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## Abstract

At the City Bridge of Stavanger, or Bybrua, there have been recorded vibrations of stay cables that does not have a clear explanation. Due to the age of the structure and this occurrence, Statens Vegvesen and University of Stavanger are cooperating on various research topics to gain further knowledge on the behaviour of the bridge. In this thesis I create a model of the City Bridge of Stavanger using the software Abaqus FAE that I perform a frequency analysis of. From the bridge there is also acquired acceleration data at two different locations within the deck, so I perform a spectral analysis of this data too. From the analyses I can observe the how the motion of the bridge is based on the relative displacement for different eigenfrequencies. There is also found some correlation between the data sets, and the effect of the stay cables is visible on the data. The model and analysis in this thesis are simplified, so there are many opportunities for further improvements and research, and there is still more to learn about this bridge.

# Preface

This master's thesis was carried out between January and July of 2020 at the University of Stavanger, Department of Mechanical and Structural Engineering and Materials Science, in cooperation with Statens Vegvesen.

The work with this thesis has been a good challenge and learning opportunity, both in terms of the academic content, but also the circumstances in which it was written. I've gotten the opportunity to test many of my skills and acquire new ones when working to the thesis. It is also worth acknowledging that this thesis was written during the outbreak of COVID-19, which caused the university to close down and disrupted work with the thesis for a period, but things were eventually allowed to operate as relatively normal. This project has been a great experience and I'm glad for it to be the result of my master's degree.

I would like to thank my supervisors, Jasna Bogunovic Jakobsen and Ibuki Kusano, for all their support, help and guidance during the work with this thesis. I would like to thank Statens Vegvesen for allowing this opportunity and cooperating to make this thesis happen. I would also like to thank Sander Albin Felberg who has done his separate thesis on this topic and who has been helpful in figuring out how to do this. Lastly, thank you to the University of Stavanger and the staff there who have been helpful during this period, but also thank you to everyone who has educated me and gotten here.

Pål Ruben Hovind

University of Stavanger, July 13th, 2020

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#### Notations Roman

Roman	
Acs	Area of cross-section
d	Diameter
$d_i$	Distance, local neutral axis to cross-sectional neutral axis
E	Modulus of elasticity
G	Shear Modulus
J	Torsional constant
Ι	Second moment of area
Jcb	Second moment of area, cable cross-section
L	Length
h <sub>cs</sub>	Height of cross-section
b <sub>x</sub>	Width along x-axis
bz	Width along z-axis
m	Mass
Μ	Moment
Р	Applied load
$\mathbf{P}_{\mathrm{H}}$	Horizontal component of load
$P_{\rm V}$	Vertical component of load
q	Distributed load
V	Volume
t	Longitudinal direction, local axis element
n1	First normal direction, local axis element
n2	Second normal direction, local axis element

## Greek

γeq	Equivalent density (for stay cables)
Е	Strain
ν	Poisson's ratio
ρ	Density
ρ <sub>c</sub>	Density of reinforced concrete
ρ <sub>s</sub>	Density of steel
σ	Stress

## Abbreviations

SVV	Statens Vegvesen
NA	Neutral axis

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# **1** Introduction

In the eastern part of Stavanger city on the south-western coast of Norway, a bridge stretches across the bay. This is Bybrua in Stavanger, which translates to "the City Bridge", though it also has been referenced to as Strømsteinbrua after the bay that it spans over, Strømsteinsundet. It is 1067 meters long and connects the nearby islands with the city of Stavanger on the mainland via a short drive. It's hard to miss it when you're here, with the central concrete tower looming 78 meters over the water, making it visible from a large part of the surrounding area. The bridge has been there for over 40 years now as an important part of the city's landscape, and it will likely be there for many more years to come.



Figure 1: The City Bridge / Bybrua, Stavanger

As is expected for a bridge in a coastal region of the eastern part of the Atlantic Ocean, it can be subjected to strong wind loads and varying weather conditions. It was designed with this in mind in the 1970s, with the structural calculation and design being led by dr. Arne Selberg and engineer Johs. Holt. This resulted in the first cable-stayed bridge in Norway, using then stateof-art methods and materials.

Since when the bridge was designed and constructed there have been multiple developments when it comes to analysis of structures. In the past 20 to 30 years computational software has gotten more and more widespread, and today it is an essential part of the design of structures

comparable to the city bridge. These developments make it easier to examine how a structure might behave when exposed to different conditions, as well as giving a greater understanding of the internal reactions and interactions of the structure, or just a single part, when under said conditions. And such it is great value to perform analyses today of the city bridge with the aid of modern technology.

#### 1.1 Aim

The aim of this thesis is to examine the properties of Bybrua using computer software and compare it with acceleration measurements done with equipment that is installed on the bridge. What I hope this analysis can achieve is to give some explanation to the motion of the highlighted structure, ie. the part of the bridge that is under effect of the stay cables. I also hope that it is possible to look closer at the cables themselves and see what vibration occurs there under loads.

Why this is being done is so that one can easier can explain certain behaviors of the bridge. There have been observed significant vibrations of the stay cables under certain conditions, first

and foremost under strong wind and precipitation. With the current data and knowledge there is no clear explanation for this occurrence, so the University of Stavanger (UiS) and Public Road Administration (Statens Vegvesen, or SVV) are cooperating on multiple research topics surrounding the bridge and cause for these vibrations.



Figure 2: Stay cables of bybrua, North side of tower

This cooperative research is done to examine the various properties of the bridge, as well as the effect of various conditions. As the bridge has stood there for over 40 years, it is important to know why the bridge acts the way it does. There have been installed various monitoring devices throughout the bridge, such as accelerometers on the stay cables and within the bridge deck,

and these devices records the motion of the different parts of the bridge, which then can be used for this research.

In this thesis I will create a simplified model of a specified section of the bridge in the FEMmodelling software Abaqus FEA by Dassault Systèmes, and here I will be able to represent the loads and reactions that takes place throughout the structure. I will perform a frequency analysis that looks at the eigenfrequencies of the structure, which in turn can be used to look at relative displacement throughout the bridge. From this model I hope that it will be possible to explain the behavior of the bridge, such as the obtained acceleration data from two different locations within the bridge deck.

By the end of this thesis I hope that one will have a better degree of understanding when it comes to the motion of the bridge due to loads, where one can see for example how the steel deck behaves in relation to the concrete deck. I also what the model to be a good representation of the city bridge itself that in turn can be of use for future research.

### 1.2 Limitations and Methods

Through my study and this thesis, I want to give a realistic representation of bridge and actual conditions, but there are certain shortcomings one need to be aware of when reading this thesis and examining its results.

The design and calculation of the bridge took place in the 1970s, and a lot of the documentation that I had to rely on were calculations done by Selberg and Holt, as well as drawings to give dimensions and distances across the bridge. There is no overview of dimensions and properties, so there had to be made assumptions based on the calculations and drawings, as well as use of approximation where needed. The properties of cross-sections were calculated manually as those values were not available.

For this thesis I modelled the bridge using input-files to be processed by Abaqus FEA. This will provide a fairly simplistic model where it is defined by nodes, elements, element properties, and interaction between the elements. The interactions are limited to connection of the degrees of freedom, so there is no surface or contact modelling that might be more representative of

reality. There might better methods to model the bridge or to analyze its behavior, but the use of input-files and Abaqus FEA were decided to be used due to its functionality and simplicity.

As for the scope of the thesis, it is focused on modelling a representative version of the bridge that is affected by the stay cables, and also examining the motion of the bridge deck. Any part of the bridge that extends north or south of the designated section is not taken into consideration in this thesis.

Variable loads such as traffic load is not taken into consideration. The acceleration data is produced presumably due to the specified wind load, but this is not taken into consideration with the modelling. Neither is other weather conditions such as temperature, humidity or precipitation.

Calculations done manually by hand or Microsoft Excel; Abaqus FEA and a text editor software used for modelling and frequency analysis; Matlab used for spectral analysis and plotting of results.

Abaqus FEA is a software used for finite element modelling. It will consider each element of the model and compute how the elements interact with each other.

# 2 Theory

#### 2.1 Cable Stayed Bridges

Cable supported bridges is a method of bridge building that has evolved over the past 200 years, allowing for larger bridges with longer and longer bridge spans. It mainly involves using cables to hold up and stabilize bridge decks through various methods; an example of this is suspension



bridges where the deck is connected to two larger main cables through multiple smaller cables that are spread evenly across the bridge span (see figure 3 [1]).

Figure 3: Suspension bridges [1]

Another variety of a cable supported bridge that I want to examine now is the cable stayed bridge. The City Bridge in Stavanger is an example of this, so it is helpful to explain some of the concepts behind the design of this bridge.

What a cable stayed bridge is comprised of is the bridge deck, one or multiple pylons/towers, and cables that connects those two parts. From the pylon(s) the cables can be distributed in various configurations to suit whatever the load case for the bridge in question. For cable stayed bridges there are three prevalent configurations: fan system, semi-fan system, and harp system (see figure 4[1]).



Figure 4: Cable stayed bridges [1]

#### 2.1.1 Cable System

In the harp system the cables are parallel to each other, positioned at the same angles relative to the deck/pylon(s), as seen in figure 4. The cables are distributed along the pylon(s), similarly to how it is along the bridge deck, which allows the load distribution to be evenly spread along the pylon(s).

The alternative to this is the fan system where the cables are connected at one point on the pylon, usually near the top, and the cable system is spread along the bridge deck at varying angles from that point, like the fan it is named after. Following this, the configuration can also either be symmetrical or asymmetrical about the pylon(s) based on what would be deemed necessary.

#### 2.1.2 Cables

The cables that are used for these bridges as steel cables consisting of multiple wires strands forming a thicker, resilient cable. The type of cable depends on the placement of the multiple strands within the cable, whether they are parallel to each other or placed in a helical pattern. Some of the types that are worth notice include helical bridge strands, parallel-wire strands and the locked-coil strand. With the helical bridge strands the strands are located in helical layers, each layer spiraling in alternating directions; and for parallel wire strands they all lie in the same direction, longitudinal with the cable and no spiraling.



Figure 5: Cable cross-section, locked-coil strands [1]

The cable that is most relevant for this thesis is locked-coil strands, where the outer layers are in a twisting z-pattern of interlocking wires and the innermost being in a helical pattern (figure 5[1]); based on given information this is the cable type used in the city bridge in Stavanger. A unique capability of this cable type is the low void ratio, that is the amount of hollow space within the cable, so it is packed tighter than other types. Combined with the low amount of corrosion resistance used, due to galvanizing, the equivalent density  $\gamma_{eq}$  ends up being about 8800kg/m<sup>3</sup>. The strength and stiffness are lower compared to other cable types used today.

