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Abstract

During periods of downward turns, and high volatility, there is an associated increase in individual asset risk, as well as effects stemming from the volatility of other assets. This spillover effect is well studied for equites and portfolio assets. In this thesis we investigate the presence of return spillover for fixed income, commodities, and currencies (FICC assets) over the past 20-years, using Norway as a case study. We develop a general framework using Archimedean copulas as a statistical method for modelling the tail-dependencies between Brent oil price, the exchange rate of the U.S. Dollar (USD) to the Norwegian Krone (NOK) and the terms of the Norwegian Inter-Bank Offered Rates (NIBOR). Further, we investigate whether the discovered spillover and dependence structure varies across sub-samples of four-year periods.

The results of the analysis indicate that there are some dependency structures present between these markets, and there are cases of significant return spillover. Concretely, the analysis found a negative tail-end correlation between Brent oil prices and USD/NOK exchange rate, a relatively small dependence between NIBOR terms and Brent oil prices, and lastly, varying dependence structures between the NIBOR terms and the USD/NOK exchange rate, both over periods of varying economic movements and the terms themselves. The analysis also found the strongest interactions to occur in times associated with high volatility and global economic turmoil.

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Table of Abbreviations

AD	Anderson-Darling Test
ADF	Augmented Dickey-Fuller Test
AIC	Akaike Information Criterion
API	American Petroleum Institute
bbl	Unit volume for crude oil (blue barrels)
CDF	Cumulative Density Function
CME Group	Chicago Mercantile Group
EDF	Empirical Distribution Function
EEX	European Energy Exchange
EM	Emerging Markets
ERR	Exchange Rate Regimes
EURIBOR	Euro Inter- Bank Offered Rate
FICC	Fixed Income Currency and Commodities
FNO	Finans Norge (Finance Norway)
FOREX	Foreign Exchange Market
FX Instrument	Foreign Exchange Instrument
GDP	Gross Domestic Product
GRSS	Global Rate Set Systems
GSP	Government Selling Price
i.i.d.	Identically Independently Distributed
ICE	Intercontinental Exchange
JB	Jarque-Bera
KR	Key Rate
KS	Kolmogorov Smirnov Test
LIBOR	London Inter-bank Offered Rate
NIBOR	Norwegian Interbank Offered Rate
NOK	Norwegian Krone
NoRe	Norske Finansielle Referanser AS
NOWA	Norwegian Overnight Weighted Average
NYMEX	New York Mercantile Exchange
OPEC	Organization of Petroleum Exporting Countries
OSP	Official Selling Price
OTC	Over the Counter
PDF	Probability Density Function
USD	United States Dollar
USSR	Union of Soviet Socialist Republics
WTI	West Texas Intermediate

1. Introduction

This thesis was written during the spring of 2020, and as it happened, the infamous corona crisis struck. Now, along with other major global economic downturns, this crisis affected to an extent as of now yet to be determined, crude oil prices and currency exchange rates, which along with Inter-bank offered rates are important aspects of this thesis. This thesis aims to combine these three markets in a copula analysis to find dependency structures. These structures are then used to infer about how these markets are related to one another, and how this relation may affect aspects such as market return risk.

The value of a particular currency is dependent on a number of factors, but of course the most important indicator is the productivity of the underlying economies of the currency. The productivity of these economies may be dependent on particular commodities. Case in point, the petroleum industry and Norway: Norway's petroleum industry constitutes around 18% of total GDP, and 62% of exports (European Commission, 2020). Hence, we have a nation whose economy greatly depend on a specific industry. Naturally, one might then consider the relation between a nations currency, and volatilities associated with an industry, e.g. one can consider spillover-effects between commodity prices and currency exchange rates. Both volatility- and return spillover is considered to be the effect of an economic dependence: Some seemingly unrelated event in a particular context, e.g. large change in oil price, affects the outcome of a supposedly unrelated activity, e.g. textile industry productivity. Going back to our Norway case, an interesting relation might be the spillover effect between Brent oil and the Norwegian Krone (NOK) exchange rate to some other globally essential currency. Spillover-effects between currencies, and assets have already been widely discussed for several different nations and markets (Roesch and Schmidbauer, 2014, Antonakakis and Kizys, 2015, Katusiime, 2018). In general, one might even say that the spillover effects surrounding the crude oil market is widely studied and understood. Concerning the exchange rate for the NOK in particular, one can consider the findings of T. Ellen (2016), which indicates strong nonlinear correlations between NOK exchange rate and oil price when the latter experiences larger movements away from the mean.

Interbank rates are the interest rates charged on short term unsecured loans between banks in a nation. These rates are usually set by the banks themselves within nation-specific regulation bounds. Banks needs to lend or borrow money to cover their liquidity needs generated by the daily withdrawal and deposit activity of their customers. The interbank offered rate is typically based on the domestic key rate, and banks usually utilize it to generate interest on interbank and customer lending. Volatility on interbank rates and how they relate to a country's fixed income market, has been previously reported. Rossetti et al. (2017) finds that "bad news" relates to a higher volatility of the interbank rates of their respective countries. One might also find that in periods of downward economic turns, the regulating bodies of a nations elects to reduce the key rate in hope of increasing spending, as a boost to the economy. On the other hand, fear of inflation or market destabilization may cause the central bank to increase key rate, resulting in more restrictive spending (Olsen, 2018).

1.1 Problem Formulation

> *In this thesis we aim to explore the return spillover between Brent oil price, USD/NOK exchange rate, and NIBOR term rates, by means of an Archimedean-copula-based methodology.*

We aim to identify non-linear dependence structures in the whole output of our selected markets for a 20-year period, as well as this period differentiated into 4-year spanning non-overlapping sub-periods, with a particular focus on tail-end correlations. From these tail-end correlations we can identify incidences of return spillover, as well as the strength and direction of the manifestation of these effects in resent global economic history.

A copula-based approach was implemented for the empirical calculations of this thesis. A copula is in general terms a function describing the dependence relation between two or more randomly distributed variables. From this approach we are able to model dependency structure between markets and marginal return models separately. This further lets us infer the type of dependency for the non-linear relationships and from the empirical results, make inferences about the type of relationship these assets experience.

1.2 Structure

First, we identify and present the relevant markets and their structures in chapter 2. Secondly in chapter 3 Methodology, we present the underlying statistical theory needed to get an understanding of the performed technical analysis. Chapter 4 summarizes data, and how it was appropriately filtered, tested, and handled to suit the main analysis. Empirical results are summarized and inferences about the results are made in chapter 5. Finally, this leads into the conclusion of chapter 6.

2. Background

The chapter presents the markets of Brent oil, the currency exchange rate between U.S. Dollar and the Norwegian Krone (USD/NOK) and the Norwegian Inter-Bank Offered Rate (NIBOR). It can be argued that in the end, each of these are affected by the same fundamental values resulting from the global or domestic economy, but the individual reactions can also influence and provoke changes in the inter-market stability and development. Each of these markets have their own economic function in the financial society and can be related to different systems and benchmarks. This chapter briefly introduces the crude oil market and describe its importance and the most influential factors. Further, the currency market, and how the trades are conducted and what drives the foreign exchange market system is discussed. We also introduce the inter-bank market, specifically the Norwegian inter-bank market, and discuss its function and how it is determined by panel banks and the role of the central bank. Finally, we present the market interactions and intuitively discuss how dependencies might occur.

2.1 Crude Oil Market

The crude oil market covers the entirety of the wide range of types and qualities available. Crude oil is the unprocessed product resulting from geological formations and is primarily used in fuel related products for either transportation or as a central energy source. As of 2018, crude oil and petroleum products acts as the world leading energy source, according to BP PLC (2019) covering 85% of all the total global consumption.

2.1.1 Importance

It is safe to say that the crude oil market has a large influence on many of the world's major economies, and therefore the global economy as well. Through several recessions and periods of economical flourish, it has been the consensus that the fluctuations in the crude oil price may be solely or partially to blame. Based on the study of Hamilton (2008), we observe that significant reductions (shocks) in global oil production has resulted in global recessions. Viewing Rogoff (2006) study of crude oil related recessions in the post-war era, it illustrates that four out of five major drops in oil price from 1970-2005 resulted in a global recession. Recession expressed as a significant decrease in world GDP growth compared to earlier years. The International Monetary Fund (Hesse and Poghosyan, 2009) published a paper in 2009 studying the effect of real oil price shocks on bank activity and profitability. The study indicated that the shocks, appearing from 1994-2008, did not have a direct impact on the central profitability of the banks. The indirect effect however proved significant, primarily through country-specific macroeconomic factors and institutional variables. The commercial banks were less impacted compared to the investment banks, due to the heavy financial instruments concentrated in the petroleum sector.

Speaking specifically about the industrial aspect, and how the market changes compared to the oil price. In recent years, much research has been concentrated around the correlation between the stock market returns and the fluctuations in the crude oil price. The stock market gaining high returns is obviously also correlated with the state of the national and global economy. In the study of Elyasiani et al. (2011) there is presented convincing evidence of a volatility spillover. Nine out of thirteen industrial sector returns in the U. S. shows a statistical relationship with oil-futures returns and/or oil futures volatility. The study indicates that the industry's most vulnerable to the oil price fluctuations are the ones based on the consumption, not the industrial oil producers themselves. The effect of the fluctuations tends to be protracted and is probable to affect the future return over a substantial period of time.

It is justifiable to think that a flourishing industrial economy creates new jobs and significantly reduces the nation's unemployment rate. We have seen from earlier studies how global economy and industrial returns are affected by shock and movement in the real oil price and production, but what about the unemployment rate? According to the study of Karaki (2018) on effects by shocks in oil price, we see that negative shocks in the oil supply have an increasing effect on the unemployment rate. Norway was one of the victims of the oil crisis of 2014. From 2014-2016 the price fell from a year high 115.01 USD/bbl in 2014 to a year low of 30.89 USD/bbl in 2016 (Markets Insider, 2020). This makes over a 73% fall in real oil price over a two-year period. It can be argued that the fall in oil price over two years cannot be view as a "shock", but it certainly had its effect on the employment within the sector and the nation. In the same period of the reduced oil price, the number of workers in the petroleum sector in Norway fell by over 19%. Counting all relevant fields like extraction, service and pipelines (Norwegian Petroleum, 2020). According to the "Labor force survey" by Statistics Norway, the unemployment rate of the total Norwegian workforce (age 15-74) climbed 35% in the period (Hvinden, 2016). Based on these numbers there are reasons to believe that heavy movement in the real oil price do affect the unemployment rate.

2.1.2 Crude Oil Pricing Systems

As any product that is for different reasons considered a necessity by individuals, governments or corporations, the fundamental laws of supply and demand are destined to fully or partly decide the market value. The crude oil market is considered oligopolistic, which means that there are only a few capital-strong global producers supplying the market. On the demand side there are billions of costumers, making the market imperfect. The oligopolistic structure complicates the "natural" phenomenon of supply and demand balance. The market for crude oil is not only in the form of "over-the-counter" commerce but comprises several types of structural agreements between a supplier and a customer. Petroleum-based financial instruments, that are tradeable, includes assets like petroleum equities and different types of derivatives. The value of these assets is obviously linked to the market value of crude oil. Since oil is what we call heterogeneous, the logic of having a standardized price for a specific quantum of crude oil is challenged. It exists in different types and qualities, produced all over the world. Today, to handle this problem there has been created oil benchmarks containing the determined prices given the individual types and qualities.

Pre 1960, the oil industry consisted mainly of a handful large corporations. These companies where known as the *seven sisters*, or the majors, and contributed with over 85% of the global crude oil production from United States, China, USSR and Canada (Danielsen, 1982). Acting as the global leader of both exploration and production activities, and at least partially *downstream* operations, the *seven sisters* had control of the majority of the market crude oil supply. Given the tight link between the companies, they were successful in preventing the situation of secondhand trader accumulation, that could cause a downforce in the crude oil market price. This way there were reduced speculative reactions resulting in fluctuating market prices. The relevant governments were in no direct position of the crude oil production or pricing, but handled the national trades of oil related licenses and concessions (Adams, 1970).

The primary pricing model up to the mid-1970s was based on the concept of *posted price*. The *posted price*-model was intuitively a price posted by a seller or a buyer to enlighten the public of at which terms they are willing to make a trade. The OPEC, Organization of Petroleum Exporting Countries, used the posted price model to indicate the companies crude oil selling price. A governments tax and royalty income were estimated based on the model, since the systems of spot and long-term contract prices were victim to international tax speculations by the production companies (Mabro, 1984). As the governments wanted a bigger portion of the cake, the oil industry underwent a transformation in the early 1970s. The states within the OPEC region became restrictive of approving new concessions, and forwardly demanded equity shares in the current, and prospected, affairs. The government equity claims grew from the early 1960s but did not catch producer's attention until later that decade. The customer base had now evolved to third-party buyers. Adapting to the new situation of government ownership and the introduction of the official selling price (OSP), also known as government selling price (GSP), were due. Due to lack of experience in marketing and crude oil processing, most of the governments share of the crude oil were sold back to the companies that initially produced it at a "buyback" price. In this period, the crude oil price system consisted of three factors: the posted price, OSP and GSP, and the buyback price. The complexity of the system resulted in an early collapse in the mid-1970s (Fattouh, 2011).

After the collapse of the pricing system in 1975, the OPEC had now practically all the authority of crude oil pricing. The general base of pricing was now the marker of the crude Arabian Light, produced in Saudi Arabia. All other member was now pricing their oil, the OSP, in reference to the marker of Arabian Light. The underlying price difference to the marker was reflected in relative changes in regional and global supply and demand, and the value of the refined products. Entering the 1980s, non-OPEC countries advanced in their production, creating competition based on different crude oils. These market changes resulted in a collapse of the OPEC pricing system and the rise of the current global market system. OPEC is still producing the majority of the global crude oil supply, which means they today also have a large influence on the global prices, adjusting their production to maintain a sustainable equilibrium between supply and demand (Fattouh, 2011).

2.1.3 Benchmarks

The benchmarks are the reference prices used by traders, sellers and buyers, of the different types of crude oil. We can divide the market into three primary benchmarks: West Texas Intermediate (WTI), Brent Blend and Dubai Crude. The WTI oil originates from the U.S. and is considered light and sweet, measured after the API-gravity system. The factor of sweetness is reflected on the amount of sulfur the oil contains. Sweet means low-sulfur, light means low density. Given the typical characteristics, the WTI oil is well suited for fuels like gasoline and diesel. Brent Blend is extracted in the North Sea, and is considered light, sweet as well, but slightly heavier than WTI. The Brent Blend is the primary benchmark for crude oil in Europe. The Dubai Crude (Fateh) is middle eastern heavy, sour crude, extracted from Dubai in the United Arab Emirates. The Dubai Crude benchmark is essentially used for petroleum export in the Asia region (1995).

Table 1 Crude Oil Benchmarks - (PSA Management and Services BV, 1996)

	WTI Crude	**Brent Crude**	**Dubai Crude**
Location	United States	United Kingdom	United Arab Emirates
API Gravity	40.6	37.5	31.4
Sulfur Content	0.22%	0.40%	1.96%
Acid Number	0.10	0.03	0.25

2.1.4 Crude Oil Trades

Compared to the world of equities, including common stocks etc., the financial products of crude oil profitability can also be viewed as expectations of future market conditions. The physical dimension of crude oil presents added possibilities for trade and speculation. The markets can be distinguished as over-the-counter (OTC) markets or exchange-traded markets. The trade market for crude oil can be divided into three platforms, (1) spot market, (2) futures and (3) contract transactions (Inkpen and Moffett, 2011).

2.1.4.1 Spot Market

The spot market for crude oil reflects a tradeable opportunity for a delivery given a determined price. This can either be of a bilateral OTC nature or at exchangeable grounds. There exist several forms of the spot market, i.e. the “auction market”, where both sellers and buyers express their terms. As of most cases, primary oil and gas standards, there is the “market clearing price” which often represents the result of supply and demand at that given time. Different qualities and crude oil types have different spot prices, presented by the relevant benchmarks. The time of product delivery can often vary, but usually occur in the instant future. The spot market is often the basis of derivative contract speculation. Given the extreme volatility in the real oil price, as seen in the figure 1, price “speculators” attempt to establish arguments for shifts in different directions from today’s reality. This extreme uncertainty in the market gave birth to the crude future market (Burger et al., 2014).

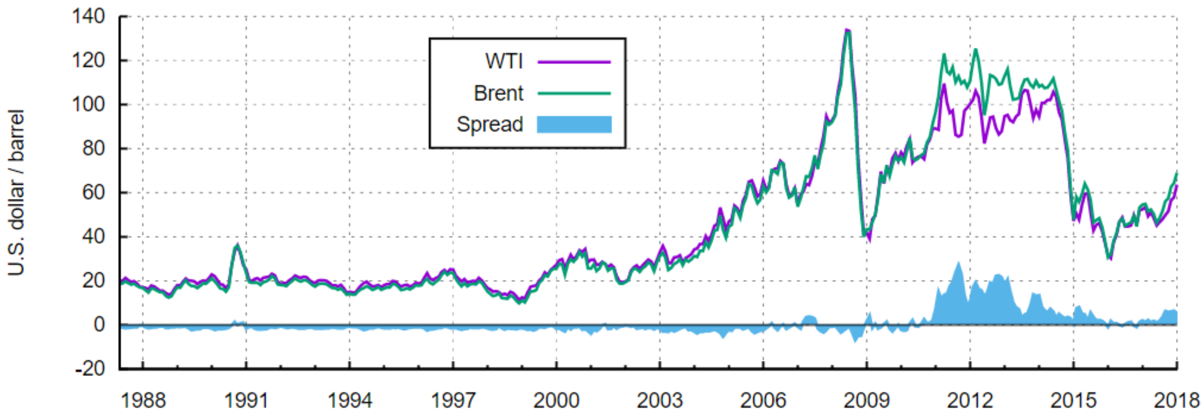


Figure 1 WTI and Brent Spread Graph (Wikipedia, 2020)

The spot price for crude oil serves other purposes than just OTC trading. Term contracts can contain a variable product pricing clause, that either includes a continuous update of contract terms connected to immediate shifts in spot price, or quarterly/periodically re-negotiations based on previous and/or expected market trends. There might be a price differential between contract price and spot trough all negotiations, but the spot rate is always grounds for the evaluation. Since the 1980s, the spot prices were used by companies in the possession of their own refineries to calculate the profitability of processing their own quantities, versus buying finished products on the market at current spot. Historically, the spot price was also often used to set the different companies selling price, or the OSP as mentioned earlier. Posted price tended to lag behind the spot, but this was because it did not reflect any current trades; it was only used as a reference for negotiations. For the governments, the lack of suitable measures for internal price transfers, and the growing need for domestic price regulations, caused the spot to be used in retail price control (Razavi, 1989).

In recent years the concept of spot price has been heavily criticized; the different benchmarks are not tied to a real-time price reported marketplace for crude, but rather independently traded between informed and speculative buyers/sellers. Considering the WTI Benchmark, the spot is determined from a trader survey each day (Cameron and Gijbert, 1992). The market raises both political and technical complaints. The fact that the oil prices can be used as a political weapon and influence balance of power both domestically and globally, makes the system victim to major speculation. In the late 1970s, contract prices were increased due to the high spot prices created by OPEC, either implicitly or explicitly. The technical concerns are considered more systematic, connected to the trading organization and the spot assessment (Razavi, 1989).

Further, the study of Fattouh (2006) raised concerns about the thin trading of benchmarks and argued for the dissolving of the Brent marker.

2.1.4.2 Futures Market

Futures are standardized contract-based agreements between a buyer and a seller/exchange. The contract is specified to the commodity selling amount, delivery date and agreed price of trade. The profitability of the contract is determined by the spot price at the delivery date and the price of commodity agreed in the contract itself (Burger et al., 2014). The contracts are highly speculative due to the underlying volatility, which creates a platform for both long positions and short positions, dependent on the traders current and futuristic view of the market. In risk management, the use of futures is often connected to hedging activity. Hedging can provide good margin protection, balance the incoming cash flow and help gaining a market advantage. This can be done either by taking the long or short positions in crude oil, respectively gaining or losing money from an increase in crude market value. The short position only proves profitable if the value declines in the future compared to the current price (Roncoroni et al., 2015). The holder of the futures contract has two options, either roll over the contract or close on before maturity. The roll option is what we call a swap, which at a specified cost prolongs the maturity of the contract. Implying that one is closed and another one initiated, or swapped if you will. The contract rarely results in any physical delivery of the underlying product since it is often closed by trades near maturity. As opposed to a forward contract, which is directly between a seller and a buyer, futures are traded through a futures-exchange platform (Mack, 2014).

Given the similarities between futures and forwards the method of pricing is basically the same concept. Forwards and futures calculated on the basis of equities or FX-instruments is somewhat different than for commodities. Equities can often consider dividend payouts and bears no cost of storage or arbitrage possibilities. Commodities, which historically has proven a victim to arbitrage, offers other costs and yields due to a physical dimension, which must be added to the base of calculation in the pricing process (Marroni et al., 2013). Other factors that need considering is the convenience yield. The convenience yield speculates in the future market expectations and gives a valuable option by storing the commodity. I.e. if it is justifiable to believe there will occur a shortage in the future, an opportunity to store the product and sell it when the value rises may present itself. This way the futures contracts can consider two scenarios, as seen in figure 2, (1) an increase in the future spot price relatively to the current spot (contango), (2) a decrease in future spot relative to the current (backwardation). In a situation where the convenience yield is higher than the cost of storage, plus the risk-free rate, we have contango (Fabozzi et al., 2008)

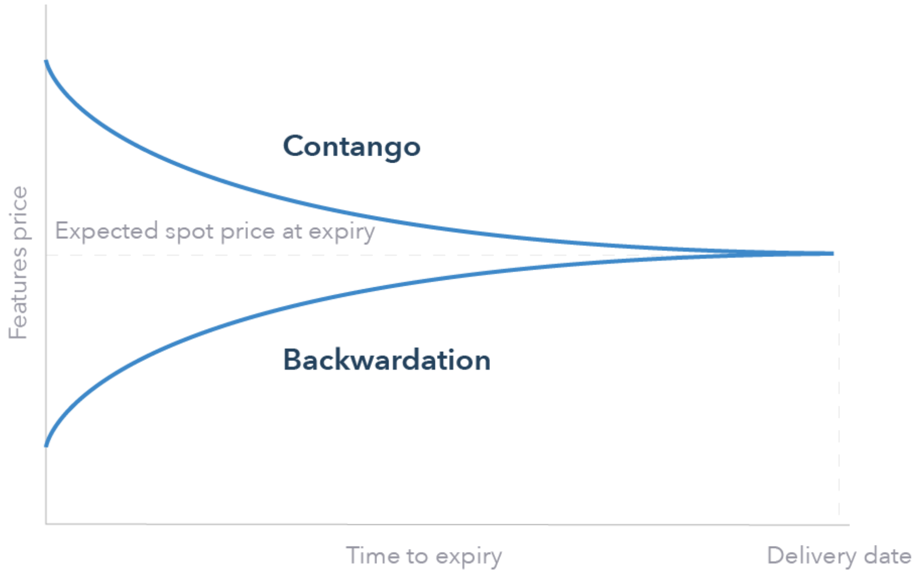


Figure 2 Contango and Backwardation Illustration (IG, 2020)

Futures contract pricing is based on the value of the underlying product. For crude oil, the pricing happens on the basis of the relevant benchmark spot price. We mentioned earlier that the forwards and futures are separated by the platform of trade. Futures are standardized and traded through an exchange, of which there exist several. The most important ones are the CME group's New York Mercantile Exchange (NYMEX). It offers a wide range of contract types for energy products like electricity, coal, natural gas and crude oil. This exchange introduced the NYMEX Light Sweet crude oil futures in 1983 and is now considered the most important energy benchmark in the U.S. ICE, the Intercontinental Exchange, was introduced in 2000 and had the vision of creating an OTC energy trading platform. ICE have key crude oil benchmarks as ICE Brent futures, which is important to the oil cargo pricing in Europe. Other important energy futures exchanges are the NASDAQ OMX and the European Energy Exchange (EEX) (Burger et al., 2014).

