Conventionalists, Pioneers and Criminals Choosing Between a National Currency and a Global Currency

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Received: 27 September 2021 / Revised: 24 November 2021 / Accepted: 3 December 2021 / Published online: 30 December 2021

ABSTRACT

The article analyzes how conventionalists, pioneers and criminals choose between a national currency (e.g. a central bank digital currency) and a global currency (e.g. a cryptocurrency such as Bitcoin) that both have specific characteristics in an economy. Conventionalists favor what is traditional and historically common. They tend to prefer the national currency. Pioneers (early adopters) tend to break away from tradition, and criminals prefer not to get caught. They both tend to prefer the global currency. Each player has a Cobb-Douglas utility with one output elasticity for each of the two currencies, comprised of backing, convenience, confidentiality, transaction efficiency, financial stability, and security. The replicator equation is used to illustrate the evolution of the fractions of the three kinds of players through time, and how they choose among the two currencies. Each player's expected utility is inverse U-shaped in the volume fraction of transactions in each currency, skewed towards the national currency for conventionalists, and towards the global currency for pioneers and criminals. Conventionalists on the one hand typically compete against pioneers and criminals on the other hand. Fifteen parameter values are altered to illustrate sensitivity. For parameter values where conventionalists go extinct, pioneers and criminals compete directly with each other. Players choose volume fractions of each currency and which kind of player to be. Conventionalists go extinct when criminals gain more from criminal behavior, and when the parameter values in the conventionalists' expected utility are unfavorable, causing competition between pioneers and criminals.

JEL Classification: C60; E50

Keywords: Bitcoin, digital currencies, currency competition, money, evolution, replicator dynamics, cryptocurrencies, central bank digital currencies.

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1.1. Background

This article considers a national currency operational within a country, and a global currency operational within the same country and also outside the country. We do not model the characteristics of more than one country, but do model the characteristics of the global currency assumed operational beyond the country under analysis. We require the two currencies to operate as media of exchange (means of payment). We do not specify whether the two currencies are non-digital or digital, paper currencies combined with physical coins, etc. The comparison of a national currency and a global currency has become more relevant with the emergence of digital currencies. At the time of writing this article most countries still allow paper currencies. For some countries most transactions are digital, conducted e.g. through debit and credit cards, electronic funds transfers, etc. We expect currencies to become increasingly digital in the future, to transform the financial system in ways that are still unclear, but with more competitors. Most central banks are in the process of launching CBDCs (central bank digital currencies), e.g. the People's Bank of China, the European Central Bank, the Bank of England, and the US Federal Reserve. The transformation is partly impacted by the emergence of blockchain technology and the cryptocurrency Bitcoin, with a genesis block mined² on January 3, 2009 at 18:15:05 UTC. Bitcoin is increasingly considered to have value (Kelleher, 2021). On November 22, 2021, 14,641 cryptocurrencies contribute to a marketcap of \$2.5 trillion. Among these, 1,039 are coins (not tokens) which are our main interest in this article (coinmarket.com).

When the global currency is conceptualized as a cryptocurrency such as Bitcoin, which allows 5-7 transactions per second, we account for the presence of Layer 2 solutions for scaling such as the lightning network where transactions are faster, less costly and more readily confirmed (Frankenfield, 2021).³ The lightning network introduces off-ledger transactions, and disintermediates central institutions such as banks. The off-ledger transactions are updated on the main blockchain on the base Layer 1 only when two parties open and close a payment channel on the lightning network. Two examples of Bitcoin payments on the lightning network are the El Salvador Chivo wallet, which on October 16, 2021 recorded 24,076 remittance requests, which added up to \$3,069,761.05 in one day (Sarkar, 2021), and Twitter tipping applying various third party operators such as the Strike Bitcoin lightning wallet service (Rodriguez, 2021). El Salvador's acceptance of Bitcoin as legal tender, and Tesla's on-and-off acceptance of Bitcoin for car payments (Zainab Hussain & Balu, 2021) means that goods and services in principle can be priced in Bitcoin. Hence, to the extent the global currency is a cryptocurrency combined with a Layer 2 solution, the global currency functions as a medium of exchange and a unit of account. It may also function as a store of value and a standard of deferred payments, which are beyond the scope of this article.

A plethora of different kinds of digital currencies emerge, tentatively classified into CBDCs, cryptocurrencies, digital currencies issued by private companies such as Meta's Diem, which is a stablecoin, digital currencies issued by political jurisdictions such as Miami's MiamiCoin, etc. As digital currencies become more common, these can be expected to compete with each other and with non-digital currencies. Hence it becomes relevant to assess which factors affect the market share of each currency over time, the implications of different market shares, and which

² Mining is how new Bitcoins enter circulation and how transactions are confirmed by the network on the blockchain ledger. Bitcoins are awarded through mining to the first computer to solve mathematical problems to verify blocks of transactions, applying hardware and energy known as "proof of work" (Hong, 2021).

³ The Bitcoin base Layer 1 requires "proof of work" to ensure decentralization, which costs energy. See Willms (2021) regarding energy consumption. Bitcoin mining enables locating stranded energy sources, favorable technology, politically favorable jurisdictions, and financially favorable circumstances; grows its network optimally, and operates optimally through space and time. Layer 2 usually does not require proof, which causes more centralization.

kinds of users apply the various currencies. Each currency's market share may depend on various factors such as backing, convenience, confidentiality, transaction efficiency, financial stability, and security, as perceived by users, contributors, regulators, governments, etc., and as elaborated upon in this article.

Competition between currencies implies different market shares for the various currencies. The implications of changes in the shares of the various currencies, from an economic point of view, are that the various actors involved in the various currencies benefit differently and incur different costs depending on the success of each currency. Examples of actors are currency producers, users, borrowers, lenders, stakers, and miners.

For example, central banks and their associated governments can expect to benefit from the success of CBDCs. Users may benefit if the CBDC is stable with low transaction costs, but may experience a cost if they value privacy and all their transactions get centrally recorded. The success of a cryptocurrency such as Bitcoin can be expected to benefit libertarians and actors preferring decentralized currencies less controlled by central actors, and not to benefit middlemen such as banks and others enabling, facilitating and negotiating transactions. The success of Meta's Diem can be expected to benefit Meta's stakeholders and users. The success of Miami's MiamiCoin can be expected to benefit Miami.

1.2. Contribution

This article considers an economy with a national currency and a global currency. The national currency offers the most common usage, such as buying goods, paying taxes, etc. A global currency may offer more limited usage, e.g. for buying goods and paying taxes, but may offer other opportunities such as tax evasion, user autonomy, etc. Three kinds of players are assumed, i.e. conventionalists, pioneers, and criminals. These are believed, first, to represent all societal players and, second, to have different preferences for the national currency and a global currency. Conventionalists favor what is traditional and historically common, which is often the national currency. Pioneers (early adopters) tend to depart from tradition and search for new ways of transacting, which may involve a global currency. Criminals search for currencies ensuring that they do not get detected and caught, which may also involve a global currency. Conventionalists typically compete against pioneers and criminals. When conditions for conventionalists are unfavorable causing their extinction, pioneers and criminals compete more directly with each other. All the three kinds of players can in principle choose some degree of criminal behavior, but criminals are assumed to have preferences explicitly focused on criminal behavior. The three groups are assumed to be mutually exclusive and jointly exhaustive to represent all possible kinds of market participants. If a player is empirically determined to fall somewhere between two kinds of players, a choice has to be made one way or the other. A player can over time choose to change from being of one kind to being of another kind.

Each player has a Cobb-Douglas utility with one output elasticity for each of the two currencies, split into backing, convenience, confidentiality, transaction efficiency, financial stability, and security, as perceived by the player. Factors such as usability and technological potential are assumed present in most of these six subelasticities, perhaps especially in convenience and transaction efficiency.⁴ These six subelasticities are assumed to comprise the main concerns relevant for each player's preferences regarding which of two currencies to choose. Each player makes two strategic simultaneous choices to maximize its expected utility which is shown to be inverse U-shaped in the volume fraction of transactions in each currency. The first choice is the volume fraction of its transactions in each currency. This choice depends on what kind of player the player is, but does not depend on how many players exist of this player's kind, and hence does

⁴ A factor such as investment profitability is more relevant for the function of a cryptocurrency as a store of value rather than a medium of exchange and a unit of account.

not depend on time. Each player's second choice is which kind of player it should be at each point in time. Hence this second choice depends on time, through replicator dynamics.

Applying replicator dynamics, the research questions are how the volume fractions of the two currencies and the fractions of the three kinds of players evolve through time, and are sensitive to various characteristics. A further research question is to determine society's expected utility to account for welfare at the societal level. Scenarios are illustrated where the output elasticities and other characteristics cause some of the three kinds of players to become dominant or inferior over time. For the stationary solution after sufficiently much time has elapsed, sensitivity analysis is conducted to show how the fractions of the three kinds of players depend on variation in parameter values relative to a benchmark. Applying credible specific functional forms, an exact analytical solution is produced for the fraction of each player's transactions in the national currency , and replicator dynamics becomes applicable to determine the fractions of how the three kinds of players evolve.⁵

The world population is 7.9 billion, of which 74% is above 15 years old (Szmigiera, 2021) and 66.8% is above 20 years old (Ang, 2021). Assume that 69.7% is above 18 years old, i.e. 5.5 billion. The World Bank (2017) estimates that 1.7 billion adults lack a bank account, which is subtracted from 5.5 billion to give 3.8 billion adults with a bank account. Howarth (2021) estimates 300 million cryptocurrency users on October 25, 2021, i.e. 5.5% of adults and 7.9% of adults with a bank account. The authors expect these percentages to increase in the future. Without knowing which digital currencies may succeed as global currencies, the authors believe that players may increasingly sort themselves into conventionalists, pioneers, and criminals.

1.3. Literature

Limited literature exists on this topic. The following literature review is intended to cover and extend beyond this article's topic, usefully divided into four groups as an overview, i.e. competition between fiat currencies and cryptocurrencies, CBDC and cryptocurrencies, the cryptocurrency market, and game theoretic analyses.

1.3.1. Competition between fiat currencies and cryptocurrencies

The following articles that have been identified are the closest relative to the current article and somehow consider competition between fiat currencies and cryptocurrencies, with various implications. Schilling and Uhlig (2019) enable agents to choose between two kinds of currencies, i.e. a cryptocurrency and a fiat currency. They explore how asymmetry in transaction costs and exchange fees decreases currency substitution. This exploration corresponds to the generally different transaction efficiencies considered for the national and global currencies in the current article. For payments of certain goods, cryptocurrencies are more suitable or cost less than fiat money, due to censorship resistance, tax evasion and anonymity. However, exchanging cryptocurrencies to fiat money is costly, and some goods are more easily purchased using fiat money. The condition under which agents are indifferent between purchasing with Bitcoin or US dollars depends on the amount of the value-added tax and transaction fees to miners. These assessments correspond to some extent to different backing, convenience, confidentiality, financial stability, and security for the national and global currencies in the current article.

Fernández-Villaverde and Sanches (2019) build a model of competition among privately issued fiat currencies. Based on the Lagos-Wright environment, they identify a price stable equilibrium for multiple currencies, comparable to two coexisting currencies in the current article,

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DOI: 10.7172/2353-6845.jbfe.2021.2.6

⁵ In return for sacrificing generality, a successful specification through functional forms demonstrates internal consistency and is illuminating. For example, the Cobb-Douglas function has enhanced our understanding of consumer preferences. Functional forms facilitate determining ranges of parameter values within which solutions are possible.

and various less desirable equilibria. In the current article society's expected utility is a weighted sum, by the fraction of players of each kind, of each player's expected utility.

Almosova (2018) extends her model by assuming that the circulation of private currencies involves costs, i.e. verification of transactions, mining costs, etc. She points out that cryptocurrency competition will not cause price stability. But when the costs of private currency circulation are sufficiently low, competition will impose a downward pressure on the inflation of the public currency.

