Teachers' Beliefs about Mathematical Horizon Content Knowledge

Reidar Mosvold[1] and Janne Fauskanger[2]

In this article, we present and discuss an example of how teachers' discussions of mathematical knowledge for teaching (MKT) items elicited their beliefs about the knowledge needed to teach mathematics. One category of MKT is "horizon content knowledge," and this can be described as mathematical knowledge not directly deployed in instruction—or knowledge behind as well as ahead of the pupils in an actual teaching situation. Since teachers' beliefs may influence their teaching and how they approach a professional development course, it is important to study teachers' epistemic beliefs. The transcripts were analyzed in a three-step content analysis approach, and the results indicate that teachers disregard important aspects of knowledge at the mathematical horizon from their teaching knowledge.

Keywords: teacher knowledge, mathematical knowledge for teaching, horizon content knowledge, epistemic beliefs

Introduction

Teachers play an important role as far as the quality of students' learning is concerned, and teachers' knowledge is a factor of particular importance (e.g., Askew, 2008). Several frameworks have been developed to capture the content of teachers' knowledge (e.g., Petrou & Goulding, 2011). One of these frameworks is referred to as "mathematical knowledge for teaching" (MKT). This particular kind of professional knowledge is defined as "the mathematical knowledge used to carry out the work of teaching mathematics" (Hill, Rowan, & Ball, 2005, p. 373). Researchers at the University of Michigan have developed the framework (e.g., Ball, Thames, & Phelps, 2008), as well as measures of teachers' MKT. These measures have been used, among other things, for the planning and evaluation of professional development for teachers. In an attempt to adapt the MKT measures for use in a Norwegian school context, group interviews with teachers were conducted (e.g., Fauskanger & Mosvold, 2010). In these interviews the teachers discussed which knowledge they regarded as relevant (and irrelevant) for their teaching, and the rationale behind their claims were articulated (Fauskanger, 2012). Although the interviews were primarily conducted as part of the overall adaptation process (Fauskanger, Jakobsen, Mosvold, & Bjuland, 2012), we realized that the interview data provided an interesting glimpse into the teachers' beliefs about aspects of MKT.

The teachers' discussions of MKT items informed us about their beliefs about the structure,

[1] R. Mosvold, Associate Professor in the Department of Education and Sports Science at the University of Stavanger, Norway. Email: reidar.mosvold@uis.no

[2] J. Fauskanger, Assistant Professor in the Department of Education and Sports Science at the University of Stavanger, Norway. Email: janne.fauskanger@uis.no

certainty and source of knowledge; these types of beliefs are often referred to as epistemological (Hofer, 2002) or epistemic (Buehl, 2008) beliefs. As an example, the analysis of the interviews revealed which parts of the large mathematical and curricular landscape the teachers considered relevant, or their mathematical perspective on what lies in all directions, behind as well as ahead, for their pupils. This type of teaching knowledge is referred to as 'horizon content knowledge' (HCK) (e.g., Ball & Bass, 2009).

Prior research emphasizes the importance of studying teachers' epistemic beliefs as these beliefs may influence how and what they learn from participating in professional development (e.g., Ravindran, Greene, & Debacker, 2005); epistemic beliefs may also influence teaching practices (e.g., Sinatra & Kardash, 2004). In this article, we aim at gaining insight into practicing teachers' professed beliefs about MKT, and we address the following research question:

> What beliefs about mathematical horizon content knowledge become visible when teachers discuss MKT items?

Building on Fives and Buehl (2008) and Buehl and Fives (2009), teachers' epistemic beliefs about central aspects of the content of teaching knowledge constitute the focus of attention in this article. We analyzed transcripts from group discussions of in-service teachers in order to gain insight into these beliefs.

Theoretical background

Researchers have addressed different aspects regarding the learning and development of mathematics teachers—e.g., knowledge, practices and identity—and it is not necessarily fruitful to argue which aspect is more fundamental than the other (cf. Ponte, 2011). Identity can be seen as the embodiment of different aspects like beliefs, knowledge and affect, and research on teacher beliefs can thus be seen as addressing an important aspect of teacher identity (Ponte, 2011). Although the overarching aspects—such as identity—are important, research on specific components is still relevant (Philipp, 2007), and our focus in this article is on one such specific component: teachers' beliefs about the knowledge needed for teaching mathematics. The term 'belief' is widely used in contemporary education literature, but it is often used without proper definition (Philipp, 2007). In the following section we present the theoretical background and the definition of beliefs that we use in the present article. This is followed by an extended presentation of the MKT framework.

Epistemic beliefs

In his overview of literature on beliefs and affect, Philipp (2007) described beliefs as lenses used by the observer to interpret and understand various phenomena in the world. He followed up by defining beliefs as "psychologically held understandings, premises, or propositions about the world that are thought to be true" (ibid., p. 259). According to this definition, beliefs are connected with both knowledge and other related concepts such as attitude and emotion. For the purpose of this article, we follow a slightly adapted version of Philipp's definition. We specify our focus by replacing "the world" with "MKT", and we then end up with a working definition where beliefs are defined as "psychologically held understandings, premises, and propositions about *MKT* that are thought to be true". In this adapted definition, there is an embedded shift of focus from beliefs about the world to a particular aspect of personal epistemology.