Comparing the futures contracts to the crude oil spot market, there is often speculation in the relationship with the daily current spot price. Since the contracts are based on a future delivery, there should be correspondence between spot and added factors of cost calculation. A study by

Bekiros and Diks (2008) examines the linear and non-linear correlation between daily crude oil spot and futures prices. The results of the study indicate statistical similarities and asymmetric mirrored properties between them. The tests considering nonlinearity is also conclusive about there being no significant lags or leads patterns in the markets, implying that there is some direct effect of volatility in daily spot prices and futures prices. A rolling window study by Liu and Wan (2011) shows that the correlation between spot and futures exist mainly in the contracts with longer maturity.

2.1.4.2 Contract Transactions

It is essential that the oil producers find customers to buy their oil, often being refineries that further process the crude oil into refined product like heating oil, jet fuel and gasoline. The transactions are contract-based and considers both the spot and futures market in their pricing models. The trades can often be conducted by the producers themselves or by specialized agencies which are industrially engaged (Inkpen and Moffett, 2011). What separates these agreements from futures is the purpose of product delivery. The contract specifications themselves are quite similar, containing product, quantity, quality, delivery location and price (Nossa et al., 2016).

2.2 FX Market

Most countries, with the exception of some participants in the European Union and smaller nations, bases their domestic money on different currency. Since each individual country create different investing appeals, have their own rate of inflation/deflation, and present a varied opportunity of economic growth, the global currency demand is highly speculative. The foreign exchange market, or FX market, is where the trade of the different national currencies is conducted. This market is considered an over-the-counter (OTC) market, and the platform of spot trading has no official exchange, rather trading centers located in the major cities such as New York, London and Tokyo (Roncoroni et al., 2015). The FX market becomes relevant on many occasions, either if there are travelers in another country, looking to exchanges money from your domestic currency to the local currency, or a company that wants to buy products or services internationally.

The function of the so-called FX rate is basically to compare to different currencies. Roncoroni et al. (2015) (p. 499) defines the FX rate as:

FX Rate. An exchange (FX) is the price of one currency in terms of another currency; the two currencies make a pair. The pair is named by a label comprising two tags of three characters: each currency is identified by its tag. The first tag in the exchange rate is the base currency, the second is the numeraire currency. So the FX is the price of the base currency in terms of the numeraire currency.

I.e. if attending the FX rate between the Norwegian Krone (NOK) and the U.S. Dollar (USD) we get the expression USD/NOK. USD being the base currency and NOK being the numeraire, also known as the domestic. The rate will explain the relative value of the U.S. Dollar compared to the Norwegian Krone. This USD/NOK rate will be defined as a five-digit number. The selection of the base currency is primarily done by fiscal judgement, as which currency makes profit or losses in terms of the domestic currency. To avoid any traveler's confusion, the operator of the relevant trading center will define the currencies informatively displayed next to the FX rate (Roncoroni et al., 2015). The FX rate can be referred in two overviews (1) the 10 most developed countries called G10 or (2) EM, which contains all other countries (Strumeyer and Swammy, 2017).

Table 2 Table of Illustrative Exchange Rates (Bloomberg, 2013)

Currency	USD	EUR	JPY	GBP	CHF	CAD	AUD	HKD
HKD	7.7736	10.2976	0.0928	12.2853	7.9165	7.6987	7.6584	–
AUD	1.015	1.3446	0.0121	1.6042	1.0337	1.0053	–	0.1306
CAD	1.0097	1.3376	0.0121	1.5958	1.0283	–	0.9948	0.1299
CHF	0.9819	1.3008	0.0117	1.5519	–	0.9725	0.9674	0.1263
GBP	0.6328	0.8382	0.0076	–	0.6444	0.6267	0.6234	0.0814
JPY	83.735	110.9238	–	132.3348	85.2751	82.9281	82.4949	10.7718
EUR	0.7549	–	0.009	1.193	0.7688	0.7476	0.7437	0.0971
USD	–	1.3247	0.0119	1.5804	1.0184	0.9904	0.9852	0.1286

2.2.1 FX Trades

Much like in crude oil trading there are several instruments in the FX market that stimulate hedging, arbitrage and speculation. The spot market, futures and forwards, options and swaps are all trading alternatives with the same structural properties as in a stock or a commodity.

2.2.1.1 Spot Rates

The currency spot rate is for immediate trades, as with crude oil, and involve that the transaction must be made within a business day, or two if there are international agreements.

The spot rate is determined by supply and demand, and there is no defined physical trading floor for spot currency. A common participant of trades are the banks and foreign exchange dealers, i.e. FOREX, and other financial corporations. The rates that the relevant institutions are willing to sell currency at is called ask or offer rate. This is when the institutions are selling a foreign currency to someone in exchange for local currency. When the institution is buying foreign currency, it is called a bid rate. The value gap between ask and bid rate is called a spread and usually reflects cost of transaction and conversion. The spread is often a fixed amount for small transactions, and a percentage of the exchanged amount in big transactions (Clark et al., 2004). It is fair to say that the value of all foreign currency is relative, and we cannot establish just one global rate for a local currency to others. The volatility in the spot market is reflected by the fluctuations in the individual currencies. Taking the USD/NOK example, if the FX rate were to increase, we would say that the U.S. Dollar had been appreciated or strengthened, or the Norwegian Krone had been devalued, or both. If there was a decrease in the FX rate it would have been the other way around (Weithers, 2013).

2.2.1.2 Forwards and Futures

Futures and forwards contracts in currency have been available for trading since 1972, when the Chicago Mercantile Exchange was opened. Similar to the FX rate, or spot rate, the futures contracts explains the value of one currency compared to another. The contracts are quoted in the same way and uses the same terminology. Currency futures, unlike the spot market and forwards, are traded on a defined and regulated exchange, similar to commodity futures. The contracts are specified in the same way as any commodity future, with a certain delivery, time and place of delivery and cost of contract. There is rarely a need for any cash delivery, since most of the currency futures are closed before the last trading day, which is defined in the contract. The reason of the closing could either be to take a profit or to cut their losses. The settlement is somewhat different given the long or short position in the future. To close the standard open long position, the trader will have to sell all the contracts holding the long position, while the short position trader will have to buy back all contracts holding his position (Butcher, 2011). All futures trades demand a margin account with licensed brokers, which will trade according to your orders. In the event of a leveraged trading position, where the trader has borrowed money to enhance his deposit, or shorting, the decrease in portfolio value will demand the trader to increase his margin account. This is informed by a “margin call” (Kinahan, 2016).

2.2.1.3 Swaps and Options

In addition to futures, swaps and options are popular instruments in the currency markets. Swaps reflects a trade where currency are bought and sold simultaneously but have different delivery dates. In swaps trading, the value of instruments bought is the same as the ones sold. It is an exchange in positions with the same amount invested. The difference between a currency spot rate and a forward exchange rate is called a swap rate. The swap rate is displayed as a percentage, either a discount or premium conditional on the underlying rate difference (Poniachek, 2012).

A currency option, often used to hedge a company’s exposure to risk, is a derivative in many ways similar to futures and forwards. The difference lies in the option, since the option contract have no obligation to strike. The deal is still specified in the contract, with the same parameters, but the owner has the choice of acting on the contract or not to do so. This could either be a put (sell) or a call (buy) option. Since the buyer of the option can choose to not strike, the contract has a price called a premium to ensure the issuer (Butler, 2016).

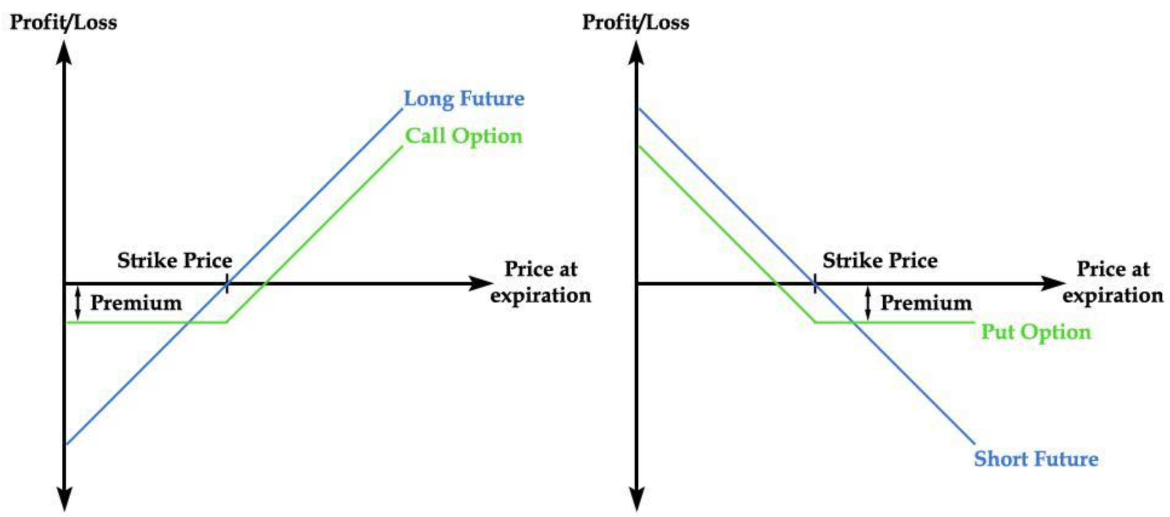


Figure 3 Options and Futures (Cryptarbitrage & MrJozza, 2019)

2.2.2 Currency Demand

As stated earlier, the currency exchange spot rate is determined by supply and demand. Given that the exchange rate between two currencies decreases, for different reasons, the natural phenomenon of equilibrium will balance and increase the demand, and vice versa. But it does not necessarily have to be a rate drop to increase currency demand. One of the most important factors of increasing demand is connected to the relevant country's interest rates. As the domestic interest rates increase, many foreign investors may consider the nation's financial investment opportunities to be attractive. To be able to invest in the instruments investors need to exchange to local currency, creating a currency demand (Simpson, 2014). Interest rates, in a macroeconomic sense, is a tool to regulate the balance between inflation and deflation. Being that none of them are an attractive feature in a country's development, the regulation should happen on the count of domestic sustainability, rather than a pure exchange rate case (Fama, 2013). Given that countries have different monetary policies and uses their tools individually, there are opportunities for inter-currency investor speculation.

2.2.3 Exchange Rate Regimes

The choice of a country's exchange rate regimes (ERR) have different effects on the important economic factors. By applying governmental regulations on the key actions to create volatility and economic growth, central banks can achieve the desired results on platforms of international trade and finance. The actions are often conducted by the central banks and directed towards one of a three-way scheme of operations: (1) Fixed regime, (2) Floating regime and (3) intermediate regimes (Caprio and Caprio, 2012).

The fixed exchange rate regime is described by a country without its own independent exchange or monetary policy. There are two examples of the absolutely fixed system, dollarization and monetary union. A country can choose to operate with another currency, which monetary policy is regulated by a foreign central bank. This is called dollarization. With a monetary union there is a common currency and a common central bank. What separates the two are the power of influence. In dollarization, the country must get behind the monetary policy of the selected currency owner, but in monetary union there are positions of shared influence and split governmental control. There is also a possibility to obtain an advantage with the monetary union, inheriting the trust and reputation of the shared currency, but with dollarization you may have to sacrifice a substantial amount of resources to obtain another currency (Corden, 2002). The fixed regime will allow the central banks to manipulate the exchange rate, trading foreign currency to keep the nominal rate at a stable level (Caprio and Caprio, 2012).

The floating regime is more a natural policy that allows reduced intervention in the foreign exchange market by the central bank. In this regime the currency rate is market-determined, only affected by balance of supply and demand. This is what separates floating (flexible) and fixed regimes. There can exist at least two types of floating regimes, (1) pure floating and (2) managed (dirty) floating. These are separated by rate of intervention. The managed float is categorized as intermediate variates of fixed and float. The flexible regime became increasingly popular in the post WW2 era, sharing the currency control from the central banks alone, to the people as well. The theory behind this regime was to protect the economy towards international shocks, thereby stabilizing foreign effects on domestic markets (Dellas and Tavlas, 2013). Today, almost all countries have a form of managed floating exchange rate regime, to some extent allowing central banks interventions in the markets (Simpson, 2014).

2.3 Inter-Bank Offered Rates

The money market is a platform where financial institutions and wealthy traders can borrow or lend out capital surplus over a short-term structure (less than one year). These loans are usually unsecured, and includes a wide coverage of financial assets as commercial papers, treasury bills, discount papers, federal funds etc. Both banks and private investors can be buyers of the available securities. Loans between banks are a part of the inter-bank market, where the banks regulate their current capital. The loans have reference rates that defines the loan terms, which are often directly affected by the relevant domestic interest rates. Examples of these reference rates are LIBOR (London Inter-Bank Offered Rate), EURIBOR (Euro Inter-Bank Offered Rate) and NIBOR (Norwegian Inter-Bank Offered Rate) which are connected to the London, Europe and Norwegian money markets respectively (Fabozzi et al., 2003).

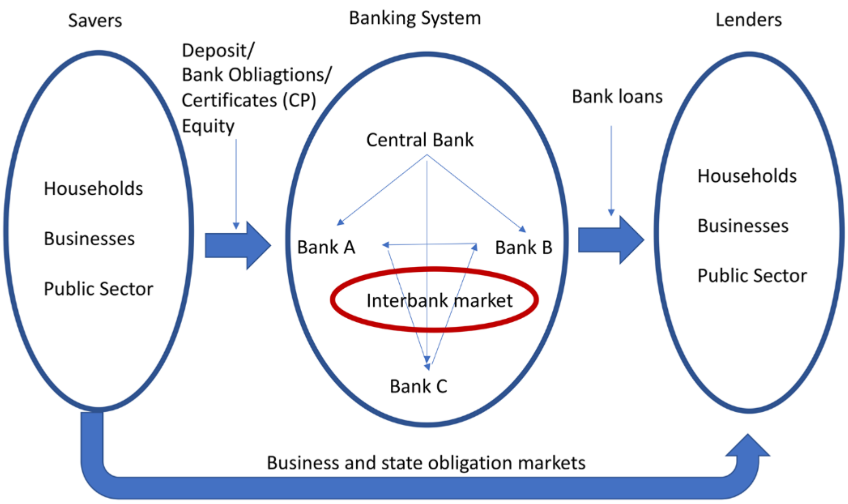


Figure 4 Interbank Market Illustration (Norges Bank, 2012)

2.3.1 Norwegian Inter-Bank Offered Rate (NIBOR)

The inter-bank market in Norway bases its terms on the Norwegian Inter-bank Offered Rate (NIBOR). The current system was initiated in 2011 after the consensual agreement between the Norwegian central Bank (Norges Bank) and FNO (Finans Norge). Its main purpose is, according to FNO, to reflect the rate level that the lender demands for a unsecured loan in Norwegian Kroner (NOK), based on what the banks will demand from lending money to other leading banks active in the Norwegian money market. The use of the NIBOR reference rates are traditionally applied when the banks are setting their lending rates, when setting rates for bonds and the pricing of several types of financial derivatives. The bank's lending rate is often the reference rate plus credit risk and term premiums. The current NIBOR reference rates mainly exist in five terms: one week, one month, two months, three months and six months after the changes 1st of January 2014 when Oslo Stock Exchange inherited the responsibility. Before this, there were ten different terms available in the NIBOR system (Bernhardsen et al., 2012). Today, the responsibility for the NIBOR benchmark have been shifted back to FNO-owned NoRe (Norwegian financial References) after the agreement January 2017. The panel banks that presents daily NIBOR reports to calculation agents are:

- DNB ASA
- Danske Bank
- Svenska Handelsbanken AB
- Nordea Bank
- SEB AB
- Swedbank AB

These banks send in their rate estimation for NIBOR to GRSS (calculation agent), The Global Rate Set Systems, where they calculate the weighted NIBOR based on the banks contribution and stated rules of estimation (Norske Finansielle Referanser AS, 2019c).

2.3.1.1 Rate Calculation

When calculating the interbank reference rates there are basically two factors of consideration: (1) the expected central bank key rate and (2) the risk premium. The weighting of these two factors varies according to the amount of market distress. In troubled times (in a macroeconomic sense), the risk premium may be considered the most important since there are many factors that can cause payback interruption or reduced liquidity. Historically, the risk premium has been stably low until the financial crisis of 2008, when the investment bank "Lehman Brothers" went out of business. This created a hoarding tendency in the liquidity market, with spread panic and fear of bankruptcy, increasing the premium. Another factor that may influence the risk premiums are the capital requirements demanded from the banks, which often affects the rate of overnight loans. When the market is in a stable growth position, with limited risk and uncertainty, the important factor in rate calculation is in the key rate expectation. In these times, the key rate normally decides the overnight loans rate. To eliminate the theoretical opportunity of arbitrage, the cost of these loans should be the same, excluding risk premiums (Bernhardsen et al., 2012).

Every open market day the panel banks sends in their NIBOR rate suggestions, for all the relevant terms, to the calculation agent in form of a two-decimal number. To prevent any individual unwanted market-influence, the highest and the lowest suggested rates are omitted if six or seven banks submitted rates. If fewer than five banks deliver, all rates are used. The confirmed and calculated NIBOR rates are then the simple average of the sample (Norske Finansielle Referanser AS, 2019b). In a discontinuous event, where the reference rate of NIBOR suffers, there is a fallback rate to replace it called NOWA. NOWA, Norwegian Overnight Weighted Average, is administrated by the Norwegian central bank (Norges Bank) and not only seen as a replacement for NIBOR, but also to co-exist as a secondary base rate for financial contracts and instruments. The difference from NIBOR is that NOWA presents the average rate of current overnight loans between banks active in the market (DNB ASA, 2020).

Prior to the 1st of January 2020, the panel banks submitted expert estimations of the rates they are exposed to in the market. These rates in Norwegian Kroner (NOK) was estimated based on the spread between borrowing and lending rates and the information of spot and terms in the exchange market. Traditionally being quoted as a currency swap rate, the NIBOR was calculated as:

$$NIBOR = USD\,rate + Forward\,premium$$

The USD rate in the equation is reflecting the cost of unsecured USD interbank market loans. This rate was prior to the financial crisis in 2008 the USD LIBOR but since there was skepticism about their conservatism, NIBOR banks switched to the "real rate" set by Carl

Kliem in Frankfurt. This was more applicable to European banks operating with USD in the interbank market. The forward premium presents the difference between the spot rate and the forward exchange rate. This difference shows the cost of swapping the currency today and then later reversing the same amount at a given term. This will basically give an interest rate differential between the countries (Norges Bank, 2013).

Today, the method of *weighted* calculation is slightly different. Adapting the reference rate to meet the requirements of the European Union's benchmark regulations (BMR), NoRe proposed a waterfall methodology considering different aspects than earlier. What the new system presents is a priority line of three steps, (1) rates from current unsecured interbank loans to a leading bank, (2) rates from sales of certificates of deposit and (3) rates based on quotations, market data and expert opinions. If the information in step 1 is available it will be the grounds of calculations, if not, step 2 will determine the rate etc. In the event of absent sufficient submitted data, the last day fix will be base of calculation (Norske Finansielle Referanser AS, 2019a).

2.4 Market Dependencies

Even though each of the markets operates as independent factors affecting the Norwegian economy, there are certain degrees of which they are correlated and dependent. Considering the exchange rate between the U.S. Dollar and the Norwegian Krone, we know that the attractiveness of the Norwegian currency is highly dependent with the outlook for growth in domestic economy. In 2019, almost a quarter of the total Norwegian national income came from the petroleum related industry, making it a bigger contributor to the national budget than value added tax on products and services (Det Kongelige Finansdepartement, 2020). Therefore, it is easy to understand the significance of Brent oil prices when considering the outlook on domestic industry growth prospects. In a study by Bernhardsen and Røisland (2000) they found obvious correlation between the strength of NOK currency and the real oil price, and evidence that justify the *rand-currency hypothesis*: That the NOK is a less attractive currency in times of global financial volatility. Most oil-related investments are international, demanding foreign trades, and therefore heavily dependent on the relative strength of the Norwegian Krone. A weak NOK generates increasing costs of global operations and investment financing.

Being that NIBOR can be described as a USD swap rate, the terms of NIBOR is destined to be affected by U.S. and otherwise international economy. Prior to the financial crisis, the NIBOR terms were dependent on the LIBOR benchmarks for U.S. Dollar, not Carl Kliem as of today. LIBOR represented a less realistic rate, undershooting the real rates of unsecured USD loan in the interbank market, which perhaps at that time were more correlated with the calculated NIBOR. The longer NIBOR terms (3 months and 6 months), can in some cases experience more volatility from day-to-day, indicating that there might be regulations in the key interest rates resulting from a rate meeting in the central bank. This is perhaps more relevant in times of financial distress when there are expectations of major changes in the domestic key rates (Tafjord, 2015).