#### 2.1.3 Anchoring System

An important part of the cable supported bridge is maintaining balance, so for a harp system there is an equal amount of cables to each sides of the pylon for example. Under this one also need to consider anchoring that is there to prevent the pylon from experiencing too large displacements. The anchoring cables are there to transfer the loads onto parts not as subjected to displacements. For cable supported bridges there are two common methods for anchoring, and they are the earth-anchored system and the self-anchored system. Whereas the earthanchored system transfers the vertical and horizontal loads onto an anchor block separate from the bridge deck, the self-anchored system transfers vertical loads to an anchor pier underneath the deck and the horizontal forces becomes compression within the deck itself.

### 2.2 The City Bridge of Stavanger, Bybrua

The City Bridge of Stavanger began planning in the early 1970s due to a want to connect to the island of Hundvåg and other nearby islands to the north with the city center. After year of planning and construction it opened in early 1978 and has since been an important connection for many people who commute every day.



Figure 6: On the City Bridge

When it opened in 1978 it was the first cable stayed bridge in concrete and steel built in Norway, and one of the largest bridges in the country at the time with 1067 meters length divided over 24 bridge spans. The main span of the bridge consists of a steel deck, with box-girder design, supported by stay cables and is 185 meters as it crosses Strømsteinsundet; the remainder of the bridge deck is of reinforced concrete with a box section. The bridge is 15.5 meters wide, with three lanes for automotive traffic and two lanes on each side for pedestrians.



Figure 7: Main span of bridge, seen from underneath

#### 2.2.1 Components of the bridge

The City Bridge of Stavanger is a cable stayed bridge with cables in an asymmetrical fan system with a total of 40 cables of the type locked-coil strands. At one end the cables are attached to the top of the central concrete tower; the tower is 76.75 meters tall, and the anchoring for the cables are at approximately 73 meters height, i.e. 45 to 50 meters above the bridge deck. Along the deck the cables are connected to crossbeams at six different sections; to the south is the main span where stay cables distributed at 39, 83 and 132 meters from the tower; on the north

side at 82 and 38 meters from the tower. The cables to the north acts as the self-anchoring system where the loads are transferred into the concrete deck and onto the support pillars underneath the deck. The length of the cables varies between approx. 60 and 142 meters.



Figure 8: Bridge seen from Buøy

#### 2.2.2 Status of the bridge

In 2016 and 2017 it had a daily average of almost 17,000 vehicles driving across, though that number has decreased some recently and is expected to continue to do so; in 2019 the average was 15,523 vehicles[2]. In April of 2020 there opened up a new tunnel, Hundvågtunnelen, that goes underneath the city and the bay before it resurfaces on Hundvåg, which in essence allow traffic from the highway E18 and the south of Stavanger to entirely bypass the City Bridge. This new tunnel is part of a larger road construction project called Ryfast to connect the areas of Ryfylke with roads and allow easier travel without the use of ferries. Beside Hundvågtunnelen, this project also includes the tunnel Ryfylketunnelen between Hundvåg and the area of Solbakk, which at 14.3 kilometers is the longest subsea tunnel for automotive traffic when it opened in December of 2019[3]. The City Bridge is not prepared for the amount of extra traffic Ryfylketunnelen will introduce, so a noticeable percentage of the traffic will likely be redistributed to Hundvågtunnelen starting this year (2020). Most recent recordings done by SVV shows a daily average of 8000 vehicles in the months of May and June.

Table 1: Traffic on the City Bridge[2]

Year	Volume, daily average
2016	16895
2017	16963
2018	16361
2019	15523

# **3** Methodology and Procedures

#### 3.1 Modelling of the bridge

At the start of this thesis there were limited data available when it came to the properties and dimensions of the bridge. For me what was initially made available where drawings of the overall structure, including cross-sections of some of the various parts of the bridge, as well as various drawing to give an overview of the bridge. Eventually there were more drawings made available to show more cross-sections and the reinforcement of different part of the bridge.

The other documentation that was made available and was very helpful for figuring out properties and dimensions were some of the calculations done by Arne Selberg and Finn Holst[4]. These calculations showed what values were used for calculations, as these were mainly checks for the different sections to ensure that they were able to sustain the design loads. By searching through the calculations, using the available drawings[5] and making some approximations, I was able to determine the distances, dimensions and properties to use for my model. I will now go through how the design of the model came about.



Figure 9: Section of bridge considered in this thesis [5]

For my thesis I concentrate about the section of the bridge that is affected by the stay cables, that is main span in steel that is located between what is referenced as axis 12 and axis 13, and concrete deck between axis 9 and axis 12, in addition to the tower. This means that I will model 305 meters of bridge deck; the remaining sections are supported by concrete pillars the effect of the cables and tower are assumed to be negligible. At both axis 9 and axis 13 in each their end the deck sections are separated from the rest of the deck so that motion and displacement is allowed without providing further motion throughout the structure.

#### **3.1.1** Defining the geometry

To model the bridge in Abaqus FEA, the relevant components of the bridge are represented as elements that are located between two nodes; so, there are placed nodes at points of interest throughout the bridge, and then elements are defined based on which nodes it is located between. For all elements that are defined in this thesis, the locations of nodes that represents the position of the elements are placed at the neutral axes for the specific elements.

One of the first things that were calculated was the location of the bridge deck, as that was something the location of the other parts could build upon in subsequent calculations. From drawing 7208/1C (Figure 9, Appendix A) I read of the height of the top of the bridge deck at the locations I could and formulated an equation that would give the height along the bridge at the locations with no height given (figure 10). I assume that the deck rises linearly between two set of points indicated as "tangentpkt." both to the north and south, while it curves in middle section between these sets, and it reaches its maximum height of 29.110 meters at "topp pkt. pel 61+7.72". Equation for locations of nodes in Appendix B.



Figure 10: Elevation of road, top of deck

Similarly, I calculated the shape and distances of the central tower. From the given documentation I had values for some of the cross-sections at different heights, but again I had to formulate equations so that I knew the positions of neutral axes and the different dimensions along each leg of the tower. The crossbeam between the two legs of the tower that the bridge deck rests upon was also calculated, and from the documentation it was assumed to be a hollow cross-section with the same dimensions throughout its longitudinal direction; in reality the top section of the crossbeam is thicker at the point(s) where the bridge deck is resting upon. Position of crossbeam was decided based upon the available drawings.

Something that had lacking documentation and drawings was the top of the tower where the cables are anchored. As there are no clear indication of the interior of this part, I assumed it to be a solid cross-section. The location of the anchoring points where assumed to be at the center of the top section at heights deduced from the original calculations that were given.

The stay cables of the bridge are connected between the top and the deck, and at the deck the cables are anchored to crossbeams that extrudes from the deck at the sections where the cables connect. In the main span these cross beams are of steel and concrete in the concrete section. Using drawings and observations from visiting the bridge I performed some calculations to get a better understanding of these crossbeams, since the only available data were the locations of a few points on the crossbeams at three out of the six sections.

From calculations the neutral axes of these beams are off from the neutral axes of the decks by some centimeters, but they are assumed to be at the same height when modelling for simplicity purposes.



Figure 11: Top of tower



Figure 12: Crossbeams for anchoring at concrete deck

Along the entire bridge structure there are pillars evenly spaced out that supports the bridge deck. For this thesis only the pillars at axis 10 and axis 11 are considered, as they are located at the connection points for 16 of the stay cables. Though there are two pillars at each of these two axes, they are modelled as one at axis 10 and axis 11 and given properties and constraints to ensure that it behaves similarly to real-life behavior. The locations of pillars were read from drawings (Appendix A), and the position of the top and bottom of these pillars where measured and calculated from said drawings.

From all of the calculations and measurements, the following model can be displayed in Abaqus CAE, which is the graphic user interface of Abaqus FEA.

#### 3.1.2 Defining element properties

When modelling this bridge and performing an analysis it is essential to know the properties of the elements one creates, but there were lacking information here too. To ensure that the properties complies with drawings, everything were calculated manually, either by hand or use of Microsoft Excel. Properties that were calculated were the neutral axis  $y_{cs}$ , cross-sectional area  $A_{cs}$ , second moment of area about the elements two normal axes  $I_{11}$  and  $I_{22}$ , and where possible torsional constant *J*. For all elements the product moment of area is assumed to be zero due to symmetry about at least one axis.

Properties such as modulus of elasticity *E* and Poisson's ratio are either given or assumed based on what material the element is. From the given documentation, the concrete that is used through the entire bridge is assumed to be of type C35, so properties can be found from NS-EN 1992-1-1:2004[6]; steel properties are found in NS-EN 1993-1-1:2005[7] (see table 1). All concrete is assumed reinforced, which gives a density of 2.5 tons/ m<sup>3</sup>; density of steel assumed to be 8.0 tons/m<sup>3</sup>. Meanwhile for the stay cables the material properties are gathered from Cable Supported Bridges: Concept and Design by Niels J. Gimsing and Christos T. Georgakis[1], where locked coil strands are described; density that is used is the equivalent density of 8.8 tons/ m<sup>3</sup>. The material parameters that are used throughout this thesis will be the following:

Material	E [GPa]	v[-]	G [GPa]	$\rho[(10^3 \times kg)/m^3]$
Concrete, C35	34.000	0.2	14.000	2.5
Steel	210.000	0.3	81.000	8.0
Locked-coil strands	180.000	0.3	69.231	8.8

Table 2: Material Properties, elements

Beside the material properties, one also need to consider the cross-sectional properties. First, I find the area and neutral axis of the cross-section by dividing it into n multiple smaller areas with their own neutral axis and then add it together using the following formulas:

$$A_{cs} = \sum_{i=1}^{n} A_i$$

Equation 1: Summation of cross-sectional area

$$y_{cs} = \frac{\sum_{i=1}^{n} (A_i \times y_i)}{A_{cs}}$$

Equation 2: Determination of cross-sectional neutral axis

Using the same principles, I calculate the second moment of area using the same n areas and sum it together, which also entails use of the parallel axis theorem. The local neutral axes are at distance  $d_j$ , where j is the axis which the second moment of area is calculated about and i is the numbering of the specified area.

$$I_{jj} = \sum_{i=1}^{n} (I_{jj,i} + A_i \times (d_{j,i})^2)$$

Equation 3: Second moment of area about axis j, j=[x,y,z]

For the torsional constant J the methods of calculations are a bit more uncertain due to the lack of any generalized equations to be used for most cross-sections. For rectangular and rectangular hollow-sections the following formulas were used:

$$J_{T,rectangular} = a \times b^3 \times \left(\frac{1}{3} - 0.21 \times \frac{b}{a} \times \left(1 - \frac{b^4}{12 \times a^4}\right)\right)$$

Equation 4: Torsional constant for a rectangular section

$$J_{T,thin-wall} = \frac{4 \times A_o^2}{\int_S \frac{ds}{t}}$$

Equation 5: Second moment of area for a thin-walled cross-section

For a cross-section to be considered thin-walled the ratio between the thickness and length of side should be less than 10, but this requirement is ignored due to simplification. Other assumptions and simplifications that are made in terms of the torsional constant include considering sections as doubly axisymmetric which would give  $J = I_{11} + I_{22}$ , though this is inaccurate, and for the steel section it is set as  $1m^4$ .

