| Faculty of Sci MASTE | 1 <br> $y$ of ger nd Technology <br> THESIS |
| :---: | :---: |
| Study program/ Specialization: <br> Petroleum Technology/Well Engineering | Spring semester, 2020 <br> Open access |
| Writer: <br> Caetano Pinheiro Saramago de Andrade |  |
| Faculty supervisor: <br> External supervisor(s): | ui <br> zej Tunkiel |
| Thesis title: <br> Rotary Steerable System modelling and simulator |  |
| Credits (ECTS): 30 ECTS |  |
| Keywords: <br> Rotary Steerable System, ROP Composition, Directional Drilling, Natural Displacement |  |

Frontpage for master thesis
Faculty of Science and Technology

## Acknowledgements

I would like to express my gratitude to Dan Sui, Professor at the University of Stavanger, as well as to Andrzej Tunkiel and specialists from Canrig Drilling Technology Norway for their guidance and support. They have always been available for discussion aiming to enrich this project.


#### Abstract

Nowadays, there is a fact in the oil industry: directional drilling is a common and essential procedure in major drilling operations. With the development of technologies that allows horizontal and multilateral wells, the percentage of recoverable oil has increased. This fact represents the main motivation of this master thesis. In other words, this master thesis aims to contribute to the development and further understanding of directional drilling, more specifically on the Rotary Steerable System (RSS) technology. Working in partnership with the industry, this thesis aims to model a Rotary Steerable System.

Whenever a directional drilling process has started, there is a point called the kickoff point, where a vertical well intentionally deviates from the vertical direction. The RSS is a method, among several, that can be used to give an angle to the drilling path. Classical definitions from fundamental physics, as Newton's third law and beam-bending analysis, and original definitions developed by this thesis, as the ROP composition and force on the bit analysis, will be used to model and foresee the behavior of the RSS system. Every step of modelling will be described starting from the first simple idea until the most sophisticated model simulator.

The RSS system, used as a foundation of the mathematical model, is a commercial system from Canrig Drilling Technology Norway. Costs and specific characteristics of the system will not be exposed. The main physical concepts and behavior will be studied and modelled in this thesis.


Keywords: Rotary Steerable System, ROP Composition, Directional Drilling, Natural Displacement

## Nomenclature

## Resultant Force on the bit

| Fbit | Resultant force on the bit on the 2D model |
| :--- | :--- |
| Fbit' | Resultant force on the bit on the North and East plane of <br> coordinates |
| Fbit ${ }^{\prime \prime}$ | Resultant force on the bit on the horizontal displacement (HD) and true vertical depth <br> (TVD) plane of coordinates |
| $\theta_{\text {long }}$ | Arctangent of the slope on the created line from the bit to the upper stabilizer |
| $\theta_{\text {short }}$ | Arctangent of the slope on the created line from the bit to the upper stabilizer |

## Natural displacement definition/force

| $m_{\text {long }}$ | Slope of the created line from the upper stabilizer to the bit |
| :---: | :---: |
| $c_{\text {long }}$ | Independent coefficient of the created line from the upper stabilizer to the bit |
| $m_{\text {short }}$ | Slope of the created line from the actuator to the bit |
| $c_{\text {short }}$ | Independent coefficient of the created line from the actuator to the bit |
| $m_{3}$ | Slope of the created line from the upper stabilizer to the bit (simulated tool) |
| $c_{3}$ | Independent coefficient of the created line from the upper stabilizer to the bit (simulated tool) |
| $m_{0,5}$ | Slope of the created line from the actuator to the bit (simulated tool) |
| $c_{0,5}$ | Independent coefficient of the created line from the actuator to the bit (simulated tool) |
| Xo | Location of the bit X axis |
| Yo | Location of the bit Y axis |
| $X_{3}$ | Well location of the upper stabilizer on the X coordinates |
| $Y_{3}$ | Well location of the upper stabilizer on the Y coordinates |
| $X_{0,5}$ | Well location of the offset on the X coordinates |
| $Y_{0,5}$ | Well location of the offset on the Y coordinates |
| X | Distance from the bit to the offset location |
| $Y p_{0.5}$ | Point behind of the bit on the distance x on the short line created from the bit to the offset location (Y coordinates) |
| $Y p_{3}$ | Point behind of the bit on the distance x on the long line created from the bit to the upper stabilizer location ( Y coordinates) |
| $X p_{0.5}$ | Point behind of the bit on the distance x on the short line created from the bit to the offset location (X coordinates) |
| $X p_{3}$ | Point behind of the bit on the distance x on the long line created from the bit to the upper stabilizer location (X coordinates) |
| $H_{\text {Normal }}$ | Natural displacement in meters on the 2D model |
| Fbit $_{H}$ | Force on the bit caused by the natural displacement on the 2D model |


| $H_{A z i}$ | Natural displacement in meters on the North and East coordinates plane (3D) |
| :--- | :--- |
| $H_{I n c}$ | Natural displacement in meters on the horizontal displacement (HD) and true vertical <br> depth (TVD) plane of coordinates (3D) |
| Fbit $_{H}{ }^{\prime \prime}$ | Force on the bit caused by the natural displacement on the HD and TVD coordinates <br> plane |
| Fbit $_{H}{ }^{\prime}$ | Force on the bit caused by the natural displacement on the North and East plane of <br> coordinates |

## Force caused by the offset

Fbit $_{\text {offset }}$
Fbit $_{\text {offset }}{ }^{\prime \prime}$

Fbit $_{\text {offset }}{ }^{\prime}$

Offset $_{\text {Normal }}$
Offset ${ }_{\text {Inc }}$
Offset ${ }_{A z i}$

Force on the bit caused by the offset on the 2D model
Force on the bit caused by the offset displacement on the HD and TVD plane of coordinates

Force on the bit caused by the offset displacement on the North and East plane of coordinates

Offset defined by the offset controller on the 2D model
Offset calculated by the offset controller on the plane TVD x HD
Offset calculated by the offset controller on the plane North x East

## Rate of Penetration (ROP) model

| $R O P_{\text {Normal }}$ | ROP that controls the inclination (2D) |
| :--- | :--- |
| $R O P_{I n c}$ | ROP that controls the inclination (3D) |
| $R O P_{\text {Azi }}$ | ROP that controls the azimuth (3D) |
| $R O P_{\text {Axial }}$ | ROP that controls the axial velocity (2D and 3D) |
| $\mu$ | Sliding friction |
| $N$ | Revolutions per minute |
| $E s_{m i n}$ | Rock compressibility |
| $E F F_{m}$ | Efficiency of the rock compressibility |
| $E s$ | Specific Energy |
| $\mu^{\prime}$ | Sliding friction for the ROPRormal |
| $N^{\prime}$ | Specific Energy for the ROP $P_{\text {Normal }}$ |
| $E s^{\prime}$ | Weight on Bit |
| $W O B$ | Area of the borehole |
| $A_{b}$ | Diameter of the Bit |
| $D$ | ROP from the Bourgoyne et al. 1986 ROP model |
| R | Constants from the Bourgoyne et al. 1986 ROP model |
| $f_{1}, f_{2}, f_{3}, f_{4}, f_{5}, f_{6}, f_{7}, f_{8}$ | Cormal |


| $a b$ | Constant from the Bingham ROP Model |
| :--- | :--- |
| $b b$ | Constant from the Bingham ROP Model |
| $T$ | Torque |
| $\Delta$ Tsonic | Time that is analyzed on the neutron log |
| neutron porosity | Porosity that is analyzed on the neutron log |

## Beam Bending

| $W$ | Load on the beam |
| :--- | :--- |
| $a$ | Length of the shorter segment (from the bit to the offset location) |
| $b$ | Length of the longer segment (from the upper stabilizer to the offset location) |
| $E$ | Elasticity modulus |
| $I$ | Inertial coefficient |
| $l$ | Length of the beam |
| Do | Outside diameter from the internal body of the tool |
| Di | Deflection on the load application location |

## Survey parameters

| $M D_{V e r t i c a l(T)}$ | Measured depth on a vertical well on the analyzed time T |
| :--- | :--- |
| $M D_{V e r t i c a l(T-1)}$ | Measured depth on a vertical well on the previous timestep T-1 |
| $M D_{2 D(T)}$ | Measured depth on an analyzed time T on a 2D model |
| $M D_{2 D(T-1)}$ | Measured depth on a previous timestep T - 1 on a 2D model |
| $M D_{3 D(T)}$ | Measured depth on an analyzed time T on a 3D model |
| $M D_{3 D(T-1)}$ | Measured depth on a previous timestep T - 1 on a 3D model |
| $T V D_{T}$ | True vertical depth on an analyzed time T |
| $T V D_{T-1}$ | True vertical depth on the previous timestep T - 1 |
| $H D_{T}$ | Horizontal displacement on an analyzed time T |
| $H D_{T-1}$ | Dogleg severity |
| $D L S$ | Inclination on the previous timestep T - 1 |
| $I n c_{(T-1)}$ | Inclination on an analyzed time T |
| $I n c_{(T)}$ | Azimuth on the previous timestep T - 1 |
| $A z i_{(T-1)}$ | Azimuth on an analyzed time T |
| $A z i_{(T)}$ |  |

## Table of Contents

Acknowledgements ..... ii
Abstract ..... iii
Nomenclature ..... iv
Table of Contents ..... vii
List of Figures ..... ix
1 Introduction ..... 1
1.1 History ..... 1
1.2 Motor Versus RSS ..... 2
1.3 Future Opportunities ..... 2
2 Directional Drilling Tools ..... 4
2.1 Directional Drilling Concepts. ..... 4
2.2 RSS Systems ..... 4
2.3 Canrig Equipment ..... 5
Disclaimer ..... 7
3 RSS Modelling Approach ..... 8
3.1 2D and 3D Logic ..... 8
3.1.1 2D Model ..... 9
3.1.2 3D Model ..... 14
3.2 Dynamic Mathematical Model ..... 18
3.3 Improvements on the 3D Model ..... 19
4 Modelling Theory and Concepts ..... 21
4.1 Survey Calculations and Trajectory Definitions ..... 21
4.1.1 Survey Points 2D ..... 22
4.1.2 Survey Points 3D ..... 25
4.2 Force Calculations ..... 27
4.2.1 Beam Bending ..... 28
4.2.2 Natural Displacement 2D ..... 30
4.2.3 Natural Displacement 3D ..... 35
4.2.4 Force Caused by the Natural Displacement on the Bit ..... 36
4.2.5 Force Caused by the Offset on the Bit ..... 37
4.2.6 Resultant Force on the Bit ..... 39
4.3 ROP Modelling ..... 42
4.3.1 Bourgoyne et al. ROP Model ..... 43
4.3.2 Bingham Model ..... 44
4.3.3 R.Teale ..... 44
4.4 Offset and Offset Controller ..... 48
4.4.1 Theoretical Definition ..... 48
4.4.2 Modelling ..... 49
4.4.3 Offset Controller 3D ..... 50
5 Case Study ..... 55
5.1 Basic 2D Modelling - Vertical Well ..... 55
5.2 2D Modelling Disregarding Natural Displacement ..... 56
5.3 2D Modelling Considering Natural Displacement ..... 61
5.4 Final 2D Model ..... 65
5.5 Final 3D Model ..... 68
6 Result Errors ..... 72
6.1 Data Issues ..... 72
6.2 Model Uncertainties ..... 72
6.3 Errors ..... 72
6.4 RSS Challenges and Improvements ..... 73
8 References ..... 75
Appendixes ..... 77
Appendix 1. Functions ..... 77
Appendix 2. Program process ..... 77
Appendix 3. Three Dimension Program ..... 79
Appendix 4. 2D Program ..... 91

## List of Figures

## Chapter 1

Figure 1. 1 - Drilling tools throughout time3
## Chapter 2

Figure 2. 1: ..... 4
Figure 2.2 ..... 5
Figure 2.3 ..... 6
Figure 2. 4: Forces felt by the RSS tool ..... 7
Chapter 3
Figure 3. 1 ..... 8
Figure 3.2 ..... 9
Figure 3.3 ..... 10
Figure 3. 4 - ROP Composition ..... 10
Figure 3.5 ..... 12
Figure 3.6 ..... 12
Figure 3.7 ..... 14
Figure 3.8 ..... 14
Figure 3.9 ..... 15
Figure 3.10 ..... 16
Figure 3. 11 - ROP composition 3D ..... 16
Figure 3.12 ..... 17
Figure 3.13 ..... 17
Figure 3. 14 ..... 18
Figure 3. 15: 2D Functions of the Mathematical model - Every function has an input and output used on the model ..... 18
Figure 3. 16: 3D Functions of the Mathematical model - Every function has an input and output used on the model ..... 19
Figure 3. 17: 3D improved functions of the Mathematical model ..... 20
Figure 3. 189: 3D improved variables - Possible location of each displacement ..... 20
Chapter 4
Figure 4.1 ..... 21
Figure 4. 2: Inclination ..... 22
Figure 4. 3: Inclination modelling ..... 23
Figure 4. 4: Horizontal displacement - View from above of well path ..... 24
Figure 4.5 ..... 25
Figure 4. 6: Azimuth ..... 26
Figure 4.7 ..... 27
Figure 4.8 ..... 27
Figure 4. 9: Beam bending scenarios ..... 28
Figure 4. 10: Beam bending scenario that represents the RSS tool ..... 29
Figure 4. 11: Drillstring on a hypothetical situation ..... 30
Figure 4. 12: Natural displacement ..... 31
Figure 4. 13 ..... 32
Figure 4. 14: Process of calculating the natural displacement ..... 32
Figure 4. 15: Simulation collecting the geometry at approximately 3 meters behind the bit ..... 33
Figure 4. 16: Simulation collecting the geometry at $0,5 \mathrm{~m}$ behind the bit ( $1,64 \mathrm{ft}$ ) ..... 34
Figure 4.17 ..... 35
Figure 4.18 ..... 37
Figure 4.19 ..... 38
Figure 4. 20: Forces evaluations ..... 40
Figure 4. 21: Forces evaluations 2 ..... 40
Figure 4.22 ..... 41
Figure 4.23 ..... 42
Figure 4.24 ..... 47
Figure 4. 25: Offset controller 2D ..... 50
Figure 4. 26: Offsets 3D model ..... 51
Figure 4.27 ..... 52
Figure 4.28 ..... 54
Chapter 5
Figure 5. 11: TVD x Horizontal displacement in a straight well ..... 55
Figure 5. 22: TVD x Inclination in a straight well ..... 56
Figure 5. 33: DLS x Inclination in a straight well ..... 56
Figure 5.4: TVD x Horizontal displacement in a well without the effect of natural displacement57
Figure 5. 5: TVD x Inclination in a well without the effect of natural displacement ..... 57
Figure 5. 6: MD x DLS in a well without the effect of natural displacement ..... 58
Figure 5. 7: MD x OffsetNormal in a well without the effect of natural displacement ..... 58
Figure 5. 8: Force on the bit caused by an offset x MD in a well without the effect of natural displacement ..... 59
Figure 5. 9: Force on the bit caused by the natural displacement x MD in a well without the effect of natural displacement ..... 59
Figure 5. 10: Resultant Force on the bit caused by the natural displacement x MD in a well ..... 60without the effect of natural displacement
Figure 5. 11: MD x DLS in the Volve data ..... 60
Figure 5. 12: TVD x Horizontal Displacement in a well with the effect of natural displacement ..... 61
Figure 5. 13: Inclination x MD in a well with the effect of natural displacement ..... 61
Figure 5.14: DLS x MD in a well with the effect of natural displacement ..... 62
Figure 5. 15: Offset x MD in a well with the effect of natural displacement ..... 62
Figure 5. 16: Resultant Force on the bit caused by the offset x MD in a well with the effect of natural displacement ..... 62
Figure 5. 17: Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement ..... 63
Figure 5. 18: Resultant Force on the bit caused by the natural displacement and offset x MD ina well with the effect of natural displacement63
Figure 5.19: Zoomed -Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement ..... 64
Figure 5. 204: Zoomed - Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement ..... 64
Figure 5. 21: Zoomed - Resultant Force on the bit caused by the natural displacement and offset $x$ MD in a well with the effect of natural displacement ..... 65
Figure 5. 225: TVD x Horizontal Displacement in a well with the effect of natural displacement - Long well ..... 66
Figure 5. 236: Inclination x MD in a well with the effect of natural displacement - Long well ..... 66
Figure 5. 24: DLS x MD in a well with the effect of natural displacement - Long well ..... 66
Figure 5. 25: Offset x MD in a well with the effect of natural displacement - Long well ..... 67
Figure 5. 26: Force on the bit caused by the offset $x$ MD in a well with the effect of natural displacement - Long well ..... 67
Figure 5. 27: Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement - Long well ..... 68
Figure 5. 28: Resultant Force on the bit caused by the natural displacement and offset x MD in a well with the effect of natural displacement - Long well ..... 68
Figure 5. 29: Horizontal displacement $x$ true vertical depth ..... 69
Figure 5. 30: North coordinates $x$ East coordinates ..... 70
Figure 5. 31: DLS x MD ..... 70
Figure 5. 32: North coordinates x East coordinates x TVD coordinates ..... 71

## 1 Introduction

The goal of this thesis is to model an RSS system. This challenging task enables the development of original concepts that will be explained in this thesis such as natural displacement caused by the formation on the RSS system, ROP composition, analysis of forces on the bit by beam bending physics, and original behavior of offset controllers. The development of these concepts was needed to develop a reliable model to foresee the position of the bit before actual drilling. It is believed that the development of such concepts enriches the knowledge about RSS systems and contributes to a new approach to model RSS systems. The work that will be exposed in this thesis shows innovation on modelling, creativity in defining original drilling concepts and high quality on coding to develop a drilling simulator prototype.

RSS system is a technology among others that are used on directional drilling. This thesis aims to share knowledge about the RSS system, develop a mathematical model for it, and program a prototype of a simulator. In this introduction chapter, the context of the RSS technology in the industry will be exposed. In the second chapter, the focus will be on the functionalities and characteristics of the RSS system. The third chapter introduces the mathematical model that will be explained in detail on the fourth chapter. On chapter 5, the results of the developed simulator will be shown followed by chapter 6 exposing the main problems and possible improvements of the simulator. The thesis' conclusion will take place in the seventh chapter.

In this project, several concepts will be defined and the main focus will be the Rotary Steerable System, but a small brief on the background of directional drilling and the main reason that this technology is being used nowadays will be exposed in this introductory chapter. The main goal of this first introduction is to emphasize the importance of this technology and how it has helped to develop the oil and gas industry

### 1.1 History

The development of directional drilling represented a great achievement for the oil industry in the middle '30s. Initially, the tools were simple and did not have a proper tracking system. However, the initial directional drilling tools inspired the development of the tools that are in the market nowadays.

The oil companies have changed their operational and technical standards with the development of directional drilling. This technology has allowed the development of several operations that are common nowadays, as sidetracking and relief wells [1]. The development of multilateral wells, allowed by directional drilling, can increase production and extend the productive life of a field. Directional drilling also can be used to avoid undesired formations, such as salt domes, where corrosion and washouts may occur.

According to Verteuil R. and McCourt I. [2], intentional directional drilling began in intending to build relief wells in blowout situations and/or correct well paths back to the vertical direction [2]. Considering this point, it is possible to wonder how interesting engineering can be, since one of the goals that have created the technology is the opposite of what it aims to achieve nowadays. The goal of directional drilling nowadays, in several cases, is to give an angle and deviate the well path from the vertical direction. Following the history, the first directional well was drilled in American soil, in Huntington Beach, California, USA in 1930 [2]. It was an onshore well deviated into offshore area. In 1934, the technology was used to kill a well in

Texas, USA [2]. This new application increased interest from the companies and contracts about directional drilling.

### 1.2 Motor Versus RSS

Since the firsts directional drilling methods, two main tools have shared space out of the market share of directional operations. Both technologies are more advanced than the initial whipstock used at the beginning of the development of directional drilling, but there is a clear difference in the limitation of each one. The technologies that are being referred to are the steerable motor and the Rotary Steerable System. For a matter of context, this part of the project will expose the limitations and opportunities of each technology.

According to T. Warren [3], two different modes of operations are expected for the steerable motor: sliding and rotating operations [3]. Sliding operations aim to achieve an inclination and azimuth. The drill string does not rotate and the driver responsible for drilling is the bottom hole motor, in other words, the desired inclination will be achieved depending on the ROP of the motor and the configurated tool face applied on the motor. The friction on these operations is a point of concern and the ROP usually decreases $50 \%$ whenever the tool is in sliding operations [3] and eventually there is a depth where sliding is no longer possible. Common problems on these operations are situations of buckling, high tortuosity, poor hole cleaning, differential sticking, among others [3].

As already mentioned, the second operational mode of the steerable motor is the rotating mode. When the tool is on this mode, the entire drill string rotates and usually generates an enlarged wellbore. The problems in this situation usually are accelerated bit wear and poor hole quality as mentioned by [3].

The RSS system uses a different approach to give inclination and azimuth to the well trajectory. The system usually is composed of a steering sleeve. These sleeves are responsible to steer the bit meanwhile rotating the drill string. This movement and respective changes are continuously dynamic and recalculated as the tool drills. This method brings several improvements, compared to the steerable motor. Considering operational improvements, the horizontal limitation of steerable Motor assemblies was 16.000 ft on measured depth with a control of +5 ft feet on true vertical depth [4] in fields of the North Sea. This horizontal limitation was extended to 28.000 ft with full azimuth control and precision of +-1 ft on true vertical depth whenever rotary steerable technology was first used in these fields in 1998 [4]. Beside of this extension of the horizontal limit, the RSS system has allowed the drill string design that causes fewer vibrations, it has improved hole cleaning and ROP, it has reduced tortuosity and it has allowed more precise guidance [3].

In summary, if the economical evaluation is not considered, the RSS approach represents an improvement among the directional drilling technologies. In the late '90s, commercial RSS systems have been developed and further in this thesis, the increase of the use of this technology will be exposed along to the associated technologies developed using RSS systems.

### 1.3 Future Opportunities

According to T. Warren [3], since the invention of this revolutionary technology, the RSS systems were not an attractive economical option. Even with better and more efficient work, this technology was limited to just a small niche market. As the RSS system depends on dynamic control systems and computer capability, the mud motors have occupied a bigger space
on the market and have developed a better price standard than the RSS, since the computer capability needed for the RSS was not available when directional drilling was developed. Consequently, for a simple horizontal well, the steerable motor was the most chosen technology, holding around $80 \%$ of the directional drilling projects in the market in 2006 [3]. The presentation of Probert [5], at IADD Rotary Steerable Forum, exposed the development of several technologies throughout the years. Figure 1.1 focuses on steerable motors and RSS systems development over the years.


Figure 1. 1 - Drilling tools throughout time
Nevertheless, T. Warren [3] has pointed the possibility of using several technologies whenever the RSS system is applied, such as 3D seismic, geo-steering, and extended reach drilling [3]. The possibilities of developing new technologies are unlimited after the creation of a commercial RSS system. The possibility of using newer technologies that generates better results has increased the usage of RSS systems in the late '90s and beginning of 2000. In 2005 an increase of $43 \%$ per year on the usage of RSS systems was registered [3]. A most recent study from Chip Alvord et al. [6] exposes a solution that is capable of reaching smaller targets and extending horizontal sections. The RSS technology is capable to extend the life of producing fields and avoid huge investments for new structures [6]. Therefore, this fact reaffirms how attractive and economically viable the RSS system has become to a wider niche market.

