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MSC ENGINEERING STRUCTURES AND MATERIALS  
MASTER THESIS

**Structural, environmental and  
economy study of different  
timber-concrete slabs/beams and  
reinforced concrete slabs/beams**

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# Abstract

This thesis discusses two case studies, where case 1 is compared Timber-concrete composite (TCC) beam and Reinforced concrete beam (RC beam), while case 2 is compared Cross laminated timber (CLT) slab and Reinforced concrete slab (RC slab). Both cases are analyzed with respect to structural design, greenhouse gas emissions, and cost analysis. The effects of beam/slab span on design values, carbon dioxide emissions and cost are examined in terms of structural design, environmental impact and economy. Different span lengths are chosen for this purpose.

The comparison shows that, the ability of TCC and CLT elements to store carbon is far greater than reinforced concrete elements, therefore, TCC and CLT elements have significantly lower carbon dioxide emissions and less impact on the climate.

From economic point of view, reinforced concrete slab/beam are cheaper than TCC and CLT. But CLT and TCC element are competitive in the construction market because of their rapid production process which saves money for construction in longer run. The difference of material cost for TCC and reinforced concrete is less for shorter spans, but becomes greater as the span length is increased, the same applies between CLT and RC slab. On the other hand, the difference for cost of ready to assemble becomes narrower for TCC beam and reinforced concrete than CLT slab vs reinforced concrete slab are. This study shows that TCC and CLT elements can compete with concrete elements for long spans. Moment capacity is a critical factor for concrete slabs with shorter spans, while CLT experience difficulty to meet the design requirements of natural frequency for longer spans. The structural design for TCC beam can also compete with concrete beam with same cross section dimensions, natural frequency criterion is the main design issue for both beams with spans above 6m.

Keywords: Timber-Concrete composite, Cross laminated timber, Cost analysis, Eurocode 2 and 5, Environmental Impact.



# Preface

This thesis is the final work of my master study at the University of Stavanger. It serves as documentation of my research, which has been made from August 2020 until December 2020. The main objective was to compare two case studies of a beam and a slab using different building materials such as reinforced concrete, cross laminated timber and timber-concrete composite.

It was really true learning experience and I had fun reading into many books and standards on this subject. I would first like to thank my thesis supervisor Dr. S M Samindi M.K. Samarakoon of the Mechanical and Structural Engineering and Material Science at university of Stavanger (UiS). Dr. Samindi was always there when I needed help, regardless of her own deadlines. She consistently allowed this paper to be my own work, but steered me in the right the direction whenever she thought I needed it. I also would like to thank Dr. Sudath Sirwadane of the institute Mechanical and Structural Engineering for helping me throughout my studies.

Finally, I must express my very profound gratitude to my parents, my family and to my spouse for providing me with unfailing support and continuous encouragement throughout my years of study and through the process of researching and writing this thesis. This accomplishment would not have been possible without them.

Thank you.  
Stavanger, Desember 2020  
Ali Osman



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# Abbreviations

TCC	.....	Timber Concrete Composite
ULS	.....	Ultimate Limit State
SLS	.....	Serviceability Limit State
$(EI)_{ef}$	.....	Effective Bending Stiffness
CLT	.....	Cross Laminate Timber
RC	.....	Reinforced Concrete
$A_s$	.....	Cross sectional area of reinforcement
$A_{s,min}$	.....	Minimum cross sectional area of reinforcement
$E_{c,eff}$	.....	Effective modulus of elasticity of concrete
$E_{cD}$	.....	Design value of modulus of elasticity of concrete
$G_k$	.....	Characteristic permanent action
$I$	.....	Second moment of area of concrete section
$L$	.....	Length
$M$	.....	Bending moment
$M_{Ed}$	.....	Design value of the applied internal bending moment
$V$	.....	Shear force
$V_{Ed}$	.....	Design value of the applied shear force
$b$	.....	Overall width of a cross-section
$bw$	.....	Width of the web on T, I or L beams
$d$	.....	Diameter ; Depth
$d$	.....	Effective depth of a cross-section
$f_c$	.....	Compressive strength of concrete
$f_{cd}$	.....	Design value of concrete compressive strength
$f_{ck}$	.....	Characteristic compressive cylinder strength of concrete at 28 days
$f_{cm}$	.....	Mean value of concrete cylinder compressive strength
$f_{ctk}$	.....	Characteristic axial tensile strength of concrete
$f_{ctm}$	.....	Mean value of axial tensile strength of concrete
$f_y$	.....	. Yield strength of reinforcement
$f_{yd}$	.....	Design yield strength of reinforcement
$f_{yk}$	.....	Characteristic yield strength of reinforcement
$h$	.....	Overall depth of a cross-section
$h_w$	.....	Height of the web
$h_f$	.....	Thickness of flange
$\alpha$	.....	Angle ; ratio
$\beta$	.....	Angle ; ratio; coefficient
$\gamma$	.....	Partial factor
$\gamma_G$	.....	Partial factor for permanent actions
$\gamma_M$	.....	Partial factor for a material property
$\gamma_C$	.....	Partial factor for concrete
$\rho$	.....	Density of concrete in kg/m <sup>3</sup>
$\rho_c$	.....	Steel density
$\rho_{mean}$	.....	Mean density of timber in kg/m <sup>3</sup>
$\phi(t, t_0)$	.....	Creep coefficient, related to elastic deformation at 28 days
$\phi(\infty, t_0)$	.....	Final value of creep coefficient
$\psi$	.....	Factors defining representative values of variable actions

# Chapter 1

## Introduction

### 1.1 Background

The civil engineering domain is continuously searching for new improvements in construction systems or building materials. Impact of climate change, increasing cost of energy and water, sustainable design buildings has increasingly become a factor to be considered. The objective is to get a sustainable design, it is important to combine technical design with economic development, social development, and environmental protection [1]. The main concern in this context is the choice of building materials. This thesis considers two of the main building materials that are commonly used in constructions, namely wood and concrete.

Wood has been used as a building material for thousands of years. After a period of in-activeness, this material has increasingly become a popular over the past several decades because of the certain advantages of wood such as high mechanical properties and low environmental effects, and aesthetics, therefore it is becoming gradually more competitive in the construction market. Unfortunately, timber structures are limited by the deformability of wood and variability of its properties. Because of lower Young's modulus of wood, the deformation is greater than that of concrete under equivalent loads [2].

Different methods have been exploited to make wood more practical and effective means of improving material behavior of wooden structures. Regularly applied technique is that wood is combined with other materials such as steel or a concrete. The goal is to combine wood with a stiffer material and therefore increase the rigidity of the structure and lower its deformability.

On the other hand, concrete is one of the most commonly used as a building material probably because concrete has a good workability, durability, longevity and high compressive strength. But at the same time, production of cement for concrete has its own environmental effects, as cement can constitute about 5-7% of the total  $CO_2$  emission in the atmosphere [3]. Despite advantages and disadvantages of concrete there is still high demand for concrete and due to the extensive usage and demand of this material, many researchers are deeply investigating its engineering properties. The addition of new materials in concrete

such as basalt fiber, glass fiber, basalt ribber etc. improved significantly mechanical properties of concrete [4].

## 1.2 Scope And Limitations

The compared beams and slabs are simply supported frames, beams and slabs are supported by load bearing columns and walls respectively, on each end. For case 1, the design of TCC beam the choice is limited to service class 1, strength class C24 and C30 for timber part, where top concrete part has strength of class of B25 and B30. When designing the reinforced concrete beam, the choice is limited to concrete strength class of B25 and B30. A similar T-shaped cross section is considered for this case.

For Case 2, the designing of slabs a symmetrical structure is considered in which the neutral axis is located in the center of the section. Number of layers for CLT are limited to values between 3 to 7 layers with strength class C24 and C30, safety class of 3 and service class 1. Two extreme values of imposed loads are considered for both cases, which are category A and C5 imposed load [5]. Snow load and other load effects are not considered. The cost analysis is based on the Norwegian Market.

## 1.3 Methodology

The structural design requirements of the two studied cases are based on the European standards specially Eurocode 2 [6] and Eurocode 5 [7]. For the economy, the material cost and ready to assemble for both cases are calculated using the assessment on the Norwegian construction market. For environmental evaluation, the effect on the environment with respect to the emission of carbon dioxide is considered. The data used in calculation of the environmental assessment are based on earlier findings of research on life cycle analysis of the two studied cases, as discussed in chapter 4.

The comparison methodology is to study and analyse the two cases: Case (1) Timber concrete composite beam and reinforced concrete beam. Case (2) Cross laminated timber slab and reinforced concrete slab. Different beam and slab dimensions on the structural design values, cost and carbon dioxide emission that are emitted during lifetime of the two cases.

A series of different design parameter is executed: Only two vertical imposed loads on the structure are considered, The dimensions studied in case (1) is a t-shaped cross section as shown in figures 3.3 and 3.4 with different slab lengths (L) such as :2m; 2.5m; 3m; 3.5m; 4m; 4.5m; 5m; 5.5m; 6m; 6.5m and 7m. For case (2) studied slab dimensions (L x B), where L is the slab length and B is the width of the slab as shown figure 3.7 and ??in are as follows: 3m × 5m; 3,5m × 5.5m; 4m × 6m; 4.5m × 6.5m; 5m × 7m; 5.5m × 7.5m; 6m × 8m; 6.5m × 8.5m; 7m × 9m.

## 1.4 Aim And Objectives

The main objective of this thesis is mainly on three criteria:

- To find out the influence of static and dynamic design can have the choice between beams: TCC and Reinforced concrete beam and slabs: CLT and Reinforced concrete slab
- Find out financial consequences of choosing type beam and slab.
- The amount of greenhouse gas emissions, in terms of carbon dioxide equivalents emitted during the lifetime of structures (two cases).

## 1.5 Project Outline

The study outline includes:

1. Chapter One: Introduction to the research work.
2. Chapter Two: Literature review on Timber-concrete composite, Reinforced concrete and Cross laminated timber.
3. Chapter Three: Structural Design.
4. Chapter Four: Environmental impact.
5. Chapter Five: Economy.
6. Chapter Six: Results and Discussion of the research work.
7. Chapter Seven: Conclusion on what has been studied.





## Chapter 2

# Literature Review

### 2.1 Timber Concrete Composite (TCC)

Wood is one of the oldest and most used building materials, wood has still a key role in the construction sector. After the end of World Wars I and II, there was a shortage of steel as a construction material. This led to initiation of Timber Concrete Composite TCC systems Specially in Europe. In the last 50 years, the use of TCC systems has increased mainly in bridge construction (United States, New Zealand, Australia, Austria, Switzerland, and many Scandinavian countries) and maintaining existing timber floors. Different methods have been exploited to make wood more practical and effective inters of improving material behavior of wooden structures. Regularly applied technique is that wood is combined with other materials such as steel or concrete. The goal is to combine wood with a stiffer material and therefore increase the rigidity of the structure and lower its deformability.

A light timber frames suffer from excessive deflection, low capacity of fire resistance, inadequate acoustic separation, and sensitivity to vibrations. All these drawbacks of timber can be solved by using TCC floors. There are many advantages of TCC over only timber or reinforced concrete floors.

Some of the advantages of using TCC are:

- Increase of stiffness compared to only timber floors
- Much improved the acoustic separation
- Thermal mass is increase and the energy consumption needed to heat and cool the building, is reduced.

The lower part of reinforce concrete beams or slabs are insufficient, due to the cracking induced by the tensile stress of concrete, therefore replacing the lower part with timber joist or solid timber deck can have several advantages:

- Reduce imposed load on the foundation.
- Lower embodied energy.
- Rapid erection of the timber part, due to its low weight, and function as

permanent formwork for the concrete topping.

Nevertheless, despite the advantages of timber concrete composite structures abovementioned, the use of wet concrete has its limitations; notably:

- The time needed the concrete to set, which adds the time required on site before the next action can be carried out.
- Low stiffness and high creep while the concrete cures.

### 2.1.1 Standard and Design Methods

The Design process of TCC structure is not well complemented by an adequate regulatory framework. As a matter of fact, some guidelines are developed, mostly guidelines related to designing of TCC specially bridges [8]. Timber beam or deck is connected to an upper concrete flange using different types of shear connectors. The shear connectors are not fully rigid; therefore, the assumption of plane section remain plane does not apply to the composite section due to relative slip between the bottom fiber of concrete and the top part of timber.

The Design guidelines are very important to the industrial acceptance of TCC systems. For beams and slaps, both serviceability (SLS) and ultimate limit state (ULS) under short-term and long-term need to be considered for design procedure. Some sectional properties are needed for the designing purpose e.g., moment of inertia, are dependent on the rigidity of individual parts are connected to each other.

Today's practitioner use mainly two methods for designing of TCC systems:

**$\gamma$  Method:** This method is described in Eurocode 5 [7], and accounts for the semi-rigidity of timber-concrete connectors and make use of an effective bending stiffness  $(EI)_{ef}$ . For simplification, it is assumed that spacing of shear connectors are equally spaced along the span, instead of the actual case where the shear connectors are placed according to the shear force and are placed closer to the supports and spaced further until the mid-span. In this method, the composite is quantified through the parameter  $\gamma$ , where  $\gamma = 1$  is fully rigid connection and  $\gamma = 0$  is no connection between timber and concrete at all. This is a simple method and allowed practitioners to design a TCC system. This method is adopted in Eurocode 5 in the Annex B.

**Other guidelines:** A similar method is used in the Australian and New Zealand design guidelines [9]. This method takes in to account the influence of floorboards to the stiffness of the system and therefore needs to be accounted.

### 2.1.2 Connections Types

Timber and concrete need to be connected through a shear connection, this enables the component to work as a composite system, therefore the choice of connections is critical, as it determines the system's effectiveness and economic competitiveness. Fasteners are very critical component in the designing and performance of TCC systems. Due to the indeterminate of Connections are usually positioned along the beam and counteract the shear force. Therefore, placement of connections is concentrated near the support where the internal shear stress is greatest and spaced out gradually to the mid span for simply supported beams.

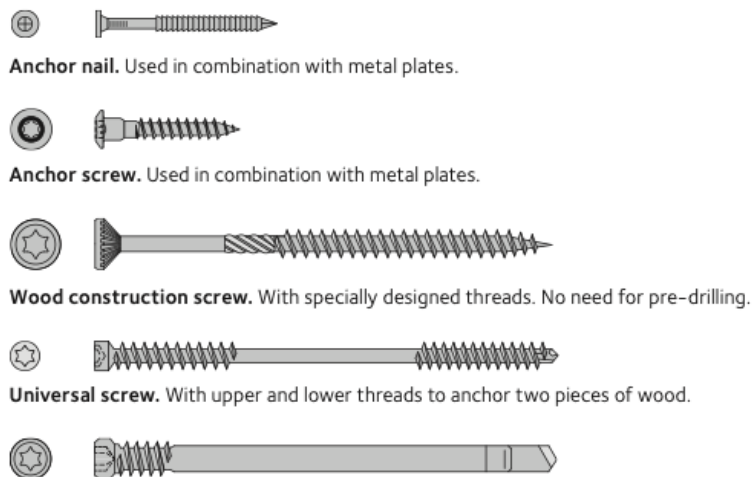
In general, from the mechanical performance point of view, the ideal connection should be:

1. Strong enough to transmit the shear force developed between concrete and timber.
2. Stiff enough to transmit the load with a limited slip at the interface.
3. Ductile enough to allow full load distribution and avoid failure on the fasteners.

An overview of different types of fastener is given in Dias [2015] [10].

The connection systems available today only fulfill part of the mechanical performance for an ideal connection system. Fasteners are mainly divided into four types:

- Dowel type fasteners
- Notches
- Notches combined with steel fasteners
- Other connection systems



**Figure 2.1:** Some commonly used connector systems used for CLT and TCC [11].

Based on the arrangement along the beam and the way they are inserted in the timber, the connectors can be divided in either glued/non-glued and prestressed or non-prestressed. Dowel type fasteners such as nails, screws, bolts and stables are the most used connection system in timber construction and is also used normally in TCC connection systems. Different types of Dowel and screw fasteners are shown in figure 2.1.

Notches cut in the timber and reinforced with some type of Dowel fasteners or steel screws as shown in the picture figure 2.2, is the best connection type for TCC systems regarding strength and stiffness performance, but this is not economical if the cut is made manually. Some factors affect the parameters of the connection system e.g., the length of the notch, presence of leg screw, and its depth into the timber.



(a) Notche cut in timber.

(b) Dowel type fastener

**Figure 2.2:** Some commonly used connector systems. [12]

To determine the strength and stiffness of connections systems a push out tests is carried out, this provides the load-slip response under shear load according to EN26891 [13]. Strength of connection is quantified as maximum load applied when failure occurs in the specimen, while stiffness if also quantified by the slip modulus and different loads corresponding the service, ultimate and near collapse load levels. Provides A comparison of shear force-slip relationships for different types of connections is shown in figure 2.3.

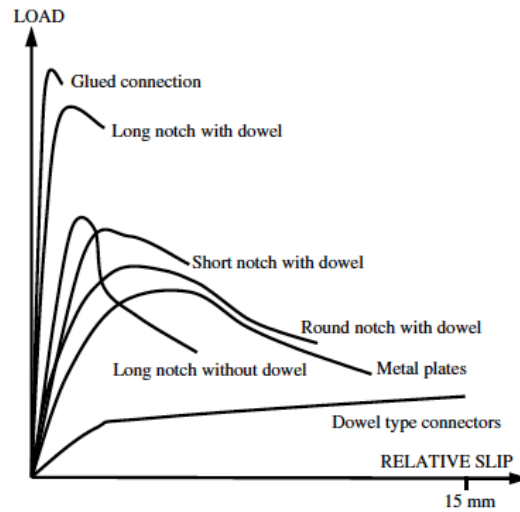


Figure 2.3: Comparison of shear force-slip relationships. [14]

A number of investigations has been done on the influence of concrete properties on timber-concrete composite. Some push out tests [14] used a lightweight concrete with density of  $1.6 \text{ kN/m}^3$  instead of using a normal concrete with density of  $2.5 \text{ kN/m}^3$ . They concluded timber-lightweight concrete composite is affected by modulus of elasticity of light weight concrete. This leads to lower effective bending stiffness of the system. And furthermore, spacing of connectors are placed closer.

Curing of concrete in the early days, shrinkage of concrete will result a gap between concrete and timber, causing initial permanent deflection of the composite beam or slab. To reduce this issue, it is recommended to use concrete with a low shrinkage [15].

### 2.1.3 Fire, Acoustic and Vibrations

The issues that are associated with low fire resistance of timber, inadequate acoustic separation, and prone to vibrations of timber floors are significantly minimized in TCC. TCC system is more competitive with that of reinforced concrete slabs or beams. A research made by Natterer [16] described that self-weight of structures are significantly reduced with the use of TCC systems, and also fire resistance is increased from 60min to 90 min when compared to that of common reinforced concrete slabs. The effect of temperature is reduced and delayed by the top concrete part which acts as a cladding to timber joist at the bottom. On the other hand, concrete and connectors are protected from high temperatures by the char that is developed on the timber in the burning process [17].

A simplified method of calculating fire resistance of TCC is developed by Frangi and Fontana [18] proposed a function of the relationships for the reduction of stiffness and strength of connection cover. With regards of TCC vibrations and

acoustic, the mass of the system is increased by the top concrete slab and stiffness of timber joist, and therefore, improves the overall the impact sound insulation and vibration behavior[19].

## 2.2 Reinforced Concrete

Concrete is one of the most used as a building material probably because concrete has a good workability, high compressive strength, durability, and longevity.

Concrete is a composite material which consists of a mixture of a fine and coarse materials such as cement, aggregate and water, in some cases admixtures are added. Admixtures allows the mix to be more workable and reduce water content in the mix.

Concrete can be placed and molded into almost any shape and reproduce a variate of surface textures. But at the same time, concrete has some serious shortcomings, such as a low tensile strength and low crack resistance and difficult to recycle it. Production of cement for concrete has its own environmental effects. Despite advantages and disadvantages of concrete there is still high demand for concrete and due to the extensive usage and demand of this material, researchers are deeply investigating its engineering properties. Use of concrete has improved lately, the use of high-performance concrete is one of the new trends in concrete structures . Polymer bars and basalt fiber-reinforcement are some of the new types of reinforcement [4].

The design of concrete beam and slab is performed using the Euro-Code [6], accounting all the requirements for Serviceability limit State (SLS) and Ultimate Limit State (ULS)

### 2.3 Cross Laminate Timber (CLT)

CLT is becoming more popular as a construction material and also for multistory constructions. CLT consists of several (more than two) timber board layers that are glued to each other at a certain angle as shown in figures 2.4 and 2.5. Normally hydraulic and vacuum press is used, resulting in a symmetric cross section. Apparently, a standard CLT element consists of an odd number of layers (3-7 layers), this gives a better bearing capacity than normal homogeneous wood [20]. The dimensions of a single piece of lumber varies from 15 mm to 50 mm in thickness and the width from 60 mm to 250 mm.

The longevity of CLT depends on the types of timber, glue, application methods and how it is preserved. CLT is suitable for internal and external walls, for durability purpose CLT is also used for swimming pool roofs, this is an environment where the levels of humidity, corrosive and chloride content is high and therefore, CLT provides a low maintenance and durable solution.

For beams the cross-sectional area is normally large, therefore, the rate of charring of the surface of the beam is low. Consequently, CLT can carry loads longer when fire is exposed. Fire and flame resistance can be increased by surface coating or pressure impregnation with fire retardants.

Timber is a locally available material for many countries, and no need to be mined and subjected to high energy demand through the manufacturing process that steel and cement require. The consumption of energy for timber is  $0.76 \text{ GJ/m}^2$  where steel and concrete structures are  $3.24 \text{ GJ/m}^2$  and  $2.13 \text{ GJ/m}^2$  respectively [21].

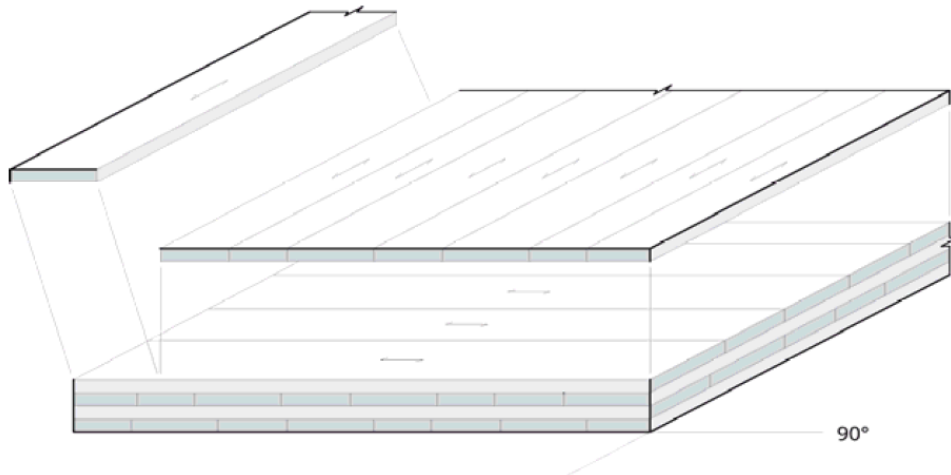
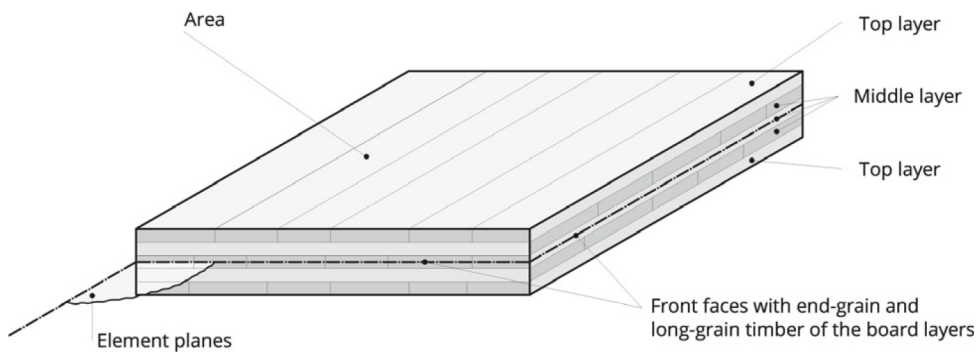


Figure 2.4: Cross laminated timber board layers [22].



### 2.3.1 Design of CLT

Some design regulations for CLT are still missing in the European standards, apart from some application documents for EC5, for example DIN EN 1995-1-1/NA [23]. Shear flexibility is so high in the transverse layers in CLT; therefore, it should be considered the influence of shear. Because of this limitation of beam theory by Euler Bernoulli in respect shear and other theories are considered, such as  $\gamma$  method [6] and transverse shear-flexible beam according to Timoshenko beam theory [7]. The design of CLT is accounted all the requirements for Serviceability limit State (SLS) and Ultimate Limit State (ULS).



**Figure 2.5:** Cross laminated timber board layer definition. [22].

## Chapter 3

# Structural Design

To satisfy requirements of structural performance of structures, two main criteria should be fulfilled. The first criteria is safety of the structure, and expressed in terms of load-bearing capacity in other words ultimate limit state (ULS). The second criterion is serviceability limit state (SLS), this means structures should perform its intended function throughout its working life.

For the analysis of the two cases, loading of beams and slabs are chosen to represent values that structures experience in practice.

### 3.1 Loading Actions

The loading on the investigated cases are characteristic permanent loads: self-weight of beams and slabs and also characteristic imposed loads on structures. The analysis and designing of both cases, two categories of imposed load are considered [5]. Category A are areas for domestic and residential activities occur, with value  $q_k = 2kN/m^2$ . Category C5 are areas where a large gathering happens e.g. concert and sports halls, has value of  $q_k = 5kN/m^2$

#### 3.1.1 Ultimate limit state (ULS)

The ultimate limit state is the design for safety. ULS is used to void collapse of the structure. The design value  $q_{Ed}$  is found by the load combinations of interest as for equations 6.10a and 6.10b in clause 6.4.3.2 of EN 1990 [Annexes A1 A2].

$$q_{Ed} = \gamma_G g_k + \gamma_Q q_k + \sum_i \gamma_{Q,i} \psi_{0,i} q_{k,i} \quad (3.1)$$

Where,  $\psi_0$  is the factors for the combination value of variable load. Partial factors of ultimate limit state are  $\gamma_G=1.35$  and  $\gamma_Q=1.5$ .  $g_k$  is the characteristic self weight of structure.

### 3.1.2 Serviceability limit state (SLS)

Serviceability limit state, is the second design requirement for structure to satisfy and therefore structures to perform its intended function throughout its working life. For this study, only deflection and vibrations limit state will be analysed.

$$q_{Ed} = \gamma_G g_k + \gamma_Q q_k + \sum_i \gamma_{Q,i} \psi_{0,i} q_{k,i} \quad (3.2)$$

For quasi-permanent load combinations:

$$q_{Ed} = \gamma_G g_k + \sum_i \psi_{2,i} q_{k,i} \quad (3.3)$$

Where, partial factors for serviceability limit states are  $\gamma_G=1.0$  and  $\gamma_Q=1.0$ .  $\psi_2$  is the factor for quasi permanent load of variable action.

### 3.2 Case 1: TCC beam and RC beam

In case (1) timber concrete composite (TCC) beam is analysed and compared with reinforced concrete (RC) beam.

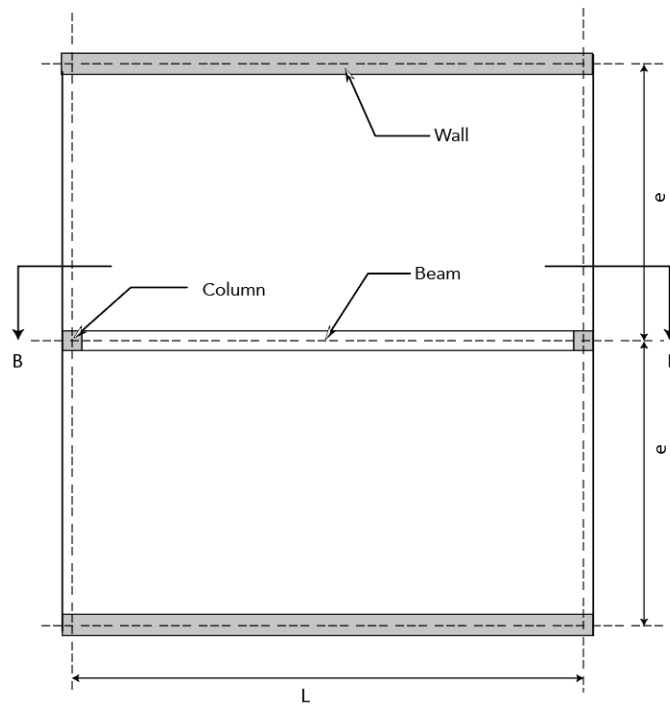
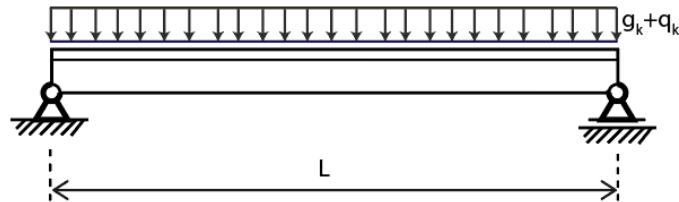


Figure 3.1: Simply supported T-beam supported by two columns.

### 3.2.1 Structural model

For the static analysis of case (1), a simply supported T-beam is chosen as shown in figure 3.2. The beam is supported by two columns on each end of the span length  $L$ .



**Figure 3.2:** Simply supported T-beam loaded with uniformly distributed load (Section B-B in figure 3.1) .

The design bending and shear force for simply supporter beam is given in equations equations (3.4) and (3.5):

$$M_{Ed} = \frac{q_{Ed}L^2}{8} \quad (3.4)$$

$$V_{Ed} = \frac{q_{Ed}L}{2} \quad (3.5)$$

### 3.2.2 TCC beam

Timber-concrete composite consists of a combination of timber and concrete. Timber acts as beams for the concrete slab on the top as mentioned section 2.1. Calculation of structural capacities such as moment, shear, deflection and vibration is considered along the design span  $L$ . Dimensions for cross section of TCC beam is shown in figure 3.3.

Some of the used input design data are: Imposed load on the slab  $q_k(kN/m^2)$ . Mass of the composite structure  $M(kg/m^2)$ .

**Concrete:** Slab part consists of a concrete grade C25/30 with characteristic tensile capacity of 500 (MPa), Compressive strength of concrete  $f_{ck}(MPa)$ , steel wire mesh reinforcement  $V_r(m^3/m^2)$ , concrete and steel density  $\rho_c(kg/m^3)$ ,  $\rho_s(kg/m^3)$  respectively, Width of concrete slab  $b_2(mm)$ , thickness of the floor ( $h_1$ ),

**Timber:** Flexural strength of timber  $f_{mk}(MPa)$ , spacing of timber beams  $e(m)$ . width of timber beam  $b_2(mm)$ , height of timber  $h_2(mm)$ , distance between beams  $e(mm)$ .

**Inter-layer:** Width  $b_i(mm)$ , thickness ( $t_i$ ). density of inter-layer plate:  $p_p(kg/m^3)$ ,

**Fasteners:** Tensile strength of fasteners  $f_u(MPa)$ , diameter of fastener  $d(mm)$  Spacing of fastener  $s(mm)$ , Screw length  $l_s(mm)$ , slip modulus of screw  $K_{ser}(N/mm)$ , number of fasteners ( $n$ )

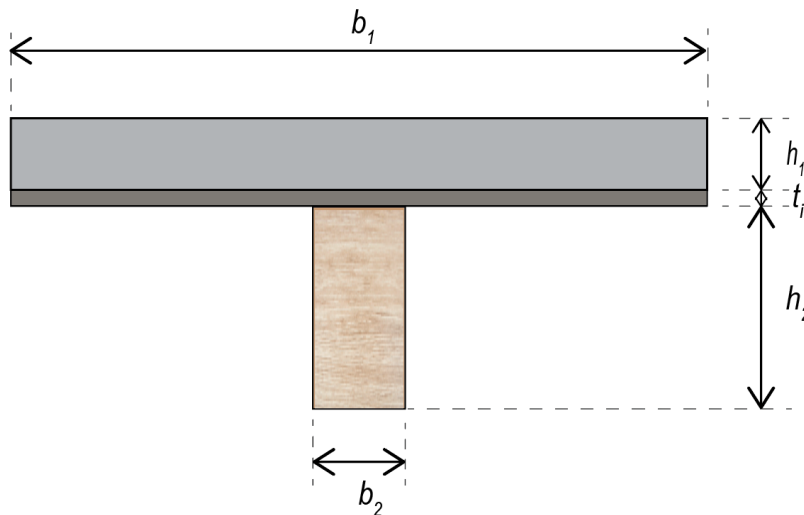


Figure 3.3: Definition of dimensions for cross section of T-beam TCC.

