# ARTICLE

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# The evolution of fixed-supply and variable-supply currencies

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Competition is analyzed between a fixed-supply currency (e.g. Bitcoin) and a variable-supply currency (e.g. a fiat currency). Two kinds of players support the currencies differently and choose their volume fractions of transactions in each currency. The variable-supply currency enables money printing/withdrawal and inflation/deflation, which counteract each other in each player's utility. The exponentially increasing 1959–2021 US M2 money supply and the positive inflation cause this utility to increase over time with high weight assigned to money printing/withdrawal, and decrease otherwise. Three replicator equations determine each player's volume fraction of transactions in each currency, and which kind of player each player prefers to be. High weight assigned to money supply relative to inflation induces players to prefer the variable-supply currency. A player's utility of transacting in each currency is proportional to the player's support of that currency, the volume fraction of all players' transactions in that currency, and the fraction of players of the same kind as the given player. A player's utility of transacting in the variable-supply currency is additionally proportional to two ratios. The first is the initial money supply plus the accumulative money printing/withdrawal divided by the initial money supply. The second is the inverse of the accumulative inflation/deflation. The players' fractions of transactions in each currency may be inverse U shaped or U shaped before typically converging towards preferring one or the other currency. If each player can choose which kind of player to be, it may choose to be the kind with the highest support of a given currency. If a player's utility of transacting in a given currency depends more on the fraction of players being of one kind than the other kind, the player prefers to be of the first kind, thus assigning less weight to its support of that currency and the volume fractions of transactions in that currency.

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## Introduction

**Background**. Humans have used cash currencies for 40,000 years, which evolved from natural objects to coins to paper to digital versions (Kusimba, 2017). The Mesopotamian shekel emerged nearly 5000 years ago, and silver and gold mints emerged in Asia Minor 650–600 B.C, expanding to lead and copper coins in the first millennium A.D. Currencies commonly have a central authority and usually emerged for certain geographic areas and nations. Sometimes the expansion is global, e.g. as a world reserve currency. Fiat currencies have more recently expanded to also become digital. Digital currencies such as Bitcoin have no central authority and easily expand globally. Nakamoto (2008) shows how a decentralized currency such as Bitcoin can be built on a blockchain. He applies the proof of work technology to secure the ledger and avoid the double spending problem. Today 17,834 cryptocurrencies exist with a market cap of $1.8 trillion (https://coinmarketcap.com/, retrieved February 26, 2022). These vary substantially regarding fixed versus variable supply, consensus mechanisms (e.g. proof of stake), degree of decentralization, ownership, regulation, confirmation of transactions, etc. New digital currencies suggest competition between these and conventional currencies. Understanding this competition can be expected to be essential in the coming years.

**Contribution**. This article's purpose, motivation, objectives, research hypotheses, and research questions are as follows: First, competition between one fixed-supply and one variable-supply currency is analyzed to determine the evolutionary dynamics of each currency and which currency survives. Second, each player maximizes its utility by choosing which volume fraction of transactions to conduct in each currency, and which of two kinds of player to be, depending on various preferences. Third, the variable-supply currency enables money printing/withdrawal which impacts inflation/deflation which impacts each player's utility and strategic choices and thus how each currency evolves.

Being a certain kind of player means supporting one or the other currency to a certain extent. Such support is expressed by a currency's backing, convenience, confidentiality, transaction efficiency, financial stability, and security. A player's utility of transacting in the fixed-supply currency depends on the player's support of that currency, the volume fraction of all players' transactions in that currency, and the fraction of players of the same kind as the given player. A player's utility of transacting in the variable-supply currency depends on the same kinds of factors, and additionally depends on the variable money supply and inflation/deflation. That latter dependence is expressed on the Cobb Douglas form multiplying two ratios, i.e. the initial supply plus the accumulative money printing/withdrawal divided by the initial supply, and the inverse of accumulative inflation/deflation. If both ratios are valued equally and multiply to 1, money printing/withdrawal and inflation/deflation counteract each other. A product higher (lower) than 1 suggests higher (lower) weight to money printing/withdrawal.

Fixed-supply currencies have been historically uncommon. Gold viewed as a currency (Mitchell, 2021) is the best example, with 1.5% additional gold mined in 2020 (197,576 metric tons has been mined (gold.org, 2022). 3030 metric tons were produced in 2020 (Basov, 2022)). As a comparison, as of January 2022, 18.9 million Bitcoin out of 21 million coins have been mined, i.e. 90% (Hayes, 2022). The process will continue at a decreasing speed until approximately 2140. Both gold and Bitcoin are durable and fungible (Learn, 2021). Gold has more established history, with more entrenchment in cultures, central banks, and institutions, but falls short of Bitcoin on portability, divisibility, censorship resistance, verifiability, and scarcity (Ikkurty, 2019).

Whereas fixed-supply currencies eliminate inflation/deflation caused by money printing/withdrawal, variable-supply currencies do not. Variable-supply currencies offer added flexibility and possibilities not possible for fixed-supply currencies, e.g. funding wars and critical events, and Roosevelt's 1933–1939 New Deal for economic recovery. Money printing during such events suggests subsequent contraction to avoid inflation. Many economies have not exhibited the sufficient fiscal discipline. Even a traditionally fiscally responsible economy like the US has experienced that $1 in 2022 buys 1.22% of what it would buy in 1695.

Using the 1959–2021 US M2 money supply and inflation data, we show how a player's utility of exchanging in the fixed-supply currency is constant over time. The player's utility of exchanging in the variable-supply currency increases over time if more weight is assigned to money printing/withdrawal, and otherwise decreases over time.

One replicator equation expresses each kind of player's transaction volume in each currency. A third replicator equation expresses how each player prefers to be of one or the other kind. Each player's fractions of transactions in each currency may be inverse U shaped or U shaped before converging towards preferring one or the other currency, depending on the player's support of each currency. If a player can choose which kind of player to be, thus changing its support for a certain currency, it may choose to be of the kind which supports a certain currency highly. If a player is additionally impacted by how many players exist of each kind, it may choose to be of the kind that is most common.

Understanding how players choose between competing currencies is useful for consumers, traders, policy makers, regulators, institutions designing and issuing currencies, and institutions adjusting and impacting money supply and inflation/deflation.

**Literature**. Four groups of literature have been identified, i.e. competition between fiat currencies and cryptocurrencies, central bank digital currencies and cryptocurrencies, the cryptocurrency market, and game theoretic analyses.

*Competition between fiat currencies and cryptocurrencies*. Schilling and Uhlig (2019) evaluate how agents choose between a cryptocurrency and a fiat currency. Cryptocurrencies may enable tax evasion, anonymity, and censorship resistance, impacted by transaction fees to miners. Fiat currencies are currently useful for most purchases, impacted by value-added-taxes. They argue that substitution decreases as the asymmetry in exchange fees and transaction costs increase. This finding relates to how players in the current article choose volume fractions of transactions in two currencies, depending on their support for each currency which in turn depends on each currency's transaction efficiency, and depending on other factors.

Fernández-Villaverde and Sanches (2019) specify a price stable equilibrium, and some less desirable equilibria, for multiple competing privately issued fiat currencies in a Lagos-Wright environment. Their approach has a linkage to the analysis of two coexisting currencies in the current article.

Almosova (2018) evaluates costly circulation of private currencies, impacted by verification of transactions, mining costs, etc. She finds that sufficiently low costs of private currency circulation (mining costs) are needed to put downward pressure on the inflation for the public currency. Cryptocurrency competition may not cause price stability. These insights relate to the current article where players may choose a fixed-supply currency to avoid the inflation in the variable-supply currency.

Benigno et al. (2019) evaluate a global cryptocurrency and two national currencies. They find that different interest rates may cause the national currency to be abandoned or the zero lower bound may be approached. They argue that ensuring an independent monetary policy, free capital flows, and a fixed exchange rate may become even less possible. As a comparison, the current article evaluates various other conditions that may cause a currency to be abandoned.

Rahman (2018) considers how monetary policy is impacted by fiat and digital currency competition. He argues that a purely private arrangement of digital currencies cannot cause socially efficient allocation, and that optimal monetary policy at the Friedman rule will be socially inefficient. These insights suggest the need to understand the nature of currency competition.

Verdier (2021) analyzes how competition in the deposit and lending markets is impacted by a digital currency. She finds that the digital currency crowds out bank deposits causing increasing bank lending rates. That insight furthermore illustrates how currency competition can cause substantial disruption, which suggests a need to understand the evolutionary dynamics.

*Central bank digital currencies and cryptocurrencies*. Caginalp and Caginalp (2019) analyze how the wealthy divide their assets between a cryptocurrency and a home currency, similarly to how the current article analyzes players choosing how to transact in two currencies. Additionally they evaluate how a government can confiscate some of the players' assets.

Blakstad and Allen (2018) evaluate various conditions for issuing central bank digital currencies, and risks and possibilities associated with cryptocurrencies. Their analysis relates to the current article where two currencies may be supported differently, and the variable-supply currency may be designed with different characteristics related to facilitating money printing/withdrawal and inflation/deflation.

Masciandaro (2018) analyzes the evolution of different media of payments depending on individual preferences, similarly to this article modeling this evolution. They assess the implications for monetary policy, addressing the zero lower bound constraint for interest rates, and banking policy, e.g. risks of bank disintermediation when the opportunity-cost discrepancies between currencies decrease. That latter focus is partly or indirectly present in the current article in the sense that the abandonment of a variable-supply currency may cause banks to change how they operate.

Benigno (2021) argues that competing currencies may cause central banks to lose control of the nominal interest rate and inflation which depend on structural factors. Cryptocurrencies may set lower bounds on interest rates and inflation. The implication of that insight may be the kind of coexistence of two currencies, or one currency going extinct, as analyzed in the current article.

Asimakopoulos et al. (2019) evaluate substitution between a government currency and a cryptocurrency, depending on preferences, technology and monetary policy shocks, akin to how the current article considers players' substitution between currencies.

*The cryptocurrency market*. ElBahrawy et al. (2017) analyze the 2013–2017 evolutionary dynamics of market shares of cryptocurrencies. They find several stable statistical properties, e.g. the market share distribution, turnover, and number of active cryptocurrencies. The current article confines attention to the evolutionary dynamics of two currencies.