Based on the mentioned data, certainly, the RSS technology is a technology that is going be used more and more by the industry. The possibilities of using additional technologies are higher and, generally, life extension of production wells is a reality whenever RSS technology is used. The improvements that the RSS technology bring, certainly, are attractive to the oil industry and this is the main motivation of this thesis. Any study regarding RSS systems is a study that fits the industry demand for more efficient technology and contributes to further development of directional drilling technology.

## 2 Directional Drilling Tools

### 2.1 Directional Drilling Concepts

There are two main concepts regarding directional drilling. Such concepts are point the bit and push the bit. The nomenclature represents a description of the functionality of each concept. In the push the bit concept, there is a force on the directional drilling tool that pushes the bit to the target location. Additionally, in the point the bit concept, the directional drilling tool bends the bit to achieve the same target location [6].

The mud motor is an example of point the bit concept. Located downhole, the motor is responsible to rotate the bit when building a curvature [3]. The motor bends downhole and points the bit to the target location. The attack angle of the point the bit concept allows this concept to perform well even in hard formations. The push the bit concept is exposed in the next section. Both concepts, point the bit, and push the bit, are exposed in detail by Saeverhagen [7].


Figure 2. 1:
In the next section, a push the bit RSS system will be exposed, followed by the tool from Canrig Drilling Technology Norway. A partnership between Canrig Drilling Technology Norway and UiS has been built to develop a mathematical RSS simulator. The RSS system developed by Canrig will be exposed and used as a foundation for the development of such a simulator. It must be noted that the simulator does not represent necessarily the RSS tool from Canrig.

### 2.2 RSS Systems

Nowadays, there are different RSS systems that assure a high quality and constant well path. New technologies of RSS systems have developed tools with hybrid concepts of point and push the bit. The explanation of the push the bit concept is given by explaining a push the bit RSS system. The first RSS system exposed contains three pads that guide the system [8]. The three pads are installed at the same distance from the drill bit around the body of the RSS tool. The
entire system (drill bit, RSS tool, and drill string) rotates. The rotation is assured from the surface.

A view from above the drill string is going to be considered. From this perspective, a vertical well is seen constantly as a dot as the bit drills vertically. At the kickoff point, the activation of the pads occurs. A hypothetical situation proposes a target location in the South direction. At an RSS tool, one pad faces the North direction, a second pad faces the Southeast direction and a third pad faces the Southwest direction. In this situation, the only pad that is creating a force against the formation is the pad facing North. This pad pushes the formation and a reactive force is felt by the RSS tool to the South direction. This reactive force is transferred to the bit and the entire system goes to the South direction. It must be noted that the RSS system bends by the reactive force from the formation.

On the point the bit, the tool bends the bit from inside of its shaft [9], and on the push the bit, the RSS tool bends by the reaction force from the formation. This initial movement of pushing the formation can be done with the use of pads or different actuators.


Figure 2.2

For this master thesis, the equipment that is going to represent a foundation for the mathematical model development is designed by Canrig Drilling Technology (Norway) AS. Further on, this thesis will explain how this equipment works and how the mathematical model will be developed.

### 2.3 Canrig Equipment

The RSS System developed by Canrig is called OrientXpress® Rotary Steering System and can be observed in figure 2.3. The system has two eccentric cylindrical parts when drilling vertically. This is a similar situation as the previous RSS system, where the offset (red cylinder) has the same function as the pads. The offset can dislocate in any direction and push the formation. In figure 2.3, the entire system is being pushed to the East direction, as the offset is activated to the West direction. It must be kept in mind that the offset does not rotate with the drill string as it guides the bit.


5 mm is a generic value and it does not reflect the geometry from Canrig tool
Figure 2.3
The push the bit concept still takes place. The offset dislocates and pushes the formation gradually to the West. The reactive force bends the RSS tool and transfers the force to the bit. The bit pushes the formation to the East. The essential part of this process is to understand the bending of the RSS tool. Some effects, as natural displacement, impact on the bending of the tool. The natural displacement effect is an original concept developed by this master thesis and it will be explained it the next chapter.

One of the advantages of the tool from Canrig is that the offset can dislocate its center and push the formation in any direction with less moving parts than the 3 pads. This assures better downhole maneuverability according to Canrig [10]. The offset activation is realized by electric motors. The entire equipment is self-powered by a generator that uses kinematic energy from the mud to supply the RSS tool. Additional advantages from the Canrig tool are versatile operational conditions, best build-up rates in the US, and closest sensors to the bit [10].

At this point, an introduction to the modelling of the tool is exposed. It is already known that the offset generates a force that pushes the formation and the formation pushes back the RSS tool. This reactive force will bend the RSS tool and generate two other forces in the body of the tool. A force is generated on the bit and the second force appears on the upper stabilizer of the RSS equipment. The forces on the tool can be seen in figure 2.4.


Figure 2. 4: Forces felt by the RSS tool
In the example of figure 2.4, the blue arrows represent the forces felt by the tool when the offset is activated in the upper direction. In that situation, the formation causes a reactive force on the RSS tool. This reactive force bends the tool and generates a force on the drill bit. This drill bit force is going to be responsible to push the bit to the downside of the tool. This can be observed on the reactive force upwards felt on the bit. This thesis considers the $2,5 \mathrm{~m}$ from the upper stabilizer to the offset and $0,5 \mathrm{~m}$ from the upper stabilizer to the bit. Different RSS systems can be simulated with minor changes in the program.

The exposed tool from Canrig nowadays has no simulator to foresee its location while drilling. The goal of this thesis is to model an RSS system and propose an approach to foresee the location of the bit.

## Disclaimer

It must be noted that the presented methods, dimensions, and performance are not representative of Canrig's tool, and it was only used as a general basis for the study. Values and some concepts differ from the OrientXpress design. This was realized due to the confidentiality of this equipment.

## 3 RSS Modelling Approach

Developing a model of an RSS system is a challenging task. This thesis has brought new concepts like natural displacement, original offset controllers, composition of ROPs on the bit and trajectory calculations based on this composition of ROPs. This contribution is an original work supervised by professors from UiS and professionals from Canrig Drilling Technology Norge AS. Original concepts will be explained to enrich the knowledge about RSS models and build a reliable model of this system.

The main idea of the mathematical model is to derive the forces on the bit caused by the weight on the bit and by the bending of the RSS system. Depending on each force applied to the system, the traditional ROP definition will be decomposed in different ROPs. This is an original approach developed by this thesis. The forces present on the bit will decompose an ROP on the axial direction of the tool and a second ROP on a normal direction from the body of the tool for the 2D model. Definitions of measured depth, plan of trajectory inclination, and azimuth based on this ROP composition are also required and developed. Formulas and details will be exposed in the fourth chapter. The goal of this chapter is to introduce the connection between the variables and the process of calculation based on modelling an RSS system.

In the first part of the chapter, the 2 D and 3 D modelling will be explained. Considering the complexity of the model, this approach aims to explain the logic used on the RSS modelling, expose required knowledge, and point the connections between functions and variables of the model. This chapter does not aim to expose deeply the peculiarities of each function or to explain the calculations behind it, this will be realized in the fourth chapter. The goal here is to build up a flow of understanding for the modelling chapter.

### 3.1 2D and 3D Logic

The logic, developed by this master thesis, on both models (2D/3D) can be observed in the following process. It is important to keep in mind that the blue numbers will be used as references in this thesis.


Figure 3.1
Both models, 2D and 3D, follow the same logic. The main logic developed in this thesis can be analyzed in figure 3.1. The first step is to define a target point. This is done by the well planner depending on each well design and, on the 2D model, the target point requires two parameters: the target inclination for every point of the trajectory and the location of the kickoff point
considering measured depth. On the 3D, additional information about target azimuth is needed as an input of the system. Once the kickoff point is reached, the offset from the RSS system and natural displacement are calculated and updated dynamically. Both parameters will be explained, but for now, it is important to keep in mind that each one of these two parameters is responsible for developing two forces on the bit normal to the drill string for the 2D model ( 90 degrees to the drill string). Resultant forces on the bit will be calculated and each resultant force will create its respective ROP in each direction. This is the original composition of ROP, as mentioned previously. Based on the ROPs composition created, the outputs of the system are calculated, such as horizontal displacement (HD), true vertical depth (TVD), azimuth, inclination, dogleg severity (DLS), and North and East coordinates. Furthermore, some differences and characteristics will be exposed depending if the model is 2D or 3D.

### 3.1.1 2D Model

After the logical process of the mathematical model was explained in the previous section, in this section some characteristics of the process will be exposed:

1. The 2D development considers a constant azimuth, a target inclination, and a kickoff point defined by the well planner design.
2. The 2D model has 4 processes of calculations are used in the mathematical model: Offset controller model, ROP Axial model, ROP Normal model, and Geometry model to calculate the natural displacement (natural displacement will be defined).
3. The model depends on input variables from the ROP model.
4. The model is divided into two behaviors - vertical well path and inclined well path.
5. The model is built timestep per timestep to calculate its respective outputs.

In the following, the importance of each point will be explained in detail.
The first point shows that the 2 D model has prioritized the understanding of the inclination parameter over the azimuth parameter. This decision took place after conversations with professionals at Canrig and colleges from the master at UiS. It was exposed that it is hard to control the first meters and direction of the azimuth. The paper "Effects of Magnetic Interference on Directional Surveys in Horizontal Wells" from 1992 written by C.A. Cheatham et al. [11] reaffirms that the errors on inclinations are usually smaller than the errors on azimuth since the inclinometers are not affected by the magnetic interference. The 2D model will consider just the inclination variation that is responsible for the dogleg severity of the well path. The inclination generates the horizontal displacement and the true vertical depth, and this is the coordinate plane that the well path will be exposed in the 2D model.


Analyzing the second point, the 2D model has four main calculations that interact with each other that are responsible to maintain the logic of the system, as following: ROP Axial, ROP Normal model, Geometry Function, and Offset Function. The ROP Axial model represents the
rate of penetration on the axial direction of the drill string $\left(R O P_{\text {Axial }}\right)$. The ROP Normal model represents the rate of penetration 90 degrees to the axial direction of the drill string $\left(R O P_{\text {Normal }}\right)$. The geometry model calculates the current displacement of the bending of the RSS tool caused by the well path geometry/curvature (known as natural displacement, $H_{\text {Normal }}$ ) and the Offset controller model calculates the current offset (known as offset, Offset Normal ) given the bit position and the target inclination. $R O P_{\text {Axial }}, R O P_{\text {Normal }}, O f f s e t_{\text {Normal }}$, and $H_{\text {Normal }}$ are dynamic variables that are calculated and updated for every new point drilled. It must be noted that $\beta$ represents additional possible variables depending on the ROP model considerations.


Figure 3.3


Figure 3. 4 - ROP Composition

$$
\begin{gather*}
R O P_{\text {Axial }}=f(R P M, W O B, \text { Rock proprieties, bit size, } \beta)  \tag{3.1}\\
R O P_{\text {Normal }}=f(\text { Force on the bit }(\text { Fbit }), \text { RPM }, \text { geometry of the wellpath }, \beta) \tag{3.2}
\end{gather*}
$$

The $R O P_{\text {Normal }}$ will depend on both offset model and geometry model. The offset and the natural displacement are responsible to create a resultant force on the bit caused by the RSS system (Fbit). On the 2D model, the $R O P_{\text {Normal }}$ can be called as $R O P_{I n c}$, as the inclination will be the only angular variable responsible for DLS variations. Choosing a more accurate ROP
model will produce a more reliable output of the mathematical model. The main idea is to choose a defined function of ROP and convert it to calculate the $R O P_{\text {Normal }}$ generated by the RSS system. On the previous $R O P_{\text {Axial }}$, the weight on the bit is the main driver of the force on the bit. On the converted $R O P_{\text {Normal }}$ function, the resultant force on the bit caused by the RSS system will be the main driver of the force on the bit (Fbit). To have reliable data, a correction should be done, evaluating the resultant values of DLS depending on the defined steerability of the RSS system. If no correction is done, unreal values of DLS and steerability can be found.

Furthermore, the ROP normal function will depend on the input variables of the chosen ROP model and instead of weight on the bit (WOB) a force on the bit from the RSS system will be calculated (Fbit). Even though this approach was made in this master thesis, other approaches are also possible to include the resultant force on the bit on the $R O P_{\text {Normal }}$. This brings up the necessity of calculation of the resultant force on the bit composed by the force from the geometry function $\left(F b i t_{H}\right)$ and the force from the offset function ( $F_{\left.b i t_{o f f s e t}\right)}$ ). The resultant force on the bit (Fbit) will depend on how the forces and reactive forces are present on the system bit/RSS body. In this master thesis, the Fbit $_{\text {Offset }}$ and Fbit $_{H}$ are going to be calculated using a beam bending logic. Generally, it is hard to expose general guidance for the bending of a system like this. There are several RSS tools in the industry and each tool will bend differently and will have different geometry between stabilizers. The main idea is to understand how the offset/actuator ( $O f f$ set $_{\text {Normal }}$ ) impacts on the force on the bit (force caused by the offset, Fbit $_{\text {Offset }}$ ) and how the natural displacement of the bending of RSS tool generated by the curvature of the well ( $H_{\text {Normal }}$ ) will impact on the force on the bit, (force caused by the natural displacement, $\mathrm{Fbit}_{H}$ ). These concepts will be modeled in the fourth chapter. In the following picture, the offset (Offset ${ }_{\text {Normal }}$ ) and the natural displacement ( $H_{\text {Normal }}$ ), the resultant force on the bit (Fbit) and the consequent $R O P_{\text {Inc }}$ can be observed. The Fbit is a function of the force caused natural displacement $\left(\mathrm{Fbit}_{H}\right)$ and the force caused by the offset $\left(\right.$ Fbit $\left._{\text {Offset }}\right)$.

## Calculate Forces on the bit




Figure 3.5
On the schematics of figure 3.5, it is possible to note the dependency of the variables and the possible position and orientation of it. It is possible to observe that if the natural displacement and the offset are on the downside of the RSS system, this is going to generate a force felt by the bit downwards. This force that is felt by the bit has a reactive force that is felt by the formation with the same intensity and opposite orientation. This force will drive the ROP upwards. If the natural displacement and the offset are on the upside, the opposite logic applies. It must be noted that there is no possible force on the left or right direction of the drill string since there is no variation on azimuth in the 2D. The following image expose the possible locations of each term considering a section cut of the RSS tool.


Figure 3.6
The first force in the analysis is the force caused by the natural displacement $\left(F b i t_{H}\right)$. The first step to calculate $\mathrm{Fbit}_{H}$ starts with the geometry function. The goal of the geometry function is the calculation of the natural displacement ( $H_{N o r m a l}$ ). The input of the geometry function considers the geometry of the well path behind the bit for the length where are forces from the
formation that results in forces in the bit. Exemplifying, if an RSS tool has a length of 10 ft , any bending generated by the formation curvature on this 10 ft will result on a force on the bit. The geometry function will calculate how much the tool has been bent (natural displacement, $H_{\text {Normal }}$ ) by the formation along the well path. This measure displacement caused by the well path curvature formation is called natural displacement ( $H_{\text {Normal }}$ ). This natural displacement generates a force on the bit that will be calculated through beam bending calculations known as $\mathrm{Fbit}_{H}$, which is one of the two possible forces to be used by the $R O P_{\text {Normal }}$ model to calculate the ROP composition.

The second possible force is the force on the bit caused by the offset (Fbit ${ }_{\text {offset }}$ ). To calculate the $R O P_{\text {Normal }}$, the calculation of the offset (Offset Normal ) and its consequential force on the bit (Fbit ${ }_{\text {Offset }}$ ) are needed. The calculation of the offset will be realized by the offset function, which represents an offset controller that analyses the target point defined by the well planner and the current location of the bit as the drilling takes place. It is good to keep in mind that each RSS tool has a maximum offset that the tool can offer. For this reason, the value of offset is given as a percentage and the maximum offset of the simulated RSS tool. Once the offset is defined, beam bending equations define the Fbit $_{\text {offset }}$. Offset and natural displacement will generate $F b i t_{\text {offset }}$ and Fbit $_{H}$ respectively. The resultant of $F b i t_{O f f s e t}$ and $F b i t_{H}$ generates the resultant force on the bit (Fbit) that will be used to calculate the $R O P_{\text {Normal }}$.

Beam bending knowledge is needed to understand how this transfer from the measurement displacement (offset and natural displacement) to bending force takes place (Fbit offset ${ }_{\text {ond }}$ Fbit $_{H}$ ). Information on the geometry of the tool to calculate inertia (I) and the material of the tool to calculate the elasticity modulus (E) are needed on the beam bending calculation. Once the beam bending calculates the resultant force on the bit (Fbit), the ROP Normal function calculates the $R O P_{\text {Normal }}$ to the axial body of the tool, 90 degrees from the previous $R O P_{\text {Axial }}$. On the 2D model, this ROP model will be called the $R O P_{\text {Normal }}$ (for the 2D it can also be called $R O P_{\text {Inc }}$ ) and it will control the variation of inclination.

The third point regards the dependency of the model to the inputs from the ROP model. The goal of the project is to define and model an RSS system, therefore, no development was made to develop data for the input of the ROP model. If extra functions are added to the ROP mathematical model, more reliable output from the RSS modelling can be found. A more accurate ROP model results in more accurate outputs.

The fourth point is analyzed in the following. The well has two behaviors, the vertical behavior, and the inclined behavior. The transition of behavior depends on the kickoff point. Before the first kickoff point, the well has a vertical behavior and each output parameter (true vertical depth, inclination, measured depth, horizontal displacement...) is calculated with simpler equations regarding a vertical situation. After the kickoff point, the inclination variable takes place on the mentioned equations. The well planner must define the point of kickoff depending on the measured depth and the target inclination that the RSS is going to start to pursue after the moment of kickoff. There are no limitations on the number of target inclinations defined along a well path on this mathematical perspective. The well planner must keep in mind that the tool needs some meters of measured depth to achieve the target inclination. This thought is important because, on the several simulations, the second target inclination was defined before the bit achieving the first target inclination, so the well did not achieve the first target inclination.

The estimation of how many meters of measured depth is needed to achieve a target inclination can be done based on the definition of dogleg severity of the used RSS system.

The last point deals with the fact that the system is calculated step by step. Every timestep is calculated based on the previous point. The simulations can be limited to the number of timesteps, a good timestep needs to be defined to achieve a good resolution of outputs.

> Calculate TVD, MD, DLS
> Inclination, Azimuth, Horizontal displacement, North and East coordinates


For the 2D: True Vertical Depth (TVD), Measured Depth (MD) Inclination, DLS and Horizontal displacement

Figure 3.7

### 3.1.2 3D Model

After exposing the mathematical dynamic model for the 2D modelling, this section of the master thesis is dedicated on showing the developments and differences from the 2D model to the 3D model. As the 3D model was developed based on the 2D model, a lot of similarities are present. The main points of development of the 3D modelling were on the geometry function inputs, azimuth definition, and offset controller. The characteristics that differ from the 3D to the 2 D model are mainly the following:

1. Azimuth is not constant, and it varies according to the time
2. The North and East coordinate plane will compose the 3D model
3. Instead of having just two ROPs $\left(R O P_{\text {Axial }}\right.$ and $\left.R O P_{\text {Normal }}\right)$ the $R O P_{\text {Normal }}$ will be divided into the rate of penetration that controls the azimuth, known as $R O P_{A z i}$ and rate of penetration that controls the inclination as defined here previously as $R O P_{I n c}$.
4. The offset controller must be able to define the best direction to set the offset, considering the azimuth and inclination target at the same time.
5. There will be a natural displacement that will be present on the bending of the RSS tool that is more complex than the 2 D model.


Figure 3.8

On the 3D system, the azimuth is not constant. This means that the horizontal displacement can deviate in different directions. The variation of inclination and azimuth happens simultaneously after the kickoff point. This generates the necessity of defining an additional coordinates plane.


Figure 3.9
Previously, as the azimuth was constant, there was no need to define an ROP that controls the azimuth. On a 3D modelling, there will be two ROPs normal to the axis of the tool in addition to the $R O P_{\text {Axial }}$. One of the two ROPs normal is the same from the 2 D model, called $R O P_{\text {Inc }}$ that controls the inclination and the second ROP normal to the axis of the RSS tool is the $R O P_{\text {Azi }}$ that controls the azimuth.

As exposed in the previous 2D system, the well planner is responsible to calculate the kickoff point of the well. After the kickoff point, the 3D model aims the bit to reach the target azimuth and the target inclination established by the well planner for each meter drilled. This movement to reach the azimuth and inclination is simultaneously developed by the offset model. The resultant offset of the tool is defined by the offset controller depending on the offset limitations of the RSS system, the target inclination and azimuth, and the current location of the bit. After the calculation of the resultant offset, the model calculates the offset components ( Offset ${ }_{A z i}$ and $O f f s e t_{I n c}$ ). Posteriorly, the natural displacements ( $H_{A z i}$ and $H_{I n c}$ ) are calculated by the geometry function. Based on beam bending, offsets and natural displacements generate forces on the bit that impact the $R O P_{A z i}$ e $R O P_{I n c}$. It is important to keep in mind that the $O f f s e t_{A z i}$, Offset ${ }_{\text {Inc }}, H_{A z i}$, and $H_{\text {Inc }}$ represent the same meaning from the 2D model, but each component is evaluated in a different coordinates planes. The $O f f s e t_{A z i}$ and $H_{A z i}$ are calculated depending on the North and East coordinates plane and $O f f \operatorname{set}_{I n c}$ and $H_{I n c}$ are calculated depending on the true vertical depth x horizontal displacement coordinates plane.

## Calculate Offset



Calculate ROP composition

Offset model calculates Offset $_{\text {Azi }}$ and $O f f$ set $_{\text {Inc }}$

Geometry model calculates
$H_{A z i}$ and $H_{I n c}$

$$
R O P_{\text {Axial }}, R O P_{\mathrm{Inc}} \text { and } R O P_{\mathrm{Azi}}
$$

## ROP Composition 3D



Figure 3. 11 - ROP composition 3D
The $R O P_{\text {Axial }}$ will be a function of the energy displaced on the axial direction. The same logic will be used for each $R O P_{\text {Normal }}$.

$$
\begin{gather*}
R O P_{\text {Axial }}=f(R P M, W O B, \text { Rock proprieties, bit size }, \beta)  \tag{3.3}\\
R O P_{\text {Inc }}=f\left(\text { Force on the bit }{ }^{\prime \prime}\left(\text { Fbit }^{\prime \prime}\right), R P M, \text { geometry of the wellpath }, \beta\right)  \tag{3.4}\\
R O P_{A z i}=f\left(\text { Force on the bit }{ }^{\prime}\left(\text { Fbit' }^{\prime}\right), R P M, \text { geometry of the wellpath }, \beta\right) \tag{3.5}
\end{gather*}
$$

It must be noted that $\beta$ represents additional possible variables depending on the ROP model considerations. The equations of the ROPs present on the mathematical model depend on which general ROP model is considered. The chosen ROP model for the simulator is the Teale [12] ROP model, but other considerations and ROP models are possible.

The Offset ${ }_{A z i}$ and $H_{A z i}$ evaluate the forces (respectively Fbit $_{o f f s e t}{ }^{\prime}$ and Fbit $_{H}{ }^{\prime}$ ) on the North and East quadrant (right-left side of the bit). On the other hand, the $H_{\text {Inc }}$ and the Offset ${ }_{\text {Inc }}$ evaluate the forces (respectively Fbit $_{H}{ }^{\prime \prime}$ and Fbit $_{\text {Offset }}{ }^{\prime \prime}$ ) present on the TVD and horizontal displacement plane of coordinates (upper-down side of the bit).


