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Gain-scheduled observer-based consensus for linear parameter varying multi-agent systems[☆]

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ABSTRACT

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1. Introduction

In recent years, consensus problems in multi-agent systems (MASs) have been researched for potential application in flocking, formation, and synchronization (Oh, Park, & Ahn, 2015). Earlier works considered agents with single or double integrator dynamics (Ferrari-Trecate, Galbusera, Marciandi, & Scattolini, 2009). However, since low-order models could not describe accurately the agents' dynamics, the case of higher-order dynamics (Rezaee & Abdollahi, 2015) and nonlinear MASs (Wang, Xu, & Ji, 2016) was later investigated. In particular, adaptive control has shown to be a successful tool for dealing with nonlinear MASs, as it has been applied for dealing with transmission (Shen, Shi, Zhu, & Zhang, 2019), saturations and dead-zone (Shen, Shi, Shi, & Zhang, 2016) nonlinearities. More recently, adaptive control protocols have been combined with sliding mode observers (Shen, Shi, & Shi, 2015). However, in spite of its doubtless appeal and effectiveness, adaptive control does not distinguish between predictable and unpredictable variations in the dynamics, as it is generally assumed that the adaptation mechanism will take care of them regardless of their nature.

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the real-time operation. The main advantage of relying on the LPV framework is that it enables controller synthesis for nonlinear systems by extending LTI techniques with the use of appropriate Lyapunov functions that allow converting the design problem into a set of linear matrix inequalities (LMIs) (Rotondo, Sánchez, Nejjari, & Puig, 2019). The LPV paradigm provides an alternative

This paper investigates the gain-scheduled observer-based consensus of linear parameter varying (LPV)

multi-agent systems (MASs). The main contribution of the paper is that, differently from LPV observer-

based consensus design methods previously reported in the literature, the controller and observer

gains are allowed to be functions of some time-varying parameter vector, which can be measured in

real-time. It is shown that, under the assumption of synchronized varying parameter trajectories, the design conditions can be described as a feasibility problem involving a finite number of linear matrix

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inequalities (LMIs). The obtained results are illustrated by means of a numerical example.

simpler framework for assessing/enforcing consensus in nonlinear MASs, which is why some works related to LPV MASs have been reported, such as Gonzalez, Hoffmann, and Werner (2015), where formation control was carried out within the framework of linear fractional transformations (LFT), and Chadli, Davoodi, and cent works have started translating the mathematical conditions for the design of consensus protocols for LPV MASs into LMIs, see e.g., Chen, Zhang, Cao, and Chu (2017), ur Rehman, Rehan, Iqbal, and Ahn (2018). However, the above papers do not provide a discussion about the conditions under which an LPV MAS reaches consensus, and consider robust control laws which do not exploit possible online information about the value of the time-varying parameters.

Motivated by these limitations, this paper aims at investigating the gain-scheduled consensus of LPV MASs. Similar to Chen et al. (2017), we assume that the state variables are not fully

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Meskin (2016), where the problem of distributed state estimation and fault detection/isolation in MASs was considered. Some re-

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Linear parameter varying (LPV) systems can be considered as an attractive alternative for controller design to handle predictable variations in time (Mura & Lovera, 2015). LPV systems are

linear models whose behavior depends on some time-varying pa-

rameters, which are unknown a priori but they are known during









Brief paper

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available, so that the estimated states provided by local state observers must be used instead to feed the control law. However, we assume that the state-space matrices which describe the agents' dynamics are functions of time-varying parameters which can be measured in real-time, so that a gain-scheduled consensus control can be used (instead of the robust consensus control reported in the literature). For this case, we describe a necessary condition for achieving consensus, and we show that it holds in the case of synchronized trajectories of the time-varying parameters. Under such assumption, we provide sufficient LMI-based conditions for designing gain-scheduled observer and controller gains that ensure consensus.

Note that this work assumes that the time-varying parameters that schedule the agents' state-space matrices are exogenous signals, i.e., they are not functions of state/input variables hiding some underlying nonlinearities. This assumption is satisfied, for instance, by networks of electrical circuits (Ding & Li, 2016) and in spacecraft flying formation problems involving a virtual Ref. Li, Duan, Chen, and Huang (2009). In the first case, the varying parameter would be the surrounding temperature (then, synchronization would correspond to the circuits operating at the same temperature), whereas in the second case it would be the instantaneous angular rate of the virtual reference (in this case, synchronization would occur naturally as the virtual reference is shared by all the agents). Another scenario where the above assumption would be satisfied is when the varying parameter is introduced artificially by the designed, e.g., for enabling online modification of the closed-loop performance (Rotondo, Nejjari, & Puig. 2015). Note that the case of quasi-LPV agent can still be considered using the results in this manuscript with slight modifications following the theoretical remarks in Bruzelius, Pettersson, and Breitholtz (2003), although special care should be put in the case where the time-varying parameters were functions of non-measured state variables. However, this latter case, in which the observer design procedure should be modified to account for inexact scheduling parameters (Sato, 2012), goes beyond the scope of this paper and will be considered in future work.