Caporale et al. (2018) find that cryptocurrencies' past and future values are positively correlated, with changing degree over time. They argue that this constitutes market inefficiency, enabling the generation of abnormal profits. Partly related, the current article shows how players' utilities change over time depending on how they transact in two currencies.

ElBahrawy et al. (2019) evaluate the interplay between online Wikipedia attention and market performance of cryptocurrencies. They find that tightly knit editors impact Wikipedia and that trading based on Wikipedia views mostly performs better than baseline strategies, apart from buying and holding during explosive market expansion. This also illustrates how players' utilities change over time depending on various strategies, and analyzed in this article.

White (2014) evaluates the market shares of Bitcoin and altcoins, similarly to this article evaluating players' volume fractions of transactions in two currencies.

Sapkota and Grobys (2021) identify market inefficiency where privacy coins exhibit market equilibrium unrelated to non-privacy coins. They suggest that the result may be due to criminals preferring non-privacy coins with high liquidity and anonymity. Their approach shows how players consciously choose between currencies with different properties, as in the current article.

Milunovich (2018) determines weak connectedness between six major asset classes and five cryptocurrencies, and mostly strong connectedness within each of these two groups. If such weak connectedness proves to be common for multiple currencies, that suggests the need to understand how players choose between multiple currencies with different characteristics, as in the current article.

Gandal and Halaburda (2016) characterize recent cryptocurrency competition as winner-take-all, and early competition as no winner-take-all. That more recent insight may reflect the finding in this article of players gradually moving towards favoring one or the other currency.

*Game theoretic analyses*. Imhof and Nowak (2006) consider a stochastic frequency dependent Wright–Fisher process to determine the survival of two strategies. They specify two absorbing states for the Markov process, where homogeneous populations choose either strategy A or strategy B. Players typically abandon a strategy occurring less frequently than 1/3 in an unstable equilibrium. That corresponds partly to this article's finding of players often preferring one or the other currency.

Lewenberg et al. (2015) apply cooperative game theory to determine that Bitcoin mining pools may find it challenging to distribute rewards in a stable way, causing players to switch pools frequently. That, in turn, may cause fluctuations which suggests the importance of applying evolutionary dynamics to assess players preferences over time.

**Article organization**. Section "The model" presents the model. Section "Analyzing the model" analyzes the model. Section "Discussion and future research" discusses the results. Section "Conclusion" concludes.

## The model

### Nomenclature. *Parameters*

$g$ Fixed-supply currency

$n$ Variable-supply fiat currency

$t_0$ Initial time, $t_0 \geq 0$

$T$ Final time, $T \geq t_0$

$j$ Time counting variable, $t_0 \leq j \leq T$

$i$ Player of kind $i$, $i = 1,2$

$s_{it}$ Player $i$'s support of currency $g$ relative to currency $n$ at time $t$, $0 \leq s_{it} \leq 1$

$\mu_i$ Scaling proportionality parameter in player $i$'s utilities $u_{igt}$ and $u_{int}$, $\mu_i \geq 0$

$m_i$ Scaling exponent in player $i$'s utilities $u_{igt}$ and $u_{int}$, $m_i \geq 0$

$S_j$ Supply at discrete time $j$ of the variable-supply fiat currency $n$, $S_j \in \mathbb{R}$

$\pi_j$ Inflation at time $j$, $\pi_j \in \mathbb{R}$

$\alpha_i$ Player $i$'s Cobb Douglas elasticity for money supply $S_j$, $0 \leq \alpha_i \leq 1$

$k_i$ Player $i$'s process sensitivity for the fraction $p_{it}$ in the replicator equation, $k_i \geq 0$

$h$ Process sensitivity for the fraction $q_{1t}$ in the replicator equation, $h \geq 0$

*Independent variables*

$t$ Time, $t \geq t_0$

$p_{it}$ Volume fraction of player $i$'s transactions in currency $g$ at time $t$, $0 \leq p_{it} \leq 1$

$q_{it}$ Fraction $q_{it}$ of players of kind $i$ at time $t$, $0 \leq q_{it} \leq 1$, $q_{1t} = 1 - q_{2t}$

*Dependent variables*

$p_t$ Volume fraction of all players' transactions in currency $g$ at time $t$, $0 \leq p_t \leq 1$

$u_{igt}$ Player $i$'s utility of transacting in the fixed-supply currency $g$ at time $t$, $u_{igt} \geq 0$

$u_{int}$ Player $i$'s utility of transacting in the variable-supply currency $n$, $u_{int} \geq 0$

$u_{it}$ Player $i$'s weighted utility of transacting in both currencies, $u_{it} \geq 0$

$u_t$ Society's utility weighing the utilities of all players of both kinds, $u_t \geq 0$

**Overview of the model.** Section "Simplified player utilities" presents the simplified player utilities where two kinds of players receive a fixed utility depending on their support of a fixed-supply currency to two different extents. They also receive a variable utility of transacting in the variable-supply currency depending on money printing/withdrawal of that currency and inflation/deflation. Section "More realistic player utilities" generalizes so that the two kinds of players' utilities also depend on their support of a given currency, the volume fraction of all players' (of both kinds) transactions in the given currency, and the fraction of players of the same kind as the player being analyzed. Section "Replicator dynamics" introduces three replicator equations specifying each player's volume fraction of transactions in each currency, and which kind of player each player prefers to be.

**Simplified player utilities.** Consider two kinds of players referred to as kind $i$, $i = 1, 2$. Assume that player $i$ (i.e. player of kind $i$) earns a simplified utility $u_{igst}$ of transacting in the fixed-supply currency $g$ proportional to player $i$'s support $s_{it}$, $0 \leq s_{it} \leq 1$, of currency $g$ relative to currency $n$ at time $t$, i.e.

$$u_{igst} = 0.5 s_{it} \qquad (1)$$

where the scaling 0.5 is chosen to ensure comparison with the generalization in the next section. Assume further that player $i$'s utility $u_{inst}$ of transacting in the variable-supply currency $n$ is proportional to its support $1 - s_{it}$ of currency $n$. Player $i$'s utility $u_{inst}$ also depends on the variable money supply $S_j$ and inflation/deflation $\pi_j$ expressed on the Cobb Douglas form with elasticities $\alpha_i$ and $1 - \alpha_i$, respectively, $0 \leq \alpha_i \leq 1$. We assume money supply $S_j$, $S_j \in \mathbb{R}$, at the discrete times $j = t_0, t_0 + 1, \ldots, T$, where $t_0 \geq 0$ is the initial time and $T$ is the final time. Any time interval of length 1 applies, e.g. year, month, week, day, etc. Thus $S_{j+1} - S_j$ is the changed supply from time $j$ to time $j+1$, $\sum_{j=t_0}^{t-1}\left(S_{j+1} - S_j\right)$ is the changed supply from $j = t_0$ to $j = t-1$, and $\frac{S_{t_0} + \sum_{j=t_0}^{t-1}\left(S_{j+1} - S_j\right)}{S_{t_0}}$ is the supply at time $t$ divided by the supply at time $t_0$ which

expresses player $i$'s purchasing power at time $t$ relative to its purchasing power at time $t_0$ without inflation. With inflation $\pi_j$, $\pi_j \in \mathbb{R}$, at time $j = t_0, \ldots, T$, an asset valued as 1 at time $j = t_0$ is valued as $\frac{1}{\prod_{j=t_0+1}^{t}(1+\pi_j)}$ at time $j = t$, thus degrading the asset value due to accumulative inflation if $\prod_{j=t_0+1}^{t}\left(1 + \pi_j\right) > 1$, and increasing the asset value otherwise. Thus player $i$'s simplified utility of transacting in the variable-supply currency $n$ is

$$u_{inst} = 0.5\left(1 - s_{it}\right)\left(\frac{S_{t_0} + \sum_{j=t_0}^{t-1}\left(S_{j+1} - S_j\right)}{S_{t_0}}\right)^{\alpha_i}\left(\frac{1}{\prod_{j=t_0+1}^{t}\left(1 + \pi_j\right)}\right)^{1-\alpha_i} \qquad (2)$$

If $\alpha_i > 0.5$, player $i$ assigns more weight to purchasing power than to inflation/deflation, and conversely if $\alpha_i < 0.5$. Equal weights $\alpha_i = 0.5$ can theoretically be conceptualized as equating the two last Cobb Douglas terms in Eq. (2) with 1 where player $i$'s adjusted purchasing power from adjusted money supply $S_{j+1} - S_j$ is exactly offset by inflation/deflation $\pi_j$ through time.

**More realistic player utilities.** A fraction $q_{it}$ of the players are of kind $i$ at time $t$, where $q_{1t} = 1 - q_{2t}$, $0 \leq q_{1t} \leq 1$. Player $i$ chooses a volume fraction $p_{it}$ of its transactions in currency $g$, and the remaining volume fraction $1 - p_{it}$ of its transactions in currency $n$, see Fig. 1 which exemplifies with $p_{1t} > p_{2t}$ and $q_{1t} < q_{2t}$, but generally $0 \leq p_{it} \leq 1$, $0 \leq q_{it} \leq 1$, $i = 1, 2$.

