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DEDICATION

This thesis is dedicated to my beloved mother Abiti Chande whose love, support and guidance saw me through to this level of education. I also dedicate my thesis to my wife Evelyn, children Atupele, Zikomo and Upile with deepest gratitude and reverence.

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ABSTRACT

This study explores the cognitive opportunities to learn mathematics provided by grade 11 Malawian mathematics textbooks through the topic of quadratic equations. Four textbooks which were approved by the Ministry of Education Science and Technology as core textbooks for grade 11 mathematics were analyzed using the two frameworks: Mathematics Discourse in Instruction analytical tool for textbook analysis (MDITx) (Ronda & Adler, 2017) and Mathematics Task analysis (Stein & Smith, 1998). A total of 98 examples and 532 tasks were analyzed in this study from the four textbooks. Findings show that the four textbooks contain examples and tasks of all levels of cognitive demand in the topic of quadratic equations. However, the tasks which dominate in all the four textbooks are those of low level of cognitive demand. Tasks requiring high cognitive demand, are poorly represented in all the textbooks, for instance 6% in Textbook A, 11% in Textbook B, 9% in Textbook C and 7% in Textbook D. These findings reveal that the textbooks are providing very little opportunity to engage learners with high cognitive demand tasks. This illuminates textbooks' potential in limiting opportunities to engage learners in high cognitive demand mathematics. Although this study explored one topic, this may suggest that grade 11 Malawi textbooks limit the opportunity of learners to learn cognitive demanding mathematics. It is necessary to find out more about the opportunities to learn cognitive demanding mathematics in the other topics in these textbooks to make the generalized conclusion. It could also be interesting to investigate how the teachers use the textbooks in their classrooms to establish if the cognitive level is maintained, upgraded or downgraded.

Keywords: Mathematics, textbooks, examples, tasks, cognitive demand.

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ACRONYMS

APA	American Psychological Association
ICME	International Congress on Mathematics Education
MIE	Malawi Institute of Education
MSCE	Malawi Schools Certificate of Education
MDI	Mathematics Discourse in Instructional
MDITx	Mathematics Discourse in Instructional analytic framework for Textbook analysis
MoEST	Ministry of Education Science and Technology
NCTM	National Council of Teachers of Mathematics
OBE	Outcome-based education
OTL	Opportunity to learn
PSLC	Primary school leaving certificate
QUASAR	Amplifying Student Achievement and Reasoning
SSCAR	Secondary school curriculum assessment review
TIMSS	Trends in International Mathematics and Science Study
USA	United States of America

CHAPTER 1
INTRODUCTION

This study was about exploring opportunities to learn cognitive demand mathematics provided in form 3 (grade 11) Malawian mathematics textbooks. In this chapter, I will present the background, statement of the problem, purpose of the study, research questions, and significance of the study. I will conclude by presenting an overview of the subsequent chapters of the thesis.

1.1 Background

A textbook is defined as a printed and published resource designed to be used by teachers and students in the learning process (Van Steenbrugge, Valcke, & Desoete, 2013). These authors further argue that textbooks provide explanations and exercises for students to complete and offer instructional guides for teachers. Johansson (2006) argues that textbooks play an important role in mathematics education because of their close relationship to classroom instruction. She further argues that textbooks contribute to the preservation and transmission of knowledge and skills from generation to generation. She gave an example of how the knowledge of mathematicians such as those from Euclid's "The elements" has been used for 2000 years now. A Textbook is one of the most important tools for the implementation of a new curriculum in many countries (Valverde, Bianchi, Wolfe, Schmidt, & Houang, 2002). Fan (2013) argued that textbooks have the potential to influence the teachers' pedagogy as he found out that teachers using different textbooks employ different teaching strategies. They further argue that textbooks convoy pedagogical messages and may provide necessary or unnecessary conditions for the teachers to use a particular teaching strategy. Textbooks can also be regarded as serving the purpose of reflecting the agreement and support for the consistency within the school system since it is often structured in such a way that it contains the required subject matter that should be covered by learners during a particular school year (Johansson, 2006). In addition, textbooks facilitate learners' engagement with specific contents and skills, as stipulated by educational guidelines (Houang & Schmidt, 2008).

Malawi's education system is 8-4-4 (5) where the first eight years is for primary school, the next four years is for secondary school and the tertiary education may take four or five years depending on the program one is pursuing. Currently starting age for primary education in Malawi is six years. This means that the first age bracket for primary school is from 6 to 13 years old, whereas secondary school serves 14 to 17-year-olds. However, in reality, due to non-strictness in the starting age of primary education, and the fact that other learners repeat a class, children of various ages are found both at primary and secondary schools. At the end of primary school, learners take Primary school leaving certificate (PSLC) national examination which are used to select learners to secondary school. Those learners who fail PSLC national examination are not allowed to proceed to secondary school. At the last year of secondary school, students take Malawi Schools Certificate of Education (MSCE) national examinations, equivalent to the Ordinary level (General Certificate of Education). Only learners who pass MSCE examinations are allowed to go for tertiary education.

In Malawi, the textbooks are designed following the content provided in the national curriculum as suggested by the Ministry of Education Science and Technology (MoEST). Therefore, what is contained in Malawian school textbooks at all levels is determined by the national curriculum. This may mean that Malawian textbooks are expected to mediate both the intended and the implemented curriculum (Mwadzaangati, 2019). Textbooks can be regarded as potential implemented curricula (Valverde et al., 2002). Textbooks for primary school are written and published by Malawi Institute of Education (MIE), a Malawi government institution responsible for the development of school curriculum materials. Secondary school textbooks are written and published by public publishers but then vetted and approved by the Ministry of Education Science and Technology (MoEST) through the Malawi Institute of Education. Once the textbooks are approved by the Ministry of Education Science and Technology through Malawi Institute of Education, all schools and learners use these approved textbooks. However, the schools are free to choose any textbook(s) of their preference to use from the approved textbooks. The criteria used by schools to choose the preferred textbook to use at their schools is not well known in Malawi. However those schools with capacity to purchase textbooks, usually keep in stock all the approved textbooks. This enables teachers and learners to have access to all the approved textbooks.

In Malawi, mathematics is a compulsory subject for both primary and secondary education and textbooks are the most important resource used by both teachers and students in mathematics classrooms. Textbooks determine and affect what is to be taught and to be learnt and most of the times how it should be taught (Kolovou, van den Heuvel - Panhuizen, & Bakker, 2009). Malawi follows a centralized curriculum where content put in the textbooks are determined by the curriculum which the Ministry of Education Science and Technology recommends, meaning that in a Malawian context mathematics textbooks contain the intended curriculum. All schools follow the same curriculum and use the same types of textbooks approved by the Ministry of Education Science and Technology. So what teachers teach and what learners learn are regulated by the mathematics textbooks (Stein & Smith, 1998). Just like in many other developing countries where by teaching and learning resources are scarce, teachers and students use textbooks as the main content resource (Ronda & Adler, 2017). Therefore, the knowledge presented by textbooks are important for facilitating the students' opportunities to learn (van Zanten & van den Heuvel-Panhuizen, 2018). Floden (2002) asserts that the way content and the cognitive domains are presented in textbooks has the potential to determine and influence students' opportunities to learn.

Research on textbooks has been conducted in the past two decades on textbook analysis. For instance, Fan (2013), Johansson, (2006), Jones and Tarr (2007), Tornroos, (2005) just to mention some but a few. Fan (2013) describes textbook research as a systematic investigation into issues about textbooks and the relationship between textbooks and other factors in education. He further observed that research in mathematics school textbooks has attracted many researchers in the recent years, giving instances such as: The 10th International Congress on Mathematics Education (ICME-10), which was held in Denmark in July 2004, under the theme ''Focus on the development and research of mathematics textbooks''. Another conference was ICME-11, which was held in Mexico in July 2008, under the theme, ''the changing nature and roles of mathematics textbooks: form, use, access''. Despite these studies, Fan (2013) argues that, in general, mathematics textbook research as a field of research is still at an early stages of development as compared with many other fields of research in mathematics education. However more research on textbooks followed up, for instance in July 2016, at ICME-13 which was held in Hamburg where the Topic Study Group (TSG) 38 meeting textbook was given attention as one of the most important teachers' resources in mathematics teaching and learning worth discussing. (Fan et al., 2018). A timely response to this call were many such as Hadar (2017) who conducted research on mathematics textbooks and students' achievement in Israel. Also Mellor and Mellor (2018) conducted a research on textbooks, where they looked at a comparative study of two textbooks from South

Africa and Germany, and most recently Hadar and Ruby (2019) conducted a study where they explored breadth and depth of understanding as addressed in mathematics textbooks certified as aligned to Israeli national mathematics curricula. The research on textbooks could be interesting and beneficial to a country like Malawi because textbooks are one of the most reliable resources used by both teachers, students and parents for teaching and learning.

In Malawi, textbook analysis research is very scarce. To my knowledge, Phiri (2018) conducted a comparative analysis of Malawian and Japanese primary mathematics textbooks to establish how textbooks are designed to help learners achieve the noble objective of critical skills acquisition. Most recently, Mwadzaangati (2019) examined similarities and differences between circle geometric proof development tasks set up in the Malawian Grade 11 mathematics textbook, and those that are designed and worked out by teachers in the classroom. She found out that the textbook presented the geometric proof development tasks at a high level, but during implementation some teachers tends to low the cognitive level.

Inspired by the revelation that different textbooks are designed to implement the same curriculum can present the same topic in different ways, resulting in the inequity affordance of students' opportunities to learn (Hadar, 2017; Hadar & Ruby, 2019; Mellor & Mellor, 2018; van Zanten & van den Heuvel-Panhuizen, 2018). The present study has investigated the cognitive opportunities to learn mathematics provided by Malawian grade 11 mathematics textbooks through quadratic equations. Four approved grade 11 core mathematics textbooks for senior secondary in Malawi were selected to be analyzed.

1.2 Statement of the problem

The Malawi Secondary Education Curriculum was reviewed in 2013 under secondary school curriculum assessment review (SSCAR) and its implementation started in 2015/2016 academic year. This curriculum puts emphasis on student-centred teaching and learning approaches, including continuous assessment and focus on student achievement (Ministry of Education, Science and Technology (Ministry of Education, Science and Technology (MoEST), 2013). In this outcome-based education (OBE) curriculum, teaching is now putting the student at the center and allowing them to take responsibility for their learning (Ozer & Sezer, 2014). In the Malawi mathematics curriculum, some of the developmental outcomes to be realized are that, “students should be able to apply mathematical concepts in socio-cultural, political, economic, environmental, scientific, and technological contexts to solve problems; apply research skills for problem-solving” (MoEST, 2013, p. VIII). Furthermore, one of the secondary outcomes in the curriculum states that after having finished secondary education, students should be creative and resourceful in using problem-solving techniques to solve practical problems. In addition, one of the aims of teaching secondary school mathematics in Malawi is to promote logical reasoning, critical thinking and problem-solving skills in students. This is also echoed in the rationale for teaching mathematics in Malawi which states in part that “… Learners develop such skills as computational, reasoning, critical thinking, and problem solving through the learning and application of mathematics…” (MoEST, 2013, p. XI). These type of skills can be acquired through learning of school mathematics which is contained in mathematics textbooks, because textbooks have the potential to mediate between the intended curriculum and the implemented curriculum (Ronda & Adler, 2017). Mathematics textbooks are one of the main teaching and learning resources used by both teachers and students in the classroom in Malawi. An important question could be, do the mathematics textbooks used in Malawian classrooms present mathematics in the way that could promote students’ opportunities to learn mathematics so that they are able to reason logically, think critically, and solve practical problems as demanded by the curriculum? To answer this question, there is a need to analyze the textbooks used in Malawian classrooms. In addition to this, in my twenty one years of working in Malawian classrooms, both as a mathematics teacher and as a teacher trainer of in-service teachers, I have observed that mathematics textbooks are used by teachers in what may be referred to as offloading level (Fan, Zhu, & Miao 2013). This suggests that teachers use” textbooks in a literal manner following the established contents as closely as possible, without flexibility and content adjustment”, (Fan, Trouche, Qi, Rezat, & Visnovska, 2018, p. 34). It could be much beneficial for these teachers to be aware of what Malawian textbooks are capable and not capable of offering that could promote opportunities to learn cognitive demanding mathematics. Furthermore, learners also rely on textbooks to learn mathematics both in classroom and outside classroom in Malawi due to scarce of other technology dependent teaching and learning resources. Therefore, it is important to analyze the Malawian textbooks. This study aims to explore the cognitive opportunities to learn mathematics provided to learners by grade 11 Malawian mathematics textbooks through quadratic equations.

The research questions that will be addressed in this study are as follows:

i. What opportunities to learn mathematics are provided by examples and tasks in the topic of quadratic equations in Malawian grade 11 mathematics textbooks?

ii. To what extent are the examples and tasks in Malawian grade 11 mathematics textbooks engaging learners in cognitive demanding mathematics in the topic of quadratic equations?

It is hoped that this research will be a contribution to the scarcity of textbook analysis research in Malawian context. The study may also contribute to the research community in mathematics education, since there is still a need for further studies on mathematics textbooks analysis (Chang & Salalahi, 2017: Fan, 2013). Furthermore, the findings of the study will hopefully benefit the following stakeholders in the education system such as: teachers, teacher educators, curriculum developers, textbook reviewers and evaluators, and researchers.

1.3 Chapter Summary and overview of the thesis

This first chapter has presented the introduction of the whole thesis. It has discussed the background to the study in relation to mathematics textbooks. The chapter has also presented a statement of the problem, purpose of the study, research questions, and significance of the study.

The thesis has been structured in such a way that chapter two reviews related literature and discusses the theoretical frameworks which guides the study. This is followed by Chapter three which discusses the methodology of the study. Chapter four presents and gives a discussion of the research results and findings. Finally, Chapter five draws conclusions and discusses implications of the findings and proposes some recommendations based on the findings.

CHAPTER 2
THEORY

This chapter is divided into six sections. The first section gives an overview on textbooks; the second section presents an overview of examples: the third section will present an overview of tasks; the fourth section will look at cognitive demand as related to tasks while the fifth section will look at some studies that have looked into opportunity to learn cognitive demanding mathematics. The sixth section will look at the theoretical frameworks that will guide this study and the conclusion of the chapter will be presented as well.

2.1 Textbooks

Textbooks are artefacts that transform policy into pedagogy and represent connectors between the intended and implemented curriculum, that is to say, they usually reflect the potentially implemented curriculum (Valverde et al., 2002). Remillard, Harris, and Agodini, (2014, p. 71) state that as one of the curriculum resources "textbook is a print or digital artefacts designed to support a program of instruction and student learning over time". This view is supported by Remillard (2005) who argues that textbooks are printed and published resources containing explanations and tasks for learners to work on. He further says that textbooks are designed to serve both teachers and learners in the learning process. Fan et al., (2018) argue that in most cases a textbook is regarded as curriculum material.

Many researchers ascertain that textbooks are one of such curriculum resources which is most important in teaching and learning in many countries (Fan & Zhu, 2007; Van Steenbrugge et al., 2013). Textbooks have been shown to have a great impact on classroom work and to form the backbone of mathematical teaching globally (Törnroos, 2005; Valverde et al., 2002). The textbook has the potential to mediate between the officially developed curriculum and the implemented curriculum by the teachers in their classroom (Valverde et al., 2002). Thompson and Fleming (2004) argue that research shows that many teachers, as they are planning and implementing the teaching activities, follow the prescribed textbook. This could be the case because, by following the curriculum, textbooks provide the content to be covered and normally the content is presented sequentially with suggested activities for engaging learners in them (Reys, Reys, &

Chavez, 2004). However, Reys et al., (2004) recommends the need for careful and wise selection of textbooks to be used in the classrooms in order to develop learners' mathematics learning and to realize learning outcomes.

