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## DEDICATION

I dedicate this work to my Husband, George, and my children Precious, Geoffrey, Ester, Annie, Favour, and Joshua with the deepest gratitude and reverence for their great support, advice, and prayers. God bless you.

To my mum, Ms Elizabeth Mofolo Kazembe for your encouragement and support.

In memory of my Father, Mr. Dyson Kazembe and my uncle, Mr. Daniel Mpharu who emphasized the value of education and influenced my life.

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## ABSTRACT

In Malawi, the current performance of students in early-grade mathematics is low due to lack of focus on developing number concepts, as well as the reliance on rote learning in the existing OBE curriculum. To address these issues, a new curriculum has been introduced in 204 pilot schools, aiming to revise the mathematics curriculum for lower primary grades (Standards 1-4). The revised curriculum focuses on teaching various calculation strategies within the core elements of numbers, operations, and relationships, with the goal of equipping students with a wider range of problem-solving skills and a deeper understanding of mathematical concepts. This study investigated how mathematics teachers teach calculation strategies in the new National Numeracy Curriculum. The research aimed to identify the calculation strategies taught, how teachers teach them and their views on teaching of calculation strategies The following questions guided the study: 1) What calculation strategies do mathematics teachers teach in the NNP curriculum? 2) How do mathematics teachers teach calculation strategies in the NNP curriculum? 3) How do mathematics teachers view the teaching of calculation strategies in the NNP curriculum? The study used a mixed methods approach, employing the Mathematical Discourse of Instruction (MDI) analytical framework (Adler & Ronda, 2015) for data analysis. The findings indicated that mathematics teachers teach a diverse set of calculation strategies, including building up and breaking down, number lines, physical modeling, counting on and counting down, addition bubbles, estimation, number pyramid, doubling and halving, commutativity, completing 10s, rounding, and compensation. Teachers illustrated these strategies using examples and tasks, either selecting one question from workbooks as an example or formulating their own examples that involve multiple calculation strategies. The remaining tasks assigned to learners came from the workbooks. Teachers used various teaching aids such as workbooks, charts, tables, chalkboards, and physical models to demonstrate and explain the calculation strategies. They encouraged learner participation through whole-class discussions, group work, pair work, and individual work. Teachers emphasized the importance of naming, explaining, and justifying the chosen strategies and highlighted the contextual differences in applying calculation strategies to solve different types of questions. However, teachers perceived the allocated time for mathematics lessons as not enough and suggested either doubling the time or reducing the workload per page to facilitate more effective teaching and learning.

**Keywords:** *Teaching, number operations and relationships, calculation strategies, examples and tasks, the National Numeracy Program curriculum, MDI framework,*

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## ABBREVIATIONS

**DfES**: Department for Education and Skills

**MIE:** Malawi Institute of Education

**MDI:** Mathematics Discourse for Instruction

**MoE:** Ministry of Education

**MoEST** Ministry of Education, Science and Technology

**NNP:** National Numeracy Program

**OB**E: Outcome-Based Education

# CHAPTER 1:  INTRODUCTION

The main aim of this study is to investigate Malawi primary school mathematics teachers' teaching of the mathematics calculation strategies of the national numeracy program (NNP) curriculum (Malawi Institute of Education, MIE, 2021). Calculation strategies refer to the various techniques and methods used to perform mathematical calculations efficiently and accurately (Stein et al., 2014). These strategies can vary depending on the type of calculation, the complexity of the problem, and the personal preferences of the mathematician (Torbeyns, & Verschaffel, 2016). This chapter therefore presents the background, statement of the problem, purpose of the study, research questions and significance of the study.

## 1.1 Background

Improving education is one of the Malawi Government's priority areas in its Growth and Development Strategy (2017–2022) (Ministry of Education Science and Technology (MoEST), 2019). The government wants more children to progress through and succeed in their schooling, so that they graduate with the right skills to enable them to participate in and contribute to Malawi's workforce (MoEST, 2018). Like in many other countries, Mathematics is a core subject in primary and secondary schools in Malawi. It is a very important subject because it plays an important role in developing reasoning skills and meeting the demands of everyday living (MIE, 2005). As such, 50% of the time is allocated to mathematics lessons on the timetable in Malawi primary school curriculum (MIE, 2009).

The Malawi primary school education comprises eight Standards[1] (Ministry of Education, (MoE), 2004). For most children, schooling begins at Standard 1. The first two years (Standards 1–2) are infant primary, the next two (Standards 3–4) are junior primary, and then the next four (Standards 5–8) are senior primary. Primary school education has eight subjects in the infant and junior sections and the senior section has ten subjects. One academic year has three terms.

From January 2007 to May 2008, the primary education curriculum was revised through the Primary Curriculum and Assessment Reform which emphasize Outcome-Based education

[1] In Malawi, Standards is a word used for grades

(OBE) (MIE, 2009). For simplicity, this current curriculum is called the OBE curriculum. The implementation of the OBE curriculum started early 2008 which replaced the 1991 to 2007 curriculum, the fourth version since the first curriculum was established in 1961 – (three years after Malawi got its independence). The mathematics part of the OBE curriculum in Malawi aims at developing learners' critical awareness of the mathematical relationships in social, cultural, and economic context (MoE, 2004). The curriculum has six core-elements which are:

(i) number operations and relationships.
(ii) patterns, functions, and algebra.
(iii) space and shape.
(iv) measurement.
(v) data handling
(vi) accounting and business studies (MIE, 2009).

The core element of number operations and relationships takes up to more than 50% of the intended lessons planned for Mathematics. The expectation of OBE curriculum is that within the first two years, learners should be able to count and perform basic mathematical operations within the number range of 0 to 1000 (MoE, 2009).

However, despite the emphasis on Mathematics in primary schools in Malawi, the learners' achievement continues to be a concern (Brombacher, 2019; Ravishanker, 2016). Both international and national assessments confirm that learners in Malawi are struggling with mathematics and lagging where they should be, both in terms of their own learning and in relation to neighbouring countries (Brombacher, 2019). For instance, in the international assessments by the Southern and Eastern Africa Consortium for Monitoring Educational Quality (SACMEQ), which assesses the achievement of Standard 6 level learners in 15 countries in South-east Africa, Malawi ranked at the bottom as one of two least performing countries in mathematics in all the assessments, SACMEQ I in 1995, SACMEQ II in 2000 (Njora, 2010). The SACMEQ III data also showed that Malawian learners perform below the regional average, with only 1.7% of Standard 6 learners achieving competent numeracy (Level 5 out of 8 levels in the SACMEQ classification) and 59.9% of learners fail to achieve even basic numeracy (Level 3). The majority of the standard 6 learners were classified as reaching the lower-level competencies of emergent numeracy and basic numeracy, but not the higher competencies of mathematically skilled, concrete problem solving, and abstract problem solving (Milner et al., 2011).

National assessments have also revealed that many learners in Malawi achieve below the expectation of what students should be able to do in one of the core elements of the mathematics curriculum: number, operation, and relationship (Kazima et al., 2021). The underachievement was evidenced in the 2010 Early Grade Mathematics Assessment (EGMA) data that indicated that learners are behind in their learning. For example, the EGMA data showed that 61.8% of Standard 4 learners were only confident with Standard 2 Mathematics content (Brombacher, 2011). The EGMA also revealed that nearly 56% of the 500 participating Standard 2 learners could not perform basic addition of single-digit numbers with a sum of less than 10, which is a curriculum expectation at the end of Standard 1. Likewise, findings were also evidenced from the Malawi Longitudinal School Survey (2016) assessments which were administered to Standard 4 learners (Brombacher, 2019). The assessment contained a mixture of items from subjects like Mathematics, Chichewa, and English ranging from Standard 1 to Standard 4. The overall average score for mathematics was 43%, with 50% average score for the Standard 1 item and 33% average score for the Standard 4 items. The survey revealed that only 33% of the standard 4 learners could add simple two- and three-digit numbers correctly (Brombacher, 2019). Along with these findings, Maganga et al. (2009) explained that Similar findings were also reported in earlier assessment studies such as the 2006 Primary Curriculum and Assessment Reform (PCAR) Standard 1 Baseline Study, the 205 study on Monitoring Learner Achievement in Lower Primary School, and the 2008 study on Assessing Learner Achievement in Standards 2 and 5.

Furthermore, reports from Malawi National Examination Board (MANEB, 2016) also indicate that learners in primary school fail to reach the minimum levels of mathematical proficiency specified in the National Examination. Learners in upper primary school classes fail to accomplish the rationale and the criterion-referenced measurement of the mathematics curriculum in primary school. This is observed through low performance of learners during the primary school national examinations (MANEB), 2006–2016) (Eliya, 2016). The item performance data indicated that when an item is presented in a less-standard format, the had trouble increases, and learners are not confident in applying their mathematical knowledge. The performance is therefore not satisfactory. According to Brombacher (2019), learners rely on rote learned methods that are only applicable in limited and familiar situations. As a result, learners fail to apply them to unfamiliar situations that are found in MANEB assessments. The assessment of the primary school national examinations draws topics in mathematics from standard 5 to standard 8 (Eliya, 2016).

Kazima, Jakobsen, and Kasoka (2016) describe many factors that contribute to Malawian students' low achievement, including large class sizes, limited teaching and learning resources, and the teacher's poor quality of teaching.

To address and improve the quality of teaching and learning of mathematics, there have been curriculum reforms which revised the Content Objective Based curriculum to learner Outcome Based curriculum and aimed at shifting teaching from traditional teacher centred to more learner centred teaching approaches (MoE, 2007). In addition, over the years, there have been interventions in the teaching and learning of mathematics; some interventions targeted mathematics teachers in schools to implement new teaching strategies, while other interventions targeted entire schools and communities. However, it is surprising that learners still face difficulties in performing basic mathematical operations such as addition and subtraction and the problem of learner underachievement persists (Brombacher, 2019; Ravishanker, 2016).

In response to the assessments and the reports above the MoEST in Malawi and the UK government conducted a scoping study to better understand the key bottlenecks that are preventing learners' satisfactory performance in mathematics in lower primary school and find the best approach to improve numeracy outcomes (Standards 1–4) (MoEST, 2019). The study revealed that current performance in early grade mathematics in Malawi has much to do with perceptions of what it means to do mathematics as it does with doing the mathematics (Brombacher, 2019). In addition, the study revealed that the Malawian mathematics, especially in the early grades, is also insensitive to the interrelated way in which learners develop number concepts (MIE, 2021). The OBE mathematics curriculum is also characterized by few learning opportunities to learn and that the teaching and learning focused on rote learning rather than knowing understanding, reasoning, and applying mathematics (Brombacher, 2019). As a result, learners fail to apply their understanding or apply their knowledge when they face assessment items in a format that is unfamiliar to them (Brombacher, 2019).

According to Brombacher (2019), the scoping study recommended that there should be a program that will provide the opportunity of developing a modernized vision of what it means to do mathematics in Malawi primary schools. The study suggested that the program should develop a vision of mathematics in which students experience mathematics as a meaningful, sense-making, and problem-solving activity. In addition to that, the study suggested that the program should also develop a vision of mathematics teaching and learning that expects students not only to know mathematics but also to understand the mathematics they know, be able to apply the mathematics they know to solve unfamiliar problems and be able to reason with the mathematics that they know.