### 3.2.3 RC beam

Reinforced concrete T-beam is a solid concrete and consist of concrete with grade C25/30 with characteristic tensile capacity of steel equal to 500 (MPa). Calculation of structural capacities considered along the design span "L". Dimensions for cross section of reinforced concrete beam is shown in figure 3.4.

Used input design loads are the same as loads on TCC section mentioned in section 3.1.

Compressive strength of concrete  $f_{ck}$ (MPa), steel reinforcement  $A_s$ (mm<sup>2</sup>), concrete density  $\rho_c$ (kg/m<sup>3</sup>), width of concrete flange  $b_{eff}$ (mm), thickness of the flange  $h_f$ (mm), thickness of the web  $b_w$ (mm), height of the web  $h_w$ (mm)

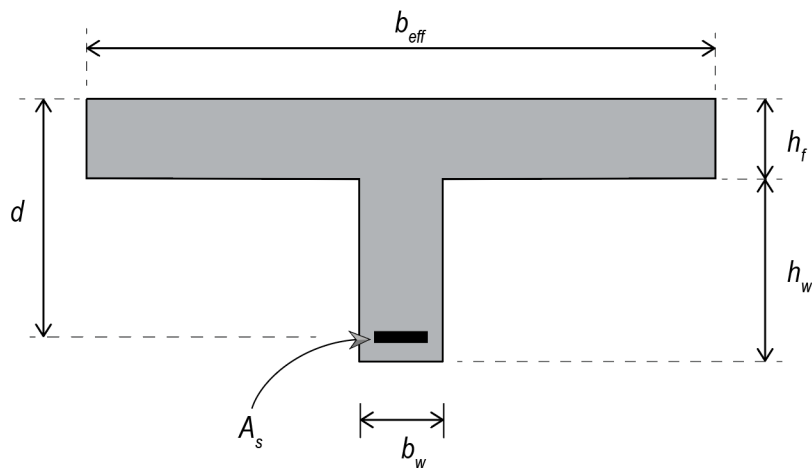


Figure 3.4: Definition of dimensions for cross section of T-beam RC.

### 3.3 Ultimate limit state check for Case (1)

Ultimate limit conditions should be fulfilled.

#### Effective flange width

For both TCC and RC has to satisfy with the following condition:

$$b_{case1} \leq b_{eff} \quad (3.6)$$

Where:

$$b_{case1} = e \quad (3.7)$$

$$b_{eff} = L/4 \quad (3.8)$$

$b_{case1} \rightarrow$  is for both  $b_1$  and  $b_{eff}$

### 3.3.1 TCC: Normal stresses in concrete section

#### Compressive stress of concrete (Top)

This check is satisfied when the calculated design normal stress of concrete  $\sigma_{cd}(MPa)$  is less than the design compressive strength of concrete  $f_{cd}(MPa)$

$$\sigma_{cd} \leq f_{cd} \quad (3.9)$$

Where:

$$\sigma_{cd} = \frac{\gamma_1 E_1 \alpha_1 M_{Ed}}{(EI)_{ef}} + \frac{0.5 E_1 h_1 M_{Ed}}{(EI)_{ef}} \quad (3.10)$$

For the  $\gamma$  method the effective bending stiffness  $(EI)_{ef}$  for simply supported composite beam is calculated in equation (3.59), according to [EN 1992-1-1, Eq. B.5 and B.4]

$$(EI)_{ef} = \sum_{i=1}^2 E_i I_i + \gamma_2 E_2 A_2 \alpha_2^2 + \gamma_1 E_1 A_1 \alpha_1^2 \quad (3.11)$$

Where;

$$\gamma_1 = \left[ 1 + \frac{\pi^2 E_1 A_1 s_{ef}^2}{K_i L^2} \right]^{-1} \quad (3.12)$$

Slip modulus of elasticity for (ULS)  $K = (2/3) * k_{ser}$

$$\gamma_2 = 1 \quad (3.13)$$

Distance between the centroid of concrete slab and centroid gravity:

$$\alpha = \frac{h_1}{2} + \frac{h_2}{2} + t_i \quad (3.14)$$

$$\alpha_2 = \frac{\gamma_1 E_1 A_1 \alpha}{\gamma_1 E_1 A_1 + \gamma_2 E_2 A_2} \quad (3.15)$$

Distance between the centroid of the timber and centroid gravity  $\alpha$  is given in equation (3.64)

$$\alpha_1 = \alpha - \alpha_2 \quad (3.16)$$

Design compressive strength of concrete  $f_{cd}(MPa)$

$$f_{cd} = \frac{0.85 f_{ck}}{1.5} \quad (3.17)$$

$E_i$  is modulus of elasticity,  $E_1$  is modulus of elasticity for concrete and  $E_2$  is modulus of elasticity of timber,  $I_i$  is the second moment of inertia,  $A_i$  is the cross sectional area.

Ratio of design normal stress of concrete and compressive strength is given as %:

$$\frac{\sigma_{cd}}{f_{cd}} \cdot 100 \quad (3.18)$$

### Compressive stress of concrete (Botom)

The design of normal stress in concrete should be less then design tensite strength of concrete

$$\sigma_{td} \leq f_{ctd} \quad (3.19)$$

Where:

$$\sigma_{td} = \frac{0.5E_1h_1M_{Ed}}{(EI)_{ef}} - \frac{\gamma_1E_1\alpha_1M_{Ed}}{(EI)_{ef}} \quad (3.20)$$

$$f_{ctd} = \frac{f_{ctk0.05}}{1.5} \quad (3.21)$$

$f_{ctd}$  is the design tensile strength of concrete,  $f_{td}$  is the stress at the bottom of concrete,  $f_{ctk0.05}$  is the characteristic axial tensile strength.  $M_{Ed}$  is given in equation (3.4)

Ratio of design normal stress of concrete and tensile strength is given as %:

$$\frac{\sigma_{td}}{f_{ctd}} \cdot 100 \quad (3.22)$$

### 3.3.2 TCC: Check for shear stress

The calculated shear stress (the stress perpendicular to the grain)  $\tau_{max}(MPa)$  should not be greater than the design shear strength of timber  $f_{vk}(MPa)$ . For the sake of simplicity, the sear force  $V_{Ed}$  is considered to be totally carried by the timber beam. The design shear stress  $V_{Ed}$  is given in equation (3.5).

$$\tau_{max} \leq f_{vk} \quad (3.23)$$

Where:

$$\tau_{max} = \frac{1.5 \cdot E_2 \cdot h^2 \cdot V_{Ed}}{(EI)_{ef}} \quad (3.24)$$

$$h = 0.5 \cdot h_2 + \alpha_2 \quad (3.25)$$

$$f_{vd} = \frac{k_{mod} \cdot f_{vk}}{1.3} \quad (3.26)$$

$f_{vk}$  is the characteristic shear strength of timber joist,  $k_{mod}$  is modification factor.

The ratio of shear stress (stress perpendicular to the grain) and design shear strength given as %:

$$\frac{\tau_{max}}{f_{vk}} \cdot 100 \quad (3.27)$$



### 3.3.3 TCC: Check for combined axial and bending of cross section

This is where check for combined axial and bending cross sectional resistance must be satisfied.

$$\left[ \frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} \right] \leq 1 \quad (3.28)$$

Where:

$$\sigma_{m2d} = \frac{\gamma_2 E_2 \cdot \alpha_2 \cdot M_{Ed}}{(EI)_{ef}} \quad (3.29)$$

$$\sigma_{m2d} = \frac{0.5 \cdot E_2 \cdot h_2 \cdot M_{Ed}}{(EI)_{ef}} \quad (3.30)$$

$$f_{t0d} = \frac{k_{mod} \cdot f_{t0k}}{1.3} \quad (3.31)$$

$$f_{md} = \frac{k_{mod} \cdot f_{mk}}{1.3} \quad (3.32)$$

### 3.3.4 TCC: Capacity of fasteners

Design applied load  $F_{1d}(kN)$  on the fastener is lower than the calculated design strength of fastener  $F_{Rd}(kN)$ .

$$F_{1d} \leq F_{Rd} \quad (3.33)$$

Where:

$$F_{1d} = \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot s \cdot \alpha_1 \cdot V_{Ed}}{(EI)_{ef}} \quad (3.34)$$

$$F_{Rd} = \frac{n_{ef} \cdot k_{mod} \cdot F_{rk}}{\gamma_M} \quad (3.35)$$

$$F_{rk} = \min(F_{rk1}, F_{rk2}) \quad (3.36)$$

([ETA 013/0029] Tab. 2.3)  $F_{rk}$  is the characteristic load bearing capacity of screws:

$$F_{rk1} = \cos(a) \cdot f_{ax.a.Rk} \quad (3.37)$$

$$F_{rk2} = \cos(a) \cdot f_{tens.k} \quad (3.38)$$

(a) is the angle of screws.

$$f_{ax.a.Rk} = \frac{f_{ax.k} \cdot d \cdot l_{ef}}{1.2 \cdot (\cos(a))^2 + (\sin(a))^2} \cdot \left( \frac{\rho_k}{350} \right)^{0.8} \quad (3.39)$$

### 3.3.5 RC beam: Bending Moment Capacity

In this section, the design criterion for moment capacity is checked. Moment capacity should be enough to carry the design bending moment capacity.

$$M_{Ed} \leq M_{Rd} \quad (3.40)$$

Where,  $M_{Ed}(kNm)$  is the applied moment at the limit state of strength of beam,  $M_{Ed}$  is given in equation (3.122),  $M_{Rd}(kNm)$  is the ultimate moment capacity of concrete slab.

Some of design values of concrete material properties according to EN1992-1-1 is given as,

$$f_{cd} = \frac{a_{cc} \cdot f_{ck}}{\gamma_c} \quad (3.41)$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \quad (3.42)$$

Where  $f_{ck}$  is the characteristic cylinder compressive strength,  $f_{yk}$  is the characteristic yield strength of the steel reinforcement ( $f_{yk} = 500MPa$ ),  $\gamma_c$  and  $\gamma_s$  are the partial safety factor for concrete and steel respectively.  $a_{cc}$  Coefficient takes into account the long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied, it is taken here  $a_{cc} = 0.85$ . partial safety factor are taken as  $\gamma_c = 1.5$  and  $\gamma_s = 1.15$ .

The required steel area  $A_s(mm^2)$  of tension reinforcement for singly reinforced concrete slab is given as:

$$A_s = \frac{M_{Ed}}{f_{yd} \cdot z} \quad (3.43)$$

Where  $z(mm)$  is the lever arm distance between moments  $f_s$  and  $f_c$ , it is given as:

$$z = \min \left[ d \left( 0.5 + \sqrt{\left( 0.25 - \frac{k}{1.134} \right)} \right), 0.95 \cdot d \right] \quad (3.44)$$

Where  $d$  is the effective depth of section, it is measured from the top of the beam to the center of the area of reinforcement For singular reinforced concrete section the term  $k$  is given as:

$$k = \frac{M_{Ed}}{b_w \cdot d^2 \cdot f_{ck}} \quad (3.45)$$

where  $b_w$  is the web thickness of the beam.

Minimum steel area of reinforcement for the section is:

$$A_{s,min} = 0.26 \cdot b \cdot d \cdot \frac{f_{cmt}}{f_{yk}} \quad (3.46)$$

Where  $f_{ctm}$  (MPa) is the mean tensile strength  
**Moment capacity** of the slab is given as:

$$M_{Rd} = 0.168 \cdot f_{ck} \cdot b_w \cdot d^2 \quad (3.47)$$

The utilization ratio for moment capacities can be expressed as % :

$$\frac{M_{Ed}}{M_{Rd,c}} \cdot 100 \quad (3.48)$$

### 3.3.6 RC beam: Shear Check

Calculated design shear force should be lower than allowable shear force capacity, therefore the beam should fulfill this criterion:

$$V_{Ed} \leq V_{Rd,c} \quad (3.49)$$

where  $V_{Ed}$  (kN) is the design value of transverse shear force of the beam,  
 $V_{Rd,c}$  (kN) is Design shear resistance of the slab.

$V_{Rd,c}$  is given as:

$$V_{Rd,c} = [C_{Rd,c} \cdot k (100\rho_l \cdot f_{ck})^{1/3}] \cdot b_w \cdot d \quad (3.50)$$

where  $C_{Rd,c}$  is coefficient driven from testes (recommended value=0.12) and expressed as:

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = 0.12 \quad (3.51)$$

where value of k is expressed as

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \quad (3.52)$$

Percentage of tensile reinforcement is:

$$\rho_l = \frac{A_{sl}}{b_w \cdot d} \quad (3.53)$$

Where  $A_{sl}$  is the Area of tensile reinforcement,  $A_{sl} = A_s$

The design shear should not be less than

$$V_{Rd,c} = v_{min} \cdot b_w \cdot d \quad (3.54)$$

where

$$v_{min} = 0.035 \cdot \sqrt{k^3 \cdot f_{ck}} \quad (3.55)$$

The utilization ratio for shear capacities can be expressed as % :

$$\frac{V_{Ed}}{V_{Rd,c}} \cdot 100 \quad (3.56)$$

### 3.4 Serviceability limit state check for Case (1)

In this section, a check is made on whether the calculated deflection of the beam is lower than recommended value.

#### 3.4.1 TCC: Deflection

(1) [SLS At The **Beginning** Of Life Time]

$$u_{inst} \leq \frac{L}{250} \quad (3.57)$$

Where: Deflection of beam:

$$u_{inst} = \frac{5(g_k + q_k)L^4}{384(EI)_{ef}} \quad (3.58)$$

$$(EI)_{ef} = \sum_{i=1}^2 E_i I_i + \gamma_2 E_2 A_2 \alpha_2^2 + \gamma_1 E_1 A_1 \alpha_1^2 \quad (3.59)$$

Where;

$$\gamma_1 = \left[ 1 + \frac{\pi^2 E_1 A_1 s_{ef}}{K_i L^2} \right]^{-1} \quad (3.60)$$

Slip modulus of elasticity for (SLS)  $K = n * k_{ser}$ , n is number of screws in a row (n=2).

$$\gamma_2 = 1 \quad (3.61)$$

Distance between the centroid of concrete slab and centroid gravity:

$$\alpha = \frac{h_1}{2} + \frac{h_2}{2} + t_i \quad (3.62)$$

$$\alpha_2 = \frac{\gamma_1 E_1 A_1 \alpha}{\gamma_1 E_1 A_1 + \gamma_2 E_2 A_2} \quad (3.63)$$

Distance between the centroid of the timber and centroid gravity  $\alpha$  is given in equation (3.64):

$$\alpha_1 = \alpha - \alpha_2 \quad (3.64)$$

(2) [SLS At The **End** Of Life Time]

$$w_{fin} \leq \frac{L}{200} \quad (3.65)$$

Where:

$$w_{fin} = w_{qp} + w_q \quad (3.66)$$

$$w_{qp} = w_{qp} + w_q \quad (3.67)$$

### Deflection of beam by quasi-permanent load

Consider creep :

$$w_{qp} = \frac{5f_{qp}L^4}{384(EI)_{ef,fin}} \quad (3.68)$$

Deflection caused by the rest of load:

$$w_q = \frac{5f_qL^4}{384(EI)_{ef,inst}} \quad (3.69)$$

Quasi permanent load:

$$f_{qp} = g_{0k} + g_{1k} + \psi_2 q_k \quad (3.70)$$

$g_{0k}$  is the characteristic value of self weight,  $g_{1k}$  is the characteristic value of dead load,  $q_k$  is characteristic value of variable load.

The rest of variable load:

$$f_q = (1 - \psi_2)q_k \quad (3.71)$$

### Effective bending stiffness

$$(EI)_{ef,inst} = E_{1,inst}I_1 + \gamma_{1,inst}E_{1,inst}A_1\alpha_{1,inst}^2 + E_{2,inst}I_2 + \gamma_2E_{2,inst}A_2\alpha_{2,inst}^2 \quad (3.72)$$

$$(EI)_{ef,fin} = E_{1,fin}I_1 + \gamma_{1,fin}E_{1,fin}A_1\alpha_{1,fin}^2 + E_{2,fin}I_2 + \gamma_2E_{2,fin}A_2\alpha_{2,fin}^2 \quad (3.73)$$

Distance between the centroid of concrete slab and centroid gravity:

$$\alpha = \frac{h_1}{2} + \frac{h_2}{2} + t_i \quad (3.74)$$

$$\alpha_{2,inst} = \frac{\gamma_{1,inst}E_{1,inst}A_1\alpha}{\gamma_{1,inst}E_{1,inst}A_1 + \gamma_2E_{1,inst}A_2} \quad (3.75)$$

$$\alpha_{2,fin} = \frac{\gamma_{1,fin}E_{1,fin}A_1\alpha}{\gamma_{1,fin}E_{1,fin}A_1 + \gamma_2E_{2,fin}A_2} \quad (3.76)$$

Distance between the centroid of concrete slab and centroid gravity:

$$\alpha_{1,inst} = \alpha - \alpha_{2,inst} \quad (3.77)$$

$$\alpha_{1,fin} = \alpha - \alpha_{2,fin} \quad (3.78)$$

Slip modulus and  $\gamma$  factor

$$\gamma_{1,inst} = \left[ 1 + \frac{\pi^2 E_{1,inst} A_1 s}{K_{1,inst} L^2} \right]^{-1} \quad (3.79)$$

$$\gamma_{1,fin} = \left[ 1 + \frac{\pi^2 E_{1,fin} A_1 s}{K_{1,fin} L^2} \right]^{-1} \quad (3.80)$$

$$\gamma_2 = 1 \quad (3.81)$$

$$K_{1,inst} = nK_{ser,1} \quad (3.82)$$

$$K_{1,fin} = \frac{nK_{ser,1}}{(1 + K_{def})} \quad (3.83)$$

Modulus of elasticity for Concrete

$$E_{1,Inst} = E_{cm} \quad (3.84)$$

$$E_{1,fin} = \frac{E_{cm}}{(1 + \varphi)} \quad (3.85)$$

Modulus of elasticity for Timber

$$E_{2,Inst} = E_{0mean} \quad (3.86)$$

$$E_{1,fin} = \frac{E_{0mean}}{(1 + K_{def})} \quad (3.87)$$

**The rest of calculation for deflection and Vibration is given in Appendix A (Additional material).**

### 3.4.2 RC beam: Deflection

Deflection calculation, it is necessary to consider deflection by creep and shrinkage, so that future problems are minimized. Eurocode 2 [6], provides to methods to calculate design deflections. (1) Deflection for limit to span/depth ratio and (2) calculation by theoretical design guidelines provided by the Code. The design deflection calculations used in this thesis is by checking deflection using theoretical calculation.

Total deflection can be calculated using curvature. Where the total curvature is equal to (i) Average curvature due to load considering cracked and un-cracked section at SLS (quasi-permanent combination) and (ii) Average curvature due to shrinkage considering cracked and un-cracked section at SLS.

The final deflection for load combinations can be written as

$$v = \zeta \cdot \alpha_{II} + (1 - \zeta)\alpha_I \quad (3.88)$$

where  $\alpha_I$  and  $\alpha_{II}$  are the values for the parameter calculated for the un-cracked and fully cracked section respectively. The term  $\zeta$  is a distribution coefficient given by expression [EC 2 eq.7.19]

$$\zeta = 1 - \beta + \left( \frac{\sigma_{sr}}{\sigma_s} \right)^2 \quad (3.89)$$

$\zeta = 0$  for un-cracked section,  $\beta$  is the coefficient taking account the influence of duration of loading or repeated duration on average strain:

$\beta = 1.0$  for short term loading while

$\beta = 0.5$  for long term loading.

$\sigma_s$  is the stress in tension reinforcement of the cracked section,

$\sigma_{sr}$  is the stress in tension reinforcement of cracked section under loading conditions causing first cracking.

The reduction factor  $\zeta$  can also be written as

$$\zeta = 1 - \beta + \left( \frac{M_{cr}}{M_{Ed,SLS}} \right)^2 \quad (3.90)$$

where  $M$  is the design moment of the slab,  $M_{cr}$  is the cracking moment and is given as

$$M_{cr} = \frac{f_{ctm} \cdot I_{uc}}{(y)} \quad (3.91)$$

Where  $f_{ctm}$  is the mean value of cracking axial tensile strength of concrete.  $h$  is the height of cross section. Moment is given as in equation (3.4)

$$M_{ED,SLS} = q_{Ed} \cdot \frac{L^2}{2} \quad (3.92)$$

where  $q_{Ed}$  is for quasi permanent load combinations given in equation (3.3)

if the  $M_{cr} > M$  then the section will not crack, but if the  $M_{cr} < M$  the section will crack.

$$y = \frac{b_{eff} \cdot h_f \cdot y_1 + b_w \cdot (h - h_f) \cdot y_2}{b_{eff} \cdot h_f + b_w \cdot (h - h_f)} \quad (3.93)$$

where  $y$  is the natural axis depth,  $\alpha_e$  is the effective modulus ratio:

$$\alpha_e = \frac{E_s}{E_{c,eff}} \quad (3.94)$$

$E_{c,eff}$  is the effective modulus of elasticity of concrete, it is given as

$$E_{c,eff} = \frac{E_c}{1 + \varphi} \quad (3.95)$$

Where  $E_c$  is the secant modulus of elasticity of concrete denoted as  $E_c = (W_{cm})$ .

The moment of inertia of **uncracked section** is given as:

$$I_{uc} = I_1 + I_2 \quad (3.96)$$

Where  $I_1$  is given as:

$$I_1 = \frac{1}{12} \cdot b_{eff} \cdot h_f^3 + b_{eff} \cdot h_f \cdot (y - y_1)^2 \quad (3.97)$$

where  $I_2$  is given:

$$I_2 = \frac{1}{12} \cdot b_w \cdot (h - h_f)^3 + b_w \cdot (h - h_f) \cdot (y - y_2)^2 \quad (3.98)$$

where  $y_1 = h_f/2$  and  $y_2 = h_f + h/2$

The moment of inertia of **cracked section** is given as:

$$I_{cr} = \frac{(b - b_w) \cdot h_f^3}{12} + \frac{b_w \cdot (kd)^3}{3} + (b - b_w) \cdot h_f \cdot \left( kd - \frac{h_f}{2} \right)^2 \quad (3.99)$$

$$kd = \frac{\sqrt{C \cdot (2 \cdot d + h_f \cdot f) + (1 + f)^2} - (1 + f)}{C} \quad (3.100)$$

$$C = \frac{b}{n \cdot A_s} \quad (3.101)$$

$$f = \frac{h_f(b - b_w)}{n \cdot A_s} \quad (3.102)$$

where  $n = \alpha_e$

**1 - Curvature due to Load** is given as:

Curvature Due to load:



$$\alpha_{load} = \zeta \cdot \alpha_{II} + (1 - \zeta)\alpha_I \quad (3.103)$$

■ – *Uncracked – section* :

$$\alpha_I = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{uc}} \quad (3.104)$$

■ – *Cracked – section* :

$$\alpha_{II} = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} \quad (3.105)$$

**2- Curvature due to Shrinkage** is given as:  
Curvature Due to shrinkage:

$$\alpha_{shrinkage} = \zeta \cdot \alpha_{crII} + (1 - \zeta)\alpha_{ucI} \quad (3.106)$$

■ – *Uncracked – section* :

$$\alpha_{ucI} = \varepsilon_{cs} \cdot \alpha_e \cdot \frac{S_{uc}}{I_{uc}} \quad (3.107)$$

$$S_{uc} = A_s \cdot (d - X) \quad (3.108)$$

where  $\varepsilon_{cs}$  is the strain in steel compression.

■ – *Cracked – section* :

$$\alpha_{crII} = \varepsilon_{cs} \cdot \alpha_e \cdot \frac{S_{cr}}{I_{cr}} \quad (3.109)$$

$$S_{cr} = A_s \cdot (d - x) \quad (3.110)$$

where  $x = kd$

Deflection due to load and shrinkage:

$$v_{total} = \alpha_{load} + \alpha_{shrinkage} \quad (3.111)$$

Maximum deflection of the simply supported slab can be written as

$$v_{max} = K \cdot v_{total} \cdot L^2 \quad (3.112)$$

where  $K = 0.104$  for simply supported frame.

Allowable deflection

$$v_{allow} = \frac{L}{250} \quad (3.113)$$

Deflection ratio:

$$\frac{v_{allow}}{v_{max}} \cdot 100 \quad (3.114)$$

### 3.4.3 RC beam: Vibration

Check the structure vibrations are above the allowable natural frequency (8Hz)

$$f_1 \geq 8Hz \quad (3.115)$$

Lowest acceptable vibration frequency  $f_1$  for floor structures (RC slab) is calculated as:

$$f_1 = \frac{18}{\sqrt{\sigma_{tot.max}}} \quad (3.116)$$

$$\sigma_{tot.max} = \sigma_x + \sigma_y \quad (3.117)$$

$$\sigma_x = \frac{5 \cdot q_{slab} \cdot L^4}{348 \cdot EI_{slab}} \quad (3.118)$$

$$\sigma_y = \frac{5 \cdot q_{beam} \cdot B^4}{348 \cdot EI_{beam}} \quad (3.119)$$

where  $\sigma_x$  is the vertical displacement of the slab in x-axis, while  $\sigma_y$  is the vertical displacement of the beams (if any) caused by the weight of slab in y-axis, for this case  $\sigma_y = 0$  because slab rests on load carrying walls.  $L$  is the length of slab and  $B$  is strip of the width of section.

where

$$q = g + q_{dyn} \quad (3.120)$$

where  $g$  is the self-weight of the structure including flooring materials,  $q_{dyn}$  is the dynamic fraction of life load, which is chosen to be 10% of life load ( $q_k$ ).

For concrete (beam and slab) have same  $E_{cm}$ ; therefore  $E_{c,dyn}$  is give as:

$$E_{c,dyn} = E_{cm} \cdot 1.1 \quad (3.121)$$

Where  $E_{cm}$  is the secant modulus of elasticity of concrete.

### 3.5 Case 2: CLT Slab and RC Slab

In this section (Case 2), cross laminated timber slab is analyzed and compared with a reinforced concrete slab. Ultimate limit state and also serviceability limit state is check for both slabs. Loading action is mentioned in section 3.1. For the static analysis of case (2), a simply supported slab is chosen as shown in figures 3.5 and 3.6. The beam is supported by two load bearing walls on each end of the span length  $L$ .

Load combinations for ultimate limit state and serviceability limit state is motioned in section 3.1.1, and section 3.1.2 in page 14.

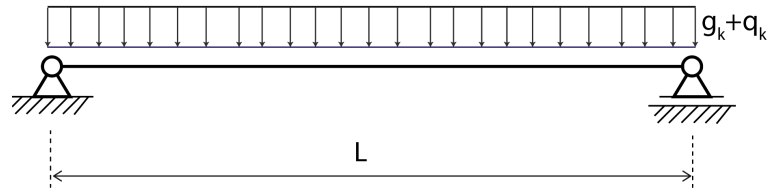


Figure 3.5: Simply supported slab loaded with uniformly distributed load.

(for case 2) The design bending moment ( $M_{Ed,2}$ ) and shear force ( $V_{Ed,2}$ ) for simply supporter beam is given in equations equations (3.122) and (3.123):

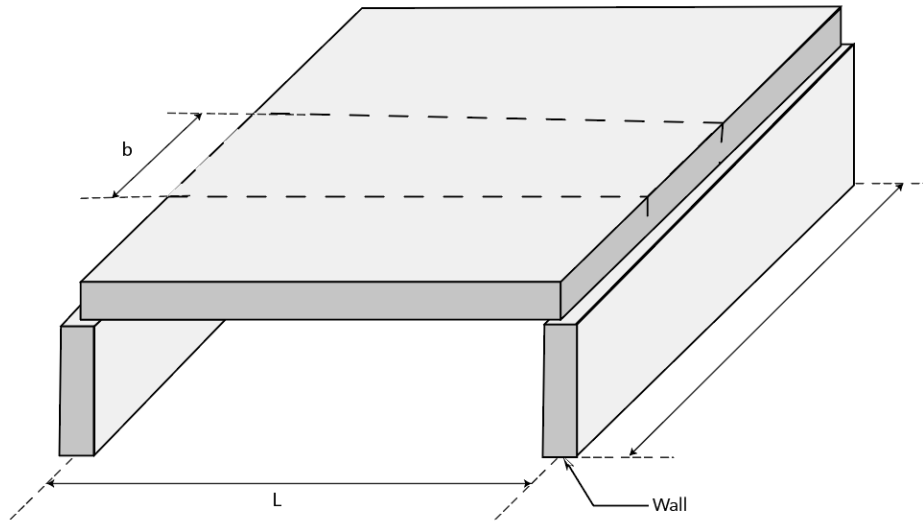
$$M_{Ed,2} = \frac{q_{Ed}L^2}{8} \quad (3.122)$$

$$V_{Ed,2} = \frac{q_{Ed}L}{2} \quad (3.123)$$

#### 3.5.1 CLT slab

As mentions in section 2.3, a standard CLT element consist of an odd number of layers. The direction of the top layers is placed in load bearing direction  $L$ . Layer thicknesses or heights are symbolized by  $t_1, t_2 \dots t_i$  from the bottom to top layer. Moment of inertia for the top layer (parallel to the grain direction) ( $I_{net,x}$ ) is stronger than layers that are perpendicular to the grain direction ( $I_{net,y}$ ). For this reason, only layers perpendicular to the grain direction is considered as load bearing layers. All layers of boards have the strength class C24 with service and safety class 1 and 3 respectively.

CLT is considered as a composite element, therefore  $\gamma$  method [23] is used to calculate the effective bending stiffness  $(EI)_{eff}$ . Horizontal shear deformations that are perpendicular to the design span length is considered. Therefore longitudinal layers are not taken into account in the analysis of this CLT-element. Exposed CLT slab is not sufficient to meet the design requirements, therefore, CLT need to be provided with flooring materials such as insulation layer to protect from sound,



**Figure 3.6:** Case 2: Simply supported slab supported by two walls .

plaster to protect from fire. in the designing process additional dead load of  $50 \text{ (kg/m}^2\text{)}$  is added [11].

In the analysis, To types of categories of variable loads are used as mentioned in section 3.1. Different number of layers with different thicknesses is examined.

Charecteristic value of self-weight  $g_k \text{ (kN/m}^3\text{)}$  for CLT is given: in equation equation (3.124):

$$g_k = (\rho_{mean} \cdot h_{CLT} + 80)g \quad (3.124)$$

Where:  $h_{CLT} \text{ (mm)}$  is the total height of CLT,  $\rho_{mean} \text{ (kN/m}^3\text{)}$  is the average density of layers, ( $g = 9.81 \text{ m/s}^2$ ) is gravitational force.

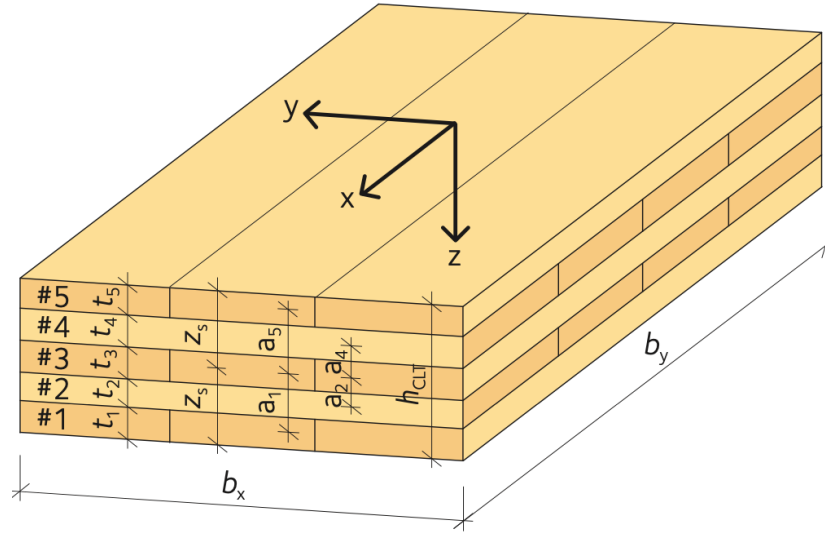


Figure 3.7: Definitions, directions and measurements of CLT. [11]

### 3.5.2 RC slab

Reinforced concrete slab is a solid concrete and consist of concrete with grade B25 and B30. The characteristic yield strength of steel is 500 (MPa). Calculation of structural capacities considered along the design span "L". Dimensions and the notations used for cross section of reinforced concrete slab are shown in figure 3.8.

