FACULTY OF SCIENCE AND TECHNOLOGY

MASTER'S THESIS

Study program/ Specialization: Risk Management/Risk Assessment and Management	Spring semester, 2020 Open / ~~Confidential~~
Author: Gulay Tahmiscioglu	
Faculty Supervisor: Roger Flage Company Supervisors: Baris Aslan, Thomas Solberg Fylking	
Title of master's thesis: ON THE EFFECTS OF ASSET LIFETIME EXTENSION FOR SAFETY INSTRUMENTED SYSTEMS (SIS)	
Credits: 30	
Keywords: Aging and Lifetime Extension Safety Instrumented System Safety Integrity Level Bayesian Methodology	Number of pages: 105 + supplement material/other:0 Stavanger, 15.07.2020

ON THE EFFECTS OF ASSET LIFETIME EXTENSION FOR SAFETY INSTRUMENTED SYSTEMS (SIS)

By

Gulay Tahmiscioglu

This thesis is submitted to the Faculty of Science and Technology

University of Stavanger

In Fulfillment of the Requirements for the degree of

Master of Science

(MSc)

Study Program: Risk Management

FACULTY OF SCIENCE AND TECHNOLOGY

University of Stavanger

2020

ABSTRACT

A number of the offshore facilities in Norway have either been approaching their design life or exceeded them while there is still a considerable amount of oil and gas extractable in the field. Therefore, aging and life extension (ALE) has become a strong topic in Norway's offshore activities. The principles of aging and life extension activities have been addressed by numerous standards and implementation guidelines published in years. As offshore structures change with time (age), the question of how these changes could affect safety systems beyond their pre-defined useful lifetimes arises and becomes one of the crucial concerns for the offshore industries.

Safety Instrumented Systems (SISs) are critical to ensure optimal and compliant industrial operations due to their capability of preventing hazardous and catastrophic situations. They are formed of one or more Safety Instrumented Functions (SIFs) and designed to endure and work with critical processes with a low failure rate. It is a common practice to analyze their effectiveness based on specified failures rates of individual components of each SIF. The failure rates are treated both with generic failure rates and with plant-specific failure rates during the design and operation stages of facilities. Practically, the required risk reduction provided by SIFs must be ensured as long as the facility is in operational phases. In the recent past, it has been encountered that most of the components forming the SIFs have been used longer than their originally stipulated useful lifetimes within the scope of ALE. This raises the question of whether the failure rates remain as expected or not throughout the extension period.

Although there are various studies in the literature addressing the behavior of components based on probability of failure while they are aged, none of them provides a computational estimation to re-quantify their Safety Integrity Levels (SILs) which is based on the estimated failure rates. The purpose of this thesis is to investigate the changes in failure rates of the components over time, thus the SILs of SIFs and safety of SISs, when their operation times exceed their pre-defined useful lifetimes. The estimations of failure rates are obtained by taking the expert knowledge about component failure rates and data on hand into account. For this purpose, Bayesian Methodology is generated and the model for failure rate estimation over time is created and implemented in this thesis. The discussion about whether the safety systems maintain the expected safety levels set by the national/international/company standards is presented based on the results of performed analysis.

The restricted time of this thesis does not allow analyzing the behavior of all prevalent safety systems throughout ALE period. For this reason, only 3 SISs (Emergency Shutdown Systems, Pressure Shutdown Systems (PSD), Fire & Gas Detection Systems) are chosen as the basis of this thesis. It is assumed that understanding the changing trend of failure rates for the most common SIFs belonging to these 3 SISs under aging and life extension will give overall insight about the performance of other SIFs and corresponding SISs in time. For this reason, High Pressure Protection System (HIPPS) from PSD SIS, Emergency Shutdown (ESD) Segregation Function from ESD SIS, and Gas Detection Function from Fire & Gas Detection SIS are chosen to be the SIFs whose components' failure rates are estimated by the proposed Bayesian Methodology.

The Bayesian Methodology is executed for the failure rates are estimations of 9 components forming these 3 SIFs. After the estimated failure rates are obtained for the lifetime period of 40 years, the PDS[1] method is used to quantify the SILs of SIFs based on estimated failure rates. The focus is given to deviations of failure rates of components over the years, and thus, to Probability of Failure on Demand (PFD) of SIFs and the overall reliability prediction of SIFs.

The results of the Bayesian Analysis conducted in this thesis have shown that the failure rate estimations of components, as well as PFD values of the functions, start increasing at the end of the useful lifetime of the components. According to the calculations based on the estimated failure rates for the components of the HIPPS function of PSD SIS, the SIL obtained from generic failure rates cannot be met (SIL3) when aging is considered. Yet, the system remains in SIL2 from the installation to year 17. Similar to HIPPS, the SIL based on generic failure rates (SIL2) also cannot be met for ESD Segregation Function and the function stays on SIL1 level until the end of year 7. The inexistence of SIL based on the generic failure rates (SIL2) is also the case for Gas Detection Function, while it remains in the SIL1 level for the first 13 years of operation before it goes out of the SIL compliance range.

Although SIL could be kept the same for some years of the initial operations, still; the consideration should be given to the deviations from initially anticipated PFD values and potential consequences of loss of safety from the safety and reliability point of view. If existing SIFs are deemed to be inadequate, it is recommended to take action to change the components or maintenance strategies.

It must be kept in mind that ALE could cause deviations from related standards and project requirements. Therefore, the asset operators, as well as the decision-makers, should be well prepared before and during the ALE period. Proper actions should be taken before the unintended consequences arise due to loss of safety. This thesis proposes practical measures and roadmap for asset operators to prevent to overlook potential safety and reliability losses due to aging.

Recommendations for possible future works are included in the final part of the thesis.

---

[1] Norwegian acronym for reliability of computer-based safety systems

ACKNOWLEDGEMENTS

This master thesis is written, as a requirement for my master's degree in Risk Management at the University of Stavanger during the spring semester of 2020. The title of the thesis is "On the Effects of Lifetime Extension for Safety Instrumented Systems (SISs)".

The main objective is to develop a structured approach to quantify SIL performances of SIFs beyond their useful time/in the period of life extension.

First, I wish to thank my supervisor Professor Roger Flage at the Department of Safety, Economics and Planning at the University of Stavanger for his invaluable guidance throughout the entire master thesis period.

I would also like to thank the team members in ORS Consulting and my external supervisor Baris Aslan for giving me the possibility to write my Master`s thesis with their collaboration.

Moreover, I would also like to thank Murat Korkmaz and Tamas Petkovic for their contributions.

Stavanger, June 2020
Gulay Tahmiscioglu

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

ABBREVIATIONS

Definition	Description
ALE	Aging and Life Extension
CMBL	Combined Lifecycle
CCF	Common Cause Failures
DU	Dangerous Undetected
E/E/PE	“electrical/electronic/programmable electronic
ESD	Emergency Shutdown
ESV	Emergency Shutdown Valve
F&G	Fire and Gas
FT	Flow Transmitters
HIPPS	High Pressure Protection System
IEC	The International Electrotechnical Commission
ISO	International Standards Organization
LE	Life Extension
LS	Logic Solver
NCS	Norwegian Continental Shelf
NOROG	Norwegian Oil and Gas Association
ORS	ORS Consulting AS
PFD	Probability of Failure on Demand
PSA	Petroleum Safety Authority
PSD	Process Shutdown
PT	Pressure Transmitter
RBD	Reliability Block Diagram
SIF	Safety Instrumented Function
SIL	Safety Integrity Level

1 INTRODUCTION

Aging and Life Extension (ALE) studies are conducted in order to maximize the utilization of engineering systems and assets in operation. ALE seems essential in today's challenging offshore industry which struggles with oil price fluctuations and demand threats from alternative energy sources.

The most important aspect of life extension assessment is to ensure that the systems exposed to aging are still safe enough to continue their operation. Although changes related to aging are visible since the first day, they tend to become significant once the components'/systems' useful lifetimes are exceeded. The best practices of standards ensure the safety of the components/systems during the expected useful lifetime, however; it must be ensured as long as the facility is in operational phases, even though the design life is exceeded.

1.1 BACKGROUND AND PROBLEM PRESENTATION

Offshore facilities in Norwegian Continental Shelf (NCS) are intended to be operated for a certain period called "Design Life". Today, several facilities in operation are beyond their design life. In case of possible oil recovery beyond this period (which is economically more profitable compared to decommissioning activities), structural improvement studies are taken in place.

With over thirty years of oil and gas production in NCS, a significant number of platforms are approaching or have exceeded their original design life, which was specified as typically 25 years on average. Figure 1-1 shows the age distribution of existing installation on the NCS where over 100 platforms being installed (1). This means that; as of today, all platforms build before 1995 have reached the 25-year limit (more than 50 platforms).

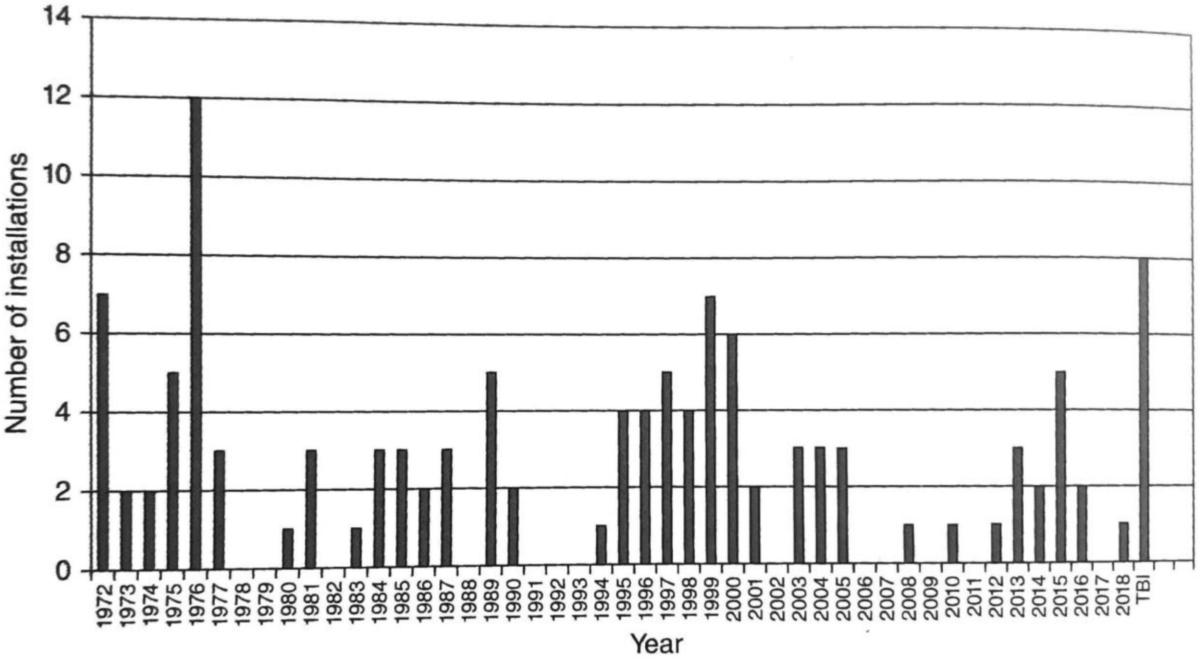


Figure 1-1 Age distribution of existing installations in Norwegian Continental Shelf from (1)

With the concern of economically viable operations, offshore facilities are operated to serve in the oil or gas industry beyond their designed lifetime, either as the original production fields or to serve as a host for subsea completions as long as possible. It is desired that they stay in operations for a significant period of time in the foreseeable future even their designed lifetimes are exceeded. Indeed, in some cases, there are plans to extend the operational life to up to several times of the original design life. These plans are called life extension (LE) studies and they require planned maintenance, repair and inspection analysis. While LE is conducted, safety should not be compromised in any of its identified aspects (1).

Focusing on safety considerations of aging facilities, it is likely that not only structural conditions but also safety systems may not be in the acceptable range of safety levels during the extended operations. There are multiple safety systems in the oil and gas industry that utilize Safety Instrumented Systems (SISs). SISs are formed of one or more Safety Instrumented Functions (SIFs). Each part of the SIF is called subcomponents which are also designed to operate within a certain lifetime. The end-user, (typically the asset operator) is responsible to ensure that the entire SIF fulfills the required Safety Integrity Level (SIL) compliance determined by industry or standards as long as the SIFs are in the operational phase, this could also happen beyond the useful lifetime.

SILs are quantified by the Probability of Failure on Demand (PFD) of SIFs, and PFD of SIF is quantified by PFDs of individual SIF components, and those depend on the failures rate of individual SIF components. Figure 1-2 shows the steps related to SIL calculation. One should reach out to the individual failure rates of SIF components first to determine the SIL compliance. This process is visited before the SIFs are put in operation based on the vendor determined or generic failure rates of SIF components, however; age-dependent changes in failure rates could occur over time during operations. In this case, the probability that the SIFs perform satisfactorily shall be re-questioned based on the given state of the lifetime. This results in the need for re-visiting the integrity and reliability requirements for the SIFs and an SIS of interest, and re-quantification of SILs.

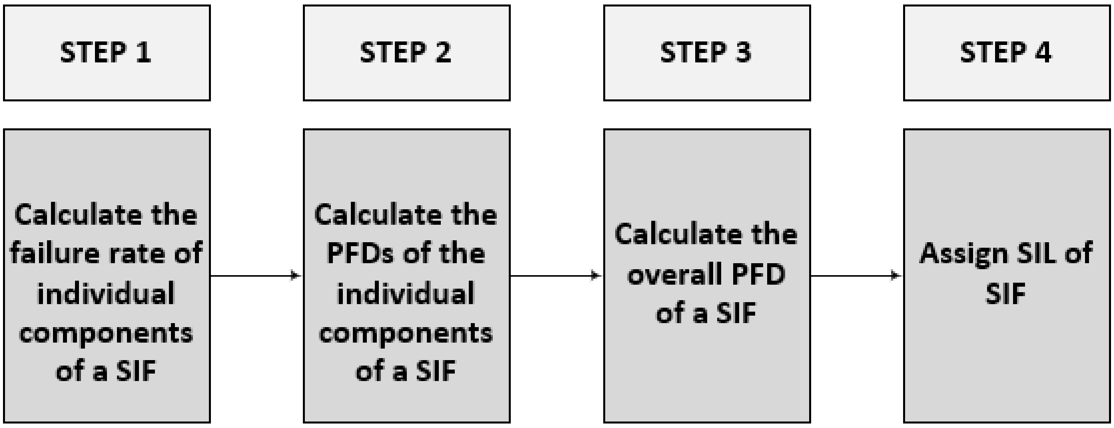


Figure 1-2 Quantification of SIL

The literature on ALE mostly is about assessment and management of structural integrity of offshore facilities which are beyond their design lifetime. On the other hand, studies concerning components of SIFs, therefore SILs beyond useful lifetime are limited.

The aim of this Master thesis is to discuss, investigate and understand if the safety levels (individual failure rates of components, overall PFDs of SIFs, and thus; their SILs) of the aging facilities are still in acceptable (or originally accepted) range for extended lifetimes. The contribution of aging on the changes in SIL compliance can be determined with the use of a suitable tool. This thesis attempts to develop a structured methodology in order to assess the quantification of how overall SIF performance change beyond useful lifetime due to the effect of time (aging) on components` failure rates.

1.2 “DESIGN LIFE” AND “USEFUL LIFETIME”

The term design life has different interpretations in different guidelines and standards. For example, in NORSOK[2] N-001-Structural Design (2), it is defined as “Structures shall be designed to withstand the presupposed repetitive actions during the lifespan of the structure. This lifespan is called design life”. In International Standards Organization’s book (ISO) 2394-General Principles on reliability for structures (3), it is defined as “The assumed period for which a structure is to be used for its intended purpose which anticipated maintenance but without substantial repair being necessary”.

Figure 1-3 illustrates the designed lifecycle of a structure (or a component/system). The useful lifetime is defined as the time interval within the design life where the failure rate is slowly declining or completely steady (maturity phase). However, ideally, the design life should include the time interval where also the failure rate of a structure/component declines. It must be noted that, since the structures/components could have passed through life extension just after the maturity phase, in the context of this thesis the (original) design life, as well as useful lifetime is ideally defined as the time when the structures/components are still safely be used, or where it reached the maturity phase but not aging phase on Figure 1-3. Therefore, the terms “beyond useful lifetime” and “beyond design life” refer to the same time span and both could be used in the context of this thesis.

---

[2] Norwegian acronym for Norwegian shelf's competitive position

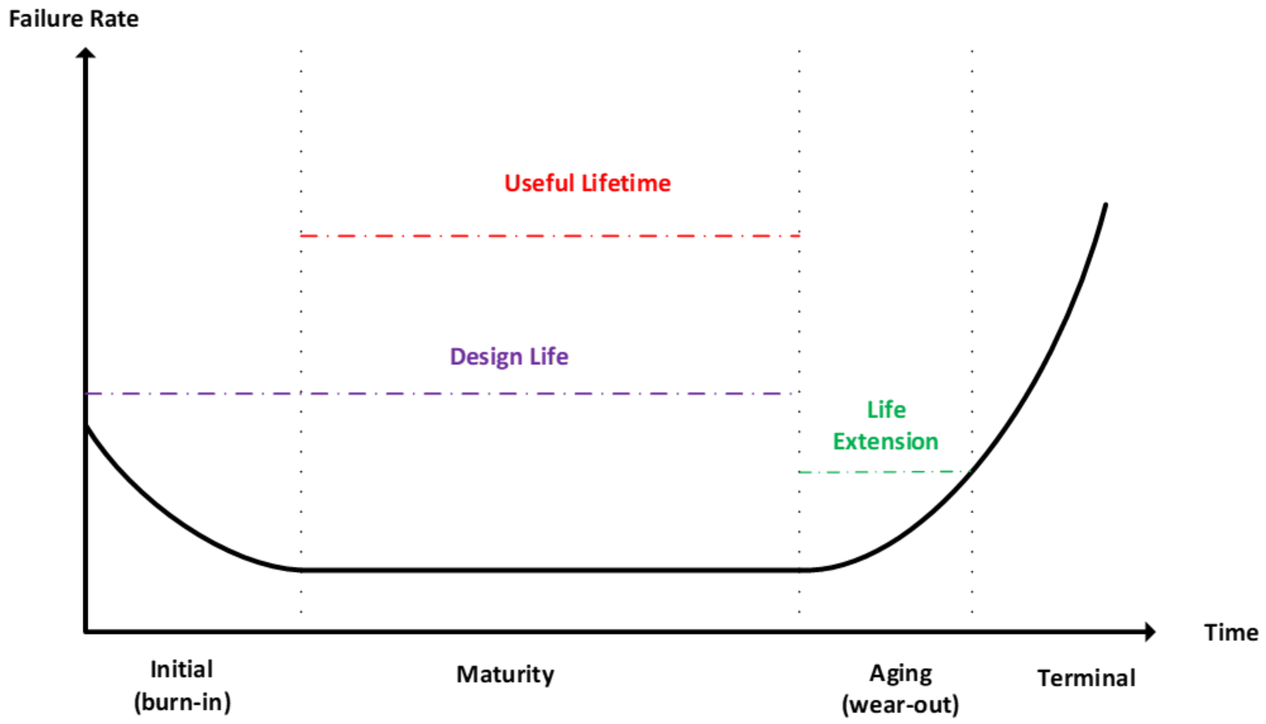


Figure 1-3 Bathtub curve representing the failure rate of a structure/component through its lifecycle adapted from (1)

1.3 THE ROLE OF SIF(S) IN RISK REDUCTION

A process can include several possible hazardous situations that can be identified with the associated risk levels. Without any safety functions, it is more likely that operations are exposed to unacceptable risk levels. The difference between unacceptable risk and acceptable risk levels (determined by company policies, standards, etc.) is called the required amount of risk reduction. However, the provided (actual) risk reduction always aims to reduce the risk as much as reasonably possible. Actual risk reduction is provided by the combination of external reducing measures (such as organizational nature), SIFs, and other technical safety-related systems (such as escape and evacuation system). Figure 1-4 illustrates the risk reduction achieved by different measures. The various risks included in Figure 1-4 are defined in the standard of the International Electrotechnical Commission (IEC) 61511 (4) as follows:

- Acceptable (Tolerable) risk: The risk level which is accepted in a given context based on the current values of society.

- Residual risk: The risk of a hazardous event that could still remain after the introduction of safety measures.

The required risk reduction from a SIF is a key to prevent accidents and/or to mitigate their consequences together with other risk-reducing measures, therefore, it must be ensured anytime during the operations. The partial risk reduction measures covered by other technology safety-related systems and by external risk reduction facilities can be exemplified as firewalls, personnel in the field, transmitters installed in process units detecting low level indicating a leak, etc.

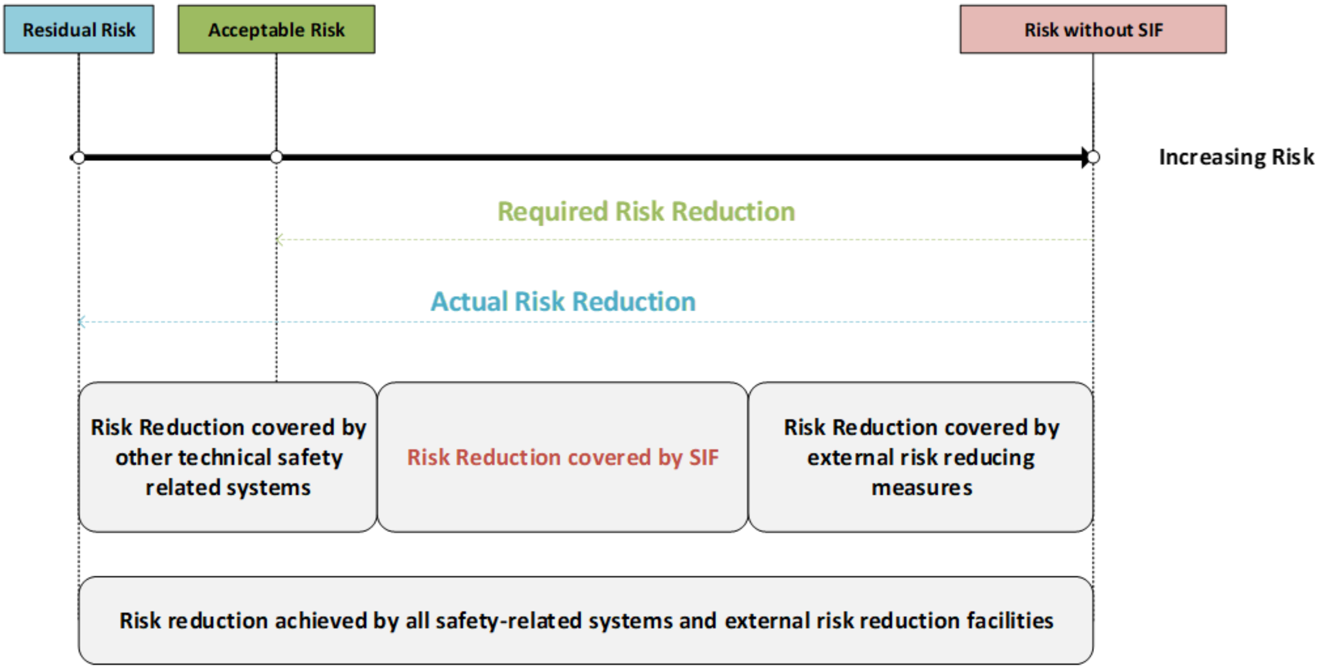


Figure 1-4 SIFs role in risk reduction adapted from (5).

1.4 WHAT IS SIL OF SIF?

An SIS, which is formed by one or more SIFs, is a system that is responsible for safe operations whose contributions to the achievement of a required risk reduction are defined by SILs.

In principle, the safety integrity of an existing structure must be ensured according to the standards. The standard IEC 61508 (5) provides sufficient guidance in safety aspects and widely accepted in the field of reliability of SISs. The quantification of safety is addressed as SILs of SIFs forming SISs. Four discrete levels of safety are described in IEC 61508. Each level represents the different amount of risk reduction in the related equipment as explained in Section 2.4.1 in detail. IEC standards require that the SIS design, as well as operation and maintenance choices, must be verified against the SIL. SIL is not a measure of risk, it is the quantified reliability level of a safety function/system that is required to achieve the necessary amount of risk reduction (6).

However, it is unclear how SIL compliance of SIFs can be ensured for facilities operated beyond their designed life. It is a common understanding that the aging of the SIF components through LE may have a negative impact on the operator’s target SIL (7).

For operations beyond the lifetime, LE regulations require the submission of a revised safety case identifying all hazards with the potential to cause a major accident, as well as demonstrating adequate control of major risks arising from the decreasing performance of SIF through LE (7). Although there is no specific requirement in order to quantify the changes in safety performance, the author believes that a formal assessment /structural approach is still the best way to ensure the SIL compliance of SIFs through LE.

1.5 PROBLEM FORMULATION AND RESEARCH QUESTIONS

The hypotheses that have led to the development of this thesis are:

1) ALE must be discussed and studied for offshore installations at SIL of SIFs perspective because SIF performance must be supported by solid evidence as long as the facility is in operation;
2) Operating beyond lifetime may compromise SIF performance, thereby SIL compliance;
3) Careful and periodic assessment of SIF Performance / SIL compliance is paramount in order to ensure that SIFs provide the required risk reduction continuously.

The topic of this thesis is developed with the cooperation of ORS Consulting (ORS). The business partners of ORS have increasing interest in the validity of originally accepted SIL compliance while the facilities become aged. Therefore, ORS is aiming for a deeper understanding of the ALE concept in order to offer improved consultancy on this topic. This thesis will form the core of the corresponding consultancy services of the company.

The main questions that are answered within the scope of this thesis are:

1) To what extent the current ALE studies cover SIFs and SISs?

2) What should the structured approach to estimate failure rates of components be when the useful lifetime is exceeded? How does the change in failure rates affect the SIL of SIF beyond the useful lifetime based on?

3) What are the practical measures/road map for asset operators can resort to in order to monitor SIF performance beyond the useful lifetime based?

For these purposes, the author of this thesis gives an outline of how the performances of SIFs can be affected by age (time), generates of a structured approach (Bayesian Model) that merges solid evidence (i.e. failure rates) with available information (i.e. opinion from experts) to capture the change in failure rates of SIFs components over years, presents estimated failure rates obtained from the Bayesian Model together with the deviations from required SILs and draws a conclusion of possible changes in overall safety system reliability over time.

1.6 METHODOLOGY

The main reasons for the need for an approach to estimate SIF performance in the overall lifecycle of components have been discussed in previous chapters. Once the failure rate of a component is known, there are various methods referred to in IEC61508 (5) to calculate PFD and SIL of a SIF. One of the most reliable methods is the PDS method presented in PDS Method Handbook (8). However, in the current literature, there are no defined methods/distributions that can be used for modeling the failure rates of SIFs components over time, therefore; the SIL performance.

In this thesis, the author presents a methodology that estimates age-dependent failure rates of components beyond their useful lifetime. The methodology has been generated based on Bayesian Analysis due to the recommendation and preference of ORS. The experts from ORS believes that there is a valuable prior knowledge from their experiences about SISs and there is the data of failure events related to SISs. Since Bayesian Methodology is a proper way of combining current beliefs (experiences) with the evidence (data), the mutual agreement between the author and ORS has become to construct the basis of this research on Bayesian Analysis.
Figure 1-5 explains the new approach, which is the updating process of initial information about the failure rates. First, a prior distribution is assigned based on expert knowledge before any observation has taken

into consideration. When data is available, the prior distribution is updated to the posterior distribution. The observed values, therefore, change the initial belief regarding the distribution parameter(s). Then, updated SIL is calculated based on failure rate estimations of the posterior distribution. This process may be repeated if the new prior distribution is chosen based on observed values and/or change in initial belief. When additional data is available, this process may lead to the new posterior distribution of failure rates and associated SILs. The main objective of this methodology is to present a quantitative framework to estimate the changes in failure rates and thus SIF performance quantified by SIL.