It is against this background that the NNP was launched as a project to develop the visions. The aim of the project was to improve the children's mathematics learning outcomes by laying a solid Mathematics foundation in the six core elements. The success of the project has led to the development of a new curriculum that I chose to call the NNP curriculum. It is being piloted in 2022 and 2023 in 203, standard 1 to standard 4 primary schools in Malawi.

## 1.2 The National Numeracy Curriculum in Malawi

The NNP is a four-year (2020–2023) Government of Malawi program. It is led by the MoEST and is funded by the UK Government (MIE, 2021). One of the objectives of the program is to revise the OBE mathematics curriculum for lower primary (Standards 1–4) and develop the teaching and learning materials aligned to the revised curriculum. The NNP curriculum has six core elements like the OBE curriculum (MIE, 2021). The NNP curriculum is advocating for understanding, reasoning, and application of mathematics concepts in unfamiliar situations.

The NNP curriculum therefore expects learners to perform calculations, recognize and make patterns, recognize shapes, measure, and solve problems involving length, mass, capacity, and volume, collect and organize data, develop problem solving skills and apply to problems in real life. The approach to teaching builds learners' mathematical knowledge with understanding, supports them to confidently apply mathematical knowledge in an unfamiliar situation, and develops the ability to reason. After the successful assessment and completion of piloting phase, the curriculum will be expanded to all the thirty-four education districts (MIE, 2021).

## 1.3 Statement of the Problem

Currently, Malawi has introduced the new curriculum to 204 pilot schools, and it is covering all the six revised mathematics core elements (MIE, 2022). One of core element is number operation and relationships which is also the foundation of all mathematics (Tyobeka, 2010). It is therefore not surprising that it is also the first core element that the National Numeracy Curriculum in Malawi is dealing with (MIE, 2021). This study will therefore be limited this core elements of numbers, operation, and relationships. The core element has been arranged by three underpinning key elements: counting, problem-solving and manipulating numbers (calculating flexibly).

One of the new concepts in the core element of numbers, operation, and relationships is the teaching of calculation strategies. Various calculation strategies have been included in the curriculum materials as one way of helping children understand, reason, and apply mathematical concepts. Learners are introduced to a range of different calculation strategies with the expectation that they will develop the ability to apply these strategies fluently and flexibly in unfamiliar situations. They are also expected to apply fluently and flexibly what they already know and makes sense of the situation when solving mathematical problems (MIE, 2022). The examples of the calculation included in the NNP curriculum are physical modelling (concrete apparatus or drawing), number lines, breaking down and building up numbers doubling and halving, estimation, number bubbles, number pyramid, rounding and compensating, and commutativity. Several definitions of calculation strategies and what it means to calculate fluently and flexibly exist and this will be further discussed in Chapter 2 Teachers in the pilot schools have been trained and are teaching the calculation strategies in their classes.

Mathematics researchers have recommended teachers to teach a range of calculation strategies to learners for them to calculate flexibly. According to Tyobeka, (2010). teaching a wide range of calculation strategies, to learners gives them the opportunity to choose the strategies they understand and feel comfortable with. He goes on to say that, in doing so learners discover and explore new calculation strategies. Tony (2013) noted that human brains do not all function in the same way, hence teaching a range of calculation strategies increases learners opportunity to choose and calculate flexibly, confidently, and accurately. Straker (2010) also recommended that teachers need to raise their awareness and understanding of the range of strategies, develop their confidence and fluency. Straker further explains that the teacher’s awareness and understanding is raised by teaching and allowing learners to explore, discover and explain their choice and understanding of the calculation strategies. However, Grouws (2007) noted that many teachers teach mathematics to learners procedurally. They rarely provide learners with opportunities to explore, discover new strategies and justify their reasoning. As a result, learners struggle when they meet unfamiliar mathematics questions.

What Grouws (2007) noticed is similar to the research findings done in Malawi that show that one of the reasons for the underachievement of learners in mathematics in primary schools is also the use of procedural teaching (Brombacher 2019). The teaching focuses on rote learning and not on understanding, reasoning, and application. Brombacher (2019) further reported that learners are not given the opportunity to discover, explore and explain their understanding of the mathematics concepts. They follow exactly the procedure that the teacher teaches them. However, Bruce (2007) argue that this type of teaching is not effective, and learners do not experience meaningful mathematics. Bruce recommends effective teaching as the teaching that require teachers create a classroom culture that is conducive to effective mathematics teaching and learning. The teachers can create such environment by deciding and selecting appropriate examples, tasks, terminologies, questions, and give learners opportunities choose, discover, explore, and explain the ways of solving mathematics. By doing so, learners learn and experience mathematics as meaningful.

As stated earlier, the NNP curriculum is advocating for this type of teaching where learners should understand, reason, and flexibly apply calculation strategies in solving mathematical problems in unfamiliar situations and experience mathematics as meaningful. The researcher, therefore, seeks to investigate how the primary school teachers in Malawi, teach various calculation strategies in the NNP curriculum to learners.

## 1.4 Purpose of the Study

The purpose of this study is to explore how primary school teachers teach the calculation strategies that are in the NNP curriculum and their views on the calculation. Lesson observations, interviews, and document analysis were used to collect data and the focus was on classroom activities especially on examples, tasks, explanations, and learner participation.

## 1.5 Research Questions

The research questions have been developed from the research topic to facilitate the investigation on how teachers teach the calculation strategies of the NNP curriculum in primary schools in Malawi. The main research question for the study is: How do primary school teachers in Malawi teach the calculation strategies in the NNP curriculum?

To answer this question, three specific research questions must be investigated, and these are.

a. What calculation strategies do mathematics teachers teach in the NNP curriculum?
b. How do mathematics teachers teach calculation strategies in the NNP curriculum?

c. How do mathematics teachers view the teaching of calculation strategies in the NNP curriculum?

## 1.6 Significance of the Study

This study is significant to mathematics teachers in a way that the findings may help to bring to light what they are doing right and need to be encouraged in their teaching or where they need to improve to make teaching and learning of mathematics meaningful and enjoyable. But most importantly, the findings of this research may benefit the learner who is at the centre of the learning process. This study is also important for teacher educators as they are the ones responsible for training teachers. If teacher educators become aware of how teachers teach calculation strategies in the NNP, they may need to check their own teaching if it helps their student teachers or not. Thus, I as one of the teacher educators, this study is helpful in a way that it brings insights as to where I am failing and how best can I improve and help student teachers in their initial training become better mathematics teachers.

## 1.7 Chapter Summary

The chapter has introduced the research study. It has discussed the aim, the background and context of the study, the statement of the problem, the purpose of the study, the research questions and finally the significance of this research study.

Following this chapter is the literature review.

# CHAPTER 2: LITERATURE REVIEW AND THEORETICAL FRAMEWORK

## 2.1 Introduction

Mathematics researchers have studied and developed various calculation strategies over time. They have also investigated how students develop the calculation strategies, and how they can be taught effectively in educational settings. This chapter, therefore, provides an overview of the definition, importance, and examples of calculation strategies from the literature. It also gives an overview of how students develop the calculation strategy skills, how calculation strategies are taught and how the teaching of calculation strategies has been implemented in other countries. The chapter also presents the context in Malawi. Finally, the mathematics Discourse for Instruction (MDI) framework, which forms the basis for the study is presented.

## 2.2 Definitions of Calculation Strategies.

In the literature, mathematics educators have discussed the concept of calculation strategies, providing various definitions. Burnett and Irons (2009) define calculation strategies as a combination of strategies used to obtain accurate or approximate answers. Threlfall (2002; 2009) describes calculation strategies as informal solution methods developed by children to solve arithmetic problems. Eather (2016) defines calculation strategies as the methods students use to solve mathematical problems involving numbers or quantities. Maclellan (2001) explains calculation strategies as the application of number facts and properties of the number system to find solutions to unknown calculations.

While these definitions have slight variations, they all agree that calculation strategies involve the knowledge and selection of different solution methods to solve specific problem structures, even in unfamiliar situations. When students engage in calculations, they choose strategies that are efficient and effective for them, applying these strategies fluently and flexibly in various mathematical problems (Clark, 2008).

Flexibility in calculation refers to an individual's ability to select appropriate strategies from a range of available options based on the specific features of the numbers involved (Threlfall, 2009). Star and Newton (2009) define flexibility in calculation strategies as having knowledge of multiple solutions and the ability to selectively choose the most suitable approach for a given problem. Additionally, Brombacher (2019) defines fluency in calculating as the confident use of a variety of calculation strategies. These perspectives highlight the importance of fluency and flexibility in effectively utilizing calculation strategies.

### 2.3 The Importance of Calculation Strategies

The inclusion of calculation strategies in mathematics curricula, particularly in the early years, is highly important and has been recommended by numerous mathematics researchers (Anghileri, 2001; Clarke, 2005; Heirdsfield & Lamb, 2005; Treffers, 1998). Research conducted by Anghileri (2001) and Treffers (1998) highlights the significance of teaching calculation strategies early on as it contributes to the development of number sense. Number sense entails understanding the structure of numbers, comprehending how numbers function, and fostering cognitive and metacognitive thinking. Clarke (2005) supports this notion by asserting that the early development of calculation strategies enables children to make informed decisions regarding the procedures they employ to solve mathematical problems. Additionally, Heirdsfield and Lamb (2005) emphasize that calculation strategies enhance learners' mathematical reasoning abilities, which involve representing, communicating, and connecting mathematical ideas. Through exploration, discussion, and justification of solutions, learners develop their thinking processes and gain a deeper understanding of mathematics. Furthermore, learners become aware of and rectify misunderstandings, bridge gaps in their knowledge, and acquire new strategies. Consequently, the teaching of calculation strategies plays a vital role in fostering mathematical competence among learners (Kilpatrick et al., 2001). Considering these reasons, this study focuses on the early teaching of calculation strategies.

### 2.4 Examples of Calculation Strategies

A wide variety of examples of calculation strategies have been identified in research literature by several mathematics teachers. For example, Beishuizen (1993) described calculation strategies for addition and subtraction of single-digit and multi-digit number problems called the splitting calculation strategy. In this strategy, numbers are split into tens and ones and the parts are treated separately from left to right. For example, the addition 46 + 38, can be split in 40 + 30 which is 70, and 6 + 8 which is 14, and then 70 + 14 is 84. This can also be solved using the associative law of addition and commutativity of addition. For example, 46 + 38 = 38 + 46 = 84. Beishuizen (1993) further explains that the splitting can also be done by dividing a number into tens and ones and adding the tens of the second number to the first number, followed by the ones. For example, the addition 46 + 38, can first be viewed as 46 + 30 is 76. Then added on the remaining 8, 76 + 8 = 84.

