FACULTY OF SCIENCE AND TECHNOLOGY

BACHELOR THESIS

Study programme / specialisation:
The Lektor program in mathematics.

The spring semester, 2022

Open

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Thesis title: “The reasons for the collapse of the Tacoma Narrows Bridge and the lessons for the classroom”

Credits (ECTS): 10

Keywords: Bridge collapse, resonance, aerodynamics, physics textbokk,

Pages: ....61............

Stavanger, 15.5.2022
date/year

The reasons for the collapse of the Tacoma Narrows Bridge and the lessons for the classroom

Figure 0.1 Collapse of the Tacoma Narrows Bridge at 11:15 AM on 7 November 1940 (Scott, 2001)

Preface

We knew we had to write a bachelor thesis like every other teacher-student by our third year. Little did we know that we were allowed to write about an event that we were used to as early as our first semester here at the University of Stavanger. While attending the course FYS100 and learning about oscillatory motion, we were introduced to and shown a video of this peculiar bridge oscillating in a torsional matter. Our lecturer told us this was a topic we would have to understand before our upcoming exams. However, he also questioned if the bridge's torsional motion was caused by Resonance or something else. The book also mentioned that this view is still contested by mathematicians and physicists today. But should such an example be used if it only brings confusion around the understanding of Resonance? The case of Tacoma Narrows Bridge shows that not everything one reads has to be a definitive truth and that one should always question what one reads, even if the source is an academic book or article. When we researched which topic of our thesis we should request, we were struck by nostalgia and curiosity over the bridge we remembered from our earlier course. And we are both happy and satisfied with the thesis we have written. We want to thank Alex Bentley Nielsen for showing us this bridge back in FYS100 and bringing us this topic for our thesis. We also want to thank him for helping us throughout this semester when working on the thesis. We also want to thank Jasna Bogunovic Jakobsen for finding time to answer some of our questions regarding the Tacoma Narrows Bridge and how she and the professional community view the bridge's collapse today. We would also like to thank Stian Penev Ramsens for helping us with the program Tracker and tracking the video data.

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Abstract

On 7 November 1940, a historical event occurred for suspension bridge construction and aerodynamic engineering around suspension bridges. Engineers investigating the event concluded the bridge collapsed due to high winds but did not explain how. Later lab tests by other engineers and scientists demonstrated that the collapse happened either due to forced oscillations with Resonance or aeroelastic flutter. Forced oscillations with Resonance treated the bridge as an object being periodically pushed by the winds in Resonance with its natural frequency. And aeroelastic flutter treats the bridge as a wingspan in a fluid stream where the winds would alternate the pushing of the bridge span as it enters above and under the plate. Due to historical similarities, some believed the collapse occurred due to Resonance. However, later articles would discuss the aeroelastic flutter and criticize the resonance argument. One of these articles would be written by Billah and Scanlan, criticizing the use of the bridge as an example of Resonance in physics books and showing an alternative interpretation of the collapse. After discussing the collapse with an expert in aerodynamics on bridges from the University of Stavanger, we were informed that the Billah and Scanlan article is considered the modern explanation by the professional community. However, there are still physics books today that still misrepresent the circumstances around the collapse. We, as teaching students, agree with the Billah and Scanlan article and the opinion of the professional community that the collapse was most likely due to aeroelastic flutter. And that the collapse being represented as Resonance simplifies and misrepresents a more complicated and comprehensive problem around the Tacoma Narrows Bridge.

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Introduction

On the morning of 7 November 1940, the Tacoma Narrow Bridge in Washington, USA, collapsed due to high winds, making it move up and down and oscillate in a torsional motion until it tore itself up and fell into the narrows. But what phenomena caused the collapse? There are many leading theories as to why this happened, but physicists and engineers are still in debate about what is the real cause that led to the bridge collapse.

The theories for the collapse are many, but the most common theories are forced oscillations with Resonance or aeroelasticity and flutter on the bridge. The argument for forced oscillations and Resonance builds upon the thought of wind forces hitting the bridge with a frequency like that of the bridge's natural frequency, making it oscillate in the torsional matter. This theory is the most common explanation in physics books used in schools and universities, as shown in section 8 of this thesis. However, is it correct to show early physicists this as the cause of the collapse when the explanation is still up for debate? Another common explanation is that aeroelasticity and aeroelastic flutter and is the explanation favored by most engineers. This theory builds upon wind theory and how the span of the bridge worked as a wingspan in high winds. The steady flow of the wind and the internal damping force of the bridge created the torsional motion, which caused the bridge to collapse.

An example of these textbooks and one that we are familiar with from one of our earlier courses is a figure from the FYS100 book used at the University of Stavanger. This figure in the resonance part of the book shows the bridge's collapse. It states:

"In 1940, turbulent winds set up torsional vibrations in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. Once established, this resonance condition led to the bridge's collapse. (Mathematicians and physicists are currently challenging some aspects of this interpretation)". (Serway & Jewett, 2017)

We will also show more examples of this type of representation of the collapse in section 8.

As mentioned earlier, the main objective of this thesis is to summarize the different mathematical explanations of what caused the collapse and the strengths and weaknesses of the arguments. But, again, to emphasize that this problem is not as simple as a forced oscillation and resonance problem shown in the different physics books. But there are quite a few articles that have been written since the collapse in 1940. And these articles all have other arguments for why the bridge collapsed or criticism for earlier explanations in earlier articles. So, we will try to present these different articles and understand their views from a mathematical standpoint and what they believed caused the collapse of the Tacoma Narrows Bridge. And show that the problem is more complicated and comprehensive than a simple forced oscillation and resonance problem.

1. Suspension bridges

A suspension bridge comprises two towers, a deck and curved cables, as you can see on the picture of the Lysefjord bridge below. The cables carry tension forces. These forces are transferred to the two towers who leads the forces to the foundation in the form of compression. Since the start of the 20th century deflection theory has been used to design suspension bridges. Deflection theory says that when the structure becomes heavier, and the spans become longer the stiffness needed from the deck is decreased. This was especially important for the design of suspension bridges in the 1930s. They wanted to make the bridges thinner and more elegant. (The Editors of Encyclopaedia Britannica, 2021) Maybe this contributed to the failure of the Tacoma Narrow Bridge? Because when a bridge gets slimmer it is more susceptible to oscillations. Therefore, you can relate this to the Tacoma Narrows bridge because it collapsed due to oscillations.

Figure 1.1 Picture of the Lysefjord suspension bridge. Photo: Fredrik M. Pedersen

1.1 Theory about suspension bridges

As we will see later in this thesis, there are certain types of bridges with a medium- and long span that can be influenced by aerodynamic forces that will then generate a motion in the structure. The motion that the bridge will be induced is self-exited, which will, in turn, be affected by the aerodynamic force that it produces self, hence the name. The actions that are associated with self-excited motion are also known as aeroelastic.

The aeroelastic phenomena on bridges are of interest to different fields, like engineering, physics, and mathematics (Simiu & Miyata, 2006). However, engineers who study bridges and those who look at aeronautics differ in three ways. First, bridge structures are typically bluff, although, in modern suspension and cable-stayed bridges, streamlined box-like deck shapes are being used increasingly. Second, bridge decks have appurtenances such as handrails, curbs, parapets, center barriers, grillages, and so on that can significantly affect the flow and the aeroelastic forces. Third, unlike flows typically considered in aeronautical engineering, the flows in which civil engineering structures are immersed are, in most cases, turbulent (Simiu & Miyata, 2006).

Atmospheric turbulence comes from the thermal stratification of the flow. So, if there were high wind velocities, mechanical turbulence is the dominant turbulence. Therefore, the airflow may be assumed to be neutrally stratified. But the atmospheric flow is not necessarily neutrally stratified, even at relatively high wind velocities. Hence, the actual flow turbulence may differ substantially from, in some cases, be considerably weaker than the turbulence inherent in standard models. There is a chance that this affects bridge behavior unfavorably, especially under vortex-induced excitation, which is typically stronger in smoother flow (Simiu & Miyata, 2006).

Suspension bridges have also been a standard construction method for Norwegian bridges. They used the knowledge obtained by the Americans and other countries and made adaptions to Norwegian conditions. For example, are the Norwegian suspension bridges often slimmer than in other countries because there are usually only two-field roads and not four-field ones, so you do not need the bridge to be that wide? (Øderud & Nordhal, 2021)

2. The Story of Tacoma Narrows

In the 1930s, there was a surge of suspension bridges in the USA. Bridges like the Golden Gate Bridge and the Bronx-Whitestone Bridge finished construction in 1937 and 1939 and were significant successes for the engineering world. Golden Gate Bridge was the largest suspension bridge globally built, and Bronx-Whitestone Bridge was considered the most graceful suspension bridge to be made at that time. Another suspension bridge that was proposed in 1929 was the Tacoma Narrows Bridge. This bridge would connect the city of Tacoma to the sparsely developed Olympic Peninsula in northwest Washington, and a couple of assorted designs were petitioned over the years that followed. However, none of the designs would follow through until 1938, when the Toll Bridge Authority petitioned a design under the direction of engineer Clark H. Eldridge. This design would later be compromised to form a 792-m span flanked by longer 396-m side spans. There would also be a 12m wide floor truss stiffened by a 12m deep stiffening truss. They applied for funding of $3.3 million but only received $2.7 million. Something that would spark a later independent review of the construction plans (Scott, 2001).

After being denied proper funding for the suspension bridge, The Washington State Toll Bridge Authority obtained an independent review of its plans by Leon Moisseiff, arguably the most prolific suspension bridge designer of the era. This review would again alter the bridge's design, where the stiffening truss would be changed to a solid plate girder that would be both lighter and cheaper than the trusses. However, because of Moisseiff's working theories for suspension bridges, the change would not raise suspicion; instead, it would be praised for its economic and graceful design (Scott, 2001). The Toll Bridge Authority's Advisory Board of Engineers also stated, "*We have full confidence in Mr. Moisseiff and consider him to be among the highest authorities in suspension bridge design*" (Scott, 2001). And would not provide cause for suspicion on the design or potential grounds for disaster. There were some skeptics about the layout, like the septuagenarian engineer T.L Condron, who had done examinations of the design and was concerned with the width to span ratio of the design. However, despite Condron's concerns, the design would remain unaltered (Scott, 2001).

The construction of The Tacoma Narrows Bridge was not a smooth occurrence either. During the construction of the two caissons, box-like structures were used to build the underwater foundation for the bridge. Unfortunately, a crane boom buckled and dropped a load of steel into one caisson, injuring two and delaying work for ten days (Scott, 2001). One of the caissons also sank further down than estimated because of poor rock, causing further work and unintentionally making the caissons one of the deepest bridge peers ever at 68m depth below the span, which prompted the contractors to sue the Toll Bridge Authority (successfully) over faulty tidal information. Other signs of unease came during the final stages of construction when an unusual rhythmic vertical motion began to grip the main span in only moderate winds. These motions had plagued the construction workers with nausea and the engineers with concern. It was becoming apparent that a potentially dangerous phenomenon was around the corner. By early June, the bridge had been fitted with additional wires to support the midspan cable band to the plate girder. By the end of June, it was also fitted with hydraulic buffers like the ones on the similar Bronx-Whitestone Bridge. Tacoma Narrows Bridge then opened on 1 July 1940.

Despite all these measures, the vertical motion continued and became the subject of much local interest. Motorists and tourists gathered by the bridge to spectate and drive over the bridge during these motions. Because of this, the bridge earned the nickname Galloping Gertie, and a local bank even planted a billboard near the bridge, proclaiming it "as safe as the Tacoma Narrows Bridge." The bridge also proved to be financially stable; commuting tolls were reduced by 25% within a month. However, the bridge's motional behavior continued to worry Toll Bridge Authority engineers. By the end of July, it was devised to provide additional restraining measures on the bridge. Therefore, markers were planted on the lampposts along the span and a camera on top of one of the toll towers to document the angle and amplitude of the oscillations. And throughout August, observations and photographs were taken of the bridge and its oscillations. These videos and observations would prove that the span would also start to move during small winds, but since there was little evidence for side sways, the engineers saw little cause for concern. However, mindful of autumn winds, the engineers added hold-down cables to restrain the east side span and were repeated three days later for the west span.