Rahman (2018) applies the Friedman rule to investigate the implications of digital and fiat currency competition for monetary policy. He finds that a monetary equilibrium with a purely private arrangement of digital currencies cannot deliver a socially efficient allocation. Rahman's (2018) article is linked to the current article, which considers society's expected utility as a weighted sum of the three kinds of players' expected utilities.

Benigno, Schilling, and Uhlig (2019) consider a two-country economy with complete markets, two national currencies and a global cryptocurrency. They propose that the deviation from interest rate equality implies the risk of approaching the zero lower bound or the abandonment of the national currency, which they call Crypto-Enforced Monetary Policy Synchronization (CEMPS). Consequently, the impossibility of simultaneously ensuring a fixed exchange rate, free capital flows and an independent monetary policy (the classic Impossible Trinity) becomes even less reconcilable.

Verdier (2021) examines how issuing a digital currency impacts competition in the deposit and lending markets. She assumes that a digital currency can be issued or managed by a central bank, a regulated entity, or a non-bank operator, and that a digital currency issued by a nonbank operator does not enable offering loans to individuals. This assumption gradually seems ready for revision as decentralized finance increasingly allows loans, e.g. of cryptocurrencies, to individuals. Verdier (2021) assumes that depositors decide how much money to store in a bank account or in a digital currency account. Thus, issuing a digital currency generates a crowding-out effect on commercial deposits. The author concludes that the lending rate of banks increases when a digital currency crowds out a higher amount of bank deposits.

1.3.2. CBDCs and cryptocurrencies

The following articles that have been identified are the closest relative to the current article and compare CBDCs and cryptocurrencies, where we interpret CBDC as the national currency and cryptocurrencies as the global currency. Caginalp and Caginalp (2019) determine Nash equilibria for how players divide their assets between a home currency and a cryptocurrency, similarly to the focus in the current article. Additionally they assume that the government seizes fractions of the players' assets with certain probabilities.

Blakstad and Allen (2018) review opportunities for central banks and individuals presented by cryptocurrencies for central banks and individuals, together with the risks. They assess possible impacts on financial systems and structures which may challenge CBDC issuance.

Masciandaro (2018) proposes a function of a store of information for cryptocurrencies and central bank digital currencies as new media of payments emerge over the next years, supplementing a medium of exchange and a store of value. Thus, the evolution of the different media of payments may depend on individual preferences.

Benigno (2021) points out that the presence of multiple currencies can jeopardize the primary function of central banking. In addition, in a world of multiple competing currencies issued by profit-maximizing agents, the nominal interest rate and inflation are both determined by structural factors, i.e. the intertemporal discount factor, the exit rate and the fixed cost of entry, and are thus not subject to manipulation.

Asimakopoulos, Lorusso, and Ravazzolo (2019) present a Dynamic Stochastic General Equilibrium (DSGE) model to evaluate the economic repercussions of cryptocurrencies. They

estimate the model with Bayesian techniques. They document a sturdy substitution effect between the real balances of government currency and cryptocurrencies, in response to technology, preferences and monetary policy shocks. Similarly, the current article shows how the three kinds of players strike balances between the two currencies.

1.3.3. The cryptocurrency market

The following articles analyze multiple currencies in the cryptocurrency market, which relates to the current article since the two currencies may also be two cryptocurrencies which evolve over time with fluctuating volume fractions of transactions. ElBahrawy, Alessandretti, Kandler, Pastor-Satorras, and Baronchelli (2017) assess the evolutionary dynamics of the cryptocurrency market. They illustrate the fluctuating market shares of 1,469 cryptocurrencies between April 2013 and May 2017, akin to fluctuations.

Caporale, Gil-Alana, and Plastun (2018) implement a rescaled range analysis and a fractional integration method to analyze the persistence in the cryptocurrency market. They identify a positive correlation between cryptocurrencies' past and future values.

ElBahrawy, Alessandretti, and Baronchelli (2019) investigate the relationship between online attention to digital currencies on Wikipedia and market dynamics across multiple digital currencies.

White (2014) points out, based on empirical observation, that as a first-mover monopolist in the market for cryptocurrencies, Bitcoin is surrounded by effective competitors. The introduction of various altcoins, if successful, decreases Bitcoin's market share. The current article similarly shows how the market share of two currencies may change over time.

Sapkota and Grobys (2021) analyze the top ten cryptocurrencies ranked by market capitalization in 2016–2018. They find that the submarket equilibria of privacy coins and the submarket equilibria of non-privacy coins are unrelated. This contrasts with the current article where players strike balances between which currencies to choose, and what kind of player to be.

Milunovich (2018) applies Granger causality tests to five popular cryptocurrencies and six major asset classes. He estimates weak connectedness between the two groups and strong connectedness within each group. A few exceptions exist. Out of 80 cross-pairs, six statistically significant relations are shown from non-digital to digital assets (e.g. from Monero to US\$), and two statistically significant relations are shown from digital to non-digital assets (e.g. from the SPGSCI commodity index to Litecoin).

Gandal and Halaburda (2016) explore how network effects impact competition in the cryptocurrency market. They identify no winner-take-all effects in the early stages, but strong network effects and winner-take-all dynamics more recently. Similarly, the current article shows how two currencies and three kinds of players may coexist, and also that one kind of players, e.g. conventionalists, may go extinct.

1.3.4. Game theoretic analyses

The following articles are game theoretic analyses, which are linked to this group since the three kinds of players, while choosing among two currencies, interact with each other through time modeled by game theory and replicator dynamics. Imhof and Nowak (2006) propose that a frequency dependent, stochastic Wright-Fisher process can be used to describe the evolutionary game dynamics in finite populations to determine which of two strategies survives. This article similarly determines how the fractions of the three kinds of players, and the volume fraction of transactions in each currency, evolve over time.

Lewenberg, Bachrach, Sompolinsky, Zohar, and Rosenschein (2015) develop a cooperative game theoretic model to explore the dynamics of pooled Bitcoin mining and rewards. They show that it is difficult or even impossible to distribute rewards in a stable way. Players are always

incentivized to switch between pools. This is partly linked to the current article where players switch between which of three kinds of players to be, and which volume fraction of transactions in each currency to choose.

1.4. Article Organization

Section 2 presents the model. Section 3 analyzes the model. Section 4 explains the implications of the results. Section 5 concludes.

2. THE MODEL

2.1. Nomenclature

Parameters

- *j* Currency of kind j, j = n, g
- *n* National currency
- g Global currency
- *i* Player of kind i, i = x, y, z
- *x* Conventionalist player
- *y* Pioneer player
- *z* Criminal player
- b_{ii} Output subelasticity for backing of currency *j* at time *t* as perceived by player *i*, $b_{ij} \ge 0$
- c_{ij} Output subelasticity for convenience of currency *j* at time *t* as perceived by player *i*, $c_{ij} \ge 0$
- d_{ij} Output subelasticity for confidentiality of currency j at time t as perceived by player i, $d_{ij} \ge 0$
- e_{ij} Output subelasticity for transactional efficiency for currency *j* at time *t* as perceived by player *i*, $e_{ij} \ge 0$
- f_{ij} Output subelasticity for financial stability of currency j at time t as perceived by player $i, f_{ij} \ge 0$
- ^{*s*}_{*ii*} Output subelasticity for security of currency *j* at time *t* as perceived by player *i*, $s_{ii} \ge 0$
- w_i , Fraction of player *i*'s transactions which is criminal, $0 \le w_i \le 1$
- k_i Scaling exponent for what player *i* retains after criminal behavior, $k_i \ge 0$
- ω_i Probability that the government detects and prosecutes player *i*'s criminal behavior, $0 \le \omega_i \le 1$
- m_i Scaling exponent for how player *i* gets increased/decreased expected utility, $-\infty \le m_i \le \infty$
- μ_i Scaling proportionality parameter for how player *i* gets increased expected utility, $\mu_i \ge 0$
- α_i Parameter for the rapidity of change or sensitivity of the replicator equation, $\alpha_i > 0$
- t Time, $t \ge 0$

Free choice variables

- p_i Volume fraction of player *i*'s transactions in currency $n, 0 \le p_i \le 1, i = x, y, z$
- $1-p_i$ Volume fraction of player *i*'s transactions in currency $g, 0 \le 1-p_i \le 1$
- *p* Volume fraction of all players' transactions in currency $n, 0 \le p \le 1$
- 1–*p* Volume fraction of all players' transactions in currency $g, 0 \le 1 p \le 1$
- q_i Fraction of players of kind $i, 0 \le q_i \le 1, i = x, y, z, q_x + q_y + q_z = 1$
- q_x Fraction of conventionalists
- $\vec{q_v}$ Fraction of pioneers
- q_z Fraction of criminals, $q_z = 1 q_x q_y$

Dependent variables

 $U_i(p_i, q_i)$ Player *i*'s expected utility, i = x, y, zU Society's expected utility

2.2. Two Currencies *n* and *g*

Consider an economy with two available currencies. The first currency n is national and offers the most common usage, and especially legal usage, within the economy. Examples of usage are to make various purchases or pay taxes. For simplicity, we can think of this currency as a CBDC (central bank digital currency). The second currency g is a global currency which on the one hand offers more limited usage (e.g. cannot be used for all kinds of purchases), but on the other hand offers other opportunities, e.g. tax evasion, payment on the black market, user autonomy, discretion, peer-to-peer focus, no banking fees, low transaction fees. For simplicity, we can think of this currency as a cryptocurrency such as Bitcoin or Monero, a privately issued currency such as Meta's Diem, or some future hypothetical currency operating globally.

2.3. Three Kinds of Players x, y, z

Assume three kinds of players which we can think of as households, referred to as player *i*, i = x, y, z. We can think of the three kinds of players as conventionalists, pioneers and criminals, respectively. Conventionalists tend to do what is traditional and historically common, and tend to prefer the national currency *n* more than the global currency *g*. Pioneers (early adopters) tend to break away from tradition and prefer the global currency *g* more than the national currency *n*. Criminals prefer not to get caught and tend to prefer the global currency *g* more than the national currency *n*. Criminals prefer not to get caught and tend to prefer the global currency *g* more than the national currency *n* if the global currency *g* offers confidentiality and user autonomy, e.g. through a privacy coin such as Monero. Assume that q_i , $0 \le q_i \le 1$ is the fraction of players of kind *i*. We assume that q_x is the fraction of conventionalists, that q_y is the fraction of players of kind *i*. We assume that fraction of criminals. As time progresses, what used to be conventional may become oldfashioned, and what pioneers do may become conventional. Hence q_x and q_y may change over time. All players of the same kind *i* are equivalent. Player *i* (i.e. player of kind *i*) conducts a volume fraction p_i , $0 \le p_i \le 1$ of its transactions in currency *n*, and the remaining volume fraction $1 - p_i$ of its transactions in currency *g*, as shown in Figure 1 which assumes $p_x > p_y > p_z$, but generally $0 \le p_i \le 1$, i = x, y, z.

Figure 1

Three kinds of players. Player *i* (i.e. player of kind *i*), i = x, *y*, *z*, conducts a volume fraction p_i of its transactions in currency *n*, and the remaining volume fraction $1 - p_i$ of its transactions in currency *g*, $0 \le q_i \le 1$, $q_x + q_y + q_z = 1$. The illustration assumes $p_x > p_y > p_z$, but generally $0 \le p_i \le 1$, i = x, y, z.

