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DEDICATION

I dedicate this work to my children Wongani, Wantwa, Wonani and Waliko with deepest gratitude and reverence for their great support, advice, and prayers. God bless you.

To my brother Austin Musukwa for your encouragement and support

And

In memory of my parents, Sophie Luhanga and Asayile Musa Musukwa, who emphasized the value of education and had a great influence on my life.

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Abstract

This study investigated the opportunities for learning number concepts and operations in mathematics that are available to learners in upper primary classes in Malawi. Learners' textbooks for grade 5 to grade 8 were analyzed using the Mathematical Discourse of Instruction (MDI) framework developed by Adler and Ronda (2015). The analysis was based on exemplification as one of the interacting components of a mathematics lesson that help to illuminate what is made available to learn. The study specially sought to understand the opportunities to learn mathematics that are created by the textbooks in upper primary classes for learners to participate in mathematical discourses. The MDI framework helped the investigator to analyze the four textbooks thereby answering the following questions: What opportunities to learn number concepts and operations are provided through the examples and task in the textbooks?, How do the tasks enable enactment of the learning objects that are stated in the textbooks? And To what extent do tasks allow learners to apply and connect critical features of the mathematical content? The study used mixed methods approach in which MDI analytical tool for textbook analysis (Ronda & Adler, 2017) was used for data collection. The findings suggest that the textbooks do not provide a balanced range of examples and tasks as such, the textbooks offer few high-level thinking examples and few high-level cognitively demanding tasks. The textbooks emphasize on computations and procedures without connection, which are of low-level of thinking and low-level cognitive demands of tasks. The textbooks in upper primary school in Malawi offer few tasks with presentation and modeling, interpretation, argumentation, reflective-thinking and reasoning competence that will challenge learners and help them to develop their understanding. In general, the textbooks in upper primary classes in Malawi offer lower level thinking examples and lower level cognitively demanding tasks that provide few opportunities for learner to mediate several learning outcomes in a set of examples.

Keywords: Textbook analysis, number concepts and operations, opportunity to learn, learners, Exemplification, cognitive demands, and MDI framework

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ABBREVIATIONS

MANEB Malawi National Examination Board

MDI Mathematical Discourse in Instruction

MDITx Mathematical Discourse in Instruction Analytical Tool for Textbook Analysis

MoEST Ministry of Education, Science and Technology

NCTM National Council of Teachers of Mathematics

NOP Numbers, Operations and Relationships

OBE Outcome-Based Education

OEM Objective Education Model

OTL Opportunity to learn

SACMEQ Southern And Eastern Africa Consortium for Monitoring Education Quality

Chapter 1: INTRODUCTION

The aim of this study is to investigate the opportunities for learning mathematics that are provided in upper primary textbooks in Malawi. The chapter presents the background of the study, problem statement, and purpose of the study, research questions and significance of the study.

1.1 Background

Mathematics is one of humanity's achievements (Mckenzie, 2001). It is a tool for practical utility as well as a discipline that develops reasoning and analytical abilities. People who have basic knowledge in mathematics participate fully in a society (Mckenzie, 2001) because mathematics plays an important role in meeting the demand of everyday life. Most of the job industries and also the world of business require knowledge of mathematics. In Malawi, mathematics is one of the core subjects in primary schools as well as secondary schools.

Learning mathematics introduces learners to concepts, skills and thinking strategies that are essential in their everyday life and support learning across curriculum. Mathematics learning stimulates curiosity in learners, fosters creativity and equips learners with the skills they require in life beyond school. However, mathematics by nature is abstract and all themes in mathematics are ideas that develop in our mind. As a result, for learners to understand these abstract ideas, textbooks in primary school should provide affordances to learn mathematics that will assist the learners to reach their full potential in mathematical proficiency in the course of learning.

The most important years of a learner's school life are the primary school years. In connection with that, primary school mathematics at this stage is regarded as crucial as it serves as foundation knowledge for the secondary school and beyond. In Malawi, the developers of primary school textbooks had learners in mind when they developed the activities; an understanding of what children already know and need to know. As a result, the development of each topic is based on the principle of moving from known to unknown and simple to the more complex. The primary outcomes (indicated in the textbooks) focus on the expanded opportunities of ensuring that educators are finding multiple ways of exposing the learners to learning opportunities that will help them to demonstrate their full potential of mathematical competence (Malawi Institute of Education (MIE), 2007). However, examining what textbooks intend to provide to learners when learning mathematics to assist them acquire full potential of concepts and mathematical proficiency remains relatively unanswered. It is important to examine the opportunity to learn (OTL) mathematics in textbooks since it helps to understand how much attention is given to that specific topic (Hong & Choi, 2018).

Primary school education in Malawi comprises 8 grades. The infant classes include grade 1 and grade 2, junior primary includes grade 3 and grade 4 and lastly upper primary (senior) is grade 5 to grade 8. The official entry age for primary school level of education is 6 years though variations appear. The rationale for learning mathematics in primary schools focus on "developing the learner's critical awareness of how mathematical relationships are used in social, environmental, cultural and economic context" (MIE, 2008 p. x). In line with this, learners in infant and junior primary school must be able to count and carry out mathematical operations at the end of grade 4. And in upper primary school, learners are required to make inferences using manipulated data and apply mathematics to solve practical problems (MIE, 2008). However, learners in upper primary school classes fail to accomplish the rationale and the criterion-referenced measurement (assessment standards and success criteria of OBE) of the mathematics curriculum in primary school. This is observed through low performance of learners during the primary school national examinations (Malawi National Examination Board (MANEB), 2006– 2016) as many learners fail to reach minimum levels of mathematical proficiency as specified in the National curriculum (Eliya, 2016). The assessment of the primary school national examinations draws topics in mathematics from grade 5 to grade 8.

According to the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ, 2010), primary school learners in Malawi only master the lower level of mathematical numeracy because following analysis of the SACMEQ results, the MoEST observed that most teachers did not have sufficient training and/or experience (MoEST, 2011). This was evidenced by poor performance by the learners in primary schools and continues to secondary mathematics (Malemya, 2019). Mathematics as a subject is a hub for development of other sciences which collectively install the much needed reflective and critical thinking skills in learners (Isoda & Katagiri, 2012) which is essential in socio-economic development of the country as a whole. Primary education in Malawi is characterized by teaching as opposed to learning (Malemya, 2019). Malemya explained that teachers are treated as the source of almost all the information with learners being the recipient. Malemya further explained that, "this is attributed by the textbooks which do not provide enough platforms for independent study by the learners themselves; such as lack of enough practice problems, examples and applicability in real life" (Malemya, 2019, p. 172). That is, how textbooks provide opportunities to learn (OTL) mathematics to learners in order to access the object of learning in mathematical discourse in instruction has a bearing on learning outcomes.

Opportunity to learn is regarded as an important contributing factor in learning outcomes (Tornroos, 2005). As a result, in mathematics, textbook lessons or teacher's lessons need to create opportunities for learners to participate in mathematical discourse. Carroll, (1963) defined opportunities to learn mathematics as the amount of time allocated to the learner for the learning of a specific task. In addition, Banicky (2000) defined OTL as what the education system does to enable students meet the expectations set by the content and performance standards. Furthermore, Floden, (2000) defined opportunities to learn (OTL) as related to content domains or cognitive skills provided in the curriculum materials or textbooks which depict learners' engagement with different aspects and features of mathematical discourse. Thus, these are learning moments that the textbooks provide for learners to experience mathematics made available to learn in textbooks and including certain practices as the learners interact with them. OTL is the main determinant of learners' content and cognitive achievement because it is a mechanism for improving mathematics teaching and learning. McDonnell (1995, p. 305) suggested that OTL is one of the small sets of generative concepts that "had changed how researchers, educators and policy makers thought about the determinant of student learning". In support, Ronda and Adler (2017) state that OTL is one of the mathematical practices that is also needed to be factored in when describing the mathematics made available to learn in textbooks. In textbooks, opportunities to learn mathematics can be determined through examples, tasks, word and legitimations and learners' participation (Adler & Ronda, 2016). However, Ronda and Adler (2017) argued that the way the author uses examples, tasks, words and legitimations affords or constrains opportunities for learning mathematics.

In mathematics, textbooks are thought to characterize the teaching process more than in other subjects (Fan et al., 2013). Valverde et al. (2000, p. 2) explained that, "textbooks are designed to translate the abstractions of curriculum policy into operations that teachers and students can carry out. They are intended as mediators between the intentions of the designers of curriculum policy and the teachers that provide instruction in classrooms.” The textbooks reflect the intended curriculum by translating it into a sequence of contents and defining the content to be discussed during mathematics lessons (Tornroos, 2005). That is, the choice of what to put into textbooks shapes schooling by providing certain opportunities to learn (Houang & Schmidt, 2008). Textbooks provide examples, activities, explanations, and exercises for learners to complete and offer instructional guides to both teachers and learners. In support, Apple (1986, p.81) explains that, “it is the textbook which establishes so much of the material conditions for teaching and learning in classrooms …. and …. often defines what is elite and legitimate culture to pass on.” Mathematics textbooks have a significant influence on students’ opportunities to learn because they influence what students learn, how they learn and the cognitive level at which these students learn. Adler and Ronda (2015) add that, “textbooks create particular opportunities for engaging the different aspects and features of mathematics discourse” (p. 1100). Therefore, it is necessary to study OTL for learning mathematics in upper primary classes through textbook analysis, particularly, on number concepts, operations and relationships.

Studies in mathematics learning, particularly in area of number knowledge possessed by children in the early years of elementary school, have increased; some of which were done by Wright (1991), Payne and Huinker (1993), Rumiati (2010) and (Eliya, 2016). NCTM (2000) noted that studies like that are vital because the effective teaching of mathematics needs an in-depth understanding of what children already know and need to know. NCTM argues that without good early instruction, progress to higher-order skills is more difficult. Alongside these studies, however, other research done in Malawi, such as SACMEQ (2011) and Mulera et al. (2017) show that many learners fail to achieve levels of competence expected of their grades in number concepts and operations. This type of report is discouraging, especially in mathematics, because number concepts and operations form the core content area that is regarded as foundational for later progress where mathematics learning is based. Thus, its teaching and learning are important due to larger impact for learners’ later mathematics knowledge. When learners fail to comprehend the basic concepts in mathematics, they struggle to continue. As a result, some learners drop out of school or repeat classes. The contributing factors to learners having difficulties to learn mathematics is that textbooks and teachers ignore the prerequisite knowledge that learners bring to class with them as they start formal learning. To add on that, textbooks mostly do not encourage the learners to use their own mathematics procedures in order to develop their critical and sophisticated thinking instead textbooks use imaginary examples which are sometimes ambiguous in nature (Malemya, 2019).

Mathematics textbooks play a decisive role in the development of the concepts of mathematics thereby conveying the curriculum to the teacher and learners in the process of teaching and learning. Rohitaille and Travers (1992) argued that, “a great dependence upon textbooks is perhaps more characteristic of the teaching of mathematics compared to any other subject. The activities the mathematics textbook is involved in do not only give insight into learners’ utilization of mathematics textbooks but also give an idea of what the learning mathematics is all about for learners” (p. 107). It is essential, therefore, to explore the opportunities to learn mathematics that the textbooks provide for learners to access the object of learning.

1.2 Problem Statement

Mathematics education in Malawi has passed through different stages of development, which have all left their mark on the teaching and learning context in schools. According to assessment done by Malawi Teacher Professional Development Support (MTPDS) (2010), primary school learners only mastered the lower level of mathematics skills below the expectations of the curriculum in Malawi. Surveys done by SACMEQ in 1998, 2002 and 2007 also showed that the performance of grade 6 learners measured by the learners’ mean score in mathematics, especially in number concepts and operations had declined between 1998 and 2002 from four hundred and sixty-three to four hundred and twenty-nine points and Malawi ranked 13th in mathematics out of 14 Southern African countries. The mean improved to four hundred and thirty-three in 2007 (SACMEQ, 2010). The average performance of Malawi learners was lowest as compared to 15 other Southern African countries that participated in SACMEQ III survey in 2007 and Malawi was ranked 14th out of 15. In addition, reports from MANEB (2016) also indicate that learners in primary school fail to reach minimum levels of mathematical proficiency specified in the National Examination. Studies and reports reveal several factors that contribute to this poor performance in primary school mathematics. The factors include large classes which hinder teacher-learner interaction, inadequate teaching and learning resources, teachers’ lack of knowledge in mathematical proficiency, teachers’ limited knowledge of learner-centred approaches, inadequate qualified teachers and long period without revising the curriculum to meet present societal needs (International Mathematical

Union, 2014; National Education Sector Plan, 2008). As one way of improving learners' achievement, Malawi government and its developing partners tried to implement a number of interventions. These interventions include, among others, the development of education plans at central and district levels, construction of classrooms, provision of teaching and learning materials, recruitment and training of teachers, revision of curriculum, conducting social/community mobilization campaigns, and the implementation of support activities such as mother groups, school feeding programme, among others (Mulera et al., 2017).

The primary school curriculum was reviewed in 2007. The Ministry of Education proposed a shift from the objective education model (OEM) to outcome-based education model (OBE). The underlying argument of the shift was that OEM focused too much on the teacher, hence treated learners as recipients who could only play a passive role in achieving learning objectives. As a result, OBE was adapted as a vehicle to improve and promote learners' active involvement and performance in mathematics and also other learning areas. The new curriculum focuses on a learners' achievement that is fostered through active participation in classroom activities and beyond that promotes independent learning and critical thinking (MIE, 2008). In OBE, textbooks are major conveyors of curriculum content because they influence the implemented curriculum by shaping the instruction in the classroom thereby defining the content to be discussed during mathematics lessons (Törnroos, 2005). As a result, the value of a textbook is determined by the extent to which it contributes to learners' achievement of learning outcomes and hence achieving the goal of OBE (Chang & Salalahi, 2017).

Textbooks are considered as the basis of instruction and the most fundamental and popular teaching medium at school which facilitates mathematics communication besides the teacher and has been identified as an important factor that affect learners' learning outcomes (Pang, 2008). Teachers' decisions about the selection of content and teaching strategies are often directly set by the textbooks that teachers use (Freeman & Porter, 1989; Reys et al., 2004). As a result, textbooks are considered to determine largely the degree of students' opportunities to learn mathematics in primary schools (Schmidt et al., 1997; Törnroos, 2005). Studies of textbooks are thus important because of their influence on both teachers and learners.

However, despite the review of primary school curriculum to OBE in 2007 where learners' achievement through active participation in classroom activities and learner-centred, MANEB

(2014–2016) indicates that most learners perform poorly in number concepts and operations. This is particularly disturbing in the case of mathematics as number concepts and operations are considered prerequisites for learning school mathematics since they form the foundation upon which later progress in mathematics learning is based. In addition, studies examining how mathematics textbooks influence instruction generally agrees that textbooks have a significant influence on students' opportunities to learn mathematics (Stylianides, 2009). In Malawi, little is known about the research studies that have been conducted on analyzing textbooks for upper primary classes to explore the opportunities that these materials are providing for learners to learn number concepts and operations in mathematics. As such, the investigator aims to explore the opportunities that the curriculum materials are providing for the learning of number concepts and operations in mathematics in upper primary classes in Malawi.

1.3 Purpose of the study

The purpose of the study is to investigate the opportunities for learning number concepts and operations in mathematics that are provided in upper primary textbooks in Malawi. The textbooks are expected to provide the learners with the content, procedures, resources, methods, examples, explanations and the tasks that will involve the learners to participate in mathematical discourse as they interact with them. The research intends to find out what OTL mathematics textbooks are providing in order to make object of learning (number concepts and operations) accessible to the learners.

1.4 Research Questions

The research questions of this study will be divided into two parts: main question and the subsidiary research questions.

1.4.1 Main Research Question

What opportunities for learning number concepts, operations and relationships in mathematics are provided in upper primary textbooks in Malawi?

1.4.2 Subsidiary Research Questions

1. What opportunities to learn number concepts and operations are provided through the examples and tasks in the textbooks?

2. How do the tasks enable enactment of the learning objects that are stated in the textbooks?
3. To what extent do tasks allow learners to apply and connect critical features of the mathematical content?

1.5 Significance of the study

The study will provide insight to the curriculum developers, teacher educators and teachers on the importance of creating opportunities for engaging the different aspects and features of mathematics discourse in the textbooks The findings will also add literature in the field of knowledge specifically on how to create opportunities for learners to participate in mathematics discourse. Lastly information on opportunities to learn mathematics will be used to evaluate the appropriate teaching strategies.

1.6 Chapter summary

This chapter introduces the whole thesis. The section has introduced the thesis and discussed the background to the study in relation to opportunity to learn mathematics that textbooks provide in upper primary classes. The chapter has also presented problem statement, purpose of the study, research questions and significance of the study.

Chapter 2: LITERATURE REVIEW

This chapter gives a brief explanation of the concept of opportunity to learn, the MDI framework, the OTL and the textbooks, textbooks in Malawi, Number operations and relationships, How students learn mathematics, previous studies on OTL, and lastly the chapter summary.

2.1 Introduction

Learning mathematics has become one of the foremost topics focuses of interest in mathematics education (Phiri, 2011). There have been studies in mathematics education that aimed at studying the processes involved in mathematics learning and the ways in which learners may be assisted to become more proficient in mathematics some of which were done by authors (e.g., Davis, 1984; Phiri, 2011; Silver, 1985; Schoenfeld, 2007). The primary focus for this study is on what opportunities for learning number concepts and operations in mathematics are provided in textbooks for learners in upper primary classes in Malawi.

2.2 Opportunity to learn and Mathematical Discourse in Instruction (MDI)

Effective mathematics teaching engages students in discourse to advance the mathematical learning of the whole class. Mathematical discourse includes the purposeful exchange of ideas through textbooks explanations, as well as through other forms of verbal, visual, and written communication (NCTM, 2000). Research study by the World Bank (Abadzi, 2007) reveals that successful learning outcomes should not be expected without sufficient teaching and practice opportunities. And Ronda and Adler (2017) add that teaching depends on creating opportunities for learners to participate in mathematical discourse. That is, learners should be able to participate and interact with the cultural tools such as examples, tasks, words, legitimations and algorithms made available in mathematical teaching and learning in the mathematical discourse. In order for the learners to develop a deep understanding of certain concepts or to nurture certain capabilities, curriculum materials should avail a variety of opportunities for learners to access the crucial features of the object of learning. Thompson and Senk (2009) suggest that one of the most critical variables in determining students' learning and achievement is opportunity to learn.

2.3 Opportunity to learn and Textbook lessons

According to Stein and Smith (2010), textbooks determine what teachers teach and what students learn in mathematics education. Textbooks can influence students' learning as they contain different types of mathematical tasks that require student engagement with the mathematics content embedded in them. A textbook is defined as a printed and published resource designed to be used by teachers and students in the learning process (Van Steenbrugge et al., 2013). The role of the textbook is to provide a structural scheme of ideas, organize teaching and learning and the ability to develop thinking and conceptual understanding of the content. A textbook lesson is “a segment of text materials devoted to a single main mathematical topic intended to correspond to a teacher’s classroom lesson on that topic taught over one to three instructional periods” (Valverde et al., 2002, p. 139). Textbook lessons use a variety of examples, tasks, and accompanying explanatory text such as definitions, analogies, illustrations and much more to mediate the mathematics (Ronda & Adler, 2017). Using each of these specialized tools of MDI in a conducive manner, textbook lessons create opportunities for learners to participate in the discourse when interacting with the textbooks. Therefore, textbook lessons need to be logically structured so as to encourage learners’ interaction, address gaps in learners’ understanding, and help learners to express mathematical concepts more precisely. Textbook lessons should be user friendly thereby being written in such a way that mathematics concept is well explained to learners even in the absence of the teacher. This will give learners a chance to practice and expand their ideas.

2.4 Curriculum materials (textbooks) in Malawi

The Malawi Outcome-Based Education curriculum focuses on learners’ achievement through active participation in classroom and out of classroom activities that promote independent learning and critical thinking (Malawi Institute of Education, 2008). In Malawi, curriculum materials consist of syllabi, teachers’ guides and learners’ books. The content and the structure of the syllabus, teachers’ guide and learners’ book in primary school are determined by the content and focus of the curriculum, as a result, the core role of a textbook is determined by the degree to which it contribute to learners’ achievement of the learning outcomes (Chang & Salalahi, 2017). The role of the learners’ book is to highlight and achieve the goals of the curriculum (OBE) thereby mediating between the intended and implemented curriculum. That is, learners’ books tie the intended curriculum and the implemented curriculum together (Schmidt et al., 2001).

A curriculum is a resource that attends to sequencing or mapping learners’ learning over a period of time. Curriculum focuses on prescribing the objectives while the syllabus describes the means to achieve the intended objectives. As a result, a syllabus is just an outline of the content that has to be learnt by children that include topics to be covered, their order and other relevant information. The content from the syllabus is converted into teachable units in the teachers' guide that are designed to offer information, instructions and suggestions that will aid in construction of curriculum in the classroom. The learners' book consists of tasks that are meant for learners to practice. This implies that studying textbooks is important because of their influence on both teachers and learners.

2.5 Numbers, Operations, and Relationships

When analyzing the textbooks, this study will focus only on number concepts and operations. Number concepts and operations are key elements in mathematics because they define numeracy and they are considered as prerequisites for the learning of school mathematics beyond literacy level (Kasoka, Jakobsen, & Kazima, 2017). Concepts in mathematics are hierarchical in structure. That is, each idea is contained in the idea that follows it. Number concepts and operations in primary textbooks in Malawi build on each other. For example, numbers, operations and relationships follow the sequencing from counting of concrete objects, counting using number line, ordering, order of operations, cumulative, associative, fractions and many more.

Numbers, operations, and relationships is the first core element in primary mathematics. A core element is a component of a curriculum that is maintained without alteration in order to ensure the program's effectiveness. A core element consists of a learning outcome that describes significant and important learning that learners will achieve and will be able to demonstrate at the end of a unit/lesson or concept. Learning outcomes for numbers, operations and relationship for primary school in mathematics in Malawi focus on seeing that the learners are able to use numbers and their relationships to solve practical problems (MIE, 2008).

According to Chin and Zakaria (2015), number concepts and number operations form the basic mathematics that learners need to master before they can pursue advanced mathematics learning. Chin and Zakaria further explained that numerical skills are considered as skills that enable an individual to control his/her daily life effectively. Mastery of mathematics number concepts and operations at the early stage of schooling is really important and becomes the indicator of achievement and mastery of advanced mathematics. Studies show that mastery of number concepts and operations at the early stage plays an important role in the development of mathematical achievement in primary schools and secondary school level (Jordan, Kaplan, Ramineni &

Lacuniak, 2009; Lacuniak & Jordan, 2008). This is due to the process of mathematics that is hierarchical in nature in which higher level skills can only be mastered after the low-level skills have been mastered and involves a combination of understanding in terms of concepts and procedures (Aonola, Leskin, Lerkkanen, & Nurmi, 2004). In primary mathematics in Malawi, numbers, operations and relationships consists of more topics compared to other core elements. For example, it consists of eight topics in grade 5, fourteen topics in grade 6, ten topics in grade 7 and eight topics in grade 8.

2.6 How students learn number concepts and operations

Children come to school with some rudimentary skills of counting (Bass, 2015; Kilpatrick et al., 2001) which they acquire in their everyday activities before starting school. According to Reuben (2009), learners develop number concepts and operations at different levels before they start schooling. Learners come to school with knowledge of number concepts that blossom from informal experiences acquired from their community settings such as home, playground, grocery store, shopping malls and games. Such knowledge is usually represented non-verbally or verbally and it is often learned incidentally (Baroody et al., 2006). Learners progress with construction of number knowledge using their existing knowledge that later become an essential basis for understanding school taught mathematics. In formal schooling, learners develop number concepts and operations through modeling using sets of objects, role playing and through games. Findings found by Reuben agrees with earlier studies by Ginsburg (1977) where the researcher discovered that learners' understanding of number concepts and operations such as addition and subtraction evolves from their early counting experiences. Kilpatrick et al. (2001) gave an example of children getting involved in sharing cookies and candies with their sisters and noticing that their sisters get more cookies than them; also children count stairs, and divide cakes with peers, before coming to school.

Early understanding of natural numbers was influenced by Piaget's logical operations framework that include classification, seriation and conservation as the foundation of understanding the natural numbers (Verschaffel et al., 2007). However, educators believed that it was not possible for children to develop understanding of natural numbers rationally before concrete-operational stage and that logical operation was part of the integration of cognitive structure of the child (Kilpatrick et al., 2001; Verschaffel et al., 1996). Many scholars had questioned the centrality of logical operations in the development of rational understanding of natural numbers and instead, they have attributed the development to the importance of children's declarative and procedural knowledge of counting. Verschaffel et al., (2007) explain that the development of counting ability is interwoven with the development of understanding counting principles. Counting ability is one of the precursors for a good development of the later arithmetic ability and this ability can be acquired by children through mastery of essential counting principles. Gelman and Galistel(1978) described these early counting principles as the five conceptual principles in counting such as, the one-one-correspondence principle (number word can only be attributed to one counted object), the stable-order principle (number words must be invariant across counted sets), the cardinality principles (value of the last number word represent the quantity of the counted objects), the abstraction principle (objects of every kind can be counted) and lastly the order-irrelevance principle (object in a set can be counted in any sequence). Kilpatrick et al., (2001) explain that by the time children start elementary school, they understand the rules that underlie counting such as, performing conventional counting of sets of objects even greater than 10, use counting to solve some simple mathematical problems and even know some Arabic symbols up to 10. Kilpatrick and others further explained that from counting ability and counting principles, children learn basic facts—this include the addition and multiplication of single-digit numbers and subtraction and division. However, researchers emphasize on the gradual learning of these number facts from children's own constructed strategies and prior knowledge. The children should start progression of mastery of orally stated single-digit addition such as counting-all-with sets of objects (materials), counting-all-without materials and then facts on multi-digit numbers- children use algorithms.

Bass (2015) conducted a study in the United States of America, and his findings correlated with earlier studies by Gelman and Galistel (1978) and Ginsburg (1977) that learners' understanding of numbers evolve from their counting experience. Kilpatrick et al. (2001) report that, children begin to learn mathematics from infancy and continue throughout their preschool period developing basic skills, concepts and misconceptions. At all levels of development, learners encounter quantitative situations where they learn a variety of things about numbers outside of school. Baroody (2001) argues that children from different social backgrounds differ in the rate they acquire informal mathematics levels because of the different amount of stimulation available in their environment. He further explains that the rate acquisition of mathematics skills can be influenced by the opportunities provided to children in their society. In support Guberman (1996) argues that many parents in Brazil's northeast coast a few times sent their children to the local shops to buy goods such as food or drinks. In these types of scenarios children were able to participate in activity that contributed to informal mathematics development. This informal knowledge is a critical basis for understanding formal mathematics, mastering basic skills, and developing mathematical proficiency (Baroody et al., 2006). To add on, Reuben (2009. p. 4) emphasizes that, for learners to learn basic operations, "they must know how to count; they must understand how to simultaneously count and keep track of objects; and then they must continue with this progression, and develop automaticity as the foundation of success with future number operations such as addition, subtraction, multiplication, and division through the following years."

A whole number is the core content, which is regarded as foundational for later mathematics learning; its teaching and learning is essential due to larger impact for later mathematics knowledge. Reuben (2009) in his study noted that, after learning to count numbers, learners develop their understanding of number concepts and become more proficient with skills such as single-digit addition and subtraction and later with multiplication and division. These basic number combinations include mastery of basic facts, for example addition basic facts (5 + 4 = 9). Learners then move to double-digit addition and subtraction, and also learn place value. For example, 58 + 31 = 89 or 58 – 31 = 27. And finally, learners move to the mastery stage where they work with greater computational, modeling, representation interpretation and problem-solving competence (Fuson, 2003). Thus, learners are advanced in the way they integrate skills into simple word problems. At this stage, the learners are efficient, which means they are fast and accurate in production of answers.

Learners in school learn mathematics with understanding, actively building their new knowledge from experience and prior knowledge. In Malawi, OBE curriculum follows this trend thereby introducing number concepts and operations from informal mathematical knowledge which the learners bring from their society. This agrees with one of the key findings of the project of the US National Science Foundation about how people learn. Bransford, Brown and Cocking (2000, p. 14) state that if the learners' "initial understanding is not engaged, they may fail to grasp the new concepts and information that are taught". In grade 1 the first topic is pre-number activities where learners are involved in different skills such as sorting out objects, classifying, comparing, matching and ordering objects according to different attributes like shape, size, color, kind and use. From pre-number activities, learners advance to counting, naming, and writing numbers, ordering and order of operations (advancing from single-digit operations to mastery phase). Acquisition of formal mathematical knowledge by learners is done through modeling using objects, number lines, regions, games, songs, and role playing (MIE, 2008). The way MIE has structured and sequenced the topics on number concepts and operations in primary school mathematics curriculum materials (MIE, 2007 - 2009) confirm what earlier studies by (Gelman & Galistel, 1978; Guberman, 1996; Fuson, 2003; Reuben, 2009) found that mathematical concepts are hierarchical in structure and each idea is contained in the idea that follows it. In support, Stols (2013) agrees that learners build their understanding on pre-knowledge. As a result, the starting point of each topic in a grade should build on pre-knowledge from the previous grade level. The activities must also be selected in such a way that they will help the learner to form connections between concepts.

2.7 Other Studies on opportunity to learn (OTL)

Textbooks are important resources for teaching and learning mathematics. In most cases, a textbook is the only resource that all learners will have access to during the lesson. Li, Zhang, and Ma (2009) pointed out the importance of textbooks to learners as follows:

> The textbook should arouse students' interest in learning mathematics, help students to study mathematics actively, develop students' potential in creativity through the process of learning basic knowledge, improve students' mathematical thinking when trying to understand the essence of mathematics knowledge, and raise students' awareness to apply mathematics knowledge in everyday lives (p. 173).

This crucial role of textbooks is also reflected in a number of studies that are devoted to analyzing and understanding the potential effect of the different features of the textbooks on mathematical learning (e.g., Fan et al., 2013; Mikk, 2000; Pepin & Haggarty, 2001; Remillard, 2005; Valverde et al., 2002). In support, Li et al. (2009) explained that the analysis made by these studies reveal nuanced insights into variation in what the learners need to learn in textbooks in order to achieve the desired learning outcomes and how this object of learning is made available to learners. And the value of textbooks is determined by the extent to which it contributes to student' achievement of the learning outcomes (Chang & Salalahi, 2017). This shows that the goals of the curriculum (on number concepts and operations) are expected to be highlighted and achieved through the textbook.

Studies worldwide examining how mathematics textbooks influence instruction generally agree that textbooks have a significant influence on learners' opportunities to learn mathematics (Stylianides, 2009). This means that mathematics textbooks play a decisive role in daily teaching practice and therefore in the learning opportunities that students are offered in these textbooks. For example, it is observed that learners look for information in the textbooks that can be directly applied to the assigned task or exercise in order to be assisted to solve the task. Opportunities to learn mathematics are therefore generated by how instruction is structured and delivered by the textbooks.

The concept of opportunity to learn has a long history but in mathematics, it dates back to the 1960s when Carroll (1963) contrasted OTL mathematics with the amount of time the student actually spent engaged in the process of learning. Similar studies were done that used the concept of OTL to determine or quantify conditions within school or classroom that hinder or promote learning such as time on, task coverage, curriculum content and instructional strategies (e.g., Carroll, 1963; Floden, 2002; Stols, 2013). In addition, these researchers explained that when learners are provided with greater OTL, mostly when engaged time is maximized, the learners' achievement increases. Research studies were also carried on textbook analysis by different researchers. Several studies investigated and analyzed different series of textbooks to find out OTL provided by textbooks on tasks demanding different cognitive levels of understanding (Hadar, 2017; Van Zanten & Van den Heuvel-Panhuizen, 2018; Wijaya, 2015). The textbooks analysis revealed that there is a positive relation between OTL provided in textbooks and learners' achievement.

Glasnovic Gracin, (2018) investigated what students should do in a particular textbook task in order to be provided with OTL to compute, interpret, and to use argumentation to experience the object of learning when interacting with them. The study revealed that incorporating mathematical activities into the multidimensional framework of textbook tasks may help to better understand the opportunities to learn which are afforded students by using mathematics textbooks (Gracin, 2018).

Textbook tasks (examples and exercises)

Textbooks are important because they contain tasks which are considered as devices for initiating activity and for creating opportunities to learn mathematics. The research conducted by Fan et al. (2013) indicate that textbooks are used by learners as a source of tasks in the form of practice exercises. Therefore, the nature of textbook tasks influence and structure the way learners think and can either serve or limit their views of the content matter with which they are engaged (Gracin, 2018). Thus, it is important for the textbooks that are used by learners to provide rich and worthwhile mathematical tasks, which are fit into the core of the curriculum, authentic, thought-provoking, appropriate for the learners and should have more than one answer or procedure.

Studies indicate that most textbook tasks that are provided in textbooks had few non-routine problems which led to learners experiencing lower procedural complexity in mathematics textbooks (Brändström 2005; Dole & Shield, 2008; Gracin, 2018; O'Keeffe & O'Donoghue, 2011; Vincent & Stacey, 2008). For example, Brändström (2005) examined the differentiation of tasks in textbooks in grade 7 in Sweden. The results indicated a low level of challenging tasks in textbooks because the textbooks emphasized on low-level thinking and tasks with lower cognitive demand. And also, the comparative studies done by Fan et al. (2013) and Li (2000) on textbook tasks in US and China indicated that routine, closed and traditional exercises with no application to real life situations were dominating in the textbooks in both countries. This shows that formulation of suitable textbook tasks is a challenge in many countries.

Examples play an important role in learning about mathematical concepts, techniques, reasoning and in the development of competences in mathematics. Examples are key features in any instructional explanation as well as one of the principle tools used to illustrate and communicate concepts between teachers and learners (Peled & Zaslavsky, 1997). Examples in form of worked examples are used to demonstrate methods, indicate relationships, and to explain difficult tasks. Worked examples act as templates to assist learners to have general devices for solving classes of problems. And textbooks need to provide examples with explanations of how to carry out procedures appropriately. Ronda and Adler (2017) argue that examples highlight features of the mathematical concept that is exemplified in a lesson. Several studies point to the contribution of worked examples for learning to solve mathematical exercises. Studies that investigated the role of textbooks in secondary schools in Swedish and Finnish mathematics found that students work with textbook tasks most of their allocated time in the lesson and that student often seek for help from the examples presented in the textbook in support of solving tasks (e.g., Johansson, 2006; Viholainen et al., 2015). In addition, examples presented by teachers in the classrooms mostly come from the textbooks. Several scholars have also argued on the advantages of using different types of examples rather than only one example (e.g., Bills et al., 2006; Gentner, 2005;; Rittle-Johnson, & Star, 2009; Rohrer & Pashler, 2010; Watson & Mason, 2005; Zhu & Fan, 2006). The scholars explained that the use of examples of different types are effective and facilitate student learning better than using same types of examples. They further argued that when multiple examples are mixed, learners are forced to discern the variations between them and thus get better at sense making of the novel examples. Watson and Mason (2005) suggest that it is the structure of the set of examples (exercise) as a whole that promotes common mathematical sense-making. They explain that carefully designed sequences of examples with systematic variation are likely to result in progress and can make certain aspects of the object of learning noticeable for the learners. In line with this, variation theory claims that learning implies seeing or experiencing different aspects of an object of learning (Marton, 2015). Variation theory emphasizes on the discernment as the necessary condition of learning (Kullberg et al., 2017) and one of its specific principles is that seeing differences in examples/tasks precedes seeing sameness (Marton & Pang, 2006, 2013). Marton and Pang (2013) explain that when assisting learners to understand new concepts, the authors/teachers point to examples that share the aimed-at meaning but with a difference. Marton and Pang also argue against the view of developing new meaning from the experience of sameness in support of variation theory.