On the night of 6 November 1940, a storm began to assert high winds on the bridge. The first signs of concern came from the Toll bridge Authority employee stationed overnight on a scow (wide beamed sailing boat) beneath the west side span. Some of the temporary hold-down cables had loosened by the bridge's motion, but the side spans had remained placid. By 05:00 AM, the motion had subsided. However, by 08:00 AM, the wind had increased to 17 m/s, and the main span had started to move up and down again. Then around 09:30 AM, a contractor's employee noticed the center ties on the north cable "alternately tightening and loosening, causing considerable snapping at this point." His motion picture film would later provide a lot of evidence for the events around the bridge on this day. In addition, Professor Farquharson of the Department of Civil Engineering at the University of Washington reportedly rushed to a local camera shop to borrow a camera to document the chaos unfolding on the bridge. The same man also tried to walk out on the span to save a dog stuck in one of the stranded vehicles on the bridge but had to retreat due to the increasing motion. Two truck occupants were out on the span during the large oscillations, but luckily, they managed to escape before the truck fell over on its side. By 10:00 AM, the winds had increased to 19m/s. And shortly after, astonished onlookers and aghast engineers watched as the bridge's behavior started oscillating in a violent motion so that the span seemed to almost roll completely over (Scott, 2001). At 10:30 AM, a floor panel broke and fell into the narrows. These events were the heaviest sign for Farquharson and the curious onlookers that the bridge would collapse on this day. And by 11:00 AM, some of the main span by the west tower peeled away and fell into the narrows. By 11:10 AM, the rest of the span plunged into the water, taking both the abandoned vehicles and the dog. The most spectacular bridge collapse ever was over. The Tacoma narrows bridge lasted just four months, but an entire era of engineering philosophy went with its demise. (Scott, 2001)

2.1 The Aftermath of Tacoma Narrows bridge

What transpired after the bridge collapse was a thorough investigation and attempt to explain the events at Tacoma Narrows Bridge. How could a span designed to withstand 45m/s winds and static horizontal wind pressure of 146 kg/m^2 succumb under the wind of less than half that velocity imposing a static force one-sixth the design limit? And how could horizontal wind forces be transformed into dynamic vertical and torsional motion? A baffled Moisseiff was quoted to be "completely at a loss to explain the collapse." The collapse was well documented when one accounted for the cameras at the scene. The camera installed on top of the spans toll booth and the ones brought onto the scene by Farquharson and the onlookers would provide incalculable scientific value to the subsequent investigations.

In the wake of the collapse, The Board of Engineers (known as the Carmody Board after Public Works Administration (PWA) head John M. Carmody) was established to investigate the collapse and find a solution to make sure this would not happen again. The board included big names from the industry like:

- Othmar Ammann known for being the chief engineer during the construction of the Bronx-Whitestone Bridge and George Washington Bridge.
- Glenn B. Woodruff, Engineer of design for Transbay.
- Theodore von Kármán (1881-1963), director of the Guggenheim Aeronautical Laboratory, was a leading figure in airfoil aerodynamics and had pioneered the study of wake turbulence since 1911.

In March of 1941, the Carmody board reported its findings. They pointed out three main flaws that caused the bridges to collapse. First was its flexibility, and second, was the plate girder and deck acting like an airfoil creating drag and lift. And third, was the aerodynamic forces acting on the bridge from the storm and how engineers did not have enough understanding of how these forces could influence the bridge.

The years that followed Gertie's demise would also prove to be difficult for the citizens of Tacoma. The bridge had been a lifeline for many as it connected their rural area to commercial hotspots like Seattle and the rest of Washington state. The new bridge had to be thoroughly planned and researched to prevent another disaster. This new bridge would not see the light of day until a decade later after salvaging, World War 2, and material shortages. The new Tacoma Narrows Bridge would stand completed in October 1950 and was nicknamed "Sturdier Gertie" by local promoters. The bridge was wider with four lanes, Heavier, and more robust with a deep stiffening truss, and with it came a new era for bridge design and aerodynamics around bridges. (Washington State Department of Transportation, 2020) (Scott, 2001)

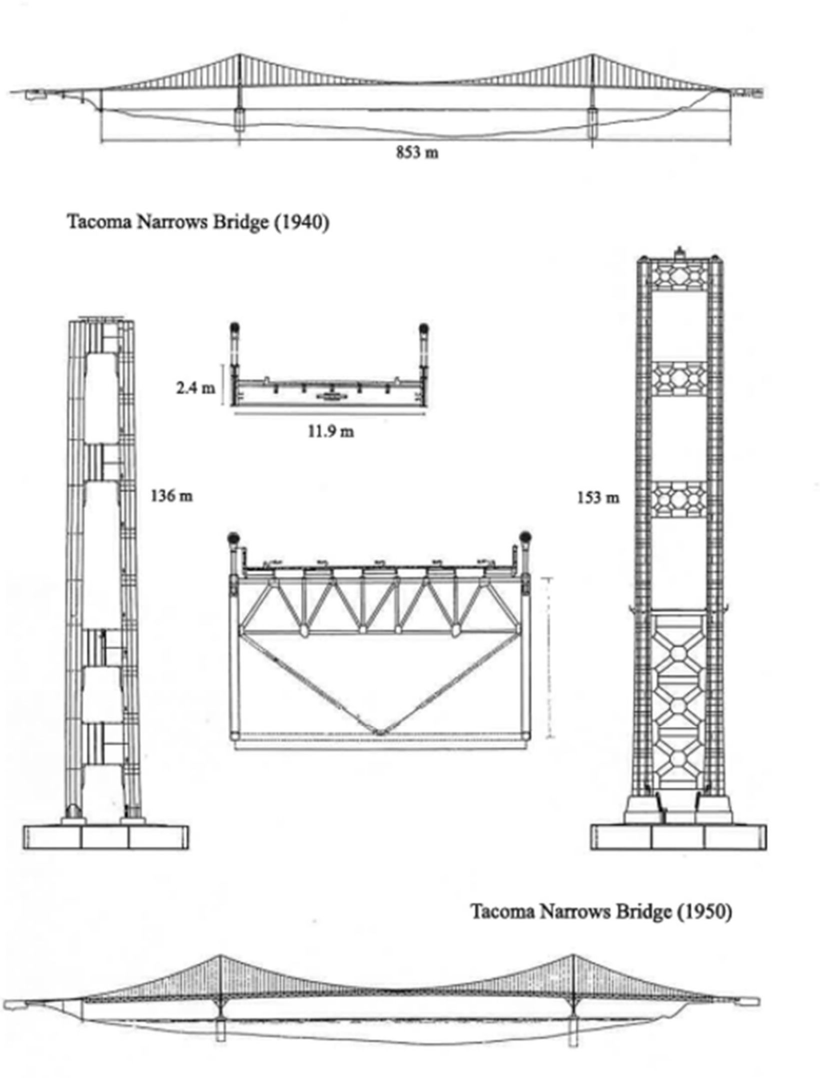

Figure 1.1 A picture with both designs of the Tacoma Narrows bridge before the collapse (1940) and the design after (1950) which still stands today. (Scott, 2001)

2.2 Tacoma's Influence on later bridges

As the Tacoma narrow span hit the narrow waters bridge, engineers had to reevaluate the use of long and thin suspension bridge design. The design that Leon Moisseiff had proposed was up to the standards of the time, but that was the main problem. The standards at the time failed to consider aerodynamics on bridges. As a result, former bridges that were similar in scale and design prevented the motions that occurred at the collapse and were thought to be constructed due to preference, not the ingenuity around bridges and their stiffness and ability to resist the winds. An example of this would be the Brooklyn Bridge, constructed by John Roebling in 1883. He consciously reinforced his spans with trusses and tower stays not because of innovative ideas but because it was already established after the introduction of deflection theory in the late 19th century trusses and reinforced spans faded into memory. In other words, the bridge engineering society had forgotten the problems with the aerodynamics of suspension bridges in favor of cheap and majestic bridges with long and thin spans (Scott, 2001).

So, what happened with the suspension bridge design after the Tacoma Bridge collapsed? Stiffening trusses quickly became more popular again in bridge design. Engineers soon realized that the trusses from earlier works like The Brooklyn bridge worked to prevent the torsional motion that had torn up The Tacoma Narrows Bridge. Therefore, it was not unusual to fit new suspension bridges with stiffening trusses after the collapse. A notable example of this would be the new Tacoma Narrows Bridge fitted with stiffening trusses like those proposed in the original design for the bridge, which would be the same as the trusses used at Mackinac in 1957 seen in figure 2.2. And as trusses evolved, so would the aerodynamic stability of the bridges become with them. An invention used at Severn Bridge to reduce aerodynamic instability was an enclosed box girder where the surface would run over the box of an airfoil shape where the winds could run through the box. A similar design can be seen in figure 2.2, as Tsing Ma (1997) also used an enclosed box girder. Not only did this improve stability, but it also reduced construction costs. The box girder would prove to be a new favorite for stability in suspension bridges next to the use of trusses. Bridges that would become significant accomplishments for suspension bridge history would later be the Mackinac Bridge and The Verrazano-Narrows Bridge, building upon the failures and lessons learned from the Tacoma Narrows Bridge disaster.

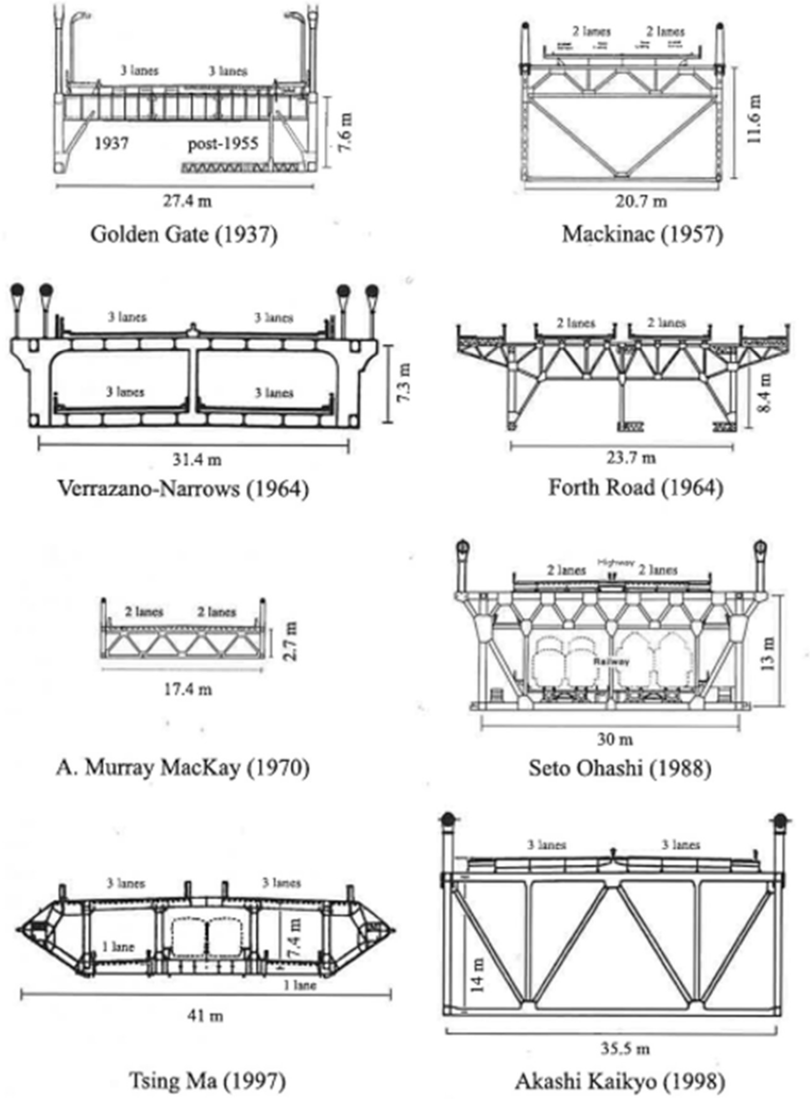

Figure 2.2 Different truss designs and which bridge they were applied to. (Scott, 2001)

Another invention that proved the structure's ability to prevent the deck from oscillating was the introduction of Inclined Hangers. As shown in figure 2.3, the hanger suspended from the main cables to the deck would be hung in an inclined fashion. This simple employment would assist the bridge, together with the longitudinal movements and the distribution of weight due to traffic, improve the damping potential of the structure (Severn Bridge Trust, 2019).