The market correlation between Brent oil prices and NIBOR terms are more difficult to conclude. According to theory, low interest rates opens for more corporate spending, making it more feasible to maintain operations, profitability, and continue any further investments (Olsen, 2018). This is probably more likely to be the case for non-petroleum industries since low rates often correlates with low Brent oil prices in the Norwegian markets. These low oil prices make little room for sector investments and may often cancel out the stimuli factors of the low interest rates. Since oil price shocks are extremely difficult to foresee, and heavily connected to the attractiveness of Norway as an investment case, one might be right in thinking that there exists a degree of spillover from Brent oil to NIBOR terms.

3. Methodology

This chapter presents and summarizes some background theory in general statistics, as well as the theory needed to understand Copulas. Further, this theory is utilized in a description of how the analysis is performed, and how the results may be interpreted.

3.1 Primer on Probability Distributions.

This thesis utilizes copulas as a tool for statistical modelling of multivariable distributions. Inherently, copulas can be described as types of multivariate distributions, and as such, a quick primer on probability distributions and multivariate distributions is appropriate.

3.1.1 Probability Density Functions.

Consider a random variable X. This variable can in principle take on any value, but some values may have different probabilities. X may take on some values more frequent than others. Illustratively, one might consider a set of two dice, one of which is fair, the other biased towards some values. As an example, let figure 5 represent the histogram for the outcomes of these dice, where the x-axis represents the die value, and y-axis represents the proportion of outcomes.

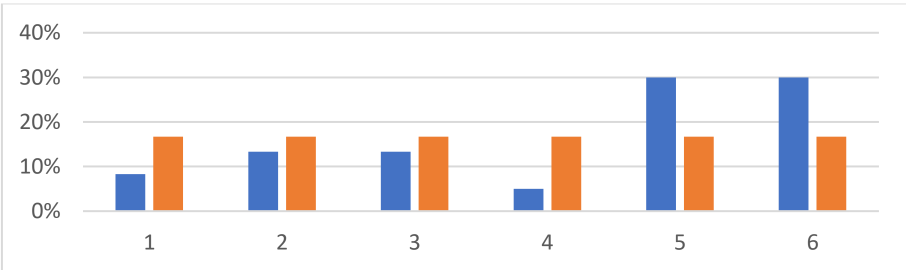


Figure 5 Dice Outcomes

The orange columns represent the fair die's outcomes, and conversely the blue represent the biased die. Summing up the column values for a die, say 1 to 3 represents the probability of the die taking the value from one to three. Summing up every column then correspond to all possible outcomes, and as such will take value 1 (100%).

This example illustrates *discrete* outcomes, meaning that X can only take on certain discrete values. Generally, we are often more interested in the case where X can take on *any* value in an interval. More strictly: We want the outcome-space of X to be continuous. If X can take any value in a continuous space, we can ask the question of whether X lands in a given interval or not. One is due to note that in this definition it is meaningless to ask if X lands on a specific value, (one could try to ask the probability of finding X in an arbitrarily small interval but would also then end up with an arbitrarily small probability). Within this system, probability of finding X within some interval can be expressed as an integral of some function over the interval (Area under a curve, if one considers the Riemann integral). This function is known as the probability density function and is usually denoted $f(x)$, and as the name suggest, describes the density of probability which can vary across the domain of X. The integral of f(t) from the lower bound to the variable x is commonly known as cumulative probability density function and its value at point x represents the probability of finding X between the lower bound and x.

Definition.
$f(x)$ is a probability density function for the continuous random variable X if:

1. $f(x) \geq 0, \forall\, x \in \mathbb{R}$
2. $\int_{-\infty}^{\infty} f(x)dx = 1$
3. $P(a \leq X \leq b) = \int_{a}^{b} f(x)\,dx$

Once the probability density if defined, one can also define the cumulative density function:

$$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(t)\,dt \tag{1}$$

3.1.2 Gaussian Distribution

The most common and arguably most important continuous distribution function is the Gaussian (Normal) distribution. Which has the following density function:

$$f(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{2}$$

The gaussian distribution is frequently seen in statistics, often because of the central limit theorem. This theorem states that if $X_1, \ldots X_n$ are samples form a distribution of any type with population mean μ and variance σ^2, the value $Z = \frac{X-\mu}{\sigma/\sqrt{n}}$ will be normal N(0,1) distributed as $n \rightarrow \infty$.

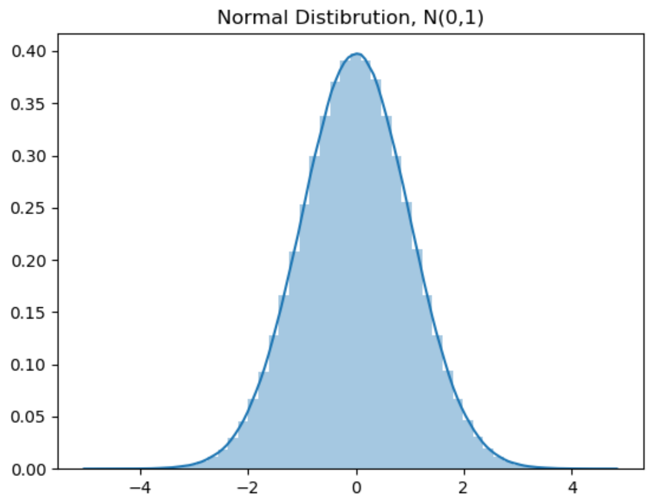


Figure 6 Normal Distribution

3.1.3 Uniform Distribution

An important distribution to this thesis is the so named uniform distribution. This distribution can be demonstrated as a continuous analogue for the fair die. That is, for a defined interval any number of sub-intervals of equal length is equally likely. We can define random variable U to be uniformly distributed between any interval $[a, b]$, but in most cases $[0,1]$ is utilized, and one can then write out: $U \sim Uniform(0,1)$. The probability density and cumulative density for such a distribution can be summarized as follows:

$$f(x) = \begin{cases} 0, & x < 0 \\ 1, & x \in [0,1] \\ 0, & x > 1 \end{cases} \tag{3}$$

$$F(x) = \begin{cases} 0, & x < 0 \\ x, & x \in [0,1] \\ 1, & x > 1 \end{cases} \tag{4}$$

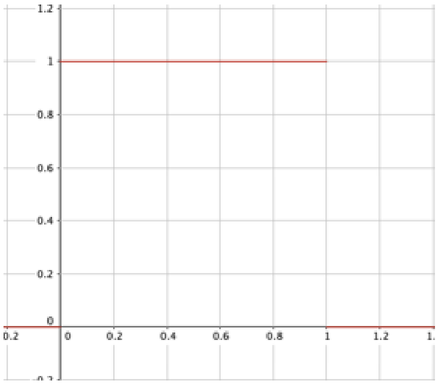


Figure 7 Uniform PDF

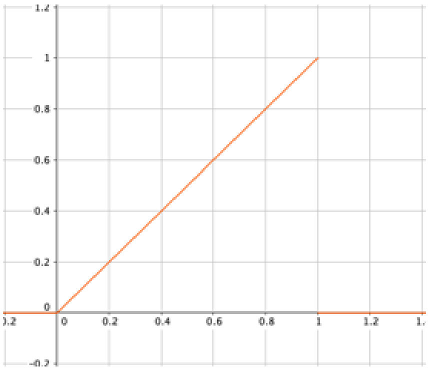


Figure 8 Uniform CDF

3.1.4 Laplace Distribution

Financial return data often display a tendency to be (close to) Laplace-distributed (Kotz et al., 2001). The Laplace distribution (or double exponential as it is often known) is a distribution that is highly leptokurtic, meaning it is highly concentrated around the mean, and with large tails. The general formula for a Laplace distribution is a two-sided exponential:

$$f(x;\mu,b) = \frac{1}{2b} e^{\left(-\frac{|x-\mu|}{b}\right)} \tag{5}$$

Where μ, b are the mean and shape-parameter, with variance $\sigma^2 = 2b^2$.

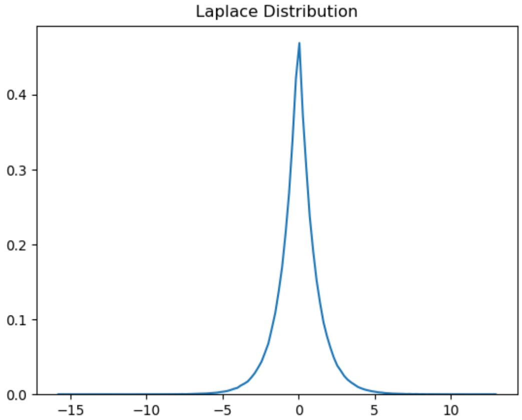


Figure 9 Laplace Distribution

3.1.5 Other Common Distributions

Weibull Distribution:

$$f(x;\lambda,k) = \begin{cases} \frac{k}{\lambda}\left(\frac{x}{\lambda}\right)^{k-1} e^{-(x/\lambda)^k}, & x \geq 0 \\ 0, & x < 0 \end{cases} \tag{6}$$

The Weibull distribution is a highly applicable distribution, often seen in applications of reliability analysis and extreme value theory.

Gamma Distribution:

$$f(x;\theta,k) = \frac{1}{\Gamma(k)\theta^k} x^{k-1} e^{-\frac{x}{\theta}} \tag{7}$$

Where $\Gamma(k)$ denotes the gamma function, a continuous analogue to the factorial $k!$. The gamma distribution is a general distribution where examples such as the *exponential distribution* and *chi-squared distribution* (also common distributions) are special cases of the gamma distribution.

3.2 Multivariate Distributions

The concept of probability distributions can be expanded to incorporate more than one variable. Multivariate density functions are as the name suggests, probability distributions defined for more than one variable. Bivariate distributions, that is, distributions defined for two variables, are commonly envisioned as sheets or surfaces with varying height. The double integral over some domain, which can be interpreted as a volume, gives the probability of both variables landing within the domain. Although multivariate distributions can also be defined discretely, we will concern ourselves with the continuous case.

Definition: If X and Y are two continuous random variables, they have joint probability distribution $f(x,y)$ if

1. $f(x,y) \geq 0 \ \forall \ [x,y] \in \mathbb{R}^2$
2. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)dxdy = 1$
3. $P\big((X,Y) \in A\big) = \iint_A f(x,y)dxdy$

3.2.1 Marginals and Independence.

By integrating all other variables but one from a multivariate distribution, one obtains the marginal distribution of the remaining variable. That is, a univariate probability distribution. Illustratively: $f_x(x) = \int_{-\infty}^{\infty} f(x,y)dy$.

An important aspect of multivariate distributions are dependences between marginals. Consider the formula for conditional probability:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{8}$$

Similarly, the conditional probability distribution becomes:

$$f(x|y) = \frac{f(x,y)}{f(y)}, \qquad f(y) > 0 \tag{9}$$

Also, the following definition holds for independent marginals.
Two random variables are independent if, and only if:

$$f(x,y) = f_x(x) \cdot f_y(y) \tag{10}$$

Meaning that the joint distribution is a product of its two marginals.

3.2.2 Surfaces

Bivariate distributions can be illustrated as surface plots. As an example, figure 10 illustrates the bivariate gaussian distribution, which is a multivariate distribution consisting of normal distributed marginals. If one were to view this plot purely from one slide, or “slice” the distribution along the X or Y axis, one would obtain a univariate marginal. An important aspect to note about multivariate distributions is that the marginal distributions can be any univariate distribution, thus one is not limited to symmetric cases where both marginals are, as in the example figure 10, Gaussian distributed.

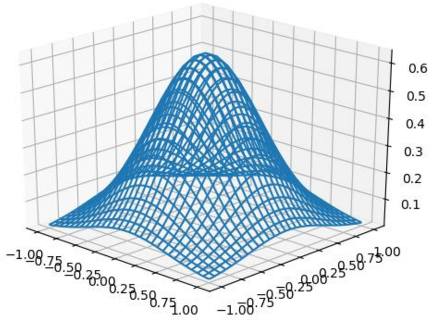


Figure 10 Bivariate Gaussian Distribution Plot

3.3 Copula

A copula is a cumulative multivariate distribution consisting of marginals of uniform univariate distribution and are used to describe dependence structures between random variables. This is achieved through Sklar's theorem (Sklar, 1959) which dictates that using the *probability integral transform*, one can transform any random variable with continuous and strictly increasing cumulative distribution function, into a variable that is uniformly distributed. A copula can be constructed from the transformed uniform variables, and a dependence structure between the original variables can then be described.

3.3.1 Bivariate Copula:

Although copulas can be defined for any number of variables greater than one, we focus particularly on *bivariate copulas*. These are copulas of two input variables, meaning that it is a function taking in two univariately distributed variables and returning a probability measure. For any copula, the inputs and outputs are both restricted to the domain of [0,1].

Definition:

The function C is a bivariate copula if the following criteria holds:

$$C(u_1, u_2): [0,1]^2 \rightarrow [0,1] \tag{11}$$

$$C(u_1, 0) = C(0, u_2) = 0 \tag{12}$$

$$C(u_1, 1) = u_1 \wedge C(1, u_2) = u_2 \tag{13}$$

A bivariate copula must also be strictly 2-increasing, meaning that

$$C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) - C(u_1, v_1) \geq 0 \tag{14}$$

For all $0 \leq u_1 \leq u_2 \leq 1 \; and \; 0 \leq v_1 \leq v_2 \leq 1$

3.3.2 Sklar's Theorem

Sklar's theorem is arguably the most important theorem of copulas, dating back to Sklar (1959). The theorem states that any multivariate cumulative distribution function

$$H(x, y, \dots) = Pr\,[X \leq x, Y \leq y, \dots] \tag{15}$$

With marginal distributions:

$$F_x(x), F_y(y), \dots$$

Can be expressed as a copula with the marginals as arguments. In the bivariate case:

$$H(x, y) = C(F_x(x), F_y(y)) \tag{16}$$

The fact that the marginals are uniformly distributed (3,4) can be seen more easily by considering the *probability integral transform*:

$$U = F_x(x) \tag{17}$$

1. Let X be a random variable with a continuous cumulative distribution function $F_x(x) = Pr\,[X \leq x]$.

2. Then define a new variable U, defined as $U = F_x(X)$.

3. U will then be uniformly distributed.

This can be showed relatively straight forward:

$$U = F_x(X)$$

$$F_U(u) = \Pr[U \leq u]$$

$$F_U(u) = \Pr[F_x(X) \leq u]$$

$$F_U(u) = \Pr[X \leq F_x^{-1}(u)]$$

$$F_U(u) = F_x\left(F_x^{-1}(u)\right)$$

$$F_U(u) = u$$

As we recall from definition (4) for uniform distributions, this is the cumulative distribution function for a univariate uniform variable U. Following Sklar's theorem one can write out the cumulative bivariate distribution as follows:

$$H(x,y) = C(u,v) = C\left(F_x(x), F_y(y)\right) \tag{18}$$

Where $U,V \sim iid.uniform(0,1)$. Utilizing the chain rule of derivation, we can write out the bivariate probability density function:

$$h(x,y) = \frac{\partial^2 C(u,v)}{\partial x \partial y} = c(u,v) f_x(x) f_y(y) \tag{19}$$

Where $c(u,v)$ is known as the copula density:

$$c(u,v) = \frac{\partial^2 C(u,v)}{\partial u \partial v} \tag{20}$$

As one can see from (19), an appropriate (multi-) bivariate distribution can be described by the product of the marginals and the copula. As previously stated, the product of two marginals are known as an independent distribution. Hence, then entire dependence structure is encapsulated in the copula density. It is this very property that allows us to model the marginals and dependence structure separately.

3.3.3 Fréchet-Hoeffding Bounds

The Fréchet-Hoeffding bounds (Fréchet, 1960, Nelsen, 2007) are the upper and lower bounds for any copula. These bounds can be stated as follows:

For any copula C: $[0,1]^d \rightarrow [0,1]$ and any $(u_1, \dots u_d) \in [0,1]^d$

The copula is bounded between the lower and upper Fréchet-Hoeffding bounds:

$$W(u_1, \dots u_d) \leq C(u_1, \dots u_d) \leq M(u_1, \dots u_d) \tag{21}$$

Where W is the lower Fréchet-Hoeffding bound defined as:

$$W(u_1, \dots u_d) = max\left\{1 - d + \sum_{i=1}^{d} u_i, 0\right\} \tag{22}$$

Likewise, the upper bound M is defined as:

$$M(u_1, \dots u_d) = min\,\{u_1, \dots u_d\} \qquad (23)$$

These bounds can be (but not always) copulas in and of themselves. As such, the upper bound would be the copula in the case of perfectly positive dependent variables, but only in the bivariate case is the lower bound a copula. Then, in this case, the lower bound corresponds to the independence copula. Any real copula exists within the confines of these bounds.

3.3.4 Archimedean Copulas.

An important and popular subset of multivariate copulas is the family of Archimedean copulas. Archimedean copulas are characterized by consisting of one dependence parameter which accounts for the dependence structure of the copula. Since there is only one dependence parameter to be estimated, this type of copula has grown popular. Archimedean copulas are then defined by one expression. As a result, they become somewhat easier to model as opposed to more involved copulas such as the gaussian copula, which has no direct analytical solution for its density function (Nelsen, 2007).

Definition
A copula is said to be Archimedean if it can be described as follows:

$$C_\psi(u_1, u_2, \dots; \theta) = \psi^{-1}(\psi(u_1;\theta) + \psi(u_2;\theta) + \cdots; \theta) \qquad (24)$$

Where θ is known as the dependence parameter, and ψ is the generator function where: $\psi: [0,1] \rightarrow [0, \infty)$ is a continuous, convex, strictly increasing function, such that $\psi(1, \theta) = 0$.

The most used Archimedean copulas are Clayton, Gumbel, and Frank copulas, and it is these copulas that have been utilized in this thesis. Each of these copulas model dependency differently. The clayton copula describes dependences in the lower tail ends of distributions. Likewise, the Gumbel copula can model any level of upper tail dependence. The Frank copula is good for modelling relative upper and lower tail dependence.

Some of the most important characteristics of these copulas are summarized in table 3

Table 3 Archimedean Copulas and Generator Functions

Name	$C_{\psi}(u,v)$	$\psi_{\theta}(t)$	θ
Clayton	$max\ [0, \{u^{-\theta} + v^{-\theta} - 1\}^{-\frac{1}{\theta}}]$	$\frac{1}{\theta}(t^{-\theta} - 1)$	$\theta \in [-1, \infty)\backslash\{0\}$
Gumbel	$exp\ [-((-log\ (u)^{\theta} + (-log\ (v))^{\theta})^{-\theta}]$	$(-\ log\ (t)\ )^{\theta}$	$\theta \in [1, \infty)$
Frank	$-\frac{1}{\theta}\left[1 + \frac{(e^{-\theta u} - 1)(e^{-\theta v} - 1)}{(e^{-\theta} - 1)}\right]$	$-log\ \left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}\right)$	$\theta \in \mathbb{R}\backslash\{0\}$

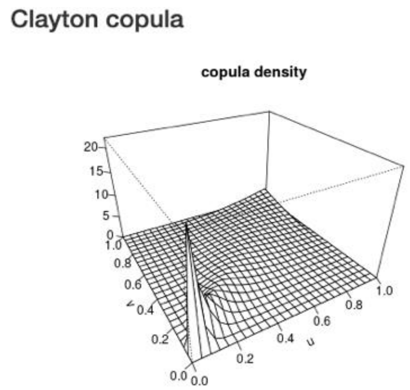


Figure 11 Clayton Density plot (Theta = 1.4)

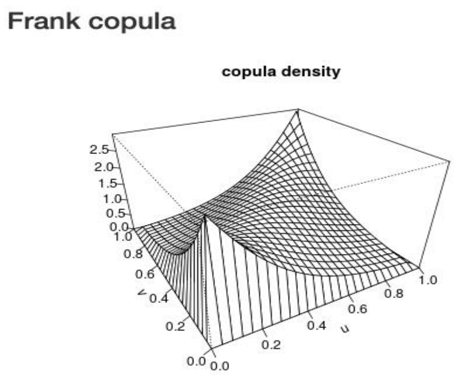


Figure 12 Frank Density Plot (Theta = 3)

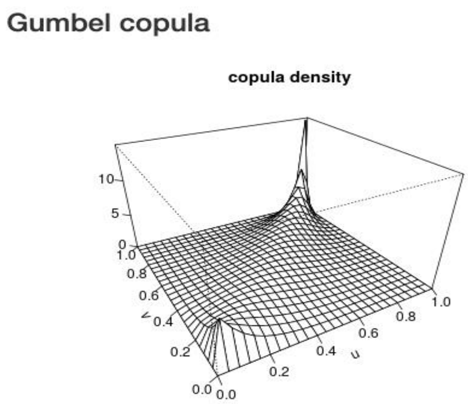


Figure 13 Gumbel Density Plot (Theta = 2)

3.3.5 Kendall's Tau

Kendall's rank correlation coefficient, or Kendall's tau (Kendall, 1938), is a statistic for measuring the relationship between ranked data. This statistic is useful in several ways, especially in copula estimation as one can show a direct relationship between Kendall's tau and the generator function for an Archimedean copula. The general method for estimating Kendall's tau is given as follows.

Let $(x_1, y_1), (x_2, y_2), .. (x_n, y_n)$ be the observations of two random variables X and Y. Such that any value of (x_i) and (y_i) are unique. A pair of observations $(x_i, y_i), (x_j, y_j)$ where $i < j$ is said to be concordant if the rank of both elements agrees. By agreeing ranks, we mean that for the pair of observations, both $x_i, y_i > x_j, y_j$ or conversely $x_i, y_i < x_j, y_j$. Pairs of observations are said to be discordant if $x_i > x_j$ and $y_i < y_j$, or $x_i < x_j$ and $y_i > y_j$. If there are non-unique observations i.e. if $x_i = x_j$,or $y_i = y_j$ the observations are said to be neither concordant nor discordant.