Studies conducted in Malawi

In Malawi, Malemya (2019) in a comparative study of textbooks between Malawi and Japan analyzed the differences and similarities between Japanese and Malawian mathematics textbooks for the first 9 years of school. His focus of study based on curriculum information for the textbooks, their content structural organization, depth, breadth and presentation. The analysis revealed that Malawian mathematics education has many things to be desired, and one of the findings is that textbooks contain low level cognitive demanding tasks, and that the textbooks do not encourage independent study for the learners. Malemya gave an example of learners' books from grade 5 to grade 8 where he noted that tasks lacked high level questioning techniques that can help to induce learners' thinking and stimulate their interest and curiosity. He further states that activities in the learners' books are usually too slow to make learners graduate to fast and sophisticated thinking such as reflective thinking (too much repetition of same content and procedures) that will enable learners to develop deep understanding of the tasks (Malemya, 2019). That is, the textbooks dwell on one concept for a long time, and the concept takes a long time to develop.

Other studies done in Malawi focused on the factors that contribute to poor performance of learners in mathematics also reported that 98% of learners in primary schools failed to acquire skills beyond basic numeracy (level 3) in mathematics and that no learner in grade 6 had skills beyond competent numeracy (level 5) in number concepts and operations (e.g., World Bank, 2010; SACMEQ I, II, 2005). This trend did not change in learners' achievement when SACMEQ III was implemented with Malawian learners being ranked fourteenth out of fifteen countries that participated (SACMEQ, 2010). Based on SACMEQ survey reports, Mulera et al. (2017), in their analysis of factors affecting learner performance in Malawi's primary schools, also revealed low achievement in number concepts and operations in mathematics. All these studies were conducted in the classroom, but, until now, no study has been done on textbooks as they are the implementers of the intended curriculum in the classroom. And hence, it is important to investigate what impact the textbooks have on learners' achievement in primary mathematics.

Analyzing the content on numbers, operations and relationships, Malemya (2019) noted that counting numbers start from grade 1 up until grade 7. Malemya explains that counting is the most important basic concept for the other concepts that follow and need to be introduced fast. Malemya's study also reveals that in upper primary textbooks, examples lack patterns of variation across examples and tasks are of low cognitive level that makes the learners fail to grasp the learning outcomes (e.g., averages in grade 7). However, textbook analysis is relatively new in Malawi, and, as a result, no study has been conducted on the opportunity to learn mathematics in number concepts and operations. This study will investigate opportunities for learning number concepts and operations in mathematics that are provided in upper primary curriculum materials in Malawi.

2.8 Chapter summary

This chapter gives an overview of the importance of providing opportunities to learn mathematics in textbooks, how opportunity to learn relate with MDI, textbook lessons, how students learn numbers and previous studies. The next chapter will present the theoretical framework.

Chapter 3: THEORETICAL FRAMEWORK

This chapter presents the theoretical framework that is guiding this study and the components of the theory that were selected: object of learning and exemplification. Lastly, chapter summary.

3.1 Introduction

A theoretical framework is a "blueprint", which serves as a guide on which to build and support the research study (Grant & Osanloo, 2014). This study has adapted Mathematics discourse in instruction (MDI) theoretical framework developed by Ronda and Adler (2017). The MDI framework was developed from another theory known as variation theory. The variation theory is a theory of learning that emphasizes alternation as a necessary condition for learners to be able to discern new aspects of an object of learning (Kullberg et al., 2017).

3.2 Mathematical Discourse in Instruction (MDI) Framework

The study is guided by the Mathematical Discourse in Instruction analytic framework for textbook analysis (MDITx) developed by Ronda and Adler in 2017. This analytic framework for the textbook analysis was adapted from MDI framework that was developed to analyze opportunities made available for learners to learn mathematics (Ronda & Adler, 2017) and it is rooted from the socio-cultural perspective that foregrounds the importance of mathematics in a coherent manner. The MDI framework allows for nuanced description of mathematics teaching and interpretations of differences in what is mathematically made available to learn (Ronda & Adler, 2017). Ronda and Adler further explain that the MDI framework is a framework that characterizes teaching across classroom contexts and practices and foregrounds the importance of generality in mathematics made available to learn. The aim of the MDI framework is to capture the complexity of mathematics teaching by concentrating on the discourse involved in the work of teaching mathematics (Adler & Ronda, 2015). MDITx framework is constituted by two commonplaces of mathematics pedagogy, namely exemplification and accompanying explanations/legitimations of mathematics which characterizes the teaching of mathematics about mediating an object of learning (Marton et al., 2004). These mediatational means of MDI are the commonplace in teaching that work together with the opportunities provided for the learners to participate in mathematical discourse (Ronda & Adler, 2017). Therefore, the quality of MDI is reflected in levels of coherence and connection within and between the example and explanation spaces teachers/textbooks set up within and across lessons, and how these mediate and connect learners to the mathematical object(s) of the lesson (Shortino-Buck, 2017). Mathematical discourse provides an effective way of facilitating learners' conceptual understanding and the acquisition of mathematical knowledge which is vital for learners in upper primary classes.

The MDI framework focuses on five interacting components in the teaching of a mathematics lesson such as: object of learning, examples, tasks, naming/legitimations, and learner participation for mediating the object learning during the lesson. However, this study focuses on two elements of MDITx which are examples and tasks. This is because tasks and examples are the raw material upon which instruction is overlaid (Watson & Mason, 2006), and they play a significant role in the development of mathematics as a discipline (Olteanu, 2018). Figure 1 shows each component of the MDI framework. In the following section, the investigator explains in detail each of the two selected components of MDITx.

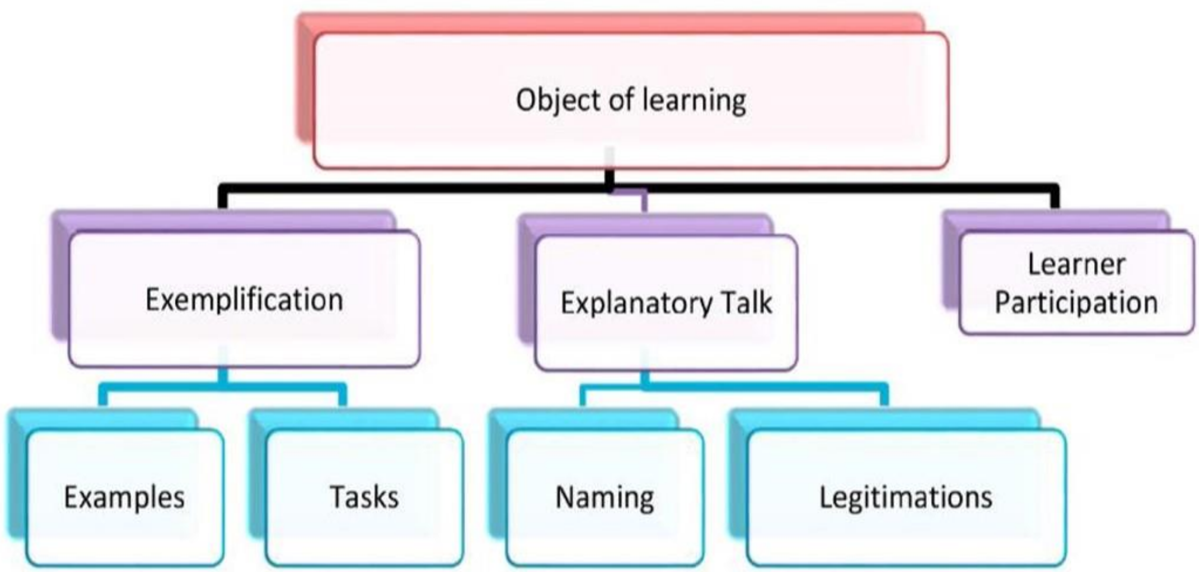


Figure 1: Mathematics Discourse in Instruction framework (Adler & Ronda, 2015, p. 239).

Object of learning

Learning has a purpose and bringing that purpose into focus is central to the work of teaching (Adler & Ronda, 2015). The object of learning is often announced explicitly and relates to the mathematical content and skills that students are expected to learn in a given lesson (content and capability/competence) (Adler & Ronda, 2015). Ling Lo (2012) defines object of learning as what the student needs to learn in order to achieve the desired learning objectives. Ronda and Adler

(2015) describe object to learn (the lesson goal) as what the learners are expected to know and they are able to do, such as concept, procedure, algorithm or meta-mathematical practice. It focuses on what is introduced by the textbook lesson at the start, often the title or topic. For example: “adding fraction”. The choice of object of learning affects the quality of learners’ learning. It is therefore vital that the object of learning should be made known to learners at the beginning of the lesson.

Exemplification

Examples

Mathematical content is made visible through examples because they are a common place in primary mathematics lessons (Ronda & Adler, 2017). Zodik and Zaslavsky (2008) define examples as a particular case of a larger class, from which one can reason and generalize as an instantiation of the content in focus. The example which the textbook selects should enable the learners to determine and understand important concepts in the lesson. The examples may be written in symbolic form ($7 > 4x - 2$), or in visual form like drawings of rectangles and graph of functions. The examples might be worked to demonstrate the procedure, or not worked to act as learners’ exercises. Marton and Pang (2006) explain that the key to better learning involves bringing attention to patterns of variation amidst invariances. Thus, textbook lessons need to provide a variety of examples that will enable learners to learn. It is therefore important for the author/teacher to provide opportunities for learners to discern key features of the content so as to enable them to learn mathematics.

To determine the opportunities to learn mathematics through example, Adler and Ronda (2015) formed and described a set of progressive indicators for the example spaces in the textbook. They came up with three necessary aspects for a pattern of examples that constitute a basis for generalization. These included contrast, generalization, and fusion. Contrast (C) is when a set of examples bring attention for seeing one instance of what the object of learning is not. Generalization (G) is when the sequence of examples displays a similar aspect of object of learning across all sets of examples. Fusion (F) is when more than one aspect of an object of learning is simultaneously varying across an example space (Adler & Ronda, 2015). Coding of the example space was done by assigning three different levels of exemplification of textbook lessons. The example space is coded Level 1 when only one pattern of variation is used: either contrast or generalization. Level 2 is assigned when two patterns of variation are applied, and the patterns may be contrast and generalization or contrast and fusion and or generalization and fusion. Level 3 is assigned when there is (fusion) simultaneous variation of more than one aspect of the object of learning. If there are no patterns of variation that can be detected in the example space, then the code is NONE. “NONE does not mean that the author did not provide any examples. It means that the author did not provide opportunities for learners to discern the key feature of the content” (Ronda & Adler, 2017, p. 1102).

Tasks

Tasks are designed to mediate the competence of learners with respect to content. Ronda and Adler (2017) define tasks as what learners are asked to do with the examples. Examples make the goal of the content to be visible thereby drawing attention to relevant features within the task. In order to increase the opportunity to learn, tasks need to address not only the capability stated in the object of learning but also whether the task has a potential to engage learners in different experiences of the content which will enable learners to make connections among features of the mathematical content (Ronda & Adler, 2017). In their analysis, Ronda and Adler assigned tasks into three different levels of cognitive demand namely KPF (known procedures and facts), CTP (current topic procedures) and AMC (application/making connection tasks). Level 1 was coded KPF the tasks required a learner to carry out a known operation or procedure and facts. Level 2 was coded CTP as the tasks required a learner to use a new method of solving the task that was introduced in the current lesson and level 3 was coded AMC as the task required learners to involve multiple concepts and make connections between or among concepts.

3.3 Theoretical framework as applied in the study

In this study, Mathematical Discourse in Instruction (MDI) framework guided the investigator to formulate questions and analyze the textbooks in order to collect data. The MDITx assisted the investigator particularly, to divide the textbook lessons into blocks (example spaces) and coding of each example space using patterns of variation found in the examples and assigning levels to both example spaces and tasks. In addition, the MDITx framework guided the investigator in the discussion of the findings and conclusion.

3.4 Chapter Summary

The chapter gives the introduction of the theoretical framework. The section has discussed the background of the MDITx framework, its components and how the framework is going to guide the study to analyze and discuss the findings.

Chapter 4: METHODOLOGY

4.1 Introduction

This chapter presents the research design/methodology and data collection techniques used in the study. It also describes how sample and sampling was carried out. It further justifies how data analysis procedures were conducted.

4.2 Research Design

Research designs are procedures for collecting, analyzing, interpreting, and reporting data in research studies (Creswell, 2012). Thus, there are plans and actions for research that include detailed methods of data collection and analysis. There are several factors that guide the research design. These include the nature of research problems, personal experiences of a researcher, audience the research is writing for, and the research sample (Creswell, 2009). Research designs are useful, because they help guide the methods and decisions that researchers must make during their studies and provide the analytical framework at the end of the study (Creswell, 2012).

The study was guided by mixed method approach to answer the research questions and collect data related to opportunities for learning mathematics that are provided in textbooks in upper primary classes. Mixed methods research is defined as an approach to inquiry that combines or associates both qualitative and quantitative forms (Creswell, 2009). In mixed methods approach, the investigator follows both the principles and assumptions of qualitative and quantitative study. This approach was selected because looking at the research questions, the use of a single research approach would not have been enough to answer all the specific questions for this research. The goal was to combine both approaches in creative ways that utilize the strengths of each within a single study (Creswell, 2009).

Mixed method design comprises different strategies that are used to collect data. These strategies include sequential explanatory, sequential transformative, concurrent triangulation, concurrent embedded and concurrent transformative (Creswell, 2009). In particular, this study employed sequential explanatory strategy. In this approach, the investigator expands on the findings of one method with another (Ary et al., 2014; Creswell, 2009). That is, data collection starts with quantitative approach followed by qualitative data in the second phase (Creswell, 2009). The results for quantitative phase are the ones that inform the qualitative part of data collection.

Quantitative research designs are designs that are characterized by data collection which is expressed in numerical forms and analyzed using suitable statistical methods (Ary et al., 2014).

Qualitative research designs are designs that are a means of understanding the meaning individuals or groups give to a problem (Creswell, 2009).

The study collected quantitative data by coding the examples and the tasks from the learners' book using the MDITx analytical tool adapted from Ronda and Adler (2017). The assigned codes were examined against the ability to provide a set of examples and tasks that were able to foreground critical features of the object of learning in the lesson. The codes for each example space were later analyzed and interpreted using qualitative design approach in order to get an in-depth understanding of the content.

4.3 Sampling procedures

Sample Size

A sample is any group from which a researcher get information; and sampling is the statistical process of selecting a subset (called a "sample") of a population of interest for purposes of making observations and statistical inferences about that population (Bhattacherjee, 2012).

The study targeted mathematics textbooks for upper primary classes from grade five to grade eight. The sample size for the study included four teachers' guides, four learners' books and four syllabi.

Sampling Technique

Mathematics textbooks for the upper primary classes were selected by using convenience sampling technique. According to Ary, Jacobs, Sorensen and Walker (2014), convenience sampling is "choosing a sample based on availability, time, location or ease of access" (p. 459). The investigator's choice of sampling technique was based on availability and ease of access of the textbooks since primary schools in Malawi use one prescribed mathematics textbook per class. Malawi Institute of Education (MIE) is the sole publisher of these curriculum materials and MIE is a body within the Ministry of Education, Science and Technology.

From the curriculum materials sampled in the study, the investigator purposefully selected the learners' book since the aim of the study was to investigate opportunity for learning mathematics in the upper primary. Thus, the study focused on the learners and how much the textbooks were providing opportunity for them to access the object of learning in mathematics. According to Gall, Gall and Borg (2007), purposeful sampling aims at selecting cases that will provide rich information in respect to the purpose of the study. And the only textbook that learners interact with is the learners' book.

In the learners' book, the investigator selected the first learning outcome (core element), which is numbers, operations, and relationships. The assumption for selecting numbers, operation and relationships was that the learning outcome comprises more topics per textbook more than the other learning outcomes. Therefore, this provided the investigator a wide range of coverage of the example spaces. And also, numbers, operations and relationships is the foundation for understanding other mathematical concepts. There were 184 example spaces where data was collected from the four learners' book. Table1 summarizes targeted topic and example spaces.

Table 1: summary of topics and example spaces per book

Class	Number of Topics	Example spaces (examples/tasks)
Grade 5	8	48
Grade 6	14	49
Grade 7	10	54
Grade 8	8	33
Total number	40	184

The first column indicates the class, column two shows the number of topics per class, and column three indicates the example spaces where the data was collected. The last row indicates the total number of topics and total number of example spaces for the four textbooks.

However, the syllabi and the teachers' guides were part of the sample because they provided the investigator with background information for the analysis since introduction of the units/topics and worked examples were found in the teachers' guide. And the teachers' guides extract information from the syllabi.

4.4 Data collection

The instrument used to collect the data was Mathematical Discourse in Instruction Analytic Tool (MDITx). The analytical tool was used to code the examples and tasks in order to foreground different elements of mathematical discourse of instruction in the textbooks. In the study, the components of explanatory talk and learner participation were excluded in the analysis because the investigator was only focusing on the exemplification part of the MDI framework.

4.5 Data analysis techniques

In order to analyze the mathematics offered in the textbooks, the study employed MDI analytic tool (Ronda & Adler, 2017). This MDI analytical tool assisted the investigator to highlight the possible OTL mathematics which the curriculum materials provided in the textbooks in order to mediate the object of learning to the learners. The study focused on the author's use of examples and tasks which afford opportunities for learning mathematics. In order to carry this study, the investigator adapted the method of TIMMS textbook study which, according to Ronda and Adler (2017), parsed (computed) the textbook lessons into blocks. Each block represented the author's pedagogical focus such as introduction of the topic, worked examples and exercises and then coded the example spaces according to each element of instruction.

4.6 Data collection procedures

The collection of data began by identifying different separated blocks of the lesson (content) in the learners' book and coded them by looking at patterns of variations provided in the textbooks. Each block comprises worked examples and practicing exercises (tasks) and coding. The coding was guided by MDI Tool for Textbook analysis (MDITx) developed by Rhonda and Adler (2017). The main focus of the analysis was on exemplification (examples and tasks).

4.7 Data Analysis

The purpose of data analysis was to make sense out of data that was collected. Data analysis is the process of making sense out of text and image data (Creswell, 2009). In this study, data analysis was done both quantitatively and qualitatively for the reason that this study adopted a mixed methods approach. The analysis started with quantitative analysis followed by qualitative one.

Table 2: MDI analytic Tool for Textbooks lesson (MDITx)

Examples	Tasks
Examples are coded as follows: **Level 1**: one pattern of variation used; either contrast (C) or Generalization (G)	Tasks are coded as follows:
Level 2: any two patterns of variation used; C, G/ C, F and/ G, F **Level 3**: - all patterns of variation used Other description **Contrast** (noticing difference) **Generalization** (noticing similarity) **Fusion** (discerning simultaneous dimensions of variation – add previous experience)	**Level 1** – carry out known procedures or known concepts related to the object of learning (**KPF only**) **Level 2** – carry out procedures involving the object of learning (**includes CTP**) **Level 3** – carry out level 2 tasks plus tasks that involve multiple concepts and connections (**includes CTP and AMC**)

Table 2 describes code for analyzing textbook examples and tasks. Column one shows the codes for examples and column two shows coding for the tasks.

4.8 Chapter Summary

This chapter has described the design of the study, the strategies that were used to collect data, the instruments that were used, sampling of textbooks and how data was analyzed.

Chapter 5: Findings

This chapter presents details of the findings of the study. The chapter begins with a presentation of findings following two components of the MDI framework that have been discussed in methodology. Furthermore, the findings are presented through document analysis on each of the four learners' books. Finally, the chapter summarizes what has been discussed in the entire chapter.

5.1 Introduction

The analysis was done in the four learners' books on the first core element: numbers, operations, and relationships for grade 5 to grade 8 – the upper primary classes. There are six core elements in primary school mathematics namely, numbers, operations and relationships, accounting and business studies, space and shape, measurement, patterns, functions and algebra and data handling. The investigator focused on the first core element as it forms the basis for the other core elements. The purpose of the analysis was to collect data on how examples and tasks are providing opportunities for learner to learn mathematics in upper primary curriculum materials following MDITx framework.

Overview of the chapter

The results from the document analysis of the four learners' books for the four grades (grade 5 – grade 8) indicate several observations:

First, the introductory explanations of the object of learning are not included in the learners' books for grade 5, 6 and 7 instead textbook lessons are introduced in the teachers' guide. In the learners' books there are only the object of learning, worked examples and exercises. The instructions on how the learners are going to do the tasks are written in the teachers' guides instead of the learners, book. This shows that the teachers give the instruction to the learners on the way they should do the tasks.

Second, the textbooks are providing low-level examples and low-level cognitive demanding tasks. This shows that the textbooks are offering few opportunities for learners to participate in mathematical discourse in upper primary school. The low-level cognitively demanding tasks encourage rote learning where the learners memorize the rules or definitions thereby using known procedures and facts instead of using procedures with connections which initiate conceptual understanding that will help the learners to acquire mathematical proficiency.

Third, there are few high-level examples in the textbooks. The textbooks do not provide opportunities for learners to discern simultaneous dimensions of variations which incorporate all the three patterns of variations. The textbooks also have few high-level cognitively demanding tasks. Gracin (2018) argues that, procedures with connections develop deeper levels of understanding of mathematical concepts and ideas. This shows lack of mathematical proficiency in the textbooks that can assist learners to access critical features of object of learning.

Four, the textbooks lack multiple modes of representation of concepts. Tasks in the textbooks encourage computation and procedure without connection. Most of the procedures provided in the textbooks encourage memorization through use of formulas. There are very few representations such as modeling, games, role play, diagrams/illustrations, graphs, or tables.

Five, there are few worked examples in the learners' books. Most of the example spaces in all learners' books have one worked example and in other cases, no worked example is given to guide the learners when working on the tasks. Sometimes worked examples do not match with some of the problems given in the learners' task. Most of the worked examples provided in the textbooks are simple examples, and they only show one procedure in most cases. Thus, the textbooks limit the learners to experience patterns of variation that will help them to learn.

Six, the analysis reveals that all the learners' books contain a lot of repeated activities. For example, counting numbers begin from grade 1 up to grade 7. The only difference which is done when counting number is that values of numbers vary from grade to grade. The textbooks also contain activities which have same type of questioning techniques.

Seven, the textbooks dwell on the same skills for a long time, and this leads to slow development of concepts in the learners. For example, the concept of factors and multiples trade all the way from grade 4 through grade 8 and on almost the same content and procedures. The textbooks limit learners from widening their knowledge as they only use low level cognitive demand (use known procedures and facts). That is, the learners reproduce the rules, definitions and procedures without applications and making connections across the concepts and operations.

Lastly, textbooks are teacher-centred. Thus, all the important information and instructions that are supposed to be written in the learners' books are written in the teachers' guide and the teacher is mandated to give instruction to the learners for them to follow. Teacher-centredness of the textbooks makes learners to find problem when they are using the textbooks on their own, at school or in their homes.

Analyzing the textbooks

Analyzing the four learners' books in upper primary school on the first learning outcome, the investigator found out that the textbooks provide only one worked example per example space in most textbooks. For example, in the first core element, in grade 5 all example spaces have only one worked example. Grade 6 has fourteen example spaces that contain two worked examples per example space; grade 7 have twelve example spaces that contain two worked examples per example space and grade 8 have only four example spaces that contain two worked examples per example space. The analysis combined worked examples with the practice exercises in order to capture good coverage of examples for coding and this also is in line with MDITx analytical framework. Ronda and Adler (2017) state that, "practice exercises following worked examples are regarded as in the same block" (p. 1106). The investigator coded the tasks by analyzing the practice exercise that the learners' books provide. However, review exercises were not coded in all the learners' book as they were treated as concluding exercises for the unit for assessment purpose and they were not used to develop the topic.

The analysis expects to answer the following research questions:

- What opportunities to learn number concepts and operations are provided through the examples and task in the textbooks?
- How do the tasks enact/disclose the learning objects that are stated in the textbooks?
- To what extent do tasks allow learners to apply and connect critical features of the mathematical content?

5.2 Textbook 1: Grade 5 learners' mathematics textbook

Grade 5 learners' book contains units and topics. The six units that are found in grade 5 learners' book stands for the six learning outcomes/core elements for primary school mathematics. With contrast in that for grade 6, 7 and 8, the units represent topics and there are not learning outcomes for primary school. With regards to this explanation, analysis of grade 5 learners' book will concentrate much on topics.

Object of learning

The textbook starts with the concepts and capabilities (object of learning) on every block (example spaces) and then followed by worked examples and practice exercises. However, the analysis reveals that most of the example spaces do not start with the introductory explanations which could assist learners to understand the object of learning. That is, the author does not elaborate to the learners how they should do the tasks. Instead the content area commences with worked example followed by practice exercise. In other cases, the tasks do not state the capability. For example, Topic 1A and topic 1C, the author just wrote "Exercise 1D and "Exercise 3" respectively.

The first core element, numbers, operations and relationships, contains 48 example spaces that were analyzed and coded. The codes were assigned to different examples and tasks and the results were quantitatively recorded using the table. Table 3 presents summary of coded findings from analysis of the examples and tasks from unit 1.

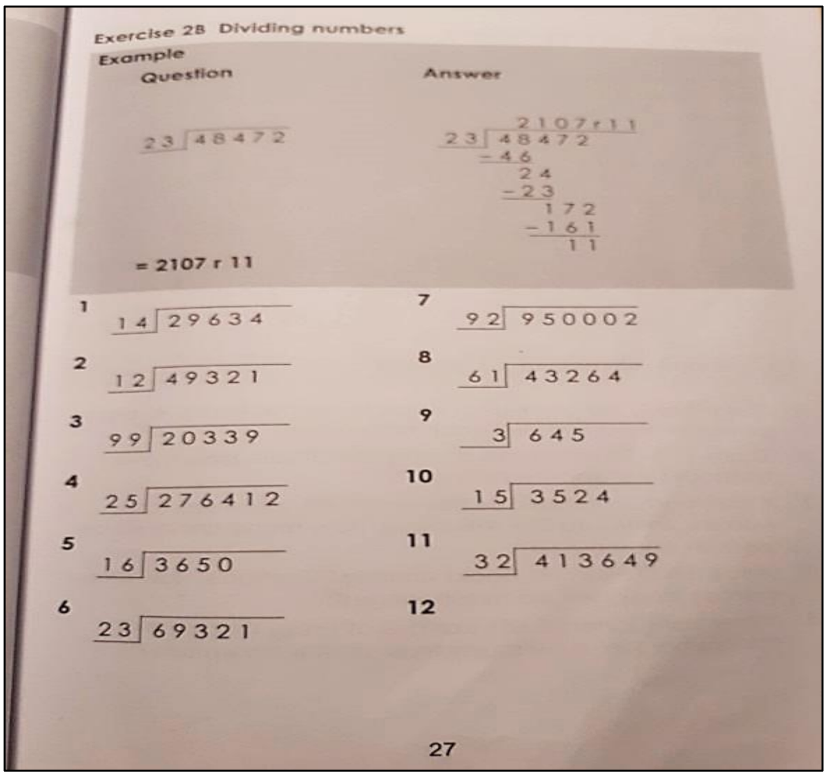
Table 3: summary of coded examples and tasks for learners' book 5

Core Element/ Learning outcomes	**Examples**		**Tasks**	
	Codes	Example spaces	Levels of tasks	Number of Tasks
Number, operations, and relationships (8 topics),	C	11	Level 1	24
	G	15	Level 2	21
	C and G	19	Level 3	3
	C, G & F	3		
	Levels			
	Level 1	26		
	Level 2	19		
	Level 3	3		

Table 3 shows core element/ learning outcome for the topics, examples and tasks that were coded.

Findings from examples

The summary for the analysis for grade 5 textbook shows that out of 48 example spaces, 11 were coded contrast (C) and they belong to level 1. These example spaces provide opportunity for learners to see some differences thereby helping them to compare and contrast. This shows that the example space is able to provide opportunity for learners to discern contrast. The example spaces are applying one pattern of variations across examples and hence belong to level 1. In order to code the examples as contrast, the following example space was used to illustrates the analysis (see figure 2).


Exercise 2B Dividing numbers

Example

Question

23) 48472

= 2107 r 11

Answer

2107 r 11
23) 48472
−46
24
−23
172
−161
11

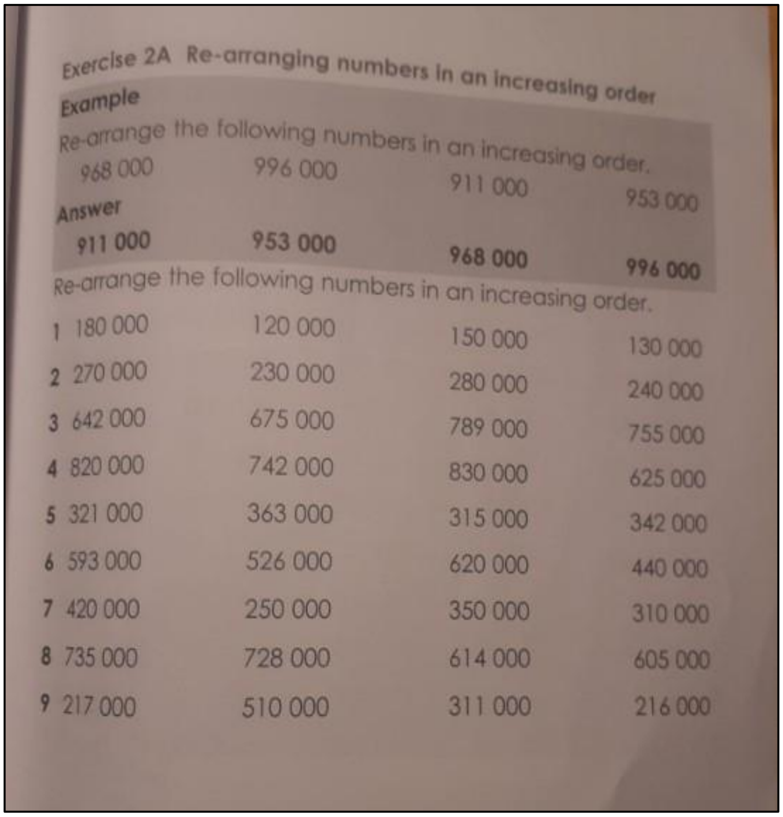
1. 14) 29634
2. 12) 49321
3. 99) 20339
4. 25) 276412
5. 16) 3650
6. 23) 69321
7. 92) 950002
8. 61) 43264
9. 3) 645
10. 15) 3524
11. 32) 413649
12.

27

Figure 2: Exercise 2B. Dividing numbers (MIE, 2007, p. 27*)*

The Examples 1–11 and worked examples have the same structure: dividing numbers by two-digit divisors with a remainder (as seen in the worked example and also after solving the practice exercise); with contrast in that example 9 is divided by 1-digit divisor without a remainder. The example space provides opportunity for learners to notice the different between and within examples in the example space.

Fifteen example spaces were coded generalization (G), and they belong to level 1. These example spaces enable the learners to discern similarity which provide opportunity for generalizing the features of the problems. And they are also a low-level example spaces as they provide only one pattern of variation (see figure 3).



Exercise 2A Re-arranging numbers in an increasing order

Example

Re-arrange the following numbers in an increasing order.

968 000	996 000	911 000	953 000

Answer

911 000	**953 000**	**968 000**	**996 000**

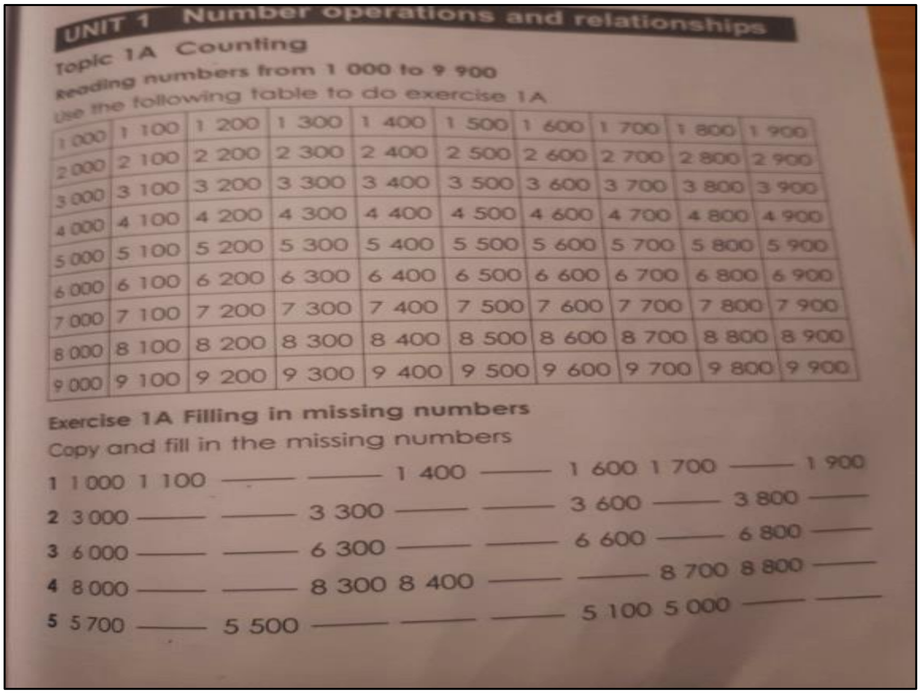
Re-arrange the following numbers in an increasing order.

1	180 000	120 000	150 000	130 000
2	270 000	230 000	280 000	240 000
3	642 000	675 000	789 000	755 000
4	820 000	742 000	830 000	625 000
5	321 000	363 000	315 000	342 000
6	593 000	526 000	620 000	440 000
7	420 000	250 000	350 000	310 000
8	735 000	728 000	614 000	605 000
9	217 000	510 000	311 000	216 000

Figure 3: Exercise 2A. Re-arranging numbers in an increased order (MIE, 2007, p. 5).

In this example space, examples 1–9 and worked example have similar structures, ordering numbers from largest to smallest. As the learners' order numbers from largest to smallest in all examples, they will notice same pattern —similar structure and sequence in all the examples. The learners will see similarity as they follow logical sequence of ordering numbers.

Out of 48 example spaces, 19 example spaces belonged to level 2 and they were coded contrast and generalization (C, G). These example spaces have two patterns of variation that are used in the same example space. They enable the learners to notice certain features that are not object of learning and at the same time see similarity in the same example space which enable them to generalize. To clarify this pattern of variation, exercise 1A was used to shows how the investigator analyzed and coded the example space (see figure 4).