It is worth remarking the existence of similarities between the results in this paper and LMI-based results concerning the consensus of switched MASs, see e.g. Zhang, Xu, Karimi, Wang, and Yu (2017). As discussed by Bertolin, Oliveira, de Oliveira, and Peres (2018), when it comes to techniques for investigating the stability of LPV and switched systems, most approaches can handle both cases, although there are a few exceptions. The main difference between an LPV and a switched system is that in the former class of systems the varying parameters are allowed to undergo variations within a continuous set, whereas in the latter such a set contains a discrete number of elements. As a consequence, to ensure that the stability holds for any variation of the varying parameters, the controller must be chosen as LPV or switched, respectively, and in the former case constrained to be polytopic to obtain a finite number of design conditions. One might argue that given the similarities between the socalled vertex matrices of polytopic LPV systems and the mode matrices of switched systems, one could apply controller design conditions for switched systems to LPV systems. However, this would be guaranteed to work only in the particular case where the input matrix is constant, and would be tempting fate in the case where such a matrix is parameter-varying, due to the appearance of a double polytopic sum in the design conditions, that must be handled appropriately as discussed in Rotondo et al. (2019). Motivated by such discussion, this paper discusses the observer-based consensus protocol for LPV MASs in the most general case, in which the agents' input and output matrices can undergo variations, and therefore the design conditions are obtained through an application of Polya's theorem on definite quadratic forms (Sala & Arino, 2007).

Notation. For a matrix M, M > 0 ($M \ge 0, M < 0, M \le 0$) denotes a symmetric positive definite (positive semidefinite, negative definite, negative semidefinite) matrix, and M^T, M^{-1}, M^{\dagger} denote its transpose, its inverse and its generalized inverse, respectively. The symbol \otimes denotes the Kronecker product. For simplicity, the symbol \star within a symmetric matrix represents the symmetric entries. The shorthand notation He {M} = $M + M^T$ represents the Hermitian part of a square matrix M. Given $s \in \mathbb{N}$, the symbols \mathbb{P}_s and \mathbb{P}_s^+ denote the following sets:

$$\mathbb{P}_{s} = \left\{ \vec{p} = \left[\vec{p}_{1}, \dots, \vec{p}_{s} \right]^{T} \in \mathbb{N}^{s} | 1 \le \vec{p}_{k} \le s \; \forall k = 1, \dots, s \right\}$$
(1)

$$\mathbb{P}_{s}^{+} = \left\{ \vec{p} \in \mathbb{P}_{s} | \vec{p}_{k} \le \vec{p}_{k+1}, k = 1, \dots, s-1 \right\}$$
(2)

whereas $\mathcal{P}(\vec{p}) \subset \mathbb{P}_s$ denotes the set of permutations, with possible repeated elements, of the multi-index \vec{p} .

2. LPV observer-based consensus

Let us consider a MAS with *N* LPV agents in an undirected and connected graph \mathcal{G} described by a Laplacian matrix $\mathcal{L} \in \mathbb{R}^{N \times N}$, so that the dynamics of the *i*th individual node is given by:

$$\dot{x}_i(t) = A\left(\theta_i(t)\right) x_i(t) + B\left(\theta_i(t)\right) u_i(t) \tag{3}$$

$$y_i(t) = C\left(\theta_i(t)\right) x_i(t) \tag{4}$$

where i = 1, ..., N, $x_i(t) \in \mathbb{R}^{n_x}$ represents the state vector, $u_i(t) \in \mathbb{R}^{n_u}$ denotes the control inputs and $y_i(t) \in \mathbb{R}^{n_y}$ are the sensor outputs of the *i*th agent, respectively. The matrix functions $A(\theta_i(t)) \in \mathbb{R}^{n_x \times n_x}$, $B(\theta_i(t)) \in \mathbb{R}^{n_x \times n_u}$ and $C(\theta_i(t)) \in \mathbb{R}^{n_y \times n_x}$ are scheduled by the vector of time-varying parameters of the *i*th agent $\theta_i(t) \in \mathbb{R}^{n_\theta}$, which is available in real-time to its agent, and it is assumed to vary in a closed set Θ .

For the purpose of further deliberations, it is assumed that the parameter-varying matrices $A(\theta_i(t))$, $B(\theta_i(t))$, $C(\theta_i(t))$ have been expressed as a convex combination of *S* vertex matrices through parameter-varying coefficients, which can be done using available methods in the literature, such as the bounding box (Sun & Postlethwaite, 1998):

$$\begin{pmatrix} A(\theta_{i}(t)) \\ B(\theta_{i}(t)) \\ C(\theta_{i}(t)) \end{pmatrix} = \sum_{h=1}^{S} \alpha_{h}(\theta_{i}(t)) \begin{pmatrix} A_{h} \\ B_{h} \\ C_{h} \end{pmatrix}$$
(5)

$$\sum_{h=1}^{S} \alpha_h \left(\theta_i(t) \right) = 1, \quad \alpha_h \left(\theta_i(t) \right) \ge 0 \quad \forall \theta_i(t) \in \Theta$$
(6)