Volume fraction $1 - p_x$ of currency g Volume fraction p_x of currency n	Volume fraction $1 - p_y$ of currency g	Volume fraction $1 - p_z$ of currency g	
	Volume fraction p_y of currency n	Volume fraction p_z of currency n	
Fraction q_x of players of kind x	Fraction q_y of players of kind y	Fraction q_z of players of kind z	

2.4. Volume Fraction p of All Players' Transactions in Currency n

The volume fraction *p* of all players' transactions in currency *n* is the weighted sum of each player *i*'s volume fraction in currency *n*, weighted by the fraction of each kind of player *i*, i = x, y, z, i.e.

$$p = \sum_{i=x,y,z} p_i q_i.$$
(1)

2.5. Cobb-Douglas Utility With Two Output Elasticities

Assume that player *i* has a risk-neutral Cobb-Douglas utility in net terms, hereafter referred to as utility, described by

$$U_{iCD}(p_i) = p_i^{b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in}} (1 - p_i)^{b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig}}$$
(2)

with one output elasticity $b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in}$ for the national currency *n*, and one corresponding output elasticity $b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig}$ for the global currency *g*. Player *i*'s Cobb-Douglas utility $U_{iCD}(p_i)$ in (2) is multiplied with a penalty described in the next section 2.6 if player *i*'s criminal behavior is detected and prosecuted by the government, and multiplied with the impact of the fractions q_x , q_y , q_z of the three kinds of players in the subsequent section 2.7. When $S = b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in} + b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig} = 1$, S > 1, S < 1, (2) expresses constant, increasing, and decreasing returns to scale, respectively. The 12 output subelasticities a_{ij} , $a_{ij} = b_{ij}$, c_{ij} , d_{ij} , e_{ij} , f_{ij} , s_{ij} in (2), for currency j, j = n, g, at time t as perceived by player i, i = x, y, z, are as follows:

First, b_{ij} expresses how currency *j* has various forms of backing from actors, systems or characteristics that users of currency *j* respect and trust, as perceived by player *i*. Examples of backing for currency *j* are central banks for CBDCs, and various decentralized characteristics such as a distributed ledger technology for cryptocurrencies. The variable b_{ij} is not objective, but depends on player *i*'s subjective judgment. The parameter b_{ij} expresses the weighted average backing of currency *j* by its users, i.e. within each of the three kinds *x*, *y*, *z* of players. For example, legitimate lawful users preferring transparency and allegiance to a certain country, may back the CBDC (central bank digital currency) of that country, which may be currency *n*, whereas illegitimate users may not back that currency, but back the global currency *g* instead. Criminal users may, for example, back a privacy cryptocurrency such as Monero, which may also be backed by many legitimate users. Currently, after the gold standard collapse (June 5, 1933 in the US), no fiat currency is backed by gold. The extent to which a player backs currency *j* may depend on a variety of factors. For example, a central bank may back its CBDC in the hope of obtaining a broader tax base, reduced tax evasion, a backstop to the private sector which may fail, and enhanced financial inclusion.

Second, c_{ij} expresses the convenience of using currency *j* as perceived by player *i*. One example of convenience is ease of use, e.g. few and easily comprehensible operations when purchasing at the supermarket or online, when transferring funds nationally or globally, or when incurring and paying back a loan. Other or related examples are how electronic wallets operate, how transfers between one's own and other wallets operate, and how offline transactions are processed when offline and getting back online. Furthermore, for some digital currencies users may not need to open a bank account with required identifications, but may instead install a digital currency wallet, and transact and pay via a digital currency address.

Third, d_{ij} expresses the confidentiality of using currency *j*, as perceived by player *i*, which expresses well-known balances to be struck between privacy, availability or accessibility for

oneself and various other players, and discrimination. For example, privacy cryptocurrencies such as Monero, Dash, and Zcash⁶ offer enhanced privacy for users since transactions are harder to track, which also may make it harder to rectify, correct, or reverse undesirable transactions. For example, paying ransom money in Monero may preserve the anonymity of the recipient and the provider, but may make it harder for law enforcement to reverse or prosecute the transaction. A CBDC, properly designed, may offer confidentiality for player *i* with respect to many other players if the central bank can be trusted, but may not offer confidentiality for player *i* if the central bank cannot be trusted, or a court orders the confidentiality to be broken. The output subelasticity d_{ij} thus also expresses discrimination regarding in what sense and for whom and towards whom confidentiality is honored.

Fourth, e_{ii} expresses the transaction efficiency of currency *j*, as perceived by player *i*, operationalized as low cost, fast speed, affordability, and finality. Fast speed refers to how quickly the transaction is executed, which for cryptocurrencies is impacted by how many confirmations are needed for execution and how quickly the miners can mine blocks. Wire transfers have historically had a certain speed, and may be held up over weekends. Affordability refers to a fee or cost of executing the transaction, which is usually positively correlated with how quickly the transaction is executed. Finality refers to the extent to which the transaction is final, or can somehow be reversed or negotiated. Cryptocurrency transactions are usually irreversible, which is the common logic of smart contracts on the blockchain. Non-cryptocurrency transactions, exemplified by traditional wire transfers are usually reversible, e.g. if a court of law determines that the transaction was illegal. Costs of transactions have historically varied substantially across different kinds of transactions. Affordability may depend on size, recipient, sender, whether the transaction is recurring, etc. Costs may range from the common no costs, e.g. for grocery purchases, to high costs for international money transfers. Costs of transacting cryptocurrencies have usually been low, and often beneficial when transacting high amounts, with variation across different cryptocurrencies. Speed of transfers also vary. At the time of writing, the speed of CBDC transactions is unknown. For Bitcoin the average time for mining one block is 10 minutes. For two confirmations, the transaction may take 20 minutes. The initiator of a cryptocurrency transaction is usually requested to specify a transaction fee (e.g., low, medium, high), which impacts how quickly it gets processed by the miners. For Ethereum the average time for mining one block is 10–15 seconds, which may cause one transaction after two confirmations to require 20-30 seconds. In 2019 Bitcoin processes ca 4.6 transactions per second, while Visa processes ca 1700 transactions per second. The lightning network may speed up the transaction time for Bitcoin. Credit card transactions typically require around 48 hours to settle. The finality of transactions also pertains to efficiency. Some cryptocurrency exchanges may require three confirmations, six confirmations for large transactions, and 60 confirmations for very large transactions. Different central banks may develop different procedures for finality and confirmations depending on the characteristics of transactions, senders, recipients, etc., which impacts the efficiency e_{ii} .

Fifth, f_{ij} expresses the financial stability of currency *j*, as perceived by player *i*. The financial stability of the national currency *n* depends on the conditions in the given country. A variety of indicators exist for the financial stability of countries and currencies. Some currencies such as the Swiss franc, the Japanese yen, and the Norwegian krone are relatively stable (Protska, 2021b), while some, such as the Venezuelan bolivar, the Iranian ria and the Vietnamese dong (Protska, 2021a) can be more unstable than many cryptocurrencies. For CBDCs the central bank adjusts interest rates (which can be negative for digital currencies), and can be expected to be able to adjust a variety of factors to adjust the financial stability of currency *j*, within the constraints of the country's conditions. One hypothetical possibility is to adjust the tax rate for households or individuals depending on their characteristics (e.g. in understanding with tax authorities and

⁶ https://www.investopedia.com/tech/five-most-private-cryptocurrencies/, retrieved November 22, 2021.

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others) to ensure financial stability. Fast response time when faced with crises, and activities to curtail or prevent money laundering and terrorist financing may impact the financial stability of currency *j*. Most cryptocurrencies, and especially altcoins, have traditionally varied substantially in value, caused partly by their novelty and limited usage, but also by the absence of a governing authority. One exception is stablecoins, e.g. Tether, USD Coin, TrueUSD, Dai, Paxos Standard, Binance USD, which have the stated purpose of being stable in some sense. The top ten list of countries adopting Bitcoin typically contains countries in the western world, but also countries which struggle to ensure financial stability, e.g. Venezuela (Lanz, 2020).

Sixth, s_{ij} expresses the security of currency *j*, as perceived by player *i*. A variety of security possibilities exist for digital currencies, see e.g. Allen et al. (2020) and Kiff et al. (2020). The security of the blockchain supporting Bitcoin has not collapsed since the first block was mined on January 3, 2009 at 18:15:05, although controversies and forks have occurred. Considering that 7,594 cryptocurrencies exist (https://coinmarketcap.com), 51% attacks are relatively rare.⁷

Each of the two output elasticities consists of six summed subelasticities as expressed above. Each of the six output subelasticities for the national currency *n* is of the form $p_i^{a_{in}}$, where p_i is the volume fraction of player *i*'s transactions in the national currency *n*. Each of the six corresponding output subelasticities for the global currency *g* is of the form $(1 - p_i)^{a_{ig}}$, where $1 - p_i$ is the volume fraction of player *i*'s transactions in the global currency *g*. The parameter a_{ij} , $a_{ij} = b_{ij}, c_{ij}, d_{ij}, e_{ij}, f_{ij}, s_{ij}$ is the output subelasticity in the Cobb-Douglas function, $0 \le a_{ij} \le 1$, which is a characteristic of currency *j*, *j* = *n*, *g*, as perceived by player *i*. The output subelasticity a_{ij} may sometimes be objectively specified, and may occasionally be mutually agreed upon by the players *x*, *y*, *z*, allowing the removal of the subscript *i* from a_{ij} . Since objective specification, and mutual agreement, may not be generally possible, and player *i* may perceive the output subelasticity a_{ij} subjectively, we keep the subscript *i* on a_{ij} .

2.6. Detection and Prosecution of Criminal Behavior

Examples of criminal behavior are tax evasion, money laundering, theft, terrorist financing, corruption, and financial crimes. Although we expect criminals to be more criminal than conventionalists and pioneers, all these three kinds of players can in principle engage in criminal behavior, through both the national currency n and the global currency g. This reflects that in our societies no groups of citizens can be expected to be 100% non-criminal. We thus assume that a fraction w_i , $0 \le w_i \le 1$ of player *i*'s transactions is criminal and is detected and prosecuted by the government with probability ω_i , $0 \le \omega_i \le 1$. The product $\omega_i w_i$ multiplies player *i*'s fraction w_i of criminal behavior with its detection and prosecution probability ω_i . Hence $1 - \omega_i w_i$ expresses the joint probability of neither engaging in criminal behavior nor being detected and prosecuted. We introduce a scaling exponent k_i , $k_i \ge 0$, on the fraction w_i and express player *i*'s expected utility as

$$U_{iC} = 1 - \omega_i w_i^{k_i} \tag{3}$$

which is a fraction between 0 and 1. When $k_i = 1$, player *i*'s expected utility U_{iC} decreases linearly in the fraction w_i of player *i*'s transactions which is criminal. When $k_i > 1$, U_{iC} decreases concavely in w_i , which economically means that a higher fraction w_i (compared with when $k_i = 1$) of player *i*'s criminal transactions is needed in order to decrease player *i*'s expected utility U_{iC} . In contrast, when $0 < k_i < 1$, U_{iC} decreases convexly in w_i , which economically means that a lower fraction w_i (compared with when $k_i = 1$) of player *i*'s criminal transactions is sufficient in order to decrease

DOI: 10.7172/2353-6845.jbfe.2021.2.6

⁷ The most well-known 51% attacks among cryptocurrencies occurred for Verge, Ethereum Classic, Bitcoin Gold, Feathercoin, and Vertcoin (Attah, 2019). A 51% attack means that a majority of miners impact mining to their advantage, including preventing other miners from completing blocks, and channeling funds from each block to themselves. Changing historical blocks is difficult due to the hard coding of past transactions into the Bitcoin software.

player *i*'s expected utility U_{iC} . When $k_i = 1$, $U_{iC} = 1 - \omega_i$ is independent of w_i . Player *i*'s expected utility U_{iC} in (3) expresses what is probabilistically retained for potential criminal behavior, and is multiplied with player *i*'s Cobb-Douglas utility $U_{iCD}(p_i)$ in (2) to determine what player *i* keeps of its utility when accounting for criminal behavior being probabilistically detected and prosecuted.

