Studying dilemmas of mathematics teaching in Southern Africa

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Background: Learners in many countries in sub-Saharan Africa are underperforming in important subjects such as mathematics, and research in these contexts tends to focus on the lack of resources, insufficient teacher knowledge or poor quality in teaching as explanatory factors. This study has taken a different approach.

Aim: The study aimed at exploring how analysis of dilemmas that teachers encounter in the work of teaching mathematics may provide a productive approach to studying mathematics teaching in the African context.

Setting: The study was conducted in a rural Malawian Grade 1 classroom, where a teacher was teaching arithmetical notation to young learners.

Methods: A case study approach was applied, and data were gathered through video observations and interviews. Inductive analysis of observation data was applied to identify and unpack dilemmas of mathematics teaching.

Results: Two inherent dilemmas of the complex work of teaching mathematics have been identified and discussed. One dilemma was to decide when and how to present arithmetical notations in different modalities without losing the mathematical meaning. A second dilemma was to decide how to deal with unexpected learner errors while maintaining the planned focus of the lesson.

Conclusion: Considering dilemmas of teaching shifts the emphasis from evaluating the teacher to understanding and developing shared understanding of teaching as professional practice.

Contribution: The suggested shift in focus acknowledges the challenges of the local context without reverting to deficit views, and it contributes to developing a shared professional language.

Keywords: mathematics; teaching; arithmetical notation; early years; dilemmas; teacher education; Southern Africa; Malawi.

Introduction

Research on mathematics teaching in sub-Saharan Africa often emphasises the low performance of learners, and there is a tendency to look at teachers' knowledge or the quality of teaching as explanatory factors (e.g. Graven 2016; Johnson, Hayter & Broadfoot 2000; Moloi & Chetty 2011; Nilsen & Gustafsson 2016). This study takes a different approach. Instead of evaluating teachers' knowledge or the quality of their teaching, we consider teaching as a complex work that teachers are faced with (Ball 2017; Mosvold 2016), and we seek to better understand this work by identifying types of situations that might occur where teachers are faced with a difficult choice. We refer to these situations as *dilemmas*, and we argue that considering dilemmas of teaching and their entailments might be a productive approach to research on mathematics teaching in the African context. Our emphasis on the importance of studying dilemmas is based on the understanding that most of the decisions made by the teacher during lesson enactment are based on the professional judgements of the teacher that are often made with little or no time to think. Using the analogy of a court of law, plausible judgements are often based on a good knowledge of how similar cases were handled in the past after a thorough examination of the context. As such, we also argue that the focus on dilemmas can contribute to the preparation and development of teachers in the African context.

Theoretical background

Trends in research on mathematics teaching

Research on mathematics teaching has often focused on identifying certain behaviours of teachers and considering their effectiveness. These tendencies are not surprising if we consider the history of research on teaching. Already in the first *Handbook of Research on Teaching*, Gage (1963:97) defined research on teaching as 'research in which at least one variable consists of a behavior or characteristic of teachers'.

Research on teacher behaviour has a long history, and numerous instruments have been developed and used to study instructional quality in mathematics (e.g. Charalambous & Praetorius 2018). Observation instruments are often used in combination with outcome measures such as tests of learner learning. The purpose of such instruments is to identify the kind of behaviours or instructional practices that constitute teaching of high quality.

The most commonly used measures of instructional quality in mathematics, such as the Mathematical Quality of Instruction (MQI) instrument (Learning Mathematics for Teaching Project 2011), have been developed in affluent Western contexts. The use of such instruments in developing countries may be problematic. To account for these differences, frameworks have been developed for studying mathematics teaching in contexts like that of Southern Africa. Two prominent examples are the Mathematics Discourse in Instruction (MDI) framework (Adler & Ronda 2015) and the Mediating Primary Mathematics (MPM) framework (Venkat & Askew 2018). Both of these frameworks have been developed to provide sensible tools to evaluate the quality of mathematics teaching in a context of traditional and teacher-driven instruction.

Another strand of research that has been prominent considers content knowledge that matters for teaching. The latter strand has been particularly influenced by the work of Shulman (1986). In a review of research on mathematical knowledge for teaching, Hoover et al. (2016) identified that almost half of the studies in this area focused on improvement of teachers' knowledge. For instance, many studies investigated how teacher education or professional development contributes to the development of mathematical knowledge for teaching. Only seven of the 190 studies identified in this review were conducted in Africa. Jakobsen and Mosvold (2015) reviewed the African studies on mathematical knowledge for teaching in more detail and called for further studies. Following this, several more recent studies have been conducted in the African context – and in particular in Southern Africa. Below is a brief overview of some of the more recent studies on mathematics teaching and mathematical knowledge for teaching in Southern Africa.