For the multi-agent system (3)-(4), let us consider the following consensus protocol of the MAS based on using estimated states:

$$u_i(t) = K\left(\theta_i(t)\right) \sum_{j \in \mathcal{N}_i} a_{ij}\left(\hat{x}_i(t) - \hat{x}_j(t)\right), \quad i = 1, \dots, N$$
(7)

where the matrix function $K(\theta_i(t)) \in \mathbb{R}^{n_u \times n_x}$ is the LPV controller gain to be designed, and \mathcal{N}_i denotes the set of agents adjacent to *i*. The estimated states $\hat{x}_i(t) \in \mathbb{R}^{n_x}$ are provided by local state observers of the form:

$$\hat{x}_{i}(t) = A\left(\theta_{i}(t)\right)\hat{x}_{i}(t) + B\left(\theta_{i}(t)\right)u_{i}(t)$$

$$+L\left(\theta_{i}(t)\right)\left[C\left(\theta_{i}(t)\right)\hat{x}_{i}(t) - y_{i}(t)\right]$$
(8)

where $L(\theta_i(t)) \in \mathbb{R}^{n_x \times n_y}$ is the LPV state observer gain matrix to be designed. Let us choose the gains to be designed $K(\theta_i(t))$, $L(\theta_i(t))$ to be polytopic as well, with the same coefficients as in (5), which means that:

$$\begin{pmatrix} K(\theta_i(t))\\ L(\theta_i(t)) \end{pmatrix} = \sum_{h=1}^{S} \alpha_h(\theta_i(t)) \begin{pmatrix} K_h\\ L_h \end{pmatrix}, \quad i = 1, \dots, N$$
(9)

. .

After connecting the agent dynamics (3)–(4), the consensus protocol (7) and the local state observers (8), by defining the estimation errors $e_i(t) = \hat{x}_i(t) - x_i(t)$, one gets the following equations:

$$\dot{e}_i(t) = [A(\theta_i(t)) + L(\theta_i(t))C(\theta_i(t))]e_i(t)$$
(10)

$$\dot{x}_{i}(t) = A(\theta_{i}(t)) x_{i}(t) + B(\theta_{i}(t)) K(\theta_{i}(t)) \cdots$$
(11)

$$\cdot \sum_{j \in \mathcal{N}_i} a_{ij} \left(x_i(t) - x_j(t) + e_i(t) - e_j(t) \right)$$

Define the multi-agent vector of time-varying parameters $\theta(t) = \left[\theta_1(t)^T, \theta_2(t)^T, \dots, \theta_N(t)^T\right]^T$ and:

$$\alpha_{ij}(\theta(t)) = \operatorname{diag}\left(\alpha_{i\star j}(\theta_1(t)), \dots, \alpha_{i\star j}(\theta_N(t))\right)$$
(12)

with $\alpha_{i\star j}(\theta_l(t)) \triangleq \alpha_i(\theta_l(t)) \alpha_j(\theta_l(t)), l = 1, ..., N$. Then, by defining $e(t) = [e_1(t)^T, e_2(t)^T, ..., e_N(t)^T]^T$, the following is obtained from (10), taking into account the polytopic assumption in (5) and (9):

$$\dot{e}(t) = \sum_{h=1}^{S} \sum_{l=1}^{S} \left[\alpha_{hl} \left(\theta(t) \right) \otimes \left(A_h + L_h C_l \right) \right] e(t)$$
(13)

Let us define a synchronizing state error as $\delta_i(t) = x_i(t) - (1/N) \sum_{j=1}^N x_j(t) = x_i(t) - \overline{x}(t)$, where $\overline{x}(t) = (1/N) \sum_{j=1}^N x_j(t)$ denotes the instantaneous average state vector for the agents. Then, one can verify that the vectors $\delta(t) = [\delta_1(t)^T, \delta_2(t)^T, \dots, \delta_N(t)^T]^T$ and $x(t) = [x_1(t)^T, x_2(t)^T, \dots, x_N(t)^T]^T$ are related by:

$$\delta(t) = \begin{bmatrix} \frac{N-1}{N}I_{n_{x}} & -\frac{1}{N}I_{n_{x}} & \cdots & -\frac{1}{N}I_{n_{x}} \\ -\frac{1}{N}I_{n_{x}} & \frac{N-1}{N}I_{n_{x}} & \cdots & -\frac{1}{N}I_{n_{x}} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{N}I_{n_{x}} & -\frac{1}{N}I_{n_{x}} & \cdots & \frac{N-1}{N}I_{n_{x}} \end{bmatrix} x(t) = \Upsilon x(t)$$
(14)

which means that the following can be written:

$$\dot{\delta}(t) = \sum_{h=1}^{S} \sum_{l=1}^{S} \Upsilon \left\{ \left[\alpha_{hl} \left(\theta(t) \right) \mathcal{L} \otimes B_{l} K_{h} \right] e(t) \cdots + \alpha_{hl} \left(\theta(t) \right) \left[I_{N} \otimes A_{h} + \mathcal{L} \otimes B_{l} K_{h} \right] x(t) \right\}$$
(15)

