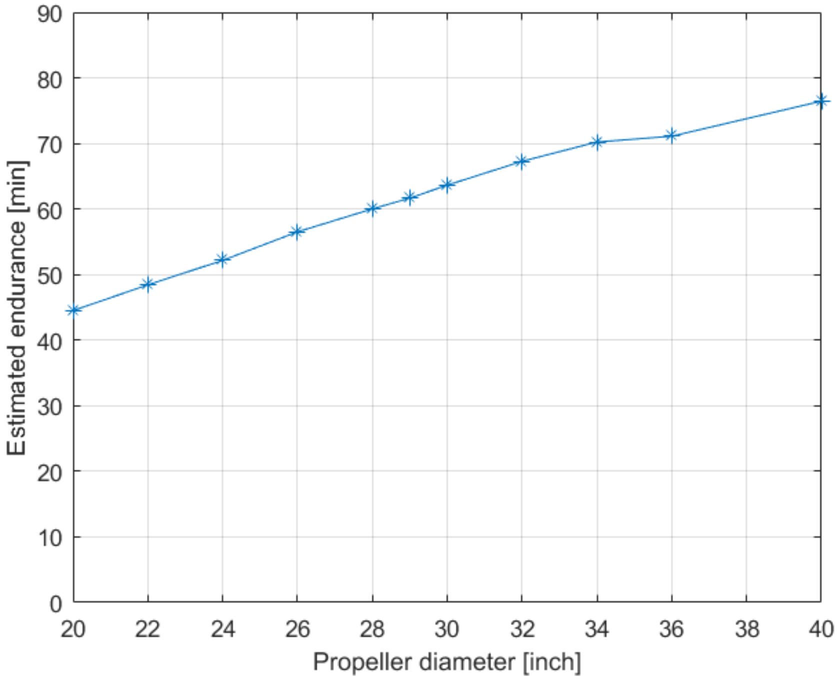


MAIN CONFIGURATION COMPARISON

Coaxial 8 Rotors

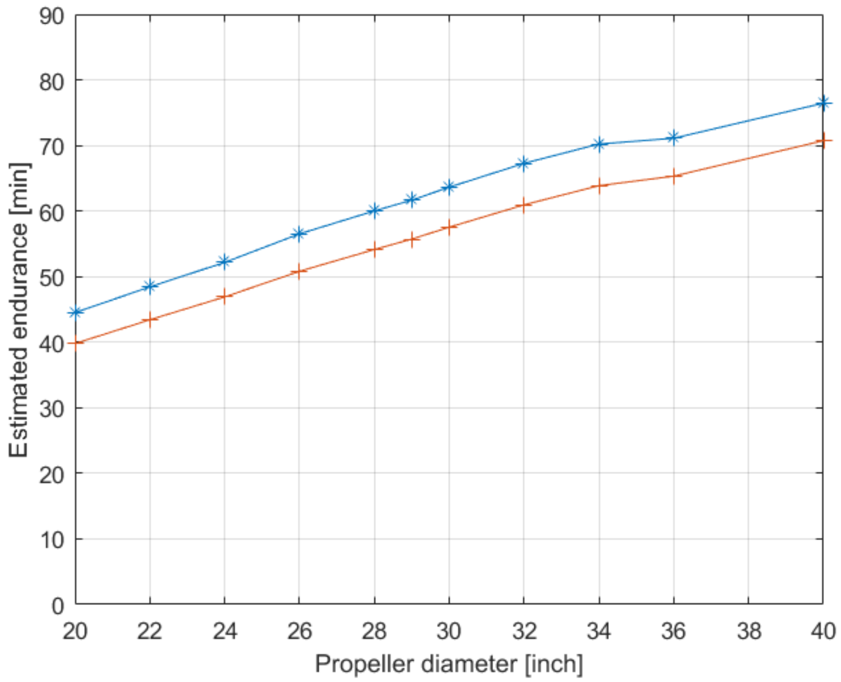
```
NR10      = 8;          % Number of rotors
dP10      = [20, 22, 24, 26, 28, 29, 30,  32,  34,  36,  40];  %
 Propeller diameter in inches
isCoax10  = 1;          % 1 if UAV has coaxial rotors, 0 if otherwise

% Calcucation of endurance
m10 = mass(NR10, dP10, isCoax10);
P10 = PMulti(NR10, dP10, m10, etaPS, isCoax10);
t10 = kB * E ./P10;

% Calculation of size
L10 = DroneSize(NR10,dP10,isCoax10);

minutes10 = t10*60;     % Endurance in minutes, from hours

figure(10)
plot(dP10,minutes10,'-*')
xlabel('Propeller diameter [inch]');
```

```
ylabel('Estimated endurance [min]');
ylim([0,90]);
configurationLegendArray{1} = 'Coax 8';

grid on
hold on
```



