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ABSTRACT

The focus in this master's thesis was to describe and compare the two long-term analysis methods: All Sea States and Peak-Over-Threshold. Both methods are equitable and commonly approved methods to estimate extreme waves. Hence, the motivation of this study was to compare and discuss similarities and differences between the extreme waves estimated by these two long-term analysis methods. The analysis aimed for the estimation of the extreme wave crest heights and wave heights. The extreme waves within this study are defined as waves corresponding to an annual exceedance probability of, respectively, $q = 10^{-2}$ and $q = 10^{-4}$.

In this thesis the metocean contour line method is also introduced. This method is useful to establish preliminary estimates of the extreme waves during initial phases of design. Then at a later stage, estimates from the metocean contour line method can typically be verified by a long-term analysis using one of the two methods stated above.

A general conclusion of this thesis is that the extreme wave crest heights obtained with the Peak-Over-Threshold are less conservative than those estimated using the All Sea States approach. The practical implication is a more optimized design of offshore structures without compromising the safety aspect.

KEY WORDS: Long term analysis, Metocean modelling, Wave characteristics, All Sea States, Peak-Over-Threshold, Metocean contour lines

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SYMBOLS

α_F	Parameter Forristall distribution
β_F	Parameter Forristall distribution
η	Weibull distribution form parameter
σ	Weibull distribution scale parameter
μ	Weibull distribution location parameter
ξ	Wave crest height realization
Ξ	Wave crest height random variable
Φ	Cumulative distribution function for a standard normal random variable
d	Water depth
$g(\quad)$	Limit state function
H_s	Significant wave height
$F_\nu(\nu)$	Cumulative distribution function for ration ν
$F_{X_{3h}}(x)$	Cumulative distribution function for the 3-hours extreme value
$F_{\Xi_{3h}}(\xi)$	Cumulative distribution function for the 3-hours extreme wave crest height
$F_{\tilde{\Xi}}(\tilde{\xi})$	Cumulative distribution function for the most probable storm maximum
$f_{H_sT_p}(h,t)$	Joint probability density function for simultaneous combination of H_s and T_p
$F_{H_s}(h)$	Cumulative distribution function of Significant wave height H_s
$f_{T_p\|H_s}(t\|h)$	Conditional probability density function of the spectral peak period T_p given H_s
n_{3h}	Expected number of global maxima in 3 hours
n_T	Expected number of global maxima in T years
p_f	Probability of failure
T_1	Mean wave period
T_p	Spectral peak period
$\overline{\nu_0^+}$	Long term average zero-up-crossing frequency
σ	Standard deviation

σ^2	Variance
v	Ratio storm maximum realization and most probable storm maximum
ω	Angular frequency

ABBREVIATIONS

ALS	Accidental Limit State
NCS	Norwegian Continental Shelf
POT	Peak-Over-Threshold
ULS	Ultimate Limit State

1 INTRODUCTION

1.1 BACKGROUND

The design of offshore structures and vessels includes among others two very important aspects: safety, and cost-efficient, optimized design. The safety aspect in this context is related to underestimated design loads. If a design has underestimated design loads, then the likelihood of a fatal event will most probably be high, and can in its most severe form lead to collapse of structure. On the other hand, using very conservative values will lead to high cost and poorly optimized construction. In other words, there needs to be a trade-off between safety regulations and costs options, to ensure a cost-efficient and maybe most importantly, safe structure. So, which loads are important to take into consideration?

The loads acting on the offshore structures are usually loads from wind, current and waves acting simultaneously(Haver and Nyhus,1986). However, the major loads are generally those caused by the waves and hence an accurate prediction of the wave-induced loads is of major importance for a safe and cost-efficient design (Haver and Nyhus,1986). The focus in this work will be on the wind-generated ocean waves and the stochastic methods for predicting the *extreme waves*. The extreme waves in this context are waves with an *annual exceedance probability* of, respectively, $q = 10^{-2}$ and $q = 10^{-4}$. Hence, a properly designed offshore structure must be able to endure the extreme waves without or with a limited damage.

In order to meet the safety conditions and locally set demands, there have been developed acceptance criteria. For the Norwegian Continental Shelf (NCS) and according to the requirements in the Norwegian rules and regulations (NORSOK 2007 & 2008) (Haver,2013), the offshore structure must sustain overload failures at two levels: Ultimate Limit State (ULS) and Accidental Limit State (ALS). The ULS criterion is set to ensure that the structure resists, without damage, a load induced by a wave with $q = 10^{-2}$ probability. Similarly, the ALS criterion is set to ensure that the structure survives, with minor damages accepted, a load induced by a wave with $q = 10^{-4}$ probability. So, how can we ensure that a design fulfils the acceptance criteria and safety demands? To ensure this, we need reliable estimates for the extreme waves corresponding to the ULS and ALS annual exceedance probabilities. In order to do so, a stochastic long-term evaluation of the wave conditions is necessary. The long-term evaluation is herein referred to as long-term analysis of the wave characteristics. The focus of this work is the study and comparison of two long-term analysis methods: All Sea States and Peak-Over-Threshold. Furthermore, the focus will be on the estimation of the q-probability wave heights and q-probability wave crest heights. Before introducing the two long-term analysis methods, it is worth considering some basic concepts and assumptions of the ocean wave random process.

The wave profile in a given sea state is irregular in time and space, and for that reason we resort to the use of stochastic methods for analysis of wave random process(Ochi,1998). Furthermore, since the ocean climate is continuously changing, it implies that the wave processes are non-stationary over a long time span, even on the scale of hours(Næss,2012). One of the assumptions in this work is to approximate the long-term periods of the wave conditions as a sequence of stationary short-term periods(Haver,2013). The short-term periods are herein defined as *sea states*. The term stationary in this context means that the statistical properties (e.g. mean and standard deviation) of the process are independent of time(Ochi,1998), (Næss,2012). For the Norwegian Continental Shelf (NCS), we typically refer to 3- or 6-hours stationary short-term period. In this document, the sea state duration is defined as 3-hours during which we assume stationary wave conditions characterized by H_s and T_p (Haver and Nyhus,1986). The significant wave height H_s and the corresponding spectral peak period T_p are in many practical applications the primary quantities of interest (Haver,2013). This will be the assumption throughout this study.

Now, let us get back to the long-term analysis methods introduced above: All Sea State and Peak-Over-Threshold. Both methods take into account the sources of inherent randomness: the short-term variability of the extreme value in a given sea state and the long-term variability of the sea state characteristics (Haver,2008). The result within the context of this study is the long-term distribution of the extreme waves.

For the NCS, the most common long-term analysis is the All Sea State approach. The aim with this method is to establish the long-term distribution of the largest wave during an arbitrary 3-hours sea state. The long-term evaluation consists of determining the joint probability density function for all possible sea states (i.e. all possible combinations of H_s and T_p)(Haver,2014). The short-term evaluation yields the probability of exceedance of certain value (e.g. wave crest height) within the stationary sea states. The All Sea State is most convenient in case the number of slowly varying metocean characteristics (e.g. H_s, T_p) is limited to 2-3 (Haver,2015). However, for problems that are more complex and with more parameters involved, this method becomes inconvenient(Haver,2015).

The storm base approach ("Peak-Over-Threshold") applies very much to the hurricane dominated areas. The basic idea is to establish the long-term distribution of the largest wave during an arbitrary storm(Haver,2004). This method includes the sea states corresponding to storms exceeding a chosen threshold. The long-term evaluation consists of determining the distribution of the most probable storm maximum value. The short-term evaluation consists of determining the conditional distribution of the largest wave in an arbitrary storm, given the most probable storm maximum. The POT method is well suited for cases where more than 2-3 metocean characteristics should be included(Haver,2015).

In this thesis the metocean contour line method is also introduced. The metocean contour line method permits to estimate the long-term extremes (i.e. extremes corresponding to a given annual probability of exceedance) using selected short-term sea states. In this study, the application of this method will be limited to establishing the contour lines based on the metocean modelling. In addition, the percentile levels suggested in NORSOK (N-003 2015) will be verified using the metocean contour lines and the estimates for extreme waves obtained from the All Sea State approach. Hence, the intention is rather for the author to get acquainted with the relevance of the metocean contour lines and their relation to the long-term analysis methods presented above. It should also be noted that the estimation of extreme waves using the metocean contour line method and as stipulated in NORSOK (N-003 2015) is not part of this study.

The database used for the analyses in this thesis is the hindcast data for the Heidrun field (NCS). As per NORSOK (N-003 2015), the duration of data sampling of simultaneous metocean characteristics should be sufficiently long to capture the characteristic action levels corresponding to long return periods we are aiming for. We will assume that this condition is fulfilled.

1.2 OBJECTIVES AND SCOPE

The main objective of this thesis is the study and comparison of two long-term analysis methods: All Sea States and Peak-Over-Threshold. As already introduced in section 1.1, we consider the extreme waves i.e. waves having annual probabilities of exceedance of, respectively, $q = 10^{-2}$ and $q = 10^{-4}$. The primary focus will be on the on extreme values of the wave crest height ξ and wave height H.

The All Sea States long-term analysis of 3-hours extremes using the Gaussian sea surface assumption was performed during the project work autumn 2014 (Pekovic,2014). Before proceeding to further analysis within the scope of this thesis, we will recapitulate the methodology with assumptions and recall the results of the metocean modelling and long-term 3-hours extremes determined by the All Sea States approach.

As part of this thesis, the above introduced All Sea States analysis will be extended under the assumption of a second order sea surface process. Firstly, the second model for long-crested waves (2D) will be used to estimate extreme waves and compared to the estimates obtained with the Gaussian. Following that, the effect of wave directionality on the extreme wave crest heights will be evaluated using the second order model for the short-crested waves (3D).

Furthermore, an alternative approach within the All Sea States method will be presented. Namely, the extreme waves will be estimated by considering all the individual global maxima (wave crest height) instead of the largest global maxima within each 3-hours sequence of stationary sea state. The purpose is to study the effect on the estimated extreme crest heights.

The chapter related to the All Sea States long-term analysis will be rounded by application of the metocean contour line method, with the aim to demonstrate the method and verify the percentile recommendations stipulated in NORSOK (N-003 2015).

The Peak-Over-Threshold methodology will be presented and the application demonstrated using the Heidrun hindcast data. The results from the Peak-Over-Threshold analysis will be compared to those obtained with the All Sea States approach, followed by a discussion on findings.

1.3 THESIS OUTLINE

built up on a theoretical part and methodology presentation followed by the practical application, result presentation and discussion. The Matlab scripts and additional figures are attached to the Appendix section.

Chapter 2 includes review of metocean modelling and the results performed during project work

Chapter 3 includes the All Sea States approach, with review and comparison of the results.

Chapter 4 contains the Peak-Over-Threshold (POT) method with application and review of results. The comparison between the estimates obtained with the All Sea States and POT are presented, followed by a discussion.

Finally, Chapter 5 contains a summary with the main conclusion including discussions presented throughout the respective sections of the document, and suggestions for further work.

2 METOCEAN MODELLING

The focus in this chapter will be the review of the long-term modelling of the sea states. This analysis was performed within the project for the subject OFF600 Marine operations during autumn 2014(Pekovic,2014). In section 2.1, the main assumptions related to the stochastic wave process will be reviewed. The short-term and long-term modelling of the sea states characteristics will be introduced in, respectively, sections 2.2 and 2.3.

2.1 MAIN ASSUMPTIONS

The basic assumptions for the stochastic wave process will be briefly recalled in the following lines:

1. The sea surface of an irregular sea at a fixed location is, for short time periods, a stationary stochastic wave process $\Xi(t)$. Under this assumption, the mean and the variance of the process $\Xi(t)$ are constant within the given time period (Haver,2013).
2. A stochastic process can be defined as: The quantity $X(t)$ is called a stochastic process if $X(t)$ is a random variable for each value of t in an interval $[a, b]$. (Næss,2012)
3. For each short term sea state, assumed to be stationary, the wave process $\Xi(t)$ is ergodic (Haver,2013). By definition, a stochastic process is ergodic if the expected values (means) of all ensembles can be replaced by a time average over a single realization. The term “ensemble” in this context can be described as an infinite number of thought-constructed (or laboratory) sample time histories (Næss,2012)
4. For longer periods, the wave process $\Xi(t)$ is a sequence of the stationary (short time periods) processes with no transition periods between the sea states(Næss,2012). The reasoning behind this assumption is that the weather characteristics (e.g. H_s and T_p) change much slower in comparison with the wave process $\Xi(t)$ (Haver,2013).
5. The wave process $\Xi(t)$ is assumed to be Gaussian, meaning that the Gaussian (normal) density function describes the probability density function of the surface elevation at an arbitrary point in time (Haver, 2013). Furthermore, it means statistical symmetry i.e. the expected value (mean) of the vertical elevation of the free sea surface over the sea level is equal to zero with variance σ^2 (Borge 2014). The

probability density function of the surface elevation at an arbitrary point in time reads (Haver,2013):

$$f_{\Xi}(\xi) = \frac{1}{\sigma_{\Xi}\sqrt{2\pi}} \cdot exp\left\{-\frac{1}{2} \cdot \left(\frac{\xi}{\sigma_{\Xi}}\right)^2\right\} \tag{1}$$

Where σ_{Ξ} is the standard deviation of the surface process.

It should be noted that the Gaussian assumption for the surface elevation process is sufficiently accurate for a number of practical cases. However, for many other situations it needs to be taken into account that the real sea surface process deviates somewhat from the Gaussian assumption(Haver,2013). In that case, one should appeal to a second or higher order surface model.

In the next sections, we will briefly review the short term description of the sea surface elevation (wave process $\Xi(t)$) and thereafter the long term description.

2.2 SHORT TERM MODELLING OF SEA STATES

The main assumptions and definitions related to the short term modelling will be repeated in this section, prior to presenting the long term modelling of sea states in section 2.3.

The stochastic wave process $\Xi(t)$ is considered stationary within duration of 3 hours (DNV-RP-C205,2007), and characterized by the significant wave height H_s and the spectral peak period T_p. The most recent definition of the significant wave height H_s is as per equation (3).

The spectral peak period T_p is the wave period determined by the inverse of the frequency at which a wave energy spectrum has its maximum value(DNV-RP-C205,2007).

The wave spectrum describes the distribution of wave energy as a function of wave frequency in short term wave conditions. The area under the spectrum is equal to the variance of waves(Ochi,1998). The Pierson-Moskowitz (PM) and JONSWAP spectrum are frequently applied for wind seas, describing conditions that often occur for the most severe sea states(DNV-RP-C205,2007). Another important definition is the spectral moments m_n of general order (DNV-RP-C205,2007):

$$m_n = \int_0^\infty f^n \cdot S(f) df \tag{2}$$

Where $f\ [Hz]$ is the frequency, $S(f)$ the wave spectrum and $n = 0,1,2..$

The significant wave height H_s is defined from the wave spectrum by (DNV-RP-C205,2007):

$$H_s = 4\sqrt{m_0} \tag{3}$$

Under the assumption of a stationary Gaussian stochastic process, the wave spectrum $S_{\Xi\Xi}(\omega; h, t)$ describes fully the wave process $\Xi(t)$ (Haver,2013). In other words, the short term sea state is characterized in statistical sense by H_s and T_p (Haver,2014). Furthermore, the variance $\sigma_\Xi^2(h,t)$ and the expected zero-up-crossing frequency, $v_{0,\Xi}^+(h,t)$ for a given stationary sea state characterized by H_s and T_p (Haver,2013) are defined as:

$$\sigma_\Xi^2(h,t) = m_\Xi^{(0)}(h,t)\ \ [Hz] \tag{4}$$

$$v_{0,\Xi}^+(h,t) = \sqrt{\frac{m_\Xi^{(2)}(h,t)}{m_\Xi^{(0)}(h,t)}} \tag{5}$$

Finally, by combining equations (3) and (4) the following relation is obtained:

$$H_s = 4\sigma_\Xi \tag{6}$$

2.3 LONG TERM MODELLING OF SEA STATES

The long term variation of the wave conditions (sea states) can be described by the joint density function of the parameters describing a sea state, significant wave height H_s and the spectral peak period T_p (Haver,2013):

$$f_{H_sT_p}(h,t) = f_{H_s}(h) f_{T_p|H_s}(t|h) \tag{7}$$

To be able to create the joint probabilistic model, we need to create a joint frequency table (scatter diagram) for the significant wave height H_s and spectral peak period T_p. We will for the purpose of this study use the corrected hindcast data for the Heidrun field (NCS). The Matlab script (see Appendix A.1) used to create the scatter diagram permits to sort the significant wave height H_s and the spectral peak period T_p into classes with intervals of $\Delta h_s = 0.5m$ and $\Delta t_p = 1s$, creating a 50x25 matrix. The resulting scatter diagram shown in Figure 1 describes the frequency of occurrence of the various of sea states (i.e. combinations of H_s and T_p).

		0< Tp ≤1																										
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	Sum	Cumulative
0< Hs ≤0,5	0,5	0	0	0	5	71	94	73	109	93	79	39	14	13	2	3	1	0	0	0	0	0	0	0	0	0	596	596
	1	0	0	3	147	972	1835	3063	2930	2395	1720	888	439	255	100	52	38	27	15	12	5	2	0	0	0	0	14898	15494
	1,5	0	0	0	16	990	2879	3774	5380	6049	4171	2673	1766	865	440	229	147	87	39	29	10	11	3	3	0	0	29561	45055
	2	0	0	0	0	177	1815	3367	3637	4966	4893	3508	2511	1688	833	427	254	137	92	33	18	18	10	4	0	0	28388	73443
	2,5	0	0	0	0	9	383	2146	2828	3241	3785	3478	2483	1763	1093	592	337	149	72	48	19	18	3	4	0	0	22451	95894
	3	0	0	0	0	0	61	849	2138	2496	2783	2934	2540	1772	1036	637	427	186	94	69	13	6	3	5	0	0	18049	113943
	3,5	0	0	0	0	0	4	211	1079	1991	2150	2291	2141	1634	968	641	425	188	91	65	12	13	6	3	0	0	13913	127856
	4	0	0	0	0	0	0	46	321	1359	1840	1875	1628	1425	849	550	353	208	104	64	12	10	0	2	0	0	10646	138502
	4,5	0	0	0	0	0	0	3	71	649	1184	1477	1392	1052	762	507	314	152	80	72	7	8	0	0	0	0	7730	146232
	5	0	0	0	0	0	0	0	14	234	733	1168	1158	822	580	415	259	118	73	47	5	2	0	1	0	0	5629	151861
	5,5	0	0	0	0	0	0	0	1	62	353	801	1012	760	405	280	187	89	49	41	1	4	1	0	0	0	4046	155907
	6	0	0	0	0	0	0	0	0	23	137	430	746	670	378	251	161	89	42	28	4	1	1	1	0	0	2962	158869
	6,5	0	0	0	0	0	0	0	0	6	48	206	462	574	323	194	126	73	47	23	2	0	0	0	0	0	2084	160953
	7	0	0	0	0	0	0	0	0	0	9	104	307	493	302	137	87	41	37	23	1	2	0	0	0	0	1543	162496
	7,5	0	0	0	0	0	0	0	0	1	2	32	136	363	296	148	74	41	31	19	0	0	0	0	0	0	1143	163639
	8	0	0	0	0	0	0	0	0	0	2	10	67	182	258	142	51	17	21	22	0	1	0	0	0	0	773	164412
	8,5	0	0	0	0	0	0	0	0	0	1	2	27	91	235	108	53	15	14	12	0	1	0	0	0	0	559	164971
	9	0	0	0	0	0	0	0	0	0	0	0	6	39	145	88	61	13	10	6	1	0	0	0	0	0	369	165340
	9,5	0	0	0	0	0	0	0	0	0	0	0	1	15	83	69	45	17	11	8	0	2	0	0	0	0	251	165591
	10	0	0	0	0	0	0	0	0	0	0	0	0	7	42	60	44	17	4	8	0	0	0	0	0	0	182	165773
	10,5	0	0	0	0	0	0	0	0	0	0	0	0	4	19	37	27	14	7	6	0	0	0	0	0	0	114	165887
	11	0	0	0	0	0	0	0	0	0	0	0	0	0	5	20	17	7	10	3	0	0	0	0	0	0	62	165949
	11,5	0	0	0	0	0	0	0	0	0	0	0	0	0	4	18	4	4	5	3	0	0	0	0	0	0	38	165987
	12	0	0	0	0	0	0	0	0	0	0	0	0	0	0	11	8	3	2	1	0	0	0	0	0	0	25	166012
	12,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	6	2	2	1	0	0	0	0	0	0	14	166026
	13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	4	3	1	0	0	0	0	0	0	0	8	166034
	13,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	2	2	1	0	0	0	0	0	0	0	6	166040
	14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	2	0	0	0	0	0	0	4	166044
	14,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	3	1	0	0	0	0	0	0	4	166048
	15	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	2	166050
	15,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166050
	16	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166050
	16,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0	0	2	166052
	17	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	1	166053
	17,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	18	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	18,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	19	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	19,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	20	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	20,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	21	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	21,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	22,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	23	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	23,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	24	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	24,5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
	25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	166053
																											166053	
		0	0	3	168	2219	7071	13532	18508	23565	23890	21916	18836	14487	9158	5620	3512	1701	960	648	110	99	27	23	0	0	166053	

Figure 1 Scatter diagram (Heidrun hindcast 1957-2014)

The marginal probability density function $f_{H_s}(h)$ and the conditional probability density function $f_{T_p|H_s}(t|h)$ are fitted to the hindcast data. The joint probabilistic model fitted to the hindcast data is obtained by combining these two density functions as per equation (7) (Haver,2014).

The significant wave height H_s, considered as the most important characteristic in design applications, can be modelled by the marginal distribution $F_{H_s}(h)$ and using a 3-parameter Weibull distribution (Haver,2014):

$$F_{H_s}(h) = 1 - exp\left\{-\left(\frac{h-\mu}{\sigma}\right)^{\eta}\right\} \tag{8}$$

The method of moment was used to fit the probability distribution to data and estimate the distribution parameters μ, σ and η from the first three statistical moments: mean, variance and skew. The estimated parameters are presented in Table 1 below.

Form parameter η	1,2219
Scale parameter σ	2,0743
Location parameter μ	0,7203

Table 1 Weibull distribution parameters, $F_{H_s}(h)$

The adequacy of the fitted model has been verified by plotting both the sample distribution (hindcast data) and the fitted distribution in a Weibull probability paper, as shown in Figure 2.

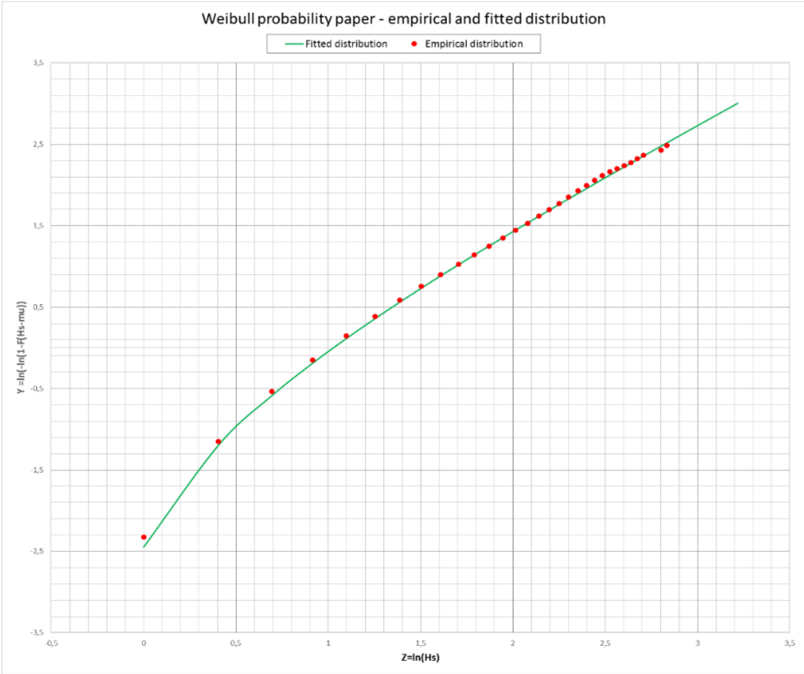

Figure 2 Weibull (3-p) fitted model vs. empirical data

The fitted model corresponds well to the data above the threshold set by the location parameter, i.e. $\mu = 0.7202$.

The conditional distribution of the spectral peak period T_p given a significant wave height H_s can, based on experience, be modelled with a reasonable degree of accuracy by the log-Normal distribution (Haver,2013). The conditional probability density function of the spectral peak period T_p is:

$$f_{T_p|H_s}(t|h) = \frac{1}{\sigma_{ln_{T_p(h)}} \cdot t \cdot \sqrt{2\pi}} \cdot exp\left\{-\frac{1}{2}\left(\frac{lnt - \mu_{ln_{Tp}(h)}}{\sigma_{ln_{T_p(h)}}}\right)^2\right\} \quad (9)$$

In order to define the conditional probability density function $f_{T_p|H_s}(t|h)$, the equation (9) was fitted to the data of each row in the scatter diagram i.e. for each class of H_s (Haver,2014). By doing this, the fitted curves for, respectively, functions of $\mu_{ln_{Tp}(h)}$ and ${\sigma^2}_{ln_{T_p(h)}}$ vs. H_s read:

$$\mu_{ln_{Tp}} = a_1 + a_2 \cdot {h_s}^{a_3} \quad (10)$$

$$\sigma^2_{ln_{tp}} = b_1 + b_2 \cdot exp\{-h_s \cdot b_3\} \quad (11)$$

The coefficients of the function $\mu_{lnT_p(h)}$, calculated by using the least square method, are shown in Table 2.

Mean		
Variables		Sum squared differences
a_1	1,4487	0,014943218
a_2	0,6095	
a_3	0,3101	

Table 2 Parameters for $\mu_{lnT_p(h)}$

The Figure 3 shows the function $\mu_{lnT_p(h)}$ fitted to the hindcast data.

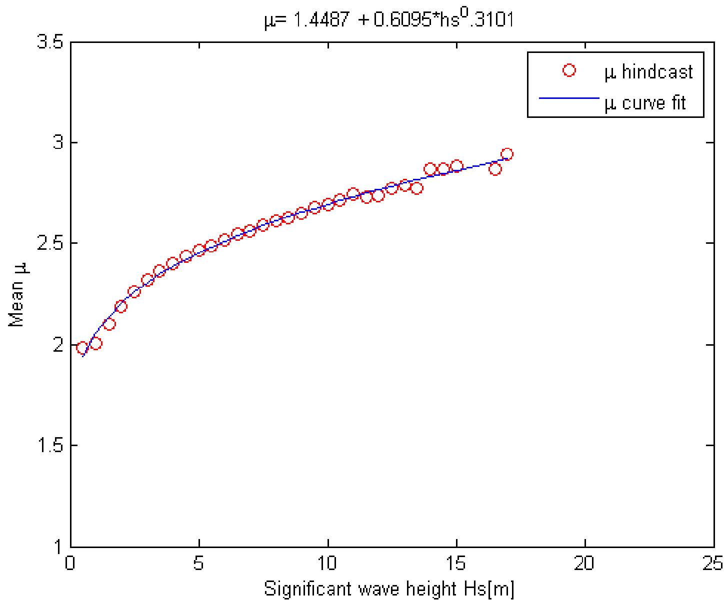

Figure 3 Fitted function mean lnTp, $\mu_{lnT_p(h)}$

The parameters of the function ${\sigma^2}_{lnT_p(h)}$ are shown in Table 3 and Table 4. Two cases are presented, one with coefficient $b_1 = 0.005$ (fixed value) and the second with coefficient b_1 obtained by iteration. The reason and effect for evaluating these two cases will be discussed later on in this section.

Variance (b1=0.005)		
Variables		Sum squared differences
b_1	0,0050	0,0005
b_2	0,0961	
b_3	0,2866	

Table 3 Parameters for ${\sigma^2}_{lnT_p(h)}(b_1 = 0.005)$

Variance (b1 varying parameter, LSM)		
Variables		Sum squared differences
b_1	0,0000	0,0002
b_2	0,0973	
b_3	0,2420	

Table 4 Parameters for ${\sigma^2}_{ln_{T_p(h)}}(b_1 = 0)$

The Figure 4 shows the function ${\sigma^2}_{ln_{T_p(h)}}$ fitted to the hindcast data.

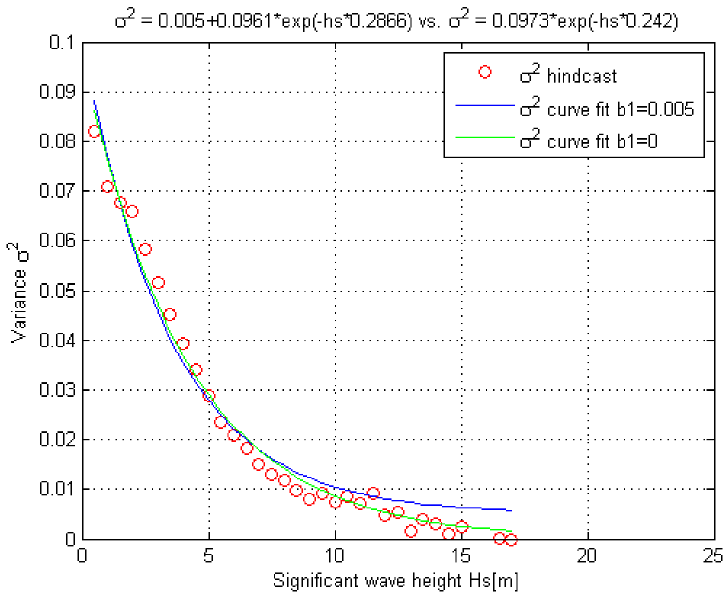

Figure 4 Fitted function variance ${\sigma^2}_{ln_{T_p(h)}}$

With all the elements in place i.e. the marginal distribution of H_s and the conditional distribution of the spectral peak period T_p, the joint distribution of sea state characteristics can be determined by using the equation (7). The resulting tables with joint probabilities for both $b_1 = 0.005$ and $b_1 = 0$ are presented in Appendix A.2 and Appendix A.3.

In addition to results presented above and as part of the work performed within this thesis, it is of interest to introduce the methodology for calculating the conditional mean spectral peak period T_p and the conditional 90% band. The conditional mean spectral peak period T_p and the conditional 90% band are calculated using the fitted functions of $\mu_{ln_{Tp}(h)}$ and ${\sigma^2}_{ln_{T_p(h)}}$(equations (10) and (11)). The mean (expected value) for a nonnegative random variable (in this case T_p given H_s) having a lognormal distribution is(Devore and Berk,2007):

$$E(T_p|H_s) = \exp(\mu_{ln_{T_p}} + \tfrac{1}{2}\sigma^2_{ln_{T_p}}) \quad (12)$$

It has been shown that ln_{T_p} has a normal distribution(Pekovic,2014) and the cumulative distribution function of the spectral peak period T_p can be expressed as:

$$F\left(T_p;\ \mu_{ln_{T_p}}, \sigma_{ln_{T_p}}\right) = \Phi\left(\frac{ln_{T_p} - \mu_{ln_{T_p}}}{\sigma_{ln_{T_p}}}\right) \tag{13}$$

In order to define the 95th percentile of the T_p distribution, we wish to determine the value of T_p for which:

$$0.95 = P\left[1.645 \leq \frac{ln_{T_p} - \mu_{ln_{T_p}}}{\sigma_{ln_{T_p}}}\right] \rightarrow \mathrm{T_p} = \exp(\mu_{ln_{T_p}} + 1.645\sigma_{ln_{T_p}}) \tag{14}$$

The 5th percentile of the spectral peak period T_p distribution can be obtained in similar manner as explained above. The calculation was performed for, respectively, $b_1 = 0.005$ and $b_1 = 0$. The corrected hindcast data, 5th, mean and 95th percentile curves for both values of b_1 are presented in Figure 5 and Figure 6.

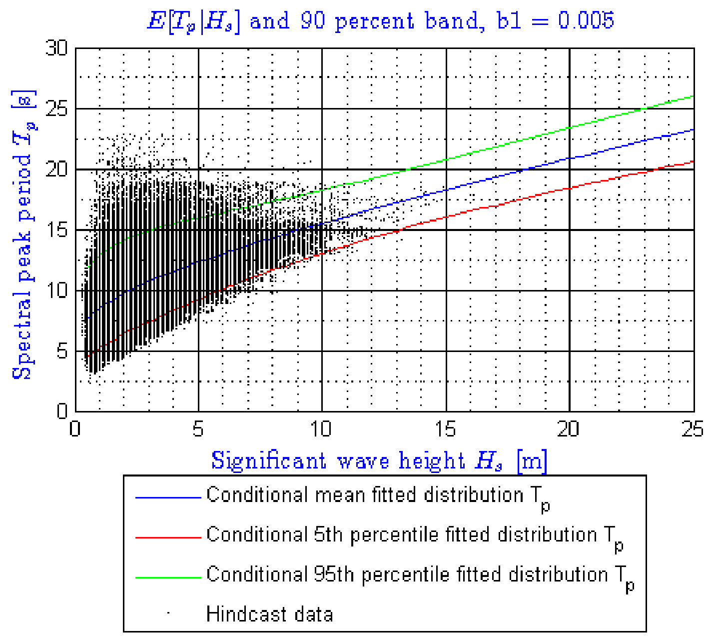

Figure 5 Conditional mean and 90% band of T_p given H_s ($b_1 = 0.005$)

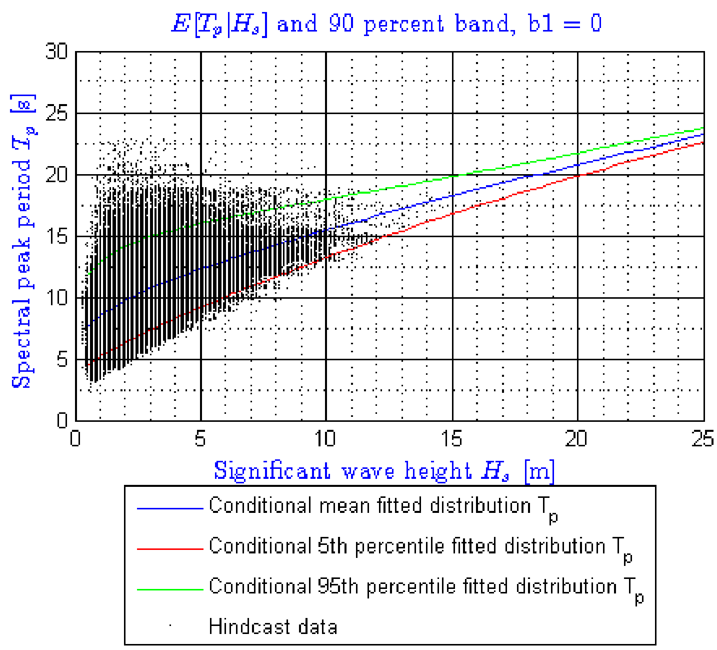

Figure 6 Conditional mean and 90% band of T_p given H_s ($b_1 = 0.005$)

2.3.1 Discussion

It has been demonstrated how the short term description of the surface elevation process can be extended to a long term statistical description of the surface elevation process. In chapter 3, it will be shown how the long term description can be used in for estimating the extreme waves.

The marginal distribution of the significant wave height H_s was fitted to the hindcast data using the 3-parameter Weibull distribution. Based on evaluation of the probability plot on Figure 2, the 3-parameter Weibull distribution seems to be an adequate model in this case.

The conditional distribution of the spectral peak period T_p was fitted to the hindcast data using the log-normal distribution. There are two functions of $\sigma^2{}_{ln_{T_p(h)}}$ shown in Figure 4, depending on the coefficient b_1:

- $b_1 = 0.005$ while the parameters b_2 and b_3 are fitted to data by the least square method, approach used by Statoil (Haver,2012). In that sense, the coefficient b_1 represents a lower bound for the conditional variance of T_p given H_s for very large significant wave height H_s.(Haver,2012)

- Fit b_1 in the same manner as the parameters b_2 and b_3 i.e. by the least square method. This methodology can be considered as the correct one in the mathematical sense but it doesn't set the lower bound (as stated above).

From Figure 4 presented above, it seems that the value of $b_1 = 0$ may be a better choice than $b_1 = 0.005$ from a mathematical point. However, the optimal choice of b_1 should be evaluated more into depth as suggested in (Haver,2012) and will not be elaborated on in this thesis.

Furthermore, it can be expected that the value of b_1 has no major influence on the long term distribution of extremes. This consideration is based on comparison of the joint distributions presented in Appendix A.2 and Appendix A.3. The differences in the probabilities for the respective sea states are small. In addition, as it will be shown in chapter 3, the conditional short term distribution of the 3-hour maxima is not a function of b_1. Hence, the only influence on the long term distribution of extremes comes from the joint distribution of sea states where as mentioned, the differences are relatively small. Following this, the calculations of the long-term distribution for the extreme waves will not be performed for $b_1 = 0$. Consequently, only the value $b_1 = 0.005$ will used throughout the rest of this document.

However, for the calculation of the conditional mean spectral peak period T_p (with the 90% band) and metocean contour lines, b_1 has a significant influence. From Figure 5 and Figure 6, it can be seen that the coefficient b_1 influences the width of the 90% band for increasing H_s values. The second term of equation (11) decreases with increasing H_s. For $b_1 = 0.005$, the 90% band is approximately constant and governed by the fixed value of b_1. For $b_1 = 0$, the 90% band get narrower with increasing H_s values. In other words, a higher value of b_1 leads to a higher confidence band and hence more conservative approach since more severe sea states are involved.

The effect of coefficient b_1 on the environmental contour lines will be discussed in section 3.3.

3 LONG TERM ANALYSIS: ALL SEA STATES

The results for the long term analysis of 3-hours extremes for Gaussian sea surfaces will be presented in section 3.1. The work will then be extended to the long term analysis of the 3-hours extremes assuming second order sea surface. In section 3.2, the long term distribution considering all the individual global maxima will be introduced followed by a discussion on the results. Finally, in section 3.3, the metocean contour line method and application will be presented.

3.1 LONG TERM DISTRIBUTION - 3-HOURS LARGEST GLOBAL MAXIMA

Complementing the assumptions introduced in section 2.1 and prior to presenting the methodology and results of the long term analysis, we will briefly recall the following:

1. Given the assumption of Gaussian distribution for the surface elevation and in addition assuming a narrow-banded wave spectrum, the Rayleigh distribution is a good model for the distribution of the global maxima (largest extreme value between adjacent zero up-crossings) (Haver,2013).

$$F_{X_{\Xi}}(x;h,t) = 1 - exp\left\{-\frac{1}{2}\left(\frac{x}{\sigma_{\Xi}(h,t)}\right)^2\right\} \tag{15}$$

   The amplitude and length (period) of subsequent cycles of a narrow banded process vary slowly, implying that there is only one peak value between an up-crossing and subsequent down-crossing at any given level (Næss,2012).

2. The wave directionality is at first not taken into account i.e. this assumption is related to analyses in chapter 3. Hence, the assumption is that the waves propagate in the same direction and are long-crested (Haver and Nyhus,1986).

The long term distribution of the 3-hours extreme value X_{3h} is given by the following expression (Haver,2014)

$$F_{X_{3h}}(x) = \int_h \int_t F_{X_{3h}|H_sT_p}(x;h,t) f_{H_sT_p}(h,t)dhdt \tag{16}$$

In other words, the long term distribution of the 3-hours extreme value X_{3h} is obtained as a weighted sum of the short term distributions, where the weights are the probabilities of occurrence for the short term sea states (Haver,1980).

The joint density function $f_{H_sT_p}(h,t)$ of the significant wave height H_s and the spectral peak period T_p has been presented in chapter 2. The expression for the short term (conditional) distribution of the 3-hours extreme value will depend on the wave characteristic of interest and will be presented accordingly in the next sections.

Finally, the 3-hours extreme corresponding to a q-annual probability is determined by (Haver,2014):

$$1 - F_{\mathrm{X}_{3h}}(x_q) = \frac{q}{2920} \tag{17}$$

3.1.1 Wave crest height – Gaussian sea surface

Assuming that crest heights are statistically independent and identically distributed, the distribution function for the largest maximum Ξ_{3h} within 3-hours stationary sea state is (Haver,2014):

$$F_{\Xi_{3\mathrm{h}}}(\xi) = \left[1 - exp\left\{-\frac{1}{2}\left(\frac{\xi}{\sigma_{\Xi}}\right)^2\right\}\right]^{n_{3h}} \tag{18}$$

Introducing equation (6) into equation (18), the expression of the short-term (conditional) distribution for the maximum wave crest height, $F_{\Xi_{3h}|H_sT_p}(\xi;h,t)$, given H_s and T_p becomes(Haver,2014):

$$F_{\Xi_{3h}|H_sT_p}(\xi;h,t) = \left[1 - exp\left\{-8\cdot\left(\frac{\xi}{h_{\mathrm{s}}}\right)^2\right\}\right]^{n_{3h}} \tag{19}$$

The expected number of global maxima in 3-hours is defined by (Haver,2014) (Haver,2013). :

$$n_{3h} = \frac{10800}{T_2} \tag{20}$$

Where T_2 is the mean wave period, defined as (Faltinsen,1990):

$$T_2 = \sqrt{\frac{m_0}{m_2}}\ [s] \tag{21}$$

For a Pierson-Moskowitz and JONSWAP spectrum we have, respectively, the following relations(Faltinsen,1990):

$$T_2 = 0.7102 \cdot T_\mathrm{p} \tag{22}$$

$$1.0372 \cdot T_2 = 0.834 \cdot T_\mathrm{p} \Rightarrow T_2 = 0.7773 \cdot T_\mathrm{p} \tag{23}$$

The assumption for this study is a JONSWAP spectrum and hence, the following approximation is adopted in the analysis:

$$T_2 = 0.78 \cdot T_\mathrm{p} \tag{24}$$

The equation (20) becomes then:

$$n_{3h} \cong \frac{10800}{0.78 \cdot T_p} = \frac{13846}{T_p} \tag{25}$$

The expression for the extreme of crest height corresponding to a q-annual probability reads(Haver,2014):

$$1 - F_{\Xi_{3h}}(\xi_q) = \frac{q}{2920} \tag{26}$$

The long term distribution of the 3-hours maximum crest height Ξ_{3h} is obtained by applying equation (16). The result is shown in Figure 7.

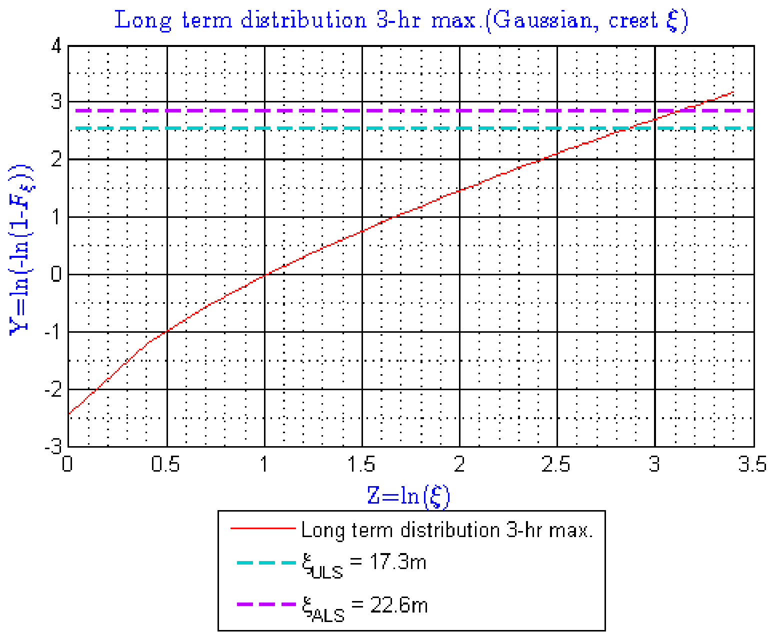

Figure 7 Probability plot for long term distribution of 3-hr extreme crest height ξ

The methodology steps for estimating the extremes applied in (Pekovic,2014) have been implemented in Matlab. Although the main steps are identical to those already used, the procedure will be repeated for the sake of completeness by presenting an example. Starting with the equation (17), the calculation of the 10^{-2}(ULS) extreme crest height ξ_q is as following:

$$1 - F_{\Xi_{3h}}(\xi_{0.01}) = \frac{10^{-2}}{2920} = 3{,}425 * 10^{-6} \Rightarrow Y = \ln\left(-\ln\left(\frac{10^{-2}}{2920}\right)\right) = 2{,}532 \qquad (27)$$

The extreme crest height is obtained by using the ordinate value Y and finding the intersection with the long term distribution on Figure 7. The estimate for the extreme is:

$$Z = ln(\xi) = 2{,}8443 \Rightarrow \xi_q = \exp(Z) = 17{,}3m \qquad (28)$$

The estimates for the extreme crest heights with probability of exceedance of 10^{-2}(ULS) and 10^{-4} (ALS) are summarized in Table 5:

ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
ξ_{ULS} [m]	ξ_{ALS} [m]
17,3	22,6

Table 5 Extreme crest height ξ_q

The Matlab script is presented in Appendix B.1.

3.1.2 Significant wave height – Gaussian sea surface

The estimates for the extreme significant wave height H_s can be calculated using equations (8), (26) and the parameters from Table 1:

$$1 - F_{H_s}\left(h_{s_q}\right) = \frac{q}{2920} \Rightarrow h_{s_q} = \mu + \sigma * \left[-\ln\left(\frac{q}{2920}\right)\right]^{1/\eta} \tag{29}$$

The spectral peak period T_p corresponding to the extreme H_s is found by applying equations (10), (11) and (12). The results for the extreme H_s and corresponding T_p are shown in Table 6.

			b_1=0.005	b_1=0
	$(\frac{q}{2920})^{-1}$	H_{s_q} [m]	T_{p_q} [s]	T_{p_q} [s]
ULS	292000	17,20	18,62	18,58
ALS	29200000	21,99	20,92	20,87

Table 6 Extreme significant wave height H_s with T_p

3.1.3 Wave height – Gaussian sea surface

According to(Haver,2004), a possible approach in estimating the extreme wave crest height is to estimate the wave height and thereafter introduce this estimate (wave height) into a deterministic 5th order Stoke profile. Depending on the water depth and wave steepness, the wave crest height of the 5th order Stokian wave profile is typically within the range of 58-62% of the wave height(Haver,2004). These considerations will be, within the limit of this thesis, limited to applying the 2-parameter Weibull model for the short term distribution of the wave height(Haver,2004):

$$F_{H|H_sT_p}(h|h_s, t_p) = 1 - exp\left\{-\left(\frac{h}{\alpha_H}\right)^{\beta_H}\right\} \tag{30}$$

For extremely narrow banded Gaussian sea, we can apply $\alpha_H = 0.707 \cdot h_s$ and $\beta_H = 2$. The short-term (conditional) distribution for the maximum wave height $F_{H_{3h}|H_sT_p}(h|h_s, t_p)$ within 3-hours stationary sea state, given H_s and T_p becomes(Haver,2004):

$$F_{H_{3h}|H_sT_p}(h|h_s, t_p) = \left[1 - exp\left\{-\left(\frac{h}{0.707 \cdot h_s}\right)^2\right\}\right]^{\frac{13846}{T_p}} \tag{31}$$

The long term distribution of the 3-hours maximum wave height H_{3h} is obtained by applying equation (16). The result is shown in Figure 8.

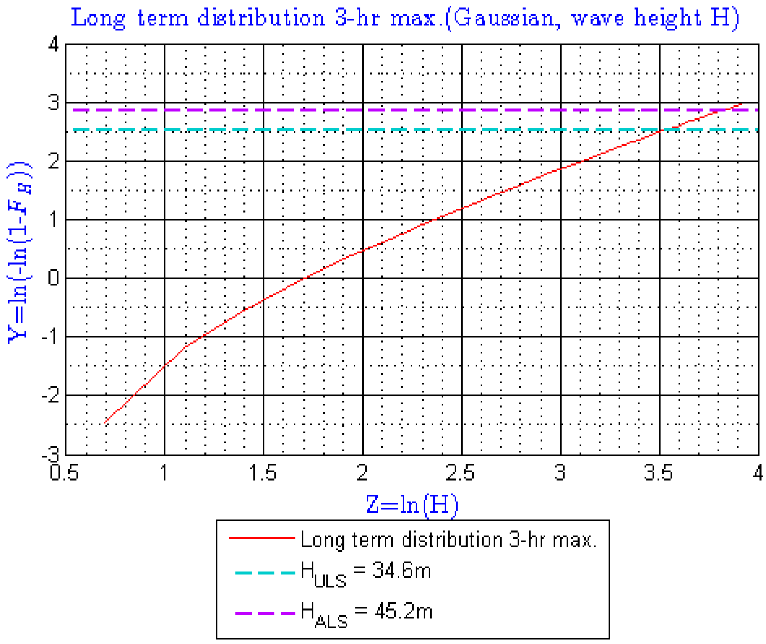

Figure 8 Probability plot for long term distribution of 3-hr maximum wave height H_{3h}

Using the methodology presented in section 3.1.1 and reading data from Figure 8, the estimated extreme wave heights with probability of exceedance of 10^{-2} (ULS) and 10^{-4} (ALS) read:

ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
H_{ULS} [m]	H_{ALS} [m]
34,6	45,2

Table 7 Extreme wave height $H_{3h,q}$

The Matlab script is attached in Appendix B.2.

3.1.4 Wave crest height – second order sea surface (2D)

The long-term analysis of extreme wave crest heights using the Gaussian model were presented in section 3.1.1. In this section, the analysis will be extended by applying a 2-parameter Weibull model based on a number of second order time domain simulations of various sea state steepness and water depths(Haver,2004).

We have until now assumed that the surface elevation wave process $\Xi(t)$ is a narrow-banded Gaussian process with zero mean and variance σ_{Ξ}^2, implying that the distribution of the global crests (largest maximum between adjacent zero up-crossings) is described by the Rayleigh distribution. However, the real sea surface process deviates from this assumption, giving higher crests and shallower troughs than expected under the Gaussian assumption(Haver,2013). Alternative models have been proposed to take account of this. The Forristall model which appeals to the second order surface model will be used within the scope of this work.

The long-crested second order model(DNV-RP-C205,2007) for the wave process $\Xi(t)$ is composed of the Gaussian process and a second order correction(Haver,2013):

$$\xi(t) = \xi_G(t) + \xi_2(t) \tag{32}$$

The important terms in the second order correction $\xi_2(t)$ are the quadratic transfer functions for the sum frequency correction and the difference frequency correction(Haver,2013). Furthermore, we can assume that the sum frequency correction term adds to the Gaussian crest while it tends to reduce the Gaussian troughs(Haver,2013). The Figure 9 illustrates the first-order (Gaussian) component, the sum frequency correction component and the resulting second order process (the difference frequency correction component omitted) (Haver,2013).