Taking into account that $\mathcal{L} \otimes B_l K_h x(t) = \mathcal{L} \otimes B_l K_h \delta(t)$ and that:

$$\alpha_{hl}(\theta(t)) [I_N \otimes A_h] \mathbf{x}(t) = \cdots$$

$$\cdots [\alpha_{hl}(\theta(t)) \otimes A_h] \delta(t) + f_{hl}(\theta(t), \overline{\mathbf{x}}(t))$$
(16)

with:

$$f_{hl}(\theta(t), \bar{x}(t)) = \begin{bmatrix} \alpha_{h\star l}(\theta_1(t)) & \dots & \alpha_{h\star l}(\theta_N(t)) \end{bmatrix}^T \otimes A_h \bar{x}(t)$$
(17)

then (15) can be rewritten as:

$$\dot{\delta}(t) = \sum_{h=1}^{S} \sum_{l=1}^{S} \Upsilon \left\{ \left[\alpha_{hl} \left(\theta(t) \right) \mathcal{L} \otimes B_{l} K_{h} \right] e(t) \cdots + \alpha_{hl} \left(\theta(t) \right) \left[I_{N} \otimes A_{h} + \mathcal{L} \otimes B_{l} K_{h} \right] \delta(t) + f_{hl} \left(\theta(t), \overline{x}(t) \right) \right\}$$
(18)

Let $z(t) = [e(t)^T, \delta(t)^T]^T$, then by considering (13) and (18), we obtain (19) (see Box I).

3. LMI-based consensus analysis and design

Let us provide an interpretation of the value z = 0. Given the above definitions, z(t) = 0 corresponds to e(t) = 0 and $x_i(t) = \frac{1}{N} \sum_{j=1}^{N} x_j(t) = \bar{x}(t)$, which means that consensus is achieved. However, by looking at (19), it is clear that in cases where $\Upsilon f_{hl}(\theta(t), \bar{x}(t)) \neq 0$, the dynamics for z(t) is not described by an autonomous system, so that consensus would be achieved only in cases where $\Upsilon f_{hl}(\theta(t), \bar{x}(t)) = 0$. Although by looking at (17) this condition depends on the vertex state matrices and the value of $\bar{x}(t)$, it is possible to state that a sufficient condition for $\Upsilon f_{hl}(\theta(t), \bar{x}(t)) = 0$ is that $\theta_i(t) = \theta_j(t) \forall i, j = 1, ..., N$. Under this condition, which will be referred to as *synchronization* in contrast to the case where the varying parameters trajectories are different (*non-synchronization*), it is possible to provide LMI-based analysis and design conditions for reaching consensus, which is done in the next theorem.

Theorem 1. Consider the closed-loop augmented system (19), obtained as the interconnection of the agent dynamics (3)–(4), the consensus protocol (7), and the local state observers (8), under the polytopic assumptions (5) and (9). For any $s \in \mathbb{N}$, with $s \ge 2$, and for given eigenvalues α_j , j = 2, ..., N, of the Laplacian matrix \mathcal{L} , if there exist symmetric matrices $P_{11} \succ 0$ and $P_{22} \succ 0$ and a matrix P_{12} of compatible dimensions with:

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \succ 0$$
 (20)

such that (21) and:

$$\sum_{\vec{r} \in \mathcal{P}(\vec{p})} \begin{bmatrix} \operatorname{He} \left\{ P_{11} \left(A_{\vec{r}_{1}} + L_{\vec{r}_{1}} C_{\vec{r}_{2}} \right) + \alpha_{j} P_{12} B_{\vec{r}_{2}} K_{\vec{r}_{1}} \right\} & \star \\ \Omega_{\vec{r}_{1} \vec{r}_{2}, 12}^{j} & \operatorname{He} \left\{ P_{22} \left(A_{\vec{r}_{1}} + \alpha_{j} B_{\vec{r}_{2}} K_{\vec{r}_{1}} \right) \right\} \end{bmatrix} \\ \prec 0 \tag{21}$$

$$\sum_{\vec{r} \in \mathcal{P}(\vec{p})} \operatorname{He} \left\{ P_{11} \left(A_{\vec{r}_1} + L_{\vec{r}_1} C_{\vec{r}_2} \right) \right\} \prec 0$$
(22)

hold $\forall \vec{p} \in \mathbb{P}_s^+$, with:

~

$$\Omega^{j}_{\vec{r}_{1}\vec{r}_{2},12} = P^{T}_{12}(A_{\vec{r}_{1}} + L_{\vec{r}_{1}}C_{\vec{r}_{2}})\cdots$$

$$\cdots + (A_{\vec{r}_{1}} + \alpha_{j}B_{\vec{r}_{2}}K_{\vec{r}_{1}})^{T}P^{T}_{12} + \alpha_{j}P_{22}B_{\vec{r}_{2}}K_{\vec{r}_{1}}$$
(23)

then consensus is achieved if $\theta_1(t) = \theta_2(t) = \cdots = \theta_N(t) = \overline{\theta}(t)$.