Hence the volume fraction $p_t$ at time $t$ of all players' transactions in currency $g$ is the weighted sum of each player $i$'s volume fraction $p_{it}$ in currency $g$, weighted by the fraction $q_{it}$ of each kind of player $i$, $i = 1, 2$, i.e.

$$p_t = p_{1t}q_{1t} + p_{2t}q_{2t} \qquad (3)$$

More realistically than the previous section "Simplified player utilities", assume that player $i$ earns a utility $u_{igt}$ of transacting in the fixed-supply currency $g$ proportional to three factors, i.e. its support $s_{it}$ of currency $g$ relative to currency $n$, the volume fraction $p_t$ of all players' (of both kinds) transactions in currency $g$, and the fraction $q_{it}$ of players of kind $i$. We operationalize the latter as $1 + \mu_i q_{it}^{m_i}$, where $\mu_i$, $\mu_i \geq 0$ is a scaling proportionality parameter, and $m_i$, $m_i \geq 0$, is a scaling exponent. Thus a negligible fraction $q_{it} \approx 0$ causes the proportionality parameter $\approx 1$, and a dominant fraction $q_{it} = 1$ causes the proportionality parameter $1 + \mu_i$. Generalizing Eq. (1), player $i$'s utility of transacting in the fixed-supply currency $g$ is

$$u_{igt} = s_{it}\left(p_{1t}q_{1t} + p_{2t}q_{2t}\right)\left(1 + \mu_i q_{it}^{m_i}\right) \qquad (4)$$

Analogously, player $i$'s utility of transacting in the variable-supply currency $n$ is proportional to the same three factors, i.e. its support $1 - s_{it}$ of currency $n$, the volume fraction $1 - p_t$ of all
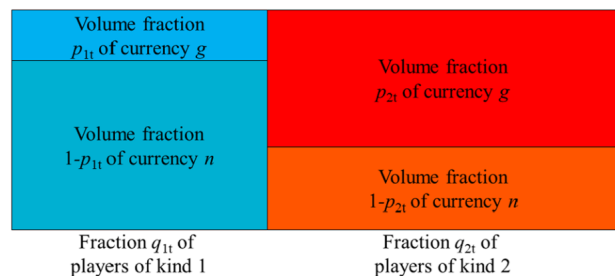


**Fig. 1 Volume fractions $p_{1t}$ and $p_{2t}$ of transactions in currencies $g$ and $n$ for two kinds of players of different fractions $q_{1t}$ and $q_{2t}$.** Player $i$, $i = 1, 2$, chooses a volume fraction $p_{it}$ of its transactions in currency $g$, and $1 - p_{it}$ in currency $n$, $0 \leq p_{it} \leq 1$, $0 \leq q_{it} \leq 1$, $q_{1t} + q_{2t} = 1$, $i = 1, 2$.

players' transactions in currency $n$, and $1 + \mu_i q_{it}^{m_i}$. Generalizing Eq. (2), player $i$'s utility of transacting in the variable-supply currency $n$ is

$$
\begin{aligned}
u_{int} = & \left(1 - s_{it}\right)\left(1 - p_{1t}q_{1t} - p_{2t}q_{2t}\right)\left(1 + \mu_i q_{it}^{m_i}\right) \\
& \times \left(\frac{S_{t_0} + \sum_{j=t_0}^{t-1}\left(S_{j+1} - S_j\right)}{S_{t_0}}\right)^{\alpha_i}\left(\frac{1}{\prod_{j=t_0+1}^{t}\left(1+\pi_j\right)}\right)^{1-\alpha_i}
\end{aligned}
\tag{5}
$$

Equations (4), (5) simplify to Eqs. (1), (2) when $p_{it} = q_{it} = 0.5$ and $\mu_i = 0$. Player $i$'s utility at time $t$ is the weighted combination of its volume fraction $p_{it}$ of transactions in the fixed-supply currency $g$, and its remaining volume fraction $1 - p_{it}$ in the variable-supply currency $n$, i.e.

$$
u_{it} = p_{it}u_{igt} + \left(1 - p_{it}\right)u_{int}
\tag{6}
$$

Society's utility, comprising all players of both kinds, is

$$
u_t = q_1 u_{1t} + \left(1 - q_1\right)u_{2t}
\tag{7}
$$

### Replicator dynamics

*Player $i$'s volume of transactions in the fixed-supply currency $g$.* To analyze the evolution of the fraction $p_{it}$ of player $i$'s volume of transactions in the fixed-supply currency $g$, causing $1 - p_{it}$ to be in currency $n$, the replicator equation (Taylor and Jonker, 1978; Weibull, 1997)

$$
\frac{\partial p_{it}}{\partial t} = k_i p_{it}\left(u_{igt} - u_{it}\right) = k_i p_{it}\left(1 - p_{it}\right)\left(u_{igt} - u_{int}\right)
\tag{8}
$$

is applied, inserting Eq. (6), where $k_i > 0$ is the process sensitivity, i.e. how rapidly the fraction $p_{it}$ changes. Intermediate $k_i$ causes a stable process, while high and low $p_{it}$ give quick and slow changes, respectively. The right-hand side of Eq. (8) is proportional to the difference $u_{igt} - u_{it}$ between player $i$'s utility of transacting in the fixed-supply currency $g$ and the weighted combination of both utilities in Eq. (6), and also proportional to the difference $u_{igt} - u_{int}$ between player $i$'s utility of transacting in the fixed-supply currency $g$ and the variable-supply currency $n$. When $u_{igt}$ exceeds $u_{it}$ or $u_{int}$, the fraction $p_{it}$ increases, and decreases otherwise. The right-hand side of Eq. (8) is furthermore proportional to $p_{nt}\left(1 - p_{nt}\right)$ which is inverse U shaped with a maximum at $p_{it} = 0.5$ and minima at $p_{it} = 0$ and $p_{it} = 1$. The fractions $p_{it}$ and $1 - p_{it}$ change most quickly when equally large, and most slowly when one fraction dominates the other.

*The fraction $q_{1t}$ of players of kind 1.* If we allow each player of kind 1 to change its preferences so as to be of kind 2, and each player of kind 2 to be of kind 1, we can analyze the analogous evolution of the fraction $q_{1t}$ of players of kind 1, causing $q_{2t} = 1 - q_{1t}$ to be of kind 2, i.e.

$$
\frac{\partial q_{1t}}{\partial t} = h q_{1t}\left(u_{1t} - u_t\right) = h q_{1t}\left(1 - q_{1t}\right)\left(u_{1t} - u_{2t}\right)
\tag{9}
$$

where Eq. (7) is inserted and the process sensitivity $h > 0$ is interpreted analogously to $k_i > 0$ in Eq. (8).

### Analyzing the model

**The US 1659–2021**. Figure 2a, b plots the US M2 money supply $S_j$ (Federal Reserve, 2022) and the US inflation $\pi_i$ (CPI Inflation Calculator, 2022) from time $t_0 = 1959$ to time $T = 2021$. Figure 2c uses Eqs. (4), (5) and the empirics in Fig. 2a, b to plot player $i$'s utilities $u_{igt}$ and $u_{int}$ of transacting in both currencies, assuming support $s_{it} = 0.5$, equal volume fractions $p_{it} = 0.5$ of transactions in both currencies, equal fractions $q_{it} = 0.5$ of both kinds of players, scaling proportionality parameter $\mu_i = 0$, and Cobb

Douglas elasticities $\alpha_i = 0.6, 0.5, 0.35, 0.2$. Player $i$'s utility is constant at $u_{igt} = 0.25$ since currency $g$ has no changes in supply and no inflation. High and intermediate weights $\alpha_i = 0.6$ and $\alpha_i = 0.5$ for changes in money supply $S_j$ causes player i's utility $u_{int}$ to increase. Low weight $\alpha_i = 0.35$ causes $u_{int}$ to oscillate slightly above and below $u_{int} = 0.25$. Very low weight $\alpha_i = 0.2$ causes $u_{int}$ to decrease overall. Figure 2c uses Eqs. (6) and (7) to plot player $i$'s weighted utility $u_{iAt}$ of transacting in both currencies and society's utility $u_{At}$ weighing the utilities of all players of both kinds. These two utilities $u_{iAt} = u_{At}$ are equal since $p_{it} = q_{it} = 0.5$. Since $u_{igt} = 0.25$, the weighted utilities $u_{iAt} = u_{At}$ increase less for $\alpha_i = 0.6$ and $\alpha_i = 0.5$ and decrease less for $\alpha_i = 0.2$.

**Replicator dynamics with simplified utilities $u_{igst}$ and $u_{inst}$ in Eqs. (1) and (2).** Figure 3 applies the simplified utilities $u_{igst}$ and $u_{inst}$ in Eqs. (1), (2) and the replicator equation in Eq. (8) to plot player $i$'s fraction $p_{it}$ 1959–2021 with the same assumptions as in Fig. 2, i.e. $q_{it} = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. Player $i$'s process sensitivity and initial condition are $k_i = p_{it_0} = 0.5$. Figure 3a assumes the high weight $\alpha_i = 0.6$ for money supply $S_j$. With low support $s_{it} \leq 0.5$ for the fixed-supply currency $g$ relative to the variable-supply currency $n$, the fraction $p_{it}$ of transactions in currency $g$ decreases towards zero. With higher support $s_{it} = 0.6$, the fraction increases to a maximum $p_{it} = 0.59$ in 1972, and thereafter decreases towards $\lim_{t \to T} p_{it} \approx 0$. That eventual decrease occurs because of the high weight $\alpha_i = 0.6$ assigned to money supply $S_j$, which for the US 1959–2021 has meant preferable money printing, which is impossible for the fixed-supply currency $g$. With higher support $s_{it} = 0.7$, the fraction increases to a maximum $p_{it} = 0.84$ in 1990, and thereafter decreases. With very high support $s_{it} = 0.99$, the fraction increases towards $\lim_{t \to T} p_{it} \approx 1$. Hence sufficiently high support $s_{it}$ for currency $g$ can cause player $i$ to prefer it even with high weight assigned to money supply $S_j$. Figure 3b assumes the low weight $\alpha_i = 0.2$ for money supply $S_j$. High support $s_{it} \geq 0.6$ then causes the fraction $p_{it}$ to quickly increase towards $\lim_{t \to T} p_{it} \approx 1$. Intermediate support $s_{it} = 0.5$ causes the fraction $p_{it}$ to decrease marginally to $p_{it} = 0.498$ in 1968, and thereafter increase towards $\lim_{t \to T} p_{it} \approx 1$. Support $s_{it} = 0.4$ causes $p_{it}$ to decrease to $p_{it} = 0.32$ in 1979, and thereafter to increase. Support $s_{it} = 0.3$ causes $p_{it}$ to decrease to $p_{it} = 0.115$ in 2000, and thereafter to increase marginally to $p_{it} = 0.126$ in 2021. Negligible support $s_{it} = 0.01$ causes $p_{it}$ to decrease quickly to $\lim_{t \to T} p_{it} \approx 0$.