Recent research in Southern Africa

When considering research on mathematics teaching in Southern Africa, it is noticed that several studies investigate the effectiveness of different kinds of interventions, approaches or resources. For instance, Ubah (2021) explored different approaches to the teaching of fractions in Grade 5. This study was conducted in the KwaZulu-Natal province of South Africa and involved three experienced Mathematics teachers and their learners. The study indicates that 'good practices and appropriate use of multiple representations by the teacher' (Ubah 2021:1) were beneficial in terms of learner achievement. Some studies focus on the use of technology in mathematics teaching. An example of this is the study by Joubert, Callaghan and Engelbrecht (2020), which applied lesson study as a tool to support the development of mathematics teaching with technology. Other studies investigate challenges related to the use of local languages in mathematics teaching (e.g. Mashige, Cekiso & Meyiwa 2019). This is a common challenge in many African countries (Kazima 2008) and requires continued attention.

There have been numerous studies on mathematics teacher knowledge in Southern Africa in recent years. Some studies report on inadequate or weak knowledge among teachers (e.g. Ngema & Lekhetho 2019; Ramaligela 2021). Other studies concentrate on the knowledge teachers use in teaching (e.g. Chikiwa, Westaway & Graven 2019) or how teachers understand mathematical knowledge for teaching (e.g. Jacinto & Jakobsen 2020). Several studies in the African context also focus on the development of mathematical knowledge for teaching (e.g. Feza 2018; Jita & Ige 2019; Msimango, Fonseca & Petersen 2020; Siyepu & Vimbelo 2021). Recently, there have also been some studies that follow the suggestions from Ball (2017) and investigate what is involved in the mathematical work of teaching. One example is the study by Mwadzaangati and Kazima (2019), which explores tasks of teaching involved in the work of teaching geometric proofs in secondary school. This appears to be a promising route, and the present study follows in a similar direction when we explore dilemmas entailed in the work of teaching mathematics in early years.

Early mathematics teaching and learner errors

In this study, we consider a small slice of mathematics teaching as a starting point for discussing a different approach to research on mathematics teaching. Our focus here is on early mathematics teaching and situations that involve some kind of learner errors. When learners are introduced to new mathematical concepts and notation during the early years of primary school, they often make errors. The teacher's reaction to a learner's error made during whole-class activities has implications for the individual learner and the whole class (Bass & Mosvold 2019). For instance, when a learner has made a writing error, the teacher may just compare the wrong inscription or notation made by the learner with the correct one presented on a chart or workbook. However, as observed by Venkat and Askew (2018), young learners may not have yet developed the mental faculties for distinguishing features of seemingly related representations and may require appropriate teacher's mediating talk and gesture to make these features apparent.

Handling learner errors is complex (Sapire et al. 2016). For persistent errors and misconceptions, learners may need to be equipped with strategies for checking the correctness of their work, even in the absence of the teacher. One possible strategy is to embody some mathematical concepts and processes that learners often find difficult to remember. The embodiment of mathematical concepts enables learners to view the subject as an activity involving physical actions and gestures (eds. Edwards, Moore-Russo & Ferrara 2014). This makes the association of mathematical concepts and processes with their corresponding physical actions essential, especially to young and inexperienced learners, who are just being inducted into school mathematics. Eventually, the teacher is supposed to help the learners progress from the embodied physical representations to their corresponding abstract mental structures (Venkat & Askew 2018). If the teacher sticks to the physical representations, the learners may no longer see the need to look for mental conceptual structures to make some necessary connections and generalisations (Askew 2019; Wilson 2002). This implies that teaching can either enhance or constrain what is made available to learn in a lesson – thus constituting numerous dilemmas.

