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FRP pipeline performance in tensional and torsional S-lay installation loads

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Abstract. The loading conditions of a composite pipeline is the main factor for its dimensioning. During S-lay offshore installation of multilayered FRP pipelines, severe tensional and torsional loads take place in the above sea part of the pipeline. Since the wall pipe is multilayered and the material properties of the laminae and the laminate is anisotropic, the maximum stresses depend on the stacking sequence. In the present work, an analytical model is proposed for calculating the capacity of multilayered FRP pipelines to carry axial and torsional loads. Numerical results for typical multilayered filament wound E-Glass/Epoxy pipelines under axial tension and torsion are provided and discussed.

1. Introduction

Since in past the unit price of composite materials was high, steel pipes are still mainly used for oil and oil product transmission. Nowadays, the high maintenance cost due to corrosion of the old steel pipelines as well as the reduction of the unit price of composite materials have changed the key parameters of the optimum design of offshore pipelines. In the last decade the offshore industry has been benefited from the development of Fiber Reinforced Polymeric (FRP) pipelines and more multi-layered anisotropic pipelines are used for offshore applications.

The final cost is strongly affected by the material strength, density and performance in fatigue and corrosion. Since FRP materials have significantly lower density and much higher strength than carbon steel, the final cost of such materials is today comparable to the cost of carbon steel. Moreover, the lower maintenance cost of composite pipelines due to their excellent resistance in corrosion and fatigue we can conclude that the use of composite materials for pipeline applications is today advantageous comparing to the use of carbon steel pipelines.

As FRPs are anisotropic materials, the methods, and theoretical tools for mechanical design of composite pipelines are completely different than the design procedures of steel pipelines [1-6]. The existing design standards are rather semi-empirical and cover simple loading cases.

During offshore S-lay installation of FRP pipelines, the above level part of the pipeline is subjected to bending and axial tension due to the weight of the pipeline, and torsion during the turning of the vessel (Fig. 1). A model for calculating the bending capacity of FRP pipelines has been presentment in [3, 7]. In the present work, a model for estimating the axial tension and torsional capacity is proposed. The models are based in the classical lamination theory.
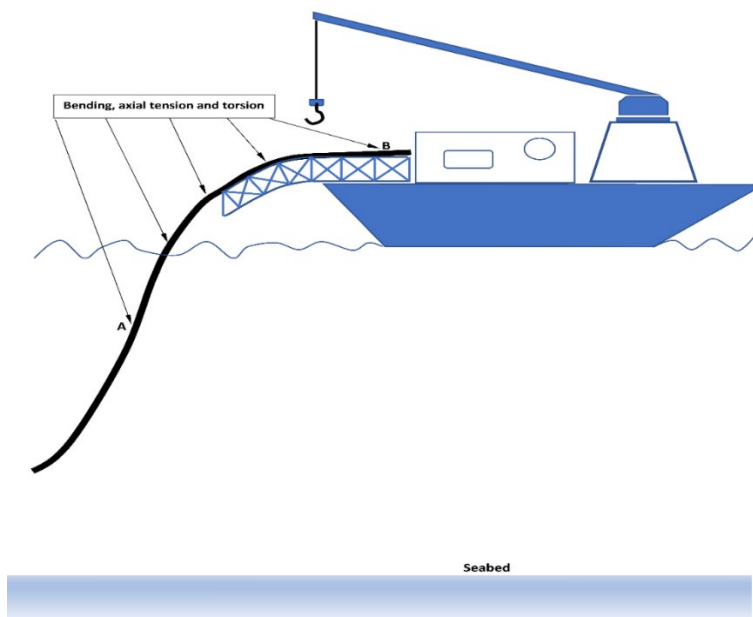
Figure 1. Schematic representation of the FRP pipeline part subjected to S-lay installation loads.