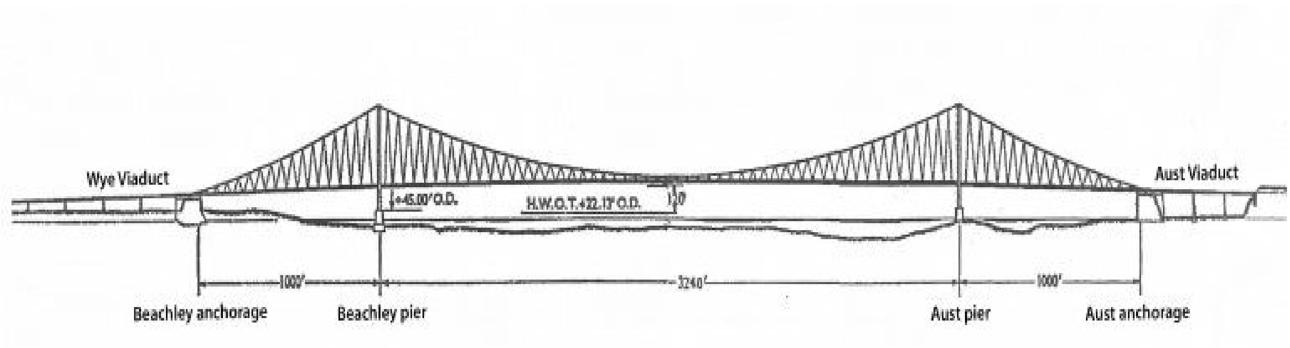

Figure 2.3 A model of the Severn Bridge and its use of both an enclosed box girder and inclined hangers. (Scott, 2001)

2.3 The Norwegian difference

An exciting contender in bridge history and theory is our own sparsely populated country of Norway. Throughout the 19th and 20th centuries, Norway has strived to make traversing the fjords a more straightforward and shorter task. The first recorded suspension bridge dated to 1844, the Bakke bridge in Sira Flekkefjord, and remained standing for 110 years. The nation continued to construct many suspension bridges throughout the century. What makes this part interesting is that during the 1920s and 1930s, the American trend of constructing long, thin, and cheap suspension bridges was also observed in Norway. A natural evolution of suspension bridge design in the times of economic instability and post wartimes. Norway, however, would continue to construct bridges that were indeed thin and long with depth to span ratios of 1:458 like the Elverum bridge. A feat that would perplex American investigators since it proved to be immune to aerodynamic oscillations. The most crucial Norwegian suspension bridge history case comes from the Fyksesund Bridge. The bridge experienced the same kind of oscillations as the ones observed at the Tacoma narrows. However, this was back in 1937, three years before the Tacoma collapse, and Fyksesund Bridge did not meet its demise by these oscillations. But this event influenced the up-and-coming bridge engineer Arne Selberg, a man who would pioneer aerodynamic suspension bridge design in Norway in the years to come. He would get his doctorate thesis on Design of Suspension Bridges accepted in 1946 and later the post of Professor of structural Mechanics in 1949 at NTH (Norwegian Institute of Technology). And after the completion of a wind tunnel at NTH, he would investigate the aerodynamic behavior of suspension bridges. His research would earn him the reputation of one of the world's leading experts in the field.

3. Local suspension bridges in the Stavanger area

To look at how suspension bridges are built later than Tacoma Narrow Bridges, we have visited two local suspension bridges, the Lysefjord bridge and the Bybru in Stavanger.

3.1 The Lysefjord bridge

The Lysefjord bridge is a suspension bridge that was opened in 1997. It has a main span of 446 meters, about half of the main span of the Tacoma Narrow Bridge. The bridge is constructed with two towers, one on each side of the fjord (NAF, u.d.). So overall, it is pretty similar to the Tacoma narrows bridge, just a bit shorter.

Figure 3.1 The Lysefjord bridge from the road. Photo: Fredrik M. Pedersen.

Figure 3.2 Tacoma Narrows Bridge. (Ann, 2018)

As you see from the pictures above, there are similarities between the Lysefjord bridge and the Tacoma Narrow Bridge when you see the bridges from above. They are both much like a standard suspension bridge. In the pictures below, you can observe the Lysefjord from below. The deck is shaped like a trapeze and is not rectangular as it was for the Tacoma Narrows bridge. You can see the profile of the Tacoma Narrow Bridge in figure 2.1. This is probably to make the bridge more aerodynamic so an accident like the Tacoma Narrow Bridge collapse will not happen in the future.

Figure 3.3 The Lysefjord bridge from below. Photo: Fredrik M. Pedersen.

3.2 Stavanger Bybru

The Bybru in Stavanger was opened in 1978 and is on the national protection plan for roads and bridges. The bridge is 1067 meters long with a main span of 185 meters. (Stor norske leksikon, 2018) What differentiates this bridge from many other suspension bridges is that the bridge is only made of one tower with wires. This is when the tower crosses the Straumsteinsundet. When the bridge crosses over land, it is founded to the ground with columns to support it. So, the bridge is a hybrid when it comes to construction methods.

Figure 3.4 The Bybru in Stavanger. Photo: Fredrik M. Pedersen

Figure 3.5 The Bybru in Stavanger. Photo: Fredrik M. Pedersen

4. Differential equations.

The Tacoma Narrows bridge collapsed due to oscillations. Mathematical oscillations can be expressed as differential equations. More generally, differential equations describe all kinds of continuous motion. Differential equations are equations of motion. Different people have tried to explain the collapse with several types of differential equations.

To discuss the different mathematical models for the Tacoma collapse, you need to distinguish different differential equation types from each other. There are various terms to describe the equations. For example, are the ordinary or partial differential equations linear or nonlinear, and are they homogenous or nonhomogeneous? And what order does the equation have? First, second, or maybe higher. These are different terms you can use to describe what type of differential equation you are dealing with. (Adams, 2018)

The difference between an ordinary and a partial differential equation is that an ordinary differential equation only has derivatives with respect to one variable. A partial differential equation includes the derivatives with respect to at least two variables. The highest order derivative determines the order of a differential equation in the equation. For example, if the highest order of derivatives in the equation is a second-order derivative. The equation is called a second-order differential equation. This can include the zeroth-order derivatives too. If one or more of the derivatives in a differential equation has power, we say it is a nonlinear equation. So, if none of the derivatives has power it is a linear equation. If we have a linear ordinary differential equation it is homogeneous if the function f(x) = 0 and inhomogeneous if f(x) is nonzero. For example, the equation

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = 0 \tag{4.1}$$

is homogenous because it is equal to zero. An example of a nonhomogeneous equation looks like this. (Adams, 2018)

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + x = F(t)\sin(\theta\, t) \tag{4.2}$$

The equation is not equal to zero.

One of the explanations for the bridge collapse is forced oscillation and Resonance. This is described by the second-order nonhomogeneous ordinary linear differential equation:

$$\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = Fsin\omega t \qquad (4.3)$$

You can see that the highest order derivative is of second-order ($\frac{d^2x}{dt^2}$) therefore, it is a second-order equation. $F\ sin\omega t$ is a nonzero term, so the equation is nonhomogeneous. The derivatives are only with respect to one independent variable t. Therefore it is an ordinary differential equation. In the equation, you do not take the power of any of the derivatives; consequently, it is a linear equation. (Adams, 2018)

5. Forced oscillations and Resonance

A frequent example used in the textbooks which mention the Tacoma narrows bridge collapse is that of harmonic oscillations. An oscillation where the object in question oscillates with a constant frequency in a harmonic motion and where the restoring force is proportional to the displacement in the opposite direction. However, although one of the collapse's leading theories is that of type oscillation, the theory is that of forced oscillation and Resonance.

But what are forced oscillations? Forced oscillations can be described as a repetitive motion where an external force does positive work on a system, making the system oscillate with respect to its central axis or a certain point. Examples of this can be a pendulum, a swing, or a string on an instrument. A typical example forced oscillator is a damped oscillator driven by an external force that varies periodically, such as

$$F(t) = F_0 sin\omega t \qquad (5.3)$$

where F_0 is a constant, and ω is the angular frequency of the driving force. Where the frequency of ω is generally the driving variable of the equation. And the natural frequency ω_0 of the system is fixed by values of k (stiffness) and m (Mass of the object). By modeling a forced oscillator with both a driving and retarding force, Newton's second law gives us the equation

$$F_0 \sin \omega\, t - b\frac{dx}{dt} - kx = \frac{d^2x}{dt^2} \Longrightarrow \quad \frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0 \sin \omega\, t \tag{5.4}$$

where b is a constant known as the damping coefficient.

When an external force starts working on the initially stationary system. The oscillation will start, and the amplitude will increase. The system in question and the surrounding medium in question is an isolated system. This means the work done by the driving force will make the vibrational energy (kinetic energy of the object, elastic potential energy in a spring) and the internal energy of the object and the medium increase (Serway & Jewett, 2017). After a sufficiently long period of time when the energy input per cycle from the driving force equals the amount of mechanical energy transformed into internal energy for each cycle, a steady-state condition is reached in which the oscillations continue with a constant amplitude. In this situation, the solution of the equation transforms into that of

$$x = A\cos(\omega t + \varphi) \tag{5.5}$$

$$A = \frac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}} \tag{5.6}$$

And were

$$\omega_0 = \sqrt{\frac{k}{m}}$$

Is the natural frequency of the undamped oscillator. The equations show that the forced oscillator vibrates at the frequency of the driving force and that the amplitude of the oscillator is constant for a given driving force because it is in a steady-state by an external force. When the damping is small, and the frequency of the driving force is the same or close to that of the natural frequency of the oscillation $\omega \approx \omega_0$. The amplitude will dramatically increase, as shown in the equation and figure 5.1.

$$A = \frac{F_0/m}{\sqrt{\left(\omega^2-\omega_0^2\right)^2+\left(\frac{b\omega}{m}\right)^2}},\ when\ \omega \approx \omega_0 \ \rightarrow \ A = \frac{F_0}{b\omega} \quad (5.7)$$

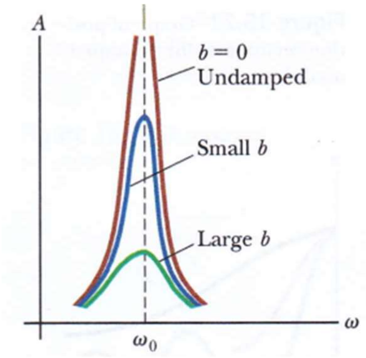

Figure 5.1 Graph of the amplitude versus frequency for a damped oscillator when a periodic force is present. (Serway & Jewett, 2017)

This is what we call Resonance. And what does this have to do with the Tacoma narrows collapse? We will look at this and the theories of how this worked in section 8 of the thesis.

But what does Resonance have to do with forced oscillations? Resonance is a particular case of forced oscillations. There are also several types of Resonance in forced oscillation systems like:

- Acoustic Resonance: A branch of mechanical Resonance that deals with the vibrations produced within the frequency range of 20 Hz and 20kHz. Instruments with strings under tension have resonant frequencies directly related to mass (m), length (L), and tension of the string (T). This can be expressed in the equation:

$$f = \frac{n\sqrt{\frac{T}{m/L}}}{2L} \quad (5.8)$$

  Where n is a positive integer multiple of the fundamental frequency (Vendantu, 2022).

- Electrical Resonance: Is the phenomenon of Resonance in electrical circuits seen in wireless communication. Transmitting and receiving signals in phones, television, and radio. This happens because of capacitors and inductors and can be expressed in terms of the equation:

$$\omega = \frac{1}{\sqrt{LC}} \tag{5.9}$$

Where $\omega = 2\pi f$, in which f is the resonant frequency in Hertz (Hz), L inductance in Henries (H), and C is capacitance in farads (F) (Vendantu, 2022)

- Parametric Resonance: This is when oscillations are driven by some varying parameter of the system's frequency, typically different from the natural frequency of the oscillator. Resonance occurs when the system is parametrically excited with one of its resonant frequencies. Unlike forced oscillating Resonance, the action appears as a time-varying modification on a system parameter. The parametric oscillator can be expressed in terms of the equation:

$$\frac{d^2x}{dt^2} + \beta(t)\frac{dx}{dt} + \omega^2(t)x = 0 \tag{5.10}$$

A linear equation in x(t). And by assumption, the parameters ω^2 and β depend only on the time and not by the state of the oscillator. If the parameters vary roughly twice the natural frequency of the natural frequency the oscillator phase locks to the parametric variation and absorbs energy at a rate proportional to the energy it already has. Without compensation for the energy-loss mechanism of β the amplitude will grow exponentially (Wikipedia, 2022).

However, in the case of Tacoma and the physics at play, we will focus on mechanical Resonance. At least before section 8, where this last Resonance is mentioned. Mechanical Resonance is when a mechanical system absorbs more energy when the frequency of the external force working on the system matches or is close to matching that of the natural frequency of the object, causing the amplitude of the oscillation to increase in great magnitude. Hence the name of Resonance as the two frequencies start to resonate with each other. Forced Oscillation and Resonance is one of the leading theories why Tacoma Narrows Bridge is often referenced in physics and university books when giving an example of real-life mechanical Resonance, as seen in section 7.

6. Aeroelastic flutter

This section describes aerodynamic and aeroelastic phenomena affecting the bridge's structure. First, we will look closer at aeroelastic behavior like different types of vortices, galloping, flutter, and a two-dimensional problem of the Tacoma Narrow Bridge.