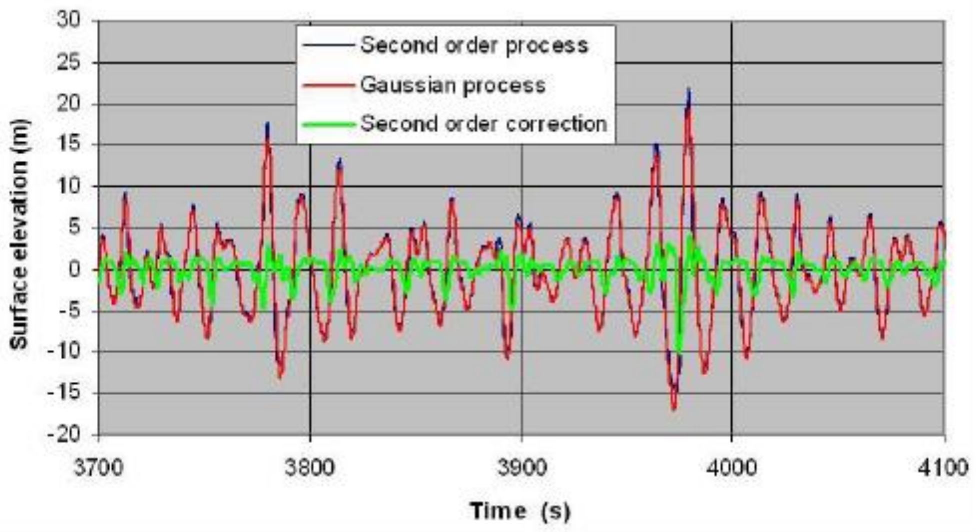

Figure 9 First- and second-order components in time domain simulation of second-order process(Haver,2013)

Based on the second order theory, Forristall suggested a probabilistic model for the short term distribution of crest heights based on a 2-parameter Weibull model(Haver,2013), (Haver,2004). This model was established by performing second order time domain simulations of various sea state steepness (see equation (37)) and water depths(Haver,2004):

$$F_{\Xi|H_sT_1}(\xi|h_s,t_1,d) = 1 - exp\left\{-\left(\frac{\xi}{\alpha_F \cdot h_s}\right)^{\beta_F}\right\} \tag{33}$$

The short-term distribution for the maximum wave crest height $F_{\Xi_{3h}|H_sT_1}(\xi|h_s,t_1,d)$ within 3-hours stationary sea state, given H_s and T_p reads:

$$F_{\Xi_{3h}|H_sT_1}(\xi|h_s,t_1,d) = \left[1 - exp\left\{-\left(\frac{\xi}{\alpha_F \cdot h_s}\right)^{\beta_F}\right\}\right]^{\frac{13846}{T_p}} \tag{34}$$

The parameters α_F and β_F are functions of steepness, s_1, and the Ursell number, U_r (Haver,2004):

$$\alpha_F = 0.3536 + 0.2892 * s_1 + 0.1060 * U_r \tag{35}$$

$$\beta_F = 2 - 2.1597 * s_1 + 0.0968 * {U_r}^2 \tag{36}$$

The average wave steepness s_1 reads(Haver,2004):

$$s_1 = \frac{2\pi \cdot h_s}{g \cdot {T_1}^2} \tag{37}$$

Where h_s is the significant wave height, g is the gravity acceleration and T_1 is the mean wave period calculated from the two first moments of the wave spectrum(DNV-RP-C205,2007):

$$T_1 = \frac{m_0}{m_1}\,(S(f)) \tag{38}$$

For a Pierson-Moskowitz spectrum, the mean wave period T_1 can be approximated by(Faltinsen,1990):

$$T_1 \approx 0.77 * T_p \tag{39}$$

Assuming a JONSWAP spectrum, the following relation is obtained(Faltinsen,1990):

$$T_1 \approx 0.83 * T_p \tag{40}$$

The equation (40) will be used for further calculations.

The Ursell number, U_r, which is a measure of the impact of water depth on the non-linearity of waves(Haver,2004), is defined as:

$$U_r = \frac{h_s}{{k_1}^2 \cdot d^3} \tag{41}$$

The unknown in equation (41) is k_1, the wave number corresponding to the wave period T_1. The variable k_1 can be determined from the linear dispersion relation (see equation (42)), since the linear dispersion relation holds for the second-order Stokes waves(DNV-RP-C205,2007). The linear dispersion relation in finite water depth, d, reads (DNV-RP-C205,2007):

$$ {\omega_1}^2 = k_1 \cdot g \cdot \tanh(k_1 \cdot d) \tag{42}$$

The assumption for the purpose of this analysis is that the water depth for the Heidrun field is 350m. Furthermore, the angular frequency $\omega_1 \left[\frac{rad}{s}\right]$ is defined as:

$$\omega_1 = \frac{2\pi}{T_1} \cong \frac{2.3981\pi}{T_p} \tag{43}$$

Introducing equation (43) into the left side of equation (42), the variable k_1 is solved by iteration for each class of T_p:

$$\left(\frac{2.3981\pi}{T_p}\right)^2 = k_1 \cdot g \cdot \tanh(k_1 \cdot d) \tag{44}$$

Furthermore, for each class of H_s and T_p the parameters s_1 and U_r are calculated as per equations (37), (41). The resulting values of these parameters are presented in matrices and classified per classes of H_s and T_p. See Appendix B.3 and Appendix B.4.

Once the values of s_1 and U_r are determined, the parameters α_F and β_F for the Weibull model describing the short distribution of the crest height are calculated (equation (34)). In a similar way as described above, the resulting matrices containing the parameters α_F and β_F corresponding to the respective classes of H_s and T_p. See Appendix B.5 and Appendix B.6.

Finally, with all the unknown parameters defined the short term distribution for the crest heights is calculated as per equation (34). The long term distribution of the 3-hours maximum crest height Ξ_{3h} is determined by applying equation (16). The result is shown in Figure 10.

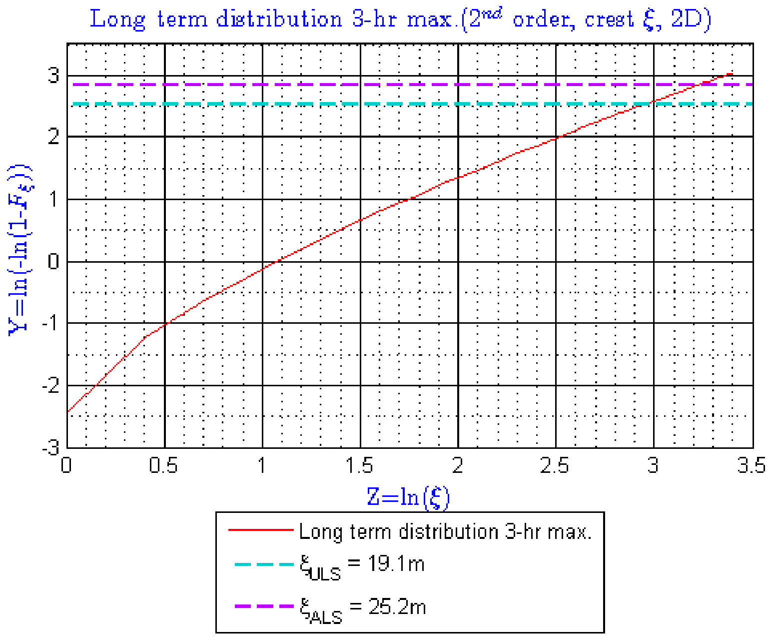

Figure 10 Probability plot for long term distribution of 3-hr maximum crest height ξ

Using the methodology presented in section 3.1.1 and reading data from Figure 10, the estimated extreme wave crest heights with probability of exceedance of 10^{-2}(ULS) and 10^{-4} (ALS) read:

ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
ξ_{ULS} [m]	ξ_{ALS} [m]
19,1	25,2

Table 8 Extreme crest height $\xi_{q,3h}$

The Matlab scripts are presented in Appendix B.7.

3.1.5 Wave crest height – second order sea surface (3D)

In section 3.1.2 we have established the long term distribution of the 3-hours maximum crest height Ξ_{3h} and estimated the extremes assuming long-crested waves. As presented in section 1.1, we will now evaluate the impact of including the wave directionality by assuming short-crested waves. The second order sea surface model presented in section 3.1.4 will be used for this calculation. The theory and methodology are identical, with the exception for the calculation of parameters α_F and β_F. These can be found from the following equations(Forristall,1999):

$$\alpha_F = 0.3536 + 0.2568 * s_1 + 0.0800 * U_r \tag{45}$$

$$\beta_F = 2 - 1.7912 * s_1 - 0.5302 * U_r + 0.284 * {U_r}^2 \tag{46}$$

The long term distribution of the 3-hours maximum crest height Ξ_{3h} is obtained by applying equation (16) and is shown in Figure 11:

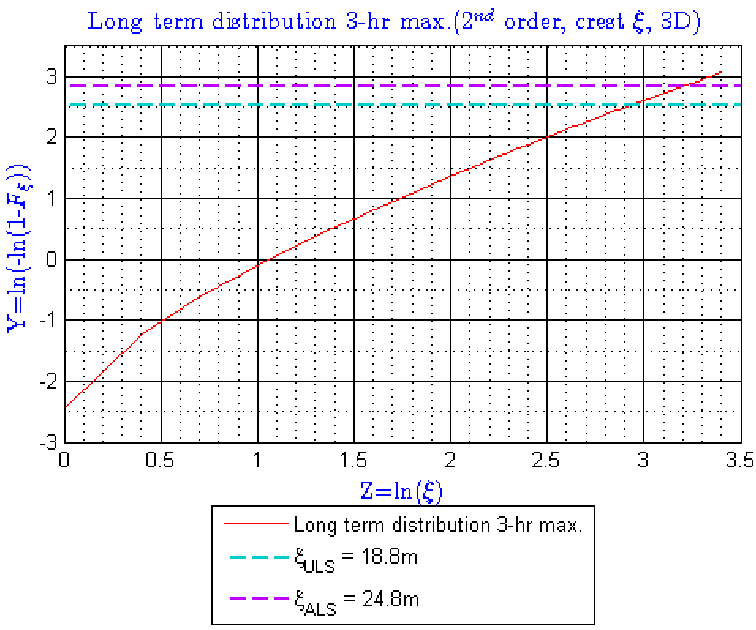

Figure 11 Probability plot for long term distribution of 3-hr maximum crest height ξ

Using the methodology presented in section 3.1.1 and reading data from Figure 11, the estimated extreme wave crest height with probability of exceedance of 10^{-2}(ULS) and 10^{-4} (ALS) read:

ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
ξ_{ULS} [m]	ξ_{ALS} [m]
18,8	24,8

Table 9 Extreme crest height $\xi_{q,3h}$

The Matlab script is presented in Appendix B.8.

3.1.6 Wave height – empirical (Forristall) model

An empirically based short term wave height distribution is the Weibull distribution(DNV-RP-C205,2007):

$$F_H(h) = 1 - exp\left\{-\left(\frac{h}{\alpha_H \cdot H_s}\right)^{\beta_H}\right\} \tag{47}$$

Using the Forristall empirical model, we can apply $\alpha_H = 0.683 \cdot h_s$ and $\beta_H = 2.13$ (Haver,2004). The short-term (conditional) distribution for the maximum wave height $F_{H_{3h}|H_sT_p}(h|h_s, t_p)$ within 3-hours stationary sea state, given H_s and T_p becomes:

$$F_{H_{3h}|H_sT_p}(h|h_s, t_p) = \left[1 - exp\left\{-\left(\frac{h}{0.683 \cdot h_s}\right)^{2.13}\right\}\right]^{\frac{13846}{T_p}} \tag{48}$$

The long term distribution of the 3-hours maximum wave height H_{3h} is obtained by applying equation (16). The result is shown in Figure 12.

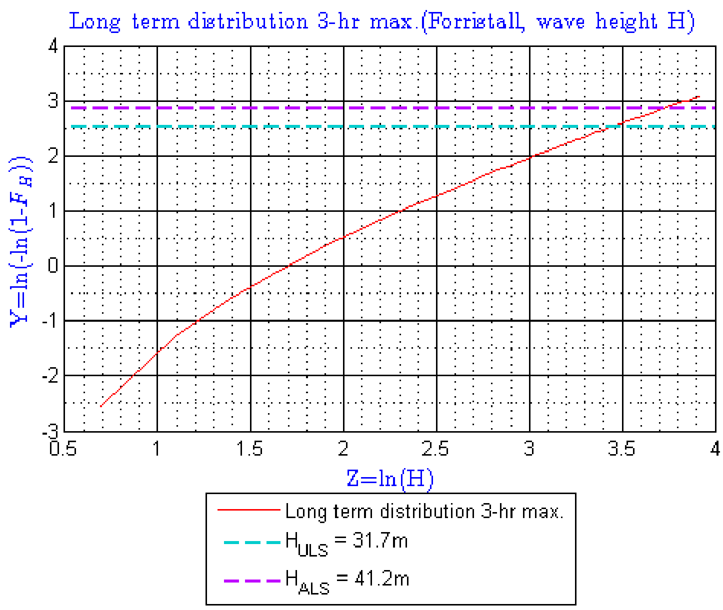

Figure 12 Probability plot for long term distribution of 3-hr maximum wave height H

Using the methodology presented in section 3.1.1 and reading data from Figure 12, the estimated extreme wave heights with probability of exceedance of 10^{-2} (ULS) and 10^{-4} (ALS) read:

ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
H_{ULS} [m]	H_{ALS} [m]
31,7	41,2

Figure 13 Extreme wave height H_q

The Matlab script is presented in Appendix B.9.

3.1.7 Discussion

The results obtained in section 3.1 are shown in Table 10 and Table 11

	ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
	ξ_{ULS} [m]	ξ_{ALS} [m]
All sea states max. crest 3hr (Gaussian)	17,3	22,6
All sea states max. crest 3hr (2nd, 2D)	19,1	25,2
All sea states max. crest 3hr (2nd, 3D)	18,8	24,8

Table 10 Summary all sea states results – extremes (3hr) for wave crest height

	ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
	H_{ULS} [m]	H_{ALS} [m]
All sea states max. wave height 3hr (Gaussian)	34,6	45,2
All sea states max. wave height 3hr (empirical, Forristall)	31,7	41,2

Table 11 Summary all sea states results – extremes (3hr) for wave height

Crest height - Gaussian vs. 2nd order model (2D)

The theory introduced in section 3.1.4 shows that the 2nd order model waves are asymmetric with wave crest height > wave trough height. In other words, the crests are higher and sharper than expected from a summation of sinusoidal waves with random phase, and the troughs are shallower and flatter(Forristall,1999). The wave crest height estimates obtained with the 2nd order model (2D) are larger than the values obtained with the all sea states approach, which confirms the theory in section 3.1.4.

Directional spreading of waves

According to (Forristall,1999), the 3D model that account for the directional spreading of waves produce crests that are about 2% lower than the 2D model in deep water. The crest heights obtained with the 3D model are approximately 2% lower than with the 2D model for both ULS and ALS criteria, which confirm the statement above.

Wave height - Gaussian vs. 2nd order model

The wave heights estimated using the empirical model are lower than the values obtained with the Gaussian model. For the Gaussian model, the wave crest height is equal to the wave trough height and hence the wave height is equal to $2 \cdot \xi$. This can be deducted by looking at the results presented in Table 10 and Table 11.

For the empirical model, the waves are asymmetric i.e. the crests are steeper and troughs are shallower and flatter. This leads to a lower wave height than the value obtained using the Gaussian model.

3.2 LONG TERM DISTRIBUTION - ALL GLOBAL MAXIMA

We have until now considered the largest global maxima within each 3-hours sequence of stationary sea state as the quantity of interest. A different approach is to establish the long term distribution for global maxima (largest extreme value between adjacent zero-up crossings) by considering all the individual global maxima(Haver,2013):

$$F_X(x) = \frac{1}{\nu_0^+} \int_h \int_t \nu_0^+(h,t) \cdot F_{X|H_sT_p}(x|h,t) \cdot f_{H_sT_p}(h,t)dtdh \tag{49}$$

The following figure illustrates a surface process with global maxima and the 3-hour extreme are marked with a red dot and green circle, respectively. The $200s$ window is the window with the 3-hour maximum value(Haver,2014):

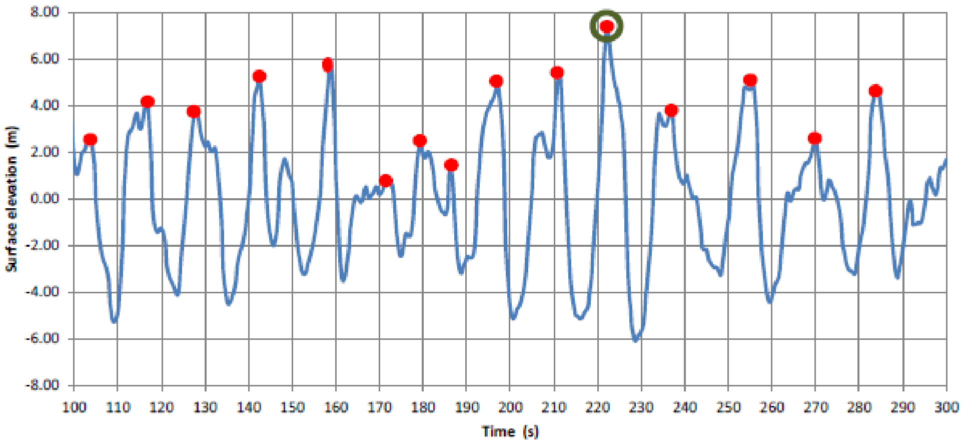

Figure 14 Illustration of global maxima and 3-hours extreme (Haver,2014)

The long term average zero-up-crossing frequency is given by(Haver,2013):

$$\overline{v_0^+} = \int_h \int_t v_0^+(h,t) \cdot f_{H_sT_p}(h,t) dt dh \tag{50}$$

Furthermore, the expected number of global maxima in T years is defined as(Haver,2013):

$$n_T = T \cdot 365 \cdot 24 \cdot 3600 \cdot \overline{v_0^+} \tag{51}$$

The value which is expected to be exceeded only once during T years i.e. value having an annual probability of exceedance $\frac{1}{T}$ can be determined using the following equation(Haver,2013):

$$1 - F_X(x_T) = \frac{1}{n_T} \tag{52}$$

In the next two sections, we will consider the wave crest height for second order sea surface for both long-crested (2D) and short-crested waves (3D).

3.2.1 Wave crest height – second order surface (2D)

We first need to determine the zeroth and second spectral moments for H_s range $0.25 \rightarrow 24.75m$ and T_p range $0.5 \rightarrow 24.5,$ using the mid-intervals for both H_s and T_p . Introducing the spectral moments into equation (5) results into a matrix containing expected zero-up-crossing frequencies. The next step is then to determine the long term average zero-up-crossing frequency using the equation (50), followed by the expected number of global maxima in T years. The results are shown in Table 12.

$\overline{\nu_0^+}$ [Hz]	n_T (ULS)	n_T (ALS)
0,162	5,0975E+08	5,0975E+10

Table 12 Expected number global max. and average zero-up-crossing frequency

The Matlab script is presented in Appendix B.10.

The next step is to calculate the short term (conditional) distribution for the global maximum wave crest height. In a similar manner as presented in section 3.1.4, the first step is to calculate the parameters s_1 and U_r using the mean wave period as defined by equation (38). Once the matrices for s_1 and U_r are obtained, the parameters α_F and β_F corresponding to the respective classes of H_s and T_p are determined.

The short-term (conditional) distribution for the global maximum wave crest height $F_{\Xi|H_sT_1}(\xi|h_s, t_1, d)$ given $H_s, T_p,$ is obtained using equation (33). The Matlab script is presented in Appendix B.11.

The long term distribution of the global maximum wave crest height is determined by integrating the short term distribution over all sea states, weighting for the number of individual wave cycles within each sea state(DNV-RP-C205,2007). This sequence is performed by using the Matlab script presented in Appendix B.12. The probability plot is obtained by using Matlab script presented in Appendix B.13 and the result is shown in Figure 15.

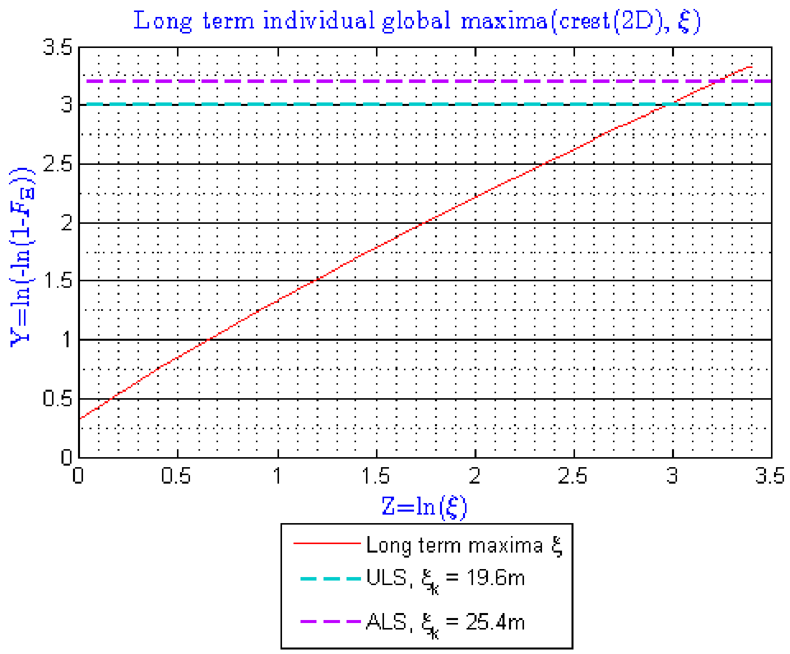

Figure 15 Probability plot for long term distribution of global maximum crest height ξ

Using the methodology presented in section 3.1.1 and reading data from Figure 15, the estimated extreme wave crest heights with probability of exceedance of 10^{-2}(ULS) and 10^{-4} (ALS) read:

ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
ξ_{ULS} [m]	ξ_{ALS} [m]
19,6	25,4

Table 13 Extreme crest height ξ_q

The sequence for estimating the extremes is included in the Matlab script presented in Appendix B.13.

3.2.2 Wave crest height – second order surface (3D)

The methodology presented in section 3.2.1 is valid for the short-crested sea and the calculations steps almost identical. The only difference is the calculation of the parameters α_F and β_F. The equations ((45), (46)) are used to calculate the parameters α_F and β_F for short-crested sea, in a similar way as explained in section 3.1.5.

The resulting long term distribution of the global maximum wave crest height is shown in Figure 16.

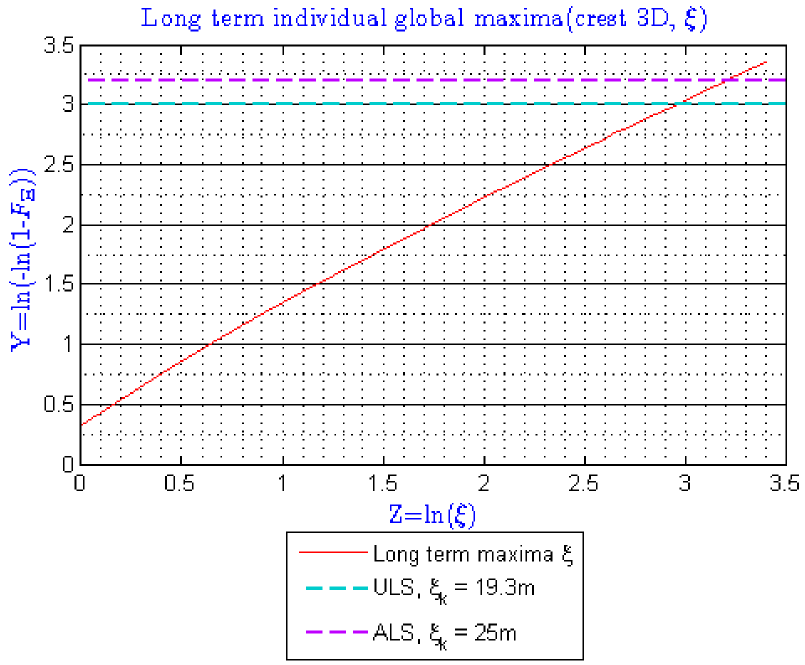

Figure 16 Probability plot for long term distribution of global maximum crest height ξ

The estimated extreme wave crest heights with probability of exceedance of 10^{-2}(ULS) and 10^{-4} (ALS) read:

ULS $(q = 10^{-2})$	ALS $(q = 10^{-4})$
ξ_{ULS} [m]	ξ_{ALS} [m]
19,3	25,0

Figure 17 Extreme crest height ξ_q

The Matlab scripts are presented in Appendix B.14, Appendix B.15 and Appendix B.16.

3.2.3 Discussion

The results obtained in sections 3.1 and 3.2 are summarized in Table 14.

	ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
	ξ_{ULS} [m]	ξ_{ALS} [m]
All sea states max. crest 3hr (Gaussian)	17,3	22,6
All sea states max. crest 3hr (2nd, 2D)	19,1	25,2
All sea states max. crest 3hr (2nd, 3D)	18,8	24,8
All sea state All global max. crest (2nd,2D)	19,6	25,4
All sea state All global max. crest (2nd,3D)	19,3	25,0

Table 14 Summary all sea states results – extremes (3hr & all maxima) for wave crest height

Comparison 3-hours largest global maxima vs. all global maxima

The estimated extreme crest heights using the all global maxima approach are higher than the values from the 3-hours largest global maxima approach. This can be attributed to the correlation between the adjacent maxima when adopting the all global maxima approach. The correlation effect can be eliminated by adopting the 3-hours largest global maxima approach (Haver,2013). Hence, the largest global maxima approach leads to less conservatism of the extreme waves which explains why the estimated extreme values are lower.

Effect of short-crested waves

As discussed in section 3.1.7, the effect of short-crested waves has the effect that the extremes are lower than when considering the long-crested waves. This consideration can be seen from the results presented in Table 14.

Statistical independence of maxima

The assumption of independent maxima is generally not fulfilled and the actual extremes are likely to be overestimated(Haver and Nyhus,1986). However, the effect is expected to be very small for large values of number of years (i.e. low q-probability) while more pronounced as the number of years approaches 1 (i.e. high q-probability). This consideration is verified by applying the 2nd order model (2D) for 1-year ($q = 1$) and 10-year ($q = 10^{-1}$) return periods. The Figure 18 and Figure 19 show the probability plots of the long term distributions with the 1- and 10-year return periods.

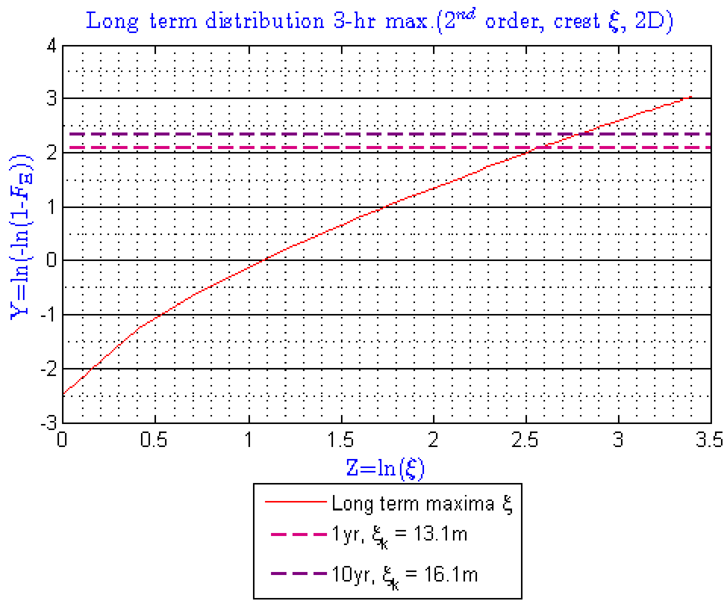

Figure 18 Probability plot, 3-hr largest maxima, crest height ξ (1- and 10-yr return period)

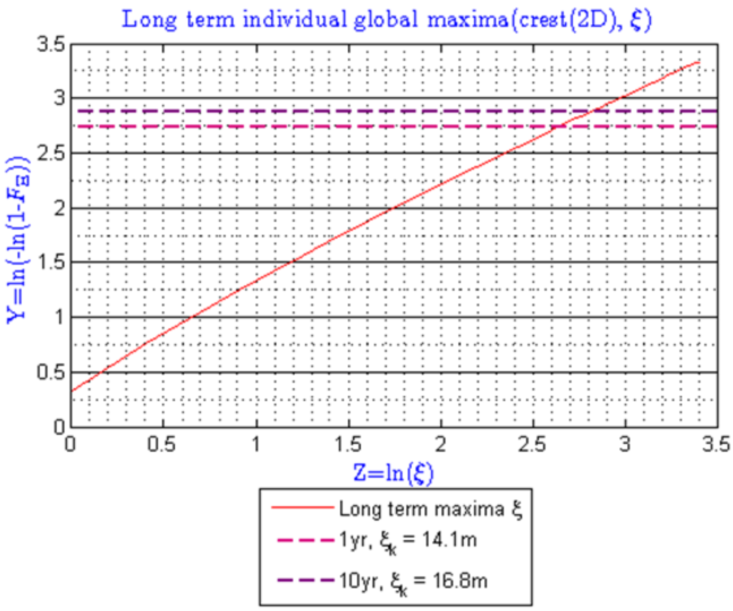

Figure 19 Probability plot, all global maxima, crest height ξ (1- and 10-yr return period)

The extremes estimates are presented in Table 15.

All sea states max. crest 3hr (2nd, 2D)		All sea state All global max. crest (2nd,2D)	
1 yr ($q = 1$)	10yr ($q = 10^{-1}$)	1 yr ($q = 1$)	10yr ($q = 10^{-1}$)
ξ_{1yr} [m]	ξ_{10yr} [m]	ξ_{1yr} [m]	ξ_{10yr} [m]
13,1	16,1	14,1	16,8

Table 15 Comparison extremes (3hr & all maxima), wave crest height, 1yr & 10yr

By comparison of values in Table 14 and Table 15, it can be seen the overestimation is more pronounced with the number of years (return period) decreasing.

3.3 METOCEAN CONTOUR LINES

The basic idea is to, using the joint distribution of sea state characteristics from section 0, define contours of combinations of H_s, T_p corresponding to a constant annual probability of exceedance, in our case $q = 10^{-2}$ (ULS) and $q = 10^{-4}$ (ALS). This method permits to estimate the long term extremes (i.e. extremes corresponding to a given annual probability of exceedance) using selected short term sea states(Haver,2012), depending on the type of problem to analyse. We will adopt the approach that the metocean contours are determined such that the probability of exceedance in an arbitrary 3-hour event reads $\frac{q}{2920}$, implying that all 3-hour events are statistically independent(Haver,2013).

The metocean contour line method is very convenient for analysis of complex problems where a full long term analysis would be costly and time consuming(Haver,2006). As previously stated, we will not consider the response problem but rather demonstrate the metocean contour line method and compare with the results of the long term analyses from sections 3.1 and 3.2. We will assume that the sea state with the peak significant wave height $H_{s_{peak}}$ is the most unfavourable sea state along the contour lines.

The main theoretical concepts of the metocean contour line method are presented in section 3.3.1. In section 3.3.2, the metocean contour line method are established using the fitted functions from the metocean modelling (see section 2.3). Furthermore, the influence of the coefficient b_1 on the metocean contour lines is demonstrated. Finally, the percentile levels will be estimated using the extreme values obtained in section 3.1. The estimated percentile levels will be compared with the recommended percentiles levels stipulated in(N-003,2015) and (N-003,2013).

3.3.1 Methodology

The metocean contour line method relies on the theoretical concepts from the field of structural reliability engineering (see e.g. (Madsen,1986)). The long term distribution of extreme value $F_{X_d}(x)$ involves three basic random variables: the d-hour maximum value X_d(i.e. extreme value within a stationary sea state), the significant wave height H_s and the spectral peak period T_p (Haver,2008). Defining a limit state function permitting to estimate the probability of exceeding a given threshold x_{crit} (Haver,2008), the following expression is obtained:

$$g\left(X_d, H_s, T_p;\ x_{crit}\right) = x_{crit} - X_d(H_s, T_p) \tag{53}$$

The definition of failure is when the function $g < 0$ (equation (53)). The failure boundary is defined for $g = 0$ and the failure probability can be estimated using the following expression(Haver,2008):

$$p_f(x_{crit}) = \iiint\limits_{g(\ \ )<0} f_{X_d|H_sT_p}(x|h,t) f_{H_sT_p(h,t)} dx dt dh \tag{54}$$

By applying the Rosenblatt transformation within the First-Order Reliability Method (FORM), the integral of equation (54) is transformed into the u-space consisting of independent, standard Gaussian variables U_1, U_2, U_3 as following(Haver,2008):

$$F_{H_s}(h) = \Phi(u_1) \Leftrightarrow u_1 = \Phi^{-1}[F_{H_s}(h)] \tag{55}$$

$$F_{T_p|H_s}(t|h) = \Phi(u_2) \Leftrightarrow u_2 = \Phi^{-1}[F_{T_p|H_s}(t|h)] \tag{56}$$

$$F_{X_d|H_sT_p}(x|h,t) = \Phi(u_3) \Leftrightarrow u_3 = \Phi^{-1}[F_{X_d|H_sT_p}(x|h,t)] \tag{57}$$

These equations conserve the probabilities between the two spaces (i.e. condition imposed by the Rosenblatt transformation) and the transformation is a unique two-way transformation between the u-space and space defined by h, t, x since the involved functions increase monotonically(Haver,2008). Furthermore, by following the FORM methodology the estimate for the failure probability reads(Haver,2008):

$$\hat{p}_f(x_{crit}) = \Phi(-\beta) \tag{58}$$

The probability $\hat{p}_f$ represents an approximation of the long term probability obtained from equation (17) as(Haver,2012):

$$\hat{p}_f = 1 - F_{X_{3h}}(x_q) \tag{59}$$

$$\Phi(-\beta) = \hat{p}_f \Rightarrow \beta = -\Phi^{-1}(\hat{p}_f) \tag{60}$$

In the present study, we will consider two slowly varying parameters: significant wave height H_s and the spectral peak period T_p, corresponding to, respectively, U_1, U_2. Applying the Inverse First Order Reliability Method (IFORM) (Haver,2008), it can deducted that the contour lines in the u-space (Gaussian space) are circles with radius β (DNV-RP-C205,2007):

$$\sqrt{u_1^2 + u_2^2} = \beta \tag{61}$$

As an example, the radius for the $q = 10^{-2}$ (ULS) contour is(DNV-RP-C205,2007):

$$\beta = -\Phi^{-1}\left[\frac{1}{100 * 365 * 8}\right] = 4.5 \tag{62}$$

The values of u_1 and u_2 can be determined by(Baarholm,2010):

$$u_1 = \beta \cdot cos\theta \tag{63}$$

$$u_2 = \beta \cdot sin\theta \tag{64}$$

The pairs of (u_1,u_2) can be calculated by varying the angle θ and all the combinations of (u_1,u_2) along the circle with radius β correspond to the q-annual exceedance probability (see Figure 20). The transformation of (u_1,u_2) to the (h_s, t_p) space defines the closed contour lines in the $h_s - t_p$ space (see Figure 21). Since the transformation is unique and conserves probability, all the combinations along the $h_s - t_p$ contour line will under the assumption of linearized failure boundary (FORM) correspond to a q-annual exceedance probability(Baarholm,2010).

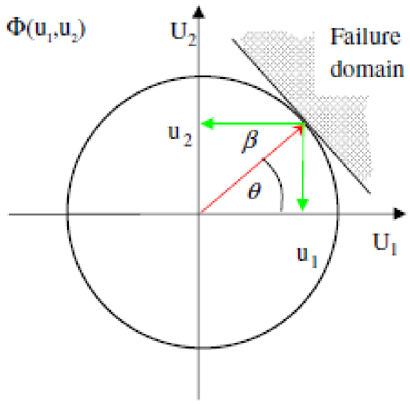

Figure 20 Definition of U-space (Baarholm et. al.)

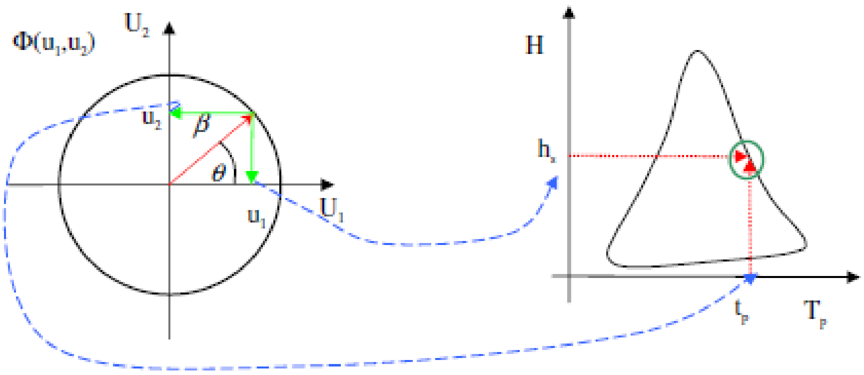

Figure 21 Transformation from standard Gaussian space to (Hs -Tp) space (Baarholm et.al.)

In order to estimate the long term extremes, both short-term and long-term randomness in metocean conditions must be taken into account(Haver,2008). The long-term randomness is characterized by H_s and T_p and their joint distribution (see section 2). The short-term randomness is reflected in the short-term variability within a stationary metocean state (3-hours in our case). If the short-term distribution of the extreme value is very narrow, it means that there is very little variability and that we can use the median of the extreme value distribution as the short-term characteristic. However, in most of the cases the short-term variability within a stationary sea state (3-hours in our case) has to be accounted for. We then have to select a short-term characteristic somewhat larger than the median (Haver,2004) i.e. determine the cumulative level, α, for the 3-hours extreme value that accounts properly for the short-term variability(Baarholm,2010). As an illustration, the Figure 22 shows distributions with small and large short-term variability.

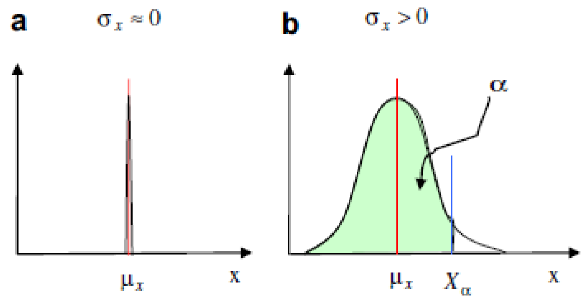

Figure 22 Small short-term variability ($\sigma_x \approx 0$) and large short-term variability ($\sigma_x > 0$) (Baarholm,2010)

There are several approaches to deal with the uncertainties in the short-term distribution of the extreme value: *(i)* "inflation" of the metocean contour lines and based on FORM omission factors (Haver,2004); *(ii)* use of correction factor on the extreme estimate(Haver,2004); *(iii)* application of a percentile value (cumulative probability level, α), method consisting of replacing the median value with a higher percentile value, p (Haver,2008).

The following three sections will be dedicated to, respectively, demonstrate the metocean contour line method (section 3.3.2) and, verify the recommended percentile levels for Gaussian (section 3.3.3) and 2nd order sea surfaces (section 3.3.4). The purpose of the verifications in sections 3.3.3 and 3.3.4 is to identify which percentiles of the 3-hours extreme value yield values equal to (or closest possible) the long term extremes obtained in in section 3.1.

3.3.2 Metocean contour lines - application

The two values of coefficient b_1 discussed in section 0 will be used for demonstrating the metocean contour line method. The Matlab scripts are presented in Appendix B.17 and Appendix B.18. To begin with, a vector of values θ_i with sufficiently small resolution steps is created, permitting to define vectors of Gaussian variables u_1,u_2. Using the equations (8) and (55), and introducing the estimated parameters from Table 1 , the expression giving a vector of values h_{s_i} reads:

$$F_{H_s}(h) = \Phi(u_1) \Rightarrow h_s = \mu + \sigma * \left[-\ln\left(1 - \Phi(u_1)\right)\right]^{1/\eta} \tag{65}$$

The next step is to calculate $\mu_{lnTp(h)}$ and $\sigma^2{}_{ln_{Tp(h)}}$ (see equations (10) and (11)) for the calculated values of h_{s_i}. Finally, the vector of values t_{p_i} corresponding to the respective h_{s_i} can be determined by using equations (13) and (56):

$$F_{T_p|H_s}(t|h) = \Phi(u_2) = \Phi\left(\frac{ln_{T_p} - \mu_{ln_{T_p}}}{\sigma_{ln_{T_p}}}\right) \Rightarrow t_p = \exp(\mu_{ln_{T_p}} + u_2 \cdot \sigma_{ln_{T_p}}) \tag{66}$$

The metocean contour lines for coefficient $b_1 = 0.005$ are shown in Figure 23.

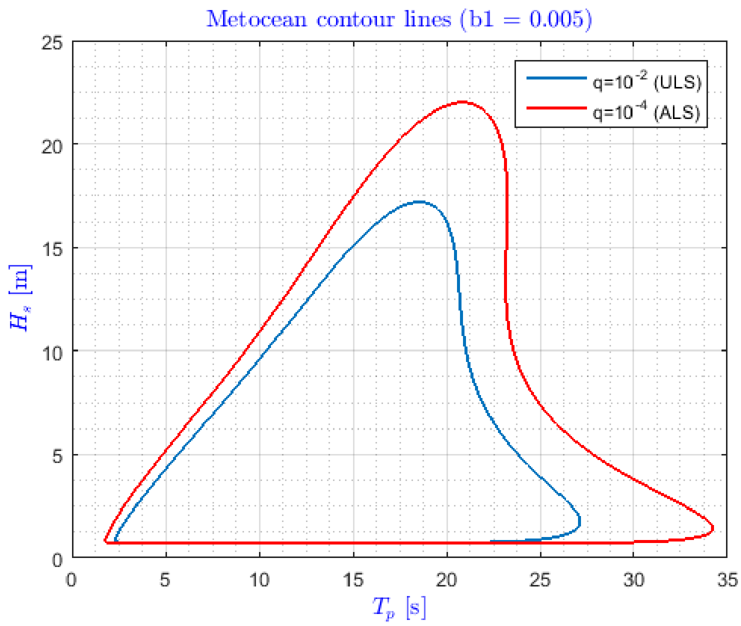

Figure 23 Metocean contour lines ($b_1 = 0.005$)

The metocean contour lines for coefficient $b_1 = 0$ are shown in Figure 24.

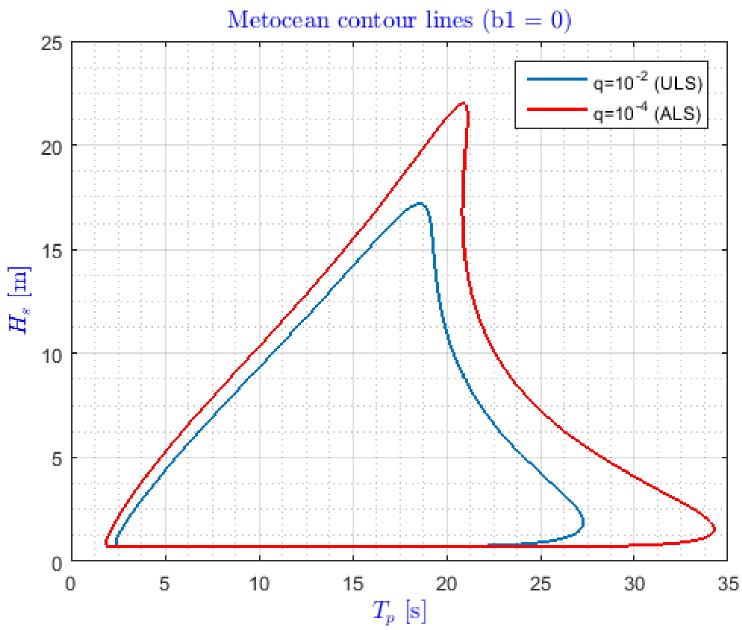

Figure 24 Metocean contour lines ($b_1 = 0$)

3.3.3 Verification of percentile level – Gaussian process

The first task is to plot the distribution function of the 3-hour maximum crest height for six sea states (combinations of H_s and T_p) along the $q = 10^{-2}$ (ULS) and $q = 10^{-4}$ (ALS) contour lines. The sea state with peak significant wave height H_s is of main interest within the context of this study, and the other five sea states are chosen from both sides of the peak H_s value. As previously discussed, the coefficient $b_1 = 0.005$ is used for the analysis. The Matlab script is presented in Appendix B.19.

In order to obtain smooth curves of the cumulative distributions, the probability range is set from $0 \rightarrow 0.999$ with resolution steps of 0.001. The next step is to obtain a vector of realizations of 3-hours crest maxima by using equation (19) for each sea state. The result is a matrix of 6 vectors containing 1000 realizations 3-hours crest maxima. The value in the vector of peak significant wave height $H_{s_{peak}}$ which closest to the extreme value obtained in section 3.1 is used as the estimate of the 3-hours crest maximum. The cumulative probability (percentile level) corresponding to the estimated extreme 3-hour crest maximum is marked on Figure 25 and Figure 26.

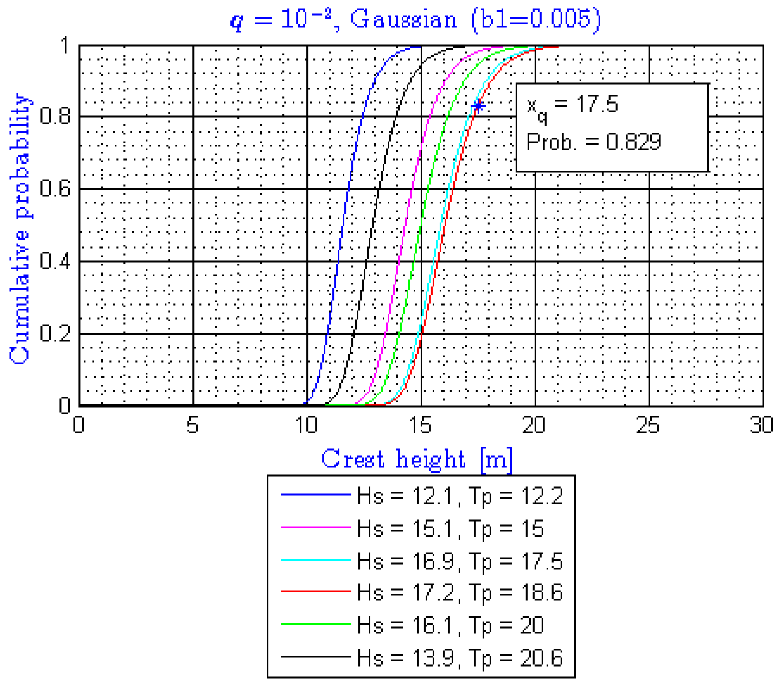

Figure 25 Percentile level, Gaussian process, ULS ($b_1 = 0.005$)

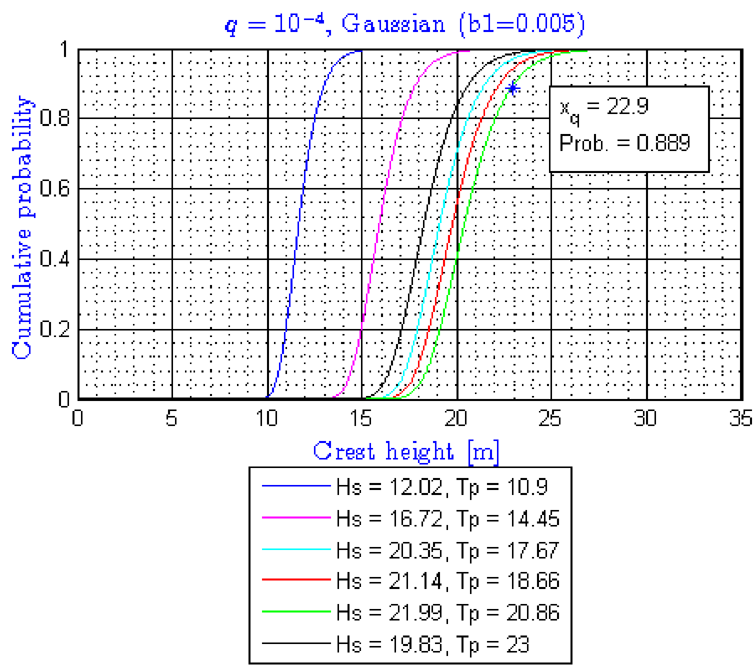

Figure 26 Percentile level, Gaussian process, ALS ($b_1 = 0.005$)

3.3.4 Verification of percentile level – second order process

The methodology is similar to the one described for Gaussian process, except that equation (34) is used to create vectors of realizations of 3-hours crest maxima. The Matlab script is presented in Appendix B.20. The resulting plots and estimated percentile levels are shown in Figure 27 and Figure 28.

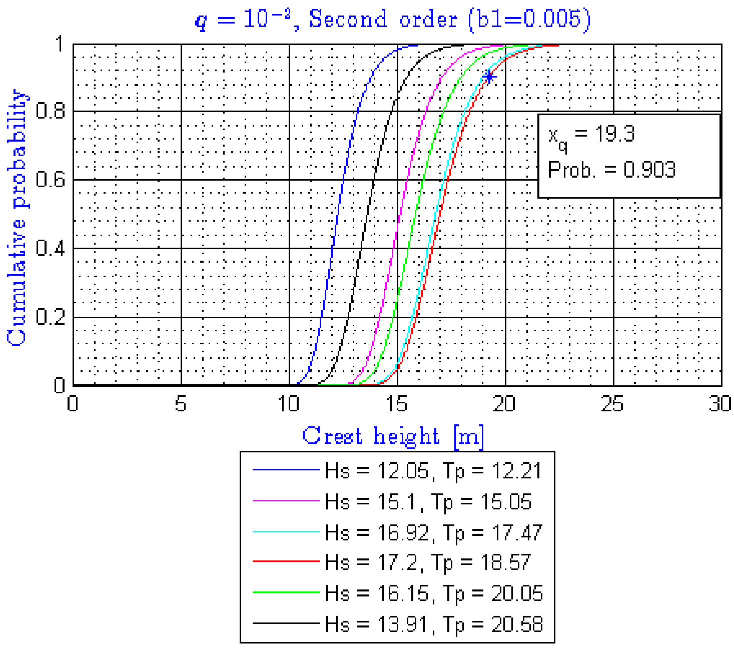

Figure 27 Percentile level, second order process, ULS ($b_1 = 0.005$)

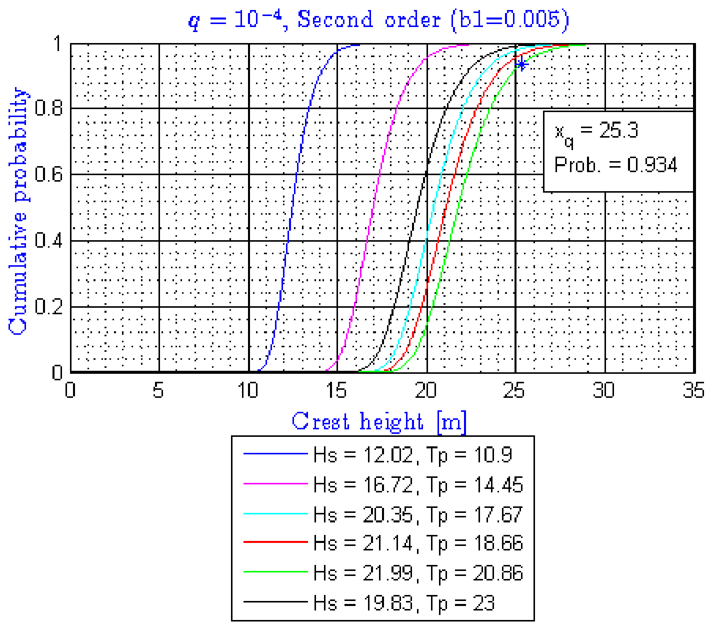

Figure 28 Percentile level, second order process, ALS ($b_1 = 0.005$)

3.3.5 Discussion

Impact of coefficient b_1

The impact of the variations of the coefficient b_1 on the contour lines is that the spectral peak period T_p band with $b_1 = 0$ is narrower than in the case of $b_1 = 0.005$. In other words, using $b_1 = 0$ covers less combinations of H_s and T_p and hence narrower contour lines, in comparison to $b_1 = 0.005$. The values of H_s are not influenced by the coefficient b_1 and hence identical in both cases.