The practice in mathematics classrooms is somehow influenced by the mathematics textbooks even though it may vary from teacher to teacher using the same textbook (Fan, 2013; Stylianides, 2009; Valverde et al., 2002). Nico and Crespo, (2006) identified a three level model reached by teachers when using the textbooks, thus adhering, elaborating and creating levels. They argue that teachers who are new in the profession commonly reach at adhering level while using textbooks while more proficient teachers, mostly reach the elaborating and creating levels (Nicol & Crespo, 2006). By adhering level, they mean using the textbook with no or minor adjustment and modification so that what is taught and how it is taught is largely dictated by the textbook. In the elaborating level, the teachers use the textbook as a guide and with the aid of other supplementary resources, the teachers can make changes in some parts of the content such as questions, tasks and exercises. Whereas at the creating level, the teachers use the textbooks critically and innovatively by bringing a lot of changes such as setting up appropriate problems.

Similarly, Brown (2009) identified three levels at which teachers use the textbooks, thus offloading, adapting, and improvising. He argues that offloading level is using textbook in such a way that the content is followed without any modification. The teacher rigidly sticks to what is written in the textbook. The adapting level is when the teacher shows flexibility in using the textbook by making some adjustment where necessary, and lastly the improving level is when the teacher uses the textbook innovatively and flexibly by incorporating his or her own changes during the teaching process. Teacher's knowledge and experience may affect how he/she uses textbooks in classroom practices (Nico & Crespo, 2006).

Furthermore, Johansson (2006) observed that some teachers regard textbooks as more superior than themselves. She argues that textbooks control these teachers on the type of tasks, examples they present to learners in the classroom as well how mathematics concepts and features are described. On the contrary, she says that some teachers use the textbooks as the guideline for teaching even though the educational policy makers expect the teaching to be as it is presented in textbooks, being itself a representation of the intended curriculum.

Van Steenbrugge, et al., (2013) in their study titled “Teachers’ views of mathematics textbook series in Flanders: Does it (not) matter which mathematics textbook series schools choose?” (p. 322). They studied teachers’ views of mathematics textbook series, They found that with regard to teachers’ views of mathematics textbook series, the textbook teachers chose to use has an influence in the teachers’ classroom practice. They concluded that it matters which mathematics textbook series schools choose to use in their classroom because textbooks influence the way they teach. These authors also added that teachers prefer mathematics textbook series which provide them with support in terms of additional teaching materials, detailed explanation of the content, suggested teaching and assessment strategies.

Since textbooks contain explanations, examples and tasks which are used by both teachers and learners (Remillard, 2005: Van Steenbrugge et al., 2013), the nature of examples and tasks in a textbook may influence the teaching and learning process. The textbook influences the type of tasks and examples the students are engaged with during the lessons, as well as type of mathematics concepts and how these concepts are discussed (Johansson, 2006). TIMSS data has shown that students from high achieving countries are engaged in more mathematical activities than those from low achieving countries. This is attributed to dynamically and adaptively implementation of classroom teaching by their teachers as they are using curriculum materials such as textbooks (Hiebert, Gallimore, Garnier, Givvin, Hollingsworth, Jacobs, et al., 2003).

Johansson (2006) argues that in a mathematics classroom, the teaching and learning is greatly influenced by textbooks. In her study on textbooks and their use in mathematics teaching in Swedish classrooms, she looked at the authorization of a textbook and the role of the textbooks as links between the national guiding lines and the teaching of mathematics in schools. She found out that the textbook influences the type of tasks the students are engaged with during the lessons, examples the teacher presents to learners, type of mathematics concepts to bring into the classroom and how these concepts are discussed among other things. Her study analysis further discloses that the tasks and how they are constructed have an influence on the effectiveness of teacher-student interaction. Moreover Tornroos (2005) in the study of textbooks, opportunity to learn, and student achievement found out that textbooks have the potential to influence the opportunity to learn mathematics and that there is correlation between opportunity to learn and student achievement.

The impact of textbooks in mathematics teaching was also echoed by Hadar (2017) who conducted research on opportunities to learn: Mathematics textbooks and students' achievements in Israel. She examined the correlation between opportunities facilitated by mathematics textbooks to participate in tasks demanding various levels of understanding and students' performance on tasks demanding equivalent levels of understanding on a standardized examination. She found out that students using a textbook which offers the opportunity to engage in tasks demanding higher levels of understanding, perform better than those using textbook containing tasks of low level of understanding. Her study also shows that textbooks have the capacity to facilitate opportunities to learn mathematics (Hadar, 2017). Due to the great influence textbooks have on learners' opportunities to learn cognitively demanding mathematical content, textbook analysis was an ideal study. The next two sections will discuss examples and tasks.

2.2 Examples

Zodik and Zaslavsky (2008, p. 165) define an example as "a particular case of a larger class, from which one can reason and generalize". They further assert that to realize mathematical conceptualization, generalization, abstraction, argumentation, and analogical thinking, examples play a vital role in the classroom. In addition the two authors explain that examples are different in nature as well as the purpose they serve in the learning process, some examples could be in a form of worked out solutions while others could be in a form of activities to be done by learners. Furthermore, they argue that examples are used to demonstrate and communicate concepts or to demonstrate how to carry out a procedure.

Examples are frequently used in mathematics education (Bills, Dreyfus, Mason, Tsamir, Watson & Zaslavsky, 2006). These authors argue that teaching which is based on examples offers both a practically useful and an important theoretical perspective on the design of teaching activities. They further stated that examples provide an analytical window into what is made available to learn in ways that have both theoretical importance and practical purchase. To support this idea, Marton and Pang (2006) explain that the key to better learning involves bringing attention to patterns of variation amidst invariance. It is argued that multiple examples with varying features have the potential to support deep understanding of the concept being discerned in the examples (Atkinson, Derry, Renkl & Wortham, 2000). Furthermore if teachers want learners to attend to a particular feature crucial to the object of learning, then they need to give a set of examples that will foreground this feature in the lesson (Adler & Ronda, 2015). Moreover, Marton (2015) suggests that in an example, learners should first know the object of learning and then through variation of key features be allowed to experience the contrasting, generalization and at last fusion for them to discern what is being taught. However, the quality of the examples used is largely influenced by many factors such as knowledge of the target audience, intention of the instructions, as well as the perception and its relation to mathematical generalities (Watson & Chick, 2011).

In a longitudinal study conducted by Olteanu (2018) on a series of examples, the aim was to create variation patterns that would enable learners to discern the use of the four basic arithmetic operations in different situations. Findings from the study demonstrated that well-thought-out examples facilitate the right mathematical operation in numerous instances and offer a basis from that learners will make out the relationship between the text and also the use of operation in mathematical examples.

Rittle-Johnson and Star (2009) found out that using multiple examples is beneficial for mathematics learners more especially when at least two examples are presented together than presenting them separately. They suggest that when learners are dealing with mixed examples, they can be able to distinguish between them and thus get better at making sense of mathematics communicated to them. This is also supported by Atkinson et al., (2000, p. 181) who argues that "effective instruction employs multiple examples for each conceptual problem type, varies example formats within problem type, and employs surface features to signal deep structure".

Watson and Mason (2005) argue that tasks or sets of examples can make certain aspects noticeable for the learner if they display well-guided variation than those which show uncontrolled variations. Kullberg, Runesson Kempe, & Marton, (2017) ascertain that when guided by variation theory in planning for learning, the teacher must know how to bring out variation for the learners to notice them and being able to decide on the importance of those variation are with respect to the object of learning. These authors further argue that the specific choice of examples may facilitate or impede students' learning, and this must be carefully taken into consideration by the teacher. This is also echoed by Zodik and Zaslavsky (2008) who argue that teachers ought to weigh several concerns before bringing examples into the classroom because selection of examples could facilitate or impede students' learning. They further submit to the idea that examples are always attached to their representations, and so they are meant to assist making mathematics understandable to learners. This could be a challenge to some teachers, and this scenario may also apply to examples presented in the textbook and all others learning instruction materials. Examples are selected to mediate the object of learning by making visible the features of the content, while tasks are designed to mediate the capabilities with respect to the content (Ronda & Adler, 2017). In a textbook learners usually depend on worked examples to understand mathematical concepts. Atkinson et al., (2000) argues that worked examples contain a question and a procedure for solving a problem which is meant to show how other comparable problems might be solved. They further state that worked examples "provide an expert's problem- solving model for the learner to study and emulate", (p. 182). In addition they argue that by providing the model for learners to study and emulate, worked examples encourage learners to actively train on their own. There is often a good link between the worked examples and tasks in textbooks. The section to follow will discuss the tasks.

2.3 Tasks

Ronda and Adler, (2017 p. 1102) define tasks as "what students are asked to do with the examples". They further state that selection of examples is aimed to mediate the object of learning by making visible the important feature(s) of the content that are vital in mediating meaning of the object of learning, whereas tasks are designed specifically to mediate the capabilities of learners with respect to the content. This is also echoed by Doyle, (1983), who states that tasks guide learners' thinking towards specific aspects of content and defining ways of processing information. Tasks are the basis of learners' work with mathematics as they may reveal what learners are capable or not capable of doing in mathematics (Sidenvall et al., 2014). Summing all arguments stated above support the idea expressed by Stein, Remillard, and Smith, (2007) that the mathematics learnt by learners in a classroom, and how they learn, is mostly determined by the tasks they are engaged in from their mathematics textbooks. Learners' different levels of mathematical thinking and learning can be inspired by the tasks that are included in the textbooks they are using (Stein, Smith, Henningsen, & Silver, 2000). Moreover, according to Hiebert et al (2003), students learn what they are given the opportunity to learn, this idea being one of the most reliable findings that the research on teaching and learning has concluded.

Tasks are important because working on tasks constitutes what students do during the majority of their time in the classroom (Gracin, 2018). Students in the seven countries analyzed in the Trends in International Mathematics and Science Study (TIMSS) Video Study, including the United States, spent over 80% of their time working on tasks (Tekkumru-Kisa, Stein, & Schunn, 2015). Henningsen and Stein (1997) posit that the way learners become to think and do mathematics is to a certain extent impacted by the nature of textbook tasks they are engaged with. They suggest that the nature of textbook tasks may hinder or wide open their views of the subject matter with which they are involved in. Therefore, rich and worthwhile mathematics tasks must be included in curricular materials such as textbooks (Gracin, 2018). Tasks can be categorized as 'complex', 'rich' or 'authentic' (Shimizu, Kaur, Huang & Clark, 2010). According to Stenmark (1991), rich tasks are those tasks which are essential, authentic, engaging, active, feasible, equitable, and open. He argues that essential tasks are those that are suitable for the curriculum; authentic tasks are those that use relevant procedures, while engaging tasks are those that challenge learners' thinking. Active tasks are those that will enable learners to make meaning out of them and develop understanding of the concepts. Feasible tasks are those that are proper and matching the level of the learners, equitable tasks are those that promote multiple ways of thinking in learners, and open tasks are those that can be approached in many ways and may have more than one solution. When students are engaged in these types of tasks, they may be challenged to reason and this could possibly afford opportunities for developing understanding as mostly rich tasks are connected to high cognitive demands (Gracin, 2018). In the Amplifying Student Achievement and Reasoning (QUASAR) project, Stein, Smith, Henningsen, and Silver, (2000) found out that, it is a requirement for learners to be given opportunities more often to engage with tasks that lead to deeper, more generative understandings of the nature of mathematical concepts, processes, and relationships. These tasks in which learners engage with are the ones which provide the contexts where they learn to think about subject matter (Doyle, 1983). Lappan and Briars (1995) argue that:

> There is no decision that the teacher makes that has a greater impact on students' opportunities to learn and on their perceptions about what mathematics is than the selection or creation of the tasks with which the teacher engages students in studying mathematics (p. 138).

In addition, Gracin (2018) agrees that tasks in the textbooks greatly influence mathematics teaching and learning. He therefore recommends the need to find out about the nature and demand of tasks in textbooks, whether they help to promote mathematical understanding and to what extent they can be regarded as high-level cognitive tasks.

2.4 Cognitive demand

According to Hadar and Ruby (2019, p. 57) cognitive demand in mathematics is defined as "thinking and understanding in the process of learning mathematics". They further argue that such thinking processes are the main focus of curricula in many countries. Floden (2002) argued that the way content and the cognitive demand are presented in textbooks have the potential to determine and influence students' opportunities to learn. Elliott and Bartlett (2016) explain that opportunity to learn (OTL), are inputs and processes within a school context required for producing learner achievement of intended learning outcome. They argue that OTL is enhanced when learners are allowed to experience cognitive demanding tasks. Hadar and Ruby (2019) argues that students need to learn how to think mathematically, this includes understanding and solving problems in new contexts, solving problems that differ from those solved before, reasoning, connecting and having access to a broad range of strategies in order to solve those problems This is also echoed by Idris (2009), who suggest that understanding mathematics requires reasoning processes that should surpass recalling of facts, remembering mathematical concepts or being conversant with procedures to follow without understanding the concepts.

In working with tasks, Stein and Smith (1998) argue that cognitive demand is the different kinds of thinking required in the process of working out the solution of the task. They further argue that cognitive demand is the complexity and difficulty of a task. They categorized the tasks into four levels, according to the thinking processes required to complete the tasks: memorization, procedures without connections to concept, procedures with connections to concept and doing mathematics. Tasks requiring memorization involve reproducing rules or definitions, and procedures without connections require conducting algorithms which have no connections to the concepts or meaning that underlie the procedure being used. These two categories were classified as low-level tasks because they can be worked out with or without procedures, and they usually have no connection to the concepts or meaning that underlie the procedure, facts, rules, formulas, or definitions being learned or reproduced. These authors further argue that tasks at low level require limited cognitive demand in terms of what needs to be done and how to do it.

Furthermore Smith and Stein (1998) assert that tasks belonging to procedures with connections to concepts and meaning require learners to develop deeper levels of understanding of mathematical concepts and ideas. Those belonging to doing mathematics require learners to think with considerable cognitive effort in order to successfully complete them. Tasks under procedure with connection to concept and meaning, and doing mathematics were classified as high-level tasks. They argued that this is because these tasks require students to access relevant knowledge and experiences, flexibly apply knowledge and skills, and assemble information from several different sources to use in working through the tasks. Along the same line, Wijaya, van den Heuvel-Panhuizen and Doorman (2015) suggest that learners must be exposed to real world context tasks that also include implicit mathematical procedures in order for them to have an opportunity to learn high cognitive demanding mathematics.

Hiebert and Wearne (1993) argues that tasks of high cognitive demand support learners to make connections between mathematical procedures and their underlying concepts, as opposed to the tasks which just encourages memorization of procedures. They also argue that high-cognitive demanding tasks have been found to be related with greater improvements in learners' understanding, acquisition as well as retention of mathematical procedure. According to Stigler and Hiebert (2004), the Trends in International Mathematics and Science Study (TIMSS), showed that high-performing countries engage learners with instructional tasks involving connections between concepts and procedures. This was also found by Tornroos, (2005) in the study of textbooks, opportunity to learn and student achievement. In this study it was found out that textbooks have the potential to influence the opportunity to learn mathematics and that there is correlation between opportunity to learn and student achievement. In agreement to this idea Hadar (2017) also found out that learners using a textbook which provides the opportunity to engage in tasks demanding higher levels of understanding, scores higher than those using a textbook which provides learners opportunity to engage with tasks of low cognitive demanding levels.

Since learners are of mixed abilities, they are supposed to be presented with balanced curriculum experience that considers both low and high-achieving learners by exposing them all to the full range of task types (Vincent & Stacey, 2008). However, it should be noted that the competences learners will acquire depend largely on the cognitive demands of mathematics tasks they have been engaged in (Stein & Smith, 1998). Jones and Tarr (2007) also support the idea that opportunity for learners to develop a deep understanding of mathematics content depends on the cognitive demand presented in the textbook being used. The next section highlights some of the research conducted on cognitive demand.