Proof. First of all, let us note that, by exploiting the symmetry property of undirected topology graphs, one finds out that the dynamics of z(t) when $\theta_1(t) = \theta_2(t) = \cdots = \theta_N(t) = \overline{\theta}(t)$ is driven by (24).

$$\dot{z}(t) = \sum_{h=1}^{5} \sum_{l=1}^{5} \alpha_h \left(\bar{\theta}(t) \right) \alpha_l \left(\bar{\theta}(t) \right) \\ \times \begin{bmatrix} I_N \otimes (A_h + L_h C_l) & \mathbf{0} \\ \mathcal{L} \otimes B_l K_h & I_N \otimes A_h + \mathcal{L} \otimes B_l K_h \end{bmatrix} z(t)$$
(24)

For the autonomous system (24), let us choose the following Lyapunov function candidate (from now on, dependency of variables on time is dropped to ease the notation):

$$V(z) = \begin{bmatrix} e \\ \delta \end{bmatrix}^T \left(I_N \otimes \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \right) \begin{bmatrix} e \\ \delta \end{bmatrix}$$
(25)

where P_{11} , P_{12} , P_{22} are matrices to be determined such that (20) holds.

Let us consider the spectral decomposition of $\mathcal{L} = \Pi \Lambda \Pi^T$, where the orthogonal matrix $\Pi \in \mathbb{R}^{N \times N}$ contains the eigenvectors of \mathcal{L} , while $\Lambda = \text{diag}(\alpha_1, \ldots, \alpha_N) \in \mathbb{R}^{N \times N}$ where the eigenvalues are ordered so that $\alpha_1 < \alpha_2 \leq \ldots \leq \alpha_N$, with $\alpha_1 = 0$ due to $\mathcal{L} \succeq 0$.

Let us compute the derivative of V(z) by defining the following change of coordinates:

$$\vartheta = (\Pi^T \otimes I_n) e \qquad \zeta = (\Pi^T \otimes I_n) \delta \qquad (26)$$

thus obtaining (27).

$$\dot{z}(t) = \sum_{h=1}^{S} \sum_{l=1}^{S} \left\{ \begin{bmatrix} \alpha_{hl} \left(\theta(t)\right) \otimes \left(A_{h} + L_{h}C_{l}\right) & 0\\ \Upsilon \alpha_{hl} \left(\theta(t)\right) \mathcal{L} \otimes B_{l}K_{h} & \Upsilon \alpha_{hl} \left(\theta(t)\right) \left[I_{N} \otimes A_{h} + \mathcal{L} \otimes B_{l}K_{h}\right] \end{bmatrix} z(t) + \begin{bmatrix} 0\\ \Upsilon f_{hl} \left(\theta(t), \overline{x}(t)\right) \end{bmatrix} \right\}$$
(19)

Box I

$$\dot{V}(z) = 2 \sum_{h=1}^{S} \sum_{l=1}^{S} \alpha_{h}(\bar{\theta})\alpha_{l}(\bar{\theta}) \left\{ \vartheta^{T} \left[(I_{N} \otimes P_{11}(A_{h} + L_{h}C_{l})) \vartheta \right] \right. \\ \left. + \zeta^{T} \left[\left(I_{N} \otimes P_{12}^{T}(A_{h} + L_{h}C_{l}) \right) \vartheta \right] + \cdots \right. \\ \left. \cdots + \vartheta^{T} \left[(\Lambda \otimes P_{12}B_{l}K_{h}) (\vartheta + \zeta) + (I_{N} \otimes P_{12}A_{h}) \zeta \right] \right. \\ \left. + \zeta^{T} \left[(\Lambda \otimes P_{22}B_{l}K_{h}) (\vartheta + \zeta) + (I_{N} \otimes P_{22}A_{h}) \zeta \right] \right\}$$
(27)

By exploiting the fact that $\zeta_1 = 0$ and $\alpha_1 = 0$, and through the definition of the augmented vector $\eta_j = [\vartheta_j^T, \zeta_j^T]^T$, we obtain eventually the expression in (28)–(30).

$$\dot{V}(z) = \sum_{h=1}^{S} \sum_{l=1}^{S} \alpha_{h}(\bar{\theta}) \alpha_{l}(\bar{\theta})$$

$$\times \left\{ \sum_{j=2}^{N} \eta_{j}^{T} \Omega_{hl}^{j} \eta_{j} + \vartheta_{1}^{T} \operatorname{He} \left\{ P_{11}(A_{h} + L_{h}C_{l}) \right\} \vartheta_{1} \right\}$$
(28)

$$\Omega_{hl}^{j} = \begin{bmatrix} He\left\{P_{11}(A_{h} + L_{h}C_{l}) + \alpha_{j}P_{12}B_{l}K_{h}\right\} & \star \\ \Omega_{hl,12}^{j} & He\left\{P_{22}(A_{h} + \alpha_{j}B_{l}K_{h})\right\} \end{bmatrix}$$
(29)

$$\Omega_{hl,12}^{j} = P_{12}^{T}(A_{h} + L_{h}C_{l}) + (A_{h} + \alpha_{j}B_{l}K_{h})^{T}P_{12}^{T} + \alpha_{j}P_{22}B_{l}K_{h}$$
(30)

