**Assessment of Shallow Water Random Wave-Induced Scour at the Trunk Section of Breakwaters using Deep Water Wind and Wave Conditions**

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**ABSTRACT**

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This article provides a simple analytical method giving estimates of random wave-induced scour at the trunk section of vertical-wall and rubble-mound breakwaters in shallow water from deep water wind and wave conditions. Results are exemplified by using a Pierson-Moskowitz model wave spectrum for deep water wind waves with the mean wind speed at the 10 m elevation above the sea surface as the parameter. The significant value of the scour depth within a sea state of random waves is provided and an example typical for field conditions is given. The method should serve as a useful tool for assessing shallow water random wave-induced scour based on input from deep water wind and wave conditions.

**ADDITIONAL INDEX WORDS:** Breakwater, vertical wall, rubble-mound, scour depth, random waves, Pierson-Moskowitz spectrum.

**INTRODUCTION**

Scour around breakwaters may lead to failure of these structures, which is caused by the complicated flow structure due to the interaction between the incoming combined wave-current flow, the breakwater and the sediments at the seabed (see *e.g.* Sumer and Fredsøe (2002)). The results will depend on the relative magnitude between waves and currents, the seabed features and its material, and the wave-induced fluid particle excursion amplitude relative to a characteristic dimension of the breakwater. Assessment of scour under field conditions needs to take into account that waves are stochastic, which implies that the flow structure around breakwaters is more complex. The background and review of scour around marine structures in general and specifically around breakwaters are given in *e.g.* Whitehouse (1998) and Sumer and Fredsøe (2002).

The purpose of this study is to demonstrate how deep water wind and wave conditions can be utilized to estimate random wave-induced scour at the trunk section of vertical-wall and rubble-mound breakwaters in shallow water. The approach is based on adopting the formulae for regular wave-induced scour in finite water depth by Xie (1981) (for vertical-wall breakwaters) and Sumer and Fredsøe (2000) (for rubble-mound breakwaters).

Previous result on random wave-induced scour around breakwaters are available in the literature; Hughes and Fowler (1991) provided random wave-induced scour data for vertical-wall breakwaters; Sumer and Fredsøe (2000) gave random wave-induced scour data for rubble-mound breakwaters. Myrhaug *et al.* (2004) derived the scour depth and protection layer width around the head of vertical-wall breakwaters, the scour and deposition depths as well as the protection layer widths at the round head of rubble-mound breakwaters exposed to random waves by using a stochastic method. Myrhaug and Ong (2009) used a similar stochastic method to derive the random wave-induced scour at the trunk section of both vertical-wall and rubble-mound breakwaters in finite water depth based on the regular wave-induced formulae by Xie (1981) and Sumer and Fredsøe (2000) used in this article. Thus, the present study is supplementary to that in Myrhaug and Ong (2009).

The paper is organized as follows. The introduction contains a subsection giving the background and a summary of the maximum scour depth at the trunk section of vertical-wall and rubble-mound breakwaters exposed to normally incident regular waves. The section on the methods presents the random wave-induced scour in shallow water by transforming deep water waves to shallow water. The section on the results provides two subsections; the first gives an example for a Pierson-Moskowitz model wave spectrum for deep water wind waves, with the mean wind speed at the 10 m elevation above the sea surface as the parameter; the second includes an example representing realistic field conditions. The discussion section addresses the present method versus common procedure. Finally, some conclusions are provided.

**Background of scour for regular waves**

By following Myrhaug and Ong (2009), the maximum scour depth at the trunk section of a breakwater exposed to normally incident regular waves can be summarized as (see Figures 1 and 2)

 (1)

where

 (2)

 (3)

 (4)

Here *H* is the wave height of incident waves with wave length *L*, *h* is the water depth,  is the wave number determined from the dispersion relationship ,  is the angular wave frequency, *T* is the wave period, and  is the breakwater slope with the unit of degrees.

