Depth as a key issue for implementing DEM: The case of a teacher

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University of Stavanger, Norway; asmund.l.gjere@uis.no

This case study examines one Norwegian teacher's enactment of an innovative system for mathematics teaching called developmental education in mathematics (DEM). The findings show that despite appropriate textbooks, high motivation, a belief that its principles are effective for mathematics teaching and learning, and indications of a shift of ownership of DEM, the teacher did not follow the fundamental principle of appropriate mathematical challenges for the students. Based on the findings, this paper identifies challenges regarding qualitative aspects, such as depth, of the implementation of the DEM project, while suggesting a path forward for a type of scaling-up process that does not necessarily include spreading to the largest possible number of schools.

Keywords: Developmental education in mathematics, Zankov, Vygotsky, scale of implementation, depth of implementation

Introduction and aim

In Norway, some teachers have had success using a system for teaching elementary mathematics called *developmental education in mathematics* (DEM). DEM consists of mathematics textbooks adapted from Russia and a didactical theory developed by Russian psychologist Leonid V. Zankov (1901–1977), who was a student of Lev S. Vygotsky. Inspired by the excellent results of the pilot class (Melhus, 2015), around 100 schools across Norway have now adopted DEM.

However, a recurring issue in mathematics education is turning small-scale successes into improvements of practice on a larger scale (e.g., Jankvist et al., 2021). A main problem of the DEM project is a lack of systematic knowledge about the teaching practices of the various schools since much of the effort so far has focused on curriculum development and dissemination. In addition, some schools simply use the textbooks without consulting DEM facilitators, leading to an even wider knowledge gap. In this sense, DEM remains an innovation at an early stage despite the history of the first teacher going as far back as 2009 (Gjære & Blank, 2019). To begin to address this knowledge gap, a PhD project seeks to characterize both the potential for DEM to support students' mathematical development and the challenges that some teachers face along the way.

The aim of this paper is to analyze one teacher's enactment of DEM to answer the following research question: How is the main principle of teaching at an optimal level of difficulty realized in a 4th Grade mathematics classroom? The findings will form a basis for discussing more general challenges pertaining to scale, specifically the *depth* of implementation (Coburn, 2003) of the DEM system.

A short introduction to DEM

DEM builds on the didactical theory developed by L. V. Zankov. Its use in mathematics education in Norway depends on a series of textbooks written in the 1990s under the guidance of Iren Arginskaya, a mathematician and member of Zankov's research group. These books follow Zankov's principles and have been translated and adapted for Norwegian schools. The main goal of DEM is not only to increase the mathematical abilities of the students, but more so to stimulate their general development (Melhus, 2015; Zankov, 1977). The didactical principles of DEM are as follows (Zankov, 1977):

1. Teaching at a high (optimal) level of difficulty
2. The leading role of theoretical knowledge
3. Proceeding at a rapid pace
4. Promoting students' awareness of the learning process
5. Systematic development of each student in the classroom

The five principles form a whole; they are interconnected and augment each other (Zankov, 1977). Nevertheless, this paper focuses on the first principle. Zankov's system has been called "implementing the zone of proximal development" (Guseva & Solomonovich, 2017), since this concept lies at its core and its realization is fundamental and necessary to promote students' development. Zankov (1977) built on Vygotsky when he wrote that the ZPD

> is identified by noting the kinds of problems that the child is unable to cope with himself, but can solve with the aid of grownups, in collaborative activity, or through imitation. But what a child can do in cooperation with someone else today, he will be able to do alone tomorrow (p. 18).

This can be contrasted with problems students can do by themselves, in their actual zone of development. Zankov (1977) underlined the importance of the students' emotional engagement needed to spend the intellectual effort to cooperate with others and overcome difficult problems. DEM teachers said that they saw "challenge-as-fun" as a central characteristic of DEM and something that had changed their views about teaching and learning mathematics (Gjære & Blank, 2019).

Framing the study within a discussion of *scale* in implementation research

An increasing number of schools now use the DEM textbooks. However, a discussion of the scale of an implementation cannot rely on numbers alone. Addressing this issue, Coburn (2003) suggested four major dimensions for assessing both quantitative and qualitative aspects of *scale* of educational implementations: Depth, sustainability, spread, and a shift in ownership of the innovation (Figure 1).