UNIT 1 Number operations and relationships

Topic 1A Counting

Reading numbers from 1 000 to 9 900

Use the following table to do exercise 1A

1 000	1 100	1 200	1 300	1 400	1 500	1 600	1 700	1 800	1 900
2 000	2 100	2 200	2 300	2 400	2 500	2 600	2 700	2 800	2 900
3 000	3 100	3 200	3 300	3 400	3 500	3 600	3 700	3 800	3 900
4 000	4 100	4 200	4 300	4 400	4 500	4 600	4 700	4 800	4 900
5 000	5 100	5 200	5 300	5 400	5 500	5 600	5 700	5 800	5 900
6 000	6 100	6 200	6 300	6 400	6 500	6 600	6 700	6 800	6 900
7 000	7 100	7 200	7 300	7 400	7 500	7 600	7 700	7 800	7 900
8 000	8 100	8 200	8 300	8 400	8 500	8 600	8 700	8 800	8 900
9 000	9 100	9 200	9 300	9 400	9 500	9 600	9 700	9 800	9 900

Exercise 1A Filling in missing numbers

Copy and fill in the missing numbers

1 1 000 1 100 ___ ___ 1 400 ___ 1 600 1 700 ___ 1 900

2 3 000 ___ ___ 3 300 ___ ___ 3 600 ___ 3 800 ___

3 6 000 ___ ___ 6 300 ___ ___ 6 600 ___ 6 800 ___

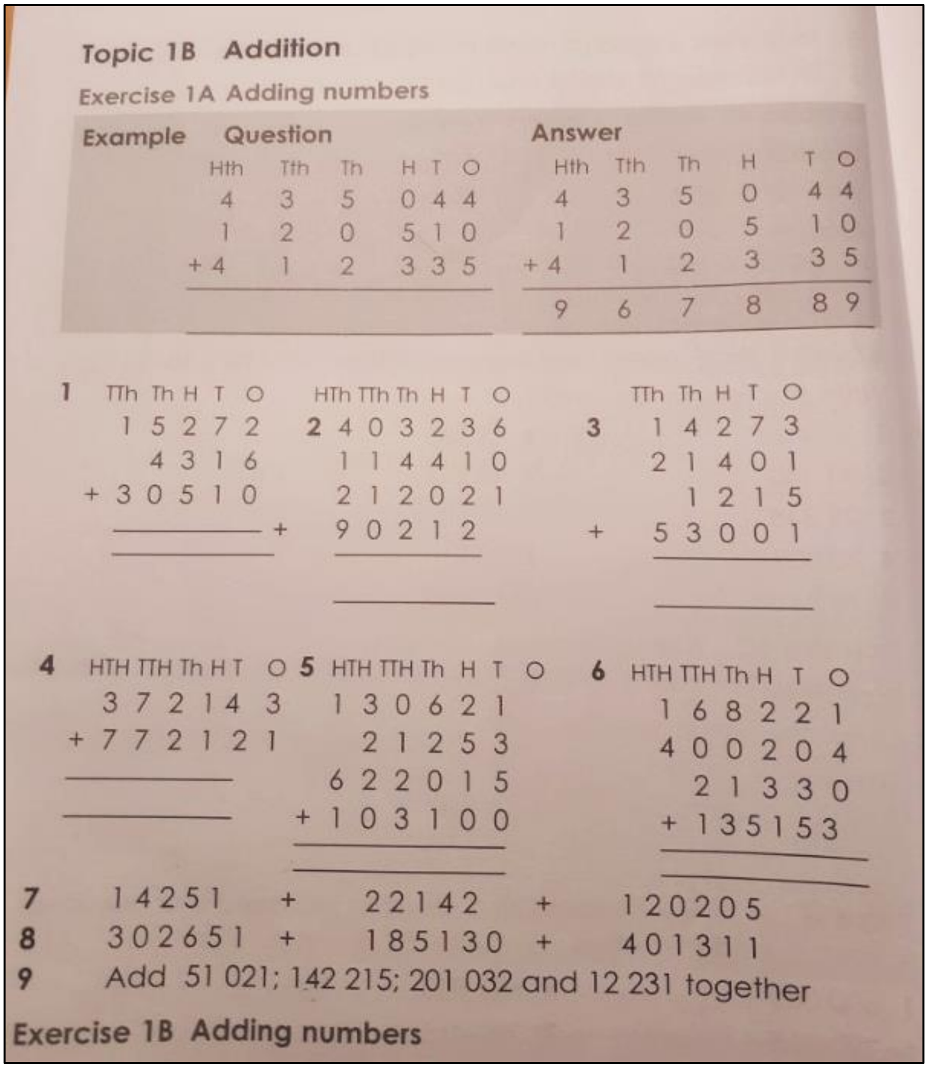
4 8 000 ___ ___ 8 300 8 400 ___ ___ 8 700 8 800 ___

5 5 700 ___ 5 500 ___ ___ ___ 5 100 5 000 ___ ___

Figure 4: Exercise 1A. Filling in the missing numbers (MIE,2007, p. 1*)*

The example space shows that examples 1–4 involve filling in the numbers in ascending order and example five involves filling in numbers in descending order. The learners will notice that all the numbers are following a certain sequence where the difference between successive terms is the same – 100. Thus, providing opportunity for learners to see similarity and be able to generalize. On the other hand, learners will notice contrast as the fill in numbers in descending order – having the opportunity to notice difference. Therefore, the example space is a level 2 example as they apply two patterns of variation across examples.

And lastly, 3 example spaces out of 48 example spaces were coded contrast, generalization and Fusion (C, G & F) and these belong to level 3. The example spaces in level 3 help the learners to discern different critical features of object of learning at the same time and they also use knowledge from their previous experience. To illustrate the findings in this level, exercise 1B was used to show how the investigator analyzed and coded the example space (see figure 5).



Topic 1B Addition

Exercise 1A Adding numbers

Example	Question						Answer					
	Hth	Tth	Th	H	T	O	Hth	Tth	Th	H	T	O
	4	3	5	0	4	4	4	3	5	0	4	4
	1	2	0	5	1	0	1	2	0	5	1	0
	+ 4	1	2	3	3	5	+ 4	1	2	3	3	5
							9	6	7	8	8	9

1 TTh Th H T O
1 5 2 7 2
4 3 1 6
+ 3 0 5 1 0

2 HTh TTh Th H T O
4 0 3 2 3 6
1 1 4 4 1 0
2 1 2 0 2 1
+ 9 0 2 1 2

3 TTh Th H T O
1 4 2 7 3
2 1 4 0 1
1 2 1 5
+ 5 3 0 0 1

4 HTH TTH Th H T O
3 7 2 1 4 3
+ 7 7 2 1 2 1

5 HTH TTH Th H T O
1 3 0 6 2 1
2 1 2 5 3
6 2 2 0 1 5
+ 1 0 3 1 0 0

6 HTH TTH Th H T O
1 6 8 2 2 1
4 0 0 2 0 4
2 1 3 3 0
+ 1 3 5 1 5 3

7 14251 + 22142 + 120205

8 302651 + 185130 + 401311

9 Add 51 021; 142 215; 201 032 and 12 231 together

Exercise 1B Adding numbers

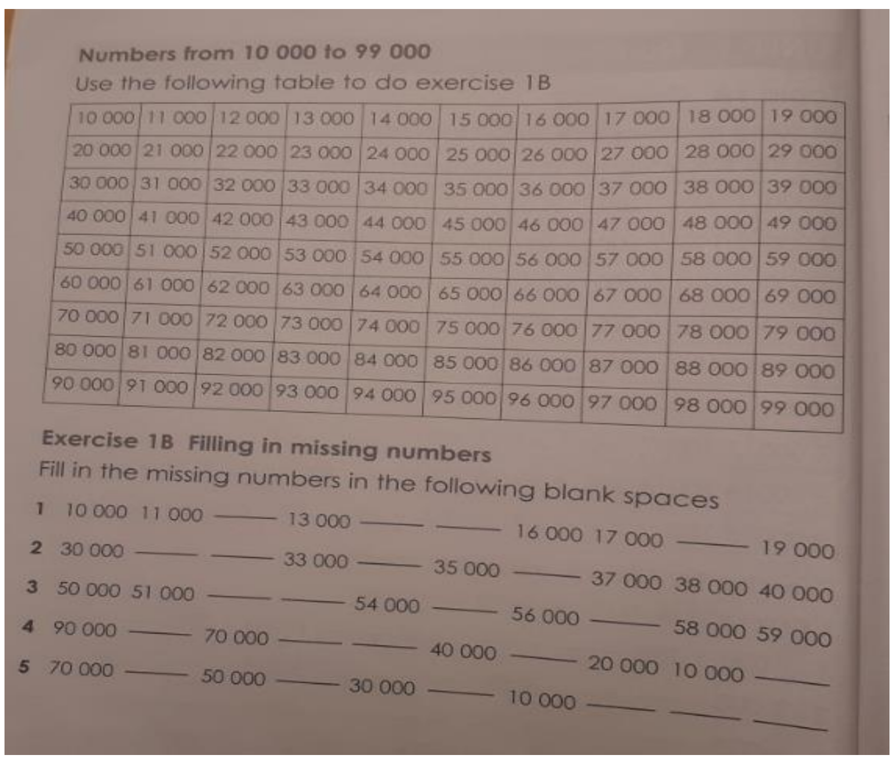
Figure 5: Exercise 1B. Adding numbers (MIE, 2007, p. 14)

The example space shows different ways of presenting addition of numbers such as vertical addition, horizontal addition, addition without and with regrouping and adding numbers up to four addends. Thus, learners will be able to compare and contrast among the examples. As the learners add numbers vertically or horizontally, they will notice some similarity and they will be able to generalize. Moreover, in examples 7–9, involves learners to identify place value positions, addition sign, ways of presenting addition and identifying key word that shows addition (number 9) – this will require learners to use previous knowledge (fusion). This example space provides opportunity for discerning simultaneous dimension of variations in order to mediate the critical features of object of learning.

The analysis of the example spaces indicate that most example spaces belong to level 1 (26 out of 48) because they have only one pattern of variation; that is, either contrast or generalization. And the example spaces also lean partly on level two with two patterns of variations such as C and G (19 out of 48). Very few example spaces belong to fusion, which is level three (3 out of 48). This implies that grade 5 learners' book provide low level examples that enable learners to experience only one pattern of variation. Therefore, few opportunities for learners to experience features that are essential for particular learning in a single example space in order to develop certain capabilities.

Findings from the tasks

The other part of exemplification is task. The textbook provided a set of exercises that enable the learners to practice the aspect being illustrated in the worked examples. The overall analysis of the tasks for the 8 topics in unit 1 shows (portray) that, 24 tasks out of 48 tasks were coded under KPF. Thus, the tasks required the learners to use the known procedures and facts that they already have. Figure 6 was used to demonstrate the analysis.



Numbers from 10 000 to 99 000

Use the following table to do exercise 1B

10 000	11 000	12 000	13 000	14 000	15 000	16 000	17 000	18 000	19 000
20 000	21 000	22 000	23 000	24 000	25 000	26 000	27 000	28 000	29 000
30 000	31 000	32 000	33 000	34 000	35 000	36 000	37 000	38 000	39 000
40 000	41 000	42 000	43 000	44 000	45 000	46 000	47 000	48 000	49 000
50 000	51 000	52 000	53 000	54 000	55 000	56 000	57 000	58 000	59 000
60 000	61 000	62 000	63 000	64 000	65 000	66 000	67 000	68 000	69 000
70 000	71 000	72 000	73 000	74 000	75 000	76 000	77 000	78 000	79 000
80 000	81 000	82 000	83 000	84 000	85 000	86 000	87 000	88 000	89 000
90 000	91 000	92 000	93 000	94 000	95 000	96 000	97 000	98 000	99 000

Exercise 1B Filling in missing numbers

Fill in the missing numbers in the following blank spaces

1 10 000 11 000 ____ 13 000 ____ ____ 16 000 17 000 ____ 19 000

2 30 000 ____ ____ 33 000 ____ 35 000 ____ 37 000 38 000 40 000

3 50 000 51 000 ____ ____ 54 000 ____ 56 000 ____ 58 000 59 000

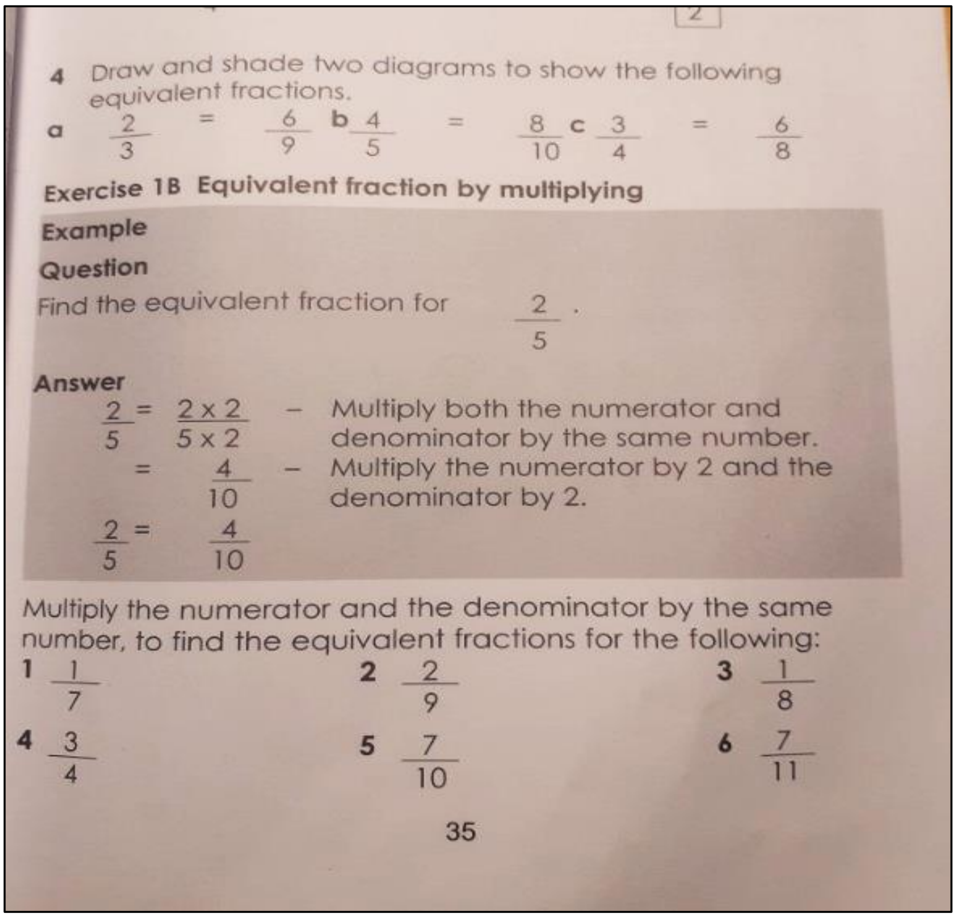
4 90 000 ____ 70 000 ____ ____ 40 000 ____ 20 000 10 000 ____

5 70 000 ____ 50 000 ____ 30 000 ____ 10 000 ____ ____ ____

Figure 6: Exercise 1B. Filling in missing numbers (MIE, 2007, p. 2)

The example space allows learners to recall and use previously learned procedures in order to fill in the numbers in ascending and descending order. For example, the learner will identify the difference between the two numbers in each example and then follow the sequence. The cognitive level of this task is low level task because the learners will just use the knowledge that they already have from the previous lessons.

There were 21 tasks out of 48 tasks that were recorded under CTP as they needed learners to use the methods or knowledge that was introduced in the textbook lessons through worked examples. The tasks require learners to use the procedures carried in the worked example within the example spaces. The example space that follow illustrate how the tasks were coded (see figure 7).


4 Draw and shade two diagrams to show the following equivalent fractions.

a $\frac{2}{3} = \frac{6}{9}$ b $\frac{4}{5} = \frac{8}{10}$ c $\frac{3}{4} = \frac{6}{8}$

Exercise 1B Equivalent fraction by multiplying

Example

Question

Find the equivalent fraction for $\frac{2}{5}$.

Answer

$\frac{2}{5} = \frac{2 \times 2}{5 \times 2}$ – Multiply both the numerator and denominator by the same number.

$= \frac{4}{10}$ – Multiply the numerator by 2 and the denominator by 2.

$\frac{2}{5} = \frac{4}{10}$

Multiply the numerator and the denominator by the same number, to find the equivalent fractions for the following:

1 $\frac{1}{7}$ 2 $\frac{2}{9}$ 3 $\frac{1}{8}$

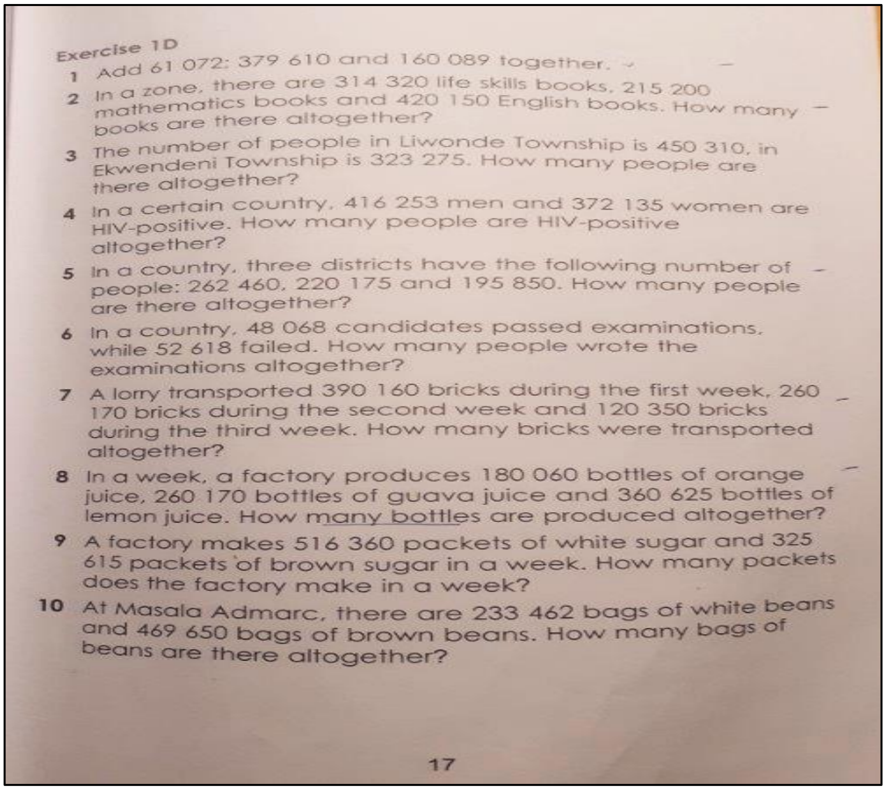
4 $\frac{3}{4}$ 5 $\frac{7}{10}$ 6 $\frac{7}{11}$

35

Figure 7: Exercise 1B. Equivalent fraction by multiplying (MIE, 2007, p. 35).

To find the equivalent fractions in the task, learners will solve the exercise by following the instruction given in the task and also the worked example. That is, the example space is providing opportunity for the learners to use the new method that is being currently introduced.

Only 3 tasks out of 48 tasks belonged to AMC because the tasks required the learners to involve KPF, CTP and more concepts and connections in order to solve the problems. Thus, the tasks needed learners to make decision on how to solve them in terms of algorithms and concepts and sometimes required the learners to apply and make connections between concepts. Exercise 1D illustrate how the investigator coded the tasks (see figure 8).



Exercise 1D

1 Add 61 072; 379 610 and 160 089 together.

2 In a zone, there are 314 320 life skills books, 215 200 mathematics books and 420 150 English books. How many books are there altogether?

3 The number of people in Liwonde Township is 450 310, in Ekwendeni Township is 323 275. How many people are there altogether?

4 In a certain country, 416 253 men and 372 135 women are HIV-positive. How many people are HIV-positive altogether?

5 In a country, three districts have the following number of people: 262 460, 220 175 and 195 850. How many people are there altogether?

6 In a country, 48 068 candidates passed examinations, while 52 618 failed. How many people wrote the examinations altogether?

7 A lorry transported 390 160 bricks during the first week, 260 170 bricks during the second week and 120 350 bricks during the third week. How many bricks were transported altogether?

8 In a week, a factory produces 180 060 bottles of orange juice, 260 170 bottles of guava juice and 360 625 bottles of lemon juice. How many bottles are produced altogether?

9 A factory makes 516 360 packets of white sugar and 325 615 packets of brown sugar in a week. How many packets does the factory make in a week?

10 At Masala Admarc, there are 233 462 bags of white beans and 469 650 bags of brown beans. How many bags of beans are there altogether?

17

Figure 8: Exercise 1D (MIE, 2007, p. 17).

The author did not specify the object of learning for this example space. However, to carry out the task, learners will apply previously learned knowledge and procedures together with new procedures that are introduced in the current lesson. Learners also need to identify the key words that show the operations and make connections between the concepts.

The analysis reveals that, most of the tasks in the learners' book belong to level 1 (KPF) since there were 24 tasks that were coded KPF out of 48 tasks in unit. That is, most of the tasks required the learners to use previously learned knowledge and procedures in order to interpret the capabilities associated with object of learning. The analysis also reveals that some tasks which were coded level 2 (CTP) involved concepts and capabilities which were being introduced in grade 5 (e.g.

finding common factors, HCF, LCM and some fractions) and hence the tasks required learners to use present topic procedures in order to solve them.

5.3 Textbook 2: Grade 6 learners' mathematics textbook

In grade 6, there are 14 units (topics) on the first learning outcome and all the units were analyzed. The object of learning is stated for every unit and capabilities at the beginning of example spaces. However, some example spaces do not have the capability. For example, exercise 5C, p. 19: "Practical problem" and also exercise 2C, p. 9: "answer the following questions", followed by worked example and then a task. These types of example spaces were also coded as they were also part of developing the topic. There is no introduction of the example space which could help the learner to study the tasks on their own. Three topics are introduced grade 6 namely, rate, ratio, Approximation and estimation. Assessments in form of review exercises, are also present in grade 6 but the investigator did not code them because they are not used to develop the topic but allow learners to practice the concepts and capability. Table 4 shows the results from the analysis.

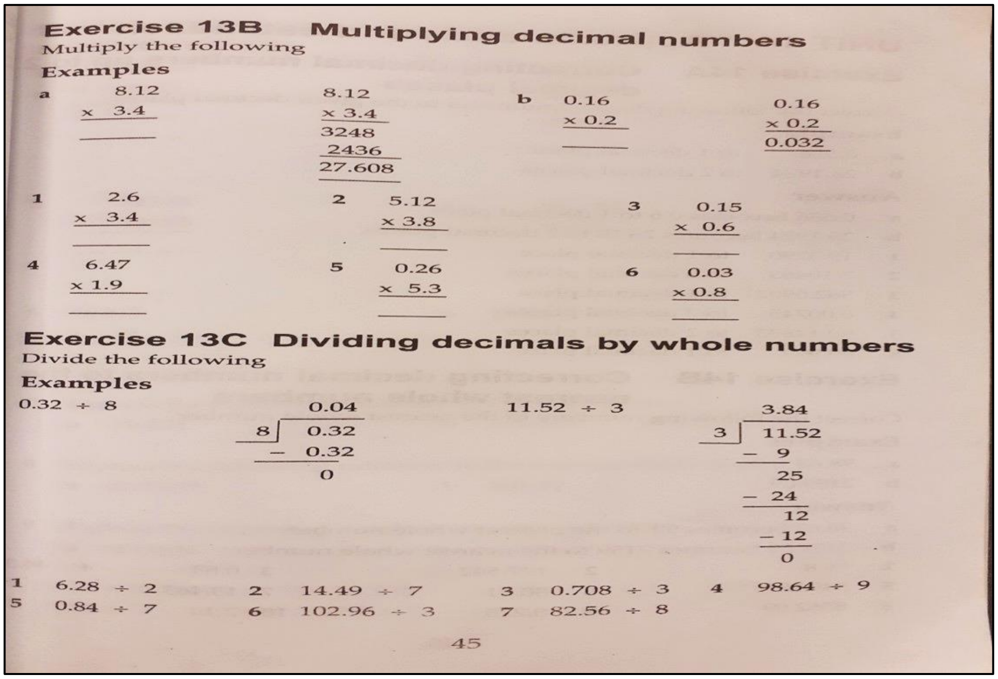
Table 4: summary of coded examples and tasks for learners' book 6

Core Element/ Learning outcome	**Examples**		**Tasks**	
	Codes	Example spaces	Levels of tasks	Number of Tasks
Numbers, operations and Relationships (14 topics)	C	10	Level 1	21
	G	17	Level 2	27
	C and G	21	Level 3	1
	C, G and F	1		
	Levels			
	Level 1	27		
	Level 2	21		
	Level 3	1		

Table 4 shows the core element/learning outcome for the topics, examples and tasks that were coded.

Findings from the examples

The summary of the analysis for the examples on the first fourteen topics of grade 6 learners' book indicate that, 10 example spaces out of 49 example spaces were coded contrast and the belong to level 1. The example spaces provide opportunity for comparing and contrasting. That is the learners will be able to notice the difference within examples in the example space (see figure 9 & 10).



Exercise 13B Multiplying decimal numbers

Multiply the following

Examples

a 8.12 × 3.4 = ; 8.12 × 3.4: 3248, 2436, 27.608

b 0.16 × 0.2 = ; 0.16 × 0.2 = 0.032

1 2.6 × 3.4 2 5.12 × 3.8 3 0.15 × 0.6

4 6.47 × 1.9 5 0.26 × 5.3 6 0.03 × 0.8

Exercise 13C Dividing decimals by whole numbers

Divide the following

Examples

0.32 ÷ 8: 8) 0.32 = 0.04; − 0.32; 0

11.52 ÷ 3: 3) 11.52 = 3.84; − 9; 25; − 24; 12; − 12; 0

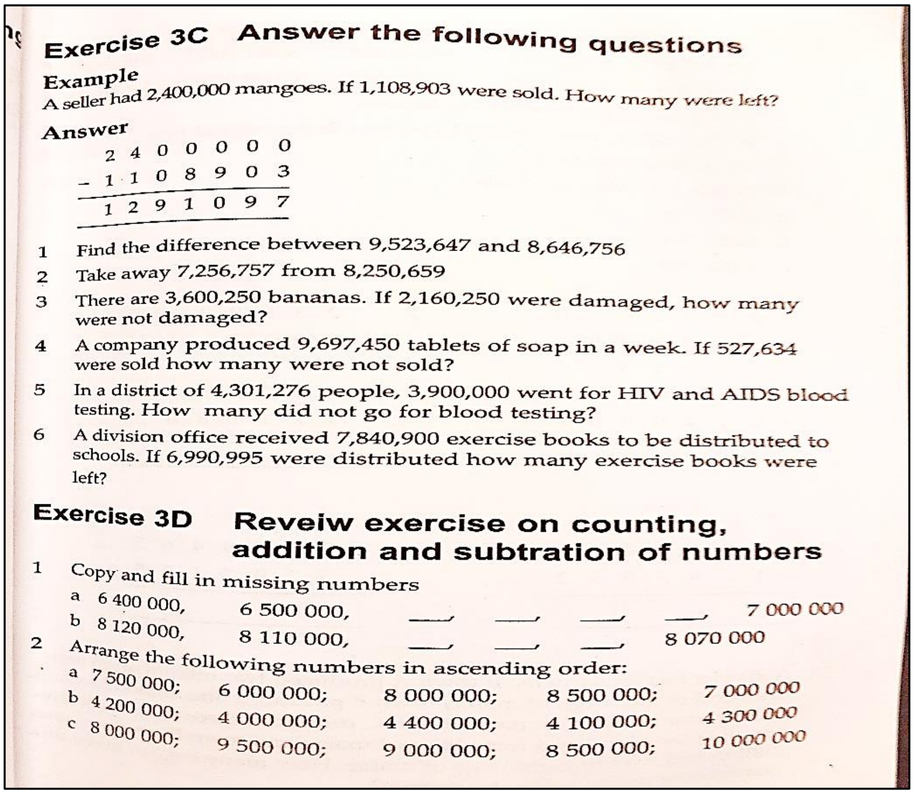
1 6.28 ÷ 2 2 14.49 ÷ 7 3 0.708 ÷ 3 4 98.64 ÷ 9

5 0.84 ÷ 7 6 102.96 ÷ 3 7 82.56 ÷ 8

45

Figure 9:Exercise 13 B. Multiplying decimal numbers (MIE, 2007, p. 45).

The investigator coded this example space as contrast (C) because the example space helps learners to notice the difference within the example space. There are two different decimal numbers involved in the examples. That is multiplication of decimal numbers that combine a whole number and multiplication of decimal numbers without a whole number. For example: 8.12 x 3.4 and 0.16 x 0.2. The solutions also differ in the way that the first example does not need a zero as a place holder whereas the second example need zero as the place holder because when multiplying decimal numbers, the sum of the decimal places in the two factors is equal to the number of decimal places in the product (0.032). Thus, learners have to compare and contrast between or among the decimal numbers involved and then understand different decimal numbers used in the examples.



Exercise 3C Answer the following questions

Example
A seller had 2,400,000 mangoes. If 1,108,903 were sold. How many were left?

Answer

```
  2 4 0 0 0 0 0
- 1 1 0 8 9 0 3
---------------
  1 2 9 1 0 9 7
```

1 Find the difference between 9,523,647 and 8,646,756
2 Take away 7,256,757 from 8,250,659
3 There are 3,600,250 bananas. If 2,160,250 were damaged, how many were not damaged?
4 A company produced 9,697,450 tablets of soap in a week. If 527,634 were sold how many were not sold?
5 In a district of 4,301,276 people, 3,900,000 went for HIV and AIDS blood testing. How many did not go for blood testing?
6 A division office received 7,840,900 exercise books to be distributed to schools. If 6,990,995 were distributed how many exercise books were left?

Exercise 3D Reveiw exercise on counting, addition and subtration of numbers

1 Copy and fill in missing numbers
a 6 400 000, 6 500 000, ___, ___, ___, ___, 7 000 000
b 8 120 000, 8 110 000, ___, ___, ___, 8 070 000
2 Arrange the following numbers in ascending order:
a 7 500 000; 6 000 000; 8 000 000; 8 500 000; 7 000 000
b 4 200 000; 4 000 000; 4 400 000; 4 100 000; 4 300 000
c 8 000 000; 9 500 000; 9 000 000; 8 500 000; 10 000 000

Figure 10:Exercise 3C. Answer the following question (MIE, 2007, p. 13).

The investigator coded the example space 3C as contrasting because there are two different ways of subtracting the numbers in the same example space. The first way is by “take away” and the second way is by “how much more”. These two procedures will enable learners to see difference hence providing opportunity for learners to compare and contrast between the procedures. The example space also helps the learners to see the contrast as they identify the key expressions which show subtraction among other expressions in the sentences.

There are 17 example spaces out of 49 that were coded generalization and they belong to level 1. These example spaces provide opportunity for seeing similarity and therefore allowed learners to generalize. The example spaces that follows were used to illustrate how the examples were coded (see figure 11 & 12)

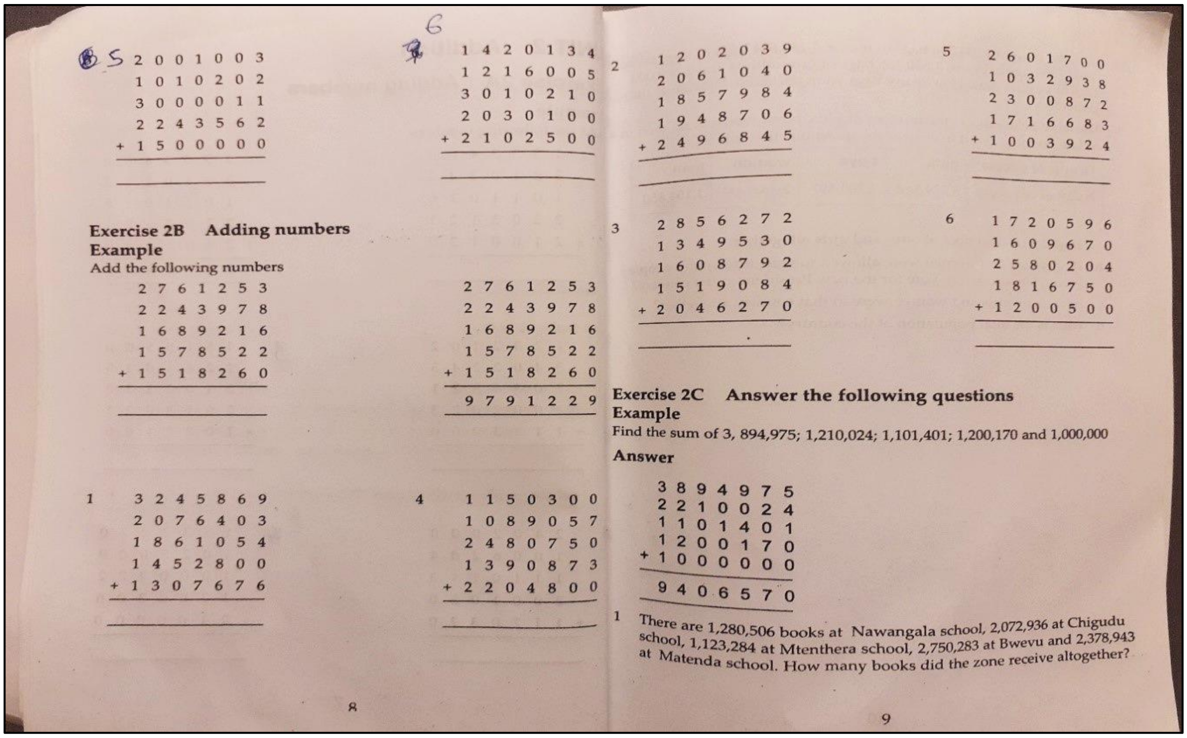
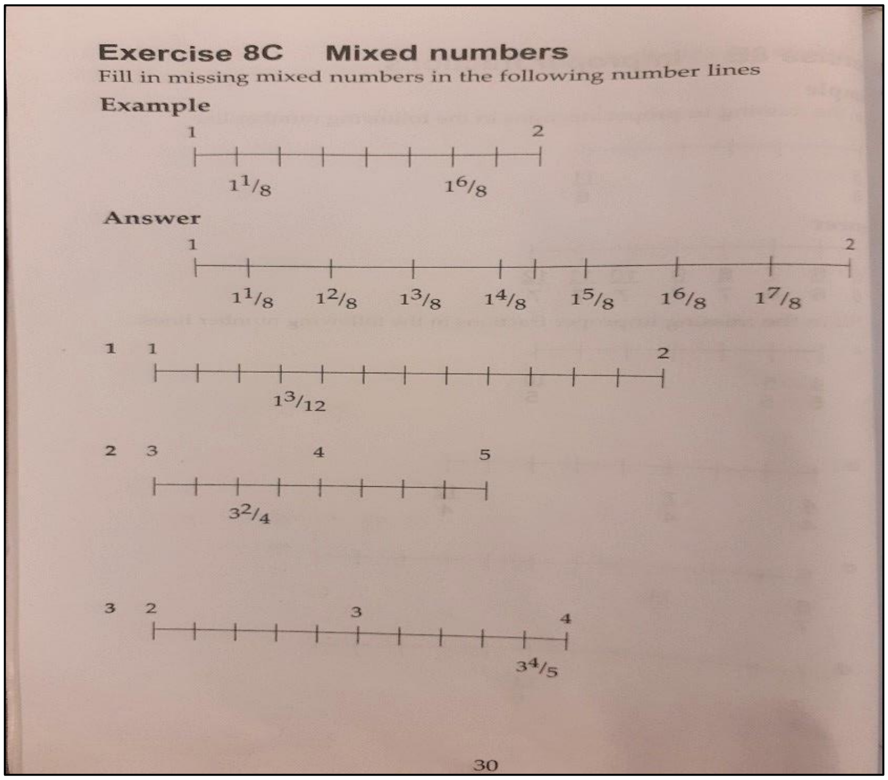


Figure 11: Exercise 2B. Adding numbers (MIE, 2007, p. 8).