Dilemmas in the teaching of mathematics

Observation of classroom practice indicates that handling dilemmas constitutes an integral aspect of the work of teaching mathematics. These dilemmas have also been referred to as tensions or conflicts in literature (Roth & Lee 2007; Rouleau & Liljedahl 2017). Some dilemmas may result from a teacher's internal conflict between their sense of good professional practice and policy requirements. For instance, Baxter and Williams (2010) discuss one of the most common dilemmas faced in the work of teaching mathematics; that is, how to enable learners to reach understanding of some mathematical concepts without necessarily *telling* them what to do or how to do it. Using the case of Katherine, a fifth-grade science teacher at a rural school, Roth and Lee (2007) demonstrate the tensions experienced by a teacher who professionally believes in letting learners reach an understanding of concepts through discovery learning but is forced just to tell learners what to do in order to meet the requirements set by her school board that put emphasis on enabling the learners to pass high-stakes examinations. On the other hand, Rouleau and Liljedahl (2017) discuss the case of Naomi, a primary school mathematics teacher, who is required by policy to teach mathematics through discovery learning approaches, yet her mathematics learning during her school years was mostly by rote.

Ball (1993) reflects on her own experience as a third-grade mathematics teacher to highlight three dilemmas that are faced by an elementary teacher. The first dilemma involves deciding the appropriate representation of mathematical concepts that may not inadvertently lead to misconceptions, considering the limited background knowledge of the young learners. Ball also faced the second dilemma of respecting children's ideas, considering them as mathematical thinkers despite their limitations in mathematical language on which to base their reasoning. In the early classroom, the third dilemma is associated with developing the classroom as a learning community. The teacher must empower the learners to be able to make plausible judgements on the correctness or incorrectness of learner offers, without necessarily relying on the teacher. The third dilemma by Ball (1993) can be related to the dilemma discussed by Bass and Mosvold (2019) on how a teacher may need to quickly understand the potential value in a learner's offer and decide whether to make the response private or public for the advancement of the classroom discourse.

Conceptual framework and research question

In this article, we share Ball's (1993) definition of dilemmas as paradoxical situations where the teacher has several alternative choices to make. Each of these choices might have different consequences, and the decisions are often required instantly. We also follow Lampert (1985), who argues that the emphasis should be more on the deliberation about the alternatives than on the decisions made. When we focus on dilemmas of teaching, we consider these dilemmas as components of the special work of teaching mathematics (Ball 2017). Studying teaching as *work* involves a shift from considering behaviours of teachers to unpacking common tasks, challenges or dilemmas that teachers have to face in mathematics teaching. Instead of focusing on the decisions teachers make when faced with a dilemma, the focus is more on identifying and deliberating about what is involved (Lampert 1985). Tasks of teaching constitute one set of constituent aspects of the work of teaching mathematics (e.g. Mwadzaangati & Kazima 2019); dilemmas constitute another. A key difference is that tasks of teaching involve some kind of problem that teachers routinely have to solve in teaching, like 'asking productive mathematical questions' (Ball, Thames & Phelps 2008), whereas dilemmas of teaching are situations that involve problems that cannot be solved on the spot, but the alternative responses or ways of dealing with them inevitably include some negative or unsatisfactory aspects. As a site for exploring such dilemmas, we use the case of a Malawian Grade 1 classroom. We approach the following research question: what are some potential dilemmas of teaching mathematical notation to young learners in a Malawian context?

To answer this question, we identify and discuss dilemmas entailed in this work. Like Bass and Mosvold (2019), we focus only on a small slice of the work of teaching mathematics here, namely what may be involved in attending to learners' errors.

Research design and methodology

A qualitative case study design was adopted, which enabled an in-depth inquiry into the complex work of teaching mathematics to young children. Our case is a Grade 1 teacher with an overall teaching experience of seven years and six months after graduating from a two-year teacher training programme. The teacher had been teaching mathematics to different cohorts of Grade 1 learners (aged six to nine) for four consecutive years. The teacher was selected as a paradigmatic case (Flyvbjerg 2006), exemplifying outstanding learner achievement in resource-limited settings. The school consistently outperformed other primary schools in the same geographical area, both during the standardised end of primary school examinations and during quiz competitions with nearby schools. The school was based in a remote village where learners mostly relied on the teacher as the sole source of mathematical instruction, as learners had limited access to extra tuition through books, parents and relatives or educational television programs. The rural setting increased the possibility of attributing the school's exemplary learner achievement to the classroom practices of its teachers.

Data collection was scheduled for the week when the teacher was introducing the addition of whole numbers to Grade 1 learners. This was done across six lessons that were observed and video recorded. Unstructured interviews were conducted at the end of each lesson to seek clarification on some observations made in the classroom. An in-depth video-stimulated recall interview was conducted with the teacher after a preliminary analysis of the lesson transcripts. Interview data were analysed thematically. Themes were centred around the choices that the teacher was faced with and the teacher's reasoning in response to these situations. A previous study applied variation theory and the MPM framework for analysis of data (Gobede 2021; Gobede & Mosvold 2022).