> Fbit $^{\prime}=f\left(\right.$ Fbit $_{H}{ }^{\prime}$, Fbit $\left._{\text {offset }}{ }^{\prime}\right)$
> Fbit $^{\prime \prime}=f\left(\right.$ Fbit $_{H}{ }^{\prime \prime}$, Fbit $\left._{\text {offset }}{ }^{\prime \prime}\right)$


Figure 3.12
The following figure 3.13 explains the force calculations. The offset and natural displacement generate forces on the bit in each direction. On the right-hand side, it is possible to observe the $R O P_{\text {Inc }}$ and $R O P_{\text {Azi }}$ calculated based on the previous force in each direction. Details about the calculation of this process can be observed in the modelling chapter. The following image expose the possible locations of each term considering a section cut of the RSS tool.


Figure 3.13
The last step on the logic of the mathematical model is the trajectory calculation of the model. The 3D model is more complete and generates a bigger amount of outputs needed for a proper trajectory.

Figure 3.14

### 3.2 Dynamic Mathematical Model

The goal of this section is to expose the developer's point of view of the dynamic mathematical model. All the models (small blue circles), inputs, and outputs will be explained by the developer's point of view that includes all the details of the model. It must be noted that the first small circle is the required well design realized by the well planner on each independent well, therefore, it is not modeled by the mathematical model.

The first schematics to be exposed here are the functions of the dynamic model. An important observation is the dependency between the functions. The functions follow the model logic exposed in the light blue numbers. The following figure 3.15 also indicates when each model will be further explained.


Figure 3. 15: 2D Functions of the Mathematical model - Every function has an input and output used on the model
The offset function receives the bit location, the target inclination, and generates the current offset. The Geometry function defines the natural displacement for the current drilling point. Both values are constantly updated as the drilling takes place. Beam bending calculates the forces on the bit. Each ROP function receives the inputs of the ROP model and generates the current ROP composition. In the case of the 2D model, there is only one $R O P_{\text {Normal }}$, the $R O P_{\text {inc }}$. In the case of the 3D model, there are two normal ROPs, $R O P_{I n c}$ and $R O P_{A z i}$. Finally, the trajectory is updated, and the new position of the bit is simulated.


Figure 3. 16: 3D Functions of the Mathematical model - Every function has an input and output used on the model

### 3.3 Improvements on the 3D Model

The 3D model can be improved, but, on the other hand, a complete remodelling of the programming logic is necessary. The previous 3D model can be inaccurate for very tortuous wells with simultaneous variations of azimuth and inclination. This inaccuracy can be caused by the fact that the beam bending calculation considers two 2D analyses for the 3D model. To avoid this possible error, either a better study about 3D beam bending can be used or the following logic can be used:


Figure 3. 17: 3D improvement process


Figure 3. 17: 3D improved functions of the Mathematical model
It is possible to notice that the steps in orange ( 4,5 , and 6 ) are added to the previous 3D modelling. These extra processes are responsible to calculate a unique displacement that can be used on the beam bending equation. The step number 4 calculates the displacement on the upper-downside of the RSS tool on the variable "Displacement Inc". The variable "Displacement Azi" does the same process regarding the left-right side of the drill string. On step 5, a "Resultant Displacement" is calculated considering both displacements. This "Resultant Displacement" can be used on the beam bending formulas and it generates a "Resultant Force" on the bit that would be used on the ROP composition. This process requires extra programming but assures that the beam bending logic can use simpler equations. This is possible since a "Resultant Displacement" assures a bidimensional beam bending schematic, while on the previous case the beam bending was realized on theoretical displacements on the upper-downwards orientation and left-right orientation separately. The following image expose the possible locations of each term considering a section cut of the RSS tool.


Figure 3. 189: 3D improved variables - Possible location of each displacement

## 4 Modelling Theory and Concepts

Modelling a complex system as an RSS is at the same time challenging and interesting, as a master student specialized on well engineering. This project has brought enrichment of knowledge, considering the contact with professionals from the industry to better understand how the RSS system works. At the beginning of this chapter, the modelling strategy will be organized and explained.

Programing is necessary. This decision was made since programming can help to evaluate the mathematical model in a faster manner. Considering the programming skills needed, a simple programing language helped in investigating and sharing programming approaches needed to achieve the results.

The inspiration in this modelling came from a conversation regarding the definition of ROP composition. Such a definition or idea was not exposed throughout the master of well engineering at UiS and at the same time, it was an original and simple definition that fits RSS modelling. The paper from Marck, J. and Zalluhoglu, U. [13] exposes the kinematic relationship at the bit. Even though this definition is interesting, the paper does not contain a definition of ROP composition.


Figure 4.1

### 4.1 Survey Calculations and Trajectory Definitions

As already mentioned in this master thesis, the modelling is going to consider realistic limitations of the RSS tool and natural reactions that happen while drilling. In this chapter, the goal is to explain and deeply describe some concepts that will be involved in the modelling to facilitate understanding.

Original concepts like beam bending schematics, natural displacement by the formation, dogleg severity limitations, offset controller and its limitations, inclination, and azimuth based on the original ROP composition will be described in this chapter. On this initial topic, this thesis will briefly refresh some concepts well known by the industry that are needed to define a trajectory of a well. These concepts will receive an original definition depending on the ROP composition established by the mathematical model developed in this thesis. This section will be divided into 2D and 3D survey calculations. Mostly all the concepts from the 2D calculation will be used on the 3D.

### 4.1.1 Survey Points 2D

### 4.1.1.1 Inclination (2D and 3D)

To have a clear understanding of the modelling, it is needed to have a solid description of coordinates, since the RSS system is a dynamic model and variations of its coordinates are constants. The first step of the next chapter will be modelling a 2 D perspective where inclination will be the only responsible for any angular variation on the well path. The chosen approach aims to facilitate its model and evaluates correction of programming and drilling concepts. A clear definition of inclination and a good understanding of it are essential for the development of this work.

Inclination is the deviation from the vertical direction [14]. Its values are measured by accelerometers while drilling. It must be noted that if a well is being drilled vertically the inclination will be zero and if there is a horizontal situation the inclination is 90 degrees.


Figure 4. 2: Inclination
The definition of inclination is the same regardless of 2D or 3D modelling. The inclination before the kickoff point is defined as zero as the simulated well has a vertical behavior before kickoff. After the kickoff point, the inclination is calculated and updated for each new point drilled based on the previous inclination and the ROP composition on the system. Figure 4.3 illustrates this idea.


Figure 4. 3: Inclination modelling
In the previous figure 4.3, it is possible to notice that the angle formed between the $R O P_{\text {Axial }}$ and the "Resultant ROP" is responsible for the increase or decrease of the inclination. By simple geometry, the "angle ROP" is calculated by the arctangent of the $R O P_{\text {Normal }}$ divided by the $R O P_{\text {Axial }}$. This interaction must be updated for each time step that has the duration of a delta T period. The resultant formula responsible for describing the inclination is the following:

$$
\begin{equation*}
\operatorname{Inc} c_{(T)}=\operatorname{Inc}(T-1)+\arctan \left(\frac{\operatorname{RoP~Inc}_{(T)}}{\operatorname{ROP~Axial}_{(T)}}\right) * \operatorname{deltaT} / 3600 \tag{7}
\end{equation*}
$$

It must be noted that the "deltaT/3600" represents the value of the timestep divided by 3600 . The purpose of this parameter is to calculate the angular variation during the chosen timestep value. At the models exposed here, the "deltaT" has a value of 5 seconds. This value of five seconds was chosen because it shows a good relationship between programming time versus output resolution. Thus, in the remaining part of this work, each timestep simulates the bit position after 5 seconds. The reason for the division by 3600 is that the ROP formula has an output of $\mathrm{ft} /$ hour and this division passes the hour term to seconds.

### 4.1.1.2 Measured Depth, True Vertical Depth and Horizontal Displacement (2D and 3D)

In the 2D modelling, this master thesis exposes the variations on true vertical depth (TVD) and horizontal displacement (HD). True vertical depth is the vertical depth or vertical distance from the surface to the drilling point where the bid is located. The surface point of reference is usually the rotary kelly bushing (RKB). This measure is mainly used for pressure calculations since the pressure increases with vertical depth. In the other hand, measured depth (MD) represents the actual distance from the surface to the bit location. In this thesis, the measured depth will be driven by the resultant force on the bit, which can be composed of the weight on the bit and/or the force from the RSS tool. The weight on the bit impacts on the $R O P_{\text {Axial }}$ and the force from the RSS tool impacts on the $R O P_{\text {Normal }}$. Finally, the horizontal displacement represents the horizontal component on the drilling of the well. For practical reasons, it is easier to understand if a view from above of the well is analyzed.

These three variables, TVD, HD, and MD are connected. As said before the change in MD is driven by the resultant force on the bit. Whenever there is a section with inclination, the mathematical model considers the equation 4.1:

$$
\begin{equation*}
M D_{2 D(T)}=M D_{2 D(T-1)}+\operatorname{sqrt}\left(R O P \operatorname{Inc}_{(T)}{ }^{2}+\operatorname{ROPAxial}_{(T)}^{2}\right) * \operatorname{deltaT} / 3600 \tag{4.1}
\end{equation*}
$$

Using the certainty that the $R O P_{\text {Axial }}$ and the $R O P_{\text {Normal }}$ (also known as $R O P_{\text {Inc }}$ in the 2 D ) will have a degree of 90 in between them, as it is defined in this master thesis on the ROP composition, the resultant force on the bit will be achieved by the Pythagoras theorem. It must be noted that $M D_{2 D(T)}$ represents the measured depth at the time T and $M D_{2 D(T-1)}$ represents the measured depth at the previous time step (T-1), or five seconds before, whenever the "deltaT" is equal to 5 seconds.

On the 3D model, the formula of MD is slightly different. It still represents the same logic of the previous 2D, but it considers the three ROPs present on the 3D model as shown in equation 4.2.

$$
\begin{equation*}
M D_{3 D(T)}=M D_{3 D(T-1)}+\operatorname{sqrt}\left(R O P_{A z i}(T){ }^{2}+R O P_{I n c(T)}{ }^{2}+\operatorname{ROP}_{\operatorname{Axial}_{(T)}}{ }^{2}\right) * \frac{\operatorname{deltaT}^{3600}}{} \tag{4.2}
\end{equation*}
$$

The value of TVD, in a vertical well, will be the same value as for the MD. In an inclined well, the formula looks like it is shown in equation 4.3. It must be noted that the formula follows the exact definition of the inclination where the variation of TVD divided by the variation of MD is equal to the cosine of inclination:

$$
\begin{equation*}
T V D_{(T)}=T V D_{(T-1)}+\cos \left(\operatorname{Inc} c_{(T)}\right) * \operatorname{sqrt}\left(\operatorname{ROP}_{\operatorname{Inc}(T)}{ }^{2}+\operatorname{ROPAxial}_{(T)}{ }^{2}\right) * \operatorname{deltaT}^{2} / 3600 \tag{4.3}
\end{equation*}
$$

where TVD is the true vertical depth at the time T, $T V D_{T-1}$ is the true vertical depth at the time $\mathrm{T}-1$ and the $I n c{ }_{(T)}$ is the calculated inclination at the time T . This is basic geometry since the inclination represents the angle between the TVD and MD. The horizontal displacement will follow the same logic as the TVD calculation, the only difference will be from cosine to sine since the horizontal displacement is the horizontal component and the TVD is the vertical component. The equation follows the same logic as previously:

$$
\begin{equation*}
H D_{(T)}=H D_{(T-1)}+\sin \left(\operatorname{Inc} c_{(T)}\right) * \operatorname{sqrt}\left(R O P \operatorname{Inc}_{(T)}{ }^{2}+\operatorname{ROP~Axial}_{(T)}{ }^{2}\right) * \operatorname{deltaT} / 3600 \tag{4.4}
\end{equation*}
$$

where HD is the horizontal displacement at the time T and the $H D_{T-1}$ is the horizontal displacement at the previous timestep (T-1). Considering the unit of the ROP (ft/hour), the horizontal displacement is displaced in feet. In figure 4.4, it is possible to observe the horizontal displacement by the red line with variations on azimuth.


Figure 4. 4: Horizontal displacement - View from above of well path

### 4.1.1.4 Dogleg Severity (DLS) (2D and 3D)

It is important to define the dogleg severity limitations for this study. Based on [1], dogleg severity is the measure that verifies the change of inclination and/ or direction on a well path. There are several studies and evaluations as the paper "Bending rules with high build up rates

RSS" written by Imran Tipu et al. [15] where the authors describe the challenges of a well trajectory plan in the Emirate of Abu Dhabbi (UAE) and the development of a hybrid tool that can achieve a DLS of 18 degrees for every 100 ft . Another example of high dogleg severity can be observed in the paper "Results of extensive RSS testing with PDC bits" where it is possible to observe DLS above 10 degrees per 100 ft [16]. This technology aims to drill faster and expose a bigger area of the reservoir.


Figure 4.5
For a matter of developing a realistic simulator and showing updated values of drilling, this study will follow a steerability of an RSS system capable to achieve 18 degrees for every 100 ft on DLS.

DLS is a well-known variable in the drilling industry. The DLS reflects the angular variance on the angle for each meter drilled. The formula 4.5 is used on the 2D model:

$$
\begin{equation*}
D L S=\frac{\left(I n c_{T}-I n c_{(T-1)}\right) * 180}{\left(M D_{T}-M D_{T-1}\right) * \pi} * 100\left[\frac{\text { degrees }}{f t}\right] \tag{4.5}
\end{equation*}
$$

where the inclination is given in radians and the measured depth in feet. This model follows the same description as the concept of the dogleg severity.

For the 3D model the formula 4.6 is considered [17]:

$$
\begin{equation*}
D L S_{T}=\frac{\left(\arccos \left(\cos \left(\operatorname{Inc} c_{T}\right) * \cos \left(\operatorname{Inc} c_{T-1)}\right)+\sin \left(I n c_{T}\right) * \sin \left(\operatorname{Inc} c_{(T-1)}\right) * \cos \left(A z i_{T}-A z i_{T-1}\right)\right)\right) * 100}{\left(M D_{T}-M D_{T-1}\right)}, \tag{4.6}
\end{equation*}
$$

where DLS is given in degrees per feet, inclination in degrees, azimuth in degrees, and MD in feet. This method is based on the radius of curvature method.

### 4.1.2 Survey Points 3D

### 4.1.1.1 Azimuth (3D)

Azimuth has a similar definition compared to inclination. Azimuth is the angular variation on the left-right dimension when the RSS system is defined as a reference point. If there is no
horizontal displacement, the well is vertical and azimuth calculation has no relevance. At the kickoff point, where the well gets a deviation, a horizontal displacement takes shape. The angular variation from the north direction of the horizontal displacement is called the azimuth. If, from a perspective from above, the well path has a horizontal displacement directed to the north, the azimuth is zero. Any other direction will result in a horizontal displacement that will create an angle with the north direction. This angle is the azimuth.


Figure 4. 6: Azimuth
On the 2D model, azimuth variations are not considered in this thesis. That is the reason why the azimuth definition was not prioritized on the thesis earlier on. On the 3D modelling, the azimuth has similar modelling to the inclination with some small, but essential, differences. The main difference is that whenever a change on the azimuth is defined, the initial value of azimuth does not need to start from zero. Whenever a well increases its inclination from its kickoff point, it starts from zero until the desired inclination. With azimuth, it is possible to start the drilling with the desired azimuth from the kickoff point. The equation 4.7 defines the azimuth modelling and it is possible to note similarities to the previous inclination equation:

$$
\begin{equation*}
A z i_{(T)}=A z i_{(T-1)}+\arctan \left(\frac{\operatorname{ROP~Azi}_{(T)}}{\operatorname{ROP~axial}_{(T)}}\right) * \operatorname{deltaT} / 3600 \tag{4.7}
\end{equation*}
$$

where $A z i_{(T)}$ is the azimuth at the time T and $A z i_{(T-1)}$ represents the azimuth at the previous time step T-1.

Another difference between azimuth and inclination are the projection coordinates. Azimuth is projected on the North and East quadrant while inclination is projected on the TVD and horizontal displacement quadrant (In this model, if there are negative values for the North or/and East coordinates, the bit is on the South or/and West quadrant). The equations of TVD and horizontal displacement were already exposed, therefore, the definitions of displacements on the North and East directions are also exposed at this point.

$$
\begin{gather*}
\operatorname{North}_{(T)}=\operatorname{North}_{(T-1)}+\cos \left(A z i_{(T)}\right) *\left(H D_{(T)}-H D_{(T-1)}\right)  \tag{4.8}\\
\operatorname{East}_{(T)}=\operatorname{East}_{(T-1)}+\sin \left(\operatorname{Azi}_{(T)}\right) *\left(H D_{(T)^{-}} H D_{(T-1)}\right) \tag{4.9}
\end{gather*}
$$

where $\operatorname{North}_{(T)}$ is the location of the bit on the North (positive values) or South (negative values) direction at the time T and the $\operatorname{North}_{(T-1)}$ is the location of the bit on the North or

South direction at the time T-1. For the East displacement, East $_{(T)}$ is the location on the East or West direction at the time T and the East ${ }_{(T-1)}$ is the location of the bit on the East or West direction at the time T-1. Both parameters depend on horizontal displacement. If there is no horizontal displacement, the azimuth does not change.

### 4.1.1.2 Toolface (3D)

Toolface is a well-known concept in 3D drilling. Toolface is the angle where the tool is directed. In a case of an RSS system, the change of this toolface can be done without tripping out of the hole. Whenever using a motor, the drill string must be rotated to set up a new toolface. The representation of toolface can be seen in figure 4.7, where a cut on the body of the tool is made and the point of view is from the inner axis of the tool.


Figure 4.7
This project will not use the tooface definition to guide the bit on the 3D model. The offset controller will use a similar angular concept, as it can be observed in figure 4.8.


Figure 4.8
The model to guide the 3D orientation of the bit depends on the offset controller. The final offset and the "angle Beta" guide the bit position. By this means, it is possible to develop the model to extract the information of the toolface out of the system, even without using such a definition to guide the trajectory. The discussion of how the orientation of the 3D modelling is realized can be found at the section "4.4.3 Offset Controller 3D".

### 4.2 Force Calculations

The goal of chapter 4.2 is to define some forces that were exposed in the previous section. The first objective is to define the beam bending system used in this project. Further on, concepts like natural displacement and offset are going to be defined in this section. The natural displacement and offset generate two forces on the bit. The mentioned two forces will need to be defined: the force on the bit generated by the natural displacement and the force on the bit generated by the offset.

### 4.2.1 Beam Bending

### 4.2.1.1 Theoretical Definition

Establishing a clear definition of a beam bending scenario is essential to understand the modelling of the RSS system, since the RSS tool can be treated as an analogy to a steel pipe being bent constantly. This analogy will be useful to model the system and its clear definitions are obligatory for further understanding.

According to Erik Oberg et al. at the Machinery's Handbook 27th Edition [18], there are several beam bending equations that reflect the beam bending theory and a clear understanding of which equation best describes the real scenario is important to avoid errors on the modelling.

On the beam bending scenarios, the behavior of a bar/pipe regarding its material, geometry, and length is studied depending on the loads and the point of contact of these loads. That translates exactly the reality of RSS operations. The force/load on the beam bending is generated by the offset and/or the natural displacement and the point of contacts of the beam bending schematic are the stabilizer, the offset and the bit.

Aiming to exemplify the different scenarios [18] and the main idea behind the beam bending this thesis exposes the different situations.


Figure 4. 9: Beam bending scenarios
Each beam bending scenario has its peculiarities. The main idea behind it is the balance of force and momentum. It is important to select the correct situation, given the reality of the RSS tool. The beam bending situation, in which the application and transmission of forces from the body of the RSS tool to the bit are best described, is shown in figure 4.10.


Figure 4. 10: Beam bending scenario that represents the RSS tool
Some errors can be found in this consideration since a 3D distribution of forces fits the best on a 3D model. The offset has an area of contact and the load is not present in a single point on the beam bending on a real situation. It must be noted that the peculiarities about the beam bending schematics [18] are exposed in the equation 4.10.

$$
\begin{equation*}
\text { Deflection at load }=\frac{W a^{2} b^{2}}{3 E I l} \tag{4.10}
\end{equation*}
$$

Let $\boldsymbol{a}$ be the length of the shorter segment ( 500 mm on the simulated tool) and $\boldsymbol{b}$ the length of the longer one ( 2500 mm on the simulated tool).

### 4.2.1.2 Modelling

Once the theoretical explanation of the bending scenario is done, the goal of this topic is to model this event. Observing the previous model (eq. 4.10), the following equation will be adapted on the RSS modelling:

$$
\begin{equation*}
\text { Maxdeflection @load }=\frac{W_{\text {Load }} a^{2} b^{2}}{3 E I l} \tag{4.11}
\end{equation*}
$$

Discriminating the elements of the equation, $\boldsymbol{l}$ is the length of the beam, that represents the body of the tool with approximately 3 meters, $\boldsymbol{W}$ represents the load caused by the offset displacement and/or natural displacement, $\boldsymbol{E}$ represents the elasticity modulus and $\boldsymbol{I}$ represents the inertia coefficient. Considering the real scenario, the bit is on the left side of the beam, so the distance $\boldsymbol{a}$ represents the distance between the bit and the offset location and the distance $\boldsymbol{b}$ represents the distance between the offset location and the upper stabilizer.

For the elasticity modulus, this thesis considers the steel elasticity value of 210 GPa . Regarding the inertia coefficient, as the location analyzed is at the load on the beam, the respective measures of the tool should be used in the formula 4.12:

$$
\begin{equation*}
I=\frac{\pi}{32}\left(D o^{4}-D i^{4}\right) \tag{4.12}
\end{equation*}
$$

where Do and Di represent the outside and inside diameters of the body of the tool at the offset location.

### 4.2.2 Natural Displacement 2D

### 4.2.2.1 Theoretical Definition

For proper organization, this thesis defines natural displacement as the name of the distance of the displacement caused by the formation curvature on the tool. This natural displacement will be one of the two drivers that will rule the RSS system. This deformation created by the formation will bend the tool. The bending generates a force on the bit that impacts the $R O P_{\text {Normal }}$. The natural displacement depends on the geometry of the well path until the point of installation of the flex joint. The flex joint is responsible to avoid undesired forces on the bit from above its point of installation according to specialists at Canrig.

The other force that drives the RSS is the offset force (Fbit offset on the 2 D and Fbit $_{\text {Offset }}$ ' and Fbit ${ }_{\text {offset }}$ " on 3D) that will be modeled further on the thesis. The natural displacement is symbolized as $H_{\text {Normal }}$ on the 2D. On the 3D, there will be two natural displacements $H_{A z i}$ and $H_{\text {Inc }}$, one on each coordinate plane (North/East and TVD/HD). To understand the natural displacement, an example of such a situation is given below.

Hypothetically, a vertical well is being drilled. If considered a 2D analysis, the TVD would be equal to MD meanwhile the inclination is equal to zero. On this hypothetical well trajectory plan, there is a kickoff point, at which the inclination increases, a horizontal displacement takes shape and the TVD does not match the values of MD anymore. At first, the normal force to the axis of the drill string on the bit is generated only by the offset. To understand the natural displacement, an evaluation of the force on the bit on the sequence moment is required. As the drilling proceeds, the well path is being curved and the inclination is gradually increasing to reach the target inclination. This process of increasing the inclination generates a curved well path that impacts the force on the bit and ROP of the tool. This takes place since the RSS tool has stabilizers that are in contact with the formation and as the formation is being curved, these stabilizers are being pushed by the formation and this generates an additional force on the bit.