In terms of the cables, the area was given as 4,272mm<sup>2</sup> and diameter as 79mm, which implies a void-ratio of 0.12846. Using equations from Cable Supported Bridges: Concept and Design[1], calculation of second moment of area and torsional constant becomes possible. Due to the arrangement of the cables, the second moment of area that is used in the model is that of a single cable, which is:

$$J_{cb} = \frac{A_{cb} \times d^2}{16} = \frac{0.004272m^2 \times (0.079m)^2}{16} = 1.666347 \times 10^{-6}m^4$$

Equation 6: Equation for second moment of area for a cable, calculation for stay cables

The stay cables were initially attempted to be modelled as beam-elements, but due to unexpected effects on the model, the decision was made to model them as truss-elements. These elements are not the most realistic method of depicting the cables, but in terms of transferring the axial load it works well.

Other elements that are altered are the crossbeams where the stay cables are anchored. For simplification purposes, these are considered as massless elements with high stiffness so as to not deform or be the cause of any large deformation. The crossbeams within the steel deck have lacking data so it would have been difficult to represent these components accurately. A similar approach was done at the top of the tower to connect the tower top to the two legs of the tower, since this is a non-existing component, but is a convenient way to transfer loads and displacements between the sections.

#### 3.1.3 Defining pre-stress within concrete deck section

Before the analysis can be performed, it will also be useful to apply the prestress that is located within the concrete deck. In the deck between axis 9 and axis 12 there are cables in the walls that provide equivalent loads and moments that counteracts the effect of self-weight. There are

other cables that provides prestress in the deck, e.g. in the upper section of the concrete crosssection close to the tower, but this not used in this analysis.

By looking at the drawing 7208/B005 (Appendix A), one gets a general idea of the location and shape of the cables, as well as an indication of the pre-stress that was applied. From the table on this drawing it says: "minimum sum of tensile forces after release", and gives the values in Mp, where 1Mp is 1000 kilograms. These values are what I used when applying pre-stress, not considering that sum of forces could be higher or that there is a loss of pre-stress since it was applied.

To find the accurate location and shape of the pre-stressing cables I use drawing B006, B007 and B008 where the height within the deck is given at multiple points. One sees that there are three groups of cables; group 1 is curved and between axis 9 and 12 with two splices; group 2 is straight and located above axis 10; group 3 is also considered to be straight and located at the bottom of the wall between axis 11 and axis 12. There are more cables indicated in 7208/B005, but only those aforementioned ones are given enough data to be for further use.

The shape of the cables is then used to calculate the equivalent loads. Along the deck there will be a distributed load in the vertical direction, horizontal point loads at the ends of the cables and at the splices, and applied moments due to the eccentricity of cables relative to the neutral axis the elements.

#### 3.1.4 Other components of the model

Beside the nodes, elements and prestress, there are a few other components that are incorporated into the model.



*Figure 13: Connection, deck with support pillars and crossbeam* 

There are constraints at some of the end points of the bridge. At the bottom of the tower and the support pillars there are fixed supports, i.e. there not allowed movement in any of the six degrees of freedom. The other constraints are at the two ends of the deck; at axis 9 it is considered to be a pin support, so d.o.f. 1, 2 and 3 are constrained; at axis 13 it is considered a roller support.

To connect the various elements, I used the command "\*EQUATION", which sets up an equation where it can equate the motion in one d.o.f. at one point with that

at another one. This was used to set the displacement of the deck to equal that of the support pillar it rests upon at axis 10 and axis 11, in addition to at the crossbeam at axis 12 (figure 13).

Lastly, beside the self-weight, there are other permanent loads to be added. These are nonstructural and are hence added as non-structural mass. The loads in question are the asphalt, any steel on top of the deck such as railing and poles, and steel plates that hang on the sides of the box-girder deck of the main span. They are added as mass/meter onto each deck element.

### 3.2 Analysis of bridge in Abaqus FEA

Once all node locations and the properties of elements are determined, then it is necessary to convert the data to a format that can be interpreted by Abaqus FEA. For this thesis I will be using input files, so I create files for the nodes, each element set, and for other commands.

Central in this will be the one input files that will be used for the computation and references all the other input files that are active, so by using a command-window within the folder containing all the input-files, and Abaqus commands, it will run the necessary analyses to obtain the results.

Input files can be found in Appendix C.



Figure 14: Model, placement of nodes



Figure 15: Model, numbering of nodes



Figure 16: Model, numbering of elements

This frequency analysis is done in four steps, where the first three steps are necessary to calculate the state of the bridge, and the fourth one is the frequency analysis itself. The first three are static analyses that calculates displacements, where it applies the loads in step increments. After the last increment, one gets the final displacement after the application of the given loads. For this model I will look at the horizontal displacements in the x-direction at node 143, which is where all the cables are anchored at the top of the tower, and at vertical displacement (y-direction) at nodes 39, 49 and 60 which are where the cables connect to main span (see figure 15).

#### 3.2.1 Application of self-weight

The first step is to calculate displacements of certain nodes when the bridge is subjected to purely self-weight, i.e. that one does not assume any loads in the stay cables at the start and there are no other loads beside pre-stress in the concrete deck. After the analysis has been ran through by the Abaqus command functionality it will return .dat-files within that folder that contains the procedure and results of the analysis. I record the nodal displacements I need and move onto the next step.



Figure 17: Model, displacement U when self-weight applied

#### 3.2.2 Application of unit stress

During the next part there will not be any gravitational load, but each cable will be introduced to a unit stress. For this model there are four points where we record the displacements, so that would mean four equations; from this there are four unknowns and that are the stresses in the stay cables. Therefore, I apply the unit stress four separate times, with the first one being the stay cables that connects to the concrete deck and are part of the self-anchoring system. Here the cables connect at places where there are support pillars, so it would not be possible to record the vertical displacement, hence why the displacement at the top of the tower is being recorded. In this first part the unit stress is applied onto all of these cables, three element sets with 20 cables in total, and it is the same in each. The unit stress that is used is 1000 kN/m<sup>2</sup>.

Following this the unit stress is removed and applied onto the three remaining element sets of cables that are connected to the main span, one by one. After the displacements caused by unit stress in one set of cables, e.g. the set of the four cables that connect to the main span at 39 meters from the tower, I move onto the next set, at 83 meters, and record the displacements there.

#### 3.2.3 Assembly of model

The third step of this process is to calculate what the accurate values for the unit stresses should be. The displacements that have been recorded due to self-weight is the vector B, while displacements due to unit stress becomes the 4-by-4 matrix A. By the logic that the stay cables are subjected to the stress in vector X when the self-weight applies a force onto the cables, one can solve the equation  $A \times X + B = 0$ , and with A and B recorded we find the stresses that exists in the stay cable. I insert the values for A and B into the software Matlab, and use the command "linsolve" to produce the vector X. This X needs to be multiplied with the unit stress from the previous step to give the amount of kilonewtons that are used. These calculated stresses, together with the previous input files, will then give the final representation of the bridge within Abaqus FEA.



Figure 18: Model, displacement U of final model

Shape at node 20 due to error in node numbering for this print of model.

## 3.3 Frequency Analysis

Now that the model is complete, it is time to perform the frequency analysis using Abaqus FEA, step four. As was done for calculating the displacements, I once again use the command window and Abaqus commands within, this time using the "run\_freq.inp"-file. Here it instructs Abaqus to run a frequency analysis of 30 modes, i.e. 30 eigenvalues and eigenfrequencies, which will be printed to the .dat-file, and also motion in degrees of freedom of the nodes along the bridge deck; in this thesis the d.o.f. of interest are translation in all axes as well as rotation about the longitudinal bridge axis.

With all the analysis complete and the modes and motion at nodes recorded, the results can be processed and presented. Provided to me was a Matlab-file, coded by Ibuki Kusano (Appendix D), that gather the necessary information from the .dat-file and prints graphs for each mode that displays the motion at the nodes. The four motions in these graphs are lateral, vertical, longitudinal and torsional. These graphs show which motion is dominant at the different modes and the relative displacement for the nodes.

The graphs produced by the frequency analysis and the Matlab-code will be compared to the spectral-analysis in the next part of the thesis, where acceleration data at two locations provides us with which frequencies causes the largest relative motion.

#### 3.4 Spectral Analysis

#### 3.4.1 Gathering of data



Figure 19: Accelerometer within concrete deck, between axis 10 and axis 11

At the bridge there are two accelerometers located within the deck; one in the concrete deck between axis 10 and axis 11 at approximately 60 meters away from the tower to the north; the other is in the main span approximately 20 meters from the tower to the south. These record accelerations in the three principal axes, before storing and sending the data. In this thesis I will examine recordings done on August 28<sup>th</sup> in 2019 at 7:00am. These recordings are for a 10-minute interval when the average wind speed was 8 m/s; sampling rate was 50Hz, which results in 180,000 data points for acceleration.

#### 3.4.2 Acceleration data

These data have been converted into the file "dataAccDeckBybrua\_28082019\_0700.mat", where there are the (2x180000) acceleration matrices: Ax, Ay and Az. The first row of these matrices are the accelerations of the concrete deck, while the second row are those of the steel deck. Using the sampling rate of 50Hz and the size of the matrices, it is possible to obtain a vector for the time. The time interval between each data point is  $dt = \frac{1}{fs} = \frac{1}{50} = 0.020s$ , and thus the vector t will be (1x180000) starting at dt and ends at 3,600 seconds. With the acceleration matrices and time vector, I plot the time-series for the accelerations which will indicate the motion of the bridge deck at the two different locations.


Figure 20: Acceleration of deck, vertical direction



Figure 21: Acceleration of deck, lateral direction



Figure 22: Acceleration of deck, longitudinal direction

### 3.4.3 Spectral analysis in Matlab

With the acceleration data one can perform a spectral analysis through Matlab. Matlab has a function called "pwelch", that performs Welch method to output an estimate of the power spectral density, as well as a frequency vector.