Video recordings from the lessons were divided into instructional episodes, and these episodes were investigated inductively to identify dilemmas of teaching. We consider an episode as a stand-alone section of the lesson where the teacher and the class completed a task. As such, we marked the beginning and the end of each episode with a change in the task being worked on or a shift in the way tasks were done by the class, such as from class work to individual work (Venkat & Askew 2018). Episodes that constituted dilemmas were associated with cases where the teacher handled errors from learners' offers. Our assumption is that learner errors create a situation or 'teachable moment' (Muir 2008) where the teacher has to make a responsive move, thus constituting a dilemma. During the coding of lesson transcripts, these episodes were identified using the MPM framework's 'Talk for building learning connections' that focuses on how the teacher handles learner errors (Askew 2019; Venkat & Askew 2018). The least desirable response is where the teacher simply ignores the offer or just evaluates the offer as correct or incorrect. The most desirable response from the teacher involves advancing the offer to the class or making it public (Bass & Mosvold 2019), followed by a discussion of the justification for accepting or rejecting the offer. Each of the six observed lessons had instances where learners made errors and the teacher had to decide how to advance the discourse. For the purpose of this study, we use two episodes as illustrative examples of situations where the teacher is challenged to handle the evaluation of something the learners offer. We selected episodes that constituted multiple dilemmas from which a rich analysis could be made for the benefit of the discussion being advanced in this article. Our analysis aims at unpacking the entailments of these situations and the dilemmas that they constitute.

During data analysis, coding and management were done using ATLAS.ti (ATLAS.ti GmbH, Berlin, Germany) qualitative data analysis software. Figure 1 shows a screenshot of how the coding was done in ATLAS.ti.

Data used for this study are from a doctoral study by Gobede (2021). Ethical issues were handled in compliance with the procedures that were acceptable by the University of Malawi at that time, before the establishment of the University of Malawi Research Ethics Committee (UNIMAREC) that began providing approval numbers. Data were collected after obtaining proper clearance using a top-down approach, beginning from the highest office in the district before any contacts were made with the study school. At the school, consent was sought from the participants before data of any kind were collected from them. As the study was conducted at a rural school, the cultural requirement of verbal communal approval and communal consent in African rural settings was observed (Tindana, Kass & Akweongo 2006). This included seeking verbal consent from the heads of sections at the school, as well as the chairperson of the parents–teachers' association and the village headman. Participants kept a signed copy of the consent form detailing the voluntary

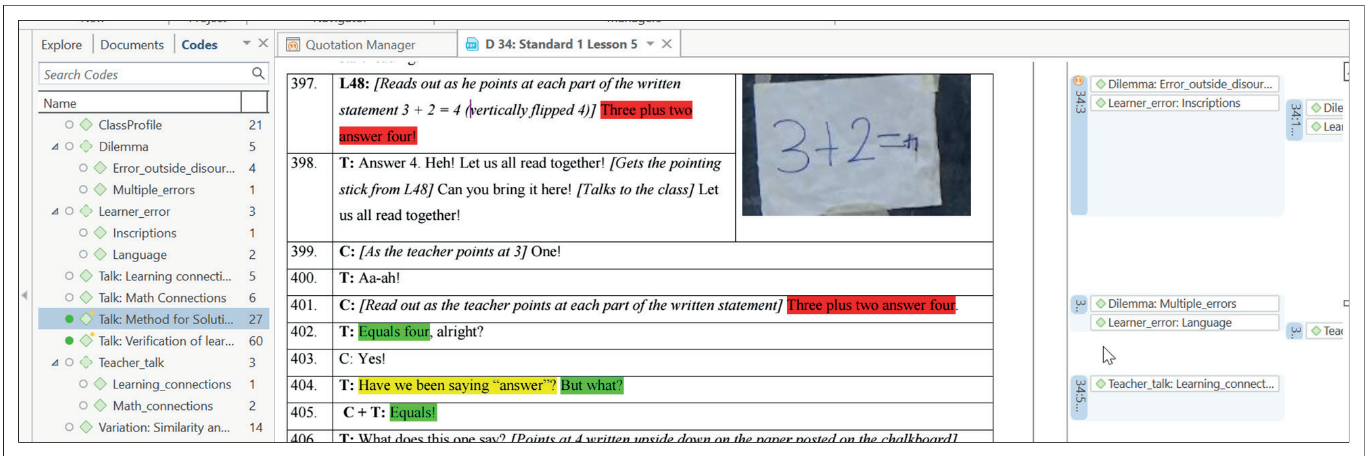


FIGURE 1: Coding in ATLAS.ti.

nature of their participation and how the data would be used. Identities of individuals and the school in the write-up, as well as on photos, were concealed.