2.7. How a Fraction q_i of Players of Kind *i* Impacts Expected Utilities

Players of kind *i* may get increased or decreased expected utility if their fraction q_i increases or decreases. We operationalize this with the term $1 + \mu_i q_i^{m_i}$, where μ_i , $\mu_i \ge 0$ is a scaling proportionality parameter, and m_i is a scaling exponent. The term $1 + \mu_i q_i^{m_i}$ is multiplied with the Cobb-Douglas utility and what is probabilistically retained for potential criminal behavior.

Conventionalists prefer to do what others do and what is common, which gives them increased expected utility. Hence conventionalists get increased expected utility if the fraction q_x of conventionalists increases, i.e. $m_x \ge 0$. The positive exponent m_x scales the strength of how conventionalists get multiplicatively increased expected utility when the fraction q_x increases.

In contrast, pioneers prefer to do what others do not do, what is uncommon, and what breaks ground beyond what is conventional, which gives them increased expected utility. When pioneers become a majority, they are no longer pioneers, but conventionalists. Hence pioneers get decreased expected utility if the fraction q_y of pioneers increases, i.e. $m_y \leq 0$. The negative exponent m_y scales the strength of how pioneers get multiplicatively decreased expected utility when the fraction q_y increases.

Criminals focus on what is criminally lucrative, what they can get away with, and what does not get detected and prosecuted. Whether what they do is common or uncommon may be irrelevant. What criminals have in common with pioneers is that they prefer to be few so that they can operate under the radar. As criminals become more numerous, the benefits for each in most stable and relatively lawful societies can be expected to decrease since they compete with each other, and non-criminals adapt to defending against them. Exceptions, such as the Italian mafia in Italy, or the cartels in Colombia, operate according to another logic not considered in this article, where subsections of societies follow different norms. At the extreme, a society with only criminals will not function since everyone will prey on everyone causing breakdown. Hence criminals, just as pioneers, get decreased expected utility if the fraction q_z of criminals increases, i.e. $m_z \leq 0$. The negative exponent m_z scales the strength of how criminals get multiplicatively decreased expected utility when the fraction q_z increases.

The three paragraphs above enable us to operationalize player *i*'s expected utility as

$$U_{iF}(q_{i}) = 1 + \mu_{i} q_{i}^{m_{i}}$$
(4)

which is multiplied with player *i*'s Cobb-Douglas utility $U_{iCD}(p_i)$ in (2) and player *i*'s expected utility U_{iC} in (3). When $m_i = 1$, player *i*'s expected utility $U_{iF}(q_i)$ increases linearly in the fraction q_i of players of kind *i*. When $m_i > 1$, $U_{iF}(q_i)$ increases convexly in q_i , which economically means that a higher fraction q_i (compared with when $m_i = 1$) of players of kind *i* is needed in order to increase player *i*'s expected utility $U_{iF}(q_i)$. In contrast, when $0 < m_i < 1$, $U_{iF}(q_i)$ increases concavely in q_i , which economically means that a lower fraction q_i (compared with when $m_i = 1$) of players of kind *i* is sufficient in order to increase player *i*'s expected utility $U_{iF}(q_i)$. When $m_i = 0$, $U_{iF}(q_i) = 1 + \mu_i$ is independent of q_i .

Equation (4) means that player *i*'s expected utility $U_{iF}(q_i)$ depends explicitly on the fraction q_i of players of kind *i*, i = x, y, z, which is a measure of the number of players of kind *i*. This dependence of $U_{iF}(q_i)$ on q_i implicitly means that $U_{iF}(q_i)$ depends on the fraction $1 - q_i$ of players which is not of kind *i*, since $q_x + q_y + q_z = 1$. That is, more players of one kind mean fewer players of the two other kinds. In the next section 3 on the replicator equation the interdependence of

the numbers of players of each kind, and thus the interaction between the three kinds of players, becomes clearer.

2.8. The Players' Expected Utilities

This section combines multiplicatively player *i*'s expected utilities $U_{iCD}(p_i)$ in (2), U_{iC} in (3), and $U_{iF}(q_i)$ in (4), which gives player *i*'s expected utility

$$U_{i} = U_{i}(p_{i},q_{i}) = U_{iCD}(p_{i})U_{iC}U_{iF}(q_{i})$$

$$= p_{i}^{b_{in}+c_{in}+d_{in}+s_{in}}(1-p_{i})^{b_{ig}+c_{ig}+d_{ig}+e_{ig}+f_{ig}+s_{ig}}(1-\omega_{i}w_{i}^{k_{i}})(1+\mu_{i}q_{i}^{m_{i}}).$$
(5)

Equation (5) assumes that player *i* is risk neutral and abstracts away other factors such as player *i*'s consumption preferences concerning goods, and player *i*'s preference for work versus leisure, which are beyond the scope of this article. Such factors are to some extent implicitly or indirectly present in (5). For example, player *i*'s convenience c_{ij} of using currency *j* and transaction efficiency e_{ij} of currency *j* may play different roles for different goods, and may impact player *i*'s preference for work versus leisure.

2.9. Society's Expected Utility

Society's expected utility $U(p_x, p_y, p_z, q_x, q_y)$ is the weighted sum of each player's expected utility $U_i(p_i, q_i)$, weighted by the fraction of players of kind i, i = x, y, z, i.e.

$$U = U(p_x, p_y, p_z, q_x, q_y) = \sum_{i=x,y,z} q_i U_i(p_i, q_i), q_z = 1 - q_x - q_y.$$
 (6)

2.10. The Players' Strategic Choices

Assume that player *i* at time *t* makes two strategic simultaneous choices to maximize its expected utility $U_i(p_i, q_i)$ in (5). First, it chooses its volume fraction p_i of its transactions in currency *n*, causing the remaining volume fraction $1 - p_i$ of its transactions to be in currency *g*. Player *i*'s choice of p_i to maximize $U_i(p_i, q_i)$ in (5) does not depend on time *t*, and does not depend on the fraction q_i of player *i* in the population, since $1 + \mu_i q_i^{m_i}$ appears proportionally in (5), without impacting the shape of $U_i(p_i, q_i)$ as a function of p_i , and without impacting which value of p_i causes $U_i(p_i, q_i)$ to have its maximum. Hence no dynamic considerations for player *i*'s choice of volume fraction p_i of its transactions in currency *n* are needed. Second, player *i* chooses which kind *i* of player it should be, i = x, y, z. That choice depends strongly on time *t*, as described by the replicator equation in the next section. When player *i* switches from being of one kind to another kind, i = x, y, z, its first choice of the optimal volume fraction p_i of its transactions in currency *n* as player *i* such as player in also changes. In other words, as long as player *i* remains of a specific kind, its optimal volume fraction p_i does not depend on time *t*, which reflects real life, but if it switches to be of another kind according to the replicator equation described in the next section, then it also changes its optimal volume fraction p_i at time *t* to what is optimal for this new kind *i*, *i* = *x*, *y*, *z*.

2.11. The Replicator Equation

To determine the evolution of the fraction q_i of players of kind i, i = x, y, z, we consider the replicator equation (Taylor & Jonker, 1978; Weibull, 1997)

$$\begin{aligned} \frac{\partial q_i}{\partial t} &= \alpha_i q_i \left(U_i \left(p_i, q_i \right) - U(p_x, p_y, p_z, q_x, q_y) \right) \end{aligned} \tag{7}$$

$$\Leftrightarrow \begin{bmatrix} \frac{\partial q_x}{\partial t} \\ \frac{\partial q_y}{\partial t} \end{bmatrix} = \begin{bmatrix} \alpha_x \left(U_x \left(p_x, q_x \right) - U(p_x, p_y, p_z, q_x, q_y) \right) & 0 \\ 0 & \alpha_y \left(U_y \left(p_y, q_y \right) - U(p_x, p_y, p_z, q_x, q_y) \right) \end{bmatrix} \begin{bmatrix} q_x \\ q_y \end{bmatrix}$$

where α_i , $\alpha_i > 0$, is the rapidity of change or sensitivity of the process. The process is stable when α_i is intermediate. If α_i is high, the process changes rapidly. If α_i is low, a negligible change occurs. The right hand side of (7) multiplies the fraction q_i of players of kind *i* with the difference $U_i(p_i, q_i) - U$ between player *i*'s expected utility $U_i(p_i, q_i)$ and the average expected utility *U* of the three kinds i = x, y, z of players. If the right hand side of (7) is positive (negative), player *i*'s expected utility $U_i(p_i, q_i)$ is higher (lower) than the average expected utility *U*, which causes the fraction q_i of players of kind *i* to increase (decrease).

The economic interpretation of (7) is that the three kinds of players over time continuously move towards becoming the kind of player where the expected utility U_i , i.e. U_x , U_y , U_z , is highest. In doing so, player *i* accounts for both the income effect (i.e., the absolute value of player *i*'s expected utility U_i) and the substitution effect (i.e., which kind of player is optimal for player *i* to be or become). As a player changes from being of one kind to becoming of another kind, the fraction q_i of players of kind *i*, i.e. the fractions q_x , q_y , $q_z = 1 - q_x - q_y$, change. The prominent presence of q_i in (7) on the left hand side, multiplicatively on the right hand side, and in $U_i(p_i, q_i)$ and $U(p_x, p_y, p_z, q_x, q_y)$, means that the replicator equation is quite sensitive to changes in q_i . The expected utilities $U_i(p_i, q_i)$ and $U(p_x, p_y, p_z, q_x, q_y)$ also depend on the volume fractions p_i and $1 - p_i$ of player *i*'s transactions in the currencies *n* and *g*, respectively. Hence the replicator equation reflects how the three kinds of players perceive the two currencies *n* and *g* as they choose which kind of player they want to be to maximize their expected utility $U_i(p_i, q_i)$.

The limiting behavior (the evolutionary outcome) of the replicator equation in (7) is a Nash equilibrium. We determine a pure-strategy Nash equilibrium where each player *i*, i = x, y, z, maximizes its expected utility $U_i(p_i, q_i)$. This equilibrium is a set of strategies q_i^* for the three players, i = x, y, z, such that

$$U_i(p_i, q_i^*) \ge U_i(p_i, q_i) \forall 0 \le q_i \le 1, i = x, y, z; q_z = 1 - q_x - q_y.$$
(8)

For research on the equilibrium properties of replicator dynamics see (Duong & Han, 2020) and the references therein.

If $\alpha_i (U_i(p_i, q_i) - U(p_x, p_y, p_z, q_x, q_y))$ in (7) had been constant, (7) would have been a linear time-invariant system for which well-known techniques illustrated by Khalil (2002, p. 46), or Laplace and Fourier transforms, are applicable. Since $\alpha_i (U_i(p_i, q_i) - U(p_x, p_y, p_z, q_x, q_y))$ is not constant, (7) is a time-variant system which is more challenging to analyze theoretically. We thus proceed over to the next sections to analyze (7) with simulations.

3.1. Analyzing As a Function of p_i When q_i Is Exogenously Fixed

This section assumes that the fraction q_i of players of kind *i* is fixed, and analyzes how player *i* chooses its volume fraction p_i of currency *n*, implying volume fraction $1 - p_i$ for currency *g*. Differentiating player *i*'s expected utility $U_i(p_i, q_i)$ in (5) with respect to p_i and equating with zero gives

$$\frac{\partial U_{i}(p_{i},q_{i})}{\partial p_{i}} = \left(\frac{b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in}}{p_{i}} - \frac{b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig}}{1 - p_{i}}\right) p_{i}^{b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in}} (1 \qquad (9)$$

$$- p_{i})^{b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig}} (1 - \omega_{i} w_{i}^{k_{i}}) (1 + \mu_{i} q_{i}^{m_{i}}) = 0$$

which is solved to yield

$$p_{i} = p_{iopt} = \frac{b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in}}{b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in} + b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig}}.$$
 (10)

Property 1. $\partial p_{iopt} / \partial a_{in} \ge 0$, $\partial p_{iopt} / \partial a_{ig} \le 0$, $a_{ij} = b_{ij}$, c_{ij} , d_{ij} , e_{ij} , f_{ij} , s_{ij} , j = n, g.