The equations that are used are:

$$[pxxc, fxc] = pwelch(Ax(1,:), [], [], [], fs)$$
  

$$[pxxs, fxs] = pwelch(Ax(2,:), [], [], [], fs)$$
  

$$[pzzc, fzc] = pwelch(Az(1,:), [], [], [], fs)$$
  

$$[pzzs, fzs] = pwelch(Az(2,:), [], [], [], fs)$$
  

$$[pyyc, fyc] = pwelch(Ay(1,:), [], [], [], fs)$$
  

$$[pyys, fys] = pwelch(Ay(2,:), [], [], [], fs)$$

Equation 7: Use of "pwelch" in Matlab, calculation of power spectral density

Each equation produces two vectors of size (32769x1), one is the frequencies and the other is the correlating power spectral density with the units [( $m/s^2$ )/Hz]. To refine the results it is possible to use a low-pass bandwidth filter, such as the filter provided by the function "butter" in Matlab, but they are not taken advantage of in this thesis.

From the output I can plot the power spectral density to the frequencies, and one can then see the correlation between acceleration and frequency. This will be used to compare the measured acceleration with the modes from the model.

## 4 **Results**

### 4.1 Frequency analysis

After the frequency analysis is performed and plotted, there are 30 graphs that depicts the motion of the four different degrees of freedom. The frequency for each mode is displayed in table 3.

The plots, which can be found in Appendix E, shows the what motion occurs at the frequencies and the relative magnitudes along the bridge deck between x=[-120,185]. One of the mode plots that can be used as an example is that of mode 18 (figure 23).



### Figure 23: Mode 18

The x-axis is the location along the deck, so we see it starts at x=-120 and ends at x=185. The y-axis is the motion/displacement that is caused at that mode. Another thing to observe is that even though motion in all of the d.o.f. we use is plotted, i.e. lateral, vertical, torsional and longitudinal motion, only vertical is impacted for this node. This varies for each mode, as the frequency analysis is done by impacting one section at the time. There is some longitudinal displacement as well in

Mode	Frequency
1	0.711
2	0.9711
3	1.3950
4	1.4571
5	1.5623
6	2.3182
7	2.4381
8	2.4778
9	2.6088
10	2.6763
11	3.2061
12	3.6768
13	3.7331
14	4.0354
15	4.1339
16	4.5790
17	4.6258
18	5.3648
19	5.8719
20	5.8838
21	6.3031
22	6.8780
23	7.3482
24	7.3484
25	7.7569
26	7.9808
27	8.9396
28	9.2727
29	9.3338
30	10.3280

Table 3: Modes and eigenfrequencies

this plot, but its magnitude is significantly less than that of the vertical motion. One more thing worth paying attention to is the difference in behavior between the concrete deck (negative x) and steel deck (positive x). Whereas the steel deck swings between  $\pm -0.7$ , the concrete deck does not move to the same extent; the vertical displacement is only positive to a max of approx. 0.50, indicating that at least for this mode the steel deck is more subjected to movement in the vertical direction.

Another one to look at is the plot for mode 14 (figure 24). It has similarities with the plot for mode 18, but there are some noticeable differences. As a reminder, the stay cables connect at x=-82, x=-38, x=39, x=83 and x=132, and there seems to be some correlation between those positions and the shape of the plot. From x=-82 until x=0 the vertical motion is only positive, whereas from x=0 to x=132 it is negative and 5 times the magnitude. The



other motion that is of significance here is that of the longitudinal direction. At the points where the stay cables are connected, we see that the shape of the plot changes; at 82 it starts to increase and increase even more at 38; decrease starts at 0 but lessens at 83 and 132. From the plot we can see that the stay cables do have an impact on the overall motion of the structure.



To show how the lateral displacement is affected, we can take a look at mode 24 (figure 25). There is a difference in the magnitude between concrete and steel decks, and there is also a noticeable discontinuity at x=0, indicating the different properties and components of the two sections.

## 4.2 Spectral analysis

By using the pwelch-function on the acceleration data, I obtain six power spectral densitydiagrams, three for each location, then plot them against each other to see the relation between the two locations, but also to compare motion in the three axes against each other.



Figure 26: Power spectral density, vertical, concrete vs. steel



Figure 27: Power spectral density, lateral, concrete vs. steel



Figure 28: Power spectral density, longitudinal, concrete vs. steel

The three diagrams above (figure 26-28) compare the power spectral density between the acceleration data from the concrete section (blue) with that from the steel section (red). In all three the red is dominant, though the peaks can be at different frequencies.

From these diagrams it is also possible to pick out which frequencies that causes these spikes in PSD. From them we get the following frequencies (approximate readings):

	1	2	3	4	5	6	7	8	9	10	11	12	13
Vertical	0.60	1.20	2.05	2.95	3.55	4.15	4.80	5.90	7.15	8.50	8.65	9.8	11.05
Lateral	0.00	1.20	2.95	3.50	4.15	4.45	6.0	7.10	8.50	9.90	10.45	11.10	-
Longitudinal	0.00	0.85	1.60	2.50	4.20	5.95	9.00	11.00	-	-	-	-	-

Table 4: Frequencies of notice, PSD

# 5 Discussion

### 5.1 Model of bridge

The model of the bridge that I modelled did end up working well and gave the values that expected values. There were a lot of trial-and-error to get the model right, and for the most part of the modelling there were made attempts to make a model that approached the actual bridge where it was possible. Sadly, these versions of the model produced either errors or inaccurate values, so in the end I decided to with a more simplified model.



Figure 29: Top of tower, cables connect at node 143

Some of these simplifications were to model the stay cables as truss-elements and connecting all the cables at a singular point at the top of the tower (figure 29). Connecting the cables should still be a good approximation that works but using truss-elements instead does have some negative effects. The truss elements only transfer axial loads and do not deform like the cables does in reality, so the elements act purely for transferring loads through the system. The effect of the cables might be small in reality, but it is a loss of data and accuracy that would've been appreciated.



Figure 30: Final model, displacement U, all loads and pre-stress applied

The final version of the model with all loads and pre-stress applied appears to function as planned, where the loads and displacements are distributed well throughout the structure. The one area with a noticeable displacement U is at the end of the main span towards axis 13 (figure 30). At axis 13 to the rightmost of the model, it is modelled as a pin support that allows rotation about two axes.

### 5.2 Comparison and correlation, analyses

In general, the analyses show that the main span is easier to have displacement, which is to be expected. The various figures show an approximation of the relation to other parts of the structure when it comes to motion that is either lateral, vertical, longitudinal or torsional.

By looking at the frequency analysis and the spectral analysis, we can see that there is some correlation between the frequencies that give motion in the structure.

From the analysis there is also an indication of the effect that the stay cables have on the bridge. The diagrams show how the relative displacement changes due to the influence of the cables.

### 5.3 Sources of error

- Lacking data to create accurate model
- Assumptions made can be wrong
- Error when calculating values
- Error in code
- Altercations done by software when performing
- Simplification of model

### 5.4 Possible improvements and future research

- Gather more data
- Model bridge in a different configuration
- Model different section of bridge
- Expand analyses
- Look at other loads on bridge
- Examine stay cables further

## 6 Conclusion

At the start of this is thesis what I set out to do was to model the City Bridge of Stavanger and perform both a frequency analysis of the bridge, in addition to a spectral analysis recorded acceleration data. I believe I have in part achieved this with the model and the analyses that were performed, though I had hoped that more data could've been both gathered and produced. The model was able to be a good representation of the bridge, and from using the analyses we get values that can be seen as similar to those of the recorded acceleration data. The frequency analysis of the model also gives some information on the how the cables effect the relative displacement for the rest of the structure. It is a simplified model, so it is not accurate, but it does give a good idea for some of the actions happening with the bridge.

From the data of this thesis we have not yet acquired an answer for exactly why the stay cables vibrates at certain conditions or gathered all properties for the City Bridge, but I hope that this thesis will be of help for future research when examining the bridge. The bridge is an important part of the city of Stavanger, and with this work that I have been a part of, I believe it will sustain for several more years.

APPENDIX A: Original drawings



Figure 31: Overview, bridge







Figure 33: Cross-section, steel deck







Figure 35: Deck between axis 9 and axis 10, tapering box cross-section



Figure 36: Cables pre-stress, 9-10



Figure 37: Cables pre-stress, 10-11



Figure 38: Cables pre-stress, 11-12



Figure 39: Support pillars, axis 9



Figure 40: Cross-section axis 10, outline of support pillars

## **APPENDIX B: Node and element definition**

Units: meter

#### Elevation of deck **B**.1

Coordinate system for model, not same as acceleration data. x: longitudinal (for deck) y: vertical z: lateral (for deck)

Equations for finding the elevation of deck. y<sub>road</sub> is assumed to be top of cross-section. In relation to coordinates of model, axis 12 is located at x=513 in equations, x=0 in model.

 $y_{road,1} = \left(\frac{1607}{60158}\right) \times x + 14.74627321$ Equation 8: Elevation of road, valid for x=[156.73,457.52]

 $y_{road,2} = (-8.350803011 \times 10^{-5}) \times x^2 + (0.1031691607) \times x - 2.753673555$ Equation 9: Elevation of road, valid for x=[457.52,675.95], relation to model: x\_local=0 at x=513

$$y_{road,3} = \left(-\frac{4.378}{451.05}\right) \times x + 35.3889336$$

Equation 10: Elevation of road, valid for x=[675.95,1127.0]

Calculation of neutral axis. Calculate Acs; calculate relative neutral axis for element at ycs.

$$A_{cs} = \sum_{i=1}^{n} A_i$$
$$y_{cs} = \frac{\sum_{i=1}^{n} (A_i \times y_i)}{A_{cs}}$$

For deck cross-section, y of neutral axis is at:

 $y = y_{road} - (h_{cs} - y_{cs})$ 

x=0, z=0

#### B.2 Tower

Position of z for leg of tower in relation to height y

$$\pm z_{tower} = \pm \left( -78 \times \frac{(y_{tower} - 2)}{473} + 13.421 \right)$$

Dimensions, cross-section b<sub>z</sub>=2.25

$$b_x = 2 \times \left(\frac{618}{307} - 2 \times \frac{y_{tower}}{307}\right)$$

Cross-sectional area

$$A_{cs,tower} = A_o - A_i = b_x \times b_z - (b_x - 1.2) \times (b_z - 1.2)$$

Top of tower, cross section, between y<sub>tower</sub>=[70.5,78.75]

$$b_{z} = 2 \times \left(-78 \times \frac{(y_{tower} - 2)}{473} + 2.125 - 68.5 \times \left(\frac{-78}{473}\right) + \frac{2.25}{2}\right)$$
$$b_{x} = 2 \times \left(\frac{618}{307} - 2 \times \frac{y_{tower}}{307}\right)$$
$$A_{cs} = b_{x} \times b_{z}$$