2.5 Previous research on cognitive demand mathematics

Globally there is growing emphasis on textbooks which are able to facilitate deep learners' mathematical concepts beyond the routine use of mathematical procedures (Hadar & Ruby, 2019). Schoenfeld (2004) suggests that the aims of mathematics instruction should be much broader than mastery of procedural content in a textbook. He asserts that learners need to be exposed to mathematical instructions which will make them reason mathematically and be able to solve non-routine problems, think logically, make connections, and access a wide range of problem-solving strategies. Hadar and Ruby (2019) argue that cognitive demand is a construct employed to characterize the cognitive processes involved in learning mathematics, be it in classroom activities, assessment materials, and textbook tasks.

Some research has been conducted on mathematics analysis in cognitive demand over the past two decades. For instance, Jones and Tarr (2007) examined the levels of cognitive demand required by probability tasks in middle grades mathematics textbooks. They analyzed mathematics textbooks from a historical perspective from four recent eras of mathematics education that is: New Math, Back to Basics, Problem Solving, and Standards of the two series, one popular and the other alternative. Their research was guided by the questions:

> What is the nature of the treatment of probability topics in middle grades mathematics textbooks? How has the nature of the treatment of probability changed over the past 50

> years and across popular textbooks series and alternative (or innovative) textbook series? More specifically, what levels of cognitive demand are required by tasks and activities related to probability, and what are the trends in the required level of cognitive demand over the past 50 years? (p. 6).

In this study among other results, they found out that over 85% of tasks for six series required low levels of cognitive demand (Jones & Tarr, 2007).

Vincent and Stacey (2008) examined nine Australian 8th-grade textbooks in terms of their procedural complexity, types of solving processes, degree of repetition, and proportion of 'application' problems and proportion of problems requiring deductive reasoning. They found that most textbooks include a high percentage of problems of low procedural complexity with sizable repetition and with non-presence of logical reasoning. Also in the United States of America (USA), Stylianides (2009) analyzed US mathematics textbooks to investigate the opportunities provided for learners to engage in reasoning-and-proving. The study found that on average approximately 40% of textbooks' tasks afforded learners with an opportunity to engage in reasoning and proving while more than 50 % with no opportunity at all for learner's engagement in reasoning and proving. Ding and Li (2010) also conducted a comparison on textbooks in the United States and China on cognitive demand and they found that high cognitive demand was made on students using Chinese mathematics textbooks. In the USA, Polikoff (2015) analyzed the alignment of three textbooks produced by major publishers in the US to curriculum standards and found that textbooks systematically stress much on procedures and memorization and underrate goals in the cognitive domain.

Wijaya et al., (2015) conducted a study where they investigated the opportunity-to-learn offered by Indonesian textbooks for solving context-based mathematics tasks and the relation of this opportunity-to-learn to students' difficulties in solving these tasks. They investigated the characteristics of tasks in textbooks from four perspectives: the type of context used in tasks, the purpose of context-based tasks, the type of information provided in tasks, and the type of cognitive demands of tasks. Among other things, they found that;

> Out of the context-based tasks, 45% are reproduction tasks requiring performing routine mathematical procedures, 53 % are connection tasks requiring linking different

mathematical curriculum strands, and only 2 % are reflection tasks, which are considered as tasks with the highest level of cognitive demand (p. 41).

Hadar (2017) conducted a study to examine whether opportunities provided in math textbooks to engage in tasks demanding different levels of understanding correlate with students' achievements on tasks demanding equivalent levels of understanding on a standardized exam. She evaluated two 8th grade mathematics textbooks used by students in the Arab community in Israel. She found out that cognitive demand in the textbooks are correlated to the students' achievement. Her study revealed that learners using the textbook which provide the opportunity to engage in tasks demanding higher levels of understanding, perform better than those using textbook providing opportunity to engage learners with low cognitive level. These findings clearly illuminate textbooks' ability to provide opportunities to learn mathematics.

Recently, Hadar and Ruby (2019) conducted a study to explore breadth and depth of understanding as addressed in mathematics textbooks where they compared opportunities for students to engage with mathematics, requiring different types and levels of understanding provided by the tasks in mathematics textbooks. Their study showed significant differences in the opportunities to learn in the cognitive domain that each mathematics textbook provides. These authors established that teachers using mathematics textbooks that place greater emphasis on routine tasks may not be able to realize the cognitive aims suggested in the curriculum. They suggested that it is important for teachers to be cognizant that the level of cognitive demand may broadly differ from one textbook to the other. This will enable them to consider bringing into the classroom activities that may engage learners with a complete spectrum of cognitively demanding mathematics. Hadar and Ruby (2019) concluded by recommending for a speedy discussion on textbook adjustment and arrangement to reflect both content and cognitive goals, including attention to both level of understanding and task complexity.

The studies above seem to support the idea that engaging learners with high cognitive demanding tasks has the potential to facilitate learners' deep understanding of mathematics and consequently being able to think mathematically and solve different problems. As discussed already, TIMSS, showed that high-performing countries engage learners with instructional tasks involving high cognitive demand, (Stigler & Hiebert, 2004).

In Malawi, textbook analysis research is very scarce. To my knowledge, Phiri (2018) conducted a comparative analysis of Malawian and Japanese primary mathematics textbooks to establish how best are textbooks designed to help learners achieve the noble objective of critical skills acquisition. In his study, he compared Malawian and Japanese textbooks in primary school level, learners aged 9. He found out, among other things, that Malawian textbooks have a teacher centered perspective, most of the activities that are described in them are designed to be carried out by the teacher while learners are just recipients of the information and not actively participating in them. In addition he found out that Japanese textbooks contain more sophisticated mathematics that enable learners to solve their daily life problems than the Malawian textbooks. Furthermore he found out that Japanese textbooks are designed to promote learners' independence and development of critical thinking skills than the Malawi textbooks. Moreover and most recently, Mwadzaangati (2019) examined in her study similarities and differences between circle geometric proof development tasks as they are presented in the Malawian Grade 11 mathematics textbook, and those that are designed and implemented by teachers in the classroom. She analyzed a textbook titled Strides in Mathematics 3 (Hau & Saiti, 2002) and found out that the textbook presented the geometric proof development tasks at a high level but the procedures that were used during task implementation by teachers in the classroom resulted in reduction of the cognitive level of the proof tasks. For example, she found that only one teacher of the three teachers involved in her study maintained the high cognitive level of the tasks during implementation while the other two teachers reduced the cognitive level of the tasks through the methods they used in the classroom during task implementation. Furthermore her study revealed that in addition to quality of textbooks, teachers' conceptual ability to make effective use of textbook content affected the set up and implementation of high cognitive level tasks that promote learners' understanding and discovery of deductive geometric proofs in the classroom (Mwadzaangati, 2019). Inspired by the revelation that different textbooks implementing the same curriculum can present the same topic in different ways, resulting in the inequity affordance of students' opportunities to learn (Hadar, 2017; Hadar & Ruby, 2019a; Mellor & Mellor, 2018; van Zanten & van den Heuvel-Panhuizen, 2018). The present study analyzed Malawian grade 11 mathematics textbooks in order to investigate the cognitive opportunities to learn mathematics provided by these textbooks through quadratic equations.

2.6 Theoretical frameworks

The present study was guided by the Mathematics Discourse in Instructional analytic framework for Textbook analysis (MDITx) developed by Ronda and Adler, (2017) and Mathematical analysis framework by Stein and Smith (1998). The MDITx framework is adapted from the Mathematical Discourse in Instruction (MDI) framework (Adler & Ronda, 2015). The MDITx framework has been chosen because it contains different levels for analyzing cognitive level of examples and tasks, thereby giving an insight of affordance/limitation of cognitive opportunities to learn in the quadratic equations.

The framework developed by Stein and Smith (1998) was chosen to establish the cognitive demand levels of tasks made available in the textbooks in the topic of quadratic equations. They define cognitive demand of a task as the complexity of the task that learners may experience in answering it. This framework was chosen to help answer the second research question; ‘to what extent are the examples and tasks in Malawian grade 11 secondary school mathematics textbooks engaging learners in cognitive demanding mathematics in the topic of quadratic equations?’ This is because the framework characterizes the cognitive demand of tasks as the required learners’ thinking processes in the course of solving a task. The framework further categorizes these thinking processes into memorization, procedure without connection to a concept, procedure with connection to concept and doing mathematics (Stein & Smith, 1998). This framework gives specific attention to tasks as they are present in textbooks and represent the relationship between students' learning and task implementation (Jones & Tarr, 2007).

The MDI Framework is a socio-cultural framework which focuses on a project of developing teachers’ mathematical discourse in instruction (Adler & Ronda, 2015). The MDI framework builds on variation theory which is a theory of learning that emphasizes variation as a necessary condition for learners to be able to discern new aspects of an object of Learning (Kullberg et al., 2017). The variation theory considers classroom lessons as enacted by the teacher as well as exploring variation in sets of instructional examples in mathematics lessons. The framework characterizes the teaching of mathematics as about mediating an object of learning (Marton & Tsui, 2004) via exemplification, and explanatory talk, while taking care of learner participation in mathematics discourse (Ronda & Adler, 2017. The MDI framework is characterized by four interacting components in the teaching of a mathematics lesson. Figure 1 shows these four components:

(i) Object of learning
(ii) Exemplification
(iii) Explanatory talk
(iv) Learner participation

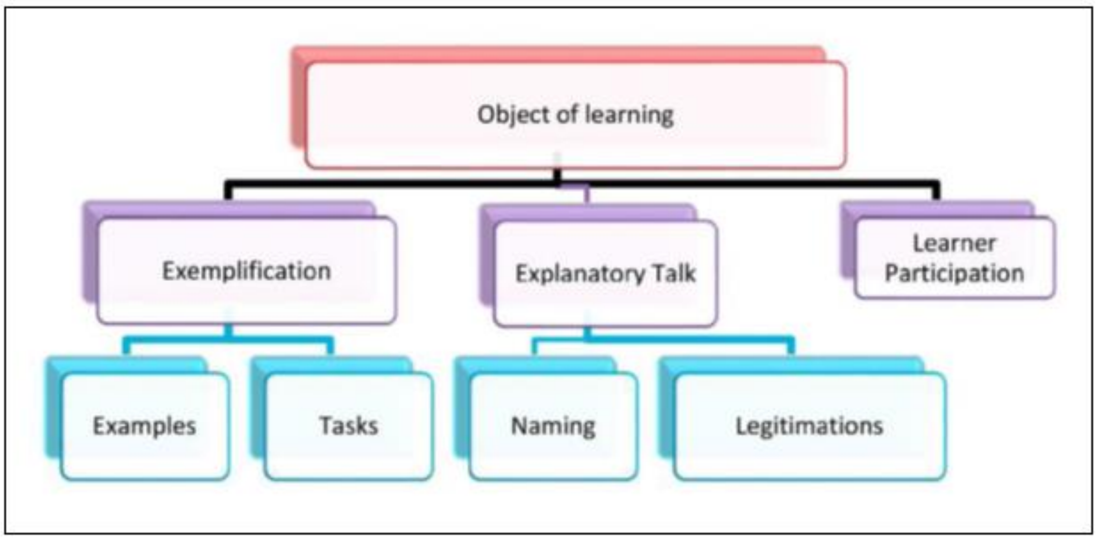


Figure 1: Mathematics Discourse in Instruction framework (Adler & Ronda, 2015, p. 239)

Ronda and Adler (2017) adapted the Mathematics Discourse in Instruction analytical tool for textbook (MDITx) but excluded the learner participation. They argue that exemplification (examples and tasks) and explanatory talk mediate mathematics in textbook lessons, but learner participation cannot be noticed through the textbook lesson. In the following section, I will explain the four components of the MDI framework.

2.6.1 Object of learning

Learning is always about conceptualizing something and making learner to aware of that thing. That things is what is referred to as, the 'object' of learning, and it is essential to the work of teaching (Adler & Ronda, 2015). Adler and Ronda, (2015) describe the object of learning, also known as the lesson goal as the focus of the lesson. They further explain that the object of learning has both content and capability components. The content in a lesson goal can be a mathematical concept, relationship or procedure. The object of learning may be determined from the intended lesson title. In this study, the object of learning is quadratic equations, and the capabilities expected to be developed in learners is factoring quadratic equations, solving quadratic equations and solving practical problems related to quadratic equations.

2.6.2 Exemplification

The MDI framework categorizes exemplification into two parts, examples and tasks. Watson and Mason (2005) highlight that tasks and examples are the raw material upon which instruction is overlaid. As already expressed above, an example is a particular case of larger class, from which one can reason and generalize and an instantiation of the content in focus Zodik and Zaslavsky (2008). For instance, solving the equation $q^2 - 5q - 2 = 0$ is an example of a quadratic equation. Examples in an example space can highlight features of the concept that is exemplified in a lesson (Adler & Ronda, 2015). In this study, the example space will be defined as the set of examples related to a particular object of learning (Ronda & Adler, 2017) contained in the learners' textbooks. Learners are supposed to experience a particular feature of the object of learning through examples contained in an example space (Ronda & Adler 2017). Mathematics textbooks can contain two types of example spaces; worked example spaces and example exercise spaces (Adler & Ronda 2015). Additionally these authors came up with three necessary aspects for a sequence of examples that constitute a basis for generalization including similarity, contrast and fusion. They also argue that similarity is when all aspects of an object of learning are the same across all example sets in a sample space. Usually similarity on its own, does not draw attention to the boundaries around a concept, and so does not demonstrate what the concept is not (Adler & Ronda, 2015). Contrast is when a set of examples bring attention to different classes of examples with some aspects of the object of learning varied and others kept invariant and make available opportunity for generality, while fusion is when more than one aspect of an object of learning is simultaneously varying across an example set (Adler & Ronda, 2015).

Ronda and Adler (2017, p.1102) define tasks "as what students are asked to do with the examples". They explain that tasks and examples are different but linked to each other in such a way that examples are selected to mediate the object of learning by making visible the features of the content that are key in mediating meanings of the object of learning. Tasks are designed to mediate the capabilities with respect to the content. Besides they also argue that an opportunity to learn is enhanced when learners are engaged with various tasks related to the object of learning. In addition they also argue that tasks should not only target the capabilities required in the object of learning, but also whether the tasks have the potential to engage the learners to make connections among features of mathematical content. Ronda and Adler (2017) categorize tasks into three level of cognitive demand. Tasks in the first level were those that require learners to carry out known procedures or facts which the learners already know at the time of engaging with the tasks. Tasks at the second level were those that required application of what is known in the current topic in relation to the object of learning. Tasks at the third level were those that demand the learners to make decisions as to the procedures and concepts that need to be used to answer the task. These tasks require learners to make connections with other mathematical concepts including the previous knowledge.

2.6.3 Explanatory talk

The MDI framework identifies name and legitimation as the components of explanatory talk. According to Adler and Ronda, (2015), naming and legitimation focus on what is done, how and why? All these aspects are related to examples and tasks. They also argue that the specific words that we use for naming mathematical concepts and the way we name the procedures or actions carried out on them affect learners' attention in particular ways. Legitimations are the mathematical and non-mathematical criteria that are communicated to substantiate the key steps in procedures or in statements about the object of learning. Adler and Ronda (2015) argue that the way learners come to think about the mathematical concepts is specifically determined by how the procedures and actions carried out on them have been named. In support of this argument, Wagner

(2015) explains that pointing things out and naming them draws one's attention to something in particular and gives that thing a signifier to facilitate communication about it.

Moreover, Ronda and Adler (2017) also add that word use shows the manner and extent of formal mathematics talk in the text particularly the talk on procedures and talk of concepts. They further argue that in their analysis of teachers' lessons, from the empirical data, they were able to identify types of word use during instruction and assigned corresponding levels to show the degree of appropriate and more formal uses of mathematical words and phrases. Non-mathematical words in this case include everyday language (e.g. 'over' in division, and/or ambiguous pronouns such as this, that, thing, to refer to objects acted on) during a mathematical procedure and action.

