Maths003

LEARNING TO NOTICE LEARNERS' MATHEMATICAL THINKING WHILE CO-ENACTING INSTRUCTION

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Abstract

The study critically examines in-service teachers' opportunities to develop their ability to learn professional noticing when co-enacting instruction. It focuses particularly on exploring what as well as how teachers notice when the participants pause the instruction by initiating a Teacher Time Out (TTO) in the enactment phase of learning cycles of enactment and investigation. Fourteen primary school in-service teachers collaborate with teacher educators and participate in TTO discussions. A framework of noticing was applied in the analyses with the aim of shedding light on the ways in which these discussions enable teachers to collectively learn to notice learners' thinking. Findings from the analyses of all the 189 TTOs reveal that in 45 of the TTOs, teachers made sense of individual learners' mathematical thinking (focused noticing), and also used evidence from the situation in the lesson to reason and elaborate on important teaching and learning issues based on learners' thinking (extended noticing). These are examples of higher-level noticing.

Introduction

A critical role of teacher education and professional development (PD) is to equip teachers with teaching practices that support learners from diverse backgrounds. Such practices have been referred to as ambitious because they support the learning of all learners "across ethnic, racial, class and gender categories" and because they "aim to deepen learners' understanding of mathematical ideas" (McDonald et al., 2013, p. 385). One core activity of ambitious teaching practices is sizing up learners' ideas and responding (Ball et al., 2001). Novice teachers are, however, found to have had limited exposure to interpreting learners' mathematical thinking during teacher education (Jacobs et al., 2010). They are often able to talk about ambitious teaching practices, but the enactment of such practices is more challenging (e.g. Sleep & Boerst, 2012; Thompson et al., 2013). Learning ambitious teaching practices takes time (e.g. Kinser-Traut & Turner, 2020) and the enactment of such practices is thus important, also for mathematics teachers' PD (Kavanagh et al., 2019).

In this study, *learners' thinking* refers to the strategies, representations and reasonings they use. Learners' thinking is a coherent and logical approach to mathematical reasoning that often differs from what mathematicians and other adults use. For teachers, noticing learners' thinking is essential and research has suggested that developing the ability to notice can be learned through scaffolded support and collaboration (e.g. Star et al., 2011). In the *Mastering Ambitious Mathematics teaching* project (MAM), in-service teachers collaborated in learning cycles of enactment and investigation (for learning cycles, see methods section) so they could develop their ability to notice learners' thinking and build on this in their teaching. For the purpose of this study, the analysis aims to shed light on the ways in which learning cycles enable teachers

to collectively learn to professionally notice learners' thinking. *What* as well as *how* teachers notice (van Es, 2011) is the focus here. Our work is grounded on the underlying assumption that "teacher noticing is worthy of study because teachers can be responsive only to what has been noticed" (Jacobs & Spangler, 2017, p. 192). The present study builds on findings from an exploration of the co-planning discussions in the learning cycles in MAM, suggesting that the teachers focused on particular learners' thinking (focused noticing) and on both learners' thinking and teacher's pedagogy (extended noticing) (Fauskanger & Bjuland, in press).

Theoretical background

Professional noticing builds on the concept of professional vision (Mason, 2002; Sherin & van Es, 2003) as a process through which teachers make sense of what occurs during instruction and make plans to respond to learners' thinking (Sherin et al., 2011). Mason (2002, 2011) presents noticing as a discipline and as a collection of practices. Ball (2011, p. xii) sees noticing "as a practice essential to attending to learners, to the domain for which the teacher is responsible, and to connections between the learners and the domain". Noticing is consequential, it is an awareness that enables action (Mason, 2011) and skilled teachers are quicker to identify situations that require intervention (Miller, 2011) or action. Noticing has consequences for what a teacher observes and does not observe, and for what a teacher does and does not do. Because it can lead to changed practices, noticing is thus "a key component of teaching expertise and of mathematics teaching expertise in particular" (Sherin et al., 2011, p. 79).

Teacher noticing is conceptualised in a variety of ways (Miller, 2011), but the two interrelated and cyclical processes of attending to and making sense of particular events in an instructional setting are often involved (Sherin et al., 2011). For example, Star et al. (2011) include what a teacher attends to as well as what the teacher decides not to attend to in their conceptualisation of noticing. For the purposes of this paper, the term noticing is considered to include: a) attending to learners' thinking throughout learning cycles, b) reasoning about learners' thinking, and c) making informed teaching decisions according to an analysis of these observations (e.g. Jacobs et al., 2011; van Es, 2011).