The examples on this example space have same structure; addition of five addends, addition with regrouping at all place value position and no place value position titles which depict learners' mastery of place value position of digits in a number. The investigator coded this example space under generalization because there is only one pattern of variation used and hence showing similarity. That is the example space enables learners to generalize as they add the numbers since there is transfer of number from one place value position to another place value position throughout the place value positions. Learners will focus much on regrouping of number throughout the example space and be able to generalize.



Exercise 8C Mixed numbers

Fill in missing mixed numbers in the following number lines

Example

1 2

$1^1/_8$ $1^6/_8$

Answer

1 2

$1^1/_8$ $1^2/_8$ $1^3/_8$ $1^4/_8$ $1^5/_8$ $1^6/_8$ $1^7/_8$

1 1 2

$1^3/_{12}$

2 3 4 5

$3^2/_4$

3 2 3 4

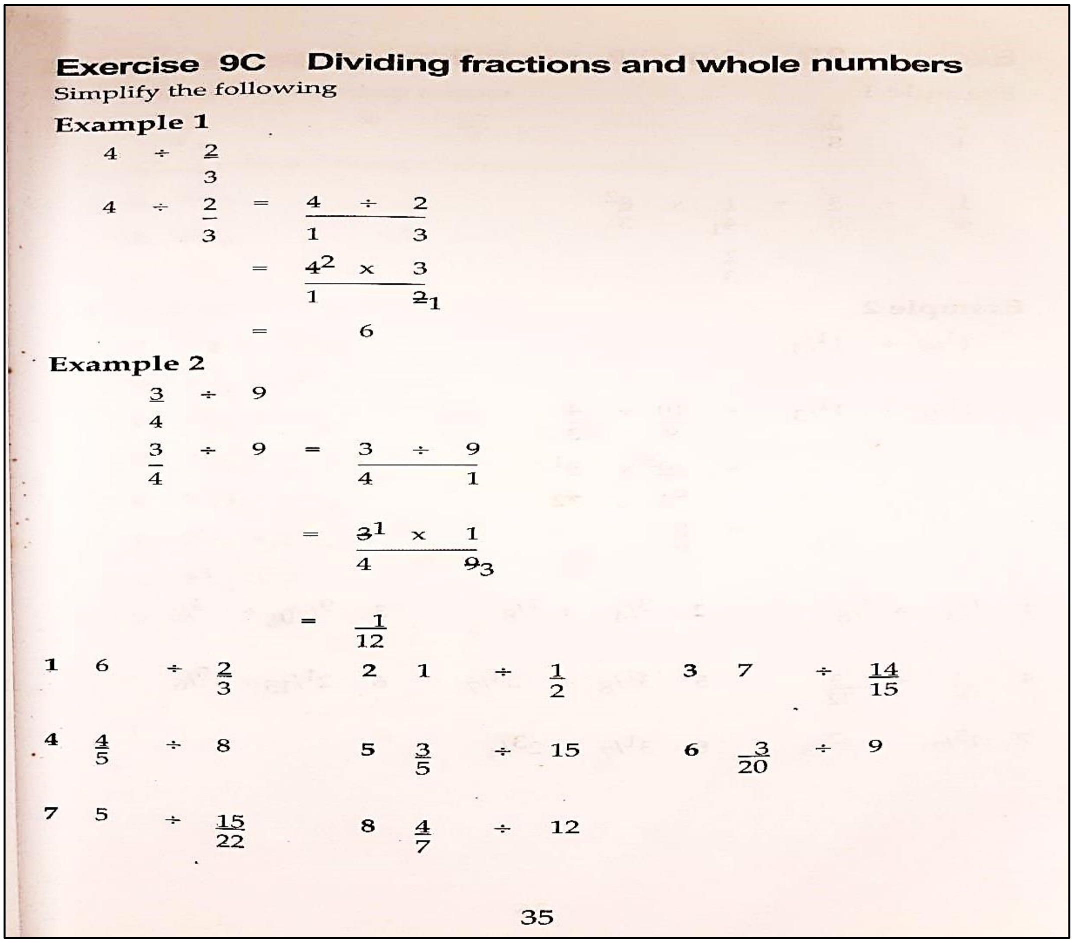
$3^4/_5$

30

Figure 12: Exercise 8C. Mixed numbers (MIE, 2007, p. 30).

The example space was coded Generalization because the examples are the same and follow similar sequence of arrangement and have same procedures. Learners will model the mixed numbers in the number line following a general sequence as seen in the worked example. When they reach the whole number, learners will be able to see the relationship between the whole number provided on the number line and the mixed numbers that they are modeling. This will enable the learners to notice similarity as they move through all the examples in the example space.

There are 21 example spaces out of 49 example spaces that were coded C and G and belonged to level 2. Thus, the example spaces enable the learners to discern similarity and differences within the example space (see figure 13 & 14).



Exercise 9C Dividing fractions and whole numbers

Simplify the following

Example 1

$4 \div \frac{2}{3}$

$4 \div \frac{2}{3} = \frac{4}{1} \div \frac{2}{3}$

$= \frac{4^2}{1} \times \frac{3}{\cancel{2}_1}$

$= 6$

Example 2

$\frac{3}{4} \div 9$

$\frac{3}{4} \div 9 = \frac{3}{4} \div \frac{9}{1}$

$= \frac{\cancel{3}^1}{4} \times \frac{1}{\cancel{9}_3}$

$= \frac{1}{12}$

1 $6 \div \frac{2}{3}$ 2 $1 \div \frac{1}{2}$ 3 $7 \div \frac{14}{15}$

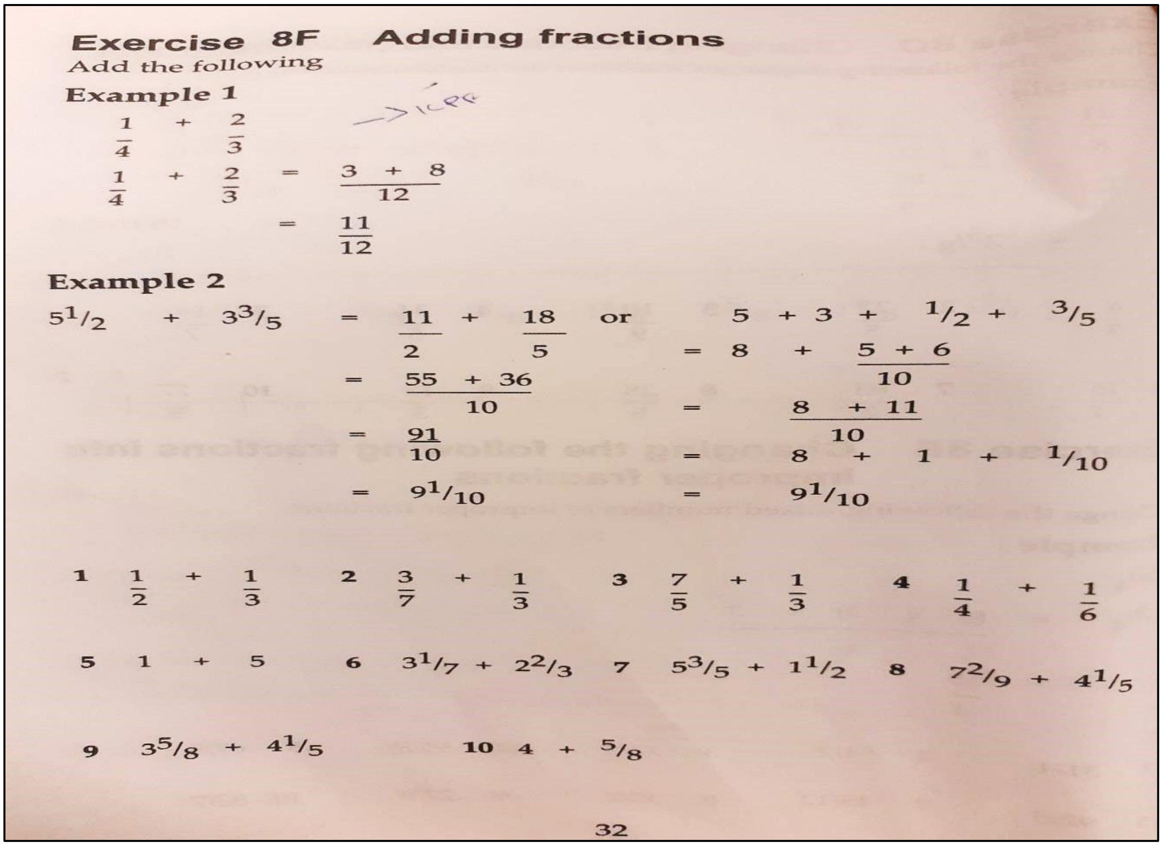
4 $\frac{4}{5} \div 8$ 5 $\frac{3}{5} \div 15$ 6 $\frac{3}{20} \div 9$

7 $5 \div \frac{15}{22}$ 8 $\frac{4}{7} \div 12$

35

Figure 13: Exercise 9C. Dividing fractions and whole number (MIE, 2007, p. 35).

The investigator coded this example space under contrasting and generalization as it allows learners to see similarities and difference in the same example space. In the example space, examples 1–3, 7 and worked example 1 have same structure, dividing a whole number by a proper fraction and examples 4–6, 8 and worked example 2 have similar structure, dividing a proper fraction by a whole number. Learners will notice similarity between examples and be able to generalize. With contrast in that there are a division of a whole number by a proper fraction and a division of a proper fraction by a whole number. These are two different capabilities that are required for the learners to acquire in this example space. Thus, the example space provides opportunity for learners to seeing difference. At the same time, the learners will also notice similarity as they see the whole numbers in both examples (i.e.,$4 \div \frac{2}{3}$ and $\frac{3}{4} \div 9$) being changed into improper fractions (4/1 and 9/1). With the contrast in that, 4/1 will remain the same but 9/1 will be turned upside down as $\frac{1}{9}$ (nine becoming a denominator).



Exercise 8F Adding fractions

Add the following

Example 1

$\frac{1}{4} + \frac{2}{3}$

$\frac{1}{4} + \frac{2}{3} = \frac{3 + 8}{12}$

$= \frac{11}{12}$

Example 2

$5^1/_2 + 3^3/_5 = \frac{11}{2} + \frac{18}{5}$ or $5 + 3 + ^1/_2 + ^3/_5$

$= \frac{55 + 36}{10}$ $= 8 + \frac{5 + 6}{10}$

$= \frac{91}{10}$ $= \frac{8 + 11}{10}$

$= 9^1/_{10}$ $= 8 + 1 + ^1/_{10}$

$= 9^1/_{10}$

1 $\frac{1}{2} + \frac{1}{3}$ 2 $\frac{3}{7} + \frac{1}{3}$ 3 $\frac{7}{5} + \frac{1}{3}$ 4 $\frac{1}{4} + \frac{1}{6}$

5 $1 + 5$ 6 $3^1/_7 + 2^2/_3$ 7 $5^3/_5 + 1^1/_2$ 8 $7^2/_9 + 4^1/_5$

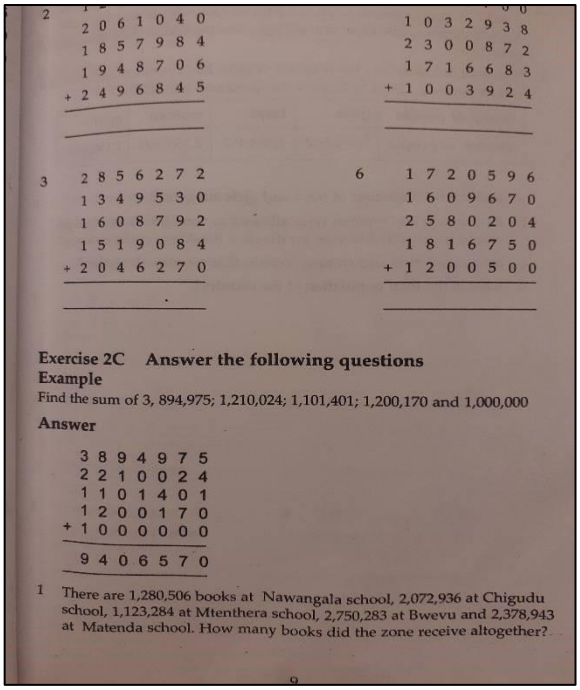
9 $3^5/_8 + 4^1/_5$ 10 $4 + ^5/_8$

32

Figure 14: Exercise 8F. Adding fractions (MIE, 2007, p. 32).

The investigator coded this example space under C and G because it involves contrast and generalization. There are variations in this example space in the way that there is addition of proper fraction and mixed numbers, addition of whole numbers and addition of a whole number and a proper fraction; for example: ¼ + ⅔; 5½ + 3⅗ ,1 + 5 and 4 + ⅝, in the same example space. Addition of fractions (proper fractions and mixed numbers) are in line with object of learning and this will help learners to see similarities as they identify the lowest common multiple (LCM) with contrast in that the example space has included addition of whole numbers (1 + 5) and addition of a whole number and a proper fraction (4 + ⅝) enabling them to see difference. The working procedures will also differ as learners solve the examples.

And lastly, one example space out of 49 example spaces was coded C, G, and F, and it belonged to level 3. This example space provides opportunity for learners to discern simultaneous dimensions of variation within the example space (see figure 15).



2

2 0 6 1 0 4 0	1 0 3 2 9 3 8
1 8 5 7 9 8 4	2 3 0 0 8 7 2
1 9 4 8 7 0 6	1 7 1 6 6 8 3
+ 2 4 9 6 8 4 5	+ 1 0 0 3 9 2 4

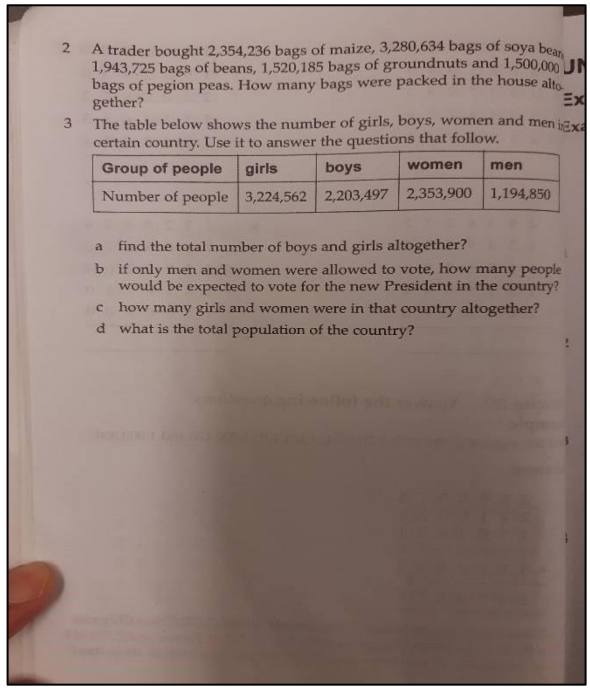
3 / 6

2 8 5 6 2 7 2	1 7 2 0 5 9 6
1 3 4 9 5 3 0	1 6 0 9 6 7 0
1 6 0 8 7 9 2	2 5 8 0 2 0 4
1 5 1 9 0 8 4	1 8 1 6 7 5 0
+ 2 0 4 6 2 7 0	+ 1 2 0 0 5 0 0

Exercise 2C Answer the following questions
Example
Find the sum of 3, 894,975; 1,210,024; 1,101,401; 1,200,170 and 1,000,000
Answer

3 8 9 4 9 7 5
2 2 1 0 0 2 4
1 1 0 1 4 0 1
1 2 0 0 1 7 0
+ 1 0 0 0 0 0 0
9 4 0 6 5 7 0

1 There are 1,280,506 books at Nawangala school, 2,072,936 at Chigudu school, 1,123,284 at Mtenthera school, 2,750,283 at Bwevu and 2,378,943 at Matenda school. How many books did the zone receive altogether?



2 A trader bought 2,354,236 bags of maize, 3,280,634 bags of soya beans, 1,943,725 bags of beans, 1,520,185 bags of groundnuts and 1,500,000 bags of pegion peas. How many bags were packed in the house altogether?

3 The table below shows the number of girls, boys, women and men in a certain country. Use it to answer the questions that follow.

Group of people	girls	boys	women	men
Number of people	3,224,562	2,203,497	2,353,900	1,194,850

a find the total number of boys and girls altogether?
b if only men and women were allowed to vote, how many people would be expected to vote for the new President in the country?
c how many girls and women were in that country altogether?
d what is the total population of the country?

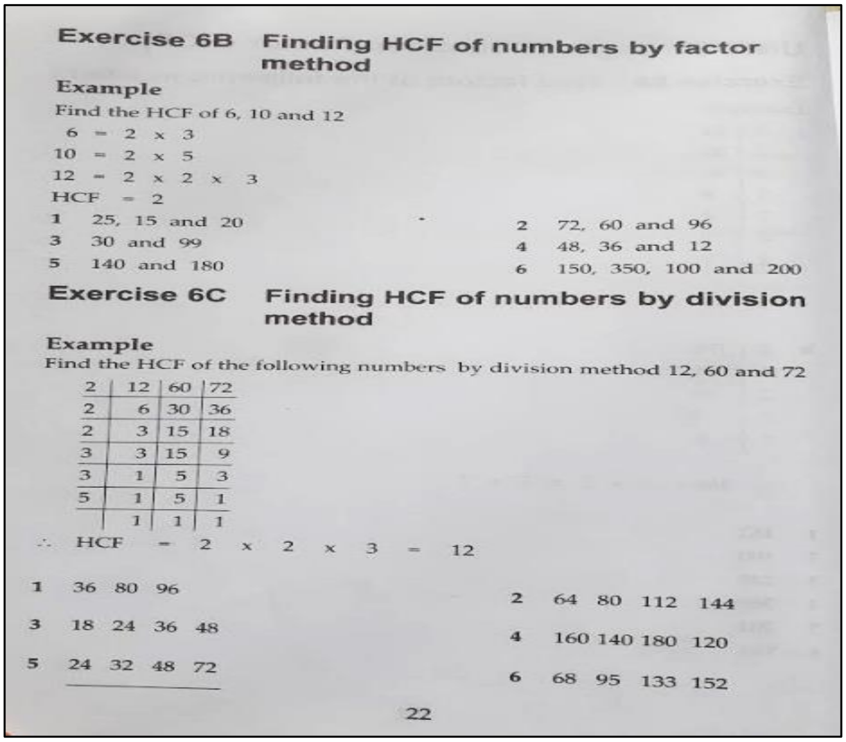
Figure 15: Exercise 2C: Answer the following questions (MIE, 2007, p 9–10).

The investigator coded the example space as C, G and F because it discerns simultaneous aspects of variation. All examples in the example space involve addition with regrouping as a result; learners will see similarity and then be able to generalize. The learners will identify expressions that indicate addition in the sentence which will assist them to compare and contrast among the expressions. And example 3 has a different structure as it is represented in a table form. Moreover, to solve example 3a – 3d, learners will require applying concepts of data handling and experiencing fusion.

The analysis reveals that, most of the example spaces belong to level 1 (27 example spaces out of 49) since they have only one pattern of variation applied. That is, either contrast or generalization. This also implies that the units under the first core element do not provide high level exercises that provide enough platforms for learners to discern different features that are essential for a particular learning outcome in grade 6.

Findings from Tasks

Analysis of the tasks indicates that, out of 49 tasks, 21 tasks were coded under KPF which belong to level 1 as they needed the learners to use known procedures and facts. That is, the tasks require learners to use previously learned knowledge and /or procedures associated with object of learning (see figure 16).



Exercise 6B Finding HCF of numbers by factor method

Example

Find the HCF of 6, 10 and 12

6 = 2 x 3
10 = 2 x 5
12 = 2 x 2 x 3
HCF = 2

1 25, 15 and 20
2 72, 60 and 96
3 30 and 99
4 48, 36 and 12
5 140 and 180
6 150, 350, 100 and 200

Exercise 6C Finding HCF of numbers by division method

Example

Find the HCF of the following numbers by division method 12, 60 and 72

2	12	60	72
2	6	30	36
2	3	15	18
3	3	15	9
3	1	5	3
5	1	5	1
	1	1	1

∴ HCF = 2 x 2 x 3 = 12

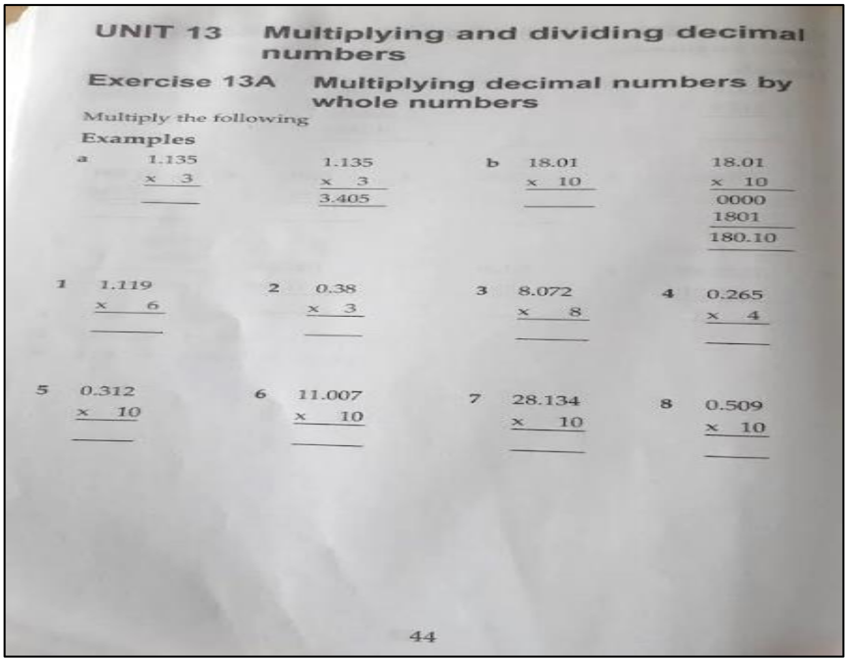
1 36 80 96
2 64 80 112 144
3 18 24 36 48
4 160 140 180 120
5 24 32 48 72
6 68 95 133 152

22

Figure 16: Exercise 6B. Finding HCF of numbers by factor method (MIE, 2007, p. 22)

Exercise 6B was coded level 1 task (KPF) because learners will solve the task by using known procedures and facts. The learners are going to use previously learned concepts such as knowledge of factors, prime factors and common factors in order to solve the HCF.

There were 27 tasks which were coded CTP and belong to level 2 as they required learners to apply knowledge and procedures that were being introduced in the current lessons (see figure 17).


UNIT 13 Multiplying and dividing decimal numbers

Exercise 13A Multiplying decimal numbers by whole numbers

Multiply the following

Examples

a
```
 1.135      1.135
×    3    ×    3
-------   -------
           3.405
```

b
```
 18.01      18.01
×   10    ×    10
-------   -------
            0000
            1801
          -------
           180.10
```

1
```
 1.119
×    6
```

2
```
 0.38
×   3
```

3
```
 8.072
×    8
```

4
```
 0.265
×    4
```

5
```
 0.312
×   10
```

6
```
 11.007
×    10
```

7
```
 28.134
×    10
```

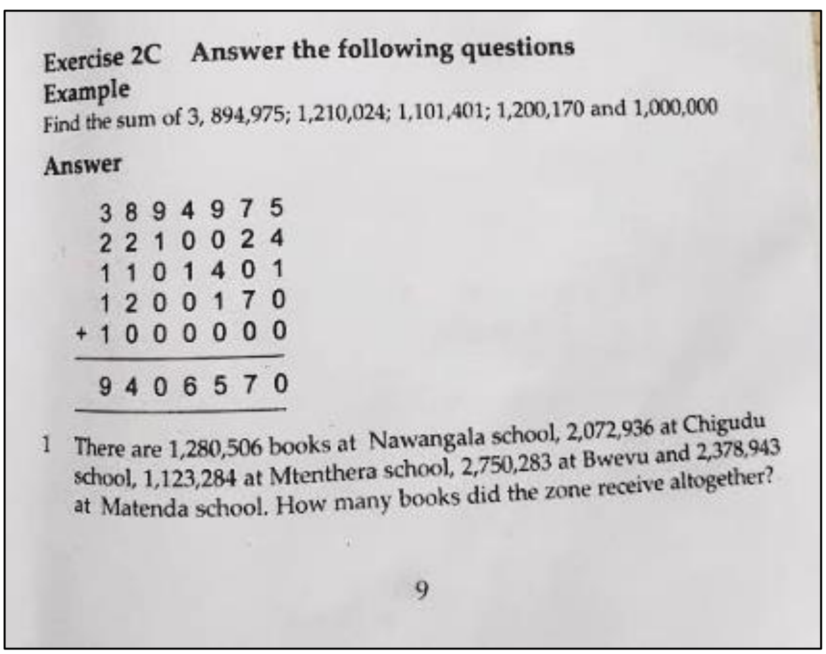
8
```
 0.509
×   10
```

44

Figure 17: Exercise 13A. Multiplying decimal numbers by whole number (MIE, 2007, p.44).

This task was coded level 2 task because the topic is new, and the learners are required to use the current topic procedure to solve the problem thereby following the given example.

Lastly, 1 task out of 49 tasks was coded AMC as it required learners to make decision as to what procedures and concepts were needed to answer the task or required connections between concepts. Learners also required using knowledge acquired from everyday life (see figure 18).


Exercise 2C **Answer the following questions**

Example

Find the sum of 3, 894,975; 1,210,024; 1,101,401; 1,200,170 and 1,000,000

Answer

```
  3 8 9 4 9 7 5
  2 2 1 0 0 2 4
  1 1 0 1 4 0 1
  1 2 0 0 1 7 0
+ 1 0 0 0 0 0 0
---------------
  9 4 0 6 5 7 0
```

1 There are 1,280,506 books at Nawangala school, 2,072,936 at Chigudu school, 1,123,284 at Mtenthera school, 2,750,283 at Bwevu and 2,378,943 at Matenda school. How many books did the zone receive altogether?

9

2 A trader bought 2,354,236 bags of maize, 3,280,634 bags of soya beans, 1,943,725 bags of beans, 1,520,185 bags of groundnuts and 1,500,000 bags of pegion peas. How many bags were packed in the house altogether?

3 The table below shows the number of girls, boys, women and men in a certain country. Use it to answer the questions that follow.

Group of people	girls	boys	women	men
Number of people	3,224,562	2,203,497	2,353,900	1,194,850

a find the total number of boys and girls altogether?

b if only men and women were allowed to vote, how many people would be expected to vote for the new President in the country?

c how many girls and women were in that country altogether?

d what is the total population of the country?

Figure 18: Exercise 2C. Answer the following questions (MIE, 2007, p. 9–10).

Learners are required to make decision on how to use the table provided in question 3 and hence they will need the idea of data handling which is another learning outcome (core element) in primary mathematics. That is, it is a task that requires connections between concepts.

The results from the analysis of tasks show that the first 14 units in grade 6 learners' book mainly contain level 2 tasks (CTP). That is, most of the tasks are designed to mediate learners' capabilities with regard to current topic and / or apply the procedures that are being introduced in the current topic/lesson (Ronda & Adler, 2017).

5.4 Textbook 3: Grade 7 learners' mathematics textbook

The first learning outcome (core element) for grade 7 has 10 units and all the units were analyzed. The textbook contains three units which are just being introduced namely, Roman numerals, averages and percentages and three units which combine two or more basic operations on same example. And these units include basic operation on whole numbers, basic operation on fractions and basic operation on decimal numbers.

The textbook starts with concepts on every unit and then capability on most of the example spaces followed by worked example and the practice exercises. However, some of the example spaces lack background information on the procedures needed to solve the tasks. As a result, it does not provide enough grounds for independent study by the learners themselves. For example, on three units (unit 3, 6 and 7) that deal with combination of two or more basic operation on same example, the textbook is silent on the order of operations following BODMAS until at a later stage. Below is the summary of the analysis of the textbook:

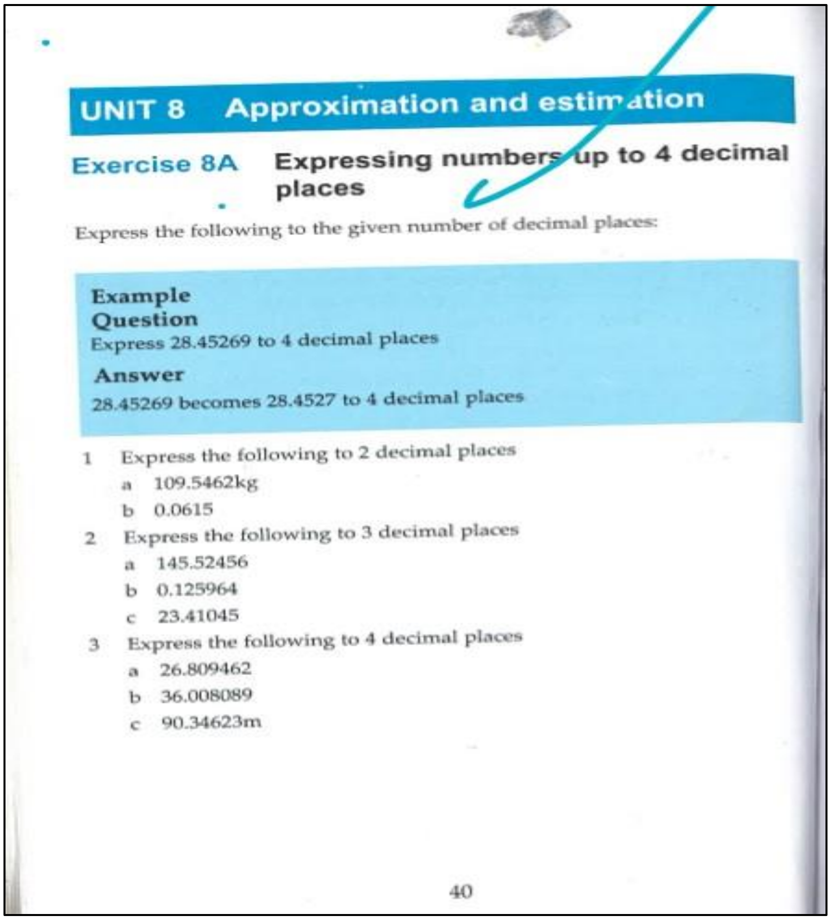
Table 5: summary of coded examples and tasks for learners' book 7

Core Element/ Learning outcome	**examples**		**Tasks**	
	Codes	Example spaces	Levels of tasks	Number of Tasks
Numbers, Operations and Relationships (10 topics)	C	6	Level 1	14
	G	15	Level 2	33
	C & G	18	Level 3	7
	C & F	6		
	C, G & F	9		
	Levels			
	Level 1	21		
	Level 2	24		
	Level 3	9		

Table 5 shows the core element/learning outcome for the topics, examples and tasks that were coded.

Findings for examples

There are 54 example spaces for the first learning outcome (core element) in grade 7. 6 example spaces out of 54 were coded C and they belong to level 1. These example spaces provide opportunity for learners to notice different features between or among examples thereby experiencing something that is not a critical feature of that object of learning (Marton et al., 2004). The two examples that follows are used to illustrate the coding (see figure 19 & 20).



UNIT 8 Approximation and estimation

Exercise 8A Expressing numbers up to 4 decimal places

Express the following to the given number of decimal places:

Example
Question
Express 28.45269 to 4 decimal places

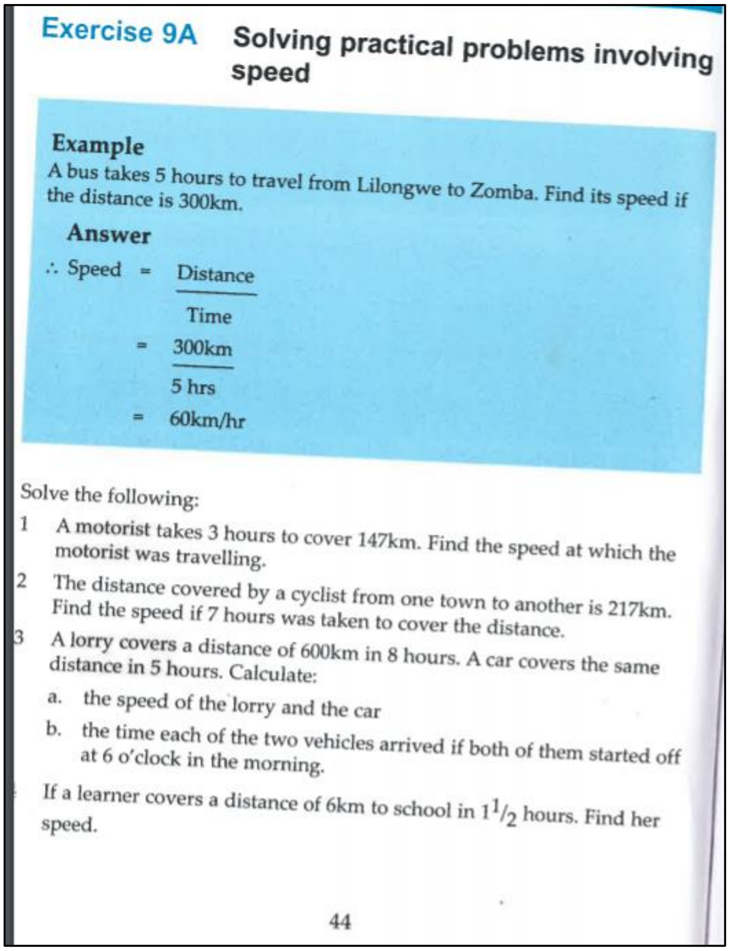
Answer
28.45269 becomes 28.4527 to 4 decimal places

1 Express the following to 2 decimal places
  a 109.5462kg
  b 0.0615
2 Express the following to 3 decimal places
  a 145.52456
  b 0.125964
  c 23.41045
3 Express the following to 4 decimal places
  a 26.809462
  b 36.008089
  c 90.34623m

40

Figure 19: Exercise 8A. Expressing numbers up to 4 decimal places (MIE, 2008, p. 40).