2.6.4 Learner participation

Adler and Ronda (2015) argue that learner participation in the MDI framework allows us to describe what learners are invited to say apart from the tasks being assigned to them. They suggest that learner participation is important in multilingual contexts. In addition to the mediational means discussed so far (examples, tasks, naming, legitimating criteria), learner participation reveals what learners are invited to say, and specifically whether and how learners have the opportunity to speak mathematically and to verbally display mathematical reasoning. Learner participation is an important aspect as it plays so many crucial roles in enhancing learners' understanding in a lesson (Adler & Ronda, 2015). However in MDI analytical tool for textbook analysis, learner participation is not included because learner participation cannot be noticed in textbook lessons (Ronda & Adler, 2017).

2.6.5. How the theoretical Frameworks are used in the study

This study focuses on some of the components of the MDITx framework when analyzing the content of the topic, quadratic equations in the four selected textbooks. The examples and tasks in the textbooks are analyzed since the research questions being addressed in this study are about exemplification. For instance, the first question addresses what cognitive opportunities to learn mathematics are provided by examples and tasks in the topic of quadratic equations in Malawian grade 11 secondary mathematics textbooks. The second question puts emphasis on to what extent the examples and tasks in Malawian grade 11 secondary school mathematics textbooks engage learners in cognitive demanding mathematics in the topic of quadratic equations. The MDITx framework has been chosen because it contains different levels for analyzing cognitive level of examples and tasks, thereby giving an insight of affordance/limitation of cognitive opportunities to learn in the quadratic equations.

The MDITx framework was used to analyze the example spaces, which is the unit of analysis, in the textbooks. In the example spaces, examples and tasks were analyzed separately to establish the answer to the research questions. The examples and tasks were coded according to the MDITx framework in Table 1

Table 1: MDI analytic tool for textbooks lessons (MDITx) (Adapted from (Ronda & Adler, 2017, p. 1106).

Examples	Tasks
Level 1—at least one of the pattern of variation	Level 1—carry out known procedures or use known concepts related to the object of learning (KPF only)
Level 2—any two of C, G, or F)	Level 2—carry out procedures involving the object of learning (includes CTP but no AMC codes)
Level 3—all the patterns of variation	Level 3—carry out Level 2 tasks plus that involve multiple concepts and connections (includes CTP and AMC codes)
Legend: C –Contrast G- Generalization F-Fusion)	KPF – Known procedure/fact CTP - Current topic procedure AMC – Application making connections

The examples were analyzed to establish the similarities and differences offered in the worked examples. Worked examples which could offer contrasting features were coded (C), while those offering contrasting and similarities leading to generalization were coded (G). If many features are changing simultaneously in an example space, it was coded (F). According to Ronda and Adler (2017), the example spaces whose examples are showing only one pattern of variation are categorized as level 1, providing little opportunities for learners to discern key features of the content, while the example spaces which are providing two different patterns of variations, are categorized as level 2 textbook. They further argue that the example spaces showing all the three patterns of variation are categorized as level 3. They say that level 3 textbooks provide the potential for learners to discern key features of the content. Finally, if the example spaces across the lesson in a textbook do not use any pattern of variation, Ronda and Adler argue that opportunities to discern key features of the content is not provided to learners. Then the code of this textbook is NONE.

The tasks were analyzed to establish which ones require procedures or facts already known by students at the time they are working on the solution to the tasks. These tasks were coded KPF (known procedures/facts). Tasks which required learners to use the procedures which they are currently learning and clearly shown in the worked examples were coded CTP (current topic procedures) and tasks which required the learners to apply the previously learned knowledge and connect to the topic under discussion were coded AMC (application and making connection) (Ronda & Adler 2017, p. 1106). The method chapter will further explain how different tasks were categorized in the topic of quadratic equations.

Cognitive demand of tasks

Guided by the mathematical task analysis framework by Smith and Stein (1998), tasks were further analyzed. Stein and Smith's (1998) framework was chosen to establish the cognitive demand levels of tasks made available in the textbooks in the topic of quadratic equations thereby enabling to answer the second research question. Stein and Smith define cognitive demand of a task as the complexity of the task that learners may experience in answering it. This framework explains four categories of tasks in terms of the cognitive processes required by learners to successfully complete a task which classifies tasks into two demand levels, lower and higher levels. The four categories of cognitive demands levels of tasks are: (i) memorization, (ii) procedures without connections to concepts or meaning, belonging to lower level, (iii) procedures with connections to concepts and meaning, and (iv) doing mathematics, belonging to higher level. In the lower level, memorization tasks involve exact reproduction of previously learnt facts without showing their algorithms while procedures without connections to concepts or meaning are tasks that require the use of an algorithm without showing understanding of how the algorithm works (Stein & Smith, 1998). These authors further argue that procedures with connections to concepts require some degree of cognitive effort because they involve thinking of how to apply a procedure to a task. Doing mathematics tasks demands considerable cognitive effort because the procedures are not known to the learners. Such tasks require that the students explore and understand the nature of mathematical concepts, processes, or relationships to be used in working out the solution to the tasks. The next chapter presents more details about how these frameworks have guided the textbook analysis. This coding process of example spaces and tasks has been explained in detail in the method chapter.

This chapter has looked at an overview on textbooks, examples and tasks. It has also presented the cognitive demand as related to tasks and some studies that have looked into the opportunity to learn cognitive demanding mathematics. Finally, the chapter has presented the theoretical frameworks that guided the study. The next chapter presents the methodology of the study.

CHAPTER 3
METHODOLOGY

This chapter presents the methodology of the study. It describes the design of the study, sampling, critical approach to analytical frameworks where approach to textbook analysis, coding of examples, coding of tasks, and tasks' level of cognitive demand were discussed. The chapter further describes ethical consideration, limitations of the study, and finally a brief summary of the main issues.

3.1 Design of the study

This study has adopted a mixed research methods approach to conducting research in order to, collect, analyze and interpret data in view of addressing the research questions. . Creswell, (2009, p .4) defines mixed methods research as "an approach to inquiry that combines or associates both qualitative and quantitative forms". I choose a mixed research method in order to get both breadth and in-depth understanding of issues surrounding the topic under study. Furthermore, mixed methods are useful in that they help the researcher to gain more understanding of the research problems, and it gives the study a greater strength than it would have if it only used a qualitative or a quantitative approach (Creswell, 2009). Content analysis was used to analyze the textbooks. The tasks and worked examples at each category and level of cognitive demand across the topic of quadratic equations in the textbooks were counted and recorded. This was not necessarily for comparing the quantity across the textbooks but to see the extent to which learners are given an opportunity to practice the mathematics in this topic.

3.2 Sampling

This study used a nonprobability sampling technique in selecting the textbooks to be analyzed. According to Cohen, Manion, and Morrison, (2007, p.113) "a nonprobability sample derives from the researcher targeting a particular group, in the full knowledge that it does not represent the wider population; it simply represents itself". There are several types of non-probability sampling such as: convenience sampling, quota sampling, dimensional sampling, purposive sampling and snowball sampling. This study employed a purposive sampling. The sample of the textbooks have been purposefully selected to best help me to understand the problem and the research questions under study, with respect to textbooks currently in use in Malawi secondary schools.

As explained previously in, in purposive sampling the researcher freely chooses the cases to be included in the sample (Cohen et al., (2007). To explore cognitive opportunities to learn mathematics in Malawian textbooks through quadratic equations, four of the eight approved core mathematics textbooks for senior secondary education in Malawi were selected to be analyzed. The four textbooks were chosen because they are among the current core textbooks which were approved by the Ministry of Education Science and Technology to be used by all schools in the country. Furthermore these four textbooks contain the topic quadratic equations, which is the topic of the study. The textbooks analyzed were:

1. Achievers Senior Secondary Mathematics Student's Book 3, (Textbook A), (Nyirenda & Okumu, 2018)
2. Arise with Mathematics Student's Book 3, (Textbook B), (Chimalizeni & Mwale, 2014)
3. Excel & Succeed Senior Secondary Mathematics Student's Book 3, (Textbook C), (Thomo, Maina, & Kirangi, 2017)
4. Target in Senior Secondary Mathematics Student's Book 3, (Textbook D), (Banda & Namakhwa, 2015)

Furthermore, only the topic "quadratic equations" was chosen for this study since this is the first topic that is covered at the first year of senior secondary education according to the Malawi mathematics curriculum providing the ground stone for senior secondary mathematics which prepares learners for the mathematics they will meet at the workplace or college levels. Furthermore, the topic of quadratic equations is related and has applications to much of the mathematics in all levels of senior secondary mathematics curriculum.

3.3 Critical discussion of analytical frameworks

Using the Mathematics Discourse in Instructional analytical tool (MDITx) (Ronda & Adler, 2017), the example spaces and tasks will be classified from level 1 to level 3 (see Table 1). Level 1 will be deemed as those promoting low cognitive demands and thereby limiting students' opportunities to learn whereas level 3 will be those promoting high cognitive demands consequently affording students' opportunities to learn mathematics. Then the example spaces and tasks in each category will be quantified to establish the level of the cognitive opportunity provided by each textbook. This is asserted by Hadar and Ruby (2019, p. 72) that "characterization of cognitive demands presented in this way has the potential as well as limitations of textbooks in terms of the opportunities to learn cognitively demanding mathematics". This thinking will be advanced, bearing in mind that there are many factors other than complexity of examples and tasks that contribute to opportunities to learn in mathematics textbooks. These factors may include instructional time, content covered in the instruction, quality of instruction and how the teachers use these resources in the enacted curriculum (Elliott & Bartlett, 2016).

3.3.1 Approach to the textbook analysis

The study was guided by Mathematical Discourse in Instructional analytical tools (MDITx) for textbooks (Ronda & Adler, 2017) and Mathematical task analysis (Stein & Smith, 1998) frameworks. The following steps were used to approach the textbook analysis: Firstly, by using the MDITx framework, the chapter in each textbook containing the topic quadratic equations was divided into blocks. The block was identified based on the three success criteria to be achieved by students after studying the topic of quadratic equations. According to MoEST (2013, p. 3-4), the following are the success criteria for the topic of quadratic equations: "Students must be able to: 1) factorise quadratic expressions; 2) solve quadratic equations, and 3) solve practical problems involving quadratic equations". The common characteristics of the block are that a block contains introduction (which includes definitions and explanations of some mathematical terms and concepts), worked examples followed by practice exercise (tasks) and some have included unit review exercises which are placed at the end of the unit.

Secondly, each block was divided into example spaces which was the unit of analysis. As previously stated the interest in this study is on exemplification, an example space was taken to be a section in the textbook containing worked examples and related tasks/exercises responding to a particular object of learning. Examples included all worked examples and explained tasks, while tasks included exercises, unit review exercises and activities which the learners are required to work on. In this study, a worked example was regarded as any example or task in which the suggested solution(s) has been explained and given by the author in the textbook. As stated by

Atkinson et al., (2000) worked examples provide an expert's problem- solving model by presenting a solution in a step by step for the learner to study and follow when confronted by a problem of similar nature. All worked examples and tasks in each example space were analysed and coded using MDITx. The example spaces and tasks were categorized accordingly using this framework from level 1 through to level 3.

Thirdly further analysis of cognitive level of tasks to establish the extent to which the textbooks are offering the opportunity to learn cognitive demanding mathematics in the topic of quadratic equations done using the Mathematical Task analysis by Stein and Smith (1998). This involved examining what each task required of the learners to do in order to successfully solve it. Then these requirements were related to the four categories of cognitive demands: memorization, procedure without connection to concepts, procedure with connection to concepts, and doing mathematics to determine their cognitive level. The tasks were then categorized into levels 1 through to 4 shown in Table 5 (see Results and Findings section, 4.3.3).

Lastly, the number of examples and tasks in each textbook were counted before the start of the analysis. If an example or a task has parts (a), (b) and (c) which can be worked out independent of each other, they were counted as three examples or three tasks. If the examples or tasks are connected to each other in such a way that the working of proceeding part depends on the results of the preceding part, they were counted as one example or one task (see Figure 2).

Example 1.12

Solve the following quadratic equations by factorisation.

(a) $x^2 - 8x = 0$

(b) $a^2 + 1 = 10$

(c) $2x^2 = 7x - 6$

(d) $100m^2 - 4 = 0$

Exercise 1.4

Solve each of the following quadratic equations by factorisation.

(a) $x^2 + 5x + 6 = 0$

(b) $x^2 - 5x + 6 = 0$

(c) $y^2 - y - 12 = 0$

(d) $y^2 + y - 12 = 0$

(e) $y^2 - y = 30$

(f) $10m^2 = m + 3$

(g) $t^2 - 8t - 65 = 0$

(h) $w^2 + 2w + 1 = 0$

(i) $a^2 = \frac{a}{16}$

(j) $b^2 + 11b = 0$

Adapted from Achievers Senior Secondary Mathematics Student's Book 3 page 10, 11

Exercise 1g

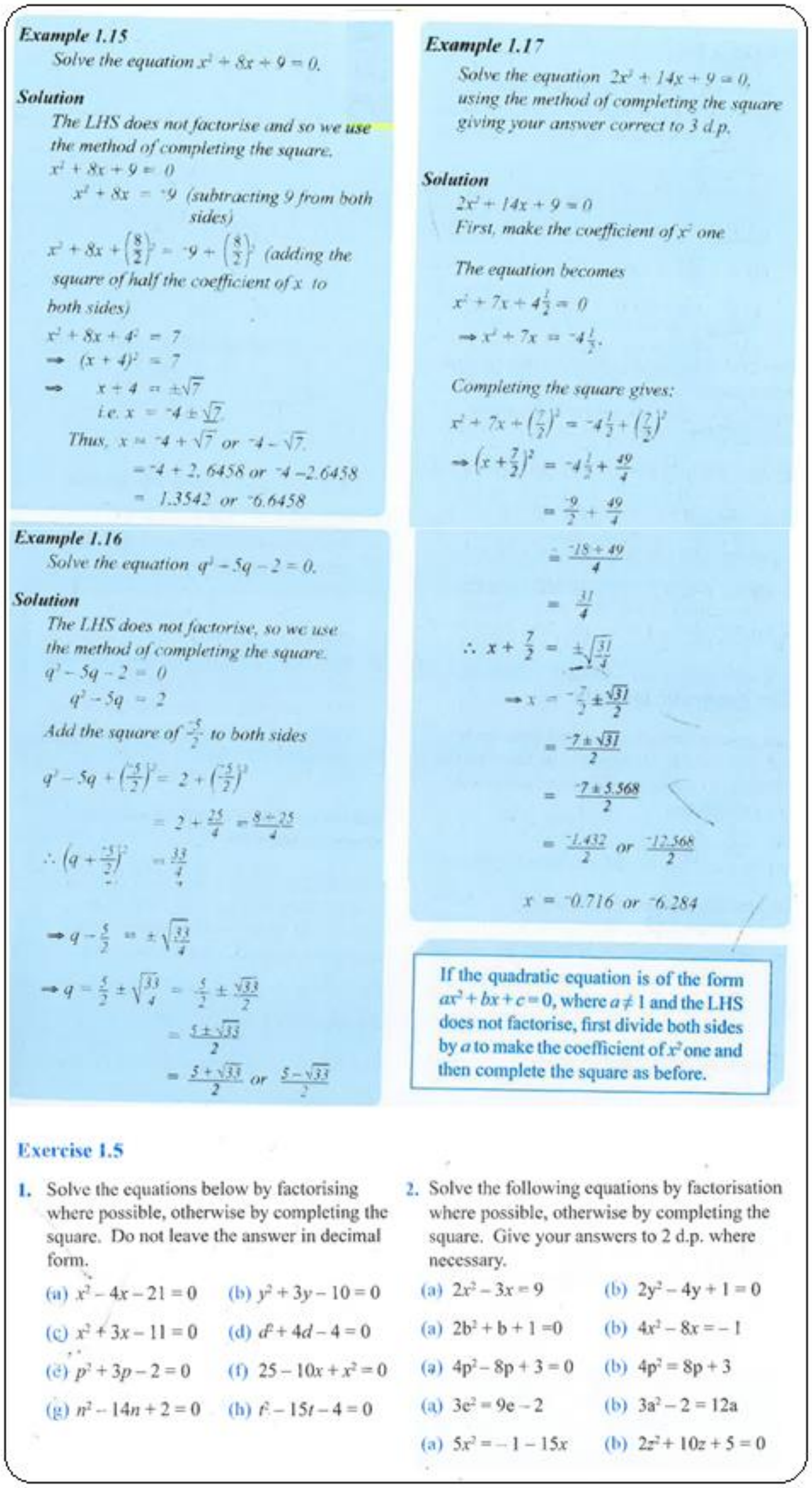
1. A right-angled triangle ABC has $<B = 90°$, $AB = x$ cm, $BC = 2$ cm longer than AB and AC is 4cm longer than AB.
   a) Illustrate this information on a diagram.
   b) Using this information show that $x^2 - 4x - 12 = 0$
   c) Solve the above equation and find the length of each of the three sides.