Clearly, if $\Omega_{hl}^j \prec 0, j = 2, \dots, N$, and He { $P_{11}(A_h + L_hC_l)$ } $\prec 0$ for all h, l = 1, ..., S, then $\dot{V} < 0$ for any $\eta_j \neq 0$. Therefore, z(t)will converge asymptotically to zero, which means that $e_i(t) \rightarrow 0$ and $x_i(t) \rightarrow \overline{x}$, so that consensus is achieved. More precisely, the condition to be assessed are:

$$\sum_{h=1}^{S} \sum_{l=1}^{S} \alpha_h(\bar{\theta}) \alpha_l(\bar{\theta}) \Omega_{hl}^j \prec 0 \qquad \forall j = 2, \dots, N$$
(31)

s

$$\sum_{h=1}^{S} \sum_{l=1}^{S} \alpha_h(\bar{\theta}) \alpha_l(\bar{\theta}) \operatorname{He} \left\{ P_{11}(A_h + L_h C_l) \right\} \prec 0$$
(32)

which correspond to the problem of verifying the negativity of double polytopic sums. By applying Polya's theorem on definite quadratic forms (Sala & Arino, 2007), (21)-(23) are obtained, thus completing the proof. \Box

Remark 1. As discussed in deep in Sala and Arino (2007), the application of Polya's theorem provides a set of sufficient conditions to assess the definiteness of double sums, which are progressively less conservative when the complexity parameter *s* increases. These conditions are asymptotically exact, i.e. there exists a finite value of s such that they become necessary and sufficient.

Remark 2. Note that the necessity for synchronization of the varying parameters is a consequence of the simplicity of the consensus protocol, which has been chosen as a linear gainscheduled law. Some works have achieved consensus in networks of heterogeneous systems, although at the cost of requiring more complex consensus protocols (Kim, Shim, & Seo, 2010; Liang, Ge, Liu, Wang, & Karimi, 2020). Whether more complex protocols can

be applied to LPV multi-agent systems to relax the synchronization assumption while still retaining the relative simplicity of an LMI-based design framework goes beyond the scope of this work and will be considered in future research.

Note that Theorem 1 provides LMI-based analysis conditions, which become BMIs when design is considered, due to the arising products between the Lyapunov matrices P_{11} , P_{12} , P_{22} and the gains to be designed K_h , L_h , h = 1, ..., S. The following theorem provides LMI-based design conditions.

Theorem 2. Consider the closed-loop augmented system (19). obtained as the interconnection of the agent dynamics (3)-(4), the consensus protocol (7), and the local state observers (8), under the polytopic assumptions (5) and (9). For any $s \in \mathbb{N}$, with $s \ge 2$, for given eigenvalues α_i , j = 2, ..., N, of the Laplacian matrix \mathcal{L} and positive scalars μ , $\xi_1, \xi_2, \ldots, \xi_S$, if there exist symmetric matrices $P_1 \succ 0, P_2 \succ 0$ and matrices K_1, K_2, \ldots, K_s , and $\Gamma_1, \Gamma_2, \ldots, \Gamma_s$ such that (33)

$$\sum_{\vec{r}\in\mathcal{P}(\vec{p})} \begin{bmatrix} \operatorname{He}\left\{P_{1}A_{\vec{r}_{1}}+I_{\vec{r}_{1}}C_{\vec{r}_{2}}\right\} & \star & \star & \star \\ 0 & \operatorname{He}\left\{P_{2}A_{\vec{r}_{1}}\right\}+I-\frac{2P_{2}}{\mu} & \star & \star \\ \xi_{\vec{r}_{2}}K_{\vec{r}_{1}} & \alpha_{j}B_{\vec{r}_{2}}^{T}P_{2} & -2\xi_{\vec{r}_{2}}I & \star \\ 0 & \frac{P_{2}}{\mu}+\alpha_{j}\mu B_{\vec{r}_{2}}K_{\vec{r}_{1}} & 0 & -I \end{bmatrix} < 0$$

$$(33)$$

holds $\forall \vec{p} \in \mathbb{P}_s^+$, and the observer vertex gains are chosen as $L_h =$ $P_1^{-1}\Gamma_h$, then consensus is achieved if $\theta_1(t) = \theta_2(t) = \cdots = \theta_N(t) =$ $\bar{\theta}(t)$.

Proof. First of all, let us note that by applying Polya's theorem on definite quadratic forms, taking into account the polytopic assumptions (5) and (9), and by defining:

$$\begin{pmatrix} \xi(\bar{\theta}) \\ \Gamma(\bar{\theta}) \end{pmatrix} = \sum_{h=1}^{S} \alpha_h(\bar{\theta}) \begin{pmatrix} \xi_h \\ \Gamma_h \end{pmatrix}$$
(34)

it is clear that (33) is a sufficient (and for high enough s, also necessary) condition for (35).