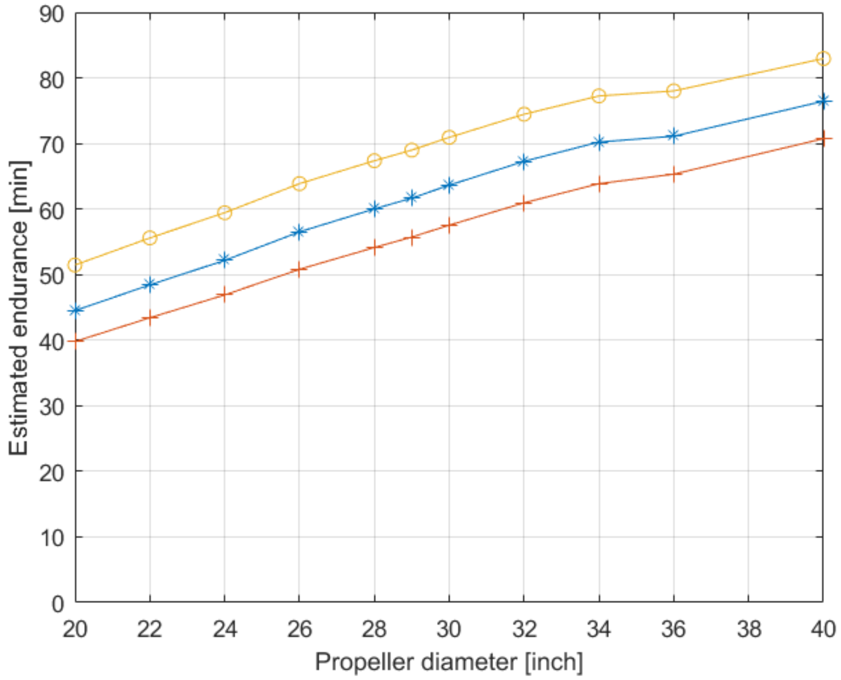
Coaxial 6 Rotors

```
NR11      = 6;
dP11      = dP10;        % Same vector of propeller diameters as
 previous case
isCoax11  = 1;

m11 = mass(NR11, dP11, isCoax11);
P11 = PMulti(NR11, dP11, m11, etaPS, isCoax11);
t11 = kB * E ./P11;

L11 = DroneSize(NR11,dP11,isCoax11);

minutes11 = t11*60;

plot(dP11,minutes11,'-+')
configurationLegendArray{2} = 'Coax 6';
```



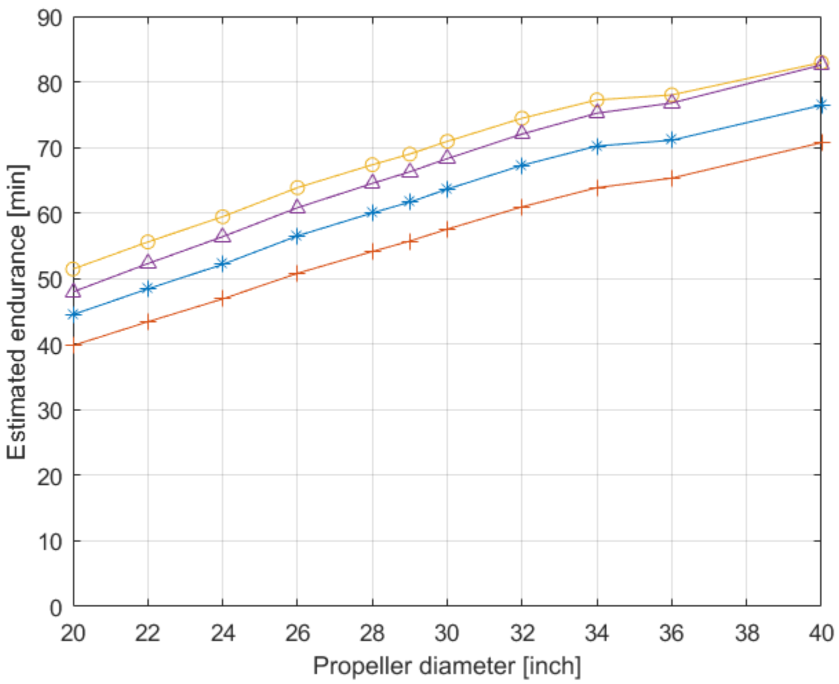
In-plane 8 Rotors

```
NR12       = 8;
dP12       = dP10;
isCoax12   = 0;

m12 = mass(NR12, dP12, isCoax12);
P12 = PMulti(NR12, dP12, m12, etaPS, isCoax12);
t12 = kB * E ./P12;

L12 = DroneSize(NR12,dP12,isCoax12);

minutes12 = t12*60;

plot(dP12,minutes12,'-o')
configurationLegendArray{3} = 'In-plane 8';
```



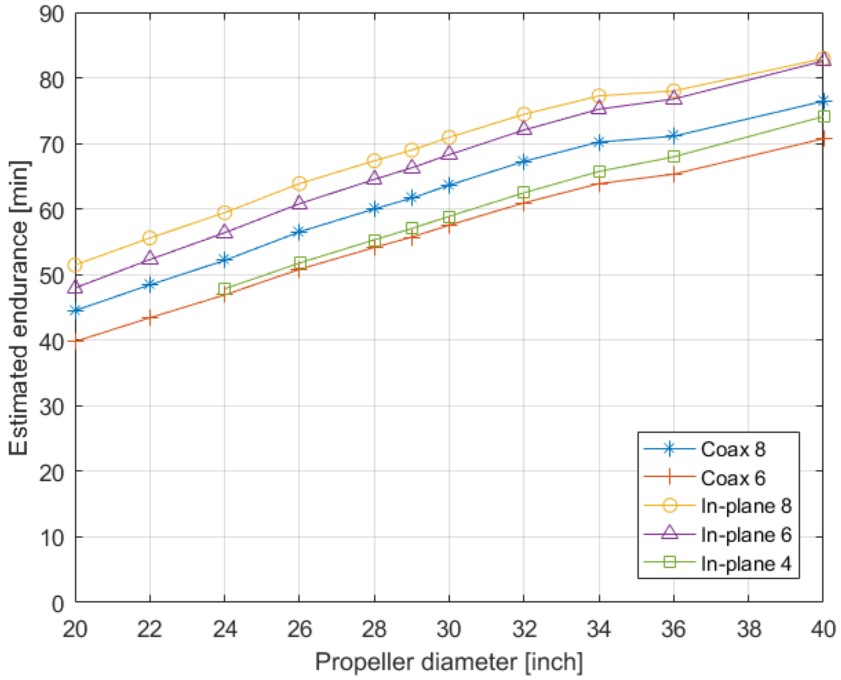
In-plane 6 Rotors

```
NR13      = 6;
dP13      = dP10;
isCoax13  = 0;

m13 = mass(NR13, dP13, isCoax13);
P13 = PMulti(NR13, dP13, m13, etaPS, isCoax13);
t13 = kB * E ./P13;

L13 = DroneSize(NR13,dP13,isCoax13);

minutes13 = t13*60;

plot(dP13,minutes13,'-^')
configurationLegendArray{4} = 'In-plane 6';
```