In this example space, the learners are required to round up decimal numbers up to 2, 3, and 4 decimal places. In order to round up the decimal numbers, the learners will be changing some decimal numbers from one place value position to another place value position if that next number is not under considerations (for example, 0.125964 to 3 decimal places = 0.126) as seen in the worked example with contrast in that there will be no transferring of number from one place value position to another place value position for some decimal numbers. For example, 90.34623m to 4 decimal places = 90.3462m. The learners will see some changes in some digits, for instance 5 changed to 6 and 2 did not change. This will enable learners to see the difference within examples and will be able to contrast.



Exercise 9A **Solving practical problems involving speed**

Example

A bus takes 5 hours to travel from Lilongwe to Zomba. Find its speed if the distance is 300km.

Answer

$\therefore$ Speed $= \frac{\text{Distance}}{\text{Time}}$

$= \frac{300\text{km}}{5\text{ hrs}}$

$= 60\text{km/hr}$

Solve the following:

1 A motorist takes 3 hours to cover 147km. Find the speed at which the motorist was travelling.

2 The distance covered by a cyclist from one town to another is 217km. Find the speed if 7 hours was taken to cover the distance.

3 A lorry covers a distance of 600km in 8 hours. A car covers the same distance in 5 hours. Calculate:

a. the speed of the lorry and the car

b. the time each of the two vehicles arrived if both of them started off at 6 o'clock in the morning.

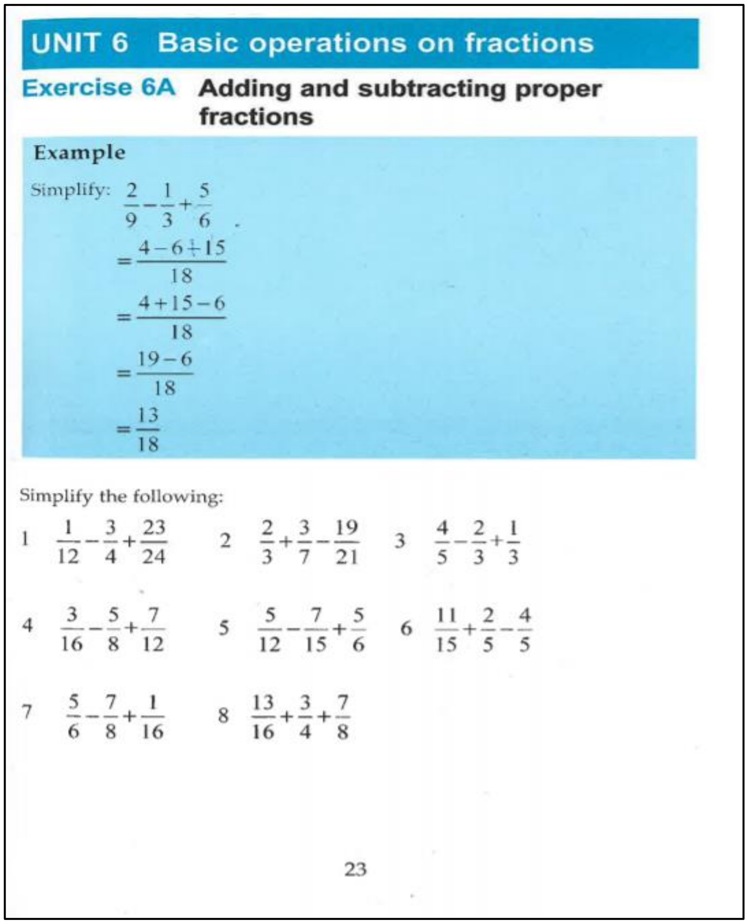
If a learner covers a distance of 6km to school in $1^1/_2$ hours. Find her speed.

44

Figure 20: Example, Exercise 9A. Solving practical problems involving speed (MIE, 2008, p. 44).

The investigator coded this example space as contrasting, because the example space is enabling the learners to compare two different features; that is speed and time. The object of learning for this example space is, "solving practical problems involving speed" but question 3b require the learners to calculate, "time" which is different to the mathematics that are supposed to be mediated in the lesson. This example space provides opportunity for learners to compare and contrast the different features.

There are 15 example spaces out of 54 example spaces that were coded under generalization and belong to level 1. The structure of the worked examples and the exercises provided on these example spaces are the same. They also use the same procedures to solve the tasks. The example space provide opportunity for learners to notice similarity and therefore enable learners to generalize in relation to one aspect of the object of learning. The two examples that follows illustrate the coding (see figure, 21 & 22).



UNIT 6 Basic operations on fractions

Exercise 6A Adding and subtracting proper fractions

Example

Simplify: $\frac{2}{9}-\frac{1}{3}+\frac{5}{6}$

$=\frac{4-6+15}{18}$

$=\frac{4+15-6}{18}$

$=\frac{19-6}{18}$

$=\frac{13}{18}$

Simplify the following:

1 $\frac{1}{12}-\frac{3}{4}+\frac{23}{24}$ 2 $\frac{2}{3}+\frac{3}{7}-\frac{19}{21}$ 3 $\frac{4}{5}-\frac{2}{3}+\frac{1}{3}$

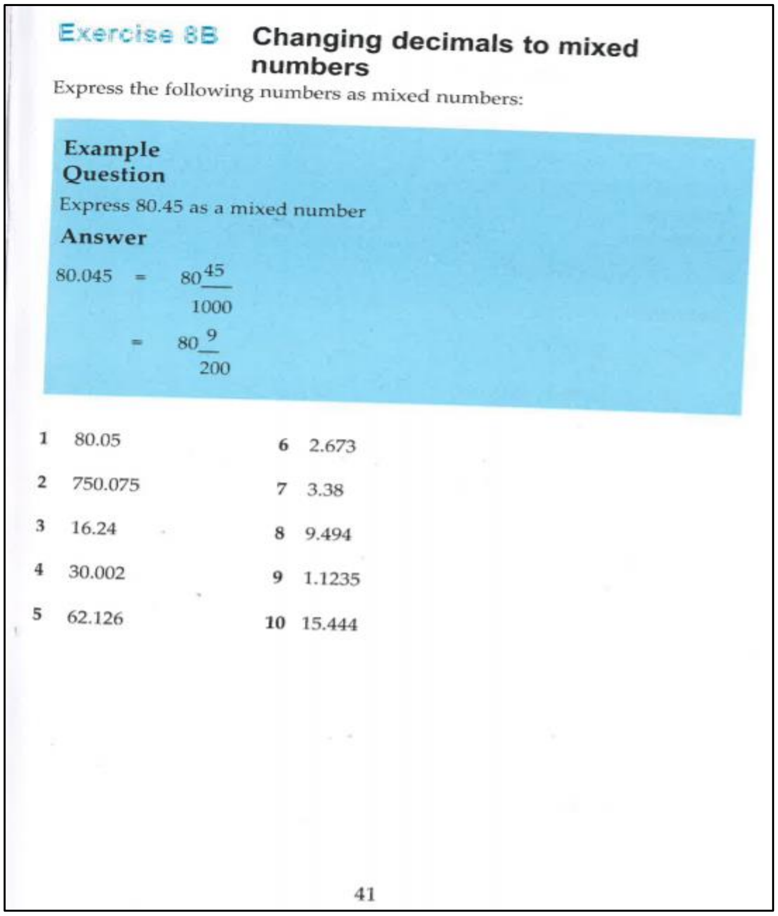
4 $\frac{3}{16}-\frac{5}{8}+\frac{7}{12}$ 5 $\frac{5}{12}-\frac{7}{15}+\frac{5}{6}$ 6 $\frac{11}{15}+\frac{2}{5}-\frac{4}{5}$

7 $\frac{5}{6}-\frac{7}{8}+\frac{1}{16}$ 8 $\frac{13}{16}+\frac{3}{4}+\frac{7}{8}$

23

Figure 21: Exercise 6A. Adding and subtracting proper fractions (MIE, 2008, p. 23).

In this example space, examples 1–8 and worked examples have similar structure and it involves addition and subtraction of proper fractions with different denominators varying from 1–2 digits ($\frac{1}{12}$ - $\frac{1}{4}$ + $\frac{23}{24}$), identifying LCM, identifying order of operations and grouping fractions with same sign. This example space helps learners to notice similarity as they are adding and subtracting proper fraction on various examples of the same type and this will enable learners to generalize.



Exercise 8B **Changing decimals to mixed numbers**

Express the following numbers as mixed numbers:

Example
Question

Express 80.45 as a mixed number

Answer

$80.045 = 80\frac{45}{1000}$

$= 80\frac{9}{200}$

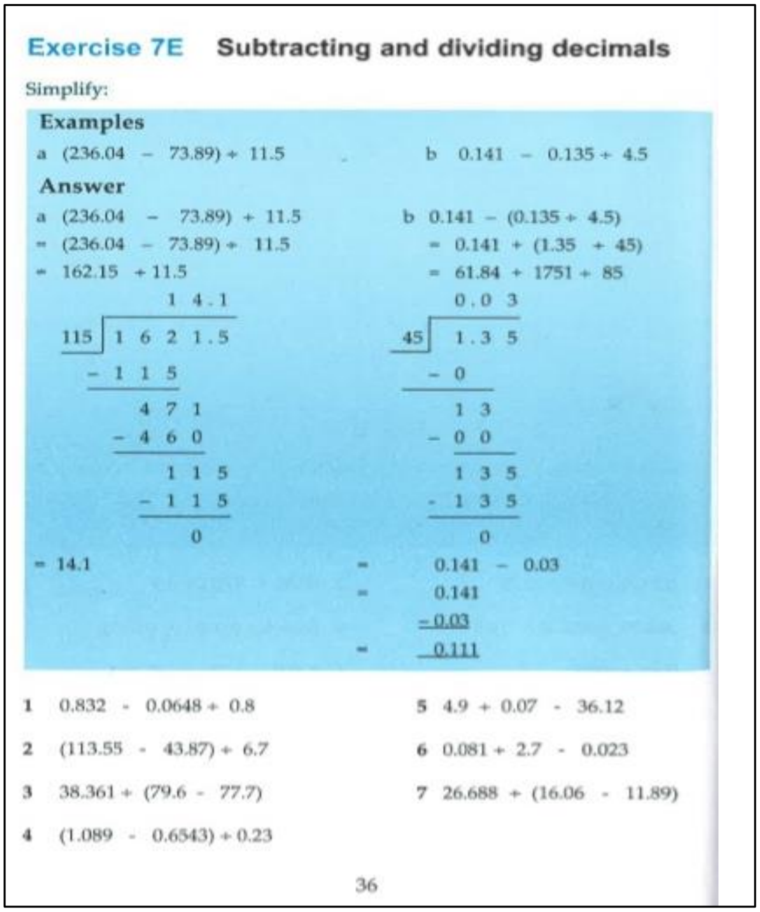
1	80.05	6	2.673
2	750.075	7	3.38
3	16.24	8	9.494
4	30.002	9	1.1235
5	62.126	10	15.444

41

Figure 22: Exercise 8B. Changing decimals to mixed numbers (MIE, 2008, p. 41).

The example space helps learners to practice different aspects of the critical features of object of learning while still focusing on changing decimals to mixed numbers. For example: 8.05; 62.126; and 1.1235 have same structures but different number of decimal places. As a result, the mixed number will also have same structure but different denominators following the place value positions of the decimal numbers. As learners change fractions from one mode of representation (decimal numbers) to another mode (mixed numbers), they will see how denominators are being identified using the place value position and then be able to generalize. And this will help them to simplify the mixed numbers.

There were 18 example spaces which were coded under contrasting and generalization, and they belong to level 2. These example spaces provided opportunity for learners to notice two variations of the aspects of the object of learning. Thus, the example spaces enable learners to noticing similarity and differences within the examples in the example space. Examples that follows were used to demonstrate the coding (see figure 23 and 24).



Exercise 7E **Subtracting and dividing decimals**

Simplify:

Examples

a (236.04 − 73.89) ÷ 11.5   b 0.141 − 0.135 ÷ 4.5

Answer

a (236.04 − 73.89) ÷ 11.5
= (236.04 − 73.89) ÷ 11.5
= 162.15 ÷ 11.5

```
        1 4 . 1
115 | 1 6 2 1 . 5
    - 1 1 5
        4 7 1
      - 4 6 0
          1 1 5
        - 1 1 5
              0
```

= 14.1

b 0.141 − (0.135 ÷ 4.5)
= 0.141 ÷ (1.35 ÷ 45)
= 61.84 ÷ 1751 ÷ 85

```
      0 . 0 3
45 |  1 . 3 5
    - 0
      1 3
    - 0 0
      1 3 5
    - 1 3 5
          0
```

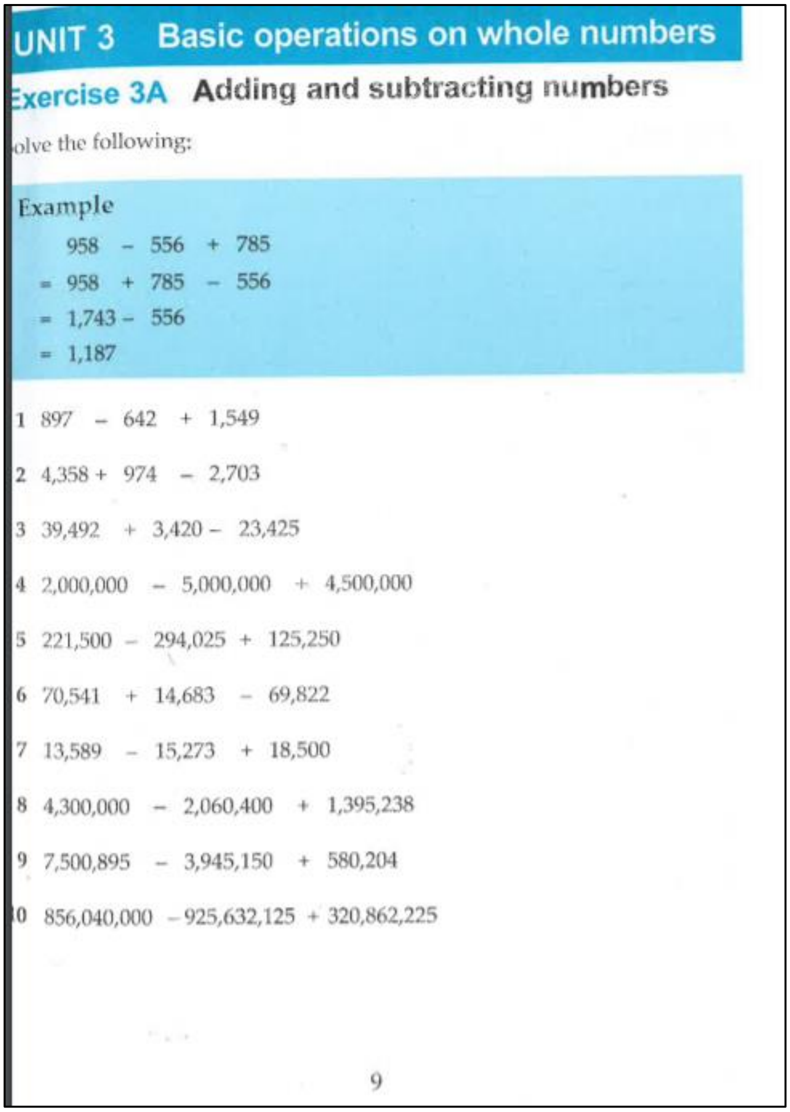
= 0.141 − 0.03
= 0.141
−0.03
= 0.111

1 0.832 − 0.0648 ÷ 0.8
2 (113.55 − 43.87) ÷ 6.7
3 38.361 ÷ (79.6 − 77.7)
4 (1.089 − 0.6543) ÷ 0.23
5 4.9 ÷ 0.07 − 36.12
6 0.081 ÷ 2.7 − 0.023
7 26.688 ÷ (16.06 − 11.89)

36

Figure 23: Exercise 7E. Subtracting and dividing decimals (MIE, 2008, p. 36).

This example space helps the learners to see some similarity and difference in that, examples 1–7 and worked examples, have same structure and the decimals vary from 1–4 decimal places in each example; with the contrast in that examples 2–4, 7 and worked example a, contain brackets which provide clarity to the order of operations. Thus, the author has clarified where to start solving the problems. Learners will also notice the difference that divisors are changed to whole number before dividing the decimal numbers. For example:



UNIT 3 Basic operations on whole numbers

Exercise 3A Adding and subtracting numbers

olve the following:

Example

958 – 556 + 785
= 958 + 785 – 556
= 1,743 – 556
= 1,187

1 897 – 642 + 1,549

2 4,358 + 974 – 2,703

3 39,492 + 3,420 – 23,425

4 2,000,000 – 5,000,000 + 4,500,000

5 221,500 – 294,025 + 125,250

6 70,541 + 14,683 – 69,822

7 13,589 – 15,273 + 18,500

8 4,300,000 – 2,060,400 + 1,395,238

9 7,500,895 – 3,945,150 + 580,204

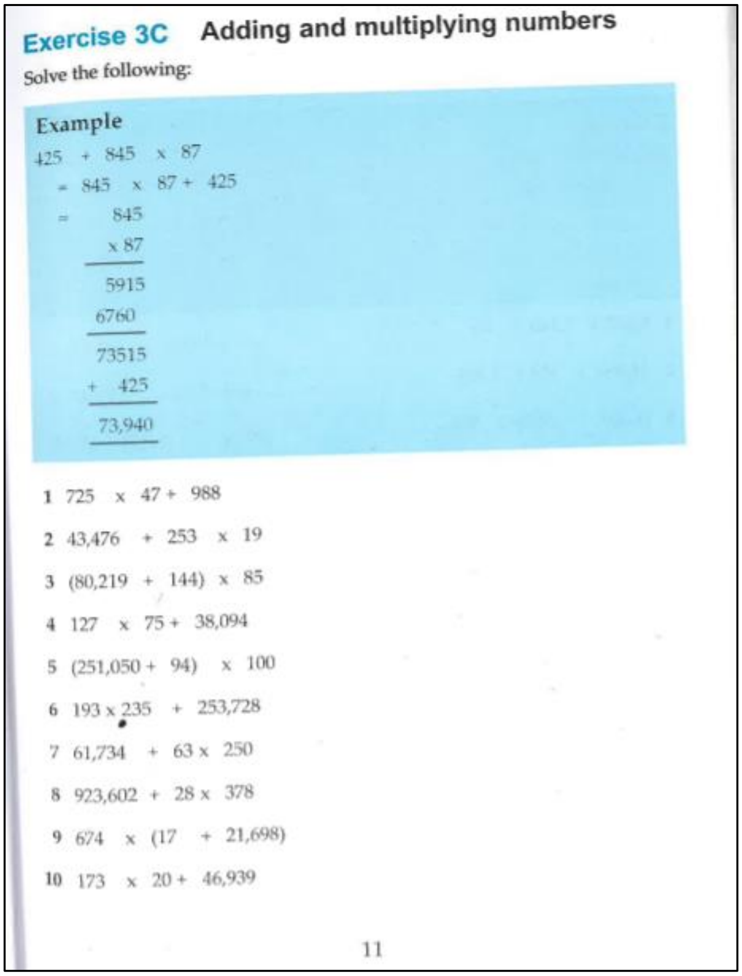
10 856,040,000 – 925,632,125 + 320,862,225

9

Figure 24: Exercise 3A. Adding and subtracting numbers (MIE, 2008, p. 9).

The investigator coded this example space as contrast and generalization because it enables learners to see similarity and difference among examples. Examples 1–10 and worked example have same structure, adding and subtracting numbers up to 9 digits. Learners will notice similarity as they add and subtract the numbers in all examples following the order of operations as seen in the worked example. With contrast in the way that there are two concepts which are involved in each example as a result, learners will identify and group numbers with same sign (897 – 642 + 1,549) and then perform the operations following BODMAS. Moreover, some examples do not need grouping numbers with same sign (4,358 + 974 – 2,703) but only identifying the operation to be carried first. Therefore, this example space provides opportunity for seeing difference.

There are 6 example spaces which were coded under generalization and fusion out of 54 example spaces. The example spaces help the learners to notice similarity as they work through the examples as a result, the learners can generalize. In addition, the space helps learners to discern simultaneous aspects of variations which provide opportunity for learners to experience fusion. (see figure 25)



Exercise 3C Adding and multiplying numbers

Solve the following:

Example

425 + 845 x 87

= 845 x 87 + 425

= 845

x 87

5915

6760

73515

\+ 425

73,940

1 725 x 47 + 988

2 43,476 + 253 x 19

3 (80,219 + 144) x 85

4 127 x 75 + 38,094

5 (251,050 + 94) x 100

6 193 x 235 + 253,728

7 61,734 + 63 x 250

8 923,602 + 28 x 378

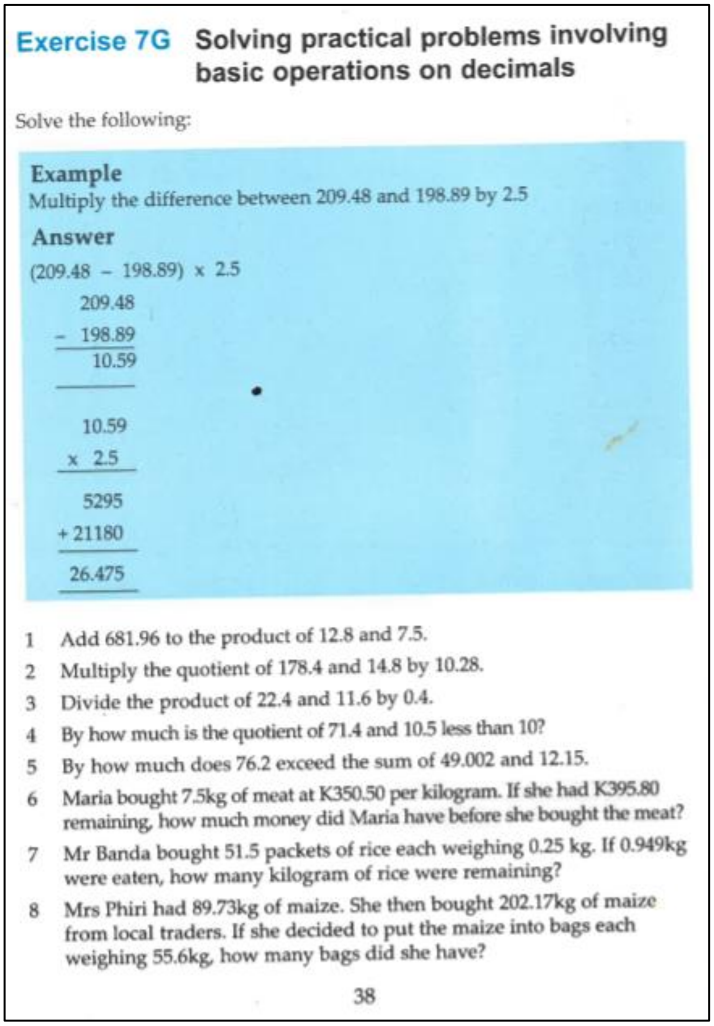
9 674 x (17 + 21,698)

10 173 x 20 + 46,939

11

Figure 25: Exercise 3C. Adding and multiplying numbers (MIE, 2008, p.11).

The example space enables learners to seeing similarity and fusion, which provides opportunity for generalizing two aspects of object of learning within the example set. Examples 1 – 10 and worked example help learners to notice similarity as they show the critical features of the object of learning in order to capture different aspects. The examples require learners to group numbers with brackets and identify order of operations according to BODMAS. Introduction of brackets in examples 3, 5 and 9 provide clarity in the order of operations and help the learners to identify the starting point when solving the problem (fusion).



Exercise 7G **Solving practical problems involving basic operations on decimals**

Solve the following:

Example
Multiply the difference between 209.48 and 198.89 by 2.5

Answer
(209.48 – 198.89) x 2.5

  209.48
– 198.89
  10.59

  10.59
x 2.5
  5295
+ 21180
  26.475

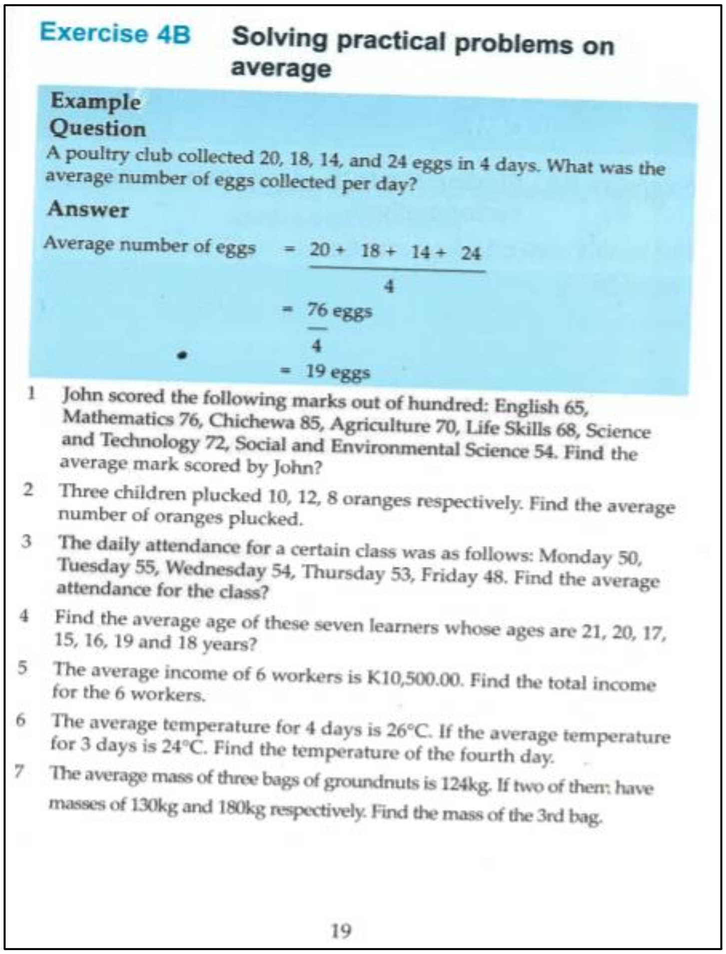
1 Add 681.96 to the product of 12.8 and 7.5.
2 Multiply the quotient of 178.4 and 14.8 by 10.28.
3 Divide the product of 22.4 and 11.6 by 0.4.
4 By how much is the quotient of 71.4 and 10.5 less than 10?
5 By how much does 76.2 exceed the sum of 49.002 and 12.15.
6 Maria bought 7.5kg of meat at K350.50 per kilogram. If she had K395.80 remaining, how much money did Maria have before she bought the meat?
7 Mr Banda bought 51.5 packets of rice each weighing 0.25 kg. If 0.949kg were eaten, how many kilogram of rice were remaining?
8 Mrs Phiri had 89.73kg of maize. She then bought 202.17kg of maize from local traders. If she decided to put the maize into bags each weighing 55.6kg, how many bags did she have?

38

Figure 26: Exercise 7G. Solving practical problems involving basic operation on decimals (MIE, 2008, p. 38).

This example space helps the learners to see similarity as they construct the mathematical sentences from the word problems, identify the key word for each operation involved in each problem and group numbers with same sign or basic operation. Moreover, the example space requires the learners to combine two basic operations on the same example following order of operations and using a brackets to provide clarity in the order of operation [e.g. multiply the difference between 209.48 and 198.89 by 2.5: (209.48 – 198.89) x 2.5]. That is, providing opportunity for learners to experience fusion as they identify where to allocate/put a bracket (discern simultaneous aspect of variation*)*.

Lastly, nine example spaces were coded under C, G and F out of 54 example spaces and belonged to level 3. The example spaces provide opportunity for learners to experience simultaneous variation of more than one aspect of critical features of object of learning and are connected with similarity and difference within the entire example set. Thus, these spaces give learners opportunity to generalize three aspect of object of learning namely: contrast, generalization and fusion. (see figure, 27 & 28)



Exercise 4B **Solving practical problems on average**

Example
Question
A poultry club collected 20, 18, 14, and 24 eggs in 4 days. What was the average number of eggs collected per day?

Answer

Average number of eggs $= \frac{20 + 18 + 14 + 24}{4}$

$= \frac{76 \text{ eggs}}{4}$

$= 19$ eggs

1 John scored the following marks out of hundred: English 65, Mathematics 76, Chichewa 85, Agriculture 70, Life Skills 68, Science and Technology 72, Social and Environmental Science 54. Find the average mark scored by John?

2 Three children plucked 10, 12, 8 oranges respectively. Find the average number of oranges plucked.

3 The daily attendance for a certain class was as follows: Monday 50, Tuesday 55, Wednesday 54, Thursday 53, Friday 48. Find the average attendance for the class?

4 Find the average age of these seven learners whose ages are 21, 20, 17, 15, 16, 19 and 18 years?

5 The average income of 6 workers is K10,500.00. Find the total income for the 6 workers.

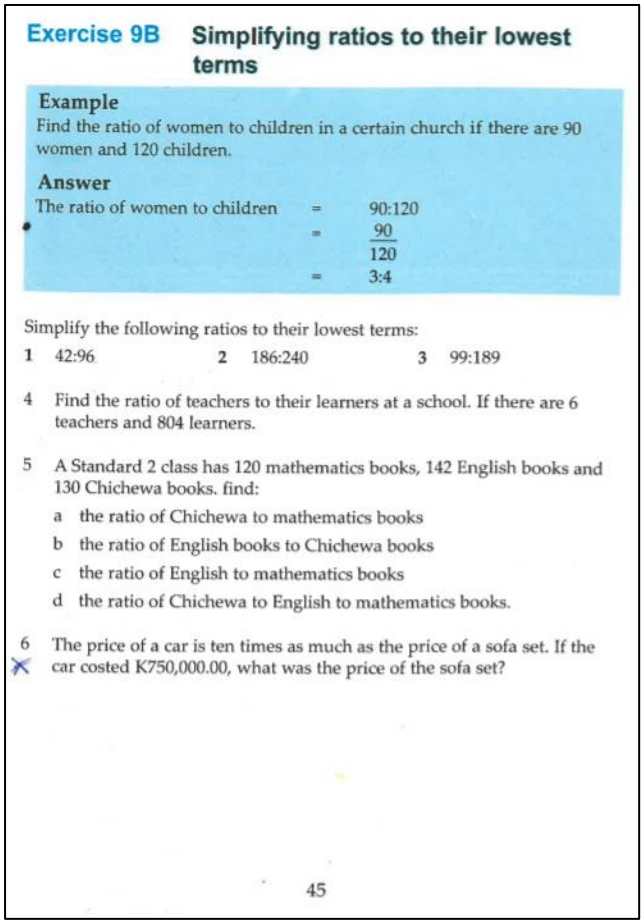
6 The average temperature for 4 days is 26°C. If the average temperature for 3 days is 24°C. Find the temperature of the fourth day.

7 The average mass of three bags of groundnuts is 124kg. If two of them have masses of 130kg and 180kg respectively. Find the mass of the 3rd bag.

19

Figure 27: Exercise 4B. Solving practical problems on average (MIE, 2008, p. 19).

In this example space, examples 1 – 4 and worked example have similar structure, a series of numerals and require learners to work out the averages. As the learners are working out the averages, they will notice similarity and be able to generalize. With contrast in that, examples 5 – 7 are already provided with averages and learners are required to find different variables. Thus, providing opportunity to see an instance of what is not the object of learning. Moreover, to solve problems 5 – 7, learners need to apply other concepts such as multiplication, addition, and subtraction that need to occur simultaneously.



Exercise 9B **Simplifying ratios to their lowest terms**

Example
Find the ratio of women to children in a certain church if there are 90 women and 120 children.

Answer
The ratio of women to children = 90:120 = $\frac{90}{120}$ = 3:4

Simplify the following ratios to their lowest terms:

1 42:96 2 186:240 3 99:189

4 Find the ratio of teachers to their learners at a school. If there are 6 teachers and 804 learners.

5 A Standard 2 class has 120 mathematics books, 142 English books and 130 Chichewa books. find:

a the ratio of Chichewa to mathematics books
b the ratio of English books to Chichewa books
c the ratio of English to mathematics books
d the ratio of Chichewa to English to mathematics books.

6 The price of a car is ten times as much as the price of a sofa set. If the car costed K750,000.00, what was the price of the sofa set?

45

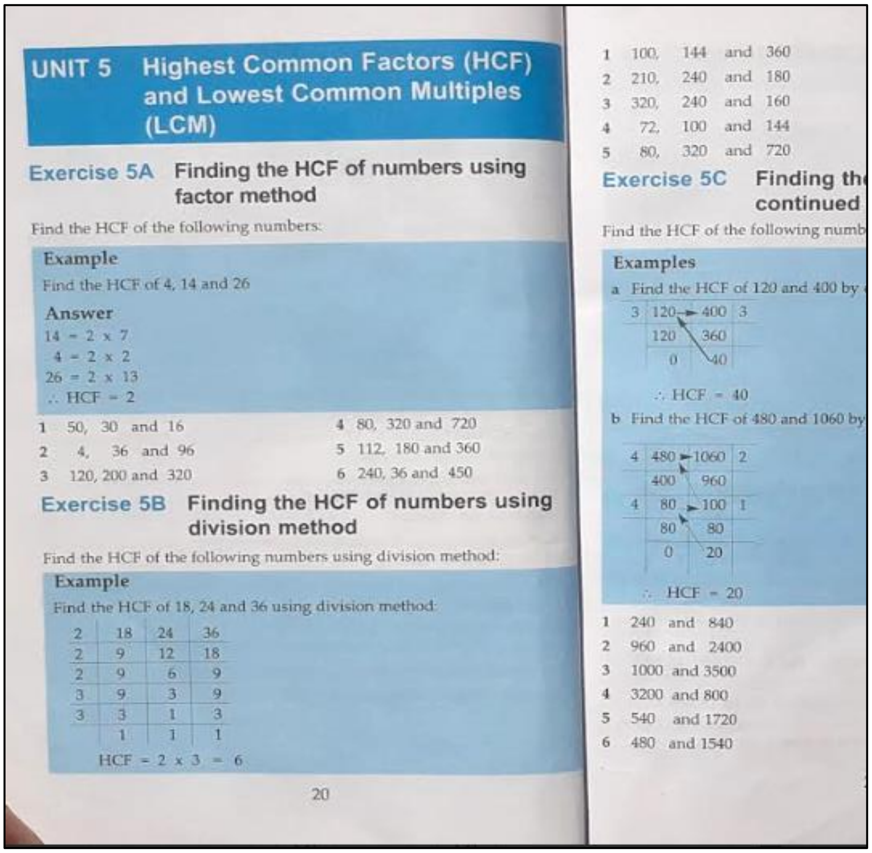
Figure 28: Exercise 9B. Simplifying Ratios to their lowest term (MIE, 2008, p. 45).