The expression of the variance of a lognormal distributed random variable i.e. T_p is(Devore and Berk,2007):

$$V(T_p) = (e^{\mu_{ln_{t_p}}+\sigma^2_{ln_{t_p}}}) \cdot (e^{\sigma^2_{ln_{t_p}}} - 1) \tag{67}$$

Using equations (11), (67) and letting $H_s \to \infty$, the following is observed:

- $b_1 \to 0 \Rightarrow \sigma^2_{ln_{t_p}} \to 0 \Rightarrow V(T_p) \longrightarrow 0$
- $b_1 = 0.005 \Rightarrow \sigma^2_{ln_{t_p}} \to 0.005 \Rightarrow V(T_p) \longrightarrow (e^{\mu_{ln_{t_p}}+0.005}) \cdot (0.005)$

In other words, $b_1 = 0.005$ gives a larger variance as contrary to $b_1 = 0$ giving less variance. The result seems more conservative i.e. requiring a larger confidence band with $b_1 = 0.005$ since more combinations of H_s and T_p are included within the contour lines. Hence, a coefficient value of $b_1 = 0$ gives narrower T_p bands in comparison to $b_1 = 0.005$.

If the most important sea state on the metocean contour line is the peak of the contour giving highest H_s then the value of b_1 is not of primary importance. This is relevant for problems that are not sensitive to the spectral peak period T_p. However, if the spectral peak period T_p is important i.e. if the region of steepest seas along the contour lines (area left to the peak H_s sea state) are of major relevance, then careful consideration should be given to the value of coefficient b_1.

Percentile level – Gaussian model

The recommended percentiles stipulated in(N-003,2013),(N-003,2015) and the results obtained in section 3.3.3 are shown in Table 16.

	α (N-003,2013)	α (N-003,2015)	$\hat{\alpha}$
$q = 10^{-2}$ (ULS)	$0.85 \rightarrow 0.95$	$0.85 \rightarrow 0.90$	0.83
$q = 10^{-4}$ (ALS)	$0.90 \rightarrow 0.95$	$0.90 \rightarrow 0.95$	0.89

Table 16 Summary of percentile levels ($b_1 = 0.005$)

The estimated percentile levels $\hat{\alpha}$ are slightly below the lower interval values recommended in (N-003,2013),(N-003,2015). However, it can be concluded that these are reasonably close to the recommended intervals. Hence, the metocean contour line method in combination with the recommended percentiles yields estimates for the 3-hour extreme values (Gaussian sea surfaces) that are very close to the values obtained by the all sea states approach.

The impact of coefficient b_1 on the long term distribution of wave crest height has been discussed in section 2.3.1. For the sake of completeness, a verification analysis has been performed with $b_1 = 0$. The differences between the extreme crest heights for, respectively $b_1 = 0.005$ and $b_1 = 0$, are to the 3rd decimal (millimetre) which is for practical purposes negligible. This also means that we can expect percentile levels for $b_1 = 0$ identical to those for $b_1 = 0.005$. The distribution curves are slightly different since the T_p values for the sea states close to the peak H_s sea state are different. The distribution curves with the percentile levels for $b_1 = 0$ are shown in Appendix B.21.

Percentile level – 2nd model

In a similar way as presented above, the results obtained in section 3.3.4 together with the recommended percentiles stipulated in (N-003,2013),(N-003,2015) are shown in Table 17.

	α (N-003,2013)	α (N-003,2015)	$\hat{\alpha}$
$q = 10^{-2}$ (ULS)	$0.85 \rightarrow 0.95$	$0.85 \rightarrow 0.90$	0.90
$q = 10^{-4}$ (ALS)	$0.90 \rightarrow 0.95$	$0.90 \rightarrow 0.95$	0.93

Table 17 Summary of percentile levels ($b_1 = 0.005$)

The estimated percentile levels $\hat{\alpha}$ are within the intervals recommended in (N-003,2013),(N-003,2015) as opposed to the Gaussian model. In other words, the estimates obtained from the second order model for wave crest height are more exact with respect to the recommendations stipulated above than the Gaussian model. As concluded above, the metocean contour line method in combination with the recommended percentiles yields reasonable estimates for the 3-hour extreme values (2nd order sea surfaces).

The discussion related the influence of coefficient b_1 and presented above is valid in this case as well. However, it should be noted that the difference (between $b_1 = 0.005$ and $b_1 = 0$) in percentiles is more pronounced in the case of second order model than with the Gaussian model. A possible reason for this could be that in the second order model, there is the influence of the parameters α_F and β_F which are both dependent on the chosen sea state (i.e. T_p) while for the Gaussian model the influence is "reduced" to T_p only. See Appendix B.22.

4 LONG TERM ANALYSIS: PEAK-OVER-THRESHOLD (POT)

The subject in this chapter will be the second long-term analysis method: Peak-Over-Threshold. Some basic statistical concepts that the Peak-Over-Threshold method is building on will be presented in section 4.1. The Peak-Over-Threshold (POT) theory and methodology, followed by application and discussion on the results will be the focus in section 4.2.

4.1 STATISTICS - THEORY

In order to show why the Gumbel distribution model is convenient when looking at extreme values (in this context "largest" values), it is worth introducing some basic theory and principles of the order statistics. Following that, the Gumbel distribution will be presented before introducing the Peak-Over-Threshold methodology.

4.1.1 Extreme order statistics

Let $X_1, X_2, \dots, X_n$ be a sample of size n drawn from a population with pdf $f(x)$ and distribution $F(x$ (Castillo,2005). By arranging the sample in an increasing order of magnitude $(X_1, X_2, \dots, X_n)$, we obtain the ordered values $X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n}$. The first order statistic $X_{1:n}$ is the minimum of $(X_1, X_2, \dots, X_n)$ while $X_{n:n}$ is the maximum(Castillo,2005), also termed extreme values(Bury,1999). In case the distribution $F(x)$ cannot be specified, asymptotic distribution can be used to model the extreme value of interest(Bury,1975). The asymptotic distribution exists if the distribution of X_i converges to a probability distribution (the asymptotic distribution) as i increases(Wikipedia,2015). There are three types of asymptotic models and each of these arises from a class of initial distributions $F(x)$ (Bury,1975). The type I Extreme Value asymptotic model will be used for the purpose of our study. This model is constructed from initial distributions that are unbounded in the direction of the extreme value and are of the exponential type(Bury,1975):

$$F(x) = 1 - \exp\{-a(x)\} \tag{68}$$

Where $a(x)$ increases monotonically with x and without limit.

The details related to the further mathematical development can be found in(Bury,1975) . The final expression of the type I Extreme Value model reads(Bury,1975):

$$F_{I,max}(x_e;\mu,\sigma) = exp\left\{-exp\left\{-\frac{x_e-\mu}{\sigma}\right\}\right\} \tag{69}$$

Where x_e is the Extreme Value variable, μ and σ are, respectively, the location and scale parameters.

For the sake of completeness, it should be mentioned that the same result can be obtained by using a more general approach from the classical extreme value theory. Without going into further details on the mathematical development that can be found in(Castillo,2005), the final result showing the expression for the only possible parametric family as a limit for maxima reads(Castillo,2005):

$$H_\kappa(x;\lambda,\delta) = \begin{cases} exp\left\{-\left[1-\kappa\left(\frac{x-\lambda}{\delta}\right)\right]^{\frac{1}{\kappa}}\right\}, & 1-\kappa\left(\frac{x-\lambda}{\delta}\right) \geq 0, \quad \kappa \neq 0 \\ exp\left[-exp\left(\frac{\lambda-x}{\delta}\right)\right], & -\infty < x \geq \infty, \quad \kappa = 0 \end{cases} \tag{70}$$

The distribution in equation (70) is known as the maximal generalized extreme value distributions (GEVD). The Gumbel distribution (for case $\kappa = 0$) is a particular case of the GEVD.

4.1.2 Gumbel distribution

The Gumbel distribution appears very frequently in practical problems when we observe data that represent maxima values. The reason for this, and as presented in previous section, is that the limit of the maxima of independent samples converges to a Gumbel distribution(Castillo,2005). The pdf of the Gumbel random variable is given by(Castillo,2005):

$$f(x) = \frac{1}{\beta} exp\left[\frac{\alpha-x}{\beta} - exp\left(\frac{\alpha-x}{\beta}\right)\right], -\infty < x < \infty \tag{71}$$

Where α and β are, respectively, the location and scale parameters.

The distribution of the Gumbel random variable is given by(Castillo,2005):

$$F(x) = exp\left[-exp\left(\frac{\alpha - x}{\beta}\right)\right] = exp\left[-exp\left(-\frac{x-\alpha}{\beta}\right)\right] \quad (72)$$

The mean and variance are defined as(Castillo,2005):

$$\mu = \alpha - 0.57772\beta \rightarrow \alpha = \mu - 0.57772\beta \quad (73)$$

$$\sigma^2 = \frac{\pi^2\beta^2}{6} \rightarrow \beta = 0.7797\sigma \quad (74)$$

Note: for the sake of consistency in this document, the naming convention for the Gumbel parameters (α, β) has been changed from the original author's references.

4.2 POT – METHODOLOGY & APPLICATION

The basic idea with the Peak-Over-Threshold method is to establish the long-term distribution of the largest wave during an arbitrary storm(Haver,2004). In this approach, the storm is treated as the random independent event (Tromans,1995) as opposed to the All Sea States where the sea state was treated as a random variable, described by H_s and T_p. The Peak-Over-Threshold method includes the sea states corresponding to storms exceeding a chosen threshold. The aim is then to establish the distribution function of the extreme during a random storm. The POT approach can accommodate any choice of variable or probability distribution; wave height or crest height with Rayleigh or empirical Forristall distribution(Tromans,1995). In this study, the focus will be on the long term distribution of the maximum wave crest height Ξ following the 2nd order model for long-crested waves (2d) and the most probable storm maximum crest height $\tilde{\Xi}$.

The common approach used in the past is based on only accounting for the observed storms(Haver,2004). This approach further implies the assumption of equal probability of occurrence $\frac{1}{n_s}$ for each observed storm, with n_s being the number of storms, and zero probability for more severe storms than those included in the observed sample(Haver,2004). A pre-requisite for yielding reasonable estimates by this approach is to have a sufficient number of extreme storms, and hence having a sample representing the population data in a reasonable manner. The disadvantage with this method is that the effect of the non-observed events is not taken into account which in most of the cases leads to underestimation of the extremes(Haver,2013).

In order to overcome the dependency on amount of data and not least, include the contribution of the non-observed events in the distribution function for the storm maximum value, an alternative methodology has been developed(Haver,2004). The alternative procedure is based on that the storm maximum value can be described conditionally upon the most probable storm maximum $\tilde{\Xi}$ (Haver,2004). According to (Haver,2013), the distribution of the storm maximum value $F_{\Xi|\tilde{\Xi}}(\xi|\tilde{\xi})$ of a random storm exceeding the pre-defined threshold converges to an asymptotic form when considered conditionally upon the most probable storm maximum $\tilde{\Xi}$. The long term distribution of the most probable storm maximum $F_{\tilde{\Xi}}(\tilde{\xi})$ is obtained by fitting a probabilistic model to the storm sample of most probable storm maximum values(Haver,2004). The distribution $F_{\tilde{\Xi}}(\tilde{\xi})$ represents the long term variability of metocean condition(Haver,2015). In other words, the variability of storm peak significant wave height H_{s_p}, associated spectral peak period T_p and the storm profile are included in the most probable storm maximum $\tilde{\Xi}$ (Haver,2015).

The long term distribution of the maximum wave crest height $F_{\Xi}(\xi)$ of an arbitrary storm accounting for non-observed storm events reads then(Haver,2013):

$$F_{\Xi}(\xi) = \int_{\tilde{\xi}} F_{\Xi|\tilde{\Xi}}(\xi|\tilde{\xi})\, f_{\tilde{\Xi}}(\tilde{\xi}) d\tilde{\xi} \tag{75}$$

The marginal and conditional distributions of, respectively, peak significant wave height $H_{s_{peak}}$ and corresponding spectral peak period $T_{p_{peak}}$ will be established in section 4.2.1. In section 4.2.2, the methodology to establish the long term distribution of most probable storm maximum $\tilde{\Xi}$ and application will be presented. The development of conditional distribution of storm maximum value, followed by application, is demonstrated in section 4.2.3. The long term distribution of the maximum wave crest height and estimates of extremes are presented in section 4.2.4. The discussion and comparison of results with the all sea states approach results is presented in section 4.2.6.

4.2.1 Long term distribution of storm peak characteristics

As a first step, the peak significant wave height $H_{s_{peak}}$ and corresponding spectral peak period $T_{p_{peak}}$ values are extracted for each storm. The Matlab script is presented in Appendix C.3. Using the vector of storm peak significant wave height $H_{s_{peak}}$ and applying the method of moments, the parameters of the Weibull distribution are estimated. As an example, the parameter values for threshold $H_s = 7m$ are shown in Table 18.

Hs threshold = 7m	
Form parameter η	1,1293
Scale parameter σ	1,6357
Location parameter μ	7,0430

Table 18 Weibull distribution parameters, $F_{H_{s_{peak}}}(h)$

The conditional distribution of corresponding spectral peak period $T_{p_{peak}}$ is determined as presented in chapter 2. The coefficients of the function $\mu_{lnT_p(h)}$, calculated by using the least square method, are shown in Table 19.

Mean		
Variables		Sum squared differences
a_1	0,9363	0,00696418
a_2	1,1247	
a_3	0,1981	

Table 19 Parameters for $\mu_{lnT_p(h)}$

The Figure 29 shows the function $\mu_{lnT_p(h)}$ fitted to the hindcast data.

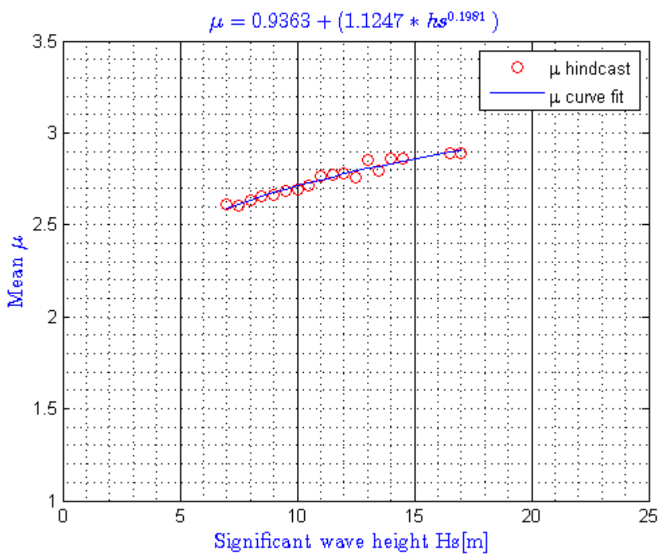

Figure 29 Fitted function mean lnTp, $\mu_{lnT_p(h)}$, POT

The parameters of the function ${\sigma^2}_{ln_{T_p(h)}}$ are shown in Table 20 and Table 21. Two cases are presented, one with coefficient $b_1 = 0.005$ and $b_1 = 0.001$. Both values of b_1 are fixed while b_2, b_3 are obtained by iteration, which is different from the approach presented in chapter 2. The reason for this is based on literature research and in order to set a lower limit for the variance.

Variance		
Variables		Sum squared differences
b_1	0,0050	0,0001
b_2	5,4676	
b_3	0,9273	

Table 20 Parameters for ${\sigma^2}_{ln_{T_p(h)}}(b_1 = 0.005)$, POT

Variance		
Variables		Sum squared differences
b_1	0,0010	0,0001
b_2	0,0595	
b_3	0,2462	

Table 21 Parameters for ${\sigma^2}_{ln_{T_p(h)}}(b_1 = 0.001)$, POT

The Figure 30 shows the function ${\sigma^2}_{ln_{T_p(h)}}$ fitted to the hindcast data.

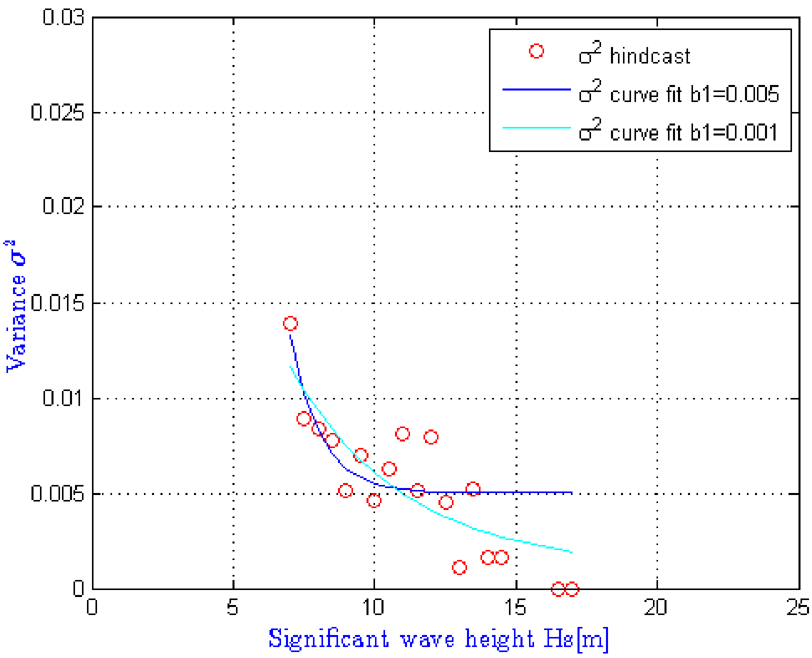

Figure 30 Fitted function variance ${\sigma^2}_{ln_{T_p(h)}}$, POT

Similar to equation (26) in section 3.1.2, the expression for the extreme corresponding to a q-annual probability reads(Haver,2014):

$$1 - F_{\Xi}(\xi_q) = \frac{q}{n_1} \quad (76)$$

Where:

$$n_1 = \frac{n_s}{Yr} = \frac{number\ of\ storms}{number\ of\ years\ (data)} \quad (77)$$

The estimated extreme significant wave heights H_s are calculated as following:

$$1 - F_{H_s}\left(h_{s_q}\right) = \frac{q}{n_1} \Rightarrow h_{s_q} = \mu + \sigma * \left[-\ln\left(\frac{q}{n_1}\right)\right]^{1/\eta} \quad (78)$$

The corresponding spectral peak period T_p are determined in identical manner as in section 3.1.2. The results for threshold $H_s = 7m$ are shown in Table 22.

	$(\frac{q}{n})^{-1}$	H_{s_q} [m]	T_{p_q} [m]
ULS	1355,1724	16,45	18,24
ALS	135517,24	21,61	20,73

Table 22 Extreme significant wave height H_{s_q} with T_{p_q} (threshold=7m)

The same procedure was applied for the remaining H_s thresholds i.e. 8, 9 and 10m. The respective Weibull distribution parameters and the extremes are shown in Appendix C.1. The summary of the results with the ULS and ALS levels obtained with the All Sea States approach are shown on Figure 31.

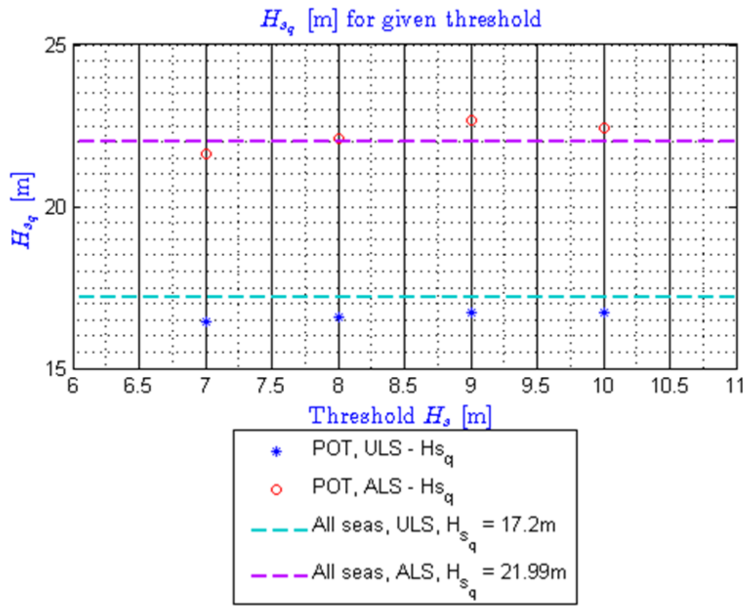

Figure 31 Peak significant wave height H_{s_q} vs. threshold

The Matlab script is presented in Appendix C.2.

4.2.2 Long term distribution of most probable storm maximum crest height

A storm sample given a threshold (e.g. significant wave height H_s level) can be either established from measurements or hindcast. As stated in introductory part of section 4.2, the assumption is that the storm sample is approximated by a sequence of stationary events with duration of 3-hours. The metocean characteristics H_s and T_p for each step i are constant, as previously assumed in the all seas states long term analysis (see section 2.2). Let $F_{\Xi_{3h|i}}(\xi|i)$ denote the distribution function for the maximum value $\Xi_{3h,i}$ for step i. Hence, if a storm include i 3-hr steps of stationary sequences, assumed statistically independent (Haver,2013) and characterized by constant H_s and T_p, then the distribution of the storm maximum value Ξ_s reads(N-003,2015):

$$F_{\Xi_s|storm}(\xi|storm) = P[\Xi_s \le \xi]$$

$$= P[(\Xi_{3h,1} \le \xi) \cap (\Xi_{3h,2} \le \xi) \cap \dots \cap (\Xi_{3h,i_{max}} \le \xi)] \overset{iid}{=} \prod_{i=1}^{i_{max}} F_{\Xi_{3h,i}}(\xi) \tag{79}$$

The most probable storm maximum $\tilde{\xi}_s$ is assumed to correspond to the median value on the density function curve $f_{\tilde{\Xi}}(\tilde{\xi})$, with a cumulative probability of ≈ 0.37. Hence it can be approximated by solving the following equation(Haver,2015):

$$\tilde{\xi}_s = F^{-1}_{\Xi_s|storm}(0.37) \tag{80}$$

The most probable maximum must be calculated for each storm, resulting eventually in a long term sample of most probable storm maximum values $\tilde{\Xi}$, $[\tilde{\xi}_{s,1}, \tilde{\xi}_{s,2} \ldots . \tilde{\xi}_{s,i_{max}}]$, reflecting the long term variability of the storm severity(Haver,2013). We treat the most probable storm maximum as a random variable , $\tilde{\Xi}$ for which the 3p Weibull model is adequate(Haver,2013):

$$F_{\tilde{\Xi}}(\tilde{\xi}) = 1 - exp\left\{-\left(\frac{\tilde{\xi} - \mu}{\sigma}\right)^{\eta}\right\} \tag{81}$$

We will now apply the methodology presented above and determine the long term distribution of the most probable maximum for a given threshold of significant wave height H_s. The analysis is split into three parts: *(i)* extract all storm and define the corresponding most probable maximum values $\tilde{\xi}$; *(ii)* estimate the three parameters for the Weibull model; *(iii)* calculate the long term distribution $F_{\tilde{\Xi}}(\tilde{\xi})$ given the threshold.

The Matlab script in Appendix C.3 permits to *(i)* extract all the 3-hr steps above the specified threshold. The result of this sequence is an array of cells containing the respective storm information i.e. the h_{s_i} and t_{p_i} for each 3-hr step within the storm s. The number of storms n_{storm} obviously varies with the threshold for the significant wave height H_s.

We now define a range of possible realizations of wave crest height Ξ. As an example, for threshold $H_s = 7m$ the range is chosen from $6.5 \rightarrow 25m$. For each value of wave crest height and each storm step i, the cumulative distribution is calculated using equation (33). The result of this sequence is then a matrix with i (storm step) rows and n_{crest} (number of wave crest height realizations) columns. By multiplying the respective distributions for each storm step i and each wave crest height realization, we obtain an array with n_{crest} cumulative distributions. In other words, we obtain the cumulative probabilities of nonexceeding the wave crest height realizations. Repeating this for all storms gives n_{storm} arrays with cumulative probabilities. From each array with n_{crest} cumulative distributions, we can deduct the most probable storm maximum. This is done by applying the equation (80) i.e. by finding the wave crest height value in the array giving a cumulative distribution closest to 0.37 we can then estimate the most probable storm maximum wave crest height.

The final part of this script permits to perform the following:

- visualize the relative frequency of $\tilde{\Xi}$ for a given threshold
- calculate the correlation between $\tilde{\Xi}$ and the significant wave height H_s

The probability histogram showing the relative frequency of $\tilde{\Xi}$ is obtained by counting the number of occurrences per classes, in a similar manner as performed for the scatter diagram in section 0. As an example, the probability histogram of $\tilde{\Xi}$ for threshold $H_s = 7m$ is shown in Figure 32. Based on visual examination, the histogram seems to fit well the Weibull model (density function) and hence it is plausible to continue the analysis with estimation of the parameters for the 3p Weibull distribution.

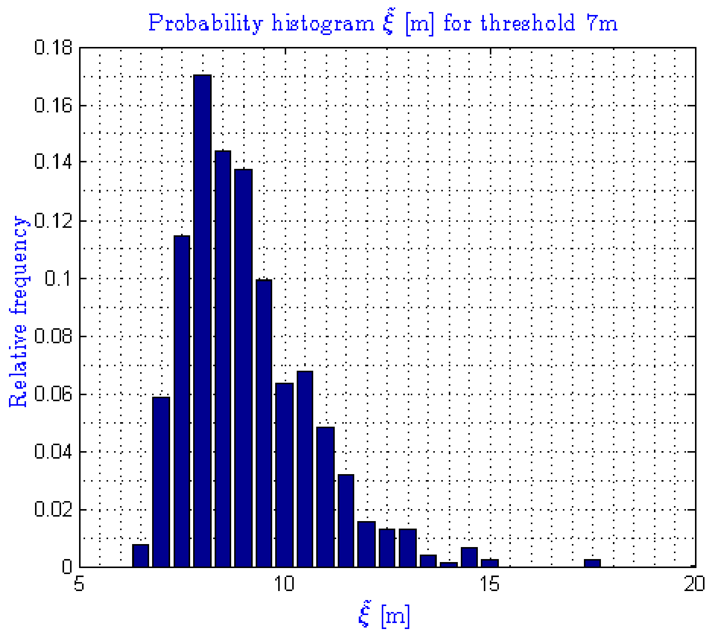

Figure 32 Probability histogram $\tilde{\xi}$, threshold 7m

The second verification consists of comparing the storm peak significant wave height $H_{s_{peak}}$ and the estimated most probable maximum $\tilde{\xi}$, for the given storm. Hence, for all the storms above threshold the maximum values of significant wave height H_s are extracted and plotted against the most probable maximum $\tilde{\xi}$. A high correlation factor between these two variables should be expected since Hs is the most important parameter and representative of the sea state. The resulting plot is shown in Figure 33.

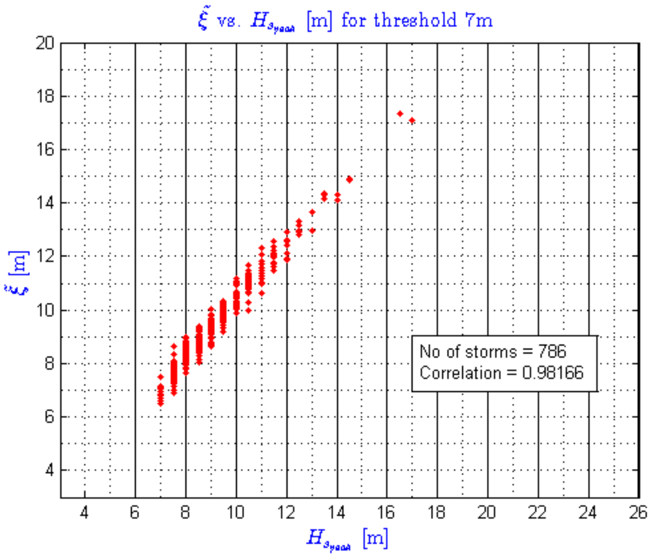

Figure 33 Correlation $\tilde{\xi}$ and $H_{s_{peak}}$, threshold 7m

The correlation factor is high indicating a strong dependence between the significant wave height H_s and the most probable maximum $\tilde{\xi}$, for a given threshold.

The same analysis is repeated for H_s threshold values of, respectively 8, 9 and 10m and the results are included in Appendix C.3.

The next step in the analysis consists of (ii) estimating the parameters in order to fit the Weibull distribution to the data. The number of storms and consequently the number of most probable storm maximum $\tilde{\xi}$ varies with given threshold. For example, given the threshold $H_s = 7m$, the number of storms is $n_s = 786$. The model is then fitted to this sample of data and we obtain four different fitted Weibull models (3-parameters) for the respective H_s thresholds i.e. 7, 8, 9 and 10m. The parameters are estimated using the method of moments. The parameters given the threshold $H_s = 7m$ are shown in Table 23.

Hs threshold = 7m	
Form parameter 'eta'	1,3155
Scale 'sigma'	2,1946
Location 'mu'	6,8317

Table 23 Weibull distribution parameters for $F_{\tilde{\Xi}}(\tilde{\xi})$, threshold 7m

The cumulative distribution $F_{\tilde{\Xi}}(\tilde{\xi})$ is calculated in a similar manner as the marginal distribution for H_S in the all sea states approach. By choosing a range of most probable storm maximum $\tilde{\xi}$ and dividing it into classes, the distributions are calculated for each class of $\tilde{\xi}$. The range of most probable storm maximum $\tilde{\xi}$ depends on the threshold and the starting value of the range is above the estimated location parameter μ. The result is a column vector of probabilities with the number of rows being the number of classes of most probable maximum $\tilde{\xi}$. The Matlab scripts used for are presented in Appendix C.5 and Appendix C.6.

The plot on Figure 34 shows the adequacy of the fitted model in comparison with the empirical data. The estimated parameters in Table 23 give a good fit of the Weibull model to the data, specifically in the upper tail of the distribution.

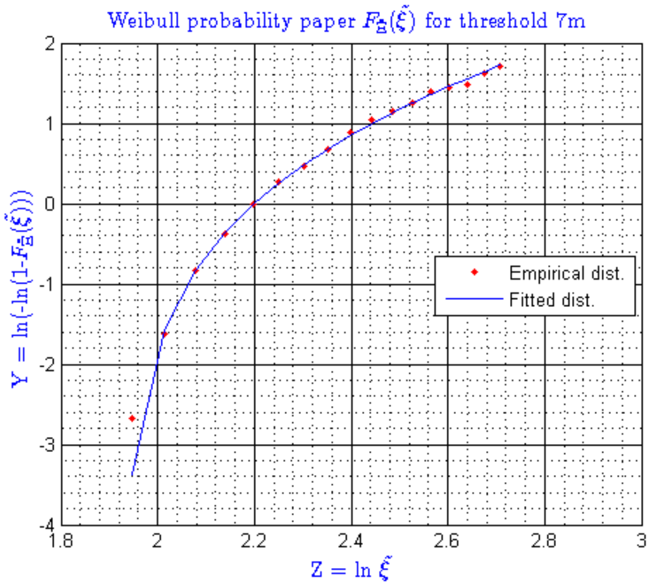

Figure 34 Probability paper, long term distribution $F_{\tilde{\Xi}}(\tilde{\xi})$, threshold 7m

The results for remaining H_S thresholds i.e. 8, 9 and 10m are obtained by repeating the exercise above and are presented in Appendix C.5.

4.2.3 Conditional (short term) distribution of storm maximum crest height

In order to take into account the short term variability of the maximum crest height around the most probable storm maximum $\tilde{\Xi}$ (Haver,2013), the conditional distribution of the storm maximum crest height Ξ given the most probable storm maximum $\tilde{\Xi}$ must be considered. The assumption is that it can be reasonably well approximated by the Gumbel distribution, see equation (72). Introducing the ratio $v = \frac{\xi}{\tilde{\xi}}$ under hypothesis that it follows a Gumbel distribution(Haver,2014) with a location parameter $\alpha_v = 1$ and scale parameter β_v constant for all storms, the distribution of V reads(Haver,2015):

$$F_V(v) = exp\left[-exp\left(-\frac{v-1}{\beta_v}\right)\right] \quad (82)$$

The hypotheses stated above must be verified for some few randomly chosen storms with different storm profiles (i.e. thresholds). One possible way to estimate the parameter β_v is by simulating possible realizations of storm maximum for each step of the storm. The possible realizations of the maximum value ξ_i for storm step i is calculated by introducing a randomly generated number r comprised in interval $[0,1]$ which replaces$F_{\Xi_{3h}|H_sT_1}(\xi|h_s, t_1, d)$, see equation (34):

$$\xi_i = \alpha_{F_i} \cdot h_{s_i} \cdot \left[-\ln\left(1 - \{r(i)\}^{t_{p_i}/_{14026}}\right)\right]^{1/\beta_{F_i}} \quad (83)$$

The Forristall parameters α_{F_i} and β_{F_i} are function of significant wave height h_{s_i} for the given storm step. From each simulation loop j comprising the realizations of all storm steps, the largest realization of wave crest height, $\xi_{i,j}$ is extracted. With a sufficient number of realizations of maximum values ξ_j and the calculated most probable storm maximum $\tilde{\xi}$ (see section 4.2.2), the mean of β_v can be estimated as the average of the values obtained from the randomly chosen storms(N-003,2015). Finally, using that(Haver,2013):

$$F_{\Xi|\tilde{\Xi}}(\xi|\tilde{\xi}) = P[\Xi \leq \xi|\tilde{\Xi} = \tilde{\xi}] = P[\mathrm{v} \cdot \tilde{\xi} \leq \xi] = P\left[\mathrm{v} \leq \frac{\xi}{\tilde{\xi}}\right] = F_v\left(\frac{\xi}{\tilde{\xi}}\right) \quad (84)$$

The expression for the conditional distribution of the storm maximum crest height Ξ given the most probable storm maximum $\tilde{\Xi}$ reads(Haver,2013):

$$F_{\Xi|\tilde{\Xi}}(\xi|\tilde{\xi}) = exp\left\{-exp\left\{-\frac{\xi - \tilde{\xi}}{\beta_v \cdot \tilde{\xi}}\right\}\right\} \quad (85)$$

Proceeding to the application of theory presented above, the first step is to estimate the Gumbel parameter β_v. For this purpose, three random storms given three different thresholds of significant wave height H_s have been extracted from the hindcast data. The chosen thresholds are, respectively, 6m (low), 10m (medium) and 14m (high). The low storm will be used as an example to demonstrate the main steps. The Matlab script is attached in Appendix C.7.

For the given storm, the most probable storm maximum $\tilde{\Xi}$ is estimated in identical manner as presented in section 4.2.1. Then, for each 3-hr step i of the given storm a random number comprised in interval $[0,1]$ and representing $F_{\xi|H_sT_1}$ is generated. Each random number is introduced into equation (83), giving a realization of the wave crest height ξ_i. The result for one run through each step of the storm is an array of i values of the realizations of ξ_i, out of which the highest values is extracted into a separate array of possible realizations of storm maximum wave crest height $\xi_{i_{max}}$. This simulation is repeated for 50 runs, giving eventually an array of 50 values of storm maximum wave crest height $\xi_{j_{max}}$. The next step is to divide each element of the array with values of $\xi_{j_{max}}$ by the most probable maximum value for the given storm $\tilde{\xi}$, giving in return an array of 50 values of variable v_k. This step is repeated for another 50 runs, giving in total a cell array of 50 cells each containing 50 values of variable v_k. The standard deviation for each of these arrays containing variables v_k is then calculated and used for estimating the Gumbel parameter β_v. The final result of these simulations is an array with 50 estimated values of $\hat{\beta}_{v_k}$. The mean value $\hat{\beta}_{v|storm_{mean}}$, calculated from the 50 estimated values of $\hat{\beta}_{v_k}$, is then used to calculate the distribution for the storm maximum wave crest height Ξ using equations (34) & (79). Then, the distribution of V is determined using equation (82). The resulting probability plots for low storm are presented in Figure 35 and Figure 36.

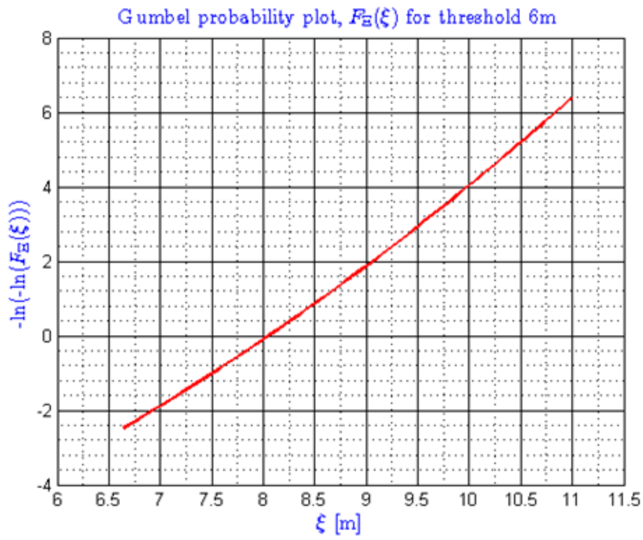

Figure 35 Distribution maximum crest height Ξ, low storm

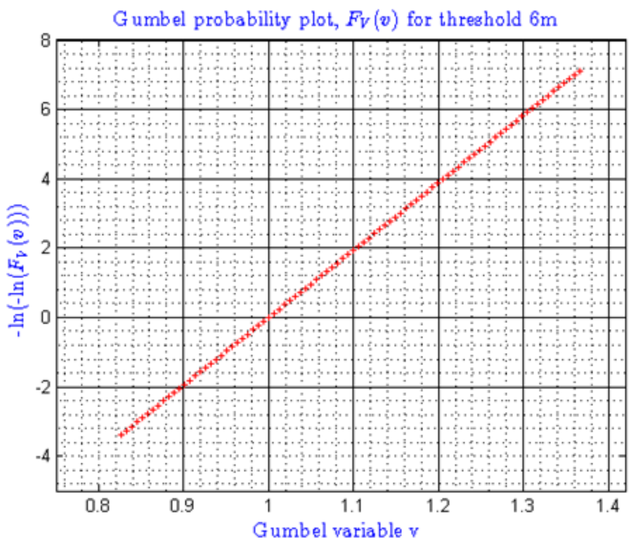

Figure 36 Distribution normalized storm maximum crest height V, low storm

The probability plot on Figure 36 shows a straight line with location parameter 1.0 and scale parameter $\hat{\beta}_{v|storm_{mean}}$. The probability plots for the medium and high storm are presented in Appendix C.8.

The mean value $\hat{\beta}_{v_{mean}}$, calculated as the mean of the respective $\hat{\beta}_{v|storm_{mean}}$, will be used for further calculations. The summary of results is presented in Table 24 .

Threshold [m]	$\hat{\beta}_{v\|storm_{mean}}$	$\hat{\beta}_{v\|storm_{min}}$	$\hat{\beta}_{v\|storm_{max}}$	$\hat{\beta}_{v_{mean}}$	$\tilde{\xi}_{storm}$
6	0,0521	0,0390	0,0697	0,0607	8,05
10	0,0637	0,0516	0,0786		12,6
14	0,0614	0,0514	0,0796		17,6

Table 24 Estimates for Gumbel parameter β_v and most probable storm maximum $\tilde{\xi}_{storm}$

The distributions of storm maximum wave crest height $F_{\Xi|\text{storm}}(\xi|storm)$, and normalized storm maximum $F_V(v)$, for the respective storms are shown on Figure 37. It should be noted that the distributions $F_{\Xi|\text{storm}}(\xi|storm)$ are on scale in order to facilitate comparison with the distributions $F_V(v)$.

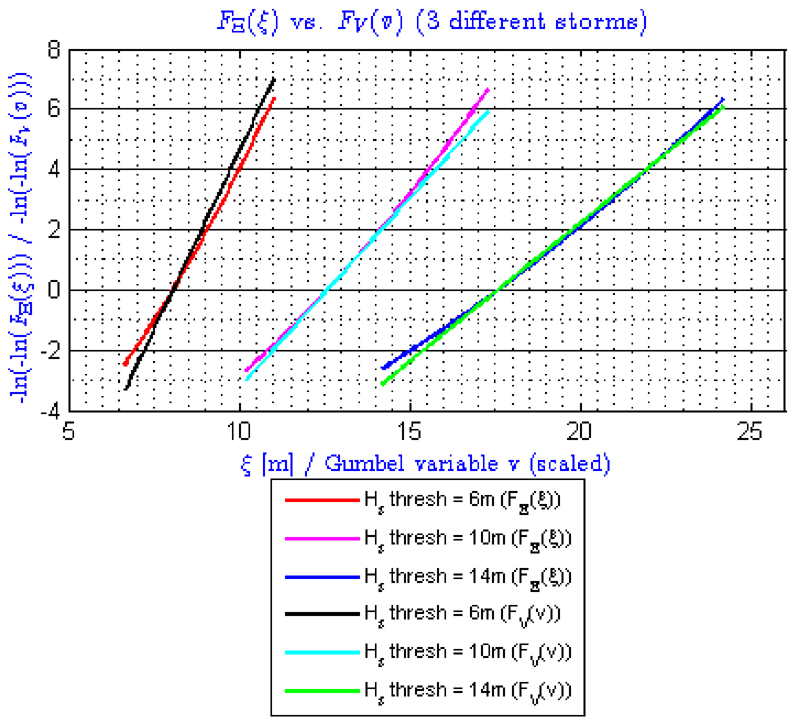

Figure 37 Gumbel probability plot $F_\Xi(\xi)$ vs. $F_V(v)$, 3 storms

The Figure 38 shows that the Gumbel parameter β_v for the three storms. The parameter β_v seems to be constant i.e. independent of the storm and threshold level, which verifies our second hypothesis.

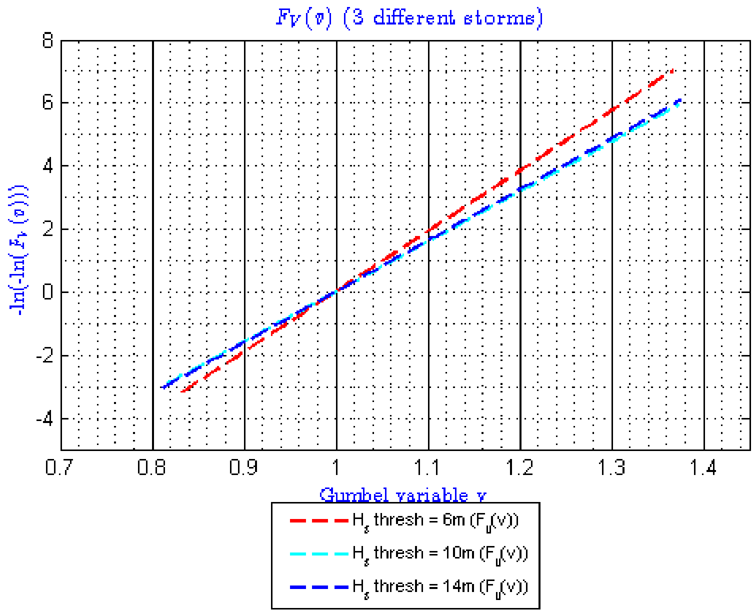

Figure 38 Gumbel probability plot $F_V(v)$, 3 storms

The Matlab script is presented in Appendix C.9.

The conditional (short term) distribution of Ξ given $\tilde{\Xi}$ is calculated using the Matlab script presented in Appendix C.10. The output is a matrix containing probabilities, with the number of rows being the number of classes of most probable maximum $\tilde{\xi}$. The number of columns corresponds to the range of possible crest height.

4.2.4 Long term distribution of storm maximum crest height

The long term distribution of storm maximum crest height $F_{\Xi}(\xi)$ is calculated in a similar way as performed with the all sea states method. The elements in column vector $F_{\tilde{\Xi}}(\tilde{\xi})$ (section 4.2.2) are multiplied with elements in matrix $F_{\Xi|\tilde{\Xi}}(\xi|\tilde{\xi})$ (section 4.2.3). This results in a matrix of long term distribution and the final step is to perform the integration (summation) over the columns (i.e. for each value of possible wave crest height ξ) giving the long term cumulative distribution $F_{\Xi}(\xi)$. The long term distribution is also represented on a Gumbel probability paper, which will be used to determine the extremes. As an example, the results for threshold $H_s = 7m$ are shown in Figure 39 and Figure 40. The plots for H_s thresholds 8, 9 and 10m are included in Appendix C.12.

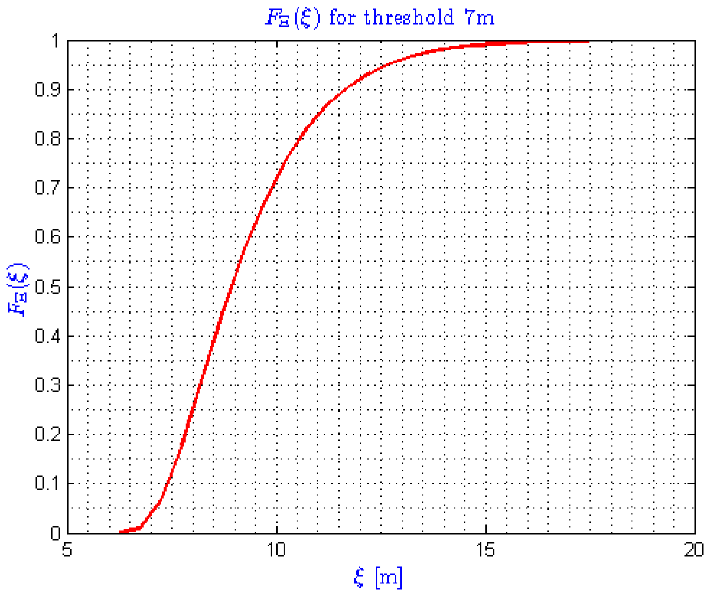

Figure 39 Cumulative distribution function $F_\xi(\xi)$, threshold 7m

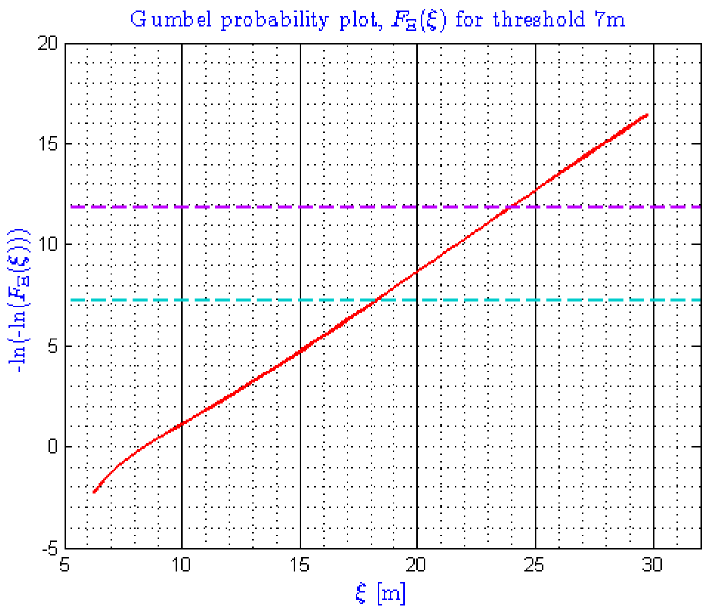

Figure 40 Gumbel probability plot $F_\xi(\xi)$, threshold 7m

In a similar manner as performed in chapter 3, the characteristic largest crest height ξ with probability of exceedance of 10^{-2}(ULS) and 10^{-4} (ALS) are determined using equations (76). As an example, the calculation of the 10^{-2}(ULS) extreme crest heights ξ for threshold $H_s = 7m$ is as following:

$$1 - F_{\Xi_{3h}}(\xi_{0.01}) = \frac{10^{-2} * 58}{786} = 7.379 * 10^{-4} \tag{86}$$
$$\Rightarrow Y = -\ln\left(-\ln\left(1 - \frac{10^{-2} * 58}{786}\right)\right) = 7.211$$
$$\Rightarrow \xi_{0.01} = 18.3m$$

The Table 25 summarizes the values obtained for the extreme of crest height, given the respective thresholds:

		ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
Threshold [m]	# storms N	ξ_{ULS} [m]	ξ_{ALS} [m]
7	786	18,3	24
8	446	18,3	24
9	226	18,5	24,4
10	109	18,6	24,5

Table 25 Extreme crest height ξ (POT)

The Figure 41 shows a summary of values obtained with the all sea states approach and Peak-Over-Threshold method, and as a function of the H_s threshold.

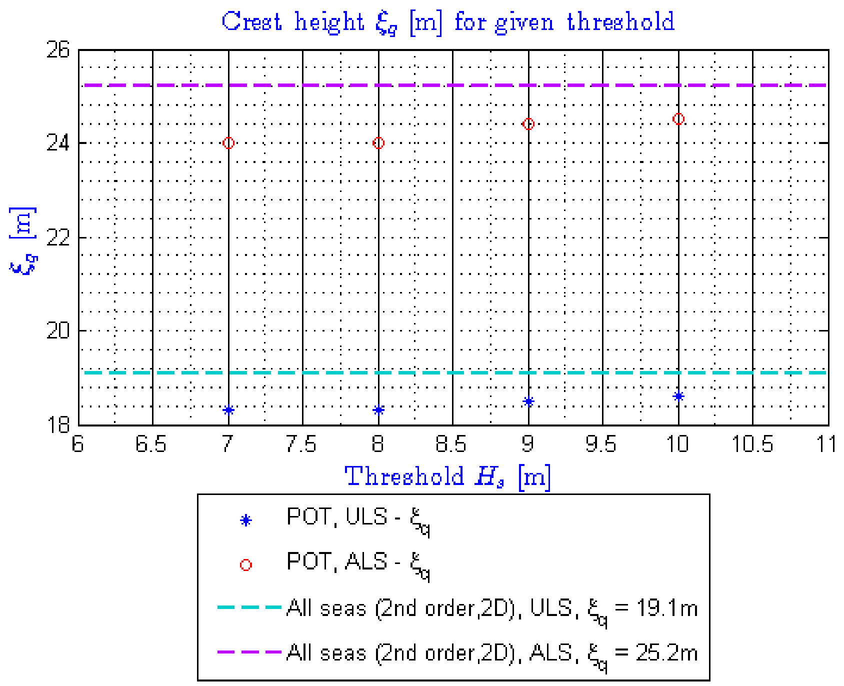

Figure 41 Extreme wave crest height ξ_q vs. threshold

4.2.5 Metocean contour lines

The metocean contour line using the fitted models for the storm peak characteristics are created in identical similar manner as in section 3.3.2. The resulting contour lines are presented in Figure 42 and Figure 43.