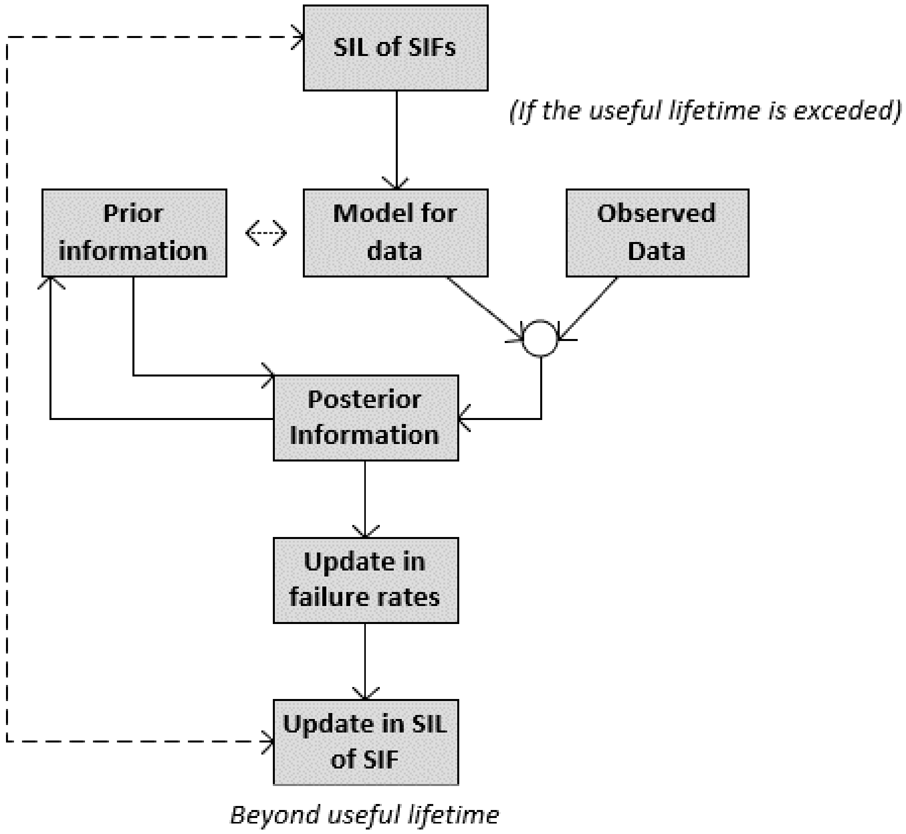


Figure 1-5 The Methodology adapted from (9).

This thesis intends to find the most relevant statistical way to estimate SIF performance beyond the useful lifetime. The methodology presented in Figure 1-5 is used to obtain the reasonable predictions of failure rates of components beyond useful lifetime under the given assumption that they increase when the useful lifetimes of components are exceeded.

The applicable model according to the component characteristics is discussed and inferences based on posterior distributions are presented. Once the estimations of failure rates are obtained by the methodology, the PDS method is implemented with updated failure rates of components forming SIFs, and new SILs are calculated. The variation of SIF performance (and hence, SIL) is observed and discussed.

1.7 LIMITATION/DELIMITATION

Since there are many SISs in the offshore installations, the restricted time of this thesis does not allow the author to analyze the behavior of all of them throughout the ALE period. SISs are divided into two main categories by IEC standards: those involving electrical/electronic/programmable electronic (E/E/PE) components those which do not.

This thesis has a specific focus on safety systems that have E/E/PE components. In the context of this thesis the most common SIFs related to the following SISs will be analyzed in order to quantify the effect of aging on the required/accepted SILs:

- Emergency Shutdown Systems (ESD);
- Process Shutdown Systems (PSD);
- Fire and Gas Detection Systems

These SISs and some related SIFs are described in Chapter 3. As mentioned before, an SIS can be realized by one or more SIFs. However, due to the time limitations of this thesis, not all possible SIFs forming these SIFs are described, identified, and quantified by their estimated SILs. Only the most common SIFs of these SISs are presented and evaluated in terms of their SILs. It is assumed that understanding the variations in SILs of the most common SIFs of the specific SISs under aging gives insights about the overall safety and reliability performance of safety systems.

The modeling and analysis in reliability engineering require historical failure event data. However, operator companies do not easily share their data and there is restricted access for generic resources. For example, Offshore Reliability Data (OREDA) (10) do not provide public availability for their failure event databases. Therefore, the data is obtained from the different reliability projects conducted by ORS Consulting for their business partners in the past. While evaluating this work, one should keep in mind the following limitations about failure event data which is presented in Appendix-C.

- Data could be incomplete: i.e. there could be missing failure events.
- Since the data collected from different sources for different components, it can vary in quality

Moreover, the Bayesian Analysis has been carried out by interval-censored data. The case of uncensored or right/left-censored data has not been considered.

1.8 STRUCTURE OF THESIS

Some initial knowledge about reliability theory and practices of safety-related systems in the oil and gas industry is beneficial while reading this thesis. For this reason, some basic terms used in reliability analysis and industry standards/guidelines are described in relevant chapters.

Chapter 1 presents the background of this research to the reader with its objectives and limitations and the brief of the proposed methodology while Chapter 2 provides the theoretical framework. Chapter 3 investigates the details of SISs related to the scope of this thesis. It also includes the methods of SIL quantification. Chapter 4 is the presentation of the proposed Bayesian Methodology for the failure rate estimations, as well as the results of the method implementation. A discussion made about the results is given in Chapter 5. Chapter 6 proposes the possible actions to be taken for the facilities going through aging and life extension processes. Finally, Chapter 7 makes a conclusion on the contribution of this research and recommendations on future works.

Appendix-A presents the Python code that is used to determine initial values of likelihood distribution parameters based on expert opinion. In Appendix-B, the Python code used to conduct the Bayesian Analysis is given. Appendix-C includes the failure event data of components. Lastly, Appendix-D presents the results of the Sensitivity Analysis related to Bayesian inferences.

2 THEORETICAL AND REGULATORY FRAMEWORK

In this section, relevant codes, standards, recommended practices and guidelines that deal with the key elements of the LE management process and safety requirements are reviewed together with the relevant theories of reliability science.

2.1 LITERATURE REVIEW OF RELIABILITY THEORY

Reliability theory states that the performance of a component cannot remain constant over time. In the observation of an unexpected outcome of safety-critical systems, it is said that the safety system fails. In other words, failure happens when a system, component or unit cannot perform its intended purpose. Some specific terms that are playing important roles for a better understanding of the reliability and safety of a component are reviewed in this section as given in the relevant chapters of (11).

Reliability. “The probability that an item will perform a required function, under stated conditions, for a stated period of time”. Simply putting, the probability of components to function for the specified time interval.

There are several different terms to quantify reliability, one of them is “time to failure “.

Time to failure. The time between two discrete failures, i.e. up-time of an item.

Mean-Time-to-Failure (MTTF). The average of the up-times which is mostly used to express overall reliability.

Failure rate. The number of failures in a unit of time, such as x failures in 10^6 hours. The expected number of failures is expressed as 1/MTTF for a given time interval.

The terms described above are only measures of success. Measures of failure also need to be reviewed when evaluating safety. The additional terms commonly used in the published literature of safety in relation to reliability are described below (11).

Failure mode. Description of the way how a unit fails. For continued safe operations, 2 failure modes are important: safe failure and dangerous failure. The former describes any failure that causes a unit to go to a safe state (state of process when safety is achieved (4) when there was actually no danger). The dangerous failure is defined as failure which causes not responding to a unit (therefore, no achievement of safety state) when there is a potential danger. Failure modes must be considered in the design of SIFs and SISs. A more detailed classification of failure modes proposed by the SIL quantification methodology followed in this thesis is given in Section 3.2.

PFS/ PFD_{avg}/ PFD. Probability of Safe Failure (PFS) is the probability of a SIF to achieve a safe state when there is no actual danger (i.e. closure of ESD valve when there is no emergency and demand for a component to operate). (Average) Probability of failure on demand (PFD_{avg}/ PFD) is the (average) probability of SIF not to respond when actual demand for it occurs.

MTTFS/MTTFD. As mentioned above, MTTF is the average time until the unit fails. This includes both safe and dangerous failure modes. While performing the reliability assessment of SIF components, including different failure modes, the mean time to fail safely (MTTFS) and mean time to fail dangerously (MTTFD) should be calculated and used separately.

The following statements are worth to remember to develop a better understanding of different concepts in the context of this thesis (12):

- A(n) component/unit/item/system is highly reliable if it works for a long time without failure;
- A system is considered to be safe if it is reliable in performing its safety function. The system may fail much more frequently in modes that are not considered to be dangerous;
- A SIF in compliance with required PFD_{avg} indicates that it will do a certain job in an SIS. Its “safety reliability” might be high, yet its “general reliability” might not be that much.

Failure Rate Function in Reliability Theory

The failure rate is the ratio of a probability of a component to fail in a given time interval to the total functioning time (operating time). In reliability science, the failure rates of the components over time are often represented by a bathtub curve as shown in Figure 1-3. This chapter explains the derivation of the bathtub shape curve.

Let $F(t)$ the probability of item that fails in the time interval $(0, t]$ and $f(t)$ is the probability density function of time to failure T. Then reliability function $R(t)$ is denoted in (9) as

$$R(t) = 1 - F(t) = 1 - \int_0^t f(u)du = \int_t^\infty f(u)du$$

$R(t)$ is the probability that a component does not fail in the time interval $(0, t]$.

Additionally, the probability that a component fails in $(t, t + \Delta \mathrm{t}]$ when it was functioning until time t is (9)

$$P(t < T \leq t + \Delta \mathrm{t} \,|\, \mathrm{T} > \mathrm{t}) = \frac{P(t<T\leq t)}{P(T>t)} = \frac{F(t+\Delta \mathrm{t})-F(t)}{R(t)} \quad \text{(Eq.1)}$$

By the definition of failure rate given above, one can obtain the failure rate function $z(t)$ of a component by dividing Eq.1 by Δt which is (9)

$$z(t) = \lim_{\Delta \mathrm{t} \to 0} \frac{P(t<T\leq t+\Delta \mathrm{t} \,|\, \mathrm{T}>\mathrm{t})}{\Delta \mathrm{t}} = \frac{F(t+\Delta \mathrm{t})-F(t)}{\Delta \mathrm{t}} * \frac{1}{R(t)} = \frac{f(t)}{R(t)}$$

When Δt is small enough, this implies;

$$z(t) * \Delta t \approx P(t < T \leq t + \Delta t \,|\, T > t)$$

Since

$$f(t) = \frac{d}{dt} F(t) = \frac{d}{dt} \left(1 - R(t)\right) = R`(t) ;$$

$$z(t) = \frac{f(t)}{R(t)} = \frac{R`(t)}{R(t)} = \frac{-d}{dt} ln\, R(t)$$ can be obtained.

Then, $R(t) = \exp(-\int_0^t z(u))du$ show that reliability function is unique for failure rate function.

To determine the shape of failure rate function, assume there is n number of identical components starting into operation at time $t = 0$, and $n(i)$ denotes the number of components that fail in small time interval i. If the functioning times are T_{1i}, T_{2i},…, T_{ni} then the total functioning time becomes $\sum_{j=1}^{n} T_{ji}$

Then the estimate of failure rate in time interval i becomes (9) $z(i) = \frac{n(i)}{\sum_{j=1}^{n} T_{ji}}$.

If $m(i)$ is the number of components that are operating at the beginning of time interval i, then (9)

$$z(i) = \frac{n(i)}{m(i) * \Delta t}$$

As $\Delta t \rightarrow 0$, and n is large enough, an estimate for the failure rate function $z(i)$ takes the following shape represented in Figure 2-1 (9).

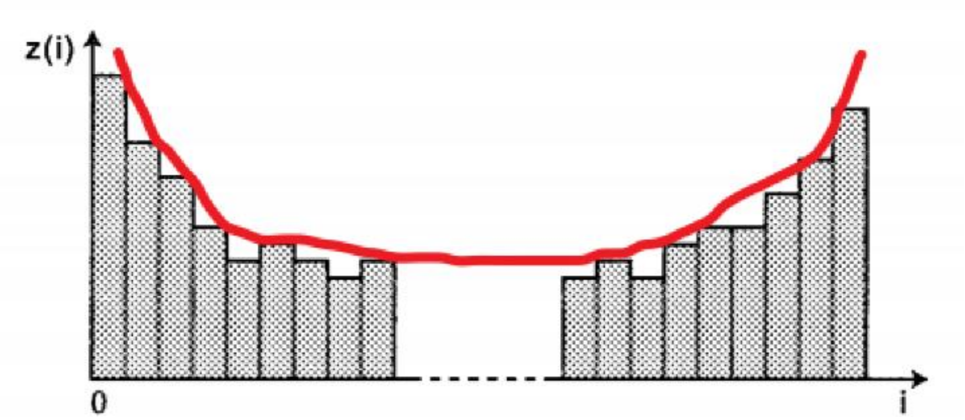


Figure 2-1 Histogram of the failure rate function

The characteristic shape of failure rate function is known as the bathtub curve, where the failure rate is often high in the beginning, then it stabilizes for a certain period (called useful lifetime) and starts increasing afterward (9).

The initial period where the failure rate is high is called the burn-in period. A component faces infant mortality problems and the left-over defects that do not expose themselves during the manufacturing processes but cause functionality problems at the beginning of the component's lifetime. It should be noted that this period can be avoided by performing "factory acceptance tests" and "site acceptance tests". The factory acceptance test is performed in the manufacturing facility to ensure that the component meets all the technical and quality requirements before it leaves the factory and is delivered to the final installation point (13). On the other hand, the site acceptance test is performed in the user's site to determine whether the component meets the criteria of the desired functionality. Even though both tests serve for the same purpose, the former is more focused on the manufacturer's criteria, where the latter takes the needs of the end-user into account. These tests should be performed for the validation of SIF components. Therefore, possible defects are identified and eliminated before the component starts its actual functioning. In reality, the high failure rates at the beginning of the lifetime are not observed for SIF components and the infant mortality period is eliminated in this way. In this thesis, all the failure rates presented (both in data and as results of analysis) are for the lifetime beyond the burn-in period. This is also emphasized in (10) that failures occurring during the initial phase is eliminated by testing procedures prior to installation. Therefore, it is assumed that data collection is started at the beginning of the useful lifetime phase where the failure rate is constant (10).

2.2 LITERATURE REVIEW OF AGE-DEPENDENT RELIABILITY MODELS

Aging brings a potential increase in failure rates of components. Due to the increasing demand for highly reliable systems, the current reliability models in the literature have widely discussed age (time) dependency that is currently applied in different industries including nuclear-power, oil and gas, etc.

In general, age-dependent reliability models consider the following parameters (14);

- The age of a component;
- Start time for the aging effects (where the time when the constant failure rate assumption does not hold anymore);
- An aging parameter (e.g. failure rate);
- A life extension parameter to represent the re-qualification (e.g. the time period of life extension).

In (15), a simple model based on adding two Weibull survival functions has presented for lifetime distributions of bathtub shaped failure rates curve as given in Figure 1-3. This model is based on classical statistics and applicable when the given data demonstrates the failure rate of bathtub shape where two distinct Weibull distributions are assumed to shape the data. Once the parameters of Weibull distributions are estimated, by the fitness of the graphical representation the failure estimates are obtained. However, the classical statistical methods have not always been found powerful enough in terms of their flexibility to update when the new data is available based on life extension parameters such as time.

When modeling time trends, Bayesian Analysis is being used by many scientists due to its power to treat uncertainties of small sample sizes and parameters of interest (time, failure rate, etc.). For example in (16), valve leakage has been modeled based on real data (prior) and a linear function of the time variable (likelihood). The future predictions of leakage probabilities are then used in probabilistic risk assessments.

The most similar work to this thesis is presented in (17). The authors have presented an age-dependent model based on Bayesian Methodology and inference which comes up with a bathtub shape failure rate. The analysis has been done for different dynamic prior distributions of failure rates of electrical instrumentation and control components and an illustrative example has been provided. Of all other alternatives, the uniform distribution shows the greatest fitness of data, therefore; specified as the prior distribution. Then Markov Chain Monte Carlo (MCMC) convergence assessment in WinBUGS[3] software has been done to obtain posterior distributions. Also, the predictive performance has been evaluated by changing the failure distributions (likelihoods) where it is assumed to be a constant rate before the defined threshold of age is exceeded.

---

[3] WinBUGS is a free statistical software developed for Bayesian Analysis.

2.3 ALE AND REGULATORY PRACTICES

Oil and gas activities worldwide are based on many regulations. In Norway, it is required to get legal approval in advance from the Petroleum Safety Authority (PSA) to keep the facilities operating beyond the planned lifetime. PSA guidelines give detailed information about the required features for the approval.

One of the requirements states that application for consent for lifetime extension should include a summary of barrier management (18). Since it is considered that aging can cause the failure of more than one barrier at the same time, the application for consent should include the identification of needs for the updated performance of barriers (1). As explained in Section 3.1 and illustrated in Figure 3-2, the barriers mentioned in regulations include internal and external technical safety-related systems and risk-reducing measures, including the SISs.

PSA also refers to the Norwegian Oil and Gas Association`s (NOROG) Guideline 122- Recommended Guidelines for the Management of Life Extension (19) for a complete assessment of requirements for life extension. According to NOROG122 (19), the life extension process should demonstrate that safe and reliable operation beyond the lifetime is performable. The following statements are taken place in NOROG122 (19) related to safety:

- Lifetime extension application should include the recommendations of the operator about maintaining the acceptable safety level throughout the extended lifetime;
- The operator must be able to monitor and control degradation and assure that the facilities are operated safely and reliably;
- The plan for safe and reliable operation should include the actions for modifications for future needs, replacement of equipment when it is necessary and strategic choices regarding maintenance.

Furthermore, NOROG122 (19) refers to NORSOK Z-008 Criticality Analysis For Maintenance Purposes (20) as a relevant standard for lifetime extension. In (20) it is stated that the availability, capacity and performance of safety-critical functions should form the basis of the plan for testing/ preventive maintenance activities. Actual failure data and system downtime are the keys for the determination and prediction of the safety system’s performance in operation.

In summary, the regulatory practices of ALE aim to ensure the correct level of safety is maintained during the life extension period. Even though no specific requirements are stated for the presentation of updated SILs of SIFs, the need for careful consideration of time-dependent performance studies related to SIFs are clear.

2.4 SIL AND REGULATORY PRACTICES

PSA specifies the requirements for safety functions in the Facilities Regulations (21) (§ 8 Safety Functions it is stated that all safety functions shall have performance requirements and design and performance of safety functions should be based on IEC 61508 (5) and Norwegian Oil and Gas’ Guideline No. 70 (NOROG070) (22) when E/E/PE systems are used in the structure of the functions.

PSA §8 Safety Functions (21) also states that the design of safety functions should be based on the standards NORSOK S-001 (NORSOK Standard on Technical Safety) (23). This document gives the definitions and descriptions of safety design in offshore industries and again refers to the following standards and guidelines that cover the principles and requirements of safety systems:

- IEC 61511-Functional Safety-Safety Instrumented Systems for Process Industry
- IEC 61508- Functional Safety of electrical, electronic and programmable electronic safety-related systems.
- NOROG 070 (NOROG guidelines no.70)- Guidelines for the application of EC 61508/61511 in the petroleum activities on the Norwegian continental shelf.

The following two sections give details of IEC standards and Norwegian Oil and Gas' Guideline No. 70 (GL070).

2.4.1 IEC

IEC 61508 (5) is the generic standard applicable to all industries and IEC 61511(4) is applicable to only the process industry. IEC 61508 defines functional safety as the discipline that studies the safety concept which depends on the correctly-functioning components or systems (i.e. functions as aimed). Some definitions from this standard that are important in the context of this thesis are summarized in Table 2-1.

Table 2-1 Related Definitions from IEC 61508 (5)

Definition	Description
EUC	equipment, machinery, apparatus or plant used for manufacturing, process, transportation, medical or other activities
High demand mode of operation	Where the safety function is only performed on demand, in order to transfer the EUC into a specified safe state, and where the frequency of demands is greater than one per year
Low demand mode of operation	Where the safety function is only performed on demand, in order to transfer the EUC into a specified safe state, and where the frequency of demands is no greater than one per year
Safe State	State of the EUC when safety is achieved
Safety	Freedom from unacceptable risk
Safety Instrumented Function (SIF)	Function to be implemented "which is intended to achieve or maintain a safe state for the Equipment under Control (EUC), in respect of a specific hazardous event"
Safety Integrity	The probability of a SIF satisfactorily performing the required safety functions under all stated conditions within a stated time period
Safety Integrity Level (SIL)	It is a discrete level (one out of a possible four) for specifying the safety integrity requirements of the safety functions to be allocated to the E/E/PE safety-related systems
Safety Instrumented System (SIS)	A distinct, reliable system used to safeguard a process to prevent a catastrophic event. It is formed by one or more SIFs.

Simplified, SIL is a quantified measure of the required performance of a SIF to maintain or achieve the safety state. The safety integrity requirements are specified by SIL. SIL is defined on a scale from SIL1 to SI 4, where SIL refers to a less reliable safety system with less stringent requirements than a SIL4 system (the highest standard). PFD is one of the criteria needed to fulfill SIL. Table 2-2 summarizes the PFD requirements for each SIL.

Table 2-2 SILs and associated PFDs

SIL level	PFD (low demand/high demand mode of operation)	Explanation (low demand/high demand mode of operation)
SIL 4[1]	10^{-5} to 10^{-4} / 10^{-9} to 10^{-8}	1 failure out of 10000 demands/ 1 failure out of 100000000 demands
SIL 3	10^{-4} to 10^{-3} / 10^{-8} to 10^{-7}	1 failure out of 1000 demands/ 1 failure out of 10000000 demands
SIL 2	10^{-3} to 10^{-2} / 10^{-7} to 10^{-6}	1 failure out of 100 demands/ 1 failure out of 1000000 demands
SIL 1	10^{-2} to 10^{-1} / 10^{-6} to 10^{-5}	1 failure out of 10 demands/ 1 failure out of 100000 demands

[1]: For the oil and gas industry, the highest SIL that can be implemented is SIL3. SIL4 is mainly used in the aviation industry.

The PDS Handbook (8) makes the differentiation between low demand and high demand (continuous) mode of operations. The low demand safety systems defined as the systems operating only when they are demanded. Typical examples are given as ESD and PSD systems. On the other hand, high demand mode systems (such as a ballast system) operates continuously. The systems analyzed in this thesis are compatible with low demand mode of operations, therefore PFD calculations are considered and evaluated in terms of SILs according to the range for low demand modes.

PFD of SIF is not the only requirement that should be fulfilled in order to comply with a certain SIL-level. In general, the IEC 61508 standard states 4 main types of requirements (5).

These are;

1) PFD (Probability of Failure on Demand) requirement: The requirement relies on the reliability of the components and subsystems used in the safety function;

2) Architectural requirement: is expressed in terms of the Safe Failure Fraction (SFF) and Hardware Fault Tolerance (HWFT), which defines the constraints to the topology of the components and subsystems which constitute a safety function;

3) Avoidance and control of systematic failure requirement. Is expressed in terms of the adequacy of the management of functional safety and the quality assurance program. This includes the techniques and measures which are implemented in order to avoid and control systematic failures;

4) Software requirement: is a qualitative requirement, expressed in terms of the adequacy of the management of functional safety and quality assurance program for the software development, testing and integration. This includes the techniques and measures which are implemented in order to avoid and control systematic failures in the software.

In the context of this thesis, the focus is given to the quantitative aspect (PFD requirement) only, due to its feasibility of quantitative analyses while performing reliability analysis and the restricted time to evaluate other requirements. Additionally, the PFD requirement takes the precedence of others, as the higher the SIL claimed for a safety function, the more stringent the requirements to PFD, architecture, systematic failures and software will become.

The IEC 61508 was the first international standard which gives the number of methods to quantify safety performance. These are basic probability calculations, reliability block diagrams approach (known as PDS method), Boolean approach, Markov modeling, Petri Nets and Monte Carlo approach, and AltaRica Data Flow approach. IEC standard does not recommend one specific approach, instead; leaves it to the user`s choice. The analysis to quantify SIL in this thesis is based on the PDS method which is the most recognized and widely used method in the offshore industry. The method is explained in Section 3.2 in detail.

2.4.2 NOROG 070

The main purpose of NOROG 070 (22) is to function as a guideline to standardize and simplify the application of IEC 61508 and IEC 61511 in the Norwegian Petroleum Industry. It provides a guideline for performance requirements to ensure minimum SIL requirements which are recommended by IEC 61508 and IEC 61511. Minimum requirements for SIFs were set based on the real data analysis collected from the industry and the risk-based approach.

The SIL calculated for each SIF forms a basis for minimum SIL requirements, which then must be verified by maintenance activities through the operational stage of the facility. This guideline helps organizations avoid documentation and calculation for SIL requirements as much as possible, however; it recommends following IEC 61508/61511 standards in case of deviations from requirements.

In summary, both IEC 61508/I61511and NOROG 070 discuss the safety systems by giving particular SIL requirements defined qualitatively based on PFDs.

3 SAFETY SYSTEMS OF INTEREST

An SIS is the combination of 3 subcomponents; input item (i.e. detectors), logic solver (LS) and a final element (actuating item) aiming to reduce the risk of some specific hazard. An SIS is formed by one or more SIF(s), and according to IEC 61508 (5) , each SIF should be allocated to a SIL. This chapter describes the safety systems which are of interest in this thesis with given performance requirements. Figure 3-1 illustrates a simple SIF.

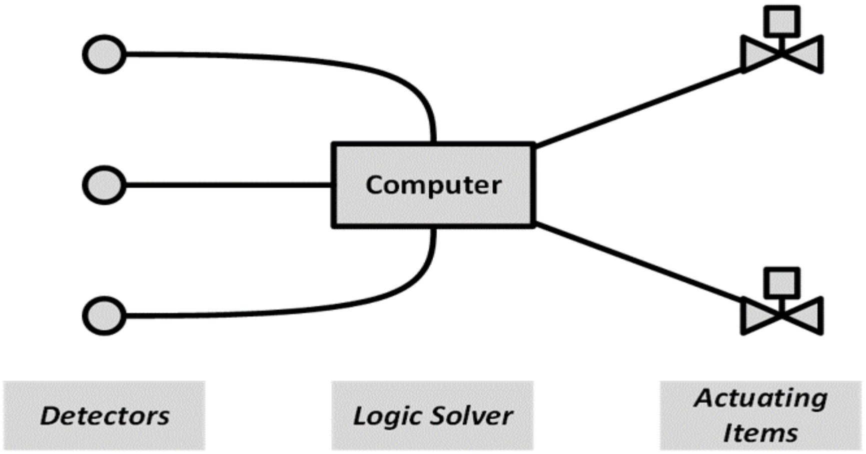


Figure 3-1 Simple SIF (9).

3.1 INTRODUCTION

NORSOK S-001(23), the standard that describes the principles and requirements of the safety design, defines the following systems as SISs:

- Emergency Shut Down;
- Gas Detection;
- Fire Detection;
- Ignition Source Control;
- Public Address, Alarm and Emergency Communication;
- Active fire protection;
- Escape and evacuation;
- Structures;
- Containment;
- Open drain:
- Process safety;
- Blowdown and flare/vent system;
- Human-machine interface;

- Natural ventilation and heating, ventilation and air conditioning, (HVAC)
- Emergency Power and Lighting;
- Passive Fire Protection;
- Fire Fighting System;
- Rescue and Safety Equipment;
- Ship Collision Barrier;
- Marine System and Position Keeping.

It is common to group safety systems according to the barrier point of view. According to PSA (24), the safety barriers are 'Technical, operational and organizational elements which are intended individually or collectively to reduce the possibility for a specific error, hazard or accident to occur, or which limit its harm/disadvantages'. Therefore, SINTEF report for LE (25) groups the above systems according to the barrier point of view as given below;

- Barrier 1: Process Control Systems
- Barrier 2: Process Protection Systems
- Barrier 3: FGD/ESD Systems
- Barrier 4: Fire Fighting Systems
- Barrier 5: Fire Protection Systems
- Barrier 6: Escape and Evacuation Systems
- Barrier 7: Rescue and Safety Equipment

Figure 3-2 shows how these barriers are positioned according to the onion model.

The main safety systems of interest of this thesis take place in Barrier 1 (Process Protection Systems- includes PSD) and Barrier 2 (FGD/ESD Systems- includes ESD and Fire & Gas Detection). This is important to know because this thesis focuses on safety systems which are the first safety instruments positioned just after the process elements itself (operator intervention) according to barrier hierarchy. It is assumed that understanding the SILs of SIFs related to these SISs beyond the useful lifetime also provides a sufficient understanding of other safety systems commonly used in the oil and gas industry.