Personal epistemology refers to individuals' beliefs about how knowledge is defined, constructed, justified, and stored (Hofer, 2002). Although several competing models are proposed, general epistemological beliefs seem to refer to "individuals' belief about the nature of knowledge and the processes of knowing" (Hofer & Pintrich, 1997, p. 112); in certain cases the term is also used with reference to beliefs about learning (Op't Eynde, De Corte, & Verschaffel, 2006). Epistemological beliefs are by some researchers defined in the same way as epistemic beliefs—as beliefs about knowledge (e.g., Buehl, 2008; Cady & Rearden, 2007). According to Buehl and Fives (2009), few studies have focused on teachers' beliefs about teaching knowledge, and they suggest that researchers should explicate the body of knowledge that is to be considered in such studies. As a result of the discussion concerning the domain specificity of beliefs, Buehl and Fives (2009) proposed that it is important to examine teachers' beliefs about the nature of teaching knowledge; teaching knowledge is then defined as all knowledge relevant to the practice of teaching. For the sake of clarity, the term "epistemic beliefs" is used in this article with reference to teachers' domain-specific beliefs about knowledge and knowing (as in Op't Eynde, et al., 2006, p. 69), and in particular teachers' beliefs about knowledge needed to teach (Fives & Buehl, 2008; Buehl & Fives, 2009) mathematics. As evidenced by the various models for teachers' knowledge that have been proposed and discussed (e.g., Petrou & Goulding, 2011), this body of knowledge is complex.

Buehl and Fives (2009) analyzed the beliefs of pre-service as well as in-service teachers regarding the source and stability of teaching knowledge. The results of their study indicated that participants hold a range of beliefs regarding these constructs. More relevant for the present study, however, is the work done by these researchers regarding teachers' beliefs about the content of teaching knowledge (Fives & Buehl, 2008). Results from that study indicated that participants valued several aspects of teaching knowledge. The teachers' conceptualization of the knowledge necessary for teaching was organized in the following themes: "1. pedagogical knowledge; 2. knowledge of children; 3. content knowledge; 4. management and organizational knowledge; and 5. knowledge of self and other" (Fives & Buehl, 2008, p. 142). The third theme refers to the need for knowledge about the specific content taught, as well as knowledge of content-specific teaching methods and strategies, and knowledge of the curriculum. This theme thus relates to Shulman's (1986) conceptualization of knowledge unique to teaching (see Figure 1 and the related discussion). There appear to be domain-general as well as domain-specific beliefs about knowledge, and some epistemic beliefs might also be specific to the academic domain (Buehl & Fives, 2009). With all of this as a backdrop, our study of teachers' beliefs about MKT can be seen as a further development of the work done by Fives and Buehl (2008).

Framework for teachers' knowledge

Since the focus of our study is on teachers' beliefs about MKT, it becomes relevant to elaborate on how this knowledge is defined. MKT belongs to a field of research that aims at acquiring further information about various aspects of teachers' professional knowledge. The study of mathematics teachers' knowledge has long been an active field of research (e.g., Sullivan & Wood, 2008), and the theories of Shulman are among the most influential (e.g. Shulman, 1986: 1987). His seminal paper (Shulman, 1986)—focusing on knowledge unique to teaching—is frequently referred to (e.g., Graeber & Tirosh, 2008), and his notions of subject matter knowledge and pedagogical content knowledge have been modified, subdivided and refined in the decades following. One of the most acknowledged refinements of Shulman's work is that of Ball and colleagues (2008). In their MKT framework, they divided subject matter knowledge into three parts (e.g., Hill, Ball, & Schilling, 2008) as represented by the left half of the oval presented in Figure 1.

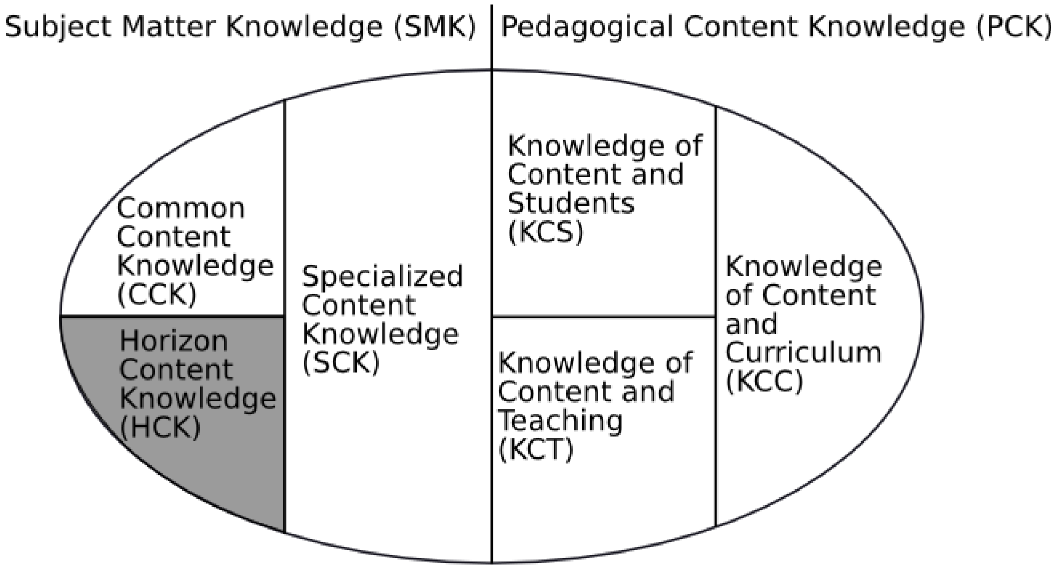

Figure 1. Domains of mathematical knowledge for teaching (adapted from Ball, Thames, & Phelps, 2008, p. 403).