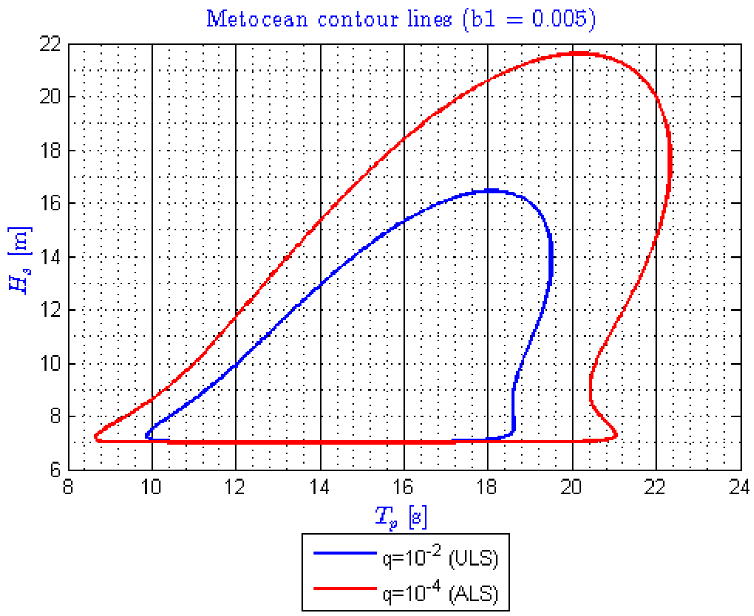

Figure 42 Metocean contour lines ($b_1 = 0.005$), POT, threshold $H_s = 7m$

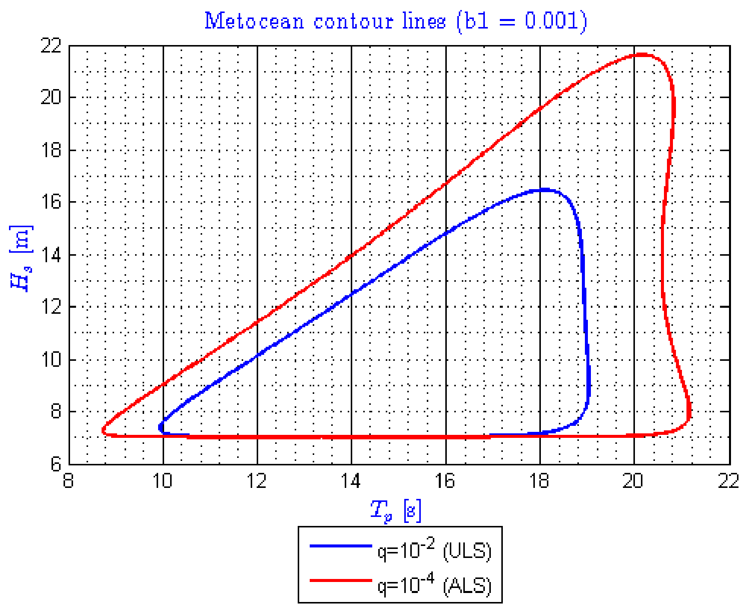

Figure 43 Metocean contour lines ($b_1 = 0.001$), POT, threshold $H_s = 7m$

The Matlab script is presented in Appendix C.13.

4.2.6 Discussion

The results obtained in sections 3.1.4 and 4.2.4 are shown in Table 26.

		Peak-Over-Threshold		All sea states max. crest 3hr (2nd, 2D)	
		ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)	ULS ($q = 10^{-2}$)	ALS ($q = 10^{-4}$)
Threshold [m]	0	ξ_{ULS} [m]	ξ_{ALS} [m]	ξ_{ULS} [m]	ξ_{ALS} [m]
7	0	18,3	24	19,1	25,2
8	0	18,3	24		
9	0	18,5	24,4		
10	0	18,6	24,5		

Table 26 Summary results crest height 2nd order 2D, All Sea States (3-hr max.) vs. POT

<u>Comparison 3-hours extreme crest height (2nd order, 2D) vs. POT extreme crest height</u>

The extreme wave crest height obtained using the 2nd order model and Peak-Over-Threshold method are shown in Table 26. The estimated extreme crest heights using the Peak-Over-Threshold approach are lower than the values from the 3-hours largest global maxima approach. This is as expected since the correlation between the respective storms is expected to be less than with the adjacent 3-hour largest maxima. The 3-hours largest global maxima approach gives results on the conservative side by 4-5% in comparison with the 2nd order model. This is in accordance with the recommendations in (N-003,2015). According to the same source(N-003,2015), the Peak-Over-Threshold approach gives the most accurate predictions but this statement cannot be confirmed based on the results available in this study. Further analysis and literature research should be done to be able to support this conclusion above.

<u>Influence of threshold level</u>

The extreme crest heights do not seem to vary much with the H_s threshold. The choice of the threshold level is the challenge with the Peak-Over-Threshold method. As per (N-003,2015), the threshold should be selected sufficiently high to ensure that the sampled values of the dominating characteristics are representative of the upper tail behaviour. The effect of the threshold level on the estimated extremes was studied and is summarized in Figure 41. The conclusion is that no distinguished trend in the estimated extreme wave crest heights could be established for threshold varying from $7 \rightarrow 10m$. However, it is suggested that for further analysis within this subject a threshold of $7 \leq H_s \leq 8m$ is adopted. Choosing a threshold above $8m$ would lead to more uncertainty since the amount of data may be insufficient. A suggestion for further work on the analysis of the influence of the threshold level would be to perform similar analysis for other locations in the NCS.

<u>Time interval between two storms</u>

The Matlab script (e.g. Appendix C.3) used for extracting the storm characteristics above a given threshold is built such that it extracts all the 3-hours stationary sea states above threshold. In other words, the length of time interval between two consecutive storms is not taken into account and all storms are treated as independent events. This is not representative of the real ocean climate since the sea states in the “second storm” depend to a high degree on the first storm. Hence, the present script may lead to erroneous estimates since the assumption of storm independence is compromised in such cases. Furthermore, the number of 3-hour sea states used for establishing the distribution model is not fully representative of the real storm conditions.

There are several suggestions found in the literature on how to approach this subject. The suggestion in(Neelamani,2009) is that a time interval of two to four days suffices to assume two independent of storms. It is understood from this suggestion that if the condition above is not fulfilled, then the two consecutive storms are considered as one. However, this should be clarified before proceeding according to this criterion.

Another approach to the consideration above would be as suggested in (Tromans,1995). If the significant wave height H_s between the two peaks is less than 80% of the lowest peak, they are broken at the trough to form two storms. Otherwise they are maintained as one storm.

<u>Storm duration</u>

Another aspect to consider in the presented Matlab scripts is to take into account the storm duration. Since the most probable maximum $\tilde{\Xi}$ is controlled by sea states within 80% of the peak significant wave height $H_{s_{peak}}$ (Tromans,1995), another suggestion is to discard time storm sea states with significant wave height H_s less than 30-40% of the peak significant wave height $H_{s_{peak}}$ (Tromans,1995) .

Comments on Gumbel parameter

The estimates of the Gumbel parameter $\hat{\beta}_v$ vary from simulation to simulation, due to the randomness introduced by Monte Carlo simulations. Sensitivity analyses were performed with increasing number of simulations to 100. The range of values kept constant and it can be concluded that the suggested methodology and value seem to be a reasonable choice within this study.

Storm peak significant wave height $H_{s_{peak}}$

The summary of peak significant wave height $H_{s_{peak}}$ using the Peak-Over-Threshold and the All Sea States approaches as a function of threshold level are shown on Figure 31 . The ULS and ALS estimates obtained from the All Sea States approach are included for comparison. The estimated peak significant wave height $H_{s_{peak}}$ obtained using the two methods are similar. Furthermore, there is no indication of a trend in the peak significant wave height $H_{s_{peak}}$ when the threshold value is changing from 7 to $10m$.

5 CONCLUSION & FURTHER WORK

The motivation of the presented work was to compare and discuss similarities and differences between the extreme waves estimated by two long-term analysis methods: All Sea States and Peak-Over-Threshold. An overall conclusion of this thesis is that the extreme wave crest heights, using the second order model for long-crested waves (All Sea States, 3-hours largest global maxima), obtained with the Peak-Over-Threshold are less conservative than estimates from the All Sea States approach. This is likely due to the higher degree of independence with the storm approach i.e. the assumption of independency of maxima seems to be fulfilled to a higher degree than with the All Sea States. This results in less conservatism with the Peak-Over-Threshold approach. The practical implication of less conservative estimates obtained with the Peak-Over-Threshold method is a more optimized design of offshore structures without compromising the safety aspect.

The effect of the threshold level on the estimated extremes was studied. The conclusion is that there is no distinguished trend in the estimated wave crest height for threshold varying from $7 \rightarrow 10m$. However, it is suggested that for further analysis a threshold of $7 \leq H_s \leq 8m$ is adopted. Furthermore, analyses for several locations in the NCS could be performed for comparison with the present study.

<u>All Sea States, 3-hours largest global maxima</u>

The 3-hours maxima extremes obtained by the All Sea States method and using the Gaussian model were compared to the 2nd order model for long-crested waves (2D). The conclusion is that the 2nd order model (2D) predicts higher wave crest heights than those obtained with the Gaussian model. The estimated 3-hours extreme crest heights using the 2nd order model for short-crested waves (3D) are lower than those obtained with the 2nd order model for long-crested waves.

<u>All Sea States, all global maxima</u>

The all global maxima approach was used to estimate the extreme crest heights with the 2nd order model, for both short-crested and long-crested waves. The values obtained with the all global maxima approach are higher than those obtained using the 3-hours largest global maxima approach. By choosing the all global maxima approach, the estimates for the extreme wave crest heights increase by 2-3% in comparison with the 3-hours extremes. The difference gets more pronounced as the return period decreases (i.e. q-probability increases). It has been concluded that the conservatism in the values with the all global maxima approach is likely due to the correlation between the adjacent maxima.

Metocean contour line method, All Sea States

The metocean contour line method was demonstrated using the metocean model of weather characteristics created Heidrun hindcast data. The effect on the metocean contours induced by the coefficient b_1 from the fitted conditional distribution of T_p was studied. The fixed value of $b_1 = 0.005$ gives a larger band for the T_p than $b_1 = 0$, and hence “inflates” the metocean contours. This results in several possible sea states. Similarly, for $b_1 = 0$ the metocean contours are narrower and hence covering a smaller area of possible sea states. The recommended percentile levels stipulated in NORSOK N003 were verified using the extreme crest heights obtained with All Sea States approach, 2nd order long-crested. It can be concluded that the suggested percentile levels correspond well to the results obtained in this study.

Metocean contour line method, Peak-Over-Threshold

The application of metocean contour line method was introduced using the storm peak characteristics. The marginal distribution for the peak significant wave height $H_{s_{peak}}$ and the conditional distribution of corresponding spectral peak period $T_{p_{peak}}$ were used to establish the contour lines. Similar findings were observed as with the All Sea States method. A larger value of b_1 has tendency of “inflating” the metocean contour lines and hence covering area with several possible sea states.

The verification of percentile levels was not performed due to lack of time and is one of the suggestions for future work within this subject.

Suggestions for further work

- Effect of wave breaking on the q-probability estimates. Due to lack of time, this subject could not be accomplished. The aim is investigate whether the wave would break before the estimated q-probability wave crest height is reached. A possible approach, suggested by Professor Haver, is to:

i) Fit a 3p Weibull distribution model to the long term distribution of 3-hours wave crest maxima obtained in section 3.1.4
ii) Simulate possible crest height realizations by using the analytical 3p-Weibull distribution obtained above
iii) Determine the distribution of the corresponding spectral peak period T_p
iv) Relate the crest height realizations with corresponding T_p to the wave breaking (steepness) criterion
v) Quantify the number of realization above the breaking criterion

- Verification of percentile levels with the storm peak characteristics obtained with the Peak-Over-Threshold method.
- Implement the changes in the Peak-Over-Threshold model (Matlab script) as suggested in section 0.
- Further analysis of optimal value of coefficient b_1 when fitting the conditional distribution of T_p

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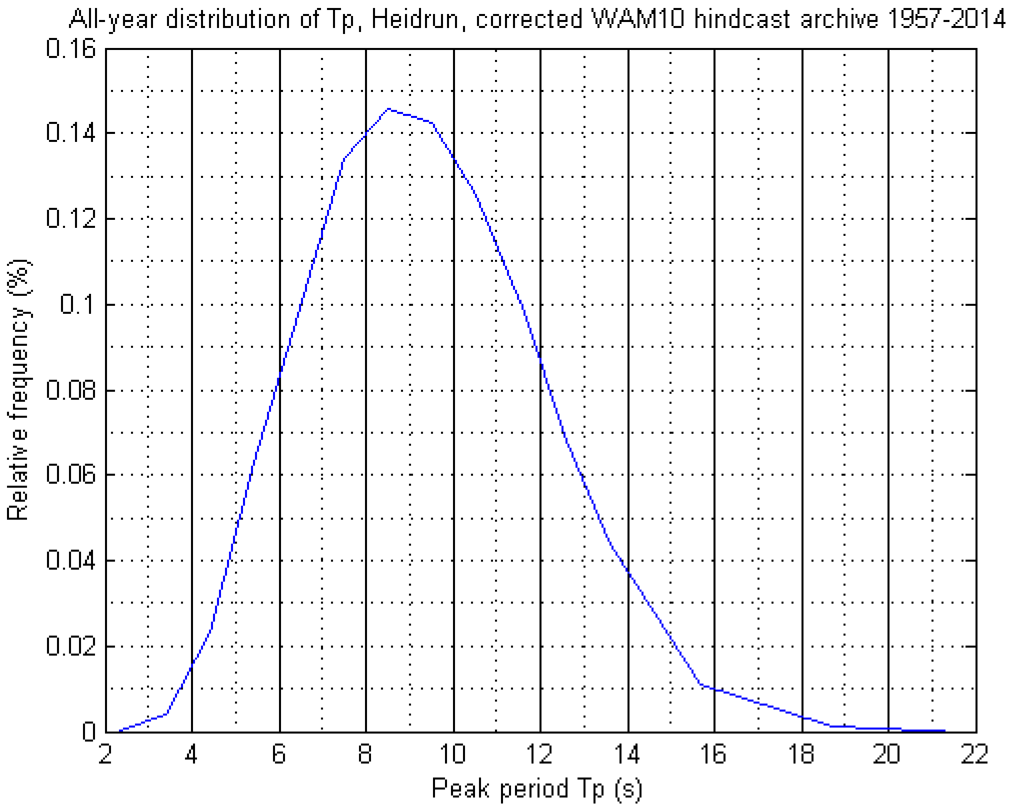
Appendix A METOCEAN MODELLING

Appendix A.1 WAVE SCATTER DIAGRAM

```
%*Correction of WAM10 hindcast peak periods tp*

clear all
close all
clc

%load data
load('WAM10.mat');
%vectors hs and tp(not corrected)
hs = Heidrun_WAM10(:,5);
tp_nc = Heidrun_WAM10(:,6);

%apply correction procedure as per "The peak period in the WAM10 hindcast
%archive" (Nygaard, 2009)
K=log(tp_nc/3.244);
I= round(1+(K/0.09525));
rand('twister',0);
%create a column vector of uniformly distributed random numbers
random = rand(size(tp_nc));
%create a column vector of corrected peak period values
tp=3.244*exp(0.09525*(I-0.5-random));

%*Relative frequency corrected WAM10 hindcast peak periods tp*

%define # of intervals
numIntervals_Q=20;
% define interval width
intervalWidth_Q=(max(tp)-min(tp))/numIntervals_Q;
%define S-axis, range of peak periods
S = 2.3601:intervalWidth_Q:22.8194;
%histogram count
ncount=histc(tp,S);
%define Q-axis, relative frequency of peak periods
relativeFrequency_Q = ncount/length(tp);
figure(3)
plot(S,relativeFrequency_Q);
title('All-year distribution of Tp, Heidrun, corrected WAM10 hindcast archive 1957-2014');
xlabel('Peak period Tp (s)');
ylabel('Relative frequency (%)');
grid on
grid minor

%*Wave scatter diagram*

%define number of columns
nBinsTp=25;
%define number of rows
nBinsHs=50;
%define range tp
```

```
maxTp=25;
%define range hs
maxHs=25;
%create zero matrix
Wave_Scatter = zeros(nBinsHs,nBinsTp);
%count occurences with resolution 0.5m(hs) and 1s(tp)
for k=1:length(hs);
    i=ceil((nBinsHs/maxHs)*hs(k));
    j=ceil((nBinsTp/maxTp)*tp(k));
    Wave_Scatter(i,j)=Wave_Scatter(i,j)+1;
end
Wave_Scatter;
%column vector of sum of occurences for a given Hs
Sum_rows=sum(Wave_Scatter,2);
%row vector of sum of occurences for a given Tp
Sum_columns=sum(Wave_Scatter,1);
```



Appendix A.2 LONG TERM JOINT DISTRIBUTION OF SEA STATES ($b_1 = 0.005$)

	$t_{p_{mid}}$																								
$h_{s_{mid}}$	0,5	1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5	9,5	10,5	11,5	12,5	13,5	14,5	15,5	16,5	17,5	18,5	19,5	20,5	21,5	22,5	23,5	24,5
0,75	1,19E-13	2,01E-07	6,52E-05	1,22E-03	0,005643559	0,01193179	0,015669	0,01521688	0,01211927	0,0084414	0,00536058	0,00319135	0,0018157	1,00E-03	5,40E-04	2,87E-04	1,51E-04	7,89E-05	4,11E-05	2,15E-05	1,12E-05	5,87E-06	3,08E-06	1,63E-06	8,64E-07
1,25	4,55E-16	1,40E-08	1,67E-05	0,000671913	0,005263534	0,01617971	0,0279121	0,03314632	0,03065012	0,02385358	0,01644119	0,0103887	0,00616419	0,00349399	0,00191562	0,00102529	0,00053943	0,00028045	0,00014466	7,43E-05	3,80E-05	1,95E-05	9,98E-06	5,13E-06	2,64E-06
1,75	4,36E-19	3,19E-10	1,56E-06	1,41E-04	0,001896012	0,00854138	0,0194189	0,02818654	0,03016244	0,02608608	0,01937748	0,01288806	0,00790226	0,00456132	0,00251695	0,00134298	0,00069893	0,00035714	0,0001801	9,00E-05	4,47E-05	2,21E-05	1,09E-05	5,40E-06	2,67E-06
2,25	1,72E-22	4,30E-12	1,01E-07	2,24E-05	0,000539641	0,00366698	0,0112139	0,02018311	0,02523567	0,02439674	0,01958912	0,01372335	0,0086853	0,00509229	0,0028177	0,00149197	0,000764	0,00038144	0,00018685	9,03E-05	4,32E-05	2,05E-05	9,69E-06	4,57E-06	2,15E-06
2,75	2,45E-26	3,25E-14	4,45E-09	2,68E-06	1,24E-04	0,00131835	0,0055848	0,01274096	0,01891807	0,02068367	0,01809618	0,0134219	0,00879274	0,00524157	0,00290675	0,00152468	0,0007661	0,00037238	0,00017645	8,20E-05	3,76E-05	1,70E-05	7,65E-06	3,42E-06	1,53E-06
3,25	1,11E-30	1,28E-16	1,27E-10	2,37E-07	2,25E-05	0,00039593	0,002408	0,00715559	0,01288207	0,01617895	0,01560159	0,01235136	0,00842134	0,00512094	0,00285045	0,00148119	0,00072943	0,00034444	0,00015739	7,01E-05	3,06E-05	1,32E-05	5,60E-06	2,36E-06	9,92E-07
3,75	1,41E-35	2,43E-19	2,26E-12	1,50E-08	3,19E-06	9,82E-05	0,0008936	0,00356916	0,00798294	0,01173558	0,01265254	0,01080537	0,00772602	0,00481711	0,00269974	0,00139179	0,0006718	0,00030784	0,00013538	5,76E-05	2,39E-05	9,73E-06	3,90E-06	1,54E-06	6,06E-07
4,25	4,45E-41	2,07E-22	2,35E-14	6,51E-10	3,45E-07	1,98E-05	2,82E-04	0,00157085	0,00448674	0,00788874	0,00966929	0,00902212	0,00682959	0,00439581	0,00249192	0,00127799	0,00060535	0,00026916	0,00011378	4,62E-05	1,82E-05	6,96E-06	2,62E-06	9,68E-07	3,54E-07
4,75	3,08E-47	7,37E-26	1,37E-16	1,88E-11	2,77E-08	3,19E-06	7,51E-05	0,00060457	0,00227288	0,0048956	0,00695222	0,00719266	0,00582959	0,00390604	0,00225313	0,0011541	0,00053769	0,00023218	9,43E-05	3,65E-05	1,36E-05	4,88E-06	1,71E-06	5,90E-07	2,00E-07
5,25	4,24E-54	1,03E-29	4,27E-19	3,48E-13	1,60E-09	4,04E-07	1,66E-05	2,01E-04	0,00102966	0,00278883	0,00468541	0,00546464	0,00480391	0,00338416	0,00200103	0,00102907	0,00047317	0,00019879	7,76E-05	2,86E-05	1,00E-05	3,39E-06	1,11E-06	3,53E-07	1,10E-07
5,75	1,06E-61	5,27E-34	6,80E-22	4,00E-15	6,56E-11	3,93E-08	3,00E-06	5,74E-05	0,00041355	0,00144842	0,00294478	0,00394302	0,00381461	0,00285763	0,00174723	0,000908	0,00041393	0,00016971	6,38E-05	2,24E-05	7,41E-06	2,34E-06	7,11E-07	2,10E-07	6,03E-08
6,25	4,54E-70	9,55E-39	5,35E-25	2,77E-17	1,85E-12	2,89E-09	4,37E-07	1,39E-05	1,46E-04	0,00068082	0,00171578	0,00268986	0,00290976	0,00234776	0,00149936	0,00079351	0,00036072	0,00014491	5,26E-05	1,76E-05	5,49E-06	1,62E-06	4,58E-07	1,25E-07	3,28E-08
6,75	3,19E-79	5,92E-44	2,03E-28	1,13E-19	3,52E-14	1,58E-10	5,08E-08	2,81E-06	4,49E-05	0,00028748	0,00092093	0,00172587	0,00212365	0,00187154	0,00126262	0,00068663	0,00031344	0,00012403	4,36E-05	1,39E-05	4,12E-06	1,14E-06	2,98E-07	7,46E-08	1,80E-08
7,25	3,68E-89	1,24E-49	3,64E-32	2,70E-22	4,46E-16	6,35E-12	4,66E-09	4,73E-07	1,19E-05	1,08E-04	0,00045249	0,00103578	0,00147616	0,00144248	0,00104088	0,00058758	0,00027156	0,00010653	3,65E-05	1,12E-05	3,13E-06	8,11E-07	1,97E-07	4,54E-08	1,00E-08
7,75	7,28E-100	8,90E-56	3,11E-36	3,71E-25	3,74E-18	1,86E-13	3,34E-10	6,57E-08	2,72E-06	3,62E-05	0,00020232	0,00057825	0,00097249	0,00107055	0,00083739	0,00049623	0,00023435	9,18E-05	3,09E-05	9,12E-06	2,43E-06	5,90E-07	1,33E-07	2,84E-08	5,73E-09
8,25	2,67E-111	2,27E-62	1,28E-40	2,98E-28	2,07E-20	3,94E-15	1,86E-11	7,48E-09	5,32E-07	1,07E-05	8,19E-05	0,00029876	0,00060423	0,00076167	0,000655	0,00041242	0,00020109	7,94E-05	2,63E-05	7,55E-06	1,92E-06	4,41E-07	9,30E-08	1,83E-08	3,38E-09
8,75	2,08E-123	2,19E-69	2,65E-45	1,42E-31	7,69E-23	6,10E-17	8,10E-13	6,99E-10	8,85E-08	2,76E-06	2,99E-05	1,42E-04	0,00035242	0,00051718	0,00049611	0,00033617	0,00017114	6,88E-05	2,27E-05	6,36E-06	1,56E-06	3,39E-07	6,70E-08	1,22E-08	2,08E-09
9,25	4,08E-136	8,73E-77	2,89E-50	4,15E-35	1,95E-25	6,95E-19	2,75E-14	5,36E-11	1,26E-08	6,28E-07	9,82E-06	6,21E-05	0,00019216	0,00033368	0,00036234	0,00026773	0,00014399	5,95E-05	1,97E-05	5,45E-06	1,29E-06	2,69E-07	5,01E-08	8,52E-09	1,34E-09
9,75	2,50E-149	1,62E-84	1,79E-55	7,81E-39	3,46E-28	5,95E-21	7,41E-16	3,39E-12	1,52E-09	1,26E-07	2,90E-06	2,49E-05	9,76E-05	0,00020377	0,00025411	0,00020749	0,00011934	5,13E-05	1,73E-05	4,75E-06	1,10E-06	2,20E-07	3,90E-08	6,22E-09	9,10E-10
10,25	6,18E-163	1,60E-92	6,81E-61	9,98E-43	4,47E-31	3,91E-23	1,60E-17	1,79E-13	1,58E-10	2,21E-08	7,68E-07	9,11E-06	4,61E-05	1,17E-04	0,00017047	0,00015584	9,71E-05	4,39E-05	1,52E-05	4,19E-06	9,57E-07	1,86E-07	3,16E-08	4,77E-09	6,53E-10
10,75	8,08E-177	9,87E-101	1,77E-66	9,25E-47	4,41E-34	2,05E-25	2,84E-19	7,95E-15	1,43E-11	3,44E-09	1,83E-07	3,05E-06	2,02E-05	6,36E-05	0,00010902	0,000113	7,71E-05	3,71E-05	1,33E-05	3,74E-06	8,50E-07	1,62E-07	2,66E-08	3,84E-09	4,97E-10
11,25	7,49E-191	4,45E-109	3,48E-72	6,71E-51	3,50E-37	8,79E-28	4,24E-21	3,04E-16	1,12E-12	4,77E-10	3,96E-08	9,38E-07	8,23E-06	3,24E-05	6,63E-05	7,88E-05	5,96E-05	3,09E-05	1,17E-05	3,36E-06	7,69E-07	1,45E-07	2,33E-08	3,24E-09	4,00E-10
11,75	7E-205	1,74E-117	5,81E-78	4,10E-55	2,35E-40	3,23E-30	5,47E-23	1,02E-17	7,88E-14	5,94E-11	7,79E-09	2,65E-07	3,11E-06	1,55E-05	3,82E-05	5,27E-05	4,47E-05	2,52E-05	1,01E-05	3,02E-06	7,05E-07	1,33E-07	2,11E-08	2,86E-09	3,40E-10
12,25	7,33E-219	6,96E-126	9,16E-84	2,29E-59	1,42E-43	1,06E-32	6,30E-25	3,06E-19	4,97E-15	6,73E-12	1,40E-09	6,92E-08	1,10E-06	6,96E-06	2,09E-05	3,37E-05	3,24E-05	2,01E-05	8,64E-06	2,71E-06	6,52E-07	1,25E-07	1,97E-08	2,64E-09	3,05E-10
12,75	1,34E-232	3,33E-134	1,52E-89	1,26E-63	8,09E-47	3,23E-35	6,68E-27	8,41E-21	2,87E-16	7,00E-13	2,33E-10	1,68E-08	3,61E-07	2,94E-06	1,08E-05	2,06E-05	2,27E-05	1,56E-05	7,27E-06	2,42E-06	6,06E-07	1,19E-07	1,89E-08	2,52E-09	2,87E-10
13,25	5,14E-246	2,20E-142	2,92E-95	7,28E-68	4,63E-50	9,53E-38	6,71E-29	2,16E-22	1,55E-17	6,77E-14	3,61E-11	3,79E-09	1,11E-07	1,17E-06	5,31E-06	1,20E-05	1,52E-05	1,18E-05	5,98E-06	2,13E-06	5,62E-07	1,14E-07	1,85E-08	2,48E-09	2,82E-10
13,75	5,02E-259	2,27E-150	7,10E-101	4,76E-72	2,78E-53	2,82E-40	6,57E-31	5,34E-24	7,89E-19	6,17E-15	5,25E-12	8,07E-10	3,23E-08	4,40E-07	2,47E-06	6,70E-06	9,82E-06	8,56E-06	4,81E-06	1,85E-06	5,18E-07	1,10E-07	1,84E-08	2,50E-09	2,86E-10
14,25	1,47E-271	4,06E-158	2,35E-106	3,70E-76	1,83E-56	8,67E-43	6,45E-33	1,29E-25	3,87E-20	5,36E-16	7,24E-13	1,62E-10	8,89E-09	1,57E-07	1,10E-06	3,56E-06	6,07E-06	6,02E-06	3,75E-06	1,58E-06	4,72E-07	1,05E-07	1,84E-08	2,57E-09	2,99E-10
14,75	1,47E-283	1,36E-165	1,12E-111	3,58E-80	1,37E-59	2,85E-45	6,48E-35	3,09E-27	1,86E-21	4,49E-17	9,56E-14	3,12E-11	2,32E-09	5,31E-08	4,62E-07	1,80E-06	3,61E-06	4,08E-06	2,84E-06	1,31E-06	4,23E-07	1,01E-07	1,84E-08	2,67E-09	3,19E-10
15,25	5,43E-295	9,04E-173	8,06E-117	4,47E-84	1,19E-62	1,02E-47	6,81E-37	7,51E-29	8,82E-23	3,67E-18	1,22E-14	5,74E-12	5,81E-10	1,72E-08	1,86E-07	8,76E-07	2,05E-06	2,66E-06	2,09E-06	1,06E-06	3,73E-07	9,50E-08	1,84E-08	2,79E-09	3,45E-10
15,75	7,79E-306	1,24E-179	9,03E-122	7,39E-88	1,24E-65	4,10E-50	7,60E-39	1,87E-30	4,19E-24	2,95E-19	1,51E-15	1,02E-12	1,40E-10	5,33E-09	7,17E-08	4,07E-07	1,12E-06	1,67E-06	1,48E-06	8,36E-07	3,21E-07	8,82E-08	1,82E-08	2,91E-09	3,76E-10
16,25	0,00E+00	3,58E-186	1,60E-126	1,64E-91	1,57E-68	1,85E-52	9,12E-41	4,85E-32	2,02E-25	2,35E-20	1,83E-16	1,76E-13	3,24E-11	1,59E-09	2,66E-08	1,82E-07	5,90E-07	1,01E-06	1,02E-06	6,39E-07	2,69E-07	8,03E-08	1,77E-08	3,02E-09	4,11E-10
16,75	0	2,18E-192	4,53E-131	4,97E-95	2,44E-71	9,58E-55	1,19E-42	1,31E-33	9,89E-27	1,88E-21	2,20E-17	2,97E-14	7,30E-12	4,59E-10	9,48E-09	7,82E-08	2,98E-07	5,92E-07	6,73E-07	4,73E-07	2,20E-07	7,15E-08	1,70E-08	3,10E-09	4,46E-10
17,25	0	2,76E-198	2,04E-135	2,05E-98	4,65E-74	5,68E-57	1,69E-44	3,75E-35	4,97E-28	1,51E-22	2,61E-18	4,91E-15	1,60E-12	1,28E-10	3,27E-09	3,24E-08	1,45E-07	3,33E-07	4,30E-07	3,39E-07	1,75E-07	6,21E-08	1,60E-08	3,13E-09	4,79E-10
17,75	0	7,12E-204	1,44E-139	1,14E-101	1,09E-76	3,87E-59	2,63E-46	1,13E-36	2,57E-29	1,22E-23	3,08E-19	7,99E-16	3,43E-13	3,48E-11	1,09E-09	1,30E-08	6,84E-08	1,81E-07	2,66E-07	2,36E-07	1,34E-07	5,25E-08	1,47E-08	3,10E-09	5,08E-10
18,25	0	3,63E-209	1,59E-143	8,57E-105	3,14E-79	3,03E-61	4,51E-48	3,62E-38	1,38E-30	1,00E-24	3,64E-20	1,29E-16	7,21E-14	9,22E-12	3,54E-10	5,04E-09	3,12E-08	9,52E-08	1,59E-07	1,58E-07	1,01E-07	4,32E-08	1,32E-08	3,01E-09	5,30E-10
18,75	0	3,54E-214	2,65E-147	8,55E-108	1,10E-81	2,73E-63	8,46E-50	1,23E-39	7,63E-32	8,38E-26	4,31E-21	2,05E-17	1,49E-14	2,39E-12	1,12E-10	1,90E-09	1,38E-08	4,85E-08	9,22E-08	1,03E-07	7,28E-08	3,45E-08	1,16E-08	2,86E-09	5,41E-10
19,25	0	6,31E-219	6,56E-151	1,12E-110	4,66E-84	2,80E-65	1,74E-51	4,44E-41	4,38E-33	7,12E-27	5,12E-22	3,25E-18	3,04E-15	6,07E-13	3,45E-11	6,96E-10	5,89E-09	2,39E-08	5,17E-08	6,51E-08	5,12E-08	2,68E-08	9,82E-09	2,64E-09	5,40E-10
19,75	0	1,98E-223	2,35E-154	1,89E-113	2,36E-86	3,25E-67	3,90E-53	1,70E-42	2,61E-34	6,17E-28	6,11E-23	5,13E-19	6,11E-16	1,51E-13	1,04E-11	2,49E-10	2,45E-09	1,15E-08	2,82E-08	3,98E-08	3,49E-08	2,02E-08	8,12E-09	2,38E-09	5,26E-10
20,25	0	1,05E-227	1,19E-157	4,05E-116	1,42E-88	4,27E-69	9,50E-55	6,86E-44	1,61E-35	5,45E-29	7,35E-24	8,08E-20	1,22E-16	3,71E-14	3,07E-12	8,68E-11	9,96E-10	5,33E-09	1,49E-08	2,37E-08	2,31E-08	1,48E-08	6,52E-09	2,08E-09	5,00E-10
20,75	0	9,00E-232	8,30E-161	1,08E-118	9,96E-91	6,26E-71	2,51E-56	2,93E-45	1,03E-36	4,91E-30	8,91E-25	1,27E-20	2,40E-17	8,98E-15	8,91E-13	2,96E-11	3,94E-10	2,42E-09	7,65E-09	1,37E-08	1,49E-08	1,05E-08	5,09E-09	1,77E-09	4,62E-10
21,25	0	1,20E-235	7,75E-164	3,56E-121	8,10E-93	1,02E-72	7,12E-58	1,31E-46	6,78E-38	4,51E-31	1,09E-25	1,99E-21	4,71E-18	2,14E-15	2,54E-13	9,91E-12	1,53E-10	1,07E-09	3,83E-09	7,67E-09	9,30E-09	7,26E-09	3,87E-09	1,47E-09	4,16E-10
21,75	0	2,39E-239	9,46E-167	1,42E-123	7,55E-95	1,84E-74	2,17E-59	6,20E-48	4,62E-39	4,21E-32	1,34E-26	3,13E-22	9,17E-19	5,05E-16	7,12E-14	3,25E-12	5,77E-11	4,62E-10	1,87E-09	4,20E-09	5,65E-09	4,88E-09	2,85E-09	1,19E-09	3,65E-10
22,25	0	6,86E-243	1,48E-169	6,68E-126	7,98E-97	3,62E-76	7,06E-61	3,06E-49	3,24E-40	4,01E-33	1,66E-27	4,91E-23	1,77E-19	1,18E-16	1,97E-14	1,05E-12	2,14E-11	1,95E-10	8,90E-10	2,24E-09	3,35E-09	3,19E-09	2,05E-09	9,30E-10	3,11E-10
22,75	0	2,75E-246	2,88E-172	3,70E-128	9,46E-99	7,75E-78	2,44E-62	1,57E-50	2,34E-41	3,88E-34	2,08E-28	7,71E-24	3,41E-20	2,72E-17	5,36E-15	3,31E-13	7,78E-12	8,06E-11	4,14E-10	1,16E-09	1,93E-09	2,03E-09	1,43E-09	7,10E-10	2,58E-10
23,25	0	1E-249	6,91E-175	2,37E-130	1,25E-100	1,79E-79	8,91E-64	8,42E-52	1,74E-42	3,82E-35	2,63E-29	1,21E-24	6,53E-21	6,23E-18	1,44E-15	1,03E-13	2,78E-12	3,26E-11	1,88E-10	5,90E-10	1,09E-09	1,26E-09	9,76E-10	5,28E-10	2,08E-10
23,75	0	1E-252	1,99E-177	1,73E-132	1,81E-102	4,45E-81	3,43E-65	4,67E-53	1,32E-43	3,81E-36	3,33E-30	1,91E-25	1,24E-21	1,41E-18	3,83E-16	3,17E-14	9,74E-13	1,29E-11	8,39E-11	2,93E-10	5,99E-10	7,65E-10	6,48E-10	3,82E-10	1,64E-10
24,25	0	1E-255	6,78E-180	1,43E-134	2,87E-104	1,18E-82	1,38E-66	2,68E-54	1,02E-44	3,85E-37	4,26E-31	3,00E-26	2,36E-22	3,18E-19	1,00E-16	9,59E-15	3,36E-13	5,04E-12	3,66E-11	1,42E-10	3,22E-10	4,52E-10	4,20E-10	2,70E-10	1,26E-10
24,75	0	1E-258	2,69E-182	1,31E-136	4,92E-106	3,31E-84	5,81E-68	1,58E-55	8,12E-46	3,95E-38	5,47E-32	4,72E-27	4,47E-23	7,09E-20	2,61E-17	2,86E-15	1,14E-13	1,93E-12	1,57E-11	6,78E-11	1,69E-10	2,61E-10	2,65E-10	1,86E-10	9,39E-11

Appendix A.3 LONG TERM JOINT DISTRIBUTION OF SEA STATES ($\boldsymbol{b_1 = 0}$)

	$t_{p_{mid}}$																								
$h_{s_{mid}}$	0,5	1,5	2,5	3,5	4,5	5,5	6,5	7,5	8,5	9,5	10,5	11,5	12,5	13,5	14,5	15,5	16,5	17,5	18,5	19,5	20,5	21,5	22,5	23,5	24,5
0,75	7,82E-14	1,67E-07	5,96E-05	1,16E-03	0,005554725	0,011921931	0,01576804	0,01534175	0,01219809	0,008461044	0,00534146	0,003157339	0,00178197	9,74E-04	5,20E-04	2,74E-04	1,43E-04	7,38E-05	3,81E-05	1,97E-05	1,02E-05	5,28E-06	2,75E-06	1,44E-06	7,54E-07
1,25	4,06E-16	1,33E-08	1,63E-05	0,000662481	0,005229258	0,0161471	0,02792533	0,03319916	0,030704167	0,023883655	0,01644573	0,010377712	0,00614788	0,00347853	0,00190347	0,00101671	0,00053379	0,00027691	0,00014252	7,30E-05	3,73E-05	1,91E-05	9,74E-06	4,99E-06	2,57E-06
1,75	5,90E-19	3,67E-10	1,68E-06	1,47E-04	0,001937831	0,008619973	0,01945066	0,02812017	0,030045987	0,025992294	0,01933795	0,012894135	0,00793154	0,00459549	0,00254641	0,0013648	0,00071364	0,00036645	0,00018572	9,33E-05	4,66E-05	2,32E-05	1,15E-05	5,71E-06	2,84E-06
2,25	3,90E-22	6,38E-12	1,26E-07	2,53E-05	0,000577831	0,003790686	0,01134104	0,02015465	0,025046159	0,024180299	0,0194566	0,013695166	0,00872574	0,00515813	0,00288095	0,00154116	0,00079786	0,00040293	0,00019974	9,77E-05	4,73E-05	2,27E-05	1,09E-05	5,20E-06	2,48E-06
2,75	1,02E-25	6,52E-14	6,59E-09	3,38E-06	1,41E-04	0,00141371	0,00575496	0,01281292	0,018773725	0,020420056	0,01788105	0,013334279	0,00881315	0,00531463	0,00298758	0,00159106	0,0008127	0,00040198	0,00019397	9,18E-05	4,29E-05	1,98E-05	9,09E-06	4,15E-06	1,89E-06
3,25	8,96E-30	3,60E-16	2,31E-10	3,38E-07	2,77E-05	0,000445432	0,00255071	0,00729442	0,012839585	0,015952045	0,01535126	0,012209271	0,00840567	0,00518172	0,00293299	0,00155357	0,00078137	0,0003774	0,00017661	8,06E-05	3,61E-05	1,60E-05	6,97E-06	3,02E-06	1,30E-06
3,75	2,25E-34	9,77E-19	5,06E-12	2,44E-08	4,30E-06	1,17E-04	0,00098007	0,00371066	0,008029476	0,011595085	0,01242185	0,010634224	0,00767414	0,00485475	0,00277214	0,00146086	0,00072269	0,00034011	0,00015387	6,75E-05	2,89E-05	1,21E-05	5,02E-06	2,05E-06	8,33E-07
4,25	1,33E-39	1,17E-21	6,49E-14	1,21E-09	5,08E-07	2,50E-05	3,22E-04	0,00167289	0,004573387	0,00783858	0,00949729	0,008854245	0,00675308	0,00440813	0,00254745	0,00133734	0,00065057	0,00029786	0,00012994	5,45E-05	2,22E-05	8,82E-06	3,44E-06	1,32E-06	5,01E-07
4,75	1,52E-45	5,46E-25	4,50E-16	3,93E-11	4,40E-08	4,26E-06	8,89E-05	0,0006602	0,002353657	0,004906965	0,00685159	0,007055155	0,00574573	0,00389763	0,00228984	0,00120031	0,00057449	0,00025563	0,00010728	4,29E-05	1,66E-05	6,19E-06	2,26E-06	8,09E-07	2,85E-07
5,25	2,60E-52	8,69E-29	1,53E-18	7,78E-13	2,68E-09	5,58E-07	2,02E-05	2,24E-04	0,001082322	0,002823708	0,0046447	0,005371631	0,0047296	0,00336371	0,00202078	0,00106126	0,00050038	0,0002163	8,72E-05	3,32E-05	1,21E-05	4,22E-06	1,43E-06	4,76E-07	1,55E-07
5,75	5,15E-60	4,02E-33	2,32E-21	8,72E-15	1,08E-10	5,42E-08	3,66E-06	6,45E-05	0,000438491	0,001478953	0,00293959	0,003893783	0,00376161	0,00283503	0,00175445	0,00092731	0,00043164	0,00018128	7,00E-05	2,53E-05	8,62E-06	2,81E-06	8,84E-07	2,70E-07	8,03E-08
6,25	8,68E-69	4,53E-38	1,38E-24	5,09E-17	2,75E-12	3,74E-09	5,14E-07	1,53E-05	1,54E-04	0,00069636	0,00172198	0,002672435	0,0028821	0,00233152	0,00149973	0,0008024	0,00036988	0,00015105	5,59E-05	1,91E-05	6,08E-06	1,84E-06	5,31E-07	1,48E-07	4,00E-08
6,75	8,83E-79	1,02E-43	2,82E-28	1,40E-19	4,05E-14	1,73E-10	5,40E-08	2,92E-06	4,58E-05	0,000290376	0,00092364	0,001723946	0,00211776	0,00186689	0,00126182	0,00068836	0,00031555	0,0001255	4,44E-05	1,43E-05	4,25E-06	1,18E-06	3,12E-07	7,87E-08	1,91E-08
7,25	3,65E-90	3,60E-50	1,70E-32	1,64E-22	3,20E-16	5,10E-12	4,04E-09	4,32E-07	1,13E-05	1,05E-04	0,00044777	0,001035668	0,00148338	0,00145043	0,00104353	0,00058564	0,00026843	0,00010423	3,53E-05	1,07E-05	2,94E-06	7,51E-07	1,80E-07	4,07E-08	8,83E-09
7,75	3,93E-103	1,55E-57	2,51E-37	7,12E-26	1,24E-18	8,85E-14	2,05E-10	4,79E-08	2,25E-06	3,26E-05	0,00019318	0,000572976	0,00098287	0,00108819	0,00084625	0,00049386	0,00022789	8,67E-05	2,81E-05	7,99E-06	2,03E-06	4,73E-07	1,02E-07	2,05E-08	3,93E-09
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8,75	9,90E-135	1,47E-75	3,50E-49	3,78E-34	1,38E-24	3,92E-18	1,27E-13	2,05E-10	4,06E-08	1,74E-06	2,35E-05	1,30E-04	0,00035107	0,0005373	0,00051683	0,00033989	0,00016342	6,06E-05	1,81E-05	4,53E-06	9,73E-07	1,84E-07	3,14E-08	4,89E-09	7,06E-10
9,25	6,75E-154	1,57E-86	2,01E-56	3,22E-39	3,03E-28	8,18E-21	1,33E-15	7,06E-12	3,39E-09	2,83E-07	6,33E-06	5,12E-05	0,00018462	0,00034762	0,00038508	0,00027587	0,00013784	5,11E-05	1,47E-05	3,45E-06	6,78E-07	1,15E-07	1,72E-08	2,32E-09	2,86E-10
9,75	9,99E-176	4,68E-99	1,04E-64	4,96E-45	1,90E-32	6,71E-24	6,95E-18	1,44E-13	1,92E-10	3,45E-08	1,38E-06	1,74E-05	8,74E-05	0,0002098	0,0002753	0,00021946	0,0001157	4,32E-05	1,21E-05	2,66E-06	4,76E-07	7,19E-08	9,38E-09	1,08E-09	1,12E-10
10,25	1,38E-200	2,33E-113	3,45E-74	1,07E-51	2,82E-37	1,88E-27	1,62E-20	1,61E-15	6,91E-12	3,01E-09	2,36E-07	4,94E-06	3,65E-05	1,17E-04	0,00018713	0,00017016	9,64E-05	3,68E-05	1,01E-05	2,08E-06	3,39E-07	4,53E-08	5,10E-09	4,98E-10	4,30E-11
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11,25	3,99E-261	2,54E-148	1,92E-97	4,21E-68	3,36E-49	2,95E-36	4,63E-27	2,18E-20	1,71E-15	6,71E-12	2,79E-09	2,08E-07	4,02E-06	2,63E-05	7,10E-05	9,19E-05	6,43E-05	2,68E-05	7,20E-06	1,33E-06	1,80E-07	1,86E-08	1,53E-09	1,03E-10	5,87E-12
11,75	8E-298	1,37E-169	1,23E-111	3,81E-78	1,59E-56	1,11E-41	4,16E-31	2,08E-23	9,70E-18	1,49E-13	1,74E-10	2,85E-08	1,00E-06	1,02E-05	3,85E-05	6,28E-05	5,09E-05	2,29E-05	6,21E-06	1,10E-06	1,35E-07	1,22E-08	8,45E-10	4,65E-11	2,11E-12
12,25	0	7,34E-194	7,22E-128	1,17E-89	6,15E-65	6,30E-48	8,90E-36	6,66E-27	2,42E-20	1,77E-15	6,89E-12	2,81E-09	1,97E-07	3,39E-06	1,87E-05	4,02E-05	3,91E-05	1,94E-05	5,42E-06	9,27E-07	1,04E-07	8,20E-09	4,74E-10	2,11E-11	7,45E-13
12,75	0	1,50E-221	1,99E-146	7,54E-103	1,34E-74	4,07E-55	3,64E-41	6,04E-31	2,31E-23	1,03E-17	1,59E-13	1,87E-10	2,93E-08	9,25E-07	8,00E-06	2,38E-05	2,88E-05	1,63E-05	4,77E-06	7,98E-07	8,26E-08	5,66E-09	2,71E-10	9,59E-12	2,60E-13
13,25	0	3,86E-253	1,20E-167	5,81E-118	1,08E-85	2,16E-63	2,21E-47	1,27E-35	7,23E-27	2,56E-20	1,96E-15	7,88E-12	3,14E-09	2,01E-07	2,93E-06	1,28E-05	2,01E-05	1,34E-05	4,23E-06	7,03E-07	6,74E-08	4,02E-09	1,59E-10	4,42E-12	9,04E-14
13,75	0	3,62E-289	6,71E-192	2,83E-135	1,98E-98	6,50E-73	1,48E-54	4,96E-41	6,24E-31	2,39E-23	1,16E-17	1,93E-13	2,29E-10	3,32E-08	8,94E-07	6,09E-06	1,32E-05	1,07E-05	3,75E-06	6,33E-07	5,69E-08	2,96E-09	9,60E-11	2,07E-12	3,14E-14
14,25	0	0,00E+00	1,29E-219	4,23E-155	4,77E-113	7,22E-84	7,94E-63	2,75E-47	1,20E-35	7,16E-27	2,91E-20	2,51E-15	1,05E-11	3,98E-09	2,19E-07	2,52E-06	7,91E-06	8,24E-06	3,31E-06	5,82E-07	4,96E-08	2,26E-09	6,00E-11	9,92E-13	1,09E-14
14,75	0	0,00E+00	2,76E-251	8,42E-178	8,03E-130	1,82E-96	2,30E-72	1,61E-54	4,10E-41	5,67E-31	2,66E-23	1,54E-17	2,80E-13	3,26E-10	4,14E-08	8,79E-07	4,28E-06	6,02E-06	2,89E-06	5,43E-07	4,49E-08	1,81E-09	3,90E-11	4,90E-13	3,84E-15
15,25	0	0,00E+00	1,83E-287	8,69E-204	4,59E-149	5,94E-111	2,34E-83	7,05E-63	1,88E-47	9,63E-36	7,55E-27	3,93E-20	3,93E-15	1,69E-11	5,73E-09	2,50E-07	2,04E-06	4,11E-06	2,47E-06	5,14E-07	4,20E-08	1,51E-09	2,66E-11	2,51E-13	1,37E-15
15,75	0	0,00E+00	0,00E+00	1,58E-233	3,92E-171	1,33E-127	5,03E-96	1,54E-72	8,49E-55	2,73E-41	5,46E-31	3,58E-23	2,58E-17	5,08E-13	5,46E-10	5,57E-08	8,35E-07	2,57E-06	2,04E-06	4,89E-07	4,08E-08	1,33E-09	1,92E-11	1,34E-13	5,04E-16
16,25	0	0,00E+00	0,00E+00	1,48E-267	1,94E-196	9,71E-147	1,28E-110	1,08E-83	2,62E-63	9,71E-48	8,02E-36	9,77E-27	6,94E-20	8,02E-15	3,32E-11	9,22E-09	2,84E-07	1,44E-06	1,60E-06	4,65E-07	4,08E-08	1,24E-09	1,46E-11	7,59E-14	1,91E-16
16,75	0	0,00E+00	0,00E+00	1,74E-306	1,91E-225	1,01E-168	2,01E-127	1,41E-96	3,66E-73	3,12E-55	1,85E-41	6,52E-31	6,50E-23	5,89E-17	1,18E-12	1,07E-09	7,69E-08	7,07E-07	1,17E-06	4,35E-07	4,21E-08	1,21E-09	1,19E-11	4,55E-14	7,58E-17
17,25	0	0,00E+00	0,00E+00	0,00E+00	1,10E-258	5,72E-194	9,10E-147	1,92E-111	1,43E-84	6,17E-64	4,94E-48	8,38E-36	1,77E-26	1,75E-19	2,22E-14	8,03E-11	1,59E-08	2,95E-07	7,88E-07	3,97E-07	4,43E-08	1,25E-09	1,04E-11	2,92E-14	3,17E-17
17,75	0	0,00E+00	0,00E+00	0,00E+00	9,02E-297	5,90E-223	4,98E-169	1,34E-128	8,97E-98	4,85E-74	1,08E-55	1,58E-41	1,13E-30	1,77E-22	1,93E-16	3,58E-12	2,34E-09	1,01E-07	4,73E-07	3,46E-07	4,72E-08	1,37E-09	9,81E-12	2,03E-14	1,41E-17
18,25	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	3,16E-256	1,22E-194	2,21E-148	4,84E-113	9,16E-86	1,30E-64	3,17E-48	1,32E-35	5,09E-26	6,75E-19	8,49E-14	2,30E-10	2,69E-08	2,48E-07	2,83E-07	5,03E-08	1,57E-09	1,00E-11	1,54E-14	6,77E-18
18,75	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	2,10E-294	4,31E-224	3,44E-171	1,08E-130	2,32E-99	5,35E-75	4,69E-56	2,10E-41	3,29E-30	7,98E-22	9,43E-16	1,37E-11	5,31E-09	1,09E-07	2,13E-07	5,28E-08	1,88E-09	1,11E-11	1,29E-14	3,55E-18
19,25	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	5,97E-258	1,79E-197	4,34E-151	4,02E-115	4,44E-87	3,34E-65	3,26E-48	3,70E-35	2,61E-25	4,22E-18	4,44E-13	7,23E-10	3,92E-08	1,43E-07	5,35E-08	2,34E-09	1,34E-11	1,19E-14	2,06E-18
19,75	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	7,32E-297	9,46E-228	1,20E-174	2,22E-133	4,00E-101	7,01E-76	3,36E-56	5,32E-41	1,86E-29	6,37E-21	6,88E-15	6,19E-11	1,08E-08	8,33E-08	5,12E-08	2,97E-09	1,74E-11	1,24E-14	1,34E-18
20,25	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	1,29E-262	7,65E-202	1,60E-154	1,92E-117	2,44E-88	1,45E-65	6,85E-48	2,21E-34	2,63E-24	4,36E-17	2,99E-12	2,17E-09	4,10E-08	4,51E-08	3,79E-09	2,46E-11	1,45E-14	9,86E-19
20,75	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	9,59E-303	3,16E-233	5,44E-179	2,16E-136	7,32E-103	1,57E-76	5,23E-56	3,17E-40	2,33E-28	9,41E-20	7,14E-14	2,87E-10	1,63E-08	3,56E-08	4,76E-09	3,72E-11	1,92E-14	8,39E-19
21,25	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	1,99E-269	2,72E-207	2,21E-158	8,85E-120	2,29E-89	1,47E-65	3,75E-47	3,31E-33	5,59E-23	7,21E-16	2,26E-11	5,00E-09	2,44E-08	5,70E-09	5,96E-11	2,88E-14	8,34E-19
21,75	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	3,6371778629	5,17E-240	6,95E-184	1,79E-139	2,26E-104	8,67E-77	2,38E-55	5,40E-39	7,08E-27	2,55E-18	9,31E-13	1,09E-09	1,41E-08	6,33E-09	9,93E-11	4,89E-14	9,79E-19
22,25	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	8,01E-278	1,92E-213	2,21E-162	6,64E-122	5,63E-90	4,84E-65	6,82E-46	1,42E-31	2,55E-21	1,71E-14	1,52E-10	6,67E-09	6,28E-09	1,68E-10	9,39E-14	1,37E-18
22,75	0	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	1,7140000000	1,10E-247	5,19E-189	2,28E-142	1,91E-105	1,76E-76	4,20E-54	3,15E-37	5,50E-25	1,16E-16	1,21E-11	2,45E-09	5,39E-09	2,78E-10	2,01E-13	2,33E-18
23,25	0	0E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	2,48E-287	6,03E-220	3,10E-166	1,41E-123	5,71E-90	7,38E-64	5,16E-44	1,89E-29	2,33E-19	4,63E-13	6,41E-10	3,88E-09	4,34E-10	4,74E-13	4,77E-18
23,75	0	0E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	7,39E-256	4,75E-194	8,35E-145	7,46E-106	1,97E-75	3,83E-52	7,17E-35	1,06E-22	7,13E-15	1,07E-10	2,30E-09	6,10E-10	1,20E-12	1,19E-17
24,25	0	0E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	1,61E-297	1,94E-226	1,23E-169	1,55E-124	3,87E-89	7,33E-62	1,95E-41	7,91E-27	3,52E-17	9,69E-12	1,09E-09	7,33E-10	3,12E-12	3,59E-17
24,75	0	0E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	0,00E+00	3,99E-264	1,19E-198	1,74E-146	2,37E-105	1,86E-73	2,31E-49	6,71E-32	4,25E-20	3,99E-13	3,81E-10	7,21E-10	8,04E-12	1,29E-16