Used input design loads are the same as loads on TCC section mentioned in section 3.1.

Compressive strength of concrete  $f_{ck}$  (MPa), steel reinforcement  $A_s$  ( $mm^2$ ), concrete density  $\rho_c$  ( $kg/m^3$ ), width of of slab is considered to be 1m , thickness of the slab  $h_c$  (mm),

Charecteristic value of self weight  $g_k$  ( $kN/m^3$ ) for RC slab is given: in equation equation (3.125):

$$g_k = (\rho_c h_c + 80)g \quad (3.125)$$

Additional load for flooring material (80 ( $kg/m^2$ )) is also added to the RC slab.  $g$  is gravitational force ( $g = 9.81m/s^2$ ).

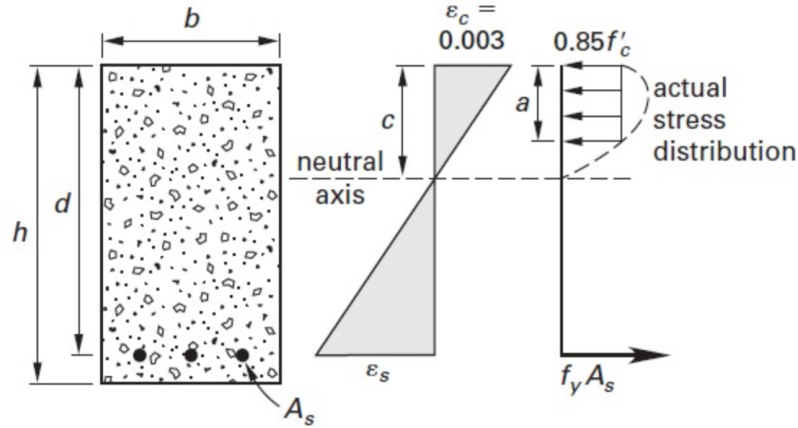


Figure 3.8: Reinforced concrete section, dimensions and the notation used [24].

### 3.6 Ultimate limit state check for Case (2)

Ultimate limit conditions should be fulfilled.

#### 3.6.1 CLT: Bending moment Capacity

The design bending stress  $\sigma_{m,d}$  (MPa) should be less than the design bending strength  $f_{m,d}$  (MPa).

$$\sigma_{m,d} \leq f_{m,d} \quad (3.126)$$

Where:

$$f_{m,d} = \frac{k_{mod} \cdot f_{m,k}}{\gamma_M} \quad (3.127)$$

$k_{mod}$  is the modification factor for duration of the load and content of moisture in the panel,  $f_{m,k}$  is the characteristic value of bending moment capacity, For strength class of C24  $f_{m,k} = 24$  (MPa), Partial factor of safety  $\gamma_M = 1.15$  in Norway for CLT.

$$\sigma_{m,d} = \frac{M_{Ed,2}}{W_{x,net}} \quad (3.128)$$

Where design bending moment  $M_{Ed,2}$  is given in equation (3.122)

$$W_{x,net} = \frac{2 \cdot I_{x,net}}{h_{CLT}} \quad (3.129)$$

$W_{x,net}$  is the panel's net moment of resistance,  $I_{x,net}$  is the net moment of inertia in the top layer direction.  $h_{CLT}$  is the height of CLT.

Calculating the net-moment of inertia. loading along the x-axis:

$$I_{x,net} = \sum \frac{E_{x,i}}{E_{ref}} \cdot \frac{b_x t_i^3}{12} + \sum \frac{E_{x,i}}{E_{ref}} \cdot b_x t_i \alpha_i^2 \quad (3.130)$$

Where  $E_{x,i}$  is the elastic modulus of the layer,  $E_{ref}$  is the elastic modulus of the reference layer. ( $E_{x,i}/E_{ref} = 1$ ),  $b_x = 1m$  strip. As mentioned in 3.5.1 longitudinal layers that are parallel to the x-axis is calculated.

The ratio of design bending stress and strength is given as %:

$$\frac{\sigma_{m,d}}{f_{m,d}} \cdot 100 \quad (3.131)$$

### 3.6.2 CLT: Shear Check

(1) **Rolling shear** force of transverse layers (Load bearing layers) should satisfy the shear condition:

$$\tau_{Rv,d} \leq f_{v,Rd} \quad (3.132)$$

Rolling shear strength  $f_{v,Rd}$  of each panel is expressed as:

$$f_{v,Rd} = \frac{k_{mod} \cdot f_{v,Rd}}{\gamma_M} \quad (3.133)$$

The design rolling shear stress is expressed as:

$$\tau_{Rv,d} = \frac{S_{Rx,net} \cdot V_{Ed}}{I_{x,net} \cdot b_x} \quad (3.134)$$

$I_{x,net} (mm^4)$  is the net moment of inertia in the top layer direction it is given in equation (3.130),  $V_{Ed} (kN)$  is the design shear force,  $S_{Rx,net} (mm^3)$  is the static rolling shear in the main direction.  $S_{Rx,net}$  is expressed as:

$$S_{Rx,net} = \sum_1^{mL} \frac{E_{x,i}}{E_{ref}} b_x t_i \alpha_i \quad (3.135)$$

Where ( $i = 1$ ),  $\alpha_i (mm)$  is the distance from center of gravity of a layer to the main center of gravity of the whole panel. ( $b_x = 1m$ ) is a strip of the panel (slab)  $t_i$  is the thickness of bottom layer.  $mL$  are layers closest to global center of gravity.

(2) For **Longitudinal shear** force of (parallel to the grain direction) should satisfy the following condition:

$$\tau_{v,d} \leq f_{v,d} \quad (3.136)$$

Design shear strength  $f_{v,d}$  of each panel is expressed as:

$$f_{v,d} = \frac{k_{mod} \cdot f_{v,k}}{\gamma_M} \quad (3.137)$$

$f_{v,d}$  is the characteristic shear strength (for strength class C24),  $f_{v,d} = 4(MPa)$

The design longitudinal shear stress  $\tau_{v,d}(MPa)$  is expressed as:

$$\tau_{v,d} = \frac{S_{x,net} \cdot V_{Ed}}{I_{x,net} \cdot b_x} \quad (3.138)$$

Where  $S_{x,net} (mm^3)$  is the static moment of longitudinal shear. if the center of gravity is located in the effected of the longitudinal layer  $S_{x,net}$  is expressed as:

$$S_{x,net} = \sum_1^{kL} \frac{E_{x,i}}{E_{ref}} \cdot b_x t_i \alpha_i + \sum_1^{kL} \frac{E_{x,i}}{E_{ref}} \cdot b_x \cdot \frac{t_k^2}{(4 \cdot 2)} \quad (3.139)$$

Where ( $i = 1$ ),  $t_k(mm)$  is thickness of effected longitudinal layer (e.g.,  $t_k$  layer in figure 3.7 is  $t_3$ ),  $kL$  are layers closest to global center of gravity.

if the center of gravity is not located in the effected of the longitudinal layer  $S_{x,net}$  is expressed as:

$$S_{x,net} = \sum_1^{kL} \frac{E_{x,i}}{E_{ref}} b_x t_i \cdot \alpha_i \quad (3.140)$$

The ratio of rolling shear force is given as %:

$$\frac{f_{v,Rd}}{\tau_{Rv,d}} \cdot 100 \quad (3.141)$$



### 3.6.3 RC Slab: Bending Moment Capacity

In this section, the design criterion for moment capacity is checked. Moment capacity should be enough to carry the design bending moment capacity.

$$M_{Ed} \leq M_{Rd} \quad (3.142)$$

Where,  $M_{Ed}(kNm)$  is the applied moment at the limit state of strength of slab,  $M_{Ed}$  is given in equation (3.122),  $M_{Rd}(kNm)$  is the ultimate moment capacity of concrete slab.

Some of design values of concrete material properties according to EN1992-1-1 is given as,

$$f_{cd} = \frac{a_{cc} \cdot f_{ck}}{\gamma_c} \quad (3.143)$$

$$f_{yd} = \frac{f_{yk}}{\gamma_s} \quad (3.144)$$

Where  $f_{ck}$  is the characteristic cylinder compressive strength,  $f_{yk}$  is the characteristic yield strength of the steel reinforcement ( $f_{yk} = 500MPa$ ),  $\gamma_c$  and  $\gamma_s$  are the partial safety factor for concrete and steel respectively.  $a_{cc}$  Coefficient takes into account the long term effects on the compressive strength and of unfavorable effects resulting from the way the load is applied, it is taken here  $a_{cc} = 0.85$ . partial safety factor are taken as  $\gamma_c = 1.5$  and  $\gamma_s = 1.15$ .

The required steel area  $A_s(mm^2)$  of tension reinforcement for singly reinforced concrete slab is given as:

$$A_s = \frac{M_{Ed}}{f_{yd} \cdot z} \quad (3.145)$$

Where  $z(mm)$  is the lever arm distance between moments  $f_s$  and  $f_c$ , it is given as:

$$z = \min \left[ d \left( 0.5 + \sqrt{\left( 0.25 - \frac{k}{1.134} \right)} \right), 0.95 \cdot d \right] \quad (3.146)$$

Where  $d$  is the effective depth of section, it is measured from the top of the slab to the center of the area of reinforcement For singular reinforced concrete section the term  $k$  is given as:

$$k = \frac{M_{Ed}}{b \cdot d^2 \cdot f_{ck}} \quad (3.147)$$

where  $b$  is a strip of the slab width  $b = 1m$

The minimum steel area of reinforcement for the section is:

$$A_{s,min} = 0.26 \cdot b \cdot d \cdot \frac{f_{cmt}}{f_{yk}} \quad (3.148)$$

Where  $f_{ctm}$  (MPa) is the mean tensile strength  
**Moment capacity** of the slab is given as:

$$M_{Rd} = 0.8 \cdot d \cdot x \cdot f_{cd} (d - 0.4 \cdot x) \quad (3.149)$$

where  $x$  (mm) is the distance from the top of the slab to the neutral axis and it is given as:

$$x = \frac{f_{yd} \cdot A_s}{0.8 \cdot b \cdot f_{cd}} \quad (3.150)$$

The utilization ratio for moment capacities can be expressed as % :

$$\frac{M_{Ed}}{M_{Rd,c}} \cdot 100 \quad (3.151)$$

### 3.6.4 RC: Shear Check

Calculated design shear force should be lower than allowable shear force capacity, therefore the slab should fulfill this criterion.

$$V_{Ed} \leq V_{Rd,c} \quad (3.152)$$

where  $V_{Ed}$  (kN) is the design value of transverse shear force of the slab,  $V_{Rd,c}$  (kN) is Design shear resistance of the slab.  $V_{Rd,c}$  is given as:

$$V_{Rd,c} = (C_{Rd,c} \cdot k (100\rho_l \cdot f_{ck})^{1/3}) \cdot b \cdot d \quad (3.153)$$

where  $C_{Rd,c}$  is coefficient driven from testes (recommended value=0.12) and expressed as:

$$C_{Rd,c} = \frac{0.18}{\gamma_c} = 0.12 \quad (3.154)$$

where value of k is expressed as

$$k = 1 + \sqrt{\frac{200}{d}} \leq 2.0 \quad (3.155)$$

Percentage of tensile reinforcement is:

$$\rho_l = \frac{A_{sl}}{b \cdot d} \quad (3.156)$$

Where  $A_{sl}$  is the Area of tensile reinforcement,  $A_{sl} = A_s$   
 The design shear should not be less than

$$V_{Rd,c} = v_{min} \cdot b \cdot d \quad (3.157)$$

where

$$v_{min} = 0.035 \cdot \sqrt{k^3 \cdot f_{ck}} \quad (3.158)$$

The utilization ratio for shear capacities can be expressed as % :

$$\frac{V_{Ed}}{V_{Rd,c}} \cdot 100 \quad (3.159)$$

## 3.7 Serviceability limit state check for Case (2)

### 3.7.1 CLT: Deflection

In this section, a check is made on whether the calculated deflection of the beam is lower than recommended value

$$w_{fin} \leq \frac{L}{250} \quad (3.160)$$

Where: Short-term deformation of characteristic load:

$$w_{inst} = w_{g,k} + w_{q,k} \quad (3.161)$$

#### Deflection of beam by quasi-permanent load

Consider creep :

$$w_{g,k} = \frac{5 \cdot g_k \cdot L^4}{384 \cdot E_{x,meam} \cdot I_{x,ef}} \quad (3.162)$$

$$w_{q,k} = \frac{5 \cdot q_k \cdot L^4}{384 \cdot E_{x,meam} \cdot I_{x,ef}} \quad (3.163)$$

**Final** deformation because of creep on quasi permanent load:  $k_{def} = 0.85$  for service class 1.

$$[w_{fin} = w_{inst} + w_{creep}] \quad (3.164)$$

$$w_{creep} = w_{fin,g} + w_{fin,q} \quad (3.165)$$

$$w_{fin,g} = w_{g,k}(1 + k_{def}) \quad (3.166)$$

$$w_{fin,q} = w_{q,k}(1 + \Psi_2 \cdot k_{def}) \quad (3.167)$$

Deflection ratio is given as %:

$$\frac{w_{fin}}{\frac{L}{250}} \cdot 100 \quad (3.168)$$

### 3.7.2 CLT: Vibration

Check the structure vibrations are above the allowable natural frequency (8Hz)

$$f_1 \geq 8Hz \quad (3.169)$$

Lowest acceptable vibration frequency  $f_1$  for floor structures (CLT) is calculated as:

$$f_1 = \frac{\pi}{2 \cdot L^2} \sqrt{\frac{(EI)_L}{m}} \quad (3.170)$$

Where  $(EI)_L$  (MPa) is the mean value of modulus of elasticity. For strength class C24  $(EI)_L = 11000$  (MPa),  $m$  (kg/m<sup>2</sup>) is mass per unit area,  $L$  is the span length of floor.

### 3.7.3 RC slab: Deflection

For deflection calculation it is necessary to consider deflection by creep and shrinkage, to that future problems are minimized. Eurocode 2 [6], provides to methods to calculate design deflections. (1) Deflection for limit to span/depth ratio and (2) calculation by theoretical design guidelines provided by the Code. The design deflection calculations used in this thesis is by checking deflection using theoretical calculation.

The total deflection can be calculated using curvature. Where the total curvature is equal to (i) Average curvature due to load considering cracked and un-cracked section at SLS (quasi-permanent combination) and (ii) Average curvature due to shrinkage considering cracked and un-cracked section at SLS.

The final deflection for load combinations can be written as

$$v = \zeta \cdot \alpha_{II} + (1 - \zeta)\alpha_I \quad (3.171)$$

where  $\alpha_I$  and  $\alpha_{II}$  are the values for the parameter calculated for the un-cracked and fully cracked section respectively. The term  $\zeta$  is a distribution coefficient given by expression [EC 2 eq.7.19]

$$\zeta = 1 - \beta + \left( \frac{\sigma_{sr}}{\sigma_s} \right)^2 \quad (3.172)$$

$\zeta = 0$  for un-cracked section,  $\beta$  is the coefficient taking account the influence of duration of loading or repeated duration on average strain:

$\beta = 1.0$  for short term loading while

$\beta = 0.5$  for long term loading.

$\sigma_s$  is the stress in tension reinforcement of the cracked section,

$\sigma_{sr}$  is the stress in tension reinforcement of cracked section under loading conditions causing first cracking.

The reduction factor  $\zeta$  can also be written as

$$\zeta = 1 - \beta + \left( \frac{M_{cr}}{M_{ED,SLS}} \right)^2 \quad (3.173)$$

where  $M$  is the design moment of the slab,  $M_{cr}$  is the cracking moment and is given as

$$M_{cr} = \frac{f_{ctm} \cdot I_{uc}}{(h - X)} \quad (3.174)$$

Where  $f_{ctm}$  is the mean value of cracking axial tensile strength of concrete.  $h$  is the height of cross section. Moment is given as in equation (3.4)

$$M_{ED,SLS} = q_{Ed} \cdot \frac{L^2}{2} \quad (3.175)$$

where  $q_{Ed}$  is for quasi permanent load combinations given in equation (3.3) if the  $M_{cr} > M$  then the section will not crack, but if the  $M_{cr} < M$  the section will crack.

$$X = \frac{h \cdot b \cdot \frac{h}{2} + (\alpha_e - 1) \cdot A_s \cdot d}{h \cdot b + (\alpha_e - 1) \cdot A_s} \quad (3.176)$$

where  $X$  is the natural axis depth,  $b$  is strip of the floor span  $b = 1m$ ,  $h(mm)$  is the height of the cross section (slab),  $A_s$  is the amount of steel area.  $\alpha_e$  is the effective modulus ratio:

$$\alpha_e = \frac{E_s}{E_{c,eff}} \quad (3.177)$$

$E_{c,eff}$  is the effective modulus of elasticity of concrete, it is given as

$$E_{c,eff} = \frac{E_c}{1 + \varphi} \quad (3.178)$$

Where  $E_c$  is the secant modulus of elasticity of concrete denoted as  $E_c = (W_{cm})$ .

The moment of inertia of **uncracked section** is given as:

$$I_{uc} = \frac{1}{12} \cdot b \cdot h^3 + b \cdot h \cdot \left( \frac{h}{2} - X \right) + (\alpha_e - 1) \cdot A_s \cdot (d - X)^2 \quad (3.179)$$

The moment of inertia of **cracked section** is given as:

$$I_{cr} = b \cdot \frac{(kd)^3}{3} + n \cdot A_s \cdot (d - kd)^2 \quad (3.180)$$

$$kd = \frac{\sqrt{2 \cdot d \cdot B + 1} + 1}{B} \quad (3.181)$$

$$B = \frac{b}{n \cdot A_s} \quad (3.182)$$

where  $n = \alpha_e$

**1 - Curvature due to Load** is given as:

Curvature Due to load:

$$\alpha_{load} = \zeta \cdot \alpha_{II} + (1 - \zeta)\alpha_I \quad (3.183)$$

■ – *Uncracked – section* :

$$\alpha_I = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{uc}} \quad (3.184)$$

■ – *Cracked – section* :

$$\alpha_{II} = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} \quad (3.185)$$

**2- Curvature due to Shrinkage** is given as:

Curvature Due to shrinkage:

$$\alpha_{shrinkage} = \zeta \cdot \alpha_{crII} + (1 - \zeta)\alpha_{ucI} \quad (3.186)$$

■ – *Uncracked – section* :

$$\alpha_{ucI} = \varepsilon_{cs} \cdot \alpha_e \cdot \frac{S_{uc}}{I_{uc}} \quad (3.187)$$

$$S_{uc} = A_s \cdot (d - X) \quad (3.188)$$

where  $\varepsilon_{cs}$  is the strain in steel compression.

■ – *Cracked – section* :

$$\alpha_{crII} = \varepsilon_{cs} \cdot \alpha_e \cdot \frac{S_{cr}}{I_{cr}} \quad (3.189)$$

$$S_{cr} = A_s \cdot (d - x) \quad (3.190)$$

where  $x = kd$

Deflection due to load and shrinkage:

$$\nu_{total} = \alpha_{load} + \alpha_{shrinkage} \quad (3.191)$$

Maximum deflection of the simply supported slab can be written as

$$\nu_{max} = K \cdot \nu_{total} \cdot L^2 \quad (3.192)$$

where  $K = 0.104$  for simply supported frame.

Allowable deflection

$$v_{allow} = \frac{L}{250} \quad (3.193)$$

Deflection ratio %:

$$\frac{v_{allow}}{v_{max}} \cdot 100 \quad (3.194)$$

### 3.7.4 RC slab: Vibration

Check the structure vibrations are above the allowable natural frequency (8Hz)

$$f_1 \geq 8Hz \quad (3.195)$$

Lowest acceptable vibration frequency  $f_1$  for floor structures (RC slab) is calculated as:

$$f_1 = \frac{18}{\sqrt{\sigma_{tot.max}}} \quad (3.196)$$

$$\sigma_{tot.max} = \sigma_x + \sigma_y \quad (3.197)$$

$$\sigma_x = \frac{5 \cdot q_{slab} \cdot L^4}{348 \cdot EI_{slab}} \quad (3.198)$$

$$\sigma_y = \frac{5 \cdot q_{beam} \cdot B^4}{348 \cdot EI_{beam}} \quad (3.199)$$

where  $\sigma_x$  is the vertical displacement of the slab in x-axis, while  $\sigma_y$  is the vertical displacement of the beams (if any) caused by the weight of slab in y-axis, for this case  $\sigma_y = 0$  because slab rests on load carrying walls.  $L$  is the length of slab and  $B$  is strip of the width of section.

where

$$q = g + q_{dyn} \quad (3.200)$$

where  $g$  is the self-weight of the structure including flooring materials,  $q_{dyn}$  is the dynamic fraction of life load, which is chosen to be 10% of life load ( $q_k$ ).

For concrete (beam and slab) have same  $E_{cm}$ ; therefore  $E_{c,dyn}$  is give as:

$$E_{c,dyn} = E_{cm} \cdot 1.1 \quad (3.201)$$

Where  $E_{cm}$  is the secant modulus of elasticity of concrete.





## Chapter 4

# Environmental impact

Today's buildings are known to be one of the main contributors of environmental emissions and has an impact on the consumers over its life cycle. But the world is moving towards achieving sustainability [25]. This encourages global communities to investigate all possibilities of reducing energy consumption and reduce the impact of environmental emissions. Environmental impact is considered in terms of the material emission of carbon dioxide, which is one of the greenhouse gases.

The Global Warming Potential (GWP) was developed in order to compare the global warming impact of different gases and measures how much energy is absorbed in a given period (Normally 100 years) against  $CO_2$ . Greenhouse gases are calculated using a measure known as "carbon dioxide equivalent"  $CO_{2e}$  and used to standardize climate effects of several greenhouse gases.

In the tables 4.1 - 4.4 shows the emission and sequestration of carbon dioxide ( $kg\ CO_{2e} / kg$ ) to and from the air from timber and concrete materials. The results are based on findings [26]. These values indicate emissions during life cycle of the materials.

### 4.1 Environmental impact of Concrete

Concrete is one of the most used building material for constructions, as mentioned in section 2.2, consisting of hard substance known as aggregate (normally made from different types of sand and gravel), and bonded together by cement, water and possibly small amounts of various chemicals known as *admixtures*.

The consumption of energy is the biggest environmental impact with cement and concrete production [27]. Most of the energy is consumed by the production of cement. This includes direct usage of fuel for mining and transporting of the material. The reliance of coal leads to higher levels of  $CO_2$  emissions, nitrous oxide and Sulphur. Besides  $CO_2$  emissions from concrete and cement production, dust is also another factor which contributes a considerable amount of air-pollutant emissions.

Carbonation process takes place during the life time of concrete, this process helps concrete to take up carbon through the reaction of  $CO_2$  with calcium hydroxide

and calcium silicate hydrate to form calcite ( $CaCO_3$ ) [28]. The amount of absorbed  $CO_2$  depends of the exposed surface area of concrete[29]. However, carbonation can have an undesired effect which is the induction of corrosion in the steel reinforcement, this may require replacement or repair of the concrete cover [29].

Life cycle assessment of concrete or broadly speaking most of building materials go through several stages [30].

$$I = I_{Extraction} + I_{Manufacture} + I_{Onsite} + I_{Operation} + I_{Demolition} + I_{Recycling} + I_{Disposal} \quad (4.1)$$

where I represents the life cycle environmental impact.

The environmental impact of concrete/materials is summarized in equation (4.1), from extraction of the raw materials to the disposal of the material at the end of life time. Emission of  $CO_2$  takes place throughout all the above mentioned stages.

At the end of life stage of concrete from demolition to the disposal is divided into four main Modules C1-C4, according to EN 15804 [31]. Modules are given as follows:

- **C1** - deconstruction of the concrete structure.
- **C2** - transportation of the recycling share of concrete debris to a recycling plant and transportation of remaining waste to final disposal.
- **C3** - Conventional recycling, including size reduction.
- **C4** - wet disposal and management of disposal site.

When concrete structure is demolished and crushed to a smaller proportion, some of the crushed concrete goes to a further recycle, while the remaining goes to landfill. Crushed concrete goes through further process and end products can be used in road construction and reuse as an aggregate for new concrete production [29]. In tables 4.1 and 4.2 shows the environmental impact of concrete, with and without stored carbon,

**Table 4.1:** Emission and sequestration of carbon dioxide to and from the air from concrete, (Excluding carbon storage) [26].

Material	Input value (kg $CO_{2e}$ / kg)	Comment
Concrete	0.3	Excl. carbon storage
	0.24	Excl. carbon storage
	0.23	Excl. carbon storage
	0.2	Excl. carbon storage
	0.21	Excl. carbon storage
	0.2	Excl. carbon storage
	0.15	Excl. carbon storage
<b>Average</b>	<b>0.22</b>	<b>Excl. carbon storage</b>

**Table 4.2:** Emission and sequestration of carbon dioxide to and from the air from concrete, (Including carbon storage) [26].

Material	Input value (kg $CO_{2e}$ / kg)	Comment
Concrete	0.25	Include. carbon storage
	0.23	Include. carbon storage
	0.2	Include. carbon storage
	0.18	Include. carbon storage
	0.16	Include. carbon storage
	0.15	Include. carbon storage
<b>Average</b>	<b>0.20</b>	<b>Include. carbon storage</b>

## 4.2 Environmental impact of Timber

To minimize environmental impact of constructions, the use of reusable materials is recommended, and wood is one the renewable materials. The advantage of wood as a construction material is that wood reduces carbon footprint through (1) carbon storage and (2) avoided greenhouse emissions (e.g.,  $CO_2$ ).

As trees get older carbon dioxide  $CO_2$  is absorbed from the atmosphere, and release oxygen ( $O_2$ ). Absorbed carbon is stored in the tree (e.g., leaves, roots and soil). When trees start to decay, stored carbon is released slowly and steadily. But when trees are burned (wildfire) the stored carbon is released instantly. Life cycle begins again as forest regenerates new small seedlings and from there start to absorb carbon.

Exposed wooden structures to the atmospheric air has the ability to capture and store  $CO_2$  from the atmosphere. This happens through a process known as "carbon sequestration" [32], where by carbon dioxide is removed from the atmosphere and store for the long term. Carbon sequestration process creates a "negative emission" as can be see in tables 4.1 and 4.2. For this study it is expected cross laminated timber and wood to have the same effects as glue laminated timber.

Many research works investigated the **potential reuse** of timber. But the mechanical properties of old timber is reasonably complex, because material properties of timber that has been in service for many years, are a consequence of several factors, namely the state of conservation, patterns and load history, the original quality of timber and the damages encountered during the service life of mounting /dismounting operations[33]. Nevertheless the material can still be reused in four stages [34] as shown in figure 4.1.

- Reprocess and reuse: parts of a timber or a complete section of timber might be salvaged for reuse in a different building. These sections can also be reused in the production of furniture.
- Recycle: Timber is chipped and shredded into smaller pieces and turned into boards or animal bedding. The store carbon in the early life continues to be stored

- Energy Recovery: The by-products and wood waste are burned in an energy recovery facility. In this process the stored carbon is returned to the atmosphere as a  $C_{O_2}$  in smaller quantities.
- Landfill: Timber stores more carbon for the long term, but decomposition of the cellulose and hemicellulose that happen in the process produces a landfill gas (LEG), which consists of 50% methane and GHG. The impact of landfill of timber is decided by the rate of decay[34].

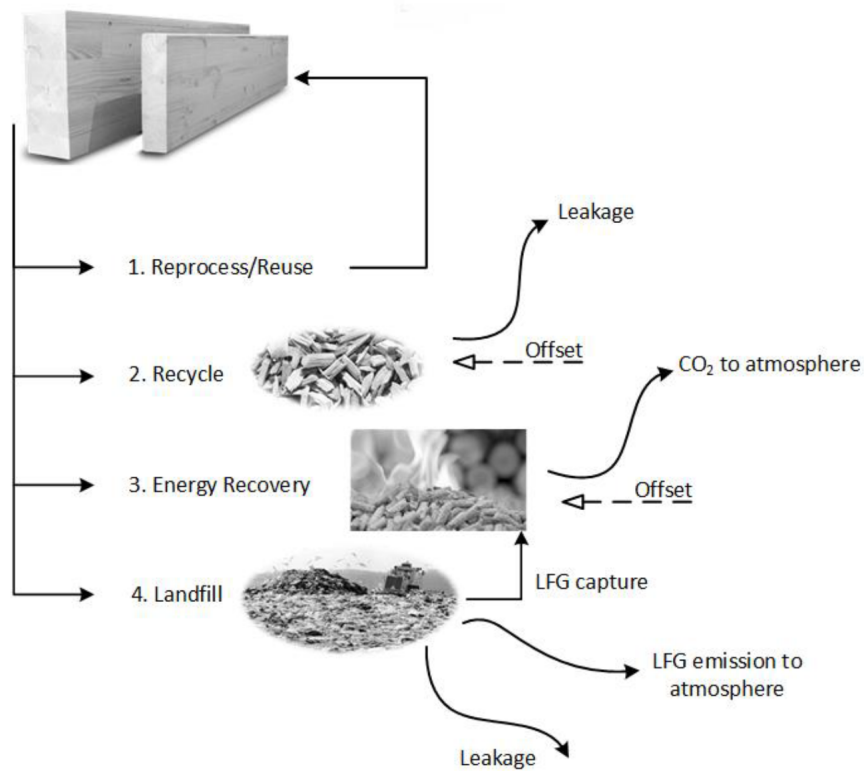


Figure 4.1: Potential reuse of timber at the end of life. [34]

Table 4.3: Emission and sequestration of carbon dioxide to and from the air from timber, (Excluding carbon storage) [26].

Material	Input value (kg $CO_{2e}$ / kg)	Comment
Clulam timber	0.6	Excl. carbon storage
	0.5	Excl. carbon storage
	0.4	Excl. carbon storage
	0.2	Excl. carbon storage
<b>Average</b>	<b>0.43</b>	<b>Excl. carbon storage</b>

**Table 4.4:** Emission and sequestration of carbon dioxide to and from the air from timber, (including carbon storage) [26].

Material	Input value (kg $CO_{2e}$ / kg)	Comment
Clulam timber	-1	Include. carbon storage
	-1.1	Include. carbon storage
	-1.2	Include. carbon storage
	-1.4	Include. carbon storage
	-1.6	Include. carbon storage
<b>Average</b>	<b>-1.26</b>	<b>Include. carbon storage</b>

### 4.3 Input values

The sequestration of carbon dioxide and to from the air from timber, wood and concrete materials shown in table 4.3 to ???. These values consider the complete life cycle emissions of the construction materials. The values depend also the emissions in the production process. In this study the mean value of several production processes has been used to calculate the total amount of  $CO_2$  emission:

#### ■ Case 1

**For RC beam and TCC beam**

$$CO_{2e} = XgL(kg) \quad (4.2)$$

where  $g$  is the weight of the beam per meter (kg/m). This is for both the T-shaped reinforced concrete beam and TCC beam. The difference is the emission value for concrete and wood.

#### ■ Case 2

**For CLT element**

$$CO_{2e} = XbhL\rho_{mean} \quad (4.3)$$

where  $\rho_{mean}$  is the mean density of the plates ( $kg/m^3$ ),  $L$  is the span length for the slab.  $X$  is the value for sequestration (emission), ( $X= 0.43$  when carbon storage is excluded and  $X= -1.26$  when carbon storage is included)

**For Concrete slab**

$$CO_{2e} = X_c bh_c L\rho \quad (4.4)$$

where  $X_c$  is the mean value of emission ( $X_c = 0.22$  when carbon storage is excluded and  $X= 0.20$  when carbon storage is included),  $h_c$  is the height of concrete slab,  $\rho$  is the density of concrete.



## Chapter 5

# Cost estimation of timber composite vs reinforced concrete beams/slabs

In the calculation of the economy, this study takes into account mainly two things. Firstly, material cost, which covers costs of the material before it is delivered to the construction site. Secondly “ready-to- assemble” cost, which include all the cost that are required to assemble and complete the construction. Logistics are not considered in the calculation, since the delivery distance of the material vary.

A number of factors affect to get an accurate price for the building materials, such as material and labor cost, logistics and also current economic crises (Covid-19), therefore prices given in this study are estimated.