"Common content knowledge" is knowledge that is used in the work of teaching, and it is used in ways that correspond with how it is used in settings other than teaching. Using an algorithm to find the answer to 29 subtracted from 146 would be an example of such common content knowledge. "Specialized content knowledge", on the other hand, is the mathematical knowledge "that allows teachers to engage in particular teaching tasks" (Hill, Ball & Shilling, 2008, p. 378). The third sub-domain of teachers' subject matter knowledge, as presented by Ball and colleagues (2008), is 'horizon content knowledge' (HCK, shaded in Fig. 1). Compared to the other MKT domains, HCK has received less attention. According to Jakobsen et al. (2012), the nature of HCK in relation to teaching is unclear. Ball and Bass (2009) situate their conception of HCK within their practice-based theory of MKT, and they describe HCK as "a kind of mathematical 'peripheral vision' needed in teaching, a view of the larger mathematical landscape that teaching requires" (p. 1). In another publication, HCK is described as "an awareness of how mathematical topics are related over the span of mathematics included in the curriculum" (Ball, et al., 2008, p. 403). It is, however, important to distinguish between HCK and knowledge of content and curriculum. "Horizon" does not refer to a curricular horizon, but rather a mathematical horizon (Jakobsen, Thames, & Ribeiro, 2013).

According to Ball and Bass (2009), HCK is an awareness of where and how the mathematics being taught fits into the structures and hierarchies of shared collective mathematical knowledge. This awareness serves both to engage students and to provide meaning to the present mathematical experience. Ball and Bass (2009, p. 6) further described HCK as consisting of four elements: "1) A sense of the mathematical environment surrounding the current "location" in instruction, 2) Major disciplinary ideas and structures, 3) Key mathematical practices, 4) Core mathematical values and sensibilities." In a more recent publication, Zazkis and Mamolo (2011) characterized HCK through the notion of viewing elementary mathematics from an advanced standpoint, and they argued that advanced mathematics is an important aspect of teaching knowledge. Even more recently, Jakobsen and colleagues (2012) argued that HCK relates to the engagement of advanced content in terms of its relevance to teaching and learning. Their claim is that:

> [Teachers] need a treatment of advanced mathematics that is conducted from an "elementary perspective," one that provides an understanding of the role of important

> topics in the discipline, an intuitive handle of concepts, and the resources needed to recognize and use such knowledge in teaching (Jakobsen, Thames, Ribeiro, & Delaney, 2012, p. 4642).

According to these researchers, HCK is about "being familiar with 'advanced' mathematics, but in a way that supports hearing, seeing, sensing, and doing for teaching" (ibid., p. 4640). They argued that HCK might support teachers in orienting instruction to the discipline, in making judgments about what is mathematically important in a teaching situation. HCK thus allows teachers to have a mathematical perspective that can serve to orient their navigation of the territory, an awareness of that which lies behind as well as ahead for their pupils (Ball & Bass, 2009). This latter perspective—especially concerning "what lies behind"—seems to be missing from the definition of Jakobsen et al. (2012). Their focus is mainly on advanced mathematics from an elementary perspective. HCK is also important for teachers in order to be able to understand their students' thinking and to plan how to challenge these students in future teaching (Ball & Bass, 2009), but it is important to emphasize that HCK is related to mathematical content knowledge and not pedagogical content knowledge. Knowledge about the content that has been taught—content that "lies behind" in the curriculum—is important, but it is considered to belong to knowledge of content and curriculum and not HCK. For the purpose of this article, we adopt the working definition of HCK developed by Jakobsen et al. (2012) when we study what beliefs about HCK—which is related to advanced mathematics from an elementary perspective—become visible when teachers discuss MKT items. The focus of attention thus also relates to Ball and Bass's (2009, p. 9) first element: "A sense of the mathematical environment surrounding the current 'location' in instruction."

Methods

There are various approaches to investigate teachers' beliefs. In this study, we used data from focus-group interviews where teachers discuss MKT items. These items were developed to measure other aspects of content knowledge than HCK—in particular CCK and SCK—and this is important to emphasize, since our aim in this article is to investigate what (epistemic) beliefs about HCK that can be found from analysis of teachers' discussions of MKT items. Focus-group interviews have the potential to facilitate discussions about issues that participants find important, and their reflections allow for examination as well as refinement of beliefs (e.g., Ambrose, 2004).

Participants and design

We selected 10 in-service teachers for participation in our study—five males and five females. Some of the participants had followed a regular teacher education program, and some of them had studied mathematics (in addition to at least one more subject) in addition to one year of pedagogical training. These two alternatives represent the possible approaches to become a certified teacher in Norway. In 2010, a new curriculum reform introduced a differentiated program for primary and lower secondary teacher education in Norway, but none of the participants in our study had followed this new teacher education program. Before the new regulation was introduced, anyone who finished teacher education would be qualified to teach mathematics (and every other subject) in grades 1–10. In order to get a wide spectrum of teachers in our study, both experienced and inexperienced teachers were selected for participation. The most experienced teacher had more than 25 years of teaching experience; the least experienced teacher had just recently finished teacher education. We wanted to analyze the teachers' beliefs, and, based on the argument that qualitative interviews "tap into beliefs, values, worldviews and the like on the part of the interviewee" (Lankshear

& Knobel, 2004, p. 198), such interviews were included as part of our study. Our focus was also to study beliefs more in depth, and such in-depth interviews are long and challenging to analyze. Fewer groups/participants are therefore recommended (Kreuger, 1998). Our selection of 10 participants for the interviews thus seemed to constitute a sensible combination of breadth and depth.

In the five interviews, we put together pairs of teachers—one male and one female in each—from the same schools (five different schools). All the participants were given fictitious names in the transcripts. The names indicate their gender, and teachers in the same group interviews have been given names that start with the same letter in order to simplify distinction between the groups.