When working closely with a group of experienced teachers in the U.S., Empson and Jacobs (2008) found that the teachers were unprepared to hear and see learners' mathematics. If a teacher is to learn to notice learners' mathematics, an interrelated and situated set of skills for attending to their mathematics is required; skills that are specialised and therefore require a significant shift in how teachers conceptualize their role (Empson & Jakobs, 2008). Although not usually developed in teacher education programmes (e.g. Ball, 1993; Fennema et al., 1996), and taking years to learn (e.g. Empson & Levi, 2011; Steinberg et al., 2004), these skills are learnable with sustained PD (e.g. van Es & Sherin, 2008). Professional noticing is thus important for cycles of enactment and investigation for PD. The present study draws to a large degree on the analysis of (student) teachers' noticing using video (i.e. Roth McDuffie et al., 2014; Star & Strickland, 2008; van Es & Sherin, 2008). However, our work augments the literature by focusing on context, i.e. situating teachers in the authentic work of teaching through learning cycles. Given this, and building on how they plan to notice (Fauskanger &

Bjuland, in press), it was important to investigate what teachers notice when they co-enact instruction.

In the co-enactment in the MAM project, the participants can pause the instruction by initiating a Teacher Time Out (TTO) so they can think out loud together in the moment, discuss how the teacher might respond to learners' contributions and determine the direction of the further instruction (Fauskanger, 2019; Fauskanger & Bjuland, 2019). After the TTO, instruction continues. TTOs allow the teachers to collectively consider in-the-moment decision-making and then try out the ideas. It can be argued that TTO discussions from participants might distract the learners as such pauses might interrupt the flow of the lessons. However, research has shown (e.g. Gibbons et al., 2017; Fauskanger, 2019) that this is not necessarily an issue. Learners have been informed that TTOs (with a duration often just up to ten seconds) might occur during lessons since such situations can be seen as a learning context for the participants.

With the aim of shedding light on the ways in which the TTOs enabled the teachers to collectively learn to notice learners' thinking by exploring *what* and *how* they notice (van Es, 2011), Fauskanger & Bjuland (in press) analysed the co-planning sessions within the MAM project. When concluding their study, they write that the co-planning sessions appear to be contexts where teachers can practise how to build on learners' mathematics and develop the ability to notice – in particular, *what* to notice (van Es, 2011). While this previous study offers the field insight into co-planning in the context of PD, the researchers point out that in order to make clearer conclusions, we need to develop our understanding of how the different elements in the learning cycles enable teachers to collectively learn professional noticing. Exploring what as well as how teachers notice (van Es, 2011, see Table 1) in the enactment phase of the learning cycles is one way to meet this call. Bearing this in mind, the present study explores TTOs in the co-enactments. The following research question is addressed: How can the participants' TTO discussions in the enactment phase of the learning cycles provide them with opportunities for learning depth of noticing?

Methodology

The present study draws on Lave's (1991) description of learning, thinking and knowing as "relations among people engaged in activity in, with, and arising from the socially and culturally structured world" (p. 67). Thus, sociocultural views on teacher learning inform the study. Learning is understood as it emerges in activities, and from this perspective, teacher learning includes developing the ability to engage in particular practices. Learning cycles (Figure 1) were designed to engage teachers in learning such ambitious teaching practices as professional noticing. In designing the learning cycles, we gave the teachers repeated opportunities to co-plan, rehearse, co-enact and reflect upon a set of intentionally selected instructional activities (e.g. choral counting, quick images, number strings) embedded in learning cycles with teacher educators as supervisors. Moreover, the activities reduced the complexity of the teachers' learning by supporting them in eliciting learners' thinking and in making judgments on how to respond in principled, instructive ways (Kavanagh et al., 2019; Lampert et al., 2013).

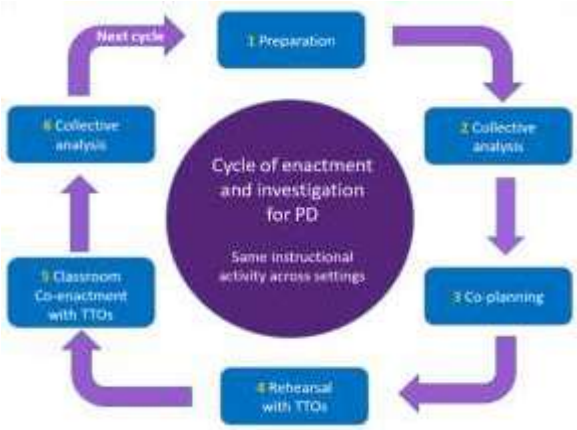

Figure 1. Cycle of enactment and investigation for professional development (adapted from Lampert et al., 2013, p. 229).