## **APPENDIX C: Input-files**

```
C.1 Boundary conditions
** _____
** Boundary conditions
** Node, first d.o.f., last
d.o.f., magnitude
** _____
*BOUNDARY
101,1,6,0
111,1,6,0
*BOUNDARY
1,2,4,0
71,1,4,0
*BOUNDARY
211,1,6,0
221,1,6,0
* *
C.2 Constraint equations
** _____
** Linear constraint equations
                                21,3,1,
** N
                                 223,3,-1,
** Node 1, d.o.f. i, A1,
                                 *EQUATION
** Node 2, d.o.f. j, A2
                                 2
                               21,4,1,
** _____
** Displacement of deck
                                 223,4,-1,
** At node 10 (deck) and node
                                 ** _____
213 (pylon)
                                  ** At node 28(deck) and node
*EQUATION
                                  118 (beam)
2
                                  *EQUATION
10,1,1,
                                  2
213,1,-1,
                                  31,1,1,
*EQUATION
                                  118,1,-1,
2
                                  *EQUATION
10,2,1,
                                  2
213, 2, -1,
                                  31,2,1,
*EQUATION
                                  118,2,-1,
2
                                  *EQUATION
10,3,1,
                                  2
213,3,-1,
                                 31,3,1,
*EOUATION
                                 118,3,-1,
2
                                  *EQUATION
10,4,1,
                                  2
213,4,-1,
                                 31,4,1,
** _____
                                 118,4,-1,
** At node 20(deck) and node
                                  **
223 (pylon)
*EQUATION
2
21,1,1,
223,2,-1,
*EQUATION
2
21,2,1,
223,2,-1,
*EQUATION
```

C *	.3 *		N	00	de	s,	,	B	ył	or	u	a			 	 			
*	*	NO	DE	ls					_				_	_	 _		_	 	
*	*	No	de	e, :	x,	У	, <sup>z</sup>	2											
*	*			- <u>-</u> -					-					-	 	 		 	
^ * * * 123456789111111	^ * * N, , , , , , , , , , 012345	NO Br 12 11 10 95 85 83 -8 -7 -7 -7 -6			×, – – EI 00 00 00 00 00 00 00 00 00 0	Y = d d = 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	-     - <td></td> <td>- 124563695123456</td> <td>693692693836103</td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td></td> <td> </td> <td></td>		- 124563695123456	693692693836103								 	
1 1 1 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	567890123*4567	-6 -5 -5 -4 -3 -3 -2 -2 -2 -2 -1	50505085009505	0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0		), ), )), )), )), )), )), )),			67901234555678		47129201658621								
2 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3 3	8,9,0,1,2,3,4,5,6,7,0	-1 -6 -5 0. 5. 10 15 20 25 30 25			00 0, 2 0, 2 0, 0, 0, 0,	22722222222	20 6. 7. 7. 7. 7. 7. 7. 7.	9 9 8 2 2 3 4 4 5	99211186395	365,00292155					1				
$3 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ 4 \\ $	9,0 1,2 3,4 5,7 8,9 0,	39 40 45 50 55 60 55 60 65 70 75 80 83 85			0, 0, 0, 0, 0, 0, 0, 0, 0, 0,	2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2	, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7, 7,	5 6 6 7 7 7 8 8 8 9 9 9	016159268122	)113154907029									

51,90.000,27.943,0.000 52,95.000,27.953,0.000 53,100.000,27.959,0.000 54,105.000,27.961,0.000 55,110.000,27.959,0.000 56,115.000,27.952,0.000 57,120.000,27.942,0.000 58,125.000,27.927,0.000 59,130.000,27.908,0.000 60,132.000,27.899,0.000 61,135.000,27.884,0.000 62,140.000,27.857,0.000 63,145.000,27.826,0.000 64,150.000,27.790,0.000 65,155.000,27.750,0.000 66,160.000,27.706,0.000 67,165.000,27.659,0.000 68,170.000,27.611,0.000 69,175.000,27.562,0.000 70,180.000,27.514,0.000 71,185.000,27.465,0.000 \*\* \_\_\_\_\_ \_\_\_\_\_ \*\* Anchoring beams (+z) 91,-83.000,25.153,8.500 92,-80.000,25.233,8.500 93,-38.000,26.330,8.500 94,39.000,27.601,8.350 95,83.000,27.922,8.350 96,132.000,27.899,8.350 \*\* \_\_\_\_\_\_ \*\* Anchoring beams (-z) 81, -83.000, 25.153, -8.500 82,-80.000,25.233,-8.500 83,-38.000,26.330,-8.500 84,39.000,27.601,-8.350 85,83.000,27.922,-8.350 86,132.000,27.899,-8.350 \*\* \_\_\_\_\_ \*\* Tower 101,0.000,2.000,13.421 102,0.000,5.500,12.844 103,0.000,15.000,11.277 104,0.000,23.200,9.925 105,0.000,40.000,7.155 106,0.000,55.000,4.681 107,0.000,70.500,2.125 111,0.000,2.000,-13.421 112,0.000,5.500,-12.844 113,0.000,15.000,-11.277 114,0.000,23.200,-9.925 115,0.000,40.000,-7.155 116,0.000,55.000,-4.681 117,0.000,70.500,-2.125 \*\* \_\_\_\_\_ \*\* Cross-beam 118,0.000,23.200,0.000 \*\* \_\_\_\_\_\_ \*\* Top B

141,0.000,70.500,0.000 142,0.000,74.750,0.000 143,0.000,75.160,0.000 144,0.000,75.350,0.000 145,0.000,78.750,0.000 151,0.000,74.750,0.750 152,0.000,75.160,0.750 153,0.000,75.350,2.100 161,0.000,74.750,-0.750 162,0.000,75.160,-0.750 163,0.000,75.350,-2.100 \*\* \_\_\_\_\_ \*\* Pylons 211,-82.000,2.500,0.000 212,-82.000,13.030,0.000 213, -82.000, 23.560, 0.000 221,-38.000,4.000,0.000 222,-38.000,14.355,0.000 223,-38.000,24.710,0.000 \* \* C.4 Elements, deck, concrete \*\* \_\_\_\_\_ \*\* Bridge deck, concrete \*\* Element, node 1, node 2 \*\* \_\_\_\_\_ \*ELEMENT, TYPE=B31, ELSET=deck c1 1,1,2 \*ELEMENT, TYPE=B31, ELSET=deck c2 2,2,3 \*ELEMENT, TYPE=B31, ELSET=deck c3 3,3,4 \*ELEMENT, TYPE=B31, ELSET=deck c4 4,4,5 \*ELEMENT, TYPE=B31, ELSET=deck c5 5,5,6 \*ELEMENT, TYPE=B31, ELSET=deck c6 6,6,7 \*ELEMENT, TYPE=B31, ELSET=deck c7 7,7,8 \*ELEMENT, TYPE=B31, ELSET=deck c 8,8,9 9,9,10 10,10,11 11,11,12 12,12,13 13,13,14 14,14,15 15,15,16 16,16,17 17,17,18 18,18,19 19,19,20 20,20,21 21,21,22 22,22,23 23,23,24

24,24,25 25,25,26 26,26,27 27,27,28 28,28,29 29,29,30 30,30,31 \*\* \_\_\_\_\_ \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=deck c1, DENSITY=2.5, POISSON=0.2 \*\*A\_a, I11\_a, I12\_a, I22\_a, J\_a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 7.753782313,6.290973823,0.0,84.69 621713,90.98719095 7.814919125, 6.45682482, 0.0, 90.406 7736,96.8635984 0.0,0.0,-1.0 3.4e+7,1.4e+7 \*\* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=deck c2, DENSITY=2.5, POISSON=0.2 \*\*A a,I11\_a,I12\_a,I22\_a,J\_a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 7.814919125, 6.45682482, 0.0, 90.406 7736,96.8635984 7.876055938,6.62267582,0.0,96.173 3,102.740006 0.0,0.0,-1.0 3.4e+7,1.4e+7 \*\* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=deck c3, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22 a,J a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 7.876055938,6.62267582,0.0,96.173 3,102.740006 7.93719275, 6.78852681, 0.0, 101.827 887,108.616413 0.0, 0.0, -1.03.4e+7,1.4e+7 \*\* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=deck c4, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22 a,J a / A b, I11 b, I12 b, I22 b, J b / n1direction / E,G 7.93719275, 6.78852681, 0.0, 101.827 887,108.616413 7.998329563,6.95437781,0.0,107.53 8443,114.492821 0.0,0.0,-1.0

```
3.4e+7,1.4e+7
                                       *ELEMENT, TYPE=B31, ELSET=deck s
* *
                                        31, 31, 32
                                       32,32,33
*BEAM GENERAL SECTION,
SECTION=GENERAL, TAPER, ELSET=deck_
                                       33, 33, 34
c5, DENSITY=2.5, POISSON=0.2
                                       34,34,35
                                       35,35,36
**A a,I11 a,I12 a,I22 a,J a /
A b,I11 b,I12 b,I22 b,J b / n1-
                                       36,36,37
                                       37,37,38
direction / E,G
7.998329563,6.95437781,0.0,107.53
                                       38,38,39
                                       39,39,40
8443,114.492821
8.059466375,7.12022288,0.0,113.24
                                       40,40,41
8999,120.369228
                                       41,41,42
                                       42,42,43
0.0,0.0,-1.0
3.4e+7,1.4e+7
                                       43,43,44
* *
                                       44,44,45
                                       45,45,46
*BEAM GENERAL SECTION,
                                       46,46,47
SECTION=GENERAL, TAPER, ELSET=deck
c6, DENSITY=2.5, POISSON=0.2
                                       47,47,48
**A a,I11 a,I12 a,I22_a,J_a /
                                       48,48,49
A_b,I11_b,I12_b,I22_b,J_b / n1-
                                       49,49,50
direction / E,G
                                       50,50,51
8.059466375,7.12022288,0.0,113.24
                                       51, 51, 52
                                       52,52,53
8999,120.369228
8.120603188,7.2860798,0.0,118.959
                                       53, 53, 54
556,126.245636
                                        54,54,55
                                       55,55,56
0.0,0.0,-1.0
3.4e+7,1.4e+7
                                       56,56,57
* *
                                       57, 57, 58
*BEAM GENERAL SECTION,
                                       58,58,59
                                       59,59,60
SECTION=GENERAL, TAPER, ELSET=deck
c7, DENSITY=2.5, POISSON=0.2
                                       60,60,61
**A a,I11 a,I12 a,I22 a,J a /
                                       61,61,62
A b,I11 b,I12_b,I22_b,J_b / n1-
                                        62,62,63
direction / E,G
                                        63,63,64
8.120603188,7.2860798,0.0,118.959
                                        64,64,65
556,126.245636
                                        65,65,66
8.181740,7.4519307977,0.0,124.670
                                        66,66,67
1123365,132.1220431
                                        67,67,68
0.0,0.0,-1.0
                                        68,68,69
3.4e+7,1.4e+7
                                        69,69,70
* *
                                       70,70,71
                                        ** _____
*BEAM GENERAL SECTION,
SECTION=GENERAL, ELSET=deck c, DENS
                                       *BEAM GENERAL
ITY=2.5, POISSON=0.2
                                       SECTION, SECTION=GENERAL, ELSET=dec
**A, I11, I12, I22, J / n1-direction
                                       k s, DENSITY=8.0, POISSON=0.3
                                        **A, I11, I12, I22, J / n1-direction
/ E,G
8.181740,7.4519307977,0.0,124.670
                                        / E,G
                                        0.508197368,0.590544323,0.0,6.958
1123365,132.1220431
0.0,0.0,-1.0
                                       329291, 53.33826599
3.4e+7,1.4e+7
                                       0.0,0.0,-1.0
* *
                                       2.1e+8,8.1e+7
                                        **
C.5 Elements, deck, steel
** _____
```