Two different cases are analyzed, where case one is compared timber-concrete composite beam against a reinforced concrete beam, both beams have the same cross section and are constructed in place. Where case 2 is compared CLT slab and Reinforced concrete slab. The whole area of slab is included in the calculation and divided by the width of the slab (B). Due to lack of enough data, labor costs are roughly estimates.

The final costs are limited to Norway and some of the neighboring countries, therefore, cost of the studied materials differ from country to another. The ratio between the costs is given in the equation equation (5.1) and represents the percentage of the amount of work and other requirements the material cost stands for. Cost ratio is given as %:

$$\frac{\text{Material} - \text{cost}}{\text{Cost of (ready - to - assemble)}} \cdot 100 \quad (5.1)$$





## Chapter 6

# Result And Discussion

### 6.1 Structural

In the calculation on the studied cases only two imposed load categories are considered as mentioned in section section 3.1. Category A are areas for domestic and residential activities occur, with value  $q_k=2\text{kN/m}^2$ . Category C5 are areas where a large gathering happens e.g., concert and sports halls, which has value of  $q_k=5\text{kN/m}^2$ . A variety of different lengths, heights and thickness are chosen for both cases, so that the ultimate (ULS) and the serviceability limit states (SLS) are fulfilled. Concrete class use in for both reinforced concrete beam and slab B25 and B30, water to cement ratio is  $W/C=0.55$ . The age of loading concrete is set to be after 28days, and service class of 50years. Cement type of R (Norcem Anleggsement FA). The available timber layers for CLT in the market are up to 8 layers but this study is limited to 7 layers. All the layers have the same strength class of C24 for imposed load of category A, and strength class of C30 for imposed load category C5. Service class 1. Safety class 3. The result shown in this chapter are exceedance threshold values, except for the length, width and vibration. If the exceedance threshold value is above 100 than this means that the value is exceeding the allowable limit, and for that reason fail the design criterion.

#### 6.1.1 TCC beam

The results for the TCC are summarized in tables 6.1 and 6.2. This beam consists of a top concrete slab part and a lower timber joist beam. The calculation of ultimate limit state, normal stresses at the bottom of concrete part are above the allowable limits (tensile stress), the utilization ratio exceeds 100% for span lengths  $L > 2.5\text{m}$  and imposed load of category C5. For the lower imposed load, all the selected beam lengths, normal stresses at the bottom of concrete exceed the tensile strength of concrete and the limit values. But for the sake of simplicity the shear force and bending moment is carried by the timber beam. Deflection is decisive for the design of the beam for the lower imposed loads when length  $L > 6\text{m}$ . In both categories A and C5, the natural frequency in less than the limit value

therefore one may consider increasing the width of the timber joist then it can be theoretically be possible so reduce natural frequency of the structure.

**Table 6.1:** Structural analysis for TCC beam, Category A, Timber strength class C24, Concrete strength class C25/30.

Length (m)	RC Top stress	RC Bottom stress	Timber Bending	Timber shear	Deflection	Natural frequency f1 (HZ)
2	11.0	104.9	3,9	12.9	6.4	66.2
2.5	15.4	130.6	7,4	16.2	10.5	42.4
3	20.3	152.8	12,0	19.6	16.0	29.5
3.5	25.8	172.8	17,6	23.1	23.0	21.7
4	31.9	191.7	24,3	26.8	31.9	16.6
4.5	38.6	210.3	32,0	30.7	42.8	13.1
5	46.1	229.1	40,6	34.6	56.0	10.6
5.5	54.2	248,5	50,2	38,6	71,8	8,8
6	63,1	268,6	60,8	53.7	90,5	7,4

**Table 6.2:** Structural analysis for TCC beam, Category C5, Timber strength class C30, Concrete strength class C30/37.

Span length (m)	RC Top stress	RC Bottom stress	Timber Bending	Timber shear	Deflection	Natural frequency f1 (HZ)
2	8.5	74.1	3.6	8.3	3.8	64.5
2.5	11.9	91.9	6.7	10.4	7.8	41.3
3	15.7	107.4	10.8	12.7	11.9	28.7
3.5	19.9	121.6	15.7	15.0	13.8	21.1
4	24.7	135.3	21.6	17.4	19.1	16.1
4.5	30.0	148.9	28.4	19.9	25.7	12.7
5	35.8	162.8	36.0	22.8	42.2	10.3
5.5	42.2	177.3	44.5	25.1	54.3	8.5
6	49.2	192.5	53.7	27.7	68.6	7.2

### 6.1.2 Reinforced concrete beam

The analyzed results for reinforced concrete beam is summarized in tables 6.3 and 6.4. This beam satisfies almost all the design criterion for ultimate and serviceability limits. The only concern is that for both higher and lower imposed loads as beam length is increased the natural frequency get closer to the design limit (8Hz).

**Table 6.3:** Structural analysis for RC beam, Category A, Concrete strength class C25/30.

Span L (m)	Steel Area (mm <sup>2</sup> )	Moment	Shear	Deflection	Natural Frequency f1 (HZ)
2	28,3	6,4	29,9	44,9	102,2
2,5	50,3	10,1	37,4	32,0	63,6
3	50,3	14,6	44,9	22,0	43,2
3,5	78,5	20,0	52,5	16,3	31,0
4	113	26,3	57,2	14,5	23,2
4,5	113	33,3	64,3	13,8	17,9
5	157	40,8	63,9	16,0	14,2
5,5	157	49,3	70,3	19,8	11,6
6	200	60,1	71,0	26,5	9,5

**Table 6.4:** Structural analysis for RC beam, Category C5, Concrete strength class C30/37.

Span L (m)	Steel Area (mm <sup>2</sup> )	Moment	Shear	Deflection	Natural Frequency f1 (HZ)
2	28,3	6,1	30,8	51,2	94,9
2,5	35,0	9,5	38,6	35,0	58,4
3	50,3	13,7	46,4	25,4	39,0
3,5	78,5	18,8	54,2	19,5	27,6
4	113,0	24,6	60,7	16,2	20,5
4,5	113,0	31,1	68,3	14,8	15,7
5	157,0	38,4	68,1	16,3	12,3
5,5	180,0	46,9	71,6	19,7	9,9
6	226,0	55,8	72,4	25,7	8,1

### 6.1.3 CLT element

The dimensions of CLT are presented in the tables 6.5 to 6.8. And results for structural analysis are presented in tables 6.9 and 6.10. Deflections and natural frequency are critical in the designing of CLT for category A imposed loads. The longer span gets deflection and natural frequency gets also higher values ( $L > 4$ ). Lengths above 6 meters, natural frequency exceeds the limit. For higher loads (category C5), deflection is grater for CLT with fewer layers despite shorter span length. this can be prevented by increasing number of layers and the slab becomes more rigid. Span lengths above 6m are critical design factor, deflections get closer to limit while the natural frequency approached the limit value.

**Table 6.5:** Dimensions of CLT category A.

Length L (m)	Transverse length (m)	Total height $h_{CLT}$ (mm)	Number of layers
3	5	90	3
3.5	5.5	100	3
4	6	120	5
4.5	6.5	140	5
5	7	160	5
5.5	7.5	180	7
6	8	200	7
6.5	8.5	220	7
7	9	240	7

**Table 6.6:** Thickness of board layers(mm) against total height of CLT category A.

Number of Layers vs total height	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$
90	30	30	30				
100	30	40	30				
120	30	20	20	20	30		
140	40	20	20	20	40		
160	40	20	40	20	40		
180	30	20	30	20	30	20	30
200	20	40	20	40	20	40	20
220	30	30	30	40	30	30	30
240	30	40	30	40	30	40	30

**Table 6.7:** Dimensions of CLT category C5.

Length (m)	L	Transverse length (B)(m)	Total height $h_{CLT}(mm)$	Number of layers
3		5	110	3
3.5		5.5	120	5
4		6	160	5
4.5		6.5	180	7
5		7	200	7
5.5		7.5	220	7
6		8	240	7
6.5		8.5	260	7
7		9	280	7

**Table 6.8:** Thickness of board layers(mm) against total height of CLT category C5, strength class C30.

Number of Layers vs total height	$t_1$	$t_2$	$t_3$	$t_4$	$t_5$	$t_6$	$t_7$
110	40	30	40				
120	30	20	20	20	30		
160	40	20	40	20	40		
180	30	20	30	20	30	20	30
200	20	40	20	40	20	40	20
220	30	30	30	40	30	30	30
240	30	40	30	40	30	40	30
260	40	40	30	40	30	40	40
280	40	40	40	40	40	40	40

#### 6.1.4 Reinforced Concrete slab

Section dimensions are shown in table 6.11 and the results of structural analysis are summarized in tables 6.12 and 6.13. For category C5 imposed load, deflections are most critical for span lengths above  $L > 5$ . Moment capacity is decisive for the design of the section for span length above  $L > 4m$ . For lower imposed loads deflections are higher for span lengths above 5.5 m and the natural frequency is with the limits although it narrows down to the limit value (8Hz).

**Table 6.9:** Structural analysis for CLT slab, Category A, strength class C24.

Span L (m)	Height (mm)	Rolling Shear	Shear Parallel to grain	Moment (Parallel to grain)	Deflection	Natural Frequency f1 (HZ)
3	90	20,1	3,8	16,5	51,9	14,1
3.5	100	21,3	4,0	19,5	65,1	11,7
4	120	20,5	3,7	20,0	57,2	11,5
4.5	140	21,6	3,9	24,8	68,1	9,8
5	160	20,9	4,0	25,4	67,5	9,2
5.5	180	19,2	3,5	23,1	59,8	9,2
6	200	18,0	3,2	21,9	58,1	8,9
6.5	220	18,9	3,4	25,9	66,2	7,8
7	240	18,8	3,4	27,5	70,5	7,2

**Table 6.10:** Structural analysis for CLT slab, Category C5, strength class C30.

Span L (m)	Height (mm)	Rolling Shear	Shear Parallel to grain	Moment (Parallel to grain)	Deflection	Natural Frequency f1 (HZ)
3	110	34,2	6,8	23,5	70,1	15,9
3.5	120	33,1	6,0	22,5	65,1	15,1
4	160	30,2	5,7	23,5	57,7	1,4
4.5	180	28,1	5,1	22,1	53,6	13,8
5	200	26,6	4,7	21,5	55,3	12,6
5.5	220	28,1	5,0	26,1	64,1	11,0
6	240	28,2	5,1	28,2	70,3	8,9
6.5	260	29,5	5,3	33,0	81,9	8,7
7	280	29,5	5,4	35,2	86,7	8,0

## 6.2 Calculation of CO<sub>2</sub> emission during life cycle

In section 4.2 is mentioned the method used to calculate the environmental impact of studied two cases.

**Case 1** it is compared TCC beam and RC beam, results are shown in figure A.2. TCC beam consists of a timber joist and a concrete part, their difference of emissions between the two materials gives the total emission of the cross section. Emissions from TCC beam is lower than that of RC beam. As the span is increased their emission difference also increases. The selected range of beams, TCC beam is the best selection for environment. By choosing a TCC beam one can achieve to reduce the emission approximately 3 times than that of RC beam.

**Case 2** is also compared CLT slab and RC slab, results are shown in figure A.3. These results indicate that carbon dioxide equivalents are lower in the CLT slab and woods products in general then concrete. Concrete values are significantly

**Table 6.11:** Dimensions of reinforced concrete slab for both categories A and C5, (Imposed load).

Length (m)	Transverse length (B) (m)	Total height (mm)
3	5	200
3.5	5.5	200
4	6	200
4.5	6.5	200
5	7	210
5.5	7.5	220
6	8	240
6.5	8.5	260
7	9	290

**Table 6.12:** Structural analysis for RC slab, Category A, Concrete strength class C25/30.

Span L (m)	Height (mm)	Steel Area (mm <sup>2</sup> )	Moment	Shear	Deflection	Natural Frequency f1 (HZ)
3	200	236	63,2	16,1	9,1	35,0
3.5	200	240	84,6	18,7	12,8	25,7
4	200	314	85,0	21,4	20,0	19,7
4.5	200	452	76,2	24,2	30,8	15,6
5	210	565	73,9	26,8	39,8	13,3
5.5	220	565	87,0	29,3	92,5	11,5
6	240	710	79,4	31,8	98,2	10,6
6.5	260	102	63,6	31,2	97,6	9,8
7	290	1130	63,4	33,2	96,5	9,5

higher even with or without carbon storage of wood products. Therefore, the potential of wood products (CLT) to store carbon is substantial and more favorable to the environment. Nevertheless, the benefits of wood products to store more carbon dioxide can be achieved if the products have a long-life span. The capacity of carbon storage can be increased through sustainable forest. Sustainable forest means that the extraction from the forest does not exceed growth, the raw material is constantly regenerated, and the wood can be returned to the eco-cycle without adding harmful greenhouse gases to the climate [11].



**Table 6.13:** Structural analysis for RC slab, Category C5, Concrete strength class C30/37.

Span L (m)	Height (mm)	Steel Area (mm <sup>2</sup> )	Moment	Shear	Deflection	Natural Frequency f1 (HZ)
3	200	314	69,9	21,5	9,8	35,2
3.5	200	393	76,5	25,1	14,0	25,9
4	200	471	83,9	28,7	21,8	19,8
4.5	200	590	86,1	32,4	33,4	15,7
5	210	700	86,9	35,5	42,8	13,4
5.5	220	820	87,3	37,3	88,6	11,6
6	240	920	87,2	39,1	96,7	10,7
6.5	260	1177	76,6	39,0	99,3	9,9
7	290	1243	78,6	41,4	98,9	9,6

### 6.3 Calculation of cost of different structural components.

Cost shown in figure A.1 is based on the given dimensions of beams and slabs shown in tables 6.2, 6.4, 6.7 and 6.11. Cost difference for case 1 and case 2 is not that much therefore, only case 2, category C5 is considered in this study. In **case1**, TCC beams are more expensive compared to RC beams for all span ranges. Both the cost material and estimated cost of "ready - to-assemble" is higher in TCC beams. As concrete that cast in place requires more workforce than that of TCC beams, for this reason, the difference for ready - to-assemble cost is not that great. For **case2**, CLT floor material cost is way more expensive than that of RC floor. Cost of ready-to-assemble for CLT is higher too for all span ranges, but the difference is so small for span lengths between 3 and 4 meters, of 3 to 4 board layers, C5 category. As the number of board layers for CLT increase sudden price raises occur, while the price of concrete increases gradually as the span length and section heights increased. Cost calculation for this study does not consider logistic cost since distance can vary.

In both case 1 and case 2 RC concrete beams and RC slabs are cheaper than timber materials, but cast in-situ concrete needs more work as concrete requires reinforcement, formwork and curing etc. Therefore, the production of concrete takes longer periods and needs more manpower. On the another hand, less time is needed to install CLT element, which allows much faster production process. CLT element weighs less than reinforced concrete, this advantage allows to reduce the overall weight of the construction and results a smaller and cheaper foundation. TCC beams/floors are also lighter compared to reinforced concrete beam/floors. Both CLT and TCC are competitive in the construction market compared when compared with an all-concrete slab or beam. This does not mean cost/square meter of the product, but there are other factors that contribute saving money on the rest of the structure and the building site. Rapid execution of structures, less

concrete formwork and less stabilization required. Both TCC and CLT elements provide above mentioned features. Concrete has the advantage of having higher natural frequency, which is more suitable for human.

Cost analysis done in this study is based in Norwegian market and prices are relative and time dependent.

## Chapter 7

# Concluding remarks

This thesis is discussed and examined the dynamic and structural design, economic and environmental impact of two different cases. Results showed for case 1, that TCC beams have a higher natural frequency for both categories A and C5, when compared with reinforced concrete beam with same cross sectional dimensions, specially spans longer than 5.5m. One critical design factor for TCC beams/slabs is the choice of connection system between concrete slab and timber joist. Type of connection allows TCC to become more competitive with traditional reinforced concrete.

In case 2, results showed that natural frequency for CLT slab is critical design factor mainly for longer spans compares with reinforced concrete slab. On the other hand, moment capacity is decisive criterion for RC slabs. Since the chosen cross sections for CLT and RC slab are fairly the same, but RC slabs have a higher risk of excessive deformations then CLT slab have, specially spans above 5.5m.

On the environmental aspect, results have shown that wooden products have environmental benefits greater than those of reinforced concrete. Both TCC and CLT have positive effect on the environment since wood has carbon storage capacity. Both concrete and wooden materials have carbon storage capacity, but results showed that wood materials store more carbon than concrete, therefore, more environmentally friendly. If the use of wooden products in construction was increased, storage of carbon would have increased with time, this would give a direct positive effect of the greenhouse effect and consequently the environment.

As mentioned earlier, both TCC and CLT are competitive in the construction market compared to all concrete slab or beam elements. As results indicate TCC and CLT cost of material is higher than all concrete. Nevertheless, TCC and CLT can contribute saving more money for the longer run, because of the rapid execution, less concrete formwork and labor cost.

Finally, a sustainable construction can be achieved by closely studying the elements of buildings such as the environmental impact and the economy. Wooden material such as TCC and CLT are there to challenge the traditional reinforced concrete materials in design and quality rather than price.

## **7.1 Recommendation for design and construction of TCC**

1. It is advisable not use wet timber. When casting try to protect timber from moisture, i.e. using plastic layer or using concrete with reduced water/cement ratio.
2. Use corrosion-protected fasteners, either zinc - coated or stainless steel.
3. Reinforce the concrete is using for thicker concrete cross sections, in order to avoid large cracks and loss of stiffness.
4. Finally, use softer connections to avoid constraint actions.

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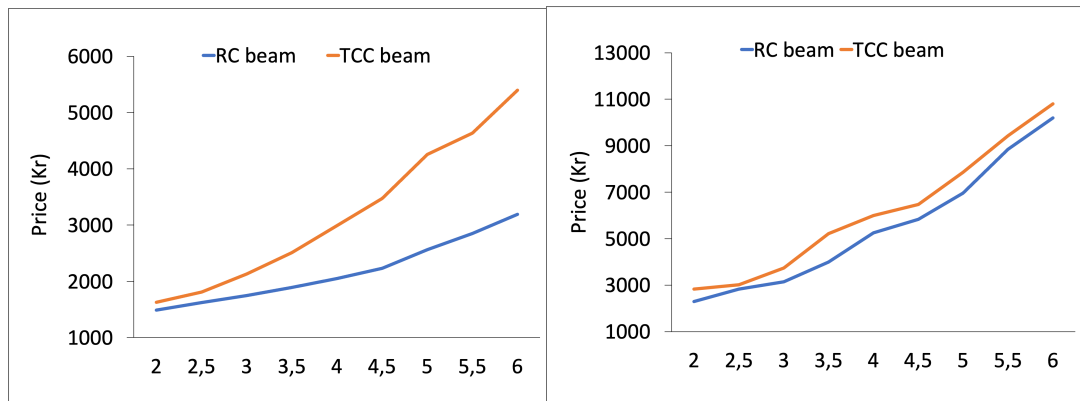
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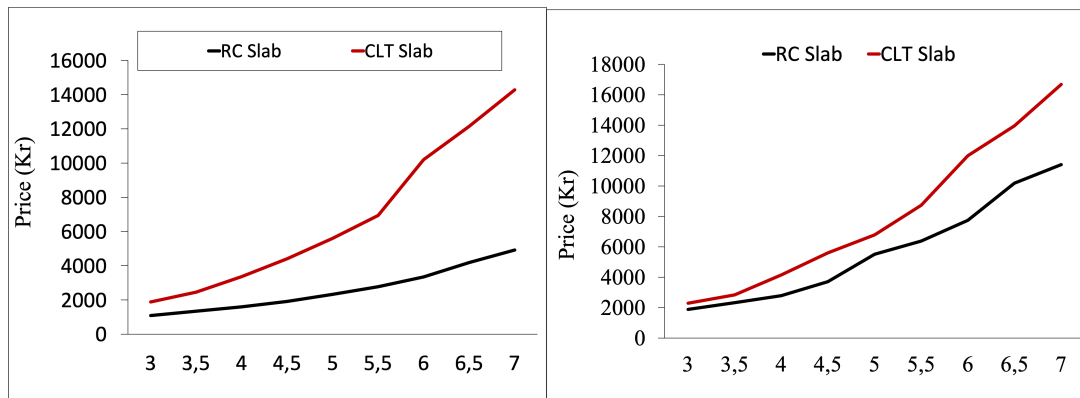


## Appendix A

### Additional Material

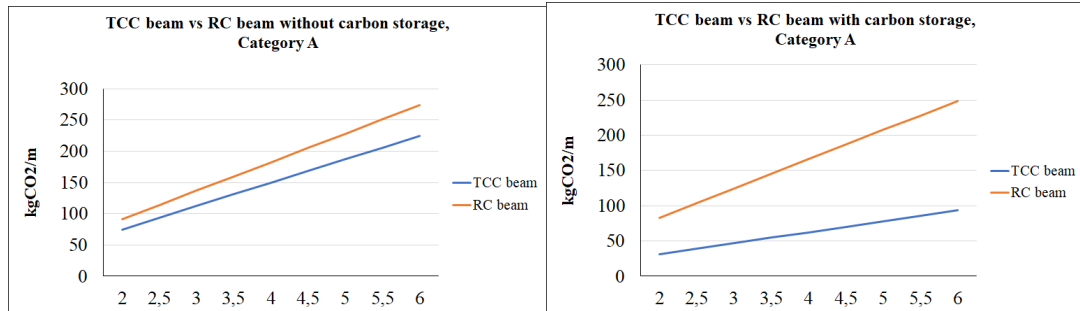


(a) Estimated cost of material, Case 1, Category C5. (b) Estimated ready to assemble cost Case 1, Category C5.

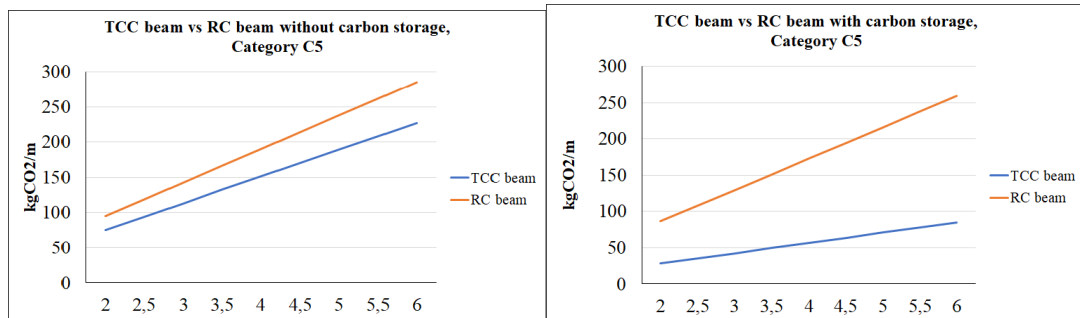


(c) Estimated cost of material, Case 2, Category C5. (d) Estimated ready to assemble cost Case 2, Category C5.

**Figure A.1:** Figures from a-d are summarized cost of material vs cost of ready to assemble for both case 1 and case 2, Category C5. (x-axis is the span length, y-axis is price in Kr)



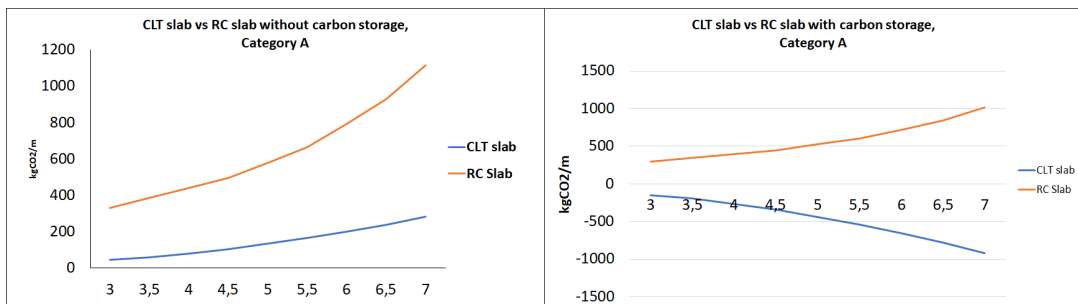
(a) TCC beam vs RC beam without carbon storage, Category A, (b) TCC beam vs RC beam with carbon storage, Category A



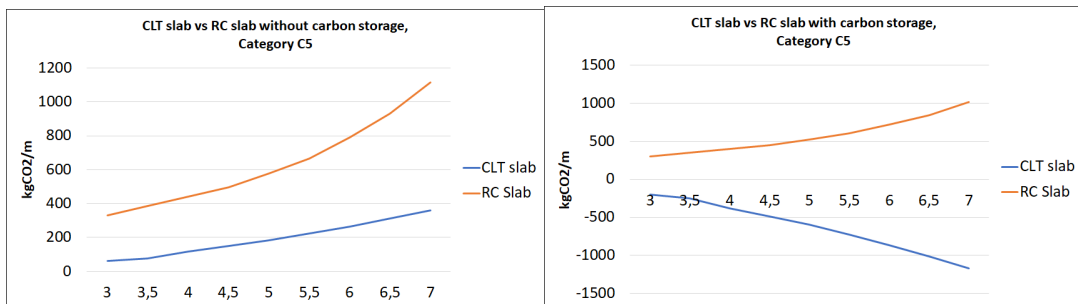
(c) TCC beam vs RC beam without carbon storage, Category C5, (d) TCC beam vs RC beam with carbon storage, Category C5

**Figure A.2:** Case 1, TCC beam vs RC beam with and without carbon storage, Category A and C5.

SECTIONAL AREAS OF GROUPS OF BARS [mm <sup>2</sup> ]										
Bar size [mm]	Number of bars									
	1	2	3	4	5	6	7	8	9	10
6	28.3	56.5	84.8	113	141	170	198	226	254	283
8	50.3	101	151	201	251	302	352	402	452	503
10	78.5	157	236	314	393	471	550	628	707	785
12	113	226	339	452	565	679	792	905	1020	1130
16	201	402	603	804	1010	1210	1410	1610	1810	2010
20	314	628	942	1260	1570	1880	2200	2510	2830	3140
25	491	982	1470	1960	2450	2950	3440	3930	4420	4910
32	804	1610	2410	3220	4020	4830	5630	6430	7240	8040
40	1260	2510	3770	5030	6280	7540	8800	10100	11300	12600



(a) CLT slab vs RC slab without carbon storage, (b) CLT slab vs RC slab with carbon storage, Category A



(c) CLT slab vs RC slab without carbon storage, (d) CLT slab vs RC slab with carbon storage, Category C5

**Figure A.3:** Case 2, CLT slab vs RC slab with and without carbon storage, Category A and C5.

Concrete Design Properties according to EN1992-1-1 ( $\gamma_c = 1.50, f_{yk} = 500 \text{ MPa}$ )

Symbol	Description	C12/15	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60	C55/67	C60/75	C70/85	C80/95	C90/105
$f_{ck}$ (MPa)	Characteristic cylinder compressive strength	12	16	20	25	30	35	40	45	50	55	60	70	80	90
$f_{ck,cube}$ (MPa)	Characteristic cube compressive strength	15	20	25	30	37	45	50	55	60	67	75	85	95	105
$f_{cm}$ (MPa)	Mean cylinder compressive strength	20	24	28	33	38	43	48	53	58	63	68	78	88	98
$f_{cm}$ (MPa)	Mean tensile strength	1.57	1.90	2.21	2.56	2.90	3.21	3.51	3.80	4.07	4.21	4.35	4.61	4.84	5.04
$E_{cm}$ (MPa)	Elastic modulus	27085	28608	29962	31476	32837	34077	35220	36283	37278	38214	39100	40743	42244	43631
$f_{cd}$ (MPa) (for $\alpha_{cc}=1.00$ )	Design compressive strength (for $\alpha_{cc}=1.00$ )	8.00	10.67	13.33	16.67	20.00	23.33	26.67	30.00	33.33	36.67	40.00	46.67	53.33	60.00
$f_{cd}$ (MPa) (for $\alpha_{cc}=0.85$ )	Design compressive strength (for $\alpha_{cc}=0.85$ )	6.80	9.07	11.33	14.17	17.00	19.83	22.67	25.50	28.33	31.17	34.00	39.67	45.33	51.00
$f_{ctd}$ (MPa) (for $\alpha_{ct}=1.00$ )	Design tensile strength (for $\alpha_{ct}=1.00$ )	0.73	0.89	1.03	1.20	1.35	1.50	1.64	1.77	1.90	1.97	2.03	2.15	2.26	2.35
$\rho_{min}$ (%)	Minimum longitudinal tension reinforcement ratio	0.130	0.130	0.130	0.133	0.151	0.167	0.182	0.197	0.212	0.219	0.226	0.240	0.252	0.262
$\rho_{w,min}$ (%)	Minimum shear reinforcement ratio	0.055	0.064	0.072	0.080	0.088	0.095	0.101	0.107	0.113	0.119	0.124	0.134	0.143	0.152

Table 3.5 Material properties for strength graded timber used for CLT.

Board properties	C14	C16	C24	C30
<b>Characteristic strength values (MPa)</b>				
Bending strength $f_{m,k}$	14	16	24	30
Tensile strength along the grain $f_{t,0,k}$	7.2	8.5	14.5	19
Tensile strength perpendicular to the grain $f_{t,90,k}$	0.4	0.4	0.4	0.4
Compressive strength along the grain $f_{c,0,k}$	16	17	21	24
Compressive strength perpendicular to the grain $f_{c,90,k}$	2.0	2.2	2.5	2.7
Shear strength $f_{v,k}$	3.0	3.2	4.0	4.0
<b>Stiffness values (MPa)</b>				
Mean value of modulus of elasticity, along the grain $E_{m,0,mean}$	7,000	8,000	11,000	12,000
Fifth percentile value of modulus of elasticity, along the grain $E_{m,0,05}$	4,700	5,400	7,400	8,000
Mean value of modulus of elasticity, perpendicular to the grain $E_{m,90,mean}$	230	270	370	400
Mean value of the shear modulus $G_{mean}$	440	500	690	750
<b>Density (kg/m<sup>3</sup>)</b>				
Fifth percentile volume of density $\rho_k$	290	310	350	380
Mean density $\rho_{mean}$	350	370	420	460

**Table 3.7** Examples of characteristic stiffness values for CLT panels based on the stiffness properties of the timber boards. About directions, see section 3.3.1, page 40.

Characteristic stiffness values		CLT panels with only C24 (MPa)	CLT panels with C30 in main direction of load and C14 across main direction of load (MPa)
Mean value of modulus of elasticity	$E_{0,x,\text{mean}}$	11,000	12,000
	$E_{90,x,\text{mean}}$	0 <sup>1)</sup> or 400 <sup>2)</sup>	0 <sup>1)</sup> or 400 <sup>2)</sup>
	$E_{0,y,\text{mean}}$	11,000	7,000
	$E_{90,y,\text{mean}}$	0 <sup>1)</sup> or 400 <sup>2)</sup>	0 <sup>1)</sup> or 280 <sup>2)</sup>
Fifth percentile value of modulus of elasticity	$E_{0,x,05}$	7,400	8,000
	$E_{0,y,05}$	7,400	4,700
Mean value of modulus of shear	$G_{090,xlay,\text{mean}}$	690	750
	$G_{090,ylay,\text{mean}}$	690	440
Mean value of modulus of rolling shear	$G_{9090,xlay,\text{mean}}$	50	50
	$G_{9090,ylay,\text{mean}}$	50	50

**Table 3.6** Examples of characteristic strength values for CLT panels based on the strength properties of the timber boards. About directions, see section 3.3.1, page 40.