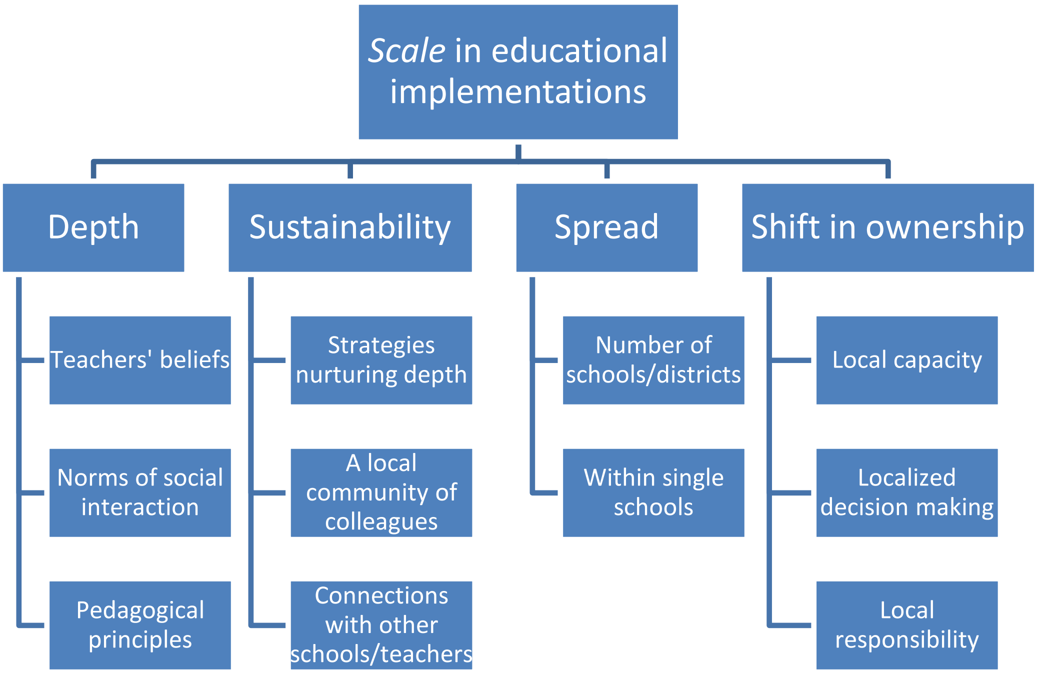

Figure 1: A conceptualization of scale of implementation, adapted from Coburn (2003) by the author of this paper.

The explicit goal of the DEM project is not to spread to as many schools as possible but to offer an alternative for teachers who are interested and find the system to suit their students' needs (Gjære & Blank, 2019). Thus, attention shifts to the other aspects of scale, namely, depth, sustainability, and a shift in ownership of the DEM system. These will form the foundation of the discussion in this paper.

Case description and method

This case study draws its data from a wider set of classroom videos, where four experienced DEM teachers participated. They were all "early movers", meaning that they were among the first to use the DEM textbooks in Norway. The 4th Grade teacher in this study works at a school that functions as a "model school" for DEM, where those who are interested can come and observe DEM lessons and talk with the teachers there. She has also participated in dissemination activities, such as writing about DEM in a journal for mathematics teachers and giving presentations about the positive experiences at her school. The same teachers also participated in focus group interviews before and after classroom videos were recorded. These interviews were analyzed separately and indicated that implementing DEM had changed their views on teaching and learning mathematics (Gjære & Blank, 2019), including the teacher in this case study. The teacher was asked to plan her lessons as usual and not think about the video cameras.

According to Eun (2019), the concept of the ZPD encapsulates Vygotsky's theory of learning and development and can profitably be used as a lens to analyze various aspects of teaching-learning situations. In this paper, the concept of the ZPD serves as an analytical frame to interpret the way the teacher engaged with the students to challenge them mathematically. While all three of this teacher's lessons were analyzed in whole in the preparation for this paper, the data presentation has been shortened to include only one task from each lesson due to space limitations.

Findings

Episode from Lesson 1: Solving an equation

The equation to be solved was $(3n + 10) : 8 = 35$. A possible solution method, introduced in the 4th Grade textbook, is based on doing opposite operations: If $a : b = c$ and a is unknown, then you can find a by multiplying c by b. Here, $(3n + 10)$ takes the role of a and you can find its value by multiplying 35 by 8. However, the teacher introduced the activity by reminding the students that they had to do the same operations on both sides of the equal sign, a slightly different and more general approach. Student 17 suggested beginning by multiplying 8 by 35, which corresponds to the textbook method, but had difficulties explaining further. Student 8 was asked to elaborate on Student 17's response:

206	S8	(*comes up to the board*) Well, you could say <u>that one</u> (*points at the left-hand side of the equation*) equals <u>that one</u> (*points at the right-hand side*).
207	Teacher:	Aha! Let me see if I understand you two correctly. It is divided by eight, <u>times eight</u>, really, on both sides (*she transforms the equation, see the second line in Figure 2)*.
208	S8:	Um, yeah.
209	Teacher:	Is that your thinking, Student 17?
210	S17:	Yeah
211	Teacher:	That you <u>multiply</u> by eight on both sides. Yes!