$$\begin{bmatrix} \operatorname{He}\left\{P_{1}A(\bar{\theta})+\Gamma(\bar{\theta})C(\bar{\theta})\right\} & \star & \star & \star \\ 0 & \operatorname{He}\left\{P_{2}A(\bar{\theta})\right\}+I-\frac{2P_{2}}{\mu} & \star & \star \\ \xi(\bar{\theta})K(\bar{\theta}) & \alpha_{j}B(\bar{\theta})^{T}P_{2} & -2\xi(\bar{\theta})I & \star \\ 0 & \frac{P_{2}}{\mu}+\alpha_{j}\mu B(\bar{\theta})K(\bar{\theta}) & 0 & -I \end{bmatrix} \\ \prec 0 & (35)$$

Using Schur complements, (35) can be rewritten as:

$$\begin{bmatrix} \operatorname{He} \left\{ P_{1}A(\bar{\theta}) + \Gamma(\bar{\theta})C(\bar{\theta}) \right\} & \star & \star \\ 0 & \Psi(\bar{\theta}) & \star \\ \xi(\bar{\theta})K(\bar{\theta}) & \alpha_{j}B(\bar{\theta})^{T}P_{2} & -2\xi(\bar{\theta})I \end{bmatrix} \prec 0 \quad (36)$$

with:

$$\Psi(\bar{\theta}) = \operatorname{He}\left\{P_{2}A(\bar{\theta})\right\} + I - \frac{2P_{2}}{\mu} \cdots$$

$$\cdots + \left(\frac{P_{2}}{\mu} + \alpha_{j}\mu B(\bar{\theta})K(\bar{\theta})\right)^{T} \left(\frac{P_{2}}{\mu} + \alpha_{j}\mu B(\bar{\theta})K(\bar{\theta})\right)$$
(37)

Now, given $\mu > 0$, the following holds:

$$\operatorname{He}\left\{P_{2}\left(A(\bar{\theta})+\alpha_{j}B(\bar{\theta})K(\bar{\theta})\right)\right\} \leq \operatorname{He}\left\{P_{2}A(\bar{\theta})\right\} - \frac{P_{2}^{2}}{\mu^{2}}\cdots + \left(\frac{P_{2}}{\mu}+\alpha_{j}\mu B(\bar{\theta})K(\bar{\theta})\right)^{T}\left(\frac{P_{2}}{\mu}+\alpha_{j}\mu B(\bar{\theta})K(\bar{\theta})\right)$$
(38)

which, taking into account that $I - 2P_2/\mu \geq -P_2^2/\mu^2$, means that:

$$\operatorname{He}\left\{P_{2}\left(A(\bar{\theta})+\alpha_{j}B(\bar{\theta})K(\bar{\theta})\right)\right\} \leq \Psi(\bar{\theta})$$
(39)

Replacing appropriately the left-hand term of (39) in condition (36), pre- and post-multiplying by:

$$\begin{bmatrix} I & 0 & K(\bar{\theta})^T \\ 0 & I & 0 \end{bmatrix}$$
(40)

and its transpose, respectively, performing the change of variables $\Gamma(\bar{\theta}) = P_1 L(\bar{\theta})$ (which means that $\Gamma_h = P_1 L_h$), and applying Polya's theorem on definite quadratic forms, (21) with $P_{11} = P_1$, $P_{12} = 0$ and $P_{22} = P_2$ is obtained. Finally, note that by replacing $P_{11} = P_1, P_{12} = 0$ and $P_{22} = P_2$ in (20), then this condition reduces to $P_1 \succ 0$ and $P_2 \succ 0$, and that (22) holds if (33) holds, since it corresponds to the upper-left diagonal block, thus completing the proof. \Box

Remark 3. The use of a block-diagonal structure for the Lyapunov matrix in Theorem 2 is rather standard in observer-based control, see e.g. Kheloufi, Zemouche, Bedouhene, and Boutayeb (2013), Lan and Patton (2016). Although it introduces conservatism, it is required in order to convert the BMIs into LMIs. Note that iterative and/or two-step design algorithms could be developed in such a way that unstructured Lyapunov matrices are used instead, following e.g. the ideas in Lo and Lin (2004). However, the development of such algorithms goes beyond the scope of this work.

Remark 4. Note that the most computationally demanding part of the proposed approach (solving the LMIs) can be performed offline on efficient hardware, and the agents' online burden is limited to the exchange of information about the locally estimated states among neighboring agents in (7) and the computation of the polytopic coefficients $\alpha_h(\theta_i(t))$ in (5) so that the gains $K(\theta_i(t))$ and $L(\theta_i(t))$ can be computed using (9).

4. Illustrative example

In this section, we present a comparison between the robust design proposed in Chen et al. (2017) and the gain-scheduled design proposed in this paper. We show that the gain-scheduled design achieves feasibility of the design LMIs given by Theorem 2 in cases where a non-scheduled protocol fails. Finally, we present simulations to demonstrate that the agents' state trajectories achieve consensus under synchronized varying parameters in contrast to non-synchronized varying parameters which lead to steady-state deviations from the average state.

