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Bringing Nordic mathematics education into the future

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Preface

This volume presents Nordic mathematics education research, which will be presented at the Ninth Nordic Conference on Mathematics Education, NORMA 20, in Oslo, Norway, in June 2021. The theme of NORMA 20 regards what it takes or means to bring Nordic mathematics education into the future, highlighting that mathematics education is continuous and represents stability just as much as change.

NORMA conferences are always organized in collaboration with the Nordic Society for Research in Mathematics Education (NoRME). NoRME is open to membership from national societies for research in mathematics education in the Nordic and Baltic countries.

Inclusive classrooms and "mathematics education for all" have traditionally been at the core of Nordic mathematics education. Currently, the digital development and possibilities for individualized learning activities widen the understanding of adaption in compulsory education. This push and pull between inclusion and adaption bring the possibility of renewing mathematics education, including pre-school and tertiary levels, while still maintaining the principle of student-centred mathematics education. Mathematics education is also changing at the level of teacher education, which is reflected in the conference papers included in this preceeding.

The International Programme Committee (IPC) of NORMA 20 represents all Nordic countries and includes one representative from the Baltic countries, with a mix of junior and senior researchers. The IPC has organized the submission and review process leading to this volume. The members of the IPC were:

- Guri A. Nortvedt University of Oslo (Chair), Norway
- Nils Buchholtz, University of Oslo, Norway, and University of Cologne, Germany
- Janne Fauskanger, University of Stavanger, Norway
- Freyja Hreinsdóttir, University of Iceland, Iceland
- Markus Hähkiöniemi, University of Jyväskylä, Finland
- Britta Eyrich Jessen, University of Copenhagen, Denmark
- Jüri Kurvits, Tallinn University, Estonia
- Yvonne Liljekvist, Karlstad University, Sweden
- Morten Misfeldt, University of Copenhagen, Denmark
- Margrethe Naalsund, NMBU, Norway
- Hans Kristian Nilsen, Universitetet i Agder, Norway
- Guðbjörg Pálsdóttir, University of Iceland, Iceland
- Päivi Portaankorva-Koivisto, Helsinki university, Finland
- Jelena Radišić, University of Oslo, Norway
- Anna Wernberg, Malmö University, Sweden

The first NORMA conference on mathematics education, NORMA 94, was held in Lahti, Finland, in 1994. Four years later, the conference was held in Kristiansand, Norway; since then, it has taken place every third year. After each conference, selected papers are published in a proceeding. Due to the

COVID-19 pandemic, the NORMA 20 conference was postponed until 2021; however, many conference papers were in progress and authors were given the opportunity to continue working on them within the original planned timespan. Traditionally, papers are presented at the conference, allowing the authors to receive feedback that is valuable towards finalizing the paper. Instead, the authors have used two rounds of reviewer feedback to substantially improve their papers. In this process, the NORMA community established in 1995, together with external reviewers who are experts in the different fields studied and presented in the papers, have played an important role in producing the Preceeding.

We believe that the NORMA 20 Preceeding is the first conference preceeding to be published, containing 36 papers from authors representing six countries.

After the conference, a traditional Proceeding will be published, containing papers written by submitting authors who decided to wait until after the conference to finalise their papers, to take advantage of feedback from both conference participants and reviewers when they revise their papers.

The IPC would like to extend our thanks to all authors and reviewers for their efforts towards this volume.

Oslo, January 2021, on behalf of the IPC

Guri A. Nortvedt

Exploring opportunities to learn mathematics in practice-based teacher education: a Norwegian case study

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This study critically examines the opportunities a group of prospective teachers—participating in learning cycles of enactment and investigation as part of their teacher education—has for developing their mathematical understanding. Using a quick image as an instructional activity, analysis of the prospective teachers' discussion in a co-planning session reveals that the following learning situations are visible: 1) when involved in learning cycles, the prospective teachers got the opportunity to experience how their own mathematical understanding matters for teaching, 2) participation in learning cycles enabled the prospective teachers to apply their own understanding in new contexts and 3) learning cycles provided an opportunity to learn mathematics in a context where the prospective teachers were motivated and convinced about the relevance of the content. Three additional areas for future research are identified and discussed.

Keywords: Learning cycles, teacher education, prospective mathematics teachers, planning.