Throughout the cycle, the teachers were encouraged to 1) ask questions, explain and justify their mathematical and instructional ideas, 2) find multiple strategies and 3) try to understand what other participants said and did. Thus, a setting was developed where teachers could be engaged together in the joint enterprise of learning professional noticing in which questions and disagreements were viewed as a productive part of the enterprise. Fourteen Norwegian primary-school teachers worked together in two groups in repeated learning cycles. Each group was guided by a supervisor. The participants met for nine full learning cycles over the course of two years, producing eighteen (18) videotaped cycles. In this paper, the analysed data material has been taken from video recordings of co-enactments in all the cycles. In these co-enactments one teacher was teaching the co-planned lesson while the other teachers and the supervisor were observing.

Van Es (2011, p. 137) identified three main areas within which noticing develops: "what stands out to teachers when they observe teaching, the strategies they use to analyze what they observe, and the level of detail at which teachers discuss their observations" (van Es, 2011, p. 137). This framework of noticing (Table 1) includes an identification of "what is noticed and how teachers' reason about what they observe" as well as "a trajectory of development in these two dimensions from Baseline to Extended Noticing" (van Es, 2011, p. 138). In the present study, this framework was used to analyse the depth and analytical stance of noticing in teachers' TTO discussions throughout the co-enactments.

Table 1. Framework for learning to notice learners' mathematical thinking (van Es, 2011, p. 139).

	Level 1 Baseline	*Level 2 Mixed Level*	*Focused*	*3Level 4 Extended*
What teachers notice	Attend to whole class environment, behaviour, and learning, and to teacher pedagogy	Primarily attend to teacher Pedagogy Begin attend	Attend to particular learners' mathematical tothinking to	Attend to the relationship between particular learners' mathematical

		particular learners'* mathematical thinking and behaviours		thinking and between teaching strategies and learners' mathematical thinking
How teachers notice	Form general impressions of what occurred	Form general impressions and highlight noteworthy events	Highlight noteworthy events	Highlight Noteworthy Events
	Provide descriptive and evaluative comments	Provide primarily evaluative comments with some interpretive comments	Provide interpretive comments	Provide interpretive comments
	Provide little or no evidence to support analysis	Begin to refer to specific events and interactions as evidence	Refer to specific events and interactions as evidence	Refer to specific events and interactions as evidence
			Elaborate on events and interactions	Elaborate on events and interactions
				Make connections between events and principles of teaching and learning
				Using interpretations, propose alternative pedagogical solutions

- Student was originally used.

What teachers notice captures both *whom* they notice and the *topic* of their analysis. Whom they notice concerns whether the participants focus on the class as a whole, learners as a group, particular learners, the teacher responsible for enacting instruction or themselves (van Es, 2011). Topic refers to issues they identify, "such as remarks focused on the pedagogical strategies, behavior or mathematical thinking, or the classroom climate" (van Es, 2011, p. 138). *How* teachers analyse what they notice includes both their *analytical stances* and *levels of depth*. Analytical stance refers to the approach teachers take to their analysis and captures whether the participants inquire into teaching and learning as well as whether they *evaluate* or

interpret what they observe. When evaluating, the participants make "uninformed judgments about what was good or bad or should have been done differently" (van Es, 2011, p. 138). Interpreting refers to the teachers' efforts "to reason about what they observe, to understand the roots of an idea, and to explain what was meant by a particular statement, drawing, gesture, or expression" (van Es, 2011, p. 138). Inspired by Karlsen and Helgevold (2019), our analytical stance has been to identify whether the teachers evaluate, describe or make claims (low-level noticing) or whether they interpret, explain and give reasons in the teaching situation (high-level noticing).

Bearing this in mind, in the analyses all 189 TTOs were transcribed and coded using the framework of noticing (van Es, 2011). This framework includes four levels of noticing – baseline, mixed, focused and extended levels (Table 1). Each level represents *what* teachers in collaboration with teacher educators notice as well as *how* they notice. Their attending to whole class observations or teacher pedagogy represents lower levels of noticing. At higher levels of noticing, the focus is on particular learners or connections between teaching and learners' learning. Descriptive and evaluative comments represent a lower level of noticing, while higher levels of noticing are characterised by a focus on learners' mathematics. Lastly, a qualitative in-depth analysis of coded TTOs was conducted to identify and explore examples of noticing on different levels. In this paper, an overview of noticing in the TTOs will be presented, followed by representative examples from selected TTOs chosen from the data material to present our findings. It is important to emphasise that teachers might be noticing in the enactment outside of TTOs as well, but for the purpose of this paper, only TTOs are analysed.