In the first interview, Claire and Cole participated. Cole was an experienced teacher, and he was teaching grades 5–7; Claire was also teaching mathematics—this year in grade 6. The teachers who participated in the second interview, David and Deborah, were both relatively inexperienced teachers. David had taken the alternative route of teacher education—with university studies of mathematics along with a year of pedagogical training—whereas Deborah had followed an ordinary teacher education program. In the third interview, Tina and Todd participated. Both were teaching grades 1–4. Todd was experienced, whereas Tina was an inexperienced teacher. The participants in the fourth interview were Frank and Fay. Frank was an experienced teacher who had administrative tasks this year, whereas Fay was an inexperienced teacher who had finished teacher education not long ago. Finally, the teachers who participated in the fifth interview—Ann and Arne—were both experienced teachers. Both of them had been teaching the lower grades before, but they were now teaching grades 8–10.

Procedure and instrument

The participants in our study first responded individually to a complete form[3] of 30 item stems (61 items) from the LMT project[4] in a testing situation, and then—after a short break—they were interviewed in pairs. The interviews lasted from 40 to 90 minutes.

The reason why the teachers took the test first, was because we wanted to learn more about their beliefs concerning the test and the content represented by the items included rather than to measure their MKT—and the teachers were informed about this. The items in the test thus constituted an important context for the focus-group interviews. We are aware that teachers do not like to be tested, and that motivation and test anxiety can influence the results (e.g., Wolf & Smith, 1995)—and these issues also has a potential to influence the interviews—but these aspects are taken into account in our discussion.

Only a few questions were asked to prompt a discussion between the teachers. In the beginning of the interviews, they were asked to share some of their initial experiences from responding to the items. They were then asked if there were some particular items they wanted to discuss to begin with. After this, they were asked to comment on each individual item stem in the test—focusing on the format of the items, the context presented as well as the mathematical content. The items have not been released for publication, but an example item is given in Figure 2 as an illustration.

[3] Elementary form A, MSP_A04.

[4] See http://sitemaker.umich.edu/lmt/home

3. Imagine that you are working with your class on multiplying large numbers. Among your students' papers, you notice that some have displayed their work in the following ways:

Student A	Student B	Student C
35 x 25 125 +75 875	35 x25 175 +700 875	35 x 25 25 150 100 +600 875

Which of these students would you judge to be using a method that could be used to multiply any two whole numbers?

	Method would work for all whole numbers	Method would NOT work for all whole numbers	I'm not sure
a) Method A	1	2	3
b) Method B	1	2	3
c) Method C	1	2	3

Figure 2. An example of a testlet (including three multiple-choice items) from the collection of released items (Ball & Hill, 2008, p. 5).

If the teachers only gave short responses—such as "that one was okay"—they were asked to elaborate on their response. The role of the interviewers was to facilitate a discussion around the items. Towards the end of the interviews, the participants were asked if they considered the items to measure mathematical knowledge that was relevant in their own work of teaching mathematics, and they were also asked if they thought something was missing. The aim of letting the teachers complete the test in our study was not to measure their knowledge as such, but to learn more about their expressed beliefs about the measure and about aspects of MKT as represented by the items in the test.

Analysis of data

The recordings from the interviews were transcribed by a group of researchers. For the purpose of this article, we analyzed the transcripts by the use of qualitative content analysis (Hsieh & Shannon, 2005). Content analysis aims at obtaining (descriptive) information about a certain issue or topic (Fraenkel & Wallen, 2006), and it can be carried out in different ways. Hsieh and Shannon (2005) distinguish between summative, conventional and directed content analysis. In our study, we wanted to investigate teachers' beliefs about HCK, and it was thus relevant to use a directed or theory-driven content analysis. Directed content analysis can be useful in a situation where investigating "a phenomenon that is incomplete or would benefit from further description" (Hsieh & Shannon, 2005, p. 1281). HCK is still a work in progress, and it thus fits with this description.

Ball and Bass (2009) described HCK as an awareness of how the location in which the mathematical content taught fits into the larger domain of mathematical knowledge. Jakobsen and colleagues (2012) also focused on this in their attempt to define HCK, and they described it as a kind of knowledge that provides teachers "with a sense for how the content being taught is situated in and connected to the broader disciplinary territory" (p. 4642).

We used the working definition(s) of HCK as a starting point for our development of codes, and we used summative content analysis in the process of developing and refining codes. When searching the concordance more closely for words related to location and situatedness, we found some words that might potentially be related to HCK: words like “relevant” (17), “teach(es)” (15) and “taught” (11), and “level” (13). We found it particularly interesting to investigate what the teachers are talking about when referring to level; after all, HCK is related to an awareness of the mathematical content in relation to the “location” of the teaching (Ball & Bass, 2009). Words like “relevant”, or “important” are also interesting, because the teachers’ reasoning about the relevance of a particular mathematical content might provide us with information about whether or not they focus on HCK.

In our directed content analysis, we ended up with two main coding categories: relevance and location. The first main category had sub-categories where we coded if the teachers argued that something was relevant in relation to: a) mathematical content level, b) grade level, c) teaching in general, or d) other factors. The second main category included the following sub-categories in order to distinguish between a focus on location in relation to: a) mathematical content, b) curriculum level, c) student level, and d) other level. The data analysis was done in three stages: 1) a manifest content analysis where we searched for occurrences of words related to the codes, 2) a latent analysis of these keywords in a micro-context, and 3) a qualitative analysis of the utterances in which these keywords occurred in a larger context. These three stages in the coding process were used to increase reliability.