The formulae in Eqs. (1) to (4) are empirical results from: (i) Xie (1981) obtained data for the maximum scour depth for suspension mode transport of fine sediments with median grain size diameter , and one data set with , represented by Eqs. (1) and (4) ( ; (ii) Sumer and Fredsøe (2000) obtained data for the maximum scour depth at the trunk section of a rubble-mound breakwater exposed to regular and irregular waves for no-suspension mode transport of coarse sand with  for the breakwater slopes 30o and 40o for regular waves and 40o for irregular waves, which for regular waves was represented by Eqs. (1) to (3). All these results are valid for live-bed scour, i.e. , where  is the undisturbed Shields parameter defined as

 (5)

where  is the maximum bed shear stress due to waves,  is the fluid density, *g* is the acceleration due to gravity,  is the sediment grain density to fluid density ratio,  is the sediment density, and  is the critical value of motion at the bed. Furthermore, the sand transport mode is determined by the fall velocity parameter  where *w* is the fall velocity of sand grains (see Soulsby (1997, Section 8.3)), and  is the friction velocity. The sand transport is in the suspension mode if  and  ; otherwise it is in the no-suspension mode. Here  is the critical value for imitation of suspension from the bed (see Sumer and Fredsøe (2002, Eq. (7.7)).

Details of the time scale and the mechanisms of the scour process are given in Sumer and Fredsøe (2000, 2002). First, it should be noted that the scour process reaches its equilibrium stage through a transition period, and thus the present results are valid by assuming that the sea state of random waves has lasted longer than the time scale of the scour. Second, normally incident waves on a vertical-wall breakwater will be reflected resulting in a standing wave, which generates a steady streaming field consisting of a system of recirculating cells where the bottom cells are related to the seabed boundary layer flow, causing the response of the bottom sediments. Thus, the scour process is a consequence of this response depending on  and . In the case of a rubble-mound breakwater, the breakwater is made of rubble and has a sloping wall, leading to a reduced reflection of waves. However, the standing wave-induced streaming is still the main mechanism leading to scour at the trunk section of the breakwater.

It should be noted that the present results will be limited to the conditions for which the empirical formulae in Eqs. (1) to (4) are valid. There are certain effects which are not explicit in these formulae, and consequently not in the subsequently derived scour depth formulae, *i.e.* Eqs. (13) and (20) together with their upper limits in Eqs. (22) and (23). For a sloped rubble mound breakwater in shallow water, the wave rundown plays an important role for the scour. The scour is a main function of the energy of the rundown water mass at the toe of the breakwater, and this energy is a main factor causing the scour. This energy depends strongly on the water depth at the toe of the breakwater, which appears to be included in the scour depth formulae in Eq. (20), but not in their upper limits in Eqs. (22) and (23). This issue will be discussed further in the results section. Moreover, the energy in the rundown of water depends strongly on wave energy dissipation due to the porosity and roughness of the breakwater. However, since these physical effects have been present in the experiments which are the basis for the empirical formulae in Eqs. (1) to (4), there are at least partly incorporated in these formulae.

As in Myrhaug and Ong (2009), it is assumed that the seabed shear stress can be calculated as for progressive wave boundary layer flow where the maximum bed shear stress within a wave cycle is taken as

 (6)

Here *U* is the near-bed orbital velocity amplitude, and  is the wave friction factor taken from Soulsby (1997, Eq. (62a)) as

 (7)

where  is the near-bed orbital displacement amplitude, and  is the bottom roughness. For linear waves *A* is

 (8)

**METHODS**

This section presents the random wave-induced scour in shallow water by transforming deep water waves to shallow water.

**Random wave-induced scour in shallow water**

For linear regular waves in shallow water  and the dispersion relationship is . Thus, by taking  where *a* is the linear wave amplitude, Eq. (1) becomes

 (9)

For a random wave component with amplitude  and frequency  the scour depth  is

 (10)

The justification of using Eq. (10) is that the random wave process is assumed to be narrow-banded, implying that the wave envelope and the associated wave amplitudes and wave heights are slowly varying, and consequently that the high waves occur in wave groups. The statistical quantity that will be deduced from Eq. (10) is the significant value given in Eq. (13), *i.e.* the mean of the one-third largest scour depths. As a consequence of the narrow-band assumption, it is reasonable that the mean of the one-third highest waves, i.e. the significant wave height, is responsible for the significant value of the scour depth, provided that the sea state of random waves has lasted longer than the time scale of the scour, i.e. that the scour process has reached its equilibrium stage. Further issues of the narrow-band assumption is given in the results section.