Baroody (1996) focused on children's development of calculation strategies for 1-digit numbers, starting with "counting-all" and progressing to "counting on." Counting-all involves grouping two sets together using fingers or physical materials and counting all items starting with those in the first set and finishing with elements from the second set. As children gain experience, they begin using the more efficient strategy of counting on, which entails starting with the total of one set (usually the larger) and counting on from there to find the answer. However, Baroody notes that while children continue to use these strategies for an extended period, they eventually transition to solving basic operation problems by relying on counting-all strategies, rather than breaking and building up numbers. For example, to find the value of 5 + 7, learners first find the more familiar sum of 5 + 5 = 10 and then add 2, using retrieval and familiar number facts to perform the addition. In subtraction, Baroody (1996) also explores "counting down," which involves starting with the initial amount and counting backward the number of times equal to the amount being subtracted, announcing the last number counted as the amount left.

Burnett and Irons (2009) described number line and count on by tens for double-digit as calculation strategy for addition and subtraction. For example, 93 + 32 is described as a process of adding on a number line with the suggestion. For example, it starts at 93 and jumps 30 to 123. Then it jumps 2 to 125. That is a difference of 32. Along these lines, Van de Walle et al. (2014) used a variety of word descriptions accompanied by number lines with arrows to describe 73 – 46. For example: 70 minus 40 is 30, take away 6 and you get 24, now add the 3 ones and you get 27. An alternative might be: 73 minus 50 is 23. That is subtraction of 4 too much, so we add 4 and get 23 and 4 is 27.

Van de Walle, (2014) studied repeated addition as a calculation strategy for multiplication. For example, 63 × 5 calculated as follows: 63 doubled is 126, this was done twice (252), and then 63 was added to 252 to give 315. He described partitioning as another calculation strategy for multiplication. For example, 27 × 4 is solved as 20 x 4 =80 and 7 x 4 = 28, 20 x 4 + 7 x 4 = 80 + 28 = 108

Ambrose et al. (2003) and Buijs (2008) conducted studies on calculation strategies for solving multidigit multiplication and division tasks. They specifically examined the use of repeated addition for multiplication and repeated subtraction for division. For example, when faced with the multiplication problem $12 \times 29$, learners employ repeated addition by adding multiples of 29. Similarly, for the division problem $736 \div 23$, they utilize repeated subtraction by subtracting multiples of 23.

Burnett and Irons (2009) examined the repetition of doubling as well as building up or building down. In the doubling strategy, the answer can be found by doubling the other number, then and then redoubling. For example, with $4 \times 3$, students might say "Double 3 is 6. Double 6 is 12. The answer is 12." In building up or building down, the answer is found by using known number facts. For example, $3 \times 6$ can be solved by starting with a factor of five ($3 \times 5 = 15$), then adding another 3 to get a total of 18. This is also using the distributive law as $3 \times 6 = 3 \times (5 + 1) = 3 \times 5 + 3 \times 1 = 15 + 1 = 16$ The $4 \times 9$ problem can be solved by starting with a factor of ten ($4 \times 10 = 40$), then subtracting 4 to get the total of 36, also using the distributive law.

Consistent with the above strategies, Fuson and Burghardt (2013) summarises the calculation strategies learners use to solve problems for single-digit and multi-digit number for addition, subtraction, multiplication, and division. These are:

a. Counting: Using fingers, objects, or images to count and perform basic arithmetic operations.
b. Decomposition: Breaking down numbers into smaller components and performing calculations on each component separately.
c. Compensation: Adjusting numbers to make calculations easier or more accurate.
d. Estimation: Using rounding, approximation, or other techniques to obtain an approximate answer to a problem.
e. Algorithms: Following a set of step-by-step instructions to perform complex calculations or solve specific types of problems.

Eather (2016) gives a table of the summary of examples of calculation strategies.

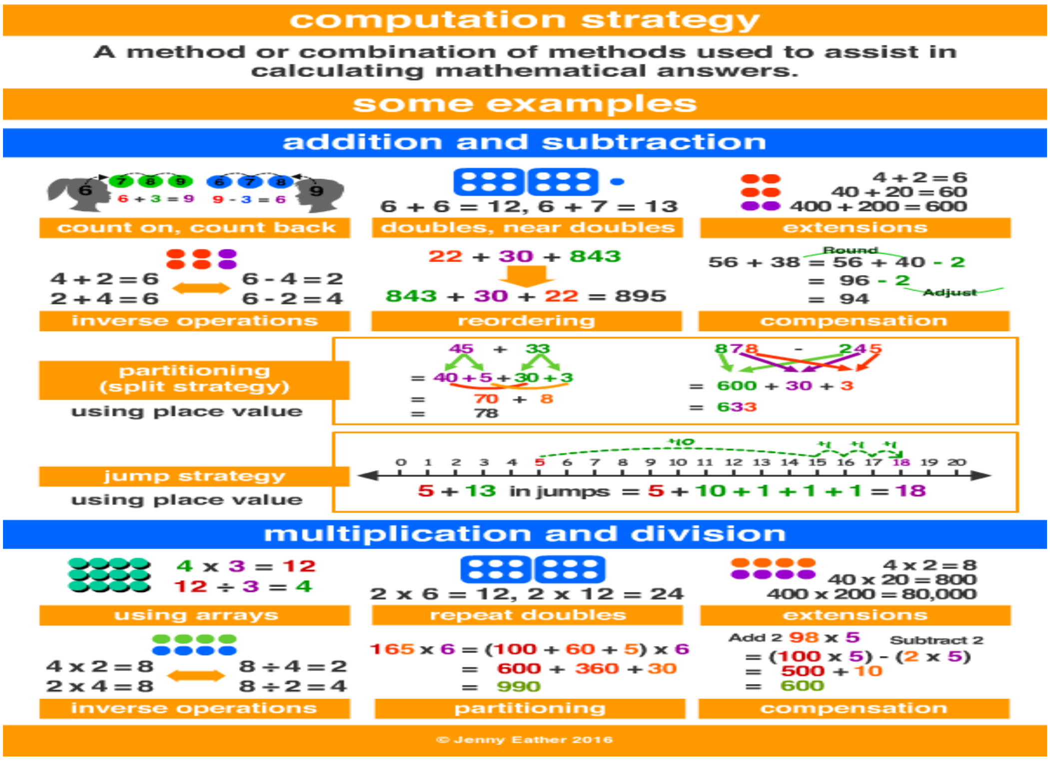


**Figure 1: Examples of calculation strategies, copy from Eather (2016, p. 2)**

Similar calculation strategies have been adopted by the NNP curriculum in Malawi.

### 2.5 How Do Students Develop the Calculations Strategies

Mathematics researchers suggest that students can develop calculation strategies by inventing them (Carpenter et al., 1998; Baroody, 2012). Inventing calculation strategies refers to using strategies that do not rely on physical materials, counting by ones, or a predefined set of steps (Van de Walle et al., 2014). These invented strategies are flexible and involve breaking apart and combining numbers in different ways to solve problems (Van de Walle et al., 2014). The flexibility is based on place value or numbers that work well together. It is important to note that these invented strategies are often personal and built upon learners' understanding (Baroody, 2012).

According to Van de Walle et al. (2014), learners should first develop a conceptual understanding of operations before they begin developing, discussing, and seeking efficient, accurate, and generalizable invented calculation strategies. The researchers recommend that teachers create learning situations and environments that encourage students to develop their own invented strategies because learners do not spontaneously invent flexible calculation methods (Van de Walle et al., 2014).

This discussion led to a debate on how teachers can create the right situations and environments for students to invent calculation strategies. Two main views emerged in mathematics education regarding the invention of calculation strategies (Murphy, 2004). The first view suggests that teachers should not directly teach calculation strategies to learners, but instead allow them to invent strategies on their own, while the teacher provides guidance and suggestions. The second view proposes that teachers should initially teach learners calculation strategies, and from those strategies, learners can invent their own.

In the United States, researchers support the first view (Baroody, 2012; Carpenter et al., 1998; Grouws & Cebula, 2000; Thompson, 2001; Yang, 2008). They believe that children should not be explicitly taught calculation strategies but should invent their own strategies to solve a variety of arithmetic problems. Grouws and Cebula (2000) suggest increasing the opportunities for invention by using non-routine problems, periodically introducing lessons that involve new calculation strategies, and allowing students to build new strategies based on their existing knowledge.

In contrast, researchers in Europe, particularly in the UK and the Netherlands, support the second view, emphasizing the need to teach learners calculation strategies to make their methods more efficient and provide access to a variety of strategies through classroom discussions (Beishuizen, 1993; Department for Education and Skills, 2006; Thompson, 2001). Threlfall (2009) suggests that teachers can promote a culture of invention among children by teaching them strategies first and then fostering the belief that there are no right or wrong ways to solve problems. Teachers should help learners work with strategies that make sense to them without worrying about how others approach the problems. Additionally, teachers should encourage learners to experiment with strategies, even if they may not always be successful.

To examine the effects of allowing learners to invent their own strategies versus teaching them strategies, Torbeyns et al. (2006) and Csikos (2012) conducted studies. Torbeyns et al. (2006) analyzed Grade 2 children with different abilities in solving addition and subtraction tasks. The researchers found that high-achieving students invented a variety of strategies, while low-achievers struggled to come up with even two strategies. This suggests that if calculation strategies are not taught, high-achievers who use a range of strategies have an advantage, while below-average students who rely on inefficient counting procedures are at a disadvantage, leading to significant differences between the two groups.

Csikos (2012) investigated the effect of teaching calculation strategies to Grade 4 learners for 1-digit and multi-digit number problems. Some students were taught the strategies, while others were not. The study included a 3-digit addition task: 176 + 135. The results showed that students who were not taught the strategies faced difficulties and made errors in solving the task. The findings also indicated that learners used a limited variety of strategies. These findings suggest that allowing learners to invent their own strategies may result in an incomplete set of strategies and hinder the availability of efficient methods that have been taught. Therefore, teachers' instruction to support students in using a variety of calculation strategies is necessary (Threlfall, 2002). In summary, by striking a balance between teaching strategies and encouraging invention, educators can foster a comprehensive development of range of calculation approaches among students

### 2.6 The Teaching of Calculation Strategies

The teaching of calculation strategies is a crucial aspect of mathematics education, and educators focus on how to effectively teach these strategies. Several key areas and recommendations have been identified in literature to support meaningful learning and the development of confidence and efficiency in calculation strategies.

Traditionally, studies have shown that many mathematics teachers have relied on repetitive exercises that emphasize following predefined rules, which may not promote meaningful learning (Burghes & Robinson, 2010). To address this, educators suggest providing learners with examples and tasks that require critical thinking and reflection, allowing them to engage in meaningful learning (Burghes & Robinson, 2010).