Proof. Follows from differentiating (10).

Property 1 states that the optimal fraction p_{iopt} of player *i*'s transactions in currency *n* increases in the six subelasticities a_{in} for currency *n*, and decreases in the six subelasticities a_{ig} for currency *g*.

Inserting $p_i = p_{iont}$ into the second order derivative gives

$$\frac{\partial^2 U_i(p_i, q_i)}{\partial p_i^2} \bigg|_{p_i = p_{iopt}} = -(b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig})p_{iopt}^{b_{in} + c_{in} + d_{in} + e_{in} + f_{in} + s_{in} - 1} (1 - p_{iopt})^{b_{ig} + c_{ig} + d_{ig} + e_{ig} + f_{ig} + s_{ig} - 2} (1 - \omega_i w_i^{k_i})(1 + \mu_i q_i^{m_i}) < 0$$
(11)

which is satisfied as negative, and hence $p_i = p_{iopt}$ is a maximum.

To illustrate the model, the following plausible benchmark parameter values are chosen. If the 12 output subelasticities a_{ij} , $a_{ij} = b_{ij}$, c_{ij} , d_{ij} , e_{ij} , f_{ij} , s_{ij} , for player *i*, i = x, y, z, for currency *j*, j = n, g, were to be given equal weight, assuming constant returns to scale as specified after (2), each output subelasticity would get weight $a_{ij} = x, y, z = 1/12$.⁸ Table 1a shows 36 output subelasticities a_{ij} , which all satisfy the requirement $a_{ij} \ge 0$, for player *i*, i = x, y, z, for currency *j*, j = n, g.

DOI: 10.7172/2353-6845.jbfe.2021.2.6

⁸ Since we have no evidence to justify increasing or decreasing returns to scale, we make the simplest and common assumption of constant returns to scale.

Player <i>i</i>	<i>i</i> =	i = x		i = y		i = z	
Currency j	j = n	<i>j</i> = <i>g</i>	j = n	<i>j</i> = <i>g</i>	j = n	<i>j</i> = <i>g</i>	
Panel a							
b_{ij}	1/4	0	0	1/4	0	1/12	
c_{ij}	1/12	0	0	1/12	0	1/12	
d_{ij}	1/12	1/12	1/12	1/12	1/12	1/4	
$e_{_{ij}}$	1/12	1/12	1/12	1/12	1/12	1/12	
f_{ij}	1/12	1/12	1/12	1/12	1/12	1/12	
S _{ij}	1/12	1/12	1/12	1/12	1/12	1/12	
Panel b							
b_{ij}	1/3	0	0	1/3	0	1/12	
c_{ij}	1/12	0	0	1/12	0	1/12	
d_{ij}	1/12	0	0	1/12	0	1/3	
e_{ij}	1/12	1/12	1/12	1/12	1/12	1/12	
f_{ij}	1/12	1/12	1/12	1/12	1/12	1/12	
S _{ij}	1/12	1/12	1/12	1/12	1/12	1/12	
Panel c							
b_{ij}	1/2	0	0	1/2	0	1/12	
c _{ij}	1/12	0	0	1/12	0	1/12	
d_{ij}	1/12	0	0	1/12	0	1/2	
e_{ij}	1/12	0	0	1/12	0	1/12	
f_{ij}	1/12	0	0	1/12	0	1/12	
S _{ij}	1/12	1/12	1/12	1/12	1/12	1/12	

Table 1 Output subelasticities a_{ii} in three panels a,b,c for currency j, j = n, g, as perceived by player i, i = x, y, z.

Table 1a assumes that player x as a conventionalist prefers at least output subelasticity $a_{ii} = 1/12$ for all the six output subelasticities backing, convenience, confidentiality, transaction efficiency, stability, and security for the national currency n, and three times higher output subelasticity $b_{xn} = 1/4$ for the backing of currency *n*, which it respects and trusts, and justifies player x as a conventionalist. Table 1a further assumes that player x prefers at most output subelasticity $a_{ii} = 1/12$ for the six output subelasticities for the global currency g, and zero output subelasticity for the backing $b_{xg} = 0$ and convenience $c_{xg} = 0$ of currency g, which also justifies player x as a conventionalist. Table 1a assumes that player y as a pioneer has the opposite preference of player x, i.e. at least output subelasticity $a_{ii} = 1/12$ for all the six output subelasticities for the global currency g, and three times higher output subelasticity $b_{yg} = 1/4$ for the backing of currency g, at most output subelasticity $a_{ij} = 1/12$ for the six output subelasticities for the national currency *n*, and zero output subelasticity for the backing $b_{vn} = 0$ and convenience $c_{vn} = 0$ of currency n. Table 1a assumes that player z as a criminal has the same preference as the pioneer player y, except that its three times higher preference is for output subelasticity $d_{z\sigma} = 1/4$ for the confidentiality of currency g. Hence it prefers output subelasticity $b_{zg} = 1/12$ for the backing of currency g.

Table 1b assumes that the three kinds of players have higher preferences $b_{xn} = b_{yg} = d_{zg} = 1/3$ for their preferred output subelasticities, i.e. backing of currencies *n* and *g* for players *x* and *y*, and confidentiality of currency *g* for player *z*. They compensate for these higher preferences by having no preferences $d_{xg} = d_{yn} = d_{zn} = 0$ for confidentiality, i.e. of currency *g* for player *x* and of currency *n* for players *y* and *z*.

Table 1c assumes that the three kinds of players have even higher preferences $b_{xn} = b_{yg} = d_{zg} = 1/2$ for their preferred output subelasticities, i.e. backing of currencies *n* and *g* for players *x* and *y*, and confidentiality of currency *g* for player *z*. They compensate for these higher preferences by having no preferences $e_{xg} = e_{yn} = e_{zn} = f_{xg} = f_{yn} = f_{zn} = 0$ for transaction efficiency and financial stability, i.e. of currency *g* for player *x* and of currency *n* for players *y* and *z*. We alternate between applying Table 1 panels a, b, c, and combinations of these for players *x*, *y*, *z*, as our benchmark, as we proceed.

The benchmark furthermore assumes that the conventionalist player x and pioneer player y choose a zero fraction $w_i = 0$ of its transactions to be criminal, i = x, y, which may be a good approximation for many countries, while the criminal player z chooses a positive fraction $w_z = 0.5$ of its transactions to be criminal, assumed as a focal intermediate between $w_z = 0.5$ and $w_z = 1$. The government is assumed to detect and prosecute criminal behavior with probability $\omega_i = 0.5$, also assumed as a focal intermediate between $w_z = 0.5$ and $w_z = 1$. We assume scaling exponent $k_i = 1$ for what player i retains after criminal behavior, which in (3) means that player i's expected utility decreases linearly in the fraction w_i of player i's transactions which is criminal. The authors believe that a linear decrease is more plausible than a convex or concave decrease. Unitary values, also assumed below to the extent possible, are assumed plausible focal points when no particular evidence seems suitable for non-unitary values.

The scaling exponent for how player *i* gets increased or decreased expected utility depending on the fraction q_i of players of kind *i* is assumed to be positive and unitary, $m_x = 1$, for conventionalists, and negative and unitary, $m_y = m_z = -1$, for pioneers and criminals.

The scaling proportionality parameter μ_i for how player *i* gets increased or decreased expected utility depending on the fraction q_i of players of kind *i*, i = x, y, z, impacts the analysis crucially. We assume the unitary $\mu_x = 1$ as a benchmark for conventionalists, which in (4) causes $U_{xF}(q_x)$ to vary between $U_{xF}(q_x) = 1$ when $q_x = 0$ and $U_{xF}(q_x) = 2$ when $q_x = 1$. For pioneers and criminals we assume $\mu_i < 1$, since $U_{iF}(q_i)$ in (4) varies between $U_{iF}(q_i) = \infty$ when $q_i = 0$ and $U_{iF}(q_i) = 1 + \mu_i$ when $q_i = 1$, i = x, y, since $m_y = m_z = -1$. More specifically, we assume the five times lower $\mu_y = 0.2$ for pioneers and the ten times lower $\mu_z = 0.1$ for criminals.

In this section, where the fraction q_i of players of kind *i* is exogenous, we assume equally large fractions $q_i = 1/3$ of the three kinds of players, i = x, y, z, thus not giving eminence to one kind of player over another kind. The values $q_i = 1/3$ are needed to determine player *i*'s expected utility $U_i(p_i, q_i)$ in (5), due to the last proportional term $1 + \mu_i q_i^{m_i}$, but do not impact the shape of $U_i(p_i, q_i)$ as a function of p_i and for which value of p_i that $U_i(p_i, q_i)$ has its maximum.

Figure 2 applies the above benchmark, including the exogenous $q_i = 1/3$, and plots player *i*'s expected utility U_i in (5) and society's expected utility U in (6) as functions of player *i*'s volume fraction p_i of currency n, i = x, y, z. The Mathematica software (www.wolfram.com) is used for plotting. Panel k assumes the output subelasticities a_{ij} in Table 1k, k = a, b, c. The two dashed vertical lines in each panel show the values of p_i where at least one expected utility U_i has its maximum value, i.e. $p_x = 2/3$ and $p_y = p_z = 1/3$ in panel a, $p_x = 3/4$ and $p_y = p_z = 1/4$ in panel b, and $p_x = 11/12$ and $p_y = p_z = 1/12$ in panel c. In panel a, society's expected utility U reaches its maximum at $p_i = 4/9$ which is the weighted sum of the p_i 's across the three kinds of players. If the weights change from $q_i = 1/3$, e.g. such that q_z increases and q_x and q_y decrease, the value p_i changes from $p_i = 4/9 \approx 0.44$ towards $p_i = 2/3$. In panels b and c, society's expected utility U reaches their maxima at $p_i = 5/12 \approx 0.42$ and $p_i = 9/25 = 0.36$, calculated analogously.

Figure 2

Player *i*'s expected utility U_i as a function of its volume fraction p_i of currency *n* when $q_i = 1/3$, i = x, y, z. Panel k assumes the output subelasticities a_{ii} in Table 1k, k = a, b, c.



In all the three panels in Figure 2 the conventionalist player x's inverse U-shaped expected utility U_x is skewed towards the right since it values the national currency *n* more than the global currency *g*. When the volume fraction p_x of the conventionalist player x's transactions in the national currency *n* is low, the conventionalist player x's expected utility U_x is intuitively low. As the fraction p_x increases, its expected utility U_x increases to its maximum when $p_x = 2/3$, $p_x = 3/4$, $p_x = 11/12$, in panels a, b, c, and thereafter decreases, as player x also assigns some, although low, output subelasticities to currency *g*.

In contrast, in all the three panels in Figure 2 the pioneer player y's and criminal player z's inverse U-shaped expected utilities U_i are skewed towards the left since they value the global currency g more than the national currency n, and thus prefer $p_i < 1/2$. As the fraction p_i increases, its expected utility U_i increases to its maximum when $p_i = 1/3$, $p_i = 1/4$, $p_i = 1/12$, in panels a, b, c, respectively, i = x, y. As p_i increases further, U_i decreases. The criminal's expected utility U_z is lower than the pioneer's expected utility U_y since its fraction $w_z = 0.5$ of transactions is criminal, detected and prosecuted by the government with probability $\omega_i = 0.5$.