This example space provides opportunity for seeing similarity, difference and experience fusion (simultaneous variation of more than one aspect of critical features of object of learning). Examples 1 – 3 have same structure, a pair of ratios, with varying digits and examples 5a – 5d and worked example have similar structure, finding a pair of ratios from given information. Both set of examples enable the learners to seeing similarity and are able to generalize. With contrast in that example 5a – 5d require learners to form ratios from given information and then simplify them to their lowest term. And again, example 6 involves finding a share (price of sofa set) which is in contrary to object of learning (experiencing contrast). Moreover, to find a share in question 6, learners require the knowledge of linear equation to solve for unknown variables. That is, the learners will be able to discern simultaneous variations and more aspect of critical features of object of learning.

The analysis of the example spaces implies that most example spaces belong to level 2 (24 out of 54 example spaces) because they contain a combination of two different patterns of variation namely, contrast and generalization and / or generalization and fusion. And 21 example spaces belong to level 1 because they contain one pattern of variation (contrast or generalization). Very few example spaces (9 out of 54 example spaces) belonged to level 3 because they combine three patterns of variations (contrast, generalization and fusion). This means that grade 7 textbook provides opportunity for learners to experience two aspects of variation to achieve learning outcome in a single example space.

Finding from tasks

Tasks are what learners are asked to do with various examples (Ronda & Adler, 2017). The textbook provides a set of exercises at every topic that enable learners to practice the concepts being illustrated in the worked examples. Analysis of the tasks shows that 14 tasks out of 54 tasks were coded KPF and they belong to level 1. These tasks required learners to use the known procedures and facts that they already know from their past experience. (see figure 29)



UNIT 5 Highest Common Factors (HCF) and Lowest Common Multiples (LCM)

Exercise 5A Finding the HCF of numbers using factor method

Find the HCF of the following numbers:

Example

Find the HCF of 4, 14 and 26

Answer

14 = 2 x 7
4 = 2 x 2
26 = 2 x 13
∴ HCF = 2

1 50, 30 and 16
2 4, 36 and 96
3 120, 200 and 320
4 80, 320 and 720
5 112, 180 and 360
6 240, 36 and 450

Exercise 5B Finding the HCF of numbers using division method

Find the HCF of the following numbers using division method:

Example

Find the HCF of 18, 24 and 36 using division method:

2	18	24	36
2	9	12	18
2	9	6	9
3	9	3	9
3	3	1	3
	1	1	1

HCF = 2 x 3 = 6

20

1 100, 144 and 360
2 210, 240 and 180
3 320, 240 and 160
4 72, 100 and 144
5 80, 320 and 720

Exercise 5C Finding th
continued

Find the HCF of the following numb

Examples

a Find the HCF of 120 and 400 by

3	120	400	3
	120	360	
	0	40	

∴ HCF = 40

b Find the HCF of 480 and 1060 by

4	480	1060	2
	400	960	
4	80	100	1
	80	80	
	0	20	

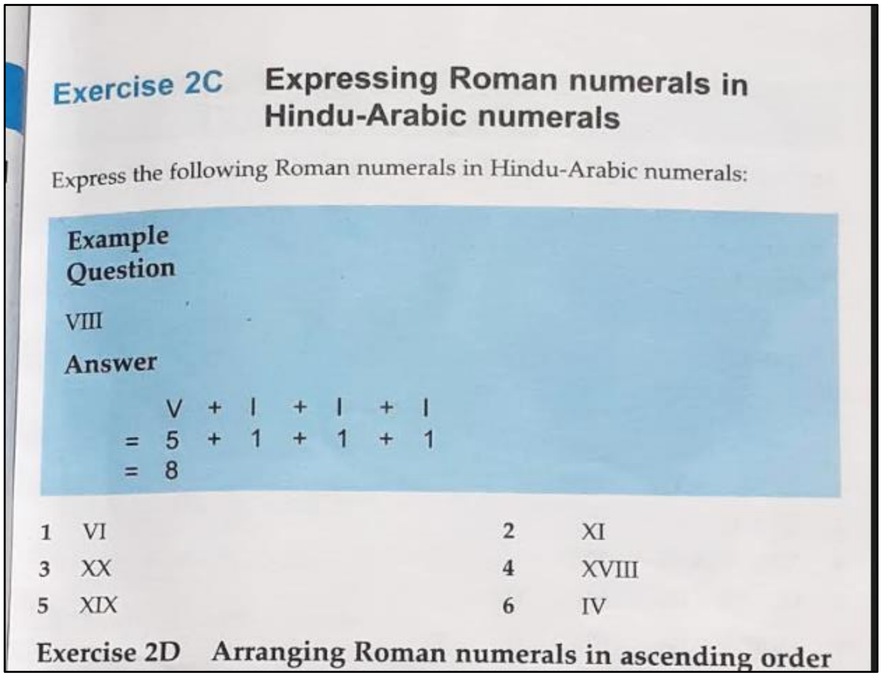
∴ HCF = 20

1 240 and 840
2 960 and 2400
3 1000 and 3500
4 3200 and 800
5 540 and 1720
6 480 and 1540

Figure 29: Exercise 5B. Finding the HCF of numbers using division method (MIE,2008, p. 20).

In this example space, learners are required to find HCF using division method thereby identifying factors which will go into the given numbers without a remainder. In order to do this task, learners require the idea of prime factors, factors and common factors which they did in the previous grades. Therefore, I coded this example space as KPF as it required learners to use known procedures and known concepts to mediate the capability in the lesson.

There were 33 tasks out of 54 tasks that were coded under CTP and they belong to level 2. The tasks require learners to use concepts and procedures that were being learnt in the textbook lessons (new methods introduced in the lesson) (see figure 30)


Exercise 2C Expressing Roman numerals in Hindu-Arabic numerals

Express the following Roman numerals in Hindu-Arabic numerals:

Example
Question
VIII
Answer
V + I + I + I
= 5 + 1 + 1 + 1
= 8

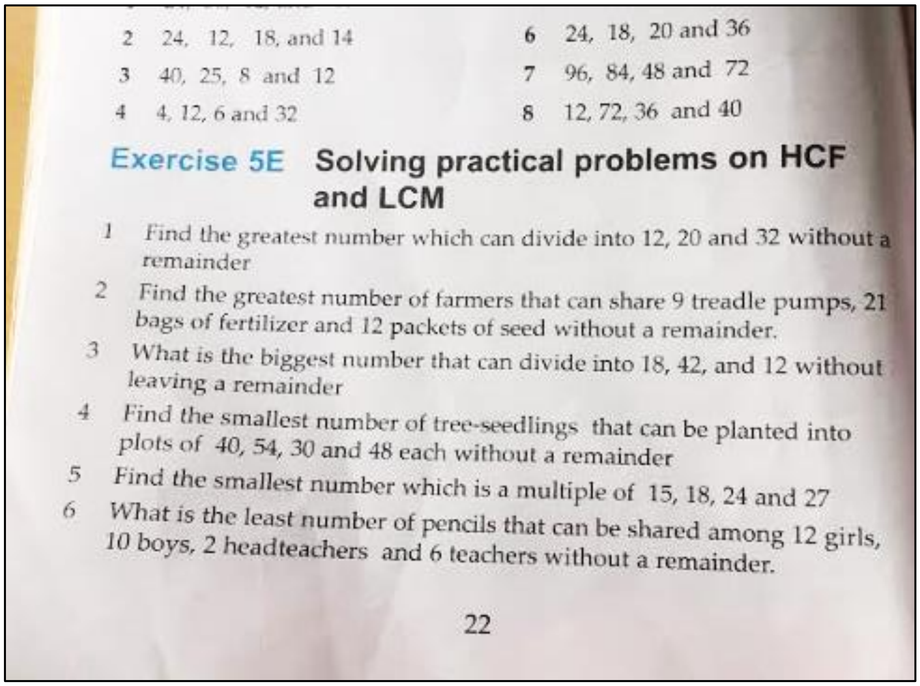
1 VI
2 XI
3 XX
4 XVIII
5 XIX
6 IV

Exercise 2D Arranging Roman numerals in ascending order

Figure 30: Exercise 2C. Expressing Roman numerals in Hindu-Arabic numerals (MIE, 2008, p.7)

The object of learning for this task is, "Expressing Roman numerals in Hindu-Arabic numerals" and learners are required to change the Roman numerals to Hindu-Arabic by following the concepts and procedures done in the worked example. The investigator coded this task as CTP because it mediates learners' capability (writing Roman numerals in Hindu-Arabic) thereby following the rules and algorithm acquired in the current topic. That is, the task requires learners to apply the procedure that is being introduced in the current lesson.

Only 7 tasks were coded under AMC and belong to level 3. These tasks require learners to make decisions concerning the concepts and procedures that needed to be called upon to answer the task. Sometimes the tasks require the learners to apply concepts or make connection between the concepts. This shows that the tasks provide learners with opportunities for further understanding of the concepts through making connections between or among the critical features of the object of learning. (see figure 31)



2 24, 12, 18, and 14
3 40, 25, 8 and 12
4 4, 12, 6 and 32
6 24, 18, 20 and 36
7 96, 84, 48 and 72
8 12, 72, 36 and 40

Exercise 5E Solving practical problems on HCF and LCM

1 Find the greatest number which can divide into 12, 20 and 32 without a remainder
2 Find the greatest number of farmers that can share 9 treadle pumps, 21 bags of fertilizer and 12 packets of seed without a remainder.
3 What is the biggest number that can divide into 18, 42, and 12 without leaving a remainder
4 Find the smallest number of tree-seedlings that can be planted into plots of 40, 54, 30 and 48 each without a remainder
5 Find the smallest number which is a multiple of 15, 18, 24 and 27
6 What is the least number of pencils that can be shared among 12 girls, 10 boys, 2 headteachers and 6 teachers without a remainder.

22

Figure 31: Exercise 5E. Solving practical problems on HCF and LCM (MIE, 2008, p. 22).

There is no worked example in this example space. As a result, learners are required to formulate their own procedures, identify the operations involved in the task, identifying the key words which shows HCF or LCM in each example (greatest number = HCF, smallest number = LCM) and factor the numbers. However, examples 5 and 6 are not similar to the worked example which precedes them but features extra questions which demands more skills than those demonstrated in the worked examples (common in most example spaces). The investigator coded this task as AMC as it required learners to make decisions as to what procedures and concepts are needed to solve the exercise.

The analysis of the textbook lessons from the first 10 topics (first learning outcome) of grade 7 learners' book reveals that, most of the tasks belong to level 2 (CTP) since there were 33 out of 54 tasks that were coded under CTP. That is, most of the tasks required the learners to use a new algorithm/method of solving that was being introduced in the lesson. However, there were some contributing factors for coding most of the tasks as CTP; and this came about because some topics were just being introduced in grade 7 and other topics combine two or more basic operation on the same example. Therefore, solving these new topics needed new procedures of solving them as introduced in the lesson (for example: following the order of operations, BODMAS). In the previous grades (grade 1 – 6) basic operation was taught separately.

5.5 Textbook 4: Grade 8 learners' mathematics textbook

There are eight units in grade 8 under the first learning outcome and all the units were analyzed. In grade 8, the units start with an introduction which gives background information of the topic that is going to be discussed. And there are also success criteria which the learners are going to achieve at the end of each unit. After success criteria, there are exercises which start with a worked example and then some practice exercises. Most of the example spaces in grade 8 comprise of one worked example and sometimes these worked examples are not similar to some of the questions in practice exercises which follow them. For example: exercise 1F; converting Roman numerals to Hindu-Arabic: example: CXIII is = 100 + 10 + 3 = 113; DCL is = 500 + 100 + 50 = 650; task: 1. XVI; 2. VIII; 3. DCLXII; 4. CDX; 5. XLVIII; 6. XCIX; 7. CML; 8. DCCC (MIE, 2008, p. 5). Learners require more information on examples 4, 6 and 7 in order to solve them and yet there are no illustrated procedures to help them tackle similar questions in the given task. This is the first textbook which includes the introduction in the learners' book in this learning outcome (core element).

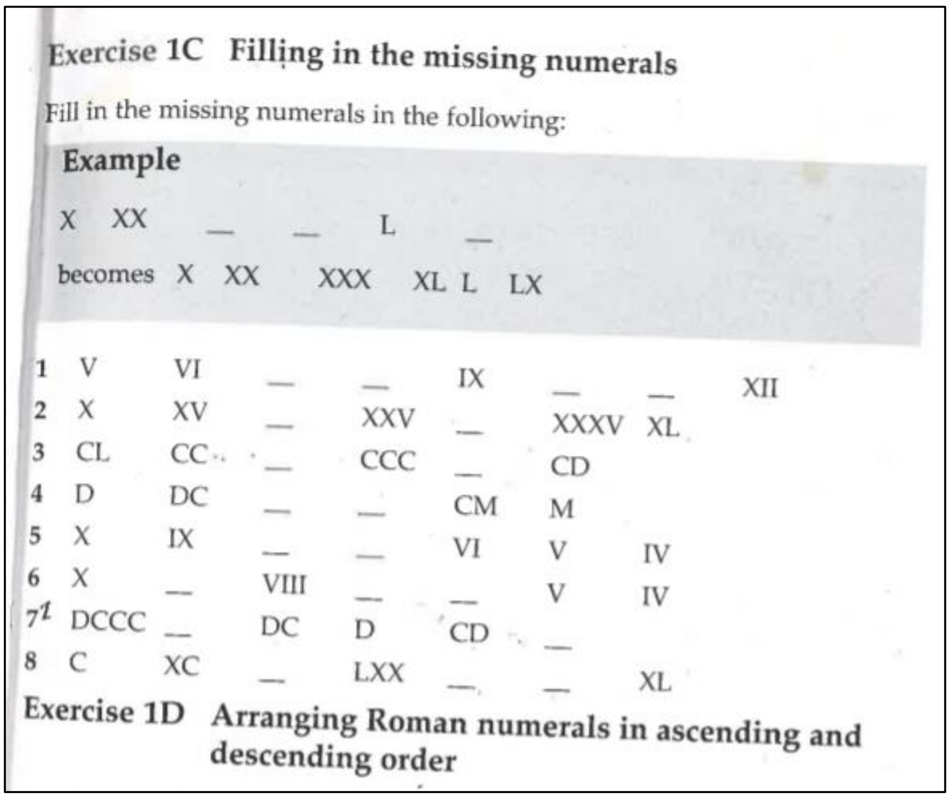
Table 6: summary of coded examples and tasks for learners' book 8

Core Element/ Learning outcome	**Examples**		**Tasks**	
	Codes	Example spaces	Levels of tasks	Number of Tasks
Numbers, Operations and Relationships (8 topics)	C	7	Level 1 (KPF)	18
	G	9	Level 2 (CTP)	11
	C and G	12	Levels 3 (AMC)	4
	G and F	1		
	C, G & F	5		
	Levels			
	Level 1	16		
	Level 2	12		
	Level 3	5		

Table 6 shows the core element/learning outcome for the topics, examples and tasks that were coded

Findings from examples

The analysis for the example spaces on the eight units of grade 8 learners' book indicate that 7 example spaces out of 33 example spaces were coded under contrast and belonged to level 1. These example spaces provide the opportunity for learners to notice the difference as they mediate on the object of learning. That is, learners had opportunity for seeing at least one instance of what object of learning was not. Figure 32 shows an extract taken from grade 8 textbook, to illustrate how the process of analysis was conducted.



Exercise 1C Filling in the missing numerals

Fill in the missing numerals in the following:

Example

X XX __ __ L __

becomes X XX XXX XL L LX

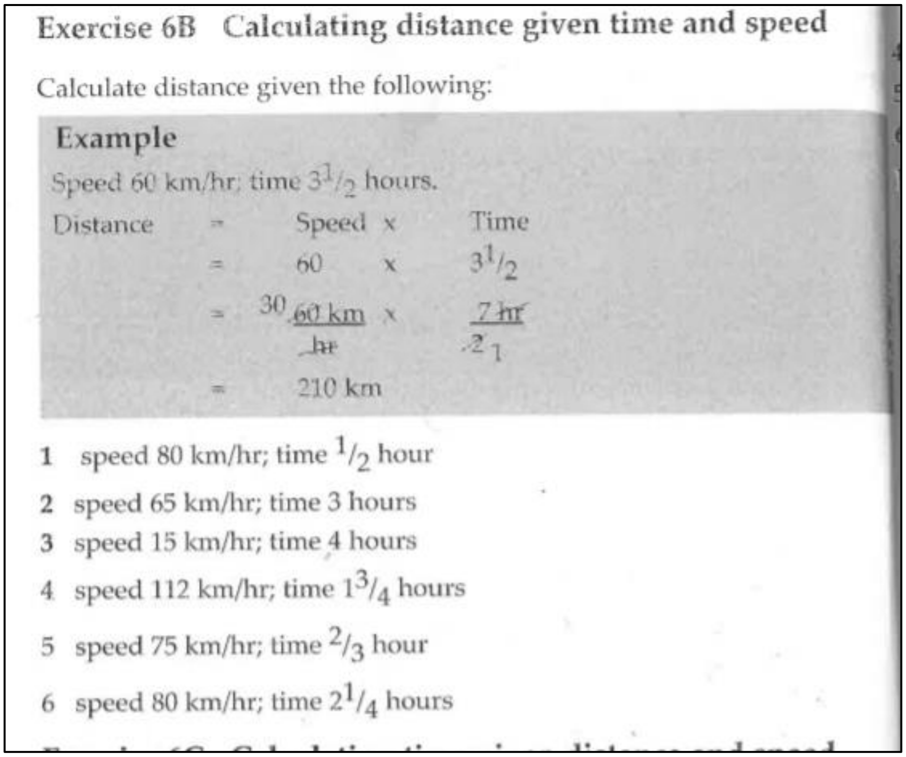
1	V	VI	__	__	IX	__	__	XII
2	X	XV	__	XXV	__	XXXV	XL	
3	CL	CC	__	CCC	__	CD		
4	D	DC	__	__	CM	M		
5	X	IX	__	__	VI	V	IV	
6	X	__	VIII	__	__	V	IV	
7[1]	DCCC	__	DC	D	CD	__		
8	C	XC	__	LXX	__	__	XL	

Exercise 1D Arranging Roman numerals in ascending and descending order

Figure 32:Exercise 1C. Filling in the missing numerals (MIE, 2009, p. 3)

This example space provides opportunity for learners to compare and contrast because the example space contains two learning outcomes: "filling in missing numbers in ascending" and "filling in the missing numbers in descending order" (MIE, 2008, p. 3). The learners will notice difference as they order numbers from the smallest to largest number and when they order numbers from the largest to the lowest number.

There were 10 example spaces out of 33 example spaces that were coded under generalization and they belonged to level 1. These example spaces provided opportunity for learners to see similarity between or among the examples. The learners were able to follow the worked examples as the examples highlighted and directed their attention to the necessary critical features of the object of learning and therefore enabled learners to generalize. (see figure 33 & 34)



Exercise 6B Calculating distance given time and speed

Calculate distance given the following:

Example

Speed 60 km/hr; time $3^1/_2$ hours.

Distance = Speed × Time

= 60 × $3^1/_2$

= $^{30}\frac{60 \text{ km}}{\text{hr}} \times \frac{7 \text{ hr}}{2_1}$

= 210 km

1 speed 80 km/hr; time $^1/_2$ hour

2 speed 65 km/hr; time 3 hours

3 speed 15 km/hr; time 4 hours

4 speed 112 km/hr; time $1^3/_4$ hours

5 speed 75 km/hr; time $^2/_3$ hour

6 speed 80 km/hr; time $2^1/_4$ hours

Figure 33: Exercise 6B. Calculating distance given time and speed (MIE, 2009, p. 28).

All examples in this example space required learners to calculate distance given time and speed. The learners will calculate the distance by using the formula that is provided in the lesson. As a result, learners will be able to see similarity between or among examples as they follow the worked example, and this will enable them to generalize

Exercise 2A Finding HCF by factor method

Find the HCF of the following numbers:

Example

175, 125 and 325

175 = 5 x 5 x 7

125 = 5 x 5 x 5

325 = 5 x 5 x 3

∴ HCF = 5 x 5 (because it appears in all the 3 numbers)

= 25

1 45, 90 and 15

2 63, 126 and 189

3 16, 64, 8 and 80

4 15, 30, 90 and 120

5 92, 115, 138 and 184

6 76, 114, 152 and 266

7 125, 225 and 350

8 80, 200, 120 and 280

9 117, 260, 468 and 585

10 105, 156 and 180

Figure 34: Exercise 2A. Finding HCF by using factor methods (MIE, 2009, p. 6).

This example space contains examples that have the similar structure, with varying numbers (3 – 4 numbers) in each example. Learners are required to find factors, common factors and then the greatest common factor in order to solve for HCF. This example space provides opportunity for learners to see similarity among various examples and then they are able to generalize as they work on them.

The example spaces that were coded under contrast and generalization were 12 out of 33 example spaces, and they belonged to level 2. The example spaces provide opportunity for generalizing two aspects of the critical features of the object of learning. Thus, the example spaces enabled learners to notice similarity and differences within the examples in the example space. For example: (see figure 35 & 36)

Exercise 2C Finding LCM using the factor method

Find the LCM of the following using the factor method:

Example

35 and 140

70 = 2 x 5 x 7

35 = 5 x 7

140 = 2 x 2 x 5 x 7

∴ LCM = 2 x 2 x 5 x 7

= 140

or

2	35	70	140
2	35	35	70
5	35	35	35
7	7	7	7
	1	1	1

∴ LCM = 2 x 2 x 5 x 7

= 140

1 117, 81 and 44

2 14, 36, 24 and 18

3 15, 27, 33 and 45

4 70, 84 and 105

5 153, 612 and 136

6 156, 105 and 180

Figure 35: Exercise 2C. Finding LCM using the factor method (MIE, 2009, p. 8).

In this example space, examples 1 – 6 and worked example have similar structure, there is a pair or 3 – 4 multiples in each example, and as a result, this shows similarity. With contrast in that two different procedures are involved to solve them (factor method and division method) while the object of learning was: “Finding LCM using the factor method”. This enabled the learners to see some variations, and that is an instance of what was not the object of learning. Therefore, I coded this example space as contrast and generalization.

Exercise 8B Increasing or decreasing quantities in given percentages

Example

a Increase 50 by 20%

$= 50 + (20\% \text{ of } 50)$

$= 50 + \frac{20}{100} \times 50$

$= 50 + 10$

$= 60$

b Decrease 60 by 10%

$= 60 - (10\% \text{ of } 60)$

$= 60 - \frac{10}{100} \times 60$

$= 60 - 6$

$= 54$

or

a Increase 50 by 20%

$= 50 \times (20\% + 100\%)$

$= 50 \times \frac{120}{100}$

$= 5 \times 12$

$= 60$

b decrease 60 by 10%

$60 \times (100\% - 10\%)$

$= 60 \times \frac{90}{100}$

$= 6 \times 9$

$= 54$

1 Increase:

a 40 by 50%

b 700 by 11%

c 65 by 20%

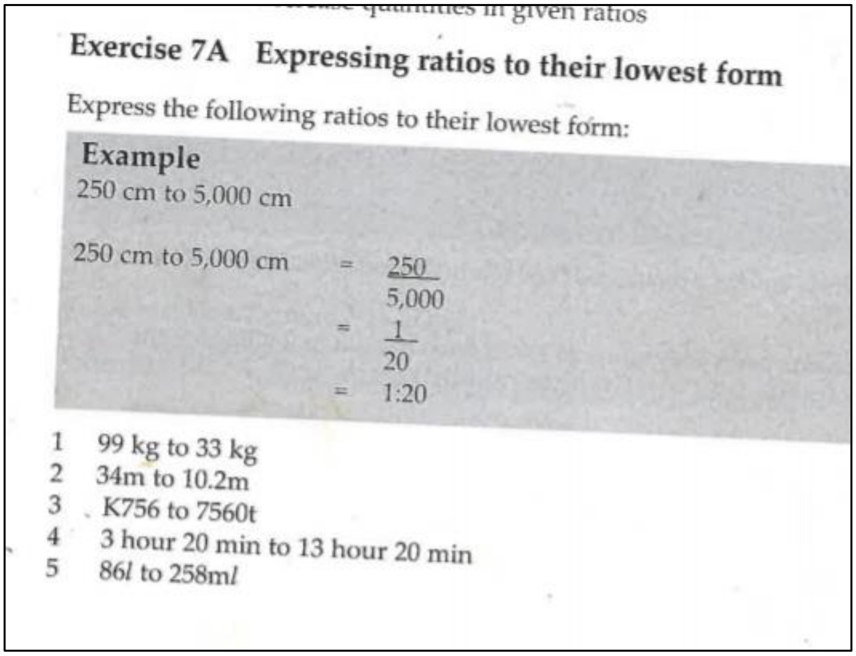
d 120 by 30%

e 88 by 200%

Figure 36: Exercise 8B. Increasing or decreasing given percentages (MIE, 2009, p. 35).

In this example space, example 1a – 1e and worked example a) have same structure, increasing the quantities in given percentages and examples 2a – 2e and worked example b also have the same structure, decreasing the quantities in given percentages; this shows similarity. However, the example space involves two different concepts; increasing and decreasing quantities in given percentages, therefore, it provides opportunity for learners to seeing some difference. The analysis made the investigator to code this example space as contrast and generalization.

Only one example space out of thirty three example spaces was coded contrast and fusion. This example space helps learners to seeing difference and discerns simultaneous dimensions of variation of more than one aspect of the object of learning. Figure 37 is used to illustrate how the analysis was conducted. (see figure 27)



... quantities in given ratios

Exercise 7A Expressing ratios to their lowest form

Express the following ratios to their lowest form:

Example

250 cm to 5,000 cm

250 cm to 5,000 cm $= \frac{250}{5,000}$

$= \frac{1}{20}$

$= 1:20$

1 99 kg to 33 kg
2 34m to 10.2m
3 K756 to 7560t
4 3 hour 20 min to 13 hour 20 min
5 86*l* to 258m*l*

Figure 37: Exercise 7A. Expressing Ratios in their lowest form (MIE, 2009, p. 30).

The example space helps learners to generalize as they simplify the ratios through division using factors in all examples (1–5 and worked example) and then, coming up with their lowest terms. However, examples 3–5 involve converting numbers from one unit to another unit (Kwachas to Tambalas, hours to minutes and liters to milliliters respectively) in order to have same units and then be able to simplify the ratios. In order to convert the units, learners needed the knowledge of measurement (which is another learning outcome) to understand the relationships between these units. I coded this example space as providing opportunity for seeing similarity and discerning simultaneous dimensions of variation.

Lastly, I coded five example spaces out of 33 example spaces under contrast, generalization and fusion, and they belonged to level 3. Thus, the example spaces provide opportunity for learners to experience simultaneous variation of more than one aspect of critical features of object of learning. That is, learners were able to discern similarity, difference and simultaneous aspects of variation for features of object of learning in the example sets. The two examples (fig. 38 & 39) that follows are used to illustrate how the process of coding was conducted.

Exercise 8C Solving practical problems involving percentages

Example

The enrolment of Standard 8 learners was 80 in term 1. If it decreased by 5% in term 2, how many learners were there in term 2?

Number of learners in term 2

$= 80 - \frac{5}{100} \times 80$

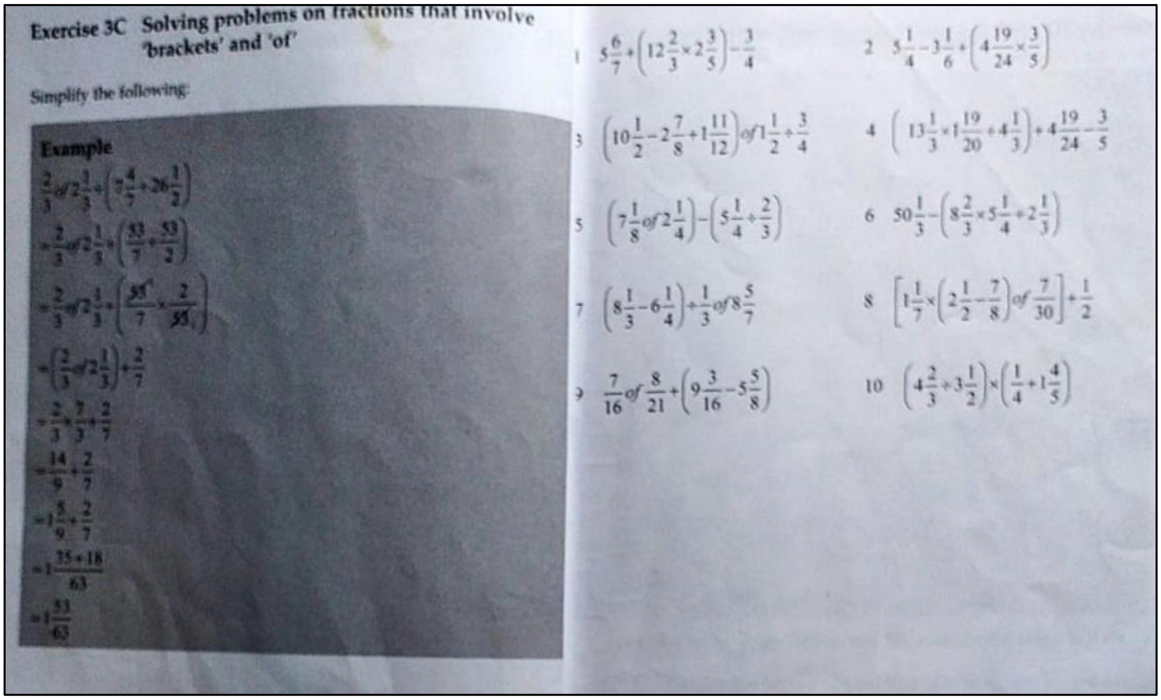
$= 80 - 4$

$= 76$ learners

1. The wages of some workers at a company was K5,500 per month. If the manager increased these wages by 12%, what are the new wages?
2. The pass rate at a certain school decreased by 5%. If in the previous year 60 learners passe, how many learners passed in the following year?
3. The population of a district was 45,000. If the population increased by 10%, find the new population.
4. A shopkeeper offered a discount of 20% for goods worthy K10,000.00. How much did a customer pay?
5. In a school there are 300 girls and 200 boys. Find:
   a. the ratio of girls to boys
   b. the percentage of girls in the school

Figure 38: Exercise 8C. Solving practical problems involving percentages (MIE, 2009, p. 36).

In this example space, examples 4–5 and worked example have similar structure, have quantities and percentage except example 5 which requires calculating the ratio and percentage. Example 5b required the learners to find the ratio, which is not in relation with the object of learning. Moreover, examples 4 require learners to apply the concept of discount from their previous experience as a result; the example space is providing opportunity for learners to discern simultaneous variations (fusion).



Exercise 3C Solving problems on fractions that involve 'brackets' and 'of'

Simplify the following:

Example

$\frac{2}{3}$ of $2\frac{1}{3} + \left(7\frac{4}{7} \div 26\frac{1}{2}\right)$

$= \frac{2}{3}$ of $2\frac{1}{3} + \left(\frac{53}{7} \div \frac{53}{2}\right)$

$= \frac{2}{3}$ of $2\frac{1}{3} + \left(\frac{53}{7} \times \frac{2}{53}\right)$

$= \left(\frac{2}{3} \text{ of } 2\frac{1}{3}\right) + \frac{2}{7}$

$= \frac{2}{3} \times \frac{7}{3} + \frac{2}{7}$

$= \frac{14}{9} + \frac{2}{7}$

$= 1\frac{5}{9} + \frac{2}{7}$

$= 1\frac{35+18}{63}$

$= 1\frac{53}{63}$

1 $5\frac{6}{7} + \left(12\frac{2}{3} \times 2\frac{3}{5}\right) - \frac{3}{4}$

2 $5\frac{1}{4} - 3\frac{1}{6} + \left(4\frac{19}{24} \times \frac{3}{5}\right)$

3 $\left(10\frac{1}{2} - 2\frac{7}{8} + 1\frac{11}{12}\right)$ of $1\frac{1}{2} \div \frac{3}{4}$

4 $\left(13\frac{1}{3} \times 1\frac{19}{20} + 4\frac{1}{3}\right) \div 4\frac{19}{24} - \frac{3}{5}$

5 $\left(7\frac{1}{8} \text{ of } 2\frac{1}{4}\right) - \left(5\frac{1}{4} \div \frac{2}{3}\right)$

6 $50\frac{1}{3} - \left(8\frac{2}{3} \times 5\frac{1}{4} \div 2\frac{1}{3}\right)$

7 $\left(8\frac{1}{3} - 6\frac{1}{4}\right) \div \frac{1}{3}$ of $8\frac{5}{7}$

8 $\left[1\frac{1}{7} \times \left(2\frac{1}{2} - \frac{7}{8}\right) \text{ of } \frac{7}{30}\right] \div \frac{1}{2}$

9 $\frac{7}{16}$ of $\frac{8}{21} + \left(9\frac{3}{16} - 5\frac{5}{8}\right)$

10 $\left(4\frac{2}{3} + 3\frac{1}{2}\right) \times \left(\frac{1}{4} + 1\frac{4}{5}\right)$

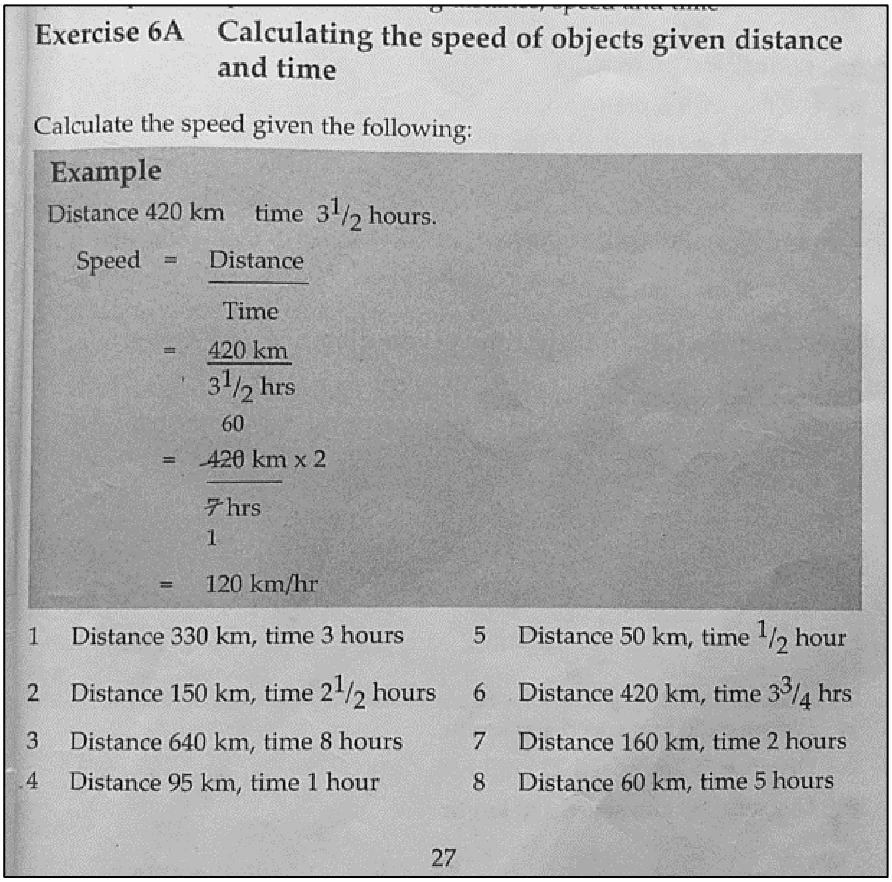
Figure 39Exercise 3C. Solving problems on fractions that involve "brackets" and "of" (MIE, 2009, p. 15)

In this example space, examples 1 – 10 and worked example have same structure, they contain fractions which involve combinations of 3 – 4 basic operations in the same problem and all examples have one to two separate brackets in each example. The example space required learners to simplify the fractions following the recommended order of operations with match focus on the bracket as it provide clarity to the order of operations. Therefore, the example space helped learners to seeing similarity and they were able to generalize. With contrast in that, example 8 has two different brackets which are combined (square bracket outside and with round bracket inside it). Moreover, the square bracket does not appear in the worked example (new). As a result, learners will apply the rules of brackets in order to solve the problem (providing the learners to discern simultaneous variation). Therefore, I coded this example space as C, G and F.