Introduction

Whereas teacher education traditionally has focused on *what* prospective teachers should learn in preparation for practice, Ball and Cohen (1999, p. 10) suggested instead to focus on *how* prospective teachers could learn "in and from practice". From their review of literature on practice-based pedagogies in mathematics teacher education, Charalambous and Delaney (2020) note that most of the research has been carried out in a US context, and they call for more studies in other countries. Although results from implementations of practice-based teacher education in the US context are encouraging, Charalambous and Delaney (2020) also report on some potential negative effects. One example is that highlighting teachers and their practices might involve a risk of ignoring students and their learning. We would like to add that highlighting teachers and their practices might also involve the risk of ignoring content. Strong and Baron (2004) also pointed out this risk. In their study of mentoring conversations with beginning teachers, only 2% of the mentors' suggestions were related to content. A lack of focus on content characterized decades of research on teaching and teacher education, until Shulman (1986) identified it as a missing paradigm. When prospective mathematics teachers engage in collective planning, enacting and analysis of teaching as part of their teacher education, there is an inherent risk of focusing more on practical issues than on the mathematics involved in that teaching. To mitigate this risk, and to call attention to the inherent danger of ignoring content in practice-based pedagogies of teacher education, we critically examine the opportunities a group of prospective teachers have to develop their own mathematical understanding when participating in one particular approach to practice-based teacher education.

Theoretical background

This study builds on a sociocultural theory of teaching and learning. Within this theory, we consider opportunity to learn in terms of Wells' (1999) spiral of knowing. This model describes four different opportunities for meaning making. The first opportunity lies in the situated *experiences* that individuals get from participation in communities of practice. Second, *information* is considered as the interpretations that people make of their experiences. Wells then describes the third opportunity as *knowledge building*, which entails active involvement in the meaning making process. The fourth and final opportunity is *understanding*, which relates to the relationship between experience and knowledge building. Understanding, then, is the pinnacle of the cycle of knowing (Wells, 1999).

When exploring what learning to teach in and from practice might be, some researchers have proposed to organize teacher education around core or high-leverage teaching practices (e.g., McDonald, Kazemi, & Kavanagh, 2013). There is not yet a common or agreed-upon definition of "core practice", but core practices are often considered to be frequently occurring practices that novice teachers can begin to learn and enact in teacher education (McDonald et al., 2013). Core practices are also research-based, and they have the potential to contribute to student learning. Although only a particular aspect of teaching is singled out in a core practice, core practices still maintain the complexity of teaching (Lampert et al., 2013). One example is the practice of leading group discussions. With the aim of making mathematics teacher education more practice-based, McDonald and colleagues (2013) developed a model where prospective teachers collectively engage in authentic instructional activities in "learning cycles of enactment and investigation". Learning cycle is a framework for learning to enact core practices in teacher education that rests on a sociocultural perspective of learning as collective activity (e.g., Wells, 1999). Under careful supervision of a teacher educator, prospective teachers collaboratively plan, rehearse, enact and analyse a particular instructional activity (McDonald et al., 2013). These instructional activities are pre-defined episodes of teaching that are found to be suitable for learning and enacting core practices.

The instructional activity we focus on in this paper is "quick images". In this activity, students are presented with an image like the one in Figure 1. After having viewed this image for three seconds, the students are asked about the total number of dots. Quick images are designed to help students visualize numbers and form mental representations of a quantity by being invited to explain how they organized and subitized quantities in order to find the total number of dots in the image. Through this activity, where students get the opportunity to learn about the commutative, associative and distributive properties of multiplication (Schumway, 2011), prospective teachers get the opportunity to enact core practices like eliciting student thinking and leading group discussions.

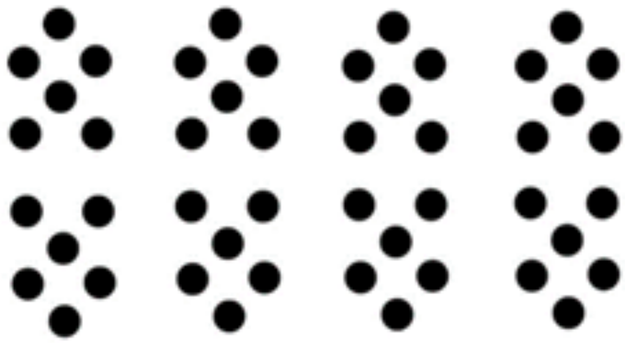

Figure 1: Quick image planned to be used (Matematikksenteret, n.d.)