From these sets of observations, we then calculate the Kendall rank coefficient as:

$$\tau = \frac{(number\ of\ concordant\ pairs) - (number\ of\ discordant\ pairs)}{\binom{n}{2}} \qquad (25)$$

3.3.6 Dependence Parameter Relations

Genest and Rivest (1993) shows that one can derive the dependence parameter for any generator function and corresponding copula from an association measure, commonly Kendall's tau and Spearman's rho (another separate dependency measure). As mentioned, there is an explicit formula linking Kendall's tau and the generator function of an Archimedean copula, which can be expressed as follows:

$$\tau_C = 1 + 4\int_0^1 \frac{\psi(t)}{\psi'(t)} dt \qquad (26)$$

For our three chosen copulas, this integral evaluates to the equations in table 4:

Table 4 Kendall's Tau Expressed in Copula Parameters

Name	$\tau_C =$
Clayton	$\frac{\theta}{\theta + 2}$
Gumbel	$\frac{\theta - 1}{\theta}$
Frank	$1 - \frac{4}{\theta}[1 - D_1(\theta)]$

D denotes the Debye function: $D_k(x) = \frac{k}{x^k}\int_0^x \frac{t^k}{e^t-1}dt$

3.3.7 Measures of Tail Dependencies

Measures of upper and lower tail dependency are defined respectively as:

$$\lambda_U = \lim_{u\to 1} \Pr\left[X \geq F_X^{-1}(u) \mid Y \geq F_Y^{-1}(u)\right] = \lim_{u\to 1} \frac{1-2u+C(u,u)}{1-u} \tag{27}$$

$$\lambda_L = \lim_{u\to 0} \Pr\left[X \geq F_X^{-1}(u) \mid Y \geq F_Y^{-1}(u)\right] = \lim_{u\to 0} \frac{C(u,u)}{u} \tag{28}$$

Where F_X^{-1} and F_Y^{-1} are the marginal quantile functions and $\lambda_U, \lambda_L \in [0,1]$. The explicit formulas for lower and upper tail dependence for the Archimedean trio is summarized in table 5.

Table 5 Tail-End Measures

Name	$\lambda_L =$	$\lambda_U =$
Clayton	$\begin{cases} 2^{-\frac{1}{\theta}}, & \theta > 0 \\ 0, & \theta < 0 \end{cases}$	0
Gumbel	0	$2 - 2^{\frac{1}{\theta}}$
Frank	0	0

As once can see from table 5, only the Clayton and Gumbel copulas have significant tail dependency measures, where Clayton describes lower tail dependencies, and Gumbel correspondingly describe upper tails.

3.3.8 Relating Copulas to Spillover

As this thesis progresses, one may see it necessary to specify the relation between the tail-end correlations one might identify with a copula, and spillover effects. In this thesis we are interested in the return-spillover effect between our chosen markets. Say we find a significant change in one market: If ripples of this change can be seen in the return data for another market, we then have a return spillover. This ripple effect can then be identified by analyzing correlations amongst the markets returns. Ordinarily, correlations in data is measured with

Pearson's correlation coefficient which measures the linear dependence in datasets. Linear dependence effectively means that the correlation effect between data is constant and independent of relative size of the deviation. When considering a spillover, a relatively large movement in one asset is more likely to affect another, e.g. a non-linear correlation structure. This notion is then not necessarily captured by Pearson correlation. This is where copulas come into the equation: Copulas allows us to model non-linear dependence structures. Hence, the tail-end correlations identified with a copula methodology can be interpreted as a spillover effect.

3.4 Maximum Likelihood Estimation

Maximum likelihood is a method of fitting a probability model to a given set of data. The model parameters are obtained by finding the parameter values for which the data and values gives the highest probability, conversely: the maximum likelihood. The formulation of maximum likelihood estimation can be surmised accordingly: Given n observations of independent random variable X: $[X_0, X_1, \dots X_n]$, and continuous probability density $f(x;\theta)$ (note that θ can be a set of parameters). The maximum likelihood function is then the product of densities for each datapoint:

$$L(X_0, X_1, \dots X_n; \theta) = \prod_{i=0}^{n} f(X_i; \theta) \tag{29}$$

One can interpret this as the density of every observation X_i in probability density f with parameter space $\theta \in \Theta^k$, (where k is the number of parameters of the density function). One then wishes to obtain the parameter values which maximize this density. Taking the logarithm of the likelihood function, one obtains the aptly named log-likelihood function:

$$l(X_0, X_1, \dots X_n; \theta) = \ln\left(\prod_{i=0}^{n} f(X_i; \theta)\right) = \sum_{i=0}^{n} \ln\left(f(X_i; \theta)\right) \tag{30}$$

Taking the derivative and solving for zero yields local maxima and minima:

$$\frac{\partial l(X_0, X_1, \dots X_n; \theta)}{\partial \theta} = 0 \rightarrow \hat{\theta}^{ML} \tag{31}$$

One then needs to check if the estimated parameter is a maximum. This is done by looking at the double derivative of the likelihood function. If the double derivative of the likelihood function is negative for $\hat{\theta}^{ML}$, it means one has obtained a maximum. $\hat{\theta}^{ML}$ is then the maximum likelihood estimator of the model parameter value.

3.5 Empirical Distribution Functions

Empirical distribution functions are often encountered in situations where the underlying distribution of a sample is unknown or yet to be determined. In basic terms, the EDF is a function one constructs from a sample by counting the amount of data points in the sample which falls below the input value x. In general:

$$F_{n,x}(x) = \frac{1}{n}\sum_{i=1}^{n} If[X_i < x, 1,0] \tag{32}$$

Where the function $If[condition, 1,0]$ yields 1 if the condition holds, and 0 otherwise, and we divide by n (sample size) to ensure the function output to be bounded in $[0,1]$.

3.6 Goodness of Fit

Goodness of fit denotes the process of summarizing and inferring several statistical tests describing whether the fitted model sufficiently describes the data. This section lists a number of measures, statistics and techniques which are involved in the process of finding goodness of fit for an estimated model.

3.6.1 Distribution Moments

Moments about a distribution yields different characteristics. The most common examples of these moments are the mean and standard deviation $[\mu, \sigma]$, which are respectively known as the first- and second-degree moments of a distribution. To fully characterize a distribution, measures of higher degree moments can be estimated and standardized. These measures can then be used as inferences in deciding what underlying distribution the data belongs to.

3.6.1.1 Pearson measure of skewness and kurtosis.

The sample skewness characterizes the tendency of the data lumping asymmetrically around the mean. That is, the tendency for longer tails on one side of the mean compared to the other. This characteristic is obtained from considering the third moment:

$$S = E\left[\left(\frac{X-\mu}{\sigma}\right)^3\right] = \frac{\mu_3}{\sigma^3} \tag{33}$$

Kurtosis is a measure of the tendency for data to be concentrated around the mean, and is obtained from considering the fourth moment:

$$K = E\left[\left(\frac{X-\mu}{\sigma}\right)^4\right] = \frac{\mu_4}{\sigma^4} \tag{34}$$

Where $[\mu_3, \mu_4]$ is the third and fourth central moments and σ is the standard deviation.

The standard normal distribution $N(0,1)$ has kurtosis of 3, and it is common to compare a distributions kurtosis to this value. Distributions with a greater amount of kurtosis are known as leptokurtic distributions, and these distributions are usually thinner, taller and have fatter tails. Examples include the Laplace distribution and student's t-distribution. This parameter can also be calculated as excess kurtosis, meaning the statistical deviation from the standard normal of 3 (Excess Kurtosis = Kurtosis - 3).

3.6.2 Jarque-Bera Test

The Jarque-Bera test (Jarque and Bera, 1980) is a test to see if the data set is matching a normal distribution. The statistic is formulated thusly:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{35}$$

Where $[S, K, n]$ denote sample -skewness, -kurtosis, and -size, respectively.

This test statistic is asymptotically chi-squared distributed: $JB(n) \sim \chi^2$

This statistic can then be used to test the hypothesis of the data belonging to a normal distribution. The null hypothesis is then the skewness and kurtosis being zero. Large confidence against the null hypothesis indicates that the samples are not normal-distributed.

3.6.3 Anderson-Darling Test

The Anderson-Darling test (Anderson and Darling, 1954) is used to test whether a given sample belongs to a particular distribution. The test is performed by optimizing a distance measure between a distribution function and the empirical distribution of the samples. The test statistic from this samples arise from the same effect of the probability integral transform (equation 17), and the cumulative distribution function of the data can be assumed to be uniformly distributed. The data can then be tested for a uniform distance measure between the empirical distribution and the hypothesized underlying distribution.

The test statistic procedure can be summarized as follow:

Let $[Y_1, \ldots, Y_n]$ be the *ordered* set of data.
A formula for assessing if A comes from a CDF of F is:

$$A^2 = -n - S \tag{36}$$

Where:

$$S = \sum_{i=1}^{n} \frac{2i-1}{n} \left[ln\left(F(Y_i)\right) + ln\left(1 - F(Y_{n+1-i})\right) \right] \tag{37}$$

Note that this procedure assumes the parameters of the hypothesized distribution to be known. If the parameters are unknown, one can use the sample std and mean, however, one must also modify the test statistic. This modification is also dependent on the distribution being estimated. A statistic can then be compared to calculated critical values. If the statistic exceeds a certain critical value, the null of the samples belonging to the hypothesized distribution can be rejected within some set confidence level.

3.6.4 Kolmogorov-Smirnov Test

The Kolmogorov-Smirnov test (Kolmogorov-Smirnov et al., 1933) is a test utilized to compare a sample to a reference distribution. This is often used to find the underlying (or closest) distribution for the sample. Being similar to the Anderson Darling test, basically, this test compares the distance between the cumulative distribution function of the sample to a given reference distribution. The Kolmogorov-Smirnov test statistic D_n is defined as follows:

$$D_n = sup_x[F_n(x) - F(x)] \tag{38}$$

Where sup_x is the supremum of the set distances, meaning that it returns the smallest element of a subset S of the set $[F_n(x) - F(x)]$, where all elements of S are greater than or equal to x.

3.6.5 Akaike Information Criterion

The Akaike information criterion (AIC) (Akaike, 1998) is a useful metric in choosing models of best fit. This test originates from the field of information theory, and the statistic commonly measures the relative amount of information lost from a given model; the less information a model loses, the better the quality of the model. Since this statistic is based on measures of information, the statistic contemplates the trade-off between goodness of fit and the simplicity of the model. The formula for the AIC statistic can be described accordingly:

$$AIC = 2k - 2\ ln\left(\hat{L}\right) \tag{39}$$

Where $\hat{L}$ is the maximum value of the log likelihood function of the model, and k is the number of estimated parameters. For a set of estimated models, the one with the lowest AIC score is the preferred model.

3.7 Stationarity

A stochastic process is said to be stationary if the underlying probability distribution does not change over time. Time series data often exhibit non-stationary properties, such as seasonality and autocorrelation. When estimating both a copula and marginal distributions, it is important that there are no such effects present in the data, as this might lead to false dependencies. It is then important to account for periodicity and autocorrelation in the data before constructing copulas. If these characteristics are present in the data, false dependences may emerge, and we obtain an incorrect model.

3.7.1 (Augmented) Dickey-Fuller Test

The Dickey-Fuller test (Dickey and Fuller, 1979) is a common test statistic to determine if given time-series data is stationary. The null hypothesis of the test is that there is a unit-root present, and the alternate is often stationarity or trend-stationarity. The test can be summarized as follows:

Consider the regression model:

$$\Delta y_t = (\rho - 1)y_{t-1} + u_t = \delta y_{t-1} + u_t \tag{40}$$

Where Δ is the first difference operator, and ρ and δ are regression coefficients. The test then is determining if there is a unit root, meaning testing for $\delta = 0$. More concretely we have: $H_0: (\rho - 1) = 0$, and $H_1: \rho < 0$.

The *augmented Dickey-Fuller test* is the same process, but based on a time series regression model of the following form:

$$\Delta y_t = \alpha + \beta t + \gamma y_{t-1} + \delta_1 \Delta y_{t-1} + \cdots + \delta_{p-1} \Delta y_{t-p+1} + \varepsilon_t \tag{41}$$

Where α is a constant β is the coefficient of time trend, and p the number of lags. The augmented models include p lags, which allows one to model and test higher order autoregressive processes. If these tests conclude that one cannot reject H_0, it suggests that effects from autocorrelation are present in the data.

3.8 Approach

The general hypothesis of this thesis is that there is some return spillover between the oil markets and the Norwegian Krone. With that in mind, interest is put on finding dependences between the oil-price, exchanges rates between NOK and USD, and the NIBOR terms. An analysis of the dependence relation between these markets can be performed by estimating and fitting bivariate copulas to selected periods. The fitted copulas give a description of the dependences present in the data, and how the assets co-vary. This sub-chapter describes the methods used for estimating and fitting copulas. All calculations of this thesis were performed in Python, and the details of this process is described in appendix A.

The general method is summarized accordingly:

1. Find and organize data
2. Test for stationarity utilizing the augmented Dickey-Fuller test.
3. Find best fit marginal distributions.
4. Fit copulas
5. Report results and make inferences.

3.8.1 Copula Pairings

The bivariate Archimedean copula-based approach demands pairings of these markets of which copulas are to be constructed. Table 6 summarizes the chosen pairings and touches on the format of how the results will be presented.

Table 6 Pairings of Datasets for a Given Time Period

Brent Oil Price	USD/NOK NIBOR 1 Week NIBOR 1 Month NIBOR 2 Months NIBOR 3 Months NIBOR 6 Months
USD/NOK	NIBOR 1 Week NIBOR 1 Month NIBOR 2 Months NIBOR 3 Months NIBOR 6 Months

3.8.2 Methods for Estimating and Fitting Archimedean Copulas

There have been found several ways to estimate and fit copula functions to a given dataset (Genest and Rivest, 1993, Bacigál and Komornıková, 2006). For the purposes of this thesis, three general methods have been identified, each with their respective advantages and disadvantages. Utilizing more than one method allows one a broader view of how a copula can be constructed. Coincidence and variance between the results of the method might also say something about the confidence of the estimations. If the results have large variations, one might conclude that any of the results are not to be trusted, and further considerations must be taken into accordance. To ensure robustness, all three methods were utilized, however, only the results of the following method is presented (the results from the remaining methods, parametric- and semi-parametric copula estimation, are not included in this thesis due to the extent of the data, but available upon request).

3.8.2.2 Non-parametric estimation

Genest and Rivest (1993) identifies a method for fitting a bivariate Archimedean copula to observed data. The first step is to define a new unobserved random variable:

$$Z_t = F(X_i, Y_i) \tag{42}$$

Which has a distribution function:

$$K(z) = \Pr[Z_i \leq z] \tag{43}$$

Genest and Rivest (1993) further show that an Archimedean copula can be uniquely defined from the relation

$$K(z) = z - \frac{\psi(z)}{\psi'(z)} \tag{44}$$

This reveals a general method for estimating the generator function and dependence parameter for any Archimedean copula:

1. Estimate Kendall's correlation coefficient using formula (25).

2. Construct a non-parametric estimation of K(z):

   a) Define pseudo-observations:

   $$Z_i = \frac{1}{n-1}\sum_{j=1}^{n} If\left[X_j < x_i \;\&\&\; Y_j < y_i, 1{,}0\right], \text{ for } i = 1, \dots, n \tag{45}$$

   Where && stands for the "and" logical operator.

   b) Construct an estimate of K, by constructing an empirical distribution as defined in subchapter 3.5:

   $$K_n(z) = \frac{1}{n}\sum_{i=1}^{n} If[Z_i < z, 1{,}0] \tag{46}$$

3. Now use relation (26), and from the expressions in table 3, construct a parametric estimate of $K_\psi(z)$.

The resulting chain of estimation can be illustrated as follows:

$$\tau_n \rightarrow \theta_n \rightarrow \psi_n(t) \rightarrow K_\psi(z)$$

Step 3 can be repeated for any of our choices of copulas. One can then determine the copula of best fit by considering the copula which minimizes the distance:

$$\left[K_n(z) - K_\psi(z)\right] \tag{47}$$

The advantage of this method stems from the fact that it is non-parametric, meaning that it is entirely independent of underlying parameters. However, one must consider the accuracy of the estimates for Kendall’s tau. Incidentally, if there is a case of much overlapping data points, this estimate might be unsuitable. The other two identified methods which involve estimating the copula by utilizing the maximum likelihood method (sub-chapter 3.4), are summarized in appendix B

4. Data

The following chapter presents the market data for all the relevant factors considered in this thesis, and the results from the marginal analysis. Here we present the data for Brent oil, the exchange rate between the U.S. Dollar and the Norwegian Krone and the Norwegian Interbank Offered rates (NIBOR) used to analyze the cross-industry dependence between them.

The data consists of 4925 spot observations for Brent oil, USD/NOK exchange rate and NIBOR rates. NIBOR is divided into terms of 1 week, 1 month, 2 months, 3 months and 6 months, which are all the terms generated by NoRe today. All observations are daily closing prices/rates, from 07.02.2000 to 31.12.2019. The data for Brent oil and USD/NOK exchange rate is gathered from Thomson Reuters Eikon, and the NIBOR data is delivered upon request from the former party responsible for the rates, Oslo Stock Exchange. The choice of spot versus futures pricing is justified by the high correlation between them, basing the prices on immediate supply and demand for both currency and Brent oil. As the different markets sometimes have different trading days, the data is filtered to rely on only common dates relevant for the analysis.

4.1 Full Data Sample

Table 7 presents the statistical properties of the closing spot prices and rates for the variables Brent oil, USD/NOK exchange rate and NIBOR rates in the full sample. As we can observe in this table, there are extreme cases of volatility in the Brent oil price, reaching the minimum of 17.00 USD/bbl in 2001 and the maximum of 143.60 USD/bbl in 2008. Studying the USD/NOK exchange rate we can see that the minimum rate occurred in the same period as the highest Brent oil price, and the maximum rate occurred around the same time as the lowest Brent oil price. Brent being the crude benchmark for the North Sea operations, and Norway being a relatively big producer of Brent oil, it is reasonable that the Norwegian Krone will perform better relative to the U.S. Dollar in times of high Brent oil prices. Low Brent oil prices may then lower the national investment prospects, resulting in a weaker NOK relative to the USD. The NIBOR, terms 1 month to 6 months, reached their peak around the same period as the Brent oil high and the USD/NOK low.

Table 7 Whole Sample Properties

	Unit	Min	Min date	Mean	Max	Max date	Std.dev
Brent oil	USD/bbl	17.00	13.08.2001	65.07	143.60	28.03.2008	30.21
USD/NOK	NOK	4.95	22.04.2008	7.07	9.58	25.10.2000	1.23
NIBOR 1 week	%	0.50	07.11.2017	2.97	9.10	27.12.2000	2.15
NIBOR 1 month	%	0.55	07.11.2017	3.01	9.13	01.10.2008	2.13
NIBOR 2 months	%	0.66	20.11.2017	3.06	8.46	01.10.2008	2.12
NIBOR 3 months	%	0.71	20.11.2017	3.13	7.91	01.10.2008	2.11
NIBOR 6 months	%	0.82	20.11.2017	3.23	7.95	01.10.2008	2.07

The relatively large base of 4925 data observations, over a 20-year period, present some statistical and observational transparency complications. When using these large samples there can be difficulties determining the distributional properties, given the span of time and amount of data. Important structural changes in the period complicates the data generating process and affects the marginal distributions. To better highlight the changes in correlation, we divided the sample into series, non-overlapping sub-samples, representing different time spans in the 20-year coverage of all data. Separating the total data into five series, giving each series approximately 4 years, can help understand how extreme events (financial crisis, oil crisis etc.) may cause shifts in market correlations. This allows testing in all five series, which will highlight how spillover differs compared to the full sample. Overall problems of data autocorrelation are also limited by the series data separation, making the log-returns data of Brent oil, NIBOR terms and USD/NOK exchange rate approach independently distributed (i.i.d.) and be more suited for testing.

Table 8 Characteristics of Time Periods

Series	**Series 1**	**Series 2**	**Series 3**	**Series 4**	**Series 5**
Date start	15.01.2016	27.01.2012	08.02.2008	12.02.2004	07.02.2000
Date end	31.12.2019	14.01.2016	26.01.2012	07.02.2008	11.02.2004
Observations	985	985	985	985	985
Events	“All-time-low” interest rates	Oil crisis	Financial Crisis	Subprime crisis	Dotcom burst
			U. S. Fracking	Oil price bubble	9/11
					War in Iraq

All factors, Brent oil price, USD/NOK exchange rate and the NIBOR terms have posted prices for each of the 985 dates in the series above. The data used in the test are log-returns, changes from the closing price to opening price for each variable. The data can be viewed as a percentage change for each day in the series, and are calculated as:

$$R = \left(\frac{P_t}{P_{t-1}} - 1\right) \text{ or equivalently: } ln\ (\frac{P_t}{P_{t-1}})$$

Table 9 presents descriptive statistics of the log-returns, for all variables. We can see that the Brent oil could on occasion vary somewhere between +-20%, while NIBOR 1 week, the most volatile term, could vary between -45% and +128% from day to day. The NIBOR terms of 2 months to 6 months was relatively similar in changes, having the same volatility range. The extreme negative returns in these terms were not occurring in the same period, unlike the extreme positive returns which are all concentrated within a two-year interval.