Figure 4. 11: Drillstring on a hypothetical situation
Based on the schematics above, some explanations can be made. The schematics on the lefthand side represent the RSS tool at the initial point of increase of inclination on $T=0$. This initial bending is realized by the offset alone. On the right-hand side, it is the following up moment when the RSS tool is bent additionally by the curved well path at $\mathrm{T}=1$. This additional bending is generated by the so-called natural displacement which is the bent distance on the RSS tool caused by the well path curvature. This is a natural event that the tool does not control. If, at this moment, the RSS system is defined to drill with zero percent of offset, the natural
displacement will be the main driver of the forces of the bit and it will be responsible to not allow an instantaneous change to straight drilling. If the offset force is defined as zero at this transitional moment, the force on the bit would be represented by the natural displacement and it would reduce gradually as the curvature of the well path decreases. This effect results in a smoother change in direction along the well path since its change is gradual and depends on the drilled geometry of the well path. This is the reason why, in some cases, it is possible to have the RSS system not activated and still have an increase or decrease of inclination for some relative time. The variations of natural displacement depend on how fast the drilling is taking place and where the flex joint is installed. According to specialists at Canrig, the flex-joint is responsible for annulate bending forces from above its installation location and it does not allow the above forces to be transferred to the drill bit. In this project, this equipment will be installed 3 meters above the drill bit, in other words, any natural bending that happens above the upper stabilizer from the RSS system will not generate a force on the bit. Any bending that happens on the space from the bit until the upper stabilizer of the RSS system will generate a natural displacement and further on an additional force on the drill bit. The definition of natural displacement is shown in figure 4.12.


Figure 4. 12: Natural displacement
Figure 4.12 shows that the most flexible point of the tool is not in the middle. The design of the tool was defined like this to protect its components. The only points of contact of the tool with the formation are on the upper stabilizer, the actuator/offset, and drill bit. The modelling of this distance is exposed and evaluated in the next section.

### 4.2.2.2 Modelling

Modelling this event was challenging and required engineering and computational concepts. As already defined here, the main idea is to model the bent distance depending on the geometry of the well and the RSS equipment used. In the case of this thesis, the natural displacement will be generated by the formation geometry that has contact with the tool from the bit until the upper stabilizer from the RSS system. That means that the mathematical model program must be able to verify at every point, how the geometry of the well path is drilled.

Some concepts of construction of lines and distance between points are required. The idea is to find the closest pointed to the upper stabilizer, the drill bit, and the offset for every new point drilled. Once these points are found, the goal is to create a line from the point of the upper stabilizer to the bit and calculate the angular coefficient (slope of the line, $m_{\text {long }}$ ) and the independent coefficient from the line $\left(c_{\text {long }}\right)$. The line from the upper stabilizer to the bit is going to be called the "long line".


Figure 4.13
Once the "long line" is defined, the closest point on the well path to the offset will be calculated and stored. The second step is to do a similar process for offset location and the bit and create the "short line". As before, the slope of the line and the independent coefficient are collected ( $m_{\text {short }}$ and $c_{\text {short }}$ ). The third step is to define the distance " x ". This distance of " x " represents the distance between the offset location and the bit on the RSS tool previously also defined as $\boldsymbol{a}$. The location of " $x$ " behind the bit considering the "long line" and the location of " $x$ " behind the bit on the considering "short line" will be calculated. The distance between the "x" location on the "long line" and the "x" location on the "short line" is the natural displacement. This process is dynamically updated for each new point drilled.

A small error should be expected since there is an inclined situation. For a clear understanding, the previous related steps are described in the following images. From the left to the right, it can be observed the process of finding the long line (green), finding the short line (blue), finding the " $x$ " point on the longest line, finding the " $x$ " point on the shortest line (blue), and calculating the distance of the natural displacement. The variable "x" represents the location where the natural displacement will bend the tool.


Figure 4. 14: Process of calculating the natural displacement

The equations for this process will be exposed with data from the simulated RSS tool. Computational knowledge is needed. Considering the simulated RSS tool, the distance from the upper stabilizer to the bit is 3 meters and the distance from the offset location to the bit is 0.5 m :

$$
\begin{gather*}
m_{\text {long }}=m_{3}  \tag{4.13}\\
c_{\text {long }}=c_{3}  \tag{4.14}\\
m_{\text {short }}=m_{0.5} \tag{4.15}
\end{gather*}
$$

$$
\begin{equation*}
c_{\text {short }}=c_{0,5} \tag{4.16}
\end{equation*}
$$

The decision of explaining the simulated RSS tool was taken aiming a clear understanding. As exposed before, the first goal is to calculate the "long line". The mathematical simulation calculates three points for this process: the location to the upper stabilizer, the offset location, and the position of the bit. These points are used to create the "long line". The offset location is required since it is considered to be the most flexible point of the tool and where the natural displacement takes place on the simulated RSS tool. If another tool is analyzed, there is the possibility that the most flexible point is not on the offset location.

In figure 4.15, it is possible to note the simulation collecting information about the upper stabilizer location (approximately 10 ft above the bit).

```
while t < t_max:
    j=t-2
    while True:
        distance = log[t-1,0] - log[j,0]
        if distance >= 10.496:
            break
        else:
            j=j-1
```

    temp_data \(1=[[\log [j, 2], \log [j, 3]]]\)
    temp_data \(2=[[\log [t-1,2], \log [t-1,3]]]\)
    temp_data1 \(=\) np.rot90(temp_data1,1)
    temp data2 \(=\) np.rot9e (temp data2,1)
    Figure 4. 15: Simulation collecting the geometry at approximately 3 meters behind the bit
Creation of the "long line" follows the equations 4.17 and 4.18, where information about the bit and the upper stabilizer location are collected and used.

$$
\begin{gather*}
X o * m_{3}+c_{3}=Y o  \tag{4.17}\\
X_{3} * m_{3}+c_{3}=Y_{3} \tag{4.18}
\end{gather*}
$$

Where Xo and Yo represent the location of the bit (known by the simulation), $Y_{3}$ and $X_{3}$ represent the upper stabilizer location (calculated by simulation on figure 4.15) and $m_{3}$ and $c_{3}$ are the unknown coefficients that are required to create the "long line". From a system of two equations (4.17 and 4.18) and two variables ( $m_{3}$ and $c_{3}$ ), the equation of "long line" can be found following the idea of figure 4.13.

The distance " $x$ " is defined as 0.5 m because this is the distance from the bit to the location where the well path curvature bends the RSS tool. To calculate the point 0.5 m behind the bit on the "long line", the following equations are defined.

$$
\begin{gather*}
\sqrt{\left(X p_{3}-X o\right)^{2}+\left(Y p_{3}-Y o\right)^{2}}=0,5  \tag{4.19}\\
Y p_{3}=X p_{3} * m_{3}+c_{3} \tag{4.20}
\end{gather*}
$$

Equations 4.19 and 4.20 find the "x" location on the "long line". Again, two equations and two variables are given, where $X p_{3}$ and $Y p_{3}$ represent the " $x$ " location of the desired point 0.5 m behind of the bit on the "long line".

Since the "x" location on the "long line" is found, the next step repeats the process for the "short line". In figure 4.16, it is possible to note the simulation collecting information about the offset location ( 1.64 ft behind the bit).

```
g=t-2
while True:
    distance = log[t-1,0] - log[g,0]
        if distance >= 1.64:
        break
    else:
        g=g-1
temp_data3 = [[log[g,2], log[g,3]]]
temp_data4 = [[log[t-1,2], log[t-1,3]]]
temp_data3 = np.rot9e(temp_data3,1)
temp_data4 = np.rot9e(temp_data4,1)
```

Figure 4. 16: Simulation collecting the geometry at $0,5 \mathrm{~m}$ behind the bit $(1,64 \mathrm{ft})$
The same procedure is done for the second line. The creation of the short line can be observed in equations 4.21 and 4.22 .

$$
\begin{gather*}
X o * m_{0.5}+c_{0,5}=Y o  \tag{4.21}\\
X_{0.5} * m_{0.5}+c_{0,5}=Y_{0.5} \tag{4.22}
\end{gather*}
$$

Where Xo and Yo represent the location of the bit (known by the simulation), $Y_{0.5}$ and $X_{0.5}$ represent the offset location (calculated by the simulation on figure 4.16) and $m_{0.5}$ and $c_{0.5}$ are the unknown coefficients that are required to create the "short line". From a system of two equations ( 4.21 and 4.22 ) and two variables ( $m_{0.5}$ and $c_{0.5}$ ), the equation of "short line" can be found following the idea of figure 4.13.

The calculation of point " $x$ " in the short line can be observed in equations 4.23 and 4.24. To calculate the point 0.5 m behind the bit on the "short line", the following equations are defined.

$$
\begin{gather*}
\sqrt{\left(X p_{0.5}-X o\right)^{2}+\left(Y p_{0.5}-Y o\right)^{2}}=0,5  \tag{4.23}\\
Y p_{0.5}=X p_{0.5} * m_{0.5}+c_{0,5} \tag{4.24}
\end{gather*}
$$

Again, two equations and two variables are given, where Xp and Yp represent the "x" location of the desired point 0.5 m behind of the bit on the "short line". If another RSS tool has a different location for its most flexible point, the distance from the bit to this point must replace the " 0.5 " term on equation 4.19 and 4.23 in addition to simple changes on the simulation code.

Finally, it is possible to calculate the distance H, or natural displacement. Equation 4.25 calculates the distance from the " $x$ " point on the "short line" to the " $x$ " point on the "long line".

$$
\begin{equation*}
H=\sqrt{\left(X p_{0.5}-X p_{3}\right)^{2}+\left(Y p_{0.5}-Y p_{3}\right)^{2}} \tag{4.25}
\end{equation*}
$$

The generic formula is represented as follows. If a different RSS tool is simulated, minor changes in its geometry must be considered.

$$
\begin{equation*}
H_{\text {Normal }}=\sqrt{\left(X p_{\text {short }}-X p_{\text {long }}\right)^{2}+\left(Y p_{\text {short }}-Y p_{\text {long }}\right)^{2}} \tag{4.26}
\end{equation*}
$$

### 4.2.3 Natural Displacement 3D

The natural displacement 3D shows significant changes, not in the function, but in the coordinate planes of analysis. Previously, the collection of data by the simulator aimed to collect the TVD and horizontal displacement of each location of interest. On 3D, further information is needed.

The points of collection of geometry are slightly different. On the 2D modelling, for every point, the TVD and horizontal displacement were collected for the offset location, upper stabilizer, and bit. For the 3D, more data collection is needed. For each point, the model still collects the TVD and horizontal displacement as the 2D modelling does. In addition to this information, the system collects location on the North axis and East axis for the upper stabilizer, offset location and bit location for every new point drilled.

The additional two points are used to calculate the natural displacement for the $R O P_{\text {Azi }}$ meanwhile the TVD and horizontal displacement are used to calculate the natural displacement for the $R O P_{I n c}$. In another words, the $H_{A z i}$ and $O f f s e t_{A z i}$ are used to calculate the resultant force on the bit Fbit $_{H}{ }^{\prime}$ that is used on the calculation of the $R O P_{\text {Azi }}$ regarding the North and East coordinate plane. Regarding the TVD and MD coordinate plane, the $H_{\text {Inc }}$ and $O f f s e t_{\text {Inc }}$ are used to calculate the resultant force on the bit $\mathrm{Fbit}_{H}{ }^{\prime \prime}$ that is used on the calculation of the $R O P_{\text {Inc }}$.


Figure 4.17
The formulas for the natural displacement that interfere on azimuth and inclination are the following:

$$
\begin{gather*}
H_{\text {Azi }}=\sqrt{\left(X p_{\text {short }}{ }^{\prime}-X p_{\text {long }}\right)^{2}+\left(Y p_{\text {short }}{ }^{\prime}-Y p_{\text {long }}\right)^{2}}  \tag{4.27}\\
H_{\text {Inc }}=\sqrt{\left(X p_{\text {short }}{ }^{\prime \prime}-X p_{\text {long }}{ }^{\prime \prime}\right)^{2}+\left(Y p_{\text {short }}{ }^{\prime \prime}-Y p_{\text {long }}{ }^{\prime \prime}\right)^{2}} \tag{4.28}
\end{gather*}
$$

Where $H_{A z i}$ analyses the coordinates on the North and East quadrant and $H_{I n c}$ analyses the coordinates on the TVD x horizontal displacement. The previously explained logic stills the same.

### 4.2.4 Force Caused by the Natural Displacement on the Bit

### 4.2.4.1 Theoretical Definition

As already mentioned, the natural displacement, that the tool feels by the formation, must have a model that represents reality. The force caused by the natural displacement is a similar event as the force caused by the offset, on a modelling perspective, but its meaning is different. While the natural displacement force is the force from the formation creating a bending on the tool and generating a force on the bit, the offset force is the force caused by the offset displacing and bending the tool. The offset force is defined as a controlled force and the natural displacement force is defined as an uncontrolled force.

### 4.2.4.2 Modelling

Modelling this force is a process similar to modelling the offset force on the tool. With the help of basic physics, it can be observed that the force on the bit caused by the natural displacement can be calculated using the following formula:

$$
\begin{equation*}
\text { Fbit }_{H}=\frac{W_{\text {Load }} b}{l} \tag{4.29}
\end{equation*}
$$

This equation comes from the momentum balance on the right point of the beam.

$$
\begin{equation*}
W_{L o a d} b+\left(F b i t_{H}\right) l=0 \tag{4.30}
\end{equation*}
$$

If equations 4.10 and 4.29 are combined and assuming that $H_{\text {Normal }}$ is defined by the natural displacement calculation, the following relationship between natural displacement and force on the bit is reached:

$$
\begin{equation*}
H_{\text {Normal }}=\frac{F b i t_{H} a^{2} b}{3 E I} \tag{4.31}
\end{equation*}
$$

So,

$$
\begin{equation*}
\text { Fbit }_{H}=3 \frac{H_{\text {Normal }} E I}{a^{2} b} \tag{4.32}
\end{equation*}
$$

It must be noted that $H_{\text {Normal }}$ represents the natural displacement calculated anteriorly in the previous sections. This is a point of difference regarding the force generated by an offset since the offset force is controlled by the RSS system and the natural force is a reaction force that cannot be controlled and depends on the well path curvature. For the 3D model the following equations are used:

$$
\begin{align*}
& \text { Fbit }_{H}^{\prime}=3 \frac{H_{A z i} E I}{a^{2} b}  \tag{4.33}\\
& \text { Fbit }_{H}^{\prime \prime}=3 \frac{H_{\text {Inc }} E I}{a^{2} b} \tag{4.34}
\end{align*}
$$

### 4.2.5 Force Caused by the Offset on the Bit

### 4.2.5.1 Theoretical Definition

Based on the beam bending exposed model, the force on the bit caused by a specific offset will be one of the two drivers of the RSS model. To have a realistic RSS model, it is important to understand its behavior. The second driver is the natural displacement that was exposed in the last section. The exposed beam bending system will be used.


Figure 4.18
The tool has three points of contact with the formation and there are two loads on the beam. Each load comes from natural displacement and offset, respectively. The idea for this chapter is to model the offset component force according to a specific offset. The definition of the offset and offset controller can be found in the chapter "4.4 Offset and Offset Controller".

### 4.2.5.2 Modelling

It must be noted that to facilitate the modelling, this thesis will consider that the lengths 1 , a, and $b$ are the exact distances of the tool. These distances would be smaller since the body of the tool is curved because of the bending and the distances 1 , a , and b consider a straight line between the defined points. This simplification is possible because after calculating the maximum bending of a tool on a real data field, this thesis has discovered that the error of this simplification is not representative neither compromises further calculations.

By using basic physics, it is possible to observe that the force on the bit can be calculated by the following formula:

$$
\begin{equation*}
\text { Fbit }_{\text {offset }}=\frac{W_{\text {Load }} b}{l} \tag{4.35}
\end{equation*}
$$

The previous equation comes from the momentum balance on the right point of the beam.

$$
\begin{equation*}
W_{\text {Load }} b+\left(\text { Fbit }_{\text {offset }}\right) l=0 \tag{4.36}
\end{equation*}
$$

If equations 4.10 and 4.35 are combined and assuming that $O f f$ set $_{\text {Normal }}$ is defined by the offset displacement according to the offset controller, the following relationship between offset and force on the bit:

$$
\begin{equation*}
\text { offset }_{\text {Normal }}=\frac{\text { Fbit }_{\text {offset }} a^{2} b}{3 E I} \tag{4.37}
\end{equation*}
$$

So,

$$
\begin{equation*}
\text { Fbit }_{\text {offset }}=3 \text { offset }_{\text {Normal }} * \frac{E I}{a^{2} b} \tag{4.38}
\end{equation*}
$$

It must be noted that the $O f f$ set $_{\text {Normal }}$ in the formula represents the offset that is driving the RSS system. Its value comes from the offset controller. The Offset ${ }_{\text {Normal }}$ calculation is explained in the chapter "4.4 Offset and Offset Controller".

Until this point, this paper has considered the lengths $\mathrm{a}, \mathrm{b}$, and 1 the lengths of the tool. This generates an unnecessary error, even if the error is considerably small. Aiming improvement, this error will be diminished at this point.


Figure 4. 19
The idea is to use a Pythagoras theorem considering that the $O f f$ set $_{\text {Normal }}$ is known and $D a$ and $D b$ represent the distance from the bit to the offset and the distance from the offset to the upper stabilizer. In this case, the updated values of $a$ and $b$ are as follows.

$$
\begin{equation*}
a=\sqrt{\text { Da }^{2}-\text { Offset }_{\text {Normal }}{ }^{2}} \tag{4.39}
\end{equation*}
$$

And b,

$$
\begin{equation*}
b=\sqrt{D b^{2}-\text { Offset }_{\text {Normal }}{ }^{2}} \tag{4.40}
\end{equation*}
$$

The following equation is achieved to the force on the bit caused by an offset:

$$
\begin{equation*}
\text { Fbit }_{\text {offset }}=3 \frac{\text { Offset }_{\text {Normal }} \text { EI }}{\left(\text { Da }^{2}-\text { Offset }_{\text {Normal }^{2}}{ }^{2} \sqrt{{D b^{2}-\text { Offset }_{\text {Normal }}}^{2}}\right.} \tag{4.41}
\end{equation*}
$$

Where $D a$ is the distance from the bit to the actuator location and $D b$ is the distance from the offset location to the upper stabilizer. Offset Normal is calculated by the offset controller (chapter 4.4) and $E$ and $I$ depend on the geometry and the material of the RSS tool. It is reasonable to say that an error in this equation is still present since the distance Da and Db are curved distances and in the previous equation these distances are considered as straight lines to compose the Pythagoras theorem. As already mentioned in this paper, the original error theoretically does not affect the results, but this simple solution should improve the model even further.

On the 3D model the following equations will be used:

$$
\begin{equation*}
\text { Fbit }_{o f f s e t}^{\prime}=3 \frac{\text { Offset }_{A z i} E I}{\left(D a^{2}-O f f \operatorname{set}_{A z i}^{2}\right) \sqrt{D b^{2}-O f f s e t_{A z i}^{2}}} \tag{4.42}
\end{equation*}
$$

$$
\begin{equation*}
\text { Fbit }_{\text {offset }}^{\prime \prime}=3 \frac{\text { Offset }_{\text {Inc }} \text { EI }}{\left(\text { Da }^{2}-\text { Offset }_{\text {Inc }}{ }^{2}\right) \sqrt{D b^{2}-\text { Offset }_{\text {Inc }}^{2}}} \tag{4.43}
\end{equation*}
$$

$D a$ should be the distance from the bit to the offset location and $D b$ represents the distance from the offset location to the upper stabilizer. The $O f f \operatorname{set}_{A z i}$ represents the offset projected on the North and East quadrant to calculate the Fbit ${ }_{o f f s e t}^{\prime}$ and the $O f f s e t_{I n c}$ is the offset projection on the TVD and horizontal displacement coordinates plane for the calculation of the Fbit $t_{f f s e t}^{\prime \prime}$. The offset calculation by the offset controller can be observed further on the thesis (chapter 4.4).

### 4.2.6 Resultant Force on the Bit

### 4.2.6.1 Theoretical Definition

As mentioned before, the correct definition of the resultant force on the bit is essential to model the RSS system. At the first stage, the modelling consists only of the force that comes from the offset. So, the first 2D model exposed in chapter 5 in this thesis considers the following:

$$
\begin{equation*}
\text { Fbit }=\text { Fbit }_{\text {offset }} \tag{4.44}
\end{equation*}
$$

It should be noted that this equation requires a definition of the sign even if only one variable is held. From this point on, a positive force is hereby defined as a force that generates an increase of the inclination angle. This definition facilitates the implementation of the natural displacement force on the bit and establishes a definition of signs for the mathematical model.

In a more evolved model, with the consideration of the natural displacement from the formation, the resultant force that will be used on the $R O P_{\text {Normal }}$ follows the equation 4.45:

$$
\begin{equation*}
F b i t=F b i t_{o f f s e t}+F b i t_{H} \tag{4.45}
\end{equation*}
$$

As mentioned previously, any force that helps the increase of inclination will be treated as positive and any force that decreases the value of inclination will be treated as negative. For a clear understanding, this paper will expose some situations and define the sign for each force. It is important to understand the axis and coordinates of the following graphs. The TVD can be observed on the Y -axis and the horizontal displacement can be observed on the X -axis. The yellow arrow symbolizes the region being evaluated.


$$
\text { Fbit }_{\text {offset }}=0 \text { and } \text { Fbit }=0
$$



Fbit $_{\text {offset }}>0$ and Fbit $_{H}>0$


Figure 4. 21: Forces evaluations 2
In the first picture 4.20, it is possible to notice that whenever a constant inclination is present, there is no normal/lateral force on the bit. On the right-hand side of figure 4.20 , there is an inclination increase, so the offset is activated on the West side of the tool and the drilling goes East. This defines a positive Fbit $_{\text {offset }}$ and the formation helps the movement to increase the inclination, so the $F b i t_{H}$ is also defined positive.

The situation in the second image in figure 4.21 is quite similar. First, the right-hand side picture will be evaluated. In this situation, there is a decrease of inclination, the offset force will be negative, and the formation helps to decrease the inclination, so it will be lower than zero. This situation is similar to the previous one. The interesting part to analyze is the transitional part, in other words, the influence of the transition on the resultant force represented on the left-hand side in figure 4.21. The natural displacement force is not controlled by the tool so its transition cannot be controlled by it. The transition of offset is whenever the offset is changed from a situation where it increases the inclination to a situation where the offset decreases the inclination on a 2D model. This transition of the offset is realized by the offset controller. At the transition of offset, there will be a point where Fbit $_{\text {offset }}$ and Fbit $_{H}$ will have opposite signs. In this case, the Fbit $_{\text {offset }}$ would change its value to negative, while the force generated by the natural displacement $\left(F_{b i t}^{H}\right)$ is still positive and slowly decreases until its value turns to be negative on a delayed behavior.

