Opportunities to learn professional noticing while co-planning, rehearsing, co-enacting and reflecting on mathematics instruction

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The study explores teachers' opportunities to learn professional noticing while co-planning, rehearsing, co-enacting and reflecting on mathematics instruction in learning cycles of enactment and investigation. Fourteen primary school in-service teachers collaborated with teacher educators in the cycles and the study focuses on exploring what and how these teachers noticed. A framework of noticing was applied in the analyses with the aim of shedding light on the ways in which the participation in learning cycles enables teachers to collectively learn professional noticing. Findings reveal that teachers were provided with opportunities to learn high-level noticing. For instance, they attended to particular students' mathematical thinking and made connections between teaching strategies and students' mathematical thinking.

Keywords: Professional development, learning cycles, professional noticing.

Introduction

Equipping teachers with practices that support students from diverse backgrounds is a critical role of professional development (*PD*). Such teaching practices "aim to deepen students' understanding of mathematical ideas" and support the learning of all students "across ethnic, racial, class and gender categories" (McDonald et al., 2013, p. 385). Teachers' professional *noticing* – a process through which teachers make sense of what occurs during instruction and make plans to respond to students' mathematical thinking (*SMT*) – has become widely accepted as a key teaching practice. What and how teachers notice, matters for student learning (van Es & Sherin, 2021). Novice teachers are often able to talk about SMT (i.e., students' strategies, representations and reasonings), but they find the enactment of practices based on what they noticed challenging (e.g., Thompson et al., 2013). Also experienced teachers are unprepared to notice SMT (Empson & Jakobs, 2008). Learning to enact practices that support all students takes time and because "teachers can be responsive only to what has been noticed" (Jacobs & Spangler, 2017, p. 192) learning such practices is important for PD (e.g., Kavanagh et al., 2019). Such learning is explored in this study.

Noticing is an awareness that enables action and skilled teachers are quicker to identify situations that require intervention (van Es, 2011). Because it can lead to changed teaching practices, noticing is "a key component of teaching expertise and of mathematics teaching expertise in particular" (Sherin et al., 2011, p. 79). Teacher noticing is conceptualised in a variety of ways (van Es & Sherin, 2021). The term is here considered to include a) attending to SMT throughout *learning cycles* of enactment and investigation (Figure 1), b) reasoning about SMT, and c) making informed teaching decisions according to an analysis of observations of SMT.

Developing the ability to notice can be learned through collaboration and scaffolded support (e.g., Star et al., 2011). In the *Mastering Ambitious Mathematics teaching* project (*MAM*), in-service

teachers collaborated in learning cycles so they could develop their ability to notice SMT and enact on what they noticed. The analysis meets the call from previous research (Stockero, 2021) when drawing on sociocultural perspectives while aiming at shedding light on the ways in which learning cycles might enable teachers to collectively learn to notice SMT. Within MAM, an exploration *what* teachers prepare to notice in co-planning discussions suggests that the teachers focused both on particular SMT and on pedagogy (Fauskanger & Bjuland, 2021b). In addition, Fauskanger and Bjuland (2021a) suggest that developing the ability to notice – both *what* to notice and *how* to notice (van Es, 2011) – can be learned through scaffolded support and collaboration in co-enactments in MAM. While these previous studies within MAM offer the field a glimpse into co-planning and co-enactment in the context of PD, they point out that in order to make clearer conclusions, we need to develop our understanding of how all the different elements in whole learning cycles provide teachers with opportunities to learn professional noticing. Bearing this in mind, the present study examines one representative learning cycle. It draws to a large degree on the analysis of teachers' learning from participation in video-based programs (for a review, see Santagata et al., 2021). However, our work augments the literature by situating practicing teachers in the authentic work of teaching (asked for by Stockero, 2021). By also exploring co-planning, this study meets limitations from previous research, namely the lack of focus on preparation to notice (Choy et al., 2017). Exploring *what* as well as *how* teachers notice in whole learning cycles is one way to meet this call. The following research question is addressed: How can teachers' engagement in learning cycles provide them with opportunities for learning *what* and *how* to notice?

Methods

Sociocultural views on teacher learning inform the presented study which draws on a description of learning, thinking and knowing as "relations among people engaged in activity in, with, and arising from the socially and culturally structured world" (Lave, 1991, p. 67). We see learning cycles as contexts for having reasoned dialogues (i.e, dialogues where everyone engages critically but constructively with each other's ideas and where everyone's ideas are treated as worthy of consideration) providing "affordances for changing participation and practice" and thus opportunities for the participants to learn (Greeno & Gresalfi, 2008, p. 172). *Opportunities to learn* is understood as emerging in activities, and from this perspective, teacher learning includes developing the ability to engage in particular practices. Learning cycles (Figure 1) were designed to engage teachers in learning professional noticing.