The analysis of the examples reveals that most of the examples in these eight units belong to level 1 (16 example spaces out of 33 example spaces). This shows that most of the examples provide opportunity for seeing one pattern of variation that is either contrast or generalization. There were few examples that required learners to discerning simultaneous aspects of variation. This shows that the eight units fail to bring attention to patterns of variation as they do not provide a set of examples that will foreground the crucial feature of object of learning in the lessons.

Findings from Tasks

The textbook provides a set of exercises for every topic that enable learners to practice the concepts being illustrated in the worked examples. Most of the tasks on the first eight units that the investigator analyzed in grade 8 belonged to level 1 (18 tasks out of 33 tasks) because they required learners to use the procedures in the author's worked example that involved previously learned procedures and facts (KPF). For example, (see figure 40).


Exercise 6A **Calculating the speed of objects given distance and time**

Calculate the speed given the following:

Example

Distance 420 km time $3^1/_2$ hours.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}}$$

$$= \frac{420 \text{ km}}{3^1/_2 \text{ hrs}}$$

$$= \frac{\overset{60}{\cancel{420}} \text{ km} \times 2}{\underset{1}{\cancel{7}} \text{ hrs}}$$

$$= 120 \text{ km/hr}$$

1 Distance 330 km, time 3 hours

2 Distance 150 km, time $2^1/_2$ hours

3 Distance 640 km, time 8 hours

4 Distance 95 km, time 1 hour

5 Distance 50 km, time $^1/_2$ hour

6 Distance 420 km, time $3^3/_4$ hrs

7 Distance 160 km, time 2 hours

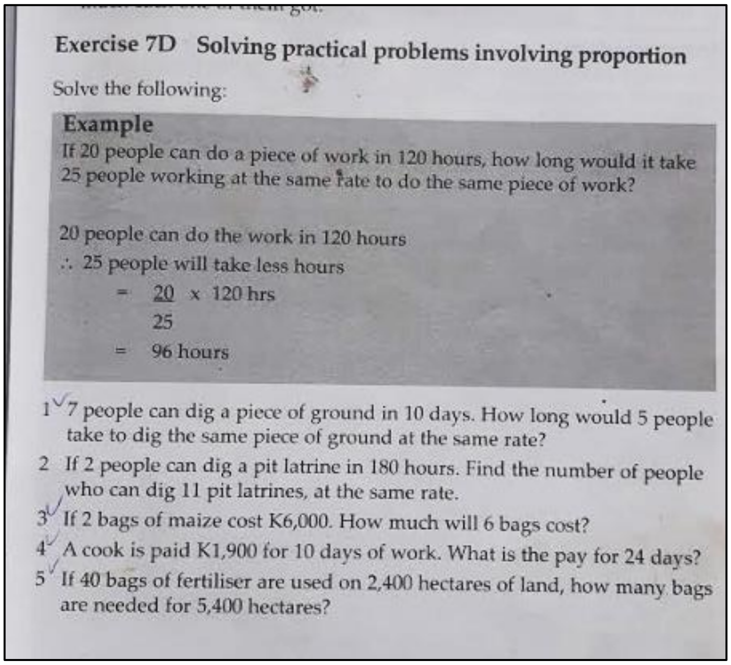
8 Distance 60 km, time 5 hours

27

Figure 40: Exercise 6A. Calculating the speed of objects given distance and time (MIE, 2009, p. 27).

In this example space, learners are required to solve the rate using the formula as illustrated in the worked example. The formula that learners are required to use is already known by the learners since they used it in grade 7, hence learners are using previously learned procedure and fact (KPF).

There were 11 tasks out of 33 tasks that were coded CTP and they belonged to level 2. The tasks require learners to apply the new methods of solving that were being introduced in the current lesson. (see figure 41)



Exercise 7D Solving practical problems involving proportion

Solve the following:

Example

If 20 people can do a piece of work in 120 hours, how long would it take 25 people working at the same rate to do the same piece of work?

20 people can do the work in 120 hours

∴ 25 people will take less hours

$= \frac{20}{25} \times 120$ hrs

$=$ 96 hours

1 7 people can dig a piece of ground in 10 days. How long would 5 people take to dig the same piece of ground at the same rate?

2 If 2 people can dig a pit latrine in 180 hours. Find the number of people who can dig 11 pit latrines, at the same rate.

3 If 2 bags of maize cost K6,000. How much will 6 bags cost?

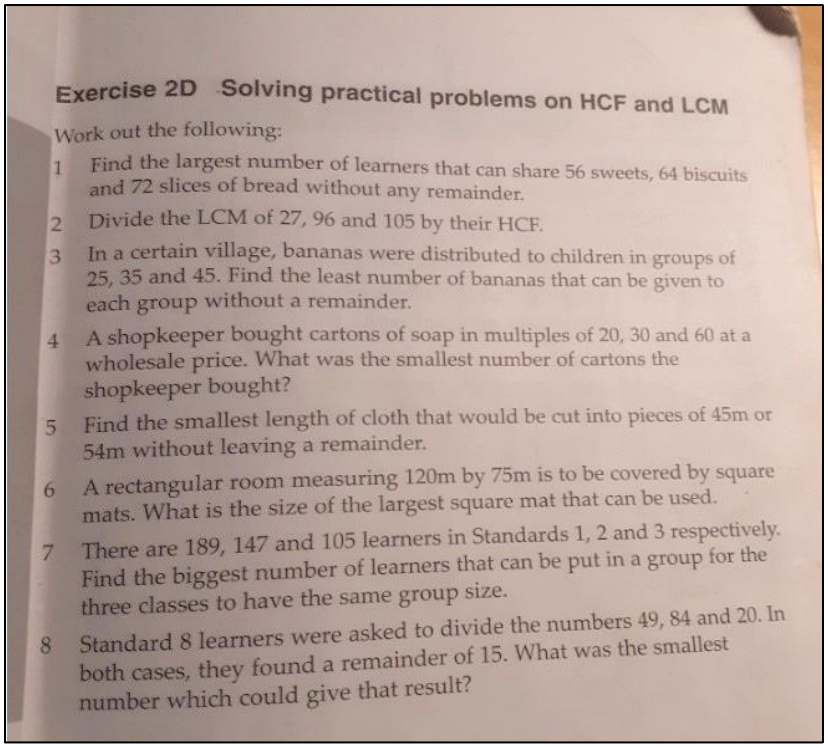
4 A cook is paid K1,900 for 10 days of work. What is the pay for 24 days?

5 If 40 bags of fertiliser are used on 2,400 hectares of land, how many bags are needed for 5,400 hectares?

Figure 41: Exercise 7D. Solving practical problems involving proportion (MIE, 2009, p. 33).

In this example space, learners are required to solve the proportions by following the new methods that are being introduced in the current lesson. The learners will follow the worked example and be able to come up with the proportion in each example.

Only four out of 33 tasks were coded under AMC and belong to level 3. These tasks required learners to make decisions concerning the concepts and procedures that needed to be called upon to answer the task. Sometimes the tasks required the learners to apply concepts or make connection between the concepts. This shows that the tasks provided learners with opportunities for further understanding of the concepts through making connections between or among the critical features of the object of learning. (see figure 42)


Exercise 2D Solving practical problems on HCF and LCM

Work out the following:

1 Find the largest number of learners that can share 56 sweets, 64 biscuits and 72 slices of bread without any remainder.

2 Divide the LCM of 27, 96 and 105 by their HCF.

3 In a certain village, bananas were distributed to children in groups of 25, 35 and 45. Find the least number of bananas that can be given to each group without a remainder.

4 A shopkeeper bought cartons of soap in multiples of 20, 30 and 60 at a wholesale price. What was the smallest number of cartons the shopkeeper bought?

5 Find the smallest length of cloth that would be cut into pieces of 45m or 54m without leaving a remainder.

6 A rectangular room measuring 120m by 75m is to be covered by square mats. What is the size of the largest square mat that can be used.

7 There are 189, 147 and 105 learners in Standards 1, 2 and 3 respectively. Find the biggest number of learners that can be put in a group for the three classes to have the same group size.

8 Standard 8 learners were asked to divide the numbers 49, 84 and 20. In both cases, they found a remainder of 15. What was the smallest number which could give that result?

Figure 42: Exercise 2D Solving practical problems on HCF and LCM (MIE, 2009, p. 9).

This task provides opportunity for learners to solve the problem on HCF and LCM using their own procedures. Learners will make decisions on that concepts and methods to use when solving the tasks. For instance, learners will use concepts such as the knowledge of factors, prime factors, and common factors and also procedures such as factor method, division method, and continued division method. Thus, using known concepts and previously learned algorithms and also the new methods that are being introduced in the current topic. Moreover, learners will also need other concepts such as the concept of area of a rectangle and basic operations as required in task 6 and 2, respectively. Sometimes, some tasks require learners to apply concepts or make connection between the concepts.

Analysis of tasks in grade 8 on the first eight units shows that most of the tasks are level 1 task. These tasks require learners to use previously learned concepts and procedures to mediate object of learning. One of the contributions of the findings is that most of the tasks are repeated, that is, the tasks were already done in lower grades. The findings also reveal that most tasks are recall type of tasks they do not require learners to apply multiple concepts or make connection between or among concepts.

5.6 Chapter summary

The chapter has presented the description of how the textbooks were analyzed and coded using MDITx analytical framework through quantitative and qualitative approach methods. The chapter discussed how each learner's book was analyzed. The next chapter present the discussion.

Chapter 6: DISCUSSION

6.1 Introduction

This study investigated the opportunity to learn mathematics that the textbooks provided on number concepts and operations in the upper primary school classes in Malawi (grade 5 – grade 8). The study aimed at investigating opportunities to learn number concepts and operations as provided through the examples and tasks, as well as how tasks enact the learning objects that are stated in the textbooks and the extent to which tasks allow learners to apply and connect critical features of the mathematical content.

Opportunities to learn number concepts and operations as provided through examples and task

The finding from the textbooks in upper primary classes reveals that, all the four textbooks have the same sequence and structure of the topics and textbook lessons, and there is no background. The textbook lessons either start with an object of learning (a concept and a capability), worked examples and exercises, or with a capability, worked examples and then exercise. Stating the object of learning at the beginning of the example space provides opportunity for learners to understand the concept in focus and the capability that they are expected to develop for the specific learning outcome. However, some example spaces only have an exercise. The study also reveals that the starting point of each topic in each textbook is building on prior knowledge from the previous textbook (previous grade). That is, the textbooks are assisting learners to develop their understanding of new concepts using their prerequisite knowledge. The activities in the textbook are selected and sequenced in such a way that they enhance understanding and provide opportunities for learners to apply and make connections between or among concepts. For example, the concept of fraction is built on the knowledge of whole numbers or the use of the ideas of lowest and highest common multiple. The finding is in line with earlier studies indicating that learners' understanding of numbers evolve from their previous experience and that each concept is contained in the idea that follows it (e.g., Bass, 2015; Fuson, 2003; Reuben, 2009; Stols, 2013).

The finding on examples reveals that mathematics textbooks for upper primary classes in Malawi contain low-order examples on number concepts and operations. Most examples across the example spaces provide opportunities for learners to experience only one pattern of variation of the critical features of the object of learning such as either contrast or generalization. For example, grade 5, grade 6 and grade 8 indicate that the textbooks are giving level 1 pattern of variation (see tables 3, 4 and 6 respectively). There are few example spaces that allow learners to discern contrast, or to combine two patterns of variation. The example spaces in the textbooks comprise various examples that allow learners to practice, but most of these examples are the same and provide opportunity for learners to notice similarities since the examples have similar structures and procedural fluency. Thus, enabling learners to experience the same pattern of variation and hence limit learners' understanding of concepts within and between examples. The finding on textbooks providing the same type of examples is in contrast with one of the principles of variation theory that argues against developing a concept from experience of sameness when using examples (Marton & Pang, 2013). Kullberg et al. (2017) also argue that it is important for teachers/textbooks to afford patterns of variation in a set of examples in order to help children to learn. Whereas the examples in the textbooks from grades 5, 6, and 8 were mostly level 1, the textbook from grade 7 was different. The findings in grade 7 shows that most of the example spaces provide opportunities for learners to experience two patterns of variation (see table 5). However, twenty-one out of fifty-four example spaces were coded level 1 in grade 7, which shows a narrow difference between level 1 examples and level 2 examples. This is also an evidence that the textbooks in upper primary in Malawi mostly contain low order examples. The finding confirms what Malemya (2019) found that mathematics examples in textbooks in Malawi are not deep enough to allow learners to study with understanding by themselves let alone lack visual of real-life examples.

In addition, the study also reveals that there are few high level (level 3) examples in the learners' books. Most of the examples found in the textbooks have few combinations of different patterns of variation. For instance, the findings from the four textbooks indicate that, in grade 5, out of forty-eight example spaces, only three were coded level 3. In grade 6, one out of forty-nine example spaces were coded level 3; in grade 7, only nine out of fifty-four example spaces were coded level 3, and in grade 8, only five example spaces were coded level 3 out of thirty-three example spaces. This shows that mathematics textbooks in upper primary classes offer few opportunities for students to learn several concepts in a single example space. The selection and sequence of examples display one form of variation, which is either contrast or generalization and hence provide the opportunity for generalizing in relationship to one aspect of the object of learning or provide the opportunity for seeing at least one situation of what is not the object of learning

(Adler & Ronda, 2015). Marton and Pang (2006) argue that the key to better learning involves bringing attention to patterns of variation amidst invariance. That is, in order for learners to attend to a particular feature crucial to the object of learning, textbooks need to give a set of examples that will introduce this feature in the lesson (Ronda & Adler, 2017). Therefore, the textbooks lack high-level thinking examples that can allow learners to discern simultaneous dimensions of more than two patterns of variations.

The finding from worked examples on number concepts and operations in all the textbooks reveal that most of the example spaces have one worked example, and, in other cases, no worked example is given to guide the learners when working on the tasks. For example, in grade 5, only one worked example is given per example space in each of the forty-eight example spaces analyzed. Worked examples are important because they are used to illustrate algorithms or procedures for tackling similar questions or tasks. However, the finding is in contrast with the study conducted by Rittle-Johnson and Star (2009) who found that two examples are better than one example, and that two examples presented together are better than two examples presented separately. They further explain that in order for multiple examples to be effective, it is essential for learners to compare different types of examples in the same example space. The reason is that multiple examples allow learners to notice similarities and differences, which will assist them to generalize.

The findings also reveal that the author provides the worked examples with no further explanations or any other conceptual support. For example, in grade 7 the author writes: $61.84 + 1.751 \div 0.85 = 61.84 + (1.751 \div 0.85) = 61.84 + (1.751 \times 100 \div 0.85 \times 100) = 61.84 + 175.1 \div 85$ (MIE, 2008). From the example, the author does not provide reasons for multiplying the decimals with 100, or why the bracket was introduced. This shows that the textbooks do not give learners examples with explanations of how to carry out procedures appropriately. In relation to this, Reimann and Schult (1996) assert that it is important to specify in a worked-out example the steps that were taken and the reasons for taking them in order to direct the attention of learners to critical features of the object of learning. The result from the analysis concur with the finding of Chi et al. (1989), who explain that learners often regard examples with no explanations as restricted patterns that do not seem applicable to them when solving problems that require a little deviation from the solution presented in the worked-out examples. This shows that it is difficult for learners to follow such worked examples, especially, when some of the examples in the given exercise deviate from the sequence or procedure of the worked example and features extra questions that demand more skills than those demonstrated in the worked example. This shows that upper primary textbooks in Malawi provide limited opportunities for learners to use the textbooks on their own when working on the tasks particularly at their own time or at home. Lack of clear instructions and explanations on the worked examples limit learners to adequately access the object of learning when they solve the tasks and hence this leads to dependence on the teachers. Worked examples should highlight the necessary features and direct the attention of learners to those features that make them exemplary. However, the finding is not in line with what Ronda and Adler (2017) explain, that worked examples should demonstrate the procedure or explain a concept so as to make the goal of mathematical content visible. Bills et al. (2006) concur with Ronda and Adler as they argue that worked examples offer insight into the nature of mathematics as they are used to demonstrate methods in complex tasks and indicate relationships, explanations and proofs in development of concepts.

A task is a set of exercises that give learners opportunity to practice the aspect being illustrated in the worked example (Leshota, 2015). The findings from the three (grade 5, grade 6 and grade 8) textbooks revealed that most of the tasks in these textbooks are low-level-tasks and belong to level 1, which enable learners to carry out known procedures and facts (KPF). There were twenty-four tasks that were coded KPF out of forty-eight tasks in the unit in grade 5, twenty-seven tasks were coded KPF out of forty-nine tasks in grade 6, and eighteen tasks were coded KPF out of thirty-three in grade 8. That is, most of the tasks required the learners to use previously learned knowledge and procedures in order to interpret the capabilities associated with the object of learning. Like with the examples, whereas the tasks in the textbooks from grades 5, 6, and 8 were mostly on level 1, the textbook from grade 7 was different. The findings for the grade 7 textbook revealed that most of the tasks belong to level 2 (CTP), since there were thirty-three tasks out of fifty-four tasks that were coded as current topic/procedures (CTP). That is, most of the tasks required the learners to use new algorithms/methods of solving that were being introduced in the lesson. However, there were some contributing factors for coding most of the tasks as CTP; this came about because some topics were just being introduced in grade 7 and other topics combine two or more basic operations on the same example. As a result, solving these new topics needed new procedures that were being introduced in the lesson (for example: following the order of operations, BODMAS).

The study, therefore, found that all textbooks contain enough tasks that give learners opportunity to practice the aspects provided in the lesson. However, the textbook tasks on number concepts and operations are low-level tasks, with low cognitive demand. Most of the tasks require learners to recall concepts, use procedures without connections and application to everyday life or sometimes the tasks require learners to use current topic procedures provided in the textbooks. That is, the tasks provide opportunity for learners to interpret concepts that are already known and follow the procedure used in the worked example provided in the textbooks. The textbooks do not provide a variety of tasks; instead, the textbooks emphasize computations. The dominance of computation in the tasks may have a negative effect on the students' understanding of mathematical ideas and may also limit their own views since the tasks encourage one answer (closed answer type of tasks).

The findings from the tasks also reveal that there are few high-level order questions in the textbooks. The four textbooks have a total number of 184 example spaces in number concepts and operations. Out of these, only sixteen example spaces are level 3 tasks (see table 1). The textbooks lack exercises that challenge learners and develop their understanding. This means that mathematics textbooks in upper primary classes lack higher-order cognitively demanding tasks that could encourage learners' reflective thinking. Aineamani and Naicker (2014) argue that textbooks should probe learners for higher mathematical reasoning by asking questions that require them to give more explanation and good justification for their responses in their mathematics classroom. Gracin (2018, p. 1004) concurs with Aineamani and Naicker by stating that, "mathematics textbooks should provide tasks that will engage students and challenge them to reason, as well as influence the quality of instruction and provide opportunities for developing understanding". The textbook analysis results indicate a predominance of low-level tasks in all topics in number concepts and operations in upper primary mathematics textbooks in Malawi.

The findings from the textbooks for upper primary classes reveals that textbook lessons are teacher-centred. That is, the introduction of the topic or unit and other explanations of important information and instructions are written in the teachers' guide. The curriculum does not give autonomy for the learners' use of the learners' book. Instead, the teacher is the one who provides all the necessary instructions to the learners on how to carry out the exercise in the learners' book. Teacher-centeredness of the textbooks cause learners to find problems when they are using the textbooks on their own, at school or in their homes and hence creating less opportunities for learners to participate in mathematical discourse. For example, in the teachers' guide, Topic 1B in addition, the author suggests that teachers should, "[l]et the learners solve practical problems involving addition with sums not exceeding 999 999. The learners should be given a chance to do exercise 1D on page 17 of their book before you move to Topic 1C" (MIE, 2007, p. 9). Whereas in the learners' book, the author writes, "Exercise 1D".

Enactment of the learning objects through tasks

The analysis of textbooks reveals that the tasks provided in the textbooks fit into the core of the learning objects that are stated in the textbooks. That is, the tasks provided in the textbooks are suitable and address the capability stated in the object of learning. But, to what extent do textbook tasks enact the learning objects that are stated? The textbooks do not provide a balance of a wide range of tasks that could constitute computation, presentation and modeling, interpretation, argumentation, and reasoning competences, which can challenge learners and help them in developing their understanding. Instead, most of the tasks are routine type of tasks that require learners to use algorithms, recall, use simple mathematics facts, formulae and make simple connections between concepts. For example, the first unit for grade 5 (1A), grade 6 and grade 7 contain tasks that require the learners to fill in the missing number, arrange numbers in ascending and descending order, and write numbers in words and figures. These types of tasks encourage rote learning as they involve learners to reproduce rules and definitions and, hence, obscuring learners from creative thinking.

The findings also reveal that some tasks in the textbooks do not depict the object of learning. That is, the capabilities in these tasks are different from the object of learning stated at the beginning of the unit/topic. For example, in grade 6, unit 8 is for "Addition and Subtraction of fractions", but exercise 8A to 8E is about types of fraction and how to change these fractions from one type to another. This type of selection and sequencing of tasks may disturb the learners.

The extent to which tasks allow learners to apply and connect critical features of the mathematical content.

The analysis reveals that there are few higher-level tasks found in the textbooks. This implies that few tasks allow learners to apply and connect critical features of the mathematical content. Looking at the level 3 tasks analyzed in the textbooks on number concepts and operations, in grade 5, three out of forty-eight tasks were coded as level 3, in grade 6, one out of forty-nine tasks was coded level 3, in grade 7, seven out of fifty-four tasks were coded level 3 and in grade 8, five out of thirty-three tasks were coded level 3 (16 tasks out of 184 tasks in 4 textbooks). The investigator noticed that the textbooks lack multiple representations of concepts on different forms such as diagrams, games, graphs and tables, verbal instructions, and modeling using objects and number lines and many more. These representations help learners acquire conceptual understanding of mathematical content. This assists learners to see connections among concepts and procedures and give arguments for them to explain. Kilpatrick, et al., (2001, p. 119) argue that, "[a] significant indicator of conceptual understanding is being able to represent mathematical situations in different ways and knowing how different representations can be useful for different purposes". That is, in order to trade or maneuver effectively around the mathematical terrain, it is essential for the learners to observe how various representations connect with each other, according to their similarities and their differences and hence, externalize their understanding of situations. The research by Chin and Zakaria (2015) indicates that games help learners to develop addition and subtraction skills in number operations, and that learning through play is one of the approaches that could nurture interest in mathematics learning among children.

This indicates that there are lower-level tasks in the textbooks that place little demand on learners' thinking and explanations. The learners use the formulas to complete the exercises quickly and accurately without showing understanding of how the formula works. For example, use of formulae (in grade 7, on the concept of rate, ratios and proportions). This means that the algorithms/procedures to be applied are given and learners do not have to identify suitable mathematics procedures to solve the tasks and hence, they are not getting enough experience to develop their own ability to transform problems with real-world context (level 3) tasks into a mathematical problem. Thus, primary school textbooks in the upper classes in Malawi encourage learners to acquire routine expertise. This lack of experience with higher-level cognitive demand tasks explains the reason why learners in primary school consistently perform very poorly on primary mathematics especially in number concept and operations (SACMEQ, 2011).

6.2 Further discussion

Textbook examples and exercises are a very important source of textbook tasks because they are used by learners in the classrooms or homework and they influence the understanding of mathematics concepts. Studies by Gracin (2018) and Vincent and Stacey (2008) say that mathematics is learned through different examples and tasks. As a result, textbooks should provide rich tasks (examples and exercises) that will engage and challenge learners to reason and provide opportunity for developing understanding. The results from the analysis of examples and tasks reveal that textbooks in upper primary classes in Malawi lack variation of the levels of examples and tasks in all the four textbooks. The textbooks offer only one pattern of variation of the aspects of object of learning in most cases for both examples and tasks. Vincent and Stacey (2008) in their study explained that all learners should be presented with a balance of curriculum experiences that will expose them to a variety of types of problems. Kullberg et al. (2017) agree that when different types of examples or practice exercises are mixed, the learners are forced to distinguish between them and thus get better at making sense of novel tasks and examples. Several studies support the variation of the levels of examples and tasks that should be included in the textbooks in order to draw attention to critical features of mathematical content (object of learning) (e.g., Bills et al., 2006; Kullberg et al., 2017; Zhu & Fan, 2006). This shows that availability of high-level examples and high-level cognitive demanding tasks in mathematics textbooks will result in instructions that promote learners' understanding of number concepts and operations development. However, the finding on the variation of the levels of examples and levels of tasks revealed in this study seem to disagree with what Watson and Mason (2006) and other studies (indicated above) claim that, examples and tasks that carefully show constrained variation are likely to result in progress and that good variations of sets of examples can make certain critical aspects of object of learning noticeable by learners and hence enhance learning.

The results from the findings indicate that tasks on number concepts and operations in all the textbooks are used to a great extent by learners as the source of examples and practice exercises. The studies conducted by several researchers (e.g., Baroody, 2001; Fuson, 2003; Reuben, 2009) found that number concepts and operations are considered as prerequisites for learning of school mathematics beyond literacy level. And that learners acquire number concepts and operations through different representations such as modeling using objects or number lines, games, drawing, graphs, tables, and others. The aim is to provide learners with a balanced variety of tasks that will challenge them and build their understanding. However, the study found that textbooks in upper primary classes in Malawi offer low-level examples and low-level cognitively demanding tasks in number concepts and operations which limit learners' performance in mathematics. This means that learners are not provided with enough opportunities that could allow them access fully the critical features of mathematical content in number concepts and operations; and hence they fail to achieve the learning outcomes expected of them by the curriculum. The textbooks should provide many opportunities for making connections between learners' emerging understanding of number concepts and operations and the structure of other representations. This finding is in line with what (Vincent & Stacey, 2008 and O'Keeffe and O'Donoghue, 2011) found in their studies that indicated lower cognitively demanding tasks.

Number concepts and operations are the foundation of other mathematical concepts in primary mathematics in Malawi (Mulera et al., 2017; Malemya, 2018; Kasoka, Jakobsen & Kazima, 2017), and the core element constitutes more topics compared to other core elements in all textbooks in primary school in Malawi. This implies that number concepts and operations play a significant part in the development of mathematics in learners in primary school. Studies in number concepts and operations emphasize the importance of good early mathematics experience for children, which focus on understanding numbers, developing meanings of operations and computing fluently (Fuson, 2003; NCTM, 2009; Reuben, 2009). This understanding of numbers allows computational procedures to be learned and recalled with ease. The researchers further explained that students learn number concepts through different representations with understanding, actively building new knowledge from experience and previous knowledge. The learners move from counting, single-digit number operations to the mastery stage where they work with greater computation, modeling, representation, interpretation, and problem-solving competence. Fuson (2003, p. 71) agrees that, "[t]he understanding of computation and integration of methods and practice with both, lead to computational fluency". Computational fluency will influence conceptual understanding in the learning of mathematics. By aligning factual knowledge and procedural proficiency with conceptual knowledge, learners will become more effective. However, for learners to be exposed to great computational fluency, the study conducted by Aineamen and Naicker (2014) discovered that, the textbook should probe learners for higher mathematical reasoning by asking questions that require them to give more explanation and good justifications for their responses in learning of mathematics. That is, there is a need for variation of examples in the textbook, which can help the learners to discern different critical features of the object of learning at the same time and this requires including more than one aspect in an example (Marton et al., 2004). The findings in this study is in contrast with what Aineamen and Naicker (2014) discovered because most of the examples in all the four mathematics textbooks in the upper primary classes in Malawi provide low-level examples in number concepts and operations, with one or two patterns of variations of aspects of object of learning that allow learners to notice similarities or differences or the combination of both patterns in order to generalize.

The finding on the examples is also in contrast with the core idea of variation theory which emphasizes discernment as the necessary condition for learning (Rullberg et al., 2017). That is, for learners to differentiate and focus on critical aspects (or dimensions of variation) of the object of learning, they have to experience variations in those aspects for them to learn. Marton (2015) points out that variation is the necessary component in the teaching in order for learners to notice what is to be learned. Textbooks in upper primary in Malawi give learners few opportunities to grasp the mathematics context in case of generalization, contrast, and fusion. Instead, the textbooks provide opportunities to notice generalization in most of the examples. Thus, the examples that help learners to notice similarities because the examples have the same structure and procedures (for example, 864975 – 221563; and 158265 – 47 144). The textbooks use multiple examples of the same type in the example space instead of mixed examples which facilitate learning much more. Marton and Pang (2013), in support of variation theory, argue against this view of developing new meaning from the same experience. Few level 3 examples in the textbooks indicate that learners are denied the opportunity to understand the object of learning as a whole and hence fail to simultaneously discern the critical aspects and relationships of number concepts and operations.

The results on worked examples also reveal that textbooks in upper primary in Malawi mostly use one worked example per example space and with no explanations. Worked examples are vital since they are problems presented with explained solutions and hence, they assist learners to see how certain examples are solved and how to direct their attention. In most cases, the worked example in the textbooks illustrate only one procedure or algorithm of solving it and there are no explanations or instructions to guide the learners on how to carry out the operations. Bills et al. (2006) argue that the worked examples become insufficient when they are provided with no further explanations or other conceptual support. They explain that learners fail to use such types of worked examples as they see the examples as not applicable to them when solving tasks and problems that require a little deviation from the solution presented in worked examples. Therefore, it is important to specify in a worked example the steps that were taken and the reasons for taking them. This finding seem to agree with the findings of Chi et al. (1989) and Renkl (2002) who on their studies emphasize the importance of learners' self-explanation of the worked-out example, and also with the work of Eley and Cameron (1993) who from their results found that learners considered an explanation to be better if it included the 'trigger' for each step. Worked-out examples that encourage explanation and reasoning may enhance learners' learning, and, in particular, their problem-solving performance.

The MDITx framework, which is guiding this study, regards worked examples as part of the examples in the example space and the examples are one of its components that provide opportunity for learners to participate in mathematical discourse in instruction. Therefore, the finding corresponds with the MDITx framework.

The findings on the cognitive level of tasks found in the four textbooks indicate that the textbooks have few high-level tasks (16 tasks out of 184) on number concepts and operations that provide opportunities for learners to engage in doing mathematics (Stein et al., 2009) and in reasoning and justifying in a coherent manner (Stylianides, 2009). That is, learners are rarely engaged in procedures with connections to concepts and meaning and doing mathematics. The categories of most tasks in the textbooks are memorization and procedure without connection to concepts or meaning. Learners' book for standard 8, for example, has the following examples: "**1**. Find the results of dividing the sum of 12.9 and 5.6 by 3.7; **2**. Divide the sum of 5.4 and 0.96 by 1.6) (MIE, 2009, p. 20). This shows that the learners will solve the exercise using routine procedure thereby starting with addition and then division following the order of operation. This finding is in line with the results that Stols (2013) found in his study of opportunity to learn that was available to Grade 12 mathematics learners in Gauteng school in South Africa. The researcher found that the lessons provided to learners lacked tasks with higher-order questions and showed that learners were rarely engaged in problem-solving activities. Therefore, lack of opportunities for learners to work on more demanding activities, resulted in low performance. This study confirmed the idea of Webb (2010) that learners need to engage in and do exercises on a range of levels of cognitive demand in order to improve learner performance. The finding of this study also corresponds with other studies indicating that there is a predominance of tasks with lower-level cognitive demand in textbooks, with very few reflective tasks to make it possible for learners to develop their ability in complex reasoning (e.g., Vincent & Stacey, 2008; Dole & Shield, 2008; O'Keeffe & O'Donoghue, 2011; Wajiya et al., 2015; Hadar, 2017; Gracin, 2018). This suggests that textbooks in primary school in Malawi contain tasks that mainly require lower levels of cognitive demand that may not support student learning since learners are rarely asked to struggle with difficult situations. Including more reflection tasks is essential because they stimulate mathematical thinking and reasoning related to authentic settings.

The predominance of tasks with lower level cognitive demand in upper primary school classes in Malawi can also be seen through the trends of performance levels of grade 6 learners as indicated in a policy brief report (World Bank, 2010). The report revealed that most of the learners failed to possess skills beyond basic numeracy (level3) in mathematics and few learners in grade 6 had skills beyond competence numeracy (level 5) in number concepts and operations. The results indicated low performance levels in mathematics as seen in table 7. This report is also in line with the finding of this study. However, the low-level achievement was also contributed by other factors that were being experienced by the learners at that time such as large classes, unqualified teachers, lack of resources and many more.

Table 7: shows percentage of Students Reaching Mathematics Competency Levels in Malawi (SACMEQ II)

Levels	Competence/skills	2007	2000
Level 1	Pre-numeracy	8.8%	12.4%
Level 2	Emergency Numeracy	51.3%	61.9%
Level 3	Basic numeracy	31.8%	23.5%
Level 4	Beginning numeracy	6.6%	2. 1%

Level 5	Competent numeracy	1.3%	0.2%
Level 6	Mathematical skills	0.4%	0%
Level 7	Concrete problem-solving	0.0%	0%
Level 8	Abstract Problem-solving	0.0%	0%

***Source*: SACMEQ I, II report 2010**

Column 1 indicates the levels, column 2 shows skills, column 3 and four shows years.

The findings from curriculum materials indicate that textbooks in upper primary in Malawi are teacher-centred. There are only the object of learning, worked examples and exercises in the learners books. The topics are introduced in the teachers' guide. The instructions on how the learners are going to do the tasks are written in the teachers' guides instead of the learners' book. The teachers' guide is provided with several worked-out examples on every example space. The worked examples have sufficient explanations and conceptual support, which are essential for their interactions with the given task. There are good instructions for every activity that the learners are going to do. The activities have some illustrations and different representations. The study found that the textbooks limit the learners' opportunity to use their textbooks effectively as they lack important information that could guide them interact with the practice exercise and be able to mediate the object of learning. Thus, the textbooks create few opportunities for learners to participate in mathematical discourse and deny learners to fully participate in and with the examples/tasks made available in the mathematical discourse (Ronda & Adler, 2017). This shows that the teacher has autonomy in the learning of the learners and learners are not given autonomy to construct their own knowledge in order to learn because the books hinder them to interact with multiple worked examples, illustrations, resources and other representations. As a result, the knowledge that the learners achieve when using these textbooks is low-level knowledge. The MDITx framework that supports this study emphasizes learning that support the acquisition of the object of learning through participation in and with the tools made available in the mathematical discourse. The finding is similar to what Malemya (2018) noticed in Malawian textbooks when he analyzed them. Malemya noticed that the textbooks are well documented and illustrated except for having a greater teacher centered approach which consequently doesn't prepare learners to be more independent to face life challenges on their own.

