Mathematics teacher educators' thinking about mutuality in teaching

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Stakeholders agree that the mathematical education of teachers needs to focus on mathematical knowledge for teaching, but the practice-based nature of this knowledge poses challenges for mathematics teacher educators — for understanding it, developing tasks that maintain its integrity in practice, and teaching it to teachers in ways that meaningfully support their learning to teach. We know little, however, about how mathematics teacher educators conceptualize the teaching that knowledge is to support. Our analysis reveals that thinking develops from a view of teaching as straightforward, where aspects can be treated in isolation, to a view of it as requiring focused attention while maintaining mutual regard for the whole. This difference has implications for how mathematics teacher educators understand specialized mathematical knowledge and for how to support their understanding and teaching of it.

Keywords: Mathematical knowledge for teaching, teaching, teacher educators.

Introduction

At present there is little doubt about the importance of specialized knowledge for teaching mathematics. Scholars have invested in supporting mathematics teacher educators (including, mathematicians, education specialists, instructional coaches, teacher leaders, and others) to develop their understanding (Even & Ball, 2009). As mathematics teacher educators are a diverse group with disparate backgrounds and uneven expertise and experiences, such work is often challenging and showcases how these differences might matter (Lloyd & Chapman, 2020). Specifically, the practice-based nature of mathematical knowledge for teaching together with uneven understanding of teaching creates challenges in the work that impede progress. A more detailed and systematic unpacking of mathematics teacher educators' thinking would help professional development efforts, however, currently we know very little about how mathematics teacher educators think about teaching or how this thinking connects to their understandings of specialized mathematical knowledge.

Theoretical Background and Research Design

For several years, the research group in which we work (based in the United States) has conducted workshops that seek to develop mathematics teacher educators' understanding of the mathematical demands of teaching. The current study is part of a larger project designed to support mathematics teacher educators' engagement in collegial development of instructional tasks for teachers that address mathematical knowledge for teaching (Ball et al., 2008). We conducted a dozen four-day workshops of approximately 30 participants each. Participants applied as teams that were composed of individuals from different professional settings engaged in the mathematical education of teachers in their local area. Workshops began with a basic introduction to mathematical knowledge for teaching, task writing, and the mathematical demands of attending to justice, where teaching is understood as management of the interactions among teacher, students, and content, occurring in

immediate and broader social environments (Ball, 2018; Brousseau, 1970-1990/2002; Jaworski, 1994), and justice is understood as an essential feature of professionally responsible teaching (Ball, 2018). Moving between whole-group and small-group work, we introduced participants to tools for developing tasks with most of the time spent on writing and reviewing tasks.

For this study, we asked, how do mathematics teacher educators think about teaching? Throughout, our focus was on differences that might matter for their thinking about mathematical knowledge for teaching. We selected 12 participants to interview, with varied demographics, professional affiliation, and experience with the ideas. Approximately one third were affiliated with university mathematics departments, one third with schools of education, and one third were in public primary and secondary schools. Roughly half of the participants had over ten years of experience teaching, and the majority were white women. (All names are pseudonyms.) Although this is a small sample, sampling was purposeful across roles and experiences, and differences in thinking (which is our focus) likely reflect differences in the wider community. We collected extensive pre-workshop applications and post-workshop surveys, including responses to specialized content knowledge tasks with detailed explanations. Our guided interview protocol was designed for multiple purposes: to gather feedback on the workshop, to gain insight into participants' understandings, and to inform future development efforts. Interviews were conducted via video conferencing, recorded, and transcribed. Open-ended questions probed four topics: (i) reasons for applying; (ii) what they learned and found useful; (iii) perspectives on a video clip of a workshop discussion; and (iv) thoughts about the mathematical-knowledge-for-teaching needs of mathematics teacher educators. The interview was not designed solely to elicit views of teaching, but teaching was a focus of the workshops and each section of the protocol touched on teaching directly and indirectly. Analysis focused on the interviews but also used applications, surveys, and observations to inform and test interpretations.