In-plane 4 Rotors

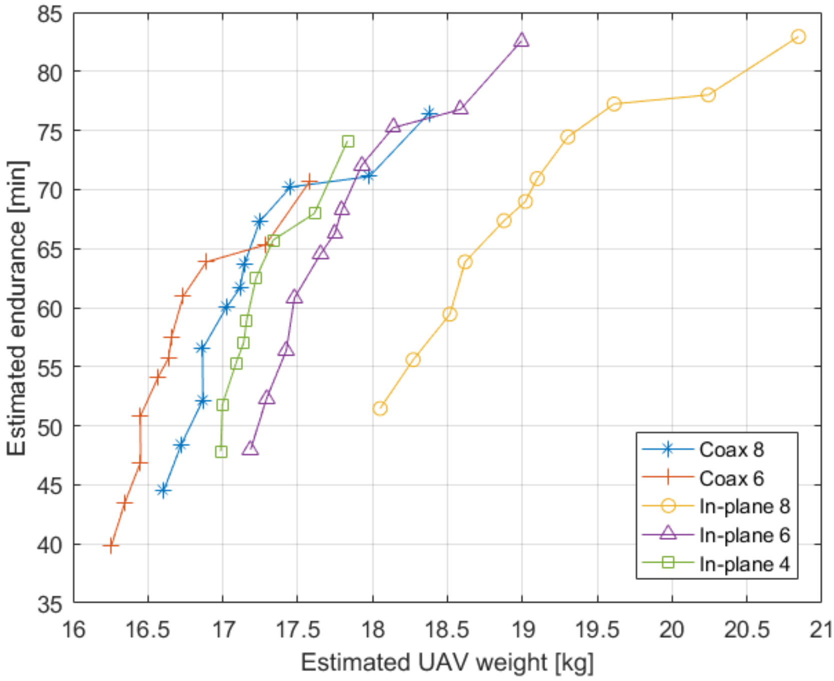
```
NR14       = 4;
dP14       = dP10(3:end);        % Excluding 20 and 22 inch as they are
 not
                                 % rated for enough thrust to reach sum
 50kg
isCoax14  = 0;

m14 = mass(NR14, dP14, isCoax14);
P14 = PMulti(NR14, dP14, m14, etaPS, isCoax14);
t14 = kB * E ./P14;

L14 = DroneSize(NR14,dP14,isCoax14);
minutes14 = t14*60;

plot(dP14,minutes14,'-s')
configurationLegendArray{5} = 'In-plane 4';

% Plot point for Staaker BG-200
%plot(dP,minutes,'o','MarkerSize',10);

legend(configurationLegendArray,'Location','SouthEast')
```



Plotting Data in Other Ways

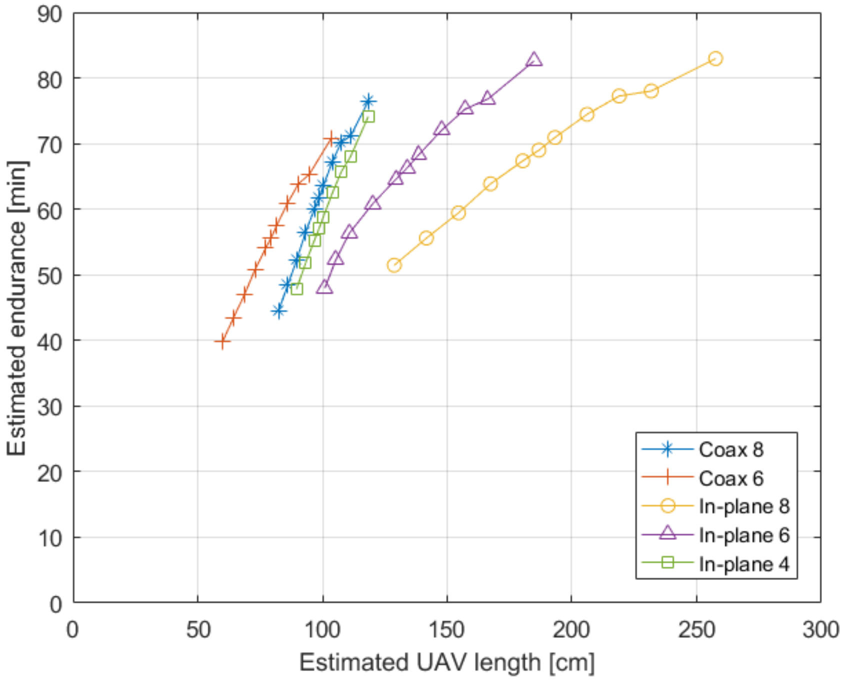
Endurance VS Mass

```
figure(11)
plot(m10,minutes10,'-*')
grid on
hold on
plot(m11,minutes11,'-+'),plot(m12,minutes12,'-o'),...
    plot(m13,minutes13,'-^'),plot(m14,minutes14,'-s')
%plot(m,minutes,'o','MarkerSize',10); % Plot point for Staaker BG-200

xlabel('Estimated UAV weight [kg]');
ylabel('Estimated endurance [min]');
legend(configurationLegendArray,'Location','SouthEast')
```