Figure 1: Cycle of enactment and investigation for PD (Wæge & Fauskanger, 2021)

In designing the cycles, we gave the teachers repeated opportunities to co-plan, rehearse, co-enact and reflect upon a set of intentionally selected instructional activities (e.g., choral counting, quick images, number strings) with teacher educators as supervisors. The activities supported the teachers in noticing SMT and in making judgments on how to respond in principled, instructive ways (Kavanagh et al., 2019). Throughout the cycles, the teachers were encouraged to 1) ask questions, explain and justify their mathematical and instructional ideas, 2) find multiple strategies and 3) try to understand what other participants said and did. Thus, a setting was developed where teachers could be engaged in the joint enterprise of learning to notice in which questions and disagreements were viewed as a productive part of the enterprise. Fourteen Norwegian primary-school teachers (divided into two groups) met for nine full learning cycles over the course of two years, producing 18 videotaped cycles. In this paper, the analysed data material has been taken from video recordings of one representative cycle (shaded in Table 1) where the instructional activity worked on was a quick image (Figure 2, more about quick images at https://tedd.org/quick-images-2/).

Table 1: Video material analysed – an overview (group 2, session 4)

MAM cycle/ group	Collective analysis (all groups)	Co-plan-ning (gr2)	Re-hearsal (gr2)	Co-enact-ment (gr2)	Collective analysis (gr2)	Discussion (all groups)	Discussion (supervisors/r esearchers)
4/2	24:51	59:21	21:29	26:46	19:56	29:42	58:18

A framework developed by van Es (2011) was used to analyse the depth and analytic stance of noticing, focusing on the teachers' discussions. This framework includes four levels. The first two levels (*baseline* and *mixed*) are considered as low-level noticing since the noticing is related to the class as a whole and teacher pedagogy (*what* teachers notice), and the teachers provide descriptive and evaluative comments from particular events throughout the learning cycle (*how* teachers notice). van Es (2011) denotes the two next levels (*focused* and *extended*) as high-level noticing since the teachers are then attending to particular SMT, or they are concerned with the connections between teaching strategies and particular SMT (*what* they notice). The extended level of noticing (*how* teachers notice) is described by van Es (2011, p. 139) as highlighting "noteworthy events", providing "interpretive comments", referring "to specific events and interactions as evidence", elaborating on "events and interactions", making "connections between events and principles of teaching and learning", "using interpretations", and proposing "alternative pedagogical solutions." The co-planning and the collective reflection session were divided into small sequences of utterances which were coded baseline, mixed, focused and extended respectively. Each Teacher Time Out (TTO) in the rehearsal as well as in the co-enactment were coded including these four levels of noticing.

Our analytical stance has primarily been to focus on high-level noticing, identifying whether the teachers are attending SMT and highlighting noteworthy events. When presenting findings from the exploration of how the teachers' engagement in this particular learning cycle provides them with

opportunities for learning *what* and *how* to notice (whether they interpret, explain and give reasons in their discussions), we will follow the learning cycle (Figure 1) step by step, starting with the co-planning session.

Findings and discussion

The focus of attention in the co-planning discussion is summarised in Table 2.

Table 2: The co-planning session

Time	Focus of the co-planning discussion
00:00-22:00	The teachers present student strategies based on student responses from trying out the quick image in their own classrooms. The supervisor writes the strategies on the smartboard. The strategies are then illustrated on a quick image before the participants discuss how to write the students' strategies by using symbols. Based on this discussion, the supervisor initiates a discussion of the commutative property (i.e., $8 \times 3 = 3 \times 8$).
22:00-51:40	The supervisor initiates a discussion about what goal for students learning they should aim at. The associative property of multiplication, as well as the idea of generalisation (extending the quick image) is discussed. About 45 minutes into the discussion, the supervisor suggests a goal from their previous discussion: "Yes, what mathematical idea should we focus on? Is it the associative property or is it more to develop a general expression for the total amount of chocolates in the boxes? [the dots in the quick image *represent* pieces of chocolate]." They agree to aim at the associative property.
51:00-59:21	Discussion of practical teaching strategies and how to structure and teach the activity for the students.