Characteristic strength values	CLT panels with only C24 (MPa)	CLT panels with C30 in main direction of load and C14 across main direction of load (MPa)
Bending strength	$f_{m,x,k}$	24
	$f_{m,y,k}$	24
Tension strength, in plane	$f_{t,0,x,k}$	14.5
	$f_{t,0,y,k}$	14.5
Tension strength, perpendicular to the plane	$f_{t,90,x,k}$	0.4
	$f_{t,90,y,k}$	0.4
Compression strength, in plane	$f_{c,0,x,k}$	21
	$f_{c,0,y,k}$	21
Compression strength, perpendicular to the plane	$f_{c,90,z,k}$	2.5
Shear strength, longitudinal shear	$f_{v,090,xlay,k}$	4
	$f_{v,090,ylay,k}$	4
Shear strength, rolling shear	$f_{v,9090,xlay,k}$	1.1 <sup>1)</sup> or 0.7 <sup>2)</sup>
	$f_{v,9090,ylay,k}$	1.1 <sup>1)</sup> or 0.7 <sup>2)</sup>

# Design Example 1 For Timber Concrete Composite TCC

Concrete material/dimensions  $F_{ck} := 25 \text{ MPa}$

$$b_1 := 750 \text{ mm}$$

$$f_{ck} := 25 \text{ MPa}$$

$$h_1 := 70 \text{ mm}$$

$$f_{cm} := 28 \text{ MPa}$$

$$A_1 := b_1 \cdot h_1 = 0.053 \text{ m}^2$$

$$f_{ctk0.05} := 1.8 \text{ MPa}$$

$$I_1 := \frac{b_1 \cdot h_1^3}{12} = (2.144 \cdot 10^{-5}) \text{ m}^4$$

$$E_{cm} := 31000 \text{ MPa}$$

Deformation factor creep  $\varphi := 2.5$

$$\gamma_c := 1.5$$

Density  $\rho_c := 25 \frac{\text{kN}}{\text{m}^3}$

## Parameter formwork (Interlayer) OBS plates

- Material/dimensions

width  $b_i := b_1$

Thickness  $t_i := 20 \text{ mm}$

Density  $\rho_p := 7 \frac{\text{kN}}{\text{m}^3}$

## Timber joist grade KVH C24

Width  $b_2 := 120 \text{ mm}$

Hight  $h_2 := 220 \text{ mm}$

$$A_2 := b_2 \cdot h_2 = 0.026 \text{ m}^2$$

$$I_2 := \frac{b_2 \cdot h_2^3}{12} = (1.065 \cdot 10^{-4}) \text{ m}^4$$

Mean value of  
modulus of  
elasticity

$$E_{0,mean} := 11000 \text{ MPa}$$



Characteristic bending strength  $f_{m,k} := 24 \text{ MPa}$

$$f_{t0k} := 14 \text{ MPa}$$

$$k_{mod} := 0.8$$

$$f_{v,k} := 4 \text{ MPa}$$

$$k_{def} := 0.6$$

$$\gamma_M := 1.3$$

Service class 1 residential building interior

Characteristic density  $\rho_k := 350 \frac{\text{kg}}{\text{m}^3}$

Density  $\rho_t := 4.2 \frac{\text{kN}}{\text{m}^3}$

## Connection properties

Type of fastener                      Wurth ASSY plus VD screws. 8x220

$$a := 45 \text{ deg} \qquad d := 8 \text{ mm}$$

Screw length  $l_s := 220 \text{ mm}$

Penetration depth in timber member  $l_{ef} := 120 \text{ mm}$

Slip modulus of one screw  $K_{ser.1} := 100 \frac{\text{N}}{\text{mm}^2} \cdot l_{ef} = 12 \frac{\text{kN}}{\text{mm}}$

Tensile capacity  $f_{tens.k} := 17 \text{ kN}$

Withdrawal parameter  $f_{ax.k} := 11 \frac{\text{N}}{\text{mm}^2}$

Spacing of the fastener  $s := 100 \text{ mm}$

Number of screw in a row  $n := 2$

## Loads

Partial factors for ultimate for actions at the ultimate limit state (ULS)

$$\gamma_G := 1.35 \qquad \gamma_Q := 1.5 \qquad \psi_2 := 0.3$$

Load width  $b := b_1 = 750 \text{ mm}$

Characteristic value of self weight

$$g_{0,k} := b_1 \cdot h_1 \cdot \rho_c + b_i \cdot t_i \cdot \rho_p + b_2 \cdot h_2 \cdot \rho_t = 1.528 \frac{\text{kN}}{\text{m}}$$

Characteristic value of dead load

$$g_{1,k} := 1 \frac{\text{kN}}{\text{m}^2} \cdot b = 0.75 \frac{\text{kN}}{\text{m}}$$

Characteristic value of variable load

$$q_k := (2.0 + 0.8) \frac{\text{kN}}{\text{m}^2} \cdot b = 2.1 \frac{\text{kN}}{\text{m}}$$

## Internal force analysis

Beam span  $L := 6 \text{ m}$

Boundary condition Single span with simple support

Design value of load

- $f_d := (g_{0,k} + g_{1,k}) \cdot \gamma_G + q_k \cdot \gamma_Q = 6.226 \frac{\text{kN}}{\text{m}}$
- $M_{Ed} := f_d \cdot \frac{L^2}{8} = 28.016 \text{ kN} \cdot \text{m}$
- $V_{Ed} := f_d \cdot \frac{L}{2} = 18.677 \text{ kN}$

## 1. Condition Check

- $b_{eff} := \frac{L}{4} = 1.5 \text{ m}$
- $b_1 \leq b_{eff} = 1$
- $b_1 = e$

$$b_1 = 750 \text{ mm}$$

Check Ok

## 2. Condition check

$$\sigma_{cd} \leq f_{cd}$$

Material properties

Concrete

$$E_1 := E_{cm} = (3.1 \cdot 10^7) \text{ kPa}$$

Timber

$$E_2 := E_{0.mean} = (1.1 \cdot 10^7) \text{ kPa}$$

Slip modulus of SLS

$$K_{ser} := n \cdot K_{ser.1} = (2.4 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

Slip modulus of ULS

$$K_u := \left(\frac{2}{3}\right) \cdot K_{ser} = (1.6 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

Slip modulus

$$K := K_u = (1.6 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

$$\text{Factor } \gamma \quad \gamma_1 := \left[ 1 + \frac{\pi^2 \cdot E_1 \cdot A_1 \cdot s}{K \cdot L^2} \right]^{-1} = [0.264] \quad ([EN 1992-1-1], Eq. B.5)$$

$$\gamma_2 := 1 \quad ([EN 1992-1-1], Eq. B.4)$$

## Effective bending stiffness

Distance between the centroid of concrete slab and centroid gravity:

$$\alpha := \frac{h_1}{2} + \frac{h_2}{2} + t_i = 0.165 \text{ m}$$

$$\alpha_2 := \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot \alpha}{\gamma_1 \cdot E_1 \cdot A_1 + \gamma_2 \cdot E_2 \cdot A_2} = 0.098 \text{ m}$$

Distance between the centroid of the timber and centroid gravity:

$$\alpha_1 := \alpha - \alpha_2 = 0.067 \text{ m}$$

Effective bending stiffness:

$$(EI)_{(ef)} = \sum_{i=1}^2 E_i \cdot I_i + \gamma_2 \cdot E_2 \cdot A_2 \cdot \alpha_2^2 + \gamma_1 \cdot E_1 \cdot A_1 \cdot \alpha_1^2$$

$$EI_{eff} := E_1 \cdot I_1 + \gamma_1 \cdot E_1 \cdot A_1 \cdot (\alpha_1)^2 + E_2 \cdot I_2 + \gamma_2 \cdot E_2 \cdot A_2 \cdot (\alpha_2)^2 = [6.553 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$

([EN 1995-1-1], Eq. B.1)

### 1– Normal stress in concrete section

The design of normal stress in concrete should be less than design tensile strength of concrete (Top)

$$\sigma_{cd} := \frac{\gamma_1 \cdot E_1 \cdot \alpha_1 \cdot M_{Ed}}{EI_{eff}} + \frac{0.5 \cdot E_1 \cdot h_1 \cdot M_{Ed}}{EI_{eff}} = [6.967] \text{ MPa}$$

$$f_{cd} := \frac{0.85 f_{ck}}{1.5} = 14.167 \text{ MPa}$$

$$\sigma_{cd} \leq f_{cd} = [1]$$

**Normal stress**

$$\frac{\sigma_{cd}}{f_{cd}} \cdot 100 = [49.178]$$

Condition satisfied Ok

### 3. Condition Check

The design of normal stress in concrete should be less than design tensile strength of concrete (Bottom)

$$\sigma_{td} \leq f_{ctmd}$$

$$\sigma_{td} := \frac{0.5 \cdot E_1 \cdot h_1 \cdot M_{Ed}}{EI_{eff}} - \frac{\gamma_1 \cdot E_1 \cdot \alpha_1 \cdot M_{Ed}}{EI_{eff}} = [2.311] \text{ MPa}$$

$$f_{ctd} := \frac{f_{ctk0.05}}{1.5} = 1.2 \text{ MPa} \quad ([EN 1992-1-1], Eq. 3.15)$$

$$\sigma_{td} \leq f_{ctd} = [0] \quad \text{Condition Not Satisfied}$$

$$\frac{\sigma_{td}}{f_{ctd}} \cdot 100 = [192.547]$$

### 4. Condition check

The calculated shear stress in timber should not exceed the design shear strength of timber

$$h := 0.5 \cdot h_2 + \alpha_2 = 0.208 \text{ m}$$

$$\tau_{max} \leq f_{vd}$$

Where;

$$\tau_{max} := \frac{0.5 \cdot E_2 \cdot h^2 \cdot V_{Ed}}{EI_{eff}} = [0.681] \text{ MPa}$$

$$f_{vd} := \frac{k_{mod} \cdot f_{v,k}}{1.3} = 2.462 \text{ MPa} \quad ([EN 1995-1-1], Eq. 2.14)$$

$$\tau_{max} \leq f_{vd} = [1] \quad ([EN 1995-1-1], Eq. 6.13)$$

$$\frac{\tau_{max}}{f_{vd}} \cdot 100 = [27.671]$$

Check Satisfied OK

## 5. Condition check Timber

This is where check for combined axial and bending cross sectional resistance must be satisfied.

$$\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} \leq 1$$

Where;

$$\sigma_{t2d} := \frac{\gamma_2 \cdot E_2 \cdot \alpha_2 \cdot M_{Ed}}{EI_{eff}} = [4.63] \text{ MPa}$$

$$\sigma_{m2d} := \frac{0.5 E_2 \cdot h_2 \cdot M_{Ed}}{EI_{eff}} = [5.173] \text{ MPa}$$

$$f_{t0d} := \frac{0.8 f_{t0k}}{1.3} = 8.615 \text{ MPa}$$

Design bending stress  
of timber

$$f_{md} := \frac{0.8 f_{m,k}}{1.3} = 14.769 \text{ MPa}$$

$$\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} = [0.888]$$

$$\frac{\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}}}{1} \cdot 100 = [88.765]$$

Varivifaction

$$\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} \leq 1 = [1]$$

**Bending strength**

$$\frac{\sigma_{t2d}}{f_{t0d}} \cdot 100 = [53.739]$$

**Tensile strength**

$$\frac{\sigma_{m2d}}{f_{md}} \cdot 100 = [35.026]$$

Check Satisfied OK

## 6. Condition check

Design applied load  $F_{1d}$  (kN) on the fastener is lower than the calculated design strength of fastener  $F_{Rd}$ (kN).

$$F_{1d} \leq F_{Rd}$$

Where;

Load of  
fastener in a  
row:

$$F_{1d} := \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot s \cdot \alpha_1 \cdot V_{Ed}}{EI_{eff}} = 8.148 \text{ kN}$$

([EN 1995-1-1], Eq. B.10)

Characteristic  
withdrawal capacity

$$f_{ax.a.Rk} = \frac{f_{ax.k} \cdot d \cdot l_{ef}}{1.2 \cdot (\cos(a))^2 + (\sin(a))^2} \left( \frac{\rho_k}{350} \right)^{0.8}$$

([ETA 013/0029] tab. 2.3)

$$f_{ax.a.Rk} := 9.6 \text{ kN}$$

Characteristic  
load bearing  
capacity of  
screws:

$$F_{rk.1} := \cos(a) \cdot f_{ax.a.Rk} = 6.788 \text{ kN}$$

$$F_{rk.2} := \cos(a) \cdot f_{tens.k} = 12.021 \text{ kN}$$

$$\bullet \quad F_{rk} := \min(F_{rk.1}, F_{rk.2}) = 6.788 \text{ kN} \quad ([ETA 013/0029] \text{ tab. 2.3})$$

Effective number of  
screws:

$$n_{ef} := n^{0.9} = 1.866$$

([EN 1992-1-1], Eq. 8.41)

Design load bearing of  
screws:

$$F_{Rd} := \frac{n_{ef} \cdot k_{mod} \cdot F_{rk}}{\gamma_M} = 7.795 \text{ kN}$$

$$F_{1d} \leq F_{Rd} = 0$$

Check Satisfied OK

## 7. Condition Deflection Check

### SLS At The **Begening** Of Life Time

Check for the calculated deflection of the floor is lower than recommended value,

$$u_{inst} \leq \frac{L}{250}$$

Concrete

$$E_1 := E_{cm} = (3.1 \cdot 10^7) \text{ kPa}$$

Timber

$$E_2 := E_{0,mean} = (1.1 \cdot 10^7) \text{ kPa}$$

Slip modulus of SLS

$$K_{ser} := n \cdot K_{ser,1} = (2.4 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

Slip modulus

$$K := K_{ser} = (2.4 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

$\gamma$  Factor

$$\gamma_1 := \left[ 1 + \frac{\pi^2 \cdot E_1 \cdot A_1 \cdot s}{K \cdot L^2} \right]^{-1} = [0.35] \quad ([EN 1992-1-1], Eq. B.5)$$

$$\gamma_2 := 1 \quad ([EN 1992-1-1], Eq. B.4)$$

$$\alpha := \frac{h_1}{2} + \frac{h_2}{2} + t_i = 0.165 \text{ m}$$

$$\alpha_2 := \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot \alpha}{\gamma_1 \cdot E_1 \cdot A_1 + \gamma_2 \cdot E_2 \cdot A_2} = 0.109 \text{ m}$$

$$\alpha_1 := \alpha - \alpha_2 = 0.056 \text{ m}$$

$$EI_{eff} := E_1 \cdot I_1 + \gamma_1 \cdot E_1 \cdot A_1 \cdot (\alpha_1)^2 + E_2 \cdot I_2 + \gamma_2 \cdot E_2 \cdot A_2 \cdot (\alpha_2)^2 = [7.071 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$



Characteristic value of the load  $f_k := g_{0,k} + g_{1,k} + q_k = 4.378 \frac{kN}{m}$

Deflection of beam  $u_{inst} := \frac{5 f_k \cdot L^4}{384 \cdot EI_{eff}} = [10.449] \text{ mm}$

Deflection limit:  $\frac{L}{250} = 24 \text{ mm}$

$u_{inst} \leq \frac{L}{250} = [1] \quad \text{Check Satisfied OK}$

### SLS Deflection At The **End** Of Life Time

Check for the calculated deflection of the floor is lower than recommended value,

$$u_{fin} \leq \frac{L}{200}$$

Concrete  $E_{1.ins} := E_{cm} = (3.1 \cdot 10^7) \text{ kPa}$

$$E_{1.fin} := \frac{E_{cm}}{(1 + \varphi)} = (8.857 \cdot 10^6) \text{ kPa}$$

Timber  $E_{2.ins} := E_{0.mean} = (1.1 \cdot 10^7) \text{ kPa}$

$$E_{2.fin} := \frac{E_{0.mean}}{(1 + k_{def})} = (6.875 \cdot 10^6) \text{ kPa}$$

Slip modulus of SLS  $K_{ser.ins} := n \cdot K_{ser.1} = (2.4 \cdot 10^4) \frac{kN}{m}$

$$K_{ser.fin} := \frac{n \cdot K_{ser.1}}{(1 + k_{def})} = (1.5 \cdot 10^4) \frac{kN}{m}$$

Slip modulus

$$K_{ins} := K_{ser.ins} = (2.4 \cdot 10^4) \frac{kN}{m}$$
$$K_{fin} := K_{ser.fin} = (1.5 \cdot 10^4) \frac{kN}{m}$$

$\gamma$  Factor

$$\gamma_{1.ins} := \left[ 1 + \frac{\pi^2 \cdot E_{1.ins} \cdot A_1 \cdot s}{K_{ins} \cdot L^2} \right]^{-1} = [0.35]$$

([EN 1992-1-1], Eq. B.5)

$$\gamma_{1.fin} := \left[ 1 + \frac{\pi^2 \cdot E_{1.fin} \cdot A_1 \cdot s}{K_{fin} \cdot L^2} \right]^{-1} = [0.541]$$

$$\gamma_2 := 1 \quad ([EN 1992-1-1], Eq. B.4)$$

$$\alpha := \frac{h_1}{2} + \frac{h_2}{2} + t_i = 0.165 \text{ m}$$

$$\alpha_{2.ins} := \frac{\gamma_{1.ins} \cdot E_{1.ins} \cdot A_1 \cdot \alpha}{\gamma_{1.ins} \cdot E_{1.ins} \cdot A_1 + \gamma_2 \cdot E_{2.ins} \cdot A_2} = 0.109 \text{ m}$$

$$\alpha_{2.fin} := \frac{\gamma_{1.fin} \cdot E_{1.fin} \cdot A_1 \cdot \alpha}{\gamma_{1.fin} \cdot E_{1.fin} \cdot A_1 + \gamma_2 \cdot E_{2.fin} \cdot A_2} = 0.096 \text{ m}$$

$$\alpha_{1.ins} := \alpha - \alpha_{2.ins} = 0.056 \text{ m}$$

$$\alpha_{1.fin} := \alpha - \alpha_{2.fin} = 0.069 \text{ m}$$

$$EI_{eff.ins} := E_{1.ins} \cdot I_1 + \gamma_{1.ins} \cdot E_{1.ins} \cdot A_1 \cdot (\alpha_{1.ins})^2 + E_{2.ins} \cdot I_2 + \gamma_2 \cdot E_{2.ins} \cdot A_2 \cdot (\alpha_{2.ins})^2$$

$$EI_{eff.ins} = [7.071 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$

$$EI_{eff.fin} := E_{1.fin} \cdot I_1 + \gamma_{1.fin} \cdot E_{1.fin} \cdot A_1 \cdot (\alpha_{1.fin})^2 + E_{2.fin} \cdot I_2 + \gamma_2 \cdot E_{2.fin} \cdot A_2 \cdot (\alpha_{2.fin})^2$$

$$EI_{eff.fin} = [3.791 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$

Quasi permanent load  $f_{qp} := g_{0,k} + g_{1,k} + \psi_2 \cdot q_k = 2.908 \frac{kN}{m}$

Rest of variable load:  $f_q := (1 - \psi_2) \cdot q_k = 1.47 \frac{kN}{m}$

Deflection of the beam by quasi-perment load

Consider creep:  $w_{qp} := \frac{5}{384} \cdot \frac{f_{qp} \cdot L^4}{EI_{eff.fin}} = [12.945] \text{ mm}$

Deflection of the beam by the rest:  $w_q := \frac{5}{384} \cdot \frac{f_q \cdot L^4}{EI_{eff.ins}} = [3.508] \text{ mm}$

Final Deflection  $w_{fin} := w_{qp} + w_q = [16.453] \text{ mm}$

Deflection limit:  $\frac{L}{200} = 30 \text{ mm}$

$$w_{fin} \leq \frac{L}{200} = [1]$$

**Deflection**  $\frac{w_{fin}}{L} \cdot 100 = [68.554]$   
250

Check Satisfied OK

## DeflectioCheck Vibration

Since the beam is placed on load bearing walls there is no need to calculate slab deflection on the beam

$$f_v \geq 8 \text{ Hz}$$

$$f_v = \frac{18}{\sqrt{\sigma_{max}}}$$

$$\sigma_{max} = \frac{5 \cdot q \cdot L^4}{384 \cdot EI_{dyn}}$$

**Only Vibration of the timber beam joist is calculated here**

$$E_{cm} = (3.1 \cdot 10^4) \text{ MPa}$$

$$EI_{dyn} := E_{cm} \cdot 1.1 = (3.41 \cdot 10^4) \frac{\text{N}}{\text{mm}^2}$$

Loading:

- Beam

$$\text{Swelf weight: } g_{beam} := (b_1 \cdot h_1 \cdot \rho_c + b_i \cdot t_i \cdot \rho_p + b_2 \cdot h_2 \cdot \rho_t) \cdot \frac{b_1}{m} = 1.146 \frac{\text{kN}}{m}$$

$$10\% \text{ of life load } q_{dyn} := 0.1$$

$$\text{Life load: } q_{slab} := (2 + 0.8) \cdot \frac{\text{kN}}{\text{m}^2} \cdot q_{dyn} \cdot b_1 = 0.21 \text{ m} \cdot \frac{\text{kN}}{\text{m}^2}$$

$$I_{beam} := \frac{b_2 \cdot h_2^3}{12} = (1.065 \cdot 10^8) \text{ mm}^4$$

$$\sigma_{beam} := \frac{5 \cdot (g_{beam} + q_{slab}) \cdot L^4}{384 \cdot EI_{dyn} \cdot I_{beam}} = 6.303 \text{ mm}$$

$$\sigma_{slab} := 0$$

$$\sigma_{max} := \sigma_{beam} + \sigma_{slab} = 6.303 \text{ mm}$$

$$f_v := \frac{18}{\sqrt{6.303}} = 7.17$$

**Model mass:**

$$Load := \frac{g_{beam}}{m} = 1.146 \frac{kN}{m^2}$$

$$Area := L \cdot (b_1 + b_2) = 5.22 \text{ m}^2$$

$$Modelmass := Load \cdot Area = 5.984 \text{ kN}$$

# Design Example 1 For Timber Concrete Composite TCC 2

Concrete material/dimensions

$$F_{ck} := 30 \text{ MPa}$$

$$b_1 := 750 \text{ mm}$$

$$f_{ck} := 30 \text{ MPa}$$

$$h_1 := 70 \text{ mm}$$

$$A_1 := b_1 \cdot h_1 = 0.053 \text{ m}^2$$

$$f_{ctk0.05} := 2 \text{ MPa}$$

$$I_1 := \frac{b_1 \cdot h_1^3}{12} = (2.144 \cdot 10^{-5}) \text{ m}^4$$

$$E_{cm} := 33000 \text{ MPa}$$

Deformation factor creep  $\varphi := 2.5$

$$\gamma_c := 1.5$$

Density  $\rho_c := 25 \frac{\text{kN}}{\text{m}^3}$

## Parameter formwork (Interlayer) OBS plates

- Material/dimensions

width  $b_i := b_1$

Thickness  $t_i := 20 \text{ mm}$

Density  $\rho_p := 7 \frac{\text{kN}}{\text{m}^3}$

## Timber joist grade KVH C24

Width  $b_2 := 120 \text{ mm}$

Hight  $h_2 := 220 \text{ mm}$

$$A_2 := b_2 \cdot h_2 = 0.026 \text{ m}^2$$

$$I_2 := \frac{b_2 \cdot h_2^3}{12} = (1.065 \cdot 10^{-4}) \text{ m}^4$$

Mean value of  
modulus of  
elasticity

$$E_{0,mean} := 12000 \text{ MPa}$$

Characteristic bending strength  $f_{m,k} := 30 \text{ MPa}$

$$f_{t0k} := 19 \text{ MPa}$$

$$k_{mod} := 0.8$$

$$f_{v,k} := 4 \text{ MPa}$$

$$k_{def} := 0.6$$

$$\gamma_M := 1.3$$

Service class 1 residential building interior

Characteristic density  $\rho_k := 350 \frac{\text{kg}}{\text{m}^3}$

Density  $\rho_t := 4.6 \frac{\text{kN}}{\text{m}^3}$

## Connection properties

Type of fastener                      Wurth ASSY plus VD screws. 8x220

$$a := 45 \text{ deg} \qquad d := 8 \text{ mm}$$

Screw length  $l_s := 220 \text{ mm}$

Penetration depth in timber member  $l_{ef} := 120 \text{ mm}$

Slip modulus of one screw  $K_{ser.1} := 100 \frac{\text{N}}{\text{mm}^2} \cdot l_{ef} = 12 \frac{\text{kN}}{\text{mm}}$

Tensile capacity  $f_{tens.k} := 17 \text{ kN}$

Withdrawal parameter  $f_{ax.k} := 11 \frac{\text{N}}{\text{mm}^2}$

Spacing of the fastener  $s := 100 \text{ mm}$

Number of screw in a row  $n := 2$

## Loads

Partial factors for ultimate for actions at the ultimate limit state (ULS)

$$\gamma_G := 1.35$$

$$\gamma_Q := 1.5$$

$$\psi_2 := 0.3$$

Load width  $b := b_1 = 750 \text{ mm}$

Characteristic value of self weight

$$g_{0,k} := b_1 \cdot h_1 \cdot \rho_c + b_i \cdot t_i \cdot \rho_p + b_2 \cdot h_2 \cdot \rho_t = 1.539 \frac{\text{kN}}{\text{m}}$$

Characteristic value of dead load

$$g_{1,k} := 1 \frac{\text{kN}}{\text{m}^2} \cdot b = 0.75 \frac{\text{kN}}{\text{m}}$$

Characteristic value of variable load

$$q_k := (5.0 + 0.8) \frac{\text{kN}}{\text{m}^2} \cdot b = 4.35 \frac{\text{kN}}{\text{m}}$$

## Internal force analysis

Beam span  $L := 2 \text{ m}$

Boundary condition Single span with simple support

Design value of load

- $f_d := (g_{0,k} + g_{1,k}) \cdot \gamma_G + q_k \cdot \gamma_Q = 9.615 \frac{\text{kN}}{\text{m}}$
- $M_{Ed} := f_d \cdot \frac{L^2}{8} = 4.808 \text{ kN} \cdot \text{m}$
- $V_{Ed} := f_d \cdot \frac{L}{2} = 9.615 \text{ kN}$



## 1. Condition Check

- $b_{eff} := \frac{L}{4} = 0.5 \text{ m}$
- $b_1 \leq b_{eff} = 0$
- $b_1 = e$

$$b_1 = 750 \text{ mm}$$

Check Ok

## 2. Condition check

$$\sigma_{cd} \leq f_{cd}$$

Material properties

Concrete

$$E_1 := E_{cm} = (3.3 \cdot 10^7) \text{ kPa}$$

Timber

$$E_2 := E_{0.mean} = (1.2 \cdot 10^7) \text{ kPa}$$

Slip modulus of SLS

$$K_{ser} := n \cdot K_{ser.1} = (2.4 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

Slip modulus of ULS

$$K_u := \left(\frac{2}{3}\right) \cdot K_{ser} = (1.6 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

Slip modulus

$$K := K_u = (1.6 \cdot 10^4) \frac{\text{kN}}{\text{m}}$$

$$\text{Factor } \gamma \quad \gamma_1 := \left[ 1 + \frac{\pi^2 \cdot E_1 \cdot A_1 \cdot s}{K \cdot L^2} \right]^{-1} = [0.036] \quad ([EN 1992-1-1], Eq. B.5)$$

$$\gamma_2 := 1 \quad ([EN 1992-1-1], Eq. B.4)$$

## Effective bending stiffness

Distance between the centroid of concrete slab and centroid gravity:

$$\alpha := \frac{h_1}{2} + \frac{h_2}{2} + t_i = 0.165 \text{ m}$$

$$\alpha_2 := \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot \alpha}{\gamma_1 \cdot E_1 \cdot A_1 + \gamma_2 \cdot E_2 \cdot A_2} = 0.027 \text{ m}$$

Distance between the centroid of the timber and centroid gravity:

$$\alpha_1 := \alpha - \alpha_2 = 0.138 \text{ m}$$

Effective bending stiffness:

$$(EI)_{(ef)} = \sum_{i=1}^2 E_i \cdot I_i + \gamma_2 \cdot E_2 \cdot A_2 \cdot \alpha_2^2 + \gamma_1 \cdot E_1 \cdot A_1 \cdot \alpha_1^2$$

$$EI_{eff} := E_1 \cdot I_1 + \gamma_1 \cdot E_1 \cdot A_1 \cdot (\alpha_1)^2 + E_2 \cdot I_2 + \gamma_2 \cdot E_2 \cdot A_2 \cdot (\alpha_2)^2 = [3.406 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$

([EN 1995-1-1], Eq. B.1)

### 1- Normal stress in concrete section

The design of normal stress in concrete should be less than design tensile strength of concrete (Top)

$$\sigma_{cd} := \frac{\gamma_1 \cdot E_1 \cdot \alpha_1 \cdot M_{Ed}}{EI_{eff}} + \frac{0.5 \cdot E_1 \cdot h_1 \cdot M_{Ed}}{EI_{eff}} = [1.862] \text{ MPa}$$

$$f_{cd} := \frac{0.85 f_{ck}}{1.5} = 17 \text{ MPa}$$

$$\sigma_{cd} \leq f_{cd} = [1]$$

**Normal stress**

$$\frac{\sigma_{cd}}{f_{cd}} \cdot 100 = [10.951]$$

Condition satisfied Ok

### 3. Condition Check

The design of normal stress in concrete should be less than design tensile strength of concrete (Bottom)

$$\sigma_{td} \leq f_{ctmd}$$

$$\sigma_{td} := \frac{0.5 \cdot E_1 \cdot h_1 \cdot M_{Ed}}{EI_{eff}} - \frac{\gamma_1 \cdot E_1 \cdot \alpha_1 \cdot M_{Ed}}{EI_{eff}} = [1.398] \text{ MPa}$$

$$f_{ctd} := \frac{f_{ctk0.05}}{1.5} = 1.333 \text{ MPa} \quad ([EN 1992-1-1], Eq. 3.15)$$

$$\sigma_{td} \leq f_{ctd} = [0] \quad \text{Condition Not Satisfied}$$

$$\frac{\sigma_{td}}{f_{ctd}} \cdot 100 = [104.886]$$

### 4. Condition check

The calculated shear stress in timber should not exceed the design shear strength of timber

$$h := 0.5 \cdot h_2 + \alpha_2 = 0.137 \text{ m}$$

$$\tau_{max} \leq f_{vd}$$

Where;

$$\tau_{max} := \frac{0.5 \cdot E_2 \cdot h^2 \cdot V_{Ed}}{EI_{eff}} = [0.319] \text{ MPa}$$

$$f_{vd} := \frac{k_{mod} \cdot f_{v,k}}{1.3} = 2.462 \text{ MPa} \quad ([EN 1995-1-1], Eq. 2.14)$$

$$\tau_{max} \leq f_{vd} = [1] \quad ([EN 1995-1-1], Eq. 6.13)$$

$$\frac{\tau_{max}}{f_{vd}} \cdot 100 = [12.949]$$

Check Satisfied OK

## 5. Condition check Timber

This is where check for combined axial and bending cross sectional resistance must be satisfied.

$$\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} \leq 1$$

Where;

$$\sigma_{t2d} := \frac{\gamma_2 \cdot E_2 \cdot \alpha_2 \cdot M_{Ed}}{EI_{eff}} = [0.46] \text{ MPa}$$

$$\sigma_{m2d} := \frac{0.5 E_2 \cdot h_2 \cdot M_{Ed}}{EI_{eff}} = [1.863] \text{ MPa}$$

$$f_{t0d} := \frac{0.8 f_{t0k}}{1.3} = 11.692 \text{ MPa}$$

Design bending stress  
of timber

$$f_{md} := \frac{0.8 f_{m,k}}{1.3} = 18.462 \text{ MPa}$$

$$\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} = [0.14]$$

Varivifaction

$$\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}} \leq 1 = [1]$$

$$\frac{\frac{\sigma_{t2d}}{f_{t0d}} + \frac{\sigma_{m2d}}{f_{md}}}{1} \cdot 100 = [14.029]$$

**Bending strength**

$$\frac{\sigma_{t2d}}{f_{t0d}} \cdot 100 = [3.938]$$

**Tensile strength**

$$\frac{\sigma_{m2d}}{f_{md}} \cdot 100 = [10.091]$$

Check Satisfied OK

## 6. Condition check

Design applied load  $F_{1d}$  (kN) on the fastener is lower than the calculated design strength of fastener  $F_{Rd}$ (kN).

$$F_{1d} \leq F_{Rd}$$

Where;

Load of  
fastener in a  
row:

$$F_{1d} := \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot s \cdot \alpha_1 \cdot V_{Ed}}{EI_{eff}} = 2.431 \text{ kN}$$

([EN 1995-1-1], Eq. B.10)

Characteristic  
withdrawal capacity

$$f_{ax.a.Rk} = \frac{f_{ax.k} \cdot d \cdot l_{ef}}{1.2 \cdot (\cos(a))^2 + (\sin(a))^2} \left( \frac{\rho_k}{350} \right)^{0.8}$$

([ETA 013/0029] tab. 2.3)

$$f_{ax.a.Rk} := 9.6 \text{ kN}$$

Characteristic  
load bearing  
capacity of  
screws:

$$F_{rk.1} := \cos(a) \cdot f_{ax.a.Rk} = 6.788 \text{ kN}$$

$$F_{rk.2} := \cos(a) \cdot f_{tens.k} = 12.021 \text{ kN}$$

$$\bullet \quad F_{rk} := \min(F_{rk.1}, F_{rk.2}) = 6.788 \text{ kN} \quad ([ETA 013/0029] \text{ tab. 2.3})$$