### 4.2.6.2 Modelling

Aiming to model this effect, an analysis of the variables available is required. The mathematical model needs to make sure that the force on the bit caused by the offset and the natural displacement have the correct signs depending on the situation. Regarding the offset force, any force that will increase the angle under analysis is positive. As the RSS control system controls
the offset, this definition of a sign is easy to be made and it will be analyzed on the offset controller topic (chapter 4.4). Regarding the natural force on the bit caused by the natural displacement, this is an event that the RSS does not control. The main challenge is to model and define the sign of the force, because, as shown before, the natural displacement calculated previously is an absolute value, since it is calculated using a distance equation. To properly model this effect, an overview of the available data about natural displacement is required. As shown here, the natural displacement modelling involves the creation of two lines based on the geometry of the tool and of the well path. From this modelling, the following variables are available:

- $c_{\text {short }} \quad$ Independent coefficient of the line from the bit to the offset
- $m_{\text {short }} \quad$ Angular coefficient of the line from the bit to the offset
- $c_{\text {Long }}$ Independent coefficient of the line from the bit to the upper stabilizer
- $m_{\text {Long }} \quad$ Angular coefficient of the line from the bit to the offset upper stabilizer

The main idea is to use the angular coefficient to define a sign for the absolute value of $H$ (whenever the $H$ nomenclature is used, it represents $H_{\text {Normal }}$ for the 2D and $H_{\text {Inc }}$ and $H_{A z i}$ for the 3D). The idea is based on the concept of angular coefficient. As it can be observed in figure 4.22.


Figure 4.22
To clarify how this data will be used to give a sign to the distance $H$ (natural displacement), the following schematics of theoretical wells are exposed in the following image. It must be noted that the orange line represents the line between the bit and the offset location (short line), and the grey line represents the line between the bit and the upper stabilizer of the RSS system (long line).



The previous images were exposed to facilitate understanding. Any movement that generates an increase in the inclination has a positive value. Figure A shows a well that starts at the inclination zero and goes for a negative inclination of -45 degrees. This sign is defined by the simulator. Due to this logic, the sign of the natural displacement force on the bit will be negative. In figure B is the opposite scenario. It represents a well that is leaving the inclination of 0 and aiming for the inclination of 45 degrees. Therefore, the force on the bit generated by the $H$ (natural displacement) must be positive.

Figures C and D are unreal due to negative values of TVD, but its analysis is important to verify the model and check if the same logic can be used by the azimuth modelling. In figure C , the well is with an inclination on 180 degrees and it is achieving an inclination of 225 degrees. According to the previous logic, the natural displacement force should be positive and finally, in figure D, the well is at 180 degrees and it is aiming to reduce its inclination to 135 degrees. Thus, its force on the bit caused by the natural displacement should be negative. The offset controller must follow the same logic to have a consistent scheme of forces.

Mathematically speaking, there is a simple loop that models this logic. As seen previously in figure 4.22 , the theta $\theta$ value represents the tangent of the slope of the lines. $\theta_{\text {short }}$ represents the tangent of the line which is built between the bit and the offset location and $\theta_{\text {long }}$ represents the tangent of the slope of the line which is built between the location of the bit and the upper stabilizer location. To model the previous logic, the following loop is programmed:

$$
\text { If } \theta_{\text {long }} \text { is bigger than } \theta_{\text {short }}:
$$

The force from the natural displacement on the bit will be positive

$$
\text { If } \theta_{\text {long }} \text { is smaller than } \theta_{\text {short }} \text { : }
$$

The force from the natural displacement on the bit will be negative
This establishes the sign for the absolute value of the natural displacement $(H)$ previously modeled.

### 4.3 ROP Modelling

In this section of the work, several ROP models will be analyzed and the simulated one from Taete [12] will be exposed.

### 4.3.1 Bourgoyne et al. ROP Model

To develop a reliable simulator, several options of ROP models were faced. The first one to be exposed here is the Bourgoyne and Young Model [19], developed by Bourgoyne and Young in 1986 and analyzed on the paper "Investigation of Various ROP Models and Optimization of drilling parameters for PDC and Roller-cone Bits in Shadegan Oil Field" from Bataee and Kamyab published in 2010 [20]. The model consists of the following equation:

$$
\begin{equation*}
\mathrm{R}=f_{1}+f_{2}+f_{3}+f_{4}+f_{5}+f_{6}+f_{7}+f_{8} \tag{4.46}
\end{equation*}
$$

Which,

$$
\begin{gather*}
f_{1=e^{a_{1}}}  \tag{4.47}\\
f_{2=e^{a_{2}(10000-T V D)}}  \tag{4.48}\\
f_{3=e^{a_{3} * D^{0,69} *(M W-67,41)}}  \tag{4.49}\\
f_{4=e^{a_{4} * T V D *\left(E M W_{P o r e}-E C D\right)}}  \tag{4.50}\\
f_{5}=\left(\frac{\frac{W}{d_{b}}-\left(\frac{W}{d_{b}}\right)_{t}}{4-\left(\frac{W}{d_{b}}\right)_{t}}\right)^{a_{5}}  \tag{4.51}\\
f_{6}=\left(\frac{N}{60}\right)^{a_{6}}  \tag{4.52}\\
f_{7}=e^{-a_{7} * h}  \tag{4.53}\\
f_{8}=\left(\frac{F_{j}}{1000}\right)^{a_{8}} \tag{4.54}
\end{gather*}
$$

Where
$a_{1}$ to $a_{8}=$ constants depending on the drilling conditions;
$\mathrm{D}=\mathrm{TVD}$ in feet;
$g_{p}=$ pore pressure gradient $\mathrm{lbm} / \mathrm{gal}$;
$p_{c}=$ equivalent circulating gradient, $\mathrm{lbm} / \mathrm{gal} ;$
$\left(\frac{w}{d_{b}}\right)_{t}=$ threshold bit weight per inch of bit diameter at which the bit begins to drill, $1,000 \mathrm{lbf} / \mathrm{in}$ $\left(\frac{w}{d_{b}}\right)=$ bit weight per inch of bit diameter, $1,000 \mathrm{lbf} / \mathrm{in}$.;
$\mathrm{h}=$ fractional tooth dullness
$F_{j}=$ hydraulic impact force beneath the bit, lbf;

Each $f_{n}$ corresponds to one empirical factor that impacts the drilling. $f_{2}$ corresponds to the normal compaction of the model. $f_{3}$ represents the compaction in abnormally pressured formations. $f_{4}$ represents the effect of the pore pressure gradient of the rock. Further information can be found in the publications mentioned above. As it can be noticed, this model depends on several rock characteristics such as rock strength and bit weight.

### 4.3.2 Bingham Model

The second model that was studied for this thesis was the Bingham Model [21] analyzed on the paper "Real-time predictive capabilities of analytical and machine learning rate of penetration (ROP) models" by Soares and Gray in 2019 [22].

$$
\begin{equation*}
R O P=a b *\left(\frac{W O B}{D_{b}}\right)^{b b} * R P M \tag{4.55}
\end{equation*}
$$

Where,
ab and $\mathrm{bb}=$ depend on the rock type
$\mathrm{WOB}=$ weight on bit in lbf $\times 10^{3}$
$D_{b}=$ bit diameter in inches
RPM in rev/min

This model was developed empirically by Bingham in 1964 and it is important to keep in mind that two variables ( $a b$ and $b b$ ) depend on the rock characteristics.

### 4.3.3 R.Teale

In "The concept of specific energy in rock drilling", Teale derived the following equation [12]:

$$
\begin{equation*}
R O P=\frac{13.33 \mu N}{D\left(\frac{E s}{W O B}-\frac{1}{A b}\right)} \tag{4.56}
\end{equation*}
$$

where $\mu$ is the coefficient of sliding friction, $N$ is the rotary speed, $D$ is the bit diameter, $W O B$ is the weight on bit, $A b$ is the borehole area and $E s$ is the specific energy of the rock. With this equation, it is possible to observe a clear relationship between $W O B$ and ROP and, most important, only one dependency from rock mechanics, which is known as specific energy. According to B. Celada et al. [23], the specific energy can be defined as the energy needed to drill a specific volume of rock in $\mathrm{GJ} / \mathrm{m}^{3}$.

According to Teale [12], some parameters rule the drilling procedure such as the drilling equipment used, the ground response from the formation, and drilling characteristics such as weight on bit and RPM. Focusing on the ground response, driven by the specific energy, Teale defines the following formula:

$$
\begin{equation*}
\mathrm{Es}=\mathrm{F} / \mathrm{A}+2 \pi \mathrm{NT} / \mathrm{A} V \tag{4.57}
\end{equation*}
$$

$\mathrm{F}=$ thrust on bit (kN).
$\mathrm{A}=$ area removed by drill bit (m2).
$\mathrm{N}=$ rotation speed (rps).
$\mathrm{T}=$ rotation torque $(\mathrm{kN} \cdot \mathrm{m})$.
$\mathrm{V}=$ drilling speed/rate of penetration $(\mathrm{m} / \mathrm{s})$.
Where the first component of the equation is known as thrust component (et) and the second component of the equation is the rotary component of energy (er). If $p=V / N$ known as penetration per revolution and rewrite the equation achieve can be the following.

$$
\begin{equation*}
\mathrm{er}=2 \pi \mathrm{~T} / \mathrm{Ap}(\mathrm{kN} / \mathrm{m} 2) \tag{4.58}
\end{equation*}
$$

where T specifies how much torque is needed to remove an amount of rock in one revolution. Since T and p do not change their value regarding changes in RPM, this ratio can be a good index for the specific energy [23].
On [23], it is possible to observe the behavior of the specific energy regarding the drilling trust " F " and the penetration per revolution " p ". It can be observed that there is a value of the trust that is limited to the specific energy. If there is trust above this value, there will be no variation in the specific energy value.

This model is the one used on the developed simulator. First of all, there is a simple definition between the drilling trust, which is the force that pushes the bit against the formation and the resultant ROP and the specific energy. Second, the ROP formula is only dependent on the system bit x formation. This created the possibility of calculating the specific energy interactively if there is no previous data about the Es. A study about the effects the Es on the RSS system and a further development of Es values in the calculation of $R O P_{\text {Normal }}$ would be interesting. That is said because, an interactive system generates an error in rock type transitions since different rocks generate different ground responses if the soils have different ground proprieties.

### 4.3.3.1 Calculation of ROP

The chosen mathematical expression of the rate of penetration (ROP), developed by Teale [12] and analyzed at the paper from Pessier and Fear [24] is shown below:

$$
\begin{equation*}
R O P=\frac{13.33 \mu N}{D\left(\frac{E s}{W O B}-\frac{1}{A b}\right)} \tag{4.56}
\end{equation*}
$$

where $\mu$ is the coefficient of sliding friction, $N$ is the rotary speed, $D$ is the bit diameter, $W O B$ is the weight on bit, $A b$ is the borehole area and $E s$ is the specific energy of the rock.
The main idea is to derive this model in two distinct equations for the 2D model, one for perpendicular displacement and a second one for vertical displacement. Respectively, the ROP responsible for the perpendicular displacement is called $R O P_{\text {Normal }}$ and the axial displacement is called $R O P_{\text {Axial }}$.

$$
\begin{equation*}
R O P_{\text {Axial }}=\frac{13.33 \mu N}{D\left(\frac{E s}{W O B}-\frac{1}{A b}\right)} \tag{4.59}
\end{equation*}
$$

$$
\begin{equation*}
R O P_{\text {Normal }}=\frac{13.33 \mu^{\prime} N^{\prime}}{D\left(\frac{E s^{\prime}}{F b i t}-\frac{1}{A b}\right)}=\frac{13.33 \mu N}{D\left(\frac{E s}{F b i t}-\frac{1}{A b}\right)} * \text { alfa } \tag{4.60}
\end{equation*}
$$

It must be noted that there is a coefficient to regulate the $R O P_{\text {Normal }}$. The idea is to take into account real values from the dogleg severity limitations of the tool and reasonable observations from field data to regulate this alfa factor. This alfa describes the steerability of the tool. A high value of alfa and DLS will be faced if a tool has high steerability.

Both equations are similar and share the same coefficients, besides the Fbit in the $R O P_{\text {Normal }}$ and $W O B$ in the $R O P_{\text {Axial }}$. The $R O P_{\text {Axial }}$ will be driven by the weight on the bit, as shown in the previous formula. Focusing on the $R O P_{\text {Normal }}$, the Fbit represents the force existent on the lateral of the bit generated by the RSS tool. In this project, two main forces compose the lateral forces on the bit. The first one would be the offset force that is caused by the offset controller. The second one is the natural displacement force which represents the impact of a curved formation on the RSS tool.

$$
\begin{equation*}
F b i t=F b i t_{o f f s e t}+F b i t_{H} \tag{4.61}
\end{equation*}
$$

The Fbit force on the bit is the sum of the force generated by the offset and the force generated by the natural displacement from the formation, but the signs of these values are not necessarily equal all the time. Each force should vary in well path transitions of trajectory.

In the 3D model, the logic that the force Fbit will be composed by a natural displacement force and an offset force is still taking place. If this line of thinking is considered in a 3D system with respective azimuth and inclination, one more equation takes place. The previous $R O P_{\text {Normal }}$ will be divided into two ROPs. Furthermore, the system is composed by three ROPs: $R O P_{\text {Axial }}$ that will be driven by the weight on bit, $R O P_{\text {Azi }}$ that will be driven by the calculated force on the bit on the North and East Cartesian plane, also known as Fbit $\left(F_{b i t}{ }^{\prime}=F_{b i t}{ }^{\prime}{ }^{+}\right.$ Fbit $_{\text {offset }}{ }^{\prime}$ ), and finally the $R O P_{\text {Inc }}$. This last one is responsible for modelling the velocity on the Cartesian plane TVD x horizontal displacement generated by Fbit" ${ }^{\prime \prime}$ Fbit ${ }^{\prime \prime}=$ Fbit $_{H}{ }^{\prime \prime}+$ Fbit $_{\text {offset }}{ }^{\prime \prime}$ ). The equations 4.59, 4.62, and 4.63 define each ROP on the 3D model.

$$
\begin{gather*}
R O P_{\text {Axial }}=\frac{13.33 \mu N}{D\left(\frac{E s}{W O B}-\frac{1}{A b}\right)}  \tag{4.59}\\
R O P_{\text {Inc }}=\frac{13.33 \mu N}{D\left(\frac{E s}{{F b i t_{H}}^{\prime \prime}+F b i t_{\text {offset }}{ }^{\prime \prime}}-\frac{1}{A b}\right)} * a l f a  \tag{4.62}\\
R O P_{A z i}=\frac{13.33 \mu N}{D\left(\frac{E s}{\text { Fbit }_{H}{ }^{\prime}+\text { Fbit }_{\text {offset }}{ }^{\prime}}-\frac{1}{A b}\right)} * \text { alfa } \tag{4.63}
\end{gather*}
$$

The method of calculations of the forces was explained previously in the chapter of force calculations. Briefly explaining the origin of some terms, the calculations of the $\mathrm{Fbit}_{H}{ }^{\prime \prime}$ and Fbit $_{\text {offset }}$ " depend on the bit trajectory regarding the North and East coordinate plane and the calculation of $\mathrm{Fbit}_{H}{ }^{\prime}$ and Fbit ${ }_{\text {offset }}{ }^{\prime}$ will depend on the trajectory and of the bit on the TVD and horizontal displacement quadrant.


Figure 4.24
After all these considerations, it is possible to calculate the $R O P_{\text {Inc }}, R O P_{A z i}$ and the $R O P_{A x i a l}$. The common terms of the equations $4.59,4.62$, and 4.63 will be analyzed first. The following dependencies can be observed from the equations: RPM, sliding friction, diameter of the bit, specific energy, weight on bit, and area of the borehole. Among these variables, some are known such as diameter of the bit, RPM, weight on bit, and area of the borehole. The sliding friction can be calculated if torque data is available as can be noted on the equation 4.64. If torque data is not available, the value of 0.23 is assumed.

$$
\begin{equation*}
\mu=\frac{36 T}{D b W O B} \tag{4.64}
\end{equation*}
$$

The last unknown variable is the specific energy. This is a variable that evaluates the interaction between the bit and the formation. According to SPE24584 [24], it can be described as follows:

$$
\begin{equation*}
E s=\frac{E s m i n}{E F F} \tag{4.65}
\end{equation*}
$$

where Esmin stands for the rock compressibility and EFF for its efficiency. By analyzing exposed data on SPE24584 [24], it can be assumed that the efficiency is about $30 \%$ for a formation made of shale and about $80 \%$ for a formation made of ground rock. Furthermore, if neutron logs are available, it is possible to calculate the rock compressibility and further on its specific energy, using the following equation observed in the [25] on equation 4.66 (the variables are auto described):

$$
\begin{array}{r}
\text { Esmin }=194.4-0.6072 \Delta \text { Tsonic }-646.1 * \text { neutron porosity }-  \tag{4.66}\\
0.01644 \Delta \text { Tsonic }^{2}+8.792 \Delta \text { Tsonic } * \text { neutron porosity }
\end{array}
$$

The ideal model should foresee the specific energy from previous drilled points in case of a lack of data. Imagining a practical situation where no geological data or logs are available, the idea is to drill not knowing the specifics parameters of the model and use its response to foresee
the $R O P_{\text {Axial }}$ and $R O P_{\text {Normal }}\left(R O P_{\text {Azi }} / R O P_{\text {Inc }}\right)$. Errors in formation transitions are expected in this case. The developed simulator requires information about the specific energy of the formation and the development of a model to foresee this variable is an opportunity for future improvements.

At this point, the specific terms of each equation (4.59, 4.62, and 4.63) will be analyzed. The $R O P_{\text {Axial }}$ uses the $W O B$ and the $R O P_{\text {Normal }}$ uses the resultant force on the bit (the second one can be divided into $R O P_{I n c}$ and $R O P_{A z i}$ in a 3D scenario). For the calculation of the $R O P_{A x i a l}$, the inputs of this equation are RPM in rotations per minute, weight on bit (WOB) in kilogramforce times $10^{3}$, specific energy (Es) of the formation in psi, the friction factor (mi) and the diameter of the bit in inches (D). The output is the $R O P_{\text {Axial }}$ given in feet per hour.

The calculation of the $R O P_{\text {Normal }}\left(R O P_{A z i} / R O P_{\text {Inc }}\right)$ is relatively complex. In a 3D model, the program calculates the ROP that will control the inclination " $R O P_{\text {Inc }}$ " and ROP that controls the azimuth " $R O P_{A z i}$ ". The $R O P_{\text {Normal }}\left(R O P_{A z i}\right.$ and $\left.R O P_{\text {Inc }}\right)$ has as input the RPM, D, mi, but this function requires extra information. There are three main extra information required: the geometry of the tool on its bending location, the geometry of the well path in 3 different points to analyze the force that the formation bends the tool and the offset controller information. This information is required to calculate the force that replace the $W O B$ in equations 4.62 and 4.63. Furthermore, the force calculation procedure can be found on chapter "4.2 Force calculations".

To finalize the ROP calculation, the model starts calculating the area of the wellbore. In this thesis, an ideal wellbore that has the same diameter as the diameter of the bit is assumed. The second step is to calculate the alfa coefficient. After testing and analyzing field data, the best alfa value is around 0.15 considering the steerability of the modelled tool. This parameter needs to be recalculated for $R O P_{\text {Axial }}$ lower than 30 ft per hour. This value of 0,15 gives a DLS of approximately 18 degrees per 100 ft that fits the dogleg severity for a tool in [15]. Further studies about the impact of the $R O P_{\text {Axial }}$ on the alfa factor can improve the simulator.

### 4.4 Offset and Offset Controller

### 4.4.1 Theoretical Definition

In the previous chapter, a brief definition of offset/actuator was made, however, a further explanation if the offset controller is interesting before modelling it.

There are several modes or configurations with which the RSS tool can be set. Usually, the most common set up is the configuration that aims to reach a target angle and hold its inclination and azimuth after the kickoff point. The kickoff point is defined on the surface and when the tool reaches this point, the RSS is activated to reach the target angle. It is important to know that the RSS system does not have control over the moment of its activation. The system depends on the sign sent by the driller.

Once the signals to activate the tool are sent, the offset and its angular direction (traditionally called toolface) will be responsible to increase/decrease the azimuth and/or inclination. Usually, there are some limitations to the variation of offset to avoid damaging the formation. Some customers limit the value of the maximum desired offset. This is also a precaution to avoid further problems with high values of dogleg severity.

The increase/decrease of azimuth and offset will be defined on the surface and an activation signal will be sent to the RSS tool. After receiving this signal, the offset system will be activated
according to the instructions of the offset controller and offset will be defined. This offset generates a force on the bit that will impact on the $R O P_{\text {Normal }}$ and will drive the bit to the desired direction. The goal of this topic is to model the controller of this force and explain how it is activated and which logic ensures that the bit achieves the desired target azimuth or/and target inclination.

### 4.4.2 Modelling

To model the controller of the offset, it was created a model for the 2D and the 3D. The logic behind this controller is $100 \%$ original and exposed for the first time in this master thesis. In the 2D offset controller, the inputs are defined by the well planner and consist of the "maxoff" and the target inclination for every point of the planned well trajectory. The maxoff represents the value of the maximum offset desired by the customer. Some customers do not want to have severe doglegs like 18 degrees per 100 feet. The value of the maxoff can vary from zero until one. It represents a percentage of the maximum offset that the tool is capable. If the percentage is decreased, the dogleg severity decreases as well. The second input is the target inclination. This target inclination is defined by the well planner of the trajectory. It must be noted that the drilling starts with inclination zero and after the kickoff point the inclination starts to increase until it achieves the target inclination. After the target inclination is achieved the tool is programmed to hold the inclination until further orders. If there is some external force or any natural fracture on the formation that generates a force on the bit that dislocates the bit from its target inclination, the offset will be automatically activated to hold the target and correct its inclination. The calculation of the Offset ${ }_{\text {Normal }}$ is updated constantly. It takes into account the target inclination defined by the well planner and the current inclination of the bit on every simulated step.

Before the offset controller modelling, it is important to realize the offset behavior of the tool. There are two main behaviors of the designed offset controller. If the target inclination is considerably far from the current inclination at which the bit is, the offset of the tool will be equal to the maximum offset allowed by the customer. When the bit achieves current inclinations closer to the target inclination, the offset will reduce its value gradually to achieve the target inclination as smooth as possible.

The point of the transition between the two behaviors is chosen to be 0,65 degrees. In other words, if the current inclination of the bit is more than 0,65 degrees further from the target inclination, the offset is automatically defined as $100 \%$ of the maximum offset. Afterward, as the current inclination becomes closer to the target inclination, the offset should decrease gradually until the value of the offset is equal to zero and the tool maintains its target inclination. The gradual decrease of the offset is ruled by the difference in inclinations. If the distance of the current inclination to the target inclination is less than 0,65 the absolute offset value will be defined as 1,5 times the maximum offset times the distance from the target inclination to the current inclination. As the distance becomes smaller over time, the offset also gets smaller and reduces its value gradually. The process of defining the $O f f$ set $_{\text {Normal }}$ is divided into two main steps, the first defines the intensity of the needed offset, the second defines its direction. The first step follows the following logic (this logic assumes maxoff equal to $100 \%$ of the capable offset of the tool):

- If the target inclination minus current inclination is bigger than 0,65 :

Offset $_{\text {Normal }}= \pm 100 \% *$ maxoff (upwards or downwards)

[^0]- If the target inclination minus current inclination is smaller than 0,65

Offset ${ }_{\text {Normal }}= \pm($ target inclination - current inclination $) * 1,5 *$ maxoff

Decreases gradually behavior (2)


Figure 4. 25: Offset controller 2D
The second step of the calculation of the Offset ${ }_{\text {Normal }}$ is the direction of the offset. This thesis defines that a positive offset (from 0 until $100 \%$ ) will work to increase the inclination of the system. Analogically, a negative offset will be responsible for decreasing the inclination (from 0 until -100\%). The following logic is used:

- If the target inclination is greater than the current inclination:

The $O f f$ set $_{\text {Normal }}$ will have a positive value

- If the target inclination is lower than the current inclination:

The $O f f^{\text {set }}{ }_{\text {Normal }}$ will have a negative value
This controller needs some development for the 3D model. On a 3D situation, there is a desired offset that controls the inclination and a desired offset that controls the azimuth. This development will be exposed in the next section.