Whereas practicalities were the focus of attention in the last part of the co-planning discussion, different levels of noticing were visible in the first two parts. When predicting student strategies and representing them in the quick image as well as by using symbols, the teachers attend both to the relationship between particular SMT (*what*, *focused*) and between teaching strategies and SMT (*how*, *extended*). As an example, the suggestion "I see four times three, twice" from one of the teachers is followed by a discussion of how this strategy might be represented in the quick image as well as by using mathematical symbols. The teachers make connections between SMT and teaching strategies (*how*, *extended*). In the mid part of the co-planning session, a discussion of the connection between student strategies and teaching is visible when the aim for the lesson is in focus. It ends by a decision that they will challenge the students to develop their strategies into three factors (e.g., $8 \times 3 = 4 \times 2 \times 3$) in order to aim for the goal for the lesson: the associative property of multiplication (e.g., $(4 \times 2) \times 3 = 4 \times (2 \times 3)$). Throughout the co-planning session the participants return to the commutative property of multiplication and some of the participants seem to mix commutativity and associativity.

However – and similar to findings from Fauskanger and Bjuland (2021b) – the participants' preparation to notice (Choy et al., 2017) seems to provide the teachers with affordances for changing practices of *how* and *what* to notice on extended levels (van Es, 2011) and thus opportunities for the participants to learn (Greeno & Gresalfi, 2008) professional noticing.

In the rehearsal as well as in co-enactment in MAM, the participants can pause the instruction by initiating a TTO (Figure 1) so they can think out loud together in the moment, discuss how the teacher might respond to students' contributions and determine the direction of the further instruction. In the rehearsals across all learning cycles in MAM, 175 TTOs were asked for (Wæge & Fauskanger, 2021), 18 in the particular rehearsal analysed here. In four of the TTOs from this particular rehearsal, the participants focused on representing anticipated students' strategies in a quick image, discussing how to write the mathematical ideas in a symbolic language (2×12 and 4×6). We will show one example from the participants' discussion in one of these TTOs to illustrate their opportunity to reason and to gain insights into SMT. This is an important component of high-level noticing (van Es, 2011). One of the teachers came up with the following strategy: "I saw another pattern: eight in each row," and went to the board and showed three groups of eight dots (Figure 2b). Based on this initiative, the teacher, who was chosen as the instructor in the enactment, wrote 3×8 on the board and said: "In which way is it possible to see the eight here [points to the first row]." Another teacher went to the board and showed two groups of four dots in the quick image. Based on this initiative, the instructor wrote $3 \times (2 \times 4)$ (Figure 2c). This example shows how predicted student strategies are discussed and represented in the quick image by the participants, gaining insights into SMT (*focused*). The instructor expressed the following utterance: "We have three factors in each and then they [the students] can talk together about what they see." This utterance indicates that the participants are attending to the relationship between SMT and between teaching strategies and SMT (*extended*). Teacher noticing is conceptualised in a variety of ways (van Es & Sherin, 2021). This example in one of these TTOs illustrates signs of the two interrelated and cyclical processes of attending and making sense of particular events (SMT and teacher strategies), often involved in teacher noticing (Sherin et al., 2011). At the end of this rehearsal, the goal set for the enactment, focusing on the associative property of multiplication was briefly mentioned.

In the co-enactments across all learning cycles in MAM, 189 TTOs were asked for (Fauskanger, 2019) and out of these 125 were identified as opportunities to learn professional noticing on various levels (Fauskanger & Bjuland, 2021b). The co-enactment in the learning cycle analysed here included three TTOs and two out of these were instances of high-level noticing (van Es, 2011). In the following, we delve into one of these TTOs to illustrate the teachers' opportunity to reason about SMT and to make informed teaching decisions based on these observations made in the moment of instruction. Just before the TTO, a student presents her strategy as seeing four six times in the quick image. The teacher circles four dots six times in the image (Figure 2d). When invited to the board, this student writes $4 \times 6 = 24$.

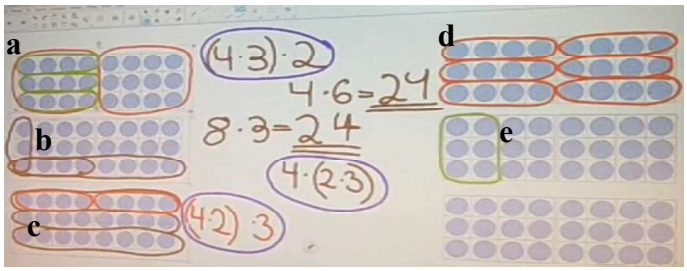

Figure 2: All student strategies from the co-enactment (The multiplication sign used is "·". This is the most common sign to use in Norway. In this paper we decided to use "×" for easier reading.)