To investigate participants' thinking, we used a conceptual analytic approach (Erickson 1986). Our approach was empirically grounded, coordinated different perspectives, and was driven by practical concerns and logical analysis. Analysis involved cycles of attuning initial conceptualizations, relevant literature, and available data with a focus on local meaning and an assumption that what people say is sensible from their perspective. We identified units of text in tandem with coding — selecting text that focused on the evidence for the code and was enough context to stand alone as evidence. We wrote memos, developed a codebook, and coded for salient distinctions in thinking from direct viewing recordings of interviews, and when uncertainty or inconsistency occasionally arose, we re-watched, explored different interpretations, imagined from the participant's perspective, and set aside to revisit later. We developed codes from sections of a subset of interviews until they could be used consistently and then applied and documented them for the full set. Our focus was on interpretive power, with attention to subjective coder reliability when reconciling our independent coding.

Analysis and results

Our analysis reveals a fundamental distinction in mathematics teacher educator's thinking about teaching — that it develops from views of teaching as *straightforward, where one gives attention to an aspect of teaching but with little regard for other aspects,* to views of it as *involved, where, while giving attention to an aspect of teaching, one maintains regard for other aspects and the whole, with*

a sense of mutuality. By *aspect* we mean a focused slice of teaching. By a sense of *mutuality*, we mean regard for the comprehensive interactions of teaching and learning, including the influence of broader social environments and the need to contend with complexity by specifying practice at a finer grain size (Ball & Forzani, 2009; Grossman & McDonald, 2008), while also considering its character as dilemma management (Lampert, 1985).

We coded units of text conveying: (i) regard for only a single aspect of teaching with no regard for other aspects; (ii) mutual regard for mathematics content and student thinking, but where these alone are privileged; and (iii) mutual regard other than this limited purview, for instance, additionally considering classroom culture, students' developing identities, or the pace and remaining time for a lesson. Viewing an aspect of teaching without regard for other aspects is common in the general public but is likely rare among mathematics teacher educators (as in our data). Instead, the second code is prominent, where teaching is considered in ways that foreground mutual regard for mathematics and student thinking to the exclusion of other aspects. Frameworks drawn from mainstream mathematics education shape, or at least reinforce, this thinking. Fuller mutual regard combines experience and more theoretically grounded consideration of the complex nature of teaching as situated human interaction.

As an example of thinking about teaching as straightforward, consider Claire, a white graduate student in mathematics education, with three years of secondary mathematics teaching experience. She describes interpreting and classifying student thinking as an independent task teachers need to do. She talks about not needing to know students' backgrounds or identities and how this can be a distraction. She acknowledges that teaching can seem complex, but in her view, teachers manage complexity by focusing on "narrow pieces" of teaching in isolation.

> *Instead of thinking about like okay there are forty individual approaches in this classroom, there are reasonably like three or four ways my students might be thinking about that, and ... based on these [students'] comments, we can try to put students into this model that we already have of how a student is thinking.*

Her comment suggests that understanding and making good use of idealized conceptual models of how students are thinking is the crux of the work in this situation, in almost complete isolation from consideration of other aspects of teaching. From this perspective, interpreting students is less an attempt to grapple with students' ideas on their own terms, and more about considering them in relation to pre-existing conceptions of content as conceived by the teacher. This view conveys that teaching is about being aware of and understanding these conceptions of content and using them to assess students and manage the presentation of mathematics going forward.

Such an understanding is limited. It reduces the work of interpreting student thinking to one where human beings and their interactions are not the primary focus, but abstract conceptions are. Content is given primacy here above all other aspects of teaching, which obscures and distorts essential mathematical work required for the multiple and interrelated concerns in teaching. In this view, we lose sight of how a student might feel when they are the subject of this kind of classification (what if the assigned classification is incorrect?) and the impact such classification might have on future interactions and the subsequent mathematical trajectory of the class. We also miss how certain

conceptions (even if accurately assigned and skillfully used in service of content goals) might be viewed as less mathematically advanced by peers and potentially reinforce stereotypes that undermine a productive learning environment. In this way, the central purpose of such an activity shifts away from understanding students or making sense of complex human interaction and toward unpacking abstract conceptualizations of content. Additionally, as teachers often interact with students across cultural and racial differences, their perceptions, unless actively interrogated and disrupted, are likely to impose norms of dominant groups that harm marginalized students.