Figure 3c, d makes the same assumptions as Fig. 3a, b except that the process sensitivity is 10 times higher, i.e. $k_i = 5$. That causes $p_{it}$ to approach $\lim_{t \to T} p_{it} \approx 0$ more quickly when $s_{it} \leq 0.3$ and approach $\lim_{t \to T} p_{it} \approx 1$ more quickly when $s_{it} \geq 0.99$. In Fig. 3c where $\alpha_i = 0.6$, $p_{it}$ when $s_{it} = 0.6$ reaches a higher maximum $p_{it} = 0.59$ than in Fig. 3a, but in the same year 1972. Also in Fig. 3c, $p_{it}$ when $s_{it} = 0.7$ reaches a maximum extremely close to 1 (determined numerically as $p_{it} = 0.9999999314$), which is higher than in Fig. 3a, and in the same year 1990, and thereafter decreases towards $\lim_{t \to T} p_{it} \approx 0$. Similarly in Fig. 3d where $\alpha_i = 0.2$, $p_{it}$ when $s_{it} = 0.5$ reaches a lower minimum $p_{it} = 0.476$ than in Fig. 3b, and in the same year 1968, and thereafter increases towards $\lim_{t \to T} p_{it} \approx 1$. Also in Fig. 3d, $p_{it}$ when $s_{it} = 0.4$ reaches a minimum extremely close to 0 (determined numerically as $p_{it} = 0.000429$), which is lower than in Fig. 3b, and in the same year 1979, and thereafter increases towards $\lim_{t \to T} p_{it} \approx 1$.
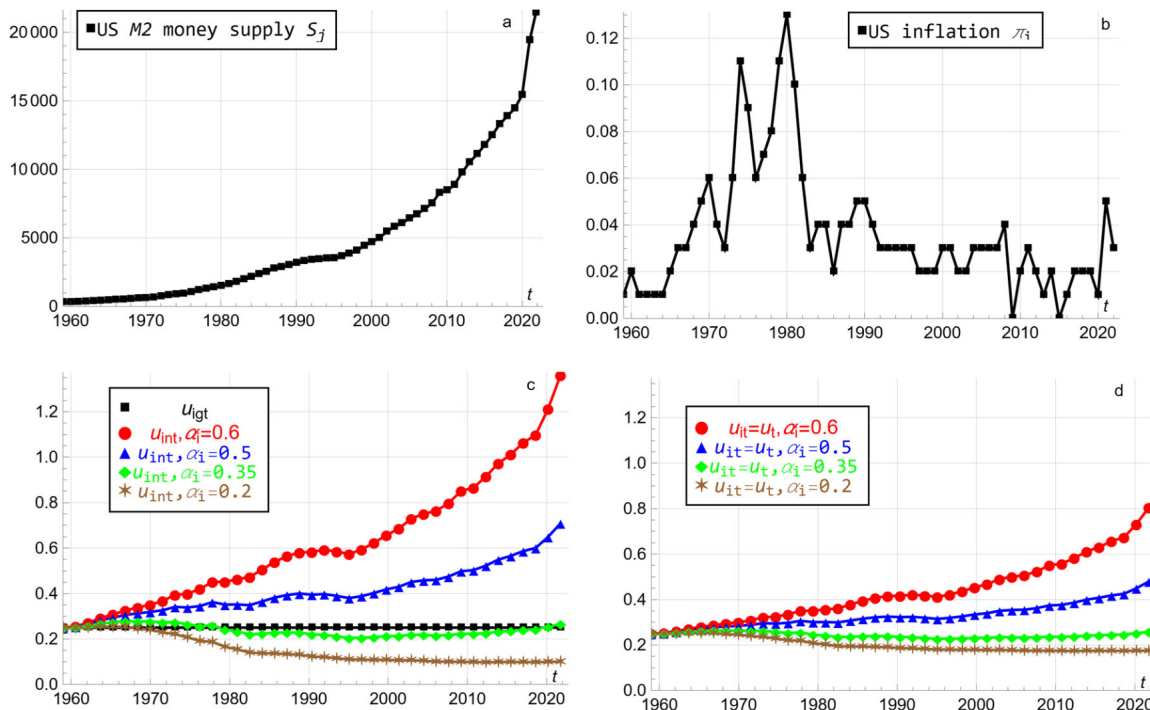
**Fig. 2 US M2 money supply, US inflation, player utilities and society's utility. a** US M2 money supply $S_j$ 1959–2021 in \$billion. **b** US inflation $\pi_i$ 1959–2021. **c** and **d** Player $i$'s utilities $u_{igt}$, $u_{int}$, $u_{it}$, $u_t$ as functions of time $t$ when $s_{it} = p_{it} = q_{it} = 0.5$, $\mu_i = 0$ and $\alpha_i = 0.6, 0.5, 0.35, 0.2$.
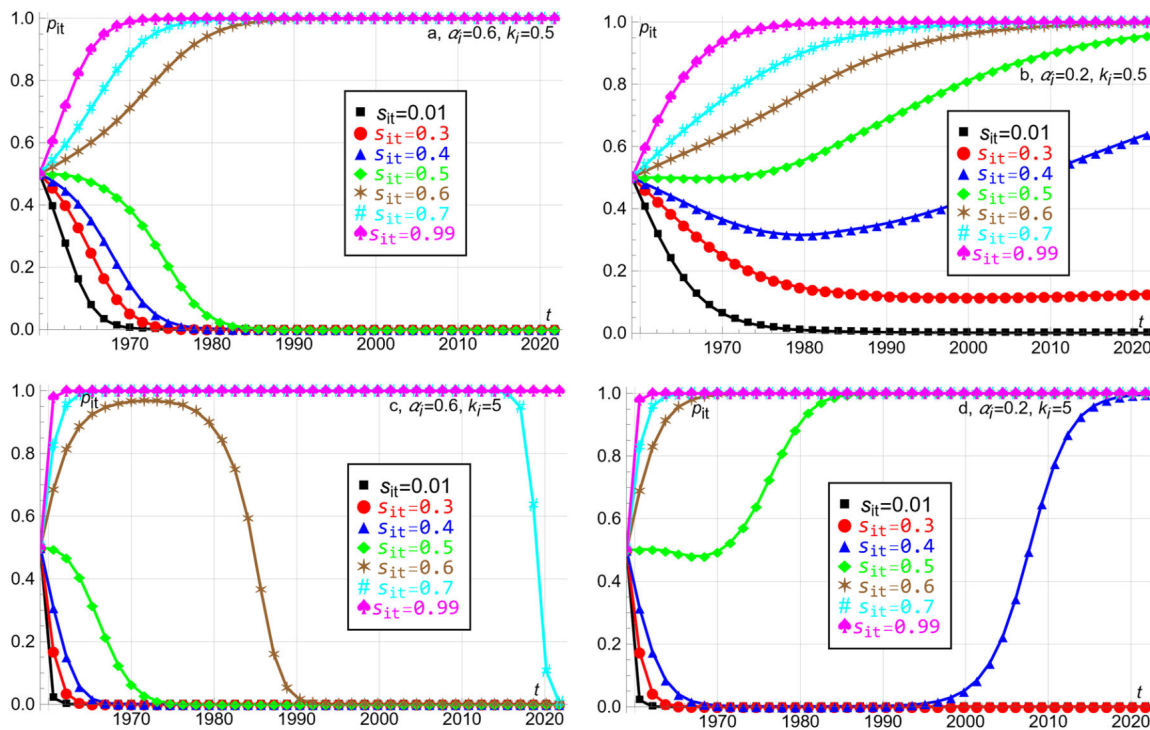


**Fig. 3 The volume fraction $p_{it}$ of player $i$'s transactions in currency $g$ at time $t$ 1959–2021 with simplified utilities $u_{igst}$ and $u_{inst}$ in Eqs. (1) and (2) when $p_{it_0} = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. a** $\alpha_i = 0.6$, $k = 0.5$, **b** $\alpha_i = 0.2$, $k = 0.5$, **c** $\alpha_i = 0.6$, $k = 5$ and **d** $\alpha_i = 0.2$, $k = 5$.

**Replicator dynamics with the utilities $u_{igt}$ and $u_{int}$ in Eqs. (4) and (5).** Figure 4 applies the utilities $u_{igt}$ and $u_{int}$ in Eqs. (4), (5), and Eq. (8) to plot $p_{it}$ with the same assumptions as in Fig. 3, i.e. $q_{it} = p_{it_0} = k_i = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. Accounting for $p_{it}$ in the utilities $u_{igt}$ and $u_{int}$ causes $p_{it}$ to approach $\lim_{t \to T} p_{it} \approx 0$ or $\lim_{t \to T} p_{it} \approx 1$ more quickly than in Fig. 3. With high weight