The teacher asks the students how to split the 6 in 4 × 6 into "another multiplication task." A student answers: "two times three" and the instructor writes 4 × (2 × 3) on the board and says that she realised that 4 × (2 × 3) could not be represented as the six fours in the image (Figure 2d). From what follows, the teacher obviously is thinking of 2 × 3 as six dots and is confused. She draws six in Figure 2e. The supervisor asks for TTO saying: "I think I see the six in the upper image (Figure 2d)" and asks the students if they "see six" in Figure 2d. They point to the six fours. This TTO invites the participants to attend to the relationship between particular SMT (i.e., that six in this quick image could be represented as six groups of four dots (Figure 2c)), SMT (i.e., six in this quick image represented as six dots (Figure 2e)) and teaching strategies such as representing SMT (*what*) and to work on alternative pedagogical solutions (*how*) as alternative ways of representing students' strategies. Whereas this TTO provides the teachers with opportunities for learning *extended* noticing, in the second TTO the teachers attended to particular students' mathematical thinking (*focused*). In the third TTO, time issues were discussed. Similar to findings from Fauskanger and Bjuland (2021a), our findings indicate that by being provided with affordances for changing practices, the participants have opportunities to learn (Greeno & Gresalfi, 2008) extended levels of noticing (van Es, 2011) in-the-moment of teaching in the co-enactment analysed.

In the collective reflection discussion following the co-enactment, all participants looked at the smartboard from the co-enactment and focused on the different student strategies represented in the quick image (Figure 2). The focus of attention is summarised in Table 3.

Table 2: The co-planning session

Time	Focus of the collective reflection after co-enactment
00:00-05:30	The participants focus particularly on two ideas introduced by two individual students from enactment. One of the students expressed that he/she "saw a box with four across and three upwards, then there were two boxes and I took [multiplied] eight times three", writing 8 × 3 = 24 on the board (see Figure 2a). Another idea was to see four six times in the quick image, writing 4 × 6 = 24 on the board (Figure 2d).
05:30-13:20	They put efforts into practical issues related to the instructional activity of the quick image. One of the participants thinks that only spending 20 minutes on this activity is too demanding and suggests that 45

	minutes would be a more realistic time frame. They are also discussing the use of talk moves and mathematical language related to mathematical terms and concepts.
13:20-19:57	The participants focus particularly on the commutative and associative property of multiplication. The teacher (instructor) posed quite a few questions related to these properties. This is exemplified by the following questions, presented in Fauskanger and Bjuland (2019, p. 126): "Why is it called commutative [property for multiplication] when there are two [factors] and associative [property for multiplication] when there are three [factors]? What's the difference? Why couldn't we just use [the word] commutative, why is another word used there? It's just the same, isn't it? It's all about the order of the factors, or are they [commutative and associative properties of multiplication] two different things?"

As seen in Table 3, the second part of the discussion after the enactment attends to the whole class environment with general descriptions and teacher pedagogy related to time frames, the use of mathematical language and the use of talk moves. This indicates low levels of noticing. Whereas a discussion related to the commutative and the associative property of multiplication was the focus of attention in the last part of this session (no levels of noticing), higher levels of noticing were visible in the first part. From one of the student strategies, seeing eight times three (*what*, *focused*), the participants agreed that it seemed to be difficult for the students to split the eight ($8 \times 3 = 24$), illustrating the mathematical representation in the quick image, $(4 \times 2) \times 3$ (*how*, *focused*). The supervisor elaborated on this particular event (SMT) and challenged the participants to consider how to use this idea with the aim for three factors which are related to their mathematical learning goal for the lesson (*how*, *extended*). In a similar way, they discussed the other student's idea ($4 \times 6 = 24$ on the board), how to "split the six", reiterating this situation from the enactment. The supervisor summarises this discussion by concluding that "the number six could be represented in the quick image in different ways, depending on whether six is seen as the number of groups or as dots within a group" (Fauskanger & Bjuland, 2019, p. 139).

Conclusion and implications

Meeting the call from Stockero (2021) – to explore noticing in-the-moment during the act of teaching – and from Chong et al. (2017) – to explore teachers' noticing while planning instruction – this study explores how teachers' engagement in learning cycles (Figure 1) provides them with opportunities for learning professional noticing. Our analysis indicates that all parts of the learning cycles are contexts where teachers can learn to notice students' ideas and respond to them. When working together in learning cycles, the participants practise how to build on SMT (Empson & Jacobs, 2008), as has been endorsed in many reform documents. In conclusion, developing the ability to notice – both *what* to notice and *how* to notice (van Es, 2011) – can be supported through collaboration (e.g., Star et al., 2011) as in the learning cycles in MAM.

While this study provides the field with a glimpse into one learning cycle in the context of PD, more research is needed. Compared to studies of teacher noticing in video clubs (Santagata et al., 2021), the learning cycles (McDonald et al., 2013) appear to provide teachers with opportunities to learn

higher levels of noticing. However, in order to be able to make clearer conclusions we need to provide systematic explorations of more learning cycles, inside as well as outside of the MAM project.

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