3.2. Analysis Applying the Replicator Equation

This section applies the replicator equation in (7) to determine the fraction q_i of players of kind *i* endogenously, while player *i* determines the volume fraction p_i of currency *n* by maximizing its expected utility U_i in (5), i = x, y, z. Figure 3 applies the output subelasticities in Table 1 and the benchmark parameter values in section 3.1, i.e. $w_x = w_y = 0$, $w_z = 0.5$, $\omega_i = 0.5$, $k_i = 1$, $m_x = 1$, $m_y = m_z = -1$, $\mu_x = 1$, $\mu_y = 0.2$, $\mu_z = 0.1$, i = x, y, z. Player *i* chooses its volume fraction p_i of currency *n* optimally to maximize its expected utility U_i , i = x, y, z. Assuming rapidity $\alpha_i = 1$ of change or sensitivity of the replicator equation, i = x, y, z, (7) is used to determine the fraction q_i of players of kind *i*, i = x, y, z. Figure 3 plots these fractions $q_x, q_y, q_z = 1 - q_x - q_y$, and the volume fraction *p* of all players' transactions in the national currency *n* from (1), as functions of time *t*.

Figure 3

Fraction q_i of players of kind *i*, i = x, y, z, and the volume fraction *p* of all players' transactions in currency *n*, as a function of time *t* for the benchmark parameter values in Table 1, $w_x = w_y = 0, w_z = 0.5, \omega_i = 0.5, k_i = 1, m_x = 1, m_y = m_z = -1, \mu_x = 1, \mu_y = 0.2, \mu_z = 0.1, \alpha_i = 1, i = x, y, z$. Panel a: Table 1a. Panel b: Table 1b. Panel c: Table 1c. Panel d: Table 1a for player *x* and Table 1c for players *y* and *z*. Panel e: Table 1c for player *x* and Table 1a for players *y* and *z*.



Figure 3 assumes initial conditions at time t = 0 equal to $q_x(0) = 0.8$ and $q_y(0) = q_z(0) = 0.1$, which means that conventionalists initially are in the majority at 80%, while pioneers and criminals are in the minority, each at 10%.

Figure 3a assumes the 36 output subelasticities in Table 1a, which according to Figure 2a gives the optimal volume fractions $p_x = 2/3$ for conventionalists and $p_y = p_z = 1/3$ for pioneers and criminals, for player *i*'s transactions in currency *n*. The fraction q_x of conventionalists decreases convexly from $q_x(0) = 0.8$ to $\lim_{t \to \infty} q_x = 0.5$, hereafter referred to as the stationary solution, after sufficiently much time *t* has elapsed. All limit values are determined numerically. The fraction q_y of pioneers increases concavely from $q_y(0) = 0.1$ to $\lim_{t \to \infty} q_y = 0.4$. The fraction q_z of criminals first decreases marginally and briefly from $q_z(0) = 0.1$, as the fraction q_y of pioneers increases rapidly. Thereafter q_z increases concavely back up towards $\lim_{t \to \infty} q_z = 0.1$. Hence the volume fraction *p* of all players' transactions in the national currency *n* decreases towards $\lim_{t \to \infty} p = 0.5$.

Figure 3b assumes the 36 output subelasticities in Table 1b, which according to Figure 2b gives the higher optimal volume fractions $p_x = 0.75$ for conventionalists and the lower $p_y = p_z = 0.25$ for pioneers and criminals, for player *i*'s transactions in currency *n*. The evolution of the fractions q_x , q_y , q_z is qualitatively similar to Figure 3a, with the same limit values $\lim_{t \to \infty} q_x = \lim_{t \to \infty} p = 0.5$, $\lim_{t \to \infty} q_y = 0.4$, $\lim_{t \to \infty} q_z = 0.1$. The reason for the similar result is that the increase in the optimum from $p_x = 2/3$ to $p_x = 3/4$ for conventionalists equals the decrease in the optimum from $p_y = p_z = 1/3$ to $p_y = p_z = 1/4$ for pioneers and criminals. These changes are in the opposite direction and equal 3/4 - 2/3 = 1/3 - 1/4 = 1/12. Furthermore, at the limit when $t \to \infty$, the fraction q_x of conventionalists equals the sum of the fractions q_y and q_z of pioneers and criminals, i.e. $\lim_{t \to \infty} q_x = 0.5 = \lim_{t \to \infty} q_y = 0.4 + \lim_{t \to \infty} q_z = 0.1$, which means that the impact in the opposite direction when determining q_x , q_y , q_z in (7) is equally strong.

Figure 3c assumes the 36 output subelasticities in Table 1c, which according to Figure 2c gives the higher optimal volume fractions $p_x = 0.92$ for conventionalists and the lower $p_y = p_z = 0.08$ for pioneers and criminals, for player *i*'s transactions in currency *n*. Also here the evolution of the fractions q_x , q_y , q_z is qualitatively similar to Figure 3a and Figure 3b, with the same limit values $\lim_{t\to\infty} q_x = \lim_{t\to\infty} p = 0.5$, $\lim_{t\to\infty} q_y = 0.4$, $\lim_{t\to\infty} q_z = 0.1$. The reason for the similar result is again that the increase in the optimum from $p_x = 2/3$ to $p_x = 11/12$ for conventionalists equals the decrease in the optimum from $p_y = p_z = 1/3$ to $p_y = p_z = 0.08$ for pioneers and criminals. These changes are in the opposite direction and equal 11/12 - 2/3 = 1/3 - 1/12 = 1/4. At the limit when $t \to \infty$, the fraction q_x of conventionalists equals the sum of the fractions q_y and q_z of pioneers and criminals, i.e. $\lim_{t\to\infty} q_x = 0.5 = \lim_{t\to\infty} q_y + \lim_{t\to\infty} q_z$, which means that the impact in the opposite direction when determining q_x , q_y , q_z in (7) is equally strong.

To illustrate results different from Figure 3a, b, c, we consider two extreme combinations of output subelasticities from Table 1, one favoring pioneers and criminals, and one favoring conventionalists. Figure 3d assumes the 12 output subelasticities in Table 1a for the conventionalist player x, which gives the minimum optimal volume fraction $p_x = 2/3$, and assumes the 24 output subelasticities in Table 1c for the pioneer and criminal players y and z, which gives the minimum optimal volume fractions $p_v = p_z = 1/12$. That both $p_x = 2/3$ and $p_v = p_z = 1/12$ are minimum optimum values for the respective players, among the alternatives in Table 1, chosen by the three kinds of players maximizing their expected utilities U_x , U_y , U_z in (5), means that all the three kinds of players choose currency *n* with minimum volume fractions p_x, p_y, p_z . That favors pioneers and criminals, who to a lower extent back and favor currency n. Consequently, the fractions q_{ij} and q_z of pioneers and criminals increase concavely and quickly from $q_y(0) = q_z(0) = 0.1$ toward $\lim_{t\to\infty} q_v = 0.85$ and $\lim_{t\to\infty} q_z = 0.15$, while the fraction q_x of conventionalist decreases convexly and quickly from $q_x(0) = 0.8$ toward $\lim_{t \to \infty} q_x = 0$, thus going extinct. This shows how a change in the output subelasticities among the alternatives in Table 1 may tilt the balance from emphasis on the national currency *n* towards emphasis on the global currency *g*. Hence the volume fraction *p* of all players' transactions in the national currency *n* decreases towards $\lim_{t \to \infty} p = 1/12$.

Figure 3e assumes the 12 output subelasticities in Table 1c for the conventionalist player x, which gives the maximum optimal volume fraction $p_x = 11/12$, and assumes the 24 output subelasticities in Table 1a for the pioneer and criminal players y and z, which gives the maximum optimal volume fractions $p_y = p_z = 1/3$. That both $p_x = 11/12$ and $p_y = p_z = 1/3$ are maximum optimum values for the respective players, among the alternatives in Table 1, means that all the three kinds of players choose currency n with maximum volume fractions p_x , p_y , p_z . That favors conventionalists, who to a higher extent back and favor currency n. Consequently, the fraction q_x of conventionalists increases concavely, quickly and marginally from $q_x(0) = 0.8$ toward $\lim_{t\to\infty} q_x = 0.835$. The fraction q_y of pioneers increases concavely, quickly and marginally from $q_z(0) = 0.1$ toward $\lim_{t\to\infty} q_y = 0.125$. The fraction q_z of criminals decreases convexly and quickly from $q_z(0) = 0.1$ toward $\lim_{t\to\infty} q_z = 0.040$. This shows how a different change in the output subelasticities among the alternatives in Table 1 may preserve the emphasis on the

national currency *n*, rather than tilting the balance towards the global currency *g*. The volume fraction *p* of all players' transactions in the national currency *n* increases marginally towards $\lim_{t\to\infty} p = 0.820$.

3.3. Sensitivity Analysis

The previous section 3.2 implies a stationary solution after sufficiently much time t has elapsed, i.e. at the limit when $t \to \infty$. This section 3.3 determines the sensitivity of that stationary solution relative to the output subelasticities in Table 1b and the 15 benchmark parameter values in section 3.1, i.e. $w_x = w_y = 0$, $w_z = 0.5$, $\omega_i = 0.5$, $k_i = 1$, $m_x = 1$, $m_y = m_z = -1$, $\mu_x = 1$, $\mu_y = 0.2$, $\mu_z = 0.1$, i = x, y, z. We choose Table 1b which has intermediate, compared with Table 1 panels a and c, optimal volume fractions $p_x = 0.75$ for conventionalists and $p_y = p_z = 0.25$ for pioneers and criminals, for player *i*'s transactions in currency *n*. In Figure 4 each of the 15 parameter values is altered from its benchmark, while the other 14 parameter values are kept at their benchmarks.

Figure 4

Fraction q_i of players of kind i, i = x, y, z, as a function of the 15 parameters w_x , w_y , w_z , ω_i , k_i , m_x , m_y , m_z , μ_x , μ_y , μ_z , relative to the benchmark parameter values in Table 1b, $w_x = w_y = 0$, $w_z = 0.5$, $\omega_i = 0.5$, $k_i = 1$, $m_x = 1$, $m_y = m_z = -1$, $\mu_x = 1$, $\mu_y = 0.2$, $\mu_z = 0.1$, i = x, y, z, assuming the stationary solution, i.e. after sufficiently much time *t* has elapsed, in section 3.2.



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In our benchmark from the previous section 3.2, Figure 3b based on Table 1b determines the stationary solution $\lim_{t \to \infty} q_x = 0.5$ for conventionalists, $\lim_{t \to \infty} q_y = 0.4$ for pioneers, and $\lim_{t \to \infty} q_z = 0.1$ for criminals, after sufficiently much time *t* has elapsed, depicted with a dashed vertical line in the 15 panels in Figure 4. As each parameter value varies, the stationary solution, hereafter for simplicity referred to as q_x , q_y , q_z , varies from $q_x = 0.5$, $q_y = 0.4$, $q_z = 0.1$ to some other values.

In Figure 4a, as the fraction w_x of conventionalists' transactions which is criminal increases above the benchmark $w_x = 0$, causing conventionalists to risk detection and prosecution if transacting criminally, the fraction q_x of conventionalists decreases from $q_x = 0.5$ to $q_y = 0$, which means extinction, due to lower expected utility. Pioneers and criminals benefit from increasing w_x . As w_x increases above $w_x = 0$, the fraction q_x of pioneers increases from $q_y = 0.4$ to $q_y = 0.85$, and the fraction q_z of criminals increases from $q_z = 0.1$ to $q_z = 0.15$, due to higher expected utilities. The fractions q_x , q_y , q_z , remain constant for $0 < w_x \le 1$ since w_x impacts only conventionalists' expected utility, and not pioneers' and criminals' expected utilities.

In Figure 4b, as the fraction w_y of pioneers' transactions which is criminal increases above the benchmark $w_y = 0$, causing pioneers to risk detection and prosecution if transacting criminally, the fraction q_y of pioneers decreases convexly from $q_y = 0.4$ to $q_y = 0.07$ when $w_y = 1$, while the fraction q_z of criminals decreases marginally and convexly from $q_z = 0.1$ to $q_z = 0.07$ when $w_y = 1$. Conventionalists benefit from increasing w_y . As w_y increases above $w_y = 0$, the fraction q_x of conventionalists increases concavely from $q_x = 0.5$ to $q_x = 0.86$ when $w_y = 1$.