### 4.4.3 Offset Controller 3D

This section focuses on the developments on the 3D offset controller. Keeping in mind that the offset value cannot exceed the value of one $(100 \%)$ is necessary to understand what is being modelled here. Inspired by the composition of ROPs as $R O P_{A x i a l}, R O P_{\text {Inc }}$, and $R O P_{A z i}$, the offset follows this concept and is divided into $O f f s e t_{\text {Inc }}$ and $O f f s e t_{A z i}$. Each offset will control the required offset on azimuth and inclination parameters to reach the target location. This requirement leads to a "Final Offset" that will drive the movement of the RSS system and define the best approximation to the target azimuth and target inclination at the same time. This controller was divided into different sections to avoid impossible offset values (bigger than $100 \%$ ). The biggest mistake of this controller would be to result in a total offset greater than $100 \%$ of the maximum offset defined by the customer. In the following image, there is an explanation of the offset present on the offset controller system in a 3D model.


Figure 4. 26: Offsets 3D model
The previous image shows all offsets in the system. The controller follows six steps to calculate the offset that will control the azimuth (in blue color) and the offset that will control the inclination (in green color).

The first step to model this group of offsets is to know how large the distances from the current azimuth and current inclination to the target azimuth and target inclination are, respectively. Depending on the distances, temporary values for the $O f f s e t_{A z i}$ and $O f f s e t_{I n c}$ are placed, and the value of beta is defined. After calculating the value of beta, the controller calculates the intensity of the "Final offset" (in black color). The logic behind this is the following: if the current azimuth minus the target azimuth and/or the current inclination minus the target inclination are bigger than 0,65 degrees, the value of the final offset should be $100 \%$ of the maximum offset defined by the customer. If both previous differences are smaller than 0,65 degrees, the value of the offset must be the arithmetic average of the two differences times the maximum offset times 1.5 . This logic assures that the value of the offset is not greater than one.

Once the value of the "Final offset" is defined, based on the previous value of beta, the value of the offset that controls inclination ( $O f f s e t_{\text {Inc }}$ ) and the offset that controls the azimuth $\left(\mathrm{Offset}_{A z i}\right)$ are redefined. In the last step, it is evaluated if the target inclination is bigger than the current inclination, if that is the case, the value of the $O f f s e t_{\text {Inc }}$ is positive, if that is not the case, the value of this $O f f s e t_{I n c}$ is negative. The same procedure is done regarding azimuth and both offsets are used for the $R O P_{I n c}(F b i t \prime)$ and the $R O P_{A z i}\left(F b i t^{\prime}\right)$. The following flowchart of this controller is defined as bellow:

Defines the intensity offset azimuth and offset inclination

2


3


5
Set up the direction of each component of offset

Use each offset on each ROP equation respectivelly

Figure 4.27
This logic is made to avoid unreal values of offset. Each step is explained in figure 4.28.

If current azimuth is further then 0,65 degrees away from the target azimuth:

$$
\text { Off Azi (abs) }= \pm 100 \%
$$

1
If current azimuth is closer then 0,65 degrees to the target azimuth:

Off Azi (abs) $=$ degreases gradually ( $\pm 1,5$ ) : (target azimuth - current azimuth)

If current inclination is fruther than 0,65 degrees from the target inclination:

Off Inc (abs) $= \pm 100 \%$
If current inclination is closer then 0,65 degrees to the target inclination:

Off Inc (abs) = decreases gradually ( target inclination - current inclination) * $\pm 1,5$ )
(Absolute values)
$2 \quad$ Beta $=\operatorname{arctangent}\left(\frac{\text { Offset }_{\text {Inc }}(\mathrm{abs})}{\text { Offset }_{\text {Azi }}(\mathrm{abs})}\right)$
(Absolute values)

If current azimuth is further then 0,65 degrees away from the target azimuth

## or

3 If current inclination is further than 0,65 degrees from the target inclination:

Resultant offset $= \pm 100 \%$
Else:
Resultant offset = decreases gradually
$\pm\left(1,5^{*}\right.$ (target inclination - current inclination) $+1,5^{*}($ target azimuth - current azimuth))/2
(Absolute values)
$\begin{aligned}{ }^{\text {Offset }} & \\ \text { Ofzi } & = \pm \text { Resultant Offset } * \cos (\text { Beta }) \\ \text { Offset }_{\text {Inc }} & = \pm \text { Resultant Offset } * \sin (\text { Beta })\end{aligned}$

- If target inclination is greater than the current inclination:

The Offset Inc will have a positive value

- If target inclination is lower than the current inclination:

The Offset Inc will have a negative value

- If target inclination is greater than the current inclination

The Offset Azi will have a positive value

- If target inclination is lower than the current inclination:

The Offset Azi will have a negative value

| 6 | Offset $_{\text {Azi }} \rightarrow$ | Fbit offset' | $\begin{aligned} & \text { Resultant } \\ & \text { Force on the } \\ & \text { bit (Fbit') } \end{aligned} \rightarrow \text { ROP }$ |
| :---: | :---: | :---: | :---: |
|  | Offset $_{\text {Inc }} \rightarrow$ | Fbit offset ${ }^{\prime \prime} \rightarrow$ | $\begin{aligned} & \text { Resultant } \\ & \text { Force on the } \rightarrow \text { ROP } \\ & \text { bit (Fbit") } \end{aligned}$ |

Figure 4.28

## 5 Case Study

Technically speaking, the first model to be exposed here is the 2D model without the implementation of the natural displacement effect. This 2D model will consider a constant azimuth to focus on inclination variances and the offset will be ruled by an offset controller. Further on, the natural displacement effect will be added on the simulator and this effect is exposed in the graphics of forces on the bit. In the last step, the offset control will be programmed and evaluated in a more complex well. The transition from this knowledge to 3D modelling is challenging. There are some analogies and some additional programming required, as the definition of the $O f f s e t_{\text {Inc }}$ (up/down direction) and the $O f f s e t_{A z i}$ (left/right direction). The approach of modelling, specializing, and improving a 2D model until a full 3D model was achieved was certainly a good decision since the programming was easier and the concepts of petroleum technology were implemented and understood more clearly.

### 5.1 Basic 2D Modelling - Vertical Well

As mentioned before, the first model will consider the 2D modelling in its simplest form, without considering the natural displacement force, constant azimuth, and constant offset.

To test the quality of the mathematical model, a straight well is drilled. The data and coding can be found in the appendix. The goal of this chapter is to explain the effects, results, and changes according to further developments of the simulator.

As can be observed in the following graphs, the simulator runs for a straight well. Whenever the well is being drilled vertically, the MD and the TVD are calculated only based on the $R O P_{\text {Axial }}$.


Figure 5. 11: TVD $x$ Horizontal displacement in a straight well


Figure 5. 22: TVD x Inclination in a straight well


Figure 5. 33: DLS x Inclination in a straight well
It is considered important to start a simulation from its simplest and easiest form. This decision was made here to introduce which data is shared in each simulation and so that the differences and developments can be analyzed. As expected, whenever a vertical well is drilled, a vertical line can be observed on the TVD x horizontal displacement graph. The TVD increases and its values are equal to the MD value, therefore, there is no angle or inclination and no horizontal displacement. Furthermore, there is no DLS, neither a $O f f$ set $_{\text {Normal }}$.

### 5.2 2D Modelling Disregarding Natural Displacement

In the previous simulation, the total time was 100000 timesteps, which is equivalent to 130 hours of drilling with the timestep of 5 seconds. To implement this first change, it is defined that after drilling 150 ft the well should reach 90 degrees and the offset controller sets a proper Offset ${ }_{\text {Normal }}$ to increase the well inclination. At 350 feet, the new target inclination is then set to 0 degrees and the offset controller should decrease the well path inclination. The analysis of respective changes will be analyzed.


Figure 5. 4: TVD x Horizontal displacement in a well without the effect of natural displacement


Figure 5. 5: TVD x Inclination in a well without the effect of natural displacement
In the previous figure, it is possible to observe the smooth and gradual behavior of the offset controller when the well is close to the second target inclination of zero degrees. Such effect is not visible on other graphics since it occurs for a short time. This effect is better seen in the inclination graphic. In figure 5.5, it is important to note that the inclination does not reach the first target inclination 90 degrees before the second definition of target inclination to zero degrees at 350 ft of MD.


Figure 5. 6: MD x DLS in a well without the effect of natural displacement


Figure 5. 7: MD x Offset $_{\text {Normal }}$ in a well without the effect of natural displacement


Figure 5. 8: Force on the bit caused by an offset x MD in a well without the effect of natural displacement


Figure 5.9: Force on the bit caused by the natural displacement $x \mathrm{MD}$ in a well without the effect of natural displacement


Figure 5. 10: Resultant Force on the bit caused by the natural displacement x MD in a well without the effect of natural displacement

Analyzing inclination, its behavior follows the offset behavior. It has a positive value whenever the offset is positive and a decreasing value on negative values of offset. This behavior follows the model established in the previous chapter. Regarding the DLS, it is possible to verify a smooth line without noise in the figure 5.6. As the true DLS, based on the Volve data [26], published by Equinor under Creative Commons (CC BY-NC-SA 4.0) license, has a considerable amount of noise, some noises on the calculated DLS would be expected in real life. The interesting point is that the DLS stays on the safety value under 18 degrees per 100 ft on the simulator in figure 5.6.


Figure 5. 11: MD x DLS in the Volve data
The last interesting point of the analysis is the forces on the bit. As explained in the previous chapter, the force on the bit Fbit (figure 5.10) will be composed by the force on the bit from the offset (figure 5.8) and the force on the bit from the natural displacement generated by the formation (figure 5.9). In the previous graphics, it is possible to notice that for this initial result,
the natural displacement is not being considered and further on the natural displacement will be added to this effect.

### 5.3 2D Modelling Considering Natural Displacement

The results of this section will implement the natural displacement effect. The expectations are an increase in the resultant force on the bit. The graphics take into account the force on the bit considering the natural displacement effect.


Figure 5. 12: TVD x Horizontal Displacement in a well with the effect of natural displacement


Figure 5. 13: Inclination $x$ MD in a well with the effect of natural displacement


Figure 5. 14: DLS x MD in a well with the effect of natural displacement


Figure 5. 15: Offset x MD in a well with the effect of natural displacement


Figure 5. 16: Resultant Force on the bit caused by the offset $x$ MD in a well with the effect of natural displacement


Figure 5. 17: Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement


Figure 5. 18: Resultant Force on the bit caused by the natural displacement and offset x MD in a well with the effect of natural displacement


Figure 5. 19: Zoomed -Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement


Figure 5. 204: Zoomed - Force on the bit caused by the natural displacement x MD in a well with the effect of natural displacement


Figure 5. 21: Zoomed - Resultant Force on the bit caused by the natural displacement and offset x MD in a well with the effect of natural displacement

The most interesting difference of this result is the analysis of the force on the bit (figure 5.21). Previously, there was no natural displacement force, now this behavior can be observed and analyzed. As can be seen from the graphics, the natural displacement force is delayed compared to the change of offset. The modelling works well since the results expose the impact of the curvature of the formation on the tool to portrait a more realistic simulation. In the graphics (figures 5.19, 5.20, and 5.21), it can be observed a logical behavior and the delayed smooth transition of force values from the natural displacement as expected.

### 5.4 Final 2D Model

In this last 2D case, the total time of drilling will be increased 10 times. This allows longer wells, where the target inclinations will be reached in time and their effects will not be overlapped by the lack of time to reach the target angle. In other words, this is the situation where the model works the best and fewer errors can be found.

For this simulation, the RSS system has received the following orders:

1) After 100 ft MD , the target inclination will be 50 degrees;
2) After 2000 ft MD , the target inclination will be 90 degrees;
3) After 4000 ft MD , the target inclination will be 110 degrees;


Figure 5. 225: TVD x Horizontal Displacement in a well with the effect of natural displacement - Long well


Figure 5. 236: Inclination $x$ MD in a well with the effect of natural displacement - Long well


Figure 5. 24: DLS x MD in a well with the effect of natural displacement - Long well

## 

Figure 5. 25: Offset x MD in a well with the effect of natural displacement - Long well


Figure 5. 26: Force on the bit caused by the offset x MD in a well with the effect of natural displacement - Long well


Figure 5. 27: Force on the bit caused by the natural displacement $x \mathrm{MD}$ in a well with the effect of natural displacement Long well


Figure 5. 28: Resultant Force on the bit caused by the natural displacement and offset $x$ MD in a well with the effect of natural displacement - Long well

In this last 2D scenario, it is possible to notice a smooth well trajectory following the desired inclination for each measured depth. It includes the offset controller and the calculation of natural displacement in a long well. It can be observed that all the inclinations are achieved, and no errors can be found.

### 5.5 Final 3D Model

A similar scenario was chosen for the 3D modelling. For this simulation, the RSS system has received the following orders:

1) After 1000 ft MD:

- The target azimuth will be 120 degrees;
- The target inclination will be 25 degrees;

2) After 2000 ft MD:

- The target azimuth will be 90 degrees;
- The target inclination will be 120 degrees;

3) After 4000 ft MD :

- The target azimuth will be 50 degrees;
- The target inclination will be 80 degrees;


Figure 5. 29: Horizontal displacement $x$ true vertical depth


Figure 5. 30: North coordinates x East coordinates


Figure 5. 31: DLS x MD


Figure 5. 32: North coordinates x East coordinates x TVD coordinates

## 6 Result Errors

### 6.1 Data Issues

The input variables must be accurate and reliable. If an input variable is set to be a constant even though a variation of this variable is expected, the output from the system carries this error. As the focus of this thesis is the development of an RSS system, the inputs of the ROP equations are not evaluated or developed in this work. The model depends on accurate inputs to provide reliable outputs. The simulator structure is open to receive functions instead of constant values and development in this area can be realized.

### 6.2 Model Uncertainties

This section of the master thesis focuses on analyzing the data uncertainties in the mathematical model. The model is based on an ROP model that has its deviations from reality, so the ROP uncertainties can be transmitted to the RSS modelling. The considered ROP model is not affected by the depth and it is often found that models with this characteristic are less reliable [19].

The derivation of the ROP compositions is not validated. This factor can generate errors compared to the actual behavior of RSS tools. The goal was to fit a ROP model that could model the conditions of the formation, the bending resultant force on the bit, and the offset controller so that the approximations presented in this thesis achieve its goal, but validations are necessary.

The ROP model needs to be further studied, as mentioned in the ROP model topic. The specific energy of the formation is an important parameter that influences the result. An interactive proposal, as suggested in this thesis, should generate errors on transitional formations that hold different ground reactions. Another point of the ROP model is the alfa calculation, as mentioned before. The bit steerability calculation (alfa on the ROP model) limits the value of the dogleg severity to assure real measures. The ideal would be a calculation of the alfa also depending on the $R O P_{\text {Axial }}$.

The survey equations that define measured depth, true vertical depth, and horizontal displacement consider the following logic: displacement is equal to velocity times time ( $\Delta S=$ $\Delta V * \Delta T$ ). This is a fundamental equation in physics, but it might be too simple for the studied variables. Expansion of the drill string due to thermal conditions is an example that would impact the calculated displacements like measured depth and true vertical depth which are not considered in the survey equations presented in this thesis.

### 6.3 Errors

Regarding general errors, the peaks/spikes in some graphics are the main errors in the calculations. In a situation where a constant value of the coordinates that are used to calculate the slope of the curve in the natural displacement $(H)$ calculation, some spikes and errors can be encountered. This error is caused by limitations in the calculation of the natural displacement which generates unreal values of $H$. This affects the $F_{b i t_{H}}\left(F b i t_{H}{ }^{\prime}\right.$ and $F b i t_{H}^{\prime \prime}$ on 3D) and respectively impacting the total force on the bit Fbit (Fbit' and Fbit' on 3D). The unreal total force on the bit is reflected in an error in the $R O P_{\text {Normal }}$ and it causes an error in the trajectory calculation. It was observed that the offset controller is activated in these cases and corrects the false trajectory. On tests realized, a well path of 5000 ft on measured depth, the error on the location of the bit encountered was approximately 1 foot. Given a real RSS system, this error is acceptable [4].

Another error in the exposed results is the gravity effect. The system does not consider the effect of gravity. The scenario for improving the model in this aspect is on the calculation of beam bending. The gravity affects the beam bending scheme and interferes with the resultant force on the bit. Including the weight of the tool as a dispersed force across the beam bending calculation is probably a good improvement and develops the model to consider gravity depending on the tool position.

The inclination defined in this thesis considers the position on the bit. As the formula of inclination depends on the forces present on the bit, the output of this variable considers the bit location. Typically, the models and sensors that measure inclination are behind of the bit on the BHA. This could cause some divergence if the model developed in this thesis is compared with downhole sensors of inclination. It is assumed that the inclination formula developed on this thesis can be interpreted as a prevision of the inclination on the BHA, therefore, the BHA will follow the well path of the bit and at some point will have the same inclination as the inclination calculated by the model developed in this thesis. This is a hypothesis, as such a comparison was not realized.

### 6.4 RSS Challenges and Improvements

The offset controller can be optimized. Variations in the offset of the proposed model follow the time between two defined time steps. In other words, the offset can be changed from a time T to a time $\mathrm{T}+1$. The difference between these two times is called delta T with a recommended value of 5 seconds. The actual activation time of a tool depends on the mechanics of the tool, the current value of $R O P_{A x i a l}$ and it should be independent of the delta T time. On the developed simulator, the activation time of the offset takes a delta T duration and there is no evaluation regarding values of $R O P_{\text {Axial }}$. The actual tool defines its activation duration as such to protect the tool from undesired and dangerous forces from its system. Undesired forces on the system can be found if there is an abrupt change of offset in high values of $R O P_{\text {Axial }}$.

The beam bending logic might be further developed. The contact area from RSS systems to the formation around the wellbore is not considered in the simple formulas used for beam bending. The study of how the force generated by the RSS system spreads over the area of the offset needs to be studied. Another possible improvement is to consider the geometry of the upper stabilizer and the gauge of the bit geometry. These factors were not considered in the exposed program and for the sake of simplicity it is assumed that they have the same diameter of the bit.

## 7 Conclusion

The main goal of this thesis is to share knowledge about RSS systems, to develop a mathematical model, and to program a simulator prototype. In this work new concepts were developed, e.g. using beam bending to evaluate the forces on the bit, defining components of ROPs, defining and calculating the natural displacement caused by the formation curvature on the bending of the tool, developing original calculations inclination, azimuth and positional coordinates of the bit and defining an offset controller behavior. Such definitions are originals, enrich the knowledge from the RSS systems, and open doors for future projects. The development of the above-mentioned original concepts was supervised by Canrig Drilling Technology Norway. The company was always available in case of doubts and helped to increase the accuracy of the mathematical model and further simulator. The simulator developed is also adaptable for different RSS tools. Furthermore, the simulator can foresee the bit location and the forces on the bit at any stage of drilling on a 2 D and 3D coordinates plane.

The work developed represents a clear enrichment for RSS knowledge. Original concepts established on this thesis allow further comprehension about RSS tools, its functionalities, and the drilling process. The original concepts developed by this thesis allowed the development of a simulator of high potential value for the industry. The simulator developed allows to identify how mechanical changes affect the drilling performance. This can be used by RSS designing engineers to evaluate the impact of possible changes in tools under planning or construction. With the mathematical model and its simulator, it is possible to simulate changes on the geometry of an RSS tool and evaluate its behavior and performance before building the actual tool.

The project had access to mechanical design details necessary for detailed analysis, testing logs to evaluate real-life performance and design of the control system of RSS tools. A great amount of initiative and commitment was needed to deliver a high-quality project. According to Canrig professionals, the results achieved show accurate behavior and theoretically fit the traditional performance of RSS systems in the industry. Certainly, improvements are possible, e.g. developing a more complex beam bending calculation, improving the duration of the offset controller activation, and considering additional effects on the inclination, azimuth, and positional coordinates equations. These possible improvements are promising avenues for future research.

Some concepts of the mathematical model still need to be validated, but the main goal of defining a mathematical model for an RSS tool was achieved. The work realized in this thesis is public to inspire further developments regarding directional drilling.