Results and discussion

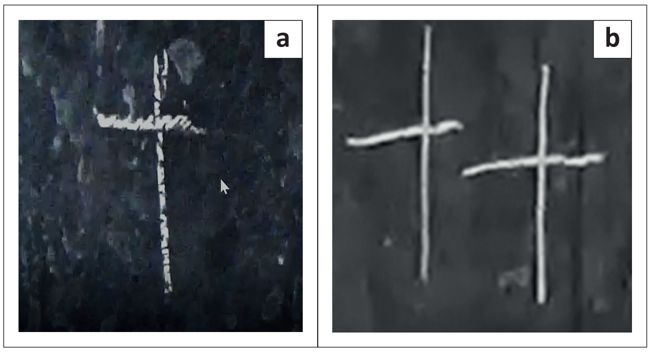
At the time of the study, the teacher had been teaching his new Grade 1 learners to write numbers from 0 to 5. The first lesson in focus introduced the addition of two whole numbers where the sum does not exceed 5. At this time, the learners had been in Grade 1 for 10 weeks. The majority of Grade 1 learners in Malawi (about 60%) have not attended any preschool education before entering school (Robertson, Cassity & Kunkwenzu 2017). The past 10 weeks would thus have been the first weeks of school experience for many of the learners. In this lesson, learners were writing solutions to given addition problems on prewritten papers or chalkboards. Below, we use two selected episodes to illustrate some of the dilemmas that teachers will often be faced with in a context like this.

Episode 1: Verbalising hand movements

When introducing the writing of the plus sign (+), the teacher demonstrated by drawing the sign in the air with verbalised hand movements: 'Dot! Down! Cut in the middle!'. The learners were then invited to do the same:

Teacher:	Aa-aah! We have not yet started writing! Just raise your hand and get ready to write [*inaudible*], alright? Everybody use your right hand! Begin!
Class and teacher:	[*Verbalise the movement of the hand while tracing the + sign in the air*] Dot! Down! Cut in the middle!
Teacher:	Again!
Class:	[*Verbalise the movement of the hand while tracing the + sign in the air*] Dot! Down! Cut in the middle!
Teacher:	Again!
Class:	[*Verbalise the movement of the hand while tracing the + sign in the air*] Dot! Down! Cut in the middle!

Following this, the teacher invited the learners to suggest similar hand movements for the equal sign. The hand movements that were demonstrated by the teacher for the plus sign served as a basis for deciding on whether or not the notations that learners made in subsequent lessons were correct. In a later lesson, one of the learners wrote a plus sign that was not accepted as correct (see Figure 2a). When rejected by the class, the teacher asked for an explanation that would convince the learner why the seemingly correct sign was wrong. In her explanation to the learner, the teacher used the original verbalised hand movement that was introduced earlier: 'Dot! Down! Cut in the middle!'. Following this, the teacher invited another learner to write the plus sign, emphasising how the downward stroke that constitutes the + sign cuts the line in the middle. This aimed at clarifying why the sign offered by the first learner was rejected, as the vertical line did not cut the horizontal line in the middle. The teacher then asked yet another learner to copy the correct notation for the plus sign to further illustrate the point (Figure 2b).



Source: Gobede, F., 2021, 'Investigating mediation strategies used by early years mathematics teachers in Malawi', PhD thesis, University of Malawi

FIGURE 2: Correction of an incorrectly written plus sign: (a) rejected + sign and (b) two accepted + signs.