Appendix B ALL SEA STATES

Appendix B.1 LONG TERM 3-HOUR MAXIMA (CREST HEIGHT, GAUSSIAN)

```
clear all
close all
clc
%load data
load('joint_pdf.mat');
%define input crest height
Xi = 1:0.5:30;
%hs range
hs = 0.25:0.5:24.75;
%tp range
tp = 0.5:1:24.5;
%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of Xi, Hs and Tp
for k = 1:length(Xi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        %create table of short term distribution (non-exceedance
        %probabilities) Fc3h|hs,tp for the given Xi
        Fc(i,j) = (1-exp(-8*(Xi(k)/hs(i))^2))^(13846/tp(j));
        %create a cell array of containing short term conditional
        %distribution for given Xi
        Fc_cell{k} = Fc;
        end
    end
end

%*Long term distribution of 3h max. F_X3h (b1=0.005)*
%cell array containing the long term distribution matrixes
Table_F_Xi3h_fit_1_cell = cell(1,k);
F_Xi3h_fit_rows_sum_1_cell = cell(1,k);
F_Xi3h_fit_1_cell = cell(k,1);
%execute loop
for k = 1:length(Xi);
    %calculate long term distribution of 3h maximum response F_X3h
    Table_F_Xi3h_fit_1 = bsxfun(@times,Fc_cell{k},Joint_pdf_norm_1);
    %create cell array with results
    Table_F_Xi3h_fit_1_cell{k} = Table_F_Xi3h_fit_1;
    %count contributions along rows (tp)
    F_Xi3h_fit_rows_sum_1 = sum(Table_F_Xi3h_fit_1_cell{k},2);
    %create cell array with results
    F_Xi3h_fit_rows_sum_1_cell{k} = F_Xi3h_fit_rows_sum_1;
    %sum along column to get total contribution for the given
    %value of the 3h response maxima Xi
    F_Xi3h_fit_1 = sum(F_Xi3h_fit_rows_sum_1_cell{k},1);
    %create cell array with results
    F_Xi3h_fit_1_cell{k} = F_Xi3h_fit_1;
end
%convert to matrix
```

```
F_Xi_3h = cell2mat(F_Xi3h_fit_1_cell);

%*Probability plot*
Z = (log(Xi))';
Y = log(-log(1-F_Xi_3h));
q_100 = (10^-2)/2920;
q_10000 = (10^-4)/2920;
Y_ULS = log(-log(q_100));
Y_ALS = log(-log(q_10000));
figure(1)
plot(Z,Y,'r');
hold on
A = line([0 3.5],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 3.5],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-
$F_{\xi}))$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term distribution 3-hr max.(Gaussian, crest
$\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
%find intersection points
[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);
%q-prob values
crest_ULS = exp(x_cs_100);
crest_ALS = exp(x_cs_10000);
%round off q-prob values
crest_ULS_round= ceil(10*crest_ULS)/10;
crest_ALS_round= ceil(10*crest_ALS)/10;
%change format to double variable
Xi_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));
%change format to double variable
Xi_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));
legend('Long term distribution 3-hr max.',['\xi_{ULS} = ',num2str(Xi_ULS_ch),'m'],['\xi_{ALS}
= ',num2str(Xi_ALS_ch),'m'],'Location','Southoutside');
grid on
grid minor
hold off

%*Save data to xl*
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'K9';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'L9';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.2 LONG TERM 3-HOUR MAXIMA (WAVE HEIGHT, GAUSSIAN)

```
clear all
close all
clc
%load data
load('joint_pdf.mat');
%hs range
hs = 0.25:0.5:24.75;
%tp range
tp = 0.5:1:24.5;
%range wave height H
hi = 2:1:50;
%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of hi, Hs and Tp
for k = 1:length(hi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        %create table of short term distribution (non-exceedance
        %probabilities) Fc3h|hs,tp for the given Xi
        Fc(i,j) = (1-exp(-2*(hi(k)/hs(i))^2))^(13846/tp(j));
        %create a cell array of containing short term conditional
        %distribution for given Xi
        Fc_cell{k} = Fc;
        end
    end
end

%*Long term distribution of 3h max. F_X3h (b1=0.005)*
Table_F_Xi3h_fit_1_cell = cell(1,k);
F_Xi3h_fit_rows_sum_1_cell = cell(1,k);
F_Xi3h_fit_1_cell = cell(k,1);
%execute loop
for k = 1:length(hi);
    %calculate long term distribution of 3h maximum response F_X3h
    Table_F_Xi3h_fit_1 = bsxfun(@times,Fc_cell{k},Joint_pdf_norm_1);
    %create cell array with results
    Table_F_Xi3h_fit_1_cell{k} = Table_F_Xi3h_fit_1;
    %count contributions along tp (rows)
    F_Xi3h_fit_rows_sum_1 = sum(Table_F_Xi3h_fit_1_cell{k},2);
    %create cell array with results
    F_Xi3h_fit_rows_sum_1_cell{k} = F_Xi3h_fit_rows_sum_1;
    %sum along column to get total contribution for the given
    %value of the 3h response maxima Xi
    F_Xi3h_fit_1 = sum(F_Xi3h_fit_rows_sum_1_cell{k},1);
    %create cell array with results
    F_Xi3h_fit_1_cell{k} = F_Xi3h_fit_1;
end

%transform cell to matrix
F_Xi_3h = cell2mat(F_Xi3h_fit_1_cell);

%*Probability plot*
Z = (log(hi))';
Y = log(-log(1-F_Xi_3h));
```

```
q_100 = (10^-2)/2920;
q_10000 = (10^-4)/2920;
Y_ULS = log(-log(q_100));
Y_ALS = log(-log(q_10000));
figure(1)
plot(Z,Y,'r');
hold on
A = line([0 4],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 4],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln(H)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-
$F_{H})$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term distribution 3-hr max.(Gaussian, wave height
H)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');

[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);

wave_height_ULS = exp(x_cs_100);
wave_height_ALS = exp(x_cs_10000);

crest_ULS_round= ceil(10*wave_height_ULS)/10;
crest_ALS_round= ceil(10*wave_height_ALS)/10;

h_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));
h_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));
legend('Long term distribution 3-hr max.',['H_{ULS} = ',num2str(h_ULS_ch),'m'],['H_{ALS} =
',num2str(h_ALS_ch),'m'],'Location','Southoutside');
xlim([0.5 4]);
ylim([-3 4]);
grid on
grid minor
hold off

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {h_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'K20';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {h_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'L20';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.3 MATRIX WITH URSELL NUMBERS

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0,5	1,09E-11	8,82E-10	6,80E-09	2,61E-08	7,14E-08	1,59E-07	3,11E-07	5,51E-07	9,09E-07	1,42E-06	2,12E-06	3,05E-06	4,25E-06	5,79E-06	7,70E-06	1,01E-05	1,29E-05	1,63E-05	2,04E-05	2,52E-05	3,08E-05	3,72E-05	4,46E-05	5,30E-05	6,25E-05
1	3,27E-11	2,65E-09	2,04E-08	7,84E-08	2,14E-07	4,78E-07	9,33E-07	1,65E-06	2,73E-06	4,26E-06	6,35E-06	9,14E-06	1,28E-05	1,74E-05	2,31E-05	3,02E-05	3,87E-05	4,90E-05	6,12E-05	7,55E-05	9,23E-05	0,000112	0,000134	0,000159	0,000187
1,5	5,44E-11	4,41E-09	3,40E-08	1,31E-07	3,57E-07	7,97E-07	1,55E-06	2,76E-06	4,55E-06	7,09E-06	1,06E-05	1,52E-05	2,13E-05	2,89E-05	3,85E-05	5,03E-05	6,46E-05	8,17E-05	0,000102	0,000126	0,000154	0,000186	0,000223	0,000265	0,000312
2	7,62E-11	6,17E-09	4,76E-08	1,83E-07	5,00E-07	1,12E-06	2,18E-06	3,86E-06	6,36E-06	9,93E-06	1,48E-05	2,13E-05	2,98E-05	4,05E-05	5,39E-05	7,04E-05	9,04E-05	0,000114	0,000143	0,000176	0,000215	0,00026	0,000312	0,000371	0,000437
2,5	9,80E-11	7,94E-09	6,12E-08	2,35E-07	6,43E-07	1,43E-06	2,80E-06	4,96E-06	8,18E-06	1,28E-05	1,91E-05	2,74E-05	3,83E-05	5,21E-05	6,93E-05	9,05E-05	0,000116	0,000147	0,000184	0,000227	0,000277	0,000335	0,000401	0,000477	0,000562
3	1,20E-10	9,70E-09	7,48E-08	2,88E-07	7,86E-07	1,75E-06	3,42E-06	6,06E-06	1,00E-05	1,56E-05	2,33E-05	3,35E-05	4,68E-05	6,36E-05	8,47E-05	0,000111	0,000142	0,00018	0,000224	0,000277	0,000338	0,000409	0,00049	0,000583	0,000687
3,5	1,42E-10	1,15E-08	8,85E-08	3,40E-07	9,29E-07	2,07E-06	4,04E-06	7,16E-06	1,18E-05	1,84E-05	2,75E-05	3,96E-05	5,53E-05	7,52E-05	0,0001	0,000131	0,000168	0,000212	0,000265	0,000327	0,0004	0,000484	0,00058	0,000689	0,000812
4	1,63E-10	1,32E-08	1,02E-07	3,92E-07	1,07E-06	2,39E-06	4,66E-06	8,27E-06	1,36E-05	2,13E-05	3,18E-05	4,57E-05	6,38E-05	8,68E-05	0,000115	0,000151	0,000194	0,000245	0,000306	0,000378	0,000461	0,000558	0,000669	0,000795	0,000937
4,5	1,85E-10	1,50E-08	1,16E-07	4,44E-07	1,21E-06	2,71E-06	5,29E-06	9,37E-06	1,55E-05	2,41E-05	3,60E-05	5,18E-05	7,23E-05	9,84E-05	0,000131	0,000171	0,000219	0,000278	0,000347	0,000428	0,000523	0,000632	0,000758	0,000901	0,001062
5	2,07E-10	1,68E-08	1,29E-07	4,97E-07	1,36E-06	3,03E-06	5,91E-06	1,05E-05	1,73E-05	2,70E-05	4,02E-05	5,79E-05	8,08E-05	0,00011	0,000146	0,000191	0,000245	0,00031	0,000388	0,000478	0,000584	0,000707	0,000847	0,001007	0,001187
5,5	2,29E-10	1,85E-08	1,43E-07	5,49E-07	1,50E-06	3,35E-06	6,53E-06	1,16E-05	1,91E-05	2,98E-05	4,45E-05	6,40E-05	8,93E-05	0,000121	0,000162	0,000211	0,000271	0,000343	0,000428	0,000529	0,000646	0,000781	0,000936	0,001113	0,001312
6	2,50E-10	2,03E-08	1,56E-07	6,01E-07	1,64E-06	3,67E-06	7,15E-06	1,27E-05	2,09E-05	3,26E-05	4,87E-05	7,01E-05	9,78E-05	0,000133	0,000177	0,000231	0,000297	0,000376	0,000469	0,000579	0,000707	0,000855	0,001025	0,001219	0,001437
6,5	2,72E-10	2,20E-08	1,70E-07	6,53E-07	1,79E-06	3,98E-06	7,77E-06	1,38E-05	2,27E-05	3,55E-05	5,29E-05	7,62E-05	0,000106	0,000145	0,000192	0,000251	0,000323	0,000408	0,00051	0,00063	0,000769	0,00093	0,001115	0,001325	0,001562
7	2,94E-10	2,38E-08	1,84E-07	7,06E-07	1,93E-06	4,30E-06	8,40E-06	1,49E-05	2,45E-05	3,83E-05	5,72E-05	8,23E-05	0,000115	0,000156	0,000208	0,000271	0,000349	0,000441	0,000551	0,00068	0,00083	0,001004	0,001204	0,001431	0,001687
7,5	3,16E-10	2,56E-08	1,97E-07	7,58E-07	2,07E-06	4,62E-06	9,02E-06	1,60E-05	2,64E-05	4,11E-05	6,14E-05	8,83E-05	0,000123	0,000168	0,000223	0,000292	0,000374	0,000474	0,000592	0,00073	0,000892	0,001079	0,001293	0,001537	0,001812
8	3,37E-10	2,73E-08	2,11E-07	8,10E-07	2,21E-06	4,94E-06	9,64E-06	1,71E-05	2,82E-05	4,40E-05	6,56E-05	9,44E-05	0,000132	0,000179	0,000239	0,000312	0,0004	0,000506	0,000632	0,000781	0,000953	0,001153	0,001382	0,001643	0,001937
8,5	3,59E-10	2,91E-08	2,25E-07	8,63E-07	2,36E-06	5,26E-06	1,03E-05	1,82E-05	3,00E-05	4,68E-05	6,99E-05	0,000101	0,00014	0,000191	0,000254	0,000332	0,000426	0,000539	0,000673	0,000831	0,001015	0,001227	0,001471	0,001749	0,002061
9	3,81E-10	3,09E-08	2,38E-07	9,15E-07	2,50E-06	5,58E-06	1,09E-05	1,93E-05	3,18E-05	4,97E-05	7,41E-05	0,000107	0,000149	0,000202	0,000269	0,000352	0,000452	0,000572	0,000714	0,000881	0,001076	0,001302	0,00156	0,001854	0,002186
9,5	4,03E-10	3,26E-08	2,52E-07	9,67E-07	2,64E-06	5,90E-06	1,15E-05	2,04E-05	3,36E-05	5,25E-05	7,83E-05	0,000113	0,000157	0,000214	0,000285	0,000372	0,000478	0,000604	0,000755	0,000932	0,001138	0,001376	0,00165	0,00196	0,002311
10	4,25E-10	3,44E-08	2,65E-07	1,02E-06	2,79E-06	6,22E-06	1,21E-05	2,15E-05	3,55E-05	5,53E-05	8,26E-05	0,000119	0,000166	0,000226	0,0003	0,000392	0,000504	0,000637	0,000796	0,000982	0,001199	0,001451	0,001739	0,002066	0,002436
10,5	4,46E-10	3,62E-08	2,79E-07	1,07E-06	2,93E-06	6,53E-06	1,27E-05	2,26E-05	3,73E-05	5,82E-05	8,68E-05	0,000125	0,000174	0,000237	0,000316	0,000412	0,000529	0,00067	0,000836	0,001032	0,001261	0,001525	0,001828	0,002172	0,002561
11	4,68E-10	3,79E-08	2,93E-07	1,12E-06	3,07E-06	6,85E-06	1,34E-05	2,37E-05	3,91E-05	6,10E-05	9,10E-05	0,000131	0,000183	0,000249	0,000331	0,000432	0,000555	0,000702	0,000877	0,001083	0,001322	0,001599	0,001917	0,002278	0,002686
11,5	4,90E-10	3,97E-08	3,06E-07	1,18E-06	3,21E-06	7,17E-06	1,40E-05	2,48E-05	4,09E-05	6,38E-05	9,53E-05	0,000137	0,000191	0,00026	0,000346	0,000452	0,000581	0,000735	0,000918	0,001133	0,001384	0,001674	0,002006	0,002384	0,002811
12	5,12E-10	4,14E-08	3,20E-07	1,23E-06	3,36E-06	7,49E-06	1,46E-05	2,59E-05	4,27E-05	6,67E-05	9,95E-05	0,000143	0,0002	0,000272	0,000362	0,000473	0,000607	0,000768	0,000959	0,001184	0,001445	0,001748	0,002095	0,00249	0,002936
12,5	5,33E-10	4,32E-08	3,33E-07	1,28E-06	3,50E-06	7,81E-06	1,52E-05	2,70E-05	4,46E-05	6,95E-05	0,000104	0,000149	0,000208	0,000283	0,000377	0,000493	0,000633	0,0008	0,001	0,001234	0,001507	0,001823	0,002184	0,002596	0,003061
13	5,55E-10	4,50E-08	3,47E-07	1,33E-06	3,64E-06	8,13E-06	1,59E-05	2,81E-05	4,64E-05	7,24E-05	0,000108	0,000155	0,000217	0,000295	0,000393	0,000513	0,000658	0,000833	0,001041	0,001284	0,001568	0,001897	0,002274	0,002702	0,003186
13,5	5,77E-10	4,67E-08	3,61E-07	1,39E-06	3,79E-06	8,45E-06	1,65E-05	2,92E-05	4,82E-05	7,52E-05	0,000112	0,000161	0,000225	0,000307	0,000408	0,000533	0,000684	0,000866	0,001081	0,001335	0,00163	0,001971	0,002363	0,002808	0,003311
14	5,99E-10	4,85E-08	3,74E-07	1,44E-06	3,93E-06	8,77E-06	1,71E-05	3,03E-05	5,00E-05	7,80E-05	0,000116	0,000168	0,000234	0,000318	0,000423	0,000553	0,00071	0,000898	0,001122	0,001385	0,001691	0,002046	0,002452	0,002914	0,003436
14,5	6,21E-10	5,03E-08	3,88E-07	1,49E-06	4,07E-06	9,09E-06	1,77E-05	3,14E-05	5,18E-05	8,09E-05	0,000121	0,000174	0,000242	0,00033	0,000439	0,000573	0,000736	0,000931	0,001163	0,001435	0,001753	0,00212	0,002541	0,00302	0,003561
15	6,42E-10	5,20E-08	4,01E-07	1,54E-06	4,21E-06	9,40E-06	1,83E-05	3,25E-05	5,36E-05	8,37E-05	0,000125	0,00018	0,000251	0,000341	0,000454	0,000593	0,000762	0,000964	0,001204	0,001486	0,001815	0,002195	0,00263	0,003126	0,003686
15,5	6,64E-10	5,38E-08	4,15E-07	1,59E-06	4,36E-06	9,72E-06	1,90E-05	3,36E-05	5,55E-05	8,65E-05	0,000129	0,000186	0,000259	0,000353	0,00047	0,000613	0,000788	0,000997	0,001245	0,001536	0,001876	0,002269	0,002719	0,003232	0,003811
16	6,86E-10	5,56E-08	4,29E-07	1,65E-06	4,50E-06	1,00E-05	1,96E-05	3,47E-05	5,73E-05	8,94E-05	0,000133	0,000192	0,000268	0,000364	0,000485	0,000633	0,000813	0,001029	0,001285	0,001587	0,001938	0,002343	0,002809	0,003338	0,003936
16,5	7,08E-10	5,73E-08	4,42E-07	1,70E-06	4,64E-06	1,04E-05	2,02E-05	3,58E-05	5,91E-05	9,22E-05	0,000138	0,000198	0,000276	0,000376	0,0005	0,000653	0,000839	0,001062	0,001326	0,001637	0,001999	0,002418	0,002898	0,003444	0,00406
17	7,29E-10	5,91E-08	4,56E-07	1,75E-06	4,79E-06	1,07E-05	2,08E-05	3,69E-05	6,09E-05	9,51E-05	0,000142	0,000204	0,000285	0,000388	0,000516	0,000674	0,000865	0,001095	0,001367	0,001687	0,002061	0,002492	0,002987	0,00355	0,004185
17,5	7,51E-10	6,08E-08	4,69E-07	1,80E-06	4,93E-06	1,10E-05	2,15E-05	3,80E-05	6,27E-05	9,79E-05	0,000146	0,00021	0,000293	0,000399	0,000531	0,000694	0,000891	0,001127	0,001408	0,001738	0,002122	0,002566	0,003076	0,003656	0,00431
18	7,73E-10	6,26E-08	4,83E-07	1,86E-06	5,07E-06	1,13E-05	2,21E-05	3,91E-05	6,46E-05	0,000101	0,00015	0,000216	0,000302	0,000411	0,000547	0,000714	0,000917	0,00116	0,001449	0,001788	0,002184	0,002641	0,003165	0,003762	0,004435
18,5	7,95E-10	6,44E-08	4,97E-07	1,91E-06	5,21E-06	1,16E-05	2,27E-05	4,02E-05	6,64E-05	0,000104	0,000155	0,000222	0,00031	0,000422	0,000562	0,000734	0,000942	0,001193	0,001489	0,001838	0,002245	0,002715	0,003254	0,003868	0,00456
19	8,16E-10	6,61E-08	5,10E-07	1,96E-06	5,36E-06	1,20E-05	2,33E-05	4,13E-05	6,82E-05	0,000106	0,000159	0,000228	0,000319	0,000434	0,000577	0,000754	0,000968	0,001225	0,00153	0,001889	0,002307	0,00279	0,003344	0,003974	0,004685
19,5	8,38E-10	6,79E-08	5,24E-07	2,01E-06	5,50E-06	1,23E-05	2,39E-05	4,24E-05	7,00E-05	0,000109	0,000163	0,000235	0,000327	0,000445	0,000593	0,000774	0,000994	0,001258	0,001571	0,001939	0,002368	0,002864	0,003433	0,00408	0,00481
20	8,60E-10	6,97E-08	5,38E-07	2,06E-06	5,64E-06	1,26E-05	2,46E-05	4,35E-05	7,18E-05	0,000112	0,000167	0,000241	0,000336	0,000457	0,000608	0,000794	0,00102	0,001291	0,001612	0,001989	0,00243	0,002938	0,003522	0,004186	0,004935
20,5	8,82E-10	7,14E-08	5,51E-07	2,12E-06	5,79E-06	1,29E-05	2,52E-05	4,46E-05	7,36E-05	0,000115	0,000171	0,000247	0,000344	0,000469	0,000624	0,000814	0,001046	0,001323	0,001653	0,00204	0,002491	0,003013	0,003611	0,004292	0,00506
21	9,04E-10	7,32E-08	5,65E-07	2,17E-06	5,93E-06	1,32E-05	2,58E-05	4,57E-05	7,55E-05	0,000118	0,000176	0,000253	0,000353	0,00048	0,000639	0,000834	0,001072	0,001356	0,001693	0,00209	0,002553	0,003087	0,0037	0,004398	0,005185
21,5	9,25E-10	7,50E-08	5,78E-07	2,22E-06	6,07E-06	1,35E-05	2,64E-05	4,68E-05	7,73E-05	0,000121	0,00018	0,000259	0,000361	0,000492	0,000654	0,000855	0,001097	0,001389	0,001734	0,002141	0,002614	0,003162	0,003789	0,004504	0,00531
22	9,47E-10	7,67E-08	5,92E-07	2,27E-06	6,21E-06	1,39E-05	2,71E-05	4,79E-05	7,91E-05	0,000123	0,000184	0,000265	0,00037	0,000503	0,00067	0,000875	0,001123	0,001421	0,001775	0,002191	0,002676	0,003236	0,003879	0,00461	0,005435
22,5	9,69E-10	7,85E-08	6,06E-07	2,33E-06	6,36E-06	1,42E-05	2,77E-05	4,91E-05	8,09E-05	0,000126	0,000188	0,000271	0,000378	0,000515	0,000685	0,000895	0,001149	0,001454	0,001816	0,002241	0,002737	0,00331	0,003968	0,004716	0,00556
23	9,91E-10	8,02E-08	6,19E-07	2,38E-06	6,50E-06	1,45E-05	2,83E-05	5,02E-05	8,27E-05	0,000129	0,000193	0,000277	0,000387	0,000526	0,000701	0,000915	0,001175	0,001487	0,001857	0,002292	0,002799	0,003385	0,004057	0,004822	0,005685
23,5	1,01E-09	8,20E-08	6,33E-07	2,43E-06	6,64E-06	1,48E-05	2,89E-05	5,13E-05	8,46E-05	0,000132	0,000197	0,000283	0,000395	0,000538	0,000716	0,000935	0,001201	0,001519	0,001897	0,002342	0,00286	0,003459	0,004146	0,004928	0,00581
24	1,03E-09	8,38E-08	6,46E-07	2,48E-06	6,79E-06	1,51E-05	2,95E-05	5,24E-05	8,64E-05	0,000135	0,000201	0,000289	0,000404	0,00055	0,000731	0,000955	0,001226	0,001552	0,001938	0,002392	0,002922	0,003534	0,004235	0,005034	0,005935
24,5	1,06E-09	8,55E-08	6,60E-07	2,54E-06	6,93E-06	1,55E-05	3,02E-05	5,35E-05	8,82E-05	0,000138	0,000205	0,000296	0,000412	0,000561	0,000747	0,000975	0,001252	0,001585	0,001979	0,002443	0,002983	0,003608	0,004324	0,00514	0,006059
25	1,08E-09	8,73E-08	6,74E-07	2,59E-06	7,07E-06	1,58E-05	3,08E-05	5,46E-05	9,00E-05	0,00014	0,00021	0,000302	0,000421	0,000573	0,000762	0,000995	0,001278	0,001617	0,00202	0,002493	0,003045	0,003682	0,004414	0,005246	0,006184

Appendix B.4 Matrix with s_1

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0,5	9,26E-01	1,03E-01	3,70E-02	1,89E-02	1,14E-02	7,65E-03	5,48E-03	4,12E-03	3,20E-03	2,57E-03	2,10E-03	1,75E-03	1,48E-03	1,27E-03	1,10E-03	9,64E-04	8,50E-04	7,56E-04	6,77E-04	6,09E-04	5,51E-04	5,01E-04	4,57E-04	4,19E-04	3,86E-04
1	2,78E+00	3,09E-01	1,11E-01	5,67E-02	3,43E-02	2,30E-02	1,64E-02	1,23E-02	9,61E-03	7,70E-03	6,30E-03	5,25E-03	4,45E-03	3,81E-03	3,30E-03	2,89E-03	2,55E-03	2,27E-03	2,03E-03	1,83E-03	1,65E-03	0,001503	0,001372	0,001258	0,001157
1,5	4,63E+00	5,15E-01	1,85E-01	9,45E-02	5,72E-02	3,83E-02	2,74E-02	2,06E-02	1,60E-02	1,28E-02	1,05E-02	8,75E-03	7,41E-03	6,35E-03	5,51E-03	4,82E-03	4,25E-03	3,78E-03	0,003383	0,003045	0,002755	0,002504	0,002287	0,002096	0,001929
2	6,48E+00	7,20E-01	2,59E-01	1,32E-01	8,00E-02	5,36E-02	3,84E-02	2,88E-02	2,24E-02	1,80E-02	1,47E-02	1,23E-02	1,04E-02	8,89E-03	7,71E-03	6,75E-03	5,95E-03	0,005292	0,004736	0,004262	0,003857	0,003506	0,003202	0,002935	0,0027
2,5	8,34E+00	9,26E-01	3,33E-01	1,70E-01	1,03E-01	6,89E-02	4,93E-02	3,70E-02	2,88E-02	2,31E-02	1,89E-02	1,58E-02	1,33E-02	1,14E-02	9,91E-03	8,67E-03	0,007654	0,006804	0,006089	0,00548	0,004959	0,004508	0,004116	0,003773	0,003472
3	1,02E+01	1,13E+00	4,08E-01	2,08E-01	1,26E-01	8,42E-02	6,03E-02	4,53E-02	3,53E-02	2,82E-02	2,31E-02	1,93E-02	1,63E-02	1,40E-02	1,21E-02	0,010601	0,009355	0,008316	0,007442	0,006698	0,00606	0,00551	0,005031	0,004612	0,004243
3,5	1,20E+01	1,34E+00	4,82E-01	2,46E-01	1,49E-01	9,95E-02	7,12E-02	5,35E-02	4,17E-02	3,34E-02	2,73E-02	2,28E-02	1,93E-02	1,65E-02	0,014316	0,012529	0,011056	0,009829	0,008795	0,007916	0,007162	0,006512	0,005946	0,00545	0,005015
4	1,39E+01	1,54E+00	5,56E-01	2,84E-01	1,72E-01	1,15E-01	8,22E-02	6,17E-02	4,81E-02	3,85E-02	3,15E-02	2,63E-02	2,22E-02	1,91E-02	0,016519	0,014456	0,012757	0,011341	0,010148	0,009134	0,008264	0,007513	0,00686	0,006289	0,005786
4,5	1,57E+01	1,75E+00	6,30E-01	3,21E-01	1,94E-01	1,30E-01	9,32E-02	7,00E-02	5,45E-02	4,36E-02	3,57E-02	2,98E-02	2,52E-02	2,16E-02	0,018721	0,016384	0,014458	0,012853	0,011501	0,010351	0,009366	0,008515	0,007775	0,007127	0,006558
5	1,76E+01	1,96E+00	7,04E-01	3,59E-01	2,17E-01	1,45E-01	1,04E-01	7,82E-02	6,09E-02	4,87E-02	3,99E-02	3,33E-02	2,82E-02	0,024138	0,020924	0,018311	0,016159	0,014365	0,012854	0,011569	0,010468	0,009517	0,00869	0,007966	0,007329
5,5	1,94E+01	2,16E+00	7,78E-01	3,97E-01	2,40E-01	1,61E-01	1,15E-01	8,64E-02	6,73E-02	5,39E-02	4,41E-02	3,68E-02	3,11E-02	0,026679	0,023126	0,020238	0,01786	0,015877	0,014207	0,012787	0,01157	0,010519	0,009605	0,008805	0,0081
6	2,13E+01	2,37E+00	8,52E-01	4,35E-01	2,63E-01	1,76E-01	1,26E-01	9,47E-02	7,37E-02	5,90E-02	4,83E-02	4,03E-02	3,41E-02	0,02922	0,025329	0,022166	0,019561	0,017389	0,01556	0,014005	0,012672	0,011521	0,010519	0,009643	0,008872
6,5	2,32E+01	2,57E+00	9,26E-01	4,73E-01	2,86E-01	1,91E-01	1,37E-01	1,03E-01	8,01E-02	6,41E-02	5,25E-02	4,38E-02	0,037046	0,031761	0,027531	0,024093	0,021262	0,018901	0,016913	0,015223	0,013774	0,012522	0,011434	0,010482	0,009643
7	2,50E+01	2,78E+00	1,00E+00	5,10E-01	3,09E-01	2,07E-01	1,48E-01	1,11E-01	8,65E-02	6,93E-02	5,67E-02	4,73E-02	0,04001	0,034302	0,029734	0,026021	0,022962	0,020413	0,018266	0,016441	0,014876	0,013524	0,012349	0,01132	0,010415
7,5	2,69E+01	2,98E+00	1,07E+00	5,48E-01	3,32E-01	2,22E-01	1,59E-01	1,19E-01	9,29E-02	7,44E-02	6,09E-02	5,08E-02	0,042973	0,036843	0,031936	0,027948	0,024663	0,021925	0,019619	0,017658	0,015978	0,014526	0,013263	0,012159	0,011186
8	2,87E+01	3,19E+00	1,15E+00	5,86E-01	3,54E-01	2,37E-01	1,70E-01	1,28E-01	9,93E-02	7,95E-02	6,51E-02	5,43E-02	0,045937	0,039384	0,034139	0,029876	0,026364	0,023437	0,020972	0,018876	0,01708	0,015528	0,014178	0,012997	0,011958
8,5	3,06E+01	3,40E+00	1,22E+00	6,24E-01	3,77E-01	2,53E-01	1,81E-01	1,36E-01	1,06E-01	8,47E-02	6,93E-02	0,057775	0,048901	0,041925	0,036341	0,031803	0,028065	0,024949	0,022325	0,020094	0,018181	0,016529	0,015093	0,013836	0,012729
9	3,24E+01	3,60E+00	1,30E+00	6,62E-01	4,00E-01	2,68E-01	1,92E-01	1,44E-01	1,12E-01	8,98E-02	7,35E-02	0,061277	0,051864	0,044465	0,038544	0,033731	0,029766	0,026461	0,023678	0,021312	0,019283	0,017531	0,016008	0,014674	0,013501
9,5	3,43E+01	3,81E+00	1,37E+00	6,99E-01	4,23E-01	2,83E-01	2,03E-01	1,52E-01	1,19E-01	9,49E-02	7,77E-02	0,064778	0,054828	0,047006	0,040746	0,035658	0,031467	0,027974	0,025031	0,02253	0,020385	0,018533	0,016922	0,015513	0,014272
10	3,61E+01	4,01E+00	1,44E+00	7,37E-01	4,46E-01	2,99E-01	2,14E-01	1,61E-01	1,25E-01	1,00E-01	8,19E-02	0,06828	0,057792	0,049547	0,042949	0,037586	0,033168	0,029486	0,026384	0,023747	0,021487	0,019535	0,017837	0,016351	0,015044
10,5	3,80E+01	4,22E+00	1,52E+00	7,75E-01	4,69E-01	3,14E-01	2,25E-01	1,69E-01	1,31E-01	1,05E-01	8,61E-02	0,071781	0,060756	0,052088	0,045151	0,039513	0,034869	0,030998	0,027737	0,024965	0,022589	0,020537	0,018752	0,01719	0,015815
11	3,98E+01	4,42E+00	1,59E+00	8,13E-01	4,92E-01	3,29E-01	2,36E-01	1,77E-01	1,38E-01	1,10E-01	9,03E-02	0,075283	0,063719	0,054629	0,047354	0,041441	0,03657	0,03251	0,02909	0,026183	0,023691	0,021538	0,019666	0,018028	0,016587
11,5	4,17E+01	4,63E+00	1,67E+00	8,51E-01	5,15E-01	3,44E-01	2,47E-01	1,85E-01	1,44E-01	1,15E-01	9,45E-02	0,078784	0,066683	0,05717	0,049556	0,043368	0,038271	0,034022	0,030443	0,027401	0,024793	0,02254	0,020581	0,018867	0,017358
12	4,35E+01	4,84E+00	1,74E+00	8,88E-01	5,37E-01	3,60E-01	2,58E-01	1,93E-01	1,51E-01	1,21E-01	9,87E-02	0,082286	0,069647	0,059711	0,051759	0,045296	0,039972	0,035534	0,031796	0,028619	0,025895	0,023542	0,021496	0,019705	0,01813
12,5	4,54E+01	5,04E+00	1,82E+00	9,26E-01	5,60E-01	3,75E-01	2,69E-01	2,02E-01	1,57E-01	1,26E-01	0,102906	0,085787	0,07261	0,062252	0,053961	0,047223	0,041673	0,037046	0,033149	0,029837	0,026997	0,024544	0,022411	0,020544	0,018901
13	4,72E+01	5,25E+00	1,89E+00	9,64E-01	5,83E-01	3,90E-01	2,79E-01	2,10E-01	1,63E-01	1,31E-01	0,107106	0,089289	0,075574	0,064792	0,056164	0,049151	0,043373	0,038558	0,034502	0,031054	0,028099	0,025546	0,023325	0,021382	0,019673
13,5	4,91E+01	5,45E+00	1,96E+00	1,00E+00	6,06E-01	4,06E-01	2,90E-01	2,18E-01	1,70E-01	1,36E-01	0,111306	0,09279	0,078538	0,067333	0,058366	0,051078	0,045074	0,04007	0,035855	0,032272	0,0292	0,026547	0,02424	0,022221	0,020444
14	5,09E+01	5,66E+00	2,04E+00	1,04E+00	6,29E-01	4,21E-01	3,01E-01	2,26E-01	1,76E-01	1,41E-01	0,115506	0,096292	0,081501	0,069874	0,060569	0,053006	0,046775	0,041582	0,037208	0,03349	0,030302	0,027549	0,025155	0,023059	0,021215
14,5	5,28E+01	5,87E+00	2,11E+00	1,08E+00	6,52E-01	4,36E-01	3,12E-01	2,35E-01	1,83E-01	1,46E-01	0,119707	0,099793	0,084465	0,072415	0,062771	0,054933	0,048476	0,043094	0,038561	0,034708	0,031404	0,028551	0,026069	0,023898	0,021987
15	5,46E+01	6,07E+00	2,19E+00	1,12E+00	6,75E-01	4,52E-01	3,23E-01	2,43E-01	1,89E-01	1,51E-01	0,123907	0,103295	0,087429	0,074956	0,064974	0,05686	0,050177	0,044606	0,039914	0,035926	0,032506	0,029553	0,026984	0,024736	0,022758
15,5	5,65E+01	6,28E+00	2,26E+00	1,15E+00	6,97E-01	4,67E-01	3,34E-01	2,51E-01	1,95E-01	1,56E-01	0,128107	0,106796	0,090392	0,077497	0,067176	0,058788	0,051878	0,046119	0,041268	0,037143	0,033608	0,030554	0,027899	0,025575	0,02353
16	5,83E+01	6,48E+00	2,33E+00	1,19E+00	7,20E-01	4,82E-01	3,45E-01	2,59E-01	2,02E-01	1,62E-01	0,132307	0,110298	0,093356	0,080038	0,069379	0,060715	0,053579	0,047631	0,042621	0,038361	0,03471	0,031556	0,028814	0,026414	0,024301
16,5	6,02E+01	6,69E+00	2,41E+00	1,23E+00	7,43E-01	4,98E-01	3,56E-01	2,68E-01	2,08E-01	1,67E-01	0,136508	0,113799	0,09632	0,082579	0,071581	0,062643	0,05528	0,049143	0,043974	0,039579	0,035812	0,032558	0,029728	0,027252	0,025073
17	6,21E+01	6,89E+00	2,48E+00	1,27E+00	7,66E-01	5,13E-01	3,67E-01	2,76E-01	2,15E-01	1,72E-01	0,140708	0,117301	0,099283	0,08512	0,073784	0,06457	0,056981	0,050655	0,045327	0,040797	0,036914	0,03356	0,030643	0,028091	0,025844
17,5	6,39E+01	7,10E+00	2,56E+00	1,30E+00	7,89E-01	5,28E-01	3,78E-01	2,84E-01	2,21E-01	1,77E-01	0,144908	0,120802	0,102247	0,08766	0,075986	0,066498	0,058682	0,052167	0,04668	0,042015	0,038016	0,034562	0,031558	0,028929	0,026616
18	6,58E+01	7,31E+00	2,63E+00	1,34E+00	8,12E-01	5,43E-01	3,89E-01	2,92E-01	2,28E-01	0,182152	0,149108	0,124304	0,105211	0,090201	0,078189	0,068425	0,060383	0,053679	0,048033	0,043233	0,039118	0,035563	0,032472	0,029768	0,027387
18,5	6,76E+01	7,51E+00	2,70E+00	1,38E+00	8,35E-01	5,59E-01	4,00E-01	3,00E-01	2,34E-01	0,187283	0,153308	0,127805	0,108174	0,092742	0,080391	0,070353	0,062084	0,055191	0,049386	0,04445	0,04022	0,036565	0,033387	0,030606	0,028159
19	6,95E+01	7,72E+00	2,78E+00	1,42E+00	8,58E-01	5,74E-01	4,11E-01	3,09E-01	2,40E-01	0,192414	0,157509	0,131307	0,111138	0,095283	0,082594	0,07228	0,063785	0,056703	0,050739	0,045668	0,041321	0,037567	0,034302	0,031445	0,02893
19,5	7,13E+01	7,92E+00	2,85E+00	1,46E+00	8,80E-01	5,89E-01	4,22E-01	3,17E-01	2,47E-01	0,197545	0,161709	0,134808	0,114102	0,097824	0,084796	0,074208	0,065485	0,058215	0,052092	0,046886	0,042423	0,038569	0,035217	0,032283	0,029702
20	7,32E+01	8,13E+00	2,93E+00	1,49E+00	9,03E-01	6,05E-01	4,33E-01	3,25E-01	2,53E-01	0,202676	0,165909	0,13831	0,117066	0,100365	0,086999	0,076135	0,067186	0,059727	0,053445	0,048104	0,043525	0,039571	0,036131	0,033122	0,030473
20,5	7,50E+01	8,34E+00	3,00E+00	1,53E+00	9,26E-01	6,20E-01	4,44E-01	3,33E-01	2,60E-01	0,207807	0,170109	0,141811	0,120029	0,102906	0,089201	0,078063	0,068887	0,061239	0,054798	0,049322	0,044627	0,040572	0,037046	0,03396	0,031245
21	7,69E+01	8,54E+00	3,07E+00	1,57E+00	9,49E-01	6,35E-01	4,55E-01	3,42E-01	2,66E-01	0,212938	0,17431	0,145313	0,122993	0,105447	0,091404	0,07999	0,070588	0,062751	0,056151	0,050539	0,045729	0,041574	0,037961	0,034799	0,032016
21,5	7,87E+01	8,75E+00	3,15E+00	1,61E+00	9,72E-01	6,51E-01	4,66E-01	3,50E-01	2,72E-01	0,218069	0,17851	0,148814	0,125957	0,107987	0,093606	0,081918	0,072289	0,064264	0,057504	0,051757	0,046831	0,042576	0,038875	0,035637	0,032788
22	8,06E+01	8,95E+00	3,22E+00	1,64E+00	9,95E-01	6,66E-01	4,77E-01	3,58E-01	2,79E-01	0,2232	0,18271	0,152316	0,12892	0,110528	0,095809	0,083845	0,07399	0,065776	0,058857	0,052975	0,047933	0,043578	0,03979	0,036476	0,033559
22,5	8,24E+01	9,16E+00	3,30E+00	1,68E+00	1,02E+00	6,81E-01	4,88E-01	3,66E-01	2,85E-01	0,228331	0,18691	0,155818	0,131884	0,113069	0,098011	0,085773	0,075691	0,067288	0,06021	0,054193	0,049035	0,044579	0,040705	0,037314	0,03433
23	8,43E+01	9,36E+00	3,37E+00	1,72E+00	1,04E+00	6,97E-01	4,99E-01	3,75E-01	2,92E-01	0,233462	0,191111	0,159319	0,134848	0,11561	0,100214	0,0877	0,077392	0,0688	0,061563	0,055411	0,050137	0,045581	0,04162	0,038153	0,035102
23,5	8,61E+01	9,57E+00	3,45E+00	1,76E+00	1,06E+00	7,12E-01	5,10E-01	3,83E-01	2,98E-01	0,238593	0,195311	0,162821	0,137811	0,118151	0,102416	0,089628	0,079093	0,070312	0,062916	0,056629	0,051239	0,046583	0,042534	0,038991	0,035873
24	8,80E+01	9,78E+00	3,52E+00	1,80E+00	1,09E+00	7,27E-01	5,21E-01	3,91E-01	3,04E-01	0,243724	0,199511	0,166322	0,140775	0,120692	0,104619	0,091555	0,080794	0,071824	0,064269	0,057846	0,05234	0,047585	0,043449	0,03983	0,036645
24,5	8,98E+01	9,98E+00	3,59E+00	1,83E+00	1,11E+00	7,42E-01	5,32E-01	3,99E-01	3,11E-01	0,248855	0,203711	0,169824	0,143739	0,123233	0,106821	0,093482	0,082495	0,073336	0,065622	0,059064	0,053442	0,048587	0,044364	0,040668	0,037416
25	9,17E+01	1,02E+01	3,67E+00	1,87E+00	1,13E+00	7,58E-01	5,43E-01	4,08E-01	3,17E-01	0,253986	0,207911	0,173325	0,146702	0,125774	0,109024	0,09541	0,084196	0,074848	0,066975	0,060282	0,054544	0,049588	0,045279	0,041507	0,038188