Table 9 Whole Sample, Returns Properties

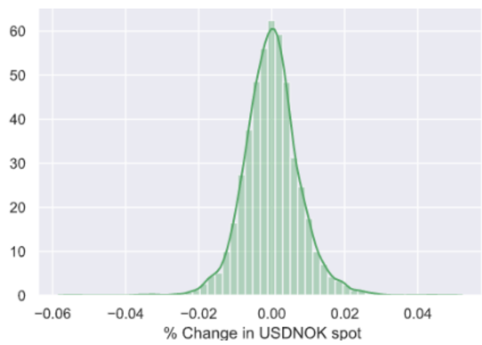
	Unit	Min	Min date	Mean	Max	Max date	Std.dev
Brent oil	%	-17.08	20.06.2001	0.046	19.68	26.09.2008	2.33
USD/NOK	%	-5.46	02.09.2015	0.004	4.83	21.10.2008	0.76
NIBOR 1 week	%	-45.08	28.12.2017	0.055	128.09	22.12.2016	4.41
NIBOR 1 month	%	-24.72	28.12.2017	-0.003	50.00	29.11.2016	2.13
NIBOR 2 months	%	-15.97	24.09.2015	-0.013	19.20	19.03.2015	1.52
NIBOR 3 months	%	-13.22	18.12.2008	-0.015	21.49	19.03.2015	1.33
NIBOR 6 months	%	-14.23	20.11.2017	-0.014	20.00	19.03.2015	1.30

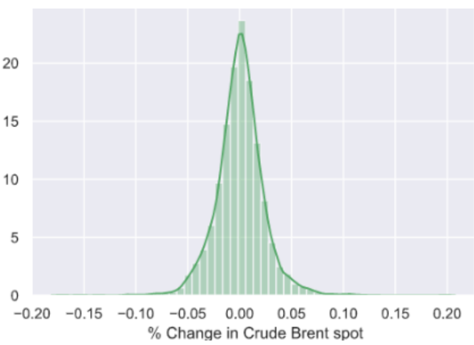
We can observe from table 7 and 9 that some peaks and lows happened in 2008. Early 2008, the U.S economy began to witness the early stages of development that later would escalate to a nation and worldwide financial crisis. Initially, the sub-prime crisis caused investors to sell off their USD currency, significantly reducing the demand. In the same period, the Brent oil price were at an all-time-high. The low demand for USD currency and high crude prices lead to a USD/NOK exchange rate low in 2008. The rate would then again rise as the recession would lead to a collapse in the oil and gas sector due to shrinkage in demand, reducing the price for Brent oil down to 34 USD/bbl within 6 months of the all-time-high. As to the start of the 4th quarter of 2008, when the investment bank Lehman brothers filed for bankruptcy, the need for capital liquidity among banks grew high. This, and the fact that banks were restrictive to approve further interbank loans, the premiums in the money market skyrocketed (Bernhardsen et al., 2012). Inevitably, this pushed the NIBOR terms to all-time-highs. The latest period, 2015-2018, there were major activity in the NIBOR rates, creating extreme volatility. The Norwegian central bank key interest rates where record low and close to 0%. The NIBOR is strongly connected to the key rates, and the Norwegian key rate has historically been changed by a minimum factor of 0.25%. The lower the value of the current rate, the larger percent change this value constitutes, thus one standard change in key rate has a larger impact on the volatility of the NIBOR when the rates are low.

Table 10 further explains the statistical properties of the full sample log-return marginals. The skewness of Brent, USD/NOK and NIBOR 2 to 6 months shows small signs of tail symmetries and normality, but the excess kurtosis shows no signs of normality (all higher than 0). The high Jarque-Bera score also indicate marginals that deviate from normality. The Augmented Dickey-Fuller test (3.7.1) confirms stationary data for all variables. The K-S test included Laplace, Weibull, exponential, normal, logistic, beta and Cauchy distributions, with best fit shown in table 10.

Table 10 Marginal Tests Summary

	Skewness	Kurtosis (Excess)	JB	K-S Test value	K-S P-value	Dist.	AD-F Test value	AD-F P-value
Brent Oil	0.133	4.361	3908	3.33e-2	1.15e-2	Laplace	-29.34	0
USD/NOK	0.183	2.518	1324	6.51e-2	1.39e-5	Weibull	-51.45	0
NIBOR 1W	10.714	272.32	15280636	3.34e-1	0	Laplace	-23.02	0
NIBOR 1M	2.877	83.627	1438947	9.93e-2	0	Laplace	-24.84	0
NIBOR 2M	-0.0132	21.971	98846	1.21e-1	0	Laplace	-12.87	0
NIBOR 3M	0.318	24.585	123848	1.12e-1	0	Laplace	-10.73	0
NIBOR 6M	-0.082	28.810	169971	1.16e-1	0	Laplace	-9.26	0





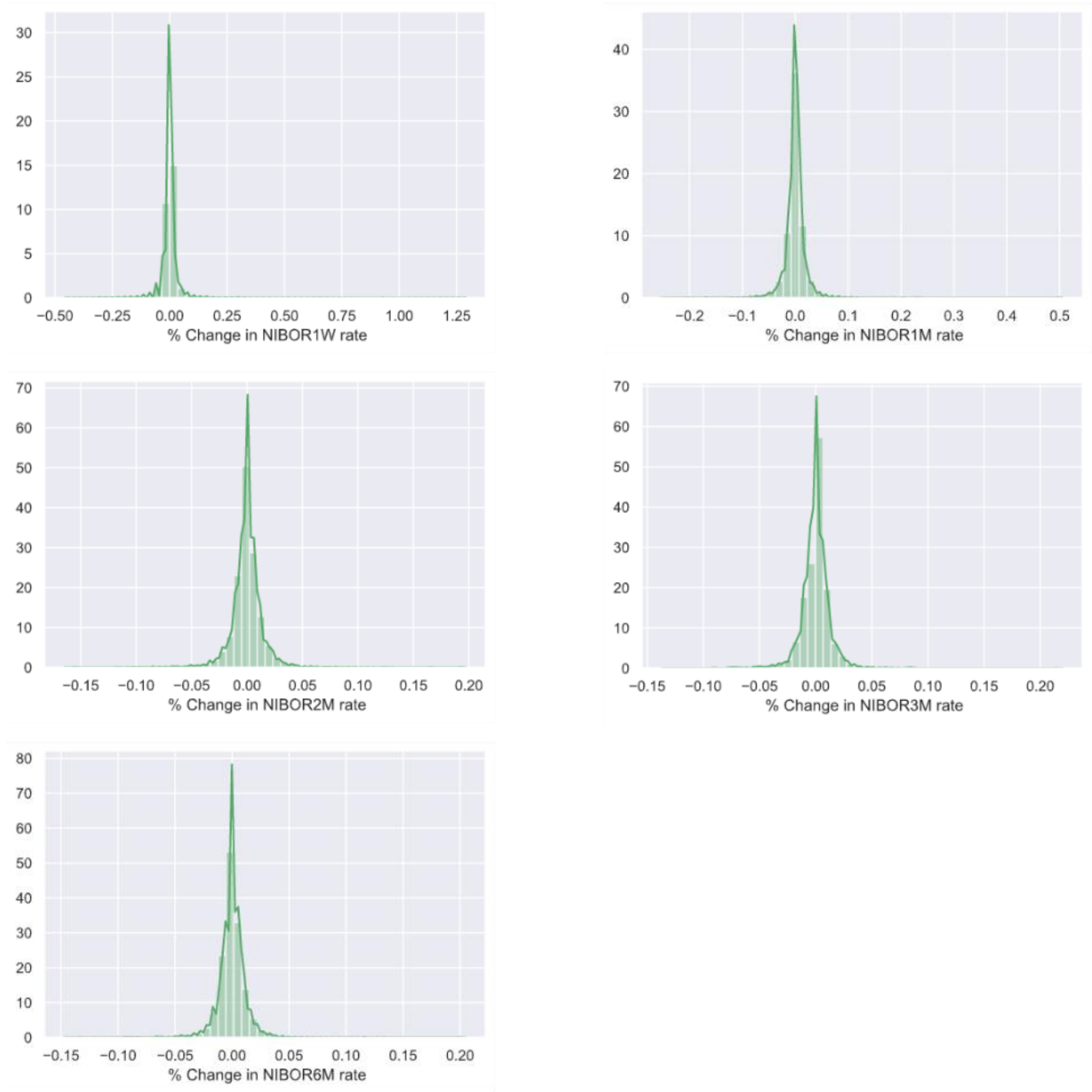


Figure 14 Marginal Plots

4.2 Series Separation Samples

To further ensure the suitability of the data when applied to later testing, we choose to separate the sample into five series, as mentioned earlier. When separated, table 11 shows how the statistical properties for the variables change dependent on the period of representation, still using daily log-returns. The separation shows that the new sub-samples often deviate from normality more than the full sample distribution. We can also see how the different periods have very different statistical behavior, and therefore may insinuate a changing dependence through the period of testing. The test results of the series can provide justifiable grounds for claiming that the data series have a non-normal distribution and demanded stationarity.

Table 11 Time Separated Marginals Properties and Test Statistics

		Min	Mean	Max	Std.dev	Skewness	E. Kurtosis	ADF
Series 1	**Brent Oil**	-6.32	0.115	11.54	2.28	0.48	2.30	-32.28
	USD/NOK	-1.89	0.002	3.08	0.57	0.19	1.09	-32.16
	NIBOR 1 week	-45.08	0.340	128.09	8.58	6.80	88.56	-11.17
	NIBOR 1 month	-24.72	0.096	50.00	3.38	2.95	55.25	-24.84
	NIBOR 2 months	-15.48	0.064	16.67	2.12	0.32	10.97	-16.45
	NIBOR 3 months	-7.14	0.057	8.64	1.52	0.39	4.15	-24.47
	NIBOR 6 months	-4.31	0.056	6.21	1.16	0.31	2.18	-32.48
Series 2	**Brent oil**	-8.60	-0.121	8.99	1.73	0.10	3.86	-8.48
	USD/NOK	-2.15	0.044	3.92	0.69	0.31	1.52	-9.33
	NIBOR 1 week	-31.3	-0.020	62.89	3.19	7.46	171.62	-18.35
	NIBOR 1 month	-16.24	-0.058	23.20	1.86	2.71	52.78	-24.61
	NIBOR 2 months	-15.97	-0.067	19.20	1.48	0.96	47.06	-23.90
	NIBOR 3 months	-12.50	-0.077	21.49	1.41	2.93	66.44	-23.87
	NIBOR 6 months	-8.97	-0.083	20.00	1.40	2.77	49.40	-15.32
Series 3	**Brent oil**	-15.39	0.054	19.68	2.57	0.40	6.51	-14.06
	USD/NOK	-5.46	0.011	4.83	1.07	0.11	1.80	-23.89
	NIBOR 1 week	-19.05	-0.056	19.17	3.06	-0.12	10.07	-27.14
	NIBOR 1 month	-17.10	-0.061	20.48	2.47	0.50	15.34	-10.16
	NIBOR 2 months	-11.66	-0.067	11.00	1.89	-0.78	7.53	-4.74
	NIBOR 3 months	-13.22	-0.061	8.81	1.78	-0.71	7.18	-5.21
	NIBOR 6 months	-14.22	-0.051	13.11	1.83	-1.15	15.02	-7.16
Series 4	**Brent oil**	-8.86	0.133	12.15	2.16	0.14	1.39	-10.26
	USD/NOK	-2.41	-0.020	2.64	0.67	0.17	0.73	-30.83
	NIBOR 1 week	-11.51	0.100	7.47	1.24	-0.71	11.73	-13.77
	NIBOR 1 month	-3.38	0.102	5.57	0.92	0.59	5.34	-30.49
	NIBOR 2 months	-4.95	0.109	7.47	0.77	0.83	13.08	-13.04
	NIBOR 3 months	-5.06	0.114	8.19	0.81	0.87	12.61	-13.66
	NIBOR 6 months	-4.68	0.120	11.59	0.90	2.07	29.98	-6.69
Series 5	**Brent oil**	-17.08	0.047	12.96	2.75	-0.34	3.37	-13.20
	USD/NOK	-2.32	-0.015	2.23	0.70	0.17	0.30	-14.27
	NIBOR 1 week	-16.06	-0.008	11.85	1.50	-1.85	35.59	-19.79
	NIBOR 1 month	-8.21	-0.095	5.43	0.93	-1.51	16.65	-6.17
	NIBOR 2 months	-8.43	-0.103	4.33	0.86	-2.54	22.02	-5.44
	NIBOR 3 months	-9.15	-0.110	4.66	0.86	-3.14	27.27	-6.11
	NIBOR 6 months	-9.29	-0.111	4.55	1.00	-3.02	22.61	-4.63

Viewing the data sets and table 12 we can observe a negative correlation between the Brent oil price and the USD/NOK exchange rate. This makes sense since the demand for the Norwegian Krone (NOK) is strongly correlated to the price of crude oil, being that the oil industry is an important source of domestic economic growth in Norway. The USD/NOK exchange rate also have negative correlations with all the terms of the NIBOR. This negative correlation can possibly be explained with the *rand-currency hypothesis*. In times of global financial turmoil, investors tend to deviate from the NOK, branding it as an uncertain currency, increasing the USD/NOK exchange rate. Again, as a result of crisis, the U.S. key rates are often reduced to stimulate the economy, which lowers the swap rate of NIBOR, perhaps creating this negative correlation. Brent oil prices and NIBOR experience shifting correlations throughout the terms. All correlations between Brent oil and NIBOR returns are relatively low and bears the resemblance of an independent relationship given the correlation matrix.

Table 12 Correlation Matrix

Correlation Matrix	Brent Oil	USD/NOK	NIBOR 1 Week	NIBOR 1 Month	NIBOR 2 Months	NIBOR 3 Months	NIBOR 6 Months
Brent Oil	1	-0.250	-0.003	-0.003	0.013	0.041	0.033
USD/NOK	-0.250	1	-0.062	-0.086	-0.123	-0.154	-0.155
NIBOR 1 Week	-0.003	-0.062	1	0.750	0.323	0.252	0.185
NIBOR 1 Month	-0.003	-0.086	0.750	1	0.688	0.601	0.469
NIBOR 2 Months	0.013	-0.123	0.323	0.688	1	0.795	0.629
NIBOR 3 Months	0.041	-0.154	0.252	0.601	0.795	1	0.750
NIBOR 6 Months	0.033	-0.155	0.185	0.469	0.629	0.750	1

4.2 Dealing with Negative Correlations

Considering the following application of data to the copula analyzes, we argue that the relatively strong negative correlation between USD/NOK and the remaining variables are problematic. The selected Archimedean copulas are sensitive to high levels of negative correlations and will fall victim to extensive generalization in the structured equations (Embrechts, 2011). To solve this, we have considered all observations of USD/NOK log-returns as reversed, multiplied with negative one (-1), thus creating a synthetic variable of the USD/NOK exchange rate (-1*(USD/NOK-log-returns)). In this case all relevant correlations, minimum, maximum, average etc. observations are flipped to opposite values, solving the negative correlation dilemma with variables. The fundamental distributional properties will not be affected to the point of re-testing, determining whether we now have a non-stationary or normally distributed sample as a result of the manipulation. These changes can be neglected. Data manipulations demands more strict interpretation of empirical results, since now all analysis outcomes will present an artificial relationship that must be handled accordingly (Appendix C).

5. Empirical Results

This chapter present the results of the copula analyzes to examine cases of return spillover between Brent oil, USD/NOK exchange rates and the NIBOR terms. We present the best fitting copula, and its resulting properties, that can most accurately explain the statistical relationship in the data for all the sub-samples (1-5) and the whole data set. Furthermore, we attempt to connect the results to real observations and events that has happened through the considered time span of 20-years and present the similarities to substantiate a conclusion.

Table 13 shows the results of the copula analysis for the whole data set. In each of the data sets we have 4925 observations, spread over 20 years. To view this whole data set, and to apply each observation, will present results without considering any changes in underlying dependency structure though the selected period. There might be times over the years where there are observational cases of extreme tail-end correlation that are watered out by times this relationship might be absent. Thus, we present the results of the whole data set, then to compare it with each individual divided time series. This will help understand how the overall correlation may vary through time, and which macro economical events that affect the relationships between the variables.

5.1 Full Sample Analysis

Table 13 Full Sample Copula Results

Data 1	Data 2	Best fit	θ	AIC	Loglik	τ	$\lambda_{L/U}$
Brent Oil Price	**USD/NOK**	Gumbel	1.170	-309.212	-155.610	0.158	0.192
	NIBOR 1 Week	Clayton	0.022	-0.240	-1.122	0.016	0
	NIBOR 1 Month	Clayton	0.035	-3.802	-2.933	0.016	0
	NIBOR 2 Months	Clayton	0.040	-5.396	-3.963	0.022	0
	NIBOR 3 Months	Clayton	0.055	-11.713	-6.841	0.031	0
	NIBOR 6 Months	Clayton	0.060	-13.889	-7.958	0.032	0
USD/NOK	**NIBOR 1 Week**	Clayton	0.115	-50.691	-26.345	0.067	0.002
	NIBOR 1 Month	Clayton	0.136	-70.063	-36.031	0.073	0.006
	NIBOR 2 Months	Clayton	0.158	-95.502	-48.751	0.077	0.012
	NIBOR 3 Months	Clayton	0.190	-133.591	-67.795	0.091	0.026
	NIBOR 6 Months	Clayton	0.196	-141.905	-71.052	0.088	0.029

Over the span of 20 year there are relatively strong tail-end correlations between Brent oil and the USD/NOK returns, resulting in a theta of 1.17 (note: 0 for Frank and Clayton, 1 for Gumbel yield independent copulas). With the AIC test score of -309, the Gumbel model best describes the return interaction. This is by far the best model fit given the whole data set. The Gumbel model gives us some evidence of an upper tail dependency, indicating sporadic events of positive co-movement, creating paths of spillover. We remember from the data chapter that the USD/NOK variable is reversed, making this interpretation of the dependency incorrect. Given this manipulation, we must consider that the USD/NOK observations is placed on the opposite value, yielding a negative correlation of the same scale. The results then present a relationship where when there are high positive returns in Brent oil, there is also high negative returns in USD/NOK exchange rates. We can observe from the data chapter that Brent oil and USD/NOK returns are significantly correlated, which can also be justified with the market theory. The full sample comprises several periods of heavy volatility in the market, probably resulting in the strong Gumbel dependence. The Gumbel primarily consider this one-way relationship between the two variables, and not when the returns are reversed. The calculated copula density can be observed in figure 15.

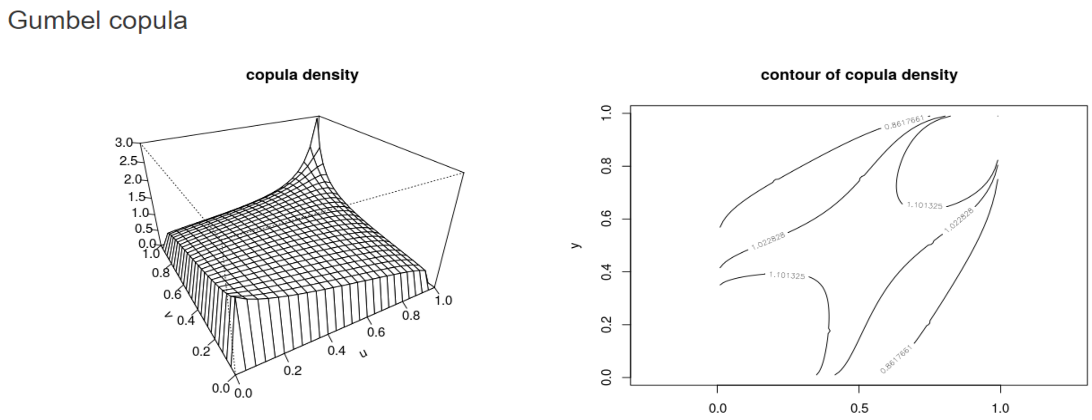


Figure 15 Full Sample, Brent Oil Price USD/NOK Gumbel Copula Density Plot

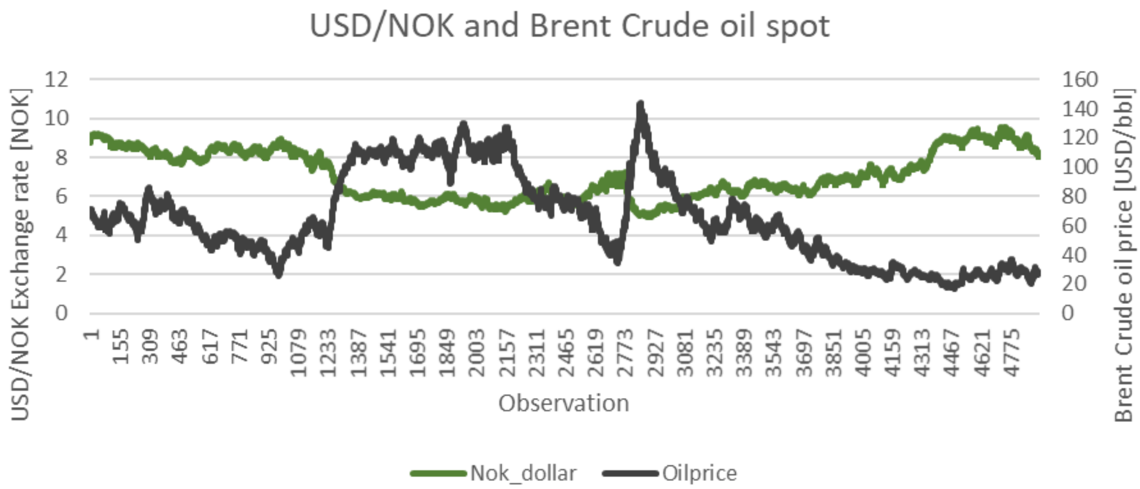


Figure 16 Exchange Rate and Crude Oil Daily Return Graph (2019-2000)

Considering the correlation between the USD/NOK exchange rate and the relevant NIBOR terms, we can see some “high negative” AIC and log-likelihood scores for Clayton copulas (AIC > 50), indicating Clayton model similarities. Starting from the NIBOR 1-week term, the tail dependence increases when considering longer terms, as seen in the associated theta. The best fitting copula, and therefore the best type that best describe the tail dependence, is the Clayton copula. Clayton copula connects lower tail dependence between USD/NOK and all NIBOR terms, meaning that when USD/NOK have “high” negative returns, often NIBOR have the same market reaction. Again, we must consider the USD/NOK variable as reversed. What we then have is a reversed tail-end dependency, which means that when USD/NOK experience high positive returns, all NIBOR terms experience high negative returns. Viewing the market theory, the relationship can be considered reasonable. NOK being a viewed as a “rand-currency”, investors sell off their NOK holdings in times of recession, rather buying “safe” currency as the U.S. Dollar. This will increase the USD/NOK exchange rates. While the economy falls victim to this recession, the central banks often reduces the key rates to stimulate industries and domestic spending, this may also reduce the NIBOR. Hence, we may experience a return spillover.