Effective number of  
screws:

$$n_{ef} := n^{0.9} = 1.866 \quad ([EN 1992-1-1], Eq. 8.41)$$

Design load bearing of  
screws:

$$F_{Rd} := \frac{n_{ef} \cdot k_{mod} \cdot F_{rk}}{\gamma_M} = 7.795 \text{ kN}$$

$$F_{1d} \leq F_{Rd} = 1$$

Check Satisfied OK

## 7. Condition Deflection Check

### SLS At The **Begening** Of Life Time

Check for the calculated deflection of the floor is lower than recommended value,

$$u_{inst} \leq \frac{L}{250}$$

Concrete  $E_1 := E_{cm} = (3.3 \cdot 10^7) \text{ kPa}$

Timber  $E_2 := E_{0,mean} = (1.2 \cdot 10^7) \text{ kPa}$

Slip modulus of SLS  $K_{ser} := n \cdot K_{ser,1} = (2.4 \cdot 10^4) \frac{\text{kN}}{\text{m}}$

Slip modulus  $K := K_{ser} = (2.4 \cdot 10^4) \frac{\text{kN}}{\text{m}}$

$\gamma$  Factor  $\gamma_1 := \left[ 1 + \frac{\pi^2 \cdot E_1 \cdot A_1 \cdot s}{K \cdot L^2} \right]^{-1} = [0.053] \quad ([EN 1992-1-1], Eq. B.5)$

$\gamma_2 := 1 \quad ([EN 1992-1-1], Eq. B.4)$

$$\alpha := \frac{h_1}{2} + \frac{h_2}{2} + t_i = 0.165 \text{ m}$$

$$\alpha_2 := \frac{\gamma_1 \cdot E_1 \cdot A_1 \cdot \alpha}{\gamma_1 \cdot E_1 \cdot A_1 + \gamma_2 \cdot E_2 \cdot A_2} = 0.037 \text{ m}$$

$$\alpha_1 := \alpha - \alpha_2 = 0.128 \text{ m}$$

$$EI_{eff} := E_1 \cdot I_1 + \gamma_1 \cdot E_1 \cdot A_1 \cdot (\alpha_1)^2 + E_2 \cdot I_2 + \gamma_2 \cdot E_2 \cdot A_2 \cdot (\alpha_2)^2 = [3.928 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$

Characteristic value of the load  $f_k := g_{0.k} + g_{1.k} + q_k = 6.639 \frac{kN}{m}$

Deflection of beam  $u_{inst} := \frac{5 f_k \cdot L^4}{384 \cdot EI_{eff}} = [0.352] \text{ mm}$

Deflection limit:  $\frac{L}{250} = 8 \text{ mm}$

$u_{inst} \leq \frac{L}{250} = [1] \quad \text{Check Satisfied OK}$

### SLS Deflection At The **End** Of Life Time

Check for the calculated deflection of the floor is lower than recommended value,

$$u_{fin} \leq \frac{L}{200}$$

Concrete  $E_{1.ins} := E_{cm} = (3.3 \cdot 10^7) \text{ kPa}$

$$E_{1.fin} := \frac{E_{cm}}{(1 + \varphi)} = (9.429 \cdot 10^6) \text{ kPa}$$

Timber  $E_{2.ins} := E_{0.mean} = (1.2 \cdot 10^7) \text{ kPa}$

$$E_{2.fin} := \frac{E_{0.mean}}{(1 + k_{def})} = (7.5 \cdot 10^6) \text{ kPa}$$

Slip modulus of SLS  $K_{ser.ins} := n \cdot K_{ser.1} = (2.4 \cdot 10^4) \frac{kN}{m}$

$$K_{ser.fin} := \frac{n \cdot K_{ser.1}}{(1 + k_{def})} = (1.5 \cdot 10^4) \frac{kN}{m}$$

Slip modulus  $K_{ins} := K_{ser.ins} = (2.4 \cdot 10^4) \frac{kN}{m}$

$$K_{fin} := K_{ser.fin} = (1.5 \cdot 10^4) \frac{kN}{m}$$

$\gamma$  Factor

$$\gamma_{1.ins} := \left[ 1 + \frac{\pi^2 \cdot E_{1.ins} \cdot A_1 \cdot s}{K_{ins} \cdot L^2} \right]^{-1} = [0.053]$$

([EN 1992-1-1], Eq. B.5)

$$\gamma_{1.fin} := \left[ 1 + \frac{\pi^2 \cdot E_{1.fin} \cdot A_1 \cdot s}{K_{fin} \cdot L^2} \right]^{-1} = [0.109]$$

$$\gamma_2 := 1 \quad ([EN 1992-1-1], Eq. B.4)$$

$$\alpha := \frac{h_1}{2} + \frac{h_2}{2} + t_i = 0.165 \text{ m}$$

$$\alpha_{2.ins} := \frac{\gamma_{1.ins} \cdot E_{1.ins} \cdot A_1 \cdot \alpha}{\gamma_{1.ins} \cdot E_{1.ins} \cdot A_1 + \gamma_2 \cdot E_{2.ins} \cdot A_2} = 0.037 \text{ m}$$

$$\alpha_{2.fin} := \frac{\gamma_{1.fin} \cdot E_{1.fin} \cdot A_1 \cdot \alpha}{\gamma_{1.fin} \cdot E_{1.fin} \cdot A_1 + \gamma_2 \cdot E_{2.fin} \cdot A_2} = 0.035 \text{ m}$$

$$\alpha_{1.ins} := \alpha - \alpha_{2.ins} = 0.128 \text{ m}$$

$$\alpha_{1.fin} := \alpha - \alpha_{2.fin} = 0.13 \text{ m}$$

$$EI_{eff.ins} := E_{1.ins} \cdot I_1 + \gamma_{1.ins} \cdot E_{1.ins} \cdot A_1 \cdot (\alpha_{1.ins})^2 + E_{2.ins} \cdot I_2 + \gamma_2 \cdot E_{2.ins} \cdot A_2 \cdot (\alpha_{2.ins})^2$$

$$EI_{eff.ins} = [3.928 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$

$$EI_{eff.fin} := E_{1.fin} \cdot I_1 + \gamma_{1.fin} \cdot E_{1.fin} \cdot A_1 \cdot (\alpha_{1.fin})^2 + E_{2.fin} \cdot I_2 + \gamma_2 \cdot E_{2.fin} \cdot A_2 \cdot (\alpha_{2.fin})^2$$

$$EI_{eff.fin} = [2.158 \cdot 10^3] \text{ kN} \cdot \text{m}^2$$



Quasi permanent load  $f_{qp} := g_{0,k} + g_{1,k} + \psi_2 \cdot q_k = 3.594 \frac{kN}{m}$

Rest of variable load:  $f_q := (1 - \psi_2) \cdot q_k = 3.045 \frac{kN}{m}$

Deflection of the beam by quasi-perment load

Consider creep:  $w_{qp} := \frac{5}{384} \cdot \frac{f_{qp} \cdot L^4}{EI_{eff.fin}} = [0.347] \text{ mm}$

Deflection of the beam by the rest:  $w_q := \frac{5}{384} \cdot \frac{f_q \cdot L^4}{EI_{eff.ins}} = [0.162] \text{ mm}$

Final Deflection  $w_{fin} := w_{qp} + w_q = [0.508] \text{ mm}$

Deflection limit:  $\frac{L}{250} = 8 \text{ mm}$

$$w_{fin} \leq \frac{L}{250} = [1]$$

**Deflection**  $\frac{w_{fin}}{L} \cdot 100 = [6.355]$

Check Satisfied OK



Since the beam is placed on load

## Beam Vibration

bearing walls there is no need to calculate slab deflection on the beam

### Vibration

**Only Vibration of the timber beam joist is calculated here**

$$f_v \geq 8 \text{ Hz}$$

$$f_v = \frac{18}{\sqrt{\sigma_{max}}}$$

$$\sigma_{max} = \frac{5 \cdot q \cdot L^4}{384 \cdot EI_{dyn}}$$

$$E_{cm} = (3.3 \cdot 10^4) \text{ MPa}$$

$$EI_{dyn} := E_{cm} \cdot 1.1 = (3.63 \cdot 10^4) \frac{\text{N}}{\text{mm}^2}$$

Loading:

- Beam

$$\text{Self weight: } g_{beam} := (b_1 \cdot h_1 \cdot \rho_c + b_i \cdot t_i \cdot \rho_p + b_2 \cdot h_2 \cdot \rho_t) \cdot \frac{b_1}{m} = 1.154 \frac{\text{kN}}{m}$$

$$10\% \text{ of life load } \quad q_{dyn} := 0.1$$

$$\text{Life load: } q_{slab} := (2 + 0.8) \cdot \frac{\text{kN}}{m^2} \cdot q_{dyn} \cdot b_1 = 0.21 \frac{\text{kN}}{m^2}$$

$$I_{beam} := \frac{b_2 \cdot h_2^3}{12} = (1.065 \cdot 10^8) \text{ mm}^4$$

$$L := 6 \text{ m}$$

$$\sigma_{beam} := \frac{5 \cdot (g_{beam} + q_{slab}) \cdot L^4}{384 \cdot EI_{dyn} \cdot I_{beam}} = 5.956 \text{ mm}$$

$$\sigma_{slab} := 0$$

$$\sigma_{max} := \sigma_{beam} + \sigma_{slab} = 5.956 \text{ mm}$$

$$f_v := \frac{18}{\sqrt{5.956}} = 7.376$$

**Model mass:**

$$Load := \frac{g_{beam}}{m} = 1.154 \frac{kN}{m^2}$$

$$Area := L \cdot (b_1 + b_2) = 5.22 m^2$$

$$Modelmass := Load \cdot Area = 6.025 kN$$

Damping of the system:  $D = D_1 + D_2 + D_3$

$D_1$	Structural dumping	2	Concrete
$D_2$	Damping due to furniture	1	<i>Home library</i>
$D_3$	Damping due to finishes	0	free floating floor

$$D_1 := 2\% \quad D_2 := 1\% \quad D_3 := 0\%$$

$$D_{tot} := D_1 + D_2 + D_3 = 0.03$$

3%

# Design for RC.Beem Case 1

# Category A

Section Dimentions

$$b_w := 120 \text{ mm} \quad b_{eff} := 750 \text{ mm} \quad h_f := 80 \text{ mm} \quad h_w := 220 \text{ mm}$$

$$\text{Asume} \quad \phi_l := 16 \text{ mm} \quad \phi_s := 8 \text{ mm}$$

Material

Steel

B500NC

$$f_{yk} := 500 \text{ MPa} \quad \gamma_s := 1.15 \quad f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \text{ MPa}$$

$$\text{Dencity} \quad \rho := 25 \frac{\text{kN}}{\text{m}^3}$$

Concrete Class: B25

$$f_{ck} := 25 \text{ MPa} \quad \gamma_c := 1.5$$

$$f_{cd} := \frac{0.85 \cdot f_{ck}}{\gamma_c} = 14.167 \text{ MPa} \quad f_{ctm} := 2.6 \text{ MPa}$$

$$\text{Cover} \quad C_{nom} := 45 \text{ mm}$$

$$\text{Dencity} \quad \rho := 25 \frac{\text{kN}}{\text{m}^3}$$

Variable load 5kN/m<sup>2</sup>

$$\gamma_Q := 1.5 \quad \gamma_G := 1.35$$

Charecteristic value of self weight

$$g_{0,k} := b_{eff} \cdot h_f \cdot \rho + (h_w - h_f) \cdot b_w \cdot \rho = 1.92 \frac{\text{kN}}{\text{m}}$$

Charecteristic value of dead load

$$g_{1,k} := 1 \frac{\text{kN}}{\text{m}^2} \cdot b_{eff} = 0.75 \frac{\text{kN}}{\text{m}}$$

Charecteristic value of variable load

$$q_k := (2.0 + 0.8) \frac{\text{kN}}{\text{m}^2} \cdot b_w = 0.336 \frac{\text{kN}}{\text{m}}$$

.....

## Internal force analysis

Beam span  $L := 6 \text{ m}$

Boundary condition Single span with simple support

Design value of load

- $f_d := (g_{0.k} + g_{1.k}) \cdot \gamma_G + q_k \cdot \gamma_Q = 4.109 \frac{\text{kN}}{\text{m}}$
- $M_{Ed} := f_d \cdot \frac{L^2}{8} = 18.488 \text{ kN} \cdot \text{m}$
- $V_{Ed} := f_d \cdot \frac{L}{2} = 12.326 \text{ kN}$

Check with ultimate moment capacity

Ultimate moment capacity  $M_{Rd} = 0.168 \cdot f_{ck} \cdot b \cdot d^2$

$$h := h_f + h_w = 300 \text{ mm}$$

$$d := h - \frac{\phi_l}{2} - C_{nom} = 247 \text{ mm}$$

$$M_f := f_{cd} \cdot h_f \cdot b_{eff} \cdot \left( h - \frac{h_f}{2} \right) = 221 \text{ kN} \cdot \text{m}$$

Natural axis with in the flange

$$M_{Rd} := 0.168 \cdot f_{ck} \cdot b_w \cdot d^2 = 30.749 \text{ kN} \cdot \text{m}$$

$$M_{Ed} < M_{Rd} = 1 \quad \text{Ok}$$

**Moment:**  $\frac{M_{Ed}}{M_{Rd}} \cdot 100 = 60.127$

$$M_f > M_{Rd} = 1$$

No need for compression reinforcement, and can carry the applied load

$$k := \frac{M_{Ed}}{b_{eff} \cdot d^2 \cdot f_{ck}} = 0.016 \quad k < 0.167$$

Singly reinforced

Calculate the lever arm Z

$$Z = d \left( 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right)$$

$$k := \frac{M_{Ed}}{b_w \cdot d^2 \cdot f_{ck}} = 0.101$$

$$Z := \min \left( d \cdot \left( 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right), 0.95 \cdot d \right) = 222.584 \text{ mm}$$

Required steel area

$$A_{s,req} := \frac{M_{Ed}}{f_{yd} \cdot Z} = 191.042 \text{ mm}^2$$

Provided steel area:  $A_{s,Prov} := 201 \text{ mm}^2$

**Steel Area**

$$A_s := A_{s,Prov} = 201 \text{ mm}^2$$

Number of bars  $\phi_{l,2} := 8 \text{ mm}$  use 2 bars  $113 \text{ mm}^2$

$$d_2 := \left( h - \frac{\phi_{l,2}}{2} - C_{nom} \right) = 251 \text{ mm}$$

Check min reinforcement

Clouse 9.3.1.1 In NS EN  
1992-1-1:2004

**Min steel Area**

$$A_{s,min} := \max \left( 0.26 \cdot b_w \cdot d \cdot \frac{f_{ctm}}{f_{yk}}, 0.0013 \cdot b_w \cdot d \right) = 40.073 \text{ mm}^2$$

$$A_{s,min} < A_s = 1 \quad Ok$$

Check minimum reinforcement Ok

## Check maximum Reinforcement

Clouse 9.2.1.1 In EC2

Cross cectional area  $A_c := b_{eff} \cdot h_f + b_w \cdot h_w = 0.086 \text{ m}^2$

$$A_{s,max} := 0.04 \cdot A_c = (3.456 \cdot 10^3) \text{ mm}^2$$

$$A_{s,req} < A_{s,max} = 1 \quad Ok$$

Check Max reinforcement Ok Less them max steel area is provided

Ultimate Limit state, Shear check  $V_{Ed} < V_{Rd,c}$

Simply supported beam shear force  $V_{Ed} := f_d \cdot \frac{L}{2} = 12.326 \text{ kN}$

Design shear resistance of beam  $V_{Rd,c} = \left( C_{Rd,c} \cdot k \left( 100 \cdot p_1 \cdot f_{ck} \right)^{\frac{1}{3}} \right) \cdot b_w \cdot d$

$$C_{Rd,c} := \frac{0.18}{\gamma_c} = 0.12$$

$\gamma_c = 1.5$  Partial factor of the material Property

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} \leq 2.0 = 1$$

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} = 1.9 \quad Ok$$

$A_{sl}$   $A_{sl} := A_s$

Percentage of tensile reinforcement  $p_1 := \frac{A_{sl}}{b_w \cdot d} = 0.007$

$$V_{Rd.c} := \left( C_{Rd.c} \cdot k \cdot \left( 100 \cdot p_1 \cdot \frac{f_{ck}}{MPa} \right)^{\frac{1}{3}} \right) \cdot b_w \cdot d \cdot \frac{N}{mm^2} = 17.359 \text{ kN}$$

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{MPa} \right)^{\frac{1}{2}} \cdot b_w \cdot d \cdot \frac{N}{mm^2} = 13.583 \text{ kN}$$

$$V_{Rd.c} := \max(V_{Rd.c}, v_{min}) = 17.359 \text{ kN}$$

$$V_{Ed} < V_{Rd.c} = 1 \quad OK \quad \text{Design shear check Ok}$$

Max Shear  $\frac{V_{Ed}}{V_{Rd.c}} \cdot 100 = 71.003$

Deflection control using span effective depth

$$p := \frac{A_{s.req}}{b_w \cdot d} = 0.006 \quad p_0 := \sqrt{\frac{f_{ck}}{MPa}} \cdot 10^{-3} = 0.005$$

$$p > p_0 = 1 \quad \text{use eq. 7.16 (b)}$$

For simply supported beam  $k := 1$

$$\left( \frac{L}{d} \right) = D \quad D := k \cdot \left( 11 + 1.5 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot \frac{p_0}{p} \right) = 16.818$$

Modification limit value  $D_2 := D \cdot \frac{A_{s.Prov}}{A_{s.req}} = 17.695$

Actual deflection  $\frac{L}{d} = 24.291$

$$D_2 > \frac{L}{d} = 0 \quad \text{Ok Deflection check satisfied}$$



## Deflection Control

$$b := b_{eff}$$

### Uncracked section

$$E_{cm} := 31 \text{ GPa}$$

$$E_s := 200 \text{ GPa} \quad (\text{Clause 3.2.7 (4)})$$

$$\varphi := 1.95$$

Loading after 7 days

$$E_{c,eff} := \frac{E_{cm}}{1 + \varphi} = 10.508 \text{ GPa}$$

$$a_e := \frac{E_s}{E_{c,eff}} = 19.032$$

$$n := a_e$$

$$C := \frac{b_w}{n \cdot A_s} = 0.031 \frac{1}{\text{mm}}$$

$$f := \frac{h_f \cdot (b - b_w)}{n \cdot A_s} = 13.175$$

$$kd := \frac{\sqrt{C \cdot (2 \cdot d + h_f \cdot f) + (1 + f)^2} - (1 + f)}{C} = 51.651 \text{ mm}$$

$$Area_1 := b_{eff} \cdot h_f = 0.06 \text{ m}^2$$

$$y_1 := \frac{h_f}{2} = 40 \text{ mm}$$

$$Area_2 := b_w \cdot (h - h_f) = 0.026 \text{ m}^2$$

$$y_2 := h_f + \frac{h}{2} = 230 \text{ mm}$$

$$y := \frac{Area_1 \cdot y_1 + Area_2 \cdot y_2}{Area_1 + Area_2} = 98.056 \text{ mm}$$

$$I_1 := \frac{1}{12} \cdot b_{eff} \cdot h_f^3 + b_{eff} \cdot h_f \cdot (y - y_1)^2$$

$$I_2 := \frac{1}{12} \cdot b_w \cdot (h - h_f)^3 + b_w \cdot (h - h_f) \cdot (y - y_2)^2$$

$$I_{uc} := I_1 + I_2 = (8.003 \cdot 10^8) \text{ mm}^4$$

Criteria:

$$M_{cr} > M_{Ed.SLS}$$

The section will not crack,  $\zeta = 0$   
Consider only uncracked section

$$M_{cr} < M_{Ed.SLS}$$

The section will crack, Consider  
cracked and uncracked section

Check is section is cracking or not.

$$\psi_1 := 1$$

$$\psi_2 := 0.3$$

Category A

NA.A1.1

$$g_k := g_{0,k} + g_{1,k}$$

$$f_{Ed} := \psi_1 \cdot g_k + \psi_2 \cdot q_k = 2.771 \frac{kN}{m}$$

$$M_{Ed.SLS} := f_{Ed} \cdot \frac{L^2}{8} = 12.469 m^2 \cdot \frac{kN}{m}$$

Cracked moment  $M_{cr} := \frac{(f_{ctm} \cdot I_{uc})}{y} = 21.221 kN \cdot m$

$$M_{cr} > M_{Ed.SLS} = 1$$

The section will not crack, Consider only uncracked section

$$\zeta = 0$$

## Deflection Long Term

Curvature Due to Uncracked Section

$$I_{cr} := \frac{(b - b_w) \cdot h_f^3}{12} + \frac{b_w \cdot (kd)^3}{3} + (b - b_w) \cdot h_f \cdot \left(kd - \frac{h_f}{2}\right)^2 + n \cdot A_s \cdot (d - kd)^2$$

$$I_{cr} = (1.852 \cdot 10^8) mm^4$$

$$\alpha_I := \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{uc}} = (1.483 \cdot 10^{-6}) \frac{1}{mm}$$

$$\left(\frac{1}{r_{uc}}\right) = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{uc}} = (1.483 \cdot 10^{-6}) \frac{1}{mm}$$

## Calculating Cracked section

$$I_{cr} = (1.852 \cdot 10^8) \text{ mm}^4$$

Carvature due to cracked section

$$\alpha_{II} := \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} = (6.406 \cdot 10^{-6}) \frac{1}{\text{mm}}$$

$$\left( \frac{1}{r_{cr}} \right) = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} = (6.406 \cdot 10^{-6}) \frac{1}{\text{mm}}$$

Carvature due to load:

For long term  $\beta := 0.5$

$$\xi = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)$$

$$\xi := \left[ 1 - \beta \cdot \left( \frac{M_{cr}}{M_{Ed.SLS}} \right) \right]^2 = [0.022]$$

Cracked Load  $\alpha_{load} = \xi \cdot \alpha_{II} + (1 + \xi) \cdot \alpha_I$

Uncracked Load  $\alpha_{load} := (1 + \xi) \cdot \alpha_I = [1.516 \cdot 10^{-6}] \frac{1}{\text{mm}}$

Calculation carvature due to shrinkage

- Uncracked section

$$S_{uc} := A_s \cdot (d - y) = (2.994 \cdot 10^4) \text{ mm}^3 \quad \varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$\alpha_{csI} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{uc}} = (1.78 \cdot 10^{-7}) \frac{1}{\text{mm}}$$

- Cracked section

$$x := kd = 51.651 \text{ mm}$$

$$\varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$S_{cr} := A_s \cdot (d - x) = (3.927 \cdot 10^4) \text{ mm}^3$$

$$\alpha_{csII} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{cr}} = (7.691 \cdot 10^{-7}) \frac{1}{\text{mm}}$$

Cracked  $\alpha_{shrink} = \xi \cdot \alpha_{csII} + (1 + \xi) \cdot \alpha_{csI}$

Uncracked  $\alpha_{shrink} := (1 + \xi) \cdot \alpha_{csI} = [1.819 \cdot 10^{-4}] \frac{1}{m}$

$$Total := \alpha_{shrink} + \alpha_{load} = [1.697 \cdot 10^{-6}] \frac{1}{mm}$$

For simply supported distributed load:

$$K := 0.104$$

$$v_{max} := K \cdot Total \cdot L^2 = [6.355] \text{ mm}$$

$$v_{allow} := \frac{L}{250} = 24 \text{ mm}$$

$$v_{max} < v_{allow} = [1] \quad Ok$$

$$\frac{v_{max}}{v_{allow}} \cdot 100 = [26.48]$$

## Check for Vibrations

$$F_v \geq 8 \text{ Hz}$$

$$f_v = \frac{18}{\sqrt{\sigma_{max}}}$$

$$\sigma_{max} = \frac{5 \cdot q \cdot L^4}{384 \cdot EI_{dyn}}$$

For concrete (beam and slab) have same  $E_{cm}$  ; therefore same  $EI_{dyn}$

$$EI_{dyn} := E_{cm} \cdot 1.1 = (3.41 \cdot 10^4) \frac{N}{mm^2}$$

For concrete B30  
 $E_{cm} = (3.1 \cdot 10^4) \text{ MPa}$

Loading

$$b_{eff} = 0.75 \text{ m}$$

- Beam:

Self weight  $g_{beam} := (b_{eff} \cdot h_f \cdot \rho) + (h_w \cdot b_w \cdot \rho) = 2.16 \frac{kN}{m}$

10% of live load  $q_{dyn} := 0.1$

**Check here** Live load:  $q_{beam} := (2 + 0.8) \cdot \frac{kN}{m^2} \cdot q_{dyn} \cdot L = 1.68 \frac{kN}{m}$

$$I_{beam} := \frac{b_w \cdot h_w^3}{12} = (1.065 \cdot 10^8) \text{ mm}^4 \quad B := b_{eff}$$

$$\sigma_{slap} := 0$$

$$\sigma_{beam} := \frac{1 \cdot (g_{beam} + q_{beam}) \cdot L^4}{384 \cdot EI_{dyn} \cdot I_{beam}} = 3.569 \text{ mm}$$

$$\sigma_{max} := \sigma_{slap} + \sigma_{beam} = 3.569 \text{ mm}$$

$$f_v := \frac{18}{\sqrt{\sigma_{max}}} = \frac{18}{\sqrt{0.03}} = 103.923$$

Damping of the system:  $D = D_1 + D_2 + D_3$

$D_1$	Structural dumping	2	Concrete
$D_2$	Damping due to furniture	1	<i>Home library</i>
$D_3$	Damping due to finishes	0	free floating floor

$$D_1 := 2\% \quad D_2 := 1\% \quad D_3 := 0\%$$

$$D_{tot} := D_1 + D_2 + D_3 = 0.03$$

3%

Model mass:

$$\text{Load} \quad \text{Load} := \frac{(g_{beam})}{1 \text{ m}} = 2.16 \frac{\text{kN}}{\text{m}^2} \quad B = 0.75 \text{ m}$$

$$\text{Area} := L \cdot B = 4.5 \text{ m}^2$$

$$\text{Modelmas} := \text{Load} \cdot \text{Area} = 9.72 \text{ kN}$$

## Design for RC.Beem Case 1

## Category C5

### Section Dimentions

$$b_w := 120 \text{ mm} \quad b_{eff} := 750 \text{ mm} \quad h_f := 80 \text{ mm} \quad h_w := 220 \text{ mm}$$

$$A_{sume} \quad \phi_l := 12 \text{ mm} \quad \phi_s := 14 \text{ mm}$$

### Material

#### Steel

#### B500NC

$$f_{yk} := 500 \text{ MPa} \quad \gamma_s := 1.15 \quad f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \text{ MPa}$$

$$\text{Dencity} \quad \rho := 25 \frac{\text{kN}}{\text{m}^3}$$

#### Concrete Class: B30

$$f_{ck} := 30 \text{ MPa} \quad \gamma_c := 1.5$$

$$f_{cd} := \frac{0.85 \cdot f_{ck}}{\gamma_c} = 17 \text{ MPa} \quad f_{ctm} := 2.9 \text{ MPa}$$

$$\text{Cover} \quad C_{nom} := 45 \text{ mm}$$

$$\text{Dencity} \quad \rho := 25 \frac{\text{kN}}{\text{m}^3}$$

#### Variable load 5kN/m<sup>2</sup>

$$\gamma_Q := 1.5 \quad \gamma_G := 1.35$$

#### Charecteristic value of self weight

$$g_{0,k} := b_{eff} \cdot h_f \cdot \rho + (h_w - h_f) \cdot b_w \cdot \rho = 1.92 \frac{\text{kN}}{\text{m}}$$

#### Charecteristic value of dead load

$$g_{1,k} := 1 \frac{\text{kN}}{\text{m}^2} \cdot b_{eff} = 0.75 \frac{\text{kN}}{\text{m}}$$

#### Charecteristic value of variable load

$$q_k := (5.0 + 0.8) \frac{\text{kN}}{\text{m}^2} \cdot b_w = 0.696 \frac{\text{kN}}{\text{m}}$$

## Internal force analysis

Beam span  $L := 6 \text{ m}$

Boundary condition Single span with simple support

Design value of load

- $f_d := (g_{0.k} + g_{1.k}) \cdot \gamma_G + q_k \cdot \gamma_Q = 4.649 \frac{\text{kN}}{\text{m}}$
- $M_{Ed} := f_d \cdot \frac{L^2}{8} = 20.918 \text{ kN} \cdot \text{m}$
- $V_{Ed} := f_d \cdot \frac{L}{2} = 13.946 \text{ kN}$

## Check with ultimate moment capacity

Ultimate moment capacity  $M_{Rd} = 0.168 \cdot f_{ck} \cdot b \cdot d^2$

$$h := h_f + h_w = 300 \text{ mm}$$

$$d := h - \frac{\phi_l}{2} - C_{nom} = 249 \text{ mm}$$

$$M_f := f_{cd} \cdot h_f \cdot b_{eff} \cdot \left( h - \frac{h_f}{2} \right) = 265.2 \text{ kN} \cdot \text{m}$$

Natural axis with in the flange

$$M_{Rd} := 0.168 \cdot f_{ck} \cdot b_w \cdot d^2 = 37.498 \text{ kN} \cdot \text{m}$$

$$M_{Ed} < M_{Rd} = 1 \quad \text{Ok}$$

**Moment:**  $\frac{M_{Ed}}{M_{Rd}} \cdot 100 = 55.785$



$$M_f > M_{Rd} = 1$$

**No** need for compression reinforcement, and can carry the applied load

$$k := \frac{M_{Ed}}{b_{eff} \cdot d^2 \cdot f_{ck}} = 0.015 \quad k < 0.167$$

Singly reinforced

Calculate the lever arm Z

$$Z = d \left( 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right)$$

$$k := \frac{M_{Ed}}{b_w \cdot d^2 \cdot f_{ck}} = 0.094$$

$$Z := \min \left( d \cdot \left( 0.5 + \sqrt{0.25 - \frac{k}{1.134}} \right), 0.95 \cdot d \right) = 226.364 \text{ mm}$$

Required steel area

$$A_{s,req} := \frac{M_{Ed}}{f_{yd} \cdot Z} = 212.543 \text{ mm}^2$$

Provided steel area:

$$A_{s,Prov} := 226 \text{ mm}^2$$

**Steel Area**

$$A_s := A_{s,Prov} = 226 \text{ mm}^2$$

Number of bars

$$\phi_{l,2} := 8 \text{ mm}$$

use 2 bars

$$113 \text{ mm}^2$$

$$d_2 := \left( h - \frac{\phi_{l,2}}{2} - C_{nom} \right) = 251 \text{ mm}$$

**Check min reinforcement**

Clouse 9.3.1.1 In NS EN 1992-1-1:2004

**Min steel Area**

$$A_{s,min} := \max \left( 0.26 \cdot b_w \cdot d \cdot \frac{f_{ctm}}{f_{yk}}, 0.0013 \cdot b_w \cdot d \right) = 45.059 \text{ mm}^2$$

$$A_{s,min} < A_s = 1 \quad \text{Ok}$$

Check minimum reinforcement Ok

**Check maximum Reinforcement**

Clouse 9.2.1.1 In EC2

Cross cectional area  $A_c := b_{eff} \cdot h_f + b_w \cdot h_w = 0.086 \text{ m}^2$

$$A_{s,max} := 0.04 \cdot A_c = (3.456 \cdot 10^3) \text{ mm}^2$$

$$A_{s,req} < A_{s,max} = 1 \quad \text{Ok}$$

Check Max reinforcement Ok Less them max steel area is provided

**Ultimate Limit state, Shear check**

$$V_{Ed} < V_{Rd,c}$$

Simply supported beam  
shear force

$$V_{Ed} := f_d \cdot \frac{L}{2} = 13.946 \text{ kN}$$

Design shear resistance  
of beam

$$V_{Rd,c} = \left( C_{Rd,c} \cdot k \left( 100 \cdot p_1 \cdot f_{ck} \right)^{\frac{1}{3}} \right) \cdot b_w \cdot d$$

$$C_{Rd.c} := \frac{0.18}{\gamma_c} = 0.12$$

$\gamma_c = 1.5$     Partial factor of the material Property

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} \leq 2.0 = 1$$

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} = 1.896 \quad \text{Ok}$$

$A_{sl}$

$$A_{sl} := A_s$$

Percentage of tensile reinforcement

$$p_1 := \frac{A_{sl}}{b_w \cdot d} = 0.008$$

$$V_{Rd.c} := \left( C_{Rd.c} \cdot k \cdot \left( 100 \cdot p_1 \cdot \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{3}} \right) \cdot b_w \cdot d \cdot \frac{N}{\text{mm}^2} = 19.249 \text{ kN}$$