$\alpha_i = 0.6$ assigned to money supply $S_j$, two curves that approach $\lim_{t \to T} p_{it} \approx 0$ or eventually decrease favoring currency $n$ in Fig. 3a, approach $\lim_{t \to T} p_{it} \approx 1$ in Fig. 4a so that player $i$ prefers currency $g$ instead. First, with high support $s_{it} = 0.7$ for currency $g$, $p_{it} > 0.5$ until 2019 in Fig. 3a which positively impacts player $i$'s utility $u_{igt}$
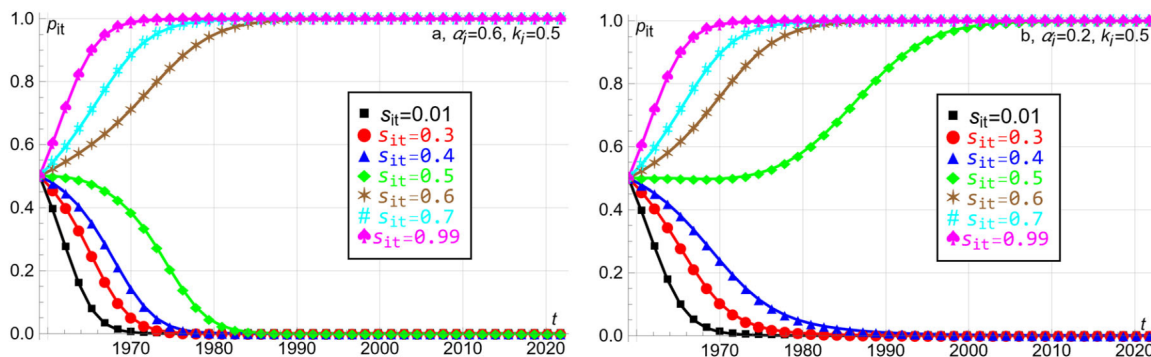
**Fig. 4 The volume fraction $p_{it}$ of player $i$'s transactions in currency $g$ at time $t$ 1959–2021 with the utilities $u_{igt}$ and $u_{int}$ in Eqs. (4) and (5) when $q_{it} = p_{it_0} = k_i = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. a $\alpha_i = 0.6$ and b $\alpha_i = 0.2$.**

causing player $i$ to favor currency g in Fig. 4a. Second, with slightly lower support $s_{it} = 0.6$ for currency g, $p_{it} > 0.5$ until 1985 in Fig. 3a which is sufficient for player $i$ to quickly favor currency g in Fig. 4a, contrary to Fig. 3a. With low weight $\alpha_i = 0.2$ assigned to money supply $S_j$, only one curve that eventually increases in Fig. 3b, with support $s_{it} = 0.4$, quickly decreases in Fig. 4b. That curve eventually increases in Fig. 3b since player $i$'s utility $u_{igt}$ does not depend on $p_{it}$. That enables player $i$ to favor currency g since low weight $\alpha_i = 0.2$ assigned to money supply $S_j$ causes player $i$ to prefer to avoid the inflation associated with currency n. The opposite result follow in Fig. 4b since $p_{it} < 0.5$ until 2008, causing $p_{it}$ to quickly decrease towards $\lim_{t \to T} p_{it} \approx 0$ where currency n is preferred.

**Replicator dynamics when players support currency g differently with $s_{1t} \neq s_{2t}$.** This section assumes that the two kinds of players support currency g differently with $s_{1t} \neq s_{2t}$. Figure 5 applies Eq. (8) to plot the volume fractions $p_{1t}$ and $p_{2t}$ of player $i$'s transactions, $i = 1, 2$, in currency g with the same assumptions as in Fig. 4, i.e. $q_{it} = p_{it_0} = k_i = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. Additionally, $s_{1t} \neq s_{2t}$. With high weight $\alpha_i = 0.6$ assigned to money supply $S_j$, negligible support $s_{1t} = 0.01$ by player 1 and more support $s_{2t} \leq 0.7$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n, though $p_{2t}$ initially experiences an inverse U shape. Although the high support $s_{1t} = s_{2t} = 0.7$ comfortably enables both players to eventually transact exclusively in currency g in Fig. 4a, $\lim_{t \to T} p_{2t} \approx 1$, the opposite result follows in Fig. 5a since player 1 supports currency g much less at $s_{1t} = 0.01$. Negligible support $s_{1t} = 0.01$ by player 1 and overwhelming support $s_{2t} = 0.99$ by player 2 cause opposite results for the two players, i.e. $\lim_{t \to T} p_{1t} \approx 0$ for player 1 and $\lim_{t \to T} p_{2t} \approx 1$ for player 2. Support $s_{1t} = 0.3$ by player 1 and more support $s_{2t} = 0.7$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n, though $p_{2t}$ initially experiences a higher inverse U shape than when $s_{1t} = 0.01$. Support $s_{1t} = 0.3$ by player 1 and overwhelming support $s_{2t} = 0.99$ by player 2 also cause opposite results for the two players, although player 1's volume fraction $p_{1t}$ approaches $\lim_{t \to T} p_{1t} \approx 0$ more slowly than when $s_{1t} = 0.01$, $\lim_{t \to T} p_{2t} \approx 1$. Support $s_{1t} = 0.4$ by player 1 and more support $s_{2t} = 0.7$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n, though $p_{2t}$ initially experiences a higher inverse U shape than when $s_{1t} = 0.3$. Support $s_{1t} = 0.4$ by player 1 and overwhelming support $s_{2t} = 0.99$ by player 2 interestingly cause both volume fractions to eventually approach

$\lim_{t \to T} p_{it} \approx 0$ favoring currency g. Although support $s_{1t} = s_{2t} = 0.4$ causes both players to eventually transact exclusively in currency n in Fig. 4a, $\lim_{t \to T} p_{2t} \approx 0$, the opposite result follows in Fig. 5b since player 2 supports currency g much more at $s_{1t} = 0.99$, which enables player 1 to also eventually support currency g. Support $s_{1t} = 0.5$ by player 1 and more support $s_{2t} = 0.6$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n. Both fractions approach $\lim_{t \to T} p_{it} \approx 0$ slowly, and $p_{2t}$ initially experiences an inverse U shape. Support $s_{1t} = 0.5$ by player 1 and more support $s_{2t} \geq 0.7$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 1$ favoring currency g. This interesting result shows that when $s_{1t} = 0.5$ for player 1, merely increasing player 2's support from $s_{2t} = 0.6$ to $s_{2t} = 0.7$ causes both players to eventually change their preferences from currency n to currency g.

With low weight $\alpha_i = 0.2$ assigned to money supply $S_j$, both players generally prefer currency g more easily. Negligible support $s_{1t} = 0.01$ by player 1 and more support $s_{2t} = 0.6$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n, though $p_{2t}$ in Fig. 5c initially experiences a lower inverse U shape than in Fig. 5a. Negligible support $s_{1t} = 0.01$ by player 1 and more support $s_{2t} \geq 0.7$ by player 2 cause opposite results for the two players, i.e. $\lim_{t \to T} p_{1t} \approx 0$ for player 1 and $\lim_{t \to T} p_{2t} \approx 1$ for player 2, so that player 2 eventually prefers currency g. This result in Fig. 5c differs from Fig. 5a when $s_{2t} = 0.7$ where $s_{2t} = 0.7$ causes both players to eventually prefer currency n. Support $s_{1t} = 0.3$ by player 1 and more support $s_{2t} = 0.5$ by player 2 cause both volume fractions to eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n. Support $s_{1t} = 0.3$ by player 1 and even more support $s_{2t} = 0.6$ by player 2 cause the fraction $p_{1t}$ for player 1 to decrease towards $\lim_{t \to T} p_{1t} \approx 0$, while the fraction $p_{2t}$ for player 2 increases overall extremely slowly towards $\lim_{t \to T} p_{2t} \approx 0.89$ in 2021, in major support of currency g. Support $s_{1t} = 0.3$ by player 1 and yet more support $s_{2t} = 0.7$ by player 2 cause player 2's fraction $p_{2t}$ to increase quickly towards $\lim_{t \to T} p_{2t} \approx 1$. Player 1's fraction $p_{1t}$ is U shaped towards a minimum, and thereafter increases slowly towards $\lim_{t \to T} p_{1t} \approx 0.30$ in 2021. Although player 1 supports currency g modestly at $s_{1t} = 0.3$, player 2's higher support $s_{2t} = 0.7$ causes player 1 to choose currency g to some modest extent. Support $s_{1t} = 0.3$ by player 1 and overwhelming support $s_{2t} = 0.99$ by player 2 cause player 2's fraction $p_{2t}$ to increase quickly towards $\lim_{t \to T} p_{2t} \approx 1$. Player 1's fraction $p_{1t}$ is first U shaped towards a minimum that is
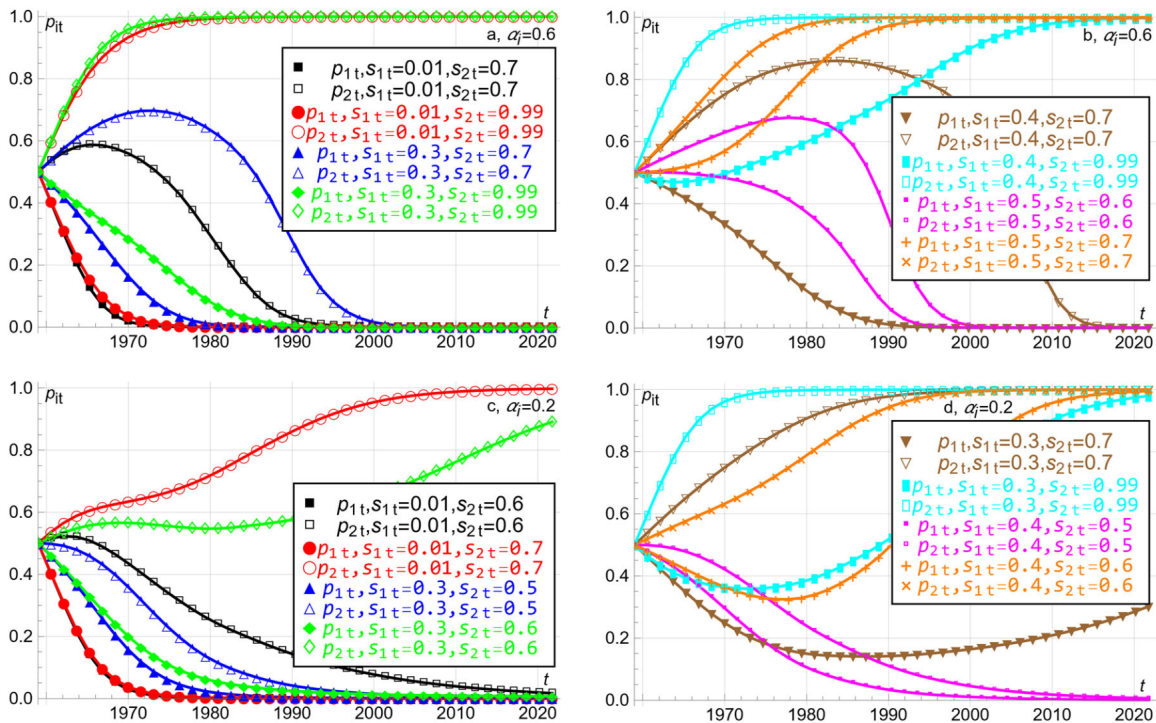
**Fig. 5 The volume fractions $p_{1t}$ and $p_{2t}$ of the two kinds of players' transactions in currency $g$ at time $t$ 1959–2021 with different support $s_{1t} \neq s_{2t}$ when $q_{it} = p_{it_0} = k_i = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. a and b** $\alpha_i = 0.6$. **c and d** $\alpha_i = 0.2$.

higher than when $s_{2t} = 0.7$, and thereafter increases logistically towards $\lim_{t \to T} p_{1t} \approx 0.98$. Despite low support $s_{1t} = 0.3$, player 1 eventually supports currency g substantially. Support $s_{1t} = 0.4$ by player 1 and more support $s_{2t} = 0.5$ by player 2 cause both volume fractions to slowly and eventually approach $\lim_{t \to T} p_{it} \approx 0$ favoring currency n. Support $s_{1t} = 0.4$ by player 1 and more support $s_{2t} \geq 0.6$ by player 2 cause player 2's fraction $p_{2t}$ to increase towards $\lim_{t \to T} p_{2t} \approx 1$, while player 1's fraction $p_{1t}$ is U shaped towards a minimum (when $s_{2t} = 0.6$) and thereafter increases towards $\lim_{t \to T} p_{1t} \approx 1$.