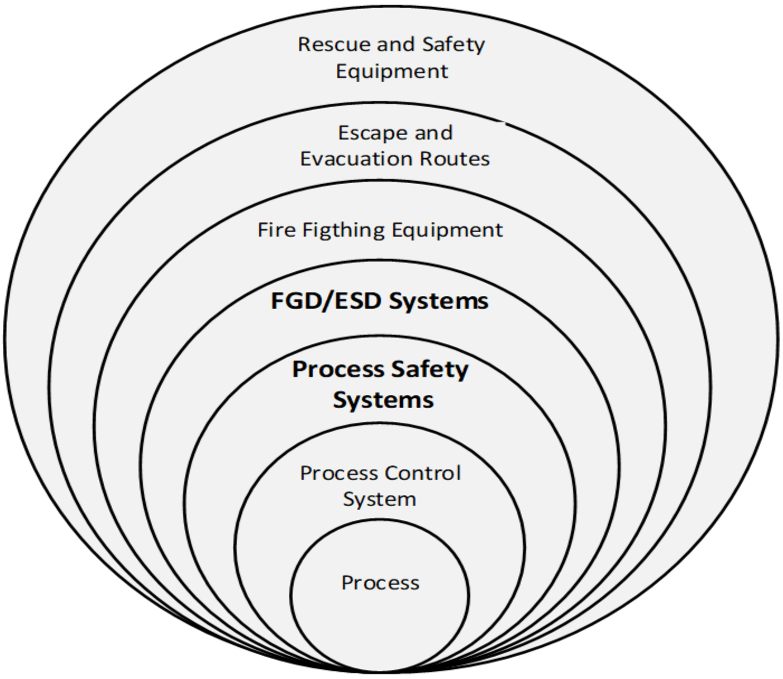


Figure 3-2 Barriers (onion model) – (25)

On the other hand, SIFs, which are parts of SISs, are independent protection layers that are intended to mitigate the hazard for specific systems which are called Equipment Under Control (EUC). The EUC may be various types of equipment, plants, apparatus or machinery. The objective of the process control system is to maintain the process in the given limits of some parameters. When any of them deviates from the preset values, process shutdown systems (PSD) are activated to close the EUC down. The actions required (for example; activation of alarms, closure of shutdown valves, etc.) are provided by the LS. While in general PSD is related to a specific EUC, the EUC of ESD system could be the entire facility when there is a potential for major accidents such as fire, gas leaks and loss of main power (9).

3.1.1 PROCESS SHUTDOWN (PSD) SYSTEM

The purpose of the PSD system is to detect abnormal operating conditions and initiate actions in order to prevent demand on the secondary process protection (e.g. relief valves), and to prevent damage and hydrocarbon release. It can be initiated by the ESD system, as well as the leak detected by such as low pressure.

If the PSD system fails on demand, the consequence will vary from affecting one piece of equipment with minor consequences to affecting larger parts of the process with large consequences for the whole installation.

A special type of PSD system is known as High Pressure Protection System (HIPPS). HIPPSs are particularly important as they are considered as a barrier between high- and low-pressure sections and upon failure, it may cause major accidents with catastrophic safety, environmental and commercial consequences. HIPPS is considered as the highest level of defence in systems facing with compressible fluids.

Figure 3-3 shows a reliability block diagram of the HIPPS which is formed of 3 Pressure Transmitters (PT) (2oo3 voting), single LS, 2 solenoids and 2 Emergency Shutdown Valves (ESV) (1oo2 voting). The PTs are the initiators and LS is a programmable logic controller (PLC). The signal from the initiator operates the solenoid and ESV out of two, that closes to protect the line against overpressure. The ESVs are installed in series; hence to protect the pipeline, it is sufficient that one of the ESVs closed.

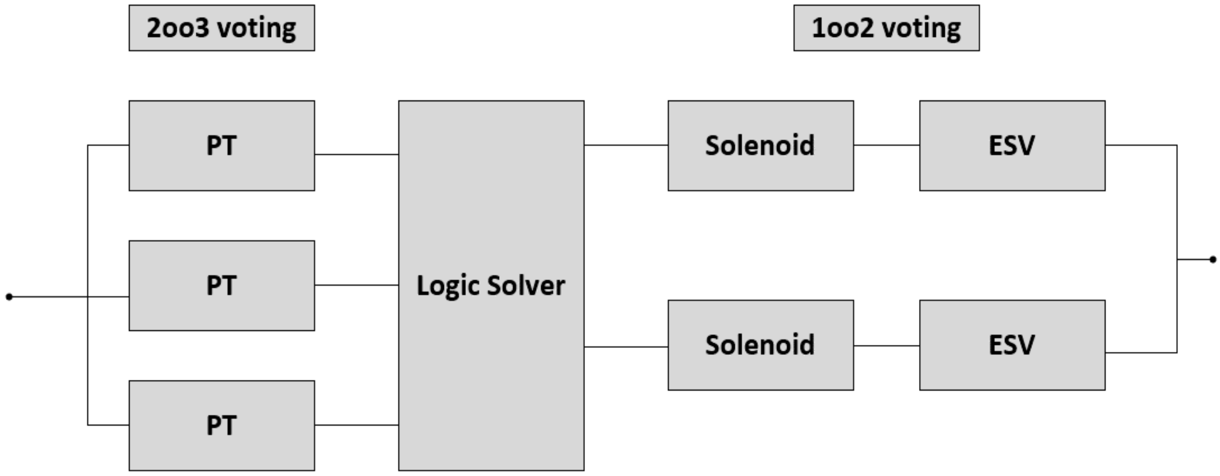


Figure 3-3 RDB of HIPPS

The calculations in this thesis give a special focus on the HIPPS system described above with predetermined component types. The updated PFD and SIL calculations for this function are given in Section 4.11.1.

3.1.2 EMERGENCY SHUTDOWN (ESD) SYSTEMS

The Emergency Shutdown (ESD) System is designed to decrease the consequence of emergencies. In other words, it is designed to detect a potentially hazardous condition and react to it by shutting the system down to protect personnel and facilities. The performance of the ESD system is determined by an uninterruptible power supply, hydraulic supply, instrumented air supply and HVAC to control pressure and temperature (23).

An ESD system as SIS may include more than one SIFs for different purposes such as “Isolation of one subsea well” function or “Immediate full platform isolation” function. Since the number of final elements to be activated upon a specified cause that differs from case to case, it is difficult to establish generic definitions for ESD SIFs.

Figure 3-4 describes the ESD node and one emergency shutdown valve (ESV) including solenoid and actuator known as an ESD segregation function. The function starts at the unit giving the demand (i.e. on detection of hydrocarbon leaks or a fire on the installation) and ends within the closure of the valve (22).

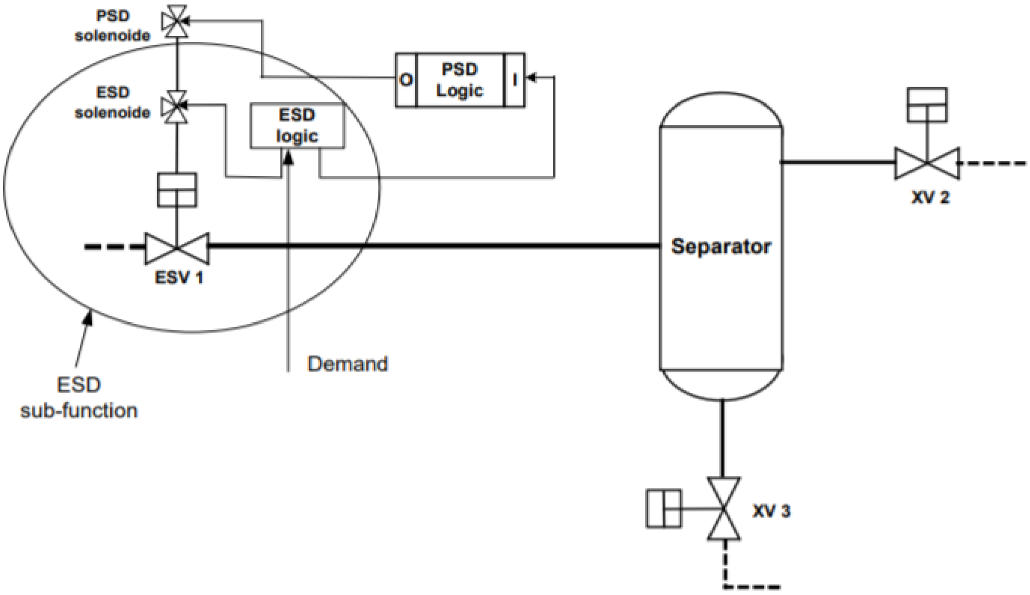


Figure 3-4: ESD segregation function

Figure 3-5 illustrates the reliability block diagram of function above (22).

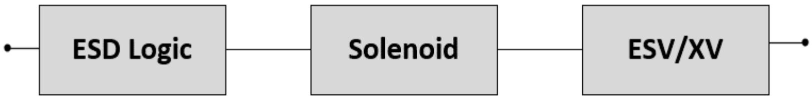


Figure 3-5 RBD of ESD segregation function

The risks associated with the failure of ESD system:

- If the ESD system is not isolating ignition sources during a gas leak, the probability of a fire/explosion will increase.
- If the ESD system fails in sectionalizing of process segments, leakage can continue over a longer period.
- If the ESD system fails in the opening, the duration of leakage/fire can increase.

The minimum SIL requirement calculated for ESD segregation function is SIL1 where a specific PFD requirement is less than 0.04 (22). The updated PFD and SIL calculations for this function is given in Section 4.11.2.

3.1.3 F&G DETECTION SYSTEM

The fire and gas detection system monitors the presence of flammable or toxic gases or fire to allow control actions to be taken either manually or automatically to prevent the explosion and fire. It requires an uninterruptable power supply to function continuously (23).

The system may include more than one function that is defined to cover the detection of fire or gas with corresponding alarm to the operators. Examples of such functions could be fire detection, gas detection, loss of overpressure detection, closure of HVAC fire damper and isolation of safety areas by the closure of fire doors.

Figure 3-6 illustrates the RBD for gas detection function with two detectors (1oo2 voting). The function includes flow transmitter (FT), 2 gas detectors and F&G logic. The function starts when a low flow of air aspiration is detected and ends when F&G actions will be activated.

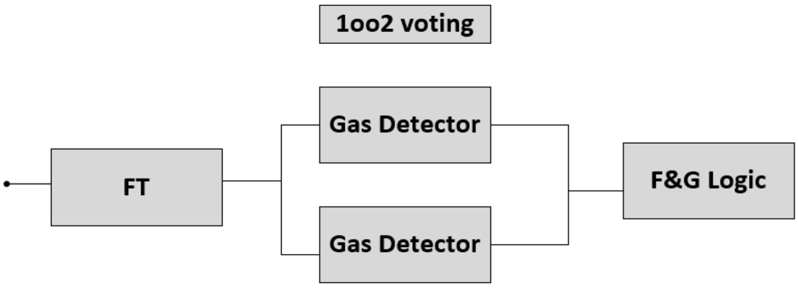


Figure 3-6 RBD for gas detection function with 2 detectors

Although there is no minimum SIL requirement defined for gas detection with two detectors, it is determined as SIL2 for the function with one detector (22). The updated PDS and SIL calculations for this function are given in Section 4.11.3.

3.2 QUANTIFICATION METHODS OF PFD

According to IEC 61508 (5) the SIL requirements to be achieved by each of the SIFs should be described in Safety Requirements Specification (SRS) documents prepared by operators. The requirements have been derived by estimating the achievable PFDs for each SIFs. Once the SIL requirement values are assessed, verification of such values is performed in order to check that the PFD for each SIF is inside the range of the PFD corresponding to the required SIL stated as in SRSs. During the operational phase, whether the SIFs meet SIL fulfill the requirements is ensured by performing some quantitative analysis based on whether generic failure data (supplied by different resources such as PDS, Exida, OREDA), vendor-specific failure data or operational data.

Compliance procedures should provide assurance that the SIS safety requirements specification has been met. As previously mentioned in Section 2.4.1, IEC 61508 provides the following methods to calculate PFDs for different configurations (votings) of SIFs.

- Basic Probabilistic Calculations;
- Reliability block diagram approach (PDS method);
- Boolean approach;
- Markovian approach;
- Petri nets and Monte Carlo approach;
- Other approaches (the AltaRica Data Flow)

However, neither the derivation nor the trustiness of these formulas has been discussed. There are different interpretations, implementations, as well as discussions of the formulas in the literature. For a detailed review of these methods see for example (26) and PDS method handbook (8). The PDS method introduced by SINTEF is chosen to quantify the safety unavailability and loss of production for SISs in this thesis due to its easy implementation and popularity in the Norwegian petroleum industry. In addition, the SIL value of the PDS method based on calculated PFDs of SIFs is conservative compared to other methods.

IEC61508 also states that since the probability of failure significantly increases with time the results of most probabilistic calculation methods are therefore meaningless beyond the useful lifetime of components. The only way of validating these models is to update failure rates beyond the useful lifetime. Therefore, after the failure rates will be re-estimated by the methodology explained in this thesis, the author will be stick to the PDS method and PFDs, as well as SILs, will be recalculated with the updated failure rate information by the PDS Method explained below.

3.2.1 PDS METHOD DESCRIPTION

The PDS method is an effective and practical approach to implementing the quantitative reliability aspects of an SIS. In the PDS method, PFD calculations are based on the typical simplified formulas from the PDS method handbook (8). The total PFD of the SIF can be calculated with the following formula, where “I” is the input device, “LS” the logic solver and “FE” the final element(s). The calculation of the total PFD of the SIF is shown as:

$$PFD_{(SIF)} = PFD_{(I)} + PFD_{(LS)} + PFD_{(FE)} \qquad \text{Eq.2}$$

Before PFD formulas for individual components are introduced, the failure classification proposed in the PDS method is reviewed.

Failure Mode Classification in the PDS method

The definitions of safe and dangerous failures are given in Section 2.1. PDS method takes these definitions to the further in component level and proposes the following classification given in Figure 3-7.

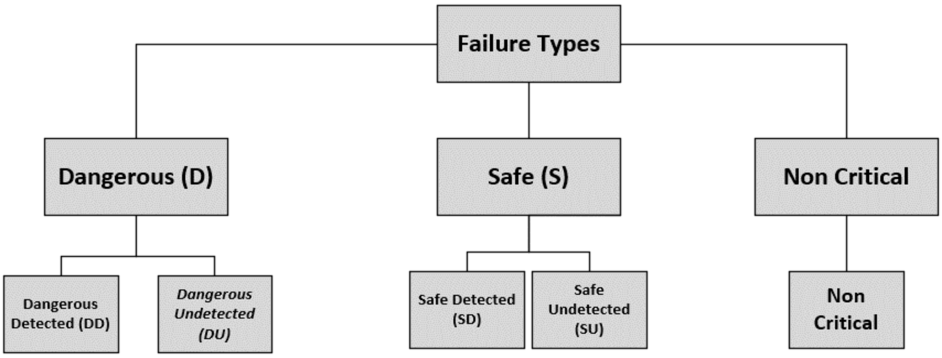


Figure 3-7 Failure mode classification in the PDS method

A dangerous failure is a failure that has the potential to put the safety-related system in a hazardous or fail-to-function state. A fraction of these failures, i.e. the “dangerous detected failures” will be revealed by diagnostics tests. The residual dangerous failures, not detected by self-test, are denoted as "dangerous undetected" (DU) failures. The reliability assessment intends to establish an estimate for DU failures.

In the PDS method, DU failures are considered important as they affect the functionality of SIFs on demand. These failures are only detected during functional tests. It is defined as " dangerous failures not detected by automatic self-testing or incidentally by personnel" in the PDS handbook (8). All the failure rates presented in this thesis are "dangerous undetected" failure rates, and all failure events are dangerous undetected failure events.

The overall reliability of SIF depends also on the configuration of the system which is expressed as the MooN (M-out-of-N). In the context of this thesis for the sake of simplicity, only 1oo1,1oo2 and 2oo3 votings of SIFs are considered. The formulas and explanation of parameters are provided in Table 3-1. See (8) for the formulas and related parameters for the various configurations.

Since the safety systems of interest in this thesis comply with the definition of low demand mode of operations in the PDS Handbook (8), the formulas provided below are valid only for low demand mode systems.

Table 3-1 PDS formulas for different votings of a component adapted from (8)

Voting	Common Cause Failures	Independent Failures	PFD$_{component}$
1oo1	-	$\lambda_{DU}T/2$	Common Cause Failures + Independent Failures
1oo2	$C_{1oo2} * \beta * \lambda_{DU}T/2$	$(\lambda_{DU}T)^2/3$	
2oo3	$C_{2oo3} * \beta * \lambda_{DU}T/2$	$(\lambda_{DU}T)^2$	

where:

λ_{DU} : DU Failure Rate

T: Proof test interval which is a periodic time interval (typically 6 or 12 months) that a test is carried out so that all critical faults of SIFs can be detected and corrected

β: Beta factor value (pre-defined value)

C_{MooN}: Modification factor value (pre-defined value)

When quantifying the reliability of redundant systems, it is important to distinguish between dependent and independent failures. In general, if a failure of one component causes failures of other identical modules in the safety system, it is called dependent. Dependent failures lead to common cause failures (CCF), i.e. simultaneous failure of more than one safety system. The way of taking CCF into account is using β, which represents a certain fraction of the failures which are common cause regardless of the configuration of the system. Since CCF may also change depending on the configuration (MooN) of the system, the modification factor value, C_{MooN}, is also defined in the PDS method. In short, β factor shows the fraction of common cause failures where C_{MooN} shows the effect of voting on the system failure rate due to the common cause. The following β and C_{MooN} values are used in the calculations in this thesis given as the PDS data handbook as presented in Table 3-2. (27).

Table 3-2 Beta and C factors for the components of interest adapted from (27)

Component	Voting	β	C_{MooN}
Gas detector	1oo2	0.06	1
PT	2oo3	0.04	2
ESV/XV	1oo2	0.03	1

Once the PFDs of individual components are calculated, PFD of SIF ($PFD_{(SIF)}$) is calculated by Eq.2 (summing up the individual PFDs of all components) and corresponding SIL is assigned as given in Table 2-2.

4 APPLICATION OF BAYESIAN METHOD FOR FAILURE RATE ESTIMATION

4.1 INTRODUCTION TO BAYESIAN METHODOLOGY

The analysis related to aging is the process of identifying the age-affected components and quantifying their impact on overall safety practices. A number of mathematical models exist for both theory and applications of aging. For example; in (28), the authors have presented several bivariate lifetime distributions forming the bathtub shape component failure. Those failure distributions have 2 change points as represented in Figure 1-3; which are the useful life phase and wear out phases are started. Family of bathtub shaped life distribution functions classified according to increasing order of sophistication, starting from quadratic models and ending with the mixed Weibull distribution. However, none of them gives the interval where the failure rate is exactly constant (which is the case for the useful lifetime). Instead, they seem to be well approximations of bathtub shape failure rate curves. Later, these distributions have been taken as useful starting points to construct a framework dealing with data at hand. Uncertainties of the model and incompleteness of data are handled by some researchers with the Bayesian approach. For example, Alztubas and Iesmantas (17) proposed a general Bayesian Methodology in which the constructed prior distribution has been selected amongst the several models presented by Lia and Xie (28).

Since Bayesian methodology can be used to generate uncertainty distributions combining different data sources including generic industry data and facility-specific data, it has an advantage when the data set consists of very low failure rates, and this is the case for SIF components.

The basis of the Bayesian model acknowledges that there is some prior knowledge about the failure distribution. Given some evidence about the distribution, the prior knowledge can be updated and produce an estimation more in line with the evidence. The evidence is given in the form of observations.

To illustrate the mathematical notation of Bayesian updates, let θ denote a parameter for a population and $p(x|\theta)$ is the probability function of a particular parameter when the experimental data is x. The problem is estimating the model parameter θ.

The relation known as Bayes Theorem is,

$$p(\theta|x) = p(x|\theta)p(\theta)/\, p(x) \propto p(x|\theta)p(\theta)$$

where

- $p(\theta|x)$ is the posterior probability (the beliefs after the evaluation of model parameters);
- $p(x|\theta)$ is the likelihood of experimental data x given “theta”;
- $p(\theta)$ is the prior probability of the event to occur (initial belief).

Bayesian method implementation starts with the evaluation of the initial probabilities before collecting new information, observing the event, conducting an experiment, etc. This is named "prior probability” and it is updated over time when new information is available. Then the final probabilities known as “posterior probabilities” are obtained (29).

This chapter is organized as follows:

- First, the construction, presentation, and corresponding assumptions of:
    - data;
    - the modeling framework;
    - computation in Python is given.
- Then, the theory is further elaborated on practical implementation. The failure rate results based on posterior distribution are presented through Section 4.5 to 4.10.
- Section 4.11 presents updated PFD/SIL performance for SIS of interest and Section 4.12 presents Sensitivity Analysis.

4.2 DATA RESOURCES

An important part of reliability analysis is to collect and choose the appropriate data. Normally, vendors are to document the failure data of their components in Safety Analysis Reports (SAR). There are also recognized data sources and the facilities may have their own failure data for selected components. Moreover, expert opinions that rely on the knowledge of experts in the specific technical field can also be used as input in reliability analysis.

There are four possible failure rate data resources to be used in this thesis:

a. generic data;

b. field data;

c. vendor data;

d. data based on experience (estimations of ORS experts).

However, according to (30), there is a need for a population of $x * t * n > 3{,}000{,}000$ hours in order to apply reliability methods based on field data alone; where x represents the number of registered DU failures, t is the aggregated time (in hours) in service of the entire population (where n is the population size). This requires wide range of data set, for example; assuming that the total aggregation time per component (the time interval where the data is collected) is 1 year and 10 months (16060 hours) and recorded number of failures is 1, the population size of 187 identical items (n=187) are needed to be observed.

The author recommends an approach where the field data is used first as the basis of the analysis if the criterion could be fulfilled. Second, if the vendor data is available and in good compliance with expert judgments, then it can be used as an input in the reliability analysis. If the vendor data is not acceptable by the experts, the failure rates recommended by experts can be used before the generic data which acquires less field-specific information compared to the experts' judgments and is more conservative. Lastly, generic data can be used as an input if the experts prefer a more conservative analysis. Of all 4 data types mentioned here, generic failure data is regarded as the most conservative by the experts to ensure satisfactory risk reduction in the industry. For the analysis conducted in this thesis, the selection method of data is illustrated in Figure 4-1.

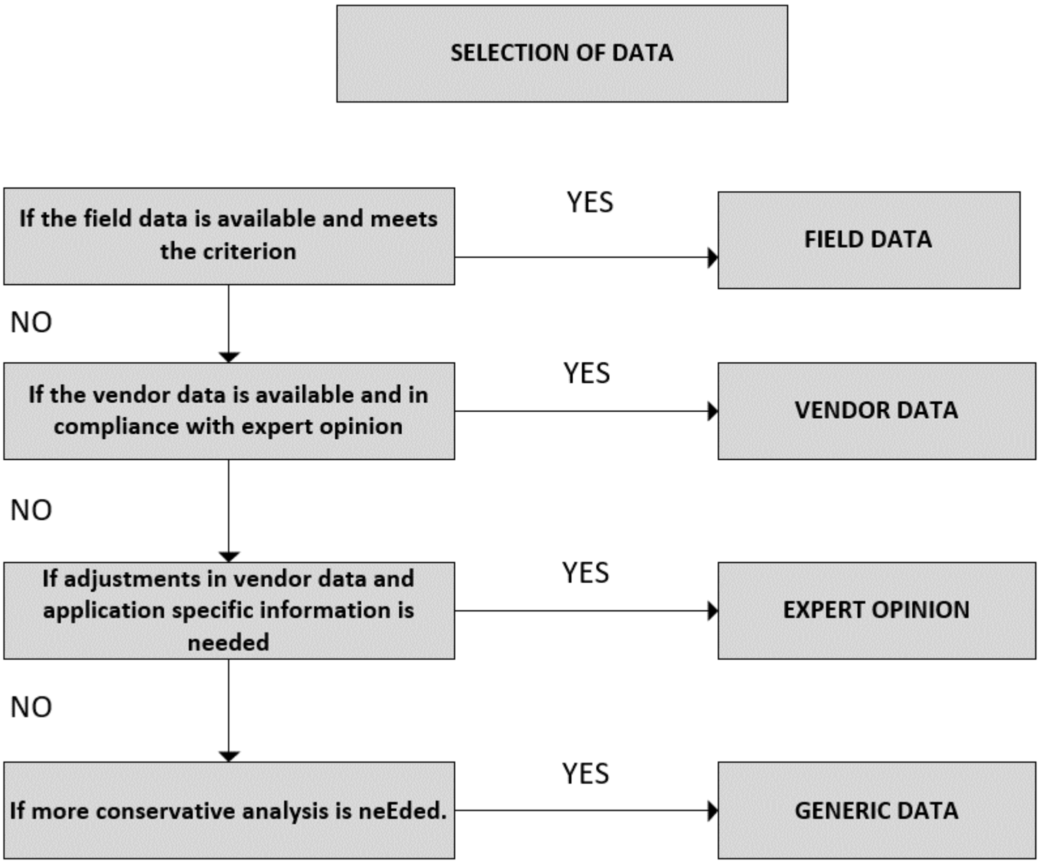


Figure 4-1 Selection of data

This selection method is also valid for other parameters considered in the analysis: i.e. proof test interval to calculate PFD values and useful lifetime of a component to set the posterior distribution parameters.

For the HIPPS system described in Section 3.1.1, ORS established a specific function with components whose vendors are industry leaders producing high-reliability safety applications. The components and their vendors are specified in Table 4-1.

For the PT component of HIPPS SIF, Emerson`s Rosemount 3051 Pressure Transmitter with 4-20mA HART (Coplanar Differential & Coplanar Gage) has been chosen. The Rosemount 3051 is a two-wire smart device programmed to send its output in case the detection of specified failure. For this component, Failure Modes, Effects and Diagnostic Analysis (FMEDA) document prepared by Exida (31) is reviewed. Exida is a well-known private company in the functional safety certification according to IEC 61511/ IEC 61508. Their report summarizes whether functional safety is achieved by the components or not. From FMEDA of Emerson`s Rosemount 3051 Pressure Transmitter (31). DU Failure Rates, Useful Lifetime and Proof Test Intervals are obtained as given in Table 4-1.

For LS component of HIPPS SIF, Yokogawa`s ProSafe SLS model has been chosen at the instigation of ORS as this is one of the most widely used and high reliable LS used in the industry. Prosafe SLS has the TUV certificate (32), which means it has been tested for safety and found to meet the minimum requirements of the German Equipment and Product Safety Act by Technischer Überwachungsverein (TUV) in German or Technical Inspection Association in English. Even though the examination report forming the basis of the TUV safety certificate of Prosafe SLS is not available online, it is stated in the certificate (32) that the solid-state, hard-wired ProSafe-SLS covers all requirements of SIL 3/4. Due to the unavailability of examination results, the DU failure rates, useful lifetime and proof test intervals are determined based on the judgments of ORS supervisors as given in Table 4-1.

For the final element component of HIPPS SIF, Mokveld`s RZD-X-SAV-S99 valve with solenoid has been chosen at the instigation of ORS as this is one of the most widely used and high reliable final elements used in the industry. Based on the Safety Manual and Safety Justification (SAM) document of final element S99 (refers to a valve and a solenoid together) (33), DU Failure Rates and Useful Lifetime are obtained as given in Table 4-1. Since SAM document (33) does not specify the proof test interval, it is determined based on the judgments of ORS supervisors (expert opinion).

Table 4-1 Specifications of the HIPPS components

Explanation	Manufacturer	λ_{DU} (per hour)	Useful Lifetime (years)	Proof Test Interval (hours)
PT (2oo3 voting)	Rosemount	3.29E-9	50[4]	8760
LS	Yokogawa Prosafe SLS	4.63E-8	12	8760
ESV with solenoid (1oo2 voting)	Mokveld	3.4E-8	6	4380

For the ESD system described in Section 3.1.2 with RBD given in Figure 3-5, the standard system with no vendor specifications is used and following DU failure rates (λ_{DU}) are obtained from generic data presented in (27). The useful lifetime and Proof Test Intervals are given by ORS based on their experience in the industry.