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{2}} \cdot b_w \cdot d \cdot \frac{N}{\text{mm}^2} = 14.957 \text{ kN}$$

$$V_{Rd.c} := \max(V_{Rd.c}, v_{min}) = 19.249 \text{ kN}$$

$$V_{Ed} < V_{Rd.c} = 1 \quad \text{OK} \quad \text{Design shear check Ok}$$

**Max Shear**     $\frac{V_{Ed}}{V_{Rd.c}} \cdot 100 = 72.449$

## Deflection control using span effective depth

$$p := \frac{A_{s.req}}{b_w \cdot d} = 0.007$$

$$p_0 := \sqrt{\frac{f_{ck}}{MPa}} \cdot 10^{-3} = 0.005$$

$$p > p_0 = 1$$

use eq. 7.16 (b)

For simply supported beam

$$k := 1$$

$$\left(\frac{L}{d}\right) = D$$

$$D := k \cdot \left(11 + 1.5 \cdot \sqrt{\frac{f_{ck}}{MPa} \cdot \frac{p_0}{p}}\right) = 17.326$$

Modification limit value

$$D_2 := D \cdot \frac{A_{s.Prov}}{A_{s.req}} = 18.423$$

Actual deflection

$$\frac{L}{d} = 24.096$$

$$D_2 > \frac{L}{d} = 0 \quad \text{Ok Deflection check satisfied}$$

## Deflection Control

$$b := b_{eff}$$

### Uncracked section

$$E_{cm} := 33 \text{ GPa}$$

$$E_s := 200 \text{ GPa} \quad (\text{Clause 3.2.7 (4)})$$

$$\varphi := 1.95$$

Loading after 7 days

$$E_{c.eff} := \frac{E_{cm}}{1 + \varphi} = 11.186 \text{ GPa}$$

$$a_e := \frac{E_s}{E_{c.eff}} = 17.879$$

$$n := a_e$$

$$C := \frac{b_w}{n \cdot A_s} = 0.03 \frac{1}{mm}$$

$$f := \frac{h_f \cdot (b - b_w)}{n \cdot A_s} = 12.473$$

$$kd := \frac{\sqrt{C \cdot (2 \cdot d + h_f \cdot f) + (1 + f)^2} - (1 + f)}{C} = 52.477 \text{ mm}$$

$$Area_1 := b_{eff} \cdot h_f = 0.06 \text{ m}^2 \quad y_1 := \frac{h_f}{2} = 40 \text{ mm}$$

$$Area_2 := b_w \cdot (h - h_f) = 0.026 \text{ m}^2 \quad y_2 := h_f + \frac{h}{2} = 230 \text{ mm}$$

$$y := \frac{Area_1 \cdot y_1 + Area_2 \cdot y_2}{Area_1 + Area_2} = 98.056 \text{ mm}$$

$$I_1 := \frac{1}{12} \cdot b_{eff} \cdot h_f^3 + b_{eff} \cdot h_f \cdot (y - y_1)^2$$

$$I_2 := \frac{1}{12} \cdot b_w \cdot (h - h_f)^3 + b_w \cdot (h - h_f) \cdot (y - y_2)^2$$

$$I_{uc} := I_1 + I_2 = (8.003 \cdot 10^8) \text{ mm}^4$$

<b>Criteria:</b>	$M_{cr} > M_{Ed.SLS}$	The section will not crack, Consider only uncracked section	$\zeta = 0$
	$M_{cr} < M_{Ed.SLS}$	The section will crack, Consider cracked and uncracked section	

Check is section is cracking or not.

$$\psi_1 := 1 \quad \psi_2 := 0.3 \quad \text{Category A} \quad \text{NA.A1.1}$$

$$g_k := g_{0.k} + g_{1.k}$$

$$f_{Ed} := \psi_1 \cdot g_k + \psi_2 \cdot q_k = 2.879 \frac{\text{kN}}{\text{m}}$$

$$M_{Ed.SLS} := f_{Ed} \cdot \frac{L^2}{8} = 12.955 \text{ m}^2 \cdot \frac{\text{kN}}{\text{m}}$$

Cracked moment  $M_{cr} := \frac{(f_{ctm} \cdot I_{uc})}{y} = 23.669 \text{ kN} \cdot \text{m}$

$$M_{cr} > M_{Ed.SLS} = 1$$

The section will not crack, Consider only uncracked section

$$\zeta = 0$$

## Deflection Long Term

### Curvature Due to Uncracked Section

$$I_{cr} := \frac{(b - b_w) \cdot h_f^3}{12} + \frac{b_w \cdot (kd)^3}{3} + (b - b_w) \cdot h_f \cdot \left(kd - \frac{h_f}{2}\right)^2 + n \cdot A_s \cdot (d - kd)^2$$

$$I_{cr} = (1.966 \cdot 10^8) \text{ mm}^4$$

$$\alpha_I := \frac{M_{Ed.SLS}}{E_{c.eff} \cdot I_{uc}} = (1.447 \cdot 10^{-6}) \frac{1}{\text{mm}}$$

$$\left(\frac{1}{r_{uc}}\right) = \frac{M_{Ed.SLS}}{E_{c.eff} \cdot I_{uc}} = (1.447 \cdot 10^{-6}) \frac{1}{\text{mm}}$$

## Calculating Cracked section

$$I_{cr} = (1.966 \cdot 10^8) \text{ mm}^4$$

Carvature due to cracked section

$$\alpha_{II} := \frac{M_{Ed.SLS}}{E_{c.eff} \cdot I_{cr}} = (5.892 \cdot 10^{-6}) \frac{1}{\text{mm}}$$

$$\left( \frac{1}{r_{cr}} \right) = \frac{M_{Ed.SLS}}{E_{c.eff} \cdot I_{cr}} = (5.892 \cdot 10^{-6}) \frac{1}{\text{mm}}$$

Carvature due to load:

$$\text{For long term} \quad \beta := 0.5 \quad \xi = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)$$

$$\xi := \left[ 1 - \beta \cdot \left( \frac{M_{cr}}{M_{Ed.SLS}} \right) \right]^2 = [0.007]$$

$$\text{Cracked Load} \quad \alpha_{load} = \xi \cdot \alpha_{II} + (1 + \xi) \cdot \alpha_I$$

$$\text{Uncracked Load} \quad \alpha_{load} := (1 + \xi) \cdot \alpha_I = [1.458 \cdot 10^{-6}] \frac{1}{\text{mm}}$$

## Calculation carvature due to shrinkage

- Uncracked section

$$S_{uc} := A_s \cdot (d - y) = (3.411 \cdot 10^4) \text{ mm}^3 \quad \varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$\alpha_{csI} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{uc}} = (1.905 \cdot 10^{-7}) \frac{1}{\text{mm}}$$

- Cracked section

$$x := kd = 52.477 \text{ mm}$$

$$\varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$S_{cr} := A_s \cdot (d - x) = (4.441 \cdot 10^4) \text{ mm}^3$$

$$\alpha_{csII} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{cr}} = (7.757 \cdot 10^{-7}) \frac{1}{\text{mm}}$$

Cracked  $\alpha_{shrink} = \xi \cdot \alpha_{csII} + (1 + \xi) \cdot \alpha_{csI}$

Uncracked  $\alpha_{shrink} := (1 + \xi) \cdot \alpha_{csI} = [1.919 \cdot 10^{-4}] \frac{1}{\text{m}}$

$$Total := \alpha_{shrink} + \alpha_{load} = [1.65 \cdot 10^{-6}] \frac{1}{\text{mm}}$$

**For simply supported distributed load:**

$$K := 0.104$$

$$v_{max} := K \cdot Total \cdot L^2 = [6.177] \text{ mm}$$

$$v_{allow} := \frac{L}{250} = 24 \text{ mm}$$

$$v_{max} < v_{allow} = [1] \quad \text{Ok}$$

$$\frac{v_{max}}{v_{allow}} \cdot 100 = [25.736]$$

Sheet 6 - Vibration - 10/11



## Check for Vibrations $F_v \geq 8 \text{ Hz}$

$$f_v = \frac{18}{\sqrt{\sigma_{max}}}$$

$$\sigma_{max} = \frac{5 \cdot q \cdot L^4}{384 \cdot EI_{dyn}}$$

For concrete (beam and slab) have same  $E_{cm}$  ; therefore same  $EI_{dyn}$

$$EI_{dyn} := E_{cm} \cdot 1.1 = (3.63 \cdot 10^4) \frac{N}{mm^2}$$

For concrete B30  
 $E_{cm} = (3.3 \cdot 10^4) \text{ MPa}$

Loading

$$b_{eff} = 0.75 \text{ m}$$

- Beam:

$$\text{Self weight } g_{beam} := (b_{eff} \cdot h_f \cdot \rho) + (h_w \cdot b_w \cdot \rho) = 2.16 \frac{kN}{m}$$

$$10\% \text{ of live load } q_{dyn} := 0.1$$

**Check here** Live load:  $q_{beam} := (5 + 0.8) \cdot \frac{kN}{m^2} \cdot q_{dyn} \cdot L = 3.48 \frac{kN}{m}$

$$I_{beam} := \frac{b_w \cdot h_w^3}{12} = (1.065 \cdot 10^8) \text{ mm}^4 \quad B := b_{eff}$$

$$\sigma_{slap} := 0$$

$$\sigma_{beam} := \frac{1 \cdot (g_{beam} + q_{beam}) \cdot L^4}{384 \cdot EI_{dyn} \cdot I_{beam}} = 4.925 \text{ mm}$$

$$\sigma_{max} := \sigma_{slap} + \sigma_{beam} = 4.925 \text{ mm}$$

$$f_v := \frac{18}{\sqrt{\sigma_{max}}} \quad \frac{18}{\sqrt{0.03}} = 103.923$$

Damping of the system:  $D = D_1 + D_2 + D_3$

$D_1$	Structural dumping	2	Concrete
$D_2$	Damping due to furniture	1	<i>Home library</i>
$D_3$	Damping due to finishes	0	free floating floor

$$D_1 := 2\% \quad D_2 := 1\% \quad D_3 := 0\%$$

$$D_{tot} := D_1 + D_2 + D_3 = 0.03$$

$$3\%$$

**Model mass:**

$$\text{Load} \quad \text{Load} := \frac{(g_{beam})}{1 \text{ m}} = 2.16 \frac{\text{kN}}{\text{m}^2}$$

$$B = 0.75 \text{ m}$$

$$\text{Area} := L \cdot B = 4.5 \text{ m}^2$$

$$\text{Modelmas} := \text{Load} \cdot \text{Area} = 9.72 \text{ kN}$$

## Case 2–1

### Design Example 3 Cross Laminated Timber CLT

#### Dimensions /Material

Simply supported floor structure of length 4.5m

Service class 1

Strength class C24

Safety class 3  $\gamma_d := 1$

Length  $L := 7.5 \text{ m}$   $a_1 := 120 \text{ mm}$

Number of layers:

CLT Consists of 5 Layers

$t_1 := 40 \text{ mm}$        $t_2 := 40 \text{ mm}$        $t_3 := 40 \text{ mm}$        $t_4 := 40 \text{ mm}$        $t_5 := 40 \text{ mm}$

$t_6 := 40 \text{ mm}$        $t_7 := 40 \text{ mm}$

$h_{CLT} := t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 = 280 \text{ mm}$

#### Material properties CLT for class 24

$E_{0.x.0.05} := 7400 \text{ MPa}$       Fifth percentile value of modulus of elasticity

$E_{x.mean} := 11000 \text{ MPa}$       Mean value of modulus of elasticity, along the grain

$G_{90.90.x.mean} := 50 \text{ MPa}$       Mean value of modulus of rollin shear

$G_{090.xlay.mean} := 690 \text{ MPa}$       Mean value of modulus shear

$f_{m.k} := 24 \text{ MPa}$       Bending strength

$f_{v.k} := 4.0 \text{ MPa}$       Shear Strength

$\gamma_M := 1.25$

$k_{mod} := 0.8$       Load – duration class for service class 1 and medium term

$\rho_{mean} := 4.2 \frac{\text{kN}}{\text{m}^3}$

Loads: (For a strip of the slab  $b_x=1\text{m}$ )  $b_x := 1 \text{ m}$

$$80 \cdot 9.82 \cdot 10^{-3} = 0.786$$

Self weight  $g_k := h_{CLT} \cdot \rho_{mean} \cdot b_x + \left(0.786 \frac{\text{kN}}{\text{m}}\right) = 1.962 \frac{\text{kN}}{\text{m}}$

Imposed load  $q_k := 5.0 \frac{\text{kN}}{\text{m}^2} \cdot 1 \text{ m} = 5 \frac{\text{kN}}{\text{m}}$

$$1.1 \cdot 0.35 = 0.385$$

Load faktors:  $\gamma_G := 1.35$   $\gamma_Q := 1.5$   $\psi_2 := 0.3$

$$\psi_1 := 0.50 \quad \psi_0 := 0.70 \quad \gamma_g := 0.89$$

$$q_d := \gamma_g \cdot g_k \cdot \gamma_G + \gamma_Q \cdot q_k = 9.857 \frac{\text{kN}}{\text{m}}$$

$$M_{Ed} := q_d \cdot \frac{L^2}{8} = 69.309 \text{ kN} \cdot \text{m}$$

$$V_{Ed} := q_d \cdot \frac{L}{2} = 36.965 \text{ kN}$$

Mean bulk Density  $\rho_{mean} := 460 \frac{\text{kN}}{\text{m}^3}$

Characteristic shear strength  $f_{v,90} := 0.7 \text{ MPa}$

## Ultimate limit state

### 1 Condition check – Shear

Rolling shear force and force should be checked – perpendicular

$$\tau_{Rv.d} \leq f_{Rv.d}$$

$$f_{Rv.d} := \frac{k_{mod} \cdot f_{v.90}}{\gamma_M} = 0.448 \text{ MPa}$$

Where;

$$\tau_{Rv.d} = \frac{S_{R.y.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x}$$

$$S_{R.y.net} = \sum_{i=1}^{ML} \frac{E_{x.i}}{E_{ref}} \cdot b_x \cdot t_i \cdot a_i$$

$$b_x := 1 \text{ m}$$

$$a_1 = 120 \text{ mm} \quad \text{Distance from the center of gravity}$$

$$t_i \quad \text{is the element thickness (dimension transverse to element plane)} \quad t_1 := 40 \text{ mm}$$

$$E_{x.i} \quad \text{modulus of layer}$$

$$E_{ref} \quad \text{Is the elastic modulus of reference layer.}$$

Since same timber quality is used for the layers

$$\frac{E_{x.i}}{E_{ref}} = 1$$

$$S_{R.y.net} := b_x \cdot t_1 \cdot a_1 = (4.8 \cdot 10^6) \text{ mm}^3$$

$$I_{x.net} = \sum \frac{E_{x.i}}{E_{ref}} \cdot \frac{b_x \cdot t_i^3}{12} + \sum \frac{E_{x.i}}{E_{ref}} \cdot b_x \cdot t_i \cdot a_i^2$$

$$I_{x.net} := b_x \cdot \left( 3 \cdot \frac{t_1^3}{12} + 2 \cdot t_1 \cdot a_1^2 \right) = (1.168 \cdot 10^9) \text{ mm}^4$$

$$\tau_{Rv.d} := \frac{S_{R.y.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x} = 0.152 \text{ MPa}$$

$$\tau_{Rv.d} \leq f_{Rv.d} = 1 \quad ([EN 1995-1-1], Eq. 6.13)$$

Rolling shear  $\frac{\tau_{Rv.d}}{f_{Rv.d}} \cdot 100 = 33.909$

*Check Satisfied OK*

For longitudinal shear (Paralell to grain)

$$\tau_{v.d} \leq f_{vd}$$

$$f_{vd} := \frac{k_{mod} \cdot f_{v.k}}{\gamma_M} = 2.56 \text{ MPa}$$

$$\tau_{v.d} = \frac{S_{x.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x}$$

$$S_{x.net} = \sum_{i=1}^{KL} \frac{E_{x,i}}{E_{ref}} \cdot b_x \cdot t_i \cdot a_i + \sum_{i=1}^{KL} \frac{E_{x,i}}{E_{ref}} \cdot b_x \cdot \frac{\left(\frac{t_k}{2} - a_k\right)}{2}$$

$a_k$  Distance from the center of gravity in the layer including the center of gravity

$t_k$  thickness of the layer including the center of gravity

$kL$  The index of the longitudinal layer closest to the the center of gravity

$$S_{x.net} := b_x \cdot t_1 \cdot a_1 + b_x \cdot \frac{t_3^2}{4 \cdot 2} = (5 \cdot 10^6) \text{ mm}^3$$

$$\tau_{v.d} := \frac{S_{x.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x} = 0.158 \text{ MPa}$$

$$\tau_{v.d} \leq f_{vd} = 1 \quad ([EN 1995-1-1], Eq. 6.13)$$

longitudenal shear  $\frac{\tau_{v.d}}{f_{vd}} \cdot 100 = 6.181$

*Check Satisfied OK*

## Moment Check CLT

Bending parallel to the grain, in the main direction

$$\gamma_{M,nor} := 1.15$$

$$\sigma_{m,d} \leq f_{m,d}$$

$$f_{m,d} := \frac{k_{mod} \cdot f_{m,k}}{\gamma_M} = 15.36 \text{ MPa}$$

$$\sigma_{m,d} = \frac{M_{Ed}}{W_{x,net}}$$

$$W_{x,net} := \frac{2 \cdot I_{x,net}}{h_{CLT}} = (8.343 \cdot 10^6) \text{ mm}^3$$

$$\sigma_{m,d} := \frac{M_{Ed}}{W_{x,net}} = 8.308 \text{ MPa}$$

$$\sigma_{m,d} \leq f_{m,d} = 1 \quad Ok$$

Bending parallel

$$\frac{\sigma_{m,d}}{f_{m,d}} \cdot 100 = 54.086$$

*Check Satisfied OK*

Deformations

$$w_{ins} \leq \frac{L}{300}$$

Short time deformation of characteristic load:

$$w_{ins} = w_{g,k} + w_{q,k}$$

$w_{g,k}$  Deformation due to permanent load action



$w_{q,k}$

Deformation due to variable load action

$$\begin{aligned} E_{x.1} &:= E_{x.mean} & G_{9090.2} &:= G_{9090.x.mean} \\ \text{Reduction factor } \gamma_1 &:= \frac{1}{1 + \frac{\pi^2 \cdot E_{x.1} \cdot t_1}{L^2} \cdot \frac{t_2}{G_{9090.2}}} = 0.942 \end{aligned}$$

$$I_{x.ef} := b_x \cdot \left( \frac{3 \cdot t_1^3}{12} + 2 \cdot \gamma_1 \cdot t_1 \cdot a_1^2 \right) = (1.101 \cdot 10^9) \text{ mm}^4$$

$$w_{g,k} := \frac{5 \cdot g_k \cdot L^4}{384 \cdot E_{x.mean} \cdot I_{x.ef}} = 6.674 \text{ mm}$$

$$w_{q,k} := \frac{5 \cdot q_k \cdot L^4}{384 \cdot E_{x.mean} \cdot I_{x.ef}} = 17.009 \text{ mm}$$

Short time deflection:  $w_{ins} := w_{g,k} + w_{q,k} = 23.683 \text{ mm}$

Deflection limit:  $\frac{L}{300} = 25 \text{ mm}$

$$w_{ins} \leq \frac{L}{300} = 1 \quad \text{Ok}$$

*Check Satisfied OK*

Final deformation because of creep on quasi permanent action:

$$k_{def} := 0.85$$

For service class 1

$$w_{fin} = w_{inst} + w_{creep}$$

$$w_{fin.g} := w_{g.k} \cdot (1 + k_{def}) = 12.347 \text{ mm}$$

$$w_{fin.q} := w_{q.k} \cdot (1 + \psi_2 \cdot k_{def}) = 21.346 \text{ mm}$$

$$w_{fin} := w_{fin.g} + w_{fin.q} = 33.694 \text{ mm}$$

$$\frac{L}{250} = 30 \text{ mm}$$

$$w_{fin} \leq \frac{L}{250} = 0 \quad \text{Ok}$$

Deflection

$$\frac{w_{fin}}{L} \cdot 100 = 112.313$$

Check Satisfied OK

Check for vibrations:

$$f_1 > 8 \text{ Hz}$$

$$E := E_{x.mean} \quad I := I_{x.ef}$$

$m$  Mass per unit area  $\text{kg/m}^2$  For walls and floor self weight is:  $110 - 150 \text{ kg/m}^2$

$$g_k = 1.962 \frac{\text{kN}}{\text{m}} \quad 1.962 \cdot 101 = 198.162$$

$$m := 198 \frac{\text{kg}}{\text{m}^2}$$

$$f_1 := \frac{\pi}{2 \cdot L^2} \cdot \sqrt{\frac{E \cdot I}{m}} = 6.906 \text{ m}^{\frac{1}{2}} \cdot \text{Hz}$$

$$13 \text{ Hz} > 8 \text{ Hz} = 1$$

Check Satisfied OK

## Case 2–1

### Design Example 3 Cross Laminated Timber CLT

#### Dimensions /Material

Simply supported floor structure of length 4.5m

Service class 1

Strength class C24

Safety class 3  $\gamma_d := 1$

Length  $L := 5 \text{ m}$   $a_1 := 90 \text{ mm}$

Number of layers:

CLT

$t_1 := 20 \text{ mm}$   $t_2 := 40 \text{ mm}$   $t_3 := 20 \text{ mm}$   $t_4 := 40 \text{ mm}$   $t_5 := 20 \text{ mm}$

$t_6 := 40 \text{ mm}$   $t_7 := 20 \text{ mm}$

$h_{CLT} := t_1 + t_2 + t_3 + t_4 + t_5 + t_6 + t_7 = 200 \text{ mm}$

#### Material properties CLT for class 24

$E_{0.x.0.05} := 8000 \text{ MPa}$  Fifth percentile value of modulus of elasticity

$E_{x.mean} := 12000 \text{ MPa}$  Mean value of modulus of elasticity, along the grain

$G_{9090.x.mean} := 50 \text{ MPa}$  Mean value of modulus of rollin shear

$G_{090.xlay.mean} := 750 \text{ MPa}$  Mean value of modulus shear

$f_{m.k} := 30 \text{ MPa}$  Bending strength

$f_{v.k} := 4.0 \text{ MPa}$  Shear Strength

$\gamma_M := 1.15$

$$k_{mod} := 0.8$$

Load – duration class for service class 1 and medium term

$$\rho_{mean} := 4.6 \frac{kN}{m^3}$$

Loads: (For a strip of the slab  $b_x=1m$ )  $b_x := 1 m$

$$80 \cdot 9.82 \cdot 10^{-3} = 0.786$$

Self weight

$$g_k := h_{CLT} \cdot \rho_{mean} \cdot b_x + \left( 0.786 \frac{kN}{m} \right) = 1.706 \frac{kN}{m}$$

Imposed load

$$q_k := 5.0 \frac{kN}{m^2} \cdot 1 m = 5 \frac{kN}{m}$$

$$1.1 \cdot 0.35 = 0.385$$

Load faktors:  $\gamma_G := 1.35$   $\gamma_Q := 1.5$   $\psi_2 := 0.3$

$$\psi_1 := 0.50 \quad \psi_0 := 0.70 \quad \gamma_g := 0.89$$

$$q_d := \gamma_g \cdot g_k \cdot \gamma_G + \gamma_Q \cdot q_k = 9.55 \frac{kN}{m}$$

$$M_{Ed} := q_d \cdot \frac{L^2}{8} = 29.843 kN \cdot m$$

$$V_{Ed} := q_d \cdot \frac{L}{2} = 23.874 kN$$

Mean bulk Density

$$\rho_{mean} := 460 \frac{kN}{m^3}$$

Characteristic shear strength

$$f_{v,90} := 0.7 MPa$$

## Ultimate limit state

### 1 Condition check – Shear

Rolling shear force and force should be checked – perpendicular

$$\tau_{Rv.d} \leq f_{Rv.d}$$

$$f_{Rv.d} := \frac{k_{mod} \cdot f_{v.90}}{\gamma_M} = 0.487 \text{ MPa}$$

Where;

$$\tau_{Rv.d} = \frac{S_{R.y.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x}$$

$$S_{R.y.net} = \sum_{i=1}^{ML} \frac{E_{x.i}}{E_{ref}} \cdot b_x \cdot t_i \cdot a_i$$

$$b_x := 1 \text{ m}$$

$$a_1 = 90 \text{ mm} \quad \text{Distance from the center of gravity}$$

$$t_i \quad \text{is the element thickness (dimension transverse to element plane)} \quad t_1 := 40 \text{ mm}$$

$$E_{x.i} \quad \text{modulus of layer}$$

$$E_{ref} \quad \text{Is the elastic modulus of reference layer.}$$

Since same timber quality is used for the layers

$$\frac{E_{x.i}}{E_{ref}} = 1$$

$$S_{R.y.net} := b_x \cdot t_1 \cdot a_1 = (3.6 \cdot 10^6) \text{ mm}^3$$

$$I_{x.net} = \sum \frac{E_{x,i}}{E_{ref}} \cdot \frac{b_x \cdot t_i^3}{12} + \sum \frac{E_{x,i}}{E_{ref}} \cdot b_x \cdot t_i \cdot a_i^2$$

$$I_{x.net} := b_x \cdot \left( 3 \cdot \frac{t_1^3}{12} + 2 \cdot t_1 \cdot a_1^2 \right) = (6.64 \cdot 10^8) \text{ mm}^4$$

$$\tau_{Rv.d} := \frac{S_{R.y.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x} = 0.129 \text{ MPa}$$

$$\tau_{Rv.d} \leq f_{Rv.d} = 1 \quad ([EN 1995-1-1], Eq. 6.13)$$

Rolling shear  $\frac{\tau_{Rv.d}}{f_{Rv.d}} \cdot 100 = 26.581$

Check Satisfied OK

For longitudinal shear (Parallel to grain)

$$\tau_{v.d} \leq f_{vd}$$

$$f_{vd} := \frac{k_{mod} \cdot f_{v,k}}{\gamma_M} = 2.783 \text{ MPa}$$

$$\tau_{v.d} = \frac{S_{x.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x}$$

$$S_{x.net} = \sum_{i=1}^{KL} \frac{E_{x,i}}{E_{ref}} \cdot b_x \cdot t_i \cdot a_i + \sum_{i=1}^{KL} \frac{E_{x,i}}{E_{ref}} \cdot b_x \cdot \frac{\left( \frac{t_k}{2} - a_k \right)}{2}$$

$a_k$  Distance from the center of gravity in the layer including the center of gravity

$t_k$  thickness of the layer including the center of gravity

$kL$  The index of the longitudinal layer closest to the the center of gravity

$$t_3^2$$

$$S_{x.net} := b_x \cdot t_1 \cdot a_1 + b_x \cdot \frac{t_3}{4 \cdot 2} = (3.65 \cdot 10^6) \text{ mm}^3$$

$$\tau_{v.d} := \frac{S_{x.net} \cdot V_{Ed}}{I_{x.net} \cdot b_x} = 0.131 \text{ MPa}$$

$$\tau_{v.d} \leq f_{vd} = 1 \quad ([EN 1995-1-1], Eq. 6.13)$$

longitudenal shear

$$\frac{\tau_{v.d}}{f_{vd}} \cdot 100 = 4.716$$

Check Satisfied OK

Moment Check CLT

Bending parallel to the grain, in the main directon

$$\gamma_{M.nor} := 1.15$$

$$\sigma_{m.d} \leq f_{m.d}$$

$$f_{m.d} := \frac{k_{mod} \cdot f_{m.k}}{\gamma_M} = 20.87 \text{ MPa}$$

$$\sigma_{m.d} = \frac{M_{Ed}}{W_{x.net}}$$

$$W_{x.net} := \frac{2 \cdot I_{x.net}}{h_{CLT}} = (6.64 \cdot 10^6) \text{ mm}^3$$

$$\sigma_{m.d} := \frac{M_{Ed}}{W_{x.net}} = 4.494 \text{ MPa}$$

$$\sigma_{m.d} \leq f_{m.d} = 1 \quad Ok$$

Bending parallel

$$\frac{\sigma_{m.d}}{f_{m.d}} \cdot 100 = 21.536$$



Check Satisfied OK

## Deformations

$$w_{ins} \leq \frac{L}{300}$$

Short time deformation of charecteristic load:

$$w_{ins} = w_{g,k} + w_{q,k}$$

$w_{g,k}$  Deformation due to permanent load action

$w_{q,k}$  Deformation due to variable load action

$$E_{x.1} := E_{x.mean} \quad G_{9090.2} := G_{9090.x.mean}$$

Reduction factor  $\gamma_1 := \frac{1}{1 + \frac{\pi^2 \cdot E_{x.1} \cdot t_1}{L^2} \cdot \frac{t_2}{G_{9090.2}}} = 0.868$

$$I_{x.ef} := b_x \cdot \left( \frac{3 \cdot t_1^3}{12} + 2 \cdot \gamma_1 \cdot t_1 \cdot a_1^2 \right) = (5.787 \cdot 10^8) \text{ mm}^4$$

$$w_{g,k} := \frac{5 \cdot g_k \cdot L^4}{384 \cdot E_{x.mean} \cdot I_{x.ef}} = 1.999 \text{ mm}$$

$$w_{q,k} := \frac{5 \cdot q_k \cdot L^4}{384 \cdot E_{x.mean} \cdot I_{x.ef}} = 5.859 \text{ mm}$$

Short time diflection:  $w_{ins} := w_{g,k} + w_{q,k} = 7.859 \text{ mm}$

Deflection limit:  $\frac{L}{300} = 16.667 \text{ mm}$

$$w_{ins} \leq \frac{L}{300} = 1 \quad \text{Ok}$$

*Check Satisfied OK*

**Final deformation because of creep on quasi permanent action:**

$$k_{def} := 0.85 \quad \text{For service class 1}$$

$$w_{fin} = w_{inst} + w_{creep}$$

$$w_{fin.g} := w_{g.k} \cdot (1 + k_{def}) = 3.699 \text{ mm}$$

$$w_{fin.q} := w_{q.k} \cdot (1 + \psi_2 \cdot k_{def}) = 7.354 \text{ mm}$$

$$w_{fin} := w_{fin.g} + w_{fin.q} = 11.052 \text{ mm}$$

$$\frac{\quad}{250} = 20 \text{ mm}$$

$$w_{fin} \leq \frac{L}{250} = 1 \quad \text{Ok}$$

Deflection

$$\frac{w_{fin}}{\frac{L}{250}} \cdot 100 = 55.261$$

Check Satisfied OK

Check for vibrations:

$$f_1 > 8 \text{ Hz}$$

$$E := E_{x.mean} \quad I := I_{x.ef}$$

$m$  Mass per unit area  $\text{kg/m}^2$  For walls and floor self weight is:  $110 - 150 \text{ kg/m}^2$

$$1.292 \cdot 101 = 130.492$$

$$g_k = 1.706 \frac{\text{kN}}{\text{m}}$$

$$m := 130.5 \frac{\text{kg}}{\text{m}^2}$$

$$f_1 := \frac{\pi}{2 \cdot L^2} \cdot \sqrt{\frac{(E \cdot I)}{m}} = 14.494 \text{ m}^{\frac{1}{2}} \cdot \text{Hz}$$

$$13 \text{ Hz} > 8 \text{ Hz} = 1$$

*Check Satisfied OK*

## Design for RC Slab Case 2-2

Simply supported floor slab structure

Category A Domestic area  
 $s_{hi} = 1$   $s_{hi2} = 0.3$

Category C Domestic area  
 $s_{hi} = 1$   $s_{hi2} = 0.6$

Section Dimensions

$$L := 6000 \text{ mm}$$

$$h := 240 \text{ mm}$$

$$\text{Length: } b := 1 \text{ m}$$

Material

Steel

B500NC

$$f_{yk} := 500 \text{ MPa} \quad \gamma_s := 1.15 \quad f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \text{ MPa}$$

$$\text{Density } \rho := 25 \frac{\text{kN}}{\text{m}^3}$$

Concrete Class: B25

$$f_{ck} := 25 \text{ MPa}$$

$$\gamma_c := 1.5$$

$$f_{cd} := \frac{0.85 f_{ck}}{\gamma_c} = 14.167 \text{ MPa} \quad f_{ctm} := 2.6 \text{ MPa} \quad E_{cm} := 31 \text{ GPa}$$

$$\text{Cover } C_{nom} := 35 \text{ mm}$$

Partial factors:

$$\gamma_G := 1.2 \quad \gamma_Q := 1.5$$

$$50 \cdot 9.82 \cdot 10^{-3} = 0.491$$

$$\text{Characteristic value of self weight } g_k := h \cdot \rho \cdot b + \left( 0.491 \frac{\text{kN}}{\text{m}} \right) = 6.491 \frac{\text{kN}}{\text{m}}$$

$$\text{Characteristic value of variable load } q_k := (2.0) \frac{\text{kN}}{\text{m}^2} \cdot b = 2 \frac{\text{kN}}{\text{m}}$$