6.1 Aeroelasticity

Aeroelastic is a complex phenomenon not thought of when the Tacoma Narrow Bridge was built in 1940. Even though there was a similar collapse in 1836 on the Brighton chain pier caused by aeroelastic flutter (Simiu & Miyata, 2006). The only difference between the bridge in Brighton and Tacoma Narrow Bridge was that Tacoma was approximately six times longer than the Brighton chain pier. In the nineteenth century, when the bridge collapsed, engineers and professors did not know about aeroelastic flutter. So, when the phenomenon occurred in Brighton, they called it "undulation" because of the lack of knowledge on aeroelastic flutter (Simiu & Miyata, 2006). To fully understand the interaction between the aerodynamics forces and the effect of structural motion on the body, its necessary to solve the equation of motion describing the flow with time-dependent boundary conditions imposed by the moving structure (Simiu & Miyata, 2006). This is not a simple problem to solve because the problem is defying analytical capabilities and therefore is hard to solve by the computational fluid dynamics method. So, solving the problem of aerodynamic force is not yet done by computing. Still, it must be tested on prototypes in the laboratory to understand how the aerodynamic force interacts with the body. This has to be done very carefully, so the results obtained from laboratory testing can be used to scale how the original bridge should be built. Therefore, experiments done at a laboratory on the model of the bridge can generally be assumed to be yielded to act like the realistic bridge and give the same results as the model in the laboratory did

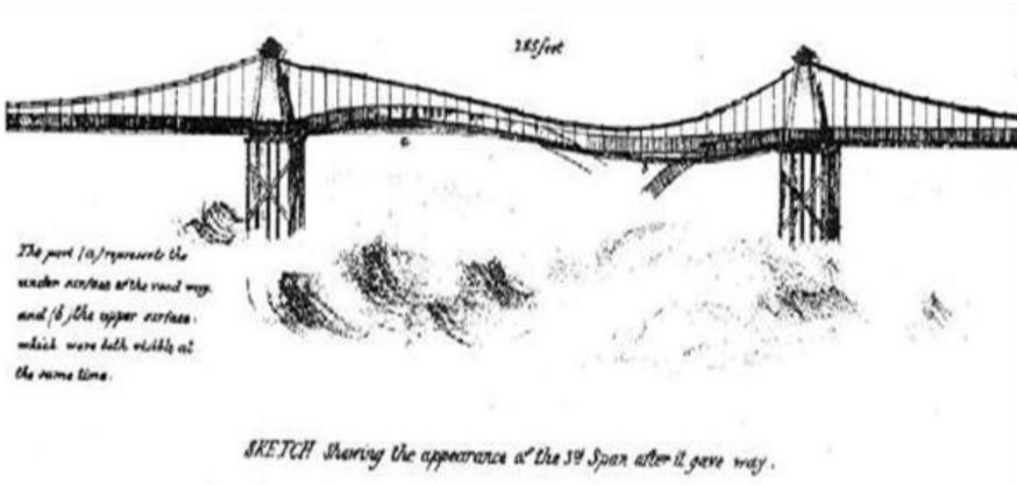

Figure 6.1 Brighton chain pier failure, 1836. (Simiu & Miyata, 2006)

When the wind flow interacts with the bridge's body, energy transfer will occur. This energy can go from the wind flow to the bridge and from the bridge to the wind flow. When discussed, the viscous damping force reduces the energy that is induced to the oscillating body (Holms, 2001). In aerodynamics, this is referred to as negative aerodynamic damping force. If a body is induced, energy from the wind flow will lead to the body's oscillation due to increasing energy and will destabilize the body (Simiu & Miyata, 2006). So, when the body has a positive aerodynamic, it will be stabilized, and the oscillatory energy will decrease and eventually come to rest. We will look at this later in the thesis in section 6.4. The motion of galloping takes place in one dimension. Because the aeroelastic behavior happens in one dimension, it is relatively simple to get a very accurate picture of how basic flow interacts with the body. This understanding of how galloping behaves in one dimension leads to how the structure interacts with wind flow (Simiu & Miyata, 2006). As we have seen, only some bodies will start galloping, and this comes from where the angle hits the body of the relative velocity of the flow with respect to the body changes. In the sense of aeroelastic phenomena, what governs galloping is similar to what governs the flutter motion. But for flutter motion, it gets more complicated due to two reasons. The first reason is that this is associated with motion in three degrees of freedom instead of one degree of freedom like galloping is. Secondly, flutter motion is affected by vorticity, which is absent in the galloping case.

When building large suspension bridges, one is interested in bridge movement and the primary interest, i.e., acceleration, deflection and wind loads, etc. The stress induced on the bridge's structure should be considered when designing the bridge. The bridge should withstand the total wind load induced on the bridge deck F_{tot} , which is found by adding up all the other forces that occur and affect the bridge's structure. Three forces are the main components induced on the bridge deck/structure. The time-averaged mean wind load, F_q , the motion-induced wind load, F_m, and the fluctuating wind load due to air turbulence, also known as buffeting F_t.

$$F_{tot} = F_q + F_t + F_m \tag{6.1.1}$$

Of the three forces acting on a suspension bridge, the force that comes from the motion-induced wind load and aerodynamic damping is the force that most impacts the bridge's structure, especially cable-supported bridges. Because these two forces are strongly dependent on the wind velocity, the bridge's vibrating frequency and the aerodynamic damping come from the wind flow. In the mathematical equation represented above, there is a clear distinction between still-air and in-wind bridge characteristics (Dyrbye & Hansen, 1999).

For flutter to occur, it will get to critical flutter wind velocity. The wind velocity inputs a certain energy from the motion-induced wind load, equal to the energy dissipated by the structural damping. When critical flutter occurs, it will lead to the energy from the wind being more significant than the energy that is dissipated by the structural damping. The dominating term in the equation will be the motion-induced wind load F_m. Finding the critical flutter wind velocity by assuming that there is zero buffeting wind load and zero mean wind load is often a mathematical abstraction used as a guideline for judging the aerodynamic behavior of the bridge (Dyrbye & Hansen, 1999). Many modes are involved in the structural vibration of the bridge. The vibration of the bridge will come from a coupling of torsional mode and vertical bending mode, where both with significant bridge-deck movement. Therefore, where both the vertical and torsional modes are vibrating, it will have the largest deflection on the bridge-deck structure. But it is not said that the first symmetrical bending mode is likely to couple with the first antisymmetrical torsional mode since the largest vertical bending mode occurs or is found where the torsional mode is vibrating small relative to the vertical mode (Dyrbye & Hansen, 1999).

We have seen that the bridge will be affected by torsional and vertical deflection, but bridges will also be affected by horizontal deflection. However, the deflection from vertical deflection/vertical load and torsional moment/ angular rotation are not coupled strongly to the horizontal deflection. There is a formulation to this type of uncoupling that is assumed. For horizontal deflection to be significant on suspension bridges, the bridge has to have a span between 1-2 km to have an impact. It is also essential to consider the shape of the modes and where vertical, horizontal, and torsional deflection couple. It is also possible to expand the formulation by adding motion-induced load terms, as shown in Wind loads on structures, where it gets 18 different aerodynamic derivatives: 3 load components, combined with 3 deflection and 3 velocity terms $\left(18 = 3 \times (3+3)\right)$. (Dyrbye & Hansen, 1999)

6.2 Vortex shedding

Generally, when talking about vortex shedding, a smooth, steady flow induces a body, typically a prism or a cylinder. But in this thesis, we are taking that the flow is the wind velocity that induces the body's motion. While the wind is flowing against the body, there will be vortices behind the body. This vortex shedding alternates with a dominant frequency n_s. This frequency has the relation

$$n_s = St\frac{V}{D} \tag{6.2.1}$$

The St is the Strouhal number depending on the Reynolds number and the cross-section of the body, D is the characteristic dimension of the body, and V is the velocity of the flow that hits the body where the flow is constant (Kaneko, et al., 2008). We describe it here in two dimensions, but vortex shedding also occurs in three dimensions. When it occurs in three dimensions, it has some conditions for the vortex. The condition is that the body has to be relatively short and/or tapered and in a nonuniform and turbulent flow. Something that can happen when the wind flows, and this is something that can occur at any given time when the wind is flowing on the body. That is when the fluctuating flow in a body makes the wind flow asymmetrical about the parallel line to the oncoming flow. This makes the pressure induced on the body asymmetrical to the flow. Therefore, the body will get a transverse fluctuating load perpendicular to the oncoming flow, which is also called the lift force. (Simiu & Miyata, 2006)

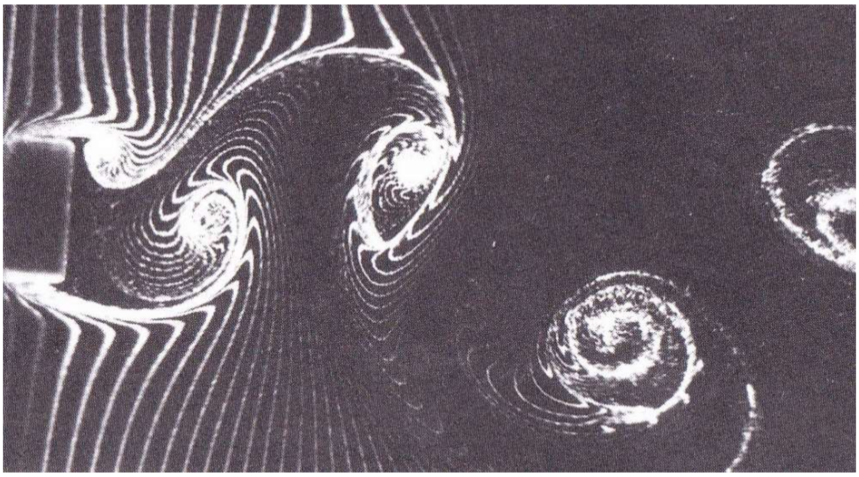

Figure 6.2 Flow around a rectangular cylinder (Re=200) (Simiu & Miyata, 2006)

6.3 Different types of vortex induced

6.3.1 Vortex-induced Lock-in

"The shedding of vortices in the wake of a body gives rise to fluctuating lift forces. If the body is flexible or if it has elastic support, it will experience motion due to aerodynamic forces and, in particular, to the fluctuating lift force" (Simiu & Miyata, 2006). So, when the motion is sufficiently small, the wind flow will not affect the vortex shedding, leading to equation 6.2.1, which remains valid. Therefore if the vortex-shedding frequency n, which is associated with the frequency of the lift force, is equal to the natural vibration frequency of the body n_{body}, resonant amplification can occur. (Simiu & Miyata, 2006). While the flow affects the body motion, the body motion affects the flow insofar as it produces lock-in, that is, synchronization of the vortex-shedding frequency with the frequency of the body's vibration.

6.3.2 Vortex-induced oscillations and lift-force

A second aeroelastic effect due to the cylinder's vibration is of interest in practice. So if we take an infinitely rigid cylinder, vortex-induced lift forces per unit span at the different stations along the cylinder are imperfectly correlated. This means they are not perfectly in tune with each other. However, the amplitudes of the cylinder oscillation will increase under excitatiton by vortex-induced lift force. (Simiu & Miyata, 2006)

6.4 Galloping

The Tacoma Narrow Bridge got the nickname Galloping Gertie because of the bridge's motion (Scott, 2001). Galloping comes from structural vibration in a perpendicular direction to the wind direction. So, if there are vibrations in the structure, these come mainly from negative aerodynamic damping. These large amplitude aeroelastic oscillations can vary from one to ten or more cross-sectional of the body. Bodies with a cylindric or prism-like shape and a certain cross-section (e.g., Ice-laden power cables, rectangular section, D-section). If there were a constant oncoming flow velocity, the oscillation would occur in a plane normal to the flow. But the frequencies will be much lower than the vortex shedding frequencies for the same section. The velocity causing a body to have a galloping motion is typically considerably larger than galloping from vortex lock-in." *Flow reattachment, which is present at vortex lock-in and at flutter, does not occur in the galloping case; fully separated flows are thus a feature of galloping motion and result in the absence of the vortex-induced effects on the body"* (Simiu & Miyata, 2006). If a body is at rest and gets induced by a flow velocity, that can either be a fluid velocity or a wind velocity; both can be expressed as a flow. Here we can take the equation from aeroelastic, that the total force or wind load, F_{tot} , is the sum of the motion of the structure, F_m, the force from the time-averaged mean wind load, F_q, and F_t from the turbulence force.