Results and discussion

When presenting and discussing the results, we illustrate the three-step approach used in our analysis by starting with word frequencies—which were analyzed in the summative content analysis. We then follow up by showing an example with analysis of micro-contexts, before we illustrate how larger contexts of following utterances were analyzed in the directed content analysis. We focus on how the teachers argue when they discuss whether or not some of the MKT items are relevant on the one hand, and on the other hand we focus on what the teachers discuss about “the mathematical environment surrounding the current ‘location’ in instruction” (Ball & Bass, 2009, p. 6). Our analysis of transcripts from the group interviews indicates a lack of agreement among the teachers in our sample. The teachers’ lack of emphasis on content knowledge not currently taught—which is related to “the broader disciplinary territory”—becomes apparent in all five interviews, but for the purpose of this article, only a few excerpts from the interviews are presented to illustrate the teachers’ arguments.

Step 1: manifest analysis of words

Content analysis—even qualitative content analysis—often starts with an analysis of the manifest occurrences of words used (Berg & Lune, 2012). One common technique is to analyze word frequencies (Mayring, 2010), and our analysis started with generating a concordance. This concordance gave us an overview of the frequencies of words used in the transcripts. A list of the most commonly used words—as presented in table 1—does not necessarily provide a lot of interesting information in itself. Sometimes, however, these common words can provide the starting point for further analyses that might turn out to be of great interest (e.g., Mosvold, Fauskanger, Bjuland, & Jakobsen, 2013). In the case of our analysis in this study, the heavy use of the personal pronoun “I” indicates that the teachers tend to emphasize their own personal beliefs.

Word	Frequency
It	1964
Is	1224
I	1159
And	839
Yes	731
So	684
At	614
Not	607
That	548
One	544

Table 1. The ten most frequently used words in the interviews.

Step 2: latent use of words in a micro-context

The next step in our analysis—after having identified relevant keywords—was to investigate these keywords in their context in order to learn more about the teachers' focus on HCK. To get a brief overview, we started with a narrow focus (small grain size) and analyzed how the words were used in a small context consisting of five words before and after the keyword.

In the interviews, we asked the teachers about whether or not they found the content of the MKT items relevant. When analyzing what the teachers said about relevance, it appeared that they almost exclusively argued about relevance in relation to grade level or the content of the curriculum at their level—never in relation to the mathematical content itself or the mathematical horizon. This lack of emphasis on HCK can be interpreted in at least two ways. On the one hand—following an assumption that HCK is an important aspect of MKT—the teachers' lack of regard for HCK might be related to low MKT. On the other hand, however, it can be seen as an indication that HCK is of less importance than researchers have assumed. According to the teachers—and this could be seen already from our analysis of the micro-contexts around this word (Figure 3)—the teachers discussed relevance in connection with pupils, teaching and the grade/level in which they were teaching.

```
split between what is most [relevant] for elementary level and what
     and that which is most [relevant] for high school level. Eventually
        (.) This is indeed a [relevant] fraction item related to our
   eh, that this should be [relevant] for me who teach, eh
        is indeed a lot more [relevant] anyway, than the way teaching
   don't think the item is [relevant] to give to students, eh
```

Figure 3. Keyword "relevant" in a micro-context.

The first two lines in figure 3 refer to relevance in relation to grades 1–7 and lower secondary school (high school level). In the next two, the teachers seem to discuss something about what is relevant for teachers or teaching, whereas the last snippet refers to an item being relevant to students. The participants in our study were told that the items were supposed to measure their knowledge as teachers. In a couple of interviews, however, the participating teachers were still discussing whether or not a particular item would be relevant for their students. One possible interpretation of this could be that the teachers had misunderstood the purpose of the measure, but a more plausible explanation would be that they were simply referring to the problems that were presented in the item stems. It is also possible that they were thinking about the mathematical content of the items.

When it comes to location, an interesting keyword to focus on is "level". In Norwegian, this word can be used both in relation to curriculum or grade level and mathematical level (e.g. advanced mathematics). When investigating this keyword in its micro-context, we observed that it sometimes referred to the level of the students, sometimes the textbook tasks, and other times it was used with reference to the level in which they were teaching. All of these are related to curriculum or grade level rather than mathematical level or "location".

```
       And in relation to the [level], it should probably have been
separate it into two different [levels], and then it would be
 seen something similar on red [level]. Interviewer: Yes, on red level.
      would have to be on the [level] I normally teach, because that's
       be used to measure the [level] of the mathematics teachers in
```

Figure 4. Keyword "level" in a micro-context.

The data excerpt in figure 4 presents examples of these different uses of the keyword. One of these micro-contexts appeared to be particularly interesting, because it included keywords like "level" and "teach" in the same micro-context. The teacher argued that something "would have to be on the level I normally teach." This might be seen as relating to HCK, but since the focus seems to be more on curriculum level than mathematical level, it can be argued that the statement is more related to knowledge of content and curriculum than HCK.

In the next step, we increase the grain-size and investigate the utterances that surround the micro-context we looked at above. The discussion that follow was initiated by the interviewer, who asked what the two participating elementary teachers thought about this way of measuring teachers' MKT. Tina, an inexperienced third-grade teacher, followed up on the question:

> 76. Tina: I think it should have been distinguished a little bit more. (...) If I were going to have [teach] any of these topics, I mean, you have to scrutinize it. For example (...), can a rectangle, which is not a parallelogram... If you were going to teach anything like that, you would have to examine it carefully for yourself, and you would have to do that all over again. I mean, I have learned this before, but not for children at such a low age [as the ones I'm teaching now]
>
> 77. Interviewer: Right.
>
> 78. Tina: So, if it [the item] is supposed to provide a realistic view of my knowledge, I think it would have to be on the level I normally teach, because that's where the teaching knowledge is displayed. I couldn't have (...) taught a lot of this without having had to re-learn it for myself.
>
> 79. Interviewer: That's right.
>
> 80. Tina: So, [I therefore think] it should have been a little bit more distinguished between grades 1–4 and 5–7 and such.
>
> 81. Todd: (...) I agree with what you say, but it is a matter of what this is going to be used for [the results of the test]. If it is going to be used to improve teacher education —that it can guide us about what to focus on—I believe it can give us some insight. If it is going to be used to measure the level of the mathematics teachers in schools, however,... I mean, it would give you some kind of impression [about that too], but as you say: I would have liked to be measured in the level I do most of my teaching.