Findings and Discussion

Table 2 summarises the four levels of noticing identified in the 189 analysed TTOs. In 64 of the TTOs there were no signs of noticing, indicating that the teachers were not concerned with themselves and their own practices (baseline noticing, Level 1), nor were they attending to particular learners' mathematical thinking (higher levels of noticing). Many of these TTOs (no noticing) were often brief, lasting only a few seconds. This dimension was often found at the end of the lessons, or when the teachers were discussing practical issues. One typical example of a TTO at the end of the lesson (TTO4, Gr2, Session 1) illustrates this. Here, one of the observing teachers (OT1) praises the learners: "Lucky teachers who get you for learners, I must say". This was followed by a brief supportive response "Yes" from the teachers before OT1 continued to praise: "You guys impress me". The following example (TT013, Gr2, Session 2) illustrates practical issues when the teacher writes on the board and says: "Oops, now it went a little too quick, but …", followed by the supervisor who responds: "That's okay".

Table 2. Overview of levels of noticing in the TTOs.

All TTOs	No noticing	Baseline	Mixed	Focused	Extended
189	64	53	27	25	20

When analysing the content of the other TTOs, there were many instances (53) where the participants appeared to be concerned with the whole class environment and teacher pedagogy (baseline noticing, Level 1). The following TTO (TTO17, Gr3, Session 4) illustrates this level when the teacher has tried to get the learners to answer why the factor order does not matter when dealing with the associative property of multiplication. She takes a TTO and asks the other participants: "How long should I stretch it before I…should I say what we're aiming for? Or should we take it…" They agreed on the teaching strategy and the teacher went on with the teaching.

In the following TTO (mixed noticing), the teachers are primarily attending to teacher pedagogy, but there are signs of being concerned with the learners' thinking. The focus of the discussion is to find patterns and connections in a quick image (TTO5, Gr2, Session 3), and the teacher has asked the learners if they see more connections. The learners are silent, but the supervisor has heard something one of the learners has said in a low voice, and she wants to include this learner's initiative in the discussion: "I want to take a time-out. Is there anyone here who can help (looks to the other teachers), who heard what she just said? I think there might be". The participants confirm that one of the learners said something in a low voice, and the supervisor suggests that the teacher should ask the learner to repeat what she said, which the teacher thinks is a good idea. This example of a TTO indicates that the discussion is general with little mathematical content, but the teachers are beginning to attend to particular learners' thinking.

As can been seen from Table 2, there were also many instances (25 focused, 20 extended) in which the teachers made sense of learner thinking and used evidence from the situation in the lesson to reason and elaborate on important teaching and learning issues (Karlsen & Helgevold, 2019; van Es, 2011). We suggest that these 45 TTOs are very important situations while co-enacting instruction in order to learn to notice learners' mathematics. In the following, we will delve into two of these TTOs (high-level noticing) to illustrate the teacher's opportunity to explain and reason about learners' mathematics and to make informed teaching decisions on the basis of these observations made in the moment of the teaching situation (e.g. Lamb et al., 2011; van Es, 2011).

In TTO2 (focused noticing, Gr2, Session 6), a word problem in which four students should run an equal distance and altogether 100 metres is introduced and one of the learners has suggested the following symbolic representation: *100 : 4 = (100 : 2) : 2)*. The teacher starts to visualise the mathematical representation on a number line by illustrating with jumps of 25 at a time. Then the supervisor asks the following question:

Supervisor: Can I ask for a time-out?
Teacher: Sure.
Supervisor: Can you draw it like she has, who was it who said it, was it Learner [learner's name]?
Teacher: Yes, I can, sure.

From this TTO, we observe that the supervisor is attending to this particular learner's thinking by suggesting that the teacher should make the visualisation on the number line in accordance

with the learner strategy. In this situation, the supervisor helps the teacher to clarify the learner strategy and to interpret and explain the link between the visual and symbolic representation in the number string. The difference between students' thinking when representing 100 : 4 as (100 : 2) : 2) and 4 × 25 respectively on a number line is clarified. The teacher continues the lesson by summing up what has been said by the learner.

In the highest level of noticing (extended), there is a clearer relationship between individual learners' mathematical thinking, and between teacher strategies and the learners' thinking. In this session (TTO1, Gr2, Session 1), the activity is choral counting in which the learners are challenged to count from the number 19 with jumps of 19 (19, 38, 57 and so on). Before the actual TTO, they have discussed solutions to the addition 190 + 19. The learners have offered several suggestions, written on the board. Two alternative solutions seem to receive the most support from the learners: 209 and 219. The teacher is uncertain about how to continue the teaching and she takes a time out:

Teacher: Can I ask for a time-out here? I'm getting a little stuck here. In relation to…in a way… Ah, you know, ah, what am I supposed to do with these suggestions now?
OT1: You can show them visually, like, how one like took 190 and then added a ten and a nine.
OT2: On a number line.
Teacher: Yes, for example on a number line, yes. That was a good idea.
The teacher continues by illustrating (190 + 10 + 9) on a number line.