S8 addressed the main concept of equality (utterance 206) but did not explicitly connect it to solving the equation. The teacher, however, went directly to the conclusion and wrote the transformation of the equation herself (207). The students' responses (208 and 210) were not convincing. This pattern persisted during the whole solution process, with the teacher leading and writing, and students only supplying short answers along the way (see Figure 2).

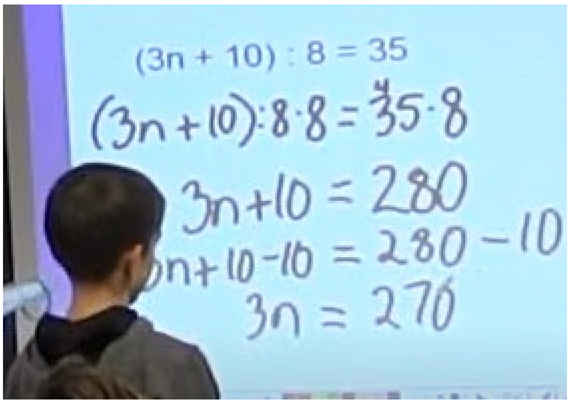

Figure 2: Student 8 watches as the teacher helps him write the solution of the equation.

This activity did not follow the DEM principle of teaching at an optimal level of difficulty. Zankov (1977) was clear that the students themselves must make the effort to solve challenging problems to develop their abilities. The beginning of this episode suggests that the task could suit the students' ZPD nicely since S17 and S8 both contribute with mathematically productive statements about the equation although their reasoning is incomplete. In utterance 207, however, the teacher completed a reasoning step for them, based on her own solution method and not the textbook method. This took away the challenge for the students to solve the equation themselves, resulting in a lack of mathematical activity.

Episode from Lesson 2: Finding the volumes of right rectangular prisms

This activity was mostly a whole-class discussion about how to find the volumes of right rectangular prisms. During the discussion, some students expressed frustration by moving about on their chairs, sighing audibly, or answering in odd voices. Two prisms, 1 and 2, were pictured on the board, along with a table to fill out with length, width, height, and volume of each prism.

277	Teacher:	OK! Let's use the formula to fill out this table and calculate the volume of these two figures. What then, can we say about the length of figure 1? (*She pauses. The students are unrestful and only a few have their hands up*) The length of figure 1? The length of figure 1, people. S11.
278	S11:	Um, it's five, I think. Yeah.
279	Teacher:	Yes, five (*writes "5" under Length, figure 1, in the table*)
280	S11:	But I found out what the whole was, too.
281	Teacher:	The length is five. What about the width? (*pause*) S17?
282	S17:	Two.
283:	Teacher:	Two. (*writes "2" in the table*) and the height? S6?
284:	S6	THREE! (*answers in an odd voice*)
285:	Teacher:	And then the volume is?
286	Students	Twenty-four / thirty (*both numbers are heard*)
287:	Teacher:	Five…? Five time two is…? (*speaks very slowly and clearly*)
288:	Students:	Ten!
290	Teacher:	Ten times three?
291	Students	THIRTY! (*shouting*)

Note that S11 was ready to provide an answer in utterance 280. This, along with some signs of unrest, suggests that the students found the progression too slow. When given a worksheet on the same topic, some students expressed a lack of challenge:

307	Teacher:	You are to find the volume of these prisms. But you see, they aren't quite filled up with cubic centimeters. Can you still find the length, the width, and the height of these prisms?
308	A student:	Um, yeah.
309	Another:	°That's easy° (*heard whispering to his desk mate*)

The whole activity, with different examples of prisms, took around 17 minutes, 10 of which were whole-class discussion. This activity also lacked the kind of challenge that characterizes a "Zankov's lesson": The students found calculating the volumes of these prisms easy and they could do it by themselves, meaning that the activity was within their actual zone of development, and the extended whole-class discussion reduced the pace of progression.

Episode from Lesson 3: A numerical pattern

The task was: “Find the pattern of the sequence and write the next number: 2, 5, 11, 23, 47, 95, …”. The students first discussed in pairs for a couple of minutes before presenting their results, and three different ways of describing the pattern were presented (Figure 3).

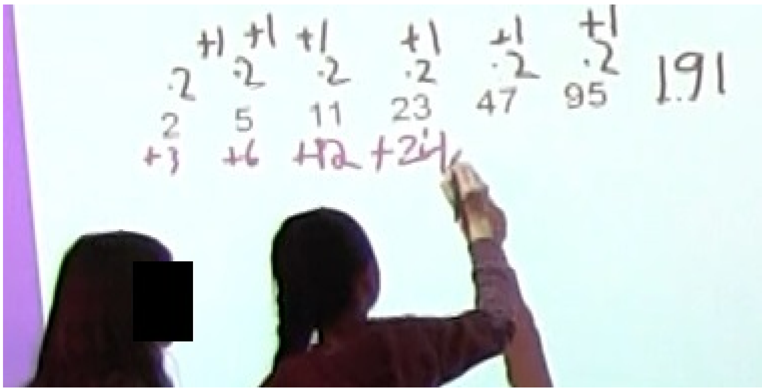

Figure 3: Students describing the number pattern.