Table 4-2 Specifications of the ESD Segregation Function components

Explanation	λ_{DU} (per hour)	Useful Lifetime (years)	Proof Test Interval (hours)
ESD logic	0.8E-06	10	8760
Solenoid	0.8E-06	7	8760
ESV/XV	2.1E-06	10	8760

Gas Detection Function described in Section 3.1.3 with RBD given in Figure 3-6, the standard system with no vendor specifications is used and following DU failure rates (λ_{DU}) are obtained from generic data

---

[4] According to IEC 61508 (5), experiences have shown that the useful lifetime is often up to 12 years for pressure transmitters. However, the tantalum electrolytic capacitors that are used in the Rosemount 3051 have increase the useful lifetime up to 50 years. In Bayesian Analysis conducted in this thesis, the useful lifetime is assumed to be 12 years to generalize the posterior results

presented in (27). The useful lifetime and Proof Test Intervals are given by ORS based on their experience in the industry.

Table 4-3 Specifications of Gas Detection Function components

Explanation	λ_{DU} (per hour)	Useful Lifetime (years)	Proof Test Interval (hours)
FT	0.6 E-06	8	8760
Gas Detectors (1oo2 voting)	0.6E-06	8	8760
F&G logic	4.9E-06	10	8760

4.3 SETTING UP THE MODELLING FRAMEWORK

In reliability analysis, the failure rate is one of the most important concepts to understand the lifetime. The bathtub shape failure rates have to be defined with nonmonotonic functions, which therefore has to be different than commonly used lifetime distributions such as Weibull or Gamma. Therefore, Combined Lifecycle (CMBL) Failure Distribution introduced by Sandia National Laboratories and studied in (34) is deemed to be one of a useful distribution for quantification and analysis of lifetime data. Different distributions for different sections of the bathtub shape failure rate curve (see Figure 1-3) are used where the useful lifetime portion distribution has been captured with exponential distribution and the wear out (aging) portion has been defined by the normal distribution. The initial mortality section is not evaluated due to the reasons explained at the end of Section 2.1 in this thesis. The Bayesian Model has been created and run in Python[5] programming and code is provided in Appendix-B.

In (34), 3 methods have proposed for updating continuous failure distributions. Method 1 treats the different phases of the bathtub shape curve separately. Method 2 evaluates all the phases as a single distribution. Lastly, Method 3 uses the posterior distribution of either Method 1 or 2, and updates it when the new data becomes available. The author of this thesis has chosen Method 1 as a starting point.

Method 1: Basics

- Each section of the bathtub curve is treated separately;
- The appropriate prior is defined based on the section;
- Bayesian Methodology has been followed.

In this study, the likelihood function for useful lifetime (the constant failure rate) section is represented by the $exponential(\lambda)$ with an expected value $1/\lambda$ and wear-out phase (aging/beyond useful lifetime) is represented by $normal\ (\mu, \sigma)$.

---

[5] Python is an interpreted, general-purpose programming language.

Then, the PDF of lifetime distribution becomes (34);

$$f(t) = \begin{cases} \lambda_c * e^{-\lambda_c t}, & 0 \le t \le t_1 \\ \dfrac{1}{\sigma\sqrt{2\pi}} * e^{\frac{-1}{2\sigma^2}*(\frac{t-\mu}{\sigma})^2}, & t_1 \le t \le t_2 \le \infty \end{cases}$$

where

t_1: useful lifetime duration

t_2: the anticipated maximum operation time (40, in this thesis)

λ_c: failure rate for the constant failure rate section (useful lifetime)

μ: mean of the normally distributed (wear-out) section

σ: standard deviation of the normally distributed (wear-out) section

Since the useful lifetime is being modeled by $exponential(\lambda)$, a typical conjugate prior becomes $gamma(\alpha, \beta)$. Then the Bayesian prior is given by (34);

$$g(\lambda|\alpha,\beta) = \frac{1}{\boldsymbol{\Gamma(\alpha)\beta^{\alpha}}} * \lambda^{\alpha-1} * e^{-\lambda/\beta}$$

the posterior becomes (34),

$$g(\lambda|t,\alpha,\beta) = \frac{1}{\boldsymbol{\Gamma(\alpha+1)\left(\frac{\beta(\alpha+1)}{\beta t+1}\right)^{\alpha+1}}} \lambda^{\alpha} e^{-\lambda/\beta} \lambda e^{-\lambda(t+\frac{1}{\beta})}$$

And the expected value can be expressed by (34):

$$E(\lambda|t,\alpha,\beta) = \frac{\boldsymbol{\beta(\alpha+1)}}{\boldsymbol{\beta t+1}}$$

which becomes the new estimate for λ_c.

For the wear-out section where the likelihood is $N(\mu_{CMBL}, \sigma_{CMBL})$ and the conjugate Bayesian prior is $N(\mu, \sigma)$, the posterior parameters become $N(\mu_p, \sigma_p)$ where (34)

$$\mu_P = \frac{\dfrac{\mu_{CMBL}}{{\sigma_{CMBL}}^2} + \dfrac{t_1}{\sigma^2}}{\dfrac{1}{{\sigma_{CMBL}}^2} + \dfrac{1}{\sigma^2}}$$

$$\sigma_P = \frac{1}{\dfrac{1}{{\sigma_{CMBL}}^2} + \dfrac{1}{\sigma^2}}$$

when the single data point t_1 is observed.

μ_{CMBL}: mean of the normally distributed (wear-out) section

σ_{CMBL}: standard deviation of the normally distributed (wear-out) section

μ_P: posterior mean (when t_1 is observed)

σ_P: posterior standard deviation (when t_1 is observed)

Since the transition of failure rate curve from constant failure rate part to normally distributed part is required to be smoother, Briand and Huzurbazar (35) have defined the following equation to be solved to obtain A (the transition parameter).

$$\lambda_c * e^{-\lambda_c t_1} = A* \frac{1}{\sigma_P\sqrt{2\pi}} * e^{\frac{-1}{2\sigma_P{}^2}*(t_1-\mu_P)^2}$$

Then, the failure rate values for the wear-out section are multiplied by A (the transition parameter) and presented in the failure rate curve.

4.3.1 DETERMINING THE INITIAL VALUES

The expert judgment approach to inform priors of the Bayesian Model relies on the knowledge of experts in the specific technical field who arrive at best estimates of the distribution of the probability of a parameter or basic event. This approach is typically used when the required information is very limited or unavailable. Such a situation is usual in studying rare events such as the failure of safety-related systems. Ideally, this approach provides a probability distribution with the parameters required to define prior distributions which represent the experts' subjective knowledge. The subjective knowledge is collected by the set of questions regarding the topic of interest. Then, these opinions related to parameters are evaluated to obtain the best estimates. The process of obtaining these estimates is typically called "expert judgment elicitation," or simply "expert judgment". Meyer and Booker (36) have provided a practical guide for the procedures of gathering, eliciting and analyzing expert judgment in Bayesian Analysis. Morris (37) has explained 3 methods for elicitation: (a) weighting schemes, (b) consensus method and (c) calibration approach. The methodologies presented in (36) and (37) and many other valuable works are applicable in many areas such as project management, marketing, investment decision making, where the experts have a good understanding of the factors. Furthermore, Noortwjik et al. (38) have described the methodology of the use of expert opinion in the Bayesian Analysis for the maintenance optimization, i.e. updating of the consensus distribution with the availability of the additional failure and maintenance data. However, it is not always easy to get accurate answers about the mathematical terms in specific areas such as maintenance optimization. There are not many qualified experts to answer a question such as "if you were going to represent your belief about the predictive maintenance interval with Weibull distribution, what would shape and scale parameters be?" (39).

Cook (40) has argued that it would be easier to capture the prior belief if the questions of distribution quantiles were asked to the experts rather than the means and/or variances. Simply, he says that it is easier to obtain Weibull distribution's shape and scale parameters out of expert comments if, for example, experts say " the 13% of identical components fail within the first 10 hours of their lifetime, while 75% of them fail within 71 hours, instead of giving their opinions about what would shape and scale parameters be. He has developed a methodology for well-known distributions to obtain the distribution parameters to satisfy two quantiles.

Suppose we want to find the mean and standard deviation of a normal distribution of parameter X. µ and σ are defined such that (40);

$P(X < x_1) = p_1$ and $P(X < x_2) = p_2$ are both satisfied where $x_1 < x_2$ and$p_1 < p_2$ where p_1 and p_2 are the lower and upper quantiles respectively.

X has the same distribution as $\sigma Z+$ µ where Z is a normal random variable with mean 0 and standard deviation 1. Let Φ denotes the CDF of Z, then the equations can be written as

$$P\left(Z < \frac{x_i - \mu}{\sigma}\right) = \Phi\left(\frac{x_i - \mu}{\sigma}\right) = p_i$$

with the solutions

$$\sigma = \frac{x_2 - x_1}{\Phi^{-1}(p_2) - \Phi^{-1}(p_1)} \text{ (40)}$$

$$\mu = \frac{x_1 \Phi^{-1}(p_2) - x_2 \Phi^{-1}(p_1)}{\Phi^{-1}(p_2) - \Phi^{-1}(p_1)} \text{(40)}$$

Similarly, suppose we want to find the shape and scale parameters of gamma distribution with shape α and scale β. Let F be the CDF of gamma (40);

$$F\ (x;\alpha,\beta) = \frac{1}{\boldsymbol{\Gamma(\alpha)\beta^{\alpha}}} \int_0^x t^{\alpha-1} * e^{-\frac{t}{\beta}} dt.$$

There are unique α and β satisfying both $F(x_1;\alpha,\beta) = p_1$ and $F(x_2;\alpha,\beta) = p_2$ where

$$\beta = \frac{x_i}{\mathrm{F}^{-1}(p_i;\alpha,1)} \text{ and } \frac{\mathrm{F}^{-1}(p_2;\alpha,1)}{\mathrm{F}^{-1}(p_1;\alpha,1)} = \frac{x_2}{x_1} \text{ (40).}$$

In this thesis, the expert opinions are used in determining the prior distributions of parameters to reflect the initial knowledge. As explained earlier, since the useful lifetime is being modelled by $exponential(\lambda)$, a typical conjugate prior chosen to be $gamma(\alpha,\beta)$. Additionally, for the wear-out section where the likelihood is $N(\mu_{CMBL},\sigma_{CMBL})$, the typical conjugate prior of the mean parameter is chosen to be $N(\mu,\sigma)$. To determine the initial values of α,β,μ and σ based on the expert opinions, the abovementioned formulas are used.

Cook (41) has provided a Python code for determining distribution parameters to satisfy two quantile conditions as given in Appendix-A.

The following opinions are shared by an expert in ORS Consulting.

Expert Opinion 1: The expert believes that 10% of failures occur within 5 years of a component's life and 20% of failures occur within 11 years of component's life during the useful lifetime. The expert also believes that this is the case for all components of interest in this thesis.

x_1 =10%,

p_1 =5,

x_2 =20% and

p_2 =11

are used to find gamma prior parameters.

Expert Opinion 2: For emergency shutdown valves, the expert believes that 40% of failures occur within 5 years and 85% of failures occur within 30 years when the component's useful lifetime is exceeded. The expert also believes that this is the case for all components of interest in this thesis.

It means that

x_1 =40%,

p_1 =5,

x_2 =85% and

p_2 =30

are used to find normal prior parameters.

The python codes given in Appendix-A are executed and the following values are obtained for the initial parameters of gamma and normal distributions.

$\alpha = 1,$

$\beta = 56,$

$\mu = 10$ and

$\sigma = 20$

Since the gamma prior is used to model the useful lifetime, the expert's opinion about gamma distribution is restricted by the useful lifetime of a component which varies by each component. Additionally, expert argues that there should be a correlation between the generic failure rates and scale parameter β of the gamma distribution. The expert believes that inverses of failure rates of each component are close enough to parameter β obtained from the python code. This claim could be easily checked and validated. For example, the yearly failure rate of ESDV is 0.018, its inverse is 55.5 which is almost 56. In compliance with the expert opinion, to make the gamma priors component-specific, the inverses of generic failure rates are used as initial values of scale parameters for each component instead of a constant value 56.

The expert has no opinion about the prior distribution of the variance parameter of the normal likelihood used to represent the wear-out section. Therefore, the Half-normal distribution, which is knowns as folded normal distribution is chosen to be the non-informative prior distribution for the variance parameter. This is suggested to be used in (42) if the data size is small. Half normal distribution is defined by a single parameter σ,which is chosen to be 10 (half of the variance parameter of normal prior for mean parameter, which previously was found to be 20.).

In summary, the prior distributions of likelihood parameters have set to be;

$$\lambda \sim \mathrm{gamma}\ (1, \frac{1}{generic\ failure\ rate(yearly)})$$

$$\mu \sim Normal\ (10{,}20)$$

$$\sigma \sim HalfNormal(10)$$

4.4 BAYESIAN COMPUTATION WITH PYTHON

PyMC3 package of Python provides tools of automatic Bayesian inference for user-defined probabilistic models based on Markov Chain Monte Carlo (MCMC) sampling with algorithms such as No-U-Turn Sampler (NUTS) and a self-tuning variant of Hamiltonian Monte Carlo (HMC). These samplers work well with complex posterior distributions by taking advantage of gradient information from the likelihood to achieve much faster convergence (43).

Python offers easy-to-read syntax and integration with scientific libraries which makes it easy to write and use custom statistical distributions, samplers and transformation functions, which are essentials of Bayesian analysis (43).

Theano, which is the linear algebra compiler library optimizes user-defined mathematical computations to produce efficient implementations (44). Since Theano also automatically optimizes the likelihood's computational graph in terms of speed, it is installed together with the PyMC3 package while performing Bayesian Analysis.

In this thesis, PyMC3 and Theano are used to solve general Bayesian statistical inference and prediction problems. The relevant steps including installation, model definition and fitting, as well as performing posterior analysis are explained step by step.

First, PyMC3 and Theano are installed using 'pip'

```
# !pip install pymc3
# !pip install theano
```

NumPy, SciPy, and Matplotlib are also required for efficient usage of PyMC3. Those, as well as optional libraries such as Pandas, are also installed using 'import'.

```
import pymc3 as pm
import numpy as np
from scipy.stats importnnorm, expon, gamma
import matplotlib.pyplot as plt
import arviz as az
import theano.tensor as tt
import theano
import pandas as pd
```

NumPy: The main library for scientific computing with Python. It supports large, multi-dimensional arrays, matrices, and complex mathematical functions to operate on the array or matrices (45).

SciPy.stats: In addition to NumPy, SciPy supplies new advanced functions such as integration, differential equation (46). SciPy.stats is the subpackage including a large number of probability distributions.

Matplotlib: Package for 2D plotting (47).

Pandas: Data analysis library (48).

Arviz: Package for exploratory analysis of Bayesian models including functions to perform posterior analysis and model fitting.

To specify the model, the required components (gamma, expon and norm) are also installed. The following code is used to implement the Bayesian Model.

```
def find_mixed(obs, rate):
    with pm.Model() as mixed:
        rate_b=pm.Gamma ('rate', alpha=1, beta=1/rate)
        expo=pm.Exponential.dist(lam=rate_b)
        mu=pm.Normal('mu', 10, 20)
        sigma=pm.HalfNormal('sigma', 10)
        normal=pm.Normal.dist(mu=mu, sigma=sigma)

        w=pm.Dirichlet('w', a=np.array([1,1]))

        like=pm.Mixture('like', w=w, comp_dists=[expo, normal], observed=obs)
        trace=pm.sample(draws=1000, cores=4)
        pm.traceplot(trace)
        ppc = pm.sample_posterior_predictive(trace, samples=500, model=mixed)
        return trace, ppc
```

The first line (def find_mixed) create a model object which is a container for the model random variables. “with” function includes specifications for mu, sigma and beta that are the priors for unknown model parameters.

The likelihood function for the useful lifetime is exponential with rate_b value, where rate_b’s (failure rates’) prior distribution is chosen to be gamma distribution with a shape parameter 1 and scale parameter ‘1/generic failure rate’.

The likelihood function for beyond useful lifetime is normal with parameters mu and sigma where mu’s prior distribution is also normal distribution with mean 10 and standard deviation 20. The mean value has chosen to be 10 based on expert opinion. Sigma’s prior distribution is chosen to be Half-normal as recommended in (42). Different choices of prior distributions and their effects on posterior have been discussed in Section 4.12.

Then, the PyMC3 mixture model, which is often used to model subpopulation heterogeneity is defined by Dirichlet function. Since the useful lifetime is modeled by exponential distribution wile beyond useful lifetime is modeled by normal distribution ensures the heterogeneity, this is where the two sub-populations get weighted equally.

Trace function draws 1000 samples from the posterior distribution and then 500 of them are used to estimate the posterior parameters.

The next set of codes groups the failure event times by checking they either belong to useful lifetime interval or beyond it. The failure rates of the exponential and normal distribution are defined and calculated accordingly.

```
def new_find_failure(alphas, betas, mus, sigmas, t, useful):
    frs=[]
    i=0
    j=0
    first=True
```

```
    for te in t:
        if te<useful:
            fr=betas[i]*np.power(np.exp(1),(-betas[i]*te))
            i +=1
        else:
            fr=1 / (np.sqrt(2 * np.pi) * sigmas[j]) * np.exp(-1 / 2 * np.square((te
- mus[j])) / (np.square(sigmas[j])))
            j +=1
        frs.append(fr)
    plt.plot(t,frs)
    return frs
```

Next, the model inputs are set.

```
useful=
maxy=
failure_r=
data=np.array([])
```

‘useful’ is the useful lifetime of the components, ‘maxy’ is the upper limit of the years until the failure rates are to be estimated (40 in this case) and ‘failure_r’ is the generic failure rate of the component. ’data’ consists of failure event data (in years) collected in a given time interval (interval-censored data).

As explained in Section 4.3, the parameter ‘A’ is defined in order to make the transitions from exponential to normal likelihood softer. After normal posterior failure rates are multiplied with ‘A’ value.

```
def find_A(useful, mu, sigma, rate):
    rc1=rate*np.exp(-rate*useful)
    rc2=1 / (np.sqrt(2 * np.pi) * sigma) * np.exp(-1 / 2 * np.square((useful - mu))
/ (np.square(sigma)))
    return rc1/rc2
A=find_A(useful, mu, sigma, rate)
A
```

Finally, the failure rate functions of posteriors distributions are set and plotted.

```
def find_rates(mu, sigma, rate, useful, A):
    tcs=np.linspace(0,maxy, maxy)
    t2=tcs[tcs>useful]
    rc1=np.repeat(rate,useful)
    rc2=(A-abs_A)*(norm.pdf(tps, loc=mu, scale=sigma)/(1-norm.cdf(tps, loc=mu, scal
e=sigma)))
    rcs=np.concatenate([rc1, rc3])
    plt.axvline(useful, color='red', linestyle='--')
    plt.plot(tcs, rcs/8760)
    return rcs, rc2
```

No-U-Turn Sampler (NUTS) Algorithm

Given data and model specifications and initial values, the NUTS algorithm which is Proposed by Hoffman and Gelman (49) and is the special case of Hamilton Monte Carlo (HMC-also known as Hybrid Monte Carlo) allows the parameters of high-dimensional target distributions to converge much more quickly than simpler methods like Metropolis or Gibbs algorithms. It avoids the random walk behavior and sensitivity to correlated parameters (49). PyMC3 package of python automatically performs NUTS.

Suppose there is a ball randomly rolling between valleys and hills with a certain kinetic energy. The area where the ball is moving is represented by the posterior distribution. Every time the algorithm wants to take the MCMC sample, it randomly picks the kinetic energy and starts rolling the ball from the previous position of where it was. This is an example of a random walk in the Metropolis-Hasting Algorithm (50). The process is repeated many times and the samples are accepted or rejected based on the acceptance or rejection criteria set initially. In the HMC method, the new position of the ball is not determined randomly, instead; it is based on some parameters shaping the posterior. This increases the chance of acceptance of samples as the next position of the ball is determined by which direction is the posterior function increases/decreases. When the HMC algorithm is implemented, human intervention is required to decide some parameters which shape the movement of the ball. An experienced user must set the parameters according to the posterior distribution. However, the NUTS algorithm, the improved version of HMC, optimizes these parameters by simulating the movements of the ball in one direction and twice as many in the other direction. The algorithm stops when U-Turn is required to avoid the ball visiting the same positions multiple times. This decreases the probability of rejection of the sample and thus, brings the faster convergence (51),(52).

For the mathematical derivation of HMC and NUTS, see Hoffman and Gelman (49). The author also recommends the reader to check the simulation of Metropolis-Hasting, HMC and NUTS algorithms from the following link created by Chi Feng (53): https://chi-feng.github.io/mcmc-demo/

Python code presented in this section has been run for the different components after the inputs (i.e 'useful', 'failure_r' and 'data') are set according to the values for each component. The results of the Python code have been presented from Section 4.5 to 4.9.

4.5 MODELLING FOR EMERGENCY SHUTDOWN VALVES (ESV/XV)

ESV/XVs are components of the ESD Segregation Function whose RBD is given in Figure 3-5. ORS has conducted a reliability assessment study for ESV/XVs in 2017. This periodic reliability assessment was based on the review of failure records in SAP revealed during scheduled assurance and maintenance and unscheduled maintenance. The collected input data from 2011 to 2016 includes 11 lifetime observations of 86 components. The criteria (explained in Section 4.2) for a population to be $x * t * n > 3000000$ hours in order to apply reliability methods based on field data has been met as 11*86*8760*6=49,721,760.

Therefore, Bayesian Methodology presented in this thesis has been applied to model the interval-censored data set of: [(1463,6359,8928,9680,9586,5413,8286,9020,6954,7613,2198)/365]. The details of the dataset are given in Table 0-1 in Appendix-C.

The useful lifetime for emergency shutdown valves is 10 years ($t_1 = 10$, see Table 4-2.) Based on this, the dataset has been divided into 2 different data sets concerning where the lifetime data observation points fall in the bathtub curve. By this, data sets of [4.01, 6.02] and [17.42, 24.46, 26.52, 26.26, 14.83, 22.70, 24.71, 19.05, 20.86] are formed with underlying distributions for the former data set, $exponential(1/\beta)$ and for the latter data set, $Normal(ɥ, \sigma)$.

Based on MCMC simulations, the posterior means of $\beta = 0.041$, $ɥ = 21.03$ and $\sigma = 5.83$ are obtained. Figure 4-2 depicts the posterior densities and Table 4-4 gives the summary of statistical values for β, ɥ and σ.

Table 4-4 Summary Statistics

Parameter	Mean	Standard Deviation	HPD_2.5%	HPD_97.5%	MCSE
м	21.026	2.898	15.265	25.533	0.079
β	0.04	0.025	0	0.083	0.001
σ	5.83	2.63	1.81	10.887	0.081

Mean: mean of the estimations of variables

Standard Deviation: standard deviation of estimations of the variables

HPD: highest posterior density; which is also known as the credible interval. In this case, the probability of the value of the posterior parameter м lies between 15.265 and 25.533 is 95%.

MCSE: Monte Carlo Standard Error. It shows the estimated accuracy of Monte Carlo posterior samples and expected to be close to zero (54).

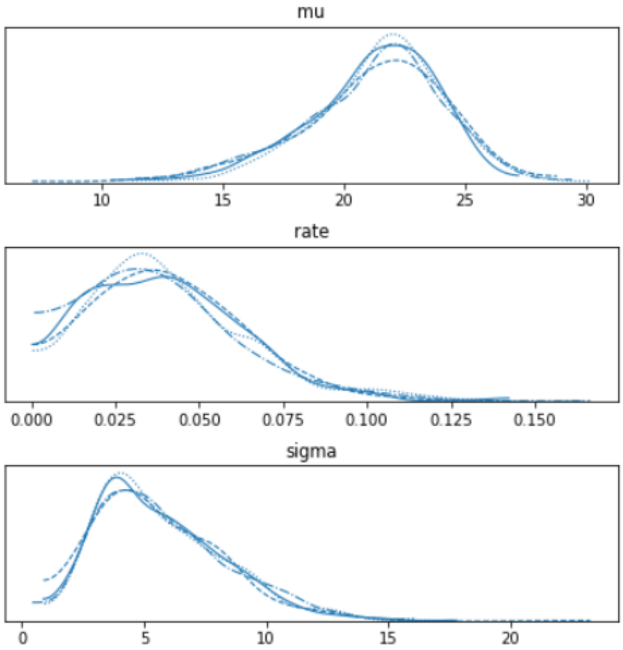


Figure 4-2 Posterior densities of м, β (rate), and σ.

The left graph of Figure 4-3 shows the posterior density until useful lifetime (blue line) which is $exponential\ (1/\ 0.04)$ and the right graph is the posterior density beyond useful lifetime (orange line) which is $Normal\ (21.406, 5.83)$.

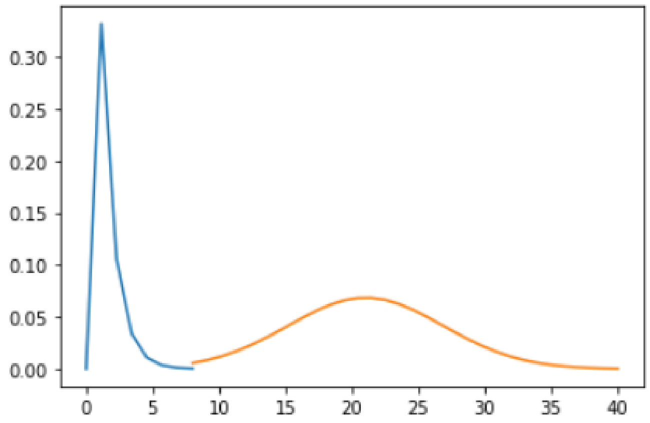


Figure 4-3 Posterior densities of ESV/XV

Figure 4-4 shows the failure rate curve for 40 years where it is constant until year 10 and then increasing beyond it while A (the transition parameter) is found to be 3.64. (Y-axis denotes the hourly failure rate while X-axis is the years).

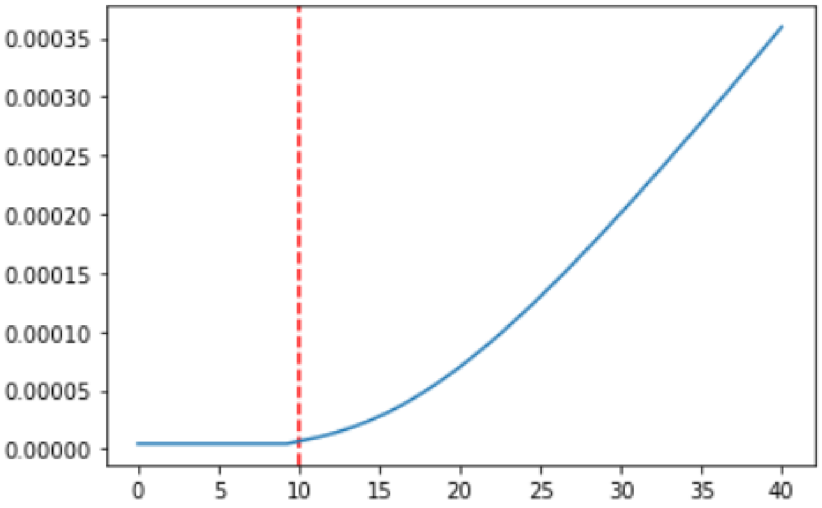


Figure 4-4 Failure rate curve (red line denotes the end of the useful lifetime)

Failure Rates

The generic failure rate for ESD valves 2.1E-6 per hour (1.8E-2 per year) (27) within the useful lifetime of 10 years. The constant failure rate obtained from the posterior model for ESD valves is 4.52E-06 per hour.