Endurance VS Size

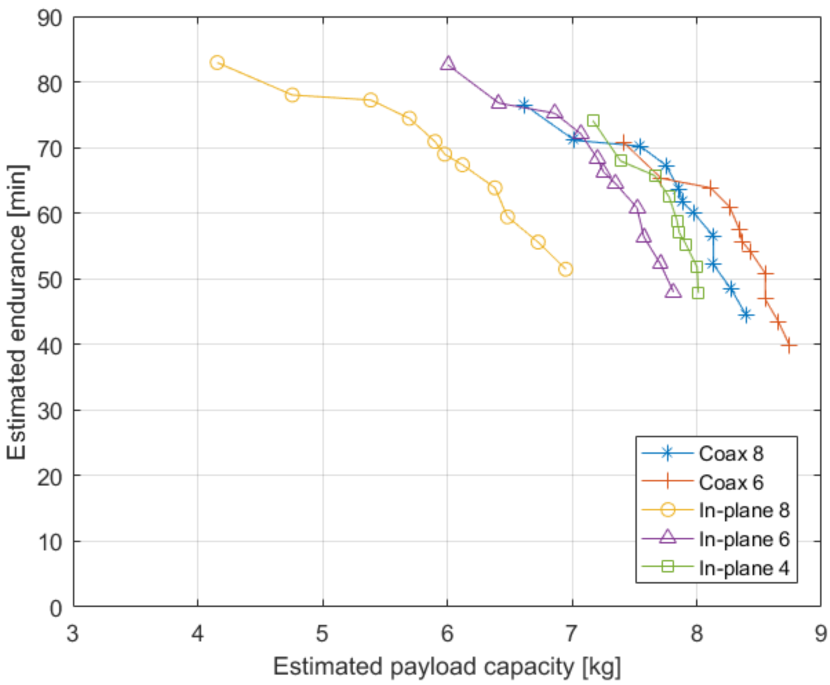
```
figure(12)
plot(L10*100,minutes10,'-*')
grid on
hold on
plot(L11*100,minutes11,'-+'),plot(L12*100,minutes12,'-o'),...
    plot(L13*100,minutes13,'-^'),plot(L14*100,minutes14,'-s')
%plot(L,minutes,'ko','MarkerSize',10); % Plot point for Staaker BG-200
ylim([0,90]);
xlim([0,300]);
xlabel('Estimated UAV length [cm]');
ylabel('Estimated endurance [min]');
legend(configurationLegendArray,'Location','SouthEast')
```



Endurance VS Payload Capacity

Payload capacity is defined as 25kg minus UAV weight due to the regulated maximum take-off weight of 25kg

```
figure(14)
plot( (25-m10), minutes10,'-*')
grid on
hold on
plot(25-m11,minutes11,'-+'),plot(25-m12,minutes12,'-o'),...
    plot(25-m13,minutes13,'-^'),plot(25-m14,minutes14,'-s')
%plot(25-m,minutes,'ko','MarkerSize',10); % Plot point for Staaker
 BG-200

xlabel('Estimated payload capacity [kg]');
ylabel('Estimated endurance [min]');
xlim([3,9]);
ylim([0,90]);
legend(configurationLegendArray,'Location','SouthEast')
```



CASE STUDY

```
% Base reference case (Staaker BG-200)
case00 = [minutes,        m,         L,        NR, dP,        isCoax];

% 13(end) Max endurance: In-plane 6 - 40inch prop
case01 = [minutes13(end), m13(end), L13(end), 6, dP13(end), 0];

% 11(4)   Min weight w/ min 80% endurance: Coax 6 - 26inch
p02 = 4; % using propeller number 4 (26 inch) from the coax 6 config
case02 = [minutes11(p02), m11(p02), L11(p02), 6, dP11(p02), 1];

case0X = [ case01; case02; case00]; % Combined matrix
% Example: case0X(:,4) gives NR for all cases

% Percentages, rounded to two decimals:
    % Endurance
Per0X_t = round( 100*( case0X(:,1)' - minutes)./minutes, 2);
    % Payload
Per0X_mp = round( 100*( (25- case0X(:,2)') - (25-m) )./ (25-m), 2);
    % Size
Per0X_L = round( 100*( case0X(:,3)' - L )./ L, 2);

disp('CASE STUDY: Original battery - value (%)');
disp(' ');
```

```
disp('Endurance optimized');
disp(['t = ',num2str(case01(1)),' (',num2str(Per0X_t(1)),')',...
    '. Payload = ',num2str(25-
 case01(2)),' (',num2str(Per0X_mp(1)),')',...
    '. L = ',num2str(case01(3)),' (',num2str(Per0X_L(1)),')']);
disp(' ');
disp('Payload optimized');
disp(['t = ',num2str(case02(1)),' (',num2str(Per0X_t(2)),')',...
    '. Payload = ',num2str(25-
 case02(2)),' (',num2str(Per0X_mp(2)),')',...
    '. L = ',num2str(case02(3)),' (',num2str(Per0X_L(2)),')']);
disp(' ');
disp('Reference design');
disp(['t = ',num2str(case00(1)),...
    '. Payload = ',num2str(25- case00(2)),...
    '. L = ',num2str(case00(3))]);
disp(' ');
```

```
CASE STUDY: Original battery - value (%)

Endurance optimized
t = 82.5792 (37.59). Payload = 6.006 (-24.7). L = 1.8478 (90.85)

Payload optimized
t = 50.8136 (-15.34). Payload = 8.5499 (7.2). L = 0.72995 (-24.61)

Reference design
t = 60.0186. Payload = 7.9758. L = 0.96817
```