These contributions of the different forces cannot be considered mutually independent. The value of the force from the motion of structure, Fm, is usually significantly influenced by force for the turbulence, which may come in beneficial use or come as a disadvantage. What is fundamental to the galloping phenomenon is the angle of where the wind attacks the relative flow velocity with respect to the body that is changing due to the flow." *The changed relative* *flow velocity creates in bodies with certain cross-sectional shapes asymmetrical pressure distributions that enhance that incipient motion, rather than suppress it, as would be the case if the body were aeroelastically stable* (Simiu & Miyata, 2006)*".*

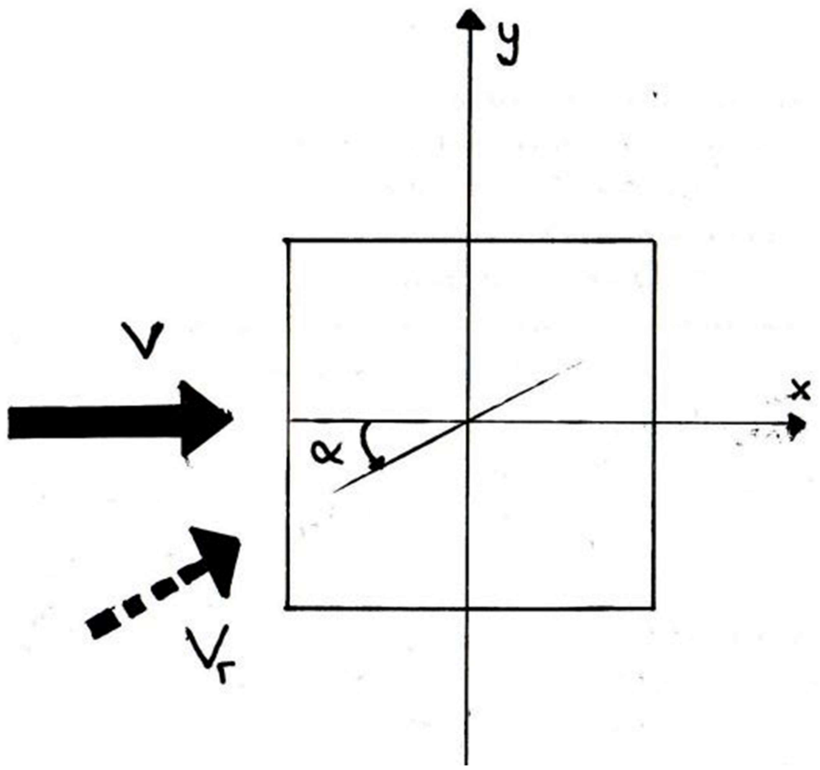

Figure 6.3 Flow velocity V and relative flow velocity V_r on a square cylinder moving upward in the y-direction. Drawn by: Anders B. Waldeland

To understand how a body is galloping and which forces are acting to make it gallop. In figure 6.3, we can see what force is acting on the body. Here we neglected the influence of turbulence. When we neglect the force of turbulence, then the lift and drag force can be expressed as

$$F_D(\alpha) = \frac{1}{2}\rho A \frac{V^2}{cos^2\alpha}\ D(\alpha) \tag{6.4.1}$$

$$F_L(\alpha) = \frac{1}{2}\rho A \frac{V^2}{cos^2\alpha}\ L(\alpha) \tag{6.4.2}$$

Where both the drag and lift forces are expressed as a force that depends on the angle α, which can be found by looking at the difference between the two velocities that hit the body V and V_r, where the V is the velocity that hits the body at a horizontal level and V_r which is the relative velocity that comes from the same direction as V but hits the body with an angle α, the difference between the two velocities are that comes from the angle α, that is obtained from the angle between the two velocities. So, the lift and drag forces are almost the same, apart from the coefficients D(α) and L(α). The other components in the equation are A, the structural area, ρ is the density of the air, and L(α) and D(α) are the coefficients. These two coefficients depend on the wind direction, geometry of the structure, and the wind's turbulence. Therefore, the only way to determine the value of the coefficients is by the wind-tunnel test of a model of the structure. (Dyrbye & Hansen, 1999) (Simiu & Miyata, 2006)

When we know the forces acting on the body, we can find how much force is acting in the y-direction. The force in the y-direction is the sum of the forces that affect the body and only depends on the angle.

$$F_y(\alpha) = F_D(\alpha) sin\alpha \; + F_L(\alpha) cos\alpha \qquad (6.4.3)$$

From this equation 6.4.3, we can write the force in the y-direction differently than a sum of two forces. We know the only difference between the drag and lift forces is the coefficient. The alternative form for the force in the y-direction is

$$F_y(\alpha) = \frac{1}{2}\rho V_r^2 d C_{F_y}(\alpha) \qquad (6.4.4)$$

Since we know that the $V = V_r \cos\alpha$, hence the coefficient in the y-direction is

$$C_{F_y}(\alpha) = \frac{C_L(\alpha) + C_D(\alpha) tan\alpha}{cos\alpha} \qquad (6.4.5)$$

As mentioned in section 3.1 about aeroelasticity, bridges have an aerodynamic damping force that will stabilize the bridge to rest if the velocity is below critical velocity. For the bridge to come to rest, the aerodynamic damping force has to be positive; otherwise, it will increase the galloping and break (Dyrbye & Hansen, 1999) (Simiu & Miyata, 2006)

6.5 Flutter

6.5.1 Basic about flutter

This section of the text looks at the aeroelastic phenomenon of flutter and classical and turbulent flutter. The last thing we are looking at in this section is how the effect of flutter on the Tacoma Narrow Bridge has been solved if it was a two-dimensional problem. This phenomenon occurs to flexible bodies with a relatively flat shape on the span. What happens to the body is that it will begin to oscillate to small degrees, but the amplitude will grow over time. If this goes over an extended period, this will result in a catastrophic failure of the structure. Flutter behaves like other aeroelastic phenomena, and therefore the solution to the equation of motion has to involve different types of force: mechanical damping, elastic restraint, inertial and aerodynamic forces. The latter force will depend on the body's shape, motion of the body, and the flow that will affect the body andnclude the self-excited forces (Simiu & Miyata, 2006).

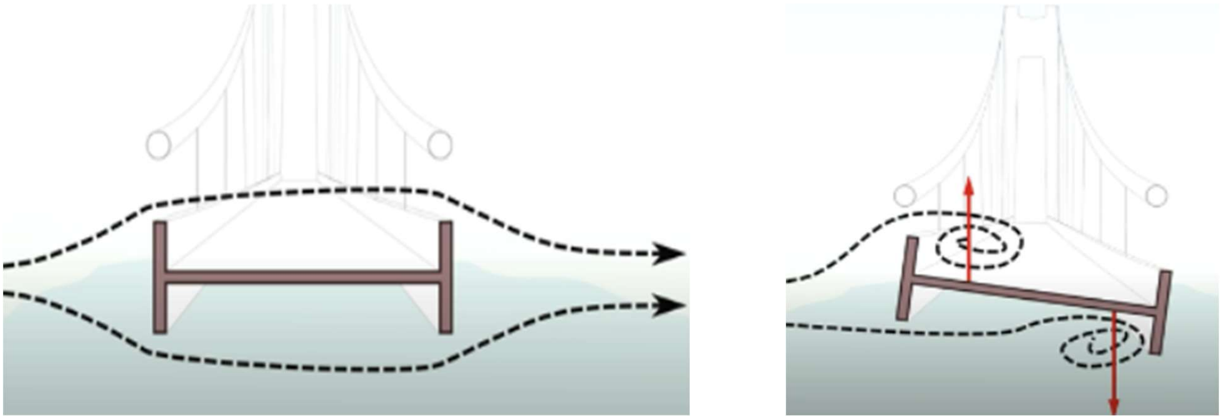

Figure 6.4 An example of the motion of aeroelastic flutter and how it affects a bridge. (Seo, 2020)

If we neglect that there is mechanical damping to a body. The body will be aeroelastic stable for motion if the body can come back to equilibrium after being affected by a small perturbation away from the equilibrium position. This comes from the self-excited force that will stabilize the force associated with perturbation (Simiu & Miyata, 2006). But if the velocity increases in the perturbation, it will change the aerodynamic forces that act on the body. So, at some point, the velocity will increase so much that it will come to a critical velocity that will cause the self-excited force to the body to be neutrally stable. Therefore, if we take a velocity that is larger than the critical velocity, the body will start to oscillate that is initiated from the small perturbation from the equilibrium point, and the oscillation will be larger over time. So, instead of having a positive aerodynamic damping effect, the self-excited force is the cause of the growing oscillation on the body and, therefore will have a negative aerodynamic damping force on the body (Simiu & Miyata, 2006).

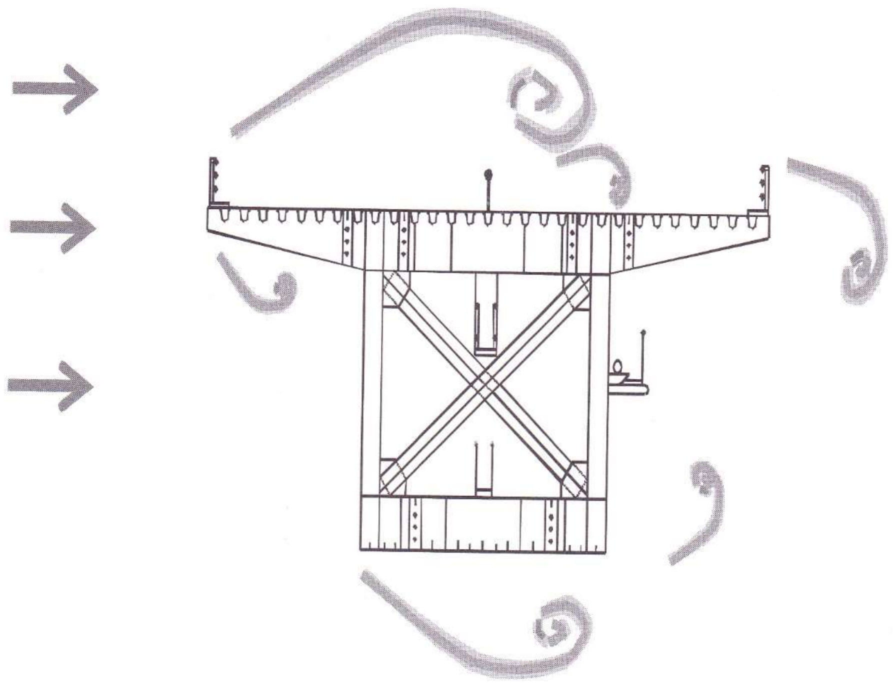

Figure 6.5 Vortex separation from sharp corners and protuberances of a typical bridge box deck. (Simiu & Miyata, 2006)

Just because the flutter is accompanied by the vortex-shedding, which has the same frequency that is always equal to the flutter frequency, this does not mean that the flutter is the same as vortex-induced oscillation but that there is a distinct difference between flutter and vortex-induced oscillation. The vortex-induced oscillation means an interaction between the body and the velocity flow where the body's structure and the vortex shedding have approximately the same frequency. If the velocity is higher than in vortex-induced oscillation, lock-in occurs. (Simiu & Miyata, 2006)

6.5.2 Torsional flutter

According to some literature, the collapse of the Tacoma Narrow Bridge was caused by torsional flutter (Simiu & Miyata, 2006). They have based their theory of the collapse on wind tunnel experiments, where there is a constant flow that hits and goes around an H-shaped section similar to the deck on the Tacoma Narrow Bridge. The wind-tunnel testing shows that the torsional response increases nonlinearly with reduced velocity (Simiu & Miyata, 2006). Figure 6.6 shows the patterns of the wind tunnel flow around an H-shaped section similar to the shape of the deck of the first Tacoma Narrow Bridge.

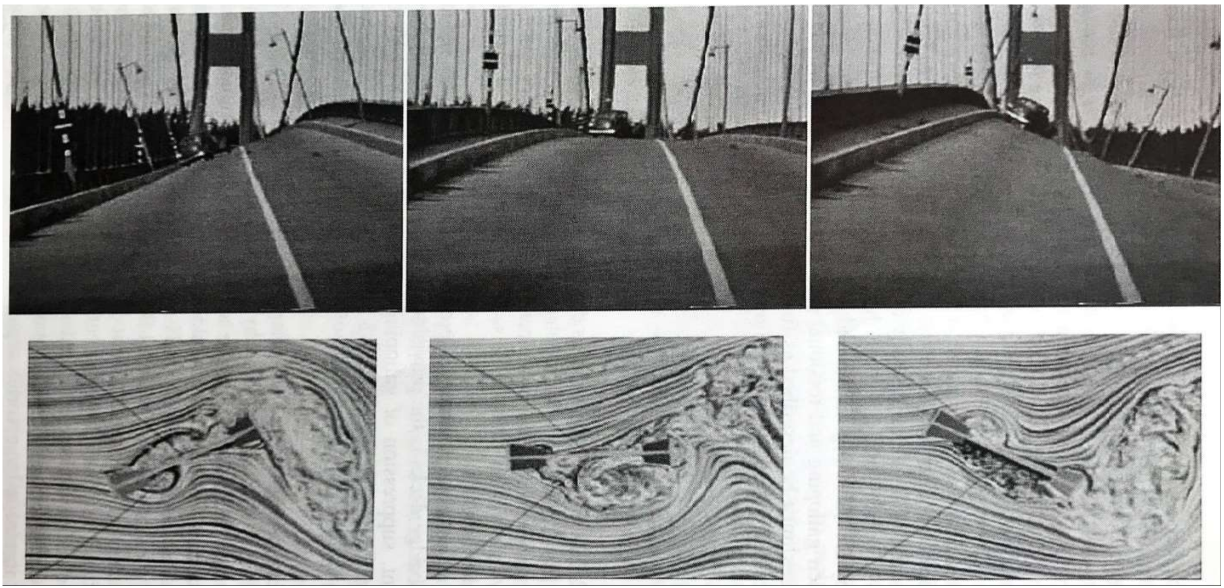

Figure 6.6 Torsional motion at the time of collapse by torsional (stall) flutter, original Tacoma Narrow Bridge, and smoke visualization of separated vortex flow. (Simiu & Miyata, 2006)

"The patterns correspond to the three instants during the one cycle of a periodic torsional motion. Also shown is a picture of the torsional displacement of the bridge at each of those three instants. Note the vortices generated at the first of the three instants at the front edge of the deck, which subsequently travel toward the deck's rear edge." (Simiu & Miyata, 2006) . At the first instant in Figure 6.6, a large vortex separated from the front edge is associated with strong negative pressure on the windward half of the lower deck surface. At the same time, a somewhat flatter flow is produced on the upper surface. In the second instance, the vortex that was started in the earlier instant, which has traveled along the underside of the deck, is broken.