Thus, Peled and Zaslavsky, (1997 ) discovered that examples and tasks play a vital role in teaching and learning mathematical concepts, strategies, and reasoning. Examples and task are therefore key features in any instructional explanation and tools used to illustrate and communicate concepts between teachers and learners. Rittle-Johnson and Star (2009) conducted research on the use of examples and tasks to teach concepts in mathematics. They focused on the effectiveness of providing learners with multiple examples versus multiple problem-solving tasks. Their research findings suggested that providing learners with multiple examples of the same concept is more beneficial for learning compared to providing them with multiple problem-solving tasks. When learners encounter multiple examples of a concept, they can notice and abstract the underlying structure or pattern. This helps them develop a deeper understanding of the concept and enables them to apply it to different problem-solving situations. On the other hand, Rittle-Johnson, and Star (2009) explained that when learners are presented with multiple problem-solving tasks that require the application of the same concept, they may focus more on the surface-level characteristics of each task. This can hinder learners' ability to generalize and transfer their learning to new situations. Rittle-Johnson and Star (2009) emphasized the importance of providing learners with well-structured examples that highlight the key features and relationships of the concept being taught. Rittle-Johnson and Star said that careful designing and presenting of multiple examples build an understanding of the calculation strategies and enhance their problem-solving abilities.

Studies have recommended teachers to select rich mathematical examples and tasks that involve real-world scenarios, support reasoning and communication, and allow for multiple solution strategies (Jackson et al., 2012; Marton & Pang, 2006). These tasks help students identify key features from past concepts and apply them to new situations, promoting deeper understanding (Watson & Mason, 2006; Marton & Tsui, 2004; Ling Lo, 2012). Teachers should also select rich mathematical tasks and pose mathematical problems that lend themselves solutions that incorporate various calculation strategies which students can demonstrate in different ways (Stein et al., 2000). According to Watson and Mason (2006), Marton and Tsui (2004), and Ling Lo (2012), teachers should also select examples and tasks that allow learners to experience variation in a new concept. These examples and tasks should help students identify key features from past concepts and use them to recognize new and critical features. In addition to that, teachers should pose math problems that lend themselves to multiple solution strategies which students can demonstrate in a variety of ways (Stein et al., 1996). Providing tasks with multiple solution strategies fosters classroom discourse and encourages students to explain their strategies and analyze others (Common Core State Standards Initiative, 2015). According to Hiebert and Wearne, (1993), teachers should also encourage students to explain their strategies and analyse other strategies rather than asking students to recall information or procedures to foster deeper mathematical understanding.

To facilitate learner explanations, studies recommend teachers to ask questions that reveal student thinking and avoid leading them towards the correct answer (Bofferding & Kemmerle, 2015). Teachers should ask learners open ended questions to explain why they chose a particular strategy and how it worked or to analyse other strategies to promote deeper mathematical understanding (Hiebert & Wearne, 1993). Anderson et al., (2015) added that asking open-ended questions encourages students to think, analyse, criticize, and solve unfamiliar problem. Hiebert and Wearne (1993) therefore encourage teachers to ask learner questions for them to name and describe an alternative strategy.

In learner participation Walshaw and Anthony (2008) encourage teachers to first monitor students' participation and explanation in mathematical dialogue to determine when intervention or support is needed in their explanation. Not only that, (Bruce, 2007). recommends that teachers should give students enough time to respond to higher-level thinking questions. Offering students adequate time can result in more detailed explanations expressed with greater confidence. Bruce added that teachers should encourage students to work together in pairs, small groups, or as a class to develop strategies and solutions and this approach enables students to learn from each other.

Furthermore, the Office for Standards in Education (Ofsted, 2oo8) in England has established principles for effective teaching and learning of calculation strategies. The first principle is that there must be a solid foundation in place value of numbers. According to Ofsted (2008) understanding how to use numbers supports the development and collection of known facts that also supports fluency and flexibility in calculating. The second principle is the principle of the worked examples. Ofsted explained that working on examples rather than studying already worked examples can lead to a heavy load on working memory which is not conducive to learning, especially for novice learners who lack proper schemas for integrating latest information into long-term memory. It is important that children should have extensive exposure to teachers demonstrating and talking about a variety of examples. Later, learners should be given time to study the examples demonstrated and developed by the teacher. The third principle is that of teaching a range of calculation strategies. The emphasis is teachers must teach a variety of calculation strategies, monitor, and evaluate the effectiveness of these calculation strategies. Teaching a variety of strategies prevents over-reliance on a single approach and empowers learners to choose and apply strategies flexibly and fluently in different situations The final principle is the principle of learner engagement. DfES (2006) recommends that teachers should give learners the opportunities to explain and justify their thinking, both to the whole class and to their peers, facilitating articulation of their calculation strategies and promoting deeper understanding and future decision-making.

It is therefore important to note that studies have shown that the teaching of calculation strategies should focus on developing strategies rather than simply acquiring them. Starting in the first grade, all students should be exposed to a range of calculation strategies to give them an advantage in their mathematical studies (Threlfall, 2009). Bransford et al. (2000, p. 14) pointed out that if the learners' "initial understanding of mathematical concepts is not fostered and taught properly, they may fail to grasp and use the concepts and information taught in the later years." Therefore, properly fostering students' initial understanding of mathematical concepts is crucial for their future learning.

In summary, effective teaching of calculation strategies involves providing meaningful examples and tasks, fostering student explanations and discussions, and creating a supportive learning environment that encourages collaboration and critical thinkin while promoting a range of strategies. By following these principles, teachers can empower students to develop a deeper understanding of mathematics and enhance their problem-solving abilities.

### 2.7 The Implementation of the Teaching of Calculation Strategies

Several countries have embraced the concept of teaching calculation strategies to learners. One such example is the United States, where subtraction has traditionally been taught using the counting down and splitting strategy. Children often find subtraction more challenging than addition, and the counting down and splitting strategy has proven to be an efficient and easier approach for subtraction (Fuson, 2004). Fuson further explains that as children progress in their understanding and proficiency in calculation strategies, they transition from single-digit addition and subtraction to more complex double-digit strategies. This includes the development of advanced strategies like the compensation strategy. Through continued practice and integration of these calculation strategies into simple word problems, learners also enhance their critical thinking skills (Fuson, 2004). The teaching of calculation strategies has been instrumental in improving learner achievement, as it lays a strong foundation for future mathematical operations in later years (Torbeyns & Verschaffel, 2013).

Similarly, in Australian schools, the use of calculation strategies is recommended in both local and international manuals (Thompson, 2009). These strategies bear resemblance to the Dutch calculation strategies (Thompson, 2001). For instance, Australian schools teach calculation strategies such as the open number line for addition and subtraction, repeated addition and doubling for multiplication, repeated subtraction for division, as well as partitioning, expansion, and grid methods (Thompson, 2009). Additionally, research conducted by Kumon (2012) indicates that Australian learners employ a combination of direct modeling, counting, grouping skills, and strategies based on addition and subtraction when solving multiplication and division problems. The study further demonstrates that early exposure to these calculation strategies instils confidence in learners, enabling them to apply their problem-solving skills effectively in various real-world contexts

In Zambia, schools conducted training for teachers who then began implementing calculation strategies in their classrooms (Tabakamulamu, 2010). These strategies focused on double-digit addition and subtraction, as well as techniques like splitting/partitioning (e.g., the 1010 method) and the use of the empty number line. To assess the effectiveness of these strategies, Tabakamulamu (2010) conducted an evaluation to measure the impact of teaching calculation strategies to grade 2 learners in selected schools in Lusaka. The results indicated that both high and low achievers demonstrated improved performance in the experimental schools, while no change was observed in the control schools. This suggests that teaching learners calculation strategies positively influenced their mathematical abilities.

Likewise, in South Africa, there was a perceived crisis in mathematics education (Fleisch, 2008), with learners consistently falling short of expectations in national, regional, and international assessments. As a response, the national curriculum underwent revisions that included the development of number sense and the implementation of calculation strategies. The curriculum encompassed various fundamental skills, such as instant recall of basic facts (fluencies) like adding ten to a number and knowing number bonds to ten. It also incorporated a range of calculation strategies, such as bridging through 10, jumping, doubling, and halving. Teachers received professional development to enhance their understanding of these strategies. Graven and Venkatakrishnan (2018) conducted a study to investigate the effectiveness of teaching calculation strategies to grade 3 learners. The study evaluated strategies like bridging through ten, jump strategies, and doubling and halving. The findings revealed that the instruction of calculation strategies contributed to improved performance in mathematics among the learners.

## 2.8 Theoretical Framework

This study will adopt the MDI framework developed by Adler and Ronda (2015). The MDI framework will be chosen for this study because it allows for a description of mathematics available for learning in a lesson (Adler & Ronda, 2015). The framework provides an opportunity to closely observe what teachers do in a lesson, how examples are offered, how explanations of what is to be taught and done are constructed and how learners are invited to participate in the lesson (Adler & Rhonda, 2015). The figure below shows the elements of the framework.

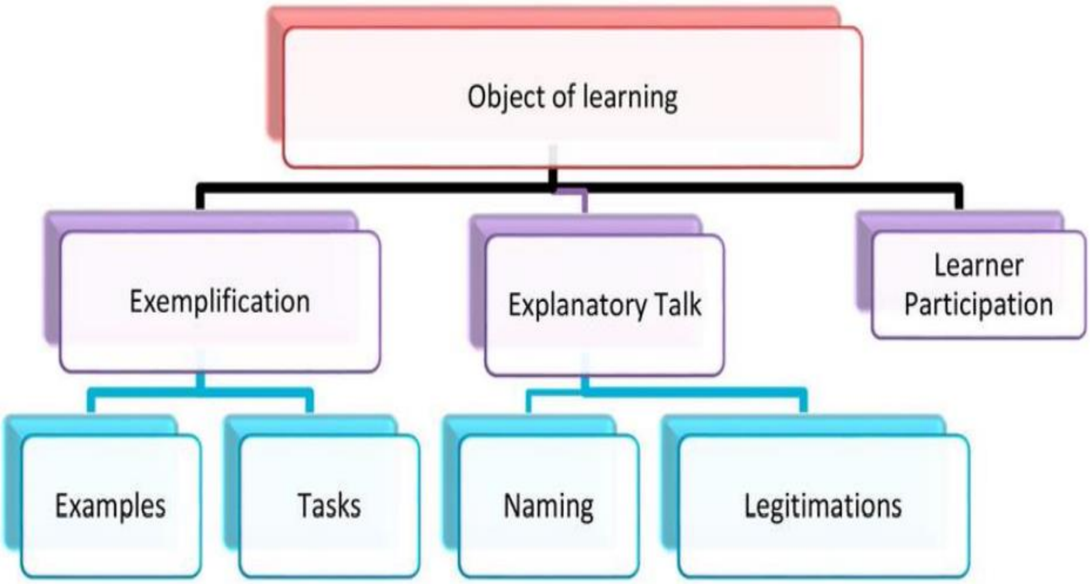


**Figure 2: Constitutive elements of the MDI framework and their interrelations, (copy from Adler & Ronda, 2015, p. 3)**

### 2.8.1 Object of Learning

Learning is about focusing on the central idea of teaching, the object of learning (Adler & Ronda, 2015). The teacher's role is to bring the object of learning clear to learners through exemplification, explanatory talk, and learners' participation (Adler & Ronda, 2015). The object of learning can be described as the lesson objective that teacher usually announces or writes on the board at the beginning of the lesson, focusing on the content and learners' skill related to that content. This is what learners need to know and be able to do. Exemplification, explanatory talk, and learner participation stand between the object and the topic and are pathways to achieve the object of learning.