2. Benjamin is x years old and his sister Susan is 5 years younger. If the product of their ages is 36, form an equation in x and solve it to find Benjamin's and Susan's age.

Adapted from Arise with Mathematics Studnets' Book 3 page 12

Figure 2: Counting of examples and tasks

Figure 2 shows how examples and tasks are appearing in the textbooks. The way they were counted was as follows: Under example 1.12, these were counted as 4 examples. Tasks under exercise 1.4 were counted as 10 tasks, while in exercise 1g, task number 1 was counted as one task even though it has parts (a), (b) and (c), because to answer question 1(b) the learner has to answer question 1(a) first which will lead to the solution of 1(b).



Example 1.15

Solve the equation $x^2 + 8x + 9 = 0$.

Solution

The LHS does not factorise and so we use the method of completing the square.

$x^2 + 8x + 9 = 0$

$x^2 + 8x = {}^-9$ (subtracting 9 from both sides)

$x^2 + 8x + \left(\frac{8}{2}\right)^2 = {}^-9 + \left(\frac{8}{2}\right)^2$ (adding the square of half the coefficient of x to both sides)

$x^2 + 8x + 4^2 = 7$

$\Rightarrow (x + 4)^2 = 7$

$\Rightarrow x + 4 = \pm\sqrt{7}$

i.e. $x = {}^-4 \pm \sqrt{7}$

Thus, $x = {}^-4 + \sqrt{7}$ or ${}^-4 - \sqrt{7}$.

$= {}^-4 + 2.6458$ or ${}^-4 - 2.6458$

$= 1.3542$ or ${}^-6.6458$

Example 1.16

Solve the equation $q^2 - 5q - 2 = 0$.

Solution

The LHS does not factorise, so we use the method of completing the square.

$q^2 - 5q - 2 = 0$

$q^2 - 5q = 2$

Add the square of $\frac{^-5}{2}$ to both sides

$q^2 - 5q + \left(\frac{^-5}{2}\right)^2 = 2 + \left(\frac{^-5}{2}\right)^2$

$= 2 + \frac{25}{4} = \frac{8+25}{4}$

$\therefore \left(q + \frac{^-5}{2}\right)^2 = \frac{33}{4}$

$\Rightarrow q - \frac{5}{2} = \pm\sqrt{\frac{33}{4}}$

$\Rightarrow q = \frac{5}{2} \pm \sqrt{\frac{33}{4}} = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$

$= \frac{5 \pm \sqrt{33}}{2}$

$= \frac{5+\sqrt{33}}{2}$ or $\frac{5-\sqrt{33}}{2}$

Example 1.17

Solve the equation $2x^2 + 14x + 9 = 0$, using the method of completing the square giving your answer correct to 3 d.p.

Solution

$2x^2 + 14x + 9 = 0$

First, make the coefficient of x^2 one

The equation becomes

$x^2 + 7x + 4\frac{1}{2} = 0$

$\Rightarrow x^2 + 7x = {}^-4\frac{1}{2}$.

Completing the square gives:

$x^2 + 7x + \left(\frac{7}{2}\right)^2 = {}^-4\frac{1}{2} + \left(\frac{7}{2}\right)^2$

$\Rightarrow \left(x + \frac{7}{2}\right)^2 = {}^-4\frac{1}{2} + \frac{49}{4}$

$= \frac{^-9}{2} + \frac{49}{4}$

$= \frac{^-18 + 49}{4}$

$= \frac{31}{4}$

$\therefore x + \frac{7}{2} = \pm\sqrt{\frac{31}{4}}$

$\Rightarrow x = {}^-\frac{7}{2} \pm \frac{\sqrt{31}}{2}$

$= \frac{^-7 \pm \sqrt{31}}{2}$

$= \frac{^-7 \pm 5.568}{2}$

$= \frac{^-1.432}{2}$ or $\frac{^-12.568}{2}$

$x = {}^-0.716$ or ${}^-6.284$

If the quadratic equation is of the form $ax^2 + bx + c = 0$, where $a \neq 1$ and the LHS does not factorise, first divide both sides by a to make the coefficient of x^2 one and then complete the square as before.

Exercise 1.5

1. Solve the equations below by factorising where possible, otherwise by completing the square. Do not leave the answer in decimal form.

(a) $x^2 - 4x - 21 = 0$ (b) $y^2 + 3y - 10 = 0$

(c) $x^2 + 3x - 11 = 0$ (d) $d^2 + 4d - 4 = 0$

(e) $p^2 + 3p - 2 = 0$ (f) $25 - 10x + x^2 = 0$

(g) $n^2 - 14n + 2 = 0$ (h) $t^2 - 15t - 4 = 0$

2. Solve the following equations by factorisation where possible, otherwise by completing the square. Give your answers to 2 d.p. where necessary.

(a) $2x^2 - 3x = 9$ (b) $2y^2 - 4y + 1 = 0$

(a) $2b^2 + b + 1 = 0$ (b) $4x^2 - 8x = -1$

(a) $4p^2 - 8p + 3 = 0$ (b) $4p^2 = 8p + 3$

(a) $3e^2 = 9e - 2$ (b) $3a^2 - 2 = 12a$

(a) $5x^2 = -1 - 15x$ (b) $2z^2 + 10z + 5 = 0$

Figure 3: Example space (Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, pages 9-10).

Furthermore, the solution for 1(c) depends on the results of 1(b). So, these examples are connected and not independent of each other. The total number of examples and tasks were recorded as shown in Table 2 (see Results and Findings section 4.1).

Figure 3 shows an example space containing three examples with proposed solutions and eighteen tasks responding to the object of learning "solving quadratic equations" by the method of completing the square. This type of structure, where tasks follow worked examples is common in all the four textbooks. Then the example spaces and tasks were coded separately according to the MDITx framework. The following sections explain how the example spaces and tasks were coded.

3.3.2 Coding of example spaces

Example space in this study is a unit of analysis and is defined as a section containing sets of worked examples and tasks related to a particular object of learning. The examples in an example space were analyzed to establish the similarities and differences offered in the worked examples. Worked examples which could offer contrasting features were coded (C), while those offering contrasting and similarities leading to generalization were coded (G) and if many features are changing simultaneously in an example space, it was coded (F). Example space in all the four textbooks contained at least two worked examples presented together except one example space in textbook B, which was on solving quadratic equations by using quadratic formula and one example space in textbook D, which was on formulating quadratic equations given roots.

Example 1.18

Use the quadratic formula to solve the equation
$2x^2 + x - 11 = 0$

Solution

We start by identifying and stating the corresponding values of a, b and c. Thus, in
$2x^2 + x - 11 = 0$, $a = 2$, $b = 1$ *and* $c = {}^-11$.

Then state the formula:

$$x = \frac{{}^-b \pm \sqrt{b^2 - 4ac}}{2a}$$

We now substitute for a, b and c in the formula so:

$$x = \frac{{}^-b \pm \sqrt{b^2} - 4ac}{2a}$$

$$= \frac{{}^-1 \pm \sqrt{(1)^2 - 4(2)({}^-11)}}{2(2)} \quad \text{...... substitution}$$

$$= \frac{{}^-1 \pm \sqrt{(1)^2 - 4(2)({}^-11)}}{2(2)} \quad \text{...... substitution}$$

$$= \frac{{}^-1 \pm \sqrt{1 + 88}}{4}$$

$$= \frac{-1 \pm \sqrt{89}}{4}$$

$$= \frac{{}^-1 \pm 9.434}{4}$$

$$= \frac{{}^-1 + 9.434}{4} \text{ or } \frac{-1 - 9.434}{4}$$

$$= \frac{8.434}{4} \text{ or } \frac{{}^-10.434}{4}$$

$$= 2.1085 \text{ or } {}^-2.6085 \text{ (4dp)}$$

Example 1.19

Find the roots of the equation $2x^2 + x - 12 = 0$, *giving your answer correct to 2 d.p.*

Solution

Comparing $2x^2 + x - 12 = 0$ *with* $ax^2 + bx + c$ *gives* $a = 2$, $b = 1$ *and* $c = {}^-12$

Now, from the formula $x = \frac{{}^-b \pm \sqrt{b^2 - 4ac}}{2a}$

$$\therefore x = \frac{{}^-1 \pm \sqrt{(1)^2 - 4 \times (2) \times ({}^-12)}}{2 \times (2)}$$

$$= \frac{{}^-1 \pm \sqrt{1 + 96}}{4}$$

$$= \frac{{}^-1 \pm \sqrt{97}}{4}$$

$$= \frac{{}^-1 \pm 9.849}{4}$$

$$= \frac{8.849}{4} \text{ or } \frac{{}^-10.849}{4}$$

$$= 2.21 \text{ or } {}^-2.71$$

Figure 4: Example space coded as G (Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, page 9)

Example space in Figure 4 consists of two examples about solving quadratic equation using the quadratic formula and was coded G, generalization, because there is similarity in the use of quadratic formula to solve the quadratic equation, and in all the two examples, the value of 'a' and 'b' are the same, which is positive 2 and positive 1 respectively. Learners may think that when using the formula the value of “a” and “b” has to be 2 and 1 respectively all the time. Furthermore, the value of ‘c’ has a negative sign in both cases. In addition, there is similarity in the way the roots of the equation in both cases have been expressed. They are all decimal fractions. In this case, there is no variation between the two examples in terms of structure and solution form, but learners are able to see similarities across the two examples. For learners to discern the object of learning and make sense of the mathematics presented to them, there is a need to see contrasting features (Marton & Pang, 2006).

3.3.3 Coding of tasks

Tasks which require procedures or facts already known by students at the time they are working on the solution to the tasks were coded KPF (known procedures/facts). Tasks which required learners to use the procedures which they are currently learning and clearly shown in the worked examples were coded CTP (current topic procedures), and finally tasks which required the learners to apply the previously learned knowledge and connect to the topic under discussion were coded AMC (application and making connection) (Ronda & Adler 2017, p. 1106).

In Figure 5, Exercise 1.5, tasks 1 through 7 were coded KPF because to find a solution to these, learners will use the known procedures/fact they already know at this level from the previously elaborated and worked examples preceding this set of tasks. Furthermore, all that is needed is just to calculate half of the coefficient of x in case of task 1 or coefficient of y in case of task 2 then square that number. This gives the solution to the task. All of these are basic skills already known by learners at this level. Similarly tasks 1 (a) and (b), 2 (a) and (b) in Figure 5 were coded KPF because to find a solution to these, learners will use the known procedures/fact they already know at this level from their junior secondary mathematics that if $ab = 0$, then either $a = 0$ or $b = 0$.

Exercise 1.5

What must be added to each of the following to form a perfect square?

1. $x^2 + 14x$
2. $y^2 - 6y$
3. $p^2 - 9p$
4. $t^2 + t$
5. $x^2 + 8xy$
6. $p^2 - 7pq$
7. $a^2 - \frac{49}{16}a$

(Adapted from Achievers Senior Secondary Mathematics Students' Book 3, page 13)

Exercise 1.3

Solve the following equations. If an equation has non-rational roots, leave the answer in the form given in Example 1.9.

1. (a) $(x + 4)(x + 5) = 0$
(b) $(x + 3)(2x - 5) = 0$

2. (a) $(2x - 3)(2x + 7) = 0$
(b) $(3x - 4)(4x - 1) = 0$

(Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, page 5)

Figure 5: Tasks coded KPF

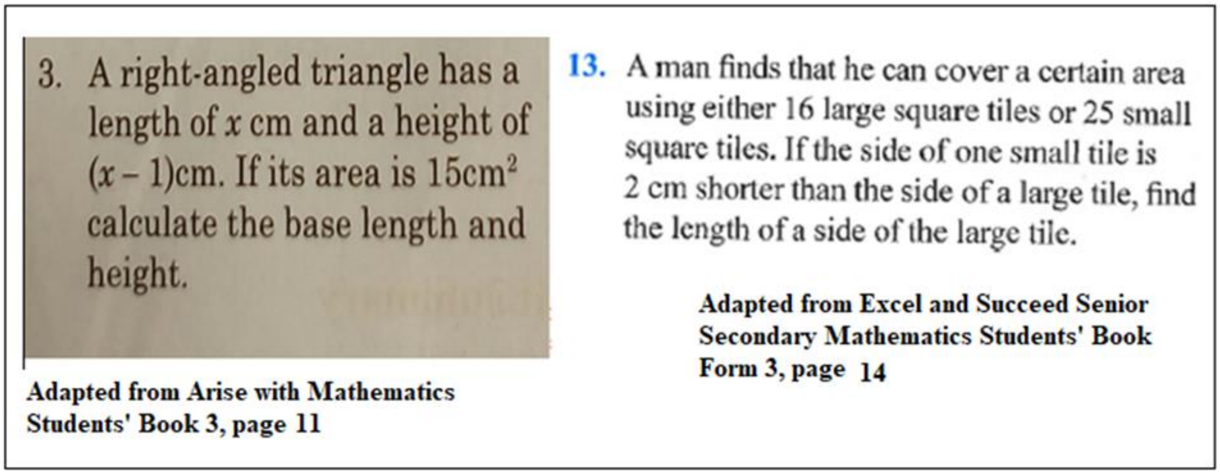
In tasks 2 through 6 in Figure 6, learners are required to solve the quadratic equations using either factor method or completing the square which are the current topic under discussion. Therefore, they were coded CTP because the learners will be required to use the currently learned procedures, as shown in the worked examples space.

Solve the following equations by factorisation where possible, otherwise by completing the square. Give your answers to 2 d.p. where necessary.

2.	(a) $2x^2 - 3x = 9$	(b) $2y^2 - 4y + 1 = 0$
3.	(a) $2b^2 + b + 1 = 0$	(b) $4x^2 - 8x = -1$
4.	(a) $4p^2 - 8p + 3 = 0$	(b) $4p^2 = 8p + 3$
5.	(a) $3e^2 = 9e - 2$	(b) $3a^2 - 2 = 12a$
6.	(a) $5x^2 = -1 - 15x$	(b) $2z^2 + 10z + 5 = 0$

Figure 6: Tasks coded CTP (Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, pages 10)

Tasks 3 and 13 in Figure 7 were coded AMC, because to work out the solution to this, learners need to apply the previously learnt knowledge not in this topic and make connections to those concepts and the current topic. For instance, there is a need to connect to concepts of area of triangles or rectangles and decide on what could represent the dimension then formulate quadratic equations. They have to choose the method to use to solve the equation formed either to use factorization, or completing the square or using quadratic formula. To successfully complete these tasks, there is a need to check the solution and verify which roots are correct according to the question. They may need to draw diagrams to visualize and understand the problem better.


3. A right-angled triangle has a length of x cm and a height of $(x-1)$cm. If its area is 15cm^2 calculate the base length and height.

Adapted from Arise with Mathematics Students' Book 3, page 11

13. A man finds that he can cover a certain area using either 16 large square tiles or 25 small square tiles. If the side of one small tile is 2 cm shorter than the side of a large tile, find the length of a side of the large tile.

Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, page 14

Figure 7: Tasks coded AMC

3.3.4 Cognitive demand level of tasks.

To establish the cognitive demand levels of tasks made available in the textbooks, in the topic of quadratic equations, thereby enabling to answer the second research question, the mathematical task analysis framework by Smith and Stein (1998) was used. The characteristics of tasks were analyzed and classified according to cognitive demand levels. According to Smith and Stein (1998), the tasks in a textbook can be categorized as low- level tasks and high-level tasks. They further argue that low-level tasks can be classified into tasks requiring memorization and tasks requiring procedure without connection to the concept, while high-level tasks can be classified into task requiring procedures with connection to the concept and those requiring doing mathematics. In this study, tasks requiring memorization will be referred to as level 1, procedure without connection, level 2, procedure with connection, level 3 and doing mathematics, level 4.

The tasks such as those in Exercise 1.5, questions 1 through 7 in Figure 5 were regarded as level 1 because they proceeded worked examples with an explanation and procedures on how to find the term to be added to the given quadratic expression to make it a perfect square. Therefore, the learners will complete this task by just following and reproducing the procedures laid down in the textbook. Thus, “to make an expression $x^2 + bx$ a perfect square the number to be added is the square of half the coefficient of x” (Thomo et al., 2017, p. 6). No ambiguity exists in the task and little cognitive demand is needed to complete it. Similarly in Exercise 1.3, questions 1 (a) and (b), and 2 (a) and (b) were regarded as those requiring learners to remember facts. For instance tasks 1(a), the learners in using the fact that that if ab = 0 then either a = 0 or b = 0, they will get$(x + 4)(x + 5) = 0\ then\ x + 4 = 0\ or\ x + 5 = 0, thus\ x = -4\ or -5.$ This is a known procedure and the learners are just required to memorize it. The tasks in Exercise 1.3 and Exercise 1.5 in Figure 5 were regarded as low cognitive demands (Level 1) as it requires limited cognitive demand for successful completion and little complexity exists about what needs to be done and how to do it (Stein & Smith, 1998).

Furthermore, tasks such as those in questions 2 through 6 in Figure 6 were regarded as level 2 because the procedure to follow has been suggested in the task (i.e. to use factoring or completing the square method). In the textbook before this set of tasks, there is a worked example with a proposed solution and explanation which shows how to solve quadratic equations by factor method or completing the square. Furthermore, the tasks such as those in Figure 5 and 6 were of low level because the focus is on getting the correct answer not necessarily developing mathematical understanding. There is little ambiguity in these tasks and they require low cognitive demand to work out the solutions (Stein & Smith 1998). The majority of the tasks in the four textbooks were of low level (see Table 4).

The tasks such as those in question 3 in Figure 7 were regarded as of level 3, procedures with connection to the concept, because a question is asked without suggesting the pathway to follow in order to find the solution and there are no proceeding worked examples that learners may refer to in working the solution to the task. Even though procedures of finding the area of a triangle may be followed, this has to be connected to the concept of quadratic equations and the procedures may not be followed mindlessly but with the aim of formulating a quadratic equation that will be solved later to find the base length and height of a triangle. The procedures will be followed for the purpose of developing deeper levels of understanding of mathematical concepts and ideas (Stein & Smith 1998). Furthermore, the task requires learners to connect the formula for finding the area of a triangle and relate it to information given in the task. After that, the learner has to form a quadratic equation and make a decision as to which method of solving the quadratic equation formed should he/she use. Since no previous procedures are given, there is a need for a learner to think of how he/she will proceed on this task. This demands some considerable reasoning.

Similarly, question 13 in Figure 7 was regarded as a level 4 task, doing mathematics because it requires complex thinking which may involve diagrams to figure out how to handle the solution to the task. It is a non-algorithmic question where by the solution procedure has not been given and learners may require to use previous acquired knowledge, connecting to the situation under discussion. Thus, a learner must probably to go through the following steps: decide what should represent the dimension, formulate quadratic equation, deciding which method to use to solve the created equation, checking the roots for the correct solution according to the concept of length and width of a rectangles. In some cases, the diagrammatic representation may be needed to aid the learner in solving these types of tasks. In addition, to complete this task, students may need to understand several mathematical concepts, such as area of shapes, formulation of quadratic equations, and diagrams may be drawn to visualize the task. These types of tasks require high cognitive demand to complete, and they were available in all the four textbooks analyzed. However, there were very few of these tasks in numbers as compared to the other categories. The tasks were counted and recorded for further discussion (see Table 5).

3.4 Ethical Consideration

According to Creswell (2009), research is a moral and ethical enterprise, and it should aim to ensure privacy and interests of research participants to avoid inflicting harm on them for taking part in the study. In view of this since I was doing textbook analysis, some ethical issues were considered as explained in this section. Johansson (2006) argues that ethics concerns discretion and truthfulness in relation to the research subjects. She further suggests that all contributors to the study must be recognized. In this study all relevant personnel and organizations that have contributed to my study have been acknowledged. Furthermore, those personnel whose papers I have used in this research have been referenced according to the general principles of American Psychological Association (APA) 6th.

Another ethical consideration is openness where all research methods and techniques should be made public so that others can scrutinize, criticize, test or reproduce (Kilpatrick, 1993). In this study, I have clearly stated the methodology and design of the study. I have also explained how the two frameworks, which have guided the study, have been used.

Furthermore, I have also been concerned with ethical consideration about the relevance and relatedness of the research to the field of mathematics education (Kilpatrick, 1993). In this research, I have included the section of implication of the findings to the practice and explained some of the relevance of the findings.

In addition, another aspect is the ethical consideration of credibility, which Kilpatrick (1993) argues that this involves grounding and justifying the findings and the conclusion in the data. He further alluded that this criteria enables others to verify or refute the conclusions drawn, the arguments and interpretations that are explicitly presented in the texts. Concerning this criteria, I have explained all the findings with the support of figures and tables wherever necessary.

3.5 Limitation of the Study

The first limitation is that this study only considered one topic in the textbooks selected. It could be better if more or all topics were to be studied to get a general view of the findings. However, the research can be replicated. Secondly, the research did not consider how the teachers use the textbooks which has a major impact on the status of cognitive level of tasks.

This chapter has given a description of the design of the study, sampling method used, how the content analyzed, and textbook analysis was approached. It has also presented how the coding of examples and tasks were done, ethical consideration, and finally some limitations of the study have been raised. The next chapter presents and discusses the results and findings of the study.

CHAPTER 4
RESULTS AND FINDINGS

In this chapter, the results and findings of the study are presented and discussed. The chapter discusses the results and findings according to exemplification as highlighted in the Mathematical Discourse in Instruction Textbook (MDITx) analysis framework, and Mathematical task analysis framework, the theoretical frameworks guiding the study. The chapter ends with a brief summary of the main issues discussed.

4.1 Number of examples and tasks in the textbooks

The findings show that all the four textbooks follow a format where the chapter under the topic of quadratic equations is divided into sections according to the learning objective. These sections are further divided into sub-section according to the teaching, learning and assessment activities. Each sub-section starts with an introduction, then some worked examples followed by tasks related to the worked out examples as already alluded to. These sub sections also contain a set of worked examples and tasks related to a particular object of learning referred to as the example space which is also the unit of analysis in this study. Examples and tasks in the example spaces were analyzed separately. The way the example spaces and tasks are structured is almost similar in all the textbooks. All the textbooks are starting with example spaces and tasks on factorization of quadratic equations, followed by example spaces and tasks on solving quadratic equations and finishing with example spaces and tasks on solving practical problems involving quadratic equations. The results reveal that moving across the lesson of quadratic equations in all the textbooks, there are many examples showing variations with respect to quadratic equations which was the object of learning. The number of examples and tasks were counted in each textbook.

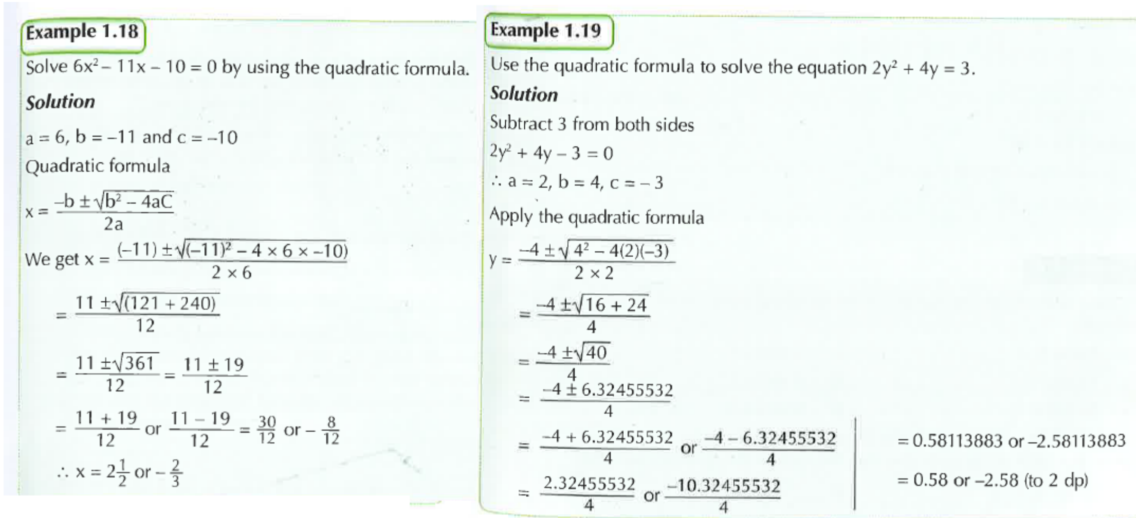
Table 2: Number of examples and tasks in each textbook

	Textbook A	Textbook B	Textbook C	Textbook D	Total
Number of examples	31	16	30	21	98
Number of tasks	109	119	167	137	532

Table 2 shows that Textbook A and C have almost the same number of examples, 31 and 30 respectively and textbook B has the least number of examples, 16, while Textbook D has 21 examples. It can also be seen from Table 2 that textbook C has a largest number of tasks (167) followed by textbook D (137), Textbook A has the smallest number of tasks (109) and Textbook B has slightly more tasks (119) than Textbook A in the topic of quadratic equation. In total 98 examples and 532 tasks were analyzed in this study from the four textbooks (see Table 2).

4.2. Number of coded examples and examples spaces

As stated previously, example spaces in all the four textbooks contained at least two worked examples presented together. Findings show that many example space contain examples in which learners can see the similarities (G) and differences (C). In few cases example spaces contained worked examples which could offer many contrasting features at the same time, fusion (F). Figure 5 shows an example space with two patterns of variation.



Example 1.18

Solve $6x^2 - 11x - 10 = 0$ by using the quadratic formula.

Solution

$a = 6$, $b = -11$ and $c = -10$

Quadratic formula

$x = \dfrac{-b \pm \sqrt{b^2 - 4aC}}{2a}$

We get $x = \dfrac{(-11) \pm \sqrt{(-11)^2 - 4 \times 6 \times -10)}}{2 \times 6}$

$= \dfrac{11 \pm \sqrt{(121 + 240)}}{12}$

$= \dfrac{11 \pm \sqrt{361}}{12} = \dfrac{11 \pm 19}{12}$

$= \dfrac{11 + 19}{12}$ or $\dfrac{11 - 19}{12} = \dfrac{30}{12}$ or $-\dfrac{8}{12}$

$\therefore x = 2\frac{1}{2}$ or $-\frac{2}{3}$

Example 1.19

Use the quadratic formula to solve the equation $2y^2 + 4y = 3$.

Solution

Subtract 3 from both sides

$2y^2 + 4y - 3 = 0$

$\therefore a = 2, b = 4, c = -3$

Apply the quadratic formula

$y = \dfrac{-4 \pm \sqrt{4^2 - 4(2)(-3)}}{2 \times 2}$

$= \dfrac{-4 \pm \sqrt{16 + 24}}{4}$

$= \dfrac{-4 \pm \sqrt{40}}{4}$

$= \dfrac{-4 \pm 6.32455532}{4}$

$= \dfrac{-4 + 6.32455532}{4}$ or $\dfrac{-4 - 6.32455532}{4}$ $= 0.58113883$ or -2.58113883

$= \dfrac{2.32455532}{4}$ or $\dfrac{-10.32455532}{4}$ $= 0.58$ or -2.58 (to 2 dp)

Figure 8: Example space coded as G, C, (Adapted from Achievers Senior Secondary Mathematics Students' Book 3, page17)

Example space in Figure 8 consists of two examples about solving quadratic equation using the quadratic formula and was coded G and C. In these two examples there is similarity in using the quadratic formula and values of 'a' being greater than 1, and the learners can notice the difference in values of 'a' as well as 'b'. This may put across the message that the values of 'a' and 'b' can be varied. In addition, the constant term in the second example is on the other side of the equal sign which brings in a variation between the two examples and learners may see the difference in structure of the two quadratic equations. The presence of this variant in the second quadratic equation in this example space has the potential of bringing to the attention of the learner about the importance of expressing a quadratic equation in the general form of $ax^2 + bx + c = 0$ before identifying the values of 'a', 'b', and 'c' to be used in the quadratic formula. Furthermore, the roots of the first example have been expressed as fraction while in the second example they have been expressed as decimal fractions. This may add on to the contrasting idea thereby allowing learners to understand different presentations of roots of a quadratic equation. This may enable learners to generalize the object of learning through seeing similarities and contrasting. These types of example spaces are categorized as level 2 because they were able to display two patterns of variation within the set of examples (Ronda & Adler, 2017).

Example 1.15

Solve the equation $x^2 + 8x + 9 = 0$.

Solution

The LHS does not factorise and so we use the method of completing the square.

$x^2 + 8x + 9 = 0$

$x^2 + 8x = {}^-9$ *(subtracting 9 from both sides)*

$x^2 + 8x + \left(\frac{8}{2}\right)^2 = {}^-9 + \left(\frac{8}{2}\right)^2$ *(adding the square of half the coefficient of* x *to both sides)*

$x^2 + 8x + 4^2 = 7$

$\Rightarrow (x + 4)^2 = 7$

$\Rightarrow x + 4 = \pm\sqrt{7}$

i.e. $x = {}^-4 \pm \sqrt{7}$.

Thus, $x = {}^-4 + \sqrt{7}$ *or* ${}^-4 - \sqrt{7}$.

$= {}^-4 + 2.6458$ *or* ${}^-4 - 2.6458$

$= 1.3542$ *or* ${}^-6.6458$

Example 1.16

Solve the equation $q^2 - 5q - 2 = 0$.

Solution

The LHS does not factorise, so we use the method of completing the square.

$q^2 - 5q - 2 = 0$

$q^2 - 5q = 2$

Add the square of $\frac{-5}{2}$ *to both sides*

$q^2 - 5q + \left(\frac{-5}{2}\right)^2 = 2 + \left(\frac{-5}{2}\right)^2$

$= 2 + \frac{25}{4} = \frac{8 + 25}{4}$

$\therefore \left(q + \frac{-5}{2}\right)^2 = \frac{33}{4}$

$\Rightarrow q - \frac{5}{2} = \pm\sqrt{\frac{33}{4}}$

$\Rightarrow q = \frac{5}{2} \pm \sqrt{\frac{33}{4}} = \frac{5}{2} \pm \frac{\sqrt{33}}{2}$

$= \frac{5 \pm \sqrt{33}}{2}$

$= \frac{5 + \sqrt{33}}{2}$ *or* $\frac{5 - \sqrt{33}}{2}$

Example 1.17

Solve the equation $2x^2 + 14x + 9 = 0$, *using the method of completing the square giving your answer correct to 3 d.p.*

Solution

$2x^2 + 14x + 9 = 0$

First, make the coefficient of x^2 *one*

The equation becomes

$x^2 + 7x + 4\frac{1}{2} = 0$

$\Rightarrow x^2 + 7x = {}^-4\frac{1}{2}$.