Table 4-5 Failure Rates of ESD Valves (per hour)

ESD Valves							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	4.52E-06	11	7.50E-06	21	7.44E-05	31	2.09E-04
2	4.52E-06	12	1.04E-05	22	8.57E-05	32	2.24E-04

3	4.52E-06	13	1.40E-05	23	9.75E-05	33	2.40E-04
4	4.52E-06	14	1.85E-05	24	1.10E-04	34	2.56E-04
5	4.52E-06	15	2.38E-05	25	1.23E-04	35	2.72E-04
6	4.52E-06	16	3.00E-05	26	1.36E-04	36	2.88E-04
7	4.52E-06	17	3.72E-05	27	1.50E-04	37	3.04E-04
8	4.52E-06	18	4.52E-05	28	1.64E-04	38	3.20E-04
9	4.52E-06	19	5.42E-05	29	1.79E-04	39	3.36E-04
10	4.52E-06	20	6.39E-05	30	1.94E-04	40	3.53E-04

Figure 4-5 Failure Rates of ESD Valves

4.6 MODELLING FOR PRESSURE TRANSMITTERS

PTs are components of HIPPS Function whose RBD is given in Figure 3-3. ORS has conducted a reliability assessment study for PTs in 2017. This periodic reliability assessment was based and review of failure records in SAP revealed during scheduled assurance and maintenance. The collected input data from 2011 to 2016 includes 34 lifetime observations of 92 components. The criteria (explained in Section 4.2) for a population of $x * t * n > 3000000$ hours in order to apply reliability methods based on field data has been met as 34*92*8760*6=164,407,680.

Therefore, Bayesian Methodology presented in this thesis has been applied to model the interval-censored data set of: [1178,7757,4958,6267,442,1915,5620,5207,6477,3443,5166,5935,5951,9146,5041,6118,4551,6651,7154,6321,4447,6347,6351,2043,6851,6072,2214,6545,7619,3649,4068,4771,7610,5228])/365. The details of the dataset are given in Table 0-5 in Appendix-C.

The useful lifetime for emergency shutdown valves is 12 years ($t_1 = 12, \text{see Table 4-1, footnote}$). Based on this, the dataset has been divided into 2 different data sets concerning where the lifetime data observation points fall in the bathtub curve.

Based on MCMC simulations, the posterior means of $\beta = 0.056, \mu = 15.56$ and $\sigma = 4.20$ are obtained. Figure 4-6 depicts the posterior densities and Table 4-6 gives the summary of statistical values for β, μ and σ.

Table 4-6 Summary Statistics

Parameter	Mean	Standard Deviation	HPD_2.5%	HPD_97.5%	MCSE
μ	15.566	1.15	13.331	17.62	0.037
β	0.056	0.031	0	0.104	0.001
σ	4.204	1.186	2.291	6.39	0.048

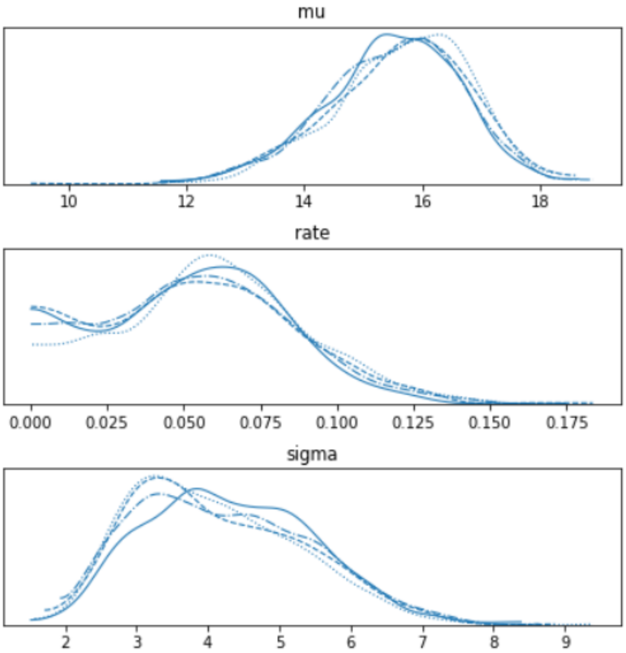


Figure 4-6 Posterior densities of м, β(rate) and σ

The left graph of Figure 4-3 shows the posterior density until the useful lifetime (blue line) which is $\mathrm{exponential}\,(1/0.056)$ and the right graph is the posterior density beyond useful lifetime (orange line) which is $Normal\,(15.56, 4.20)$.

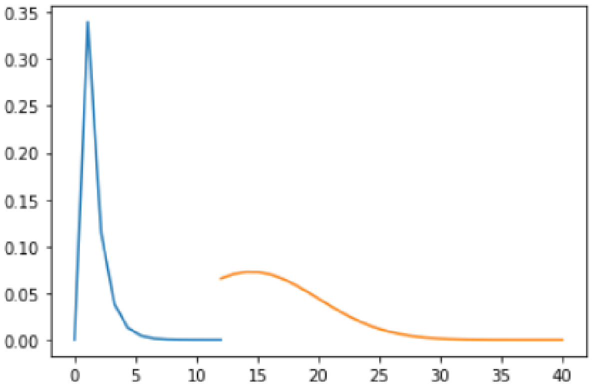


Figure 4-7 Posterior densities of PTs

Figure 4-8 shows the failure rate curve for 40 years where it is constant until year 12 and then increasing beyond it while A (the transition parameter) is found to be 0.43. (Y-axis denotes the hourly failure rate while X-axis is the years).

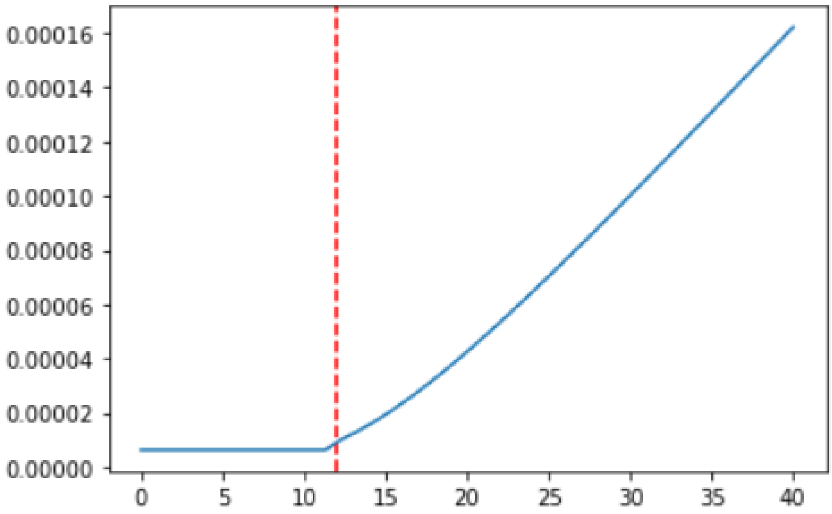


Figure 4-8 Failure rate curve

Failure Rates

The generic failure rate for PTs 3.29E-9 per hour (2.8E-5 per year) (27) within the useful lifetime of 12 years. The failure rates obtained from the posterior model for pressure transmitters valves are summarized in Table 4-7.

Table 4-7 Failure Rates of PTs (per hour)

PTs							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	6.37E-06	11	6.37E-06	21	4.53E-05	31	1.05E-04
2	6.37E-06	12	6.37E-06	22	5.08E-05	32	1.11E-04
3	6.37E-06	13	1.03E-05	23	5.65E-05	33	1.17E-04
4	6.37E-06	14	1.34E-05	24	6.23E-05	34	1.24E-04
5	6.37E-06	15	1.70E-05	25	6.81E-05	35	1.30E-04
6	6.37E-06	16	2.09E-05	26	7.41E-05	36	1.37E-04
7	6.37E-06	17	2.53E-05	27	8.01E-05	37	1.43E-04
8	6.37E-06	18	2.99E-05	28	8.62E-05	38	1.49E-04
9	6.37E-06	19	3.48E-05	29	9.24E-05	39	1.56E-04
10	6.37E-06	20	4.00E-05	30	9.86E-05	40	1.62E-04

4.7 MODELLING FOR ESD LOGIC

ESD logics are components of ESD Segregation Function whose RBD is given in Figure 3-5. ORS has conducted a reliability assessment study for ESD logics in 2020. This periodic reliability assessment was based and review of failure records in SAP revealed during scheduled assurance and maintenance and unscheduled maintenance. The collected input data from 2003 to 2019 includes 4 lifetime observations of 128 components. The criteria (explained in Section 4.2) for a population of $x * t * n > 3000000$ hours in order to apply reliability methods based on field data has been met as 4*128*8760*17=76,247,040.

Therefore, Bayesian Methodology presented in this thesis has been applied to model the interval-censored data set of: [(1178,3011,9016,13032)/365]. The details of the dataset are given in Table 0-2 in Appendix-C.

The useful lifetime for ESD logic is 10 years ($t_1 = 10$, see Table 4-2). Based on this, the dataset has been divided into 2 different data sets concerning where the lifetime data observation points fall in the bathtub curve.

Based on MCMC simulations, the posterior means of $\beta = 0.02, \mu = 15.11$ and $\sigma = 10.09$ are obtained. Figure 4-9 depicts the posterior densities and Table 4-8 gives the summary of statistical values for β, м and σ.

Table 4-8 Summary Statistics

Parameter	Mean	Standard Deviation	HPD_2.5%	HPD_97.5 %	MCSE
м	15.114	13.991	-11.322	40.095	0.39
β	0.019	0.006	0.007	0.027	0
σ	10.092	5.847	0.213	19.558	0.157

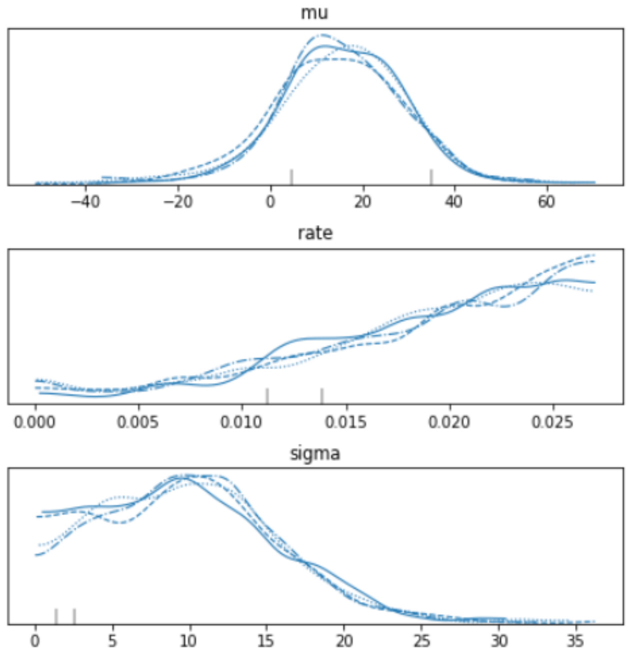


Figure 4-9 Posterior densities of м, β (rate), and σ.

The left graph of Figure 4-3 shows the posterior density until useful lifetime (blue line) which is $exponential(1/0.02)$ and the right graph is the posterior density beyond useful lifetime (orange line) which is $Normal\ (15.11{,}10.09)$.

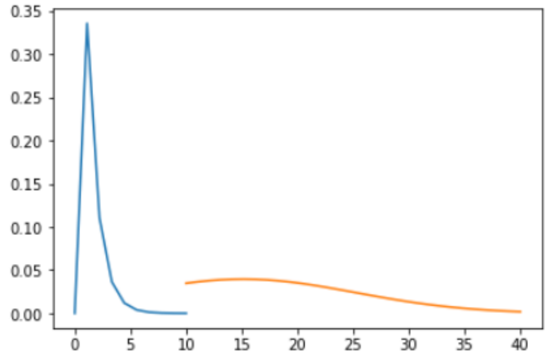


Figure 4-10 Posterior densities of ESD logic

Figure 4-11shows the failure rate curve for 40 years where it is constant until year 10 and then increasing beyond it while A (the transition parameter) is found to be 0.44 (Y-axis denotes the yearly failure rate while X-axis is the years).

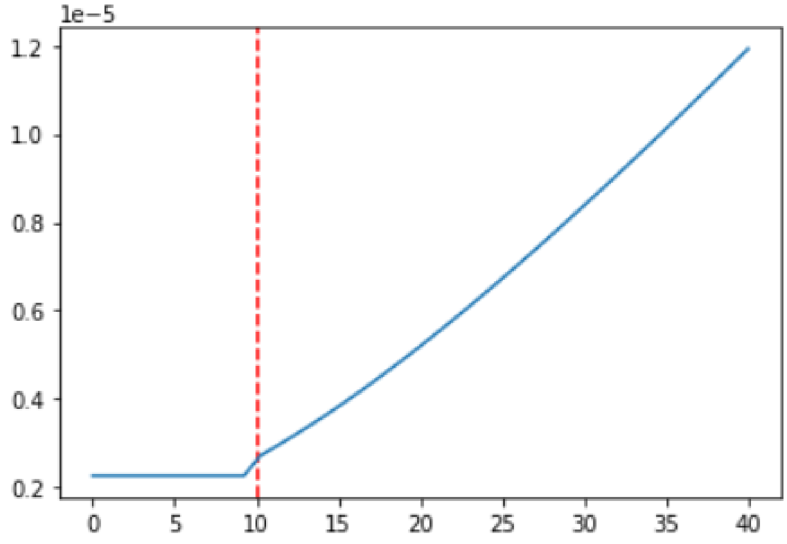


Figure 4-11 Failure rate curve (red line denotes the end of the useful lifetime)

Failure Rates

The generic failure rate for ESD logic is 8E-7 per hour (7E-3 per year) (27) within the useful lifetime of 10 years. The failure rates obtained from the posterior model and ratio-to-proportion analysis for ESD logic are summarized in Table 4-9.

Table 4-9 Failure Rates of ESD logic

ESD Logic							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	2.12E-06	11	2.60E-06	21	5.85E-06	31	9.94E-06
2	2.12E-06	12	2.87E-06	22	6.23E-06	32	1.04E-05
3	2.12E-06	13	3.16E-06	23	6.62E-06	33	1.08E-05

4	2.12E-06	14	3.46E-06	24	7.01E-06	34	1.13E-05
5	2.12E-06	15	3.77E-06	25	7.41E-06	35	1.17E-05
6	2.12E-06	16	4.09E-06	26	7.82E-06	36	1.22E-05
7	2.12E-06	17	4.42E-06	27	8.24E-06	37	1.26E-05
8	2.12E-06	18	4.76E-06	28	8.66E-06	38	1.31E-05
9	2.12E-06	19	5.12E-06	29	9.08E-06	39	1.35E-05
10	2.12E-06	20	5.48E-06	30	9.51E-06	40	1.40E-05

4.8 MODELLING FOR FLOW TRANSMITTERS

FTs are components of Gas Detection Function whose RBD is given in Figure 3-6. ORS has conducted a reliability assessment study for FTs in 2018. This periodic reliability assessment was based and review of failure records in SAP revealed during scheduled assurance and maintenance and unscheduled maintenance. The collected input data from 2011 to 2017 include 10 lifetime observations of 43 components. The criteria (explained in Section 4.2) for a population of $x * t * n > 3000000$ hours in order to apply reliability methods based on field data has been met as 10*43*8760*7=126,367,600.

Therefore, Bayesian Methodology presented in this thesis has been applied to model the interval-censored data set of: [(1390,6968,8410,9561,4846,8211,9413,6484,7041,2472]/365). The details of the dataset are given in Table 0-3 in Appendix-C.

The useful lifetime for emergency shutdown valves is 8 years ($t_1 = 8,\ \text{see Table 4-3}$). Based on this, the dataset has been divided into 2 different data sets concerning where the lifetime data observation points fall in the bathtub curve.

Based on MCMC simulations, the posterior means of $\beta = 0.008, \text{м} = 18$ and $\sigma = 7.92$ are obtained. Figure 4-12 depicts the posterior densities and Table 4-10 gives the summary of statistical values for β, м and σ.

Table 4-10 Summary Statistics

Parameter	Mean	Standard Deviation	HPD_2.5%	HPD_97.5%	MCSE
м	18.004	2.908	12.672	23.629	0.071
β	0.008	0.008	0	0.023	0
σ	7.923	2.441	3.315	12.55	0.066

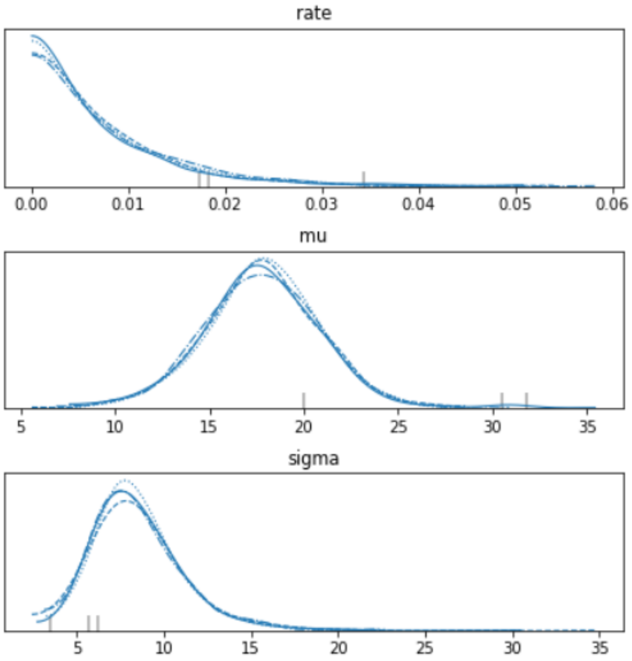


Figure 4-12 Posterior densities of β (rate), ʍ and σ.

The left graph of Figure 4-3 shows the posterior density until useful lifetime (blue line) which is $exponential\ (1/0.008)$ and the right graph is the posterior density beyond useful lifetime (orange line) which is $Normal\ (18, 7.92)$.

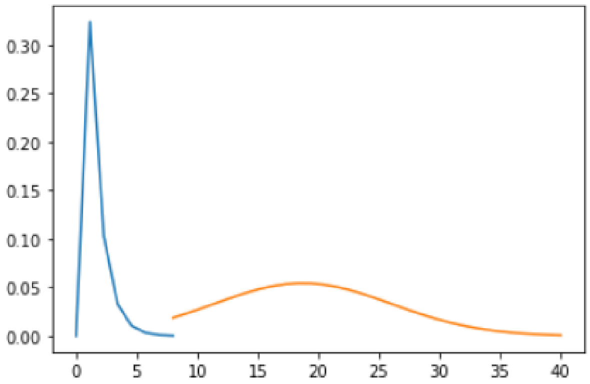


Figure 4-13 Posteriors densities of FTs

Figure 4-4 shows the failure rate curve for 40 years where it is constant until year 8 and then increasing beyond it while A (the transition parameter) is found to be 0.75. (Y-axis denotes the yearly failure rate while X-axis is the years).

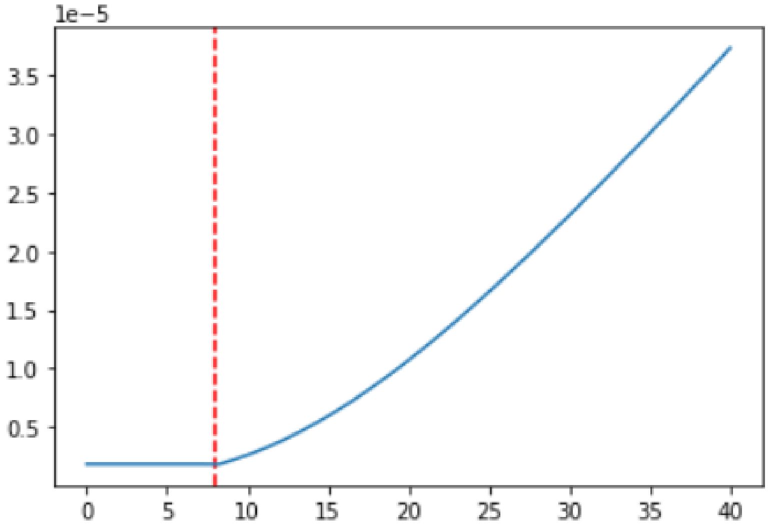


Figure 4-14 Failure rate curve (red line denotes the end of the useful lifetime)

<u>Failure Rates</u>

The generic failure rate for FT is 6E-7 per hour (5.3E-3 per year) (27) within the useful lifetime of 8 years. The failure rates obtained from the posterior model for FTs are summarized in Table 4-11.

Table 4-11 Failure Rates of FTs (per hour)

FTs							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	9.33E-07	11	1.44E-06	21	4.91E-06	31	9.88E-06
2	9.33E-07	12	1.69E-06	22	5.35E-06	32	1.04E-05
3	9.33E-07	13	1.96E-06	23	5.81E-06	33	1.10E-05
4	9.33E-07	14	2.26E-06	24	6.29E-06	34	1.15E-05
5	9.33E-07	15	2.57E-06	25	6.77E-06	35	1.21E-05
6	9.33E-07	16	2.92E-06	26	7.27E-06	36	1.27E-05
7	9.33E-07	17	3.28E-06	27	7.78E-06	37	1.32E-05
8	9.33E-07	18	3.66E-06	28	8.29E-06	38	1.38E-05
9	1.01E-06	19	4.06E-06	29	8.81E-06	39	1.44E-05
10	1.21E-06	20	4.47E-06	30	9.34E-06	40	1.49E-05

4.9 MODELLING FOR GAS DETECTORS

Gas Detectors components of Gas Detection Function whose RBD is given in Figure 3-6. ORS has conducted a reliability assessment study for Gas Detectors in 2017. This periodic reliability assessment was based and review of failure records in SAP revealed during scheduled assurance and maintenance and unscheduled maintenance. The collected input data from 2002 to 2016 include 14 lifetime observations of 1799 components. The criteria (explained in Section 4.2) for a population of $x * t * n > 3000000$ hours in order to apply reliability methods based on field data has been met as 14*1799*8760*15=3,309,440,400.

Therefore, Bayesian Methodology presented in this thesis has been applied to model the interval-censored data set of: ([2131,1282,1655,2709,2108,3137, 2700, 4673,4374,5328,5587,6955,6302,5812]/365). The details of the dataset is given in Table 0-4 in Appendix-C.

The useful lifetime for gas detectors is 8 years ($t_1 = 8, \text{see Table 4-3}$). Based on this, the dataset has been divided into 2 different data sets concerning where the lifetime data observation points fall in the bathtub curve.

Based on MCMC simulations, the posterior means of $\beta = 0.006, \mu = 10.70$ and $\sigma = 5.7$ are obtained. Figure 4-15 depicts the posterior densities and gives the summary of statistical values for м, β and σ.

Table 4-12 gives the summary of statistical values for м, β and σ.

Table 4-12 Summary Statistics

Parameter	Mean	Standard Deviation	HPD_2.5%	HPD_97.5%	MCSE
м	10.694	1.498	8.079	13.76	0.029
β	0.006	0.007	0	0.018	0
σ	5.703	1.29	3.712	8.184	0.027

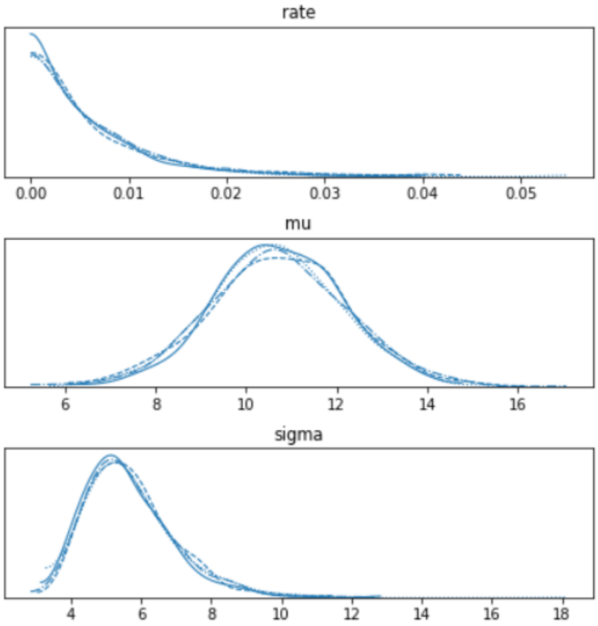


Figure 4-15 Posterior densities of м, β (rate), and σ.

The left graph of Figure 4-3 shows the posterior density until useful lifetime (blue line)which is $exponential\ (\ 1/0.006)$ and the right graph is the posterior density beyond useful lifetime (orange line)which is $Normal\ (10.70, 5.7)$.

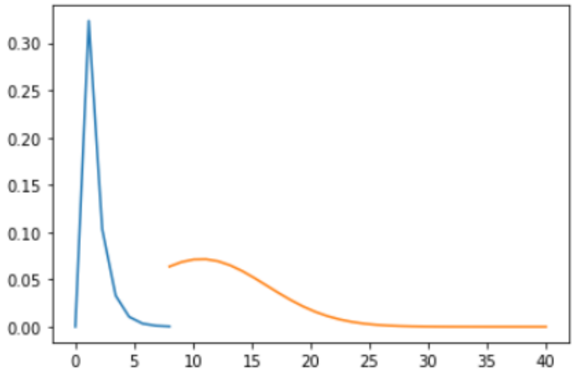


Figure 4-16 Posterior densities of Gas Detectors

Figure 4-17 shows the failure rate curve for 40 years where it is constant until year 8 and then increasing beyond it while A (the transition parameter) is found to be 0.2. (Y-axis denotes the yearly failure rate while X-axis is the years).

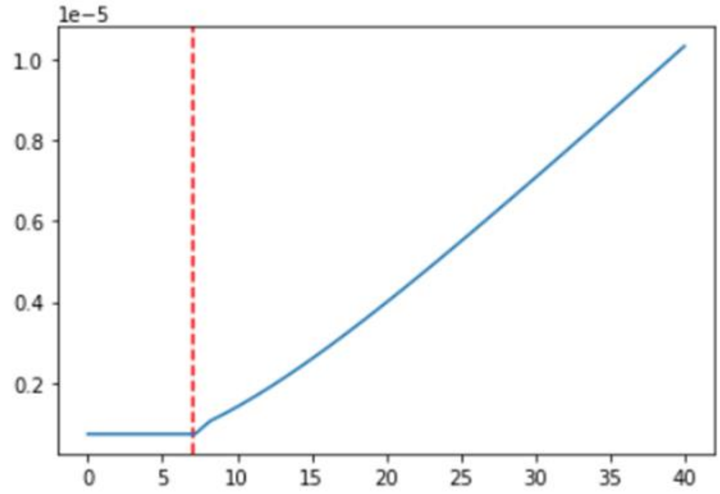


Figure 4-17 Failure rate curve (red line denotes the end of the useful lifetime)

Failure Rates

The generic failure rate for gas detectors is 0.6E-6 per hour (5.3E-3 per year) (27) within the useful lifetime of 8 years. The failure rates obtained from the posterior model for FTs are summarized in Table 4-13.

Table 4-13 Failure Rates of Gas Detectors (per hour)

Gas Detectors							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	7.29E-07	11	1.46E-06	21	4.14E-06	31	7.32E-06
2	7.29E-07	12	1.68E-06	22	4.44E-06	32	7.65E-06
3	7.29E-07	13	1.92E-06	23	4.75E-06	33	7.99E-06
4	7.29E-07	14	2.16E-06	24	5.06E-06	34	8.32E-06
5	7.29E-07	15	2.42E-06	25	5.38E-06	35	8.65E-06
6	7.29E-07	16	2.69E-06	26	5.70E-06	36	8.99E-06
7	7.29E-07	17	2.97E-06	27	6.02E-06	37	9.32E-06
8	7.29E-07	18	3.25E-06	28	6.34E-06	38	9.66E-06
9	1.05E-06	19	3.54E-06	29	6.67E-06	39	1.00E-05
10	1.25E-06	20	3.83E-06	30	6.99E-06	40	1.03E-05

4.10 FAILURE RATES ESTIMATION FOR OTHER COMPONENTS

Five datasets have been analyzed so far. However, there are 9 components whose failure rates are needed to be estimated in the context of this thesis. Due to the unavailability of failure event datasets, the Bayesian analysis could not be conducted for the real failure event data concerning 4 of the 9 components. However, the ORS expert believes that the remaining 4 components (ESD Function-Solenoid, HIPPS Function-Yokogawa LS, Gas Detection Function-F&G Logic and HIPPS Function-Mokveld-ESD valve with an actuator) have a similar aging pattern with the components having available datasets for the analysis in the light of following assumptions:

- Both the Mokveld-ESD valve and ESD Function-Solenoid are the special types of ESV/XVs. The only difference between ESD Function-Solenoid and ESV/XVs is that the word “solenoid” is used to define the operation type (electromagnetic) of the valve. On the other hand, the Mokveld-ESD valve is the type of ESV/XVs produced by a well-known vendor. These components do not differ in terms of their mechanical specifications.
- Both Gas Detection Function-F&G Logic and HIPPS Function-Yokogawa LS are solid-state LSs with mechanical, electrical and operational specifications similar to ESD Logic. It is expected that these components have the same aging pattern.