Appendix B.5 MATRIX WITH α_F VALUES

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0,5	0,6214	0,3834	0,3643	0,3591	0,3569	0,3558	0,3552	0,3548	0,3545	0,3543	0,3542	0,3541	0,3540	0,3540	0,3539	0,3539	0,3538	0,3538	0,3538	0,3538	0,3538	0,3537	0,3537	0,3537	0,3537
1	1,1571	0,4429	0,3857	0,3700	0,3635	0,3602	0,3584	0,3572	0,3564	0,3558	0,3554	0,3551	0,3549	0,3547	0,3546	0,3544	0,3543	0,3543	0,3542	0,3541	0,3541	0,3540	0,3540	0,3540	0,3540
1,5	1,6928	0,5024	0,4072	0,3809	0,3701	0,3647	0,3615	0,3596	0,3582	0,3573	0,3566	0,3561	0,3557	0,3554	0,3552	0,3550	0,3548	0,3547	0,3546	0,3545	0,3544	0,3543	0,3543	0,3542	0,3542
2	2,2285	0,5619	0,4286	0,3919	0,3767	0,3691	0,3647	0,3619	0,3601	0,3588	0,3579	0,3571	0,3566	0,3562	0,3558	0,3556	0,3553	0,3551	0,3550	0,3549	0,3547	0,3546	0,3546	0,3545	0,3544
2,5	2,7642	0,6214	0,4500	0,4028	0,3834	0,3735	0,3679	0,3643	0,3619	0,3603	0,3591	0,3582	0,3575	0,3569	0,3565	0,3561	0,3558	0,3556	0,3554	0,3552	0,3551	0,3549	0,3548	0,3547	0,3547
3	3,2999	0,6810	0,4715	0,4137	0,3900	0,3779	0,3710	0,3667	0,3638	0,3618	0,3603	0,3592	0,3583	0,3576	0,3571	0,3567	0,3563	0,3560	0,3558	0,3556	0,3554	0,3552	0,3551	0,3550	0,3549
3,5	3,8356	0,7405	0,4929	0,4247	0,3966	0,3824	0,3742	0,3691	0,3656	0,3632	0,3615	0,3602	0,3592	0,3584	0,3578	0,3572	0,3568	0,3565	0,3562	0,3559	0,3557	0,3555	0,3554	0,3552	0,3551
4	4,3712	0,8000	0,5143	0,4356	0,4032	0,3868	0,3774	0,3715	0,3675	0,3647	0,3627	0,3612	0,3600	0,3591	0,3584	0,3578	0,3573	0,3569	0,3566	0,3563	0,3560	0,3558	0,3557	0,3555	0,3554
4,5	4,9069	0,8595	0,5357	0,4465	0,4098	0,3912	0,3805	0,3738	0,3694	0,3662	0,3639	0,3622	0,3609	0,3599	0,3590	0,3584	0,3578	0,3573	0,3570	0,3566	0,3564	0,3561	0,3559	0,3558	0,3556
5	5,4426	0,9190	0,5572	0,4575	0,4164	0,3957	0,3837	0,3762	0,3712	0,3677	0,3651	0,3632	0,3618	0,3606	0,3597	0,3589	0,3583	0,3578	0,3574	0,3570	0,3567	0,3564	0,3562	0,3560	0,3558
5,5	5,9783	0,9786	0,5786	0,4684	0,4230	0,4001	0,3869	0,3786	0,3731	0,3692	0,3664	0,3642	0,3626	0,3613	0,3603	0,3595	0,3588	0,3582	0,3578	0,3574	0,3570	0,3567	0,3565	0,3563	0,3561
6	6,5140	1,0381	0,6000	0,4793	0,4297	0,4045	0,3901	0,3810	0,3749	0,3707	0,3676	0,3653	0,3635	0,3621	0,3609	0,3600	0,3593	0,3587	0,3581	0,3577	0,3573	0,3570	0,3568	0,3565	0,3563
6,5	7,0497	1,0976	0,6214	0,4903	0,4363	0,4089	0,3932	0,3834	0,3768	0,3722	0,3688	0,3663	0,3643	0,3628	0,3616	0,3606	0,3598	0,3591	0,3585	0,3581	0,3577	0,3573	0,3570	0,3568	0,3566
7	7,5854	1,1571	0,6429	0,5012	0,4429	0,4134	0,3964	0,3857	0,3786	0,3736	0,3700	0,3673	0,3652	0,3635	0,3622	0,3612	0,3603	0,3596	0,3589	0,3584	0,3580	0,3576	0,3573	0,3570	0,3568
7,5	8,1210	1,2166	0,6643	0,5121	0,4495	0,4178	0,3996	0,3881	0,3805	0,3751	0,3712	0,3683	0,3660	0,3643	0,3629	0,3617	0,3608	0,3600	0,3593	0,3588	0,3583	0,3579	0,3576	0,3573	0,3570
8	8,6567	1,2762	0,6857	0,5231	0,4561	0,4222	0,4027	0,3905	0,3823	0,3766	0,3724	0,3693	0,3669	0,3650	0,3635	0,3623	0,3613	0,3604	0,3597	0,3591	0,3586	0,3582	0,3578	0,3575	0,3573
8,5	9,1924	1,3357	0,7072	0,5340	0,4627	0,4266	0,4059	0,3929	0,3842	0,3781	0,3737	0,3703	0,3678	0,3657	0,3641	0,3628	0,3618	0,3609	0,3601	0,3595	0,3590	0,3585	0,3581	0,3578	0,3575
9	9,7281	1,3952	0,7286	0,5449	0,4693	0,4311	0,4091	0,3953	0,3860	0,3796	0,3749	0,3713	0,3686	0,3665	0,3648	0,3634	0,3623	0,3613	0,3605	0,3599	0,3593	0,3588	0,3584	0,3580	0,3577
9,5	10,2638	1,4547	0,7500	0,5558	0,4759	0,4355	0,4122	0,3976	0,3879	0,3811	0,3761	0,3723	0,3695	0,3672	0,3654	0,3640	0,3628	0,3618	0,3609	0,3602	0,3596	0,3591	0,3587	0,3583	0,3580
10	10,7995	1,5143	0,7714	0,5668	0,4826	0,4399	0,4154	0,4000	0,3897	0,3825	0,3773	0,3734	0,3703	0,3680	0,3661	0,3645	0,3632	0,3622	0,3613	0,3606	0,3599	0,3594	0,3589	0,3585	0,3582
10,5	11,3352	1,5738	0,7929	0,5777	0,4892	0,4444	0,4186	0,4024	0,3916	0,3840	0,3785	0,3744	0,3712	0,3687	0,3667	0,3651	0,3637	0,3626	0,3617	0,3609	0,3603	0,3597	0,3592	0,3588	0,3584
11	11,8708	1,6333	0,8143	0,5886	0,4958	0,4488	0,4218	0,4048	0,3935	0,3855	0,3797	0,3754	0,3720	0,3694	0,3673	0,3656	0,3642	0,3631	0,3621	0,3613	0,3606	0,3600	0,3595	0,3591	0,3587
11,5	12,4065	1,6928	0,8357	0,5996	0,5024	0,4532	0,4249	0,4072	0,3953	0,3870	0,3809	0,3764	0,3729	0,3702	0,3680	0,3662	0,3647	0,3635	0,3625	0,3616	0,3609	0,3603	0,3598	0,3593	0,3589
12	12,9422	1,7523	0,8571	0,6105	0,5090	0,4576	0,4281	0,4096	0,3972	0,3885	0,3822	0,3774	0,3738	0,3709	0,3686	0,3667	0,3652	0,3640	0,3629	0,3620	0,3612	0,3606	0,3600	0,3596	0,3592
12,5	13,4779	1,8119	0,8786	0,6214	0,5156	0,4621	0,4313	0,4119	0,3990	0,3900	0,3834	0,3784	0,3746	0,3716	0,3692	0,3673	0,3657	0,3644	0,3633	0,3624	0,3616	0,3609	0,3603	0,3598	0,3594
13	14,0136	1,8714	0,9000	0,6324	0,5222	0,4665	0,4344	0,4143	0,4009	0,3914	0,3846	0,3794	0,3755	0,3724	0,3699	0,3679	0,3662	0,3648	0,3637	0,3627	0,3619	0,3612	0,3606	0,3601	0,3596
13,5	14,5493	1,9309	0,9214	0,6433	0,5289	0,4709	0,4376	0,4167	0,4027	0,3929	0,3858	0,3805	0,3763	0,3731	0,3705	0,3684	0,3667	0,3653	0,3641	0,3631	0,3622	0,3615	0,3609	0,3603	0,3599
14	15,0850	1,9904	0,9429	0,6542	0,5355	0,4753	0,4408	0,4191	0,4046	0,3944	0,3870	0,3815	0,3772	0,3738	0,3712	0,3690	0,3672	0,3657	0,3645	0,3634	0,3625	0,3618	0,3611	0,3606	0,3601
14,5	15,6206	2,0499	0,9643	0,6652	0,5421	0,4798	0,4439	0,4215	0,4064	0,3959	0,3882	0,3825	0,3781	0,3746	0,3718	0,3695	0,3677	0,3662	0,3649	0,3638	0,3629	0,3621	0,3614	0,3608	0,3603
15	16,1563	2,1095	0,9857	0,6761	0,5487	0,4842	0,4471	0,4238	0,4083	0,3974	0,3894	0,3835	0,3789	0,3753	0,3724	0,3701	0,3682	0,3666	0,3653	0,3641	0,3632	0,3624	0,3617	0,3611	0,3606
15,5	16,6920	2,1690	1,0071	0,6870	0,5553	0,4886	0,4503	0,4262	0,4101	0,3989	0,3907	0,3845	0,3798	0,3760	0,3731	0,3707	0,3687	0,3670	0,3657	0,3645	0,3635	0,3627	0,3620	0,3613	0,3608
16	17,2277	2,2285	1,0286	0,6980	0,5619	0,4931	0,4534	0,4286	0,4120	0,4004	0,3919	0,3855	0,3806	0,3768	0,3737	0,3712	0,3692	0,3675	0,3661	0,3649	0,3638	0,3630	0,3622	0,3616	0,3610
16,5	17,7634	2,2880	1,0500	0,7089	0,5685	0,4975	0,4566	0,4310	0,4138	0,4018	0,3931	0,3865	0,3815	0,3775	0,3744	0,3718	0,3697	0,3679	0,3665	0,3652	0,3642	0,3633	0,3625	0,3618	0,3613
17	18,2991	2,3475	1,0714	0,7198	0,5751	0,5019	0,4598	0,4334	0,4157	0,4033	0,3943	0,3875	0,3823	0,3783	0,3750	0,3723	0,3702	0,3684	0,3669	0,3656	0,3645	0,3636	0,3628	0,3621	0,3615
17,5	18,8348	2,4071	1,0928	0,7308	0,5818	0,5063	0,4630	0,4357	0,4176	0,4048	0,3955	0,3886	0,3832	0,3790	0,3756	0,3729	0,3707	0,3688	0,3672	0,3659	0,3648	0,3639	0,3631	0,3624	0,3618
18	19,3704	2,4666	1,1143	0,7417	0,5884	0,5108	0,4661	0,4381	0,4194	0,4063	0,3967	0,3896	0,3841	0,3797	0,3763	0,3735	0,3712	0,3692	0,3676	0,3663	0,3651	0,3642	0,3633	0,3626	0,3620
18,5	19,9061	2,5261	1,1357	0,7526	0,5950	0,5152	0,4693	0,4405	0,4213	0,4078	0,3980	0,3906	0,3849	0,3805	0,3769	0,3740	0,3717	0,3697	0,3680	0,3666	0,3655	0,3645	0,3636	0,3629	0,3622
19	20,4418	2,5856	1,1571	0,7636	0,6016	0,5196	0,4725	0,4429	0,4231	0,4093	0,3992	0,3916	0,3858	0,3812	0,3775	0,3746	0,3721	0,3701	0,3684	0,3670	0,3658	0,3648	0,3639	0,3631	0,3625
19,5	20,9775	2,6451	1,1786	0,7745	0,6082	0,5240	0,4756	0,4453	0,4250	0,4107	0,4004	0,3926	0,3866	0,3819	0,3782	0,3751	0,3726	0,3706	0,3688	0,3674	0,3661	0,3651	0,3641	0,3634	0,3627
20	21,5132	2,7047	1,2000	0,7854	0,6148	0,5285	0,4788	0,4476	0,4268	0,4122	0,4016	0,3936	0,3875	0,3827	0,3788	0,3757	0,3731	0,3710	0,3692	0,3677	0,3664	0,3654	0,3644	0,3636	0,3629
20,5	22,0489	2,7642	1,2214	0,7964	0,6214	0,5329	0,4820	0,4500	0,4287	0,4137	0,4028	0,3946	0,3883	0,3834	0,3795	0,3763	0,3736	0,3715	0,3696	0,3681	0,3668	0,3657	0,3647	0,3639	0,3632
21	22,5846	2,8237	1,2428	0,8073	0,6281	0,5373	0,4851	0,4524	0,4305	0,4152	0,4040	0,3957	0,3892	0,3841	0,3801	0,3768	0,3741	0,3719	0,3700	0,3684	0,3671	0,3660	0,3650	0,3641	0,3634
21,5	23,1202	2,8832	1,2643	0,8182	0,6347	0,5418	0,4883	0,4548	0,4324	0,4167	0,4052	0,3967	0,3901	0,3849	0,3807	0,3774	0,3746	0,3723	0,3704	0,3688	0,3674	0,3662	0,3652	0,3644	0,3636
22	23,6559	2,9427	1,2857	0,8292	0,6413	0,5462	0,4915	0,4572	0,4342	0,4182	0,4065	0,3977	0,3909	0,3856	0,3814	0,3779	0,3751	0,3728	0,3708	0,3692	0,3677	0,3665	0,3655	0,3646	0,3639
22,5	24,1916	3,0023	1,3071	0,8401	0,6479	0,5506	0,4947	0,4596	0,4361	0,4196	0,4077	0,3987	0,3918	0,3864	0,3820	0,3785	0,3756	0,3732	0,3712	0,3695	0,3681	0,3668	0,3658	0,3649	0,3641
23	24,7273	3,0618	1,3285	0,8510	0,6545	0,5550	0,4978	0,4619	0,4379	0,4211	0,4089	0,3997	0,3926	0,3871	0,3827	0,3791	0,3761	0,3737	0,3716	0,3699	0,3684	0,3671	0,3661	0,3651	0,3644
23,5	25,2630	3,1213	1,3500	0,8620	0,6611	0,5595	0,5010	0,4643	0,4398	0,4226	0,4101	0,4007	0,3935	0,3878	0,3833	0,3796	0,3766	0,3741	0,3720	0,3702	0,3687	0,3674	0,3663	0,3654	0,3646
24	25,7987	3,1808	1,3714	0,8729	0,6677	0,5639	0,5042	0,4667	0,4417	0,4241	0,4113	0,4017	0,3944	0,3886	0,3839	0,3802	0,3771	0,3745	0,3724	0,3706	0,3690	0,3677	0,3666	0,3657	0,3648
24,5	26,3344	3,2404	1,3928	0,8838	0,6744	0,5683	0,5073	0,4691	0,4435	0,4256	0,4125	0,4027	0,3952	0,3893	0,3846	0,3807	0,3776	0,3750	0,3728	0,3709	0,3694	0,3680	0,3669	0,3659	0,3651
25	26,8701	3,2999	1,4143	0,8948	0,6810	0,5727	0,5105	0,4715	0,4454	0,4271	0,4138	0,4038	0,3961	0,3900	0,3852	0,3813	0,3781	0,3754	0,3732	0,3713	0,3697	0,3683	0,3672	0,3662	0,3653

Appendix B.6 MATRIX WITH β_F VALUES

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
0,5	-0,0002	1,7778	1,9200	1,9592	1,9753	1,9835	1,9882	1,9911	1,9931	1,9945	1,9955	1,9962	1,9968	1,9973	1,9976	1,9979	1,9982	1,9984	1,9985	1,9987	1,9988	1,9989	1,9990	1,9991	1,9992
1	-4,0006	1,3333	1,7600	1,8775	1,9259	1,9504	1,9645	1,9733	1,9792	1,9834	1,9864	1,9887	1,9904	1,9918	1,9929	1,9938	1,9945	1,9951	1,9956	1,9961	1,9964	1,9968	1,9970	1,9973	1,9975
1,5	-8,0010	0,8888	1,6000	1,7959	1,8765	1,9173	1,9408	1,9556	1,9654	1,9723	1,9773	1,9811	1,9840	1,9863	1,9881	1,9896	1,9908	1,9918	1,9927	1,9934	1,9941	1,9946	1,9951	1,9955	1,9958
2	-12,0015	0,4443	1,4399	1,7143	1,8271	1,8843	1,9172	1,9378	1,9516	1,9612	1,9683	1,9735	1,9776	1,9808	1,9834	1,9854	1,9871	1,9886	1,9898	1,9908	1,9917	1,9924	1,9931	1,9937	1,9942
2,5	-16,0019	-0,0002	1,2799	1,6326	1,7778	1,8512	1,8935	1,9200	1,9377	1,9501	1,9592	1,9660	1,9712	1,9753	1,9786	1,9813	1,9835	1,9853	1,9869	1,9882	1,9893	1,9903	1,9911	1,9919	1,9925
3	-20,0023	-0,4447	1,1199	1,5510	1,7284	1,8182	1,8698	1,9022	1,9239	1,9391	1,9501	1,9584	1,9648	1,9698	1,9738	1,9771	1,9798	1,9820	1,9839	1,9855	1,9869	1,9881	1,9891	1,9900	1,9908
3,5	-24,0027	-0,8892	0,9599	1,4693	1,6790	1,7851	1,8461	1,8844	1,9100	1,9280	1,9410	1,9508	1,9584	1,9643	1,9691	1,9729	1,9761	1,9788	1,9810	1,9829	1,9845	1,9859	1,9872	1,9882	1,9892
4	-28,0031	-1,3337	0,7999	1,3877	1,6296	1,7520	1,8225	1,8667	1,8962	1,9169	1,9320	1,9433	1,9520	1,9588	1,9643	1,9688	1,9724	1,9755	1,9781	1,9803	1,9822	1,9838	1,9852	1,9864	1,9875
4,5	-32,0035	-1,7782	0,6399	1,3061	1,5802	1,7190	1,7988	1,8489	1,8823	1,9058	1,9229	1,9357	1,9456	1,9534	1,9596	1,9646	1,9688	1,9722	1,9752	1,9776	1,9798	1,9816	1,9832	1,9846	1,9858
5	-36,0040	-2,2227	0,4798	1,2244	1,5308	1,6859	1,7751	1,8311	1,8685	1,8947	1,9138	1,9282	1,9392	1,9479	1,9548	1,9605	1,9651	1,9690	1,9722	1,9750	1,9774	1,9794	1,9812	1,9828	1,9842
5,5	-40,0044	-2,6672	0,3198	1,1428	1,4814	1,6529	1,7515	1,8133	1,8547	1,8836	1,9048	1,9206	1,9328	1,9424	1,9501	1,9563	1,9614	1,9657	1,9693	1,9724	1,9750	1,9773	1,9793	1,9810	1,9825
6	-44,0048	-3,1116	0,1598	1,0611	1,4320	1,6198	1,7278	1,7955	1,8408	1,8726	1,8957	1,9130	1,9264	1,9369	1,9453	1,9521	1,9578	1,9624	1,9664	1,9698	1,9726	1,9751	1,9773	1,9792	1,9808
6,5	-48,0052	-3,5561	-0,0002	0,9795	1,3827	1,5867	1,7041	1,7778	1,8270	1,8615	1,8866	1,9055	1,9200	1,9314	1,9405	1,9480	1,9541	1,9592	1,9635	1,9671	1,9703	1,9730	1,9753	1,9774	1,9792
7	-52,0056	-4,0006	-0,1602	0,8978	1,3333	1,5537	1,6804	1,7600	1,8131	1,8504	1,8775	1,8979	1,9136	1,9259	1,9358	1,9438	1,9504	1,9559	1,9606	1,9645	1,9679	1,9708	1,9733	1,9756	1,9775
7,5	-56,0061	-4,4451	-0,3202	0,8162	1,2839	1,5206	1,6568	1,7422	1,7993	1,8393	1,8685	1,8903	1,9072	1,9204	1,9310	1,9396	1,9467	1,9526	1,9576	1,9619	1,9655	1,9686	1,9714	1,9737	1,9758
8	-60,0065	-4,8896	-0,4803	0,7346	1,2345	1,4875	1,6331	1,7244	1,7854	1,8282	1,8594	1,8828	1,9008	1,9149	1,9263	1,9355	1,9431	1,9494	1,9547	1,9592	1,9631	1,9665	1,9694	1,9719	1,9742
8,5	-64,0069	-5,3341	-0,6403	0,6529	1,1851	1,4545	1,6094	1,7066	1,7716	1,8172	1,8503	1,8752	1,8944	1,9095	1,9215	1,9313	1,9394	1,9461	1,9518	1,9566	1,9607	1,9643	1,9674	1,9701	1,9725
9	-68,0073	-5,7786	-0,8003	0,5713	1,1357	1,4214	1,5858	1,6889	1,7578	1,8061	1,8413	1,8677	1,8880	1,9040	1,9168	1,9272	1,9357	1,9429	1,9489	1,9540	1,9584	1,9621	1,9654	1,9683	1,9708
9,5	-72,0077	-6,2231	-0,9603	0,4896	1,0863	1,3884	1,5621	1,6711	1,7439	1,7950	1,8322	1,8601	1,8816	1,8985	1,9120	1,9230	1,9320	1,9396	1,9459	1,9513	1,9560	1,9600	1,9635	1,9665	1,9692
10	-76,0081	-6,6676	-1,1203	0,4080	1,0369	1,3553	1,5384	1,6533	1,7301	1,7839	1,8231	1,8525	1,8752	1,8930	1,9072	1,9188	1,9284	1,9363	1,9430	1,9487	1,9536	1,9578	1,9615	1,9647	1,9675
10,5	-80,0086	-7,1121	-1,2803	0,3264	0,9875	1,3222	1,5147	1,6355	1,7162	1,7728	1,8140	1,8450	1,8688	1,8875	1,9025	1,9147	1,9247	1,9331	1,9401	1,9461	1,9512	1,9556	1,9595	1,9629	1,9658
11	-84,0090	-7,5566	-1,4404	0,2447	0,9382	1,2892	1,4911	1,6177	1,7024	1,7617	1,8050	1,8374	1,8624	1,8820	1,8977	1,9105	1,9210	1,9298	1,9372	1,9435	1,9488	1,9535	1,9575	1,9611	1,9642
11,5	-88,0094	-8,0010	-1,6004	0,1631	0,8888	1,2561	1,4674	1,6000	1,6885	1,7507	1,7959	1,8298	1,8560	1,8765	1,8930	1,9063	1,9173	1,9265	1,9343	1,9408	1,9465	1,9513	1,9556	1,9593	1,9625
12	-92,0098	-8,4455	-1,7604	0,0814	0,8394	1,2231	1,4437	1,5822	1,6747	1,7396	1,7868	1,8223	1,8496	1,8710	1,8882	1,9022	1,9137	1,9233	1,9313	1,9382	1,9441	1,9492	1,9536	1,9574	1,9608
12,5	-96,0102	-8,8900	-1,9204	-0,0002	0,7900	1,1900	1,4201	1,5644	1,6609	1,7285	1,7778	1,8147	1,8432	1,8656	1,8835	1,8980	1,9100	1,9200	1,9284	1,9356	1,9417	1,9470	1,9516	1,9556	1,9592
13	-100,0106	-9,3345	-2,0804	-0,0818	0,7406	1,1569	1,3964	1,5466	1,6470	1,7174	1,7687	1,8072	1,8368	1,8601	1,8787	1,8938	1,9063	1,9167	1,9255	1,9329	1,9393	1,9448	1,9496	1,9538	1,9575
13,5	-104,0111	-9,7790	-2,2404	-0,1635	0,6912	1,1239	1,3727	1,5288	1,6332	1,7063	1,7596	1,7996	1,8304	1,8546	1,8739	1,8897	1,9027	1,9135	1,9226	1,9303	1,9369	1,9427	1,9476	1,9520	1,9558
14	-108,0115	-10,2235	-2,4005	-0,2451	0,6418	1,0908	1,3490	1,5111	1,6193	1,6953	1,7505	1,7920	1,8240	1,8491	1,8692	1,8855	1,8990	1,9102	1,9196	1,9277	1,9346	1,9405	1,9457	1,9502	1,9542
14,5	-112,0119	-10,6680	-2,5605	-0,3268	0,5924	1,0578	1,3254	1,4933	1,6055	1,6842	1,7415	1,7845	1,8176	1,8436	1,8644	1,8814	1,8953	1,9069	1,9167	1,9250	1,9322	1,9383	1,9437	1,9484	1,9525
15	-116,0123	-11,1125	-2,7205	-0,4084	0,5431	1,0247	1,3017	1,4755	1,5917	1,6731	1,7324	1,7769	1,8112	1,8381	1,8597	1,8772	1,8916	1,9037	1,9138	1,9224	1,9298	1,9362	1,9417	1,9466	1,9508
15,5	-120,0127	-11,5570	-2,8805	-0,4901	0,4937	0,9916	1,2780	1,4577	1,5778	1,6620	1,7233	1,7694	1,8048	1,8326	1,8549	1,8730	1,8880	1,9004	1,9109	1,9198	1,9274	1,9340	1,9397	1,9448	1,9492
16	-124,0132	-12,0015	-3,0405	-0,5717	0,4443	0,9586	1,2544	1,4399	1,5640	1,6509	1,7143	1,7618	1,7984	1,8271	1,8502	1,8689	1,8843	1,8971	1,9080	1,9172	1,9250	1,9318	1,9378	1,9430	1,9475
16,5	-128,0136	-12,4460	-3,2005	-0,6533	0,3949	0,9255	1,2307	1,4222	1,5501	1,6399	1,7052	1,7542	1,7920	1,8217	1,8454	1,8647	1,8806	1,8939	1,9050	1,9145	1,9227	1,9297	1,9358	1,9411	1,9459
17	-132,0140	-12,8904	-3,3606	-0,7350	0,3455	0,8924	1,2070	1,4044	1,5363	1,6288	1,6961	1,7467	1,7856	1,8162	1,8406	1,8605	1,8769	1,8906	1,9021	1,9119	1,9203	1,9275	1,9338	1,9393	1,9442
17,5	-136,0144	-13,3349	-3,5206	-0,8166	0,2961	0,8594	1,1833	1,3866	1,5224	1,6177	1,6870	1,7391	1,7792	1,8107	1,8359	1,8564	1,8733	1,8873	1,8992	1,9093	1,9179	1,9254	1,9318	1,9375	1,9425
18	-140,0148	-13,7794	-3,6806	-0,8983	0,2467	0,8263	1,1597	1,3688	1,5086	1,6066	1,6780	1,7315	1,7728	1,8052	1,8311	1,8522	1,8696	1,8841	1,8963	1,9066	1,9155	1,9232	1,9299	1,9357	1,9409
18,5	-144,0152	-14,2239	-3,8406	-0,9799	0,1973	0,7933	1,1360	1,3510	1,4948	1,5955	1,6689	1,7240	1,7664	1,7997	1,8264	1,8481	1,8659	1,8808	1,8933	1,9040	1,9131	1,9210	1,9279	1,9339	1,9392
19	-148,0157	-14,6684	-4,0006	-1,0615	0,1480	0,7602	1,1123	1,3333	1,4809	1,5844	1,6598	1,7164	1,7600	1,7942	1,8216	1,8439	1,8622	1,8775	1,8904	1,9014	1,9108	1,9189	1,9259	1,9321	1,9375
19,5	-152,0161	-15,1129	-4,1606	-1,1432	0,0986	0,7271	1,0887	1,3155	1,4671	1,5734	1,6508	1,7089	1,7536	1,7887	1,8169	1,8397	1,8586	1,8743	1,8875	1,8987	1,9084	1,9167	1,9239	1,9303	1,9359
20	-156,0165	-15,5574	-4,3207	-1,2248	0,0492	0,6941	1,0650	1,2977	1,4532	1,5623	1,6417	1,7013	1,7472	1,7832	1,8121	1,8356	1,8549	1,8710	1,8846	1,8961	1,9060	1,9145	1,9220	1,9285	1,9342
20,5	-160,0169	-16,0019	-4,4807	-1,3065	-0,0002	0,6610	1,0413	1,2799	1,4394	1,5512	1,6326	1,6937	1,7408	1,7778	1,8074	1,8314	1,8512	1,8677	1,8817	1,8935	1,9036	1,9124	1,9200	1,9267	1,9325
21	-164,0173	-16,4464	-4,6407	-1,3881	-0,0496	0,6280	1,0176	1,2621	1,4255	1,5401	1,6235	1,6862	1,7344	1,7723	1,8026	1,8272	1,8476	1,8645	1,8787	1,8909	1,9012	1,9102	1,9180	1,9248	1,9309
21,5	-168,0177	-16,8909	-4,8007	-1,4697	-0,0990	0,5949	0,9940	1,2444	1,4117	1,5290	1,6145	1,6786	1,7280	1,7668	1,7978	1,8231	1,8439	1,8612	1,8758	1,8882	1,8989	1,9080	1,9160	1,9230	1,9292
22	-172,0182	-17,3354	-4,9607	-1,5514	-0,1484	0,5618	0,9703	1,2266	1,3979	1,5180	1,6054	1,6710	1,7216	1,7613	1,7931	1,8189	1,8402	1,8579	1,8729	1,8856	1,8965	1,9059	1,9141	1,9212	1,9275
22,5	-176,0186	-17,7798	-5,1207	-1,6330	-0,1978	0,5288	0,9466	1,2088	1,3840	1,5069	1,5963	1,6635	1,7152	1,7558	1,7883	1,8148	1,8365	1,8547	1,8700	1,8830	1,8941	1,9037	1,9121	1,9194	1,9259
23	-180,0190	-18,2243	-5,2808	-1,7147	-0,2471	0,4957	0,9230	1,1910	1,3702	1,4958	1,5873	1,6559	1,7088	1,7503	1,7836	1,8106	1,8329	1,8514	1,8670	1,8803	1,8917	1,9016	1,9101	1,9176	1,9242
23,5	-184,0194	-18,6688	-5,4408	-1,7963	-0,2965	0,4626	0,8993	1,1732	1,3563	1,4847	1,5782	1,6484	1,7024	1,7448	1,7788	1,8064	1,8292	1,8481	1,8641	1,8777	1,8893	1,8994	1,9081	1,9158	1,9225
24	-188,0198	-19,1133	-5,6008	-1,8780	-0,3459	0,4296	0,8756	1,1555	1,3425	1,4736	1,5691	1,6408	1,6960	1,7393	1,7741	1,8023	1,8255	1,8449	1,8612	1,8751	1,8870	1,8972	1,9062	1,9140	1,9209
24,5	-192,0203	-19,5578	-5,7608	-1,9596	-0,3953	0,3965	0,8520	1,1377	1,3286	1,4625	1,5600	1,6332	1,6896	1,7339	1,7693	1,7981	1,8218	1,8416	1,8583	1,8724	1,8846	1,8951	1,9042	1,9122	1,9192
25	-196,0207	-20,0023	-5,9208	-2,0412	-0,4447	0,3635	0,8283	1,1199	1,3148	1,4515	1,5510	1,6257	1,6832	1,7284	1,7645	1,7939	1,8182	1,8384	1,8554	1,8698	1,8822	1,8929	1,9022	1,9104	1,9175

Appendix B.7 LONG TERM 3-HOUR MAXIMA (CREST HEIGHT, 2^{ND} ORDER, 2D)

```
clear all
close all
clc
%load data
load('joint_pdf.mat');
%define input for max 3h response
Xi = 1:0.5:30;
%define range hs (mid-interval)
hs = 0.25:0.5:24.75;
%define range tp (mid-interval)
tp = 0.5:1:24.5;
%define water depth Heidrun field
d = 350;
%define matrix of zeros for the Ursell number Ur
Ur = zeros(length(hs),length(tp));
%define matrix of zeros for steepness S1
S1 = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient alpha_F
Alpha_F = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient beta_F
Beta_F = zeros(length(hs),length(tp));
%define row vector of zeros for k_1 (dispersion relation solutions)
sol_k_1 = zeros(1,length(tp));
%loop to calculate k_1, Ur, S1, alpha_F and Beta_F
for i = 1:length(hs);
    for j=1:length(tp);
    %define unknown variable(wave number)
    syms k_1;
    %solve dispersion relation for each mean wave period T1
    [sol_k_1(j)] = vpasolve([(2.3981*pi/tp(j))^2==k_1*9.81*(tanh(k_1*350))], [k_1], [0 Inf]);
    %calculate Ursell number
    Ur(i,j) = (hs(i)/(((sol_k_1(j))^2)*d^3));
    %calculate steepness
    S1(i,j) = (2*pi*hs(i))/(9.81*(0.8316*tp(j))^2);
    %calculate alpha_F
    Alpha_F(i,j) = 0.3536+(0.2892*S1(i,j))+(0.1060*Ur(i,j));
    %calculate beta_F
    Beta_F(i,j) = 2-(2.1597*S1(i,j))+(0.0968*(Ur(i,j))^2);
    end
end

%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of Xi, Hs and Tp
for k = 1:length(Xi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        %create table of short term distribution (non-exceedance
        %probabilities) Fc3h|hs,tp for the given Xi
        Fc(i,j) = (1-exp(-((Xi(k)/(Alpha_F(i,j)*hs(i)))^Beta_F(i,j))))^(13846/tp(j));
        %create a cell array of containing short term conditional
        %distribution for given Xi
        Fc_cell{k} = Fc;
        end
```

```
    end
end

%*Long term distribution of 3h max. F_X3h (b1=0.005)*
Table_F_Xi3h_fit_1_cell = cell(1,k);
F_Xi3h_fit_rows_sum_1_cell = cell(1,k);
F_Xi3h_fit_1_cell = cell(k,1);
%execute loop
for k = 1:length(Xi);
    %calculate long term distribution of 3h maximum response F_X3h
    Table_F_Xi3h_fit_1 = bsxfun(@times,Fc_cell{k},Joint_pdf_norm_1);
    %create cell array with results
    Table_F_Xi3h_fit_1_cell{k} = Table_F_Xi3h_fit_1;
    %count contributions along rows (tp)
    F_Xi3h_fit_rows_sum_1 = sum(Table_F_Xi3h_fit_1_cell{k},2);
    %create cell array with results
    F_Xi3h_fit_rows_sum_1_cell{k} = F_Xi3h_fit_rows_sum_1;
    %sum along column to get total contribution for the given
    %value of the 3h response maxima Xi
    F_Xi3h_fit_1 = sum(F_Xi3h_fit_rows_sum_1_cell{k},1);
    %create cell array with results
    F_Xi3h_fit_1_cell{k} = F_Xi3h_fit_1;
end
F_Xi_3h =  cell2mat(F_Xi3h_fit_1_cell);

%*Probability plot*
% input plot
Z = (log(Xi))';
Y = log(-log(1-F_Xi_3h));
q_100 = (10^-2)/2920;
q_10000 = (10^-4)/2920;
q_1yr = 1/2920;
q_10yr = (10^-1)/2920;
Y_ULS = log(-log(q_100));
Y_ALS = log(-log(q_10000));
Y_1yr = log(-log(q_1yr));
Y_10yr = log(-log(q_10yr));

figure(1)
plot(Z,Y,'r');
hold on
A = line([0 3.5],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 3.5],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{\xi})$)$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term distribution 3-hr max.($2^nd$ order, crest $\xi$,
2D)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);
crest_ULS = exp(x_cs_100);
crest_ALS = exp(x_cs_10000);
crest_ULS_round= ceil(10*crest_ULS)/10;
crest_ALS_round= ceil(10*crest_ALS)/10;
Xi_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));    %characteristic ULS extreme value, rounded
to 2 decimals
Xi_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));    %characteristic ALS extreme value, rounded
to 2 decimals
```

```
legend('Long term distribution 3-hr max.',['ULS, \xi_{ULS} = ',num2str(Xi_ULS_ch),'m'],['ALS, \xi_{ALS}
= ',num2str(Xi_ALS_ch),'m'],'Location','Southoutside');
%xlim([0 4]);
ylim([-3 3.5]);
grid on
grid minor
hold off

figure(2)
plot(Z,Y,'r');
hold on
D = line([0 3.5],[Y_1yr Y_1yr],'Linestyle','--','LineWidth',1.5,'Color',[0.85 0 0.5]);
E = line([0 3.5],[Y_10yr Y_10yr],'Linestyle','--','LineWidth',1.5,'Color',[0.5 0 0.5]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{\Xi}))$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term individual global maxima(crest(2D),
$\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
[x_cs_1yr,y_cs_1yr] = polyxpoly(Z,Y,[0 4],[Y_1yr Y_1yr]);
[x_cs_10yr,y_cs_10yr] = polyxpoly(Z,Y,[0 4],[Y_10yr Y_10yr]);
crest_1yr = exp(x_cs_1yr);
crest_10yr = exp(x_cs_10yr);
crest_1yr_round= ceil(10*crest_1yr)/10;
crest_10yr_round= ceil(10*crest_10yr)/10;
Xi_1yr_ch = str2double(sprintf('%.2f',crest_1yr_round));
Xi_10yr_ch = str2double(sprintf('%.2f',crest_10yr_round));
legend('Long term maxima \xi',['1yr, \xi_{k} = ',num2str(Xi_1yr_ch),'m'],['10yr, \xi_{k} =
',num2str(Xi_10yr_ch),'m'],'Location','Southoutside');
grid on
grid minor
hold off
%save workspace
save('alpha_beta_forristall','Alpha_F','Beta_F');
%%*Save data to xl*
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'M9';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'N9';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_1yr_ch};
sheet = 'Summary_Crest';
xlRange = 'M35';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_10yr_ch};
sheet = 'Summary_Crest';
xlRange = 'N35';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.8 LONG TERM 3-HOUR MAXIMA (CREST HEIGHT, 2ND ORDER, 3D)

```
clear all
close all
clc
%load data
load('joint_pdf.mat');
%define input for max 3h response
Xi = 1:0.5:30;
%define range hs (mid-interval)
hs = 0.25:0.5:24.75;
%define range tp (mid-interval)
tp = 0.5:1:24.5;
%define water depth Heidrun field
d = 350;
%define matrix of zeros for the Ursell number Ur
Ur = zeros(length(hs),length(tp));
%define matrix of zeros for steepness S1
S1 = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient alpha_F
Alpha_F = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient beta_F
Beta_F = zeros(length(hs),length(tp));
%define row vector of zeros for k_1 (dispersion relation solutions)
sol_k_1 = zeros(1,length(tp));
%loop to calculate k_1, Ur, S1, alpha_F and Beta_F
for i = 1:length(hs);
    for j=1:length(tp);
    %define unknown variable(wave number)
    syms k_1;
    %solve dispersion relation for each mean wave period T1, returning
    %array with wave numbers k_1 for given mean wave period (T1=0.8316*Tp)
    [sol_k_1(j)] = vpasolve([2.3981*pi/tp(j)==k_1*9.81*(tanh(k_1*350))], [k_1], [0 Inf]);
    %calculate Ursell number
    Ur(i,j) = (hs(i)/(((sol_k_1(j))^2)*d^3));
    %calculate steepness
    S1(i,j) = (2*pi*hs(i))/(9.81*(0.8316*tp(j))^2);
    %calculate alpha_F
    Alpha_F(i,j) = 0.3536+(0.2568*S1(i,j))+(0.08*Ur(i,j));
    %calculate beta_F
    Beta_F(i,j) = 2-(1.7912*S1(i,j))-(0.5302*(Ur(i,j)))+(0.284*(Ur(i,j))^2);
    end
end

%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of Xi, Hs and Tp
for k = 1:length(Xi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        %create table of short term distribution (non-exceedance
        %probabilities) Fc3h|hs,tp for the given Xi
        Fc(i,j) = (1-exp(-((Xi(k)/(Alpha_F(i,j)*hs(i)))^Beta_F(i,j))))^(14026/tp(j));
        %create a cell array of containing short term conditional
        %distribution for given Xi
        Fc_cell{k} = Fc;
```

```
        end
    end
end

%*Long term distribution of 3h maximum response F_X3h*
Table_F_Xi3h_fit_1_cell = cell(1,k);
F_Xi3h_fit_rows_sum_1_cell = cell(1,k);
F_Xi3h_fit_1_cell = cell(k,1);
%execute loop
for k = 1:length(Xi);
    %calculate long term distribution of 3h maximum response F_X3h
    Table_F_Xi3h_fit_1 = bsxfun(@times,Fc_cell{k},Joint_pdf_norm_1);
    %create cell array with results
    Table_F_Xi3h_fit_1_cell{k} = Table_F_Xi3h_fit_1;
    %count contributions along rows (tp)
    F_Xi3h_fit_rows_sum_1 = sum(Table_F_Xi3h_fit_1_cell{k},2);
    %create cell array with results
    F_Xi3h_fit_rows_sum_1_cell{k} = F_Xi3h_fit_rows_sum_1;
    %sum along column to get total contribution for the given
    %value of the 3h response maxima Xi
    F_Xi3h_fit_1 = sum(F_Xi3h_fit_rows_sum_1_cell{k},1);
    %create cell array with results
    F_Xi3h_fit_1_cell{k} = F_Xi3h_fit_1;
end

F_Xi_3h =  cell2mat(F_Xi3h_fit_1_cell);

%*Probability plot*
% input plot
Z = (log(Xi))';
Y = log(-log(1-F_Xi_3h));
q_100 = (10^-2)/2920;
q_10000 = (10^-4)/2920;

Y_ULS = log(-log(q_100));
Y_ALS = log(-log(q_10000));

figure(1)
plot(Z,Y,'r');
hold on
A = line([0 3.5],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 3.5],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{\xi})$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term distribution 3-hr max.($2^(N-003)$ order, crest $\xi$,
3D)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');

[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);

crest_ULS = exp(x_cs_100);
crest_ALS = exp(x_cs_10000);

crest_ULS_round= ceil(10*crest_ULS)/10;
crest_ALS_round= ceil(10*crest_ALS)/10;

Xi_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));
```

```
Xi_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));
legend('Long term distribution 3-hr max.',['\xi_{ULS} = ',num2str(Xi_ULS_ch),'m'],['\xi_{ALS} =
',num2str(Xi_ALS_ch),'m'],'Location','Southoutside');
%xlim([0 4]);
ylim([-3 3.5]);
grid on
grid minor
hold off

%*Save data to xl*
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'O9';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'P9';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.9 LONG TERM 3-HOUR MAXIMA (WAVE HEIGHT, EMP. FORRISTALL)

```
clear all
close all
clc
%load data
load('joint_pdf.mat')
%define input for max 3h value
hi = 2:1:50;
%define range hs
hs = 0.5:0.5:25;
%tp range
tp = 0.5:1:24.5;
%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of Xi, Hs and Tp
for k = 1:length(hi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        %create table of short term distribution (non-exceedance
        %probabilities) Fc3h|hs,tp for the given Xi
        Fc(i,j) = (1-exp(-2.2526*(hi(k)/hs(i))^2.13))^(13846/tp(j));
        %create a cell array of containing short term conditional
        %distribution for given hi
        Fc_cell{k} = Fc;
        end
    end
end

%*Long term distribution of 3h max. F_X3h (b1=0.005)*
Table_F_Xi3h_fit_1_cell = cell(1,k);
F_Xi3h_fit_rows_sum_1_cell = cell(1,k);
F_Xi3h_fit_1_cell = cell(k,1);
%execute loop
for k = 1:length(hi);
    %calculate long term distribution of 3h maximum value F_X3h
    Table_F_Xi3h_fit_1 = bsxfun(@times,Fc_cell{k},Joint_pdf_norm_1);
    %create cell array with results
    Table_F_Xi3h_fit_1_cell{k} = Table_F_Xi3h_fit_1;
    %count contributions along rows (tp)
    F_Xi3h_fit_rows_sum_1 = sum(Table_F_Xi3h_fit_1_cell{k},2);
    %create cell array with results
    F_Xi3h_fit_rows_sum_1_cell{k} = F_Xi3h_fit_rows_sum_1;
    %sum along column to get total contribution for the given
    %value of the 3h max value
    F_Xi3h_fit_1 = sum(F_Xi3h_fit_rows_sum_1_cell{k},1);
    %create cell array with results
    F_Xi3h_fit_1_cell{k} = F_Xi3h_fit_1;
end

%transform cell to matrix
F_Xi_3h = cell2mat(F_Xi3h_fit_1_cell);

%*Probability plot*
Z = (log(hi))';
Y = log(-log(1-F_Xi_3h));
```

```
q_100 = (10^-2)/2920;
q_10000 = (10^-4)/2920;
Y_ULS = log(-log(q_100));
Y_ALS = log(-log(q_10000));
figure(1)
plot(Z,Y,'r');
hold on
A = line([0 4],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 4],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln(H)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{H})$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term distribution 3-hr max.(Forristall, wave height
H)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');

[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);

wave_height_ULS = exp(x_cs_100);
wave_height_ALS = exp(x_cs_10000);

crest_ULS_round= ceil(10*wave_height_ULS)/10;
crest_ALS_round= ceil(10*wave_height_ALS)/10;

h_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));
h_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));

legend('Long term distribution 3-hr max.',['H_{ULS} = ',num2str(h_ULS_ch),'m'],['H_{ALS} =
',num2str(h_ALS_ch),'m'],'Location','Southoutside');
xlim([0.5 4]);
ylim([-3 4]);
grid on
grid minor
hold off

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {h_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'M20';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {h_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'N20';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.10 SHORT TERM MODELLING OF SEA STATES

```
clear all
close all
clc
%load joint pdf (b1=0.005)
load ('joint_pdf');
%generate linearly spaced vectors
f = linspace(0.025,1,196);
%hs range
hs = 0.25:0.5:24.75;
%tp range
tp = 0.5:1:24.5;
%calculate wave spectrum (JONSWAP)
wavespectrum = Jonswap_spectrum(f,hs,tp);
%0th moment
m0 = zeros(numel(hs),numel(tp));
for j = 1:length(hs);
    for k = 1:length(tp);
        m0(j,k) = trapz(f,wavespectrum(:,j,k));
    end;
end;

%1st moment
m1=zeros(numel(hs),numel(tp));
for j = 1:length(hs);
    for k = 1:length(tp);
        Response1 = f'.^1.*wavespectrum(:,j,k);
        m1(j,k) = trapz(f,Response1);
    end;
end;
%2nd moment:
m2=zeros(numel(hs),numel(tp));
for j = 1:length(hs);
    for k = 1:length(tp);
        Response2 = f'.^2.*wavespectrum(:,j,k);
        m2(j,k) = trapz(f,Response2);
    end;
end;

%Expected zero-up-crossing frequency:
zero_up_freq=sqrt(m2./m0);
%Standard deviation:
stddev=sqrt((m0));
%mean wave period T1
T1 = m0./m1;
%long term mean zero-up crossing frequency
A = bsxfun(@times,zero_up_freq,Joint_pdf_norm_1);  %elementwise product of expected zero-up freq. & long
term distribution of sea states
B = sum(A,2);   %sum along rows (tp)
mean_zero_up_freq = sum(B,1);   %sum along column

%return period ULS
T_ULS = 100;
%Expected # of global maxima in T years
n_T_ULS = T_ULS*365*24*3600*mean_zero_up_freq;
```

```
%return period ALS
T_ALS = 10000;
%Expected # of global maxima in T years
n_T_ALS = T_ALS*365*24*3600*mean_zero_up_freq;

%return period 1yr
T_1yr = 1;
%Expected # of global maxima in T years
n_T_1yr = T_1yr*365*24*3600*mean_zero_up_freq;

%return period 10yr
T_10yr = 10;
%Expected # of global maxima in T years
n_T_10yr = T_10yr*365*24*3600*mean_zero_up_freq;

save('Mean_periods','mean_zero_up_freq','zero_up_freq','n_T_ULS','n_T_ALS','T1','n_T_1yr','n_T_10yr');

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {mean_zero_up_freq};
sheet = 'Summary_Crest';
xlRange = 'W8';
xlswrite(filename,C,sheet,xlRange)
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {n_T_ULS};
sheet = 'Summary_Crest';
xlRange = 'X8';
xlswrite(filename,C,sheet,xlRange)
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {n_T_ALS};
sheet = 'Summary_Crest';
xlRange = 'Y8';
xlswrite(filename,C,sheet,xlRange)
%plot(f,wavespectrum(:,:,7));
```

Appendix B.11 SHORT TERM ALL GLOBAL MAXIMA (CREST HEIGHT, 2ND ORDER, 2D)

```
clear all
close all
clc
%load short term description data
load('Mean_periods.mat');
%define input crest height
Xi = 1:0.5:30;
%hs range
hs = 0.25:0.5:24.75;
%tp range
tp = 0.5:1:24.5;
%define water depth Heidrun field
d = 350;
%define matrix of zeros for the Ursell number Ur
Ur = zeros(length(hs),length(tp));
%define matrix of zeros for steepness S1
S1 = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient alpha_F
Alpha_F = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient beta_F
Beta_F = zeros(length(hs),length(tp));
%define row vector of zeros for k_1 (dispersion relation solutions)
sol_k_1 = zeros(1,length(tp));
for i = 1:length(hs);
    for j=1:length(tp);
    %define unknown variable(wave number)
    syms k_1;
    %solve dispersion relation for each mean wave period T1
    [sol_k_1(j)] = vpasolve([(T1(i,j)^2)==((4*(pi^2))/(k_1*9.81*(tanh(k_1*350))))], [k_1], [0 Inf]);
    %calculate Ursell number
    Ur(i,j) = (hs(i)/(((sol_k_1(j))^2)*d^3));
    %calculate steepness
    S1(i,j) = (2*pi*hs(i))/(9.81*(T1(i,j))^2);
    %calculate alpha_F
    Alpha_F(i,j) = 0.3536+(0.2892*S1(i,j))+(0.1060*Ur(i,j));
    %calculate beta_F
    Beta_F(i,j) = 2-(2.1597*S1(i,j))+(0.0968*(Ur(i,j))^2);
    end
end
%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of Xi, Hs and Tp
for k = 1:length(Xi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        Fc(i,j) = (1-exp(-((Xi(k)/(Alpha_F(i,j)*hs(i)))^Beta_F(i,j))));
        %create a cell array of containing short term conditional
        %distribution for given Xi
        Fc_cell{k} = Fc;
        end
    end
end
save('short_term_Forristall_2D','Fc_cell');
```

Appendix B.12 LONG TERM ALL GLOBAL MAXIMA (CREST HEIGHT, 2ND ORDER, 2D)

```
clear all
close all
clc
%load long term distribution sea state characteristics f_hs,tp (b1=0.005)
load('joint_pdf.mat');
%load short term crest distribution
load('short_term_Forristall_2D');
%load short term description data
load('Mean_periods.mat');
digits(40);
%input crest height range
Xi = 1:0.5:30;
%loop for elementwise multiplication zero_up_freq & short term distribution Fc
for k = 1:length(Xi);
    A = bsxfun(@times,zero_up_freq,Fc_cell{k});
    A_cell{k} = A.*(1/mean_zero_up_freq);
end;
%loop for elementwise multiplication of A with joint pdf sea states
for k = 1:length(Xi);
    B = bsxfun(@times,A_cell{k},Joint_pdf_norm_1);
    B_cell{k} = B;
    %sum along rows
    C = sum(B_cell{k},2);
    C_cell{k} = C;
    %sum along column
    D = sum(C_cell{k},1);
    %cell array with long term probabilities
    F_igm{k} = D;
end;
%convert cell to matrix
F_igm_mat = cell2mat(F_igm);
%save workspace file
save('Long_term_crest_Forristall_2D','F_igm_mat');
```

Appendix B.13 PROBABILITY PLOT, LONG TERM (CREST, 2ND ORDER, ALL GLOBAL MAX., 3D)

```
clear all
close all
clc
%load long term distribution individual global maxima
load('Long_term_crest_Forristall_2d.mat')
%load # response maxima
load('Mean_periods.mat');
% input plot
%range for crest height
xi = 1:0.5:30;
Z = (log(xi))';
Y = log(-log(1-F_igm_mat));
Y_ULS = log(-log(1/n_T_ULS));
Y_ALS = log(-log(1/n_T_ALS));
Y_1yr = log(-log(1/n_T_1yr));
Y_10yr = log(-log(1/n_T_10yr));

figure(1)
plot(Z,Y,'r');
hold on
A = line([0 3.5],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 3.5],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{\xi})$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term individual global maxima(crest(2D),
$\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);
crest_ULS = exp(x_cs_100);
crest_ALS = exp(x_cs_10000);
crest_ULS_round= ceil(10*crest_ULS)/10;
crest_ALS_round= ceil(10*crest_ALS)/10;
xi_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));
xi_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));
legend('Long term maxima \xi',['ULS, \xi_{k} = ',num2str(xi_ULS_ch),'m'],['ALS, \xi_{k} =
',num2str(xi_ALS_ch),'m'],'Location','Southoutside');
grid on
grid minor
hold off

figure(2)
plot(Z,Y,'r');
hold on
D = line([0 3.5],[Y_1yr Y_1yr],'Linestyle','--','LineWidth',1.5,'Color',[0.85 0 0.5]);
E = line([0 3.5],[Y_10yr Y_10yr],'Linestyle','--','LineWidth',1.5,'Color',[0.5 0 0.5]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{\xi})$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term individual global maxima(crest(2D),
$\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
[x_cs_1yr,y_cs_1yr] = polyxpoly(Z,Y,[0 4],[Y_1yr Y_1yr]);
[x_cs_10yr,y_cs_10yr] = polyxpoly(Z,Y,[0 4],[Y_10yr Y_10yr]);
crest_1yr = exp(x_cs_1yr);
crest_10yr = exp(x_cs_10yr);
crest_1yr_round= ceil(10*crest_1yr)/10;
```

```
crest_10yr_round= ceil(10*crest_10yr)/10;
Xi_1yr_ch = str2double(sprintf('%.2f',crest_1yr_round));
Xi_10yr_ch = str2double(sprintf('%.2f',crest_10yr_round));
legend('Long term maxima \xi',['1yr, \xi_{k} = ',num2str(Xi_1yr_ch),'m'],['10yr, \xi_{k} =
',num2str(Xi_10yr_ch),'m'],'Location','Southoutside');
grid on
grid minor
hold off

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'Q9';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'R9';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_1yr_ch};
sheet = 'Summary_Crest';
xlRange = 'O35';
xlswrite(filename,C,sheet,xlRange)