82. Interviewer: Exactly.

In this context, the teachers are discussing a content knowledge item in geometry in particular. The item is related to describing shapes that do not exist, and the teachers are asked about whether or not it is possible for a shape to be a rectangle when it is not a parallelogram. A mathematician would probably say that this is common knowledge about the mathematical content—and this was a common content knowledge item—but Tina seems to think that this is too far away from the mathematics her students are involved with to be relevant. If she were going to teach something like this, she says (76 and 78), she would have to re-learn it for herself. With this as a background, she argues that an item that is supposed to measure teachers' MKT should be within the scope of the mathematical content with which her students learn. Todd, who is a more experienced teacher than Tina, seems to agree with her view about this, but he directs the focus of the discussion towards the purpose of the measure.

The discussion between the two teachers in this excerpt can be interpreted in different ways. If we focus on Tina's argument that the content—in order to provide a more relevant view of her professional knowledge—should be on the level she normally teaches (78), we might interpret this as an expressed belief about MKT. Tina believes, we might argue, that teachers need knowledge about the mathematical content they are teaching in their particular grade level, and knowledge beyond that level is not relevant. In other words, it does not appear from the data that Tina is focusing on the mathematical location of the content she is teaching in relation to more advanced mathematics (an important aspect of HCK). Her focus—and the focus of the other teachers in our study—was more on the content of the curriculum at a particular level, and this can be seen as an indication that HCK is not an important aspect of MKT (for the teachers).

This is not, however, the only way of interpreting the discussion. Another way of interpreting it is to focus on what they say about the purpose of the measure. Todd agrees with Tina, but he then argues that this discussion is related to the purpose and use of the measures (82). Tina's argument, that the item should focus on mathematical content (only) that is directly related to what she is teaching in order to provide a realistic view of her knowledge (78), could then be interpreted as an indication that she mistakenly believes the purpose of these measures is to evaluate her qualifications as a teacher. In light of Todd's argument about the purpose of the measure (82), we can interpret this as an indication that they believe it is important for teachers to have knowledge of a mathematical content that is beyond the scope of what they are teaching in their grade level—and indeed learn about this in their teacher education—and this again might then be seen as an indication that they do believe HCK is important.

Another possible interpretation is to focus on the potential influence of affect. Tina's statements (in particular 76 and 78) indicate that she found this item difficult, and many teachers—perhaps Tina is one of them—seem to think that they should be able to answer all of these items correctly. When they fail to answer one or more items correctly, they get insecure, and an argument like the one Tina makes (78) can be seen as a kind of defense mechanism. This might be particularly relevant to consider in a study like this where the teachers took the test just before they were measured. It is interesting to observe how Todd seems to agree with Tina. He is the more experienced teacher of the two, and although he brings up the issue of different possible uses of the measure, he still ends up concluding that the content of the items should be on the level where he does most of his teaching (81). Despite all the possible interpretations then, there seem to lack of emphasis on HCK in the teachers' discussions.

Concluding discussion

The importance of knowing basic mathematics is stressed in research related to teachers' MKT, and knowledge about basic mathematics is included in all frameworks for teachers' knowledge (e.g., Petrou & Goulding, 2011). Ball, Thames and Phelps (2008) also argue that there is a specialized aspect of content knowledge that is important for mathematics teachers, and results from subsequent studies of MKT seem to support the assumption that both common and specialized content knowledge are important. In this article, however, we have focused on horizon content knowledge (HCK)—which is more related to how basic mathematics is connected with more advanced mathematics. This aspect of teacher knowledge is emphasized in the MKT framework (Ball, et al., 2008), and Ball and Bass (2009) argued that HCK includes several elements. An important element of HCK is for teachers to have a sense of the mathematical environment surrounding the current location of their instruction. Another element of HCK is related to the major ideas and structures of the discipline (Ball & Bass, 2009). This last element is discussed in the literature—from different perspectives—and emphasized as important for the pedagogical choices teachers make (Zazkis & Mamolo, 2011; Jakobsen, et al., 2012). Although HCK is included in the framework of MKT, and researchers seem to agree about its importance, there is still a lack of empirical evidence—both for the existence and importance—of this particular aspect of teacher knowledge.

When it comes to teachers' beliefs, on the other hand, previous research has often focused on teachers' beliefs about: i) the nature of mathematics, ii) mathematics teaching, or iii) mathematics learning (e.g., Beswick, 2012). More recently, an extension of these categories has also included beliefs about the knowledge for teaching mathematics (Mosvold & Fauskanger, 2013). In our analysis here, we have focused on the teachers' expressed beliefs about HCK, and as a basis for the analysis we have used arguments that teachers made concerning the relevance of the mathematical content in MKT items.

The teachers in our study seemed to be more concerned about the mathematical content at the level they were teaching than the broader (more advanced) mathematical context—which can be referred to as the mathematical horizon. Based on the results of this study, we can then argue that the teachers do not seem to emphasize HCK. Teachers like Todd and Tina did indeed seem to have a lack of regard for the connection between the mathematics they were teaching and more advanced mathematics, but it is important to be careful when attempting to interpret such findings. Tina's statements could be grounded in a feeling of insecurity due to her having found particular items difficult. When encountering difficult items in a test like this, she might have felt that her professional identity as a teacher was threatened. This could be part of an explanation for why she did not seem to consider knowing mathematics beyond the most fundamental as important. This seeming lack of focus on the surrounding mathematical location of teaching was also found among several teachers in our study. Their focus was mainly on whether or not a certain mathematical content was directly related to the curriculum content for the level in which they were teaching.