The students’ three ways to describe the pattern were: To get from a number to the next, multiply by 2 and add 1 (written ·2+1 in Figure 3); that the “add on”-numbers double for each step (+3, +6, +12, …); and finally, to get from a number to the next, add one more than the number itself (e.g., to get from 5 to 11, you add 6, which is 5 + 1). The whole task was quickly done; from thinking time in pairs to students having presented three different pattern descriptions it took about 6 minutes. The teacher did not interfere with the students’ thinking. She took the role as a discussion moderator while checking if the other students understood or agreed (they used hand signs to indicate this).

Notably, the teacher allowed her students a lot more room to discuss and present ideas in Lesson 3 than in the other two lessons. This can be gleaned from the description of the pattern activity above. However, she also refrained from interfering and did not push the students further, e.g., by directing their attention toward relationships between the three pattern descriptions. The students did the task on their own and presented their results without further explorations, and the activity therefore remained within their actual zone of development. While they were successful in solving the task, the teacher did not seize the opportunity to challenge them further. However, both this and the other two lessons demonstrated another important aspect of DEM, namely, cultivating a sympathetic and respectful community of learners (Zankov, 1977). For instance, the teacher called for an appreciative applause for the presenting students after this episode, which she also did several times during the three lessons.

Discussion

In general, the analysis showed that the teacher did not follow the didactical principle of optimal difficulty in the three observed lessons. In one sense, this finding could be interpreted as a lack of *fidelity* in the sense of “the extent to which an innovation in enacted according to its intended model” (Century & Cassata, 2016, p. 171). However, the word “fidelity” usually has meanings associated with *loyalty* or *faithfulness* (Cambridge Dictionary, 2022), so in this sense, a lack of fidelity could imply a lack of loyalty to DEM, which is clearly not the case here. On the contrary, this teacher spoke highly of DEM and showed indications of having given it a central place in her teaching practice.

According to Coburn (2003), a *shift of ownership* of the innovation is necessary to achieve a lasting impact on teaching practice at scale. At the school of this teacher, the staff have formed a local DEM community, they have decided to let DEM take a central place in the school system and influence other school subjects, and they (including the teacher in this case) have taken part in dissemination activities; all of this across an extended period of time with only limited support from university facilitators. These are indicators not only of taking ownership of DEM, but also of sustainability and spread within the school (see Figure 1, p. 3). This means that the school is well positioned to continue using DEM independently from university facilitators. However, as the findings above show, there are still challenges to work out.

Realizing pedagogical principles of the system in practice relates to what Coburn (2003) refers to as *depth* of implementation scale. In her conceptualization of scale, depth is both a key dimension in its own right and an underpinning of the other dimension. In this case, there were in fact indicators of depth of the implementation for this teacher since she reported to have changed her beliefs about mathematics education (Gjære & Blank, 2019). Also, the norms of social interaction in her mathematics classroom corresponded with the sympathetic community of learners suggested by Zankov (1977), although it is not clear how much this has changed in her practice as she was not observed prior to DEM. However, the main goal of DEM is to stimulate students' development (both general and mathematical) by engaging them in solving challenging problems and encouraging them to be persistent, analytical, investigative, self-reflecting, critical and creative. There is also the question of whether whole-class problem-solving discussions are suited for realizing the principle of optimal difficulty, since one must expect diversity among students within a class. Considering these issues, it becomes clear that depth, and in particular the realization of the principles of the system, must take center stage in any discussion of scale of the DEM project.

Concluding remarks

The discussion of this case study demonstrates the usefulness of Coburn's (2003) conceptualization of scale to bring an alternative and more varied perspective on scaling-up processes for innovation projects that do not aim for the greatest possible spread. *Scale* in educational innovations often implies spreading to many schools. The DEM project, however, is concerned with providing an alternative system for teaching elementary mathematics that could possibly improve the practice of those teachers who are interested and motivated. For assessing the scale of such projects, qualitative dimensions such as depth, sustainability and a shift in ownership becomes even more important. This could also raise the general question of whether spreading to many schools should always be implicit in scaling-up processes of innovations in mathematics education.

Challenges ahead for the DEM project include supporting a greater depth for the various DEM schools as well as sustainability and a shift in ownership of the project for the involved schools.

Addressing issues of depth requires a more nuanced understanding of the difficulties with realizing the didactical principles like those that the teacher of the case study experienced. Such research efforts could be combined with either initiating the formation of "satellite communities" in the various municipalities where DEM is used or reaching out to contact already existing communities, with the purpose of improving the scale of the DEM project across all four dimensions of Coburn's (2003) conceptualization.

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