2. Formulation of the problem

2.1. Axial tension

A multi-layered pipe with mean diameter D made by a wall composed by NP layers with stacking sequence $[\pm\theta]$ (Fig. 2) is considered.
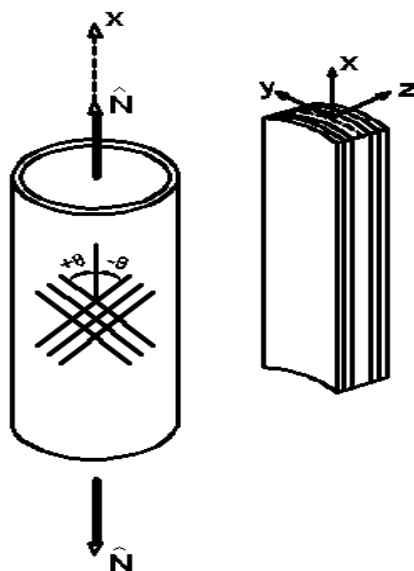

Figure 2. Geometry of the problem

We assume that the load $\hat{N}$ is distributed uniformly around the circumference of the cross section. Therefore, the load per unit length N_x of laminate is:

$$N_x = \frac{\hat{N}}{\pi D} \tag{1}$$

This is the only external load acting on the laminate constituting the wall of the pipe and thus

$$N_y = 0 \tag{2}$$

$$N_{xy} = 0 \tag{3}$$

$$M_x = 0 \tag{4}$$

$$M_y = 0 \tag{5}$$

$$M_{xy} = 0 \tag{6}$$

In order to estimate the allowable force $\hat{N}$, a failure criterion, e.g. the Tsai-Wu criterion [8] should be applied, and the principal stresses $\sigma_1, \sigma_2, \tau_{12}$ can be determined by following the general procedure shown in Fig. 3.
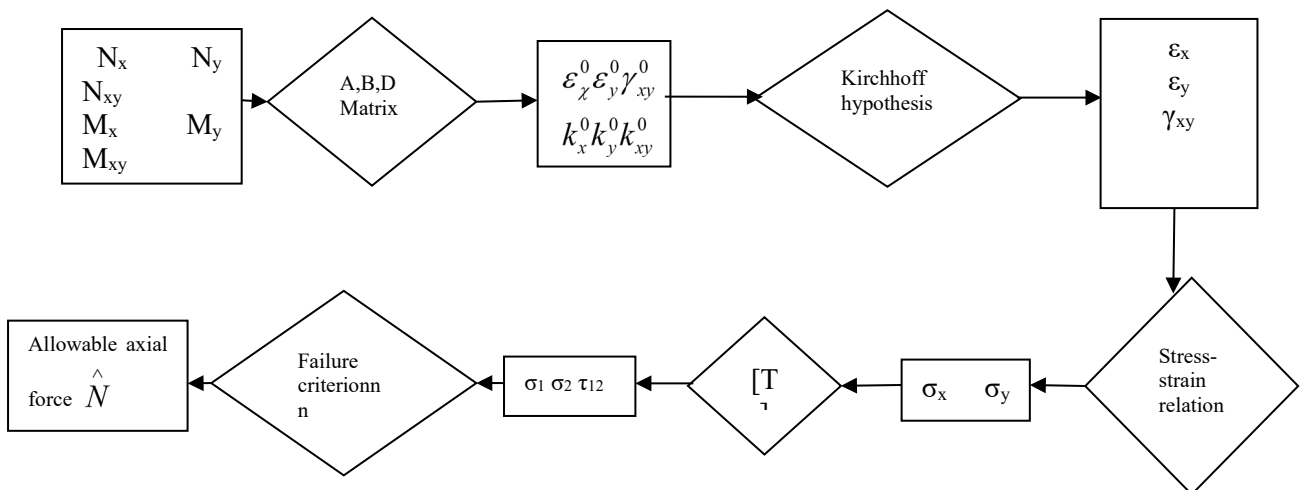