### 2.8.2 Exemplification.

Exemplification includes the examples, tasks and representations. It examines what examples are used, what tasks are associated with them, and what representations are used.

#### *Examples*

Mathematical content is made visible through examples (Ronda & Adler, 2017). In the teaching process, examples are expected to be used to enable learners understand a particular object of learning. Therefore, the example chosen by the teacher should enable the learners to have a comprehensive understanding of the various calculation strategies. There must be a sequencing of, and variation across a succession of examples as well as teachers' attention to choose and use of examples in their teaching (Adler & Ronda, 2015). Teachers need careful analysis of the examples before selecting which ones to use. Examples must be selected according to the levels and forms of variation they display. This study will focus on similar (level 1) and contrasting (level 2) examples because of the level of the class of my study, however fusion examples (level 3) will be taken into consideration if used. A set of examples is judged as level 1 if the sequence of examples displays only one form of variation, and as level 2 if at least two forms of variation are displayed. Where there are opportunities for learners to experience more than two forms of variations, a set of examples is judged as level 3.

#### *Tasks*

Tasks are designed to provide content knowledge to learners. Ronda and Adler (2017) define tasks as what learners are asked to do. To increase the understanding and mastery of calculation strategies, tasks must engage learners in a variety of content experiences that allow learners to make connections be features of the mathematical content (Ronda & Adler, 2017). Tasks require different actions, at different levels of complexity or cognitive demand, and so in this way can make available different opportunities for learning. According to Ronda and Adler, (2017), tasks must have the potential to engage the learners to make connections among features of mathematical content. According to MDI framework by Adler and Ronda (2015), there are tasks that require learners to carry out known (K) operations or procedures which are classified as level 1, tasks that require K and some application (A), and these are classified as level 2, and level 3 tasks with K and/ or A and C/PS (Adler & Ronda, 2015).

### 2.8.3 Explanatory Talk

The purpose of explanatory talk is name and legitimize the mathematical issues discussed in examples or tasks (Adler & Ronda, 2015).

#### *Naming*

Naming is defined as the use of words to refer to other words, symbols, images, procedures, or relationships (Adler & Ronda, 2015, p. 244). Naming is considered as the use of colloquial (non- mathematical) and mathematical words within and across episodes of a lesson. Naming or use of words therefore constitutes interaction between teacher and learner and learner-learner to develop a shared understanding of mathematical concepts. The function of explanatory talk in the MDI framework is to name and explain what is being focused on and talked about, that is related to examples and tasks using mathematical vocabulary (Adler & Ronda, 2015). It is categorized into levels. At level 1, there is colloquial, non-mathematical (NM) while at level 2, mathematical language is used appropriately and there is a movement between NM and MS and partially MA, while at level 3, there is movement between NM and MA.

#### *Legitimating*

This category focuses on the "criteria for what counts as mathematics that emerge over time in a lesson and provide opportunity for learning consistent with scientific concepts" (Adler & Ronda, 2015, p. 243). Legitimations are justifications given when describing the mathematical procedures and concepts. In a lesson, these justifications are given by the teacher and assist the learner in identifying what considered an important mathematical feature or an explanation for why the procedure was done. The legitimation criteria are nonmathematical (NM), if there are everyday knowledge (E), visual cues (V), assigning authority to the position (P) of the speaker of the statement, the teacher. NM in legitimation is classified as level 0. Criteria of that are considered mathematical or localized is (L) and level 1, another criterion is partial generality (PG) and full generality (FG). Level 2 is where the legitimating criteria is beyond NM, L and include PG. Level 3 is when the criteria is FG (Adler & Ronda, 2015).

### 2.8.3 Learner Participation

Learner participation is about doing maths and talking maths. It is concerned with what and how learners are invited to write and say, apart from doing the tasks assigned to them. When learners are given opportunities to answer yes/ no questions or offer single words to teachers unfinished sentence, its (Y/N), where learners answer what/ how questions in phrases/ sentences is (P/S), and opportunities for learners to answer why questions, present ideas in discussion, teacher revoices, confirms, and asks questions is (D) (Adler & Ronda, 2015). Learner participation is specifically about whether learners have opportunities to speak or non-verbally display mathematical reasoning. Learner participation also seeks to find out if learner activity builds towards the learning goal. Learner participation in the framework describes how learners are invited to participate during the lesson.

### 2.9 The Malawi Context

As mentioned earlier, formal schooling in Malawi includes eight years of primary and four years of secondary education. Primary education is free and easily accessible. At the end of primary school, learners sit for the National Primary School Leaving Certificate Examinations, which they must pass to be admitted to secondary schools. Secondary education is not free, but the fees are heavily subsidised by the government. The secondary schools are not as easily accessible as primary school. There are many more learners than the places available in secondary schools. Learners are selected to national, conventional, or community day secondary schools based on their performance in the National examinations. Secondary education lasts for four years, from Form 1 to Form 4. At the end of Form 4, learners sit for the Malawi School Certificate of Education Examinations, a national examination used for selection to universities, teacher education colleges or other institutions.

#### 2.9.1The core element number operation and relationships in Malawi

In Malawi, the core element of number, operation and relationships consists of more topics compared to other core elements. According to the analysis of the various curriculum documents (syllabuses, teacher guides and textbooks) done by Brombacher (2019), the core curriculum element Numbers, operations and relationships consists of 7 topics in standard 1, 18 topics in standard 2, thirteen topics in standard 3, and thirteen topics in standard 4. Standard 1 in particular has 145 lessons throughout the academic year. However, all the lessons in standard 1 are devoted to counting, reading, writing, and doing addition and subtraction in the number range 0 to 9. The learners are not exposed to the number 10 – the basis of the decimal system. Learners are supposed to master the low-level skills before they go to higher level skills. Bridging (regrouping, or more colloquially carrying and borrowing) is not introduced into calculations until the second half of Standard 3. The teacher’s guide approach to the core element is identical from Standard 1 through to Standard 4, the only difference is the skill levels and the increase in the range number. The approach relies only on a setting out in columns method and on a 'counting all' strategy using locally available resources like stones, leaves, and twigs to determine the sum or difference of the digits in each column.

### 2. 9.2 The Teaching of Number, Operation and Relationship in the OBE Curriculum

In the OBE curriculum, all the lesson plans in all Standards follow a very similar pattern. (MIE, 2021). The focus of teaching mathematics more on procedures and less on understanding. Typically, the teacher writes the title of the lesson on the board and explains the meaning of the title, writes two or three examples, and solves the problems with learners. The teacher then gives similar problems for groups to work and then work individually (Kazima & Jakobsen, 2013). Across all the Standards, learners do not develop their own calculation strategy (Brombacher, 2019). This type of teaching promotes rote learning because it only emphasizes on the use of procedures without understanding. This type of teaching also does not promote critical thinking and problem-solving skills in learners (Walters et al., 2014). The focus on teaching procedures has led to the observations made in the analysis of the assessment data that learners are not able to apply their mathematical knowledge in meaningful ways. In addition, and calculations are only performed using counters and a combined and "counting all" strategy across all the Standards (Brombacher, 2019).

### 2.9.3 The Teaching of Number Operation and Relationships in the NNP Curriculum.

Because NNP is the revision of the OBE curriculum, teaching the core element of "Number operations and relationships" across the two curricula develops learners' ability to recognize, describe, and represent numbers and their relations to count, estimate, and calculate, with competence and confidence in problem-solving (MIE, 2009, 2021). The core element also builds and develops learners' understanding of numbers, laying the foundation for mathematics in later years (MIE, 2009, 2021). However, in the OBE curriculum, leaners are exposed to count all calculation strategies in all the Standards and are not given the flexibility to use the strategy they know or have discovered. Learners follow exactly the calculation procedure taught to them by the teacher, while in the NNP curriculum, learners are introduced to a range of different calculation strategies that are not included in the OBE curriculum. Some of the calculation strategies that are included in the NNP curriculum are physical modelling, number lines, breaking and building up numbers, doubling and halving, estimating, compensating, and commutating (MIE, 2021).

### 2.9.4 The Teaching of Calculation Strategies in the NNP Curriculum

The idea of developing learners' ability to select from a range of calculation strategies and use these strategies fluently and flexibly in unfamiliar situation is reflected in the teaching and learning activities of the NNP curriculum. (MIE, 2021). Learners are given the opportunity to choose among the strategies that are taught by the teacher or those that are discovered by the learners themselves and do the calculations flexibly. The NNP curriculum defines fluent calculation as confidently performing calculations in an age-appropriate range of numbers and using strategies appropriate to learner's developmental level, and flexible calculation as ability to select from a range of calculation strategies available and use the strategy or strategies that is or are appropriate for the calculation (MIE, 2021).

The expectation of the NNP curriculum is that learners will develop the ability to apply these strategies frequently with confidence and flexibility across a wide range of contexts. Learners are presented with unfamiliar situations in which they are expected to apply their existing knowledge, make sense of the situation, and solve problems (MIE, 2021). In addition, learners are often explicitly or implicitly asked to reflect on their actions, which forces them to reason and develop their understanding of what they are doing. The teachers' role is to check the answers and review the activities by asking question about the similarities and differences between the activities previously learned and activities learned that day (MIE, 2022). To improve the use of these calculation strategies, classroom routines for organizing mathematics lessons are developed and written in the teachers' guide for the teachers to follow. Each lesson is divided into three phases: teacher-led activity, independent learner activity, and reflection as shown in figure 3 below.

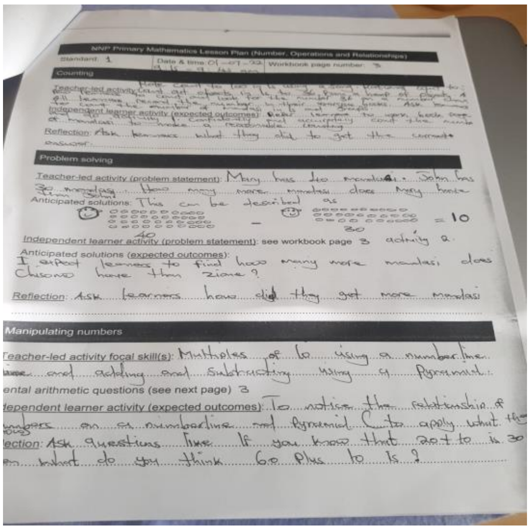


**Figure 3. A sample of a filled lesson plan.**

The teacher is expected to begin a lesson with one or more activities that prepare learners to work independently on tasks, individually, in pairs or in small groups to deepen their understanding of the teacher-led activity. The lesson concludes with a teacher-led reflection. When planning a lesson, teachers decide and plan which calculation strategies will be used in each of these components of the lesson. The role of teacher-led, and independent learner activities is to develop calculation skills and strategies. Each day, the teacher reflects on learning activities that are in the learners' workbook. Then, there is a session of mental arithmetic with the class or with groups of learners. The learners practice the calculation strategies introduced by the teacher by completing the workbook page assigned for the day (MIE, 2021).