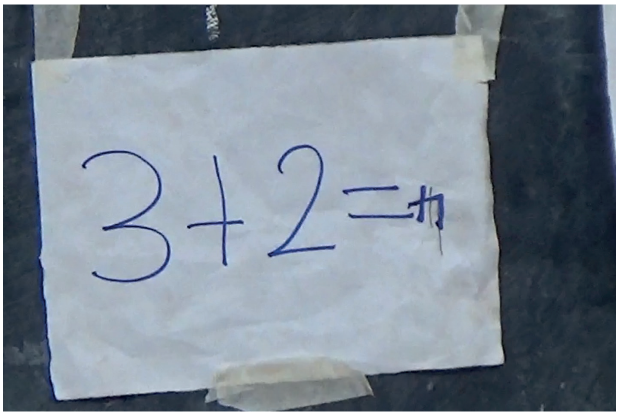
The verbalised hand movement was thus used as a rationale for justification. This approach to using verbalised hand movements was not only used for the plus sign, but the teacher used a similar approach when introducing subsequent signs. In the interview transcripts below, the teacher explains how she had used the same strategy of verbalising hand movements when teaching her learners to write numerals in previous lessons:

214. Teacher:	How to write? We have several ways. Aah, first, we start to write in the air.
215. Researcher:	OK?
216. Teacher:	If you had come when I was teaching numbers, you could see that. Because when we say: 'Let's write four!' We say: 'Dot! Then down! Then right! Then …' Those things. We first start in the air, then after in the air, it's when we go on the ground, before they write in the exercise book.

The teacher here refers to how she had introduced the writing of the number 4, and we notice how this introduction was based on the aptitude of learners. Towards the end of utterance 216 in the preceding interview excerpt, we also notice an indication of an inherent dilemma in this episode. In situations like this, teachers are faced with a dilemma of deciding when it is appropriate to present the notation verbally, through use of gestures or hand movements, in writing or by combining some of these forms. This needs to be done while maintaining the mathematical meaning of the notation. In a situation like the Malawian context, where more than half of the learners might not have attended preschool, where others might have, this dilemma can be significant. No matter how the teacher decides to act, her choice is likely to be suitable for some learners but less so for others.

The dilemma of this situation is not only about deciding on an appropriate way of presenting mathematical notation; this only constitutes the didactical perspective of the situation. The dilemma also relates to considering the mathematics involved and attending to learners and their needs. The mathematical focus of this situation revolves around notation. However important this might seem in the early grades, mathematical notations are only tools that are used to communicate important mathematical ideas. These tools are conventions, and tools and conventions are less important than the underlying mathematical ideas they are meant to communicate. At the same time, it is important for young learners to learn the notations correctly. The mathematical dilemma in the situation lies, however, in how to navigate attention to correct mathematical notation while at the same time managing to focus on the underlying mathematical ideas.

Another aspect of the dilemma in this situation relates to the learners and their needs. An important part of being a teacher is to attend to learners' mathematical thinking, stimulate this thinking and provide every learner with opportunities to develop. Attending to individual learners and their needs is particularly challenging because teachers are always faced with relatively large groups of learners with different needs. In the Malawian context this is especially pressing, since primary teachers are often faced with classes of more than 100 learners.



Source: Gobede, F., 2021, 'Investigating mediation strategies used by early years mathematics teachers in Malawi', PhD thesis, University of Malawi

FIGURE 3: A wrong answer that was written as flipped 4.

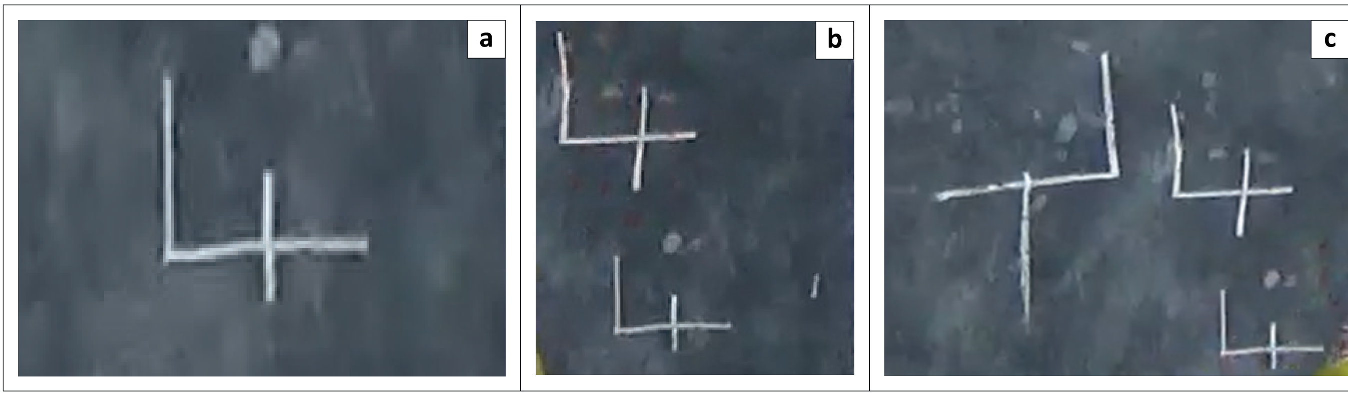
Episode 2: Remediating errors of notation

The second episode is from the fifth lesson observed. This lesson involved learner errors of writing the number 4. When faced with this error, the teacher decided to give strategic explanations to target the main source of the error. Learners had been given addition problems that were written on pieces of paper, and their task was to work in groups to find the sum. Papers with solutions were then pasted on the chalkboard. One of the groups had written the answer as shown in Figure 3.