Testing case0X With Larger Battery

```
NR50      = case0X(:,4)';    % Number of rotors
dP50      = case0X(:,5)';    % Propeller diameter in inches
isCoax50  = case0X(:,6)';    % 1 if UAV has coaxial rotors, 0 if
 otherwise

% Calcucation of endurance

m50_std = case0X(:,2)';
m50 = m50_std + 2.156; % Larger battery
E50 = VBatt*44;       % Larger battery, 44Ah

P50 = PMulti(NR50, dP50, m50, etaPS, isCoax50);
t50 = kB * E50 ./P50;

minutes50 = t50*60;

payload50 = 25 - m50;

disp('CASE STUDY: Larger batteries - value (%)')

% Percentages, rounded to two decimals:
```

```
    % Endurance
Per50_t = round( 100*( minutes50 - minutes) ./minutes, 2);
    % Payload
Per50_mp = round( 100*( payload50 - (25-m) )./ (25-m), 2);
    % Size is unchanged

disp(' ');
disp('Endurance optimized');
disp(['t = ',num2str(minutes50(1)),' (',num2str(Per50_t(1)),')',...
    '. Payload =
 ',num2str(payload50(1)),' (',num2str(Per50_mp(1)),')',...
    '. L = ',num2str(case01(3)),' (',num2str(Per0X_L(1)),')']);
disp(' ');
disp('Payload optimized');
disp(['t = ',num2str(minutes50(2)),' (',num2str(Per50_t(2)),')',...
    '. Payload =
 ',num2str(payload50(2)),' (',num2str(Per50_mp(2)),')',...
    '. L = ',num2str(case02(3)),' (',num2str(Per0X_L(2)),')']);
disp(' ');
disp('Reference design');
disp(['t = ',num2str(minutes50(3)),' (',num2str(Per50_t(3)),')',...
    '. Payload =
 ',num2str(payload50(3)),' (',num2str(Per50_mp(3)),')',...
    '. L = ',num2str(L)]);
disp(' ');

CASE STUDY: Larger batteries - value (%)

Endurance optimized
t = 96.6345 (61.01). Payload = 3.85 (-51.73). L = 1.8478 (90.85)

Payload optimized
t = 58.0834 (-3.22). Payload = 6.3939 (-19.83). L = 0.72995 (-24.61)

Reference design
t = 69.0095 (14.98). Payload = 5.8198 (-27.03). L = 0.96817
```

Balanced Designs - case1X

Criterias: Minimum 10 min increase of t, min. 7kg payload, keeping L in mind

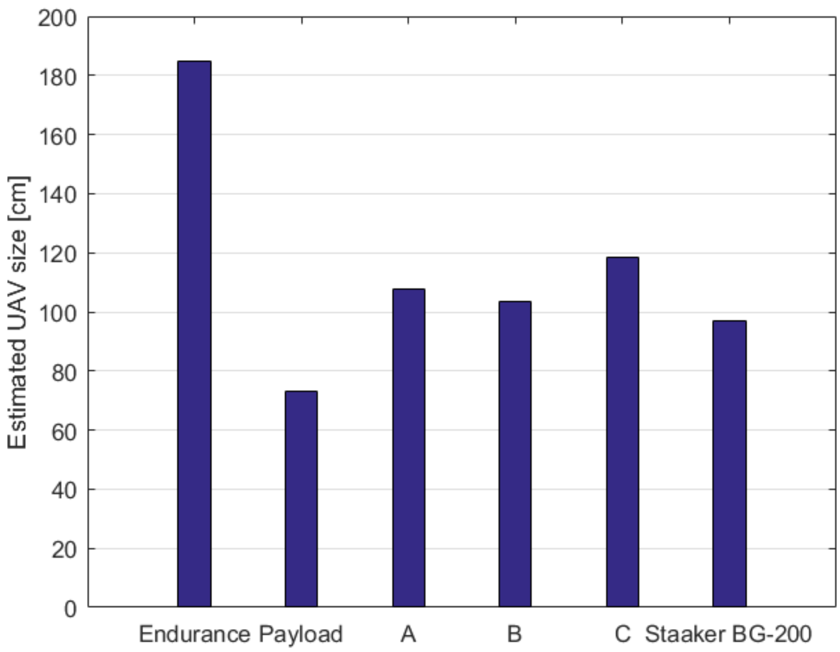
```
% caseXX:     [ t, m, L, NR, dP, isCoax ]
% caseXX(i), i= 1  2  3  4   5   6

% 10(9) Design A: Coax 8 - 34 inch prop
p11 = 9; % using propeller number 9 (34 inch) from the coax 8 config
case11 = [minutes10(p11), m10(p11), L10(p11), 6, dP10(p11), 1];

% 11(end) Design B: Coax 6 - 40 inch prop
case12 = [minutes11(end), m11(end), L11(end), 6, dP11(end), 1];

% 14(end) Design C: In-plane 4 - 40 inch prop
```

```
case13 = [minutes14(end), m14(end), L14(end), 6, dP14(end), 1];

case1X = [ case11; case12; case13; case00]; % Combined matrix

disp('CASE STUDY: Balanced designs')
disp(' ');
disp('Design A: Coax 8 - 34 inch prop');
disp(['    t = ',num2str(case11(1)),...
    '. Payload = ',num2str(25- case11(2)),...
    '. L = ',num2str(case11(3)),...
    '. Components = $6400']);
disp('Design B: Coax 6 - 40 inch prop');
disp(['    t = ',num2str(case12(1)),...
    '. Payload = ',num2str(25- case12(2)),...
    '. L = ',num2str(case12(3)),...
    '. Components = $5394']);
disp('Design C: In-plane 4 - 40 inch prop');
disp(['    t = ',num2str(case13(1)),...
    '. Payload = ',num2str(25- case13(2)),...
    '. L = ',num2str(case13(3)),...
    '. Components = $3956']);
disp(' ');
disp('Reference design');
disp(['    t = ',num2str(case00(1)),...
    '. Payload = ',num2str(25- case00(2)),...
    '. L = ',num2str(case00(3)),...
    '. Components = $5848']);
disp(' ');
```

```
CASE STUDY: Balanced designs

Design A: Coax 8 - 34 inch prop
   t = 70.2136. Payload = 7.5475. L = 1.0759. Components = $6400
Design B: Coax 6 - 40 inch prop
   t = 70.7389. Payload = 7.4164. L = 1.0344. Components = $5394
Design C: In-plane 4 - 40 inch prop
   t = 74.1176. Payload = 7.1673. L = 1.1837. Components = $3956

Reference design
   t = 60.0186. Payload = 7.9758. L = 0.96817. Components = $5848
```