In Figure 4c, as the fraction w_z of criminals' transactions which is criminal increases above the benchmark $w_z = 0.5$, the fraction q_z of criminals decreases convexly from $q_z = 0.1$ to $q_z = 0.04$ when $w_z = 1$, while the fraction q_v of pioneers decreases convexly from $q_v = 0.4$ to $q_v = 0.31$ when $w_{y} = 1$. That is because criminals and pioneers do not benefit when they or their criminal transactions become more numerous, cf (4) when $m_y = m_z = -1$ and $m_x = 1$. Conventionalists benefit from increasing w_z , while criminals and pioneers do not. As w_z increases above $w_z = 0.5$, the fraction q_x of conventionalists increases concavely from $q_x = 0.5$ to $q_x = 0.65$ when $w_z = 1$. In contrast, as w_z decreases below $w_z = 0.5$, criminals benefit from their criminal transactions becoming less numerous. That causes the expected utility U_x for conventionalists to be lower than U_{y} and U_{z} for pioneers and criminals, $U_{x} < U_{y}$ and $U_{x} < U_{z}$, regardless of the fraction q_{x} of conventionalists. That is economically detrimental for conventionalists. In such circumstances no one wants to be a conventionalist. Hence $q_r = 0$ when $w_z < 0.5$. That gives a sudden downward jump in q_x , and hence upward jumps in q_y and q_z as all the three kinds of players adapt to the disappearance of conventionalists who cannot justify their low expected utility U_r . Hence, when $w_z < 0.5$, the replicator equation in (7) strikes a balance between the fractions q_y and q_z of pioneers and criminals, which are $q_v = 0.85$ and $q_z = 0.15$ when $w_z = 0.5 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small but positive, thus excluding conventionalists. As w_z decreases below $w_z = 0.5$, the fraction q_z of criminals increases convexly from $q_z = 0.15$ to $q_z = 0.33$ when $w_z = 0$, while the fraction q_z of pioneers decreases concavely from $q_v = 0.85$ to $q_v = 0.67$ when $w_z = 0$.

In Figure 4d, as the probability ω_x that the government detects and prosecutes conventionalists' criminal behavior changes from the benchmark $\omega_x = 0.5$, the fractions $q_x = 0.5$, $q_y = 0.4$, $q_z = 0.1$ of conventionalists, pioneers and criminals remain constant and unchanged since ω_x in (5) is multiplied with the benchmark fraction $w_x = 0$ of conventionalists' transactions which is criminal. Since $w_x = 0$, ω_x has no impact.

In Figure 4e, analogously, as the probability ω_y that the government detects and prosecutes pioneers' criminal behavior changes from the benchmark $\omega_y = 0.5$, the fractions $q_x = 0.5$, $q_y = 0.4$, $q_z = 0.1$ of conventionalists, pioneers and criminals remain constant and unchanged since ω_y in (5) is multiplied with the benchmark fraction $w_y = 0$ of pioneers' transactions which is criminal. Since $w_y = 0$, ω_y has no impact.

Figure 4f, where the probability ω_z that the government detects and prosecutes the criminals' criminal behavior varies, is equivalent to Figure 4c since $k_z = 1$ in (5), and thus varying ω_z has the same impact as varying the fraction w_z of the criminals' transactions which is criminal, acknowledging that both parameters are restricted to the same interval, $0 \le \omega_z$, $w_z \le 1$ and have the same benchmark values $\omega_z = w_z = 0.5$. As in Figure 4c, as $w_z < 0.5$ so that the fraction w_z of the criminals' transactions which is criminal decreases below the benchmark $w_z = 0.5$, conventionalists cannot justify their existence due to their low utility $U_x < U_y$ and $U_x < U_z$, and hence $q_x = 0$.

In Figure 4g, as the scaling exponent k_x for what conventionalists retain after criminal behavior changes from the benchmark $k_x = 1$, the fractions $q_x = 0.5$, $q_y = 0.4$, $q_z = 0.1$ of conventionalists, pioneers and criminals remain constant and unchanged since k_x in (5) is an exponent where the base $w_x = 0$ of the conventionalists' transactions which is criminal. Since $w_x = 0$, k_x has no impact.

In Figure 4h, as the scaling exponent k_y for what pioneers retain after criminal behavior changes from the benchmark $k_y = 1$, the fractions $q_x = 0.5$, $q_y = 0.4$, $q_z = 0.1$ of conventionalists, pioneers and criminals remain constant and unchanged since k_y in (5) is an exponent with base $w_y = 0$ which expresses the fraction of the pioneers' transactions which is criminal. That is, since $w_y = 0$, k_y has no impact.

In Figure 4i, as the scaling exponent k_z for what criminals retain after criminal behavior increases above the benchmark $k_z = 1$, the expected utility U_x for conventionalists becomes lower than U_{y} and U_{z} for pioneers and criminals, regardless of the fraction q_{x} of conventionalists, and hence $q_x = 0$ when $k_z > 1$. Hence conventionalists cannot justify their existence due to $U_x < U_y$ and $U_x < U_z$, just as when $w_z < 0.5$ in Figure 4c and Figure 4f. That causes the replicator equation in (7) to strike a balance between the fractions q_v and q_z of pioneers and criminals. As k_z increases, the fraction q_v of pioneers increases from $q_v = 0.4$ when $k_z = 1$ to $q_v = 0.85$ when $k_z > 1$, and thereafter decreases convexly towards the same value as when $w_z = 0$ in Figure 4c, or when $\omega_z = 0$ in Figure 4f, i.e. $\lim_{\substack{t \to \infty, \\ k_z \to \infty}} q_y = 0.67$. The fraction q_z of criminals increases from $q_z = 0.1$ when $k_z = 1$ to $q_z = 0.15$ when $k_z > 1$, due to the disappearance of conventionalists, and thereafter increases concavely, due to successful competition with pioneers as k_z increases, eventually reaching the same value as when $w_z = 0$ in Figure 4c, or when $\omega_z = 0$ in Figure 4f, in accordance with the term $\omega_z w_z^{k_z}$ in (5), $\lim_{\substack{t \to \infty, \\ k_z \to \infty}} q_z = 0.33$. In contrast, as k_z decreases below $k_z = 1$, the fraction q_x of conventionalists increases concavely, competing successfully against pioneers and criminals, eventually reaching $q_z = 0.65$ when $k_z = 0$. As k_z decreases below $k_z = 1$, the fractions q_v and q_z of pioneers and criminals decrease convexly towards $q_y = 0.31$ and $q_z = 0.04$ when $k_z = 0$.

In Figure 4j, as the scaling exponent m_x for how conventionalists get increased (since $m_x \ge 0$) expected utility increases above the benchmark $m_x = 1$, the expected utility U_x for conventionalists becomes lower than U_y and U_z for pioneers and criminals, regardless of the fraction q_x of conventionalists, and hence $q_x = 0$ when $m_x = 1$. Hence conventionalists cannot justify their existence, just as when $w_z < 0.5$ in Figure 4c and Figure 4f and $k_z > 1$ in Figure 4i. This follows mathematically from (5) where $q_x^{m_x}$ decreases as m_x increases when $0 < q_x < 1$. That causes the replicator equation in (7) to strike a balance between the fractions q_y and q_z of pioneers and criminals are constant at $q_y = 0.95$ and $q_z = 0.15$ when $m_x > 1$. In contrast, as m_x decreases below $m_x = 1$, the fraction q_x of conventionalists increases concavely, competing successfully against pioneers and criminals, eventually reaching $q_x = 0.74$ when $m_x = 0$. This also follows mathematically from (5) where $q_x^{m_x}$ decreases when $0 < q_x < 1$. As m_x decreases below $m_x = 1$, the fractions q_y and $q_z = 0.2$ and $q_z = 0.06$ when $m_x = 0$.

In Figure 4k, as the scaling exponent m_y for how pioneers get decreased (since $m_y \le 0$) expected utility increases above the benchmark $m_y = -1$, the fraction q_y of pioneers decreases convexly, eventually going extinct, i.e. $q_y = 0$ when $m_y = 0$. This follows mathematically from

(5) where $q_y^{m_y}$ decreases as m_y increases when $0 < q_y < 1$. As m_y increases above $m_y = -1$, the fraction q_x of conventionalists increases concavely, competing successfully with pioneers and criminals, eventually reaching $q_x = 0.94$ when $m_y = 0$, while the fraction q_z of criminals decreases convexly, eventually reaching $q_z = 0.06$ when $m_y = 0$. In contrast, as m_y decreases below $m_y = -1$, the expected utility U_x for conventionalists is lower than U_y and U_z for pioneers and criminals, regardless of the fraction q_x of conventionalists, and hence $q_x = 0$ when $m_y < -1$. Conventionalists then vanish, as in several of the panels above. That causes the replicator equation in (7) to strike a balance between the fractions q_y and q_z of pioneers and criminals, which are $q_y = 0.85$ and $q_z = 0.15$ when $m_y = -1 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small but positive. As m_y decreases below $m_y = -1 - \varepsilon$, the fraction q_y of pioneers increases concavely, eventually outcompeting criminals, i.e. $\lim_{t \to \infty} m_y = 1$, while the fraction q_z of criminals decreases concavely, eventually outcompeting criminals, i.e. $\lim_{t \to \infty} m_y = -1$, while the fraction q_z of criminals decreases convexly, eventually going

extinct, i.e. $\lim_{\substack{t \to \infty, \\ m_y \to -\infty}} q_z = 0$. This follows mathematically from (5) where $q_y^{m_y}$ increases without bounds as m_y decreases towards minus infinity when $0 < q_y < 1$.

In Figure 41, as the scaling exponent m_z for how criminals get decreased (since $m_z \leq 0$) expected utility increases above the benchmark $m_z = -1$, the fraction q_z of criminals decreases convexly, eventually going extinct, i.e. $q_z = 0$ when $m_z = 0$. This follows mathematically from (5) where $q_z^{m_z}$ decreases as m_z increases when $0 < q_z < 1$. As m_z increases above $m_z = -1$, the fraction $q_{\rm x}$ of conventionalists increases concavely, competing successfully with pioneers and criminals, eventually reaching $q_x = 0.72$ when $m_z = 0$, while the fraction q_y of pioneers decreases convexly, eventually reaching $q_v = 0.28$ when $m_z = 0$. In contrast, as m_z decreases below $m_z = -1$, the expected utility U_x for conventionalists is lower than U_y and U_z for pioneers and criminals, regardless of the fraction q_x of conventionalists, and hence $q_x = 0$ when $m_z < -1$. Conventionalists then vanish, as in several of the panels above. That causes the replicator equation in (7) to strike a balance between the fractions q_v and q_z of pioneers and criminals, which are $q_v = 0.85$ and $q_z = 0.15$ when $m_z = -1 - \varepsilon$, where $\varepsilon > 0$ is arbitrarily small but positive. As m_z decreases below $m_z = -1 - \varepsilon$, the fraction q_z of criminals increases concavely, eventually outcompeting pioneers, i.e. $\lim_{\substack{t \to \infty, \\ m_z \to -\infty}} q_z = 1$, while the fraction q_y of pioneers decreases convexly, eventually going extinct, i.e. $\lim_{\substack{t \to \infty, \\ m_z \to -\infty}} q_y = 0$. This follows mathematically from (5) where $q_z^{m_z}$ increases without bounds as m_z decreases towards minus infinity when $0 < q_z < 1$.