\*\* Bridge deck, steel
\*\* Element,node 1,node 2

\*\* \_\_\_\_\_\_

```
C.6 Elements, anchoring
** Anchoring beams
** Element, node 1, node 2
** _____
** Concrete
*ELEMENT, TYPE=B31, ELSET=anch c
101,91,9
102,9,81
103,92,11
104,11,82
105,93,21
106,21,83
** Steel
*ELEMENT, TYPE=B31, ELSET=anch s
111,94,39
112,39,84
113,95,49
114,49,85
115,96,60
116,60,86
** _____
*BEAM GENERAL SECTION,
SECTION=GENERAL, ELSET=anch c, POIS
SON=0.2
**A,I11,I12,I22,J / n1-direction
/ E,G
10,10,0,10,10
-1.0,0.0,0.0
3.4E+7,1.4E+7
* *
*BEAM GENERAL SECTION,
SECTION=GENERAL, ELSET=anch s, POIS
SON=0.3
**A,I11,I12,I22,J / n1-direction
/ E,G
10,10,0,10,10
-1.0,0.0,0.0
2.1E+8,8.1E+7
* *
C.7 Elements, tower
** _____
** Tower, Alternative B
** Element, node 1, node 2
** _____
*ELEMENT, TYPE=B31, ELSET=towerB 1
201,101,102
211,111,112
* *
*ELEMENT, TYPE=B31, ELSET=towerB 2
202,102,103
212,112,113
* *
*ELEMENT, TYPE=B31, ELSET=towerB 3
203,103,104
213,113,114
* *
*ELEMENT, TYPE=B31, ELSET=towerB 4
```

204,104,105 214,114,115 \*\* \*ELEMENT, TYPE=B31, ELSET=towerB 5 205,105,106 215,115,116 \*\* \*ELEMENT, TYPE=B31, ELSET=towerB 6 206,106,107 216,116,117 \*\* \_\_\_\_\_ \*\* Cross beam \*ELEMENT, TYPE=B31, ELSET=towerB be am 251,104,118 252,118,114 \*\* \_\_\_\_\_ \*\* Тор В \*ELEMENT, TYPE=B31, ELSET=topB 1 241,141,142 \*ELEMENT, TYPE=B31, ELSET=topB 2 242,142,143 \*ELEMENT, TYPE=B31, ELSET=topB 3 243,143,144 \*ELEMENT, TYPE=B31, ELSET=topB 4 244,144,145 \*\* \_\_\_\_\_ \*ELEMENT, TYPE=B31, ELSET=top rigid 261,107,141 262,141,117 \*\* \*\* \_\_\_\_\_ \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=tower B 1, DENSITY=2.5, POISSON=0.2 \*\*A\_a,I11\_a,I12\_a,I22\_a,J\_a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 9.0,12.0,0.0,3.796875,9.85033092 8.89739414,11.5942379,0.0,3.53588 15,9.67929085 0.0,0.0,1.0 3.4e+7,1.4e+7 \* \* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=tower B 2, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22 a,J a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 6.00527687,9.76576782,0.0,3.48787 488,7.34558467 5.85674267,8.94633569,0.0,3.38232 276,6.98627529 0.0,0.0,1.0 3.4e+7,1.4e+7 \*\*

```
*BEAM GENERAL SECTION,
SECTION=GENERAL, TAPER, ELSET=tower
B 3, DENSITY=2.5, POISSON=0.2
**A_a,I11_a,I12_a,I22_a,J_a /
A b,I11 b,I12 b,I22 b,J b / n1-
direction / E,G
5.85674267,8.94633569,0.0,3.38232
276,6.98627529
5.7285342,8.27515478,0.0,3.291214
62,6.67801413
0.0,0.0,1.0
3.4e+7,1.4e+7
* *
*BEAM GENERAL SECTION,
SECTION=GENERAL, TAPER, ELSET=tower
B 4, DENSITY=2.5, POISSON=0.2
**A a,I11 a,I12 a,I22 a,J a /
A b, I11 b, I12 b, I22 b, J b / n1-
direction / E,G
5.7285342,8.27515478,0.0,3.291214
62,6.67801413
5.46586319,7.00136533,0.0,3.10455
403,6.05243046
0.0,0.0,1.0
3.4e+7,1.4e+7
**
*BEAM GENERAL SECTION,
SECTION=GENERAL, TAPER, ELSET=tower
B 5, DENSITY=2.5, POISSON=0.2
**A a,I11 a,I12 a,I22 a,J a /
A_b,I11_b,I12 b,I22 b,J b / n1-
direction / E,G
5.46586319,7.00136533,0.0,3.10455
403,6.05243046
5.2313355, 5.97489552, 0.0, 2.937892
79,5.50147176
0.0,0.0,1.0
3.4e+7,1.4e+7
* *
*BEAM GENERAL SECTION,
SECTION=GENERAL, TAPER, ELSET=tower
B 6, DENSITY=2.5, POISSON=0.2
**A a,I11 a,I12 a,I22_a,J_a /
A b,I11 b,I12 b,I22 b,J b / n1-
direction / E,G
5.2313355, 5.97489552, 0.0, 2.937892
79,5.50147176
4.98899023,5.01911862,0.0,2.76567
618,4.94079449
0.0,0.0,1.0
3.4e+7,1.4e+7
** _____
*BEAM GENERAL SECTION,
SECTION=GENERAL, ELSET=towerB beam
, DENSITY=2.5, POISSON=0.2
**A, I11, I11, I22, J / n1-direction
/ E,G
```

5.795,16.9031482,0.0,7.560675,14. 1859438 -1.0,0.0,0.0 3.4E+7, 1.4E+7\*\* \_\_\_\_\_ \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=topB 1, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22 a,J a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 20.1986971,16.2540693,0.0,71.1162 459,45.5194203 15.5606359,12.079488,0.0,33.70531 16,30.289821 0.0,0.0,1.0 3.4e+7,1.4e+7 \*\* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=topB 2, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22 a,J a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 15.5606359,12.079488,0.0,33.70531 16,30.289821 15.1214096,11.6974681,0.0,31.0395 026,28.9081075 0.0,0.0,1.0 3.4e+7,1.4e+7 \*\* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=topB 3, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22\_a,J\_a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 15.1214096,11.6974681,0.0,31.0395 026,28.9081075 14.9183556,11.5216456,0.0,29.8542 95,28.2732653 0.0,0.0,1.0 3.4e+7,1.4e+7 \*\* \*BEAM GENERAL SECTION, SECTION=GENERAL, TAPER, ELSET=topB 4, DENSITY=2.5, POISSON=0.2 \*\*A a,I11 a,I12 a,I22 a,J a / A b,I11 b,I12 b,I22 b,J b / n1direction / E,G 14.9183556,11.5216456,0.0,29.8542 95,28.2732653 11.33720093,8.50290698,0.0,13.492 5718,17.5645761 0.0,0.0,1.0 3.4e+7,1.4e+7

```
*BEAM GENERAL SECTION,
SECTION=GENERAL, ELSET=top_rigid, P
OISSON=0.2
**A,I11,I12,I22,J / n1-direction
/ E,G
10,10,0,10,10
-1.0,0.0,0.0
3.4e+7,1.4e+7
* *
C.8 Elements, cables
** _____
** New elements, cables, v5B
** Element, Node 1, Node 2
** _____
** Cable N1
*ELEMENT,
TYPE=t3d2,ELSET=cable N1W
301,91,143
*ELEMENT,
TYPE=t3d2,ELSET=cable N1E
501,81,143
** Cable N2
*ELEMENT,
TYPE=t3d2,ELSET=cable N2W
317,92,143
*ELEMENT,
TYPE=t3d2,ELSET=cable N2E
517,82,143
** Cable N3
*ELEMENT,
TYPE=t3d2,ELSET=cable N3W
333,93,143
*ELEMENT,
TYPE=t3d2,ELSET=cable N3E
533,83,143
** Cable S1
*ELEMENT,
TYPE=t3d2,ELSET=cable S1W
351,94,143
*ELEMENT,
TYPE=t3d2,ELSET=cable S1E
551,84,143
** Cable S2
*ELEMENT,
TYPE=t3d2,ELSET=cable S2W
367,95,143
*ELEMENT,
TYPE=t3d2,ELSET=cable S2E
567,85,143
** Cable S3
*ELEMENT,
TYPE=t3d2,ELSET=cable S3W
383,96,143
*ELEMENT,
TYPE=t3d2,ELSET=cable S3E
583,86,143
* *
```