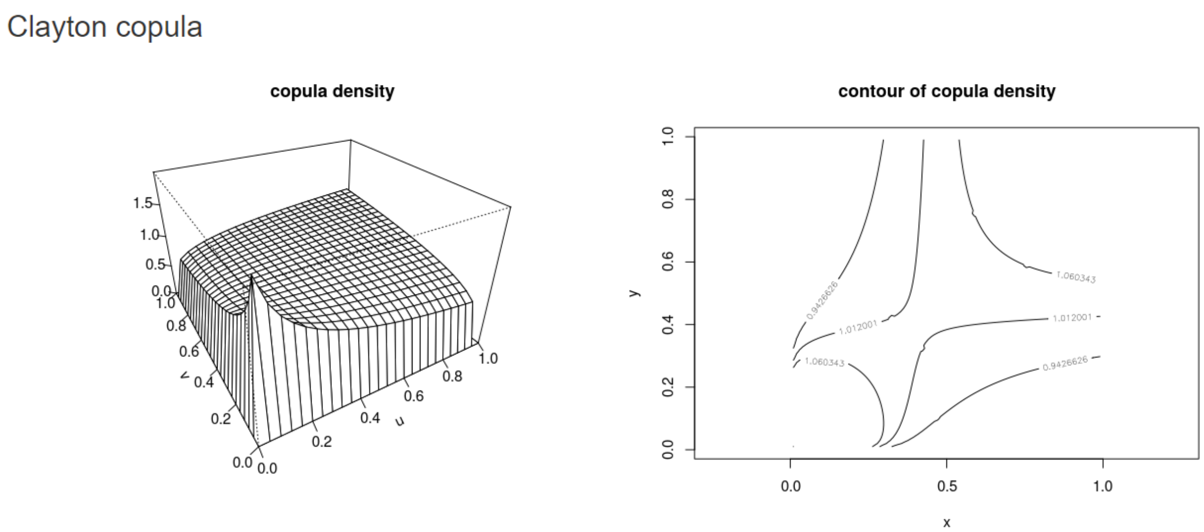


Figure 17 Full Sample, USD/NOK Exchange Rate NIBOR Clayton Density Plots

The tail dependency between Brent oil prices and the NIBOR terms are very low, as seen in the associated theta (< 0.06). Although the model is not necessarily bad fit, given the AIC (AIC = -13), the theta shows low correlation. None of the copula models can provide evidence of tail dependence. We can observe from the data chapter that there is high volatility in both variables, but the basic correlation between them was relatively low. This gives us grounds to speculate in the favor of non-spillover and tail independence over the full period, which means that there are no lasting or repeated dependence in the markets over time. The lack of return spillover is shown in figure 18, showing only insignificantly small occurrences of lower tail dependencies.

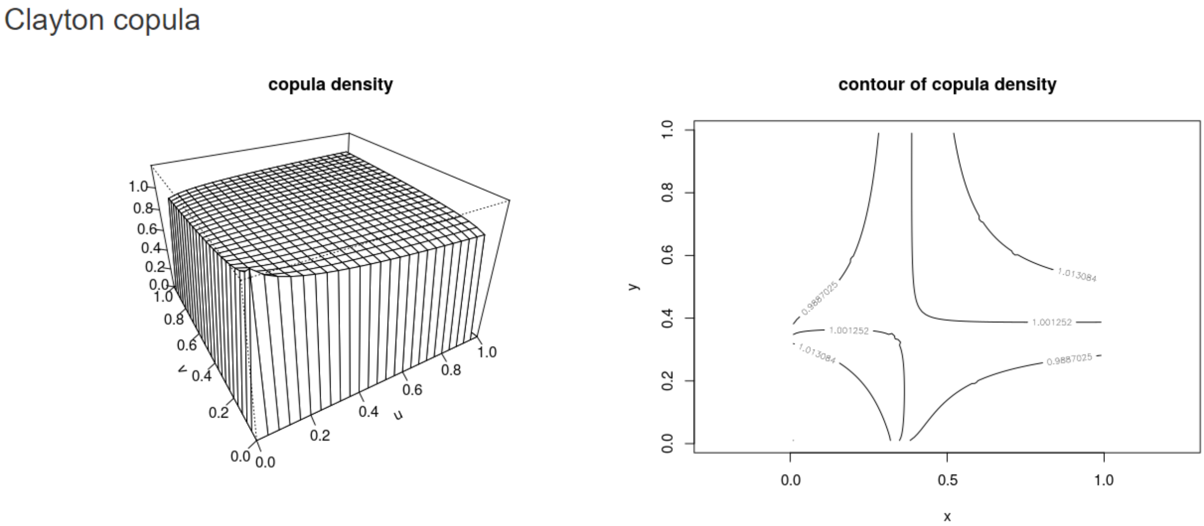


Figure 18 Full Sample, Brent Oil Price NIBOR Clayton Density Plots

5.2 Series 1 (Feb. 2016 – Dec. 2019)

Table 14 Series 1 (2016-2019) Copula Results

Data 1	Data 2	Best fit	θ	AIC	Loglik	τ	$\lambda_{L/U}$
Brent Oil Price	**USD/NOK**	Gumbel	1.177	-60.605	-30.302	0.166	0.198
	NIBOR 1 Week	Clayton	-0.036	0.741	-0.630	-0.013	0
	NIBOR 1 Month	Clayton	-0.017	1.733	-0.134	-0.015	0
	NIBOR 2 Months	Clayton	-0.044	0	-1.000	-0.039	0
	NIBOR 3 Months	Clayton	-0.006	1.964	-0.018	-0.020	0
	NIBOR 6 Months	Clayton	-0.006	1.962	-0.019	-0.030	0
USD/NOK	**NIBOR 1 Week**	Gumbel	1.028	-0.902	-1.451	0.020	0.037
	NIBOR 1 Month	Frank	0.190	1.040	-0.480	0.021	0
	NIBOR 2 Months	Clayton	0.025	1.437	-0.282	0.004	0
	NIBOR 3 Months	Gumbel	1.048	-4.612	-3.306	0.049	0.062
	NIBOR 6 Months	Gumbel	1.034	-3.702	-2.851	0.001	0.045

5.2.1 Periodical Market Spillover

From the AIC values in table 14 we can see that the only copula that produces significant results are Brent oil and USD/NOK exchange rate for this series (AIC = -60.6). The tests show that the Gumbel copula, as it was for the whole sample, is the best fit to explain the relationship between them. The high negative AIC and log-likelihood scores insinuates that there are some similarities among the variables. The Gumbel-relationship, with a moderate theta of 1.177, between Brent oil and USD/NOK indicate a significant upper tail dependency. This can also be observed in figure 19. The tail-dependence coefficient (λ) also indicate existing upper tail dependence, and the high Kendall's tau of 0.166 confirms correlating ranking of the variables. Although all these tools can be used to conclude an existing level of upper dependence between Brent oil and USD/NOK, we must consider that we have reversed USD/NOK variables. Practically, this means that all these indicators actually describe a reverse dependence, instead of the upper dependence from the Gumbel interpretation. Thus, in events where we see high positive returns in the Brent oil prices, we also see high negative returns in the USD/NOK exchange rate.

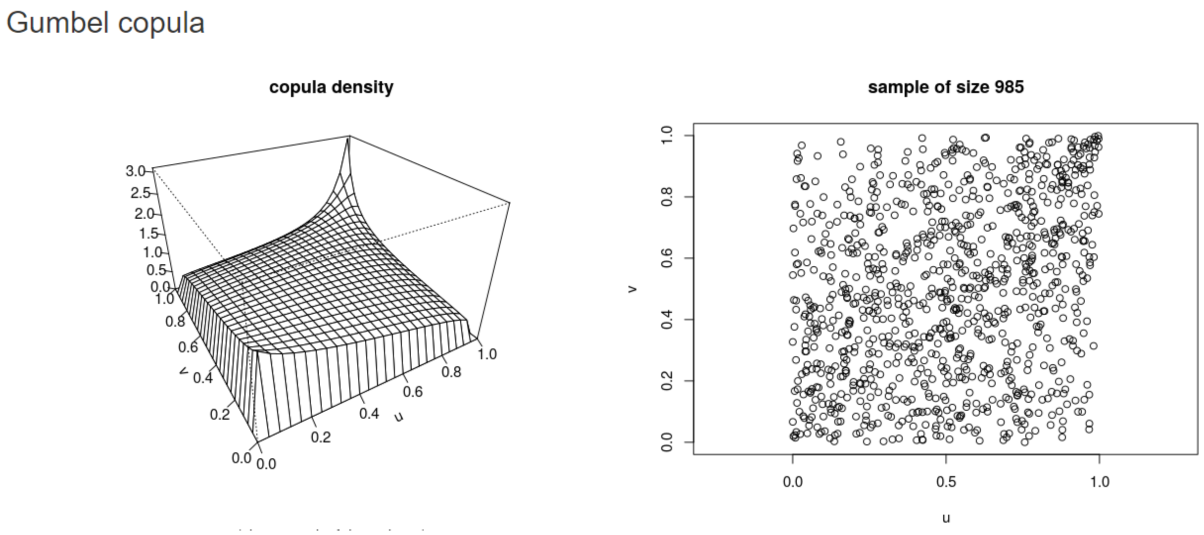


Figure 19 Series 1, Brent Oil USD/NOK Gumbel Density Plot

Connecting these results to the event in the given series, we could in this period see a recovering Brent oil price after the Oil crisis of 2015. Maybe as a reaction to the low oil prices, the exchange rate of USD/NOK was at the beginning relatively high, giving high negative returns corresponding to the following increase in the Brent oil prices. There were several days of high positive returns in the oil price in this period, giving a lift to the Norwegian economy. This increase probably resulted in investor accumulation for the Norwegian Krone and higher demand, over the period lowering the USD/NOK exchange rates. This may have created the reverse Gumbel type, and significant cases of return spillover. We primarily saw an increase in Brent oil prices from 2016-2019, on average, and this might justify why Gumbel is the best fit.

5.2.2 Periodical Market Independence

Apart from the tail-end correlation between Brent oil and USD/NOK in series 1, there are little to no connections to the rest of the variables by tail-end dependence. We are able to identify the copula that best can describe the statistical tail relationship, but good-fitting models are absent according to the AIC test, implying that there are no dependency models describing the relationships in the series. All properties, theta, log-likelihood, tau and lambda paint the picture of independent returns between the markets of Brent oil and NIBOR terms, and USD/NOK and NIBOR terms. As seen in table 9, there were cases of extreme changes in log-return for all, but seemingly independent from each other. In the U.S, the national key interest rates escalated trough the period, which would traditionally affect the NIBOR. But given the low Norwegian key rates, the premiums stayed low, possibly as a measure to ensure the corporate and public spending after the crisis. The Brent oil price primarily experienced high returns in the period, while the NIBOR was both adjusting to low domestic key rates and increasing key rates in the U.S., resulting in shifting trends. This is a possible explanation for why there are no evidence of return spillover between Brent oil prices and the NIBOR terms in this period, as illustrated by the copula density in figure 20.

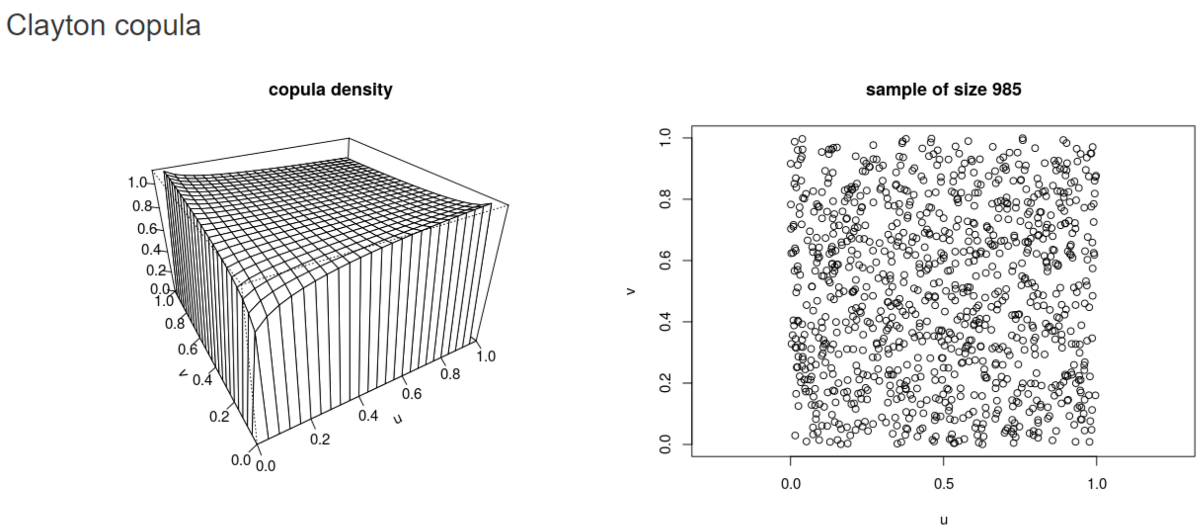


Figure 20 Series 1, Brent Oil NIBOR Clayton Density Plot

Different from the full sample analysis, there are in this period little evidence of tail dependence between USD/NOK and NIBOR terms. The best fitting copulas are mostly different from the full sample, but the goodness-of-fit parameters shows absent similarities to dependency models. In series 1, reaching from 2016-2019, the USD/NOK decreased on average, while the NIBOR terms shifted sporadically in the period, with no significant return spillover. Although some of the rates suffered some major changes in the period, the test shows that there is lack of tail dependency. Figure 20 and 21 illustrates the copula densities one obtains from these relatively independent sets.

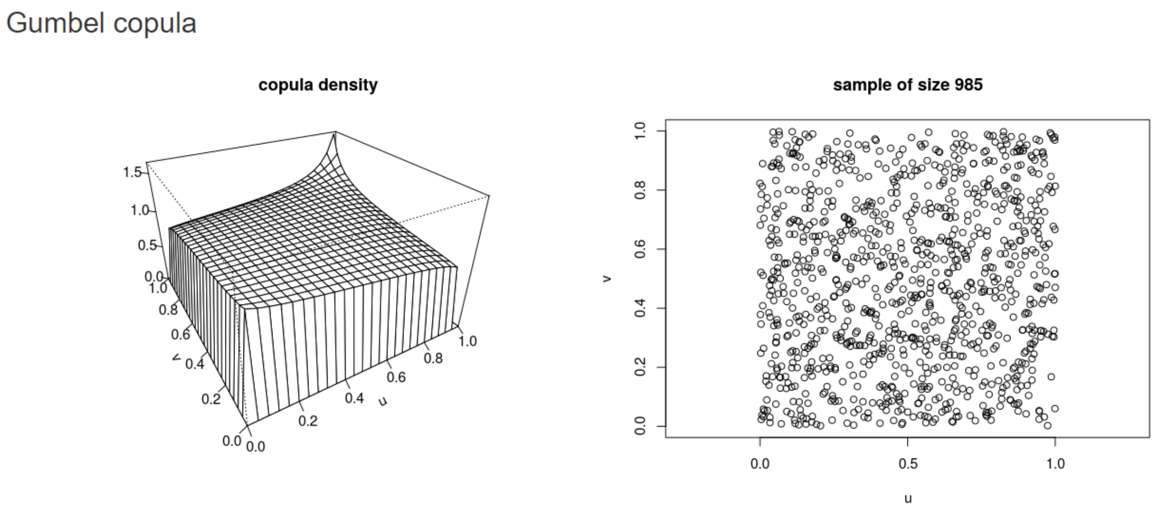


Figure 21 Series 1, USD/NOK NIBOR Gumbel Density Plot

5.3 Series 2 (Jan. 2012 – Jan. 2016)

Table 15 Series 2 (2012-2016) Copula Results

Data 1	Data 2	Best fit	θ	AIC	Loglik	τ	$\lambda_{L/U}$
	USD/NOK	Frank	1.425	-49.772	-25.886	0.151	0
	NIBOR 1 Week	Clayton	0.072	-2.228	-2.115	0.041	0
Brent Oil Price	**NIBOR 1 Month**	Clayton	0.060	-1.261	-1.630	0.016	0
	NIBOR 2 Months	Clayton	0.056	-5.990	-3.995	0.036	0
	NIBOR 3 Months	Clayton	0.077	-3.046	-2.523	0.040	0
	NIBOR 6 Months	Clayton	0.139	-12.573	-7.287	0.069	0
	NIBOR 1 Week	Clayton	0.200	-25.670	-13.834	0.104	0.031
	NIBOR 1 Month	Clayton	0.264	-43.645	-22.823	0.123	0.072
USD/NOK	**NIBOR 2 Months**	Clayton	0.290	-51.801	-26.901	0.135	0.091
	NIBOR 3 Months	Clayton	0.331	-67.218	-34.610	0.152	0.123
	NIBOR 6 Months	Clayton	0.445	-107.183	-54.591	0.204	0.211

5.3.1 Periodical Market Spillover

Like series 1 (2016-2019), series 2 (2012-2016) also shows properties that can prove tail dependence between Brent oil price and USD/NOK. What is interesting is the change in best fitting copula type. In series 1, we saw that the Gumbel copula best could describe the relationship between the two variables, showing an upper tail dependency. Here, in series 2, we see that the Frank copula best can explain the dependency, still providing significant AIC scores and log-likelihood values. A matching model with the Frank copula makes the impression of a radial symmetry, with no accumulated one-tail dependency, but rather signs of upper and lower of return correlation. What this series can provide is both an upper and lower tail dependency between changes in USD/NOK and Brent oil prices. As we have stated earlier, we must now consider the reversed USD/NOK variable, making this dependency different. We now get a negative tail dependency, which indicates that when USD/NOK experience negative returns, Brent oil will experience positive returns. What separates the Frank correlation from Gumbel is that this relationship also considers the reversed scenario. This makes the two variables match reversed from each other in both upper and lower tail dependency. Although the AIC and log-likelihood score do not reflect the perfect similarity to the Frank copula model, we see a very large Theta and Kendall’s tau. This may suggest that the variables are somewhat dependent, but only sporadically experience spillover in upper and lower tail returns, as seen in figure 22.

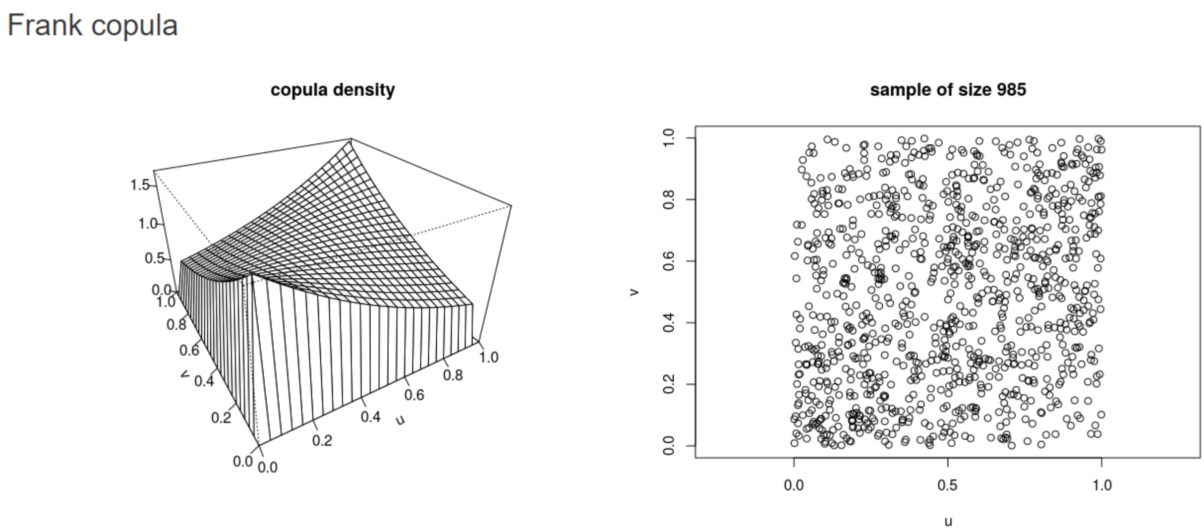


Figure 22 Series 2, Brent Oil USD/NOK Density Plot

Perhaps the biggest difference from series 1 (2016-2019) is the USD/NOK and NIBOR terms dependency in series 2 (2012-2016). In series 1, we saw almost no tail dependency between the two variables, despite the claimed correlation in the data chapter. However, in series 2 we see an increase in AIC and log-likelihood scores when watching for dependencies in higher NIBOR terms. The best fitted copula for all NIBOR terms is the Clayton copula, with theta higher than 0.3 for 3- and 6 months NIBOR. Clayton copula, as mentioned earlier, describes a lower tail dependence. Again, we consider the reversed USD/NOK variable and can interpret the result as when USD/NOK experience high positive returns, all NIBOR terms will experience high negative returns. There are significant signs of spillover, perhaps in line with what one could predict. Series 2 takes place in times of the oil crisis, and the oil and gas sector suffered major changes in profitability and work orders on the continental shelf. This situation seemingly created problems for the Norwegian economy, and as a reaction may have lowered the demand for NOK currency, and increased the USD/NOK exchange rate. The domestic key rates in Norway steadily declined, affecting the NIBOR, creating trends of Clayton dependency between NIBOR and USD/NOK. This provides good evidence of return spillover, as seen in figure 23.

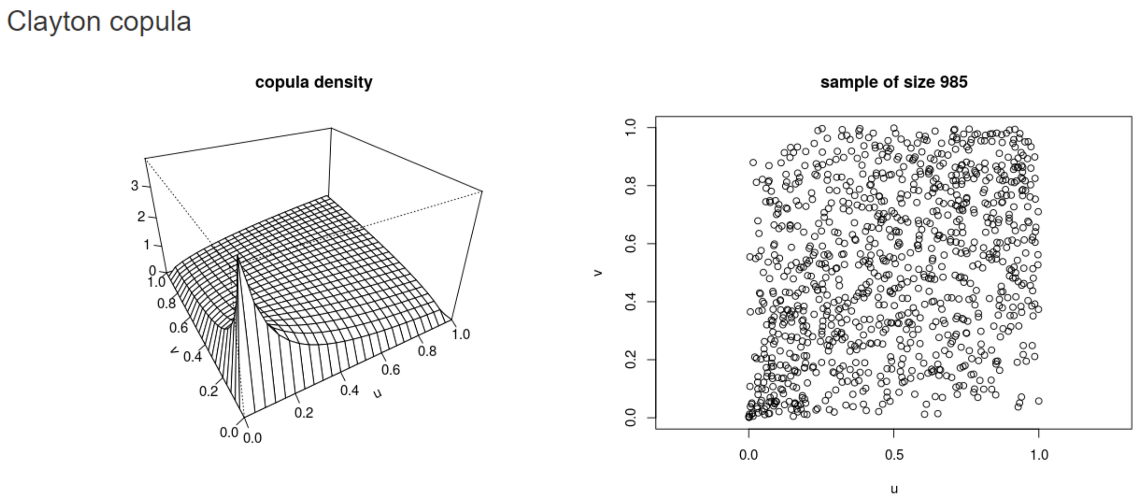


Figure 23 Series 2, USD/NOK NIBOR6M Clayton Copula Density Plot

5.3.2 Periodical Market Independence

In the full sample analysis, we saw small indications of tail dependecy between Brent oil prices and NIBOR rates, increasing along with the length of the terms. These indications are not conclusive, but may suggest that there are periods of tail dependecy somewhere in the sample. The 6 month NIBOR and the Brent oil price show some return dependence in series 2 (2012-2016), providing a Clayton theta of 0.139. Not enough to claim significant spillover, but may indicate cases of correlating market reactions. Low log-likelihood and Kendalls tau substantiates this claim, and provide no significant signs of upper or lower tail dependence. This absent spillover grows as we shorten the NIBOR terms in this series, as it was in the full sample. In series 2, Brent oil prices experienced extreme volatilty, while NIBOR terms only responded in steady reduction over the period, not mirroring the scale of the negative returns. Hence, we have problems claiming any significant spillover between them.