## 8 References

[1] Farah, Omar Farah, "Directional well design, trajectory and survey calculations, with a case study in Fiale, Asal Rift, Djibouti", Geothermal Training Programme, United Nations University Iceland, 27, 2013.
[2] Verteuil, R. and McCourt, I., "Introduction to Directional Drilling", Sugar Land learning center, Texas, EUA, 2001
[3] Warren, T. M., "Steerable Motors Hold Their Own Against Rotary Steerable Systems", presented at the SPE Annual Technical Conference and Exhibition, San Antonio, Texas, U.S.A., 2006, doi:10.2118/104268-MS.
[4] Rønnau, H.-H., Balslev, P. V., Ruszka, J., Clemmensen, C., Kallevig, S., Grosspietsch, R., \& Mader, G., "Integration of a Performance Drilling Motor and a Rotary Steerable System Combines Benefits of Both Drilling Methods and Extends Drilling Envelopes", presented at the IADC/SPE Drilling Conference, Amsterdam, The Netherlands, 2005, doi:10.2118/91810-MS.
[5] Probert, T., presentation at IADD Rotary Stearable Forum, Galveston, Texas, U.S.A., 2006
[6] Alvord, C. Noel, B. Galiunas, L. Johnson, V. Handley, R. Holtzman, K. Pulley, S. Dennis, J. and Smith, L., "RSS Application from Onshore Extended-Reach-Development Wells Shows Higher Offshore Potential", presented at the Offshore Technology Conference, Houston, Texas, U.S.A, 2007, doi:10.4043/18975-MS
[7] Saeverhagen, E. Erichsen, C. Nesheim, A. and Waldron, G., "3D RSS for 24" Section Sets Inclination Record of the Grane Field by Introduction of an Integrated Drilling and Surveying Bottomhole Assembly", presented at the SPE Annual Technical Conference and Exhibition, Colorado, Denver, U.S.A., 2008
[8] Baker Hughes, Autotrack rotary system, 2020, Available: https://www.bhge.com/autotrak-rotary-steerable-systems. Accessed on: 27 May 2020
[9] Schaaf, S. Mallary, C. R. and Pafitis, D., "Point-the-Bit Rotary Steerable System: Theory and Field Results", presented at the SPE Annual Technical Conference and Exhibition, Dallas, Texas, U.S.A., 2000, doi:10.2118/63247-MS.
[10] Nabors, Rotary Steerable Systems - RSS Drilling System | Nabors, 2020. Available: https://www.nabors.com/node/2086. Accessed on: 27 May 2020.
[11] Cheatham, C. A. Shih, S. Churchwell, D. L. Woody, J. M. and Rodney, P. F., "Effects of Magnetic Interference on Directional Surveys in Horizontal Wells", presented at the IADC/SPE Drilling Conference, New Orleans, Louisiana, U.S.A, 1992, doi:10.2118/23852-MS
[12] Teale, R "The Concept of Specific Energy in Rock Drilling", J. Rock Mech Mining Sci., vol 2, no. 1, pp. 57-73, 1965. doi: 10.1016/0148-9062(65)90022-7.
[13] Marck, J. and Zalluhoglu, U., "On the Bifurcation Behavior of RSS Steering Capabilities", presented at the 53th US Rock Mechanics/Geomechanics Symposium, New York, New York, U.S.A, 2019
[14] Schlumberger, Schlumberger Glossary, 2020. Available: https://www.glossary.oilfield.slb.com/. Accessed on: 27 May 2020.
[15] Tipu, I. Alawadhi, E. Kumar, R. Amanov, B. Gebaly, M. \& Al-Hammadi, A. Shamlan, A. Danche, A. Ganda, S. Dua, R. and Hariri, N., "Bending Rules with High Build Rate RSS", presented at the Abu Dhabi International Petroleum Exhibition and Conference, Abu Dhabi, UAE, 2015, doi:10.2118/177898-MS.
[16] Jones, S. Sugiura, J. and Barton, S., "Results of extensive RSS testing with PDC bits". J. of Petroleum Technology, vol 60, no. 6, pp. 30-37, 2008. doi: 10.2118/0608-0030-JPT
[17] Lyons, W. Carter, T. and Lapeyrousse, N., Formulas and Calculations for Drilling, Production, and Workover, $3^{\text {rd }}$ ed. Oxford: Elsevier, 2012
[18] Oberg, E. Jones, F. Horton, H. and Ryffel, H, Eds, Machinery's Handbook, 27th ed. New York: Industrial Press, Inc, 2004
[19] Bourgoyne A., Millheim K., Chenevert M. and Young F., Applied Drilling Engineering, Richardson: Society of Petroleum Engineers, 1986
[20] Bataee, M., Kamyab M., and Ashena, R. "Investigation of Various ROP Models and Optimization of Drilling Parameters for PDC and Roller-cone Bits in Shadegan Oil Field", presented at the CPS/SPE International Oil and Gas Conference and Exhibition, Beijing, China, 2010
[21] Bingham, M.G., "How rock properties are related to drilling", Oil \& Gas J., vol 62, no.50, pp. 94-101, 1964
[22] Soares, C. and Gray, K, "Real-time predictive capabilities of analytical and machine learning rate of penetration (ROP) models", J. of Petroleum Science and Engineering, vol 172, pp. 934-959, 2019. doi: 10.1016/j.petrol.2018.08.083
[23] Celada, B. Galera, J. Munoz, C. and Tardaguila, I. "The Use Of The Specific Drilling Energy For Rock Mass Characterisation and Tbm Driving During Tunnel Construction", presented at the ITA-AITES World Tunnel Congress, Madrid, Spain, 2009
[24] Pessier, R. and Fear, M, "Quantifying Common Drilling Problems with Mechanical Specific Energy and a Bit-Specific Coefficient of Sliding Friction", presented at the $67^{\text {th }}$ SPE Annual Technical Conference and Exhibition, Washington, DC, U.S.A, 1992
[25] Almalikee, H. "Predicting Rock Mechanical Properties from Wireline Logs in Rumaila Oilfield, Southern Iraq", American Journal of Geophysics, Geochemistry and Geosystems, vol 5, no.2, pp. 69-77, 2019
[26] Equinor, Volve field data set download, 2020. Available: https://www.equinor.com/en/how-and-why/digitalisation-in-our-dna/volve-field-data-villagedownload.html. Accessed on: 02 February 2020.

## Appendixes

## Appendix 1. Functions

| ROP Normal | ROP Axial | Geometry (quatorzebis) | Offsetfuntction |
| :---: | :---: | :---: | :---: |
| - Offset [ \% ] <br> - RPM [Rev. per minutes ] <br> - D [ inches ] <br> - mi [-] <br> - Es [psi] <br> - Id [ mm ] <br> - Od [mm] <br> - TVD and HD (actuator) [ ft ] <br> - TVD \& HD (upper stabilizer) [ft] <br> - TVD and HD of the bit [ ft ] <br> - Output <br> - ROP Normal [ ft/h ] <br> - Force natural Disp. [ N ] <br> - Force Offset [ N ] <br> - Total force on the bit [ N ] | - Input <br> - RPM [Rev. per minutes ] <br> - WOB [ kkgf ] <br> - D [ inches ] <br> - mi [-] <br> - Es [ psi ] <br> - Output <br> - Rop Axial [ ft/h ] | - Input <br> - TVD and HD (actuator) [ ft ] <br> - TVD \& HD (upper stabilizer) [ft] <br> - TVD and HD of the bit [ ft ] <br> - Output <br> - H [m] <br> - Slope of upper stabilizer-bit line <br> - Slope of actuator-bit line | - Input <br> - Target Inclination [degrees] <br> - Maximum offset [ \% ] <br> - Output <br> - Offset [ \% ] |

## Appendix 2. Program process




## Appendix 3. Three Dimension Program

Also available in https://github.com/caetanosaramago/RSS

```
import numpy as np
import matplotlib.pyplot as plt
import circle_fit as cf
import pandas as pd \#import pandas
import seaborn as sns \#import seaborn
from numpy import ones,vstack
from numpy.linalg import lstsq
import circle_fit as cf
In [25]:
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# ROP AXIAL [ft/hour]
def ROP_axial(RPM,WOB,D,mi,Es):\#\# units RPM - [Rotations per minute] / WOB
- [kkgf] / D - [inch] / Es - [specific energy, psi]
    \(\mathrm{Ab}=\mathrm{np} \cdot \mathrm{pi}^{*}\left((\mathrm{D})^{* *} 2\right) / 4\) \#\# Area calculation based on diameter
    return \(13.33 * \mathrm{mi}^{*} \mathrm{RPM} /\left(\mathrm{D}^{*}\left(\mathrm{Es} /\left(\mathrm{WOB}^{*} 1000 * 2.20462\right)-1 / \mathrm{Ab}\right)\right)\)
```

\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# ROP Normal [ft/hour] \#\# NB temp1,temp2,temp3 calculated
by the geometry of the well offset calculated from 'offsetfunction' - Do no
$t$ change this values
def ROP_normal(offset, RPM, WOB, D, mi, Es, Od, Id,temp1,temp2,temp3):\#\# un
its RPM - [Rotations per minute] / WOB - [kkgf] / D - [inch] / Es - [specif
ic energy, psi]
Ab = np.pi*((D)**2)/4 \#\# Area calculation based on diameter
alpha = 1 \# constant to fit the DLS to adequate values
beta $=0.15$ \# constant to fit the DLS to adequate values
$\mathrm{E}=30000000^{*}\left(10^{* *}(9)\right) /\left(0.145^{*} 10^{* *} 6\right)$ \# Elasticity modulus, considering st
eel

Hoff = offset \# do not mind - offset calculated from the 'offset functi on is a percentage from $-100 \%$ until $+100 \%$ '
maxoffset=6*10**(-3) \# Max offset of the tool [6mm]

Hoffabs = Hoff*maxoffset \# calculation of the physical offset consideri ng the max offset and the percentage value

OD = (Od*10**(-3)) \# from mm to meter
ID $=\left(\operatorname{Id} * 10^{* *}(-3)\right)$ \# from $m m$ to meter

```
    H = Geometryfunction(temp1,temp2,temp3) [0]# calculation of the natural
displacement
    mlong=Geometryfunction(temp1,temp2,temp3)[2]
    mshort=Geometryfunction(temp1,temp2,temp3)[1]
```

    if np.isnan(H):
        h = 0
    else:
        \(h=H\)
    I_natural \(=n p . \mathrm{pi}^{*}\left(\left(0 \mathrm{D}^{* *} 4-\mathrm{ID}^{* *} 4\right)\right) / 32\) \# inertia modulus
    I_OFF = np.pi*((OD**4-ID**4))/32 \# inertia modulus
    D1=((2.700)**2-Hoffabs**2)**(1/2) \# correction of the horizontal distan
    ce on the beam bending scheme
D2=((0.500)**2-Hoffabs**2)**(1/2) \# correction of the horizontal distan
ce on the beam bending scheme
\#print(h)
Fbit_Natural_abs = (h*3*E*I_OFF)/(2.7*0.5**2) \# calculation of the
force on the bit generated by the natural displacement
Fbit_OFF = (Hoffabs*3*E*I_OFF)/(D1*D2**2) \# calculation of the force o
$n$ the bit generated by the RSS offset
if (np.arctan(mlong)>np.arctan(mshort)): \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Calculates the sign of the the force from the natural disp lacement

Fbit_Natural= Fbit_Natural_abs
elif (np.arctan(mlong)<np.arctan(mshort)):
Fbit_Natural = -1*Fbit_Natural_abs
else:
Fbit_Natural = 0
FbitTOTAL = Fbit_OFF+Fbit_Natural
if (FbitTOTAL==0): \# Use to the ROP model from the SPE 24584 and the 1 oop avois errors when Fbit total is equal to zero

ROP_perpendicular $=0$
elif (FbitTOTAL>0):
ROP_perpendicular $=\left(\left(13.33 *\right.\right.$ mi*RPM*beta) $/\left(D^{*}(((E s * a l p h a) /(0.220462\right.$
*FbitTOTAL))-1/Ab)))
else:
ROP_perpendicular = ((13.33*mi*RPM*beta)/(D*(((Es*alpha)/(0.220462 *FbitTOTAL))+1/Ab)))
return ROP_perpendicular
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Calculation of H
def Geometryfunction(temp1,temp2,temp3):\# calculation of H
temp=np.concatenate((temp1,temp2), axis=1) \#tranforming the two points in one array - 3 meters behind
temp=np.rot90(temp,2).T \# rotating the array
x_coords, y_coords = zip(*temp) \#calculating the $m$ and $c$
$A=\operatorname{vstack}\left(\left[x \_c o o r d s\right.\right.$, ones $\left.\left.\left(l e n\left(x \_c o o r d s\right)\right)\right]\right) . T$ \#calculating the $m$ and $c$
$m, c=\operatorname{lstsq}\left(A, y \_c o o r d s\right)[0]$ \#calculating the $m$ and $c$
coef1 = m, $\quad$ \# calling a variable to define $m$ and $C$ [line is equal to $y=$ m*x+c]
ca=temp[0] \# ca is the evaluated point
$x o=c a[0]$ \# xo is collecting the $x$ value of the analyzed point
yo=ca[1] \# yo is collecting the $y$ value of the analyzed point
$m o=c o e f 1[0]$ \#mo is equal to $m$ of the created line
co=coef1[1] \#co is equal to the $c$ of the created line
delta $=(-2 * x o+2 *$ co*mo- $2 *$ mo*yo $) * * 2-4 *(m o * * 2+1) *(x o * * 2+c o * * 2-2 *$ yo*co + yo**2 $-0.25 * 3.28 * 3.28) \#$ calculating the delta of the bhaskara formula (sqrt( X po int on the bit - $X$ point 50 cm behind from the bit) ${ }^{* *} 2+(Y$ point on the bi $\mathrm{t}-\mathrm{Y}$ point 50 cm behind from the bit) ${ }^{* *} 2$ )) $=0.5$ meters)
\# on the created line, we are going 50 cm back because this is the dist ance from the bit to the RSS system

Xp=(-1* (-2*xo $2^{*}$ co*mo-2*mo*yo)-delta**0.5)/(2*(mo**2+1))\# calculating $X$ p 50cm behind ( the sign - is used, as this point is behind the bit)

Yp=mo*Xp+co \# calculating Yp 50cm behind
temp01=np.concatenate((temp3, temp2), axis=1) \# doing the same process for 50 cm behind the bit
temp01=np.rot90(temp01,2).T \#tranforming the two points in one array
x_coords1, y_coords1 = zip(*temp01) \#calculating the m and c
A1 = vstack([x_coords1,ones(len(x_coords1))]).T \#calculating the $m$ and C

```
    m1, c1 = lstsq(A1, y_coords1)[0] #calculating the m and c
    coef2 = m1,c1 # calling a variable to define m and C [line is equal to
y=m*x+c]
    ca1=temp01[0] # ca1 is the evaluated point, ca1=ca
    xo1=ca1[0] # xo1 is collecting the x value of the analyzed point, xo=xo
1
    yo1=ca1[1] # yo1 is collecting the y value of the analyzed point, yo=yo
1
    mo1=coef2[0] #mo1 is equal to m of the created line, mo is different to
mo1
    co1=coef2[1] #co1 is equal to c of the created line, co is different to
co1
    delta1=(-2*xo1+2*co1*mo1-2*mo1*yo1)**2-4*(mo1**2+1)*(xo1**2+co1**2-2*yo
1*co1+yo1**2-0.25*3.28*3.28)# calculating the delta of the bhaskara formula
    (sqrt((X point on the bit - X point 50 cm behind from the bit)**2 + (Y poi
nt on the bit - Y point 50 cm behind from the bit)**2)) = 0.5meters)
Xp1=(-1*(-2*xo1+2*co1*mo1-2*mo1*yo1)-delta1**0.5)/(2*(mo1**2+1))\# calcu lating Xp 50 cm behind (just the sign - is used, as this point is behind the bit)
Yp1=mo1*Xp1+co1 \# calculating Yp 50cm behind
\(H=(((n p . a b s(X p 1)-n p . a b s(X p)) * * 2+(n p . a b s(Y p 1)-n p . a b s(Y p)) * * 2) * * 0.5) * 0.30\) 48\#calculating the distance between the two points 50 cm behind of the bit
```

return $H, m o 1, m o$ \# in meter
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# CONTROLER Calculation ( to easier understa nding in advice to see the 'caetano' calculation on the end of the code)
def offsetfunction(Azimuth_target,target_inclination,maxoff):
dazi = np.abs(np.abs(Azimuth_target)*np.pi/180 - np.abs(log[t,3])) \# di stance between the desired azimuth and the azimuth where the bit is in a de termined time t
dinc $=n p . a b s\left(n p . a b s\left(t a r g e t \_i n c l i n a t i o n\right) * n p . p i / 180-n p . a b s(\log [t, 2])\right)$
\# distance between the desired inclination and the inclination where the bi
$t$ is in a determined time $t$
\#\# first we will calculate the need of offset on the azimuth and inclinat ion
if (np.abs(dinc)>0.65*np.pi/180): \# if the difference between the targe $t$ inclination and the moment inclination is greater than 0,65 , the offset $r$ egarding inclination should be $100 \%$ offset_dinc = +1*maxoff
elif (np.abs(dinc)<0.65*np.pi/180): \# if the difference between the tar get inclination and the moment inclination is less than 0,65 , the offset re garding inclination should be $1.5 *$ the difference offset_dinc $=1 * n p . a b s(d i n c) * 1.5 *$ maxoff
else:
offset_dinc=0
if (np.abs(dazi)>0.65*np.pi/180): \# if the difference between the targe $t$ azimuth and the moment azimuth is greater than 0,65 , the offset regarding azimuth should be 100\%
offset_dazi = +1*maxoff
elif (np.abs(dazi)<0.65*np.pi/180): \# if the difference between the tar get azimuth and the moment azimuth is less than 0,65 , the offset regarding azimuth should be $1.5^{*}$ the difference

```
        offset_dazi = 1*np.abs(dazi)*1.5*maxoff
```

else:
offset_dazi=0
\#\# Now, we calculate the angle between the offset azimuth and the resultan t azimuth
if offset_dazi==0: \#\# the loop avoids the division by zero value of offset azimuth

TF=90*np.pi/180
else:
TF=np.abs(np.arctan(np.abs(offset_dinc/offset_dazi))) \#\# based on t he need of offset, we will have a different angle

## \#\#\#\#\#\#\#\#\# OFFSET CALCULATION

\#\# now we calculate the intensity of the resultant offset
if (np.abs(dazi)>=0.65*np.pi/180): \# it is the same way of thinking, if any of the both differences are above 0,65 degrees, the offset will be equ al to one
offset $=+1 *$ maxoff

```
    elif (np.abs(dinc)>=0.65*np.pi/180):
```

    offset = +1*maxoff
    elif (np.abs(dazi)<0.65*np.pi/180) and (np.abs(dinc)<0.65*np.pi/180): \# \#otherwise, the offset will be the sum of the difference divided by two offset $=1 * n p . a b s(d a z i * 1.5+d i n c * 1.5) / 2 * 1 *$ maxoff
\#\#\# based on the angle calculated, we will calculate the resultant offset_ inclination and offset_azimuth
if (Azimuth_target>log[t,3]*180/np.pi): \#\# on this last part we will ca lculate the sign of the offset
offset_azi=offset*np.cos(TF)\#\# If the azimuth that is being analyze d is lower than the target azimuth, the offset will be positive
elif(Azimuth_target<log[t,3]*180/np.pi):\#\# if it is lower, the offset w ill be lower
offset_azi=-1*offset*np.cos(TF)
else:
offset_azi=0
if (target_inclination>log[t,2]*180/np.pi): \#\# the way of thinking is s
imilar to the previous azimuth
offset_inc=offset*np.sin(TF)
elif(target_inclination<log[t, 2]*180/np.pi):
offset_inc=-1*offset*np.sin(TF)
else:
offset_inc=0
return offset,offset_inc,offset_azi,TF,offset_dazi
In [25]:
\#\# defining for how long we will drill and delta $T$, the $T$ is the number of timesteps and the delta 5 is how many seconds a time step values

```
t_max = 100000
t100 = t_max/100
for i in range(100):
    print ("_", end="")
deltaT = 5
######################## Constant data defining variables for the ROP model
offset = 0# initial values required by python
offset_inc = 0# initial values required by python
offset_azi= 0 # initial values required by python
mi=0.23
RPM = 143.44375 # rotations per second - units defined on the ROP function
```

```
WOB = 7.124417 # kkgf
DLS=0
D = 12.25 # inches
ODout = 190 # mm
IDout = 175 # mm
Od = 80 # mm
Id = 37 # mm
Es = 14633.401276 # psi
```

\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# target points
\# defined target points. Notice that the initial azimuth uses the first tar get azimuth point, as it is possible to start the drilling aiming the corre ct azimuth and it is not needed to increase its value from zero as it is do ne with the inclination
\#First target points
TargetAzi=120
TargetInc=25
\#Second target points
TargetAzi2=360
TargetInc2=120
\#Third target points
TargetAzi3=180
TargetInc3=90
\#\# it is possible to add target points
\#\# defining the initial values
$t=1$
log $=n p \cdot z e r o s\left(\left(t \_m a x, 12\right)\right)$
\#\# log values and representation of the physical variables:
\#0: MD
\#1: 0 ( do not care about this variable)
\#2: inc
\#3: Azimuth
\#4: Horizontal displacement
\#5: TVD
\#6: East
\#7: North
\#8: offset
\#9: DLS
\#10: offset inc
\#11: offset azi
current_depth $=0$
while current_depth < 10+1:
$\log [t, 0]=\log [t-1,0]+$ ROP_axial(RPM, WOB, $D, m i, E s) * d e l t a T / 3600 \#$ as we ar e drilling vertically the axiāl ROP will just be composed by the axial ROP
$\log [t, 1]=0$ \#(do not care about this variable)
$\log [t, 2]=0$ \# Inclination is zero on the initial point
$\log [t, 3]=$ TargetAzi*np.pi/180 \# Azimuth is inexistent when the well is vertical, but for calculation matter here we will start with the target az imuth value
$\log [t, 4]=0$ \# horizontal displacement is zero, because the well is ver tical
$\log [t, 5]=\log [t, 0] \#$ when we are vertical, the TVD is equal to the MD
$\log [t, 6]=0$ \# no East variation, Inexistent Azimuth
$\log [t, 7]=0$ \# no North variation, Inexistent Azimuth
$\log [t, 8]=$ offset \# defined as zero previously, no offset in vertical w ells
$\log [t, 9]=$ DLS \# defined as zero previously, no DSL in vertical wells
$\log [t, 10]=$ offset_inc \# defined as zero previously, no offset on the i nclination direction in vertical wells
$\log [t, 11]=$ offset_azi \# defined as zero previously, no offset in azim uth direction in vertical wells

```
current_depth \(=\log [t, 0]\)
\#print(current_depth)
t += 1
```

\#\# this is the calculation of the geometry behind the bit while t < t max:
$j=t-2$
while True:
distance $=\log [t-1,0]-\log [j, 0]$ \# uses the Measured depth differen ce
if distance >= 10: \# for every point analyzed, we search which poin $t$ is in 3 meters behind and collect the coordinates break
else:
$j=j-1$
temp_data1 $=[[\log [j, 4], \log [j, 5]]] \#$ coordinates of approx.. 3 behind of the point regarding TVD x Horizontal displacement
temp_data $2=[[\log [t-1,4], \log [t-1,5]]] \#$ coordinates of the point TVD $x$

```
Horizontal displacement
    temp_data1 = np.rot90(temp_data1,1)
    temp_data2 = np.rot90(temp_data2,1) # do not mind... python logistics
    g = t-2
    while True:
        distance = log[t-1,0] - log[g,0] # uses the Measured depth differen
ce
        if distance >= 1.64:# for every point analyzed, we search which poi
nt is in 0,5 meters behind and collect the coordinates
            break
        else:
            g = g-1
    temp_data3 = [[log[g,4],log[g,5]]] # coordinates of 0.5 behind of the p
oint regarding TVD x Horizontal displacement
    temp_data4 = [[log[t-1,4],log[t-1,5]]] # coordinates of the point regar
ding TVD x Horizontal displacement
    temp_data3 = np.rot90(temp_data3,1)
    temp_data4 = np.rot90(temp_data4,1)
    l = t-2
    while True:
        distance = log[t-1,0] - log[l,0]# uses the Measured depth differenc
e
        if distance >= 10:# coordinates of approx.. 3 behind of the point r
egarding North x East coordinates
                    break
            else:
                    l = 1-1
    temp_data5 = [[log[1,6],log[1,7]]]# coordinates of approx.. 3 behind of
    the point
    temp_data6 = [[log[t-1,6],log[t-1,7]]]# coordinates of the point regard
ing North x East coordinates
    temp_data5 = np.rot90(temp_data5,1)
    temp_data6 = np.rot90(temp_data6,1)
    m=t-2
    while True:
        distance = log[t-1,0] - log[m,0]# uses the Measured depth differenc
e
            if distance >= 1.64:# for every point analyzed, we search which poi
nt is in 0,5 meters behind and collect the coordinates North x East
                    break
        else:
            m = m-1
```

temp_data7 $=[[\log [m, 6], \log [m, 7]]] \#$ coordinates of 0.5 behind of the po int
temp_data8 $=[[\log [t-1,6], \log [t-1,7]]] \#$ coordinates of the point regard ing North x East coordinates
temp_data7 $=$ np. $\operatorname{rot90(temp\_ data7,1)~}$
temp_data8 = np.rot90(temp_data8,1)

ROP_n_inc = ROP_normal(offset_inc, RPM, WOB, D, mi, Es, Od, Id,temp_dat a1, temp_data2,temp_data3) \# calculation of ROP's base on the previous funct ions

ROP_n_azi = ROP_normal(offset_azi, RPM, WOB, D, mi, Es, Od, Id,temp_dat a5, temp_data6,temp_data7)