\*\* \_\_\_\_\_ \*MATERIAL, NAME=cable \*ELASTIC 180e+6, 0.3 \*DENSITY 8.8, \*SOLID SECTION, ELSET=cable N1E, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable N2E, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable N3E, MATERIAL=cable 8.544e-3, \*SOLID SECTION, ELSET=cable S1E, MATERIAL=cable 8.544e-3, \*SOLID SECTION, ELSET=cable S2E, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable S3E, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable N1W, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable N2W, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable N3W, MATERIAL=cable 8.544e-3, \*SOLID SECTION, ELSET=cable S1W, MATERIAL=cable 8.544e-3, \*SOLID SECTION, ELSET=cable S2W, MATERIAL=cable 1.7088e-2, \*SOLID SECTION, ELSET=cable S3W, MATERIAL=cable 1.7088e-2, \*\*

```
C.9 Elements, support pillars
** _____
** Pylons
** Element, node 1, node 2
** _____
** Axis 10, x=-82
*ELEMENT,
TYPE=B31,ELSET=pylon a1011
411,211,212
412,212,213
**Axis 11, x=-38
421,221,222
422,222,223
** _____
*BEAM GENERAL SECTION,
SECTION=GENERAL, ELSET=pylon a1011
,DENSITY=2.5,POISSON=0.2
**A,I11,I12,I22,J / n1-direction
/ E,G
7.41,0.7307875,0.0,5.966351389,2.
0
0.0,0.0,1.0
3.4E+7, 1.4E+7
* *
C.10 Non-structural mass
** _____
** Addition of non-structural
mass onto bridge
** Asphalt + railing + Steel-
plates
** _____
* *
*NONSTRUCTURAL
MASS,ELSET=deck c1,UNITS=MASS PER
LENGTH
1.78
*NONSTRUCTURAL
MASS, ELSET=deck c2, UNITS=MASS PER
LENGTH
1.78
*NONSTRUCTURAL
MASS, ELSET=deck c3, UNITS=MASS PER
LENGTH
1.78
*NONSTRUCTURAL
MASS,ELSET=deck c4,UNITS=MASS PER
LENGTH
1.78
*NONSTRUCTURAL
MASS, ELSET=deck c5, UNITS=MASS PER
LENGTH
1.78
*NONSTRUCTURAL
MASS, ELSET=deck c6, UNITS=MASS PER
LENGTH
1.78
```

\*NONSTRUCTURAL MASS, ELSET=deck c7, UNITS=MASS PER LENGTH 1.78 \*NONSTRUCTURAL MASS, ELSET=deck c, UNITS=MASS PER LENGTH 1.78 \*NONSTRUCTURAL MASS, ELSET=deck s, UNITS=MASS PER LENGTH 1.84 \*\* C.11 Pre-stress, deck \*\* \_\_\_\_\_ \*\* Loads due to prestress within concrete cross-section \*\* \_\_\_\_\_ \*\* Group 1 (arched cable) \*\* Distributed Load \*DLOAD 1, PY, 140 2, PY, 140 3, PY, 140 4, PY, 140 5,PY,140 6, PY, 140 \* \* 7, PY, -384 8, PY, -384 9, PY, -384 10, PY, -384 11, PY, -384 12, PY, -384 \*\* 13, PY, 185 14, PY, 185 15, PY, 185 16, PY, 185 17, PY, 185 18, PY, 185 \* \* 19, PY, -457 20, PY, -457 21, PY, -457 22, PY,-457 \*\* 23, PY, 125 24, PY, 125 25, PY, 125 26, PY, 125 27, PY, 125 28, PY, 125 29, PY, 125 30, PY, 125 \*\*

\*\* Point Load

\*CLOAD 1,1,13979.25 1,2,-1972.3676 13,1,-13979.25 13,1,15009.3 13,2,3245.71586 13,2,-2440.337 23,1,-15009.3 23,1,13979.25 23, 2, 3673.72658 23,2,-1932.0522 31,1,-13979.25 31,2,-1904.7756 \*\* Moment (via applied load) \*CLOAD 1,6,4500 2,6,8127 3,6,4658 4,6,1232 5,6,-2323 6,6,-5750 7,6,-11259 \* \* 8,6,-9315 10,6,-1063 11,6,1344 12,6,9878 13,6,2500 \* \* 14,6,6860 15,6,6917 16,6,2283 17,6,-2353 18,6,-6988 19,6,-11972 \* \* 20,6,-9021 21,6,-477 22,6,2892 23, 6, 11860 \*\* 25,6,8614 26, 6, 4732 27,6,1613 28,6,-1508 30,6,-4626 31,6,-8405 \*\* \_\_\_\_\_ \*\* Group 2 (upper cable) \*\* Point Load \*CLOAD 7,1,6278.4 13,1,-6278.4 \*\* Moment \*CLOAD 7,6,-4000 13,6,4000 \*\* \_\_\_\_\_

\*\* Group 3 (lower cable) \*\* Point Load \*CLOAD 24,1,1569.6 29,1,-1569.6 \*\* Moment (via applied load) \*CLOAD 24,6,1900 29,6,-1900 \*\* C.12 Run, self-weight \*\* \_\_\_\_\_ \*\* Procedure for assembly and \*\* applying self-weight onto bridge \*\* \_\_\_\_\_ \*PREPRINT, echo=NO,model=NO,history=NO,conta ct=NO \*HEADING \*\* Include following files for assembly \*\* Nodes and Elements \*INCLUDE, INPUT=nodes bybrua.inp \*INCLUDE, INPUT=elements deckconcr ete v5.inp \*INCLUDE, INPUT=elements decksteel .inp \*INCLUDE, INPUT=elements anchoring .inp \*INCLUDE, INPUT=elements\_tower.inp \*INCLUDE, INPUT=elements cables v5 B.inp \*INCLUDE, INPUT=elements pylons.in р \* \* \*\* Node sets and element sets for output \*NSET,NSET=deck nodes,generate 1,71,1 \*NSET,NSET=deck displacement 9,11,21,39,49,60,143 \*ELSET,ELSET=cables 301, 317, 333, 351, 367, 383, 501, 517, 5 33,551,567,583 \*\* Include file for applying constraints \*INCLUDE, INPUT=constraint\_equatio ns.inp \* \* \*\* Add nonstructural mass onto bridge, ie. asphalt, road, rails \*INCLUDE, INPUT=non struct.inp \* \* \*\* \_\_\_\_\_ \*\* LOAD CASE 1 \*STEP, name=Step-1, nlgeom=yes

```
*Static
.01,1.,1e-05,1.
** Include file for applying
boundary conditions
*INCLUDE, INPUT=boundary condition
s.inp
**
** Include file for applying
prestress in concrete deck
*INCLUDE,INPUT=loads prestress de
ck.inp
* *
** GRAVITY LOAD, ie. self-weight
of structure, all elements
*DLOAD
,GRAV,9.81,0.0,-1.0,0.0
* *
** _____
** OUTPUT REOUESTS
*OUTPUT, field, VARIABLE=PRESELECT
*NODE
PRINT, NSET=deck displacement
IJ
*EL PRINT, ELSET=cables
S
*END STEP
**
C.13 Unit stress
** _____
** Determine initial stress in
cables
** Application of unit stress
** _____
** One set of cables active at
the time
** Repeat for other sets
** When BEAMGENERALSECTION -->
axial force
*Initial Conditions, type=STRESS
** Cable N1
301,1e+3,0.,0.,
501,1e+3,0.,0.,
** _____
** Cable N2
317,1e+3,0.,0.,
517,1e+3,0.,0.,
** _____
** Cable N3
333,1e+3,0.,0.,
533,1e+3,0.,0.,
* *
C.14 Run, unit stress
** _____
** Procedure for assembly and
** applying unit stress onto
```

\*\* \_\_\_\_\_\_ \*PREPRINT, echo=NO, model=NO, history=NO, conta ct=NO \*HEADING \*\* Include following files for assembly \*\* Nodes and Elements \*INCLUDE, INPUT=nodes bybrua.inp \*INCLUDE, INPUT=elements deckconcr ete v5.inp \*INCLUDE, INPUT=elements decksteel .inp \*INCLUDE, INPUT=elements anchoring .inp \*INCLUDE, INPUT=elements tower.inp \*INCLUDE, INPUT=elements cables v5 B.inp \*INCLUDE, INPUT=elements pylons.in р \*\* \*\* Node sets and element sets for output \*NSET, NSET=deck nodes, generate 1,71,1 \*NSET,NSET=deck displacement 9,11,21,39,49,60,143 \*ELSET, ELSET=cables 301, 317, 333, 351, 367, 383, 501, 517, 5 33,551,567,583 \*\* Include file for applying constraints \*INCLUDE, INPUT=constraint equatio ns.inp \* \* \*\* Add nonstructural mass onto bridge, ie. asphalt, road, rails \*INCLUDE, INPUT=non struct.inp \*\* \*\* \_\_\_\_\_ \*\* LOAD CASE 1 \*STEP, name=Step-1, nlgeom=yes \*Static .01,1.,1e-05,1. \*\* Include file for applying boundary conditions \*INCLUDE, INPUT=boundary condition s.inp \* \* \*\* Include file for applying prestress in concrete deck \*\* INCLUDE, INPUT=loads prestress dec k.inp \*\* \*\* Include file to apply unit stress in the cable set N1

bridge

```
** read uy-displacements at nodes
36,46,57
** read ux-displacements at nodes
143
*INCLUDE, INPUT=unit cable N.inp
**
** _____
** OUTPUT REOUESTS
*OUTPUT, field, VARIABLE=PRESELECT
*NODE
PRINT, NSET=deck displacement
IJ
*EL PRINT, ELSET=cables
S
*END STEP
* *
C.15 Pre-stress, stay cables
** _____
** Initial stress in cables, 1
** _____
** Apply stress calculated from
looking
** at displacement due to unit
stress
*Initial Conditions, type=STRESS
** Cable N1
301,2.70182426E+05,0.,0.,
501,2.70182426E+05,0.,0.,
** Cable N2
317,2.70182426E+05,0.,0.,
517,2.70182426E+05,0.,0.,
** Cable N3
333,2.70182426E+05,0.,0.,
533,2.70182426E+05,0.,0.,
** Cable S1
351,2.27978106E+05,0.,0.,
551,2.27978106E+05,0.,0.,
** Cable S2
367,1.70006919E+05,0.,0.,
567,1.70006919E+05,0.,0.,
** Cable S3
383,3.25098630E+05,0.,0.,
583,3.25098630E+05,0.,0.,
**
C.16 Run, model, static
** _____
** Procedure for assembly and
** applying self-weight and
prestress onto bridge
** _____
*PREPRINT,
echo=NO, model=NO, history=NO, conta
ct=NO
*HEADING
** Include following files for
assembly
```