Based on these assumptions, available data sets are matched with the remaining components and the ratios of aging obtained from the posterior analysis of available data sets have been used to calculate the estimated failure datasets for these 4 components. Table 4-14 summarizes the component matches.

Ratios of aging are calculated according to with which factor the failure rates have changed for posterior failure rates compared to the generic failure rate. For example, for ESV/XV, during its useful lifetime of 10 years the generic failure rate is 2.1E-6; and the posterior estimate is 4.52E-6 which gives the ratio of (4.52E-6/2.1E-6=) 2.15. It means that for the Mokveld-ESD valve with solenoid, the posterior estimate would be 2.15 times of its generic failure rate during its useful lifetime of 6 years. For the estimations of beyond useful lifetime (beyond year 10 for ESV/XV and beyond year 6 for Mokveld- ESD valve with a solenoid), the ratios of ESV/XVs have been shifted up to be used for Mokveld-ESD valve with solenoid component due to useful lifetime difference of 4 years between components. For example, the ratio of aging for ESV/XVs at year 11 is 3.57; it means that the failure rate of the Mokveld-ESD valve with solenoid at year 7 must be found with the multiplication of its generic failure rate and 3.57.

Following this logic, the ratio of aging for ESDV in year 40 has been used to calculate the failure rate estimation of the Mokveld-ESD valve with solenoid in year 36. For the remaining 4 years, the Python code for Bayesian Analysis has been run until year 44 to extend the estimations of failure rates and therefore ratios of aging for ESDV until year 44. The last 4 ratios of aging for the Mokveld-ESD valve with solenoid have been obtained in this way. Table 4-15 exemplifies the failure rates estimation method for the components which do not have available datasets to run the Bayesian Analysis code in Python. (The red color denotes the end of the useful lifetime for ESV/XVs while green color denotes the same for Mokveld-ESD valve with solenoid.)

Table 4-14 Dataset match

Failure Dataset	Matches with (function)	Matches with (component)
ESV/XV	HIPPS	ESD valve with solenoid - Mokveld
	ESD Segregation Function	Solenoid
ESD Logic	HIPPS	LS-Yokogawa
	F&G Detection	F&G Logic

Table 4-15 Estimation method for the components having no failure datasets

Year	Failure Rate Estimation for ESD	Ratios to generic failure rate (2.1E-6)	Mokveld generic	Mokveld estimation
6	4.52E-06	2.15	3.4E-8	7.32E-08
7	4.52E-06	2.15	3.4E-8	1.21E-07
8	4.52E-06	2.15	3.4E-8	1.68E-07
9	4.52E-06	2.15	3.4E-8	2.27E-07
10	4.52E-06	2.15	3.4E-8	2.99E-07
11	7.50E-06	3.57	3.4E-8	3.85E-07
12	1.04E-05	4.94	3.4E-8	4.86E-07
13	1.40E-05	6.67	3.4E-8	6.02E-07
14	1.85E-05	8.79	3.4E-8	7.33E-07

The methodology explained in the previous paragraph together with the component match explained in Table 4-14 have been used to obtain the failure rates of Solenoid, HIPPS Yokogawa LS and F&G Logic as well. Their failure rates are estimated as given in Table 4-16 (for HIPPS Mokveld-ESD valve with solenoid), Table 4-17 (for Solenoid), Table 4-18 (for HIPPS Yokogawa LS), and Table 4-19 (for F&G Logic).

Table 4-16 Failure rates of ESD valve with solenoid-Mokveld

ESD valve with solenoid -Mokveld							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate

1	7.32E-08	11	3.85E-07	21	2.21E-06	31	4.40E-06
2	7.32E-08	12	4.86E-07	22	2.43E-06	32	4.66E-06
3	7.32E-08	13	6.02E-07	23	2.66E-06	33	4.92E-06
4	7.32E-08	14	7.33E-07	24	2.90E-06	34	5.18E-06
5	7.32E-08	15	8.77E-07	25	3.14E-06	35	5.45E-06
6	7.32E-08	16	1.03E-06	26	3.38E-06	36	5.71E-06
7	1.21E-07	17	1.21E-06	27	3.63E-06	37	5.99E-06
8	1.68E-07	18	1.39E-06	28	3.88E-06	38	6.26E-06
9	2.27E-07	19	1.58E-06	29	4.14E-06	39	6.53E-06
10	2.99E-07	20	1.78E-06	30	2.21E-06	40	6.80E-06

Table 4-17 Failure rates of solenoid

Solenoid							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	1.72E-06	11	7.03E-06	21	4.19E-05	31	9.74E-05
2	1.72E-06	12	9.07E-06	22	4.68E-05	32	1.03E-04
3	1.72E-06	13	1.14E-05	23	5.20E-05	33	1.10E-04
4	1.72E-06	14	1.42E-05	24	5.72E-05	34	1.16E-04
5	1.72E-06	15	1.72E-05	25	6.27E-05	35	1.22E-04
6	1.72E-06	16	2.06E-05	26	6.82E-05	36	1.28E-04
7	1.72E-06	17	2.44E-05	27	7.39E-05	37	1.34E-04
8	2.86E-06	18	2.84E-05	28	7.96E-05	38	1.41E-04
9	3.95E-06	19	3.26E-05	29	8.55E-05	39	1.47E-04
10	5.34E-06	20	3.72E-05	30	9.14E-05	40	1.54E-04

Table 4-18 Failure rates of Yokogawa LS

Yokogawa LS							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	1.23E-07	11	1.23E-07	21	2.96E-07	31	5.25E-07

2	1.23E-07	12	1.23E-07	22	3.17E-07	32	5.50E-07
3	1.23E-07	13	1.51E-07	23	3.39E-07	33	5.75E-07
4	1.23E-07	14	1.66E-07	24	3.61E-07	34	6.01E-07
5	1.23E-07	15	1.83E-07	25	3.83E-07	35	6.26E-07
6	1.23E-07	16	2.00E-07	26	4.06E-07	36	6.52E-07
7	1.23E-07	17	2.18E-07	27	4.29E-07	37	6.78E-07
8	1.23E-07	18	2.37E-07	28	4.53E-07	38	7.04E-07
9	1.23E-07	19	2.56E-07	29	4.77E-07	39	7.30E-07
10	1.23E-07	20	2.76E-07	30	5.01E-07	40	7.57E-07

Table 4-19 Failure rates of F&G Logic

F&G Logic							
Year	Failure Rate	Year	Failure Rate	Year	Failure Rate	Year	Failure Rate
1	1.30E-05	11	1.59E-05	21	3.58E-05	31	6.09E-05
2	1.30E-05	12	1.76E-05	22	3.82E-05	32	6.36E-05
3	1.30E-05	13	1.93E-05	23	4.05E-05	33	6.63E-05
4	1.30E-05	14	2.12E-05	24	4.30E-05	34	6.90E-05
5	1.30E-05	15	2.31E-05	25	4.54E-05	35	7.18E-05
6	1.30E-05	16	2.50E-05	26	4.79E-05	36	7.45E-05
7	1.30E-05	17	2.71E-05	27	5.04E-05	37	7.73E-05
8	1.30E-05	18	2.92E-05	28	5.30E-05	38	8.01E-05
9	1.30E-05	19	3.13E-05	29	5.56E-05	39	8.29E-05
10	1.30E-05	20	3.36E-05	30	5.82E-05	40	8.58E-05

4.11 RESULTS (UPDATED SIF PERFORMANCE/SIL VALUES)

Experts of ORS believe that the typical lifetime of facilities can be extended up to 40 years. Taking this into consideration, the analysis in this thesis is conducted for the lifespan of 40 years for each component of SIFs to evaluate if they will still be sufficiently safe.

4.11.1 RESULTS FOR HIPPS

Table 4-20 shows the estimated PFD values for HIPPS components, as well as HIPPS function. PFD values are calculated according to Eq.2 in Section 3.2 It is seen that the SIL obtained from generic failure rates (SIL3) for HIPPS does not hold when aging is considered. SIL2 remains the same from year 1 to year 18. From year 18 to 40, the function follows SIL1.

Table 4-20 PFDs and SILs Based On Estimated λ_{DU} for HIPPS

HIPPS					
Year	PFD of PT	PFD of Yokogawa LS	PFD of Mokveld Valve	Overall PFD of HIIPS	SIL
1	1.38E-03	5.37E-04	4.91E-06	1.92E-03	SIL2
2	1.38E-03	5.37E-04	4.91E-06	1.92E-03	SIL2
3	1.38E-03	5.37E-04	4.91E-06	1.92E-03	SIL2
4	1.38E-03	5.37E-04	4.91E-06	1.92E-03	SIL2
5	1.38E-03	5.37E-04	4.91E-06	1.92E-03	SIL2
6	1.38E-03	5.37E-04	4.91E-06	1.92E-03	SIL2
7	1.38E-03	5.37E-04	8.26E-06	1.92E-03	SIL2
8	1.38E-03	5.37E-04	1.16E-05	1.92E-03	SIL2
9	1.38E-03	5.37E-04	1.59E-05	1.93E-03	SIL2
10	1.38E-03	5.37E-04	2.14E-05	1.93E-03	SIL2
11	1.38E-03	5.37E-04	2.82E-05	1.94E-03	SIL2
12	1.38E-03	5.37E-04	3.65E-05	1.95E-03	SIL2
13	2.47E-03	6.60E-04	4.65E-05	3.18E-03	SIL2
14	3.49E-03	7.28E-04	5.84E-05	4.28E-03	SIL2
15	4.81E-03	8.01E-04	7.24E-05	5.68E-03	SIL2
16	6.47E-03	8.76E-04	8.85E-05	7.43E-03	SIL2
17	8.50E-03	9.55E-04	1.07E-04	9.56E-03	SIL2
18	1.10E-02	1.04E-03	1.28E-04	1.21E-02	SIL1
19	1.38E-02	1.12E-03	1.52E-04	1.51E-02	SIL1
20	1.72E-02	1.21E-03	1.78E-04	1.86E-02	SIL1
21	2.11E-02	1.30E-03	2.07E-04	2.26E-02	SIL1
22	2.54E-02	1.39E-03	2.39E-04	2.70E-02	SIL1

23	3.03E-02	1.48E-03	2.73E-04	3.20E-02	SIL1
24	3.57E-02	1.58E-03	3.11E-04	3.76E-02	SIL1
25	4.16E-02	1.68E-03	3.52E-04	4.37E-02	SIL1
26	4.81E-02	1.78E-03	3.95E-04	5.03E-02	SIL1
27	5.51E-02	1.88E-03	4.42E-04	5.74E-02	SIL1
28	6.27E-02	1.98E-03	4.92E-04	6.51E-02	SIL1
29	7.08E-02	2.09E-03	5.45E-04	7.34E-02	SIL1
30	7.95E-02	2.19E-03	6.01E-04	8.23E-02	SIL1
31	8.87E-02	2.30E-03	6.60E-04	9.16E-02	SIL1
32	9.85E-02	2.41E-03	7.22E-04	1.02E-01	SIL1
33	1.09E-01	2.52E-03	7.87E-04	1.12E-01	SIL1
34	1.20E-01	2.63E-03	8.55E-04	1.23E-01	SIL1
35	1.31E-01	2.74E-03	9.27E-04	1.35E-01	SIL1
36	1.43E-01	2.86E-03	1.00E-03	1.47E-01	SIL1
37	1.56E-01	2.97E-03	1.08E-03	1.60E-01	SIL1
38	1.69E-01	3.08E-03	1.16E-03	1.73E-01	SIL1
39	1.83E-01	3.20E-03	1.25E-03	1.87E-01	SIL1
40	1.97E-01	3.32E-03	1.33E-03	2.01E-01	SIL1

Figure 4-18 depicts the curve of estimated PFD values for HIPPS function over 40 years.

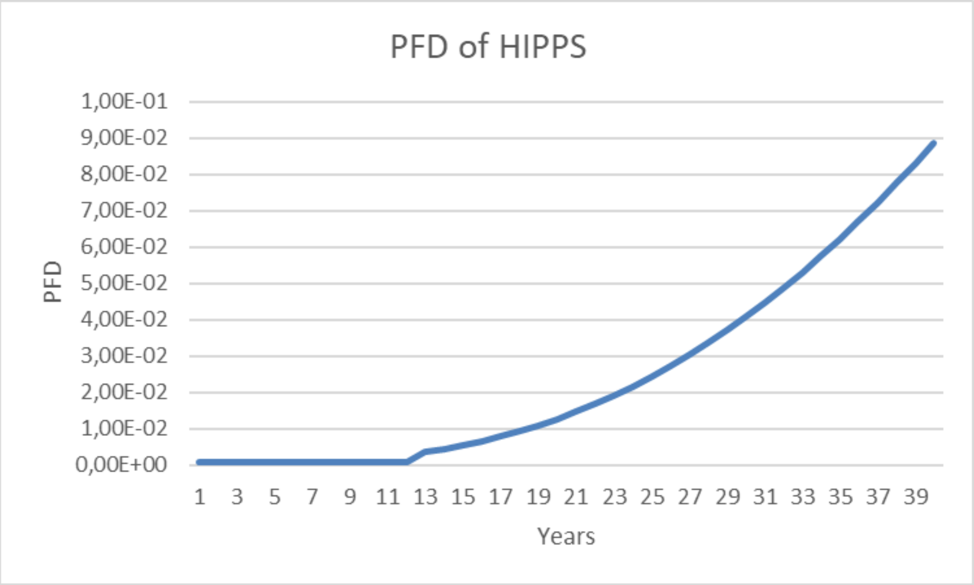


Figure 4-18 Estimated PFD values of HIPPS Function

In the initial scenario with generic failure rates given in Table 4-1, the overall PFD of HIPPS is calculated to be 1.57E-4 with SIL3.

4.11.2 RESULTS FOR ESD SEGREGATION FUNCTION

Table 4-21 shows the estimated PFD values for ESD components, as well as the ESD segregation function. PFD values are calculated according to Eq.2 in Section 3.2.

The minimum SIL requirement calculated for ESD segregation function is SIL1 where a specific PFD requirement is less than 0.04 (22). It is noticeable from Table 4-21 that in year 8 the overall PFD exceeds the specified requirement.

Table 4-21 PFDs and SILs Based On Estimated λ_{DU} for ESD

ESD					
Year	PFD of ESD Logic	PFD of Solenoid	PFD of ESV/XV	Overall PFD of ESD	SIL
1	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
2	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
3	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
4	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
5	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
6	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
7	9.28E-03	7.54E-03	1.98E-02	3.66E-02	SIL1
8	9.28E-03	1.25E-02	1.98E-02	4.16E-02	-
9	9.28E-03	1.73E-02	1.98E-02	4.64E-02	-
10	9.28E-03	2.34E-02	1.98E-02	5.25E-02	-
11	1.14E-02	3.08E-02	3.28E-02	7.51E-02	-

12	1.26E-02	3.97E-02	4.55E-02	9.78E-02	-
13	1.38E-02	5.01E-02	6.14E-02	1.25E-01	-
14	1.51E-02	6.21E-02	8.09E-02	1.58E-01	-
15	1.65E-02	7.55E-02	1.04E-01	1.96E-01	-
16	1.79E-02	9.04E-02	1.32E-01	2.40E-01	-
17	1.94E-02	1.07E-01	1.63E-01	2.89E-01	-
18	2.09E-02	1.24E-01	1.98E-01	3.43E-01	-
19	2.24E-02	1.43E-01	2.37E-01	4.03E-01	-
20	2.40E-02	1.63E-01	2.80E-01	4.67E-01	-
21	2.56E-02	1.84E-01	3.26E-01	5.35E-01	-
22	2.73E-02	2.05E-01	3.75E-01	6.08E-01	-
23	2.90E-02	2.28E-01	4.27E-01	6.84E-01	-
24	3.07E-02	2.51E-01	4.82E-01	7.63E-01	-
25	3.25E-02	2.74E-01	5.39E-01	8.45E-01	-
26	3.43E-02	2.99E-01	5.97E-01	9.30E-01	-
27	3.61E-02	3.24E-01	6.58E-01	1.02E+00	-
28	3.79E-02	3.49E-01	7.20E-01	1.11E+00	-
29	3.98E-02	3.74E-01	7.84E-01	1.20E+00	-
30	4.16E-02	4.00E-01	8.49E-01	1.29E+00	-
31	4.35E-02	4.27E-01	9.16E-01	1.39E+00	-
32	4.55E-02	4.53E-01	9.83E-01	1.48E+00	-
33	4.74E-02	4.80E-01	1.05E+00	1.58E+00	-
34	4.93E-02	5.07E-01	1.12E+00	1.68E+00	-
35	5.13E-02	5.34E-01	1.19E+00	1.77E+00	-
36	5.33E-02	5.61E-01	1.26E+00	1.87E+00	-
37	5.53E-02	5.89E-01	1.33E+00	1.97E+00	-
38	5.73E-02	6.17E-01	1.40E+00	2.08E+00	-
39	5.93E-02	6.45E-01	1.47E+00	2.18E+00	-
40	6.13E-02	6.73E-01	1.55E+00	2.28E+00	-

Figure 4-19 depicts the curve of estimated PFD values for ESD segregation function over 40 years.

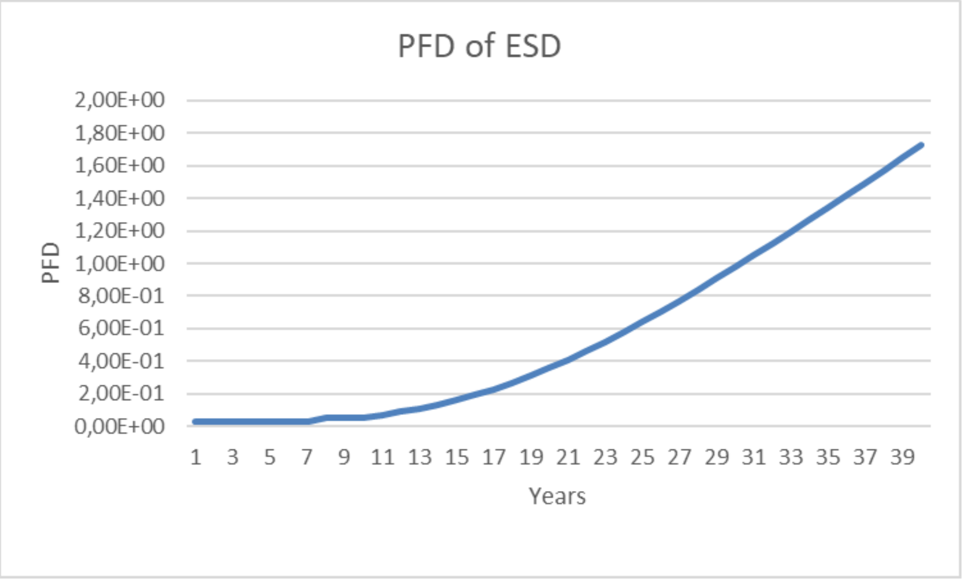


Figure 4-19 Estimated PFD values of ESD Segregation Function

In the initial scenario with generic failure rates given in Table 4-2, the overall PFD of ESD Segregation Function is calculated to be 1.58E-2 with SIL2.

4.11.3 RESULTS FOR GAS DETECTION FUNCTION

Table 4-22 shows the estimated PFD values for Gas Detection Function components, as well as gas Detection Function. PFD values are calculated according to Eq.2. Gas Detection function goes out of the SIL1 range at the end of year 13 according to the estimated failure rates.

Table 4-22 PFDs and SILs Based On Estimated Failure Rates for Gas Detection Function

Gas Detection Function					
Year	PFD of FT	PFD of Gas Detectors	PFD of F&G Logic	Overall PFD of Gas Detection Function	SIL
1	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
2	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
3	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
4	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
5	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
6	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
7	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
8	4.08E-03	2.05E-04	5.68E-02	6.11E-02	SIL1
9	4.43E-03	3.05E-04	5.68E-02	6.16E-02	SIL1
10	5.31E-03	3.68E-04	5.68E-02	6.25E-02	SIL1

11	6.30E-03	4.37E-04	6.98E-02	7.65E-02	SIL1
12	7.39E-03	5.14E-04	7.71E-02	8.50E-02	SIL1
13	8.58E-03	5.97E-04	8.47E-02	9.39E-02	SIL1
14	9.88E-03	6.88E-04	9.27E-02	1.03E-01	-
15	1.13E-02	7.87E-04	1.01E-01	1.13E-01	-
16	1.28E-02	8.92E-04	1.10E-01	1.23E-01	-
17	1.44E-02	1.00E-03	1.19E-01	1.34E-01	-
18	1.60E-02	1.12E-03	1.28E-01	1.45E-01	-
19	1.78E-02	1.25E-03	1.37E-01	1.56E-01	-
20	1.96E-02	1.38E-03	1.47E-01	1.68E-01	-
21	2.15E-02	1.52E-03	1.57E-01	1.80E-01	-
22	2.34E-02	1.67E-03	1.67E-01	1.92E-01	-
23	2.55E-02	1.83E-03	1.78E-01	2.05E-01	-
24	2.75E-02	1.99E-03	1.88E-01	2.18E-01	-
25	2.97E-02	2.15E-03	1.99E-01	2.31E-01	-
26	3.18E-02	2.33E-03	2.10E-01	2.44E-01	-
27	3.41E-02	2.51E-03	2.21E-01	2.58E-01	-
28	3.63E-02	2.70E-03	2.32E-01	2.71E-01	-
29	3.86E-02	2.89E-03	2.44E-01	2.85E-01	-
30	4.09E-02	3.09E-03	2.55E-01	2.99E-01	-
31	4.33E-02	3.30E-03	2.67E-01	3.13E-01	-
32	4.57E-02	3.51E-03	2.78E-01	3.28E-01	-
33	4.81E-02	3.73E-03	2.90E-01	3.42E-01	-
34	5.05E-02	3.96E-03	3.02E-01	3.57E-01	-
35	5.29E-02	4.19E-03	3.14E-01	3.71E-01	-
36	5.54E-02	4.43E-03	3.26E-01	3.86E-01	-
37	5.79E-02	4.67E-03	3.39E-01	4.01E-01	-
38	6.04E-02	4.93E-03	3.51E-01	4.16E-01	-
39	6.29E-02	5.18E-03	3.63E-01	4.31E-01	-
40	6.55E-02	5.45E-03	3.76E-01	4.47E-01	-

Figure 4-20 depicts the curve of estimated PFD values for Gas Detection function over 40 years.

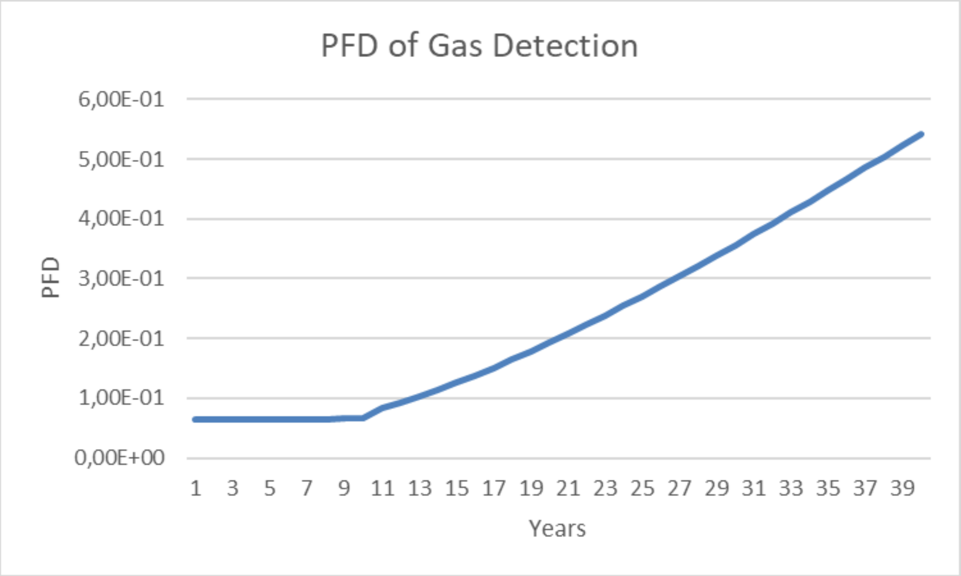


Figure 4-20 Estimated PFD values of Gas Detection Function

In the initial scenario with generic failure rates given in Table 4-3, the overall PFD of Gas Detection Function is calculated to be 2.43E-2 with SIL2.

4.12 SENSITIVITY ANALYSIS

Sensitivity analyses play an important role in Bayesian Analysis. In some cases, the lack of prior information may lead to an undesired influence on posterior results (55). Mostly, the analysts want to know which parameter selection or model specification leads to more satisfactory results. Thus, this section presents and evaluates the Bayesian results under different model specifications.

4.12.1 UNIFORM PRIOR WITHIN USEFUL LIFETIME

As presented in Section 4.3, gamma distribution with appropriate shape and scale parameters [i.e. $\lambda \sim gamma(1, \frac{1}{generic\ failure\ rate})$] has been selected to be the prior distribution for all components. The analysis is extended by selecting prior distribution as uniform with lower bound 0 and upper bound “failure rate of the component+0.02” for the components of ESD segregation function (ESD logic, Solenoid and ESDV/XV). The upper bound of the uniform distribution is chosen to be a little higher than the generic failure rate in order to not restrict the calculation errors.

The following piece of Python code is implemented while the rest has been kept the same as given in Appendix-B. (“# ” shows the inactivated part of the code)

```
def find_mixed(obs, rate):
    with pm.Model() as mixed:
        rate_b= pm. Uniform ('rate',0,rate+0.02)
        #rate_b=pm.Gamma ('rate', alpha=1, beta=1/rate
```

Based on MCMC simulations, the posterior means of emergency shutdown valves (ESDV/XV) are obtained to be $\beta = 0.04541, \mu = 21.48$ and $\sigma = 5.44$. The results are not significantly different than the results of the posterior model with gamma priors.

The posterior means of ESD Logic is obtained to be $\beta = 0.02, \mu = 8.69$ and $\sigma = 7.14$. The posterior μ under a uniform prior is found to be significantly less than μ under a uniform prior, while posterior σ remains the same.

Since there is no available dataset for solenoids, the estimated failure rates have been obtained by the approximation method presented in Section 4.10. Estimated posterior failure rates of the components of the ESD segregation function obtained from the new model are presented in Table 0-6 of Appendix-D.

Comparing Table 0-6 with the values presented in Table 4-5, Table 4-9 and Table 4-17, it can seem that the failure rate estimations from year 1 to 40 for ESV/XV component are higher with uniform priors. While it is factor 1.1 higher than the previous estimate during its useful lifetime of 10 years, it keeps accelerating until year 40 and reaches to factor 5.20 at year 40. A similar situation is not observed for ESD Logic. Its failure rate estimation under uniform priors is factor 1.77 higher than the initial case with gamma priors during its useful lifetime. The rate of increase does not change throughout 40 years of estimation. At year 40, it is factor 1.55 higher than the initial case. The average increase from year 1 to year 40 is 1.64 for ESD Logic, while it is 4.37 for ESV/XV.

The associated PFDs and SIL values are calculated and compared with the initial analysis conducted with a gamma prior. The results are presented in Table 0-7 of Appendix-D.

Comparing Table 0-7 with Table 4-21, it can be concluded that there are overall PFDs and thus, SIL values are almost the same for uniform and gamma priors. Figure 4-21 depicts the estimated PFD of ESD Segregation Function with different priors. Overall, switching prior distribution from gamma to uniform does not remarkably affect the posterior results, PFDs and thus; SIL values.