Recently, the practice-based pedagogy of learning cycles has been implemented in some professional development programs in Norway (Fauskanger & Bjuland, 2019), as well as in some initial teacher education programs (Rø, Valenta, Langfeldt, & Ødegaard, 2019). The present study from the *Practicing dialogue-based mathematics teaching* project (in Norwegian: "Øve på Dialogbasert Undervisning", ØDU) represents another attempt to implement this practice-based pedagogy in a Norwegian teacher education context. Hopefully, such a practice-based approach to mathematics teacher education will better prepare prospective teachers to carry out the complex work of teaching mathematics.

Method

In the ØDU project, 21 Norwegian prospective primary school teachers (school years 1–7) worked together in three repeated learning cycles (McDonald et al., 2013). The aim was to learn core mathematics teaching practices such as launching mathematical problems, using mathematical representations, aiming towards a mathematical goal, facilitating student talk, and eliciting and responding to students' mathematical ideas (Lampert et al., 2013). The prospective teachers were in the third year of their teacher education program and had elected a 30 ECTS[1] credit specialization course in mathematics[2] (beyond the first 30 ECTS credit course, which is mandatory). They were divided into three groups with seven prospective teachers in each group. A teacher educator guided every group. One of the groups volunteered to participate in the research study, and we videotaped all three cycles in this group. The three cycles were located at two different primary schools. Only the co-planning phase (98 minutes) of one learning cycle is analysed for the purpose of this paper. The following enactment phase was carried out with a group of seventh grade students in their own classroom.

Inspired by Wells (1999), we started the analysis by dividing the co-planning session into episodes according to different thematic foci in the prospective teachers' discussions. An episode represents a part of the session where the prospective teachers have a focused discussion. Another episode begins when there is a clear shift in the focus of the discussion illustrated by an utterance (e.g., a question or a statement). Two of the authors first identified 27 episodes individually before reconciling. There

[1] European Credit Transfer and Accumulation System (ECTS)

[2] Course literature was based around Kazemi and Hintz (2014)

was total agreement on the episodes. We have selected parts of the prospective teachers' discussions from two episodes for further analysis, aiming to illustrate the prospective teachers' opportunities to learn mathematics in a particular co-planning session. These episodes are analysed in terms of Wells' (1999) model of meaning making.

Results

The first episode considers the prospective teachers' opportunity to learn in terms of Wells' (1999) spiral of knowing, related to the third opportunity of *knowledge building.* The prospective teachers' uncertainty about the distributive and associative properties of multiplication is visible in the discussion. There is a need for clarification, discussing these properties for their own sake that entails active involvement in their meaning making process. The second episode demonstrates the fourth opportunity, *understanding*, in Wells' model of meaning making. The relationship between the prospective teachers' experiences (participation in the mathematical discussion concerning these properties) and their knowledge building illustrate their understanding of the distributive property. These discussions reveal their opportunity to apply their knowledge about the distributive property of multiplication in new task situations while planning to use in the lesson (see Figure 1).

Opportunities of knowledge building: having their own understanding of core mathematical concepts challenged

This dialogue takes place while the prospective teachers are engaged in a discussion of the mathematical content included in the quick image (see Figure 1). In the previous episode, they discussed possible suggestions aiming towards a mathematical goal for the lessons. In the continuation of the co-planning session, the teacher educator says that he is missing something about the distributive property. Astrid[3], one of the prospective teachers, responds that twelve can be written as three times four. The teacher educator challenges her by asking, "What property do you think about now?" Astrid responds, "Isn't this the distributive property [of multiplication]? Or is it maybe the associative [property of multiplication]?" We interpret this response to indicate uncertainty about her own mathematical understanding. The teacher educator follows up with a hesitant, "Ehh…", and we consider this to indicate that he has become aware of Astrid's uncertainty, and that he is considering how to follow up on it. Then, several of the other prospective teachers simultaneously start making suggestions about these properties. The teacher educator decides to intervene by giving an example of the distributive property, pointing at one of the quick images in which they have written $(5 + 1) \times 8$: "Here, I would say that it is appropriate to focus on the distributive property. The distributive property says that (3s) we can multiply (.) distribute this eight on both of these numbers. So, eight times one and eight times five." Astrid points at several copies of the quick image (Figure 1) and says: "This is the same here (2s). Well, (2s) the fours are outside [the parenthesis] and then this is inside." The teacher educator appears to realize that Astrid is mixing up associativity and distributivity, and he points at the quick image, written as $(3 \times 4) \times 4$ while responding: "Yes, I think that's more the associative property. It would be the same [answer] if we calculate that parenthesis first, or if you start with the number outside [the parenthesis]. Because we don't multiply four times four and then four times three. No, this isn't the distributive property."