**Replicator dynamics when the fraction $q_{it}$ of players of kind $i$ changes.** This section assumes that the fraction $q_{it}$ of players of kind $i$ changes through time. Figure 6 applies Eqs. (8), (9) to plot the volume fractions $p_{1t}$ and $p_{2t}$ of player $i$'s transactions, $i = 1, 2$, in currency g and the fraction $q_{1t}$ of players of kind 1 with the same assumptions as in Fig. 5 except that $q_{1t}$ varies instead of $q_{it} = 0.5$, i.e. $p_{it_0} = k_i = 0.5$, $\mu_i = 0$, $0.01 \leq s_{it} \leq 0.99$, $s_{1t} \neq s_{2t}$. Additionally, we assume the process sensitivity $h = 0.5$ for the fraction $q_{1t}$ and initial condition $q_{1t_0} = 0.5$

With high weight $\alpha_i = 0.6$ assigned to money supply $S_j$, the first three combinations of curves in Fig. 5 with support $(s_{1t}, s_{2t})$ equal to $(0.01, 0.7)$, $(0.01, 0.99)$, $(0.3, 0.7)$ eventually implying $\lim_{t \to T} p_{1t} \approx 0$, cause the fraction $q_{1t}$ of players of kind 1 to increase towards 1. According to Eq. (9), the players prefer to be of kind 1 when $u_{1t} \geq u_{2t}$, i.e. when $p_{1t} u_{1gt} + (1 - p_{1t}) u_{1nt} \geq p_{2t} u_{2gt} + (1 - p_{2t}) u_{2nt}$ according to Eq. (6), which approaches $u_{1nt} \geq u_{2nt}$ when $\lim_{t \to T} p_{1t} \approx 0$. The three support combinations $(0.01, 0.7)$, $(0.01, 0.99)$, $(0.3, 0.7)$ satisfy $s_{1t} \leq s_{2t} \Leftrightarrow 1 - s_{1t} \geq 1 - s_{2t}$ which is inserted into Eq. (5) to give $u_{1nt} \geq u_{2nt}$ when $\lim_{t \to T} p_{1t} \approx 0$. Non-mathematically, players prefer to be of kind 1 since they prefer currency n which gives higher utility $u_{1nt} \geq u_{2nt}$ when $s_{1t} \leq s_{2t}$.

That is, the players converge towards transacting in currency $n$ compatibly with kind 1 supporting currency $n$ much more than currency g. With support $(s_{1t}, s_{2t}) = (0.3, 0.99)$, player 2's volume fraction $p_{2t}$ of transactions in currency g approaches $\lim_{t \to T} p_{2t} \approx 1$ in Fig. 5, and in Fig. 6 $\lim_{t \to T} p_{it} \approx 1$, which causes the opposite result where players prefer to be of kind 2. That is, $u_{1t} \leq u_{2t}$ implies $p_{1t} u_{1gt} + (1 - p_{1t}) u_{1nt} \leq p_{2t} u_{2gt} + (1 - p_{2t}) u_{2nt}$ approaches $u_{1gt} \leq u_{2gt}$ when $\lim_{t \to T} p_{it} \approx 1$. Support $(s_{1t}, s_{2t}) = (0.3, 0.99)$ means that $s_{1t} \leq s_{2t}$ which is inserted into Eq. (4) to give $u_{1gt} \leq u_{2gt}$ when $\lim_{t \to T} p_{it} \approx 1$. Non-mathematically, players prefer to be of kind 2 since they prefer currency g which gives higher utility $u_{2gt} \geq u_{1gt}$ when $s_{2t} \geq s_{1t}$. That is, the players converge towards transacting in currency g compatibly with kind 2 supporting currency g much more than currency n.

With this insight the interpretations of the subsequent panels in Fig. 6 is straightforward. That is, $\lim_{t \to T} p_{it} \approx 0$ so that players eventually prefer to transact in currency n implies that players prefer to be of kind 1 which gives higher utility $u_{1nt} \geq u_{2nt}$ when $s_{1t} \leq s_{2t}$. In contrast, $\lim_{t \to T} p_{it} \approx 1$ so that players eventually prefer to transact in currency g implies that players prefer to be of kind 2 which gives higher utility $u_{2gt} \geq u_{1gt}$ when $s_{2t} \geq s_{1t}$.

**Replicator dynamics with positive scaling proportionality parameter $\mu_i$.** This section assumes that the scaling proportionality parameter $\mu_i$ in player $i$'s utilities $u_{igt}$ and $u_{int}$ is positive. When $\mu_i$ increases, player $i$'s utilities $u_{igt}$ and $u_{int}$ in Eqs. (4) and (5) of transacting in both currencies g and n increase equally much. The increase is proportional to the fraction $q_{it}$ of players of kind $i$ at time $t$ raised to the parameter $m_i$. If both $\mu_1$ and $\mu_2$ increase equally much, both $u_{igt}$ and $u_{int}$ increase which in the replicator Eq. (8) can be interpreted as increasing the process sensitivity $k_i$, which means quicker changes which are otherwise qualitatively similar to Fig. 6. Figure 7 makes the same
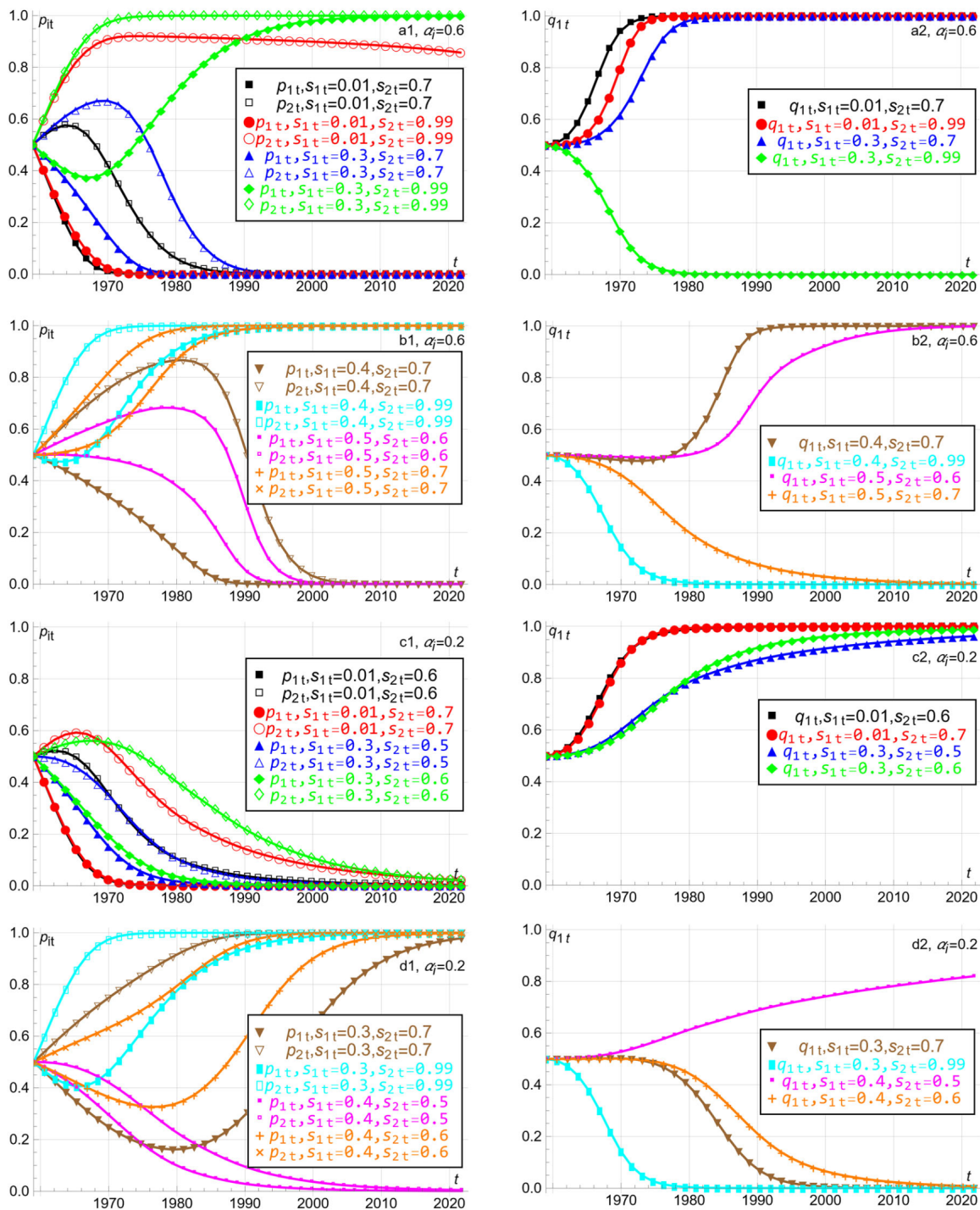
**Fig. 6 The fractions $p_{1t}$, $p_{2t}$, $q_{1t}$ at time $t$ 1959–2021 with different support $s_{1t} \neq s_{2t}$ when $q_{it_0} = p_{it_0} = k_i = h = 0.5$, $\mu_i = 0$, and $0.01 \leq s_{it} \leq 0.99$. a1, a2, b1, b2 $\alpha_i = 0.6$. c1, c2, d1, d2 $\alpha_i = 0.2$.**

assumptions as in Fig. 6 except that $\mu_2 = 1$ and $\mu_1 = 0$, i.e. $q_{1t_0} = p_{it_0} = k_i = h = 0.5$, $\mu_i = 0$, $0.01 \leq s_{it} \leq 0.99$, $s_{1t} \neq s_{2t}$. The higher $\mu_2 = 1 > \mu_1 = 0$ means that players to a higher extent than in Fig. 6 tend to prefer to be of kind 2 which gives higher utilities $u_{2gt}$ and $u_{2nt}$. Hence Fig. 7 shows three, four, two, four curves (summing to 13 curves out of 16 possible curves) for the fraction $q_{1t}$ of players of kind 1 at time $t$ approaching $\lim_{t \to T} q_{1t} \approx 0$, as compared with one, two, zero, three curves (summing to only six curves), respectively, approaching $\lim_{t \to T} q_{1t} \approx 0$ in Fig. 6. In Fig.