Despite all of this, Claire trusts that care and good intentions will sufficiently address other aspects of teaching. She has an abiding regard for mathematics and teaches prospective teachers how, for example, to set clear objectives, yet she treats this and other pedagogical tasks as separate matters that do not come into play when classifying student thinking. For Claire, the pedagogical tasks that make up teaching can each be learned relatively independently and used straightforwardly.

A second form of constrained thinking about teaching prioritizes two concerns, mathematical goals and student thinking, with these treated as interdependent but as so primary that exclusive attention to them is given. For example, when debriefing the workshop discussion from the institute, Teresa (a Latina instructional coach with over 10 years of teaching experience, whose focus is the professional development of elementary teachers) explains:

> *The hardest part that we see teachers work with is making connections, helping students connect these different ideas … What I appreciate about this video is that it makes it real time … it's almost asking teachers to write exactly what questions or what they would say to actually bring together Aniyah and Katherine's work and build on the mathematical thinking that is there to reach this goal.*

This comment highlights mutual consideration of student thinking and mathematical goals, and how their navigation requires thinking through and attending to the details of what a teacher needs to say and do. She describes Stein et al.'s (2008) five practices for leading a productive discussion as the essence of teaching — a dance between mathematical content and student thinking. When she focuses on mathematical issues, student thinking is near at hand, and when she scrutinizes student thinking, she keeps mathematical goals in mind. Relevant mathematical knowledge for teaching in this view is fundamentally shaped by the mutual consideration of student thinking in conjunction with content goals. In the quote above, Teresa expresses the need to help teachers formulate what they might actually say because she sees that specificity is needed to size up whether and how to probe student thinking in a way that maintains mathematical goals. In her interview these two aspects of teaching are often privileged — to the exclusion of others.

Another example of our second code is the following from Andy, a white high school teacher with two years of teaching experience and a graduate degree in mathematics. He talks about how challenging it is to hear the parts of student contributions that are mathematically correct and see how to leverage these to advance mathematics. For Andy, this is the heart of skillful, experienced teaching. As did Teresa, he emphasizes the challenge of evaluating student responses in real time.

> *Getting over that fact that this is so crazy, this student has one seventh, and this student is looking at fourths, and to get to the point of where their fractions are coming from … you know, you have*

five minutes basically in real time … while you're trying to manage all the other behavior issues and the other things going on in the classroom.

From his overall response, we hear Andy describe the need for mutual, interdependent regard in the work of bridging student thinking and mathematical goals, but his reference to "other things going on" posits these as independent tasks that need to be carried out simultaneously, but do not require substantial coordination. Andy does not, for example, note how the real-time nature of this moment shapes the kinds of responses that might be available to a teacher, or say how particular responses to behavior might exclude certain children from the mathematical work, both relevant concerns that significantly impact the mathematical knowledge demands of this situation.

The quotes from Teresa and Andy suggest a view of teaching as coordinated attention to mathematical goals and student thinking, but without mutual regard for other aspects, such as for students' identities, classroom culture, materials available, and practical time constraints. Such a conception of teaching does bring genuine regard for a dual attention to mathematics and student thinking but does not attend much beyond that. Indeed, in this view, teaching is still principally about content, whether student-generated or prescribed by curricular and disciplinary goals. While this conception might consider interactions between students or student ideas as mathematically relevant, they are seemingly only so if they are in service of immediate content goals — to the exclusion of other possible goals of teaching (e.g., development of a longer-term mathematical trajectory, encouraging individual and collective participation, human improvement, empathy, disrupting status hierarchies, or social change). Such a view of teaching continues to idealize content away from context and again misses important mathematical work in the interactions among the environments surrounding instruction and the multiple interacting components of the instructional triad. For example, work concerning student identity development is intertwined with and inseparable from mathematically relevant questions such as who should get to speak next, what content might be useful to surface, which examples can reasonably be done in the remaining class time, or what examples are likely to elicit unconventional responses. Mathematics teacher educators who see mathematical objectives and student identity development as isolated and disconnected concerns miss how one can shape the other. It is worth noting that this limited view of teaching is evident in much mathematics education research and many mathematics-education programs, where learning theory and mathematics are often central and integrated but other issues are treated as separate, e.g., classroom management, time management, moral and civic education, and social (in)justice.