Figure 3. Concept for estimation of allowable axial force $\hat{N}$

Taking into account this procedure as well as the eqs. (1)-(6) and the formula for the inverse *ABD* matrix, it can be written:

$$\varepsilon_x^0 = a_{11}N_x \tag{7}$$

$$\varepsilon_y^0 = a_{12}N_x \tag{8}$$

$$\gamma_{xy}^o = a_{16}N_x \tag{9}$$

$$k_x^0 = b_{11}N_x \tag{10}$$

$$k_y^0 = b_{12}N_x \tag{11}$$

$$k_{xy}^0 = b_{16}N_x \tag{12}$$

Therefore:

$$\varepsilon_x = \varepsilon_x^0 + zk_x^0 \tag{13}$$

$$\varepsilon_y = \varepsilon_y^o + zk_y^0 \tag{14}$$

$$\gamma_{xy} = \gamma_{xy}^o + zk_{xy}^0 \tag{15}$$

Thus the stresses σ_x, σ_y, τ_{xy} for each lamina with fibers orientation θ can be obtained by the well-known stress-strain relation:

$$\sigma_x = \overline{Q}_{11}(\theta) \cdot \varepsilon_x + \overline{Q}_{12}(\theta) \cdot \varepsilon_y + \overline{Q}_{16}(\theta) \cdot \gamma_{xy} \tag{16}$$

$$\sigma_y = \overline{Q}_{21}(\theta) \cdot \varepsilon_x + \overline{Q}_{22}(\theta) \cdot \varepsilon_y + \overline{Q}_{26}(\theta) \cdot \gamma_{xy} \tag{17}$$

$$\tau_{xy} = \overline{Q}_{16}(\theta) \cdot \varepsilon_x + \overline{Q}_{26}(\theta) \cdot \varepsilon_y + \overline{Q}_{66}(\theta) \cdot \gamma_{xy} \tag{18}$$

With the aid of matrix *[T(ϑ)]*, the principal stresses σ_1, σ_2, τ_{12} for every lamina can be determined by the following matrix equation:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} T(\theta) \end{bmatrix} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \tag{19}$$

By applying the Tsai-Wu failure criterion for the values σ_1, σ_2, τ_{12} of every lamina,

$$F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \tau_{12}^2 - \sqrt{F_{11} F_{22}} \sigma_1 \sigma_2 \leq 1 \tag{20}$$

where:

$$F_1 = \left(\frac{1}{\sigma_1^{\mathrm{T}}} + \frac{1}{\sigma_1^{C}} \right) \tag{21}$$

$$F_2 = \left(\frac{1}{\sigma_2^{\mathrm{T}}} + \frac{1}{\sigma_2^{C}} \right) \tag{22}$$

$$F_{11} = -\frac{1}{\sigma_1^{\mathrm{T}} \sigma_1^{c}} \tag{23}$$

$$F_2 = -\frac{1}{\sigma_2^{\mathrm{T}} \sigma_2^{c}} \tag{24}$$

$$F_{66} = \left(\frac{1}{\tau_{12}^{F}} \right)^2 \tag{25}$$

the allowable axial tension $\hat{N}$ can be obtained. Since this procedure yields different values of $\hat{N}$, the minimum one should be adopted.
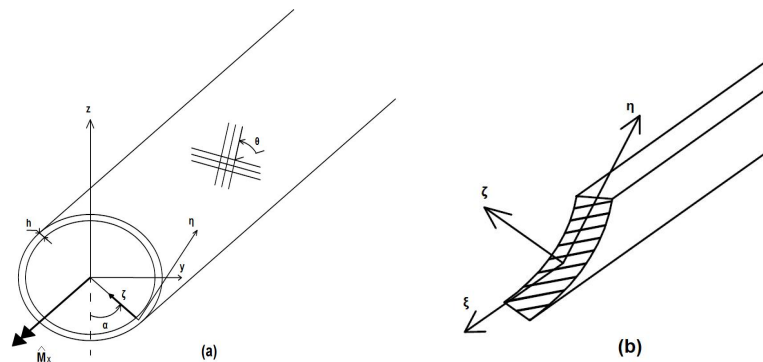
2.2. Torsion

Figure 4. Geometry of a composite pipe subjected to torsion

When a torque $\hat{M}_x$ (see Fig. 4a) is applied to a long composite pipe, a resultant $N_{\xi n}$ is acting on the cross section of the wall (see Fig. 4b). Considering the equilibrium between $N_{\xi n}$ and $\hat{M}_x$ it can be written:

$$N_{\xi n} = \frac{\hat{M}_x}{2\pi R^2} \tag{26}$$

where R is the radius of the pipe (R=D/2).