One of the learners from the group who had worked on this problem presented the solution by reading, 'three plus two answer four'. This constituted another dilemma to the teacher, because there were now two errors involved – one error related to the writing of the numeral 4 and the other related to the sum, which was the focus of this lesson. On the fly, the teacher had to decide whether to quickly dismiss the incorrectly written numeral and focus on working out the correct sum with the class or to consider this a possibility to remediate the inscription error first, even though the focus of this lesson was on addition and not the writing of numerals.

The teacher decided to approach the writing of the number 4 first, and she asked the class if 4 was written correctly. The class was split in their view. The teacher followed up by asking one of the other learners to write 4 on the chalkboard (see Figure 4a), and she then asked the class if the 4 was written correctly. Instead of appealing to logic or the learners' sentiment, the teacher reminded them about the verbalised hand movements for writing 4: 'Dot! Down! Turn right! Cut in the middle!'. She did this while simultaneously moving a pointing stick. Again, the verbalised hand movement was used as a basis for justifying the correctness of a written sign. The teacher repeated the procedure by writing another 4 above the one written by the learner and repeating the verbalised hand movement (Figure 4b).

The teacher then continued by writing an incorrect '4' that was flipped (Figure 4c), and she used this to emphasise the difference – again by using verbalised hand movements:



Source: Gobede, F., 2021, 'Investigating mediation strategies used by early years mathematics teachers in Malawi', PhD thesis, University of Malawi

FIGURE 4: Remediating errors related to the writing of 4 using similarity and contrast: (a) a correct 4 written by a learner; (b) another correct 4 written by the teacher above the one written in part (a) by a learner; (c) an incorrect 4 written by the teacher alongside the two correct 4s in (b).

'Dot! Down! Turn left! Cut in the middle!'. Like the incorrectly written number in Figure 4, this inscription also involved a common error relating to the horizontal direction of the hand. Following this, the teacher remediated the inscription error in Figure 3, before prompting the class to check whether 4 was the correct answer to 3 + 2. After having worked out the expected sum for 3 + 2, the teacher called on the next group of learners. This group had been assigned to solve the addition problem of 1 + 3, and they had written their answer as shown in Figure 5a.

Again, the teacher had to make a decision about how to respond to the offered learners' solution, but this challenge was different from the former. After all, the learners had given a correct response to the addition problem. The only error involved now concerned the writing of the numeral. The teacher again used verbalised hand movements to check whether the number had been correctly written. The teacher expressed the hand movements like this: 'Dot! Down! Go up! Cut in the middle!' At the same time, she wrote down the movements on the chalkboard (Figure 5b). Next, the teacher isolated the feature that made the just-written 4 incorrect, that is, the expected angular turn in the acceptable hand movement. This was verbalised by the teacher with an emphasis on the turn as 'Dot! Down! Turn right!' while simultaneously writing the hand movements on the chalkboard.

Much could be said about the dilemmas entailed in the situations of this episode. When faced with a situation where learners propose that 3 + 2 = 4, one might argue that the mathematical idea of addition is more important than that of notation. At the same time, there might be good reasons why a teacher would want to establish correct notation at an early stage. Balancing attention to notation over the foundational mathematical idea of addition is an important aspect of the dilemma. Another aspect is attention to learners. It might be tempting to suggest that a teacher should consider inviting learners to explain their mathematical thinking and not just evaluate their responses. Yet teachers cannot allow every learner to elaborate on their thinking in every situation – especially in a classroom with many learners – so the teacher is faced with a decision to make. Oftentimes, this decision involves dilemmas where no decision can satisfy every challenge and consideration involved. Again, following Lampert (1985), the point is not so much to evaluate the decisions made as to engage in deliberations about the dilemmas involved.