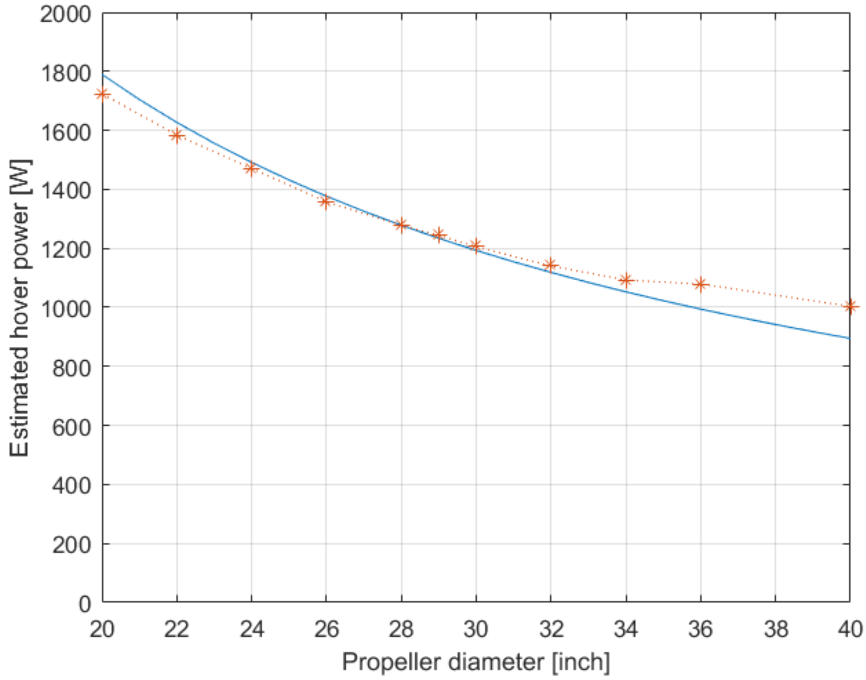
Size Comparison

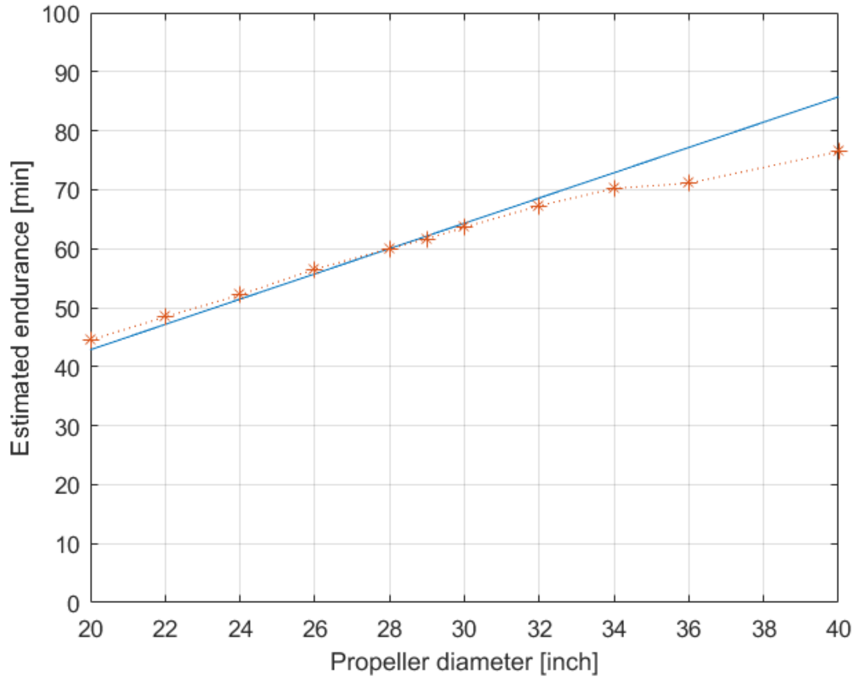
```
figure(40)
case_bar = [case01(3), case02(3), case1X(:,3)'];
bar(case_bar*100,0.3);
% colormap(cool);
set(gca, 'XTick', 1:6, 'XTickLabels', {'Endurance',...
    'Payload','A','B','C','Staaker BG-200'})
ax = gca; ax.XGrid = 'off'; ax.YGrid = 'on'; % Only grid lines in Y
ylabel('Estimated UAV size [cm]');
```



Comparing Constant Mass to Variable Mass

```
P51 = PMulti(NR4, dP4, m, etaPS, isCoax4); % Hover power at fixed mass
 m
t51 = kB * E ./P51; minutes51 = t51*60;
figure(51);
plot(dP4,P51);
ylim([0,2000]);
xlabel('Propeller diameter [inch]');
ylabel('Estimated hover power [W]');
grid on
hold on
plot(dP10,P10,':*'); % Power
figure(52)
plot(dP4,minutes51); % Endurance
ylim([0,100]);
grid on
hold on
plot(dP10,minutes10,':*');
xlabel('Propeller diameter [inch]');
ylabel('Estimated endurance [min]');
```


Estimated hover power [W]
2000
1800
1600
1400
1200
1000
800
600
400
200
0
20
22
24
26
28
30
32
34
36
38
40
Propeller diameter [inch]


Estimated endurance [min]
100
90
80
70
60
50
40
30
20
10
0
20
22
24
26
28
30
32
34
36
38
40
Propeller diameter [inch]

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APPENDIX B

Mass Function

Here follows the script for the first function, which uses the dynamic mass model (Equation 3.4) to estimate the mass of each UAV case.