Completing the square gives:

$x^2 + 7x + \left(\frac{7}{2}\right)^2 = {}^-4\frac{1}{2} + \left(\frac{7}{2}\right)^2$

$\Rightarrow \left(x + \frac{7}{2}\right)^2 = {}^-4\frac{1}{2} + \frac{49}{4}$

$= \frac{-9}{2} + \frac{49}{4}$

$= \frac{-18 + 49}{4}$

$= \frac{31}{4}$

$\therefore x + \frac{7}{2} = \pm\sqrt{\frac{31}{4}}$

$\Rightarrow x = {}^-\frac{7}{2} \pm \frac{\sqrt{31}}{2}$

$= \frac{-7 \pm \sqrt{31}}{2}$

$= \frac{-7 \pm 5.568}{2}$

$= \frac{-1.432}{2}$ *or* $\frac{-12.568}{2}$

$x = {}^-0.716$ *or* ${}^-6.284$

Figure 9: Figure 9: Example space coded as G, C, F (Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, page 9)

Figure 9 shows an example space consisting of three examples which are about solving quadratic equations by completing the square method and this was coded G, C, and F. In this example space there is similarity in the approach to the solution as well as similarity in the coefficient of x^2 in the first and second equation which is equal to 1. Moving from the first example to the second, learners may see differences in the coefficient of p in the second example. The second one is bringing in a fraction in the equation as the learner will be making the expression in the left hand a perfect square, and this variation is resulting in having factors with one term as a fraction which is different from the first example. This may enable the learners to notice the different forms of presenting solutions of quadratic equations. Furthermore moving to example three, on top of bringing in fractions into the equations, the coefficient of x^2 is also varied bringing simultaneous changes, fusion (Ronda & Adler, 2017), to the quadratic equation which may be noticed by learners. The presence of a third example in this example space is significant. By keeping the solution process constant and varying the coefficient of x^2 to a number greater than 1, this can draw the attention of learners that completing the square method should be used in a quadratic equation, $ax^2 + bx + c = 0$ when the value of ‘a’ is equal to 1 (Thomo et al., 2017). These are the types of example space that brings about the discernment of the object of learning.

Example 1.7

Solve the equation $2x^2 - 22x + 60 = 0$

Solution

We first factorise the LHS

Thus, $2x^2 - 22x + 60$

$$= \underline{2x^2 - 12x} - \underline{10x + 60}$$
$$= 2x(x-6) - 10(x-6)$$
$$= (x-6)(2x-10)$$

$\therefore (x-6)(2x-10) = 0$

If $(x-6)(2x-10) = 0$

either $x - 6 = 0$ or $2x - 10 = 0$

i.e. $x = 6$ or $x = 5$.

Alternatively

$2x^2 - 22x + 60 = 0$

$\Rightarrow 2(x^2 - 11x + 30) = 0$

$\Rightarrow 2(x-6)(x-5) = 0$

There are three factors but only $(x-6)$ and $(x-5)$ give the same answer as before.

$x - 6 = 0$ or $x - 5 = 0$

$x = 6$ $\quad x = 5$

Example 1.8

Solve the equation $(2x-5)^2 = 9$.

Solution

If $(2x-5)^2 = 9$,

$\sqrt{(2x-5)^2} = \pm\sqrt{9}$

then $2x - 5 = \pm 3$

$\therefore 2x = 5 \pm 3$

i.e. $2x = 8$ or 2 (since $5 + 3 = 8$ and $5 - 3 = 2$)

$\Rightarrow x = 4$ or 1

Note that you could also solve this equation by first expanding the LHS and rearranging the equation in the form $ax^2 + bx + c = 0$ and then factorising the LHS.

Alternative method

$(2x-5)^2 - 9 = 0$

$(2x-5)^2 - 3^2 = 0$

$(2x - 5 - 3)(2x - 5 + 3) = 0$

Example 1.9

Solve the equation $p^2 - 10p + 25 = 8$

Solution

$p^2 - 10p + 25 = 8$

$(p-5)^2 = 8$ (factorising LHS)

$\Rightarrow p - 5 = \pm\sqrt{8}$

$\Rightarrow p = 5 \pm \sqrt{8}$

Thus, $p = 5 + \sqrt{8}$ or $p = 5 - \sqrt{8}$

A result such as the above may be expressed approximately in decimals by putting $\pm\sqrt{8} = \pm 2.828$. Such roots are said to be **non-rational.**

Figure 10: Example space coded as G, C, F (Adapted from Excel and Succeed Senior Secondary Mathematics Students' Book Form 3, page 4-5)

Figure 10 shows an example space consisting of three examples which are about solving quadratic equations by factor method and this was coded G, C, and F. In this example space, there is similarity in the approach to the solution as they all use factor method. In this first example (Example 1.7), two ways of factoring the quadratic equations leading to the same solutions have been demonstrated. This brings contrasting in the handling the same equation while the solution remains the same. More importantly the second method will bring to the attention of the learners the significance of dividing all the terms of a quadratic equation by the common factor 2 in this case. Moving to example 1.8, there is contrasting in the structure of the quadratic equation from the example 1.7. This brings to attention to the learners the different appearance of quadratic equations. In this second example (Example 1.8), once again two approaches have been demonstrated leading to the same solution, allowing learners to notice the similarity in the solution and difference in the approach within the same example. This could promote learners' understanding of different approaches to solving quadratic equations by factor method, specifically learners can be made to understand the concept of "difference between two squares" which is more pronounced in the alternative method. Furthermore, in this example space, the third example (Example 1.9) was provided with the quadratic equation having constant terms on both sides of an equal sign, presenting a contrasting feature from the previous examples. The solution has also been presented as non-rational which was not the case with the other examples. This brought about multiple variations to this example space, thereby allowing learners to notice similarities (G) contrasting (C), and bringing simultaneous changes, fusion (F). Ronda and Adler (2017) argue that these types of examples in an example space bring about the discernment of object of learning, allowing learners to understand mathematics through noticing multiple variations of key features of object of learning. These types of example spaces are categorized as level 3 because they were able to display all the three patterns of variation within the set of examples (Ronda & Adler, 2017).

What was conspicuously missing in all example spaces was the graphical presentation of solutions of the quadratic equations. This could have afforded much more variation to the above given solutions. However, going through each textbook, these type examples are found in unit 15 under the topic "graphs of functions" in all textbooks except textbook D which is placed in unit 16. The number of example spaces showing one form of variation either similarity (G) or contrasting (C) only and those showing both similarity and contrasting (G, C) as well as those showing multiple variation (G, C, F) were recorded (see Table 3).

Table 3: Number of example spaces coded G, C and F

	Number of examples	Number of examples Spaces	Coding of example spaces
Textbook A	31	8	3 – G, C, F 5 – G, C
Textbook B	16	6	1 – G, C, F 5 – G, C,
Textbook C	30	9	4 – G, C, F 5 – G, C,
Textbook D	21	7	1– G, C, F 4 – G, C 2 – G
Legend: C- contrast, G- generalization, F- fusion			

Table 3 shows the number of examples and example spaces in each textbook and their codes. Textbook A has 8 example spaces containing 31 worked examples, textbook B has 6 example spaces containing 16 worked examples, textbook C has 9 example spaces having 30 worked examples and textbook D has 8 example spaces containing 21 worked examples. Three of the eight example spaces in Textbook A were coded G, C, F and five example spaces coded G, C, textbook B and D have only one example space coded G, C, F with five and four example spaces coded G, C respectively. While textbook C has four example spaces coded G, C, F and five example spaces coded G, C.

Table 3 shows that all the textbooks provided example spaces containing sets of worked examples which were providing generalization, contrasting, and fusion as well as sets of examples just providing generalization and contrast. Meaning that going through the sets of examples in an example space learners were afforded the opportunity to see similarities and differences that could lead to generalization and be able to develop deeper understanding (Ronda & Adler 2017) of how quadratic equations are solved. The next section explains findings on tasks analysis.

4.3 Number of tasks coded as KPF, CTP and AMC

Tasks were analysed to establish those requiring procedures or facts already known by students at the time they are working on the solution, those requiring learners to use the procedures which they are currently learning and clearly shown in the worked examples, requiring the learners to apply the previously learned knowledge and connect to the topic under discussion. Number of tasks coded KPF, CTP and AMC were counted and recorded (see Table 4).

Table 4: Number of tasks coded KPF, CTP, and AMC

	Number of tasks	Coding of task		
		KPF	CTP	AMC
Textbook A	134	50	76	8
Textbook B	117	15	89	13
Textbook C	167	47	104	16
Textbook D	119	50	61	8

Legend: KPF- Known procedure/fact,

CTP- current topic procedures

AMC- Application making connection

Table 4 shows that in all the four textbooks, the common appearing tasks are those coded CTP, for instance 76 in Textbook A, 89 in Textbook B, 104 in Textbook C, and 61 in Textbook D. These are tasks which require learners to apply the procedures and knowledge that is currently being learned in the topic of quadratic equation (see Figure 6). The tasks coded CTP comprised over 50% of total tasks for the topic of quadratic equations in each of the four textbooks. It can also be seen from Table 4 that tasks requiring known procedures or fact (KPF) were considerably in large numbers as well. For instance, Textbook A and D have 50 tasks each, while Textbook B and C have 15 and 47tasks respectively in this category.

Furthermore, Table 4 shows that in the textbooks analyzed, the tasks that involve application and making connections which were coded AMC are generally very few, 8 tasks in Textbook A, 13 in Textbook B, 16 in Textbook C and 8 in Textbook D. These are the tasks that require learners to make connection to other mathematical concepts and relate to the current topic in order to solve those tasks (Ronda & Adler, 2017). These are tasks which involve learners to think on their own on the procedures to follow and sometimes may require to draw diagrams to understand the problem (See Figure 7 in method section 3.3.3).

Although all textbooks had fewer numbers of AMC tasks, textbook C has a relatively larger number of tasks in the AMC category than the other textbooks. Textbook C has 16 tasks in the AMC category which is double the number of tasks in textbook A and D in the same category. So Textbook A and D may have the smallest chance of engaging students in "tasks that involve making a decision as to the procedure and concepts that need to be called upon to answer the task or requiring connections between concepts" (Ronda & Adler 2017, p. 1102). This may indicate the existence of variation in students' opportunities to learn mathematics across textbooks by different publishers even though they are responding to the same curriculum needs. Further analysis of the tasks were made to establish the cognitive demand of the tasks.

4.4 Cognitive demand level of tasks

Guided by Mathematics Task analytical tools (Stein & Smith, 1998) tasks were analysed to establish the cognitive levels of each. Tasks requiring memorization will be referred to as level 1, procedure without connection, level 2, procedure with connection, level 3 and doing mathematics, level 4.

Table 5: Number and percentage of tasks at each cognitive demand level in textbooks.

	Low level Tasks		High level tasks		
	Level 1	Level 2	Level 3	Level 4	Total
Textbook A	47 (35%)	79 (59%)	5 (4%)	3 (2%)	134
Textbook B	27 (23%)	77 (66%)	10 (8%)	3 (3%)	117
Textbook C	71 (43%)	80 (48%)	11 (6%)	5 (3%)	167
Textbook D	61 (51%)	50 (42%)	5 (4%)	3 (3%)	119

Table 5 illustrates that all the four textbooks are dominated by level 2 tasks, requiring procedures without connection to the concept in this topic of quadratic equation except in textbook D which is having level 1tasks dominating. The table shows that there are 79 level 2 tasks in textbook A, 77 tasks in textbook B, 80 tasks in textbook C and 50 tasks in textbook D. These were tasks which were algorithmic and solution methods to use for solving were suggested in the tasks. They were not complex and the procedures to follow were clearly demonstrated by the worked examples (see Figure 6). Coming second are tasks which require memorization (level 1) where textbook A has 47 tasks, textbook B, 27 tasks, textbook C, 71 tasks and textbook D, 61 tasks. These were the tasks which required memorization of facts which the learners knew already. They were also algorithmic and very simple and straightforward (see Figure 5). It can also be seen from Table 5 that high-level tasks are very small in number in all the four textbooks. For instance at level 3 (procedure with connection to the concept) textbook A has 5 tasks, textbook B, 10 tasks, textbook C has 11 tasks and textbook D has 5 tasks while level 4 (doing mathematics) there are only 3 tasks in Textbook A, B and D, and 5 level 4 tasks in textbook C. These are the tasks that required learners to use procedures which are connected quadratic equations. The procedures to follow are not suggested by the tasks, meaning that learners were to make their own decision on how to go about it. Most of these tasks were not algorithmic and required application of other concepts in relation to quadratic equations. They were complex and needed learners to think critically to work on them (see Figure 7). This shows that there is a very limited representation of high-level tasks, which are high cognitive demanding tasks, in the topic of quadratic equations for grade 11 mathematics textbooks in Malawi.

Moreover in terms of percentages of task at each level for each textbook, Table 5 shows that tasks requiring doing mathematics (level 4) is 2% in Textbook A and 3% in each of the Textbooks B, C, and D. It can also been that tasks of level 3, which requires procedure with connection to concepts is poorly represented as well. For instance, 4% in each of Textbook A and D, 8% in Textbook B and 6% in textbook C.

It can also be observed that the proportion of high-level tasks to low-level tasks in the textbooks is very small. For instance, the proportion of high-level tasks to low-level tasks in textbook A is 6:94, in textbook B is 11:89, in textbook C is 9:91 and in textbook D is 7:93. This signals the acute shortage of high-level tasks which according to Smith and Stein (1998) require students to understand several mathematics concepts, make connections, and justify their reasoning in order to answer them. Therefore these figures may suggest that learners' opportunities to engage with high cognitive demanding tasks are limited in all the textbooks.

The chapter has presented and discussed the results of the study that have revealed what type of cognitive opportunities to learn mathematics as presented in Malawian grade 11 textbook mathematics. It has also revealed to what extent the Malawian textbooks are engaging learners in cognitive demanding mathematics in the topic of quadratic equations. The next chapter presents the discussions, conclusion, implication of the study, and recommendations, as a result, of the findings presented in this chapter.

CHAPTER 5
DISCUSSION, IMPLICATION, LIMITATIONS, RECOMMENDATION, AND CONCLUSION,

This study aimed at exploring cognitive opportunities to learn mathematics provided to students in grade 11 Malawian mathematics textbooks through quadratic equations. In order to understand the cognitive opportunities to learn mathematics provided by the textbooks, the study used the following questions: what cognitive opportunities to learn mathematics are provided by examples and tasks in the topic of quadratic equations in grade 11 Malawian textbooks? To what extent are the examples and tasks in grade 11 Malawian textbooks engaging learners in cognitive demanding mathematics in the topic of quadratic equations? This chapter is giving a discussion of findings by drawing conclusions from these research questions; it also presents the implications on the practice and limitations of the study. The last two sections will outline the recommendations of the study and conclusion.

5.1 Discussion on examples and tasks

The examples and tasks in each textbook were analyzed using MDITx framework (Ronda & Adler, 2017) and Mathematics Task analytical (Stein & Smith, 1998) to establish what cognitive opportunities to learn mathematics are available in the textbooks in the topic of quadratic equation through the exemplification (examples and tasks). Research has shown that two or more examples are better and effective in regards to student learning (Rittle-Johnson & Star, 2009), because students' learning is promoted when mixed examples are used (Rohrer & Pashler, 2010). This is because "when different types of tasks or examples are mixed, the learners are forced to distinguish between them and thus get better at making sense of novel tasks and examples" (Kullberg et al., 2017, p. 561). Findings show that, in all the textbooks, the example spaces contain at least two worked examples and many tasks following the worked examples.