Taking into account that:

$$N_\xi = 0 \text{ (because of absence of axial force)} \tag{27}$$

$$N_n = 0 \text{ (because of absence of external pressure)} \tag{28}$$

$$M_\xi = M_n = M_{\xi n} = 0 \text{ (because of absence of moments)} \tag{29}$$

The corresponding strains $\varepsilon_\xi^o, \varepsilon_n^o, \gamma_{\xi n}^o, k_\xi^o, k_n^o, k_{\xi n}^o$ can be obtained by the following matrix equation:

$$
\begin{bmatrix}
a_{11} & a_{12} & a_{16} & b_{11} & b_{12} & b_{16} & -1 & 0 & 0 & 0 & 0 & 0 \\
a_{12} & a_{22} & a_{26} & b_{21} & b_{22} & b_{26} & 0 & -1 & 0 & 0 & 0 & 0 \\
a_{16} & a_{26} & a_{66} & b_{61} & b_{62} & b_{66} & 0 & 0 & -1 & 0 & 0 & 0 \\
b_{11} & b_{21} & b_{61} & d_{11} & d_{12} & d_{16} & 0 & 0 & 0 & -1 & 0 & 0 \\
b_{12} & b_{22} & b_{62} & d_{12} & d_{22} & d_{26} & 0 & 0 & 0 & 0 & -1 & 0 \\
b_{16} & b_{26} & b_{66} & d_{16} & d_{26} & d_{66} & 0 & 0 & 0 & 0 & 0 & -1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\cdot
\begin{Bmatrix}
N_\xi \\ N_n \\ N_{\xi n} \\ M_\xi \\ M_n \\ M_{\xi n} \\ \varepsilon_\xi^0 \\ \varepsilon_n^0 \\ \gamma_{\xi n}^0 \\ k_\xi^0 \\ k_n^0 \\ k_{\xi n}^0
\end{Bmatrix}
=
\begin{Bmatrix}
0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \hat{M}_x / 2\pi R^2 \\ 0 \\ 0 \\ 0
\end{Bmatrix}
\tag{30}
$$

The solution of the above equation yields:

$$\varepsilon_\xi^o = \frac{a_{16}\hat{M}_x}{2\pi R^2} \tag{31}$$

$$\varepsilon_n^o = \frac{a_{26}\hat{M}_x}{2\pi R^2} \tag{32}$$

$$\gamma_{\xi n}^o = \frac{a_{66}\hat{M}_x}{2\pi R^2} \tag{33}$$

$$k_\xi^o = \frac{b_{61}\hat{M}_x}{2\pi R^2} \tag{34}$$

$$k_n^o = \frac{b_{62}\hat{M}_x}{2\pi R^2} \tag{35}$$

$$k_{\xi n}^o = \frac{b_{66}\hat{M}_x}{2\pi R^2} \tag{36}$$

Therefore

$$\begin{Bmatrix} \varepsilon_\xi \\ \varepsilon_n \\ \gamma_{\xi n} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_\xi^o \\ \varepsilon_n^o \\ \gamma_{\xi n}^o \end{Bmatrix} + \zeta \begin{Bmatrix} k_\xi^o \\ k_n^o \\ k_{\xi n}^o \end{Bmatrix} \tag{37}$$

For symmetric lay up of fibers the maximum shear stresses $\tau_{\xi n}$ are taking place into the exterior layers of the pipe. Therefore, for $\zeta = h/2$ equations (31)-(37) yield:

$$\begin{Bmatrix} \varepsilon_\xi \\ \varepsilon_n \\ \gamma_{\xi n} \end{Bmatrix} = \frac{M_x}{2\pi R^2} \begin{Bmatrix} a_{16} + \dfrac{h}{2} b_{16} \\ a_{26} + \dfrac{h}{2} b_{62} \\ a_{66} + \dfrac{h}{2} b_{66} \end{Bmatrix} \tag{38}$$