Now  where  is the wave spectrum in shallow water (Massel (1989), Section 7.3),  is the deep water wave spectrum, and  is a constant separation between frequencies. Similarly,  where  is the spectrum associated with the random wave-induced scour depth at the trunk section of a breakwater. Thus, by substituting  and  in Eq. (10), it follows for an infinite number of frequency components that

 (11)

Thus, Eq. (11) takes the form

 (12)

where  is the zeroth spectral moment of  and  is the  spectral moment of . A commonly used statistical quantity when assessing the scour depth for random waves is e.g. the significant value of the scour depth defined as  , which from Eq. (12) is obtained as

 (13)

Thus, for a given deep water wave spectrum  is given by Eq. (13).

To the authors knowledge, no data are available in the open literature to compare with. Therefore, example of results is provided in the next section.

**RESULTS**

This section provides two subsections; the first gives an example for a Pierson-Moskowitz model wave spectrum for deep water wind waves, with the mean wind speed at the 10 m elevation above the sea surface as the parameter; the second includes an example representing realistic field conditions.

**Example for a Pierson-Moskowitz** **spectrum**

Now the results are exemplified by choosing the Pierson-Moskowitz (PM) model wave spectrum representing deep water wind waves, with the mean wind speed at the 10 m elevation above the sea surface as the parameter, given as (Tucker and Pitt, 2001)

 (14)

Here , ,  and  are the spectral peak frequency and period, respectively. Moreover, the spectral moments  for  are given by

 (15)

where  is the gamma function.

Originally, the mean wind speed at the 19.5 m elevation above the sea surface was the parameter, but the formulation with the mean wind speed at the 10 m elevation with  gives the following sea state parameters in deep water (Tucker and Pitt, 2001)

 (16)

 (17)

 (18)

 (19)

where  is the significant wave height,  is the mean zero-crossing wave period, and  is the spectral bandwidth parameter defined in terms of the spectral moments  as . It should be noted that this corresponds to that  and  Thus, the result in Eq. (13) can be expressed in terms of  as

 (20)

According to Hedges (1995) regular linear waves in shallow water is valid for the Ursell number  and the deep water wave steepness . Then, by replacing *H* with , *T* with ,  with  and *k* with , the Ursell number criterion in terms of the sea state parameters gives that (see Eq. (A2) in the Appendix).

 (21)

Similarly, for the PM spectrum in deep water the wave spectrum in terms of the sea state parameters is  (see Eq. (A3) in the Appendix), *i. e.* strictly outside the validity range, but it is considered to be acceptable for a first-order assessment of the scour depth.

Thus, by substituting Eq. (21) in Eq. (19),  in shallow water is limited by

 (22)

By combining Eqs. (17) and (22) the ratio between  and the deep water significant wave height is given by

 (23)

This means that  given by the upper limits in Eqs. (22) and (23) represents the largest value in shallow water, i.e. independent of the shallow water depth *h*, which is a consequence of the shallow water approximation. However, as discussed in Section 2, the derived scour depth formula in Eq. (20) includes the water depth *h* at the toe of the breakwater.

Longuet-Higgins (1983) provided a joint distribution of wave amplitude and wave period for individual random waves based on a narrow-band assumption by including spectral bandwidth effects in terms of . Results were presented for  = 0.1 to 0.6 in intervals of 0.1. Srokosz and Challenor (1987) found that the Longuet-Higgins distribution gave reasonable agreement with observed wave data from field measurements for  < 0.4, but poorer agreement for  > 0.5. Also Myrhaug and Kvålsvold (1995) confirmed the poorer agreement with observed wave data for  > 0.5 by comparing the Longuet-Higgins distribution with field data with  = 0.504. This suggests that the wave spectrum is narrow-banded for  smaller than about 0.4, which is the case for the PM spectrum according to Eq. (19).

**Example of results**

Further, an example with  is considered, for which the results are valid for  according to Eq. (21). Then, for :

*  from Eq. (17)
*  from Eq. (18)
*  from Eq. (16)
*  from Eq. (A4)
*  for  from Eqs. (20) and (1) to (4), *i.e.* for a vertical-wall breakwater.

For a breakwater with a slope  , Eqs. (1) to (3) give , and then

*  from Eq. (20)

Thus, these results for the significant value of the scour depth, , represent the largest values in shallow water, *i.e.* independent of the shallow water depth *h* , according to Eq. (22), *i.e.* if the Shields parameter within the sea state corresponds to that for live-bed scour, requiring that  in Eq. (A7) has to exceed 0.05.

By taking , ,  as for quartz sand, this example gives

*  from Eq. (A5)
*  from Eq. (A6)
*  from Eq. (A7), *i.e.* live-bed scour.