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Final Calculation 4


```
function [ m ] = mass( NR, dP, isCoax)

%MULTIROTOR MASS: Variable mass model
%   Total mass is calculated based on component masses, number of
%   rotors and whether or not configuration is coaxial

%   Can accept multiple cases, where the length of each input variable
%   must be equal to the number of cases or equal to one. In the
%   latter case the single value of the variable will be applied to
%   all cases.

%   Input explaination:
%   NR is number of rotors.
%   dP is propeller blade diameter in inches.
%   isCoax should be 1 if the rotors are configured coaxially,
 otherwise 0

%   Note that if both NR and dP are vectors, the function will output
%   a vector, not a cross-calculated matrix

g = 0.001;  % grams per kg, for convenience
```

Propeller Mass

```
% Propeller diameter options [inch]:
propD =     [20, 22, 24, 26, 28, 29, 30,  32,  34,  36,  40];
% Corresponding propeller mass (T-MOTOR "Glossy" CF hover propellers):
propM =     [44, 56, 71, 68, 85, 95, 97, 107, 130, 193, 237]*g;
```

Linearization

Linearization of mass per propeller diameter based on min and max in case of prop size not listed in propD above

```
% Of the form: propeller mass = a*diameter + b
% Rate of change of mass per propeller diameter:
a = (propM(end)-propM(1)) / (propD(end)-propD(1));
% Constant
b = propM(1) - a * propD(1);
```

Comparing Input Diameters to T-MOTOR Propellers

to determine the mass of the propeller

```
% Following loop runs one cycle for each input "case" of diameter in
 the vector dP
mProp = zeros(1, length(dP) ); % Preparing propeller mass vector
for i = 1:length(dP)

    % Following loop compares the input diameter dP to each of the
 stored
    % T-motor props in propD and assigns the corresponding mass to
 mProp
    % If there is no match the linearized estimate is used instead
    for j = 1:length(propD)
        if dP(i) == propD(j)
            % If the input diameter in the dP position "i" matches
            mProp(i) = propM(j);
        else
            if j == length(propD) && mProp(i) == 0
                % If we just compared dP to the final option in propD
                % AND (&&) we have still not found a match
                disp(['Given propeller size', num2str(dP(i)),' does
 not have a known mass'])
                mProp(i) = a.*dP(i) + b; % Linearized estimate
            end
        end
    end
end
% The mass of each propeller is now stored in the vector "mProp"
```

```
Not enough input arguments.

Error in mass (line 41)
mProp = zeros(1, length(dP) ); % Preparing propeller mass vector
```

Motor Mass

The original BG-200 motor U8II is rated for 7.3kg thrust each. 8 of these then provide 58.4kg of thrust, giving a thrust/weight ratio of more than 2. To maintain a thrust to weight ratio of at least 2, the U10II motor is needed if only 6 rotors are used. For only 4 rotors, the U12II is needed for the same criteria.

```
% Following loop runs one cycle for each input "case" in the vector NR
mMotor = zeros(1, length(NR) ); % Preparing motor mass vector
for i = 1:length(NR)

    if NR(i) >= 8       % 8 or more rotors
        mMotor(i) = 272*g;      % U8II  KV100

    elseif NR(i) == 6   % 6 rotors
        mMotor(i) = 415*g;      % U10II KV100
```

```
    elseif NR(i) == 4   % Quadcopter
        mMotor(i) = 778*g;      % U12II KV120

    else        % An unaccepted number of rotors
        disp(['Number of rotors (NR = ', num2str(NR(i)), ') is
 invalid'])
        mMotor(i) = 1000;       % Setting mass to 1 tonn so script can
 continue
    end
end
```

Nacelle mass

Mass of the nacelle, eg. where the motor and the ESC is connected to the arm. Depends if coaxial or not.

```
mNacelle_Coax = 210*g;  % Mass of existing nacelle packaged from T-
motor
mNacelle_Single = 150*g; % Assumed mass of nacelle for single rotor

mNacelle = zeros(1, length(isCoax) ); % Preparing nacelle mass vector
for i = 1:length(isCoax);
    if isCoax(i) == 1
        mNacelle(i) = mNacelle_Coax;
    elseif isCoax(i) == 0
        mNacelle(i) = mNacelle_Single;
    end
end
```