And the vortex flow on the upper deck is flatter than in the earlier instant. At the last instant in Figure 6.6, the flow pattern is approximately symmetrical to the first pattern. "The flow patterns of Figure 6.6 produce pressures consistent with the moments that cause the torsional motion.

6.5.3 Classical flutter

In sections 6.4 and 6.5.2, galloping and torsional flutter, we discussed the self-excited vibration of the bridge structure. These self-excited vibrations could be an interaction between separated vortices and the structure oscillating with one degree of freedom. However, classical flutter, also known as coupled flutter, is a type of self-excited motion that engages several degrees of freedom of the system. It can be argued that torsional flutter is a form of classical flutter where the dominant effect comes from the aerodynamic derivative A_2^* in the equation NUMBER.

In an attempt to explain the collapse of the Tacoma Narrow Bridge, Bleich (Simiu & Miyata, 2006) was trying to explain by assuming that coupling between vertical bending and torsion, even though it showed that the torsion was dominant by far and used similar vibration modes in vertical bending and torsion. The expression that Bleich found did not reflect correctly with the aeroelastic behavior of the H-section like the Tacoma Narrow Bridge had. There was an introduction made by Selberg (Simiu & Miyata, 2006) of a simplified empirical expression of the critical coupled flutter velocity. In other words, what is the lowest velocity that will lead to coupled flutter.

$$V_F = 0.44\omega_\alpha b\sqrt{1-\left(\frac{\omega_h}{\omega_\alpha}\right)^2}\cdot\sqrt{\frac{\sqrt{\upsilon}}{\mu}} \quad (6.5.1)$$

Where $\upsilon = \frac{8r^2}{b^2}$ and$\mu = \frac{\pi\rho b^2}{2m}$, ω_α and ω_h are natural circular frequencies of the torsional and vertical motion, m is the mass per unit length, b is the bridge's half-width, ρ is the bridge density and r is the radius of gyration of the bridge cross-section. However, this equation is independent of the aerodynamics of the bridge and can then give incorrect results.

6.6 Tacoma Narrow Bridge as a two-dimensional problem

Looking at what happened to Tacoma Narrow Bridge the hours before the collapse, we can see this as a two-dimensional problem. Some stuff needs to be assumed to solve this two-dimensional problem, which indicates that this is not valid in the real world. For this to be valid, the flow must be smooth with an unknown velocity, where the velocity can vary. Something typical for bridges is that the deck is symmetrical, which leads to the center of mass and elastic center having the same center on the bridge. The flutter derivatives on the oscillation frequency n of the fluttering body can then be expressed in terms of the nondimensional equation of reduced frequency

$$K = \frac{2\pi Bn}{V} \tag{6.6.1}$$

Where the V is the mean velocity flow, and the B is the bridge's width. If we then look at x, which is the displacement in the horizontal plane of the deck. If we take this into account, we can express the motion in a two-dimensional symmetrical bridge that has an elastic restoring force and linear viscous damping in a smooth flow as

$$m\frac{d^2x}{dt^2} + b_x\frac{dx}{dt} + k_x\,x = D_x \tag{6.6.2a}$$

$$m\frac{d^2y}{dt^2} + b_y\frac{dy}{dt} + k_y\,y = L_y \tag{6.6.2b}$$

$$I\frac{d^2\alpha}{dt^2} + b_\alpha\frac{d\alpha}{dt} + k_\alpha\,\alpha = M_\alpha \tag{6.6.2c}$$

Where x, y, and α are the horizontal displacement, vertical displacement, and torsional angle shown in figure 6.7 below. These three equations are coupled nonhomogeneous linear equations where the forces on the right side of the equation are the aerodynamic lift force L_x, drag force D_y and the mass of inertia M_α.

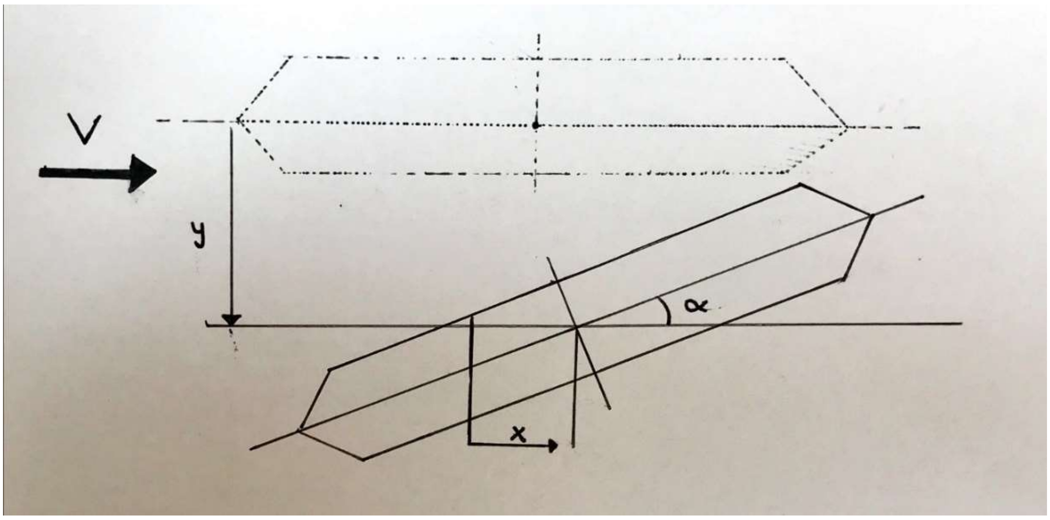

Figure 6.7 Motion of a bridge in two-dimensional with notation. Drawn by: Anders B. Waldeland

The m in the vertical and horizontal planes is the bridge section's mass. Like in forced oscillation, the equation is similar where both have the damping coefficient b, only here it depends on the direction and angle the body moves in. The latter one in the equations is the stiffness coefficient k that, like b, depends on the direction of the body. So we know that when a body is experiencing an aerodynamic phenomenon, galloping, it is like a motion of a single degree of freedom. This will lead to a small displacement in the aerodynamic force acting on the body will be linear with respect to the time change of the displacement.

On the other hand, the expression for flutter to be induced to the bridge body gets a bit more complicated. Here the motion has three degrees of freedom instead of one like in the galloping case. The concept for galloping and flutter is the same if, and only if, the wind flow comes parallel to the bridge deck. Here the wind comes with no angle difference to the bridge. Still, if the velocity comes with a different angle α, the equation gets more complex than the three nonlinear differential equations in the galloping stage in a 2-dimensional scenario. Here the equation for the lift force, drag force, and the inertia will look like this in vertical, horizontal, and torsional

$$L_y = \frac{1}{2}\rho V^2 B\left[KY_1^*(K)\frac{dy}{dt}\frac{1}{V} + KY_2^*(K) + \frac{d\alpha}{dt}\frac{B}{V} + K^2Y_3^*(K)\alpha + K^2Y_4^*(K)\frac{y}{B} + KY_5^*(K)\frac{dx}{dt}\frac{1}{V} + K^2Y_6^*(K)\frac{x}{B}\right] \quad (6.6.3a)$$

$$D_x = \frac{1}{2}\rho V^2 B\left[KX_1^*(K)\frac{dx}{dt}\frac{1}{V} + KX_2^*(K) + \frac{d\alpha}{dt}\frac{B}{U} + K^2X_3^*(K)\alpha + K^2X_4^*(K)\frac{x}{B} + KX_5^*(K)\frac{dy}{dt}\frac{1}{V} + K^2X_6^*(K)\frac{y}{B}\right] \quad (6.6.3b)$$

$$M_\alpha = \frac{1}{2}\rho V^2 B\left[K\mathrm{A}_1^*(K)\frac{dy}{dt}\frac{1}{V} + K\mathrm{A}_2^*(K) + \frac{d\alpha}{dt}\frac{B}{V} + K^2\mathrm{A}_3^*(K)\alpha + K^2\mathrm{A}_4^*(K)\frac{y}{B} + K\mathrm{A}_5^*(K)\frac{dx}{dt}\frac{1}{V} + K^2\mathrm{A}_6^*(K)\frac{x}{B}\right] \quad (6.6.3c)$$

direction for where the wind flows come from. The terms related to the second derivative of x, y and α in the nonlinear partial differential equation are neglected from the equation due to how the wind engineer applies different forces. Here, the x and y term are there to take in the change in the vibration frequency that occurs due to the aeroelastic effect on the deck. As previously mentioned in the thesis, the angle α is where the wind flow comes and attacks the deck. The first derivative in the y and a motion, $\frac{d\alpha}{dt}\frac{B}{V}and\frac{dy}{dt}\frac{1}{U}$are nondimensional quantities that imply the efficiencies of the angels of velocity. Like in the galloping case, the first derivative in the y-direction,$\frac{dy}{dt}\frac{1}{U}$ that it will represent the angle of the velocity of the relative velocity. The relative velocity has been shown previously in section 6.4. The last coefficient X_i^*, Y_i^* , and A_i^* are nondimensional coefficients that are known as Scanlan flutter derivatives.

But for small α, each equation 6.6.3 can be written in a similar form as this:

$$L = \frac{1}{2}\rho V^2 A C_L = \frac{1}{2}\rho V^2 A \frac{dC_L}{d\alpha}\alpha \quad (6.6.4)$$

7. The Theories about the collapse

7.1 The original report

The initial report of the bridge collapse was that of the Carmody board, who was the board of engineers tasked with explaining why the Tacoma narrows bridge collapsed. The report was published on 28 March 1941 (Ammann, et al., 1941). focused primarily on the bridge's structural design and did not reach a conclusive reason why the bridge tore itself apart. However, they did concur that the dynamic forces of the wind somewhat caused the collapse. The investigations showed no structural damage that could have caused the oscillations beforehand. However, they concluded that the aerodynamic instability of its design was why the bridge started to oscillate. There was no incompetence shown in designing the bridge or ill intent. It was both economic and stable against static forces. However, the slim and long design caused the wind to influence the bridge's destruction. Further testing with wind tunnels would have to be done to explain the collapse further.

One of the first critics of the explanations for the collapse was David Barnard Steinman. He argued in 1941 (that the original report by the Carmody board contained several questionable statements and conclusions. He criticized the notion that the bridge’s slim design was the cause of the collapse and set out to prove that mathematically the critical velocity and, by extension, the rigidity of the bridge was not proportional to the deck width (Scott, 2001). Steinman would continue to criticize the report by the Carmody board as he insinuated that the mid-span stays used at Tacoma Narrows Bridge had been copied by him without a word of credit or acknowledgment. This emotionalism was characteristic of Steinman in a conservative profession (Scott, 2001). Steinman would later design the Mackinac bridge building on Tacoma's failures.

An article published in the New York Times attempted to explain the collapse to be that of Resonance some days after the initial report was published. The article read like this: "*like all suspension bridges, that at Tacoma both heaved and swayed with a high wind. It takes only a* *tap to start a pendulum swinging. Time successive taps correctly and soon the pendulum swings with its maximum amplitude. So with the bridge. What physicists call Resonance was established, with the result that the swaying and heaving exceeded the limits of safety"* (Arioli & Gazzola, 2013) (New York Times, 1940). This is most likely where the argument for forced oscillations and Resonance started and what many physics books and university books later adapted as one of the leading arguments for the Tacoma Narrows Bridge collapse. Books like those that will be mentioned in section 8. However, the article was criticized for not explaining how the wind, random in nature, could produce such oscillations.

History may also be a factor in why this theory was favored by many. Earlier examples of bridges can be attributed to Resonance causing a collapse. For example, in 1831, the Broughton suspension bridge collapsed due to mechanical Resonance induced by soldiers marching on the bridge in step (Arioli & Gazzola, 2013). This is because the probability of the step being exactly in frequency with the bridge's natural frequency is zero. However, if the frequency coincides only by some margin, it can cause the structure to oscillate and destroy itself. A similar situation happened in 1850 when a battalion of soldiers marched across the Angers suspension bridge during a thunderstorm. But even though the soldiers did not march in step like in the previous example, the combination of the wind forces and the marching soldiers caused the bridge to collapse.