### 2.9.5 Structure of the workbook

A typical NNP lesson plan has teacher-led activities, independent learner activity, and teacher led reflection of learnings of the lesson. The content of the independent learner activity portion of the lesson is generally a page in the learners' workbook. The purpose of the workbook is to provide learners with the opportunity to consolidate their understanding of the content of the teacher led activity. The page of the workbook is the "lesson plan." The contents of the page and the puzzle pieces at the bottom of the page guide the teacher in the planning of the teacher led component of the lesson. Below is a sample of a page in the workbook.

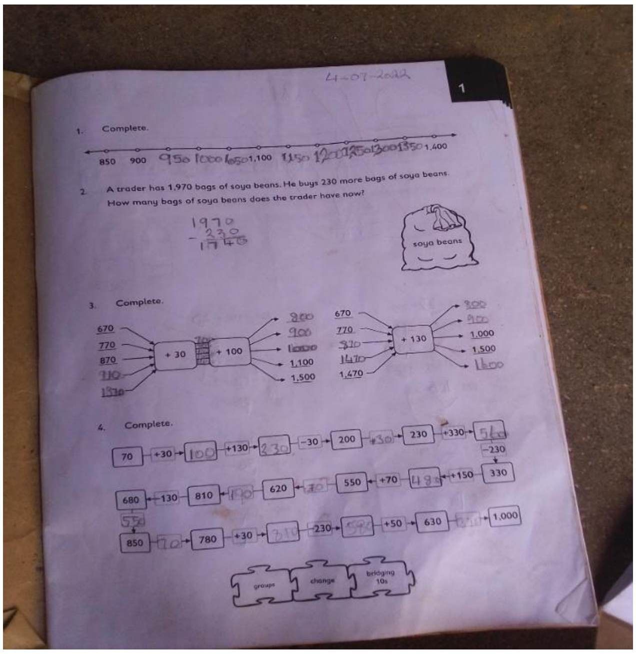


**Figure 4: Example of learner workbook page in the NNP curriculum**

## 2.10 Chapter Summary

This chapter has given a definition of calculation strategies, the examples and importance of calculation strategies. It has also looked at how calculation strategies are taught. Furthermore, the chapter has also given studies of the countries that implemented the teaching of calculation strategies. Finally, a brief literature of the teaching of the teaching of mathematics and the teaching of calculation strategies has been presented.

The next chapter is the methodology that was used in this study.

# CHAPTER 3: RESEARCH METHODOLOGY

## 3.1 Introduction

This chapter presents the methodology of the study. It describes the design of the study, data construction, instruments used, pilot testing, sampling procedures, data collection method, administration of instruments, reliability and validity, data analysis, and ethical considerations. The summary of the research methodology is presented thereafter.

## 3.2 Research Design

Research designs are procedures for collecting, analysing, interpreting, and reporting data in research studies (Creswell, 2012). Thus, there are plans and actions for research that include detailed methods of data collection and analysis. According to Creswell (2013), there are several factors that guide the research design. These include the nature of research problems, the personal experiences of a researcher, the audience the research is writing for, and the research sample. Research designs are useful because they help guide the methods and decisions that researchers must make during their studies and provide the analytical framework at the end of the study (Creswell, 2012).

This study was guided by a mixed method approach to collect data related to the teaching of calculation strategies of the NNP curriculum and answer the research questions. Mixed methods research is defined as an approach that combines or associates both qualitative and quantitative forms (Creswell, 2013). In a mixed methods approach, the researcher follows researcher the principles and assumptions of qualitative and quantitative study. This approach was selected because looking at the research questions, the use of a single research approach would not make a better argument. The goal was also to combine both approaches in creative ways that utilize the strengths of each within a single study (Creswell, 2013). Also, according to Creswell (2018), mixed methods are useful in that they to gain more understanding of the research problems, and it gives the study a greater strength than it would have if it only used a qualitative or quantitative approach.

Mixed method design comprises different strategies that are used to collect data. These strategies include sequential explanatory, sequential transformative, concurrent triangulation, concurrent embedded, and concurrent transformative (Creswell, 2013). This study employed a sequential explanatory strategy. That is, data collection starts with the quantitative approach followed by qualitative data in the second phase (Creswell, 2018), The results for the quantitative phase are the ones that inform the qualitative part of data collection. Quantitative research designs are the ones that are characterized by data collection expressed in numerical forms and analysed using suitable statistical methods (Ary et al., 2014) while qualitative research designs are designs that are a means of understanding the meaning that individuals or groups give to a problem (Creswell, 2013).

The study collected quantitative data by coding examples and tasks from lesson plans, the ones mentioned during interviews, and observed during the lesson presentation using the MDI analytical tool adapted from Ronda and Adler (2017). The assigned codes were examined against the ability to provide a set of critical features of the object of learning in the lesson. The codes for exemplification and explanatory talk were later analysed and interpreted using a qualitative design approach to get an in-depth understanding of the content.

## 3.3 Data Construction

The study aimed at answering the following research questions.

a. What calculation strategies in the NNP curriculum do teachers teach?
b. How do mathematics teachers teach calculation strategies in the NNP curriculum?
**c.** How do teachers view the teaching of calculation strategies in the NNP curriculum?

As such, various instruments and methods were used to collect data to check biases and ensure there is no misinformation (Fraenkel et al., 2012). Therefore, to answer the research questions for this study, data collection instruments such as questionnaires, lesson observation guides and interview guides were used, the method followed to collect data were surveys, lesson observations, interviews with mathematics teachers, and document analysis. These data collection techniques were chosen because they complement each other. Fraenkel et al. (2012) explain that when a conclusion is supported by data collected from several different instruments, it enhances its validity; therefore, these data collection techniques were used interdependently to valid results. Also, this combination of several data collection methods, which is called triangulation (Creswell, 2013), is helpful in supporting and shedding light on the different themes or issues that emerge from the study, thereby, minimizing bias.

## 3.4 Instruments

### 3.4.1 Questionnaire

The questionnaire was used to find out the teachers that have knowledge of the calculation strategies in the NNP curriculum. It had five questions. The first three questions aimed at finding out teachers ‘experience as teachers in general and as mathematics teachers. The other question aimed at finding out how teachers understand the calculation strategies that pilot primary teachers teach. The last questions aimed to find out how often teachers attended the NNP trainings. The questionnaire contained a section where respondents were required to write their classes because the researcher needed to know who the respondents were to follow them up for participation in the qualitative part of the study. Before answering the questionnaire, respondents were made aware of the reason why they had to include their classes.

### 3.4.2 Lesson Observation Guide

Because observation involves the researcher directly studying the participant 's behaviours by quietly watching the activities that are going on (Rea & Parker, 1997), the observation guide was chosen to be one of the instruments to make the lesson observations more focused on the required information and to record notes. It mainly looked at how lesson activities were planned and organized especially whether they were planned and how they were planned in the lesson plans, the calculation strategies taught, how they were taught (examples and tasks) and how learners were invited to participate in the activities. The observation guide was developed to suit the aim of the research, and it was designed based on the MDI framework developed by Adler and Ronda, (2015).

### 3.4.3 Interview Guide

The interview guide was developed to get insight into the calculation strategies that teachers teach, how they teach the calculation strategies. It was also developed to find out how teachers view the teaching of calculation strategies in the NNP curriculum. It sought to follow up on what the respondents answered in the questionnaire and what was observed during lesson observation. Thus, this interview guide helped to answer all questions.

## 3.5 Pilot Testing

The researcher asked for permission to conduct research from the District Education Manager for the Blantyre district. After the permission was granted, a pilot testing was conducted at one of the primary schools in the Blantyre district. This pilot testing was done in two phases, starting with pilot testing for the questionnaire, and then piloting for the lesson observation guide and interview guide.

Pilot testing was done to check the validity of the instruments. As Teijlingen and Hundley (2001) explain, a pilot study provides an advance warning about where the main research project could fail, where research protocols may not be followed, or whether proposed methods or instruments are inappropriate or too complicated. It was also done to find out whether participants understood the questions, and then make improvements based on their responses

(Fraenkel & Wallen, 2000). This pilot testing also helped to practice observation and interviewing skills. Participants in this pilot study were primary school teachers who had similar characteristics to those teachers who were going to participate in the actual study. The determining characteristics here were that they should be standard one to four NNP curriculum pilot primary school teachers.

Five teachers in the pilot school participated in answering the questionnaire. Teachers' responses to the questionnaire showed that the questions were clear because they did not have trouble answering the questions, and they answered the way they were expected to. After they had finished answering the questions, the participants were asked to comment on the questions, and they explained that the questions were easy to understand. Because of this, no modifications were made to the questionnaire. It was then used in the main study as designed

Two teachers participated in the pilot testing of the lesson observation guide and interview guide. One lesson was observed on each participant to check whether the instruments were going to give the intended results. Results of the pilot lesson observations provided information that could help in answering research questions two and three, leaving research question one unanswered. This meant that the guide was only usable for two research questions. Therefore, instruments were revised to include a section that accommodated the calculation strategies that teachers plan in the lesson plan, schemes of work, and teaching in the lesson.

On the part of the interviews, the interview questions were intended to find out what and how teachers taught the calculation strategies and view the teaching of calculation strategies. The two teachers who participated in pilot lesson observations were also the ones who participated in the interviews. The results of the pilot interview showed that the answers to the questions gave information valuable to answer the research questions. The line of questioning was controlled by rephrasing the question or probing more when necessary (Creswell, 2013). Hence the respondents showed that they understood the questions, so no modifications were made.
From the pilot testing, especially from the qualitative part, it was observed that one lesson observation was not enough but two lesson observations per participant would give a better picture of how the teachers teach mathematics. Also, it was discovered that making only one observation would not be enough to give a true picture of the calculation strategies teachers teach in their lessons because many factors affect how one teaches a particular lesson as

observed during the pilot study. This information was then used in the main study where the teachers were observed twice.

**Table 1: Research questions and how they were answered**

| RESEARCH QUESTIONS | METHODS | TOOLS |
|---|---|---|
| 1. What are the NNP calculations strategies that teachers use when teaching mathematics? | • Mathematics lesson observation<br>• Document analysis<br>• Interview | • Lesson Observation guide.<br>• Interview guide |
| 2. How do mathematics teachers teach calculation strategies in the NNP curriculum? | • Mathematics lesson observation<br>• Document analysis<br>• Interview | • Observation Guide<br>• Interview guide |
| 3. How do you view the teaching of calculation strategies in the NNP curriculum? | • Interview | • Interview guide |

## 3.6 Sampling Procedures

A sample is any group from which a researcher gets information; and sampling is the statistical process of selecting a subset (called a "sample") of a population of interest for purposes of making observations and statistical inferences about that population (Bhattacherjee, 2012). The study targeted NNP curriculum pilot schools' teachers in two sections of the primary school; the infant section, and the junior section in the Blantyre district.

### 3.6.1 Sampling of Schools

The study began with the quantitative approach in which a list of three primary schools which have been participating in the piloting of the NNP curriculum and have a high enrolment were selected from the Blantyre district. This list was obtained from the District Education Manager 's office. Simple random sampling was used to select a school that participated in the survey questionnaire. This simple random sampling was done by writing numbers on pieces of paper according to the list of schools in the Blantyre district. The pieces of paper were then folded and put in a plate. One piece of paper was picked from the plate. The number of a school that corresponded with the picked number was selected to participate. The selected school has two streams[2] which were named school 1 (S1) and school 2 (S2).