Determining Constant and Variable Mass

```
% Staaker BG-200 specifications:
mBat = 3.972;           % Weight of battery
mHub = 3.828;           % Mass sum of hub components
    mConst = mHub + 2*mBat; % Mass sum which stay constant
                        % with varying NR, measured in kg
mESC = 73*g;            % Mass of Electronic Speed Controller
    mVar = mProp + mMotor + mESC;    % Variable mass minus arms and
 clamps

mClamp = 0.128; % Mass sum of all clamps and mountings needed per arm
 in k
mpl = 0.224;    % Airframe arm mass per length [kg/m]
                % Based on carbon fiber tube used for Staaker BG-200

kP = 1.05;  % PS: Defined two places; here and in the DroneSize.m
 function
            % Propeller size factor, to limit blade tip vertices
 interaction
            % eg. 2*kP*rP is the real distance between adjacent rotor
 axels

rP = dP/2;              % Radius from diameter
```

```
rP_m = rP*25.4 /1000;   % Transforming radius from inches to metres

% The factor kCoax is used as:
% kCoax.*NR = NR        if rotors are not coaxial, meaning one rotor
 per arm
% kCoax.*NR = 1/2 NR    if drone is coaxial, to adjust for two rotors
 per arm
kCoax = 0.5.^isCoax;

% The horizontal distance from the center of the drone to
% the rotor is R:
R = kP*rP_m ./ sin( pi./(kCoax.*NR) ); % Geometric relationship

L0_hub = .171; % Distance from center to start of arm
L0_arm = .158; % Minimum distance from hub to propeller tip

Larm = max( R - L0_hub, L0_arm + rP_m );
```

Final Calculation

```
m = mConst + NR.*mVar  + kCoax.*NR .*( mpl.*Larm + mClamp + mNacelle);

end
```

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APPENDIX C

Power Consumption Function

Here follows the script for the second function, which uses Equation 3.3 to estimate the power consumption of each UAV case.

```
function [ P ] = PMulti( NR, dP, m, etaPS, isCoax )
%PMulti Calculates estmate of hover power consumtion of multirotor
 UAVs
%   Can accept multiple cases, where the length of each input
%   variable must be equal to the number of cases or equal to one.
%   In the latter case the single value of the variable will be
%   applied to all cases.

%   Input explaination:
%   NR is number of rotors
%   dP is propeller blade diameter in inches
%   m is the total Take Off Weight of the multirotor in kg
%   etaPS is the propulsion system efficiency, excluding rotor
 interaction
%   isCoax should be 1 if the rotors are configured coaxially,
 otherwise 0

g = 9.81;   % Gravitational acceleration constant, m/s^2
rho = 1.225;    % ISA standard air density at sea level, kg/m^3

%   nRI is the rotor interaction efficiency factor.
%   This is the efficiency loss due to propeller proximity.
%   In a coaxial setup there are much larger losses compared to
%   in-plane rotors

%   for-loop to accept multiple cases where each case can be coaxial
 or not
for i = 1:length(isCoax)
    if isCoax(i) == 0
        nRI(i) = 1; % for adjacent in-plane rotors
    elseif isCoax(i) == 1
        nRI(i) = 0.763; % for coaxial interference
    else
        disp('isCoax must be 1 (rotors are coaxial) or 0 (in-plane
 rotors)')
        break
    end
end
rP = dP./2;             % Radius from diameter
rP_m = rP.*0.0254;    % Transforming radius from inches to metres

P = g^(3/2) .* m.^( 3/2) ./...
    ( etaPS.*nRI .* rP_m .* sqrt(2.*NR.*rho.*pi) );

end
```

Not enough input arguments.

Error in PMulti (line 24)
for i = 1:length(isCoax)

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APPENDIX D

Size Function

Here follows the script for the final function which, uses Equations 3.11 and 3.12 to estimate the length of each UAV case.

```
function [ L ] = DroneSize( NR, dP, isCoax )

%DroneSize Calculates the length of the drone
%   Calculates the distance in one dimension, along the roll axis of
 the
%   UAV, eg. the length from the back to the front.
%   With a quadrotor as an example, L is the distance from the back
 two
%   rotors to the front two rotors.
%   The size of the motor and the propeller is neglected.

%   Input explaination:
%   NR is number of rotors
%   dP is propeller blade diameter in inches
%   isCoax should be 1 if the rotors are configured coaxially,
 otherwise 0

kP = 1.05;  % PS: Defined two places; here and in the mass.m function
            % Rotor size factor, to limit blade tip vertices
 interaction
            % eg. 2*kP*rP is the real distance between adjacent rotor
 axels

rP = dP./2;              % Radius from diameter
rP_m = rP.*25.4 /1000;   % Transforming radius from inches to metres

% The factor kCoax is used to adjust the number of arms:
% kCoax.*NR = NR        if rotors are not coaxial, meaning one rotor
 per arm
% kCoax.*NR = 1/2 NR    if drone is coaxial, to adjust for two rotors
 per arm
kCoax = 0.5.^isCoax;

L0 = .329; % Minimum length from center to propeller tip

L_kP = 2*kP.*rP_m ./ tan( pi./(kCoax.*NR) );    % limited by kP
L_L0 = 2*(rP_m + L0) .* cos( pi./(kCoax.*NR) ); % limited by L0

L = max( L_kP, L_L0 );  % Length must comply with both kP and L0
 conditions

Not enough input arguments.

Error in DroneSize (line 18)
rP = dP./2;              % Radius from diameter
```

Adjust the Size of Configurations with Only Three Arms

as the above model will underpredict the size in this case. The real lenght along the roll axis for three arms will be L/2 + R:

```
% Need R:
% The horizontal distance from the center of the drone to the rotor
R = kP*rP_m ./ sin( pi./(kCoax.*NR) ); % Geometric relationship

% Adjusting formula based on how many inputs NR and isCoax has
if length(NR) == 1 && length(isCoax) == 1
        if NR*kCoax == 3 % If number of arms = 3
            L = L/2 + R;
        end
elseif length(NR) == 1
    for i = 1:length(L);
        if NR*kCoax(i) == 3
            L(i) = L(i)/2 + R(i);
        end
    end
elseif length(isCoax) == 1
    for i = 1:length(L);
        if NR(i)*kCoax == 3
            L(i) = L(i)/2 + R(i);
        end
    end
else
    for i = 1:length(L);
        if NR(i)*kCoax(i) == 3
            L(i) = L(i)/2 + R(i);
        end
    end
end

end
```

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