There are different possible explanations for this apparent lack of regard for HCK. One possibility is that it is related to the teachers' level of MKT. Another possibility is, as we have already indicated, that it is grounded in the teachers' feeling that their identity as teachers is threatened. If teachers find an item difficult, they might argue that the MKT represented by this item is not important for them—as a kind of defense mechanism. This shows how difficult it is to investigate beliefs, and it is a potential disadvantage of having teachers take a test and then use such test items as a starting point for focus-group discussions. Care should be taken when conducting and analyzing interviews like this.

On the other hand, we would like to suggest that the design of this study have lead to some findings that might not have been found if the teachers were only interviewed about their beliefs about HCK directly without having taken the test beforehand. In such a scenario, without the presence of challenging test items, the teachers might have appeared to be more supportive of HCK. An alternative type of question could be: "If a child was learning about subtraction and then came to a situation where a larger number such as 7 was to be subtracted from a small number such as 3, what would you as a teacher say to the child?" This kind of question would probably feel less threatening to the teacher, but it would also open up to the possibility that teachers might draw upon something other than particular aspects of mathematical knowledge for teaching when responding to it. One advantage of MKT items, we argue, is that they can be seen as manifest examples of what MKT might look like; a challenge with several theoretical constructs about teacher knowledge is that they are too theoretical. We also argue that this quality of the MKT items—drawing attention towards a very specific aspect of knowledge—enables them to provide a good starting point for a focused discussion (see also Colucci, 2007) of what certain aspects of knowledge looks like, whether or not it makes sense in the course of teachers' own experiences, and in what ways it makes sense. Such a discussion has the potential to (indirectly) inform the researcher about teachers' beliefs about particular aspects of MKT as well.

In recent years, the interest in HCK has been increasing, but researchers have not yet managed to create items that have been proven to measure HCK. There is also a lack of empirical evidence for the possible effect of HCK on the quality of instruction and on students' achievement. Ball and Bass (2009, p. 10) claimed that HCK "orient the teacher to hear, to speak, and to make decisions that honor children's often surprisingly deep insights that anticipate their later mathematical journeys." Although future studies might succeed in providing evidence for this and other claims about HCK, we should keep in mind that the teachers themselves does not necessarily regard this aspect of MKT as important. In addition to these aspects, we suggest that more research is needed to investigate the possible influence of the contextual issues. The reason why teachers expressed the beliefs they do may also be related to the content of the MKT items discussed, and cultural issues might be involved; the items were translated and adapted for use in a Norwegian context, and cultural differences might have influenced what the teachers said (Fauskanger, et al., 2012). An ad hoc finding from our study is, however, that the use of MKT items as a starting point for discussions to learn more about teachers' epistemic beliefs about aspects of MKT is worthy of further investigation. In such investigations, it might also be worthwhile having a particular focus on issues related to teacher identity as discussed above.

Acknowledgements

An early version of this paper was presented at the Topic Study Group (TSG) 27 on "Motivation, beliefs and attitudes towards mathematics and its teaching" at ICME-12 in Seoul, Korea, July 2012, and we would like to thank the participants in this TSG for valuable feedback. We would also like to thank the anonymous reviewers as well as the editors of International Journal for Mathematics Teaching and Learning for valuable feedback and critical comments in the review process.

References

Ambrose, R. (2004). Initiating change in prospective elementary school teachers' orientations to mathematics teaching by building on beliefs. *Journal of Mathematics Teacher*

Education, 7(2), 91-119.
Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 1., pp. 13-35). Rotterdam, The Netherlands: Sense Publishers.
Ball, D.L., & Bass, H. (2009). *With an eye on the mathematical horizon: Knowing mathematics for teaching to learners' mathematical futures*. Paper presented at the The 2009 Curtis Center Mathematics and Teaching Conference. Retrieved February 24th 2012 from http://www.mathematik.tudortmund.de/ieem/cms/media/BzMU/BzMU2009/Beitraege/Hauptvortraege/BALL_Deborah_BASS_Hyman_2009_Horizon.pdf
Ball, D.L., & Hill, H.C. (2008). Mathematical knowledge for teaching (MKT) measures. Mathematics released items 2008. Retrieved December 12th 2011 from http://sitemaker.umich.edu/lmt/files/LMT_sample_items.pdf.
Ball, D.L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education, 59*(5), 389-407.
Beswick, K. (2012). Teachers' beliefs about school mathematics and mathematicians' mathematics and their relationship to practice. *Educational Studies in Mathematics, 79*(1), 127–147.
Buehl, M.M. (2008). Assessing the multidimensionality of students' epistemic beliefs across diverse cultures knowing, knowledge and beliefs. In M.S. Khine (Ed.), *Knowing, knowledge and beliefs: Epistemological studies across diverse cultures* (pp. 65-112). The Netherlands: Springer.
Buehl, M.M., & Fives, H. (2009). Exploring teachers' beliefs about teaching knowledge: Where does it come from? Does it change? *The Journal of Experimental Education, 77*(4), 367-408.
Cady, J.A., & Rearden, K. (2007). Pre-service teachers' beliefs about knowledge, mathematics, and science. *School Science and Mathematics, 107*(6), 237-245.
Colucci, E. (2007). "Focus groups can be fun": The use of activity-oriented questions in focus group discussions. *Qualitative Health Research, 17*(10), 1422–1433
Fauskanger, J. (2012). "For norske lærere har stort sett en algoritme" – om undervisningskunnskap i matematikk. In F. Rønning, R. Diesen, H. Hoveid & I. Pareliussen (Eds.), *FoU i Praksis 2011. Rapport fra konferanse om praksisrettet FoU i lærerutdanning* (pp. 129–141). Trondheim: Tapir Akademisk Forlag.
Fauskanger, J. & Mosvold, R. (2010). Undervisningskunnskap i matematikk: Tilpasning av en amerikansk undersøkelse til norsk, og læreres opplevelse av undersøkelsen. *Norsk Pedagogisk Tidsskrift, 94*(2), 112–123.
Fauskanger, J., Jakobsen, A., Mosvold, R., & Bjuland, R. (2012). Analysis of psychometric properties as part of an iterative adaptation process of MKT items for use in other countries. *ZDM – The International Journal of Mathematics Education, 44*(3), 387–399.
Fives, H., & Buehl, M.M. (2008). What do teachers believe? Developing a framework for examining beliefs about teachers' knowledge and ability. *Contemporary Educational Psychology, 33*(2), 134-176.
Fraenkel, J.R., & Wallen, N.E. (2006). *How to design and evaluate research in education* (6. ed.). New York, NY: McGraw-Hill.
Goldin, G., Rösken, B., & Törner, G. (2009). Beliefs – no longer a hidden variable in mathematical teaching and learning processes. In J. Maass & W. Schlöglmann (Eds.), *Beliefs and attitudes in mathematics education* (pp. 1-18). Rotterdam, The Netherlands: Sense Publishers.
Graeber, A., & Tirosh, D. (2008). Pedagogical content knowledge. Useful concept or elusive