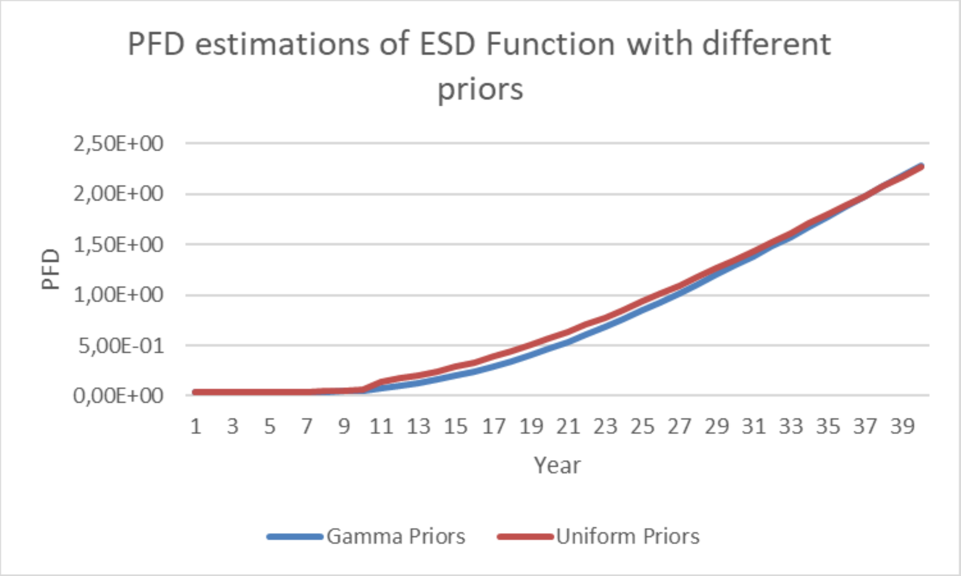


Figure 4-21 ESD Segregation Function PFD estimation with different priors

4.12.2 HALF CAUCHY PRIOR BEYOND USEFUL LIFETIME

As presented in Section 4.3, normal distribution with appropriate mean and variance parameters has been selected to be the prior distribution of the mean parameter of normal likelihood [i.e. $\mu \sim Normal\ (10{,}20)$], and half-normal distribution with variance 10 has been selected to be the prior distribution of variance parameter of normal likelihood [i.e. $\sigma \sim HalfNormal(10)$] for Bayesian modelling beyond useful lifetime.

In this section, independent and non-informative (weakly informative) prior distributions are considered for the analysis beyond the useful lifetime. Mean and variance parameters are assumed to follow a half-Cauchy distribution with scale 25 as recommended by Polson and Scott (56).

The following piece of Python code is implemented for the components of Gas Detection function (FTs, Gas Detectors and F&G Logic) while the rest has been kept the same as given in Appendix-B ("# " shows the inactivated part of the code).

```
def find_mixed(obs, rate):
    with pm.Model() as mixed:
        rate_b=pm.Gamma ('rate', alpha=1, beta=1/rate)
        expo=pm.Exponential.dist(lam=rate_b)
        #mu=pm.Normal('mu', 10, 20)
        #sigma=pm.HalfNormal('sigma', 10)
        mu=pm.HalfCauchy('mu', 25)
        sigma=pm.HalfCauchy('sigma', 25)
        normal=pm.Normal.dist(mu=mu, sigma=sigma)
```

For FTs, the posterior means of $\beta = 0.007, \mu = 17.69$ and $\sigma = 8.45$ are obtained.

For Gas Detectors, the posterior means of $\beta = 0.006, \mu = 10.63$ and $\sigma = 5.69$ are obtained. These values are not significantly different than initial ones with Normal (10,20) and Half-Normal (10) priors.

Since there is no available dataset for F&G logic, the estimated failure rates have been obtained by the approximation method presented in Section 4.10. To do this, Python code with modified priors has been run for ESD logic and failure rate estimations for F&G logic have been obtained according to calculated ratios of aging. It is worthy to note that the available dataset of ESD logic includes only 4 failure events. Figure 4-22 depicts the failure rate estimations of F&G logic with different prior distribution selections. Posterior results with Half Cauchy priors estimate almost constant failure rate beyond the useful lifetime, which is undesired. This may be closely related to too few data and indicate the wrong prior distribution selection.

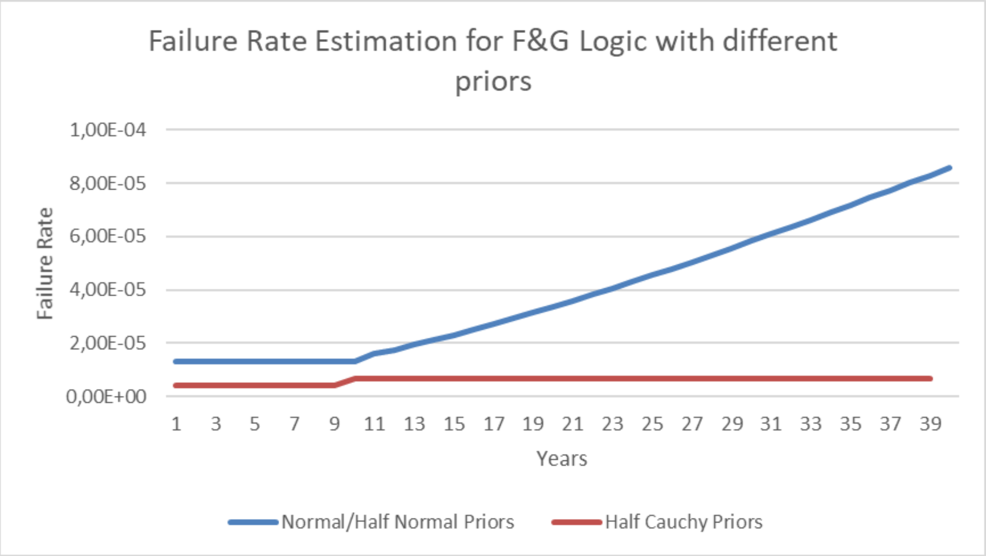


Figure 4-22 F&G Logic failure rate estimation with different priors

Estimated posterior failure rates of the components of Gas Detection function obtained from the new model is presented in Table 0-8 of Appendix-D.

Moreover, the associated PFDs and SIL values are calculated and presented in Table 0-9 of Appendix-D and Figure 4-23 depicts the estimated PFD of Gas Detection Function with different priors. Similar to Normal and Half-normal priors of mean and variance parameters, half-Cauchy priors also result in a change of SIL value in year 8. However, it gives a lower estimation of PFDs for the entire period of 40 years.

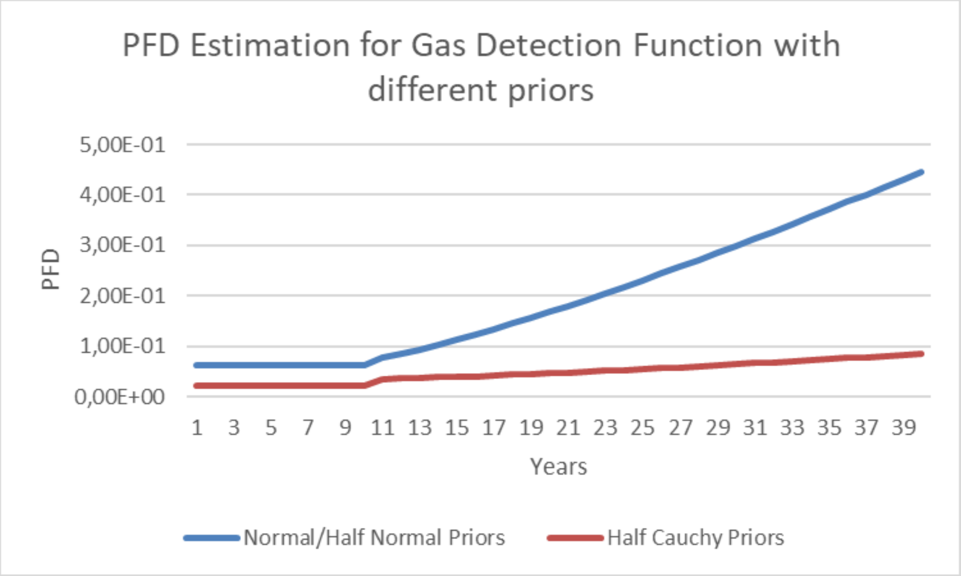


Figure 4-23 Gas Detection Function PFD estimation with different priors

4.12.3 CHANGE IN THE LENGTH OF MCMC CHAIN

With the MCMC algorithm, a huge number of draws are generated from the posterior distribution to calculate the estimated model parameters (57). During the analysis conducted in this thesis, the number of draws has been set to be 1000 for all components.

MCMC Error, calculated from posterior analysis, is an indicator of whether the specified length of the MCMC chain provides enough repeatability and thus, convergence (58). MCMC error is expected to be close to 0. MCMC Errors calculated as a result of posterior analysis are given in the tables of summary statistics of each component through Section 4.5 to 4.9. It can seem that the highest MCMC Error for the mean parameter has observed in the analysis of ESD Logic (0.39; reference is given to Table 4-8). It indicates that the Bayesian model of ESD Logic requires a higher number of draws to have an acceptable level of repeatability.

The following piece of Python code is implemented for ESD Logic while the rest has been kept the same as given in Appendix-B: (“# ” shows the inactivated part of the code)

```
        #trace=pm.sample(draws=1000, cores=4)
        #trace=pm.sample(draws=10000, cores=4)
```

The summary statistics are obtained from the modified model are presented in Table 4-23.

Table 4-23 Summary Statistics for ESD Logic with the higher number of draws

Parameter	Mean	MCSE (draw=10000)	MCSE (draw=1000)
м	15.171	0.124	0.39
β	0.015	0	0
σ	10.664	0.067	0.157

Compared to other components analyzed in this thesis, MCMC error was expected to be high for ESD Logic due to less available failure data. However, MCMC error can be reduced significantly if the number of draws is increased. There is no significant change in estimated means of model parameters, which means the posterior model converges to a similar stationary state.

5 DISCUSSION OF BAYESIAN ANALYSIS RESULTS

5.1 CHANGE IN SIF PERFORMANCE OVER TIME

The posterior failure rates are obtained and used to calculate PFDs of SIFs. The PFDs and corresponding SILs of SIFs are presented in Chapter 4.11. The estimated SIL values for HIPPS function show that SIL2 compliance can be met for the first 17 years of function's lifetime out of 40 years of anticipated operations. As expected, there is no significant change in overall PFD of the function until all 3 of its components exceed their useful lifetimes, which is at year 12 (the longest useful lifetime out of 3 components of the HIPPS function is 12 years for PTs. Although the HIPPS function meets SIL2 requirements between year 12 to 17, the PFD value of the function increases 5 times in these 5 years. From year 17 to 40, the increase in PFD value becomes significant and reaches 21 times higher than its value in year 17. The highest contribution to the overall PFD of a function comes from PTs from year 17 to 18; therefore, it might be a smart decision to act vigilance during that period and perform maintenance activities more often for PTs. Also, replacing PTs around year 18 could be a wise decision in terms of the overall reliability of HIPPS, if it seems economically viable.

The estimated PFD of ESD segregation function remains the same for the first 7 years until one of the components (solenoid) exceeds its useful lifetime. Although there is no significant increase in PFDs from year 7 to 8, the function losses its SIL1 compliance due to a specific requirement of PFD to be less than 4E-2 (22). The highest contribution to overall PFD is due to ESV/XV component, therefore; using high reliable ESV/XV during the design stage of the function could be considered by decision-makers while the vendors are specified.

The Gas Detection Function remains in the SIL1 range for the first 13 years of operations. However, overall PFD of the function only increases 20 times from year 1 to year 40. It indicates that only with slight adjustments of the components, the overall reliability could be kept unchanged if the operations are considered to be extended. For example, since the highest contribution to overall PFD of the function is because of F&G Logic, the increase in the number of maintenance activities for this component could extend the period of SIL1 compliance and; therefore, overall safety and reliability of the Gas Detection Function.

5.2 EFFECT OF PFD PARAMETERS

Since the PFD value is dependent on the values of DU failure rates (λ_{DU}), proof test intervals (T), Beta factor value (β) and Modification factor value (C_{MooN}); change in these parameters greatly influence the overall PFD of the SIFs (reference to Section 3.2). Therefore, these parameters should also be reviewed when possible in order to achieve the desired safety levels.

Test interval is one of the critical parameters in the SIS verification process. Generally, the vendor is considered to state the test interval for the subsystems (components) in the SISs. Although the vendor stated test intervals are used as a basis, the shorter test intervals should be considered in the later stages of components lifetime. When the failure rate estimates show that the targeted PFD level/SIL is not met by the chosen test interval, the shorter test intervals should be implemented. Table 5-1 shows the estimated PFD values according to the updated theoretical test interval of the EDS function for the first 13 years.

Table 5-1 PFD of ESD segregation function with shortened test intervals

Year	PFD (Test Interval=8760 hours)	SIL	PFD (Test Interval=4380 hours)	SIL
1-7	3.66E-02	SIL1	1.83E-02	SIL1
8	4.16E-02	-	2.08E-02	SIL1
9	4.64E-02	-	2.32E-02	SIL1
10	5.25E-02	-	2.62E-02	SIL1
11	7.51E-02	-	3.75E-02	SIL1
12	9.78E-02	-	4.89E-02	-

NOROG 070 (22) recommends that the highest PFD value of ESD segregation function should not exceed 4E-2. Although the estimated PFD value exceeds 4E-2 in year 12 when the test interval is 8760 hours (1 year), PFD value is also halved when the test interval is halved. Based on the results in Table 5-1 with the decrease in the length of the test interval, the lifetime of the component can be extended one more year.

Basically, any change in test intervals is directly reflected in PFD values. Decision-makers should consider making necessary adjustments on test intervals beforehand when the estimates of failure rates seem converging on unacceptable safety levels.

According to NOROG070 (22), the increase in the length of the test interval should not exceed 50%, thereby the theoretical change in test interval for the HIPPS function is restricted with 50%. The new test intervals are 6570 hours (9 months) for PTs and valves, and 13140 hours (15 months) for LS. The PFD values where the significant changes observed are presented in Table 5-2.

Table 5-2 PFD of HIPPS function with 50% longer test intervals

Year	PFD (When vendor specified test intervals are implemented)	SIL	PFD (When 50% longer test intervals are implemented)	SIL
13	3.18E-03	SIL2	9.07E-03	SIL2
14	4.28E-03	SIL2	1.29E-02	SIL1
15	5.68E-03	SIL2	1.80E-02	SIL1
16	7.43E-03	SIL2	2.47E-02	SIL1
17	9.56E-03	SIL2	3.31E-02	SIL1
18	1.21E-02	SIL1	4.34E-02	SIL1
19	1.51E-02	SIL1	5.59E-02	SIL1

Based on the results in Table 5-2, the 1.5 times expanded test intervals result in a change in SIL (from SIL2 to SIL1) at year 14, which is 4 years earlier than the initial case. Some decision-makers may trade safety over other concerns such as cost and may expand the test intervals.

Other two parameters affecting the PFD value of SIF are β and C_{MooN}.The system can also be modified by changing the voting of the SIF. Table 5-3 summarizes the increase in PFD in Gas Detection Function when gas detectors are configured to be 2oo3, instead of 1oo2.

Table 5-3 Change in PFD of Gas Detection Function when Gas Detectors have C_{2oo3} voting

Year	PFD (C_{1oo2})	SIL	PFD (C_{2oo3})	Increase in PFD (%)	Increase in PFD (%)
1-8	6.11E-02	SIL1	6.13E-02	SIL1	0.4 %
9	6.16E-02	SIL1	6.19E-02	SIL1	0.5 %
10	6.25E-02	SIL1	6.29E-02	SIL1	0.7 %
11	7.65E-02	SIL1	7.70E-02	SIL1	0.6 %
12	8.50E-02	SIL1	8.56E-02	SIL1	0.7 %
13	9.39E-02	SIL1	9.46E-02	SIL1	0.7 %
14	1.03E-01	-	1.04E-01	-	0.8 %
15	1.13E-01	-	1.14E-01	-	0.8 %
16	1.23E-01	-	1.24E-01	-	0.9 %
17	1.34E-01	-	1.35E-01	-	0.9 %
18	1.45E-01	-	1.46E-01	-	1.0 %
38	4.16E-01	-	4.23E-01	-	1.8 %
39	4.31E-01	-	4.39E-01	-	1.8 %
40	4.47E-01	-	4.55E-01	-	1.8 %

Switching from 1oo2 to 2oo3 voting, there is no significant increase in PFD values, as well as the change in SIL, which are identified during the early life of gas detectors/Gas Detection Function. However, the rate of increase becomes higher when the components get close to the end of their useful lifetimes. When all components of the function exceed their useful lifetimes (after year 10), the rate of increase keeps increasing insignificantly.

5.3 EFFECT OF STUDY LIMITATIONS

There are two main limitations mentioned in Section 1.7; time constraints and limited access to data.

Since this research is constrained by the deadline of the submission, not all SIS listed in Section 3.1 could be studied. Although the overall safety of offshore facilities is realized by many SISs, only 3 of them have been analyzed in terms of their performance during ALE. It is assumed that understanding the behavior of these SISs will give an overall picture of the safety performance under aging. However, the SISs may vary with their purposes and the components of SIFs. For example, Public Address safety systems, which are composed of microphones, amplifiers and loudspeakers, are used to address the workers in the case of danger during the operations of offshore facilities. Similar to all other SIF components, the reliability of the loudspeakers has to be also verified while the facilities are in operation. However, according to the expert of ORS, the useful lifetime of loudspeakers is generally less than 3 years if they are used outdoors, which indicates faster degradation and more possible failure events in 40 years compared to the other components analyzed in this thesis. This might indicate the higher PFD and/or loss of SIL compliance even before the anticipated duration of ALE for Public Address Systems. It means that the effects of aging can vary from component to component, therefore; for different SIFs and SIS. Nonetheless, the author believes that the results indicating the lower SILs of SIFs during ALE can be generalized for most of the SISs.

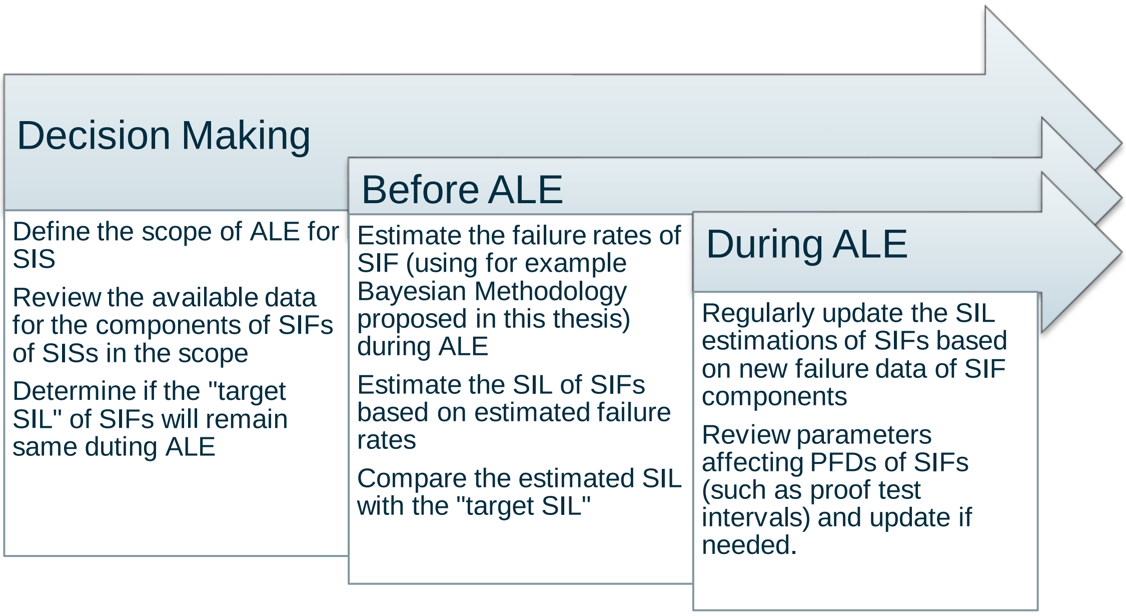
This research is subject to the limitation of access to failure event databases. OREDA (10), the unique reliability data source has restricted access for its failure event database. Although it is known as highly reliable and reputable data collected in more than 30 years in the offshore oil and gas industry, only the contributors have permission to access the failure event database. For this reason, this research has been performed with the databases provided by ORS, which were previously used in their reliability assessment projects performed for different business partners. Since the data was retrieved from various offshore facilities by different operator companies, the quality and the integrity of data collection could not be ensured. Even though it is assumed that the companies collect data in accordance with their internal guidelines, it is impossible to say whether they follow the same failure reporting system or not. Using the OREDA for this research would clear the doubt of the standardized failure reporting system. Moreover, the situation that the databases include the failure events which occurred only in given time intervals might create another limitation. For example, one can see in the first line of failure event data for ESVs presented in Table 0-1 of Appendix-C: Failure Event Data that the failure event occurred in 04.01.2011. The data is collected between 2011-2016 despite the fact that the installation date of the component is 2007. It means that the failure events between 2007-2011 are missing for this component and there is no way to know if any failure occurred during the first 4 years of its operations. If the analysis was performed with the complete failure event dataset, the different failure rate estimations might be obtained. Therefore, the results of this research must be interpreted with these limitations in mind.

The author acknowledges that this research is subject to limitations regarding time and data sources and the readers must approach the conclusions with caution.

6 PRACTICAL MEASURES AND ROADMAP

The purpose of ALE is to keep the facilities in operation beyond their designed life. There are numerous guidelines published by PSA (see for example; (59), (60),(61)) to facilitate the coordination of ALE activities. However, there is no specific guideline covering the LE period of SISs in component level and SIF level from the point of SIL compliance.

The extracted conclusion from this thesis brings the need to propose possible practical measures and roadmap to follow before and during the ALE which might seem to be a good starting point for a future guideline. Figure 6-1 shows the steps that need to be taken before and during the ALE period starts for SIS.



Decision Making

Define the scope of ALE for SIS

Review the available data for the components of SIFs of SISs in the scope

Determine if the "target SIL" of SIFs will remain same duting ALE

Before ALE

Estimate the failure rates of SIF (using for example Bayesian Methodology proposed in this thesis) during ALE

Estimate the SIL of SIFs based on estimated failure rates

Compare the estimated SIL with the "target SIL"

During ALE

Regularly update the SIL estimations of SIFs based on new failure data of SIF components

Review parameters affecting PFDs of SIFs (such as proof test intervals) and update if needed.

Figure 6-1 Roadmap for ALE

The preparation period should include the definition of scope, i.e. determination of which SIFs of which SISs are going to continue their operation beyond the components' useful lifetime. The available data of the components/systems in the scope should be reviewed in order to ensure the suitability of the components for ALE. If the failure events are already highly frequent for any components/systems the scope should be reviewed and modified accordingly. In addition, pre-determined SIL compliance should be reviewed by decision-makers. It must be acknowledged whether the target SIL is the same (or lower, higher) for the late in component's lifetime or not. This information is then to be used to assessments before ALE. Once the suitability of components/systems is ensured for ALE in the decision-making stage, the assessments for failure rate estimation for the extended period should be performed. To do this, Bayesian Methodology proposed in this thesis can be used. Later, the SIL estimations should be made based on estimated failure rates, which might be based on the PDS method as used in this thesis. If the SIL estimations of SIFs seem to be meeting the target SILs, the ALE period can be started. During ALE, SIL estimations must be updated if there are new failure events associated with SIF components. Even though there are no failure events observed, the operator should always keep in mind that the components'/systems' behavior in terms of safety could be different than it was during the useful lifetime. Therefore, some parameters such as proof test intervals should also be reviewed from the safety point of view and updated if needed. A well-planned ALE period is key to prevent the occurrence of possible unforeseen safety-critical events beyond useful lifetimes.

The author of this thesis recommends the careful consideration of SIL targets of SIFs. If the target SIL during ALE has set to a higher level compared to target SIL during the useful lifetime, the stricter maintenance plan implementation or the replacement of the components should be considered by decision-makers.

7 CONCLUSION AND FINAL REMARKS

Many safety structures in the oil and gas industry are realized by SISs which are formed of one or more SIFs. This thesis has proposed a systematic approach to the failure rate estimations of SIF`s components used beyond their useful lifetime to quantify the deviation from the required safety levels for an extended period of lifetime. A Bayesian Methodology was proposed for the measure of safety loss during ALE. The overall idea is to present a systematic approach to merging the expert opinion about the failure rates and the effect of their changes in SIL compliance with the current information on hand. The quantitative method developed in Python to have the estimates of failure rates for anticipated of the operational period of 40 years.

The proposed methodology is executed for the 3 SIFs formed by 9 components in total. Then, the estimated failure rates have been used to calculate the PFD of the components, PFD of the SIFs and finally, corresponding SILs of SIFs have been estimated.

The following main conclusions are yielded from this thesis:

- The overall PFD at year 40 is estimated to be factor 128 times higher than the value from generic data/vendor for HIPPS function, 62 times higher for ESD segregation function and 19 times higher for Gas Detection Function.
- The components having a higher estimation of failure rates may require more careful consideration in terms of maintenance and replacement activities from the risk and safety point of view.
- The other factors than failure rates affecting the PFDs (proof test intervals and component votings) must also be reviewed and modified in case they seem vital in order to achieve desired safety levels.

The author acknowledges the failure event data resources used in this analysis could be questioned in terms of its accuracy. More informative priors would result in more precise estimations in Bayesian Methodology. This could be included in the analysis as a future work when the more precise failure event data related to SIF components become available.

Although there are several other SISs applied in offshore facilities as given in Section 3.1, in the context of this thesis, only 3 SISs were examined and evaluated. However, it is important to consider the behavior of other SISs during ALE period when the overall reliability assessment of the facility is performed. The results of this thesis seem adequate to draw a conclusion that during the extended periods, the overall reliability of SISs of an offshore facility may be lower than it was anticipated during the design phase. Therefore, the required risk reduction may not be achieved without the introduction of additional measures or changes in current policies regarding safety and reliability.

There is a clear need for a guideline to perform the necessary risk evaluation and assessment for the reliability and safety of SISs during ALE. The roadmap for asset operators proposed in this thesis can be taken as a good starting point for the authorities in the development of future guidelines.

8 REFERENCES

1. Ersdal, Gerhard, Sharp, J.V., Stacey,A. Aging and Life Extension of Offshore Structures.
2. NORSOK. N-001 Structural design. 2004 Feb. Report No.: N-001.
3. Organization IS. ISO 2394: 2015: General principles on reliability for structures. ISO Geneva; 2015.
4. International Electrotechnical Commission. IEC61511-Functional safety - Safety instrumented systems for the process industry sector. Geneva, Switzerland: International Electrotechnical Commission; 2004.
5. International Electrotechnical Commission. IEC61508-Functional safety of electrical/electronic/programmable electronic safety-related systems. Geneva, Switzerland: International Electrotechnical Commission; 2010.
6. Sultana, Sharmin. A new approach of uncertainty treatment in the verification of safety integrity level of safety instrumented system. [Norway]: University of Stavanger;
7. Stacey,A., Birkinshaw, M., Sharp, J.V. LIFE EXTENSION ISSUES FOR AGEING OFFSHORE INSTALLATIONS. In: Proceedings of OMAE2008. Estoril, Portugal; 2008.
8. Reliability Prediction Method for Safety Instrumented Systems-PDS Method Handbook 2010 Edition. SINTEF Technology and Society; 2010. Report No.: A13503.
9. Rausand M, Høyland A. System Reliability Theory. 2nd ed. John Wiley & Sons; 2004.
10. SINTEF. OREDA-Offshore Reliability Data Handbook.
11. Goble, William M. Control Systems Safety Evaluation and Reliability. 3rd ed. USA: International Society of Automation; 1998.
12. Availability, Reliability, SIL. What is the difference? Cooper-Crouse Hinds AS; 2010.
13. Factory Acceptence Test [Internet]. Available from: https://www.tuv.com/world/en/factory-acceptance-test.html
14. Bäckström O. Ageing PSA – From a PSA Tool Point of View. In Paris; 2005.
15. Xie M, Lai CD. Reliability Analysis using an additive Weibull model with bathtub-shaped failure rate function. Reliability Engineering and System Safety. 1995 Nov;52:87–93.
16. Kelly DL, Smith CL. Bayesian inference in probabistic risk assessment-current state of art. Reliability Engineering & System Safety. 2009;94(2):628–43.
17. Alzbutas R, Iesmantas T. Application of Bayesian Methods for Age-Dependent Reliability Analysis. John Wiley Sons Ltd. 2013;
18. Wintle J, Sharp J. Requirements for life extension of ageing offshore production installations. Publ TWI Camb UK Rep. 2008;17554(1):08.
19. NOG122-Recommended Guidelines for the Management of Life Extension. Norwegian Oil and Gas Association; 2017 Aug. Report No.: 122.
20. NORSOK. Z-008 Criticality analysis for maintenance purposes. 2001 Nov. Report No.: Z-008.
21. Norway PSA. Regulations relating to design and outfitting of facilities, etc. in the petroleum activities (the facilities regulations). Pet Saf Auth Nor. 2015;

22. NOG070-Application of IEC 61508 and IEC 61511 in the Norwegian Petroleum Industry. Norwegian Oil and Gas Association; 2018 Jun. Report No.: 070.