Taking into account the above equation, the stress-strain relation for the exterior lamina with fiber orientation θ can be written:

$$\begin{Bmatrix} \sigma_\xi \\ \sigma_n \\ \tau_{\xi n} \end{Bmatrix} = \begin{bmatrix} Q_{ij(\theta)} \end{bmatrix} \cdot \begin{Bmatrix} \varepsilon_\xi \\ \varepsilon_n \\ \gamma_{\xi n} \end{Bmatrix} \tag{39}$$

Therefore, the principal stresses of the exterior lamina can be obtained by the following well known formula:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} T_{(\theta)} \end{bmatrix} \cdot \begin{Bmatrix} \sigma_\xi \\ \sigma_n \\ \tau_{\xi n} \end{Bmatrix} \tag{40}$$

With the aid of the above equation, the allowable torque M_x^a can be obtained by the Tsai-Wu criterion:

$$F_1 \sigma_1 + F_2 \sigma_2 + F_{11} \sigma_1^2 + F_{22} \sigma_2^2 + F_{66} \tau_{12}^2 - \sqrt{F_{11} F_{22}} \sigma_1 \sigma_2 \le 1 \tag{41}$$

where
$F_1 = 1/\sigma_1^T + 1/\sigma_1^C, \quad F_2 = 1/\sigma_2^T + 1/\sigma_2^C, \quad F_{66} = (1/\tau_{12}^F)^2, \quad F_{11} = -1/\sigma_1^T \sigma_1^C, \quad F_2 = -1/\sigma_2^T \sigma_2^C.$

3. Model implementation and results

3.1. Axial force capacity
Taking into account the derived model for axial force capacity, the allowable tensile force $\hat{N}$ is estimated for pipes made by the E-Glass/Epoxy. The algorithm has been coded in Mathematica platform [9]. In the following diagrams (Fig. 5) the allowable values $\hat{N}$ are demonstrated for pipes of diameters Dia=0.1 m - 1.2 m consisting of plies of thickness 0.150 mm, fiber orientation θ=$\pm15^0$, $\pm30^0$, $\pm45^0$, $\pm60^0$, $\pm75^0$ and number of plies NP=10, 20, 30, 40, 50.
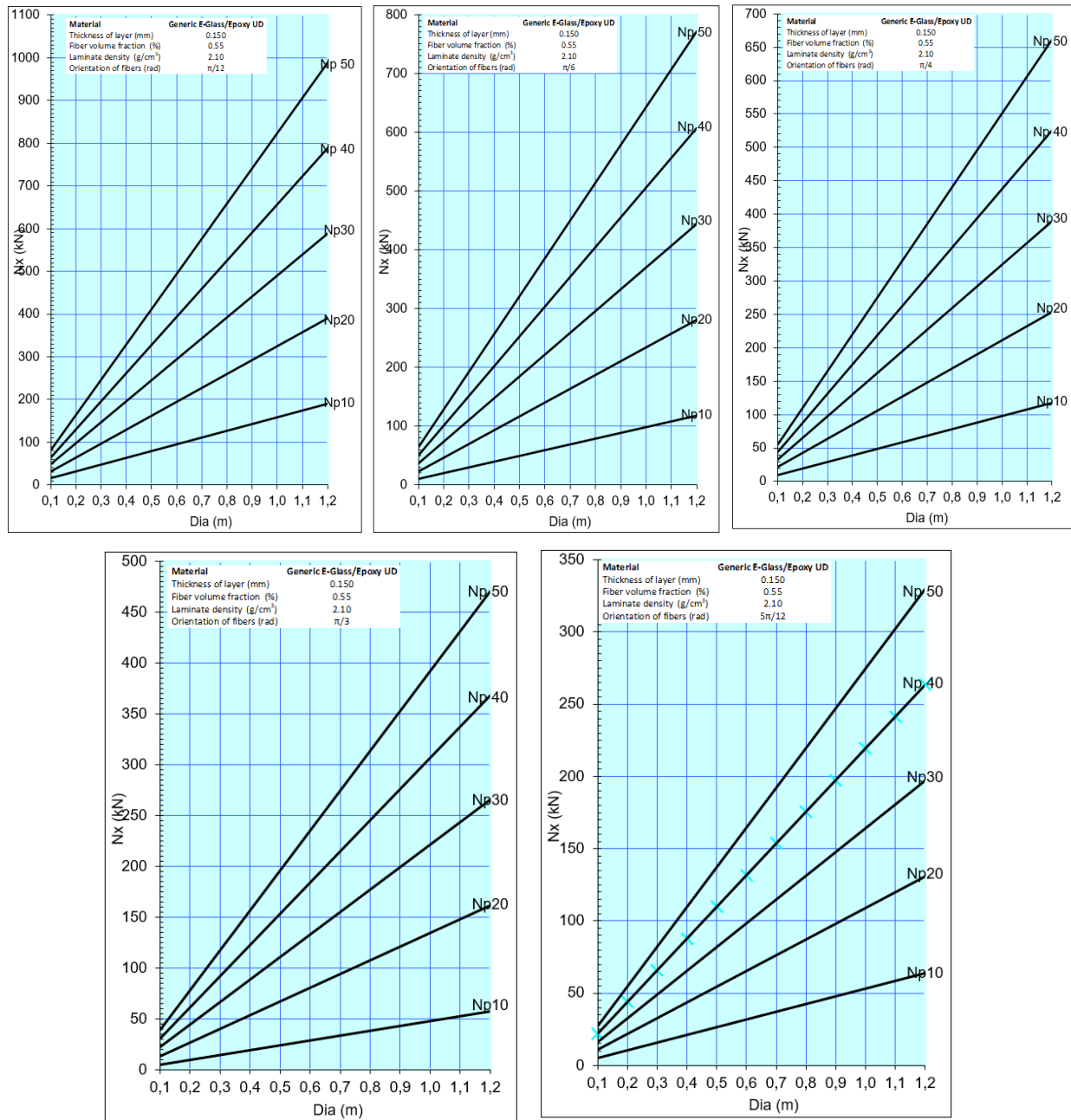
Figure 5. Axial force capacity for E-Glass/Epoxy multilayered pipelines