But how does forced oscillations and resonance work on the Tacoma Narrows bridge on the day of the collapse? To better understand this, we need to look further at the physics and the maths around Resonance; as shown earlier in section 5, we showed that the formula for forced oscillations on a stationary object could be derived from newton's second law as:

$$F_0 sin\omega t - b\frac{dx}{dt} - kx = \frac{d^2x}{dt^2} \qquad (7.1.1)$$

From this equation, we could transform x to that of $x = A\cos(\omega t + \varphi)$ where we could find the amplitude A:

$$A = \frac{F_0/m}{\sqrt{\left(\omega^2 - \omega_0^2\right)^2 + \left(\frac{b\omega}{m}\right)^2}} \tag{7.1.2}$$

The theory is that the frequency of the wind ω matched or was close to that of the natural frequency of the bridge ω_0, increasing the amplitudes of the oscillating bridge as mechanical Resonance. These oscillations combined with periodic eddies (sometimes called Karman vortices, a theory proposed by Von-Karman in the original report) have been mentioned as a possible inducement to the large oscillations that occurred on the day of the collapse.

7.2 The Billah and Scanlan article

Even though the notion of Resonance was criticized, physics books were still published where the Tacoma Narrow Bridge collapse was used as an example of forced oscillations and Resonance. This was something K. Jusuf Billah and Robert H. Scanlan could not accept. So they set out to write an article and disprove the notion of forced oscillations and Resonance. Scanlan had already criticized the notion in another document published in 1982, writing*: Others have added to the confusion. A recent mathematics text, for example, seeking an application for a developed theory of parametric Resonance, attempts to explain the Tacoma Narrows failure through this phenomenon.* (Scanlan, R.H., "Developments in low-speed aeroelasticity in the civil engineering field,"). And in June of 1990, Billah and Scanlan published an article titled "Resonance, Tacoma Narrows Bridge failure, and undergraduate physics textbooks" to disprove the notion of using the collapse as an example of Resonance in physics books. The article set out to prove that Resonance and that self-excitation were both physically and mathematically different and that one should be aware of the difference when discussing the collapse.

One of the first topics of the article is that of the vortex-induced vibration, where they discuss the topic of Strouhal vortices and the Strouhal frequency (f_s). When fixed in a fluid stream, bluff (non-streamlined) bodies generate detached or separated flow over substantial parts of their surfaces. At some critical Reynolds-number, two thin layers form to the lee of the body, where these layers act nonlinearly with each other and produce a regular periodic array of vortices. These are the Strouhal vortices, and in this case, the vortex arrays arrange themselves in two rows with opposite directions of circulation (Billah & Scanlan, 1990).

The frequency of these shedding vortices over a fixed (restrained body) is often termed the Strouhal frequency (f_s). And it follows the relation.

$$f_s D/U = S \tag{7.2.1}$$

Where U is the crossflow velocity, D is the frontal dimension, and S is the Strouhal number appropriate to the body in question. The values of D and S for the original Tacoma narrows bridge are 8 ft (2.43 m) and 0.11 (Billah & Scanlan, 1990). A frequent assumption that was made in the textbooks that Billah and Scanlan were criticizing was that the Strouhal frequency (f_s) matched the natural mechanical frequency of the bridge. Billah and Scanlan then set out to test if this was true regarding this assumption. They were working with the information given at the scene of the collapse by Professor Farquharson. They concluded the final destructive oscillation to be that of 0.2 Hz (f) while the frequency of the wind of 42 mph (68km/h) at natural vortex shedding according to the Strouhal relation to be at 1 Hz. A wholly out of sync with the actual catastrophic oscillation then going on (Billah & Scanlan, 1990).

We set out to test this theory of the frequencies ourselves. First, we acquired some of the collapse footage through YouTube (Lenz, 2008). And used the program Tracker to estimate the amplitudes of the bridge oscillations. Then by tracking the movements of one of the lampposts at the bridge in the video, we were given a trigonometric function and a set of data about the positions of the x and y planes at the given times of the video.

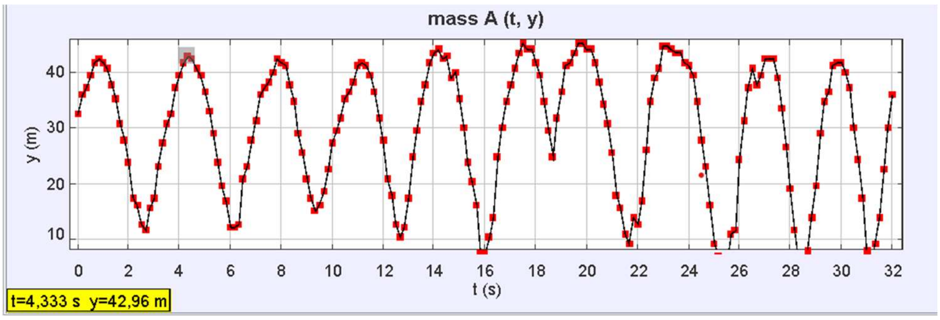

Figure 7.1 Curve showing the movement of the bridge. Taken from the Tracker file.

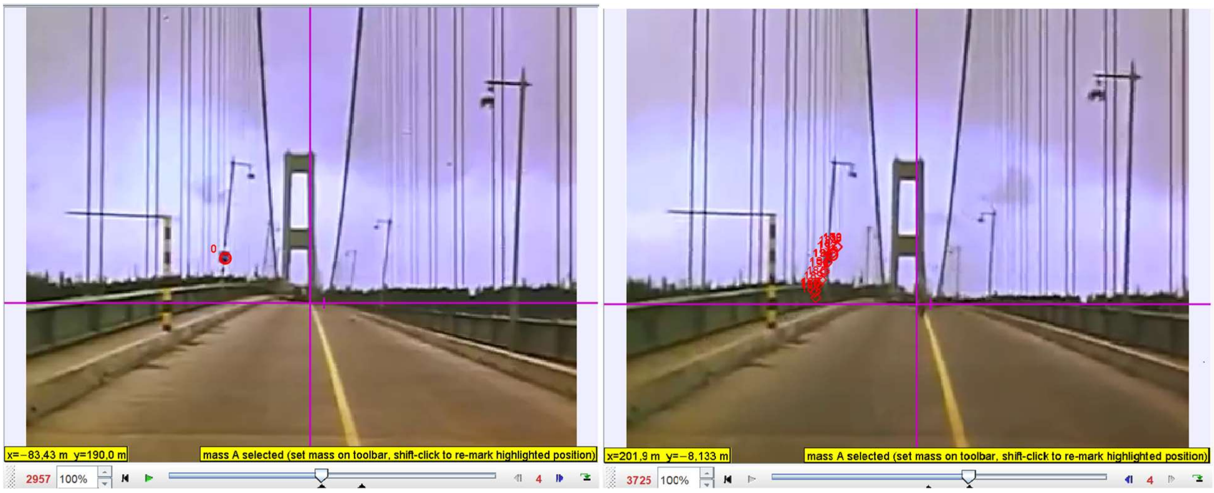

Figure 7.2 A Screenshot from Tracker.

t(s)	y(m)
0	32.511
1.5	35.414
3	17.417
4.5	42.38
6	12.192
7.5	38.316
9	20.9
10.5	35.414
12	27.866

13.5	34.833
15	35.414
16.5	24.964
18	41.8
19.5	43.541
21	25.544
22.5	34.833
24	41.219
25.5	4.644
27	42.38
28.5	5.806
30	41.8
31.5	13.933

Figure 7.3 Table from the Tracker data showing the y plots every 1.5 seconds.

By noting down the time taken per cycle and estimating the mean periods of T

$$\overline{T} = \frac{\sum_{i=1}^{n} T_i}{n} \quad (7.2.2)$$

We could estimate the frequency of the oscillations of the bridge with the frequency equation:

$$f = \frac{1}{T} \quad (7.2.3)$$

Period (T):	3.500	3.500	3.337	3.000	3.333	2.167	3.333	4.000	2.83

Figure 7.4 Periods T per max amplitude

We could also calculate the standard deviation in our data with the formulas for variance and standard deviation

$$\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{n-1} \quad (7.2.4)$$

$$\mu = \sqrt{\sigma^2} \quad (7.2.5)$$

Where σ^2 are variance and μ is standard deviation and then estimate μ for period T from figure 8.4 and find μ to be 0.514 seconds (s). Plotting this in our formula for frequency, we could then estimate the frequency at 0.313 ± 0.101 Hertz (Hz). However, these amplitudes were taken from the video sometime between 10 AM and the time of the collapse at 11:15 AM, not the final destructive oscillation observed by Farquharson. We also tracked the movement of the lamp manually at intervals of 1.5 seconds which brought uncertainties in our data than if a more advanced program did the tracking for us. The video was also old and of low quality, which could tamper with our data's precision. So even though we did not get the same frequency as the given destructive oscillation, the frequency estimated from the video would still be wholly out of sync with the frequency given by the natural vortex shedding of the wind (Billah & Scanlan, 1990).

These observations led Billah and Scanlan to conclude that the natural vortex shedding of the wind could not cause the collapse. They then discussed the potential cause by looking at experiments duplicating the final destructive oscillation observed on the original Tacoma Narrows Bridge. Observations from 1/50 scaled models from both the 1940s and 1950s showed that several modes of the Tacoma narrows bridge responded with self-limiting amplitudes, except for one node. This particular low torsional node was identified as "1-NT 2nd". Which when divided by the model frequency scaling factor of $\sqrt{50}$ defined a prototype frequency of 0.2 Hz. Farquharson observed precisely the frequency of the destructive mode at the collapse. After Scanlan and Tomoko repeated this research in 1970 and carried the work further. They concluded that the catastrophic node was a *single-degree-of-freedom torsional flutter* due to complex, separated flow. So, instead of describing the force as a single degree freedom oscillator as a purely external function of time. It was described as an aerodynamic self-excitation effect that imparted a net negative damping effect characteristic to the system. The torsional motion can then be described as:

$$I\left[\frac{d^2\alpha}{dt^2} + 2\zeta_\alpha\omega_\alpha\frac{d\alpha}{dt} + \omega_\alpha^2\alpha\right] = F\left(\alpha, \frac{d\alpha}{dt}\right) \tag{7.2.6}$$

Where I is the associated inertia, ζ is the damping ratio (simplified logarithmic decrement, $2\pi\zeta_\alpha = log\ dec$), ω_α is the natural frequency of the bridge, and α is the angle of twist. The aerodynamic force of $F\left(\alpha, \frac{d\alpha}{dt}\right)$ was postulated in the linearly self-excite form of:

$$F\left(\alpha, \frac{d\alpha}{dt}\right) = A_2\alpha + A_3\alpha \tag{7.2.7}$$

Which non dimensionally, became:

$$F\left(\alpha, \frac{d\alpha}{dt}\right) = \frac{1}{2}\rho U^2(2B^2)\left[KA_2^*\left(B\frac{d\alpha}{dt}/U\right) + K^2A_3^*\alpha\right] \tag{7.2.8}$$

Where ρ is the air density, U is wind velocity, B is deck width, ω is the circular frequency of oscillation, $K = B\omega/U$ and A_2^* , A_3^* are the dimensionless aerodynamic ("flutter") coefficients, functions of K . It is important to note that no external independent function of time is present in this formulation. This means that the equation is a homogeneous differential equation. Through Billah and Scanlan's experiments, they could determine the form coefficient of A_2^*: revealing the form plotted in figure 7.5.

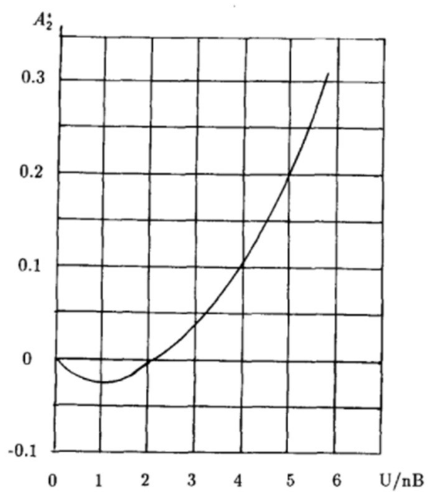

Figure 7.5 Plot of A_2^* (Billah & Scanlan, 1990)

wherein the evolution of this damping coefficient with reduced velocity exhibited a dramatic reversal in sign. When the body of the bluff body changes the angle of attack in a fluid stream, it sheds new vorticity in its wake. However, this shedding will not have anything to do with the naturally developed Karman vortex trail. Instead, bluff bodies in oscillatory motion shed wakes containing both the oscillation and the Strouhal frequencies, and under high amplitudes, it is the oscillations that predominate. The final destructive oscillation of the Tacoma Narrows bridge produced a flutter wake, not a Karman vortex street. In other words, the motion caused the vortices and led to the flutter of the bridge. This is what fundamentally brought the bridge down. However, it is important to note that this occurred 45 minutes before the collapse. A single degree driven unstable oscillation with negative damping representing an inflow of energy from the wind causing a synchrony with the motion induced pressure with the motion itself (Billah & Scanlan, 1990).