7a1 the low support $s_{1t} = 0.01$ of player 1 for currency $g$ causes both players to eventually not transact in currency $g$ when $s_{2t} = 0.7$, as explained for Fig. 6, which implies that players prefer to be of kind 1 since they prefer currency $n$ which gives higher utility $u_{1nt} \geq u_{2nt}$ when $s_{1t} \leq s_{2t}$. The corresponding curve $q_{1t}$ in Fig. 7a2 gives $\lim_{t \to T} q_{1t} \approx 1$, while the other three curves with higher support $s_{1t} + s_{2t}$ give $\lim_{t \to T} q_{1t} \approx 0$ so that the players prefer to be of kind 2. Fig. 7b1, b2 with higher support $s_{1t} + s_{2t}$ shows a clearer trend where $\lim_{t \to T} p_{it} \approx 0$ and $\lim_{t \to T} q_{1t} \approx 0$ so that players
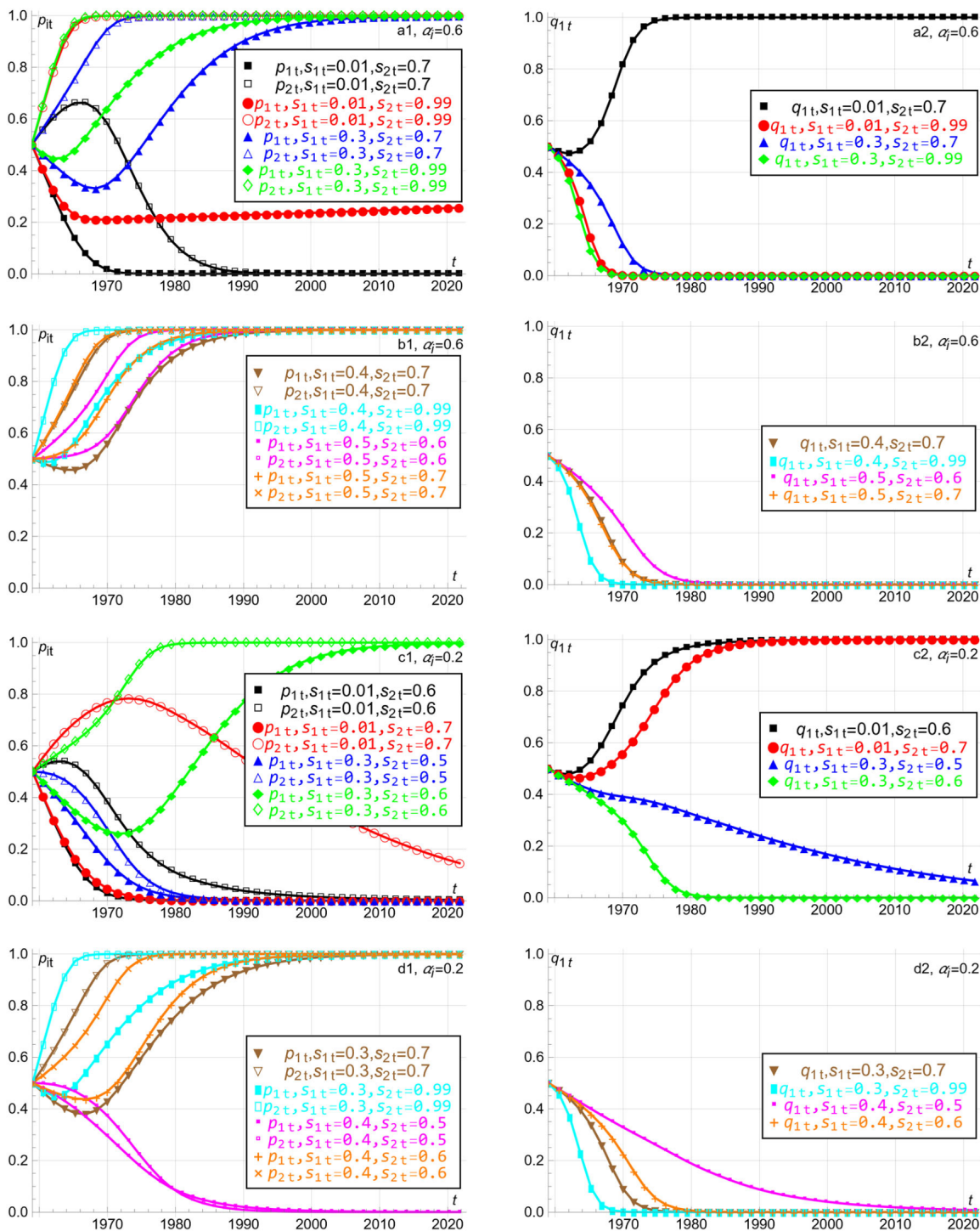
**Fig. 7 The fractions $p_{1t}$, $p_{2t}$, $q_{1t}$ at time $t$ 1959–2021 with different support $s_{1t} \neq s_{2t}$ when $q_{it_0} = p_{it_0} = k_i = h = 0.5$, $\mu_2 = 1$, $\mu_1 = 0$, and $0.01 \leq s_{it} \leq 0.99$. a1, a2, b1, b2 $\alpha_i = 0.6$. c1, c2, d1, d2 $\alpha_i = 0.2$.**

prefer to be of kind 2. Figure 7c2 shows two curves, with support $\left(s_{1t}, s_{2t}\right)$ equal to $(0.3, 0.5)$, $(0.3, 0.6)$, eventually approaching $\lim\limits_{t \to T} q_{1t} \approx 0$ so that players prefer to be of kind 2, in contrast to Fig. 6c2 which has no such curves. Figure 7d2 shows how all the four curves eventually approach $\lim\limits_{t \to T} q_{1t} \approx 0$ so that players prefer to be of kind 2. Figure 7d2 also shows how it is possible for both players to eventually prefer no transactions in currency g, $\lim\limits_{t \to T} p_{it} \approx 1$, while at the same time the fraction $q_{1t}$ of players of kind 1 slowly decreases.

## Discussion and future research

New currencies, especially these in digital format, may induce more currency competition. The competition may become especially fierce between fixed-supply and variable-supply currencies. Fixed-supply currencies rigidly avoids inflation/deflation which would otherwise be induced by altering the money supply. Variable-supply currencies allow more flexibility by allowing money printing during critical events (e.g. wars and recession), but requires fiscal discipline thereafter to avoid inflation.

To understand the competition, a player is assumed to earn a utility depending on its support of and volume of transactions in a given currency, and the fraction of players of the same kind as itself. A player may be any individual or collective unit. Essential in the article is how a player values money printing/withdrawal on the one hand versus inflation/deflation on the other hand. A time delay usually exists from the former to the latter. Batini (2006), Batini and Nelson (2001) and Friedman and Schwartz (1982) suggest that it takes over one year from money printing until inflation. Hence temptation may exist to increase money supply in the short run and postpone worrying about the subsequent inflation. The 1959–2021 US money supply and inflation data suggest that money printing and inflation indeed occur.

With high weight assigned to money supply relative to inflation, this article finds that players are more inclined to prefer the variable-supply currency. They thereby benefit from the temporarily increased purchasing power enabled by the increased money supply. Such players may not have excessively large time horizons, since then they might value the future negative consequences of inflation. This assumes that the player itself indeed can access the increased money supply. In contrast, low weight assigned to money supply relative to inflation induces players to be more inclined to prefer the fixed-supply currency, to avoid the negative impact of inflation.

When two kinds of players support two currencies differently, the players' fractions of transactions in the two currencies may exhibit substantial variation, e.g. be inverse U shaped or U shaped before converging towards preferring one or the other currency. This relates to earlier studies of how players choose between multiple currencies, see e.g. Schilling and Uhlig (2019), Fernández-Villaverde and Sanches (2019), Almosova (2018), Benigno et al. (2019). For example, assume high weight assigned to money supply, and that one player supports the fixed-supply currency much less than the other player. The first player may quickly abandon the fixed-supply currency which fails to offer additional money supply. The second player may initially support the fixed-supply currency increasingly, but may thereafter be influenced by the first player and also abandon the fixed-supply currency, thus potentially being negatively impacted by inflation. In contrast, assume low weight assigned to money supply, and that one player supports the fixed-supply currency much more than the other player. The first player may prefer the fixed-supply currency which provides a hedge against inflation. The second player may initially support the variable-supply currency increasingly, but may thereafter be influenced by the first player and also prefer the fixed-supply currency, thus potentially not benefitting from the increased money supply. The two currencies may obtain different market shares, as also analyzed ElBahrawy et al. (2017) and Imhof and Nowak (2006). These results indicate how countries or societies through various evolutionary dynamics may transform themselves into using one or another currency, or a combination of several currencies, potentially for different purposes. This in turn may impact a country's financial markets, monetary policy, and interaction with other countries.