ROP_a = ROP_axial(RPM,WOB,D,mi,Es)
\#Reminder of the meaning of the logs.... Now we will calculate its variatio ns since we are not anymore on the initial points
\#0: MD
\#1: 0 ( do not care about this variable)
\#2: inc
\#3: Azimuth
\#4: Horizontal displacement
\#5: TVD
\#6: East
\#7: North
\#8: offset
\#9: DLS
\#10: offset inc
\#11: offset azi
$\log [t, 0]=\log [t-1,0]+n p . s q r t\left(R O P \_n \_i n c * * 2+R O P \_n \_a z i * * 2+R O P \_a * * 2\right) *$ deltaT/3600 \# The MD considers the resultant ROP from the 3 existing ROPs
$\log [t, 1]=0 \#$ do not mind this one
 w inclination will be the previous inclination plus the increase caused by the ROP Normal generated for the offset_inc
$\log [t, 3]=\log [t-1,3]+n p . a r c t a n\left(R O P \_n \_a z i / R O P \_a\right) * d e l t a T / 3600 \#$ the new Azimuth will be the previous Azimuth plus the increase caused by the ROP N ormal generated for the offset_azi
$\log [t, 4]=\log [t-1,4]+n p . \sin (\log [t, 2]) * n p . s q r t\left(R O P \_n \_i n c * * 2+R O P \_a^{* *}\right.$ 2)*deltaT/3600 \# It uses the resultant of the ROP regarding inclination
$\log [t, 5]=\log [t-1,5]+n p \cdot \cos (\log [t, 2]) * n p . s q r t\left(R O P \_n \_i n c * * 2+R O P \_a * *\right.$ 2)*deltaT/3600 \# It uses the resultant of the ROP regarding inclination

```
log[t,6] = log[t-1,6] + np.sin(log[t,3])*(log[t,4]-log[t-1,4]) # the Ea
```

st variation is based on the horizontal displacement and azimuth (sin)
$\log [t, 7]=\log [t-1,7]+n p \cdot \cos (\log [t, 3]) *(\log [t, 4]-\log [t-1,4]) \#$ the Nor th variation is based on the horizontal displacement and azimuth (cos)
$\log [t, 9]=180 / n p \cdot p i * n p \cdot a b s(n p \cdot \arccos (n p \cdot \cos (\log [t, 2]) * n p \cdot \cos (\log [t-1$, $2])+n p \cdot \sin (\log [t, 2]) * n p \cdot \sin (\log [t-1,2]) * n p \cdot \cos (\log [t, 3]-\log [t-1,3]))) * 100 / n$ p.abs ((log[t,0]-log[t-1,0]))
$\log [t, 10]=$ offset_inc \# this offset is responsible to control the TVD X Horizontal displacement data
$\log [t, 11]=$ offset_azi $\#$ this offset is responsible to control the Nort h X East data
maxoff=1 \#\# definition of the max offset desired by the customer, there are some companies that do not wish to have big DLS, therefore, we must re duce the max offset here (values between 0 and 1)
\# first Target point kickoff, It must be noted that the values have been de fined before
if $\log [t, 0]>1000:$

```
function=offsetfunction(TargetAzi,TargetInc,maxoff)
offset = function[0]
offset_inc = function[1]
offset_azi= function[2]
```

\# Second Target point kickoff, It must be noted that the values have been $d$ efined before
if $\log [t, 0]>2000:$

```
function=offsetfunction(TargetAzi2,TargetInc2,maxoff)
offset = function[0]
offset_inc = function[1]
offset_azi= function[2]
```

\# Third Target point kickoff, note that the values have been defined before

```
if log[t,0] > 4000:
```

    function=offsetfunction(TargetAzi3, TargetInc3, maxoff)
    offset = function[0]
    offset_inc = function[1]
    offset_azi= function[2]
    ```
#print (offset)
```

$\mathrm{t}=\mathrm{t}+1$

```
if t%t100 == 0:
    print ('.', end="")
```

In [26]:
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# 2d Plotting
\#1 TVD x Horizontal Displacement
\#2 North X East
\#3 DLS and Measured depth

```
plt.figure(figsize=(10,10))
plt.plot(log[:,4],log[:,5])
xy0 = np.min( [ np.min(log[:,4]) , np.min(log[:,5]) ] ) - 100
xy1 = np.max( [ np.max(log[:,4]) , np.max(log[:,5]) ] ) + 100
plt.xlim((xy0,xy1))
plt.ylim((xy0,xy1))
plt.xlabel('Horizontal displacement')
plt.ylabel('TVD')
plt.show()
```

plt.figure(figsize=(10,10))
plt.plot(log[:,6],log[:,7])
xy0 $=n p \cdot \min ([\operatorname{np} \cdot \min (\log [:, 6])$, np.min(log[:,7]) ] ) - 100
$x y 1=n p \cdot \max ([n p \cdot \max (\log [:, 6]), \quad n p \cdot \max (\log [:, 7])])+100$
plt.xlim((xy0,xy1))
plt.ylim((xy0,xy1))
plt.xlabel('East')
plt.ylabel('North')
plt.show()

```
plt.figure(figsize=(10,10))
plt.plot(log[:,0],log[:,9])
plt.xlabel('MD ft')
plt.ylabel('Dogleg Severity')
plt.show()
plt.plot(log[:,0])
```


## Appendix 4. 2D Program

Also available in https://github.com/caetanosaramago/RSS

```
import numpy as np
import matplotlib.pyplot as plt
import circle_fit as cf
import pandas as pd #import pandas
import seaborn as sns #import seaborn
from numpy import ones,vstack
from numpy.linalg import lstsq
import circle_fit as cf
In [61]:
################## ROP AXIAL [ft/hour]
def ROP_axial(RPM,WOB,D,mi,Es):## units RPM - [Rotations per minute] / WOB
- [kkgf] / D - [inch] / Es - [specific energy, psi]
    Ab = np.pi*((D)**2)/4 ## Area calculation based on diameter
    return 13.33*mi*RPM/(D*(Es/(WOB*1000*2.20462)-1/Ab))
```

\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# ROP Normal [ft/hour] \#\# NB temp1,temp2,temp3 calculated by the geometry of the well offset calculated from 'offsetfunction' - Do no $t$ change this values
def ROP_normal(offset, RPM, WOB, D, mi, Es, Od, Id,temp1,temp2,temp3):\#\# un its RPM - [Rotations per minute] / WOB - [kkgf] / D - [inch] / Es - [specif ic energy, psi]
$\mathrm{Ab}=\mathrm{np} \cdot \mathrm{pi} *\left((\mathrm{D})^{* *} 2\right) / 4$ \#\# Area calculation based on diameter
alpha $=1$ \# constant to fit the DLS to adequate values
beta $=0.15$ \# constant to fit the DLS to adequate values, it fits the $r$ eality - Lateral cutting is usually $30 \%$ of axial cutting

```
    E=30000000*(10**(9))/(0.145*10**6) # Elasticity modulus, considering st
eel
```

Hoff = offset \# do not mind - offset calculated from the 'offset functi on is a percentage from $-100 \%$ until $+100 \%$ '
maxoffset=6*10**(-3) \# Max offset of the tool [6mm]
Hoffabs = Hoff*maxoffset \# calculation of the physical offset consideri ng the max offset and the percentage value
$O D=\left(0 d * 10^{* *}(-3)\right)$ \# from $m m$ to meter
ID $=\left(\operatorname{Id} * 10^{* *}(-3)\right) \#$ from $m m$ to meter

H = Geometryfunction(temp1, temp2, temp3)[0]
mlong=Geometryfunction(temp1, temp2,temp3)[2] \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Calculates the natural displacement mshort=Geometryfunction(temp1,temp2, temp3)[1]

```
    if np.isnan(H):
```

        \(h=0\)
    else:
$h=n p . a b s(H)$

I_natural = np.pi*((OD**4-ID**4))/32 \# inertia modulus
I_OFF $=n p \cdot \mathrm{pi}^{*}\left(\left(0 D^{* *} 4-\mathrm{ID}^{* *} 4\right)\right) / 32$ \# inertia modulus
D1=((2.700)**2-Hoffabs**2)**(1/2) \# correction of the horizontal distan ce on the beam bending scheme

D2 $2=((0.500) * * 2$-Hoffabs**2)**(1/2) \# correction of the horizontal distan ce on the beam bending scheme
\#print(h)
Fbit_Natural_abs = (h*3*E*I_OFF)/(2.7*0.5**2) \# calculation of the force on the bit generated by the natural displacement

Fbit_OFF = (Hoffabs*3*E*I_OFF)/(D1*D2**2) \# calculation of the force o $n$ the bit generated by the RSS offset
if (np.arctan(mlong)>np.arctan(mshort)): \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Calculates the sign of the the force ffrom the natural dis placement

Fbit_Natural= Fbit_Natural_abs
elif (np.arctan(mlong)<np.arctan(mshort)):
Fbit_Natural = -1*Fbit_Natural_abs
else:
Fbit_Natural = 0
\#print (Fbit_Natural)

FbitTOTAL = Fbit_OFF+Fbit_Natural
if (FbitTOTAL==0): \# Use to the ROP model from the SPE 24584 and the 1 oop avois errors when Fbit total is equal to zero

ROP_perpendicular $=0$
elif (FbitTOTAL>0):
ROP_perpendicular $=\left((13.33 *\right.$ mi*RPM*beta $) /\left(D^{*}(((\right.$ Es*alpha $) /(0.220462$ *FbitTOTAL))-1/Ab)))
else:
ROP_perpendicular $=\left((13.33 *\right.$ mi*RPM*beta $) /\left(D^{*}(((\right.$ Es*alpha $) /(0.220462$ *FbitTOTAL))+1/Ab)))
return ROP_perpendicular,Fbit_OFF,Fbit_Natural,FbitTOTAL
\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# Calculation of H
def Geometryfunction(temp1,temp2,temp3):\# calculation of H
temp=np.concatenate((temp1,temp2), axis=1) \# transforming the two point s in one array - 3 meters behind
temp=np.rot90(temp,2).T \# rotating the array
x_coords, y_coords = zip(*temp) \#calculating the m and c
A = vstack([x_coords,ones(len(x_coords))]).T \#calculating the $m$ and $c$
$m, c=\operatorname{lstsq}\left(A, y \_c o o r d s\right)[0]$ \#calculating the $m$ and $c$
coef1 $=m, c$ \# calling a variable to define $m$ and $C$ [line is equal to $y=$ $\left.m^{*} x+c\right]$
ca=temp[0] \# ca is the evaluated point
$x o=c a[0] \#$ xo is collecting the $x$ value of the analyzed point
yo=ca[1] \# yo is collecting the $y$ value of the analyzed point
$m o=c o e f 1[0]$ \#mo is equal to $m$ of the created line
co=coef1[1] \#co is equal to the $c$ of the created line
delta $=(-2 * x o+2 *$ co*mo- $2 *$ mo*yo $) * * 2-4 *\left(\right.$ mo $\left.^{* *} 2+1\right) *\left(x_{0} * * 2+c o * * 2-2 *\right.$ yo*co + yo**2 $-0.25 * 3.28 * 3.28) \#$ calculating the delta of the bhaskara formula (sqrt( X po int on the bit - X point 50 cm behind from the bit) ${ }^{* * 2}+$ (Y point on the bi $\mathrm{t}-\mathrm{Y}$ point 50 cm behind from the bit)**2)) $=0.5 \mathrm{~meters}$ )
\# on the created line, we are going 50 cm back because this is the dist
ance from the bit to the RSS system

Xp=(-1* (-2*xo+2*co*mo-2*mo*yo)-delta**0.5)/(2*(mo**2+1))\# calculating $X$ p 50cm behind ( the sign - is used, as this point is behind the bit)

Yp=mo*Xp+co \# calculating Yp 50cm behind
temp01=np.concatenate((temp3, temp2), axis=1) \# doing the same process for 50 cm behind the bit
temp01=np.rot90(temp01,2).T \#tranforming the two points in one array
x_coords1, y_coords1 = zip(*temp01) \#calculating the m and c
A1 = vstack([x_coords1,ones(len(x_coords1))]).T \#calculating the $m$ and C
$m 1, c 1=1 s t s q\left(A 1, y \_c o o r d s 1\right)[0]$ \#calculating the $m$ and $c$
coef2 = m1,c1 \# calling a variable to define $m$ and $C$ [line is equal to $y=m * x+c]$
ca1=temp01[0] \# ca1 is the evaluated point, ca1=ca
xo1=ca1[0] \# xo1 is collecting the $x$ value of the analyzed point, xo=xo 1
yo1=ca1[1] \# yo1 is collecting the y value of the analyzed point, yo=yo 1
mo1=coef2[0] \#mo1 is equal to $m$ of the created line, mo is different to mo1
co1=coef2[1] \#co1 is equal to $c$ of the created line, co is different to co1
delta1 $=(-2 * x o 1+2 * \operatorname{co1} *$ mo1 $-2 *$ mo1*yo1 $) * * 2-4 *($ mo1**2+1)*(xo1**2+co1**2-2*yo $\left.1 * \operatorname{co1}+\mathrm{yo} 1^{* *} 2-0.25 * 3.28 * 3.28\right) \#$ calculating the delta of the bhaskara formula
(sqrt((X point on the bit - X point 50 cm behind from the bit)**2 + (Y poi nt on the bit $-Y$ point 50 cm behind from the bit)**2)) $=0.5$ meters)

Xp1 $=\left(-1^{*}(-2 *\right.$ xo1 $+2 *$ co1*mo1-2*mo1*yo1)-delta1**0.5)/(2*(mo1**2+1)) \# calcu lating Xp 50 cm behind (just the sign - is used, as this point is behind the bit)

Yp1=mo1*Xp1+co1 \# calculating Yp 50cm behind
$H=(((n p . a b s(X p 1)-n p . a b s(X p)) * * 2+(n p . a b s(Y p 1)-n p . a b s(Y p)) * * 2) * * 0.5) * 0.30$ 48\#calculating the distance between the two points 50 cm behind of the bit

```
return H,mo1,mo # in meter
```


## \#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# CONTROLER Calculation

```
def offsetfunction(target inclination,maxoff):
```

dinc $=$ (np.abs(target_inclination)*np.pi/180 - np.abs(log[t,2]))\# dista nce between the desired inclination and the inclination where the bit is in a determined time t
if (np.abs(dinc)>=0.65*np.pi/180): \# if the difference between the targ et inclination and the moment inclination is greater than 0,65 , the offset regarding inclination should be $100 \%$
if (target_inclination>log[t,2]*180/np.pi):
offset $=+1^{*}$ maxoff $\# \#$ If the inclination that is being analy zed is lower than the target inclination, the offset will be positive else: offset $=-1 * m a x o f f \quad \# \#$ If the inclination that is being analy zed is greater than the target inclination, the offset will be negative
elif (np.abs(dinc)<0.65*np.pi/180): \# if the difference between the tar get inclination and the moment inclination is greater than 0,65 , the offset regarding inclination should be 1,5* dinc if (target_inclination>log[t,2]*180/np.pi):
offset $=1 *$ np.abs (dinc) ${ }^{*} 1 *$ maxoff $\#$ the same sign logic from the
before
else:

```
                offset = -1*np.abs(dinc)*1*maxoff
```

            return offset
    In [70]:
\#\# defining for how long we will drill and delta $T$, the $T$ is the number of timesteps and the delta 5 is how many seconds a time step values

```
t_max = 100000
t100 = t_max/100
```

for i in range(100):
print ("_", end="")

```
deltaT = 5
######################## Constant data defining variables for the ROP model
offset = 0 # initial values required by python
mi=0.23
RPM = 143.44375 # rotations per second - units defined on the ROP function
WOB = 7.124417 # kkgf
DLS=0
D = 12.25 # inches
ODout = 190 # mm
IDout = 175 # mm
Od = 80 # mm
Id = 37 # mm
Es = 14633.401276 # psi
########################### target points
# defined target points.
#First target points
TargetInc1=0
#Second target points
TargetInc2=90
#Third target points
TargetInc3=0
## it is possible to add target points
## defining the inicial values
t = 1
log = np.zeros((t_max,10))
#0: MD
#1: error
#2: inc
#3: Horizontal
#4: TVD
#5: offset
#6: DSL
current_depth = 0
while current_depth < 10+1:
    log[t,0] = log[t-1,0]+ROP_axial(RPM,WOB,D,mi,Es)*deltaT/3600 # as we ar
e drilling vertically the axial ROP will just be composed by the axial ROP
log[t,1] = 0 #(do not care about this variable)
```

```
\(\log [t, 2]=0 \#\) Inclination is zero on the initial point
```

$\log [t, 3]=\log [t, 0] \#$ when we are vertical, the TVD is equal to the MD
$\log [t, 4]=0 \#$ defined as zero previously, no offset in vertical wells
$\log [t, 5]=$ offset \# defined as zero previously, no offset in vertical w ells
$\log [t, 6]=$ DLS \# defined as zero previously, no DSL in vertical wells $\log [t, 7]=0$
$\log [t, 8]=0$
$\log [t, 9]=0$
current_depth $=\log [t, 0]$
\#print(current_depth)
t += 1
\#\# this is the calculation of the geometry behind the bit
while t < t_max:
$j=t-2$
while True:
distance $=\log [t-1,0]-\log [j, 0]$ \# uses the Measured depth differen ce
if distance >= 10: \# for every point analysed, we search which poin $t$ is in 3 meters behind and collect the coordinates
break
else:
$j=j-1$
temp_data1 $=[[\log [j, 4], \log [j, 3]]] \#$ coordinates of 3 meters behind of $t$ he point regarding TVD $x$ Horizontal displacement
temp_data $2=[[\log [t-1,4], \log [t-1,3]]] \#$ coordinates of the point TVD $x$ Horizontal displacement
temp_data1 $=$ np. $\operatorname{rot} 90($ temp_data1,1)
temp_data2 $=$ np. $\operatorname{rot90(temp\_ data2,1)~\# ~do~not~mind...~python~logistics~}$
$g=t-2$
while True:
distance $=\log [t-1,0]-\log [g, 0] \#$ uses the Measured depth differen ce
if distance >= 1.64:\# for every point analysed, we search which poi nt is in 0,5 meters behind and collect the coordinates
break
else:

$$
g=g-1
$$

temp_data3 $=[[\log [g, 4], \log [g, 3]]]$ \# coordinates of 0.5 behind of the $p$ oint regarding TVD x Horizontal displacement
temp_data4 $=[[\log [t-1,4], \log [t-1,3]]]$ \# coordinates of the point regar ding TVD x Horizontal displacement
temp_data3 = np.rot90(temp_data3,1)
temp_data4 $=$ np.rot90(temp_data4,1)

ROP_n_inc = ROP_normal(offset, RPM, WOB, D, mi, Es, Od, Id,temp_data1,t emp_data2,temp_data3)[0] \# calculation of ROP's base on the previous functi ons

ROP_a = ROP_axial(RPM,WOB,D,mi,Es)
\#Reminder of the meaning of the logs.... Now we will calculate its variatio ns since we are not anymore on the initial points
\#0: MD
\#1: error
\#2: inc
\#3: TVD
\#4: Horizontal
\#5: offset
\#6: DSL
$\log [t, 0]=\log [t-1,0]+n p . \operatorname{sqrt}\left(R O P \_n \_i n c * * 2+R O P \_a * * 2\right) * d e l t a T / 3600 \#$ The MD considers the resultant ROP from the 3 existing ROPs
$\log [t, 1]=0 \#$ do not mind thgis one
$\log [t, 2]=\log [t-1,2]+n p . a r c t a n\left(R O P \_n \_i n c / R O P \_a\right) * d e l t a T / 3600$ \# the ne w inclination will be the previous inclination plus the increase caused by the ROP Normal generated for the offset_inc
$\log [t, 3]=\log [t-1,3]+n p . \cos (\log [t, 2]) * n p . \operatorname{sqrt}\left(R O P \_n \_i n c * * 2+R O P \_a * *\right.$ 2)*deltaT/3600 \# It uses the resultant of the ROP regarding inclination
$\log [t, 4]=\log [t-1,4]+n p . \sin (\log [t, 2]) * n p . s q r t\left(R O P \_n \_i n c * * 2+R O P \_a * *\right.$ 2)*deltaT/3600 \# It uses the resultant of the ROP regarding inclination
$\log [t, 5]=$ offset
$\log [t, 6]=((n p \cdot a b s(\log [t, 2]))-(n p \cdot a b s(\log [t-1,2]))) * 180 /((n p \cdot a b s(\log$ $[t, 0])-n p . a b s(\log [t-1,0])) * n p . p i) * 100$ \#DLS
$\log [t, 7]=R O P \_n o r m a l\left(o f f s e t, R P M, ~ W O B, ~ D, ~ m i, ~ E s, ~ O d, ~ I d, t e m p \_d a t a 1, t e m ~\right.$ p_data2,temp_data3)[1]
$\log [t, 8]=$ ROP_normal(offset, RPM, WOB, D, mi, Es, Od, Id,temp_data1,tem p_data2,temp_data3)[2]
$\log [t, 9]=$ ROP_normal(offset, RPM, WOB, D, mi, Es, Od, Id,temp_data1,tem p_data2,temp_data3)[3]
maxoff=1 \#\# definition of the max offset desired by the customer, there are some companies that do not wish to have big DLS, therefore, we must re duce the max offset here (values between 0 and 1)
\# first Target point kickoff [TVD], note that the values have been defined before
if $\log [t, 0]>150:$
\#offset=1
offset $=$ offsetfunction(TargetInc1, maxoff)
\# Second Target point kickoff [TVD], note that the values have been defined before
if $\log [t, 0]>350:$
\#offset=-1
offset $=$ offsetfunction(TargetInc2, maxoff)
\# Third Target point kickoff [TVD], note that the values have been defined before

```
if log[t,0] > 1000:
    #offset=0
    offset = offsetfunction(TargetInc3,maxoff)
#print (offset)
t = t+1
if t%t100 == 0:
    print ('.', end="")
```

\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\#\# PLOT 2D
plt.figure(figsize=(10,10))
plt.plot(log[:,4], $\log [:, 3])$
$x y 0=n p \cdot \min ([n p \cdot \min (\log [:, 3]), \quad n p \cdot \min (\log [:, 4])])-100$
$x y 1=n p \cdot \max ([n p \cdot \max (\log [:, 3]), \quad n p \cdot \max (\log [:, 4])])+100$
plt.xlim((xy0, xy1))
plt.ylim((xy0, xy1))
plt.xlabel('Horizontal displacement')
plt.ylabel('TVD')

```
plt.show()
plt.plot(log[:,0],log[:,2])
plt.xlabel('MD')
plt.ylabel('Inclination')
#plt.xlim((0,1500))
plt.show()
plt.plot(log[:,0],log[:,6])
plt.xlabel('MD')
plt.ylabel('DLS')
#plt.xlim((0,1500))
plt.show()
plt.plot(log[:,0],log[:,5])
plt.xlabel('MD')
plt.ylabel('Offset')
#plt.xlim((1000,1008))
#plt.xlim((0,1500))
plt.figure(figsize=(10,10))
plt.plot(log[:,0],log[:,7])
#plt.xlim((1640,1660))
#plt.xlim((3.14325,3.151))
#plt.ylim((-20000,50000))
#plt.ylim((-2000,2000))
plt.xlim((999,1008))
plt.show()
plt.figure(figsize=(10,10))
plt.plot(log[:,0],log[:, 8])
#plt.ylim((-20000,50000))
#plt.xlim((1640,1660))
#plt.ylim((0.003,000.5))
plt.xlim((999,1008))
#plt.ylim((-20000,50000))
plt.show()
plt.figure(figsize=(10,10))
plt.plot(log[:,0],log[:,9])
#plt.xlim((1999, 2010))
#plt.ylim((-20000,50000))
#plt.xlim((350,362))
#plt.xlim((1640,1660))
plt.xlim((999,1008))
plt.show()
```


[^0]:    Constant behavior (1)