### 3.6.2 Sampling of the Participants

After the school was identified, the researcher visited the school to meet the head teacher and briefed them about the research. On the same visits, the researcher made appointments to conduct a survey questionnaire with mathematics teachers. There are ten mathematics teachers at S1 and ten mathematics teachers S2. In both schools, Standard 1 has three teachers and three classes, standard 2 has three teachers in three classes, A, B, C, standard 3 has two teachers and two classes and standard 4 has two teachers and two teachers. All the twenty teachers were notified that those who would meet the characteristics required in the study would be asked to participate in the second phase of the study, and they all agreed that if selected, they will participate freely.

### 3.6.3 Purposeful Sampling

Participants in the qualitative part of the study were purposefully selected from the twenty teachers who responded to the questionnaire. These were assigned codes randomly from PT1 to PT20. According to Gall, Gall, and Borg (2007), purposeful sampling aims at selecting cases which will provide rich information in respect to the purpose of the study. Only those who satisfied the required characteristics were selected. For the sake of confidentiality, names of teachers have not been revealed in this study; instead, the teachers are identified by teacher number they were given. The questionnaire was administered to identify mathematics teachers who attended the NNP trainings and are teaching the calculation strategies.

Selection of participants to be observed and interviewed was based on how well they understood the calculation strategies, the number of calculation strategies they indicated and how often they attended the NNP trainings, the number of years they have been teaching mathematics and the number of years they have been teaching mathematics in that class.

A total of twenty teachers, ten teachers from S1 and ten teachers from S2 responded to the survey questionnaires. The responses to the survey questionnaire were analysed statistically, and the results that were obtained from the questionnaire were the ones that guided the

[2] Stream in Malawi where learners go to school in shifts, one group come in the morning from 7 o'clock to 12 o'clock one the other group start from 12 o'clock to5 o'clock.

researcher on the selection of participants who were going to participate in the study. The questionnaire was given to each participant to fill out individually (see Appendix 2).

Before they started answering questions, they were told to ask wherever they did not understand. I waited for the participants to fill out the questionnaires and collected them on the same day. This was done to make sure that all questionnaires were returned.

Eight mathematics teachers were identified. These were four standard 1 to 4 teachers from S1 and four standard I to 4 teachers from S2. For the sake of confidentiality, the names of the teachers selected have not been revealed in this study; instead, the teachers are identified by the teacher code number they were given. Then the code numbers for the eight teachers were rearranged and were assigned code as S1TA, S1TB, S1TC, S1TD foe School 1 and S2TA, S2tb, S2TC, S2TD for School 2.

## 3.7 Data Collection Methods

### 3.7.1 Survey

This, being mixed methods research, it used a survey to collect data in the quantitative and qualitative part of the study. A survey is a process whereby data is collected by asking several people the same questions related to their characteristics, how they live, or what they think about an issue through a questionnaire (O 'Leary, 2010). In this study, the researcher used a cross-sectional survey, in which data was collected at a single point (Creswell, 2013). The survey was chosen because it allows many respondents to be reached simultaneously, thus allowing less time to collect the required data. In this survey, questionnaires were distributed to 20 pilot primary school mathematics teachers at the selected school. The aim of the questionnaire was to find how well teachers understood the calculation strategies and how often they attended the NNP in service trainings and. The of the survey was to identify the teachers that could be observed and interviewed within the qualitative part of the study.

### 3.7.2 Lesson Observation

Observation is one of the major techniques for collecting data in qualitative research (Creswell, 2013, p. 166). During observations, the researcher can be a participant observer, a non-participant observer, or a complete observer (Creswell, 2013). In this study, the researcher was a non-participant observer who did not participate in the classroom activities but just observed the participants teaching. Rolfe et al. (2001) explain that observations are useful when researchers want to understand or explain everyday behaviours or examine the effect of something on everyday behaviour. Creswell and Poth (2018) added on to say that observations help the researcher to have first-hand experience with the participants; the researcher gets an insight into the real things that people do, rather than what they say or think they do, and information can be recorded as it occurs. Susuwele-Banda (2005) claims that sometimes what teachers claim to know about their teaching is not what is reflected in their classroom practices. According to Creswell (2018), observations help the researcher to have first-hand experience with the participants; the researcher gets an insight into the real things that people do, rather than what they say or think they do, and information can be recorded as it occurs. It is therefore during observation that the researcher can notice unusual aspects which could otherwise not be noticed if the observation was not done.

However, using observation can be problematic in ways that sometimes participants might not feel comfortable with the researcher. As such, they might behave not as they would normally do (Creswell, 2013); thus, children and even the teacher in a classroom may act differently if there is an observer than they would do if there was no observer around them. Also, the researcher may lack good observation skills, and his/her expectations might influence what should be observed (Creswell, 2018). In this study, these limitations were dealt with by visiting the classes before the actual observation study day began just to familiarize the learners and the teachers. To reduce the impact of the expectation of the observer, the lesson observation guide contained a list of specific behaviours that were expected to be observed.

### 3.7.3 Interviews

In addition to surveys and observations, data were also collected using interviews. Interviews involve the researcher asking questions and getting responses from participants. Interviews were chosen because they are known to be useful data collection methods when collecting information that cannot be observed directly, such as opinions, beliefs, feelings, as well as perceptions (Merriam, 2001). There are different forms of interviews which include structured, unstructured, semi-structured, and focus group interviews (Cohen & Crabtree, 2006). As Creswell (2013) asserts, this study used one-on-one semi-structured interviews because they allow for the flexibility of the researcher to diverge from the original question to follow a response in more detail. As such, the interviews were audio recorded.

The same eight standard one to four NNP teachers who participated in lesson observations were the ones that were interviewed. In these interviews, the participants were asked to explain more about the calculation strategies they teach and how often they teach them, how they teach the calculation strategies, specifically how they chose the examples and tasks and explain them.

In addition, participants are asked to explain how they view the teaching of calculation strategies of the NNP curriculum and their suggestions on how best the teaching of calculation strategies can be improved to be successful thereby answering all questions in this study.

However, according to Creswell (2013), in interviews, the presence of the researcher may bias the responses of the participants, and again, interviews may provide information that is filtered through the views of the ones being interviewed. These limitations were dealt with by building a rapport with the respondents to make them feel comfortable. The respondents were assured that the interviews were merely for research purposes, and not for their professional evaluation. They were also assured that their names and details would not be revealed and that all reporting was going to present participants anonymously.

### 3.7.4 Document Analysis

Fraenkel and Wallen (2000) and Creswell (2013) describe documents as written or printed materials that have been produced in some form. Examples of documents that could be analysed in schools may include tests, registers, schemes of work, lesson plans, and progress books. This way of collecting data helps the researcher to get hold of the initial language and words of the participants and it can reveal the information that the participant would not want to share during interviews (Creswell, 2013; Merriam, 2001). The documents used in this study were lesson plans. Going through schemes the lesson plans was necessary for the purpose of checking if teachers indeed planned the calculation strategies, examples, and tasks to be taught in a particular lesson. Not only that, but I also wanted to find out if the teachers taught the calculation strategies according to plan. Information obtained from these documents helped to answer research questions 1 and 2 of the study.

## 3.8 Instrument Administration

After eight mathematics teachers had been purposefully selected to participate in the qualitative part of the study, the participating schools were visited to meet the head teachers and participants. The head teacher was told about the selected teachers, and I asked for permission to have a brief meeting with the selected participants. During the meeting, the teachers were once again briefed on the purpose of the study. The participants were assured that the observations and interviews were solely for research purposes, not for grading, and that confidentiality would be highly observed. They were told to voluntarily participate but they should willingly choose to do so and that they were free to withdraw at any point if they wished to do so. They were also told that before observations, they were going to sign consent forms that explained all the conditions of the study. After the meeting, the researcher asked the teachers if she could visit the classes just for a short time to familiarize herself with the learners. Permission was granted.

During lesson observation, the structured lesson observation guide was used to take down notes (see Appendix 3). T Detailed notes about what was going on during the lesson, starting from the general outlook of the class to the actual activities of the lesson were taken. It was also used to write information obtained from the lesson plans.

Interviews were done after the second round of lesson observations. They helped to obtain information that could not be obtained through lesson observation. These interviews were conducted with the same teachers who were observed. The interview guide was used to guide the flow of interview questions and record responses (see Appendix 4). The guide was with the researcher, and the participants could just answer the questions orally without necessarily looking at the questions in the interview guide. Before the beginning of the interviews, the respondents were asked if they could use a voice recorder, and each respondent had no problems with this. The voice recorder helped to capture information that was missed during the writing of responses as the interview was in progress. As soon I got home, she transcribed the voice records on paper. The files were listened to repeatedly to make sure that no important data was missed.

## 3.9 Data Analysis

The central purpose of data analysis was to make sense of the data that was collected. According to Creswell (2018), data analysis involves making sense of text and image data. Data collected in this study has been analysed using the framework analysis of the MDI framework. Framework analysis as Parkinson et al. (2016) explains, provides flexibility to use the data systematically facilitate greater familiarity and immersion in the data, and eventually a better understanding of the insights and experience of the participants. Being a method, the analysis started with quantitative analysis followed by qualitative one.

### 3.9.1 Analysis of Quantitative Data

Data derived from the questionnaires, were statistically analysed. Each teacher ‘s response to each question was carefully coded into categories depending on the questions on the instrument. Frequencies were then used to analyse the data. Thereafter, the findings were presented in form of tables to clearly show how well mathematics teachers understand the calculation strategies, how often they attended the NNP trainings. This analysis also gave the researcher some personal details of the participants such as their teaching experiences, what classes they teach, and the enrolment in their classes. This stage of analysis was the one that helped in the selection of teachers who participated in the qualitative part of the study.

### 3.9.2 Analysis of Qualitative Data

The second part of analysis was done qualitatively using inductive analysis. In inductive data analysis, the patterns, themes, and categories of analysis come from the data; they emerge out of the data rather than being imposed on them prior to data collection and analysis‖ (Patton, 1980, p. 306).

Before starting the formal analysis of data, both video audio recordings from lesson observation and the interviews that were captured as part of data collection, the general ideas were transcribed to compare instruments. As one way of getting familiar with the data, Mills and Morton (2013) suggests “immersion” as the first step of data analysis. This is where the researcher spends some time with the research material, reading and re-reading the transcripts and the field notes. All this data were then typed and saved electronically. After this, data was organized and arranged following each research instrument.

Thereafter, coding was done using the MDI framework as mentioned earlier on. Data were organized into chapters to form different units of analysis in accordance with the research questions based on the elements of the MDI framework: the object of learning, exemplification, explanatory talk, and learner participation (Adler & Ronda, 2015). All the characterization of the MDI framework—based on the selected examples and tasks were noted and aggregated

### 3.9.3 The object of learning.

According to the MDI framework, the object of learning refers to the specific content, skills, and concepts that teachers aim to teach their students. In this study, the object of learning is the calculation strategies. I focused on whether the teachers aligned their planning and teaching with the object of learning. I also focused what calculation teachers plan and teach their learners. I also focused on the sources where the examples and tasks used to plan and teach calculation strategies came from.