Our third code identifies mutuality that goes beyond regard for mathematical goals and student thinking. As an example, Naima (a black curriculum specialist and professional development facilitator with six years of teaching experience and a graduate degree in public policy) focuses on interpreting students' contributions and using them together to advance instruction, but she also stitches regard for other aspects into her comments. Discussing the mathematical content and the collective trajectory of the class, she comments about carefully choosing who should speak, with reference to each student's strengths and growth, how they are positioned, and the overall classroom culture. She attends to all parts of the instructional triad. Naima routinely focuses on a specific concern yet maintains a sensibility for teaching as complex interactional work, inserting brief asides to other aspects of teaching and offering examples that situate her specific point in an overall picture.

The participants in our study were recognized professionals. They were connected enough to hear about and attend a national workshop. The majority of our codes were of this third type (Figure 1).

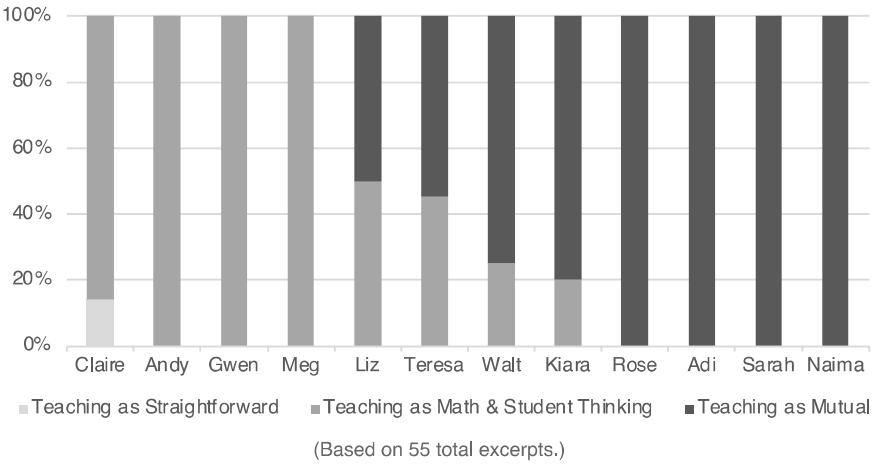

Figure 1: Percents of coded units for each way of thinking about teaching for each participant

Some participants consistently spoke of teaching as requiring mutual regard, while others did not. These two groups are visible at the two ends. The middle four were mixed. For instance, Teresa described how attention to justice issues in the workshop was making her realize she was leaving significant parts of the work implicit in her work with teachers, not only related to justice but to other mutual considerations. She reflected that, in her focus on orchestrating a productive discussion, "the characters, for lack of a better word, the little people, the kids have been absent."

> *I didn't realize how powerful it is to actually paint that context and paint the picture of who is that student with that voice. Like, as a classroom teacher I think I did attend to that, like I understood who kinda- like when you're working with an equalizer board in music, there are times where I need a tone, somebody's voice, and I was- I felt like I was in tune with that. But I didn't realize how to make that visible and explicit to teachers as they think about how they do that with their students. And now I found it seems like a simple solution, tell the whole story. Paint the picture, tell who this student is so that it is part of how they are making their decisions when they have this much time to think about it … I think is my biggest aha! Like make it visible- it can't be something you hope they think to consider in the moment.*

For Teresa, attention to justice issues became an inroad that extended her constrained thinking about teaching as navigation of dynamics only between mathematical objectives and student thinking to navigation involving a much fuller set of dynamics at play in the human interactions of teaching and learning. Similar to Teresa, the three other participants in the middle of Figure 1 all have frames that constrain their thinking about teaching (limited views of mathematics, of what is interesting or important, of students, or of teaching), but also have experiences that allow them at times to take up pedagogical concerns with fuller mutual regard.