5.1.1 Examples

Findings show that all the textbooks provided example spaces containing sets of worked examples which were providing generalization, contrasting, and fusion as well as sets of examples just providing generalization and contrast (see Table 3, Results and Findings section 4.2). The study found that most of the example spaces in the textbooks have provided two patterns of variations: similarities and generalization. By going through the sets of examples in an example space, learners were afforded the opportunity to see similarities and differences that could lead to generalization and be able to develop deeper understanding (Ronda & Adler 2017) of how quadratic equations are solved. This may suggest that the textbooks are able to provide opportunities to learn quadratic equations.

The research findings show that although many examples in the textbooks contain at least two examples, Textbook D only has two example spaces with a single example and showing one pattern of variation. One of these example spaces is about 'formulating quadratic equations given roots' and the other is about 'solving equations by using quadratic formula'. This indicates that Textbook D offers few opportunities for learners to understand the concept of formulating quadratic equations from the given roots and solving quadratic equations by using the quadratic formula.

As previously stated that most of the example spaces in the four textbooks contained at least two examples with varying patterns, this could mean that learners were able to learn mathematics. As argued by Rittle-Johnson and Star (2009) that using multiple examples is beneficial for mathematics learners more especially when at least two examples are presented together than presenting them separately. They further suggest that mixed examples, may enable learners to distinguish between them, consequently becoming aware of the mathematics being communicated to them. It was found out that most of the examples in an example space were showing two different patterns of variation which were similarity (G) and contrasting (C). For instance, Textbook A and B have five example spaces each showing two patterns of variation, while Textbook C and D have four example spaces each showing two patterns of variation, generalization and contrasting (see Table 3, Results and Findings section 4.2). This suggests that these example spaces were able to mediate the object of learning "quadratic equations" through seeing similarities and differences as Marton and Pang (2006) argue that better learning involves bringing attention to pattern of variation amidst invariants.

Furthermore, some example spaces in the textbooks were showing three patterns of variations, that is, similarity (G), contrast (C) and fusion (F). For instance, there are three example spaces in Textbook A, four in Textbook C, one in textbook B, and one in textbook D (see Table 3, Results and Findings section 4.2). According to Ronda and Adler (2017), these example spaces are regarded as having more opportunity to mediate the object of learning through simultaneity of these variations (see Figures 9 and 10, Results and Findings section 4.2). They classify these example spaces as level 3 and argue that these are the class of tasks that facilitates greater opportunities for learners to discern key features of the content. This is also supported by Atkinson et al., (2000) who argue that multiple examples with varying features have the potential to support deep understanding of the concept being discerned in the examples.

The findings of this study indicate that all the textbooks can be categorized as level 3, since moving across the lesson of quadratic equations in the textbooks, all the three patterns of variation were noticed in the set of examples in the example spaces. This could mean that the textbooks are providing opportunities for learners to discern the key features of the quadratic equations (Ronda & Adler, 2017). The results reveal that in the topic of quadratic equations in all the textbooks, there are many examples showing patterns of variation with respect to factoring quadratic equations, solving quadratic equations and solving practical problems involving quadratic equations which are components of the object of learning "quadratic equations". This suggests that learners were able to understand the concept of quadratic equations and how quadratic equations are solved. This may imply that grade 11 mathematics textbooks are offering opportunities to learn mathematics. This may agree with the idea advocated by Ronda & Adler (2017) that mathematics made available in a lesson can be revealed by patterns of variation in relation to the object of learning exposed by a set of examples across the lesson.

5.1.2 Tasks

Findings show that in the textbooks analyzed, the tasks that involve application and making connections, which were coded AMC, are generally very few: 8 in textbook A, 13 in textbook B, 16 in textbook C and 8 in textbook D. This indicates that in all textbooks analyzed, the tasks requiring students to reason critically on how to find solutions to the tasks are greatly inadequate. The textbooks contained few tasks that may involve learners to think independently on what procedures and concepts to employ in finding solutions to the tasks (Ronda & Adler 2017). Although all textbooks had fewer numbers of AMC tasks, textbook C has a relatively larger number of tasks in the AMC category than the other textbooks. Textbook C has 16 tasks in the AMC category which is double the number of tasks in textbook A and D in the same category. So Textbook A and D may have the smallest chance of engaging students in “tasks that involve making a decision as to the procedure and concepts that need to be called upon to answer the task or requiring connections between concepts” (Ronda & Adler 2017, p. 1102).

The findings also show that the majority of tasks in each of the textbooks are those that require learners to use current procedures being learned (see Table 4, Results and Findings section 4.3). These are the tasks in which the procedure to follow is already suggested in the task and they are algorithmic. These tasks are not complex and just concentrate on the correct answer (see Figure 6, Method section 3.3.3). This may imply that the textbooks give many opportunities to learners to experience tasks which are algorithmic and stress on memorizing facts and procedural fluency. Eventually this may lead to learners’ failure to understand mathematics because as already discussed understanding mathematics requires cognitive processes beyond recall of facts, remembering mathematical concepts or being able to follow a specific procedure (Idris, 2009).

Conspicuously missing in all the textbooks was the graphical representation of quadratic equations solutions. It could probably have been better to include some examples and tasks that could involve graphical representation of quadratic equations which might have enhanced conceptual understanding in learners. Since it is argued that through diagrams, graphs, and drawings, conceptual understanding of mathematics may be visually facilitated (Olteanu, 2018). Furthermore, the use of graphical representation could have afforded learners an opportunity to see multiple representations of solutions of quadratic equations. Rittle-Johnson and Star, (2009) argues that tasks encouraging different ways of presenting solutions, have the potential to facilitate student learning by prompting students to reflect on various solution methods. They further argue that these types of tasks may stimulate learners to consider their suitability in different contexts, as result this may develop learners’ competence in solving mathematical problems flexibly and creatively. This is also supported by Ni et al. (2018) who argue that engaging learners with tasks of multiple solution methods has the potential of promoting conceptual knowledge and problem solving.

5.2.3 Discussion of cognitive demand mathematics provided by exemplification

The second research question which guided this study is "to what extent are the examples and tasks in grade 11 mathematics textbooks provide learners' opportunities to learn cognitive demanding mathematics in the topic of quadratic equations"? It was found out that the four textbooks contain tasks of all levels of cognitive demand. However the tasks of high level, those requiring procedure with connection and doing mathematics were notably very few in all the four textbooks. For instance there were 8 in Textbook A, 13 in Textbook B, 16 in Textbook C and 8 in Textbook D (see Table 5, Results and Findings section 4.4). Generally these were the tasks which mediate the object of learning, 'solving practical problems involving quadratic equations' and they were found at the end of the unit. These tasks were non-algorithmic and required learners to make their own decision as to the procedure to follow in order to solve them. Furthermore these tasks are complex and require learners to make connections to other concepts, then relate to the topic of quadratic equations to work out the solution. In some cases procedures may be followed but very purposely to aid the solution strategies (see figure 7, Method section 3.3.3). It was also found out that tasks that were requiring students doing mathematics were very few: five in Textbook C and only three in each of Textbooks A, B, and D this represents about 3% of all tasks in each textbook in the topic of quadratic equations. This signals the acute shortage of high-level tasks which require students to understand several mathematics concepts, make connections, and justify their reasoning in order to answer them (Stein & Smith, 1998). This may imply that learners are denied an opportunity to develop a deeper understanding of mathematical content which promotes high reasoning (Jones & Tarr, 2007). This may mean the textbooks do not respond well to the recommendation made by Stein et al. (2000) that learners should have opportunities to "engage with tasks that lead to deeper, more generative understandings regarding the nature of mathematical processes, concepts, and relationships" (p. 15). This idea is also supported by the National Council of Teachers of Mathematics (NCTM) (2000) which calls for the use of tasks that require higher levels of cognitive demand in instruction such as textbooks to promote the development of conceptual understanding in learners.

These findings indicate that learners are not given much opportunity to experience tasks which promote reasoning, critical thinking, and problem-solving skills which according to Stein and Smith (1998) is characterizing high-cognitive demanding tasks. This suggests that the textbooks are limiting learners to engage themselves in cognitive demanding mathematics. Even though one of the aims of teaching secondary school mathematics in Malawi is to promote logical reasoning, critical thinking and problem-solving skills in students (MoEST, 2013) as stated earlier on, these findings may suggest that this could not be easily achieved.

Although this study was not aimed at comparing the textbooks, the findings show that textbook A and D has only 8 tasks of high level as compared to Textbook B and C with 13 and 16 respectively. This suggests that Textbooks A and D are affording learners with very limited opportunities to engage with tasks of high level as compared to the other textbooks. This may imply that there is a difference in the extent to which the textbooks give opportunities to learners to engage in high cognitive demand mathematics. This may create advantages or disadvantages to other learners depending on the textbook the school chooses to use.

The findings also agree with Stylianides' (2009) findings where it was found out that US mathematics textbooks, on average approximately 40% of textbooks' tasks afforded learners with an opportunity to engage in reasoning and proving while more than 50 % tasks with no opportunity at all for learner's engagement in reasoning and proving.

These results agree with the study of Wijaya et al. (2015) in Indonesian textbooks where they were investigating the opportunity-to-learn offered by Indonesian textbooks for solving context-based mathematics tasks and the relation of this opportunity-to-learn to students' difficulties in solving these tasks. They found out that textbooks offer limited opportunity to learn high-level cognitive demand tasks. They found out that from the context-based tasks analyzed only 2% are reflection tasks, which are considered as tasks with the highest level of cognitive demand. Indicating the scarcity of high cognitive demanding tasks in Indonesian textbooks.

Furthermore, the results agree with a study conducted by van Zanten and van den Heuvel-Panhuizen (2018) in Netherland on opportunities to learn problem solving in Dutch primary school mathematics. It revealed among other things that Dutch textbooks offer very limited opportunities to learn problem solving.

Recently, Hadar and Ruby (2019) conducted a study to explore breadth and depth of understanding as addressed in mathematics textbooks where they compared opportunities for students to engage with mathematics, requiring different types and levels of understanding provided by the tasks in mathematics textbooks. Their study showed significant differences in the opportunities to learn in the cognitive domain that each mathematics textbook provides.

The findings reveal that grade 11 Malawian mathematics textbooks contain few tasks of high level which may imply that the textbooks are limiting learners' opportunities to engage with cognitive demanding mathematics. This may be worrisome to Malawi because textbooks are the most reliable resource in mathematics classrooms used by both teachers and learners.

5.2 Implication on practice

Findings from this study have revealed that there are few tasks in the textbooks that engage learners in high cognitive demand mathematics. Teachers using these textbooks need to bring in additional tasks of high cognitive demand more especially in the area where the object of learning is 'solving practical problems involving quadratic equations'. Furthermore, teachers can as well scale up the level of cognitive demand of some tasks. For instance, on the task of solving the equation by factor method or completing the square method (see Figure 6, Method section 3.3.3), the teacher can scale it by asking the learners to solve the quadratic equations without suggesting to them the method to follow. Secondly, these tasks can be scaled by asking learners to show solutions of those quadratic equations graphically. This will involve learners to reason and make connections to skills of drawing and interpreting graphs.

The findings have the potential to bring to the awareness of teachers the level of cognitive demand the textbooks are providing in the topic quadratic equations. As Hadar and Ruby (2019) argue that teachers rely on textbooks to enable them to implement the intended mathematical curriculum, it is important for them to know the variation of cognitive demand in textbooks. These authors further argue that by knowing that the textbooks are not offering well balanced cognitive demand tasks, teachers may be informed of what type of supplementary materials that will fill the gap in that area. From the findings of the present study, it is important for teachers in Malawi to scrutinize the level of cognitive demand in the textbooks so as to be able to make a sound decision on the way to improve how the textbooks can be used in the classroom. In addition to this, knowledge of level in cognitive activities offered by the textbooks, can help the teachers and educators to select and recommend the best textbooks for classroom use from the pool of textbooks approved by the Malawi Institute of Education. As mentioned, the aim of this study was not to compare the four textbooks but the findings have shown that Textbooks A and B offer very little opportunities for learners to engage with high-level tasks in this topic of quadratic equation. Therefore, knowing what a textbook is capable and not capable of providing in terms of mathematics to be learnt by learners is very beneficial to teachers as it will inform them on how to use the textbook more effectively to improve classroom practices (Johansson, 2006).

In teacher training institutions, teacher educators could inform their student teachers about strengths and weaknesses in different textbooks. The student teachers could then be informed on what is available in the textbooks their students will use and may change their curriculum to include pedagogies that work effectively in situations which are presented by mathematics textbooks. For those teacher educators training serving teachers through in-services trainings (INSETs) might use the findings to plan and organize trainings. These trainings may include such issues as how to effectively improve teacher's classroom practices using the mathematics presented by our textbooks in order to facilitate students' opportunities to learn cognitive demand mathematics.

The findings from the study might inform textbook developers to improve future editions of their textbooks by incorporating some of the recommendations from this research on how the textbooks can afford or limit students' opportunities to learn cognitive mathematics. The textbook reviewers might develop some skills that can be used when reviewing textbooks and recommending mathematics textbooks to be used in our secondary education.

The findings from the study might give an insight to other researchers who may need to replicate or build on the study on other topics in the area of cognitive opportunities to learn mathematics in textbooks.

5.3 Limitations

There are eight core mathematics textbooks approved for senior secondary education in Malawi, four textbooks for grade 11 and four textbooks for grade 12 students. This study only analyzed four grade 11 textbooks. Furthermore there are sixteen topics in grade 11 Malawian mathematics textbooks, yet, only one topic 'quadratic equations' was analyzed in this study limiting the strength of generalisation of the findings to all the topics in the textbooks. Although the results of this study are not intended to be generalized to the other textbooks, analyzing all the fifteen topics would have helped to better comprehend the general picture of the study across grade 11 textbooks. In addition, analyzing all the eight core textbooks for senior secondary education in Malawi would enable more general conclusions, but as Hadar and Ruby (2019) argue, such studies may not be possible at certain times due to limitations of resources.

5.4 Recommendations

Textbooks and their use in mathematics education is drawing interest from researchers as evidenced by the contribution from ICME-13 conference held in 2016, the 2017 Congress of European Research on Mathematics Education (CERME), The first International Conference on Mathematics Textbooks (ICMT) herd in 2014, second ICMT held in 2017 (Fan et al., 2018). The major conclusion in this study is that the grade 11 Malawian mathematics textbooks give limited opportunities for learners to engage in high cognitive demanding mathematics in the topic of quadratic equations. As such there is a need to investigate opportunities for learners to engage in high cognitive demanding mathematics in the other topics in these textbooks. Furthermore there is a need to examine how teachers and students use the textbooks in the teaching and learning mathematics in the classroom.

5.5 Conclusion

This study has presented findings from textbook analysis which aimed at exploring cognitive opportunities to learn mathematics provided for grade 11 Malawian mathematics textbooks through quadratic equations. The study has found that the textbooks provide different cognitive levels of examples and tasks in the example spaces ranging from low to high level. The study found that most of the example spaces in the textbooks have provided two patterns of variations: similarities and generalization. This may suggest that the textbooks are able to provide opportunities to learn quadratic equations. However, in these textbooks, there are few example spaces which would facilitate learners' engagement with different variation patterns simultaneously, to all learners experience fusion which is the critical condition for the discerning of the object of learning. As explained already the study has also found that textbooks provide learners with opportunities to engage with tasks of all levels of cognitive demand. However the tasks requiring memorization and procedures without connection to the concept were dominant in all the textbooks. The tasks requiring high cognitive demand are significantly few in all the textbooks in the topic 'quadratic equations', 8 in Textbook A, and D, 13 in Textbook B and 16 in Textbook C. These result suggests that, the textbook might not afford learners much opportunity to experience tasks which promote reasoning, critical thinking, and problem-solving skills which is one of the attributes of high cognitive demanding tasks. These findings are only based on textbook analysis on the topic of quadratic equations, I therefore suggest further studies to focus on the other topics in these textbooks to have a general picture of all topics. Since teachers' ability to use textbooks has an impact on the cognitive level of tasks implemented in the classroom, I also suggest further studies on how teachers use the textbooks and how learners engage with the textbooks in the classroom.

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