[3] All names used in the paper are pseudonyms

The teacher educator continues by giving examples of the associative and distributive properties of multiplication and asks the prospective teachers if they are familiar with these properties or if they would like him to recapitulate the differences between associativity and distributivity. Three prospective teachers, Hedda, Viktor and Tuva, all give brief responses, indicating that there is a need to elaborate on these concepts.

Our analysis illustrates how prospective teachers get an opportunity to develop their understanding of core mathematical concepts after having been challenged by the teacher educator in the co-planning session of the learning cycle. This situation provides an opportunity for their knowledge building, entailing active involvement in their meaning making process (Wells, 1999).

Opportunities of understanding: applying knowledge about the distributive property of multiplication in new situations

The second episode takes place at the end of the planning session. The participants have been concerned with the learning goals for the lesson, how to launch the problem, how to elicit and respond to students' thinking, and how they as prospective teachers can lead a targeted discussion regarding the distributive property of multiplication. At this point, the teacher educator's initial plan was to end the planning session. Instead, he asks a final question before the rehearsal, "Would you like to include a summing up session where you illustrate the use of the distributive property (...) for instance if we are to calculate 5 times 16 [5×16], then it might be easier to calculate $5 \times (10 + 6)$?" After some discussion among the prospective teachers, Tuva suggests that, "Six times eight is not a difficult task for them [the students]. So why should we do this? But five times sixteen, for instance, is much more difficult to calculate mentally." Tuva's suggestion indicates that she recognizes the power of the distributive property of multiplication in an example that is more complex than the given task in the quick image. Asbjørn follows up on this by suggesting another example that applies the distributive property by introducing twelve times four: "So, if we have given them [the students] a task: twelve times four, do you manage to calculate this? And then it would be easier [for the students] if we write it as $(10 + 2) \times 4$". Asbjørn's suggestion demonstrates knowledge of the distributive property. Implicitly, he also appears to demonstrate knowledge of the associative property since he considers eight times six, which Tuva brings into the discussion, observing that $8 \times 6 = (4 \times 2) \times 6 = 4 \times (2 \times 6) = 4 \times 12$. After this, Astrid draws attention to larger numbers: "It could be completely different numbers as well. If we consider 1072 and show [the students] that they can split it (...) 1072 times five okay, 1000 and 70 and 2 (inaudible)." We observe that Astrid, in the first episode, seemed to be uncertain about the distributive and associative property of multiplication, but here, at the end of the planning session, she is able to apply the distributive property to a self-selected example. From this illustrative example, we notice that at least three of the seven prospective teachers correctly apply the distributive property.

At the end of this episode, the prospective teachers all agree about paying attention to the applications of the distributive property of multiplication when they are engaged with the students in the subsequent enactment of the lesson.

Discussion

Whereas practice-based teacher education aims at enabling prospective teachers' learning in and from practice (Ball & Cohen, 1999), some teacher educators are concerned that increased emphasis on practice might lead to a corresponding decrease in the emphasis on content (Strong & Baron, 2004). Our analysis shows that the practice-based approach of learning cycles might provide ample opportunities for prospective teachers to focus on mathematics. The following discussion will highlight three important aspects.

First, when prospective teachers were involved in learning cycles, they got the opportunity to experience how their own mathematical understanding matters for teaching. In a regular mathematics class at the university, they might have been tempted to move on without truly understanding the mathematics. However, in a context where they engage in planning for an instructional activity that they are going to teach themselves, they might experience, like Astrid did, that they *have to* understand the mathematical content. This situation thus provided for opportunities to meaningfully engage with content, and the prospective teachers did not have to question whether or not this knowledge of content was relevant for their own work of teaching.