Conclusion

The distinction we have unpacked here matters. It is likely a source of miscommunication and misunderstanding among mathematics teacher educators; it reflects something fundamental about the

development of mathematics teacher educators' thinking about teaching; and it also has implications for their understanding of specialized mathematical knowledge. Understanding of teaching as straightforward constrains the work that is visible and considered mathematically relevant. Consequently, such a view inhibits understanding of specialized mathematical knowledge. The tendency to idealize or abstract out of context in service of content goals distorts the purposes of teaching and severs its connections to the realities of practice. It reframes a situated activity as one that is almost entirely cognitive. Our analysis suggests that in the absence of a well-developed understanding of teaching, existing orientations and sensibilities get imported to fill the void. For mathematics teacher educators, viewing teaching through a cognitive or content lens is probably a natural step given the backgrounds and training these individuals likely bring to the work. It is perhaps no surprise then that the practice-based nature of specialized mathematical knowledge often creates difficulties for mathematics teacher educators. Writing tasks for teachers that are authentic to practice is difficult when one's understanding of practice is limited.

In addition, understanding how mathematics teacher educators think about teaching can be used in the service of professional development. In a larger study that makes use of the mutuality distinction, we found that it aligns with thinking about specialized mathematical knowledge as practice-based or resource-based, and with thinking about justice as fundamental or optional (Hoover et al., 2022).

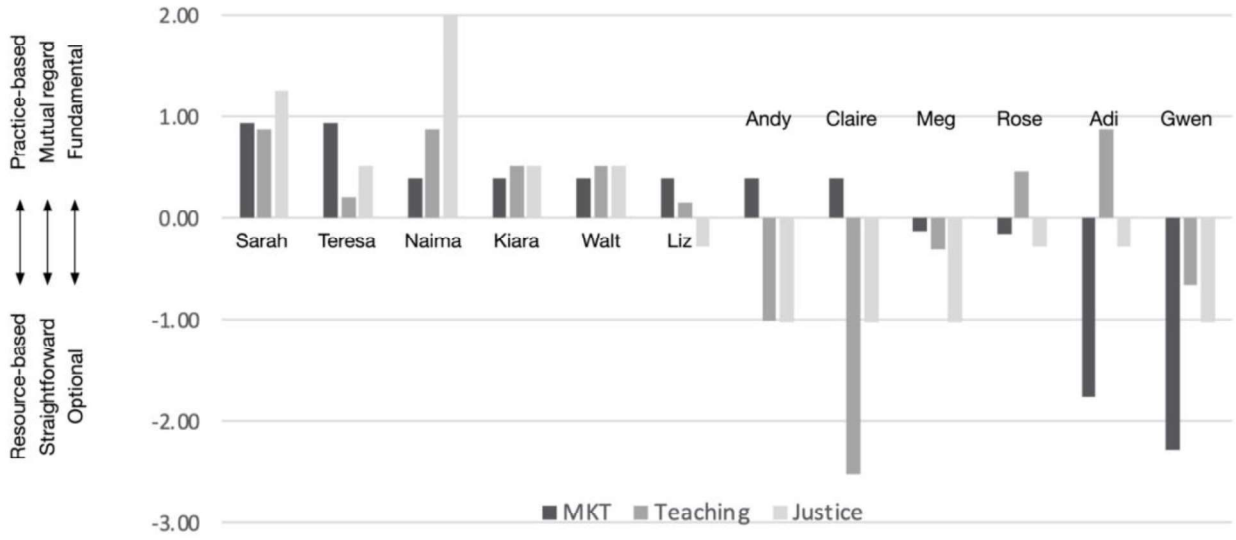

Figure 2: Profiles of the extent to which participants think of mathematical knowledge for teaching as practice-based, teaching as mutually involved, and justice as fundamental and consequential

Deeper examination suggests several clusters of participants with distinctive characterizations that might benefit from more focused professional development that takes alignment (or misalignment) into account. For example, understanding that a mathematics teacher educator thinks of teaching as straightforward allows professional development to be tailored to push on the boundaries of that view and might help them to see the practice-based nature of specialized mathematical knowledge.

While other scholars have described dynamics akin to mutuality, using different language, in different contexts, our analysis is significant in several ways. Conceptual distinctions are significant for specific purposes, and this distinction matters for how mathematics teacher educators work, including

how they think about specialized mathematical knowledge, the tasks they write, and the mathematical opportunities they provide to teachers. We recognize that our analysis is more suggestive than definitive, that our language for and elaboration of the concept of mutuality is limited, and that our data draws primarily from mathematics teacher educators working in a U.S. context, but it offers a starting point. Important questions to address are the composition of skillful regard for mutuality and whether attention to mutuality matters for teachers as well as for mathematics teacher educators.

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