Design value of load

Boundary condition: Single span with simple support

- $f_d := g_k \cdot \gamma_G + q_k \cdot \gamma_Q = 10.789 \frac{kN}{m}$
- $M_{Ed} := f_d \cdot \frac{L^2}{8} = 48.551 kN \cdot m$
- $V_{Ed} := f_d \cdot \frac{L}{2} = 32.368 kN$

Assume we are going to use  $\phi_l := 12 mm$

$$C_{min.b} := \max(8 mm, 10 mm) = 10 mm$$

$$C_{min.dur} := 15 mm \quad \text{For class XC1, Design life 50 years}$$

$$C_{dev} := 10 mm$$

$$C_{min} := \max(C_{min.b}, C_{min.dur}, 10 mm) = 15 mm$$

$$C_{nom} := C_{min} + C_{dev} = 25 mm$$

$$d := h - C_{nom} - \frac{\phi_l}{2} = 209 mm$$

$$M_{bal} := 0.167 \cdot b \cdot d^2 \cdot f_{ck} = 182.368 kN \cdot m$$

$$M_{Ed} < M_{bal} = 1 \quad \text{No Compression reinforcement needed}$$

Singly reinforced section

$$k := \frac{M_{Ed}}{b \cdot d^2 \cdot f_{ck}} = 0.044$$

$$z_1 := d \cdot \left( 0.5 + \sqrt{\left( 0.25 - \frac{k}{1.134} \right)} \right) = 200.457 mm$$

$$z := 0.95 d = 198.55 \text{ mm}$$

$$A_{s1} := \frac{M_{Ed}}{0.87 \cdot z \cdot f_{yd}} = 646.458 \text{ mm}^2$$

• **Required Steel area:**

$$A_s := 710 \text{ mm}^2$$

$$\frac{744.18}{113} = 6.586$$

Minimum reinforcement:  $A_{s.min} := 0.26 \cdot b \cdot d \cdot \frac{f_{ctm}}{f_{yk}} = 282.568 \text{ mm}^2$

Not less then:  $0.0013 \cdot b \cdot d = 271.7 \text{ mm}^2$

Maximum reinforcement:  $A_c := h \cdot b$

$$A_{s.max} := 0.04 \cdot A_c = (9.6 \cdot 10^3) \text{ mm}^2$$

$$A_{s.max} > A_{s.min} = 1 \quad Ok$$

$$A_{10} := 78.5 \text{ mm}^2$$

Spacing between bars:  $s_1 := \frac{A_{10}}{A_s} \cdot 1000 \text{ mm} = 110.563 \text{ mm}$

$$s_2 := \min(250 \text{ mm}, 2 \cdot h, s_1) = 110.563 \text{ mm}$$

$$A_{s.provid} := \frac{1000 \text{ mm}}{250 \text{ mm}} \cdot A_{10} = 314 \text{ mm}^2$$

**Check for moment capacity:**

$$M_{Ed} < M_{Rd}$$

$$M_{Rd} = 0.8 \cdot x \cdot b \cdot f_{cd} (d - 0.4 \cdot x)$$

$$x := \frac{f_{yd} \cdot A_s}{0.8 \cdot b \cdot f_{cd}} = 27.238 \text{ mm}$$

$$M_{Rd} := 0.8 \cdot x \cdot b \cdot f_{cd} \cdot (d - 0.4 \cdot x) = 61.154 \text{ kN} \cdot \text{m}$$

$$M_{Ed} < M_{Rd} = 1$$

$$Moment := \frac{M_{Ed}}{M_{Rd}} \cdot 100 = 79.392$$

Moment capacity is enough to carry the design bending moment

### Ultimate Limit state, Shear check

$$V_{Ed} < V_{Rd.c}$$

Simply supported beam shear force

$$V_{Ed} := f_d \cdot \frac{L}{2} = 32.368 \text{ kN}$$

Design shear resistance of beam

$$V_{Rd.c} = \left( C_{Rd.c} \cdot k \left( 100 \cdot p_1 \cdot f_{ck} \right)^{\frac{1}{3}} \right) \cdot b \cdot d$$

$$C_{Rd.c} := \frac{0.18}{\gamma_c} = 0.12$$

$\gamma_c = 1.5$  Partial factor of the material Property

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} \leq 2.0 = 1$$

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} = 1.978 \quad \text{Ok}$$

$A_{sl}$  Area of tensile reinforcement  $A_{sl} := A_s$

Percentage of tensile reinforcement  $p_1 := \frac{A_{sl}}{b \cdot d} = 0.003$

$$V_{Rd.c} := \left( C_{Rd.c} \cdot k \cdot \left( 100 \cdot p_1 \cdot \frac{f_{ck}}{\text{MPa}} \right)^{\frac{1}{3}} \right) \cdot b \cdot d \cdot \frac{\text{N}}{\text{mm}^2} = 101.225 \text{ kN}$$



$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{MPa} \right)^{\frac{1}{2}} \cdot b \cdot d \cdot \frac{N}{mm^2} = 101.765 \text{ kN}$$

$$V_{Rd.c} := \max(V_{Rd.c}, v_{min}) = 101.765 \text{ kN}$$

$$V_{Ed} < V_{Rd.c} = 1 \quad OK \quad \text{Design shear check Ok}$$

$$Shear := \frac{V_{Ed}}{V_{Rd.c}} \cdot 100 = 31.806$$

### Deflection control using span effective depth

$$p := \frac{A_s}{b \cdot d} = 0.003$$

$$p_0 := \sqrt{\frac{f_{ck}}{MPa}} \cdot 10^{-3} = 0.005$$

$$p < p_0 = 1 \quad \text{use eq. 7.16 (a)}$$

For simply supported beam  $k := 1$

$$\left( \frac{L}{d} \right) = D \quad D := k \cdot \left( 11 + 1.5 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot \frac{p_0}{p} + 13.2 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot \left( \frac{p_0}{p} - 1 \right)^{\frac{3}{2}} \right) = 43.429$$

$$D = k \cdot \left( 11 + 1.5 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot \frac{p_0}{p} + \frac{1}{12} \cdot \sqrt{\frac{f_{ck}}{MPa}} \right) = 0$$

Modification limit value  $D_2 := D \cdot \frac{A_{s,provided}}{A_s} = 19.207$

Actual deflection:  $\frac{L}{d} = 28.708$

$$D_2 > \frac{L}{d} = 0 \quad \text{Ok Deflection check satisfied}$$

## Deflection Control

### Uncracked section

$$E_{cm} := 31 \text{ GPa}$$

$$E_s := 200 \text{ GPa} \quad (\text{Clause 3.2.7 (4)})$$

$$\varphi := 1.95$$

Loading after 7 days

$$E_{c,eff} := \frac{E_{cm}}{1 + \varphi} = 10.508 \text{ GPa}$$

$$A_{sl} := \frac{1000 \text{ mm}}{250 \text{ mm}} \cdot A_{10} = 314 \text{ mm}^2$$

$$\alpha_e := \frac{E_s}{E_{c,eff}} = 19.032$$

$$f_{ctm} = 2.6 \text{ MPa}$$

$$X := \frac{h \cdot b \cdot \frac{h}{2} + (\alpha_e - 1) \cdot A_s \cdot d}{h \cdot b + (\alpha_e - 1) \cdot A_s} = 124.507 \text{ mm}$$

$$I_{uc} := \frac{1}{12} \cdot b \cdot h^3 + b \cdot h \cdot \left(\frac{h}{2} - X\right)^2 + (\alpha_e - 1) \cdot A_s \cdot (d - X)^2 = (1.248 \cdot 10^9) \text{ mm}^4$$

<b>Criteria:</b>	$M_{cr} > M_{Ed.SLS}$	The section will not crack, $\zeta = 0$ Consider only uncracked section
	$M_{cr} < M_{Ed.SLS}$	The section will crack, Consider cracked and uncracked section

Check is section is cracking or not.

$$\psi_1 := 1$$

$$\psi_2 := 0.3$$

Category A

NA.A1.1

$$f_{Ed} := \psi_1 \cdot g_k + \psi_2 \cdot q_k = 7.091 \frac{\text{kN}}{\text{m}}$$

$$M_{Ed.SLS} := f_{Ed} \cdot \frac{L^2}{8} = 31.91 \text{ m}^2 \cdot \frac{\text{kN}}{\text{m}}$$

Cracked moment  $M_{cr} := \frac{(f_{ctm} \cdot I_{uc})}{(h - X)} = 28.101 \text{ kN} \cdot \text{m}$

$$M_{cr} > M_{Ed.SLS} = 0$$

The section will not crack, Consider only

uncracked section

$$\zeta = 0$$

## Deflection Long Term

### Curvature Due to Uncracked Section

$$\alpha_I := \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{uc}} = (2.433 \cdot 10^{-6}) \frac{1}{mm}$$

$$\left( \frac{1}{r_{uc}} \right) = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{uc}} = (2.433 \cdot 10^{-6}) \frac{1}{mm}$$

### Calculating Cracked section

$$n := a_e \quad B := \frac{b}{n \cdot A_s} = 0.074 \frac{1}{mm} \quad kd := \frac{\sqrt{2 \cdot d \cdot B + 1} - 1}{B} = 62.848 \text{ mm}$$

$$I_{cr} := b \cdot \frac{(kd)^3}{3} + n \cdot A_s \cdot (d - kd)^2 = (3.714 \cdot 10^8) \text{ mm}^4$$

Curvature due to uncracked section

$$\alpha_{II} := \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} = (8.176 \cdot 10^{-6}) \frac{1}{mm}$$

$$\left( \frac{1}{r_{cr}} \right) = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} = (8.176 \cdot 10^{-6}) \frac{1}{mm}$$

Curvature due to load:

$$\text{For long term} \quad \beta := 0.5 \quad \xi = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)$$

$$\xi := \left[ 1 - \beta \cdot \left( \frac{M_{cr}}{M_{Ed.SLS}} \right) \right]^2 = [0.313]$$

$$\alpha_{load} := \xi \cdot \alpha_{II} + (1 + \xi) \cdot \alpha_I = [5.756 \cdot 10^{-6}] \frac{1}{mm}$$

$$\alpha_{load} = (1 + \xi) \cdot \alpha_I$$

## Calculation curvature due to shrinkage

- Uncracked section

$$S_{uc} := A_s \cdot (d - X) = (5.999 \cdot 10^4) \text{ mm}^3 \quad \varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$\alpha_{csI} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{uc}} = (2.287 \cdot 10^{-7}) \frac{1}{\text{mm}}$$

- Cracked section

$$x := kd = 62.848 \text{ mm} \quad \varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$S_{cr} := A_s \cdot (d - x) = (1.038 \cdot 10^5) \text{ mm}^3$$

$$\alpha_{csII} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{cr}} = (7.686 \cdot 10^{-7}) \frac{1}{\text{mm}}$$

$$\alpha_{shrink} := \xi \cdot \alpha_{csII} + (1 + \xi) \cdot \alpha_{csI} = [5.41 \cdot 10^{-7}] \frac{1}{\text{mm}}$$

$$\alpha_{shrink} = (1 + \xi) \cdot \alpha_{csI} = 0$$

$$Total := \alpha_{shrink} + \alpha_{load} = [6.297 \cdot 10^{-6}] \frac{1}{\text{mm}}$$

For simply supported distributed load:

$$K := 0.104$$

$$v_{max} := K \cdot Total \cdot L^2 = [23.574] \text{ mm}$$

$$v_{allow} := \frac{L}{250} = 24 \text{ mm}$$

$$v_{max} < v_{allow} = [1] \quad Ok$$

$$\frac{v_{max}}{v_{allow}} \cdot 100 = [98.227]$$

Check for Vibrations

## Check for Vibrations

$$F_v \geq 8 \text{ Hz}$$

$$f_v = \frac{18}{\sqrt{\sigma_{max}}}$$

$$\sigma_{max} = \frac{5 \cdot q \cdot L^4}{384 \cdot EI_{dyn}}$$

For concrete (beam and slab) have same  $E_{cm}$  ; therefore same  $EI_{dyn}$

$$EI_{dyn} := E_{cm} \cdot 1.1 = (3.41 \cdot 10^4) \frac{\text{N}}{\text{mm}^2}$$

$$\text{For concrete B30} \\ E_{cm} = (3.1 \cdot 10^4) \text{ MPa}$$

Loading

- Slab:

$$\text{Self weight } g_k = 6.491 \frac{\text{kN}}{\text{m}}$$

$$10\% \text{ of live load } q_{dyn} := 0.1$$

$$\text{Live load: } q_{slap} := (2) \cdot \frac{\text{kN}}{\text{m}^2} \cdot q_{dyn} \cdot 1 \text{ m} = 0.2 \frac{\text{kN}}{\text{m}}$$

$$I_{slap} := \frac{b \cdot h^3}{12} = (1.152 \cdot 10^9) \text{ mm}^4$$

$$\sigma_{tot.max} := \frac{5 \cdot (g_k + q_{slap}) \cdot L^4}{384 \cdot EI_{dyn} \cdot I_{slap}} = 2.874 \text{ mm}$$

$$f_v := \frac{18}{\sqrt{\sigma_{tot.max}}} = \frac{18}{\sqrt{2.874}} = 10.618$$

Deflection is check and doesnot exceed the limit. OK

Damping of the system:

$D_1$	Structural dumping	2	Concrete
$D_2$	Damping due to furniture	1	<i>Home library</i>
$D_3$	Damping due to finishes	0	free floating floor

$$D_1 := 2\%$$

$$D_3 := 0\%$$

$$3\%$$

## Design for RC Slab Case 2-2 B30 C5

Simply supported floor slab structure

Category A Domestic area

sh<sub>i</sub> = 1 sh<sub>i2</sub> = 0.3

Category C Domestic area

sh<sub>i</sub> = 1 sh<sub>i2</sub> = 0.6

Section Dimensions

$$L := 3000 \text{ mm}$$

$$h := 200 \text{ mm}$$

Length:

$$b := 1 \text{ m}$$

Material

Steel

B500NC

$$f_{yk} := 500 \text{ MPa}$$

$$\gamma_s := 1.15$$

$$f_{yd} := \frac{f_{yk}}{\gamma_s} = 434.783 \text{ MPa}$$

$$\text{Density } \rho := 25 \frac{\text{kN}}{\text{m}^3}$$

Concrete Class: B30

$$f_{ck} := 30 \text{ MPa}$$

$$\gamma_c := 1.5$$

$$f_{cd} := \frac{0.85 f_{ck}}{\gamma_c} = 17 \text{ MPa}$$

$$f_{ctm} := 2.9 \text{ MPa}$$

$$E_{cm} := 31 \text{ GPa}$$

$$\text{Cover } C_{nom} := 35 \text{ mm}$$

Partial factors:

$$\gamma_G := 1.2 \quad \gamma_Q := 1.5$$

$$50 \cdot 9.82 \cdot 10^{-3} = 0.491$$

Characteristic value of self weight

$$g_k := h \cdot \rho \cdot b + \left( 0.491 \frac{\text{kN}}{\text{m}} \right) = 5.491 \frac{\text{kN}}{\text{m}}$$

Characteristic value of variable load

$$q_k := (5.0) \frac{\text{kN}}{\text{m}^2} \cdot b = 5 \frac{\text{kN}}{\text{m}}$$

Design value of load

Boundary condition: Single span with simple support

- $f_d := g_k \cdot \gamma_G + q_k \cdot \gamma_Q = 14.089 \frac{kN}{m}$
- $M_{Ed} := f_d \cdot \frac{L^2}{8} = 15.85 kN \cdot m$
- $V_{Ed} := f_d \cdot \frac{L}{2} = 21.134 kN$

Assume we are going to use  $\phi_l := 12 mm$

$$C_{min.b} := \max(8 mm, 10 mm) = 10 mm$$

$$C_{min.dur} := 15 mm \quad \text{For class XC1, Design life 50 years}$$

$$C_{dev} := 10 mm$$

$$C_{min} := \max(C_{min.b}, C_{min.dur}, 10 mm) = 15 mm$$

$$C_{nom} := C_{min} + C_{dev} = 25 mm$$

$$d := h - C_{nom} - \frac{\phi_l}{2} = 169 mm$$

$$M_{bal} := 0.167 \cdot b \cdot d^2 \cdot f_{ck} = 143.091 kN \cdot m$$

$$M_{Ed} < M_{bal} = 1 \quad \text{No Compression reinforcement needed}$$

Singly reinforced Section



$$k := \frac{M_{Ed}}{b \cdot d^2 \cdot f_{ck}} = 0.018$$

$$z_1 := d \cdot \left( 0.5 + \sqrt{\left( 0.25 - \frac{k}{1.134} \right)} \right) = 166.197 \text{ mm}$$

$$z := 0.95 d = 160.55 \text{ mm}$$

$$A_{s1} := \frac{M_{Ed}}{0.87 \cdot z \cdot f_{yd}} = 260.998 \text{ mm}^2$$

$$\frac{744.18}{113} = 6.586$$

• **Required Steel area:**

$$A_s := 1469 \text{ mm}^2$$

Minimum reinforcement:  $A_{s,min} := 0.26 \cdot b \cdot d \cdot \frac{f_{ctm}}{f_{yk}} = 228.488 \text{ mm}^2$

Not less than:  $0.0013 \cdot b \cdot d = 219.7 \text{ mm}^2$

Maximum reinforcement:  $A_c := h \cdot b$

$$A_{s,max} := 0.04 \cdot A_c = (8 \cdot 10^3) \text{ mm}^2$$

$$A_{s,max} > A_{s,min} = 1 \quad Ok$$

$$A_{10} := 78.5 \text{ mm}^2$$

Spacing between bars:  $s_1 := \frac{A_{10}}{A_s} \cdot 1000 \text{ mm} = 53.438 \text{ mm}$

$$s_2 := \min(250 \text{ mm}, 2 \cdot h, s_1) = 53.438 \text{ mm}$$

$$A_{s,provided} := \frac{1000 \text{ mm}}{250 \text{ mm}} \cdot A_{10} = 314 \text{ mm}^2$$

**Check for moment capacity:**

$$M_{Ed} < M_{Rd}$$

$$M_{Rd} = 0.8 \cdot x \cdot b \cdot f_{cd} (d - 0.4 \cdot x)$$

$$x := \frac{f_{yd} \cdot A_s}{0.8 \cdot b \cdot f_{cd}} = 46.963 \text{ mm}$$

$$M_{Rd} := 0.8 \cdot x \cdot b \cdot f_{cd} \cdot (d - 0.4 \cdot x) = 95.942 \text{ kN} \cdot \text{m}$$

$$M_{Ed} < M_{Rd} = 1$$

$$\text{Moment} := \frac{M_{Ed}}{M_{Rd}} \cdot 100 = 16.521$$

Moment capacity is enough to carry the design bending moment

#### Ultimate Limit state, Shear check

$$V_{Ed} < V_{Rd,c}$$

Simply supported beam  
shear force

$$V_{Ed} := f_d \cdot \frac{L}{2} = 21.134 \text{ kN}$$

Design shear resistance  
of beam

$$V_{Rd,c} = \left( C_{Rd,c} \cdot k \left( 100 \cdot p_1 \cdot f_{ck} \right)^{\frac{1}{3}} \right) \cdot b \cdot d$$

$$C_{Rd,c} := \frac{0.18}{\gamma_c} = 0.12$$

$\gamma_c = 1.5$  Partial factor of the material Property

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} \leq 2.0 = 0$$

$$k := 1 + \sqrt{\frac{200 \text{ mm}}{d}} = 2.088 \quad \text{Ok}$$

$A_{sl}$  Area of tensile reinforcement  $A_{sl} := A_s$

Percentage of tensile reinforcement  $p_1 := \frac{A_{sl}}{b \cdot d} = 0.009$

$$V_{Rd.c} := \left( C_{Rd.c} \cdot k \cdot \left( 100 \cdot p_1 \cdot \frac{f_{ck}}{MPa} \right)^{\frac{1}{3}} \right) \cdot b \cdot d \cdot \frac{N}{mm^2} = 125.561 \text{ kN}$$

$$v_{min} := 0.035 \cdot k^{\frac{3}{2}} \cdot \left( \frac{f_{ck}}{MPa} \right)^{\frac{1}{2}} \cdot b \cdot d \cdot \frac{N}{mm^2} = 97.739 \text{ kN}$$

$$V_{Rd.c} := \max(V_{Rd.c}, v_{min}) = 125.561 \text{ kN}$$

$$V_{Ed} < V_{Rd.c} = 1 \quad OK \quad \text{Design shear check Ok}$$

$$Shear := \frac{V_{Ed}}{V_{Rd.c}} \cdot 100 = 16.832$$

### Deflection controll using span effective depth

$$p := \frac{A_s}{b \cdot d} = 0.009 \quad p_0 := \sqrt{\frac{f_{ck}}{MPa}} \cdot 10^{-3} = 0.005$$

$$p < p_0 = 0 \quad \text{use eq. 7.16 (a)}$$

For simply supported beam  $k := 1$

$$\left( \frac{L}{d} \right) = D \quad D := k \cdot \left( 11 + 1.5 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot \frac{p_0}{p} + 13.2 \cdot \sqrt{\frac{f_{ck}}{MPa}} \cdot \left( \frac{p_0}{p} - 1 \right)^{\frac{3}{2}} \right) = 16.177 - 16.264i$$

$$D = k \cdot \left( 11 + 1.5 \cdot \sqrt{\frac{f_{ck}}{\text{MPa}}} \cdot \frac{p_0}{p} + \frac{1}{12} \cdot \sqrt{\frac{f_{ck}}{\text{MPa}}} \right) = 0$$

Modification limit value  $D_2 := D \cdot \frac{A_{s, \text{provided}}}{A_s} = 3.458 - 3.476i$

Actual deflection:  $\frac{L}{d} = 17.751$

$D_2 > \frac{L}{d} = ?$  Ok Deflection check satisfied

## Deflection Control

### Uncracked section

$$E_{cm} := 31 \text{ GPa}$$

$$E_s := 200 \text{ GPa} \quad (\text{Clause 3.2.7 (4)})$$

$$\varphi := 1.95$$

Loading after 7 days

$$E_{c, \text{eff}} := \frac{E_{cm}}{1 + \varphi} = 10.508 \text{ GPa}$$

$$A_{sl} := \frac{1000 \text{ mm}}{250 \text{ mm}} \cdot A_{10} = 314 \text{ mm}^2$$

$$a_e := \frac{E_s}{E_{c, \text{eff}}} = 19.032$$

$$f_{ctm} = 2.6 \text{ MPa}$$

$$X := \frac{h \cdot b \cdot \frac{h}{2} + (a_e - 1) \cdot A_s \cdot d}{h \cdot b + (a_e - 1) \cdot A_s} = 108.07 \text{ mm}$$

$$I_{uc} := \frac{1}{12} \cdot b \cdot h^3 + b \cdot h \cdot \left( \frac{h}{2} - X \right)^2 + (a_e - 1) \cdot A_s \cdot (d - X)^2 = (7.78 \cdot 10^8) \text{ mm}^4$$

**Criteria:**

$$M_{cr} > M_{Ed, SLS}$$

The section will not crack,  
Consider only uncracked section

$$\zeta = 0$$

$$M_{cr} < M_{Ed.SLS}$$

The section will crack, Consider cracked and uncracked section

Check is section is cracking or not.

$$\psi_1 := 1$$

$$\psi_2 := 0.3$$

Category A

NA.A1.1

$$f_{Ed} := \psi_1 \cdot g_k + \psi_2 \cdot q_k = 6.991 \frac{kN}{m}$$

$$M_{Ed.SLS} := f_{Ed} \cdot \frac{L^2}{8} = 7.865 \text{ m}^2 \cdot \frac{kN}{m}$$

Cracked moment  $M_{cr} := \frac{(f_{ctm} \cdot I_{uc})}{(h - X)} = 22.005 \text{ kN} \cdot \text{m}$

$$M_{cr} > M_{Ed.SLS} = 1$$

The section will not crack, Consider only uncracked section

$$\zeta = 0$$

## Deflection Long Term

### Curvature Due to Uncracked Section

$$\alpha_f := \frac{M_{Ed.SLS}}{E_{c.eff} \cdot I_{uc}} = (9.62 \cdot 10^{-7}) \frac{1}{mm}$$

$$\left( \frac{1}{r_{uc}} \right) = \frac{M_{Ed.SLS}}{E_{c.eff} \cdot I_{uc}} = (9.62 \cdot 10^{-7}) \frac{1}{mm}$$

### Calculating Cracked section

$$n := a_e \quad B := \frac{b}{n \cdot A_s} = 0.036 \frac{1}{mm} \quad kd := \frac{\sqrt{2 \cdot d \cdot B + 1} - 1}{B} = 73.193 \text{ mm}$$

$$I_{cr} := b \cdot \frac{(kd)^3}{3} + n \cdot A_s \cdot (d - kd)^2 = (3.873 \cdot 10^8) \text{ mm}^4$$

Carvature due to uncracked section

$$\alpha_{II} := \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} = (1.932 \cdot 10^{-6}) \frac{1}{mm}$$

$$\left( \frac{1}{r_{cr}} \right) = \frac{M_{Ed.SLS}}{E_{c,eff} \cdot I_{cr}} = (1.932 \cdot 10^{-6}) \frac{1}{mm}$$

Carvature due to load:

For long term  $\beta := 0.5$   $\xi = 1 - \beta \left( \frac{\sigma_{sr}}{\sigma_s} \right)$

$$\xi := \left[ 1 - \beta \cdot \left( \frac{M_{cr}}{M_{Ed.SLS}} \right) \right]^2 = [0.159]$$

$$\alpha_{load} := \xi \cdot \alpha_{II} + (1 + \xi) \cdot \alpha_I = [1.423 \cdot 10^{-6}] \frac{1}{mm}$$

$$\alpha_{load} = (1 + \xi) \cdot \alpha_I$$

### Calculation carvature due to shrinkage

- Uncracked section

$$S_{uc} := A_s \cdot (d - X) = (8.951 \cdot 10^4) \text{ mm}^3 \quad \varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$\alpha_{csI} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{uc}} = (5.474 \cdot 10^{-7}) \frac{1}{mm}$$

- Cracked section

$$x := kd = 73.193 \text{ mm}$$

$$\varepsilon_{cs} := 0.25 \cdot 10^{-3}$$

$$S_{cr} := A_s \cdot (d - x) = (1.407 \cdot 10^5) \text{ mm}^3$$

$$\alpha_{csII} := \varepsilon_{cs} \cdot a_e \cdot \frac{S_{uc}}{I_{cr}} = (1.1 \cdot 10^{-6}) \frac{1}{mm}$$

$$\alpha_{shrink} := \xi \cdot \alpha_{csII} + (1 + \xi) \cdot \alpha_{csI} = [8.095 \cdot 10^{-7}] \frac{1}{mm}$$

$$\alpha_{shrink} = (1 + \xi) \cdot \alpha_{csI} = 0$$

$$Total := \alpha_{shrink} + \alpha_{load} = [2.232 \cdot 10^{-6}] \frac{1}{mm}$$

**For simply supported distributed load:**

$$K := 0.104$$

$$v_{max} := K \cdot Total \cdot L^2 = [2.089] \text{ mm}$$

$$v_{allow} := \frac{L}{250} = 12 \text{ mm}$$

$$v_{max} < v_{allow} = [1] \quad Ok$$

$$\frac{v_{max}}{v_{allow}} \cdot 100 = [17.409]$$

## Check for Vibrations

$$F_v \geq 8 \text{ Hz}$$

$$f_v = \frac{18}{\sqrt{\sigma_{max}}}$$

$$\sigma_{max} = \frac{5 \cdot q \cdot L^4}{384 \cdot EI_{dyn}}$$

For concrete (beam and slab) have same  $E_{cm}$  ; therefore same  $EI_{dyn}$

$$EI_{dyn} := E_{cm} \cdot 1.1 = (3.41 \cdot 10^4) \frac{N}{mm^2}$$

For concrete B30  
 $E_{cm} = (3.1 \cdot 10^4) \text{ MPa}$

Loading

- Slab:

$kN$

$$\text{Self weight } g_k = 5.491 \frac{\text{kN}}{\text{m}}$$

$$10\% \text{ of live load } q_{dyn} := 0.1$$

$$\text{Live load: } q_{slap} := (2) \cdot \frac{\text{kN}}{\text{m}^2} \cdot q_{dyn} \cdot 1 \text{ m} = 0.2 \frac{\text{kN}}{\text{m}}$$

$$I_{slap} := \frac{b \cdot h^3}{12} = (6.667 \cdot 10^8) \text{ mm}^4$$

$$\sigma_{tot.max} := \frac{5 \cdot (g_k + q_{slap}) \cdot L^4}{384 \cdot EI_{dyn} \cdot I_{slap}} = 0.264 \text{ mm}$$

$$f_v := \frac{18}{\sqrt{\sigma_{tot.max}}} \quad \frac{18}{\sqrt{2.438}} = 11.528$$

Deflection is check and doesnot exceed the limit. OK

Damping of the system:

$D_1$	Structural dumping	2	Concrete
$D_2$	Damping due to furniture	1	Home library
$D_3$	Damping due to finishes	0	free floating floor

$$D_1 := 2\%$$

$$D_3 := 0\%$$

$$3\%$$



# Creep calculations

RC beam/Slab Fcm33

$$B25 \quad f_{cm} := 33 \text{ MPa} \quad h := 220 \text{ mm} \quad b := 120 \text{ mm}$$

For design life 50years

$$\varphi_{t_0} = \varphi_{RH} \cdot \beta(f_{cm}) \cdot \beta_{t_0} \cdot \beta_c(\infty, t_0)$$

$$\varphi_{RH} = 1 + \frac{\frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \quad \text{For } f_{cm} \leq 35 \text{ MPa}$$

$$\bullet \quad \varphi_{RH} = \left[ 1 + \frac{\frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \cdot a_1 \right] a_2 \quad \text{For } f_{cm} \geq 35 \text{ MPa}$$

$$a_1 := \left[ \frac{35 \text{ MPa}}{f_{cm}} \right]^{0.7} = [1.042] \quad a_2 := \left[ \frac{35 \text{ MPa}}{f_{cm}} \right]^{0.2} = [1.012] \quad RH := 50\%$$

$$a_3 := \left[ \frac{35 \text{ MPa}}{f_{cm}} \right]^{0.5} = [1.03]$$

$$A_c := h \cdot b$$

$$\frac{RH}{100} = 0.005$$

$$U := 2 \cdot h + 2 \cdot b$$

$$h_0 := \frac{2 \cdot A_c}{U} = 77.647 \text{ mm}$$

$$h_0 := 77.647$$

$$\varphi_{RH} := \left( 1 + \frac{1 - \frac{RH}{100}}{0.1 \cdot \sqrt[3]{h_0}} \cdot a_1 \right) a_2 = 3.471$$

- $$\beta_{f_{cm}} := \frac{16.8}{\sqrt{\left(\frac{f_{cm}}{\text{MPa}}\right)}} = 2.925$$

Class R

$$t_o = t_{o,T} \cdot \left( \frac{9}{2 + t_{o,T}^{1.2}} + 1 \right)^a \geq 0.5$$

$a := 1$                        $t_{o,T} := 28$                       *days*

- $$t_o := t_{o,T} \cdot \left( \frac{9}{2 + t_{o,T}^{1.2}} + 1 \right)^a = 32.458$$

$$\beta_{t_o} := 1$$

$$\beta_{t_o} := \frac{1}{0.1 + t_o^{0.20}} = 0.475$$

$$\varphi_{t_o} := \varphi_{RH} \cdot \beta_{f_{cm}} \cdot \beta_{t_o} \cdot \beta_{t_o} = 2.289$$