Second, the context of learning cycles enabled the prospective teachers to apply the mathematical concepts they discussed earlier as part of their collective lesson planning. In the first episode, the prospective teachers found it necessary to revisit the associative and distributive properties of multiplication in order to proceed with their planning. In the second episode, we observe how they took the opportunity to apply these concepts in new and meaningful situations when planning a summing up session. Overall, these two findings could indicate a development, from the first episode to the second, of the prospective teachers' understanding of these mathematical concepts in the context of planning for teaching.

Third, the context of learning cycles provided an opportunity to learn mathematical concepts in a practice-based context. Oftentimes, mathematics teaching is located at the university campus in a situation where prospective teachers sometimes fail to see the connection between the mathematical content they are struggling with and the work they think they are being prepared for (e.g., McDonald et al., 2013). In field placement, the focus is sometimes on other aspects of the work than the mathematical content (for mentoring sessions see e.g., Strong & Baron, 2004), and prospective teachers tend to experience this as a disconnect between these two fields. In the context of learning cycles, the prospective teachers get an opportunity to bridge the gap between theory and practice in a meaningful context.

We are aware that the data material is rather limited, and some critical voices might question the importance of the findings. However, we suggest that the findings reveal some opportunities for prospective teachers' knowledge building and understanding of the distributive and associative properties of multiplication in a co-planning session.

Conclusion

This study provides existence proof (Schoenfeld, 2007) that prospective teachers might get opportunities to develop their own mathematical understanding in a practice-based context like that of the ØDU project. However, our study does not identify factors that provide these opportunities to learn in the structure of the learning cycles. Below, we point to three plausible factors for further investigation in future research.

The first factor relates to *the enactment phase*. In the ØDU project, the prospective teachers are required to carry out the lesson with actual students after the planning session. In a similar study, Rø et al. (2019) questioned the importance of the enactment phase and called for more research on this subject. We find it plausible that the close relationship to practice and the enactment with students influence the development of the prospective teachers' mathematical understanding in the context of the ØDU learning cycle. Without enactment, the prospective teachers might miss the opportunities to experience that their own mathematical knowledge matters for teaching. We therefore suggest that practice-based approaches such as the ØDU project should have parts of its instruction in the practice field, but further studies are needed to investigate how the close relationship to practice influences prospective teachers' opportunities to learn.

The second factor concerns *the structure of the planning session*. The prospective teachers are required to go through the mathematical content as there is allocated time for this. The session is divided into topics (with timeframes), and therefore they are able to focus on different parts of the work of teaching in the corresponding part of the session. They are also expected to have worked on the task prior to the session. We think these are important aspects of the structure that influence the prospective teachers' opportunities to learn, and future research should explore how such aspects of the structure might influence the opportunities for prospective teachers' learning.

A third plausible factor is *the teacher educator*. The teacher educators' own mathematical knowledge, their relationship with the prospective teachers, their abilities to orchestrate mathematical discussions, their abilities to seize pedagogical opportunities, and their abilities to assess the prospective teachers' understanding and intervene as they go along, seem important for prospective teachers' learning. Future research should investigate whether and how teacher educators might influence the prospective teachers' opportunities to learn mathematics in the context of practice-based teacher education.

The idea of making teacher education more practice-based is not new (e.g., Ball & Cohen, 1999), and progress has been made to develop and explore practice-based pedagogies in mathematics teacher education (Charalambous & Delaney, 2020). So far, a significant portion of the research on practice-based teacher education has been conducted in a US context, and it is important to study affordances and constraints of various approaches in other contexts. Our study explores how one specific practice-based pedagogy of mathematics teacher education might work in a Norwegian teacher education context. The results from our study indicate that this practice-based pedagogy might provide opportunities for prospective teachers to learn mathematics, but it also points toward a need to further investigate what might be entailed in the work of teaching mathematics in such an approach to teacher education. It could be interesting for future research to investigate if the same (or other) opportunities were present in other co-planning sessions. Another direction for future research could be to investigate how the mathematical issues addressed in the planning session played out in the classroom and in the reflection sessions.

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