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_10yr_ch};
sheet = 'Summary_Crest';
xlRange = 'P35';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.14 SHORT TERM ALL GLOBAL MAXIMA (CREST HEIGHT, 2ND ORDER, 3D)

```
clear all
close all
clc
%load short term description data
load('Mean_periods.mat');
%define input crest height
Xi = 1:0.5:30;
%hs range
hs = 0.25:0.5:24.75;
%tp range
tp = 0.5:1:24.5;
%define water depth Heidrun field
d = 350;
%define matrix of zeros for the Ursell number Ur
Ur = zeros(length(hs),length(tp));
%define matrix of zeros for steepness S1
S1 = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient alpha_F
Alpha_F = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient beta_F
Beta_F = zeros(length(hs),length(tp));
%define row vector of zeros for k_1 (dispersion relation solutions)
sol_k_1 = zeros(1,length(tp));
%loop to calculate k_1, Ur, S1, alpha_F and Beta_F
for i = 1:length(hs);
    for j=1:length(tp);
    %define unknown variable(wave number)
    syms k_1;
    %solve dispersion relation for each mean wave period T1,
    [sol_k_1(j)] = vpasolve([(T1(i,j)^2)==((4*(pi^2))/(k_1*9.81*(tanh(k_1*350))))], [k_1], [0 Inf]);
    %calculate Ursell number
    Ur(i,j) = (hs(i)/(((sol_k_1(j))^2)*d^3));
    %calculate steepness
    S1(i,j) = (2*pi*hs(i))/(9.81*(T1(i,j))^2);
    %calculate alpha_F
    Alpha_F(i,j) = 0.3536+(0.2568*S1(i,j))+(0.08*Ur(i,j));
    %calculate beta_F
    Beta_F(i,j) = 2-(1.7912*S1(i,j))-(0.5302*(Ur(i,j)))+(0.284*(Ur(i,j))^2);
    end
end
%define matrix of zeros for the short term distribution
Fc = zeros(length(hs),length(tp));
%execute loop for defined ranges of Xi, Hs and Tp
for k = 1:length(Xi);
    for i = 1:length(hs);
        for j = 1:length(tp);
        %short term distribution
        Fc(i,j) = (1-exp(-((Xi(k)/(Alpha_F(i,j)*hs(i)))^Beta_F(i,j))));
        Fc_cell{k} = Fc;
        end
    end
end
save('short_term_Forristall_3D','Fc_cell');
```

Appendix B.15 LONG TERM ALL GLOBAL MAXIMA (CREST HEIGHT, 2ND ORDER, 3D)

```
clear all
close all
clc
%load long term distribution sea state characteristics f_hs,tp (b1=0.005)
load('joint_pdf.mat');
%load short term crest distribution
load('short_term_Forristall_3D');
%load short term description data
load('Mean_periods.mat');
digits(40);
%input crest height range
Xi = 1:0.5:30;
%loop for elementwise multiplication zero_up_freq & short term distribution Fc
for k = 1:length(Xi);
    A = bsxfun(@times,zero_up_freq,Fc_cell{k});
    A_cell{k} = A.*(1/mean_zero_up_freq);
end;
%loop for elementwise multiplication of A with joint pdf sea states
for k = 1:length(Xi);
    B = bsxfun(@times,A_cell{k},Joint_pdf_norm_1);
    B_cell{k} = B;
    %sum along rows
    C = sum(B_cell{k},2);
    C_cell{k} = C;
    %sum along column
    D = sum(C_cell{k},1);
    %cell array with long term probabilities
    F_igm{k} = D;
end;
F_igm_mat = cell2mat(F_igm);
save('Long_term_crest_Forristall_3D','F_igm_mat');
```

Appendix B.16 PROBABILITY PLOT, LONG TERM (CREST, 2ND ORDER, ALL GLOBAL MAX., 3D)

```
clear all
close all
clc
% load long term distribution individual global maxima
load('Long_term_crest_Forristall_3D.mat')
%load # response maxima
load('Mean_periods.mat');
% input plot
%range for crest height
Xi = 1:0.5:30;
Z = (log(Xi))';
Y = log(-log(1-F_igm_mat));
Y_ULS = log(-log(1/n_T_ULS));
Y_ALS = log(-log(1/n_T_ALS));

figure(1)
plot(Z,Y,'r');
hold on
A = line([0 3.5],[Y_ULS Y_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
B = line([0 3.5],[Y_ALS Y_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Z=ln($\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Y=ln(-ln(1-$F_{\Xi}))$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('Long term individual global maxima(crest 3D,
$\xi$)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
[x_cs_100,y_cs_100] = polyxpoly(Z,Y,[0 4],[Y_ULS Y_ULS]);
[x_cs_10000,y_cs_10000] = polyxpoly(Z,Y,[0 4],[Y_ALS Y_ALS]);
crest_ULS = exp(x_cs_100);
crest_ALS = exp(x_cs_10000);
crest_ULS_round= ceil(10*crest_ULS)/10;
crest_ALS_round= ceil(10*crest_ALS)/10;
Xi_ULS_ch = str2double(sprintf('%.2f',crest_ULS_round));
Xi_ALS_ch = str2double(sprintf('%.2f',crest_ALS_round));
legend('Long term maxima \xi',['ULS, \xi_{k} = ',num2str(Xi_ULS_ch),'m'],['ALS, \xi_{k} =
',num2str(Xi_ALS_ch),'m'],'Location','Southoutside');
grid on
grid minor
hold off

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ULS_ch};
sheet = 'Summary_Crest';
xlRange = 'S9';
xlswrite(filename,C,sheet,xlRange)
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
C = {Xi_ALS_ch};
sheet = 'Summary_Crest';
xlRange = 'T9';
xlswrite(filename,C,sheet,xlRange)
```

Appendix B.17 METOCEAN CONTOUR LINES ($b_1 = 0.005$)

```
clear all
close all
clc
clf
%Input Weibull paramters marginal dist. Hs
eta = 1.2219;
sigma = 2.0743;
mu = 0.7203;
%define q-probabilities
q_100 = 1/(2920*100);
%q=10^-4
q_10000 = 1/(2920*10000);
%define beta, radius of sphere(circle) in u-space
%for q=10^-2
beta_1 = -norminv(q_100,0,1);
%for q=10^-4
beta_2 = -norminv(q_10000,0,1);

%define failure surface in u-space
%row vector tetha of 1000 points linearly spaced,0 to 2*pi
tetha = linspace(0,2*pi,1000);
%define u_1_1,u_2_1 q=10^-2
u_1_1 = beta_1*cos(tetha);
u_2_1 = beta_1*sin(tetha);
%define u_1_2,u_2_2 q=10^-4
u_1_2 = beta_2*cos(tetha);
u_2_2 = beta_2*sin(tetha);
%calculate Gaussian distribution Phi(u_1_1) and Phi(u_1_2)
norm_u_1_1 = normcdf(u_1_1,0,1);
norm_u_1_2 = normcdf(u_1_2,0,1);

%define physical space for hs and tp
%q=10^-2
%calculate wave height from marginal distribution F_Hs
hs_100 = (sigma*(-log(1-norm_u_1_1)).^(1/eta))+mu;
%calculate mean ln(tp) and variance ln(tp)for hs_100
mean_1 = 1.4487+0.6095*(hs_100).^0.3101;
%variance ln(tp)
var_1 = 0.005+0.0961*exp(-0.286*(hs_100));
%standard deviation ln(tp)
std_1 = sqrt(var_1);
%calculate peak period from conditional distribution
tp_100 = exp(mean_1+std_1.*u_2_1);

%q=10^-4
%calculate wave height from marginal distribution F_Hs
hs_10000 = (sigma*(-log(1-norm_u_1_2)).^(1/eta))+mu;
%calculate mean ln(tp) and variance ln(tp)for hs_10000
mean_2 = 1.4487+0.6095*(hs_10000).^0.3101;
%variance ln(tp)
var_2 = 0.005+0.0961*exp(-0.286*(hs_10000));
%standard deviation ln(tp)
std_2 = sqrt(var_2);
%calculate peak period from conditional distribution
```

```
tp_10000 = exp(mean_2+std_2.*u_2_2);

%plot results
figure(1)
plot(tp_100,hs_100,'Linewidth',1.5)
hold on
plot(tp_10000,hs_10000,'r','Linewidth',1.5)
title('Metocean contour lines (b1 =
0.005)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('$T_{p}$ [s]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('$H_{s}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
hleg1 = legend('q=10^{-2} (ULS)','q=10^{-4} (ALS)');
set(hleg1,'Location','NorthEast')
grid on
grid minor
%save workspace file
save('MCL_fixed','hs_100','hs_10000','tp_100','tp_10000');
```

Appendix B.18 METOCEAN CONTOUR LINES ($b_1 = 0$)

```
clear all
close all
clc
%Input Weibull paramters marginal dist. Hs
eta = 1.2219;
sigma = 2.0743;
mu = 0.7203;
%*define q-probabilities*
q_100 = 1/(2920*100);
%q=10^-4
q_10000 = 1/(2920*10000);
%*define beta, radius of sphere(circle) in u-space*
%q=10^-2
beta_1 = -norminv(q_100,0,1);
%q=10^-4
beta_2 = -norminv(q_10000,0,1);
%define failure surface in u-space
%row vector tetha of 1000 points linearly spaced,0 to 2*pi
tetha = linspace(0,2*pi,1000);
%define u_1_1,u_2_1 q=10^-2
u_1_1 = beta_1*cos(tetha);
u_2_1 = beta_1*sin(tetha);
%define u_1_2,u_2_2 q=10^-4
u_1_2 = beta_2*cos(tetha);
u_2_2 = beta_2*sin(tetha);
%calculate Gaussian distribution Phi(u_1_1) and Phi(u_1_2)
norm_u_1_1 = normcdf(u_1_1,0,1);
norm_u_1_2 = normcdf(u_1_2,0,1);

%*define physical space for hs and tp*
%q=10^-2
%calculate wave height from marginal distribution F_Hs
hs_100 = (sigma*(-log(1-norm_u_1_1)).^(1/eta))+mu;
%calculate mean ln(tp) and variance ln(tp)for hs_100
mean_1 = 1.4487+0.6095*(hs_100).^0.3101;
%variance ln(tp)
var_1 = 0.00+0.0973*exp(-0.242*(hs_100));
%standard deviation ln(tp)
std_1 = sqrt(var_1);
%calculate peak period from conditional distribution
tp_100 = exp(mean_1+std_1.*u_2_1);
%q=10^-4

%calculate wave height from marginal distribution F_Hs
hs_10000 = (sigma*(-log(1-norm_u_1_2)).^(1/eta))+mu;
%calculate mean ln(tp) and variance ln(tp)for hs_10000
mean_2 = 1.4487+0.6095*(hs_10000).^0.3101;
%variance ln(tp)
var_2 = 0.00+0.0973*exp(-0.242*(hs_10000));
%standard deviation ln(tp)
std_2 = sqrt(var_2);
%calculate peak period from conditional distribution
tp_10000 = exp(mean_2+std_2.*u_2_2);
```

```
%plot results
figure(1)
plot(tp_100,hs_100,'Linewidth',1.5)
hold on
plot(tp_10000,hs_10000,'r','Linewidth',1.5)
title('Metocean contour lines (b1 =
0)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('$T_'$T_{p}$  [s]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('$H_{s}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
hleg1 = legend('q=10^{-2} (ULS)','q=10^{-4} (ALS)');
set(hleg1,'Location','NorthEast')
grid on
grid minor
%save workspace file
save('MCL_free','hs_100','hs_10000','tp_100','tp_10000');
```

Appendix B.19 PERCENTILE LEVEL, GAUSSIAN, $b_1 = 0.005$

```
clear all
close all
clc
%load data
load('MCL_fixed');
%define range of cumulative probability range
F_c3h = 0:0.001:0.999;
%define matrix from column vectors hs_100 and tp_100
hs_tp_100 = [hs_100' tp_100'];
%define max value of Hs (upper limit to extract)
hsmax_100 = max(hs_tp_100(:,1));
%extract pairs Hs & Tp for 12<Hs<hsmax_100
Ext_hs_100 = hs_tp_100(hs_tp_100(:,1)>=12 & hs_tp_100(:,1)<=hsmax_100,:);
%extract pairs Hs & Tp for 12<Tp<21
Ext_tp_100 = Ext_hs_100(Ext_hs_100(:,2)>=12 & Ext_hs_100(:,2)<21,:);
%sort rows by ascending values of tp
sort_Ext_hs_tp_100 = sortrows(Ext_tp_100,2);
%find index of max Hs along 10^-2 contour line
[max_hs_100,ind_max_hs_100] = max(sort_Ext_hs_tp_100(:,1));
%extract 6 pairs of hs and tp
A = sort_Ext_hs_tp_100([1,45,90,ind_max_hs_100,165,205],:);
%round the entries in matrix A to 2 significant digits
A_round= round(10*A(:,:))/10;
%loop to calculate 3-hr max. crest height for chosen sea states
for i = 1:length(A);
    for j = 1:length(F_c3h);
        c_3h_100(i,j) = A(i,1)*(sqrt((-1/8)*(log(1-(F_c3h(j))^(A(i,2)/14026)))));
    end
end
%input crest height closest to q=10^-2 (all sea states)
x_q_100 = round(10*c_3h_100(4,830))/10;
%input corresponding probability
q_100 = F_c3h(1,830);
%plot figures
figure(1)
f1 = plot(c_3h_100(1,:),F_c3h,'Linewidth',1.3);
hold all
f2 = plot(c_3h_100(2,:),F_c3h,'m','Linewidth',1.3);
f3 = plot(c_3h_100(3,:),F_c3h,'c','Linewidth',1.3);
f4 = plot(c_3h_100(4,:),F_c3h,'r','Linewidth',1.3);
f5 = plot(c_3h_100(5,:),F_c3h,'g','Linewidth',1.3);
f6 = plot(c_3h_100(6,:),F_c3h,'k','Linewidth',1.3);
f7 = plot(x_q_100,q_100,'*b');
annotation('textbox',[0.65 0.75 0.2 0.12],'String',{['x_q = ',num2str(x_q_100)],['Prob. =
',num2str(q_100)]},'Backgroundcolor',[1 1 1]);
hold off
legend([f1,f2,f3,f4,f5,f6],['Hs = ',mat2str(A_round(1,1)),', ','Tp = ',mat2str(A_round(1,2))],['Hs =
',mat2str(A_round(2,1)),', ','Tp = ',mat2str(A_round(2,2))],['Hs = ',mat2str(A_round(3,1)),', ','Tp =
',mat2str(A_round(3,2))],['Hs = ',mat2str(A_round(4,1)),', ','Tp = ',mat2str(A_round(4,2))],['Hs =
',mat2str(A_round(5,1)),', ','Tp = ',mat2str(A_round(5,2))],['Hs = ',mat2str(A_round(6,1)),', ','Tp =
',mat2str(A_round(6,2))],'Location','southoutside');
title('$q=10^{-2}$, Gaussian
(b1=0.005)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('Crest height [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
```

```
ylabel('Cumulative probability','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0 30]);
grid on
grid minor
%display maximum Hs along 10^-2 contour line
disp(['Maximum Hs along 10^-2 contour line = ', num2str(sort_Ext_hs_tp_100(ind_max_hs_100,1))]);
%display Tp for corresponding max Hs along 10^-2 contour line
disp(['Tp for maximum Hs along 10^-2 contour line = ', num2str(sort_Ext_hs_tp_100(ind_max_hs_100,2))]);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
%*Verification percentile level,10^-4 contour(Gaussian, b1=0.005)*
%define range of cumulative probability range
F_c3h = 0:0.001:0.9999;
%define matrix from column vectors hs_100 and tp_100
hs_tp_10000 = [hs_10000' tp_10000'];
%define max value of Hs (upper limit to extract)
hsmax_10000 = max(hs_tp_10000(:,1));
%extract pairs Hs & Tp for 12<Hs<=hsmax_10000
Ext_hs_10000 = hs_tp_10000(hs_tp_10000(:,1)>=12 & hs_tp_10000(:,1)<=hsmax_10000,:);
%extract pairs Hs & Tp for 10<Tp<23
Ext_tp_10000 = Ext_hs_10000(Ext_hs_10000(:,2)>10 & Ext_hs_10000(:,2)<=23,:);
%sort rows by ascending values of tp
sort_Ext_hs_tp_10000 = sortrows(Ext_tp_10000,2);
%find index of max Hs along 10^-2 contour line
[max_hs_10000,ind_max_hs_10000] = max(sort_Ext_hs_tp_10000(:,1));
%extract 5 pairs of hs and tp
B = sort_Ext_hs_tp_10000([1,45,90,105,ind_max_hs_10000,205],:);
%round the entries in matrix B to 2 significant digits
B_round= round(100*B(:,:))/100;
%loop to calculate 3-hr max. crest height from chosen sea states
for k = 1:length(B);
    for j = 1:length(F_c3h);
        c_3h_10000(k,j) = B(k,1)*(sqrt((-1/8)*(log(1-(F_c3h(j))^(B(k,2)/14026)))));
    end
end
%input crest height closest to q=10^-4 (all sea states)
x_q_10000 = round(10*c_3h_10000(5,890))/10;
%input corresponding probability
q_10000 = F_c3h(1,890);
%plot figures
figure(2)
g1 = plot(c_3h_10000(1,:),F_c3h,'Linewidth',1.3);
hold all
g2 = plot(c_3h_10000(2,:),F_c3h,'m','Linewidth',1.3);
g3 = plot(c_3h_10000(3,:),F_c3h,'c','Linewidth',1.3);
g4 = plot(c_3h_10000(4,:),F_c3h,'r','Linewidth',1.3);
g5 = plot(c_3h_10000(5,:),F_c3h,'g','Linewidth',1.3);
g6 = plot(c_3h_10000(6,:),F_c3h,'k','Linewidth',1.3);
g7 = plot(x_q_10000,q_10000,'*b');
annotation('textbox',[0.7 0.75 0.17 0.12],'String',{['x_q = ',num2str(x_q_10000)],['Prob. =
',num2str(q_10000)]},'Backgroundcolor',[1 1 1]);
hold off
legend([g1,g2,g3,g4,g5,g6],['Hs = ',mat2str(B_round(1,1)),', ','Tp = ',mat2str(B_round(1,2))],['Hs =
',mat2str(B_round(2,1)),', ','Tp = ',mat2str(B_round(2,2))],['Hs = ',mat2str(B_round(3,1)),', ','Tp =
',mat2str(B_round(3,2))],['Hs = ',mat2str(B_round(4,1)),', ','Tp = ',mat2str(B_round(4,2))],['Hs =
',mat2str(B_round(5,1)),', ','Tp = ',mat2str(B_round(5,2))],['Hs = ',mat2str(B_round(6,1)),', ','Tp =
',mat2str(B_round(6,2))],'Location','southoutside');
title('$q=10^{-4}$, Gaussian
```

```
(b1=0.005)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('Crest height [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Cumulative probability','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0 35]);
grid on
grid minor
%display maximum Hs along 10^-4 contour line
disp(['Maximum Hs along 10^-4 contour line = ', num2str(sort_Ext_hs_tp_10000(ind_max_hs_10000,1))]);
%display Tp for corresponding max Hs along 10^-4 contour line
disp(['Tp for maximum Hs along 10^-4 contour line = ',
num2str(sort_Ext_hs_tp_10000(ind_max_hs_10000,2))]);
```

```
Maximum Hs along 10^-2 contour line = 17.201
Tp for maximum Hs along 10^-2 contour line = 18.5675
Maximum Hs along 10^-4 contour line = 21.9925
Tp for maximum Hs along 10^-4 contour line = 20.864
```

Appendix B.20 PERCENTILE LEVEL, 2ND ORDER, $b_1 = 0.005$

```
clear all
close all
clc
%load data
load('MCL_fixed');
%define range hs
hs = 1:0.5:25;
%define range tp
tp = 1:1:25;
%define water depth Heidrun field
d = 350;
%define matrix of zeros for the Ursell number Ur
Ur = zeros(length(hs),length(tp));
%define matrix of zeros for steepness S1
S1 = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient alpha_F
Alpha_F = zeros(length(hs),length(tp));
%define matrix of zeros for coefficient beta_F
Beta_F = zeros(length(hs),length(tp));
%define row vector of zeros for k_1 (dispersion relation solutions)
sol_k_1 = zeros(1,length(tp));
%loop to calculate k_1, Ur, S1, alpha_F and Beta_F
for i = 1:length(hs);
    for j=1:length(tp);
    %define unknown variable(wave number)
    syms k_1;
    %solve dispersion relation for each mean wave period T1)
    [sol_k_1(j)] = vpasolve([2.405*pi/tp(j)==k_1*9.81*(tanh(k_1*350))], [k_1], [0 Inf]);
    %calculate Ursell number
    Ur(i,j) = (hs(i)/(((sol_k_1(j))^2)*d^3));
    %calculate steepness
    S1(i,j) = (2*pi*hs(i))/(9.81*(0.8316*tp(j))^2);
    %calculate alpha_F
    Alpha_F(i,j) = 0.3536+(0.2892*S1(i,j))+(0.1060*Ur(i,j));
    %calculate beta_F
    Beta_F(i,j) = 2-(2.1597*S1(i,j))+(0.0968*(Ur(i,j))^2);
    end
end

%extract alpha_F parameters for corresponding classes of Hs and Tp
Alpha_100 = [Alpha_F(24,13); Alpha_F(30,16); Alpha_F(33,18); Alpha_F(34,19); Alpha_F(32,21);
Alpha_F(27,21)];
%extract Beta_F parameters for corresponding classes of Hs and Tp
Beta_100 = [Beta_F(24,13); Beta_F(30,16); Beta_F(33,18); Beta_F(34,19); Beta_F(32,21); Beta_F(27,21)];
%create matrix M containing respective alpha and beta parameters
M = [Alpha_100 Beta_100];

%extract alpha_F parameters for corresponding classes of Hs and Tp along
%10^-4
Alpha_10000 = [Alpha_F(24,11); Alpha_F(33,15); Alpha_F(40,18); Alpha_F(42,19); Alpha_F(43,21);
Alpha_F(38,23)];
%extract Beta_F parameters for corresponding classes of Hs and Tp
Beta_10000 = [Beta_F(24,11); Beta_F(33,15); Beta_F(40,18); Beta_F(42,19); Beta_F(43,21); Beta_F(38,23)];
%create matrix M containing respective alpha and beta parameters
```

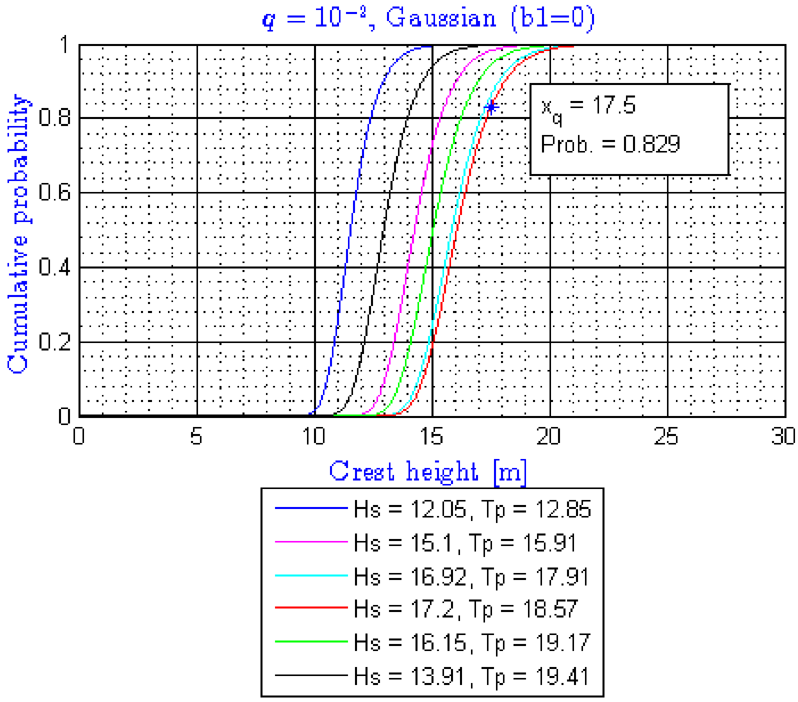
```
N = [Alpha_10000 Beta_10000];

%*Verification percentile level,10^-2 contour(second order, b1=0.005)*
%define range of cumulative probability range
F_c3h = 0:0.001:0.999;
%define matrix from column vectors hs_100 and tp_100
hs_tp_100 = [hs_100' tp_100'];
%define max value of Hs (upper limit to extract)
hsmax_100 = max(hs_tp_100(:,1));
%extract pairs Hs & Tp for 12<Hs<hsmax_100
Ext_hs_100 = hs_tp_100(hs_tp_100(:,1)>=12 & hs_tp_100(:,1)<=hsmax_100,:);
%extract pairs Hs & Tp for 12<Tp<21
Ext_tp_100 = Ext_hs_100(Ext_hs_100(:,2)>=12 & Ext_hs_100(:,2)<21,:);
%sort rows by ascending values of tp
sort_Ext_hs_tp_100 = sortrows(Ext_tp_100,2);
%find index of max Hs along 10^-2 contour line
[max_hs_100,ind_max_hs_100] = max(sort_Ext_hs_tp_100(:,1));
%extract 5 pairs of hs and tp
A = sort_Ext_hs_tp_100([1,45,90,ind_max_hs_100,165,205],:);
%round the entries in matrix A to 2 significant digits
A_round= round(100*A(:,:))/100;
%loop to calculate 3-hr max. crest height from chosen sea states
for k = 1:length(A);
    for p = 1:length(M);
        for m = 1:length(F_c3h);
        c_3h_100(k,m) = M(p,1)*A(k,1)*(((-log(1-(F_c3h(m))^(A(k,2)/14026)))))^(1/M(p,2));
        end;
    end;
end
%input crest height closest to q=10^-2 (all sea states)
x_q_100 = round(10*c_3h_100(4,904))/10;
%input corresponding probability
q_100 = F_c3h(1,904);
%plot figures
figure(2)
f1 = plot(c_3h_100(1,:),F_c3h,'Linewidth',1.3);
hold all
f2 = plot(c_3h_100(2,:),F_c3h,'m','Linewidth',1.3);
f3 = plot(c_3h_100(3,:),F_c3h,'c','Linewidth',1.3);
f4 = plot(c_3h_100(4,:),F_c3h,'r','Linewidth',1.3);
f5 = plot(c_3h_100(5,:),F_c3h,'g','Linewidth',1.3);
f6 = plot(c_3h_100(6,:),F_c3h,'k','Linewidth',1.3);
f7 = plot(x_q_100,q_100,'*b');
annotation('textbox',[0.705 0.7 0.2 0.12],'String',{['x_q = ',num2str(x_q_100)],['Prob. =
',num2str(q_100)]},'Backgroundcolor',[1 1 1]);
hold off
legend([f1,f2,f3,f4,f5,f6],['Hs = ',mat2str(A_round(1,1)),', ','Tp = ',mat2str(A_round(1,2))],['Hs =
',mat2str(A_round(2,1)),', ','Tp = ',mat2str(A_round(2,2))],['Hs = ',mat2str(A_round(3,1)),', ','Tp =
',mat2str(A_round(3,2))],['Hs = ',mat2str(A_round(4,1)),', ','Tp = ',mat2str(A_round(4,2))],['Hs =
',mat2str(A_round(5,1)),', ','Tp = ',mat2str(A_round(5,2))],['Hs = ',mat2str(A_round(6,1)),', ','Tp =
',mat2str(A_round(6,2))],'Location','southoutside');
title('$q=10^{-2}$, Second order
(b1=0.005)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('Crest height [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Cumulative probability','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0 30]);
grid on
```

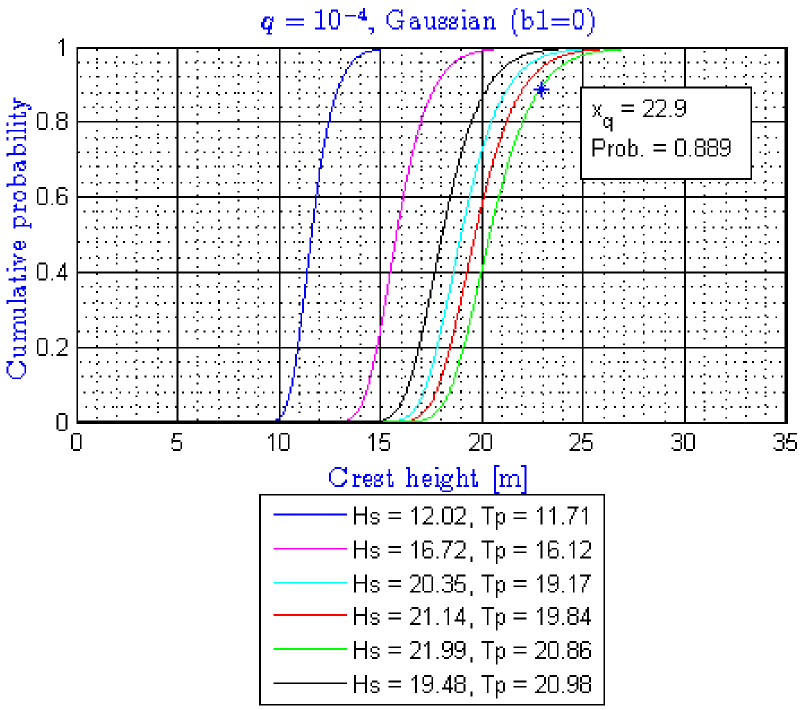
```
grid minor
%display maximum Hs along 10^-2 contour line
disp(['Maximum Hs along 10^-2 contour line = ', num2str(sort_Ext_hs_tp_100(ind_max_hs_100,1))]);
%display Tp for corresponding max Hs along 10^-2 contour line
disp(['Tp for maximum Hs along 10^-2 contour line = ', num2str(sort_Ext_hs_tp_100(ind_max_hs_100,2))]);

%*Verification percentile level,10^-4 contour(second order, b1=0.005)*
%define range of cumulative probability range
F_c3h = 0:0.001:0.9999;
%define matrix from column vectors hs_100 and tp_100
hs_tp_10000 = [hs_10000' tp_10000'];
%define max value of Hs (upper limit to extract)
hsmax_10000 = max(hs_tp_10000(:,1));
%extract pairs Hs & Tp for 12<Hs<=hsmax_10000
Ext_hs_10000 = hs_tp_10000(hs_tp_10000(:,1)>=12 & hs_tp_10000(:,1)<=hsmax_10000,:);
%extract pairs Hs & Tp for 10<Tp<23
Ext_tp_10000 = Ext_hs_10000(Ext_hs_10000(:,2)>10 & Ext_hs_10000(:,2)<=23,:);
%sort rows by ascending values of tp
sort_Ext_hs_tp_10000 = sortrows(Ext_tp_10000,2);
%find index of max Hs along 10^-2 contour line
[max_hs_10000,ind_max_hs_10000] = max(sort_Ext_hs_tp_10000(:,1));
%extract 5 pairs of hs and tp
B = sort_Ext_hs_tp_10000([1,45,90,105,ind_max_hs_10000,205],:);
%round the entries in matrix B to 2 significant digits
B_round= round(100*B(:,:))/100;
%loop to calculate 3-hr max. crest height from chosen sea states
for l = 1:length(B);
    for r = 1:length(N);
        for m = 1:length(F_c3h);
            c_3h_10000(l,m) = N(r,1)*B(l,1)*(((-log(1-(F_c3h(m))^(B(l,2)/14026)))))^(1/N(r,2));
        end;
    end;
end;
%input crest height closest to q=10^-4 (all sea states)
x_q_10000 = round(10*c_3h_10000(5,935))/10;
%input corresponding probability
q_10000 = F_c3h(1,935);
%plot figures
figure(3)
g1 = plot(c_3h_10000(1,:),F_c3h,'Linewidth',1.3);
hold all
g2 = plot(c_3h_10000(2,:),F_c3h,'m','Linewidth',1.3);
g3 = plot(c_3h_10000(3,:),F_c3h,'c','Linewidth',1.3);
g4 = plot(c_3h_10000(4,:),F_c3h,'r','Linewidth',1.3);
g5 = plot(c_3h_10000(5,:),F_c3h,'g','Linewidth',1.3);
g6 = plot(c_3h_10000(6,:),F_c3h,'k','Linewidth',1.3);
g7 = plot(x_q_10000,q_10000,'*b');
annotation('textbox',[0.735 0.7 0.17 0.12],'String',{['x_q = ',num2str(x_q_10000)],['Prob. =
',num2str(q_10000)]},'Backgroundcolor',[1 1 1]);
hold off
legend([g1,g2,g3,g4,g5,g6],['Hs = ',mat2str(B_round(1,1)),', ','Tp = ',mat2str(B_round(1,2))],['Hs =
',mat2str(B_round(2,1)),', ','Tp = ',mat2str(B_round(2,2))],['Hs = ',mat2str(B_round(3,1)),', ','Tp =
',mat2str(B_round(3,2))],['Hs = ',mat2str(B_round(4,1)),', ','Tp = ',mat2str(B_round(4,2))],['Hs =
',mat2str(B_round(5,1)),', ','Tp = ',mat2str(B_round(5,2))],['Hs = ',mat2str(B_round(6,1)),', ','Tp =
',mat2str(B_round(6,2))],'Location','southoutside');
title('$q=10^{-4}$, Second order
(b1=0.005)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
```

```
xlabel('Crest height [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Cumulative probability','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0 35]);
grid on
grid minor
%display maximum Hs along 10^-4 contour line
disp(['Maximum Hs along 10^-4 contour line = ', num2str(sort_Ext_hs_tp_10000(ind_max_hs_10000,1))]);
%display Tp for corresponding max Hs along 10^-4 contour line
disp(['Tp for maximum Hs along 10^-4 contour line = ',
num2str(sort_Ext_hs_tp_10000(ind_max_hs_10000,2))]);
```

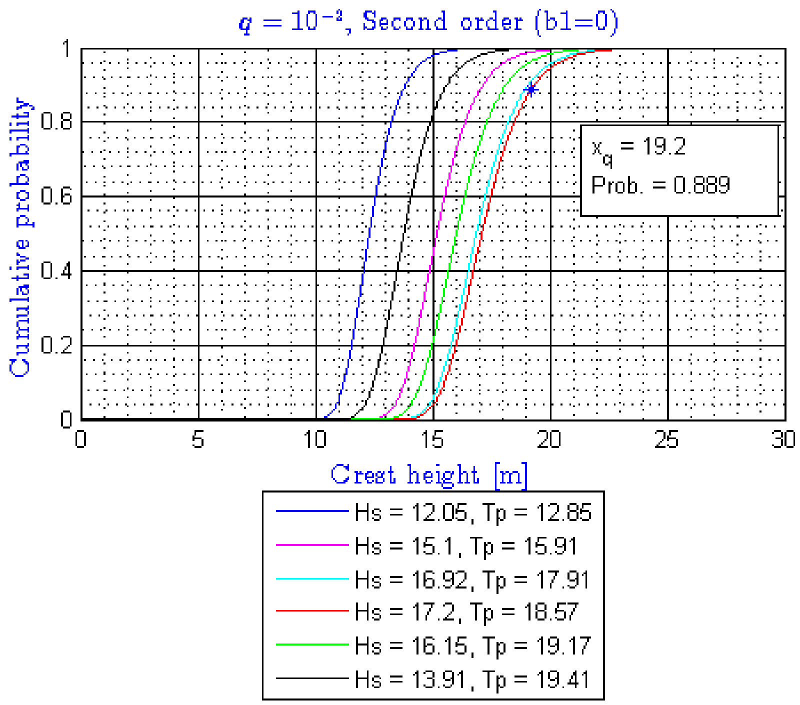
```
Maximum Hs along 10^-2 contour line = 17.201
Tp for maximum Hs along 10^-2 contour line = 18.5675
Maximum Hs along 10^-4 contour line = 21.9925
Tp for maximum Hs along 10^-4 contour line = 20.864
```

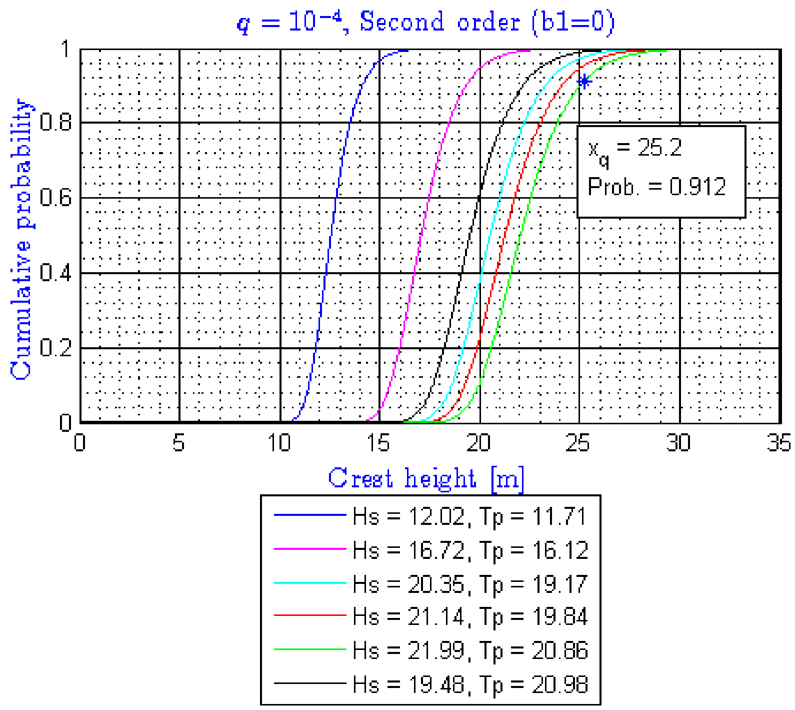
Appendix B.21 Percentile level, Gaussian, $b_1 = 0$





Appendix B.22 Percentile level, 2ND order, $b_1 = 0$





Appendix C PEAK-OVER-THRESHOLD

Appendix C.1 PARAMETERS 3P WEIBULL, $H_{s_{peak}}$, THRESHOLD $H_s = 8, 9, 10m$

Hs threshold = 8m		
Form parameter	η	1,0483
Scale parameter	σ	1,3892
Location parameter	μ	8,1097

	$(\frac{q}{n})^{-1}$	H_{s_q} [m]	T_{p_q} [m]
ULS	768,96552	16,57	18,30
ALS	76896,552	22,09	20,96

Hs threshold = 9m		
Form parameter	η	0,9803
Scale parameter	σ	1,2143
Location parameter	μ	9,1934

	$(\frac{q}{n})^{-1}$	H_{s_q} [m]	T_{p_q} [m]
ULS	389,65517	16,70	18,37
ALS	38965,517	22,65	21,22

Hs threshold = 10m		
Form parameter	η	1,0232
Scale parameter	σ	1,3358
Location parameter	μ	9,9657

	$(\frac{q}{n})^{-1}$	H_{s_q} [m]	T_{p_q} [m]
ULS	187,93103	16,70	18,37
ALS	18793,103	22,45	21,12

Appendix C.2 PLOT HS_PEAK VS. THRESHOLD LEVEL (ALL SEA STATES & POT)

```
close all
clear all
clc

%*Load Hs_peak data from xl sheet*
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9';
sheet = 'POT_Hs_peak_Weibull 3p';
%define threshold range
thresh = 7:1:10;
%ULS
xlRange_1 = 'N6';
xlRange_2 = 'N15';
xlRange_3 = 'N25';
xlRange_4 = 'N35';
hs_peak_7 = xlsread(filename,sheet, xlRange_1);
hs_peak_8 = xlsread(filename,sheet, xlRange_2);
hs_peak_9 = xlsread(filename,sheet, xlRange_3);
hs_peak_10 = xlsread(filename,sheet, xlRange_4);
%vector with Hs_peak ULS
hs_peak_ULS = [hs_peak_7 hs_peak_8 hs_peak_9 hs_peak_10];

%ALS
xlRange_11 = 'N7';
xlRange_22 = 'N16';
xlRange_33 = 'N26';
xlRange_44 = 'N36';
hs_peak_77 = xlsread(filename,sheet, xlRange_11);
hs_peak_88 = xlsread(filename,sheet, xlRange_22);
hs_peak_99 = xlsread(filename,sheet, xlRange_33);
hs_peak_1010 = xlsread(filename,sheet, xlRange_44);
%vector with Hs_peak ALS
hs_peak_ALS = [hs_peak_77 hs_peak_88 hs_peak_99 hs_peak_1010];

%*Load data from all sea states analysis*
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9';
sheet = 'Hs_Weibull 3p';
xlRange_AS_ULS = 'V8';
xlRange_AS_ALS = 'V9';
hs_ULS = xlsread(filename,sheet, xlRange_AS_ULS);
hs_ULS_round= round(100*hs_ULS)/100;
hs_ALS = xlsread(filename,sheet, xlRange_AS_ALS);
hs_ALS_round= round(100*hs_ALS)/100;

%plot Hs_peak (POT and all seas) vs. threshold
figure (1)
plot(thresh,hs_peak_ULS,'b*','Markersize',5)
hold on
plot(thresh,hs_peak_ALS,'ro','Markersize',5)
E = line([6 11],[hs_ULS hs_ULS],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
F = line([6 11],[hs_ALS hs_ALS],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('Threshold $H_{s}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('$H_{s_{q}}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title('$H_{s_{q}}$ [m] for given 
threshold','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
```

```
legend('POT, ULS - Hs_{q}','POT, ALS - Hs_{q}',['All seas, ULS, H_{s_{q}} =
',num2str(hs_ULS_round),'m'],['All seas, ALS, H_{s_{q}} =
',num2str(hs_ALS_round),'m'],'Location','Southoutside');

xlim([6 11]);
ylim([15 25]);
grid on
grid minor
hold off
```

Appendix C.3 MPM CREST (2ND ORDER, 2D) , THRESHOLD $H_s = 7m$

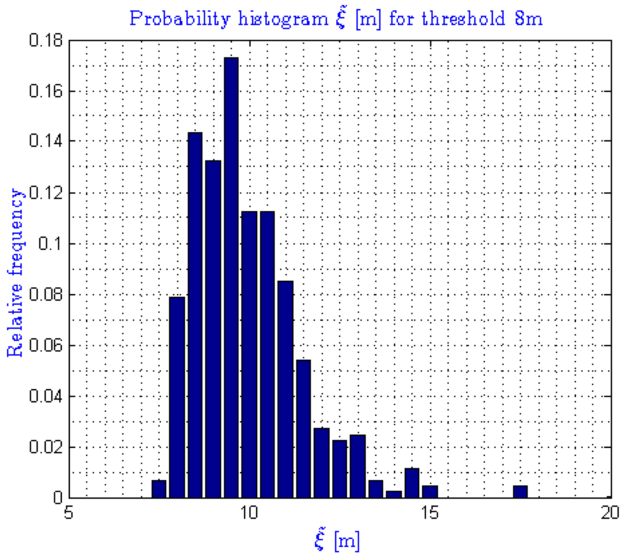
```
close all
clear all
clc

%load data
load('F:\Masters thesis\Matlab\POT\Crest height\WAM10.mat');
%load alpha_F and beta_F
load('F:\Masters thesis\Matlab\Fitted model\Forristall_Long crested\alpha_beta_forristall.mat');
%define threshold
thresh = 7;
%define matrix A of hindcast (simplify notation)
A = [Heidrun_WAM10(:,5) Heidrun_WAM10(:,6) Heidrun_WAM10(:,1)];
%extract hs and tp for chosen period (years)
Wdata = A(A(:,3)>=1957 & A(:,3)<=2014,:);
%column vector for with logical true/false index for value above/below threshold)
ind = Wdata(:,1)>=thresh;
%matrix with pairs of hs and tp above threshold
B = bsxfun(@times,ind,Wdata);

%*Column vector with Hs classes (frequency table)*
%column vector with mid-interval values for hs
hs_mid = (0.25:0.5:25)';
%column vector with class index for the mid-interval values for hs
hs_class = (1:1:50)';
%matrix with index and hs
hs_scatter = [hs_class hs_mid];

%range tp
tp_class = (0.5:1:24.5)';
%*Matrix with class indexes for Hs and Tp*
%define number of columns
nBinsTp=25;
%define number of rows
nBinsHs=50;
% define range tp
maxTp=25;
% define range hs
maxHs=25;
%create zero matrix (columns hs and tp)
M = zeros(length(Wdata),2);
for k=1:length(B);
    %find corresponding scatter index for hs
    i(k)=ceil((nBinsHs/maxHs)*B(k,1));
    %find corresponding scatter index for tp
    j(k)=ceil((nBinsTp/maxTp)*B(k,2));
    %place index in hs column
    M(k,1) = i(k);
    %place index in tp column
    M(k,2) = j(k);
end

%new matrix with hs index and class, tp index
D = [ind M(:,1) M(:,2)];
```

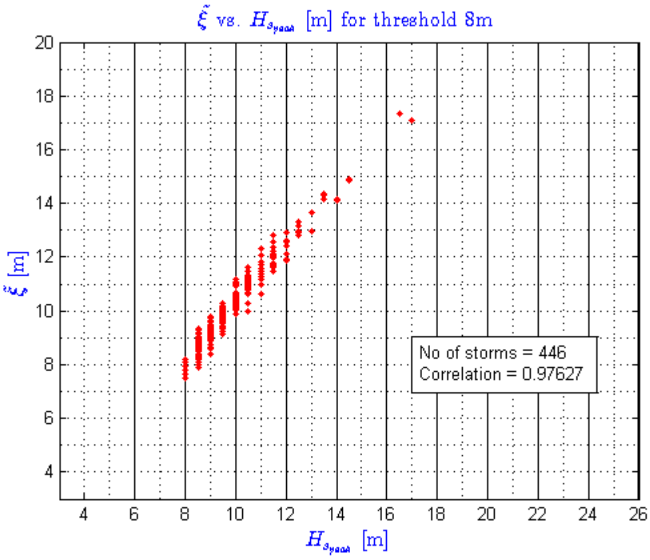
```
%*Count sequences*
%find all threshold crossings
crossings = diff(ind);
%find upcrossings i.e. start of a storm sequence
upcross = find(crossings == 1);

for l = 1:length(upcross);
    %index of the 1st 3hr sequence in a given storm
    x(l)=upcross(l)+1;
end;

%initiate index pair (hs and tp) for each 3hr sequence over threshold
seq = zeros(1,2);
%indexes for all upcrossings
data = x(1):1:length(D);

%extract storm over threshold
for q = 1:length(x);
    n = 0;
    storm_cell = {};
    for s = x(q):length(D);
        if (D(s,1) == 1);
            n=n+1;
            seq = D(s,2:3);
            storm_cell{n}=seq;
        end;
        if (D(s,1) == 0);
            break;
        end;
    end;
    storm_k{q} = (storm_cell)';
end;

%*Convert cells to matrices*
for w = 1:length(storm_k);
    storm_7m(Haver) = cell2mat(storm_k{1,w});
end;

%*Short term distribution wave crest (2nd order, Forristall, 2D)*
%range crest height
C = 6.5:0.05:25;
for z = 1:length(storm_7m);
    Fc_cell = [];
    for p = 1:length(storm_7m{:,z}(:,1));
        for m = 1:length(C);
        %calculate short term distribution for each 3hr sequence
        Fc = (1-exp(-
((C(m)/(Alpha_F((storm_7m{:,z}(p,1)),(storm_7m{:,z}(p,2)))*hs_scatter((storm_7m{:,z}(p,1)),2))))^Beta_F(
(storm_7m{:,z}(p,1)),(storm_7m{:,z}(p,2))))))^(14026/tp_class((storm_7m{:,z}(p,2))));
        Fc_cell(p,m) = Fc;
        end;
    end;
   Fc_storm(Forristall) = Fc_cell;
end;
%array of zeros, length of range for wave crest height
CDF = zeros(1,length(C));
```

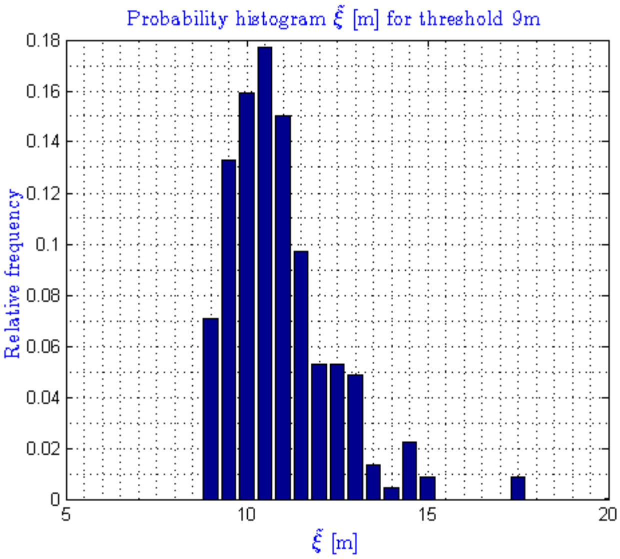
```
for t = 1:length(Fc_storm);
    for m = 1:length(C);
        CDF(m) = prod(Fc_storm{:,t}(:,m));
        CDF_storm{t} = CDF;
        end;
end;

%*Determine most probable value C_tilde*
CDF_mpm = 0.37;
%estimate MPM
for u = 1:length(CDF_storm);
    CDF_mpm_cls{u} = abs(CDF_storm{u}-CDF_mpm);
    [~,idx] = cellfun(@min,CDF_mpm_cls,'UniformOutput',false);
    C_tilda_7m{u} = C([idx{u}]);
end;

%*Create scatter with C_tilda & relative frequency*
nBins_C_tilda=50;
max_C_tilda=25;
Scatter = zeros(nBins_C_tilda,1);
C_tilda_mat = cell2mat(C_tilda_7m);
for g=1:length(C_tilda_7m);
    r=ceil((nBins_C_tilda/max_C_tilda)*C_tilda_mat(g));
    Scatter(r)=Scatter(r)+1;
end

storm_sum = numel(storm_7m);

for h = 1:length(Scatter);
    rel_freq(h) = (Scatter(h))/storm_sum;
end;
%plot probability histogram
C_tilda_class = 0.5:0.5:25;
figure (1)
bar(C_tilda_class,rel_freq);
xlim([5 20]);
xlabel('$\tilde{\xi}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('Relative frequency','Interpreter','latex','FontSize',12,'Color','b');
title(['Probability histogram $\tilde{\xi}$ [m] for threshold
',num2str(thresh),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
grid on
grid minor

%*Hs_peak vs. C_tilda. Correlation factor*
for v = 1:length(storm_7m);
    class_hs_peak(v) = max(storm_7m{1,v}(:,1));
    [~,index(v)] = max(storm_7m{1,v}(:,1));
    tp(v) = (storm_7m{1,v}(index(v),2));
    hs_peak{v} = (class_hs_peak(v))/2;
end;
%max(storm_7m{1,v}(:,1:2))
hs_p_7m = cell2mat(hs_peak);
Correlation = corr((hs_p_7m)',(C_tilda_mat)');
%plot correlation
figure (2)
plot(hs_p_7m,C_tilda_mat,'r*','Markersize',3);
xlim([3 26]);
```