Billah and Scanlan proposed that one could argue this to be a type of resonating phenomenon with the motion being self-induced with the wind supplying the power and the motion supplying the power-tapping mechanism. Although arguing this, one should emphasize it as externally forced linear Resonance since it differed strongly from normal Resonance. The mathematical difference is quite clear if one looks at equation 7.2.8 derived by Billah and Scanlan and the differential equation 4.3 for forced oscillations and Resonance. And the paper ends by remarking on physics books' lack of penetrating investigations of the Tacoma Narrows Bridge phenomena. That many of the examples of the collapse were loose descriptions, explanations, and speculations. While there was much physical and available literature collaborating on the mechanics and conditions around the bridge collapse. It is also important to note that there are more articles about the Tacoma Narrows Bridge collapse collaborating the flutter argument and some articles that try to explain the collapse with other mathematical phenomena. However, due to time constraints, we were unable to properly research these articles and discuss them here in the thesis.

7.3. Visiting an aerodynamic specialist at UiS (University of Stavanger)

We also set out to get a modern view of the collapse and its potential cause for the torsional motion, which led us to contact and visit a professor in aerodynamics and engineering at UiS (university of Stavanger) and asked her expert opinion on the matter. The Professor in question was Jasna Bogunovic Jacobsen, a wind engineer who studies structural loads and aeroelastic flutter caused by wind on bridges (Wikipedia Jasna Bogunovic Jakobsen). Her opinion on the collapse aligned with the Billah and Scanlan article. And when asked if there was a definitive modern answer to the collapse, she replied: "*In my opinion, there is no doubt that the prime cause of the bridge failure is torsional flutter, and I would say this is the well-established knowledge in the professional community. One of the reasons this is frequently misunderstood is that this particular bridge is prone to several forms of wind-induced vibrations (vortex-induced vibrations (VIV), possible galloping too) in a rather narrow interval of wind speeds, so they can appear almost together, the VIV was observed on several occasions on the bridge prior to the collapse… For modern bridges such as Hardanger bridge, these excitations are well separated, with a possible VIV (for which countermeasures are adopted) in the range around/less than 10 m/s and flutter at 78 m/s. It is the latter one (flutter) that can destroy the bridge*." (Jasna Bogunovic Jakobsen, 2022). This shows that even though several articles have been written throughout the years about the cause of the collapse. There is an established knowledge in the professional community that torsional flutter was the cause, like the ones mentioned in the Billah and Scanlan article. Some of these articles would be (Arioli & Gazzola, 2015), (Green & Unruh, 2004) and (Larsen, 2000).

8. Misrepresenting the Tacoma Narrow Bridge

8.1 Different types of academic books

In textbooks about mechanics, there has been a belief that the reason behind the Tacoma Narrow Bridge collapse is forced oscillation and Resonance. Even though several physicists and engineers have disproved this, we can still see books that say this is the reason for the collapse.

In this section, we will represent some of these textbooks and show how they represent the Tacoma Narrows Bridge as a forced oscillation and resonance problem.

The mechanic's book for the UIS course FYS100 by Serway and Jewett illustrates that the bridge collapsed due to the wind. They say that the wind made the bridge oscillate at a frequency near the bridge's natural frequency. This Resonance led to the collapse of the bridge. They say that physicists and mathematicians are challenging some aspects of the interpretation. So at least they say that this is maybe not the right answer. But they still use the Tacoma Narrow Bridge collapse as an example of Resonance even though it is wrong. (Serway & Jewett, 2017) This could be because, in many physics books, this has been used as a classic example of Resonance. People have been learning it wrong, or they trust books that have been used before and say the same thing without thinking about the physics behind it.

Another physic book by Tipler and Mosca also uses the Tacoma Narrow Bridge collapse as an example of forced oscillation and Resonance. In this book, they do not say directly that the reason for the collapse is Resonance, but they have a picture of it just below where they write about forced oscillation and Resonance. So, it is implied that there is a context. (Tipler & Mosca, 2004)

Young and Freedmann also use the Tacoma Narrow Bridge as an example in their section about forced oscillation and Resonance. They use a more diplomatic approach even though they say that the collapse is often described as Resonance driven by the wind, they say that this is debated, and some people mean there are other reasons for the collapse. (Young & Freedman, 2004)

Also, in Norwegian textbooks, Tacoma Narrow Bridge is used as an example. One example of this is the physics book for "Forkurs," a preparatory course for engineers. In this book, they take the Resonance to explain the Tacoma Narrow Bridge. They do not give any alternative explanation; they just put Resonance as the reason for the bridge's failure. They also use the bridge as the basis, for example, for calculating wavelength. So, the Resonance is presented as a fact and the reason for the Tacoma Narrow Bridge. This book is on a bit lower level than the other books. (Storelvmo & Storelvmo, 2005) "Forkurs" is more like high school level physics than university level because it is a preparatory course before starting your university education. So maybe the authors have simplified the explanation to make it easier to understand the theme.

Another Norwegian textbook is the mechanics book written by Lien and Løvhøiden. This is a book that is made for university students. On page 284, When they talk about forced oscillations and Resonance, they use the Tacoma Narrow Bridge as an example. Here Lien and Løvhøiden say that the bridge collapsed due to Resonance caused by intense winds. (Lien & Løvhøiden, 2001) They do not give any alternative explanation or mention that people are discussing the reason for the collapse. So, you can assume that they take this explanation as fact.

To summarize, many physics textbooks still use the Tacoma Narrow Bridge as an example of Resonance. Some books are more diplomatic and make room for alternative explanations, while others do not leave any room for discussion around the reason for the collapse. The reason for using the Tacoma Narrow Bridge in textbooks can be because of tradition. Textbooks use this as an example of Resonance, so the authors keep on using it because all other books are doing it. You just do the same as people have done before. This is not how it should be. The authors of the books should always check if their sources are correct and make changes if they have to.

8.2 Vitenfabrikk in Sandnes

In Sandnes, a museum is dedicated to explaining scientific phenomena and theories that occur in nature. And one of their exhibition was about Resonance, where the goal was to break a wine glass using the human voice. But as an example of Resonance, as shown in figure 8.1, Vitenfabrikken in Sandnes says that the collapse of the Tacoma Narrow Bridge is similar to the exhibition where one uses the voice to make a frequency to break the glass. When we met somebody that works there to discuss why they had the Tacoma Narrow Bridge as an example of Resonance. So, the person we discussed about the collapse of Tacoma Narrow Bridge was convinced that Resonance made the bridge oscillate and collapse. This argument comes from that the person we were discussing with had been to Tacoma and was told there that the reason for the collapse was resonance or forced oscillation. So the reason for why they have it is to explain that what's going on in the exhibition can be seen in a bigger scenario as Tacoma Narrow Bridge, as we have previously seen throughout the thesis, especially in section 7, where Jasna Bogunovic Jakobsen, a specialist in aerodynamic forces on bridges, concluded that the collapse was due to flutter and not Resonance. The impact of this will be discussed further in the conclusions.

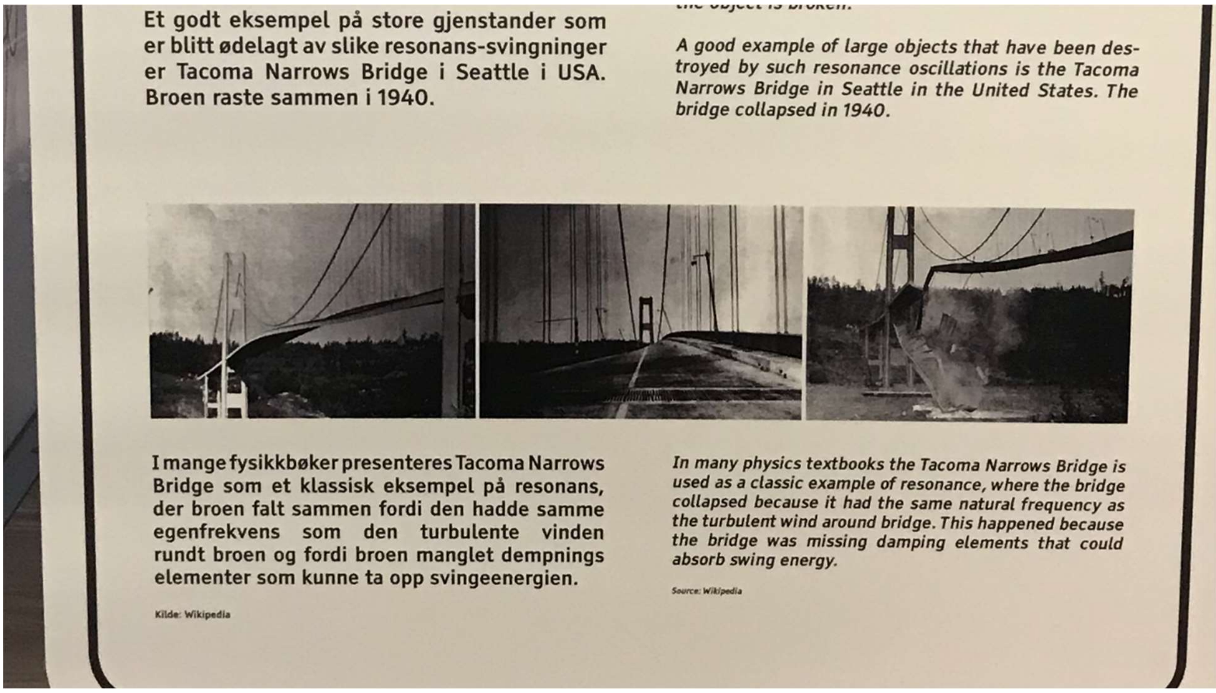

Figure 8.1 Poster from the exhibition about Resonance from the Vitenfabrikk.

Figure 8.2 Picture of the exhibition about Resonance at the Vitenfabrikk. You can crack a glass with the frequency of your voice.

9. Conclusion

Throughout this thesis we have seen that a real-life phenomenon like the collapse of the Tacoma Narrow Bridge in 1940 can be wrongly represented as a forced oscillation and resonance problem. In physics textbooks like the one used for the mechanic's course FYS100 at the University of Stavanger and the exhibition of Resonance in the Museum in Sandnes, we continue to see a trend of misrepresentation. Instead of researching many articles and studies about the Tacoma Narrow Bridge, the only recognition of uncertainty is the comments about the problem still being discussed today. However, by looking at these articles ourselves and discussing the topic with an expert in the field, it is clear that the collapse of the Tacoma Narrows Bridge does not correspond to the topic of forced oscillations and Resonance.

As lektor students studying to understand and improve how we inform and educate future students, we believe that the Tacoma Narrows Bridge being expressed as a forced oscillation and resonance problem can lead to students misconceiving physics problems out in the real world. Furthermore, oversimplifying such an aerodynamic and advanced mathematical problem to a degree of only forced oscillations and Resonance can also lead to oversimplifying other advanced and complicated problems around physics. However, we realize that the Tacoma Narrows Bridge representation in physics books is only a tiny fraction of what these books contain. Still, at the same time, it shows that it is important to question what one reads even though the book is academic.

Most people visiting the Vitenfabrikk probably never think of this peculiar bridge collapse outside the museum. Therefore, it is simpler to say that the Tacoma Narrow Bridge collapse was due to Resonance so that the person that's work which have a basic knowledge in science form school so they can explain the theory for Resonance. However, we believe that textbooks and museums are utilized to give people and students a better understanding of the world around us. In this case, they should also receive the correct interpretation around the Tacoma Narrows Bridge Collapse.

Thus it is still misrepresenting the fact of the collapse, which is not very educational. And as a museum of science, it should always represent what is true and not just What looks similar or fits the description they want. Comparing a torsional motion on a bridge at high wind velocities with a resonating voice should at least accompany some scientific basis. Instead of just comparing the two because of their similar scientific name. This also brings the problem of calling the Tacoma Narrows Bridge collapse a forced oscillation and resonance problem

So, the important part to take is that there is still a misrepresentation of why the Tacoma Narrows Bridge collapsed and that there is still a debate going on. However, there is an understanding in the professional community that the collapse was due to flutter like the one mentioned in the Billah and Scanlan report back in 1990 (Billah & Scanlan, 1990). And like the specialist from UiS, most of her peers believe that the matter is concluded with the Billah and Scanlan article. Therefore we conclude that the statement about the Tacoma Narrows Bridge given by Serway in the FYS100 book is incorrect.

"In 1940, turbulent winds set up torsional vibrations in the Tacoma Narrows Bridge, causing it to oscillate at a frequency near one of the natural frequencies of the bridge structure. Once established, this resonance condition led to the bridge's collapse. (Mathematicians and

physicists are currently challenging some aspects of this interpretation)". (Serway & Jewett, 2017)

And due to this, we believe that future representations of the Tacoma Narrows Bridge should be excluded from topics concerning forced oscillations and Resonance.

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