3.2. Torsional moment capacity

Taking into account the model for torsional moment capacity, the allowable torsional moment M_y has been estimated for pipes made by the materials E-Glass/Epoxy. In the following diagrams (Fig. 6) the allowable values M_y are demonstrated for pipes of diameter: Dia= 0.10 m - 1.20 m constituting by plies of thickness 0.150 mm, fiber orientation $\theta = \pm 15^0$, $\pm 30^0$, $\pm 45^0$, $\pm 60^0$, $\pm 75^0$ for number of plies N_P=10 - 50.
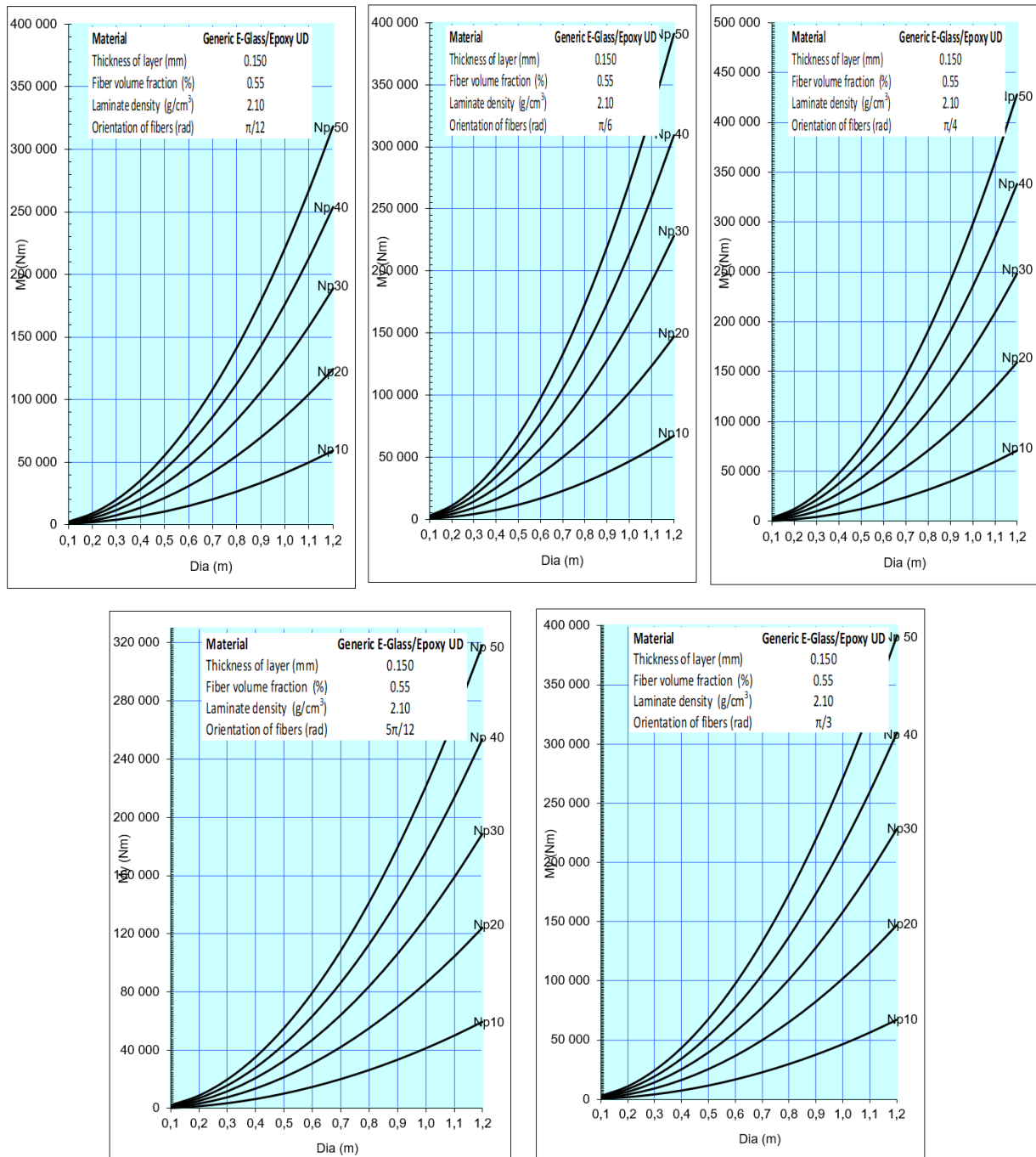
Figure 6. Torsional moment capacity for E-Glass/Epoxy multilayered pipelines

4. Conclusions

1. In the present work, stress analysis of multilayered FRP pipelines during S-lay offshore installation is performed.
2. With the aid of the classical lamination theory (CLT) of anisotropic materials, mathematical models are derived for calculating the axial tension and torsion capacity of pipelines during offshore installation.
3. Unlike existing commercial software packages, the proposed analytical models are advantageous because they provide accurate results.