In Figure 4m, as the scaling proportionality parameter μ_x for how conventionalists get increased (since $m_x = 1$) expected utility increases above the benchmark $\mu_x = 1$, the fraction q_x of conventionalists increases concavely, eventually outcompeting pioneers and criminals, i.e. $\lim_{\substack{t \to \infty \\ \mu_x \to \infty}} q_x = 1$. Thus the fractions q_y and q_z decrease concavely, $\lim_{\substack{t \to \infty \\ \mu_x \to \infty}} q_y = \lim_{\substack{t \to \infty \\ \mu_x \to \infty}} q_z = 0$. In contrast, as μ_x decreases below $\mu_x = 1$, the expected utility U_x for conventionalists is lower than U_y and U_z for pioneers and criminals, regardless of the fraction q_x of conventionalists, and hence $q_x = 0$ when $\mu_x < 1$. Conventionalists then vanish, as in several of the panels above. That causes the replicator equation in (7) to strike a balance between the fractions q_y and q_z of pioneers and criminals, which are $q_y = 0.85$ and $q_z = 0.15$ when $\mu_x < 1$.

In Figure 4n, as the scaling proportionality parameter μ_y for how pioneers get decreased (since $m_y = -1$) expected utility increases above the benchmark $\mu_y = 0.2$, the expected utility U_x for conventionalists becomes lower than U_y and U_z for pioneers and criminals, regardless of the fraction q_x of conventionalists, and hence $q_x = 0$ when $\mu_y > 0.2$. Conventionalists then vanish, as in several of the panels above. That causes the replicator equation in (7) to strike a balance between the fractions q_y and q_z of pioneers and criminals. As μ_y increases, the fraction q_y of pioneers increases from $q_y = 0.4$ when $\mu_y = 0.2$ to $q_y = 0.85$ when $\mu_y > 0.2$, and thereafter increases concavely, eventually outcompeting criminals, $\lim_{t \to \infty} q_y = 1$. The fraction q_z of criminals increases

from $q_z = 0.1$ when $\mu_y = 0.2$ to $q_z = 0.15$ when $\mu_y > 0.2$, due to the disappearance of conventionalists, and thereafter decreases convexly, due to unsuccessful competition with pioneers, eventually going extinct, $\lim_{\substack{t \to \infty, \\ \mu_y \to \infty}} q_z = 0$. In contrast, as μ_y decreases below $\mu_y = 0.2$, the fraction q_x of conventionalists increases concavely, competing successfully against pioneers and criminals, eventually reaching $q_y = 0.94$ when $\mu_y = 0$. As μ_y decreases below $\mu_y = 0.2$, the fractions q_y and q_z of pioneers and criminals decrease convexly, pioneers eventually going extinct, $q_y = 0$ when $\mu_y = 0$, while criminals enjoy some presence, i.e. $q_z = 0.06$ when $\mu_y = 0$.

In Figure 40, as the scaling proportionality parameter μ_z for how criminals get decreased (since $m_z = -1$) expected utility increases above the benchmark $\mu_z = 0.1$, the expected utility U_x for conventionalists becomes lower than U_y and U_z for pioneers and criminals, regardless of the fraction q_x of conventionalists, and hence $q_x = 0$ when $\mu_z > 0.1$. Conventionalists then vanish, as in several of the panels above. That causes the replicator equation in (7) to strike a balance between the fractions q_y and q_z of pioneers and criminals. As μ_z increases, the fraction q_y of pioneers increases from $q_y = 0.4$ when $\mu_z = 0.1$ to $q_y = 0.85$ when $\mu_z > 0.1$, and thereafter decreases convexly, eventually being outcompeted by criminals and going extinct, $\lim_{\substack{t \to \infty, \\ \mu_z \to \infty}} q_y = 0$. The fraction q_z of conventionalists, and hereafter increases convexly, due to successful competition with pioneers, eventually becoming dominant and excluding pioneers, $\lim_{\substack{t \to \infty, \\ \mu_z \to \infty}} q_z = 1$. In contrast, as

 μ_z decreases below $\mu_z = 0.1$, the fraction q_x of conventionalists increases concavely, competing successfully against pioneers and criminals, eventually reaching $q_z = 0.72$ when $\mu_z = 0$. As μ_z decreases below $\mu_z = 0.1$, the fractions q_y and q_z of pioneers and criminals decrease convexly, criminals eventually going extinct, $q_z = 0$ when $\mu_z = 0$, while pioneers are present at $q_y = 0.28$ when $\mu_z = 0$.

4. EXPLAINING THE IMPLICATIONS OF THE RESULTS

With the emergence of new currencies, each player's first choice of which volume fractions of its transactions should be in the national currency and the global currency can be expected to become more significant. The player's choice impacts both its utility, society's utility, which currencies gain traction, and which institutions and parts of society benefit from which currencies gain traction. These factors in turn can be expected to impact finance, business, markets and probably monetary policy, especially if no single currency is or becomes dominant within a given country.

Each player's second choice of whether to be a conventionalist, pioneer or criminal also impacts its utility, and impacts how society becomes composed of these three kinds of players. If conventionalists become less numerous, as illustrated for several combinations of parameter values in the previous section, society may evolve to become less conventional, with competition between pioneers and criminals.

The finding that each player's expected utility is inverse U-shaped as a function of the volume fraction of its transactions in each currency challenges each player to assess its identity as a conventionalist, pioneer or criminal. Each player is furthermore challenged to determine the impact of the subelasticities labeled as backing, convenience, confidentiality, transaction efficiency, financial stability, and security on in its Cobb-Douglas expected utility for the two currencies. This amounts to determining whether the inverse U-shape is skewed with a maximum towards the left or the right, and hence which currency should be chosen for the highest fraction of transactions, which may give fluctuations in currency markets.

5. CONCLUSION

This article analyzes conventionalists, pioneers and criminals choosing between a national currency, e.g. a CBDC (central bank digital currency) or another currency common within a nation, and a global currency, e.g. Bitcoin or Meta's Diem, which may have limited usage within a nation (e.g. for purchases and tax payments), but may offer other possibilities such as application across nations and user autonomy. Conventionalists tend to prefer the national currency, pioneers (early adopters) tend to prefer the global currency, and criminals tend to prefer the global currency if it contributes (e.g. through confidentiality) to not getting caught.

Each player has a Cobb-Douglas utility with one output elasticity for each of the two currencies. Each output elasticity is comprised of six subelasticities, i.e. which kind of backing a currency has from trustworthy actors or systems (e.g. central banks for CBDCs and distributed ledger technology for cryptocurrencies), convenience (e.g. user friendliness), confidentiality (balancing privacy, availability, accessibility, and discrimination), transaction efficiency (low cost, fast speed, affordability, finality), financial stability (e.g. resilience during crises and shocks), and security (e.g. whether funds are safe and not subject to 51% attacks). Each player's expected utility is expanded to account negatively for detection and prosecution of criminal behavior, and accounts for the fractions of the three kinds of players. Conventionalists benefit from the presence of many conventionalists. Pioneers and criminals benefit from the presence of few pioneers and criminals, respectively.

Each player makes two strategic choices to maximize its expected utility, i.e. which volume fraction of its transactions should be in the national currency (causing the remaining fraction to be in the global currency), and what kind of player it should be, i.e. a conventionalist, pioneer or criminal. The first choice becomes increasingly relevant in today's world as we expect players to have easier access to more than one currency. Hence the market share of two currencies may change over time, as illustrated in this article. The first choice depends on which kind of player the player is, but does not depend on the number of players of this kind, and hence does not depend on time. Each player's second choice is what kind of player it should be through time. Hence this second choice depends on time, through replicator dynamics.

Each player's expected utility is inverse U-shaped as a function of the volume fraction of its transactions in the national currency. Hence each player prefers not to rely exclusively on one currency. The expected utility is skewed towards the right (high fraction) for conventionalists, who prefer the national currency, and more so if the conventionalists' six output subelasticities for the national currency are high. The expected utility is skewed towards the left (low fraction) for pioneers and criminals, who prefer the global currency, and more so if the pioneers' and criminals' six output subelasticities for the global currency are high. Three examples are considered for the degree of skewness towards the right and left. Today's financial system increasingly seems to require players to assess whether the various available currencies are characterized by inverse U-shaped expected utilities skewed towards the right or the left. Players more able to assess these inverse U-shapes as functions of volume fractions, and more able to assess whether they are conventionalists, pioneers and criminals, can expect to earn higher expected utilities. Society's expected utility is the weighted sum of each player's expected utility weighted by the fraction of players of each kind.

The replicator equation is used to illustrate the evolution of the fractions of the three kinds of players through time, assuming initial conditions with conventionalists in the majority, and pioneers and criminals in the minority. We illustrate how conventionalists may become more dominant and criminals less dominant through time if all the three kinds of players' expected utilities are skewed towards the right (i.e. prefer the national currency). In contrast, pioneers and criminals may become more dominant and conventionalists may go extinct if all the three kinds of players' expected utilities are skewed towards the left (i.e. prefer the global currency). Considering the stationary solution after sufficiently much time has elapsed, the model's sensitivity with respect to 15 parameter values is analyzed. The analysis shows that, typically, conventionalists (which prefer to be in the majority) tend to compete against pioneers and criminals (which prefer to be in the minority). Hence if a change in a parameter value causes the fraction of conventionalists to increase (decrease), the fractions of both pioneers and criminals may decrease (increase). The exception is, of course, when conventionalists are extinct, which is caused by their expected utility being too low, in which case pioneers and criminals compete directly with each other, so an increasing (decreasing) fraction of pioneers causes a decreasing (increasing) fraction of criminals.

As the fraction of a player's transactions which is criminal, or the probability that the government detects and prosecutes the player's criminal behavior, increases, the fraction of that kind of players in the population decreases, causing the fraction of at least one of the other kinds of players to increase. Each player thus responds to incentives, ceasing to be a kind of player with many criminal transactions, and ceasing criminal transactions if these are detected and prosecuted.

As the scaling exponent for what criminals retain after criminal behavior increases, their fraction in the population increases. That also causes the fraction of pioneers to increase, and the fraction of conventionalists to decrease, except when conventionalists are extinct, which occurs when the scaling exponent is high, in which case the fraction of pioneers decreases due to competition with criminals.

As the positive scaling exponent for how the conventionalists get increased expected utility increases, their expected utility decreases causing their fraction in the population to decrease and eventually go extinct. That causes the fractions of pioneers and criminals to increase. As the negative scaling exponents for how pioneers and criminals get decreased expected utilities increase, their expected utilities decrease causing their fractions in the population to decrease and eventually go extinct. That causes the fraction of conventionalists to transition from extinction to increase. This illustrates how economic incentives for conventionalists can make them more numerous.

As the scaling proportionality parameter for how conventionalists get increased expected utility increases, their fraction increases, as they respond to economic incentives, causing the fractions of pioneers and criminals to decrease. As the scaling proportionality parameters for how pioneers and criminals get increased expected utility increase, both their fractions increase, also responding to economic incentives, causing the fraction of conventionalists to decrease. Eventually, conventionalists go extinct, causing more pioneers and fewer criminals if the pioneers' scaling proportionality parameter increases, and more criminals and fewer pioneers if the criminals' scaling proportionality parameter increases.

Future research should compile and assess empirical support for the six kinds of output subelasticities for national and global currencies, the relevance of each output subelasticity, whether other output subelasticities can be envisioned, or whether the focus should be on fewer output subelasticities. Such empirical support should be assessed against which volume fractions players choose for national and global currencies, and which fractions of players choose to be conventionalists, pioneers, and criminals. These assessments should be made over various time periods to determine which factors impact which national and global currencies spread and become dominant, and which currencies decline in relevance and go extinct. For a more extensive dynamic analysis, the parameters such as the 12 output subelasticities may be allowed to depend on time. Various alternatives to the players' expected utilities may be evaluated, with different risk attitudes, and more than three kinds of players may be modeled. Each kind may have different time horizons and different exchange and trading strategies, e.g. many exchanges per day versus few exchanges per decade. More than one national currency may be analyzed, with competition between multiple national and global currencies which may be generalized to national and global assets (e.g. cryptoassets). The impact of competition on inflation, interest rates, etc., may be assessed, and other players such as regulators and governments may be incorporated.

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