Let us consider an LPV MAS with four agents whose interactions are described by the following Laplacian matrix \mathcal{L} :

$$\mathcal{L} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ -1 & 0 & -1 & 2 \end{bmatrix}$$

Each agent's dynamics is described by state-space matrices with two-vertex polytopic representation given by:

$$A_1 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \qquad A_2 = R_p^{-1} \begin{bmatrix} 0 & 1+p \\ -1 & 0 \end{bmatrix} R_p$$

$$R_{p} = \begin{bmatrix} \cos(\arctan p) & -\sin(\arctan p) \\ \sin(\arctan p) & \cos(\arctan p) \end{bmatrix} \quad B_{1} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$B_{2} = \begin{bmatrix} 1 \\ 1+10p \end{bmatrix} \quad C_{1} = \begin{bmatrix} 1 & 0 \end{bmatrix} \quad C_{2} = \begin{bmatrix} 1+10p & 0 \end{bmatrix}$$

where *p* is a parameter that enables comparison, such that p = 0 corresponds to the above system reducing to an LTI system, whereas for increasing values of p > 0, the difference between the two vertices becomes bigger.

Fig. 1 compares the design performance of a non-scheduled observer-based consensus protocol, as in Chen et al. (2017), against the scheduled observer-based consensus protocol proposed in Theorem 2 of this paper. In particular, a value of s = 2 and a constant parameter $\xi_1 = \xi_2 = \xi$ (for easing the graphical representation) have been used. For each considered pair of a priori fixed parameters μ and ξ , the maximum value of p for which feasibility is maintained has been recorded. It can be seen clearly that the proposed approach improves the design performance in terms of maximum feasible p, which over the considered values of μ and ξ corresponds to p = 0.41 applying (Chen et al., 2017) and p = 0.49 for the proposed approach.

Let us now consider a specific solution to the consensus design with parameters $\mu = 0.25$, $\xi = 75$, p = 0.49, given by:

$$P_{1} = \begin{bmatrix} 198.9 & -10.7 \\ -10.7 & 193.3 \end{bmatrix} P_{2} = \begin{bmatrix} 0.29 & 0.05 \\ 0.05 & 0.25 \end{bmatrix} L_{1} = \begin{bmatrix} -0.096 \\ -0.056 \end{bmatrix}$$
$$L_{2} = \begin{bmatrix} -0.073 \\ -0.086 \end{bmatrix} K_{1} = \begin{bmatrix} -0.22 \\ -0.30 \end{bmatrix} K_{2} = \begin{bmatrix} -0.09 \\ -0.25 \end{bmatrix}$$

and let us perform simulations starting from initial conditions $x_1(0) = [1, 1]^T, x_2(0) = [1, 3]^T, x_3(0) = [3, 1]^T, x_4(0) =$ $[3, 3]^T$ under two different scenarios. The first one (Scenario 1) corresponds to synchronization, so that $\alpha_1(\theta_i(t)) = (1 + \sin t)/2$, $\alpha_2(\theta_i(t)) = 1 - \alpha_1(\theta_i(t)), i = 1, 2, 3, 4$. On the other hand, Scenario 2, the local varying parameters are such that $\alpha_1(\theta_1(t)) = (1 + t)$ $\sin 2t$ /2, $\alpha_1(\theta_2(t)) = (1 + \cos 0.1t)/2$, $\alpha_1(\theta_3(t)) = (1 + \sin 0.5t)/2$, $\alpha_1(\theta_4(t)) = (1 + \cos 0.05t)/2, \ \alpha_2(\theta_i(t)) = 1 - \alpha_1(\theta_i(t)), \ i = 1 - \alpha_1(\theta_i(t)), \$ 1, 2, 3, 4 (non-synchronization). As predicted from the theoretical discussion in the previous sections, in Scenario 1 the multi-agent LPV system reaches a consensus, as shown in the upper side of Fig. 2, whereas in Scenario 2 consensus is not reached (see lower side of Fig. 2). For the sake of completeness, Fig. 3 shows the norm of the augmented state vector $z(t) = [e(t)^T, \delta(t)^T]^T$, comprising not only the synchronization error but also the state estimation error, thus confirming the drawn conclusions.

5. Conclusions

This paper has studied the observer-based consensus in LPV multi-agent systems. It has been shown that a gain-scheduled protocol allows improving the design performance when compared to a non-scheduled protocol. Also, it has been discussed and verified through simulations that, provided a successful observer/controller design, the assumption of synchronized agents' varying parameters is a sufficient condition for reaching consensus.

The MAS model considered in this paper is not excited by external disturbances. It is known that under their presence, it is impossible to reach precise consensus, which has led to the introduction of the notion of *bounded consensus* (Li, Tang, & Karimi, 2020). Future research will aim at addressing the presence of external disturbances using rejection indexes and the concept of *quadratic boundedness*, at investigating whether more complex protocols can be applied to LPV multi-agent systems whilst retaining the relative simplicity of an LMI-based design framework, as well as employing artificial scheduling parameters that would enable the online modification of the closed-loop consensus performance.



Fig. 1. Comparison of the design performance between Chen et al. (2017) and Theorem 2 with s = 2.



Fig. 2. Trajectories of the agents' local state 1 under synchronization (Scenario 1) and non-synchronization (Scenario 2).

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Fig. 3. Norm of the augmented state $z(t) = [e(t)^T, \delta(t)^T]^T$ under synchronization (Scenario 1) and non-synchronization (Scenario 2).

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