\*\* Nodes and Elements \*INCLUDE, INPUT=nodes bybrua.inp \*INCLUDE, INPUT=elements deckconcr ete v5.inp \*INCLUDE, INPUT=elements decksteel .inp \*INCLUDE, INPUT=elements anchoring .inp \*INCLUDE, INPUT=elements tower.inp \*INCLUDE, INPUT=elements cables v5 B.inp \*INCLUDE, INPUT=elements pylons.in р \*\* \*\* Node sets and element sets for output \*NSET,NSET=deck nodes,generate 1,71,1 \*NSET,NSET=deck displacement 9,11,21,39,49,60,143 \*ELSET, ELSET=cables 301, 317, 333, 351, 367, 383, 501, 517, 5 33,551,567,583 \*\* Include file for applying constraints \*INCLUDE, INPUT=constraint equatio ns.inp \*\* \*\* Add nonstructural mass onto bridge, ie. asphalt, road, rails \*INCLUDE, INPUT=non struct.inp \* \* \*\* \_\_\_\_\_ \*\* LOAD CASE 1 \*STEP, name=Step-1,nlgeom=yes \*Static .01,1.,1e-05,1. \*\* Include file for applying boundary conditions \*INCLUDE, INPUT=boundary condition s.inp \*\* \*\* Include file for applying prestress in concrete deck \*INCLUDE, INPUT=loads prestress de ck.inp \*\* \*\* Include file to apply initial stress in cables \*INCLUDE, INPUT=initial1 v5C.inp \* \* \*\* GRAVITY LOAD, ie. self-weight of structure, all elements \*DLOAD ,GRAV,9.81,0.0,-1.0,0.0 \* \* \*\* \_\_\_\_\_ \*\* OUTPUT REOUESTS

```
*OUTPUT, field, VARIABLE=PRESELECT
*NODE
PRINT, NSET=deck displacement
IJ
*EL PRINT, ELSET=cables
S
*END STEP
**
C.17 Run, Frequency analysis
** _____
** Frequency analysis of bridge
** _____
*Preprint, echo=NO, model=NO,
history=NO, contact=NO
*HEADING
** Include following files for
assembly
** Nodes and Elements
*INCLUDE, INPUT=nodes bybrua.inp
*INCLUDE, INPUT=elements deckconcr
ete v5.inp
*INCLUDE, INPUT=elements decksteel
.inp
*INCLUDE, INPUT=elements anchoring
.inp
*INCLUDE, INPUT=elements tower.inp
*INCLUDE, INPUT=elements cables v5
B.inp
*INCLUDE, INPUT=elements pylons.in
р
*NSET,NSET=deck nodes,generate
1,71,1
*NSET, NSET=deck displacement
9,11,21,39,49,60,143
**
**Include file for applying
constraints
*INCLUDE, INPUT=constraint equatio
ns.inp
* *
** Add nonstructural mass onto
bridge, ie. asphalt, road, rails
*INCLUDE, INPUT=non struct.inp
**
** _____
_____
** Load case 1 (case 1)
** _____
_____
** STEP: Step-1
* *
*Step, name=Step-1, nlgeom=yes
*Static
.01, 1., 1e-05, 1.
** BOUNDARY CONDITIONS
```

\*INCLUDE, INPUT=boundary condition s.inp \*\* \*\* LOADS \*\* PRE-STRESS, CONCRETE DECK \*INCLUDE, INPUT=loads prestress\_de ck.inp \*\* \*\* INITIAL STRESS, CABLES \*INCLUDE, INPUT=initial1 v5C.inp \*\* \*\* GRAVITY LOAD \*DLOAD ,GRAV,9.81,0.0,-1.0,0.0 \* \* \*\* \_\_\_\_\_ \*\* OUTPUT REQUESTS \* \* \*Restart, write, frequency=0 \*\* \*\* FIELD OUTPUT: F-Output-1 \*\* \*Output, field, variable=PRESELECT \*\* \*\* HISTORY OUTPUT: H-Output-1 \*\* \*Output, history, variable=PRESELECT \*End Step \*\* \_\_\_\_\_ \_\_\_\_\_ \*\* load case 2 \*\* \_\_\_\_\_ \_\_\_\_\_ \*\* STEP: Step-2 \*\* \*Step, name=Step-2, perturbation, nlgeom=yes \*Frequency, eigensolver=lanczos, normalization=mass 30, , , , , 300 \*\* \*\* OUTPUT REQUESTS \*\* \*Restart, write, frequency=0 \*\* \*\* FIELD OUTPUT: F-Output-2 \*\* \*Output, field, variable=PRESELECT \*NODE PRINT, nset=deck nodes ΤT \*End Step

# APPENDIX D: Frequency Analysis, Matlab

Include the following files in a folder.

.dat-file produced from frequency analysis of model.

### D.1 Mod plot2.m (coded by Ibuki Kusano)

```
% -----
% This script reads Abaqus output file of frequency analysis
% .dat and plots the mode shapes.
% The mode shapes are normalized by the largest displacement value.
% Coded by Ibuki Kusano, April, 2020
%
% Input
%
      M: number of modes to plot
%
      N: number of nodes of the deck
      filename: Abaqus output file name .dat
%
%
               deck coord.txt: text file for the deck x coordinate
% Output
%
     mode shape plots
<sup>0</sup>/<sub>0</sub> -----
clear all;clc;close all;
delete('*.png')
% Read the frequencies and modes from Abaqus
        M=30; % number of modes to plot
  N=71; % number of nodes along the deck
  filename='freq v5C.dat';
  [mode,freq]=read modes2(M,N,filename);
% Read the deck x coordinate
        X=importdata('deck coord.txt');
% Find the max displacement to normalize the modes
        disp=mode(:,2:5);
        maxdisp=max(disp(:));
% plot mode shapes
        for i=1:M
               n1=N*(i-1)+1;
                n2=N*i;
                uz=mode(n1:n2,4)./maxdisp;
                uy=mode(n1:n2,3)./maxdisp;
                rx=mode(n1:n2,5)./maxdisp;
                ux=mode(n1:n2,2)./maxdisp;
                h=figure;
                axes('FontSize',20);
                title(['Mode ',num2str(i),' (',num2str(freq(i),'%.3f'),' Hz)'])
                hold on;
                plot(X,uz,'Color',[0.5,0.04,0.08],'LineWidth',2); %red
                plot(X,uy,'Color',[0,0,1],'LineWidth',2,'LineStyle','--'); %blue
```

plot(X,rx,'Color',[0.1,0.64,0.24],'LineWidth',2,'LineStyle','-.'); %green plot(X,ux,'Color',[0.95,0.36,0.13],'LineWidth',2,'LineStyle',':'); %orange legend('lat','vert','tor','long','Location','south','orientation','horizontal'); filename = ['mode\_',num2str(i),'.png']; print(h,'-dpng','-r100',filename);

end

### D.2 Read modes

function [FI,OMEGAN]=read modes(M,N,filename)

%
% MODE SHAPES AND NATURAL FREQUENCIES
°⁄0
% This function reads the matrix of mode shapes FI and the natural
% frequencies OMEGAN from the file .dat
%
% INPUT
% [M]: Total number of mode shapes
% [N]: Number of nodes of the deck
% [namefile]:Name of Abaqus file without extension
%
%
% OUTPUT
%
% [FI]: Matrix of mode shapes. Its dimension is NxM since its
% columns are the M mode shapes which are chosen to carry
% out the analysis.
% [OMEGAN]: Natural frequency vector. Its dimension is Mx1.
⁰∕₀
% i,j,n = indexes
% ux,uy,uz,rx,ry,rz = node displacements for the n mode shape.
% waux = n natural frequency.
%

fid=fopen([filename],'r');

```
% FINDING NATURAL FREQUENCIES:
% MODE NO, EIGENVALUE, FREQUENCY(RAD/TIME), FREQUENCY(Hz),GENERALIZED
MASS, COMPOSITE MODAL DAMPING
%
```

### EIGENVALUE OUTPUT '

EIGENVALUE OUTPUT ')) &

for s=1:5; linea=fgetl(fid);end;

nums=fscanf(fid,'%f %E %E %E %f %f',[6,M]);nums=nums';OMEGAN=nums(:,4);

% FINDING MODE SHAPES: %' EIGENVALUE NUMBER 1'

%%%'

### FI=[]; i=1; while (i<=M)

%'

```
% Searching for heading of mode shape.
  if i<10
                       EIGENVALUE NUMBER ',num2str(i)];
    modostr=['
  elseif i<100
    modostr=['
                       EIGENVALUE NUMBER ',num2str(i)];
  elseif i<1000
    modostr=['
                       EIGENVALUE NUMBER ',num2str(i)];
  end
  linea=fgetl(fid);
    while 0==strcmp(linea,modostr)
      linea=fgetl(fid);
    end
    for s=1:14; linea=fgetl(fid);end;
  nums=fscanf(fid,'%f %E %E %E %E %E %E %E',[7,N]); nums=nums';
  FI=[FI;nums];
```

```
i=i+1;
  end
fclose(fid);
```

end % end of function

D.3	Deck nodes, coordinates
-120	·
-115	
-110	
-105	
-100	
-95	
-90	
-85	
-83	
-82	
-80	
-75	
-73.2	
-70	
-65	
-60	
-55	
-50	
-45	
-40	
-38	
-35	
-30.4	
-29	
-25	
-20	

-120		
-115		
-110		
-105		
-100		
-95		
-90		
-85		
07		

## **APPENDIX E: Spectral Analysis**

```
E.1
      Plot, acceleration
clc; close all; clearvars;
load('dataAccDeckBybrua_28082019_0700.mat');
N=length(Az); % Number sampling points
dt=1/fs; % Time interval, sampling rate fs=50Hz
t=0:dt:dt*(N-1); % Time vector, interval dt
tiledlayout(2,1)
% Concrete deck
ax1 = nexttile;
plot(ax1,t,Az(1,:),'b')
xlim([0,3650])
ylim([-0.2, 0.2])
ylabel('a [m/s<sup>2</sup>]')
title([' Acceleration, vertical, concrete deck'])
grid on; grid minor;
% Steel deck
ax2 = nexttile;
plot(ax2,t,Az(2,:),'r')
xlim([0,3650])
ylim([-0.2,0.2])
xlabel('t [s]')
ylabel('a [m/s<sup>2</sup>]')
title([' Acceleration, vertical, steel deck'])
grid on; grid minor;
E.2
      Plot, PSD
clc; close all; clearvars;
load('dataAccDeckBybrua 28082019 0700.mat')
N=length(Az); % Number sampling points
dt=1/fs; % Time interval, sampling rate fs=50Hz
t=0:dt:dt*(N-1); % Time vector, interval dt
[pxxc,fxc]=pwelch(Ax(1,:),[],[],[],fs);
[pyyc,fyc]=pwelch(Ay(1,:),[],[],[],fs);
[pzzc,fzc]=pwelch(Az(1,:),[],[],[],fs);
[pxxs,fxs]=pwelch(Ax(2,:),[],[],[],fs);
[pyys,fys]=pwelch(Ay(2,:),[],[],[],fs);
[pzzs,fzs]=pwelch(Az(2,:),[],[],[],fs);
plot(fzc,pzzc,'b')
hold on
plot(fzs,pzzs,'r')
hold off
xlim([0,12])
xlabel('f [Hz]')
ylabel('PSD [(m/s^2)/Hz]')
title([' Power Spectral Density, vertical acceleration'])
legend(' Concrete section ',' Steel section
', 'Interpreter', 'latex', 'fontsize', 15)
grid on; grid minor;
```



# APPENDIX E: Plots, Frequency Analysis














## References

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