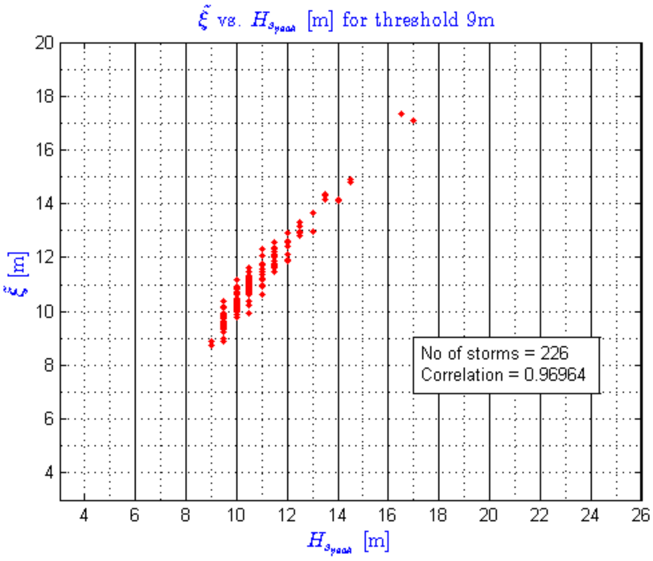
```
ylim([3 20]);
xlabel('$H_{s_{peak}}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title(['$\tilde{\xi}$ vs. $H_{s_{peak}}$ [m] for threshold
',num2str(thresh),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b')
ylabel('$\tilde{\xi}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b')
annotation('textbox',[0.6 0.3 0.25 0.1],'String',{['No of storms =
',num2str(length(C_tilda_mat))],['Correlation = ',num2str(Correlation)]},'Backgroundcolor',[1 1 1]);
grid on
grid minor

%save data
save('C_tilda_7m','C_tilda_7m','storm_7m');
save('hs_p_7m','hs_p_7m');
save('wblpdf_crest_7m','rel_freq','C_tilda_class');
Peak_ch_7m = [(hs_p_7m)' (tp)'];
save('Peak_char_7m','Peak_ch_7m');
```

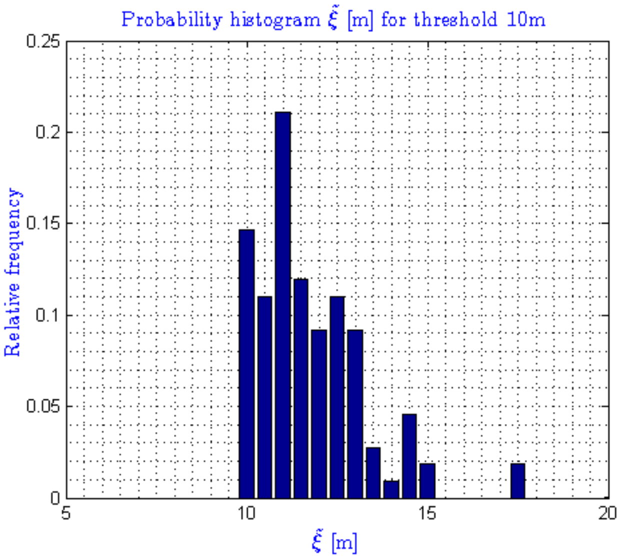
Appendix C.4 MPM CREST (2ND ORDER, 2D) , THRESHOLD $H_s = 8, 9, 10m$

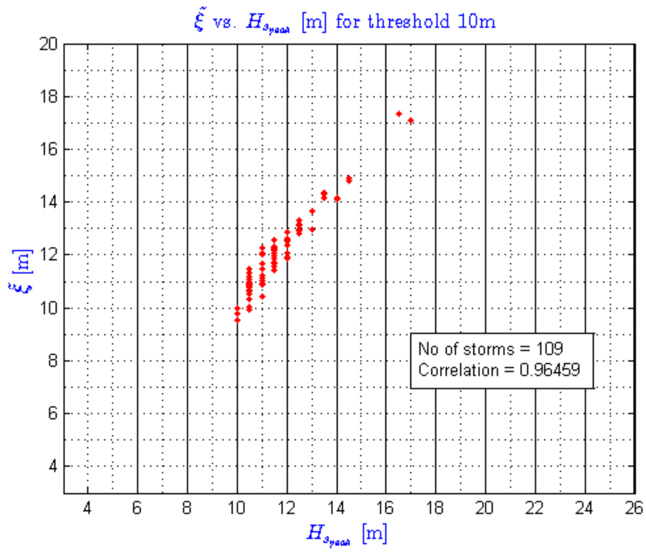






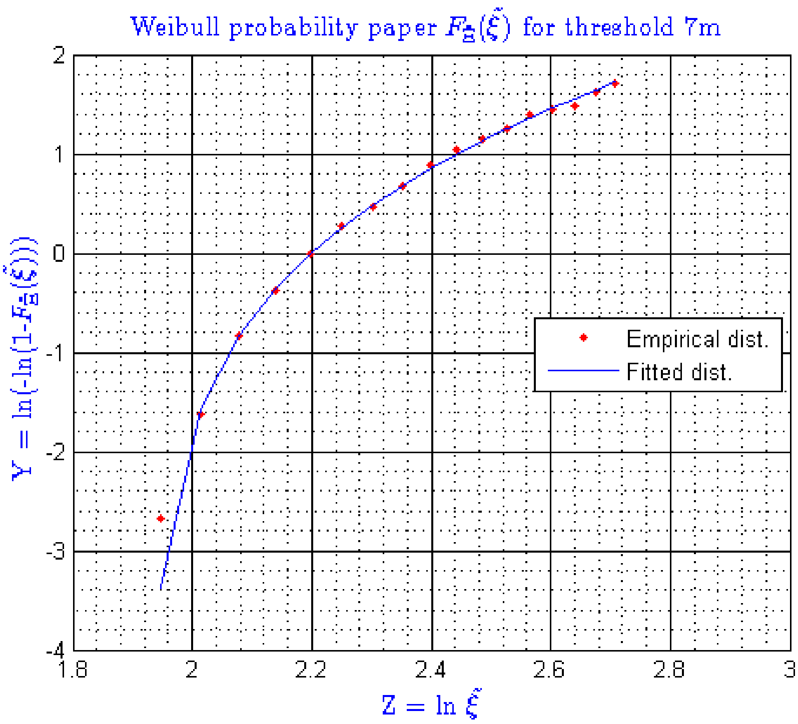




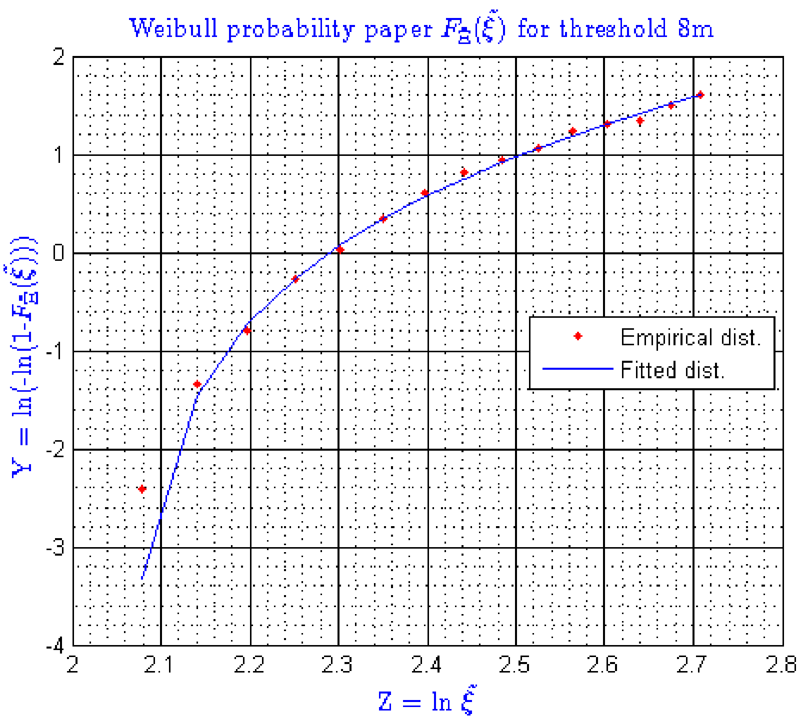


Appendix C.5 PROBABILITY PLOT, LONG-TERM MPM CREST, 2ND ORDER (2D)

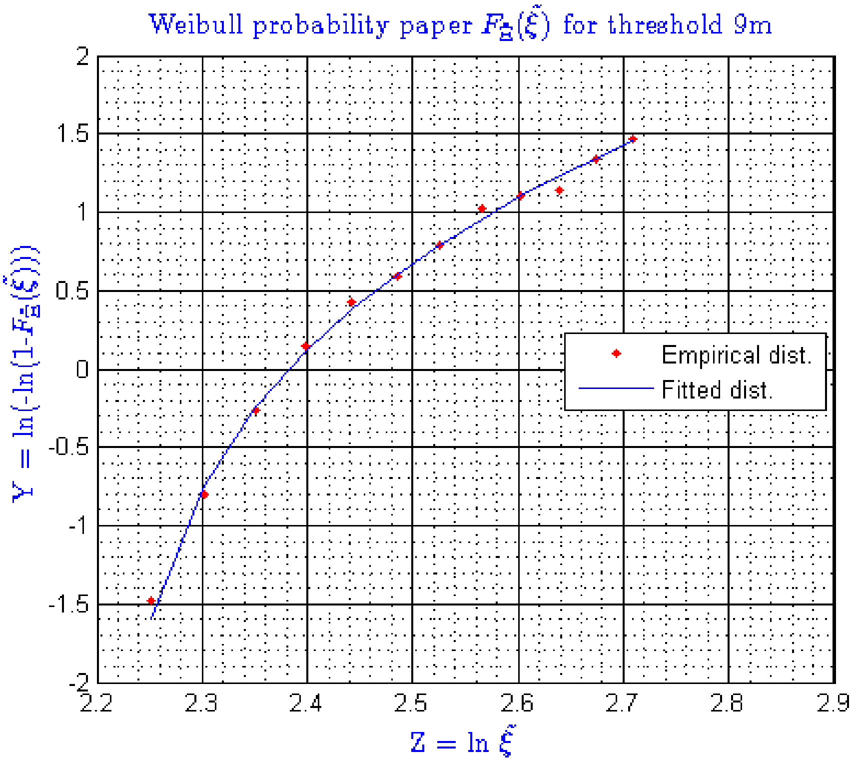
```
close all
clear all
clc
%load Weibull paramaters
load('3p_Weibull_7m.mat');
%threshold
thresh = 7;
%load data from XL
%data Z=ln(Xi_tilda)
Z = xlsread('F:\Masters thesis\XL
calculations\Wave+scatter_MSc_R9.xlsx','POT_Crest_tilda_Weibull 3p','AK29:AK45');
% empirical data Y=ln(-ln(1-F(Xi_tilda)))
Y_emp = xlsread('F:\Masters thesis\XL
calculations\Wave+scatter_MSc_R9.xlsx','POT_Crest_tilda_Weibull 3p','AL29:AL45');
% fitted data Y=ln(-ln(1-F(Xi_tilda)))
Y_fit = xlsread('F:\Masters thesis\XL
calculations\Wave+scatter_MSc_R9.xlsx','POT_Crest_tilda_Weibull 3p','AO29:AO45');

%probability plot
figure (1)
plot(Z,Y_emp,'r*','Markersize',3);
hold on
plot(Z,Y_fit,'b');
xlabel('Z = ln
$\tilde{\xi}$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title(['Weibull probability paper $F_{\tilde{\xi}}(\tilde{\xi})$ for threshold
',num2str(thresh),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b')
ylabel('Y = ln(-ln(1-
$F_{\tilde{\xi}}(\tilde{\xi})$))','Interpreter','latex','FontSize',12,'FontWeight','bold','Col
or','b')
legend('Empirical dist.','Fitted dist.','Location','East');
%xlim([3 26]);
%ylim([3 20]);
grid on
grid minor
```

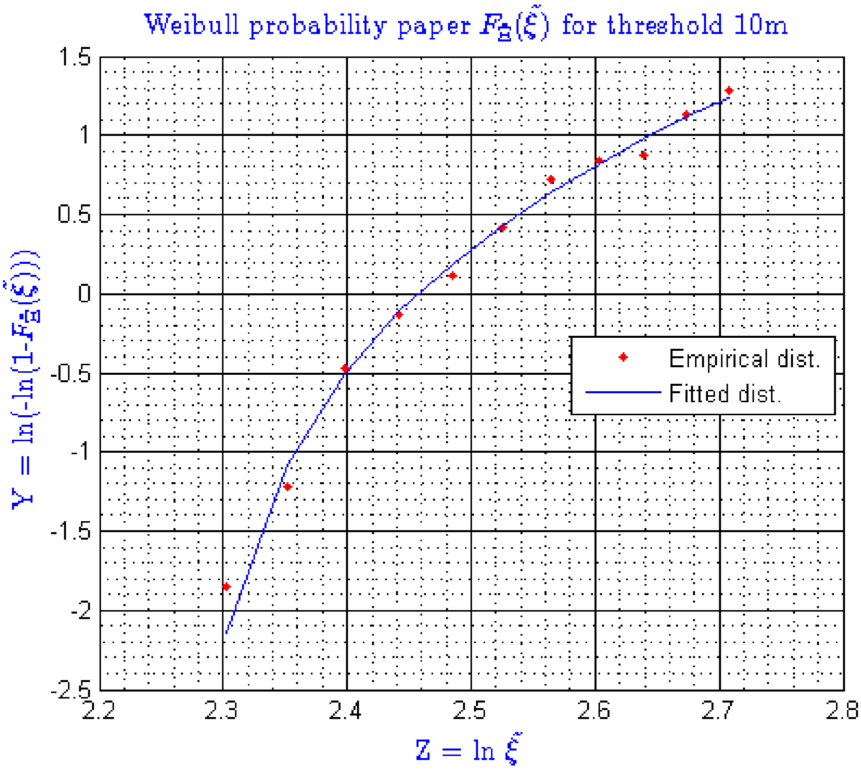


Weibull probability paper $F_{\hat{\Xi}}(\tilde{\xi})$ for threshold 7m
Y = ln(-ln(1-$F_{\hat{\Xi}}(\tilde{\xi})$))
Z = ln $\tilde{\xi}$
Empirical dist.
Fitted dist.
2
1
0
-1
-2
-3
-4
1.8
2
2.2
2.4
2.6
2.8
3



Weibull probability paper $F_{\hat{\Xi}}(\tilde{\xi})$ for threshold 8m
Y = ln(-ln(1-$F_{\hat{\Xi}}(\tilde{\xi})$))
Z = ln $\tilde{\xi}$
Empirical dist.
Fitted dist.
2
1
0
-1
-2
-3
-4
2
2.1
2.2
2.3
2.4
2.5
2.6
2.7
2.8


Weibull probability paper $F_{\tilde{\Xi}}(\tilde{\xi})$ for threshold 9m
Y = ln(-ln(1-$F_{\tilde{\Xi}}(\tilde{\xi})$))
Z = ln $\tilde{\xi}$
Empirical dist.
Fitted dist.
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
2.2
2.3
2.4
2.5
2.6
2.7
2.8
2.9


Weibull probability paper $F_{\tilde{\Xi}}(\tilde{\xi})$ for threshold 10m
Y = ln(-ln(1-$F_{\tilde{\Xi}}(\tilde{\xi})$))
Z = ln $\tilde{\xi}$
Empirical dist.
Fitted dist.
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
2.2
2.3
2.4
2.5
2.6
2.7
2.8

Appendix C.6 CUMULATIVE DISTRIBUTION MPM FOR GIVEN THRESHOLD

```
close all
clear all
clc
%load array with C_tilde for given threshold
load('F:\Masters thesis\Matlab\POT\Crest height\C_tilda_3p_Weibull\C_tilda_7m');
%load Weibull parameters for given threshold
load('F:\Masters thesis\Matlab\POT\Crest height\C_tilda_3p_Weibull\3p_Weibull_7m');

%distribution mpm
thresh = 7;
%convert cell to matrix with C_tilda values
C_tilda_mat = cell2mat(C_tilda_7m);
%sort ascending order
C_tilda_sort = sort(C_tilda_mat);
%find elements over mean value (location parameter mu)
C_tilda_over = unique(C_tilda_sort(C_tilda_sort>=mu_hat));
%round to 0.25 values
C_mpm = ceil((min(C_tilda_over))/0.25)*0.25:0.25:30;
%calculate cumulative probabilities
for j = 1:length(C_mpm);
    F(j) = (1-exp(-((C_mpm(j)-mu_hat)/sigma_hat)^eta_hat));
end;
row_pi_C_tilda_i = [F(1) zeros(1,length(C_mpm)-1)];

%calculate cumulative probabilities for each sea state
for i = 1:length(C_mpm)-1;
    row_pi_C_tilda_i(i+1) = F(i+1)-F(i);
end
%column vector
pi_C_tilda = (row_pi_C_tilda_i)';
%total probability of column vector
total_F_C_tilda = sum(pi_C_tilda(:));
%normalize column vector
F_C_tilda_norm_7m= bsxfun(@rdivide,pi_C_tilda,total_F_C_tilda);
%check total probability equal 1
total_F_C_tilda_norm = sum(F_C_tilda_norm_7m(:));
%count number of storms
N_7m = numel(C_tilda_7m);
%save data
save('F_C_tilda_norm_7m','F_C_tilda_norm_7m','N_7m');
```

Appendix C.7 GUMBEL PARAMETER β_v FOR LOW STORM

```
close all
clear all
clc
%load hindcast data
load('F:\Masters thesis\Matlab\POT\Crest height\WAM10.mat');
%load alpha_F and beta_F (Forristall CDF)
load('F:\Masters thesis\Matlab\Fitted model\Forristall_Long
crested\alpha_beta_forristall.mat');

%*Define threshold and extract data*
%define threshold
thresh_1 = 6;
disp(['Hs threshold = ', num2str(thresh_1)]);
%define matrix A of hindcast (simplify notation)
A = Heidrun_WAM10;
%extract hs and tp columns from matrix A
Wdata = [A(:,5) A(:,6)];
%logical true or false. find hs above threshold
ind_1 = Wdata(:,1)>=thresh_1;
%new matrix with hs (and corresponding tp) above threshold only
B_1 = bsxfun(@times,ind_1,Wdata);

%*Matrix with class indexes for Hs and Tp*

nBinsTp=25;
nBinsHs=50;
maxTp=25;
maxHs=25;
%create zero matrix (columns hs and tp)
M_1 = zeros(length(Wdata),2);
for k=1:length(B_1);
    %find corresponding scatter index for hs
    i(k)=ceil((nBinsHs/maxHs)*B_1(k,1));
    %find corresponding scatter index for tp
    j(k)=ceil((nBinsTp/maxTp)*B_1(k,2));
    %place index in hs column
    M_1(k,1) = i(k);
    %place index in tp column
    M_1(k,2) = j(k);
end

%*Column vector with Hs classes (frequency table)*
hs_upper = (0.5:0.5:25)';
hs_class = (1:1:50)';
hs_scatter = [hs_class hs_upper];

%*Column vector with Tp classes (frequency table)*
tp_class = (0.5:1:24.5)';

%*Short term distribution wave crest (2nd order, 2D)*
%matrix with indexes for hs & tp (storm #1)
G_1 = M_1(341:344,:);
%define crest height range
C_1 = 6.65:0.05:11;
```

```
%run loop through 1st column of matrix G (hs column)
for p = 1:length(G_1(:,1));
   %run loop through values of crest height Xi
   for m = 1:length(C_1);
     %calculate short term distribution for each 3hr sequence
     Fc_1 = (1-exp(-
((C_1(m)/(Alpha_F((G_1(p,1)),(G_1(p,2)))*hs_scatter((G_1(p,1)),2))))^Beta_F((G_1(p,1)),(G_1(p,
2)))))^(14026/tp_class((G_1(p,2))));
     Fc_cell_1(p,m) = Fc_1;
   end;
end;

%calculate probability for given crest height (class)
for m = 1:length(C_1);
    CDF_1(m) = prod(Fc_cell_1(:,m));
end;

%*Determine most probable value*
CDF_mpm = 0.37;
CDF_mpm_cl_1 = abs(CDF_1-CDF_mpm);
[~, idx_1] = min(CDF_mpm_cl_1);
C_tilda_1 = C_1(idx_1);
%display most probable max value C_tilda_1
disp(['MPM C_tilda_1 for given storm = ', num2str(C_tilda_1)]);

%*Monte Carlo simulation of realizations of Gumbel variable V. Calculate parameter beta*
for u = 1:50;
    for q = 1:50;
        c_3h = [];
        %loop to generate 3h max values (each step)
        for s = 1:length(G_1(:,1));
            r=rand;
            %calculate step max value (Forristall)
            c_3h(s) = (Alpha_F((G_1(p,1)),(G_1(p,2)))*hs_scatter((G_1(p,1)),2))*((-log(1-
(r)^((tp_class(G_1(p,2)))/14026)))^(1/(Beta_F((G_1(p,1)),(G_1(p,2))))));
        end;
        %find max value from simulated values
        c_3h_max = max(c_3h);
        %array of max values for q simulations
        c_3h_max_cell(q) = c_3h_max;
    end;
    %define array with values for variable v
    v_1{u} = bsxfun(@rdivide,c_3h_max_cell,C_tilda_1);
end;
v_1_mean = {cellfun(@mean,v_1)};
%calculate SD for v_1
v_1_sd = {cellfun(@std,v_1)};
%define input for beta_Gumbel function
v_sd = v_1_sd;
%calculate Gumbel parameter beta
beta_v_1_cell = cellfun(@beta_Gumbel,v_sd,'UniformOutput',false);
%transform cell to matrix with u values of beta
beta_v_1_mat = cell2mat(beta_v_1_cell);
%index of beta max value
[~,idx_max] = max(beta_v_1_mat);
%max beta in the range
beta_v1_max = beta_v_1_mat(idx_max);
```

```
%index of beta min value
[~,idx_min] = min(beta_v_1_mat);
%min beta in the range
beta_v1_min = beta_v_1_mat(idx_min);
%display min estimated beta
disp(['Minimum estimated beta_hat for given storm = ', num2str(beta_v1_min)]);
%display max estimated beta
disp(['Maximum estimated beta_hat for given storm = ', num2str(beta_v1_max)]);
%mean value beta
beta_hat_1 = mean(beta_v_1_mat);
%display mean estimated beta
disp(['Mean estimated beta_hat for given storm = ', num2str(beta_hat_1)]);

%*Distribution Fv*
%elementwise division
v = bsxfun(@rdivide,C_1,C_tilda_1);
%sort v_1 in ascending order
v_sort_1 = sort(v);
for t = 1:length(v_sort_1);
    Fv_1(t) = exp(-exp(-(((v_sort_1(t)-1)/beta_hat_1))));
end;

%plot distribution crest height
figure(1)
Y_1 = -log(-log(CDF_1));
plot(C_1,Y_1,'r','Linewidth',2);
xlabel('$\xi$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('-ln(-ln($F_{\Xi}(\xi)$))','Interpreter','latex','FontSize',12,'Color','b');
title(['Gumbel probability plot, $F_{\Xi}(\xi)$ for threshold
',num2str(thresh_1),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([6 11.5]);
ylim([-4 8]);
grid on
grid minor

%plot distribution Fv
figure(2)
Z_1 = -log(-log(Fv_1));
plot(v_sort_1,Z_1,'r+','MarkerSize',3);
xlabel('Gumbel variable
v','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('-ln(-
ln($F_{V}(v)$))','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
title(['Gumbel probability plot, $F_{V}(v)$ for threshold
',num2str(thresh_1),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0.75 1.42]);
ylim([-5 8]);
grid on
grid minor
%save workspace variables
save('Gumbel_beta_low_storm_plot','C_1','C_tilda_1','Y_1','Z_1','beta_hat_1','v_sort_1');
%save beta_hat to XL file
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
B = {beta_hat_1,beta_v1_min,beta_v1_max};
sheet = 'POT_Crest_Forristall';
xlRange = 'M6';
xlswrite(filename,B,sheet,xlRange);
```

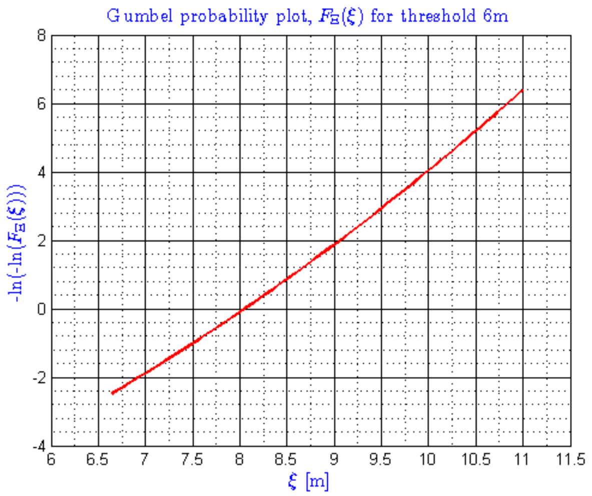
```
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
V = {C_tilda_1};
sheet = 'POT_Crest_Forristall';
xlRange = 'Q6';
xlswrite(filename,V,sheet,xlRange);
```

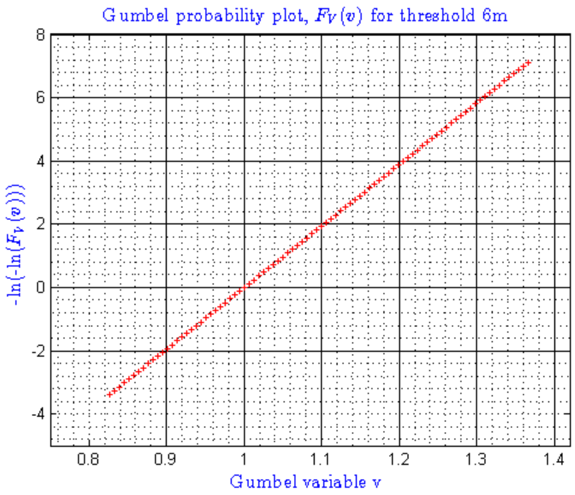
```
function [alpha_v] = alpha_Gumbel(beta_v,v_1_mean);
%Calculate Gumbel parameter beta
alpha_v = v_1_mean-0.57722*beta_v;
end
```

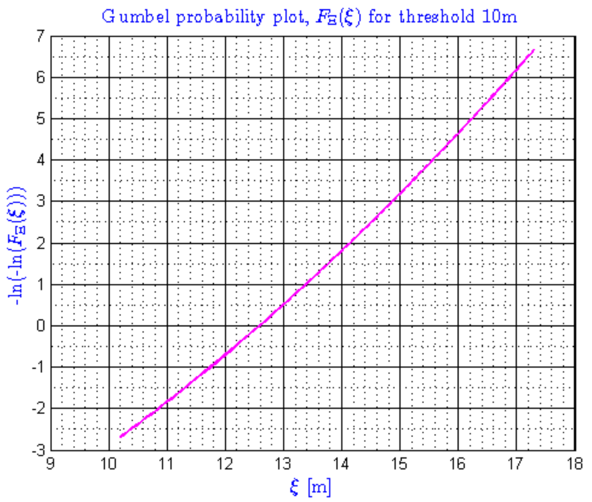
```
function [beta_v] = beta_Gumbel(v_sd)
%Calculate Gumbel parameter beta
beta_v = 0.7797*v_sd;
end
```

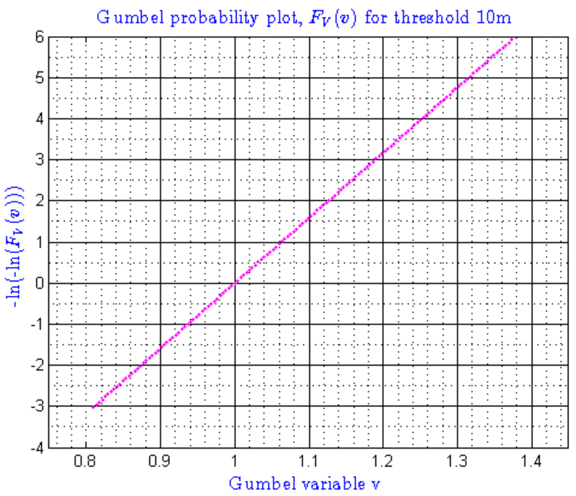
```
Hs threshold = 6
MPM C_tilda_1 for given storm = 8.05
Minimum estimated beta_hat for given storm = 0.037336
Maximum estimated beta_hat for given storm = 0.064535
Mean estimated beta_hat for given storm = 0.051442
```

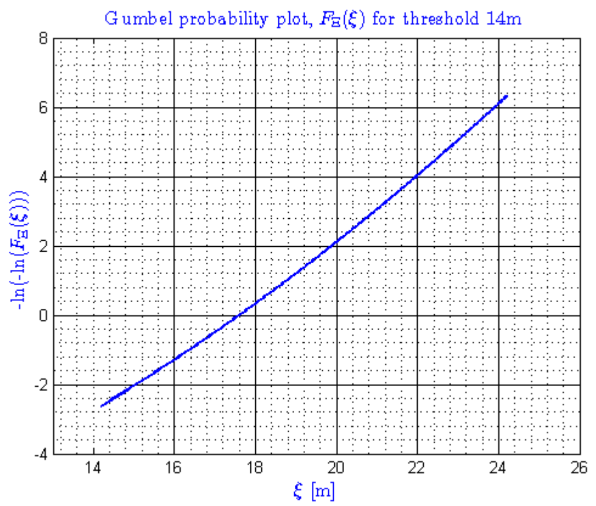
Appendix C.8 GUMBEL PROBABILITY PLOT, LOW-MEDIUM-HIGH STORM

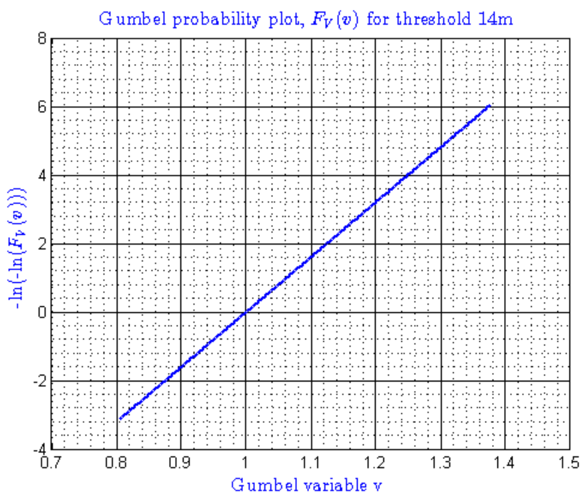












Appendix C.9 PLOT DISTRIBUTIONS V FOR 3 STORMS

```
close all
clear all
clc
%load low storm data
load('Gumbel_beta_low_storm_plot.mat');
%load medium storm data
load('Gumbel_beta_medium_storm_plot.mat');
%load high storm data
load('Gumbel_beta_high_storm_plot.mat');

thresh_1 = 6;
thresh_2 = 10;
thresh_3 = 14;

beta_hat_storms = [beta_hat_1;beta_hat_2;beta_hat_3];
beta_hat_mean = mean(beta_hat_storms);
display(['Mean estimated beta_hat for 3 storms = ', num2str(beta_hat_mean)]);

figure(1)
f1 = plot(C_1,Y_1,'r','Linewidth',2);
hold all
f2 = plot(C_2,Y_2,'m','Linewidth',2);
f3 = plot(C_3,Y_3,'b','Linewidth',2);
g1 = plot(v_sort_1*C_tilda_1,Z_1,'k','Linewidth',1.5);
g2 = plot(v_sort_2*C_tilda_2,Z_2,'c','Linewidth',1.5);
g3 = plot(v_sort_3*C_tilda_3,Z_3,'g','Linewidth',1.5);
hold off
legend([f1,f2,f3,g1,g2,g3],['H_{s} thresh = ',num2str(thresh_1),'m','
','(F_{\Xi|s}(\xi|s))'],['H_{s} thresh = ',num2str(thresh_2),'m','
','(F_{\Xi|s}(\xi|s))'],['H_{s} thresh = ',num2str(thresh_3),'m','
','(F_{\Xi|s}(\xi|s))'],['H_{s} thresh = ',num2str(thresh_1),'m',' ','(F_{V}(v))'],['H_{s}
thresh = ',num2str(thresh_2),'m',' ','(F_{V}(v))'],['H_{s} thresh = ',num2str(thresh_3),'m','
','(F_{V}(v))'],'Location','southoutside');
set(legend,'FontSize',8);
title('$F_{\Xi|storm}(\xi|s)$ vs. $F_{V}(v)$ (3 different
storms)','Interpreter','latex','FontSize',11,'FontWeight','bold','Color','b');
xlabel('$\xi$ [m] / Gumbel variable v
(scaled)','Interpreter','latex','FontSize',10,'FontWeight','bold','Color','b');
ylabel('-ln(-ln($F_{\Xi|storm}(\xi|s)$)) / -ln(-
ln($F_{V}(v)$))','Interpreter','latex','FontSize',10,'FontWeight','bold','Color','b');
xlim([5 26]);
ylim([-4 8]);
grid on
grid minor

figure(2)
h1 = plot(v_sort_1,Z_1,'r--','Linewidth',1.5);
hold all
h2 = plot(v_sort_2,Z_2,'c--','Linewidth',1.5);
h3 = plot(v_sort_3,Z_3,'b--','Linewidth',1.5);
hold off
legend([h1,h2,h3],'H_{s} thresh = 6m (F_{v}(v))','H_{s} thresh = 10m (F_{v}(v))','H_{s} thresh
= 14m (F_{v}(v))','Location','southoutside');
set(legend,'FontSize',8);
```

```
title('$F_{V}(v)$ (3 different
storms)','Interpreter','latex','FontSize',11,'FontWeight','bold','Color','b');
xlabel('Gumbel variable
v','Interpreter','latex','FontSize',10,'FontWeight','bold','Color','b');
ylabel('-ln(-
ln($F_{V}(v)$))','Interpreter','latex','FontSize',10,'FontWeight','bold','Color','b');
xlim([0.7 1.45]);
ylim([-5 8]);
grid on
grid minor


save('Gumbel_beta_hat_mean','beta_hat_mean');

filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';    %save beta_hat to
XL file
B = {beta_hat_mean};
sheet = 'POT_Crest_Forristall';
xlRange = 'P6';
xlswrite(filename,B,sheet,xlRange);
```

```
Mean estimated beta_hat for 3 storms = 0.058378
```

Appendix C.10 Conditional distribution crest | MPM for given threshold

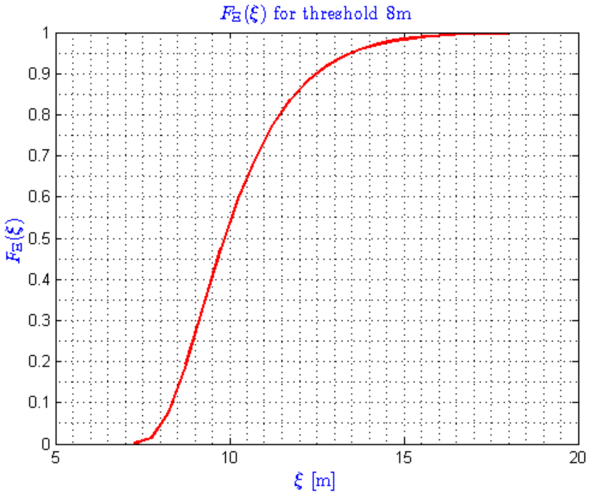
```
close all
clear all
clc
%*Load data*
%load Gumbel parameter beta
load('F:\Masters thesis\Matlab\POT\Crest height\Gumbel_Beta_3_storms\Gumbel_beta_hat_mean');
%load array with C_tilde for given threshold
load('F:\Masters thesis\Matlab\POT\Crest height\C_tilda_3p_Weibull\C_tilda_7m');
%load Weibull parameters for given threshold
load('F:\Masters thesis\Matlab\POT\Crest height\C_tilda_3p_Weibull\3p_Weibull_7m');

%define threshold
thresh = 7;
%convert cell to matrix with C_tilda values
C_tilda_mat = cell2mat(C_tilda_7m);
%range of possible crest values (midpoint of interval)
C = ((thresh-1)+0.25):0.5:30;
%sort ascending
C_tilda_sort = sort(C_tilda_mat);
%values over location parameter mu
C_tilda_over = (unique(C_tilda_sort(C_tilda_sort>=mu_hat)))';
%round off to 0.25 classes
C_mpm = ceil((min(C_tilda_over))/0.25)*0.25:0.25:30;
%calculate conditional distributions
for i = 1:length(C);
    for j = 1:length(C_mpm);
        F_Cs_C_tilde_7m(j,i) = (exp(-exp(-((C(i)-C_mpm(j))/(beta_hat_mean*C_mpm(j))))));
    end;
end;
%save workspace
save('F_Cs_C_tilde_7m','F_Cs_C_tilde_7m','C');
```

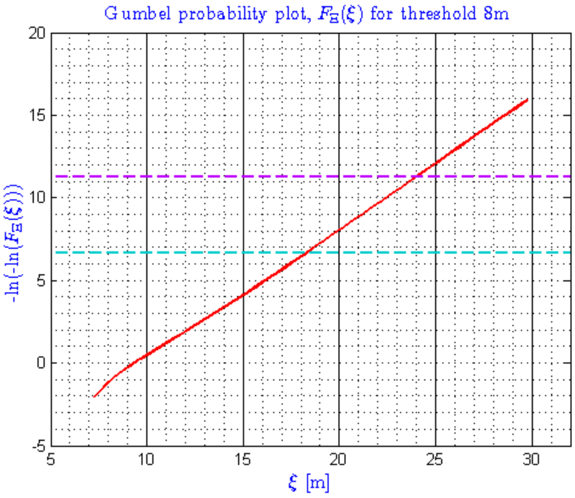
Appendix C.11 LONG TERM DISTRIBUTION CREST MAXIMUM FOR GIVEN THRESHOLD

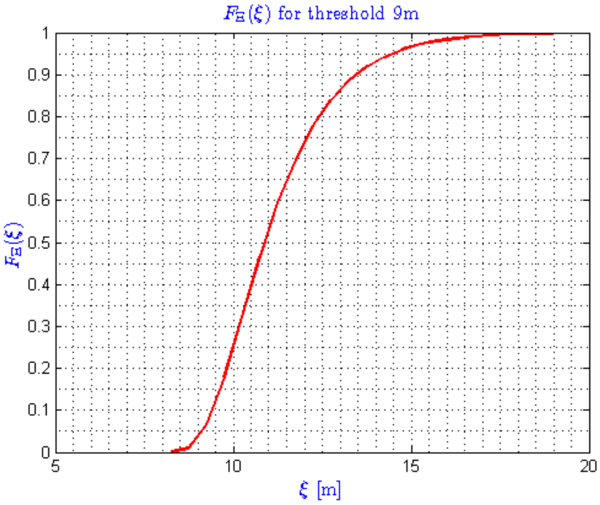
```
close all
clear all
clc

%*Load data*
load('F:\Masters thesis\Matlab\POT\Crest height\Long term\Threshold_7m\F_Cs_C_tilde_7m');
%load matrix with conditional distribution for Cs given C_tilda
load('F:\Masters thesis\Matlab\POT\Crest height\Long term\Threshold_7m\F_C_tilda_norm_7m\');
%load array with distribution for mpm and range of possible Cs (variable C)

%*Long term distribution wave crest height*
scatter_F_Cs = bsxfun(@times,F_C_tilda_norm_7m,F_Cs_C_tilde_7m);
thresh = 7;
for i = 1:length(C);
    F_Cs(i) = sum(scatter_F_Cs(:,i));
end;

%*Cumulative distribution*
figure (1)
plot(C,F_Cs,'r','Linewidth',2)
xlabel('$\xi$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('$F_{\Xi}(\xi)$','Interpreter','latex','FontSize',12,'Color','b');
title(['$F_{\Xi}(\xi)$ for threshold
',num2str(thresh),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([5 20]);
grid on
grid minor

%*Chararestic largest value*
q_100 = 10^-2;
q_10000 = 10^-4;
yr = 58;
n = N_7m/yr;
q_100_n_7m = q_100/n;
q_10000_n_7m = q_10000/n;
Y_100_7m = -log(-log(1-q_100_n_7m));
Y_10000_7m = -log(-log(1-q_10000_n_7m));

%*Gumbel probability paper*
D = -log(-log(F_Cs(1:length(C))));
figure (2)
plot(C,D,'r','Linewidth',2)
hold on
E = line([5 32],[Y_100_7m Y_100_7m],'Linestyle','--','LineWidth',1.5,'Color',[0 0.8 0.8]);
F = line([5 32],[Y_10000_7m Y_10000_7m],'Linestyle','--','LineWidth',1.5,'Color',[0.75 0 1]);
xlabel('$\xi$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('-ln(-ln($F_{\Xi}(\xi)$))','Interpreter','latex','FontSize',12,'Color','b');
title(['Gumbel probability plot, $F_{\Xi}(\xi)$ for threshold
',num2str(thresh),'m'],'Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([5 32]);
hold off
grid on
grid minor
```

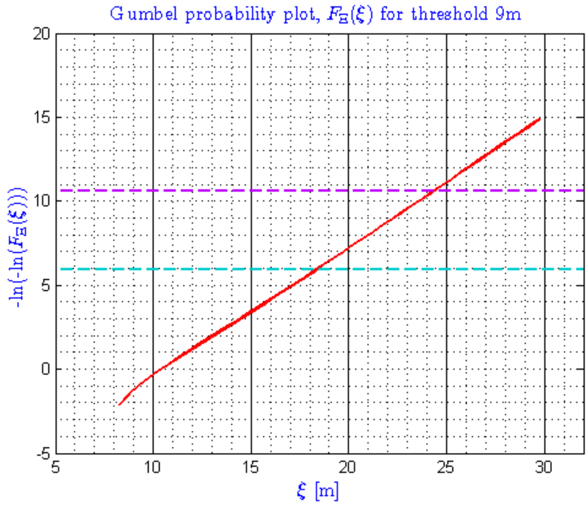
```
%find intersection point
[x_cs_100,y_cs_100] = polyxpoly(C,D,[5 25],[Y_100_7m Y_100_7m]);
[x_cs_10000,y_cs_10000] = polyxpoly(C,D,[5 25],[Y_10000_7m Y_10000_7m]);
%round off values
x_cs_100_round= ceil(10*x_cs_100)/10;
x_cs_10000_round= ceil(10*x_cs_10000)/10;
%change format to double variable
x_cs_100_7m = str2double(sprintf('%.2f',x_cs_100_round));
x_cs_10000_7m = str2double(sprintf('%.2f',x_cs_10000_round));
%save workspace
save('q_Cs_7m','x_cs_100_7m','x_cs_10000_7m');
%save in excel file
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
B = {x_cs_100_7m,x_cs_10000_7m};
sheet = 'Summary_Crest';
xlRange = 'I9:J9';
xlswrite(filename,B,sheet,xlRange)
filename = 'F:\Masters thesis\XL calculations\Wave+scatter_MSc_R9.xlsx';
G = {N_7m,yr};
sheet = 'POT_Hs_peak_Weibull 3p';
xlRange = 'I5';
xlswrite(filename,G,sheet,xlRange)
```

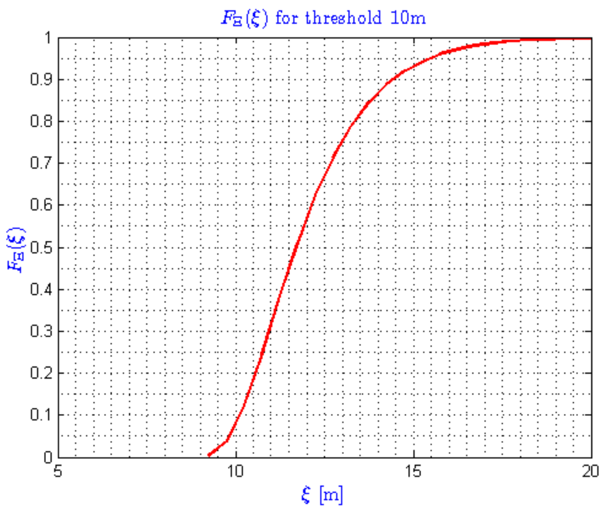
Appendix C.12 PLOT CREST MAXIMUM, THRESHOLD $H_s = 8, 9, 10m$

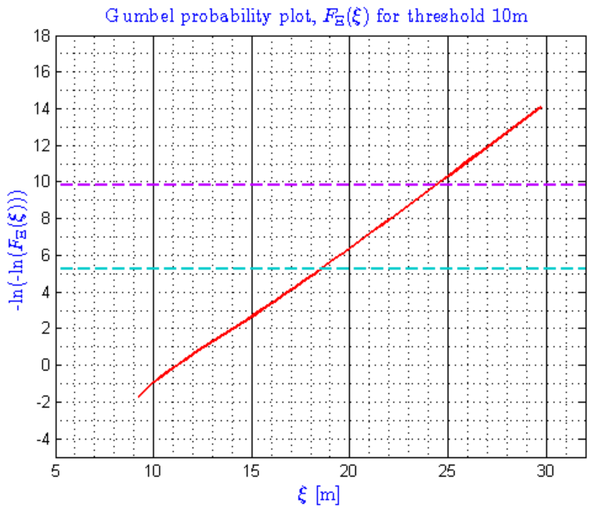












Appendix C.13 METOCEAN CONTOUR LINES ($b_1 = 0.001$), THRESHOLD 7M

```
clear all
close all
clc
clf
%Input Weibull paramters marginal dist. Hs
eta = 1.1293;
sigma = 1.6357;
mu = 7.043;
%define q-probabilities
q_100 = 10^-2/(786/58);
%q=10^-4
q_10000 = 10^-4/(786/58);
%define beta, radius of sphere(circle) in u-space
%for q=10^-2
beta_1 = -norminv(q_100,0,1);
%for q=10^-4
beta_2 = -norminv(q_10000,0,1);

%define failure surface in u-space
%row vector tetha of 1000 points linearly spaced,0 to 2*pi
tetha = linspace(0,2*pi,1000);
%define u_1_1,u_2_1 q=10^-2
u_1_1 = beta_1*cos(tetha);
u_2_1 = beta_1*sin(tetha);
%define u_1_2,u_2_2 q=10^-4
u_1_2 = beta_2*cos(tetha);
u_2_2 = beta_2*sin(tetha);
%calculate Gaussian distribution Phi(u_1_1) and Phi(u_1_2)
norm_u_1_1 = normcdf(u_1_1,0,1);
norm_u_1_2 = normcdf(u_1_2,0,1);

%define physical space for hs and tp
%q=10^-2
%calculate wave height from marginal distribution F_Hs
hs_100 = (sigma*(-log(1-norm_u_1_1)).^(1/eta))+mu;
%calculate mean ln(tp) and variance ln(tp)for hs_100
mean_1 = 0.9363+(1.1247*(hs_100).^0.1981);
%variance ln(tp)
var_1 = 0.001+(0.0595*exp(-0.2462.*hs_100));
%standard deviation ln(tp)
std_1 = sqrt(var_1);
%calculate peak period from conditional distribution
tp_100 = exp(mean_1+std_1.*u_2_1);

%q=10^-4
%calculate wave height from marginal distribution F_Hs
hs_10000 = (sigma*(-log(1-norm_u_1_2)).^(1/eta))+mu;
%calculate mean ln(tp) and variance ln(tp)for hs_10000
mean_2 = 0.9363+(1.1247*(hs_10000).^0.1981);
%variance ln(tp)
var_2 = 0.001+(0.0595*exp(-0.2462.*hs_10000));
%standard deviation ln(tp)
std_2 = sqrt(var_2);
%calculate peak period from conditional distribution
```

```
tp_10000 = exp(mean_2+std_2.*u_2_2);

%plot results
figure(1)
plot(tp_100,hs_100,'Linewidth',1.5)
hold on
plot(tp_10000,hs_10000,'r','Linewidth',1.5)
title('Metocean contour lines (b1 =
0.001)','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('$T_(Moan, Haver et al.)$
[s]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylabel('$H_{s}$ [m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
hleg1 = legend('q=10^{-2} (ULS)','q=10^{-4} (ALS)');
set(hleg1,'Location','Southoutside')
grid on
grid minor
%save workspace file
save('MCL_fixed','hs_100','hs_10000','tp_100','tp_10000');
```

Appendix C.14 EXTRACT T_p FOR A GIVEN H_s. MEAN AND VARIANCE

```
clear all
close all
clc
%load peak characteristics hs and tp
load('Peak_char_7m.mat');
M = Peak_ch_7m;
class = unique(M(:,1));
%extract for hs<=7m
extract_1 = M(M(:,1)<=7,:);
%loop to extract 7<hs<=16m
for i = 1:length(class)-1;
    extract_2{i} = M(M(:,1)>class(i) & M(:,1)<=class(i+1),:);
end;
%join results in one vector
Tp_cell_array = [extract_1 extract_2];
%calculate ln, mean and variance
for j=1:length(Tp_cell_array);
    ln_Tp_array{j} = log(Tp_cell_array{1,j}(:,2));
    mean_ln_Tp_array = (cellfun(@mean,ln_Tp_array))';
    var_ln_Tp_array = (cellfun(@var,ln_Tp_array))';
end;
save('Mean_Var_lnTp','mean_ln_Tp_array','var_ln_Tp_array');
```

```
%% *Fit curve mean ln Tp
```

```
clear all
close all
clc
%load data
load('Mean_Var_lnTp.mat');
hs=[7:0.5:14.5,16.5:0.5:17]';
%input y-axis: mean of lnTp (mu)
mu = mean_ln_Tp_array;
%parameters imported from xl (LSM fittinG)
mu_fit = 0.9363+(1.1247*(hs.^0.1981));
% plot data
figure(1)
plot(hs,mu,'ro');
title('$\mu=
0.9363+(1.1247*hs^{0.1981})$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color',
'b');
xlabel('Significant wave height
Hs[m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0 25]);
ylabel('Mean $\mu$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylim([1 3.5]);
hold on
plot(hs,mu_fit)
% insert legend
hleg2 = legend('\mu hindcast','\mu curve fit');
hold off
grid on
grid minor
```

```
%% *Fit curve variance ln Tp
```

```
clear all
close all
clc
%load data
load('Mean_Var_lnTp.mat');
hs=[7:0.5:14.5,16.5:0.5:17]';
%input variance of lnTp (sigma squared, input from XL sheet)
sigma = var_ln_Tp_array;
%calculate variance fitted function with *b1 = 0.005*
sigma_fit_fixed = 0.005+(5.4676*exp(-0.9273.*hs));
%calculate variance fitted function with *b1 = 0.001*
sigma_fit_free = 0.001+(0.0595*exp(-0.2462.*hs));
% plot data
figure(1)
plot(hs,sigma,'ro');
%title('$\sigma^2 = 0.005+(5.462*exp(-0.9272*hs))$ vs. $\sigma^2 = 0.001+(0.0595*exp(-
0.2461*hs))$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlabel('Significant wave height
Hs[m]','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
xlim([0 25]);
ylabel('Variance
$\sigma^2$','Interpreter','latex','FontSize',12,'FontWeight','bold','Color','b');
ylim([0 0.03]);
hold on
plot(hs,sigma_fit_fixed, 'b');
plot(hs,sigma_fit_free,'c');
hleg1 = legend('\sigma^2 hindcast','\sigma^2 curve fit b1=0.005','\sigma^2 curve fit
b1=0.001');
hold off
grid on
```