23. NORSOK. S-001 Technical Safety. 2018 Jun. Report No.: S-001.

24. Petroleum Safety Authority. Principles for barrier management in the petroleum industry. Pet Saf Auth. 2013;

25. Hokstad, Per, Hobrekke, Solfrid, Johsen, Roy, Sanesland, Signbjorn. Aging and Life Extension for offshore facilities in general and for specific systems. SINTEF Technology and Society; 2010 Mar. Report No.: SINTEF A15322.

26. Brombacher AC, Rouvroye JL. New quantitative safety standards: different techniques, different results? Reliability Engineering & System Safety. 1999 Nov;66(2):121–5.

27. Reliability Data for Safety Instrumented Systems-PDS Data Handbook 2010 Edition. SINTEF Technology and Society; 2010. Report No.: A13502.

28. Lai CD, Xie M. Stochastic Aging and Dependence for Reliability. Springer; 2006.

29. De Finetti B. Theory of probability: a critical introductory treatment. Chichester, UK ; Hoboken, NJ: John Wiley & Sons; 2017. 582 p.

30. Hauge S, Lundteigen MA. Guidelines for follow-up of Safety Instrumented Systems (SIS) in operating phase. 2008. Report No.: A8788.

31. Failure Modes , Effects and Diagnostoc Analysis, Emerson’s Rosemount 3051 Pressure Transmitter with 4-20mA HART. Exida; 2016 Oct.

32. Certificate- Prosafe SLS Safety System of Yokogawa Europe Solutions B.V. TUV Rheiland Industrie Service GmbH; 2017 Feb. Report No.: 968/EL 399.01/17.

33. Safety Manual and Safety Justification (SAM) Mokveld final element (S99) Closing upon demand. Mokveld; 2011 Aug.

34. Briand D, Campbell JE, Huzurbazar AV. Updating a user friendly combined lifetime failure distribution. In: 2007 Annual Reliability and Maintainability Symposium. IEEE; 2007. p. 311–6.

35. Briand D, Huzurbazar AV. Bayesian reliability applications of a combined lifecycle failure distribution. Proc Inst Mech Eng Part O J Risk Reliab. 2008;222(4):713–20.

36. Meyer MA, Booker JM. Eliciting and analyzing expert judgment: a practical guide. SIAM; 2001.

37. Morris PA. Combining expert judgments: A Bayesian approach. Manag Sci. 1977;23(7):679–93.

38. Van Noortwijk JM, Dekker A, Cooke RM, Mazzuchi TA. Expert judgment in maintenance optimization. IEEE Trans Reliab. 1992;41(3):427–32.

39. Cook JD. Parameters and percentiles [Internet]. 2010. Available from: https://www.johndcook.com/blog/2010/01/31/parameters-from-percentiles/

40. Cook JD. Determining distribution parameters from quantiles. 2010;

41. Cook JD. Finding Probability Distribution Parameters from Percentiles [Internet]. 2014. Available from: https://www.codeproject.com/Articles/56371/Finding-Probability-Distribution-Parameters-from-P

42. Gelman A. Prior distributions for variance parameters in hierarchical models (comment on article by Browne and Draper). Bayesian Anal. 2006;1(3):515–34.

43. Salvatier J, Wiecki TV, Fonnesbeck C. Probabilistic programming in Python using PyMC3. PeerJ Comput Sci. 2016;2:e55.

44. Bastien F, Lamblin P, Pascanu R, Bergstra J, Goodfellow I, Bergeron A, et al. Theano: new features and speed improvements. ArXiv Prepr ArXiv12115590. 2012;

45. Oliphant TE. A guide to NumPy. Vol. 1. Trelgol Publishing USA; 2006.

46. Bressert E. SciPy and NumPy: an overview for developers. O'Reilly Media, Inc.; 2012.

47. Tosi S. Matplotlib for Python developers. Packt Publishing Ltd; 2009.

48. McKinney W. Python for data analysis: Data wrangling with Pandas, NumPy, and IPython. O'Reilly Media, Inc.; 2012.

49. Hoffman MD, Gelman A. The No-U-Turn sampler: adaptively setting path lengths in Hamiltonian Monte Carlo. J Mach Learn Res. 2014;15(1):1593–623.

50. Haario H, Saksman E, Tamminen J. Adaptive proposal distribution for random walk Metropolis algorithm. Comput Stat. 1999;14(3):375–96.

51. Martin O. Bayesian Analysis with Python: Introduction to statistical modeling and probabilistic programming using PyMC3 and ArviZ. Packt Publishing Ltd; 2018.

52. Björn. Can somebody explain to me NUTS in english? [Internet]. 2017. Available from: https://stats.stackexchange.com/questions/311813/can-somebody-explain-to-me-nuts-in-english.

53. The Markov-chain Monte Carlo Interactive Gallery [Internet]. [cited 2020 Jun 27]. Available from: https://chi-feng.github.io/mcmc-demo/

54. Flegal JM. Monte Carlo Standard Errors for Markov Chain Monte Carlo. University of MinnesotaMajor: Statistics.; 2008.

55. Roos M, Martins TG, Held L, Rue H. Sensitivity analysis for Bayesian hierarchical models. Bayesian Anal. 2015;10(2):321–49.

56. Polson NG, Scott JG. On the half-Cauchy prior for a global scale parameter. Bayesian Anal. 2012;7(4):887–902.

57. Hahn GJ, Meeker WQ. Statistical intervals: a guide for practitioners. Vol. 92. John Wiley & Sons; 2011.

58. Liu J, Nordman DJ, Meeker WQ. The number of MCMC draws needed to compute Bayesian credible bounds. Am Stat. 2016;70(3):275–84.

59. Galbraith D, Sharp J. Recommendations for design life extension regulations. Doc Id POS-DK06-195-R02 Poseidon Febr. 2007;

60. Chockie A, Gregor F. Condition Monitoring of Passive Systems, Structures, and Components. Aging Manag Brief Rep Prep Chockie Groupe Int Inc Pet Saf Auth Nor. 2006;

61. Sørensen JD, Ersdal G. Safety and inspection planning of older installations. Proc Inst Mech Eng Part O J Risk Reliab. 2008;222(3):403–17.

APPENDIX

APPENDIX-A: PYTHON CODE FOR INITIAL PARAMETER SELECTION

The Python codes presented here are adapted from (41).

```
from scipy import stats, optimize

from math import log, pow

from functools import partial
```

gamma distribution parameters selection

```
def gamma_parameters(x1, p1, x2, p2):
    if p1 > p2:
        (p1, p2) = (p2, p1)
        (x1, x2) = (x2, x1)

    def objective(alpha):
        return stats.gamma.ppf(p2, alpha) / stats.gamma.ppf(p1, alpha) - x2/x1

    left = right = 1.0
    while objective(left) < 0.0:
        left /= 2
    while objective(right) > 0.0:
        right *= 2
    alpha = optimize.bisect(objective, left, right)
    beta = x1 / stats.gamma.ppf(p1, alpha)

    return (alpha, beta)
```

```
p1 = 0.10
p2 = 0.20
x1 = 5
x2 = 11
```

```
alpha = stats.gamma.ppf(p1, alpha, scale=beta)
```

```
beta = stats.gamma.ppf(p2, alpha, scale=beta)
(alpha, beta) = gamma_parameters(x1, p1, x2, p2)
print ("Calculated parameters: ", alpha, beta)
```

normal distribution parameters selection

```
def normal_parameters(x1, p1, x2, p2):
    denom = stats.norm.ppf(p2) - stats.norm.ppf(p1)
    sigma = (x2 - x1) / denom
    mu = (x1*stats.norm.ppf(p2) - x2*stats.norm.ppf(p1)) / denom
    return (mu, sigma)
```

```
p1 = 0.4
p2 = 0.6
x1 = 5
x2 = 15
```

```
mu = stats.norm.ppf(p1, x1, x2)
sigma = stats.norm.ppf(p2, x1, x2)
(mu, sigma) = normal_parameters(x1, p1, x2, p2)
print ("Calculated parameters: ", mu, sigma)
```

APPENDIX-B: PYTHON CODE FOR BAYESIAN ANALYSIS

```python
import pymc3 as pm
import numpy as np
from scipy.stats import weibull_min, exponweib, norm, expon
import matplotlib.pyplot as plt
import os
import arviz as az
# one of
import theano.tensor as tt
import theano
import pandas as pd
from scipy.stats import gamma
import warnings
warnings.filterwarnings('ignore')
```

```python
def find_mixed(obs, rate):
    with pm.Model() as mixed:
        rate_b=pm.Gamma('rate', alpha=1, beta=1/rate)
        expo=pm.Exponential.dist(lam=rate_b)
        mu=pm.Normal('mu', 10, 20)
        sigma=pm.HalfNormal('sigma', 10)
        normal=pm.Normal.dist(mu=mu, sigma=sigma)

        w=pm.Dirichlet('w', a=np.array([1,1]))

        like=pm.Mixture('like', w=w, comp_dists=[expo, normal], observed=obs)
        trace=pm.sample(draws=1000, cores=4)
               pm.traceplot(trace)
        ppc = pm.sample_posterior_predictive(trace, samples=5000, model=mixed)
        return trace, ppc
```

```python
def new_find_failure(alphas, betas, mus, sigmas, t, useful):
    frs=[]
    i=0
    j=0
```

```
    first=True
    for te in t:
        if te<useful:
            fr=betas[i]*np.power(np.exp(1),(-betas[i]*te))
            i +=1
        else:
            #fr=(1/(sigmas[j]*np.sqrt(2*np.pi)))*np.power(np.exp(1),(-np.power((te-mus[j]),2)/(2*(sigmas[j]**2))))
            fr=1 / (np.sqrt(2 * np.pi) * sigmas[j]) * np.exp(-1 / 2 * np.square((te - mus[j])) / (np.square(sigmas[j])))
            j +=1
        frs.append(fr)
    plt.plot(t,frs)
    return frs
```

```
useful=
maxy=
failure_r=
yearly_rate=failure_r*8760/1e6
data=np.array([])
```

```
posterior,ppc1=find_mixed(data, yearly_rate)
```

```
def find_A(useful, mu, sigma, rate):
    rc1=rate*np.exp(-rate*useful)
    rc2=1 / (np.sqrt(2 * np.pi) * sigma) * np.exp(-1 / 2 * np.square((useful - mu)) / (np.square(sigma)))
    return rc1/rc2
A=find_A(useful, mu, sigma, rate)
A
```

```
def find_rates(mu, sigma, rate, useful, A):
    tcs=np.linspace(0,maxy, maxy)
    t2=tcs[tcs>useful]
    rc1=np.repeat(rate,useful)
    rc3=A* (norm.pdf(t2, loc=mu, scale=sigma)/(1-norm.cdf(t2, loc=mu, scale=sigma)))
```

```
    rcs=np.concatenate([rc1, rc3])
    plt.axvline(useful, color='red', linestyle='--')
    plt.plot(tcs, rcs)
    return rcs
```

```
failure_rates, rc=find_rates(mu, sigma, rate, useful, A)
```

APPENDIX-C: FAILURE EVENT DATA

Table 0-1 Failure Data of ESV/XVs

Description	Failure Date	Component Installation	Time to failure (days)	Time to failure(years)
HCAA-ESDV	04.01.2011	2007	1463	4.01
HBAA-ESDV	03.06.2015	2002	6359	17.42
EC-ESDV-17027	17.06.2015	1995	8928	24.46
ESDV-38027	08.07.2016	1985	9680	26.52
ESDV-1703	05.04.2016	1985	9586	26.26
ESDV-38301	30.10.2015	2005	5413	14.83
EC-ESDV-17029	13.09.2014	1985	8286	22.70
ESDV-38401	17.09.2015	2005	9020	24.71
ESDV-38021	19.01.2015	2000	6954	19.05
ESDV-38027	08.11.2016	2001	7613	20.86
ESDV-38028	07.01.2012	2006	2198	6.02

Table 0-2 Failure Data of ESD Logic

Description	Failure Date	Component Installation	Time to failure (days)	Time to failure(years)
15.0690.280291	03.09.2003	12.06.2000	1178	3.23
15.217.8662021	30.10.2008	02.08.2000	3011	8.25
17.359.0343616	02.05.2017	25.08.1992	9016	24.70
15.927.4360107	12.09.2019	07.01.1984	13032	35.70

Table 0-3 Failure Data of FTs

Description	Failure Date	Component Installation	Time to failure (days)	Time to failure(years)
DK.GO.F.035.GOFA-FT-35050A	07.11.2017	04.09.1991	9561	26.19
DK.HB.D.036.HBDB-FIT-365103	07.01.2017	01.04.1991	9413	25.79
DK.GO.F.035.GOFA-FT-35050A	11.08.2015	01.08.1992	8410	23.04
DK.GO.F.035.GOFC-FT-35425B	22.07.2015	27.01.1993	8211	22.50
DK.GO.F.035.GOFA-FT-35050A	15.02.2015	06.11.1995	7041	19.29
DK.GO.F.011.GOFB-FT-11066	12.12.2014	14.11.1995	6968	19.09
DK.GO.F.037.GOFA-FT-35080B	12.08.2014	10.11.1996	6484	17.76
DK.GO.F.035.GOFA-FT-35050B	26.09.2013	20.06.2000	4846	13.28
DK.GO.F.057.GOFA-FT-5730	12.01.2013	07.04.2006	2472	6.77
DK.GO.F.011.GOFA-FT-11007	30.09.2012	10.12.2008	1390	3.81

Table 0-4 Failure Data of Gas Detectors

Description	Failure Date	Component Installation	Time to failure (days)	Time to failure(years)
TGS-HWC-010061	10.03.2002	09.05.1996	2131	5.84
TGS-HWC-010121	19.05.2002	14.11.1998	1282	3.51
TGS-HWC-010062	16.06.2002	04.12.1997	1655	4.53
TGS-HWC-010063	03.06.2003	02.01.1996	2709	7.42
TGS-HWC-010042	30.10.2004	22.01.1999	2108	5.78
TGS-HWC-010082	19.08.2005	16.01.1997	3137	8.59
TGS-HWC-010202	10.02.2008	19.09.2000	2700	7.40
TGS-HWC-010033	18.09.2011	02.12.1998	4673	12.80
TGS-HWC-010013	02.01.2012	11.01.2000	4374	11.98
TGS-HWC-010083	23.03.2012	21.08.1997	5328	14.60
TGS-HWC-010083	21.04.2014	03.01.1999	5587	15.31
TGS-HWC-010073	20.06.2016	05.06.1997	6955	19.05
TGS-HWC-010033	15.02.2016	14.11.1998	6302	17.27
TGS-HWC-010011	31.10.2016	02.12.2000	5812	15.92

Table 0-5 Failure Data of PTs

Description	Failure Date	Component Installation	Time to failure (days)	Time to failure(years)
HCAA-PIT-38304	26.10.2007	16.01.2011	1178	3.23
GBN-38 PT-30071	01.04.1991	26.06.2012	7757	21.25
PT-30162	22.09.1998	19.04.2012	4958	13.58
PT-30165	01.01.1996	27.02.2013	6267	17.17
GBN-38 PT-30079	20.12.2011	06.03.2013	442	1.21
GOFB-PIT-30175	21.02.2008	20.05.2013	1915	5.25
HCAB-PIT-38304	22.09.1998	10.02.2014	5620	15.40
TP-PIT-34876	27.05.2003	28.08.2017	5207	14.27
TP-PIT-34870	28.10.1996	23.07.2014	6477	17.75
GEA-PT-1589	27.05.2005	30.10.2014	3443	9.43
GEA-PT-1583	22.09.1998	13.11.2012	5166	14.15
DFE-03	22.09.1998	22.12.2014	5935	16.26
DFCM-PTS-52161	22.09.1998	07.01.2015	5951	16.30
GOFA-PT-38017	01.04.1991	15.04.2016	9146	25.06
HDAA-PIT-38408	28.06.2001	17.04.2015	5041	13.81
DFCM-PT-50790	22.09.1998	23.06.2015	6118	16.76
PIT-301102	14.01.2003	01.07.2015	4551	12.47
PIT-301107	01.09.1997	17.11.2015	6651	18.22
TEC-PT-46020	01.06.1996	02.01.2016	7154	19.60
GC-PT-0503	22.09.1998	12.01.2016	6321	17.32
GC-PT-0517	19.11.2003	22.01.2016	4447	12.18
BN-23	22.09.1998	07.02.2016	6347	17.39
BN-29	22.09.1998	11.02.2016	6351	17.40
PM-4052	28.07.2010	01.03.2016	2043	5.60
PT-300605	01.09.1997	04.06.2016	6851	18.77
PM-4078	02.11.1999	17.06.2016	6072	16.64
GC-PT-9234	06.10.2006	28.10.2012	2214	6.07
GC-PT-0519	22.09.1998	23.08.2016	6545	17.93

PM-4987	01.04.1991	09.02.2012	7619	20.87
PIT-GFN-36	06.03.2001	03.03.2011	3649	10.00
BN-46	13.01.2006	04.03.2017	4068	11.15
PIT-41035	11.03.2004	03.04.2017	4771	13.07
HDAA-PIT-381	28.10.1996	29.08.2017	7610	20.85
DFCM-PT-507	07.05.2003	29.08.2017	5228	14.32

APPENDIX-D: SENSITIVITY ANALYSIS RESULTS

Uniform Prior within Useful Lifetime

Table 0-6 ESD components' failure rates with the uniform prior

Year	ESD Logic	Solenoid	ESV/XV
1	2.36E-06	3.06E-06	3.06E-06
2	2.36E-06	3.06E-06	3.06E-06
3	2.36E-06	3.06E-06	3.06E-06
4	2.36E-06	3.06E-06	3.06E-06
5	2.36E-06	3.06E-06	3.06E-06
6	2.36E-06	3.06E-06	3.06E-06
7	2.36E-06	3.06E-06	3.06E-06
8	2.36E-06	5.30E-06	3.06E-06
9	2.36E-06	7.17E-06	3.06E-06
10	2.36E-06	9.48E-06	3.06E-06
11	1.46E-05	1.23E-05	5.30E-06
12	1.62E-05	1.55E-05	7.17E-06
13	1.79E-05	1.93E-05	9.48E-06
14	1.96E-05	2.36E-05	1.23E-05
15	2.13E-05	2.84E-05	1.55E-05
16	2.31E-05	3.37E-05	1.93E-05
17	2.49E-05	3.95E-05	2.36E-05
18	2.68E-05	4.57E-05	2.84E-05
19	2.87E-05	5.22E-05	3.37E-05
20	3.06E-05	5.92E-05	3.95E-05
21	3.26E-05	6.64E-05	4.57E-05
22	3.46E-05	7.40E-05	5.22E-05
23	3.66E-05	8.18E-05	5.92E-05
24	3.86E-05	8.98E-05	6.64E-05
25	4.07E-05	9.81E-05	7.40E-05
26	4.27E-05	1.07E-04	8.18E-05
27	4.48E-05	1.15E-04	8.98E-05
28	4.69E-05	1.24E-04	9.81E-05
29	4.90E-05	1.33E-04	1.07E-04
30	5.11E-05	1.42E-04	1.15E-04
31	5.33E-05	1.51E-04	1.24E-04
32	5.54E-05	1.60E-04	1.33E-04

33	5.76E-05	1.70E-04	1.42E-04
34	5.97E-05	1.79E-04	1.51E-04
35	6.19E-05	1.88E-04	1.60E-04
36	6.41E-05	1.98E-04	1.70E-04
37	6.62E-05	2.07E-04	1.79E-04
38	6.84E-05	2.17E-04	1.88E-04
39	7.06E-05	2.27E-04	1.98E-04
40	7.28E-05	2.38E-04	2.07E-04

Table 0-7 ESD components' PFDs and SIL with the uniform prior

Year	PFD of ESD Logic	PFD of Solenoid	PFD of ESV/XV	Overall PFD of ESD	SIL
1	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
2	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
3	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
4	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
5	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
6	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
7	1.03E-02	1.34E-02	1.34E-02	3.71E-02	SIL1
8	1.03E-02	2.32E-02	1.34E-02	4.70E-02	-
9	1.03E-02	3.14E-02	1.34E-02	5.51E-02	-
10	1.03E-02	4.15E-02	1.34E-02	6.52E-02	-
11	6.41E-02	5.37E-02	2.32E-02	1.41E-01	-
12	7.11E-02	6.81E-02	3.14E-02	1.71E-01	-
13	7.82E-02	8.47E-02	4.15E-02	2.04E-01	-
14	8.57E-02	1.04E-01	5.37E-02	2.43E-01	-
15	9.33E-02	1.25E-01	6.81E-02	2.86E-01	-
16	1.01E-01	1.48E-01	8.47E-02	3.34E-01	-
17	1.09E-01	1.73E-01	1.04E-01	3.86E-01	-
18	1.17E-01	2.00E-01	1.25E-01	4.42E-01	-
19	1.26E-01	2.29E-01	1.48E-01	5.02E-01	-
20	1.34E-01	2.59E-01	1.73E-01	5.66E-01	-
21	1.43E-01	2.91E-01	2.00E-01	6.34E-01	-
22	1.51E-01	3.24E-01	2.29E-01	7.04E-01	-
23	1.60E-01	3.58E-01	2.59E-01	7.77E-01	-
24	1.69E-01	3.93E-01	2.91E-01	8.53E-01	-

25	1.78E-01	4.30E-01	3.24E-01	9.32E-01	-
26	1.87E-01	4.67E-01	3.58E-01	1.01E+00	-
27	1.96E-01	5.04E-01	3.93E-01	1.09E+00	-
28	2.05E-01	5.43E-01	4.30E-01	1.18E+00	-
29	2.15E-01	5.82E-01	4.67E-01	1.26E+00	-
30	2.24E-01	6.21E-01	5.04E-01	1.35E+00	-
31	2.33E-01	6.61E-01	5.43E-01	1.44E+00	-
32	2.43E-01	7.02E-01	5.82E-01	1.53E+00	-
33	2.52E-01	7.43E-01	6.21E-01	1.62E+00	-
34	2.62E-01	7.84E-01	6.61E-01	1.71E+00	-
35	2.71E-01	8.25E-01	7.02E-01	1.80E+00	-
36	2.81E-01	8.67E-01	7.43E-01	1.89E+00	-
37	2.90E-01	9.09E-01	7.84E-01	1.98E+00	-
38	3.00E-01	9.50E-01	8.25E-01	2.08E+00	-
39	3.09E-01	9.94E-01	8.67E-01	2.17E+00	-
40	3.19E-01	1.04E+00	9.09E-01	2.27E+00	-

‘

Half Cauchy Prior beyond Useful Lifetime

Table 0-8 Gas Detection Function components’ failure rates with Half-Cauchy priors

Year	FTs	Gas Detectors	F&G Logic
1	8.46E-07	7.4E-07	3.99E-06
2	8.46E-07	7.4E-07	3.99E-06
3	8.46E-07	7.4E-07	3.99E-06
4	8.46E-07	7.4E-07	3.99E-06
5	8.46E-07	7.4E-07	3.99E-06
6	8.46E-07	7.4E-07	3.99E-06
7	8.46E-07	7.4E-07	3.99E-06
8	8.46E-07	7.4E-07	3.99E-06
9	9.43E-07	1.1E-06	3.99E-06
10	1.11E-06	1.3E-06	3.99E-06
11	1.29E-06	1.5E-06	6.63E-06
12	1.49E-06	1.7E-06	6.63E-06
13	1.70E-06	1.9E-06	6.64E-06
14	1.93E-06	2.2E-06	6.64E-06

15	2.18E-06	2.5E-06	6.65E-06
16	2.44E-06	2.7E-06	6.66E-06
17	2.71E-06	3.0E-06	6.66E-06
18	3.00E-06	3.3E-06	6.67E-06
19	3.30E-06	3.6E-06	6.67E-06
20	3.61E-06	3.9E-06	6.68E-06
21	3.94E-06	4.2E-06	6.68E-06
22	4.27E-06	4.5E-06	6.69E-06
23	4.61E-06	4.8E-06	6.69E-06
24	4.96E-06	5.1E-06	6.70E-06
25	5.32E-06	5.4E-06	6.71E-06
26	5.69E-06	5.8E-06	6.71E-06
27	6.06E-06	6.1E-06	6.72E-06
28	6.44E-06	6.4E-06	6.72E-06
29	6.83E-06	6.7E-06	6.73E-06
30	7.22E-06	7.1E-06	6.73E-06
31	7.62E-06	7.4E-06	6.74E-06
32	8.02E-06	7.7E-06	6.75E-06
33	8.42E-06	8.1E-06	6.75E-06
34	8.83E-06	8.4E-06	6.76E-06
35	9.25E-06	8.7E-06	6.76E-06
36	9.66E-06	9.1E-06	6.77E-06
37	1.01E-05	9.4E-06	6.77E-06
38	1.05E-05	9.8E-06	6.78E-06
39	1.09E-05	1.0E-05	6.79E-06
40	1.14E-05	1.04E-05	6.79E-06

Table 0-9 Gas Detection Function components' PFDs and SIL with the Half-Cauchy priors

Year	PFD of FTs	PFD Gas Detectors	PFD of F&G Logic	Overall PFD of Gas Detection	SIL
1	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
2	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
3	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
4	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
5	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1

6	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
7	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
8	3.70E-03	2.08E-04	1.75E-02	2.14E-02	SIL1
9	4.13E-03	3.11E-04	1.75E-02	2.19E-02	SIL1
10	4.85E-03	3.74E-04	1.75E-02	2.27E-02	SIL1
11	5.64E-03	4.45E-04	2.90E-02	3.51E-02	SIL1
12	6.51E-03	5.22E-04	2.91E-02	3.61E-02	SIL1
13	7.45E-03	6.07E-04	2.91E-02	3.71E-02	SIL1
14	8.46E-03	7.00E-04	2.91E-02	3.83E-02	SIL1
15	9.53E-03	8.00E-04	2.91E-02	3.95E-02	SIL1
16	1.07E-02	9.06E-04	2.91E-02	4.07E-02	SIL1
17	1.19E-02	1.02E-03	2.92E-02	4.21E-02	SIL1
18	1.31E-02	1.14E-03	2.92E-02	4.35E-02	SIL1
19	1.45E-02	1.27E-03	2.92E-02	4.49E-02	SIL1
20	1.58E-02	1.41E-03	2.92E-02	4.65E-02	SIL1
21	1.72E-02	1.55E-03	2.93E-02	4.81E-02	SIL1
22	1.87E-02	1.70E-03	2.93E-02	4.97E-02	SIL1
23	2.02E-02	1.85E-03	2.93E-02	5.14E-02	SIL1
24	2.17E-02	2.02E-03	2.93E-02	5.31E-02	SIL1
25	2.33E-02	2.19E-03	2.94E-02	5.49E-02	SIL1
26	2.49E-02	2.36E-03	2.94E-02	5.67E-02	SIL1
27	2.66E-02	2.55E-03	2.94E-02	5.85E-02	SIL1
28	2.82E-02	2.74E-03	2.94E-02	6.04E-02	SIL1
29	2.99E-02	2.93E-03	2.95E-02	6.23E-02	SIL1
30	3.16E-02	3.14E-03	2.95E-02	6.43E-02	SIL1
31	3.34E-02	3.35E-03	2.95E-02	6.62E-02	SIL1
32	3.51E-02	3.56E-03	2.95E-02	6.82E-02	SIL1
33	3.69E-02	3.79E-03	2.96E-02	7.03E-02	SIL1
34	3.87E-02	4.02E-03	2.96E-02	7.23E-02	SIL1
35	4.05E-02	4.25E-03	2.96E-02	7.44E-02	SIL1
36	4.23E-02	4.49E-03	2.96E-02	7.65E-02	SIL1
37	4.42E-02	4.74E-03	2.97E-02	7.86E-02	SIL1
38	4.60E-02	5.00E-03	2.97E-02	8.07E-02	SIL1
39	4.79E-02	5.26E-03	2.97E-02	8.28E-02	SIL1
40	4.97E-02	5.53E-03	2.97E-02	8.50E-02	SIL1