The teacher has observed learner solutions (what, see Table 1) written on the board. She is uncertain about how to proceed and the how-question indicates that she is asking for support from the other observing teachers. This question also includes an invitation to the other teachers to make suggestions for possible explanations, helping the learners to understand why 209 is a correct solution. Two of the observing teachers give advice (how, see Table 1) for bringing the teaching forward. OT1 suggests visualising the addition in two steps by decomposing 19 into 10 + 9 by first adding ten to the next hundred (190 + 10) and then adding 9 (190 + 10 + 9) in order to arrive at the solution (209). Building on OT1's suggestion, OT2 elaborates on the visualisation by proposing the idea of illustrating the additions on a number line. The teacher expresses that the idea of using a number line is a good teaching strategy and follows up the teaching to illustrate 190 + 10 + 9 on a number line.

In these two important TTO situations (focused and extended noticing) while co-enacting instruction, the participants are particularly concerned with the learners' mathematical thinking. We observe that different participants ask for a TTO. In the first situation, the supervisor (focused) helps the teacher to clarify a learner strategy while in the other situation the teacher (extended) asks for the TTO to discuss with the other participants how to deal with different solutions from the learners.

These results are interesting and promising. They differ from studies of teacher noticing in video clubs (e.g. Roth McDuffie et al., 2014; Star & Strickland, 2008; van Es & Sherin, 2008) and in post-lesson discussions in lesson study cycles (e.g. Karlsen & Helgevold, 2019). It seems

that the co-enactment phase in learning cycles invites teachers to learn higher levels of noticing than in the two above-mentioned approaches. At these higher levels, participants do not only attend to teacher pedagogy and learner behaviour, but also to particular learners' mathematics and to teaching strategies building on learners' mathematics (van Es, 2011). According to van Es (2011), the video clubs supported the participants in shifting from baseline to advanced levels of noticing. The present study has not focused on the differences between teachers' noticing in the first MAM sessions to the last ones but if this had been done it might have led to similar results. It seems, however, that learning cycles with co-enactments and opportunities to ask for TTOs are promising when it comes to the participants' opportunities to learn professional noticing. Compared to lesson study reflection sessions (e.g. Karlsen & Helgevold, 2019), TTOs in co-enactments in learning cycles seem to invite teachers to attend to the relationship between particular learners' mathematical thinking as well as the relationship between teaching strategies and learners' mathematical thinking. Furthermore, learning cycles seem to invite the participants to make connections between events and principles of teaching and learning and at the same time propose alternative solutions according to these interpretations. According to van Es (2011), this kind of professional noticing relates to high levels of noticing.

Conclusion and implications

Teacher Time Outs (TTOs) in co-enactments in learning cycles of enactment and investigation (Figure 1) in the MAM project have been analysed to provide insight into how these co-enactments create situations for teachers' collective learning of noticing learners' mathematical thinking. It seems that TTOs in co-enactments are contexts where teachers can learn to "size up" learners' ideas and respond to them (Ball et al., 2001). When working together in TTOs, the participants practise how to build on learners' thinking (Empson & Jacobs, 2008), as has been endorsed in many reform documents (for Norway and Malawi see Utdanningsdirektoratet, 2019; Ministry of Education, Science and Technology, 2013). Developing the ability to notice – both *what* to notice and *how* to notice (van Es, 2011, Table 1) – can be learned through scaffolded support and collaboration (e.g. Star et al., 2011) as in the learning cycles in the MAM project.

While this study provides the field with insight into co-enactments in learning cycles in the context of PD, more research is needed. Compared to studies of teacher noticing in video clubs (e.g. Roth McDuffie et al., 2014; Star & Strickland, 2008; van Es & Sherin, 2008) and in lesson study cycles (e.g. Karlsen & Helgevold, 2019), the learning cycles of enactment and investigation (Lampert et al., 2013; Mc Donald et al., 2013) appear to invite teachers to learn higher levels of noticing. However, in order to be able to make clearer conclusions we need to provide systematic descriptions of each element of the learning cycles, also in contexts outside of the MAM project, such as different African contexts and in terms of developing understanding of how the different elements enable teachers as well as student teachers to collectively learn professional noticing. Moreover, studying possible ways of learning within projects like MAM might lead to changes in teachers' classroom practices and will also be of importance for future research.

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