notion. In P. Sullivan & T. Wood (Eds.), *The international handbook of mathematics teacher education* (Vol. 1., pp. 117-132). Rotterdam, The Netherlands: Sense Publishers.

Hill, H.C., Ball, D.L., & Schilling, S. G. (2008). Unpacking "pedagogical content knowledge": Conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education, 39*(4), 372-400.

Hill, H.C., Rowan, B., & Ball, D.L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal, 42*(2), 371-406.

Hofer, B.K. (2002). Personal epistemology as a psychological and educational construct: An introduction. In B.K. Hofer & P.R. Pintrich (Eds.), *Personal epistemology: The psychology of beliefs about knowledge and knowing*. Mahwah, NJ: Lawrence Erlbaum Associates Publishers.

Hofer, B.K., & Pintrich, P.R. (1997). The development of epistemological theories: Beliefs about knowledge and knowing and their relation to learning. *Review of Educational Research, 67*(1), 88-140.

Hsieh, H.-F., & Shannon, S.E. (2005). Three approaches to qualitative content analysis. *Qualitative Health Research, 15*(9), 1277-1288.

Jakobsen, A., Thames, M.H., & Ribeiro, M. (2013). Delineating issues related to horizon content knowledge for mathematics teaching. Paper presented at the Eight Congress of European Research in Mathematics Education (CERME-8). Antalya, Turkey.

Jakobsen, A., Thames, M.H., Ribeiro, M., & Delaney, S. (2012). *Using practice to define and distinguish horizon content knowledge.* Paper presented at the 12th International Conference on Mathematical Education, Seoul, Korea (pp. 4635-4644).

Kreuger, R. A. (1998). *Developing questions for focus groups. Focus group kit 3*. Thousand Oaks, CA: SAGE.

Lankshear, C. & Knobel, M. (2004). *A handbook for teacher research: From design to implementation*. Berkshire, England: Open University Press.

Mosvold, R. & Fauskanger, J. (2013). Teachers' beliefs about mathematical knowledge for teaching definitions. *International Electronic Journal of Mathematics Education, 8*(2-3), 43-61.

Mosvold, R., Fauskanger, J., Bjuland, R., & Jakobsen, A. (2013). Who are "they"? Student teachers' beliefs. Paper presented at the Eighth Congress of European Research in Mathematics Education (CERME-8). Antalya, Turkey. Retrieved April 29th 2013, from http://cerme8.metu.edu.tr/wgpapers/WG8/WG8_Mosvold_Fauskanger_Bjuland_Jakobsen.pdf.

Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). Epistemic dimensions of students' mathematics-related belief systems. *International Journal of Educational Research, 45*(1-2), 57-70.

Petrou, M., & Goulding, M. (2011). Conceptualizing teachers' mathematical knowledge in teaching. In Rowland T. & Ruthven K. (Eds.), *Mathematical knowledge in teaching* (pp. 9-25). London: Springer.

Philipp, R.A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second Handbook of Research on Mathematics Teaching and Learning* (pp. 257-315). Charlotte, NC: Information Age Publishing.

Ponte, J.P. (2011). Teachers' knowledge, practice, and identity: essential aspects of teachers' learning. *Journal of Mathematics Teacher Education, 14*(6), 413-417.

Ravindran, B., Greene, B.A., & Debacker, T.K. (2005). Predicting preservice teachers' cognitive engagement with goals and epistemological beliefs. *Journal of Educational Research, 98*, 222–232.

Sinatra, G., & Kardash, C. (2004). Teacher candidates' epistemological beliefs, dispositions,

and views on teaching as persuasion. *Contemporary Educational Psychology, 29*(4), 483–498.
Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.
Shulman, L.S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.
Sullivan, P., & Wood, B. (Eds.). (2008). *Knowledge and beliefs in mathematics teaching and teaching development* (Vol. 1). Rotterdam, The Netherlands: Sense Publishers.
Wolf, L. F. & Smith, J. K. (1995). The consequence of consequence: Motivation, anxiety, and test performance. *Applied Measurement in Education, 8*(3), 227–242.
Zazkis, R., & Mamolo, A. (2011). Reconceptualizing knowledge at the mathematical horizon. *For the Learning of Mathematics*, *21*(2), 8-13.