Source: Gobede, F., 2021, 'Investigating mediation strategies used by early years mathematics teachers in Malawi', PhD thesis, University of Malawi

FIGURE 5: (a) A distorted 4 written by learners and (b) the teacher's emulation of the error.

Conclusion

The two episodes from this Malawian classroom illustrate some common dilemmas of teaching mathematics in early years. The first dilemma relates to deciding on how to establish correct mathematical notation in a way that is suitable for the learners' age and development (Ball 1993), while at the same time maintaining mathematical integrity. In the episode analysed, the teacher used similarity and contrast (cf. Kullberg, Runesson Kempe & Marton 2017) to help learners identify key characteristics of correct mathematical notation, and she also used verbalised hand movement. Still, there is a risk of confusing learners about the underlying mathematical idea in the process. Instead of just telling learners if the offered inscriptions were correct, the teacher in this study attempted to justify the acceptance or rejections. This may provide the learners with an opportunity to learn about the importance of justification in mathematics. However, young learners may lack the necessary understanding on which the teacher can base justifications for actions taken during lessons (Venkat & Askew 2018), and this provides a risk that the teacher must attend to. In this episode, the teacher used gestures to justify the correctness of the written arithmetic notations. This use of bodily based resources such as hands and fingers can make the learners feel competent to work out mathematical tasks anywhere, anytime (Wilson 2002).

A second dilemma relates to identifying and interpreting learner errors on the fly and deciding on which errors to attend to first when several errors are present (Muir 2008). The reasoning that is required for probing learners' errors on the fly tends to be one of the highest and complex forms of teacher knowledge (Sapire et al. 2016). In the second episode of the lesson, a learner presented an incorrectly written 4 as the sum of 3 + 2. The teacher then had to decide on whether to attend to the error or to use other pedagogic moves that do not attend directly to the errors – like assigning competence to learners and positioning them as contributors (Bass & Mosvold 2019). Deciding on whether a situation constitutes a teachable moment (Muir 2008) and deciding on how to act in ways that provide opportunities for learning constitute a common dilemma for Mathematics teachers.

Considering dilemmas of teaching involves shifting attention away from evaluating decisions that teachers make towards deliberation about the alternatives and their entailments (Lampert 1985). Such a shift of attention towards the work of teaching opens the way to understanding what is actually involved in carrying out the complex, dynamic and situated work of teaching (Ball 1993, 2017). This approach of discussing dilemmas does not simplify the picture, and it does not provide immediate solutions for how to act, but it approves of the real nature of the complex work of teaching mathematics. In addition, an approach like this involves attention to the people, contexts and cultural resources involved. The dilemmas of this special work of teaching cannot be easily solved, and their management requires professional knowledge and judgement.

Based on this, we want to highlight three reasons why focusing on dilemmas entailed in the work of teaching mathematics might be a particularly useful approach in the African context. The first reason is that the focus on dilemmas enables distinguishing between the universal and more local aspects of the work of teaching. For instance, giving explanations is a task of teaching that every mathematics teacher is faced with, no matter where. Considering dilemmas and entailments of alternative decisions in a context like this Malawian classroom might contribute to identifying more local considerations that need to be made in the work of teaching mathematics. The second reason is that the common approach to evaluating what teachers do and the choices they make often end up in deficit views – especially in the context of developing countries where resources are scant and learners are underperforming. Developing specific frameworks and measures that account for the local context might remedy some of this (e.g. Adler & Ronda 2015; Venkat & Askew 2018), but a shift of emphasis towards interpreting and understanding the entailments of common dilemmas of teaching can provide an even more positive and productive perspective. For instance, school inspectors in these contexts would focus more on building the teacher's ability to handle contextual issues arising during a lesson rather being evaluative (Venkat & Askew 2021).

The third and final reason, which is not exclusive to the context of Southern Africa, is that a focus on naming and interpreting dilemmas of teaching contributes to developing a shared professional language of teaching. This language would form part of the conceptual tools during preservice teacher preparation. Establishing a language that describes the core aspects of teaching is integral to developing teaching as a profession – in the African context and beyond.

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Authors' contributions

F.G. designed the project, collected and analysed the data as part of his doctoral study and contributed to the writing of this article. R.M. supervised the project, made a secondary analysis and contributed to the writing of this article.

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Data availability

The data that support the findings of this study are available on request from the corresponding author, F.G. The data are not publicly available due to restrictions.

Disclaimer

The views and opinions expressed in this article are those of the authors and do not necessarily reflect the official policy or position of any affiliated agency of the authors.

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