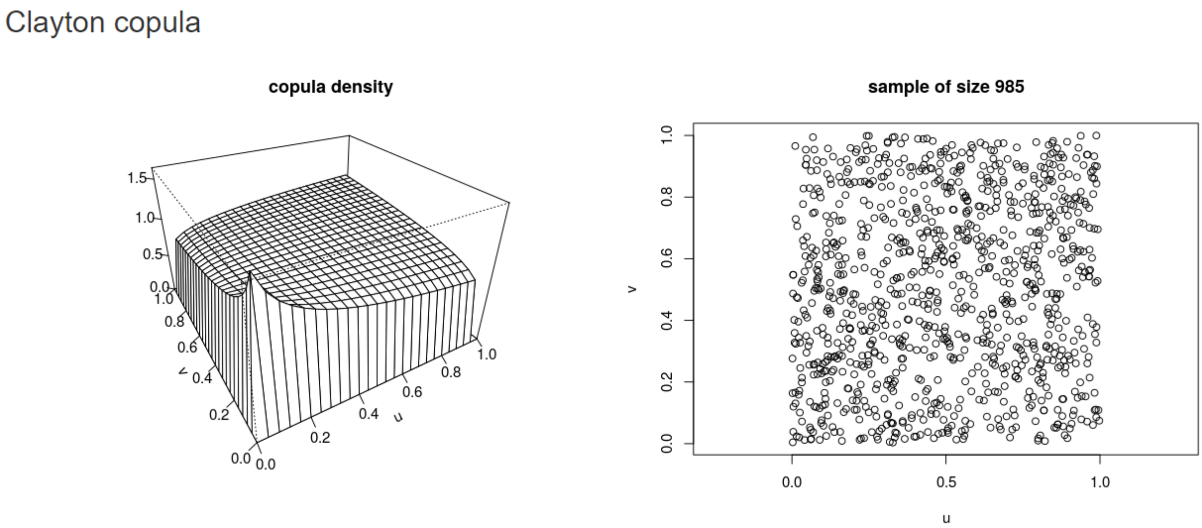


Figure 24 Series 2, Brent Oil NIBOR6M Density Plot

5.4 Series 3 (Feb. 2008 – Jan. 2012)

Table 16 Series 3 (2008-2012) Copula Results

Data 1	Data 2	Best Fit	θ	AIC	Loglik	τ	$\lambda_{L/U}$
	USD/NOK	Gumbel	1.391	-212.260	-107.130	0.298	0.354
	NIBOR 1 Week	Clayton	0.109	-7.220	-4.609	0.056	0.002
	NIBOR 1 Month	Clayton	0.116	-8.460	-5.230	0.058	0.003
Brent Oil Price	**NIBOR 2 Months**	Gumbel	1.084	-17.780	-9.891	0.077	0.105
	NIBOR 3 Months	Gumbel	1.102	-25.340	-13.670	0.093	0.124
	NIBOR 6 Months	Gumbel	1.079	-13.420	-7.712	0.082	0.099
	NIBOR 1 Week	Clayton	0.238	-34.869	-18.435	0.122	0.055
	NIBOR 1 Month	Gumbel	1.139	-41.813	-21.907	0.126	0.162
USD/NOK	**NIBOR 2 Months**	Gumbel	1.150	-51.168	-26.584	0.140	0.173
	NIBOR 3 Months	Gumbel	1.147	-47.418	-24.709	0.143	0.170
	NIBOR 6 Months	Clayton	0.286	-47.578	-24.789	0.143	0.089

5.4.1 Periodical Market Spillover

As we can see from table 16, the copula of highest significance in series 3 (2008-2012) is the Brent oil price against USD/NOK copula. This is found to be a Gumbel copula, with an AIC score of -212.26, dependence parameter $\theta = 1.391$, a very high Kendall's tau (= 0.3), and upper tail dependence λ_U= 0.354. A Gumbel copula indicates upper tail correlations. Again, remembering that the USD/NOK data is reversed, an upper tail dependence of this copula indicates a negative correlation structure in the upper quantiles. This means a downward turn in USD/NOK is more likely given a high upward turn in Brent oil price. This coincides with the notion that the NOK is strengthened by increasing oil price. This series consider the strongest recession in modern times, with high volatility in all markets. It is only logical that any underlying dependence between them is highlighted during this period. With the high AIC and log-likelihood score this copula is by far the greatest fit to its underlying data series compared the copula estimates of the other sub-samples. The theta parameter and the corresponding tail dependence signifies a somewhat strong dependence and significant return spillover. The effect of this is seen by considering the copula density plot in figure 25 where one can spot a slight thinning at the upper right tail end of the joint plot. This thinning corresponds to the upper tail correlation effect.

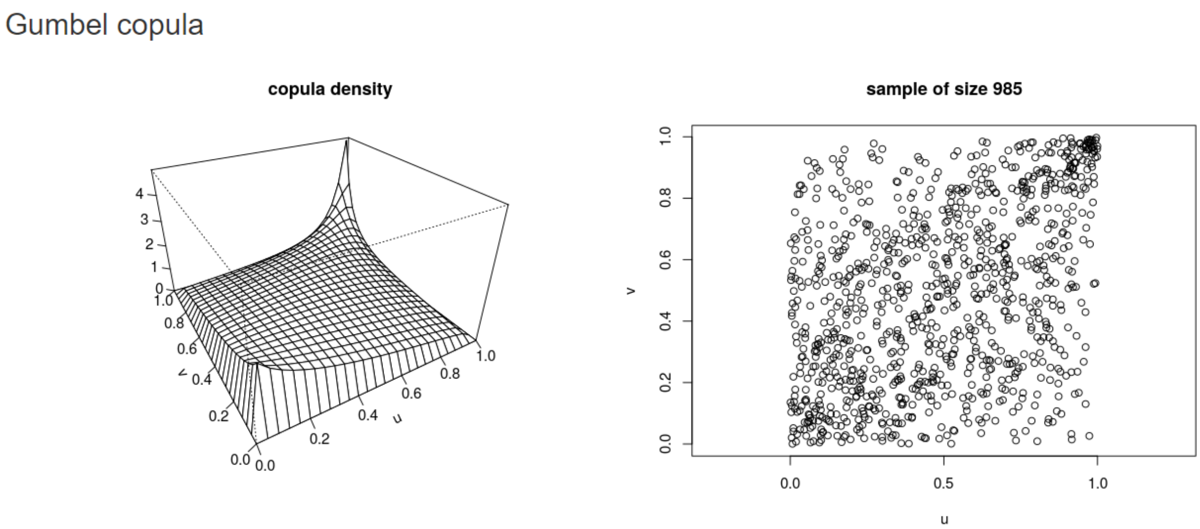


Figure 25 Series 3, Brent Oil USD/NOK Gumbel Copula Density Plot

An interesting observation of this period is the correlations related to the NIBOR terms. In particular, one can see greater dependence between the USD/NOK exchange rate and the NIBOR terms with Kendall's tau coefficients ranging between 0.12 and 0.15, whereas for the oil price, the coefficients range between 0.05 and 0.1 (closer to zero indicate less dependence). Surprisingly, this is the only series where we can see some dependence between Brent oil price and NIBOR rates, where the 2-month, 3-month and 6-month terms reach somewhat of a dependence, with Gumbel copulas of 1.08 ,1.1, and 1.08 theta parameters respectively. This does not indicate severe tail dependences, but tail dependencies, nonetheless. Curiously, this dependence is found within the time series data which also encapsulates the financial crisis. One might then infer whether a dependence between the NIBOR only occurs in periods of extreme volatility. However, one must tread carefully. The log-likelihoods range from -7.7 to -13.6, which gives AIC scores ranging from -13.4 to -25. These scores may be judged to be of low significance, which means that models are not necessarily a great fit. One explanation for the occurrence of Brent oil and NIBOR term dependency can be the financial crisis recoil. As a result of the crisis, the demand for oil and gas suffered major reduction, reducing the prices to extreme lows late in 2008. With both NIBOR and Brent oil prices at historical lows, there were due for recovery in both markets. This may explain the Gumbel dependency through this period and explain the sporadic occasions of return spillover.

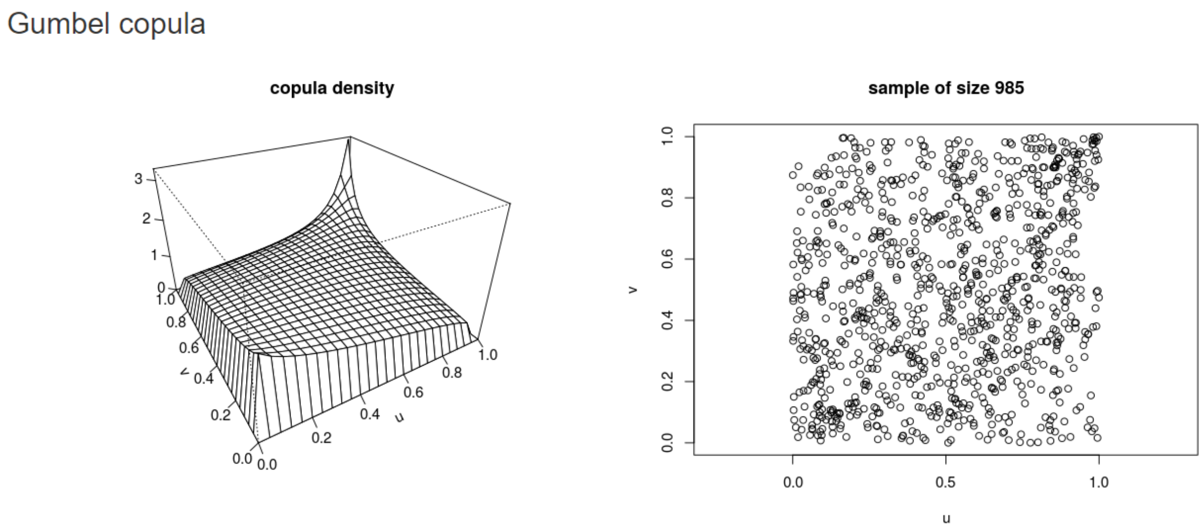


Figure 26 Series 3, USD/NOK NIBOR6M Gumbel Density Plot

Clayton copulas were found between the USD/NOK exchange rate, and NIBOR 1-week data and 6-month data, with thetas of 0.24 and 0.29, and with AIC scores of -34.9 and -47.6 respectively. Correspondingly, tail dependences of 0.055 and 0.089 were found. A curiosity about these last two copulas is the flip from best type fit across the terms. The 6-month and 1-week copulas have clayton copulas as suggested best fit. The terms inn between have Gumbel. Unlike some of the other estimates, these have some statistical significance with the lowest AIC score of -35.9 for the 1-week-maturity copula and -51.2 for the 2-month copula. The “flip” between these two types are curios, because it signifies a change in dependence relation when moving across terms. From this, one can surmise a relation between short term, medium term, and long-term rates: The short- and long-term rate have a lower tail end correlation with the *negative* movement of USD/NOK, which means a positive dependence on the exchange rate and cases of return spillover. The medium terms, however, have a negative relation, also providing evidence of return spillover. The extreme overall market volatility in series 3 created both high negative and high positive returns over the period, possibly making it rather coincidental which dependency structures that best correlates USD/NOK and NIBOR.

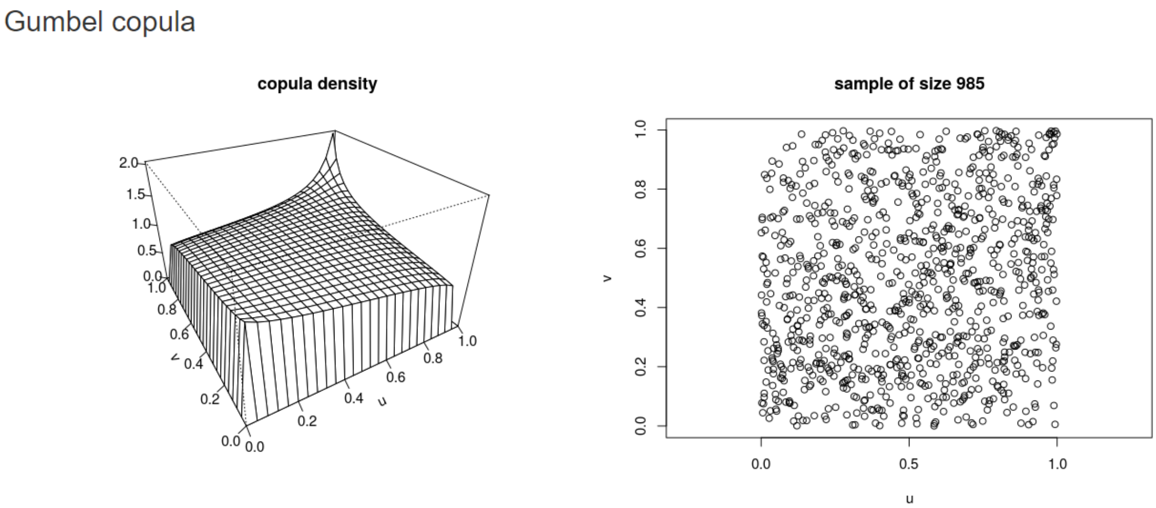


Figure 27 Series 3, Brent Oil NIBOR3M Gumbel Density Plot

5.4.2 Periodical Market Independence

Series 3 (2008-2012) stands as the period where we found the most statistically significant models. As previously discussed, dependence among Brent oil prices and the latter three NIBOR terms were found. The first two terms, however, are more akin in relation to oil price data as for the other time periods. That is to say, the NIBOR 1-week and 1-month data, together with Brent oil price, were found to have a Clayton copula, with theta parameters of 0.11 and 0.12, AIC scores of –7.2 and -8.5, and Kendall's tau coefficients of 0.056 and 0.058 respectively. A Clayton copula suggest lower tail end dependence, however, with the relatively small theta values, the tail end dependence is low, and it suggests relative independence. All NIBOR terms fell drastically trough this period, as did the Brent oil price, which may indicate that even strong volatility cannot directly connect dependence between these markets. This can provide substantial arguments for market independence in terms of return spillover.

5.5 Series 4 (Feb. 2004 – Feb. 2008)

Table 17 Series 4 (2004-2008) Copula Results

Data 1	Data 2	Best Fit	θ	AIC	Loglik	τ	$\lambda_{L/U}$
Brent Oil Price	**USD/NOK**	Frank	1.263	-39.570	-20.790	0.136	0
	NIBOR 1 Week	Clayton	-0.011	1.860	-0.068	-0.023	0
	NIBOR 1 Month	Clayton	-0.023	1.510	-0.244	-0.016	0
	NIBOR 2 Months	Clayton	0.051	-0.220	-1.110	0.019	0
	NIBOR 3 Months	Frank	0.028	1.980	-9.271	0.003	0
	NIBOR 6 Months	Clayton	0.014	1.810	-0.093	0.005	0
USD/NOK	**NIBOR 1 Week**	Clayton	-0.058	-1.370	-1.685	-0.020	0
	NIBOR 1 Month	Clayton	0.036	0.616	-0.692	-0.018	0
	NIBOR 2 Months	Clayton	0.050	-0.615	-1.308	-0.018	0
	NIBOR 3 Months	Clayton	0.050	-0.468	-1.234	-0.009	0
	NIBOR 6 Months	Clayton	0.042	0.331	-0.835	-0.004	0

5.5.1 Periodical Market Spillover

The entirety of series 4 (2004-2008) is characterized by almost no statistically significant copula estimates. On can also see from most of the Kendall's tau estimates that there is little correlation between any of the assets. The most significant dependence, as with many other series, was between Brent oil price and USD/NOK exchange rate, with a Frank copula and an AIC score of -39.6. A Frank copula then implies somewhat of a tail dependence in both the upper and lower ends, hence symmetric spillover. The theta parameter landed on a value of 1.263, which can be regarded as a moderate dependence for a Frank copula. While the Frank copula indicates degrees of both upper- and lower tail dependence, we must keep in mind the reversion of the NOK data. This means that a downward turn in Brent oil price increases the likelihood of an upward turn in NOK exchange rate, and an upward turn in oil price corresponding to a downward turn in exchange rate. Increasing Brent oil prices made a steady growth for the Norwegian economy in the period, also lowering the USD/NOK exchange rate. The results then present significant results of return spillover between Brent oil prices and USD/NOK exchange rates in the period.

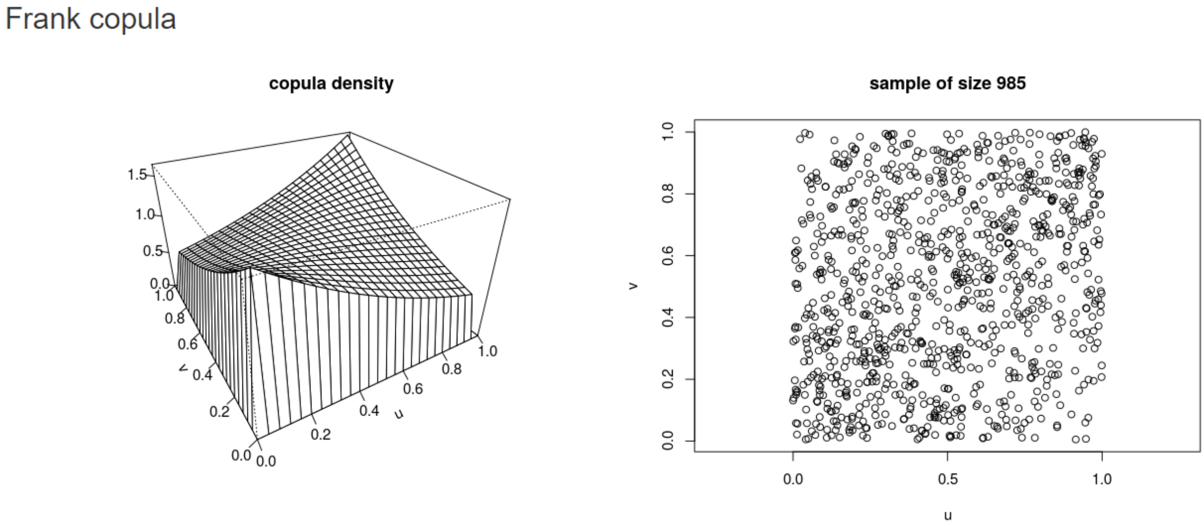


Figure 28 Series 4, Brent Oil USD/NOK Frank Plot

5.5.2 Periodical Market Independence

The results from the table indicate little to no dependence between the any of the NIBOR terms, against both the USD/NOK exchange rate and Brent oil price. One can however see, by regarding Kendall’s tau, somewhat of a larger dependence associated with the shorter maturity period. Still, the thetas of “best fit” clayton copula are all relatively close to zero, indicating an independence in the relation. One is also due to note the discrepancy in log-likelihood and hence AIC scores of low values. One might also notice a significant drop in dependencies between the NIBOR terms and USD/NOK exchange rate. In series 4, there are little to no statistically significant dependence between any of the NIBOR terms and the USD/NOK exchange rate, as seen in the density plot of figure 29.

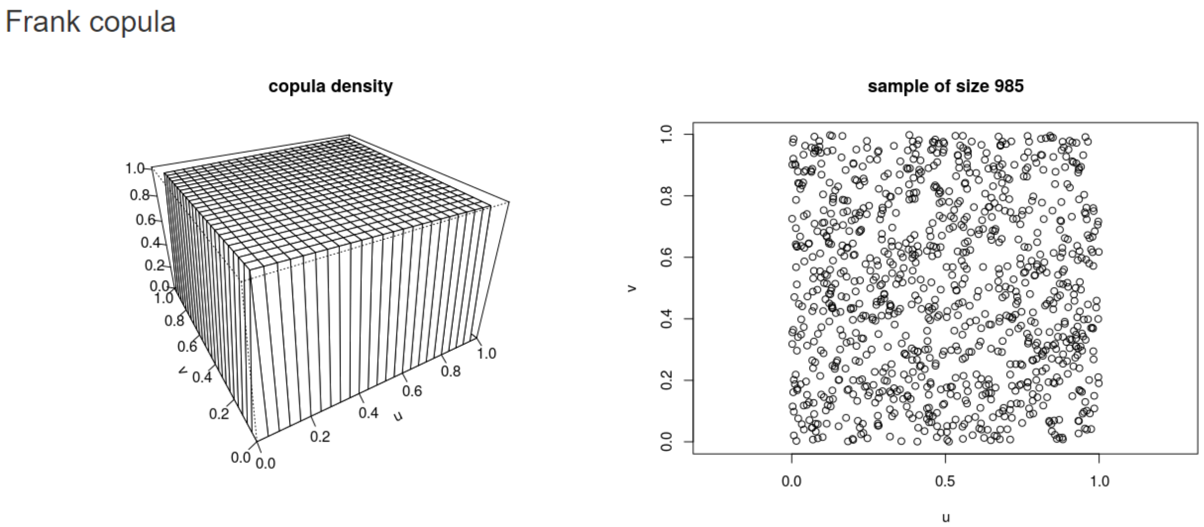


Figure 29 Series 4, Brent Oil NIBOR3M Frank Density Plot

A suggested explanation for the low dependencies and non-significance of dependence models for this period is perhaps the fact that the period is relatively calm. As we can recall, series 4 corresponds to the period prior to the financial crisis, where financial growth was steady. The steady growth seen in these terms relate to few movements far away from their marginal model means, or very few incidents of outliers. Thus, we have few outlier-type values at the marginal tail ends, ergo the dependence is low. One might also expect this to happen with a market metric such as the NIBOR, which is not likely to experience heavy volatility in periods of steady economic growth.