To summarize the discussion in these two sections, the content areas in the textbooks cover the objectives of the curriculum on number concepts and operations, and with enough content. Thus, confirming that one of the affordances of the textbook to the learner's practice is to enact the implemented curriculum (Valverde et al., 2002). However, the study reveals that, the textbooks provided (create) limited opportunities for learners to participate in mathematical discourse. The textbooks offer low-level thinking examples with one or two patterns of the dimensions of variation and low-level cognitively demanding tasks which help the learners to recall and use routine procedures. The textbooks offer one worked example in most example spaces and without instructions, explanations and illustrated representations which hinder the learners to experience fully the affords which are offered in the books. The textbooks are also teacher-centred which is in contrary to the learning outcomes of the Malawian curriculum, which emphasizes on learner-centredness of the textbooks. However, employing the MDITX framework which guided this study, the investigator was able to analyze the textbooks in order to find out the opportunities that are afforded for learners to access object of learning provided in the textbooks in upper primary classes using examples and tasks.

6.3 Chapter summary

This chapter has presented and discussed the findings of the study that have revealed how textbooks provide opportunities for learning mathematics on number concepts and operations in relation to the MDITx framework that advised this study. The findings of this study have led to the major conclusion that mathematics textbooks in upper primary classes have few high-level examples that could assist learners to discern simultaneous dimensions of variation of more than one aspect of the object of learning and connect with similarity and contrast within the example set. And also that the textbooks offer few high-level cognitively demanding tasks that provide limited opportunities to use multiple concepts and make multiple connections in order to solve problems in different ways, representations, pose problems, prove and justify the problem with reasoning. This means that the textbooks in upper primary classes in Malawi offer low cognitively demanding tasks and hence offer few opportunities for learners to achieve learning outcomes on number concepts and operations. The textbooks also lack multiple representations of concepts and procedures. The next and final chapter presents a conclusion of the study.

Chapter 7: CONCLUSION

This chapter presents the conclusion of the study. It starts by looking at the summary of the findings that are followed by implications of the study. The other chapter of the study outlines the limitations of the study and suggests areas of further research

7.1 Introduction

This study aimed at investigating opportunities for learning number concepts and operations that are provided in the upper primary curriculum materials in Malawi. In order to investigate the opportunity to learn number concepts and operations, the study used the following questions: What opportunities to learn number concepts and operations are provided through the examples and tasks in the textbooks? How do the tasks enable enactment of the learning objects that are stated in the textbooks? And to what extent do tasks allow learners to apply and connect critical features of the mathematical content?

7.2 Opportunity to learn number concepts and operation as provided through examples and tasks

The study revealed that although mathematics curriculum materials in upper primary classes are well documented and they contain enough tasks in number concepts and operations, learners' textbooks show the dominance of low-level examples and low-level cognitive demanding tasks, which is due to lack of variations of aspects of critical features of object of learning. The textbooks do not provide tasks (examples and exercise) that engage learners and challenge them to reason, as well as influence the quality of instruction and provide opportunities for developing understanding. Most of the tasks are recall types of tasks that influence use of procedures without connections to other concepts or algorithms (tasks require direct application of basic knowledge and computation skills). The tasks are tasks without context, as a result they depend on the use of symbols or formulas for learners to solve them. And most of the examples in the textbooks do not promote high-level mathematical thinking in learners which would help them to engage with various thinking patterns simultaneously. From the finding of the study, it can be concluded that the textbooks in upper primary classes provide low-level examples and low-level cognitive demand tasks. The result also agrees with what Malemya (2019) said in his comparative study of textbooks. He noted that textbooks in primary school in Malawi are of low-level cognitive demand that the learners fail even to grasp the most basic concepts in an appropriate grade in time on number concepts and operations. And that most of the tasks do not have context or in most cases, examples are imaginary.

From the curriculum materials that are used for teaching and learning in the upper primary classes in Malawi, it can be concluded that the textbooks limit the learners the opportunity to use their textbooks effectively on their own because they lack important information that could guide them interact with the examples and tasks, and be able to discern the critical aspects of object of learning. The textbooks encourage learners to depend on the teachers to provide instructions and explanations as the learners' books contain only worked examples and tasks especially grades 5–7. Textbook for grade 8 is different from the other classes as it has introductory explanations of each topic or section. Rezat, (2009) explain that learners do not use the mathematics textbooks when they are instructed by their teachers, but they use textbooks because they are self-directed. Learners use mathematics textbooks in order to look for information that can be directly applied to solve tasks and problems, acquire mathematical knowledge and for consolidation (using the summary at the end of the topic). In particular, learners expect to find useful information related to a topic (concept) at the beginning of the lesson in the textbooks. However, this is not applied in the learners' books that are present in Malawian primary schools.

The finding from worked examples presented in the textbooks indicate that most of the example spaces contain only one worked example per example space. As a result, the textbooks do not provide good ground for learners to use worked examples when solving exercises as one worked example fails to provide opportunity for learners to discern variation across the concepts or procedures within the examples. And worked examples lack sufficient instructions and explanations of the procedures to be followed. Learners utilize the worked examples in the textbook in order to get assistance with solving tasks and examples thereby following the algorithms employed, step-by-step. It can be concluded that textbooks in upper primary classes lack mixed worked examples that are different types, which will facilitate learning and allow learners to notice variations in terms of similarities and differences across the examples in the block.

Enactment of the learning object through tasks.

The finding on how the tasks enable enactment of the learning objects that are stated in the textbooks indicate that the tasks enact the learning outcomes that are provided in each topic of every book but lack application tasks that could enable learners to discern multiple concepts and make connections. The textbooks lack tasks that consist of drawings, games, presentation and modelling, interpretation, argumentation, and reasoning. Therefore, from the finding, it can be concluded that the textbooks lack rich tasks that can assist learners to develop a deep level of understanding mathematical concepts and ideas that require high level thinking with considerable cognitive effort.

The extent to which tasks allow learners to apply and connect critical features of the mathematical content.

The analysis of the tasks on the extent to which they allow learners to apply and connect features of mathematics content revealed that, textbooks in upper primary classes in Malawi provide very few tasks that help learners to apply and make connections of multiple concepts of mathematical content. Rarely did the author make connections of multiple concepts and ideas in their textbook lesson to learners' everyday life. Out of one hundred and eighty-four example spaces present in all textbooks, only sixteen tasks were coded level 3 tasks (AMC). This indicates that the textbooks offer few high-level cognitive demand tasks. It can be concluded then that the textbooks provide few opportunities for learners to carry out tasks that involve application of multiple concepts and make connections across the examples within the mathematical content in number concepts and operations. It can be argued that connecting mathematics to learners' everyday life helps to make learning more meaningful to learners. As they connect their textbook mathematics to real life situations, learners' attitude towards mathematics can change for the better and they may develop interest in mathematics because they know that they will need it even after they leave school.

7.3 Implications for practice

This study was conducted only on the first core element that forms the foundation for other concepts of mathematics, but it raises essential issues related to how textbook lessons can create opportunities for learners to participate in mathematical discourse. Textbooks are very important as they act as major conveyors of the curricula in mathematics education. Textbooks tie the intended curriculum and the implemented curriculum together (Schmidt et al., 2001). If the textbooks fail to implement the intended curriculum, it means that the textbooks are denying learners the opportunities to participate in meaningful learning of mathematics. Examples and tasks are the core components of the textbook lesson that help the textbooks to implement the intended curriculum to the learners thereby providing opportunities for learners to discern dimensions of variations of critical features of mathematical content. This implies that textbooks should incorporate high-level thinking examples and high-level cognitive demanding tasks in learners' books in upper primary classes in Malawi so as to improve teaching and learning. Engaging learners in high-order questions and high cognitive demanding tasks could improve learners' mathematical proficiency and achievement in mathematics.

It has been established that learners' books in upper primary classes do not represent mathematical concepts or ideas in multiple modes. That is, the textbooks do not have the ability to represent mathematical situations in different ways. According to literature review in chapter two, learners develop the concept of numbers and operations through different representations (modeling mathematics) such as manipulatives, pictures, games, tables, graphs, real-life context, verbal symbols, and written symbols. These multiple modes of representations provide opportunities for learners to make more meaningful connections of mathematical concepts and ideas when solving tasks and problems. This implies the importance of incorporating tasks with presentation and modeling, interpretation, argumentation, and reasoning competences in learners' textbooks to enable learners to externalize their understanding of situations in everyday life.

It has also been established that learners' textbooks do not have sufficient information that could provide opportunity for learners to mediate the mathematics made available in the textbooks with deep understanding of mathematical content. The useful information that is required by learners is written in the teachers' guide. This provides limited opportunity for learners to access important information that could assist them when solving tasks and problems or acquiring mathematical knowledge or for consolidation. Consequently, this promotes dependence on the teachers. The author needs to include introductory explanations, definitions, and other relevant information that could assist learners to interact with the textbooks independently, at school or at home.

7.4 Recommendations

Based on the findings, it can be recommended that different mathematics tasks presented in the textbooks should engage learners in challenging tasks that will help them to reason and develop their deeper level of understanding of mathematical concepts and ideas. Learners' textbooks should include more high-level cognitive demanding tasks (AMC) that will help learners to apply multiple concepts and make connections across examples. Moreover, the tasks should be reflective so as to stimulate learners' thinking and reasoning related to authentic settings. Nickson, (2002, p. 233) argues that, "[i]n order to mathematize, children need to experience mathematics in a context other than a purely mathematical one". In addition, learners' textbooks should have enough introductory explanations on concepts and procedures (should be learner-centred). This will assist learners when looking for information that can be directly applied to solve tasks and problems, acquire mathematical knowledge and for consolidation (using the summary at the end of the topic). Further, development of number concepts and operations should be done through representations such as diagrams, games, tables and graphs, presentation and modeling. These multiple modes of representations provide opportunities for learners to make more meaningful connections of mathematical concepts and ideas when solving tasks and problems. Furthermore, most of the application tasks in the textbook are fiction; they depict the author's thoughts, not real-life situations. Since mathematics ideas are abstract, for learners to understand these abstract ideas, they need to be demonstrated by applying the concepts to real life situations using problem solving skills. Lastly, the textbooks should include more context-based tasks. The learners' book in primary classes in Malawi should provide tasks which are presented within a situation that can refer to a real-life setting (relevant and essential context).

7.5 Implication for further research

In Malawi, textbooks have a central role in mathematics classes and teachers are the main mediators of textbooks to learners for them to use the textbooks in the lesson or at home. As a result, there is a need to investigate how teachers use the textbooks during preparation of lessons as well as how they use them in the classrooms. Another focus for future study could be on finding out the connections between opportunities provided in the textbooks and learners achievement. Furthermore, since the textbooks also lack multiple modes of representations that help learners to apply and make multiple connections across examples or concepts, there is a need to find out teachers' perceptions of mathematics and understanding of presentations and modeling of tasks. Lastly, the other study can also investigate the influence of other curriculum materials such as syllabus, teachers' guides, or teaching/learning resources on learners' performance.

7.6 Limitations of this study

The findings from this study are only focused on number concepts and operations as the basic core element in all textbooks. However, it would be necessary to conduct a further study on all the topics of the textbooks in order to get a strong evidence of the opportunities provided in upper primary classes. In addition, mastery of number concepts and operations is the basis of mastery of number competences and further mathematical systems (Chin & Zakaria, 2015). Therefore, it is important to conduct a similar textbook analysis study in lower grades so as to establish the extent to which opportunities to learn number concepts and operations are provided to the learners.

REFERENCES

Abadzi, H. (2007). Absenteeism and beyond: Instructional time loss and consequences. *World Bank Policy Research Working Paper No. 4376*. World Bank, Washington, DC: World Bank. Retrieved from http://openknowledge.bankband.org/handle/10986/7569

Adler, J., & Ronda, E. (2016). Mathematics discourse in instruction matters. In J. Adler & A. Sfard (Eds.), *Research for educational change: Transforming researchers' insights into Improvement in mathematics teaching and learning* (pp. 64–81). Abingdon, England: Routledge.

Adler, J., & Ronda, E. (2015). A Framework for Describing Mathematics Discourse in Instruction and Interpreting Differences in Teaching. *African Journal of Research in Mathematics, Science and Technology Education, 19*(3), 237–254. https//doi.org/10.1080/10288457.2015.1089677

Aineamani, B., & Naicker, S. (2014). Mathematics textbook analysis: *A guide to choosing the appropriate mathematics textbook*. The Association for Mathematics of South Africa-AMESA 7(2) 460–485. Retrieved from: http//www. Amesa.org.za

Apple, M. W. (1986). *Teachers and Texts: A political economy of class and gender relations in Education*. New York, NY: Routledge & Kegan.

Ary, D., Jacobs, L. C., Sorensen, C. K., Walker, D., (2014). *Introduction to Research In Education,* (9th Ed.). Australia: WADWORTH: CENGAGE Learning.

Aunola, K., Leskinen, E., Lerkkanen, M., & Nurmi, J. (2004). Developmental dynamics of mathethatics performance from preschool to grade 2. *Journal of Educational Psychology*, 96(4), 699–713.

Banicky, L. (2000*)*. Opportunity to learn. *Education Policy Brief, 7*. Retrieved from http://udspace.udel.edu/bitstream/handle/19716/2446/opp+to+learn.pdf?sequence=1

Baroody, A. J., Lai, M., & Mix, K. S. (2006). The Development of Young Children's Early Number and Operation Sense and its Implications for Early Childhood Education. In B. Spodek & O. N. Saracho (Eds.), *Hqndbook of research on the teaching of young children* (Vol. 2, pp. 187–221). London, UK: Routledge.

Baroody, A. J. (2004). The Developmental Bases for Early Childhood Number and Operations Standards. In D. H. Clements, J. Sarama, & A.-M. DiBiase (Eds.), *Engaging young children in mathematics standards for early childhood mathematics education* (pp. 173–219). Mahwah, NJ: Lawrence Erlbaum Associates.

Bass, H. (2015, June). Quantities, numbers, number names and the real number line. Paper presented at the ICMI Study 23 Conference, Macau, China. Retrieved from https://www.um.edu.mo/fed/ICMI23/PPT/HB.pdf

Bhattacherjee, A. (2012). *Social Science Research: Principles, Methods and Practices* (2nd ed.). Scholar Commons. Retrieved from https://scholarcommons.usf.edu/cgi/viewcontent.cgi?article=1002&context=oa_textbooks

Bills L., Dreyfus, T., Mason, J., Tsamir, P., Watson, A., & Zaslavsky, O. (2006). Exemplification in mathematics education. In J. Novotna (Ed.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (Vol 1., pp. 126–154). Prague, Czech Republic: PME.

Brändström, A. (2005). Differentiated tasks in mathematics textbooks: analysis of the levels of Difficulty (master's thesis). Luleå: Luleå University of Technology.

Bransford, J. D., Brown, A. L., & Cocking, R. R. (eds.), (2000). How People Learn: Brain, mind, experience and school. Washington, DC: National Academy Press.

Carroll, J., B. (1963). A model of school learning. *Teachers College Record, 64*, 723–733.

Chang, C. C., & Silalahi, S. M. (2017). A review and content analysis of mathematics textbooks in educational research. *Problems of education in the 21st century*, *75*(3), 235–251.

Chi, T. H., Bassok, M. W., Lewis, P., Reiman, P. & Glaser, R. (1989). Self-explanation: How students study and use examples in learning to solve problems. *Cognitive Science, 13*, 145–182.

Chin, L. C., & Zakaria, E. (2015). Understanding of number concepts and number operations through games in early mathematics education. *Creative Education 6*, 1306–1315. https://doi.org/10.4236/ce.2015.612130

Creswell, J. W. (2009). *Research Design: Qualitative, Quantitative and Mixed Methods Approaches*. London, UK: Sage.

Creswell, J. W. (2012). *Education Research: Planning, Conducting, and Evaluating Qualitative and Quantitative research (4th ed.).* Place: Pearson Education.

Davis B. R. (1984). Learning mathematics: The cognitive scientific approach to mathematics education. Bechenham: Croom Ltd.

Dole, S., & Shield, M. J. (2008). The capacity of two Australian eighth-grade textbooks for promoting proportional reasoning. *Research in Mathematics Education, 10*(1), 19–35.

Eley, M., & Cameron, N. (1993). Proficiency in explanation of procedures: A test of the intuitive Understanding of teachers of undergraduate mathematics. *Higher Education*, *26*, 355– 386.

Eliya, L. E. (2016). Exploring children's learning of numbers in lower classes in Malawian primary school. Master thesis, University of Malawi, Chancellor College.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. *ZDM*, *45*(5), 633–646. https://doi.org/10.1007/s11858-013-0539-x

Floden, R. E. (2002). The measurement of opportunity to learn. In A. C. Porter & A. Gamoran (Eds.), *Methodological Advances in Cross-National Surveys of Educational Achievement* (pp. 231–266). Washington, DC: National Academy Press.

Freeman, D. J., & Porter, A. C. (1989). Do textbooks dictate the content of mathematics instruction in elementary schools? *American Educational Research Journal*, *26*(3), 403–421.

Fuson, K. C. (2003). Developing mathematical power in whole number operations. In J. Kilpatrick, W. G. Martin, & D. Schifter (Eds.), *A research companion to principles and standards for school mathematics* (pp. 68–94). Reston, VA: National Council of Teachers of Mathematics.

Gall,M. D., Gall, J. P., & Borg, W. R., (2007). *Educational Research: An Introduction* (8th Ed.). USA: Pearson

Gelman, R., & Gallistel, C. R. (1978). *The child's understanding of number*. Cambridge, MA: Harvard University Press.

Gentner, D. (2005). The development of relational category knowledge. In D. H. Rakison & Gershkoff-Stowe (Eds.), *Building object categories in developmental time* (pp. 245–275).

Ginsburg, H. P. (1977). *Children's arithmetic.* New York, NY: D. Van Nostrand Co.

Glasnovic Gracin, D. (2018). Requirements in mathematics textbooks: A five-dimensional analysis of textbook exercises and examples. *International Journal of Mathematical Education in Science and Technology*, *49*(7), 1003–1024, DOI:10.1080/0020739x.

Grant, C., & Osanloo, A. (2014). Understanding, selecting, and integrating a theoretical framework in dissertation research: Creating the blueprint for your "house". *Administrative Issues Journal*, *4*(2), 12–26.

Guberman, S. R. (1996). The development of everyday mathematics in Brazilian children with limited formal education. *Child Development*, *67*, 1609–1623.

Hadar, L. L. (2017). Opportunities to learn: Mathematics textbooks and students' achievements. *Studies in Educational Evaluation*, *55*, 153–166. https://doi.org/10.1016/j.stueduc.2017.10.002

Hong, D. S., & Choi, K. M. (2018). A comparative analysis of linear functions in Korean and American standards-based secondary textbooks. *International Journal of Mathematical Education in Science and Technology*, *49*(7), 1025–1051.

Houang, R. T., & Schmidt, W. H. (2008). TIMSS international curriculum analysis and measuring educational opportunities. *The 3rd IEA International Research Conference (IRC 2008)*.

Johnsson, M. (2006). textbooks as instruments: three teachers' way to organize their mathematics lessons. *Nordic Studies in Mathematics Education, 11*(3), 5–30.

Jordan, N. C., Kaplan, D., Ramineni, C., & Locuniak, M. N. (2009). Early Math Matters: Kindergarten Number Competence and Later Mathematics Outcomes. *Developmental Psychology*, *45*, 850–867. http://dx.doi.org/10.1037/a0014939

International Mathematical Union. (2014). *Mathematics in Africa: Challenges and Opportunities report*. Commission for Developing Countries, International Mathematical Union.

Isoda, M., & Katagiri, S. (2012). Mathematical Thinking: How to Develop it in the Classroom. World Scientific.

Kasoka, D., Jakobsen, A. & Kazima, M. (2017). Pre-service primary school teachers' mathematical knowledge for teaching; Does secondary mathematics ability count? In M. K. Mhlolo, S. N. Matoti & B. Fredricks (Eds.), *Proceedings of the 25th Southern African Association for Research in Mathematics, Science and Technology Education* (pp. 77–87). Free State, South Africa: Central University of Technology.

Kilpatrick, J., Swafford, J., & Findell, B. (Eds). (2001). *Adding it up: Helping children learn mathematics.* Washington, DC: National Academy Press.

Kullberg, A., Kempe, U. R., & Marton, F. (2017). What is made possible to learn when using the variation theory of learning in teaching mathematics? *ZDM*, *49*(4), 559–569.

Leshota, M. S. (2015). The Relationship between Textbooks Affordances and Mathematics Teachers' Pedagogical Design Capacity (PDC). *PhD thesis*. South Africa, University of Witwatersrand, Faculty of Humanities.

Li, Y., Zhang, J., & Ma, T. (2009). Approaches and practices in developing school mathematics textbooks in China. *ZDM–The International Journal on Mathematics Education*, *41*, 733–748.

Lo, M. L. (2012). *Variation theory and the improvement of teaching and learning*. Göteborgs, Sweden: Acta Universitatis Gothoburgensis.

Locuniak, M. N., & Jordan, N. C. (2008). Using Kindergarten Number Sense to Predict Calculation Fluency in Second Grade. *Journal of Learning Disabilities*, *41*, 451–459. http://dx.doi.org/10.1177/0022219408321126

Malawi National Examination Board. (2016). Primary School Leaving Certificate Examinations: Mathematics Chief Examiners' Report. Zomba.

Malawi Institute of Education. (2007). *Mathematics: Teachers' guide for standard 5*. Malawi Institute of Education.

Malawi Institute of Education. (2008). *Mathematics: Teachers' guide for standard 7*. Malawi: Malawi Institute of Education.

Malawi Institute of Education. (2009). *Mathematics: Teachers' guide for standard 7*. Malawi: Malawi Institute of Education.

Malemya, P. S. (2019). A Comparative Analysis of Malawi and Japanese Mathematics Textbooks. *NUE Journal of International Educational Cooperation, 12*, 169–182.

Marton, F. (2015). *Necessary conditions of learning*. New York, NY: Routledge.

Marton, F., & Pang, M. F. (2006). On some necessary conditions of learning. *The Journal of the Learning Sciences*, *15*(2), 193–220. DO-10.1207/s1532780jls1502_2.

Marton, F., & Pang, M. F. (2013). Meanings are acquired from experiencing differences against a background of sameness, rather than from experiencing sameness against a background of difference: Putting a conjecture to test by embedding it into a pedagogical tool. *Frontline learning research 1*(1), 24–41. https://doi.org/10.14786/flr.v1i1.16

Marton, F., Runesson, U., & Tsui, B. M. (2004). The space of learning. In F. Marton & A. B. Tsui (Eds.), *Classroom discourse and the space of learning* (pp. 3–40). Mahwah, NJ: Lawrence.

McDonnell, L. M. (1995). Opportunity to learn as a research concept and a policy instrument. *Educational Evaluation and Policy Analysis*, *17*(3), 305–322. https://doi.org/10.2307/1164509

McKenzie, F. (2001). Developing children's communication skills to aid mathematical understanding, *ACE papers (Student Edition), Issue 11*, 7–16.

Mikk, J. (2000). *Textbook: research and writing*. Frankfurt am Main: Peter Lang.

Ministry of Education, Science and Technology (2011). *Education Statistics 2011*. Lilongwe, Malawi: Department of Education Planning.

Ministry of Education, Science and Technology (2008). National Education Sector plan 2008–2017; Operation Supplement. Lilongwe.

Mulera, D. Mc W. J., Ndala, K. k., & Nyirongo, R. (2017). Analysis of factors affecting pupil performance in Malawi's primary schools based on SACMEQ Survey results. *International Journal of Educational Development, 54* (C) 59–68.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston VA: NCTM.

National Council of Teachers of Mathematics (2014). *Principle to Actions: Ensuring Mathematical Success for all*. Reston, VA: NCTM. 1502 (703) 620–9840(800), 253–7566; www.nctm.org

Nickson, M. (2000). *Teaching and Learning Mathematics: A Teacher's Guide to Recent Research*. UK: Cassell.

O'Keeffe L., & O'Donoghue, J. (2011). The use of evidence-based research on mathematics Textbooks to increase student conceptual understanding. *International Journal for Cross Discipline Subjects in Education, 2*(1), 304–311.

Olteanu, C. (2018). Designing examples to create variation patterns in teaching. *International Journal of Mathematical Education in Science and Technology, 49*(8), 1219–1234.

https://doi.org/116.DOI:10.1080/0020739x.2018.1447151

Pang, M. (2008). Using the learning study grounded on the variation theory to improve students' mathematical understanding. *The 11th International Congress on Mathematical Education, Mexico.* http:hdl.handle.net/10722/63077

Payne, J. N., & Huinker, D. M. (1993). In R. J. Jensen (Ed.), *Research ideas for the classroom:Early childhood mathematics.* New York (NY): Macmillan Publishing Company.

Peled, I., & Zaslavsky, O (1997). Counter-examples that (only) and counter-examples that (also) explain. *FOCUS on Learning Problems in mathematics, 19*(3), 49–61.

Pepin, B., & Haggarty, L. (2001). Mathematics textbooks and their use in English, French and German classroom. *ZDM - International Journal on Mathematics Education, 33(5)* 158–175. DOI: 10.1007/BF02656616

Phiri L. (2011). Establishing Mathematical Proficiency of Standard 8 Learners in Fractions: A Case Study of Two Rural Primary Schools in Zomba. *Master thesis*. University of Malawi, Chancellor College.

Reimann P., & Schult, T. (1996). Turning examples into cases: Acquiring knowledge structures for analogical problem-solving. *Educational Psychologist, 31*(2), 123–132. https://doi.org/10.1207/s15326985ep3102_4

Remillard, J. T. (2005). Examining key concepts in research on teachers' use of mathematics curricula. *Review of Education Research, 75*(2), 211–246. DOI-10.3102/00346543075002211.

Renkl, A. (2002) Worked-out examples: Instructional explanations support learning by selfexplanations. *Learning and Instruction, 12*, 529–556.

Reubens, A. (2009). *EdData II early grade mathematics assessment (EGMA): A conceptual framework based on mathematics skills development in children*. Research Triangle Park: USAID.

Reys, B. J., Reys, R. E., & Chavez, O. (2004). Why Mathematics Textbooks Matter. *Educational Leadership, 61*(5), 61–66.

Rezat, S. (2009). The utilization of mathematics textbooks as instruments of learning. In V. Durand-Guerrier, S. Soury-Lavergne, & F. Arzarello (Eds.), *Proceedings of CERME 6*, Lyon France. http://www.inrp.fr/editions/cerme6.

Rittle-Johnson, B., & Star, J. R. (2009). Compared with what? The effect of different comparisons on conceptual knowledge and procedural flexibility for equation solving. *Journal of Educational Psychology, 101*(3), 529–544.

Robitaille, D. F., & Travers, K. J. (1992). International studies of achievement in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 687–709). New York, NY: Macmillan.

Rohrer, D., & Pashler, H. (2010). Recent research on human learning challenges conventional instructional strategies. *Educational Researcher, 39*(5), 406–412. doi.10.31021/0013189X10374770

Ronda, E., & Adler, J. (2017). Mining mathematics in textbook lessons. *International Journal of Science and Mathematics Education*, *15*(6), 1097–1114. DOI 10.1007/s10763-016-9738.

Rumiati, R. (2010). An investigation of the number knowledge of first and second grade children in an Indonesian school. Master Thesis, Southern Cross University, Lismore, NSW. http://epubs.scu.edu.au/cgi/viewcontent.cgi?article=1157&context=theses

USAID (2010). Evaluation of The Malawi Teacher Professional Development Support: Final Evaluation Report, Lilongwe, Malawi: US Agency for International Development.

SACMEQ II (2005). The SACMEQ II Project in Malawi: A study of the Conditions of schooling. And the Quality of Education. *Working report*. Southern and East Africa Consortium for Monitoring Educational Quality (SACMEQ). http://www.sacmeq.org/

Njora, H. (2010). SACMEQ III project results: Pupil achievement levels in reading and mathematics. Do – 10.13140/RG.2.2.27312.35841

SACMEQ (2011). Trends in achievement levels of grade 6 learners in Malawi. *Policy brief report*. Retrieved August 3, 2014 from www.sacmeq.org.

Schmidt, W. H., McKnight, C. C., Houang, R. T., Wang, A. H., Wiley, D. E., Cogan, L. S., & Wolfe, R. G. (2001). Why schools matter: A cross-national comparison of curriculum and learning. The Jossey-Bass education series. ERIC. *The Jossey-Based Education Series.* ERIC.

Schmidt, W. H., McKnight, C. C., Valverde, G., Houang, R. T., & Wiley, D. E. (1997). *Many visions, many aims: A cross-national investigation of curricular intentions in school mathematics* (Vol. 1). Springer Science & Business Media.

Schoenfeld, R. (Ed.). (2007). *Assessing mathematical proficiency.* Washington, DC: National Press Academy.

Shortino-Buck, M. M. (2017). Mathematical discourse in elementary classrooms. (Unpublished doctoral dissertation). University of Portland, USA.

Silver, E.A. (1985). Research on teaching mathematical problem solving: some under-represented themes and needed directions. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives.* Hillsdale, NJ: Lawrence Earlbaum.

Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2009). *Implementing standards-based mathematics instruction: A casebook for professional development* (2nd ed.). New York, NY: Teachers College Press.

Stols, G. (2013). An investigation into the opportunity to learn that is available to Grade 12 mathematics learners. *South African Journal of Education*, *33*(1), 1–18, http://www.sajournalofeducation.co.za

Stylianides, G. J. (2009). Reasoning-and-proving in school mathematics textbooks. *Mathematical Thinking and Learning*, *11*(4), 258–288. https://doi.org/10.1080/10986060903253954

Thompson, D. R., & Senk, S. L. (2009). Documenting curriculum implementation: A case study from UCSMP Geometry. In S. L. Swars, D. W. Stinson, & S. Lemons-Smith (Eds.). *Proceedings of the 31st Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education*, 847–854. Atlanta, GA: Georgia State University.

Törnroos, J. (2005). Mathematics textbooks, opportunity to learn and student achievement. *Studies in Educational Evaluation, 31*(4), 315–327. https://doi.org/10.1016/j.stueduc.2005.11.005

Valverde, G. A., Bianchi, L. J., Wolfe, R. G., Schmidt, W. H., & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the translation of policy into practice through the world of textbooks*. Springer Science & Business Media.

Van Steenbrugge, H., Valcke, M., & Desoete, A. (2013). Teachers' views of mathematics textbook series in Flanders: Does it (not) matter which mathematics textbook series schools choose? *Journal of Curriculum Studies, 45*(3), 322–353. https://doi.org/10.1080/00220272.2012.713995

Van Zanten, M., & van den Heuvel-Panhuizen, M. (2018). Opportunity to learn problem solving in Dutch primary school mathematics textbooks. *International Journal on Mathematics Education. ZDM, 50*(5), 827–838. https://doi.org/10.1007/s11858-018-0973-x

Verschaffel, L., Greer, B., & De Corte, E. (1996). Numbers and arithmetic. In A. Bishop, K. Clements, C. Keitel and C. Laborde (Eds.), *International handbook of mathematics education 1*, (pp. 99–138). Dordrecht, The Netherlands: Kluwer.

Verschaffel, L., Greer, B., & De Corte, E. (2007). Whole number concepts and operations. In F. Lester (Ed.), *Second handbook of Research on mathematics teaching and learning*, (pp 557–628). Charlotte NC: Information Age Publishing Inc.

Viholainen, A., Partanen, M., Piiroinen, J., Asikainen, M. & Hirvonen, P. (2015). The role of textbooks in Finnish upper secondary school mathematics: theory, examples and exercises. *Nordic Studies in Mathematics Education*, 20 (3-4), 157–178.

Vincent, J. & Stacey, K., (2008). Do mathematics textbooks cultivate shallow teaching? Applying the TIMSS video study criteria to Australian eighth-grade mathematics textbooks. *Mathematics Education Research Journal*. 20(1):82–107.

Https://doi.org/10.1007/BF03217470.

Watson, A., & Mason, J. (2006). Seeing an exercise as a single mathematical object. Using variation to structure sense-making. Mathematical thinking and learning, 8(2), 9111

Webb, N. (2010). Content complexity and depth of knowledge as applicable to research and practice. ISTE International Conference on Mathematics, Science and Technology Education, Kruger National Park, South Africa, Pretoria: UNISA Printers

Wijaya, A., van den Heuvel-Panhuizen, M., & Doorman, M. (2015). Opportunity-to-learn context-based tasks provided by mathematics textbooks. *Educational Studies in Mathematics*, 89(1), 41–65. DOI 10.1007/s10649-015-9595-1

World Bank, (2010). *The Education System in Malawi*. Working Paper No. 182. USA:

Washington D.C., UNESCO. ISSN: 1726-5878. DOI:10.1596/978-0-8213-8198-4

Wright, B. (1991). What number knowledge is possessed by children beginning the kindergarten year school? *Mathematics Education Research Journal, 3(1),* 1–14.

Zhu, Y., & Fan L. (2006). Focus on the representation of problem types in intended curriculum a comparison of selected mathematics textbooks from Mainland China and the United States. *International Journal for Science and Mathematics Education, 4(9),* 609–626. https:/doi.org/10.007/s10763-006-9036.

Zodik, I. & Zaslavsky, O. (2008). Characterirstics of Teachers' choice of examples in and for mathematics classroom. *Education Studies in Mathematics*. *69(60),* 165–182. https:/doi.org/10.1007/s10649-008-9140

APPENDICES

Appendix 1:

Table 8: MDI analytic Tool for Textbooks lesson (MDITx) analysis guide (Ronda & Adler, 2017, p. 1106)

Examples	Tasks
Examples are coded as follows: **Level 1**: one pattern of variation used; either contrast (C) or Generalization (G) **Level 2**: any two patterns of variation used; C, G/ C, F and/ G, F **Level 3**: - all patterns of variation used Other description **Contrast** (noticing difference) **Generalization** (noticing similarity) **Fusion** (discerning simultaneous dimensions of variation – add previous experience)	Tasks are coded as follows: **Level 1** – carry out known procedures or known concepts related to the object of learning (**KPF only**) **Level 2** – carry out procedures involving the object of learning (**includes CTP**) **Level 3** – carry out level 2 tasks plus tasks that involve multiple concepts and connections (**includes CTP and AMC**)

Table 2, describes code for analyzing textbook examples and tasks

In table 2, column one shows the codes for examples and column two shows coding for the tasks.