4. Implementation of the models on typical multilayered FRP pipelines made of E-Glass/Epoxy material has been carried out and useful nomographs for quick estimation of tensional force and torsional moment capacity are provided.

References
[1]   Hyer M 2009 *Stress analysis of fiber reinforced composite materials*, DEStech Publications.
[2]   Pavlou D G 2013 *Composite Materials in Piping Applications*, DEStech Publications, 2013
[3]   Bai Y and Bai Q 2005 *Subsea pipelines and risers,* Elsevier.
[4]   Reddy D V and Swamidas A S J 2014 *Essentials of offshore structures*, CRC press.
[5]   Reddy J N 2004 *Mechanics of laminated composite plates and shells,* CRC press.
[6]   Kollár L P and Springer G S 2003 *Mechanics of composite structures,* Cambridge University Press.
[7]   Pavlou D G 2021 Bending capacity of multilayered FRP pipelines during offshore installation, *Proc. of the 3rd Int Conf on Computational Methods & Ocean Technology COTech 2021*, Stavanger.
[8]   Tsai S W and Wu E M 1971 A general theory of strength for anisotropic materials, *Journal of Composite Materials*, **5**, 58-80.
[9]   Wolfram Mathematica, *Mathamatica user manul*, https://www.wolfram.com/mathematica/.