We next allow players to choose which kind of player they can be. That can be realistic when a player prefers to transact in currencies that many other players transact in, thus being less influenced by how the player individually supports each currency independently of the other players. The analysis shows that players may choose to be of a kind supporting a given currency if that support is much higher than the other kind's support of the same currency. The first kind of player may thus become more common, while the second kind player becomes less common.

We finally enable a player's utility of transacting in a given currency to be proportional to the fraction of players of the same kind as the given player. Thus players not only choose what kind of player they want to be, but they may receive higher utility for being

of one kind rather than of another kind, regardless of the players' support for each currency and their volume fractions of transactions in each currency. When the proportional impact of being a certain kind of player increases equally for both kinds of players, the players' fractions of transactions in each currency change more quickly, as if the process sensitivity in the replicator equation increases. When the proportional impact increases more for one kind of player, players increasingly prefer to be of that kind.

Future research, which implicitly indicates limitations of the current article, may extend the analysis to more features than money supply and inflation. More than two currencies and more than two kinds of players may be analyzed. Each kind of player's utility may depend on further features related to each currency's backing, convenience, confidentiality, transaction efficiency, financial stability, and security. Players may be assumed to apply different currencies for different purposes. Different kinds of players gaining different access to increased money supply, or suffering differently from money contraction, may be analyzed. Alternative player risk attitudes and time preferences may be evaluated. Empirics from other world regions may be incorporated. Additional players may be analyzed, e.g. players in different countries accessing different currencies, private versus public players, governmental agencies imposing regulation and taxation, and currency competition between countries.

## Conclusion

This article builds a model of two kinds of players who can choose between two currencies, i.e. a fixed-supply currency (e.g. Bitcoin) and a variable-supply currency (e.g. a fiat currency or a central bank digital currency). A player may be any individual or collective unit. A variable-supply currency enables money printing or money withdrawal, and may be associated with inflation or deflation. Comparing fixed-supply and variable-supply currencies has become relevant due to new currencies emerging which incorporate supply, ownership, decentralization, regulation, confirmation of transactions, geographical extension, etc. differently.

A player's utility of transacting in a given currency is proportional to three factors, i.e. the player's support of that currency, the volume fraction of all players' (of both kinds) transactions in that currency, and the fraction of players of the same kind as the given player. A currency's support depends on its financial stability, transaction efficiency, backing, convenience, confidentiality, and security. Additionally, a player's utility of transacting in the variable-supply currency is proportional to a Cobb Douglas utility of two factors. The first factor is the initial money supply plus the accumulative money printing (positive) and money withdrawal (negative) in the numerator, divided by the initial money supply in the denominator. The second factor is the inverse of the accumulative inflation (positive) and deflation (negative when measured as a percentage). If the output elasticity for the first ratio is high, money printing/withdrawal is highly valued relative to inflation/deflation, and conversely if the output elasticity for the second ratio is high.

The players' utility of transacting in the variable-supply currency is illustrated for various output elasticities for 1959–2021. The exponentially increasing US M2 money supply and the positive inflation cause this utility to increase over time with high output elasticity, and decrease with low output elasticity. Such changing utilities over time constitute policy tools for how to adjust money supply/withdrawal and inflation/deflation.

Three replicator equations are developed based on the players' utilities. Two of these model each kind of player's volume fractions of transactions in each currency over time. The third models the evolution of the fraction of each kind of player over time, i.e. how players choose to be of one or the other kind.

High weight assigned to money supply relative to inflation causes players to more likely prefer the variable-supply currency, to gain from the increased money supply, and conversely prefer the fixed-supply currency given low weight assigned to money supply. When the two kinds of players support the two currencies differently, the players' fractions of transactions in the two currencies may be inverse U shaped or U shaped before converging towards preferring one or the other currency. When players can choose which kind of player to be, players may choose to be of a kind supporting a given currency if that support is especially high. When a player's utility of transacting in a given currency is proportional to the fraction of players of the same kind as the given player, and the proportional impact is higher for one kind of player, players tend to prefer to be of that kind.

## Data availability
The article contains no associated data. All data generated or analyzed during this study are included in this published article.

## References
Almosova A (2018) A note on cryptocurrencies and currency competition. International Research Training Group 1792 Discussion Paper No. 2018-006, Technical University Berlin, Berlin.

Asimakopoulos S, Lorusso M, Ravazzolo F (2019) A new economic framework: a DSGE model with cryptocurrency. Centre for Applied Macro- and Petroleum Economics Working Paper No. 07/2019. BI Norwegian Business School, Oslo.

Basov V (2022) Top 10 largest gold producing countries in 2021—report. https://www.kitco.com/news/2022-01-31/Top-10-largest-gold-producing-countries-in-2021-report.html

Batini N (2006) Euro area inflation persistence. Empir. Econ. 31(4): 977–1002. https://doi.org/10.1007/s00181-006-0064-7

Batini N, Nelson E (2001) The Lag from monetary policy actions to inflation: Friedman revisited. Int Finance 4(3):381–400. https://doi.org/10.1111/1468-2362.00079

Benigno P (2021) Monetary policy in a world of cryptocurrencies. Centre for Economic Policy Research Discussion Paper No. DP13517, Luiss Guido Carli University, Roma.

Benigno P, Schilling LM, Uhlig H (2019) Cryptocurrencies, currency competition, and the impossible trinity. National Bureau of Economic Research Working Paper Series No. w26214. National Bureau of Economic Research, Cambridge.

Blakstad S, Allen R (2018) Central Bank digital currencies and cryptocurrencies. FinTech Revolution 87–112. https://doi.org/10.1007/978-3-319-76014-8_5

Caginalp C, Caginalp G (2019) Establishing cryptocurrency equilibria through game theory. AIMS Math 4(3):420–436. https://doi.org/10.3934/math.2019.3.420

Caporale GM, Gil-Alana L, Plastun A (2018) Persistence in the cryptocurrency market. Res Int Bus Finance 46:141–148. https://doi.org/10.1016/j.ribaf.2018.01.002

CPI Inflation Calculator (2022) CPI inflation calculator. https://www.officialdata.org/us/inflation/1850

ElBahrawy A, Alessandretti L, Baronchelli A (2019) Wikipedia and cryptocurrencies: interplay between collective attention and market performance. Front Blockchain 2:12. https://doi.org/10.3389/fbloc.2019.00012

ElBahrawy A, Alessandretti L, Kandler A, Pastor-Satorras R, Baronchelli A (2017) Evolutionary dynamics of the cryptocurrency market. R Soc Open Sci 4(11):170623. https://doi.org/10.1098/rsos.170623

Federal Reserve (2022) Money stock measures—H.6 release. https://www.federalreserve.gov/releases/h6/current/default.htm

Fernández-Villaverde J, Sanches D (2019) Can currency competition work? J Monet Econ 106:1–15. https://doi.org/10.1016/j.jmoneco.2019.07.003

Friedman M, Schwartz AJ (1982) Interrelations between the United States and the United Kingdom 1873–1975. J Int Money Finance 13–19. https://doi.org/10.1016/0261-5606(82)90002-X

gold.org. (2022) How much gold has been mined? https://www.gold.org/about-gold/gold-supply/gold-mining/how-much-gold

Gandal N, Halaburda H (2016) Can we predict the winner in a market with network effects? Competition in cryptocurrency market. Games 7(3):16. https://doi.org/10.3390/g7030016

Hayes A (2022) What happens to Bitcoin after all 21 million are mined? https://www.investopedia.com/tech/what-happens-bitcoin-after-21-million-mined/

Ikkurty S (2019) Fiat, gold, and bitcoin comparison. https://medium.com/@samikkurty/fiat-gold-and-bitcoin-comparison-e878fa2292bc

Imhof LA, Nowak MA (2006) Evolutionary game dynamics in a Wright–Fisher process. J Bank Reg 52(5):667–681

Kusimba C (2017) When—and why—did people first start using money? The conversation. https://theconversation.com/when-and-why-did-people-first-start-using-money-78887

Learn B (2021) Bitcoin vs. gold: which is a better store of value? https://learn.bybit.com/investing/bitcoin-vs-gold-store-of-value/

Lewenberg Y, Bachrach Y, Sompolinsky Y, Zohar A, Rosenschein JS (2015) Bitcoin mining pools: a cooperative game theoretic analysis. Paper presented at the proceedings of the 2015 international conference on autonomous agents and multiagent systems, Istanbul, Turkey.

Masciandaro D (2018) Central Bank digital cash and cryptocurrencies: insights from a new Baumol–Friedman demand for money. Aust Econ Rev 51(4):540–550. https://doi.org/10.1111/1467-8462.12304

Milunovich G (2018) Cryptocurrencies, mainstream asset classes and risk factors: a study of connectedness. Aust Econ Rev 51(4):551–563. https://doi.org/10.1111/1467-8462.12303

Mitchell C (2021) Gold: the other currency. https://www.investopedia.com/articles/forex/10/gold-the-other-currency.asp

Nakamoto S (2008) Bitcoin: a peer-to-peer electronic cash system. https://bitcoin.org/bitcoin.pdf

Rahman AJ (2018) Deflationary policy under digital and fiat currency competition. Res Econ 72(2):171–180. https://doi.org/10.1016/j.rie.2018.04.004

Sapkota N, Grobys K (2021) Asset market equilibria in cryptocurrency markets: evidence from a study of privacy and non-privacy coins. J Int Financial Mark Inst Money 74:101402. https://doi.org/10.2139/ssrn.3407300.

Schilling LM, Uhlig H (2019) Currency substitution under transaction costs. AEA Pap Proc 109:83–87. https://doi.org/10.1257/pandp.20191017

Taylor PD, Jonker LB (1978) Evolutionary stable strategies and game dynamics. Math Biosci 40(1):145–156. https://doi.org/10.1016/0025-5564(78)90077-9

Verdier M (2021) Digital currencies and bank competition. Université Panthéon-Assas Paris 2, Paris, Manuscript. https://doi.org/10.2139/ssrn.3673958.

Weibull JW (1997) Evolutionary game theory. MIT Press, Cambridge, MA.

White LH (2014) The market for cryptocurrencies. Cato J 35(2):383–402

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Does not apply.

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Does not apply.

## Additional information
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