An alternative to the stochastic method provided here is to use a deterministic method, *i.e.* to substitute the sea state parameters in Eq. (9). Specifically, this means to replace *a* with  and  with . Then, by using Eqs. (A4), (17) and (18) with *t* = 1.35, the deterministic method gives

 (24)

Thus, the stochastic to deterministic method ratio (*i.e.* the ratio between Eq. (20) and Eq. (24) becomes

 (25)

Then, the example with  and  gives *R* = 1.3, *i.e.* the stochastic method gives a conservative result compared with that obtained using the deterministic method.

**DISCUSSION**

For calculating the random wave-induced scour at the trunk section of a breakwater in shallow water one would commonly start with available data on wave statistics in deep water, *e.g.* joint distribution of  and ; ideally given within directional sector intervals at a (nearby) deep water offshore location. Then, to use a wave simulation model to transform these deep water joint statistics of  and  into the shallow water site, and finally to apply this as input for calculating the scour depth. This would in general include sea states with combined wind waves and swell from different directions. The present analytical method is an alternative to make first-order quick estimates of random wave-induced scour from available deep water wave conditions. Such estimates might be useful to compare with more complete computationally demanding tools; *e.g.* under field conditions such a simple method should serve the purpose as the time and access to computational tools are limited.

**CONCLUSIONS**

A simple analytical method which can be used to make first-order assessments of random wave-induced scour at the trunk section of vertical-wall and rubble-mound breakwaters in shallow water based on deep water wind and wave conditions is provided. The method is based on using scour depth formulae for regular waves in finite water depth and is used for random waves by transforming deep water wave conditions to shallow water. The significant value of the shallow water random wave-induced scour depth at the trunk section of a breakwater is given expressed in terms of deep water wave conditions. Results are exemplified by applying a Pierson-Moskowitz model wave spectrum for deep water wind waves with the mean speed at the 10 m elevation as the parameter. The validity of the result are given in terms of the Ursell number and the deep water wave steepness in terms of the sea state wave parameters. An example typical for field conditions is also given. It is demonstrated that the present stochastic method gives a conservative result compared with that obtained using a deterministic method. The present method should be useful for estimating random wave-induced scour at the trunk section of breakwaters. However, comparison with data is required before a conclusion regarding the validity this approach can be given.

**APPENDIX**

**Shallow water wave parameters**

Following e.g. Dean and Dalrymple (1984) the Ursell number is defined as

 (A1)

and gives the ratio between the nonlinearity of the waves in terms of the wave steepness *ka* and the dispersive properties of the waves in terms of *kh,* where *a* and *k* represent the values corresponding to the water depth *h*. According to Hedges (1995) linear waves are valid for  and the wave steepness in deep water . For linear harmonic waves propagating over a gently sloping bottom approaching a straight coastline at normal incidence, the Ursell number can be expressed in terms of deep water wave parameters as explained in the following. The wave amplitude is determined by using that the energy flux is constant (Dean and Dalrymple, 1984), *i.e.* = constant where the group velocity  equals the phase velocity . By taking deep water as the reference  where the index  refers to deep water), the wave amplitude in shallow water is  (by using that = constant,  in deep water and  ) . Thus, by implementing this in Eq. (A1) the Ursell number in shallow water can be expressed in terms of the deep water wave parameters as . Then, by replacing  with  and *T* with  , the Ursell number for a sea state of random waves in shallow water is obtained as

 (A2)

which is taken to be valid for  .

Similarly, the wave steepness in deep water can be expressed in terms of the sea state parameters as

 (A3)

which should satisfy  in order for linear waves to be valid.

Furthermore, by replacing *a* with  and  with , the significant wave height in shallow water is given as

 (A4)

In shallow water Eq. (8) reduces to  and consequently . Thus, in terms of the sea state parameters the corresponding quantities are given as

 (A5)

 (A6)

where  is given by Eq. (A4). Then, the Shields parameter in terms of the sea state parameters in shallow water follows as

 (A7)

**LITERATURE CITED**

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**Figure captions**

Figure 1 Definition sketch of the scour depth *S* at the trunk section of a vertical wall

breakwater with *H* and *L* as the incident wave height and wave length, respectively.

Figure 2 Definition sketch of the scour depth *S*  at the trunk section of a rubble-mound breakwater with *H* and *L* as the incident wave height and wave length, respectively.