5.6 Series 5 (Feb. 2000 – Feb. 2004)

Table 18 Series 5 (2000-2004) Copula Results

Data 1	Data 2	Best Fit	θ	AIC	Loglik	τ	$\lambda_{L/U}$
Brent Oil Price	**USD/NOK**	Gumbel	1.033	-1.017	-1.510	0.034	0.044
	NIBOR 1 Week	Gumbel	1.024	-0.480	-1.240	0.019	0.032
	NIBOR 1 Month	Frank	0.215	0.740	-0.628	0.024	0
	NIBOR 2 Months	Gumbel	1.025	-0.410	-1.206	0.024	0.034
	NIBOR 3 Months	Frank	0.282	-0.150	-1.070	0.032	0
	NIBOR 6 Months	Frank	0.203	0.900	-0.549	0.023	0
USD/NOK	**NIBOR 1 Week**	Frank	1.075	-28.413	-15.206	0.119	0
	NIBOR 1 Month	Frank	0.963	-22.326	-12.163	0.105	0
	NIBOR 2 Months	Frank	0.978	-22.991	-12.496	0.107	0
	NIBOR 3 Months	Frank	0.864	-17.872	-9.936	0.096	0
	NIBOR 6 Months	Clayton	0.133	-12.427	-7.214	0.056	0.005

5.6.1 Periodical Market Spillover

Diverging from the case in series 4, there are greater dependencies between NIBOR terms and the USD/NOK exchange rate in series 5, with Kendall's rank coefficients of approximately 0.11 for 1 week, 1 month, and 2 months terms. The 3- and 6-month terms have a somewhat decrease in effect, with coefficients of 0.096 and 0.056 respectively.

The USD/NOK on NIBOR copulas of series 5 (2000-2004) are the most significant of the period, where the strongest fit being the 1-week maturity with and AIC score of -28, which then decreases over the terms ending up with a low AIC score of -12 on the 6-month maturity. Interestingly, for all these copulas, except the 6-month maturity, all best fit was found to be

Frank copulas. Recall that a Frank copula corresponds to both upper and lower quantile dependence, but with tail indices (λ) of zero. The most significant of these, the USD/NOK on NIBOR-1-week Frank copula, with a theta parameter of 1.08, and Kendall's rank coefficient of 0.12. Figure 30 shows the copula density generated with this parameter. One can interpret this as a correlation in the upper and lower quantiles of both markets, giving a slight spillover between them. A suggested explanation for this effect is the high volatility in both NIBOR terms and USD/NOK exchange rate in the period. This period included the *.com-burst*, which may have set the stage for the conditions for both upper and lower return dependence to occur.

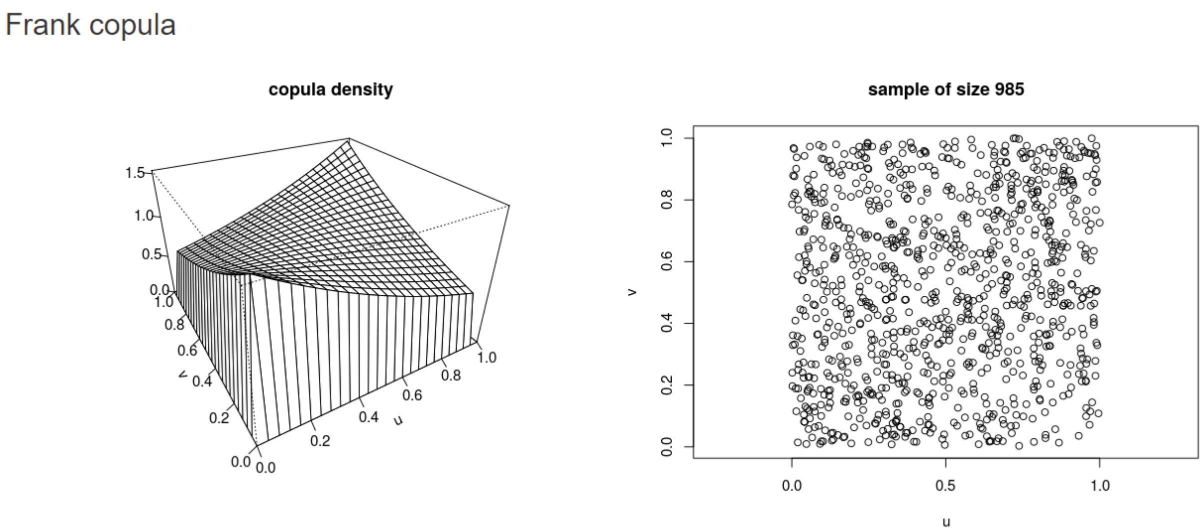


Figure 30 Series 5, USD/NOK NIBOR1W Frank Density Plot

5.6.2 Periodical Market Independence

As we saw in series 4, relatively few statistically significant dependencies were found in this sample series. Many of the estimated thetas are close to yielding independent copulas (0 for Frank and Clayton, 1 for Gumbel), copulas which in the other periods had significant dependencies. Another interesting detail is the fact that this series sees the lowest correlation between Brent oil price and USD/NOK exchange rate, with a Gumbel as the "best fit" copula, with a theta parameter close to 1, an AIC score of -1, and Kendall's tau of 0.03. These values indicate that there are no significant co-dependences between Brent oil and USD/NOK returns found in the period, and no signs of return spillover. Overall, the copula analysis struggles to find any significant dependences with the Brent oil return in this period, perhaps because both NIBOR terms and the USD/NOK exchange rate experienced growth, on average, unlike the more volatile Brent oil prices.

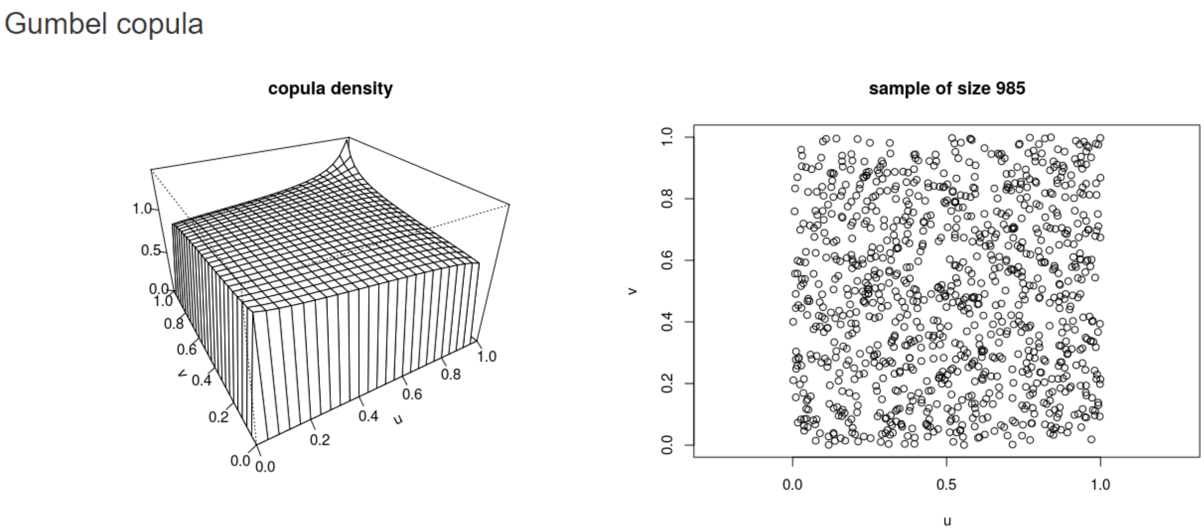


Figure 31 Series 5, Brent Oil USD/NOK Gumbel Density

5.7 Summary

Interestingly, we were able to uncover both structures that differ, and structures that are preserved throughout the different time series. Commonly, we see little to no dependence between the NIBOR terms and the Brent oil returns across the time-periods, with just some exception in series 3 (2008-2012). Further, the NIBOR terms are frequently correlated with the USD/NOK exchange rate across time periods, with the least dependence occurring in the older time periods of series 4 (2004-2008) and 5 (2000-2004). The observed dependency structures vary across both time periods and terms, which makes it difficult to extract an overall relation from the analysis. Apart from extraordinary rate meetings, longer terms of NIBOR tend to reach past planned official monetary policy meetings, creating volatility in expectation of key rate adjustments, having a larger effect on the longer terms. The full sample analysis indicates a Clayton relationship between the negative USD/NOK exchange rate and NIBOR terms. Even though the direction of dependence varies across the series, we can often see proof of return spillover between USD/NOK and all NIBOR terms.

The Brent oil spot price on USD/NOK exchange rate is usually the copula exhibiting the strongest dependence structures, with greater dependence parameters and model fit for periods of higher volatility (financial crisis; series 3, and oil crisis; series 1). The common structure between these markets is that of a Gumbel copula, which we also can see in the full sample analysis, and some instances of Frank copulas, (in series 2 (2012-2016) and 4 (2004-2008)). These structures indicate that there is a relatively stable relation between Brent oil and

USD/NOK exchange rate, where an increase in oil price increases the likelihood of the exchange rate to decrease. This dependency provides evidence of return spillover between Brent oil prices and the USD/NOK exchange rate and support the strong correlation between them.

The strength of dependence for most of the copulas are usually not to high, where we have theta parameters lying in the range of [1-1.4] for the Gumbel structures, [0-0.3] for the Claytons, and [0-1.42] for the Franks. Remembering that the lowest values for each of these structures correspond to the independence case, and that the positive theta limit corresponds to perfect dependence, the dependencies found can generally be considered as relatively low, but still present. With this in mind, it is illogical to expect any forms of perfect dependence, in any of the considered markets, due to the wide range of factors that affect each individual variable.

6. Conclusion

Through this thesis we have considered the spot rates of Brent oil prices, the USD/NOK exchange rate and NIBOR terms reaching from February 2000 to late December 2019 and analyzed the degree of return dependency and spillover. In addition to a full sample analysis, where the entirety of data is applied, we have studied the similarities, or lack of such, in divided sub-samples that each represent approximately four years. To analyze and highlight the structural dependencies between the variables we have used the Archimedean Copulas: Clayton, Frank and Gumbel. The results from the analyzes indicates varying dependencies, both in terms of degree and direction, between all variables through the period of testing. We argue that these time-varying dependencies can be justified with political and macro economical events connected to the relevant period of testing.

When considering the case between Brent oil prices and USD/NOK exchange rates, the results from the full sample analysis indicates a strong upper tail dependency described by the Gumbel model. Interpreting this on behalf of the reversed USD/NOK variable, we see a negative return dependency, when Brent oil experience high positive returns, USD/NOK is likely to experience high negative returns, creating a return spillover. This relationship is greater than the reversed scenario, which is less observable in the results. Viewing the series separately, we see that only series 1 (2016-2019) and series 3 (2008-2012) shares this result, while the rest show little evidence of return spillover. In Series 1 and Series 3, the markets are characterized by oil and financial crisis recovery, creating massive volatility in both Brent oil and USD/NOK rates. This suggests that the negative dependence and spillover between Brent oil and USD/NOK returns are primarily present in times of strong macroeconomic turmoil and volatility.

The results from Brent oil and NIBOR terms indicates small levels of tail dependence between them. The full sample analysis shows that, on a restrictive level, the Clayton copula model can describe the tail end correlation, creating a lower tail dependence structure. This fit is relatively weak, and by no means significant. Viewing each of the individual series we see that only series 3 (2008-2012) picks up a significant tail dependence. Here we see a better fit for the Gumbel copula, and the highest degree of spillover is between Brent oil and the 3-month NIBOR term. Some dependence with 6-months and 2-month NIBOR are also shown in the result. Gumbel dependency is upper tail correlated, making a return spillover between 2-6 months NIBOR terms and Brent oil prices. The grounds of increased significance in longer terms of NIBOR can possibly relate to the expectation of increasing key rates in the future, and a higher risk premium for interbank loans resulted from governmental regulations. As with Brent oil and USD/NOK, the dependence is more prominent in times of high volatility.

Overall, the effect seen between USD/NOK exchange rate and the NIBOR terms correspond to different structures throughout the sub-samples, making the results somewhat inconclusive considering the best fitting dependency. Series 2(2012-2016), 3(2008-2012) and 5(2000-2004) all present significant results, but different dependency structures. The full sample analysis, and series 2(2012-2016) indicate lower tail dependency clayton copulas between the USD/NOK exchange rate and all NIBOR terms. This means that when the exchange rate experiences an upward turn, corresponding to a decrease in value of the Norwegian Krone, the NIBOR rates decreases, creating a negative return spillover. However, these effects are not consistent when viewing the other time periods. Reasons that possibly could justify the shifting dependency structure is the current degree of financial turmoil. We interpret the results as when the markets experience recession we see an increased USD/NOK rate, in agreement with the *rand-currency hypothesis*, and the NIBOR term rates are reduced to counter-act any negative financial outcome. This will explain the Clayton dependency occurring in series 2, 3 and the full sample. Since series 3 and 5 experience both strong recession and recovery, it may be coincidental which of the events that has the strongest effect on the return dependency, which may be why we see both Gumbel and Clayton fitted for different terms of NIBOR.

The goal from this thesis lie in performing an analysis on interdependent markets and searching for structures that describe how this interdependence manifest throughout selected time periods. These results can be useful in gaining an understanding on how these markets are co-dependent and how this dependency varies across time, and in periods exhibiting certain characteristics. For this thesis, markets important for the Norwegian economy was selected as a case study. However, the methodology can be utilized for any interesting market or economy. Further, the methodology can also be expanded to include a greater number of markets to be analyzed. The interdependence structures that were found can be regarded as measure in non-linear covariation with a focus on the cases of upper and lower tail dependence, which corresponds to extreme value occurrences. The dependences that were found can help to make judgments about these markets in situations where extreme values occur, or to assess risk and make risk judgements about the included markets.

Finally, a few limitations must be addressed when regarding the results of this analysis. First off, one might regard other methodologies that could identify similar results. One often sees the method developed by Diebold and Yilmaz (2009) in finding the volatility spillover index, which is an index describing the volatility spillover across markets. This methodology might be even better suited in finding spillover and market structures than our copula methodology. Even so, there are potential improvements to be made within our own framework. One technique which is common in copula-based methodologies, but reached beyond the scope of this thesis, is to perform a time series GARCH analysis, and construct the copulas from the residuals. This ensures that there is no presence of auto-correlation type effects. Another possible limitation is the copulas chosen for the analysis. There is a multitude of other potential copulas, several of them being Archimedean as well. Examples of this can be the Joe- or Ali-Mikhail-Haq-copula, or combination copulas such as the Joe-Clayton, Joe-Gumbel etc. These examples correspond to different dependence structures which can be somewhat more advanced than the ones we have utilized. There are also even more advanced techniques related to the Archimedean copulas, such as the technique of constructing models known as copula vines. In most basic terms, a vine copula is constructed by composing Archimedean copulas into each other. These models allow one to construct more advanced dependence structures across many data sets and allows one to include different conditional structures. These techniques can be considered when expanding upon our methodology.

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8. Appendix

A. Approach in Python

Python is an open source general-purpose object-oriented programming language with a wide array of applications. As such, there are an almost endless supply of packages created in and for the language, many of which were created for scientific computation. One might in particular find the Anaconda library useful. This library is a large collection of Python packages accrued for data analysis and numerical computation, and most packages needed for this thesis was included in this library. Another important package that was utilized is the Copulae package (Bok, 2019) which is a package created for the very purpose of numerical estimation of copulas.

The central packages utilized for the computations in this thesis are:
-Copulae, package for copula specific calculations.
-SciPy, package for general scientific computation.
-Pandas, package for data manipulation and analysis
-Seaborn and matplotlib, packages for plotting
-Statsmodels, package for calculating statistical models.

Finding Marginal Distributions

The process of estimating a copula in python began with importing data, which had already been sorted and organized into columns in an Excel document. By utilizing the Pandas package, the columns can then be extracted into arrays of a data frame (a type of data structure from the Pandas package). These arrays can further be split into their respective time periods. Once the data has been accordingly separated and organized, one can find and test fit for marginal distributions. First off, the data must be tested for stationarity. This can be performed via the (augmented) Dickey-Fuller test 3.7.1, utilizing the *adfuller* function included in the Statsmodels package. Once stationarity is ensured, the data can be tested for marginal distributions. The fit and distribution tests of chapter 3.6 are included in the SciPy and Stasmodels packages.

Creating Copulas in Python.

The Copulae package requires that for a d dimensional copula with n observations, the input for the functions from the package must be of an $n \times d$ type array. Hence, for each period and since bivariate copulas are being estimated, pairs of data sets must be combined into new arrays, for each dependence structure we wish to investigate. This is easily done with the Pandas *concat* function. Table 6 illustrates the pairs constructed for a given time period.

Table 1 – Appendix Data Set Pairings for Copula Analysis

Brent Oil Price	USD/NOK
	NIBOR 1 Week
	NIBOR 1 Month
	NIBOR 2 Months
	NIBOR 3 Months
	NIBOR 6 Months
USD/NOK	NIBOR 1 Week
	NIBOR 1 Month
	NIBOR 2 Months
	NIBOR 3 Months
	NIBOR 6 Months

As one can see from table 6, one gets 11 pairs of datasets of which one can construct copulas. In Python, the copulas are constructed using the Copulae package: A copula *object* of a given type (Clayton, Gumbel, Frank) is defined for each constructed data pair. This object can then be fitted over the data with the *fit* function from the package. Essentially, this function fits a given copula via the method(s) described in section 3.8.1 and Appendix B. Properties and statistics such as the theta parameter and log-likelihood statistic from the calculation can be extracted from the fitted copula object with the *summary* function or retrieved individually with their respective call functions. This process must be repeated over all pair combinations, which culminates into a process that can be repeated over our selection of time periods.

B. Copula Estimation Methods

Semi-parametric estimation. (pseudo-log-likelihood.)

Parametric estimation of θ can be achieved by maximum likelihood estimation of the copula model:

$$L(\theta) = \sum_{i=1}^{n} \log\left(c_\theta\left(F_x(x), F_y(y)\right)\right) \tag{1}$$

However, this imposes the problem of already knowing (or having an accurate estimate of) the marginal distributions. A semi-parametric approach can then be suggested. By implementing the empirical estimates for both marginals, one can construct a pseudo log likelihood relation for the copula parameter θ:

$$L(\theta) = \sum_{i=1}^{n} \log\left(c_\theta\left(F_{n,x}(x), F_{n,y}(y)\right)\right) \tag{2}$$

Where $F_{n,x}(x)$ and $F_{n,y}(y)$ respectively denote the marginal empirical distributions. The method for obtaining these marginals are similar to the empirical distribution and is given by the following formula.

$$F_{n,x}(x) = \frac{1}{n+1}\sum_{i=1}^{n} If[X_i < x, 1,0] \tag{3}$$

Followingly, one can construct a semi-parametric estimate of the copula parameter by log likelihood estimation, by the means of calculating pseudo observations $[\bar{U}_1 \dots \bar{U}_\iota, \bar{V}_1 \dots \bar{V}_n] = [F_{n,x}(X_1), \dots F_{n,x}(X_n), F_{n,y}(Y_1) \dots F_{n,y}(Y_n)]$:

$$l(\theta; \bar{U}_1 \dots \bar{U}_n, \bar{V}_1 \dots \bar{V}_n) = \ln L(\theta; \bar{U}_1 \dots \bar{U}_n, \bar{V}_1 \dots \bar{V}_n) = \sum_{i=1}^{n} ln\ c_\theta(\bar{U}_i, \bar{V}_{i,}) \tag{4}$$

Where c_θ is the copula density $\frac{\partial C(u,v)}{\partial u \partial v}$

Method 3. Parametric estimation. (Log likelihood estimation)

This method is similar to method 2, only that instead of utilizing the empirical distributions, the marginals are beforehand estimated by maximum likelihood, and then used to fit a copula.

First, we identify the most likely type of distribution for the marginals, and estimate the parameters by maximum likelihood estimation:

$$L(\mu) = \max_{\mu} \prod_{i=1}^{n} f(X_i; \mu) \tag{5}$$

Where μ denotes the distribution parameters to be estimated, and f denotes the suggested marginal distribution function. Taking the log of this equations yields the log-likelihood:

$$l(\mu) = \sum_{i=1}^{n} \ln\left(f(X_i; \mu)\right) \tag{6}$$

Local maxima can be found by taking the derivative and solve for zero: $\frac{\partial}{\partial \mu_i} l(\mu) = 0$.

The estimated parameters then yield marginals distributions with maximum likelihood. One can then use the maximum likelihood estimated cumulative distributions ${F^{ML}}_x(x), {F^{ML}}_y(y)$ to transform the data into pseudo observations: $[F_x^{ML}(X_1), \dots F_x^{ML}(X_n), F_y^{ML}(Y_1), \dots\ F_y^{ML}(Y_n)] = [\bar{U}_1, \dots \bar{U}_n, \bar{V}_1, \dots \bar{V}_n]$

Likewise, as in method 2, we can construct the copula likelihood:

$$l(\theta; \bar{U}_1 \dots \bar{U}_n, \bar{V}_1 \dots \bar{V}_n) = \ln L(\theta; \bar{U}_1 \dots \bar{U}_n, \bar{V}_1 \dots \bar{V}_n) = \sum_{i=1}^{n} ln\ c_\theta\left(\bar{U}_i, \bar{V}_{i,}\right) \tag{7}$$

This method can be described as relatively straight forward. However, it is dependent on estimating accurate marginal distributions, which in some cases might not be achievable. This can be tested by testing for uniformity of transformed data. More concretely, one can construct a test for the uniformity condition: $F_x^{ML}(Data) \sim Uniform(0,1)$, a variation of the Anderson Darling test.

C. Clearing Confusion About Inversed Data

The modification regarding the inversion (*-1) of the USD/NOK data might create some confusion when interpreting the results of the copula analysis. Table 13 is given as an overview to help illustrate the direction of the eventual effects found in the analysis. The arrows in the table indicate both the average direction of movement in return data, and the effect a given copula would yield with these movements. As an example, consider row 2: A real upward turn in Oil price, and downward turn in exchange rate, can correspond either to an upper tail dependence Gumbel, or a Frank copula.

Table A – Appendix Direction Table

Data 1 Oil Price Movement	Data 2 USD/NOK Movement	- Data 2 (negative)	Clayton(1,-2) Effect	Gumbel(1,-2) Effect	Frank(1,-2) Effect
↑	↑	↓	-	-	-
↑	↓	↑	-	↑	↑↓
↓	↑	↓	↓	-	↑↓
↓	↓	↑	-	-	-

One may note that in the cases where there is a positive dependence between, Oil price and exchange rate, we would not be able to obtain a copula relation. There were some instances of this in the analysis, however, these instances were so small that the results may be interpreted as independence