### 3.9.4 Exemplification

According to the MDI framework, exemplification refers to the use of examples and tasks to illustrate mathematical concepts and procedures. Effective teachers use a variety of examples that are relevant and meaningful to students' lives and experiences (Adler & Ronda, 2015) I therefore began the analysis of exemplification by identifying and creating a list of examples and tasks selected and indicated in the lesson plans, observed in each lesson and narrated during the interviews, I also focused on what and how calculation strategies were highlighted through the selected examples and tasks during planned and the lesson presentation, and how teachers planned and taught the calculation strategies to learners.

To analyse the examples with regards to MDI framework, I focused on the three categories, similarity (**S),** contrast **(C),** and fusion **(F).** On similarity **(S**), I focused on how teachers taught and connected new calculation strategies to familiar ones. In Contrast **(C**), on the other hand, I focused on how teachers highlight differences between calculation strategies taught or how teachers illustrated how a calculation strategy taught t may be used to solve different questions depending on the context. Finally, in fusion **(F**) I focused on how the teachers used tasks to connect previously learned calculation strategies with new ones and to show how they work together to solve problems. On tasks, the levels were limited to high and low cognitively demanding tasks with emphasis on three categories, carry out known operations and procedures **(K),** apply skills (**A),** and use multiple calculation strategies and connections and justify the reasoning **(C/PS)**. Three categories are based on the idea that cognitive demand increases as the connections between the concepts and procedures become more complex and intertwined.

In the Table 1 below, I present the category of exemplification and its subdivisions from MDI, as well as the codes and recognition rules that informed the analysis.

**Table 2: Description of Coding of Exemplification**

| EXAMPLES | TASKS |
|---|---|
| **Examples are coded as follows.**<br>**Level 1**. One form of variation**:** similarity **S** or contrast **C**<br>**Level 2**. any two of similarity, contrast, and fusion<br>**Level 3**. Simultaneous variation fusion and generalization. Fusion of more than one object of learning and connected with similarity and contrast within a set of examples: coded **F**<br>**Level 0.** Simultaneous Variation with no similarity or/ and contrast | **Tasks are coded as follows:**<br>**Level 1**. Carry out known operations and procedures e.g., multiply, factorize, solve **(K)**<br>**Level 2**. Apply level 1 skills and learners must decide on (explain the choice of) operation and /or procedure to use e.g., Compare/ classify/match representations **(A)**<br>**Level 3.** Multiple concepts and connections. e.g., Solve problems in different ways; use multiple representations; pose problems; prove; reason. **(C/PS)** |

### 3.9.4 Explanatory Talk

The second level of analysis explored the teachers' classroom practices through the categories of explanatory talk. According to the MDI framework, Explanatory talk refers to the teacher's use of language to explain mathematical concepts and procedures. I therefore focused on how teachers named and used language that tailored to students' level of understanding of mathematical calculation strategies. I also focused on how teachers used questioning strategies to encourage students to explain their own thinking and justify their reasoning. Specifically, my main interest here is whether the teachers use Non-Mathematical (NM) language such as everyday language or uses formal mathematical (M) language to teach calculation strategies.

To analyse the data, I used the identification codes for exploratory talk presented in Table 3 below under the two subdivisions: naming and legitimations. Specifically, regarding naming, the researcher paid attention to the teacher’s discourse shifts between colloquial and mathematical word use. For example, a teacher may use colloquial language (Non-Mathematical, NM) which can include everyday language and ambiguous words like this, that, thing (Adler & Ronda, 2015). The teacher may also use specific Mathematical Language (MA) to name. On legitimating, the researcher was, interested in finding whether the criteria teachers transmit as an explanation for what counts is or is not mathematical, is particular or localized, or more general, and then if the explanation is grounded in rules, conventions, procedures, definitions, theorems, and their level of generality. For example, a teacher may use non-Mathematical (**NM)** which can be visual **(V),** Positional **(P)** and use Everyday language **(E**) or Mathematical **(M)** which can be Local Math **(L),** Partial Full **(PF),** or General Full **(GF).**

Below I have included a detailed characterization of the category explanatory talk from MDI that I used to analyse these extracts.

**Table 3: Description of Codes of Explanatory Talk**

| NAMING | LEGITIMATING |
|---|---|
| **The use of colloquial and mathematical words is coded as follows.**<br>**Level 1—NM,** Colloquial (Nonmathematical) language including ambiguous referents such as this, that, thing, to refer to objects<br>**Level 2. (MS)** String of words Formal mathematical words and phrases are used to identify an object, but no meaning is created or connected to the mathematical concept**.**<br>**Level 3—(MA)** Formal mathematics Formal language is used to create meaning and understanding of the object or mathematical concept. | **Legitimating is coded as follows.**<br>**Level 0.** Nonmathematical **(NM),** The statement made about the mathematics is mnemonic/ visual (V), Positional. (P) and use Everyday language (E) and no justification.<br><br>Mathematical **(M)**<br>**Level 1.** Local Math (**L)**, criteria stated to the learners gives specific and localised examples and justifications<br>**Level 2.** Partial full **(PF),** There is an attempt at achieving generality, but it is partial generality<br>**Level 3:** General full **(GF)**. The concept of the procedure is principled and/or derived and proved. (Full generalization). |

### 3.9.5 Learner Participation

The third level of analysis explored the teachers' classroom practices through the categories of learner participation. According to the MDI framework, Learner participation refers to the active engagement of students in the learning process. Effective teachers provide opportunities for students to explore mathematical concepts and procedures through tasks that are challenging and meaningful. Teachers encourage student collaboration and provide feedback that is specific and constructive. (Adler & Ronda, 2015). All the above categories (examples, tasks, word use, legitimating criteria) occur in a context of interaction between the teacher and learners, with learning as a function of their participation. In Learner participation the researcher was concerned with how learners were invited to participate in the lesson, whether in groups, in pairs, as individuals, or as a whole class to speak yes/ no, or single words to teachers' sentence, coded **Y/N,** or to speak some phrases and sentences in more than one episode, answering what/ how questions,**(P),** or invited in a discussion to answer why questions with teacher revoices, **(D).**

**Table 4: Description of Codes of Learner Participation**

| **Coding for Learner Participation** |
|---|
| Learners invited individually, in pairs in groups and as a whole class<br>To speak yes/ no, or single words to teachers' sentence, coded Y/N, and level 1<br>To speak some phrases and sentences in more than one episode, answering what/ how questions, coded P, and level 2<br>Some discussions why questions, teacher revoices, coded D, and level 3 |

### 3.10 Validity and Reliability

This study used triangulation of data sources as a means of ensuring validity. Document analysis (lesson plans, audio recordings were used to enrich data collected from lesson observations and interviews. Bowen (2009) describes triangulation as a combination of methodologies or information from different data sources in the study of the same phenomenon. It is further explained that triangulation helps the research guard against the accusation that a study's findings are simply facts of a single method, a single source, or a single investigator's bias. This is possible because, in triangulation, researchers make use of multiple and different sources, methods, investigators, and theories to provide corroborating evidence. As the researchers locate the evidence from different sources of data, they are triangulating the information and providing validity to their findings (Bowen, 2009). Questionnaires, lesson observation, and interview guides were also used as a way of addressing bias that would have been caused by a single observer and interviewee during observations and interviews.

### 3.11 Ethical Consideration

When dealing with people in research, there is a need to consider ethical issues because participants have their rights. This study followed ethical principles to make sure that it brings no harm or stress to participants (Robson, 2011). As such, everything related to this project followed the standards set by the Norwegian Centre for Research Data (NSD). The researchers applied for permission to NSD to carry out my research project. Permission to carry out the research project was given (see Appendix 1) Processing of personal data had been done in accordance with the principles under the General Data Protection Regulation of the NSD. No information that can be used to identify participants has been revealed. Personal names and school names have been replaced by codes or pseudonyms. After analysing the video recordings of the lessons and transcribing the audio recordings of interviews, both the video and audio recordings' files were deleted. Only anonymized text is kept after the research project. Personal data has been treated with confidentiality and in accordance with data protection legislation. In addition, participants were briefed in detail on what the study was about and were told of their freedom to withdraw from the study at any point if they wished to do so. Informed consent, according to Cohen et al. (2007), allows participants to make an informed decision about whether to participate in the study or not after understanding the aim of the study. Each participant signed a consent form before the beginning of the study. No real names were used.

## 3.12 Chapter Summary

In this chapter, I have engaged with the research approach employed for the purpose of this study. I began by describing the study's research design and data analysis. I wrapped up the chapter by discussing ethical issues.

Using the methodology outlined in this chapter, data was collected, and the findings and discussions are presented in the next chapter.

# CHAPTER 4: ANALYSIS OF DATA AND FINDINGS

## 4.1 Introduction

This chapter presents the analysis of data collected in this study on the teaching of calculation strategies. The chapter begins with a summary questionnaire data by twenty teachers. The questionnaire data was used to select eight teachers who participated in the main part of the study. Then analysis of data from the eight teachers assigned codes as S1TA, S1TB, S1TC, S1TD, S2TA, S2TB, S2TC, and S2TD is presented. The data is presented based on the specific research questions, which are:

1. What calculation strategies do teachers teach in the NNP curriculum?
2. How do mathematics teachers teach calculation strategies in the NNP curriculum?
3. How do teachers view the teaching of calculation strategies in the NNP curriculum?

The presentation and analysis of data starts with data from the lesson plans, the classroom observation, and interviews, respectively. The MDI analytical framework developed by Adler and Ronda (2015) is used to analyze the data, focusing on exemplification, explanatory talk, and learner participation.

## 4.2 Data and Analysis of the Questionnaire

The questionnaire asked for information about the teachers' age range, the teaching class, the enrolment in the teaching class, teaching experience, NNP training attendance, and their understanding of the calculation strategies.

### 4.2.1 Findings from the questionnaire

**Table 5: Age range and gender of teachers**

| | | 20-30 | 31-40 | 41-50 | Total |
|---|---|---|---|---|---|
| Number of teachers | Female | 3 | 1 | 7 | 11 |
| | Male | 1 | 4 | 4 | 9 |
| | Total | 4 | 5 | 11 | 20 |

**Table 6: Teaching class**

| | Standard one | Standard two | Standard three | Standard four | Total |
|---|---|---|---|---|---|
| Number of teachers | 6 | 6 | 4 | 4 | 20 |

**Table 7: Class Enrolment (number of learners in class)**

| | Less than 60 | 61-80 | 81-100 | More than 100 |
|---|---|---|---|---|
| Number of teachers | 0 | 4 | 11 | 5 |

**Table 8: Teaching experience in years**

| | 0-5 | 6-10 | 11-15 | 16-20 | More than 20 |
|---|---|---|---|---|---|
| Teaching experience | 5 | 2 | 5 | 3 | 5 |
| Teaching mathematics experience | 7 | 5 | 0 | 3 | 5 |
| Teaching mathematics in the current class | 16 | 4 | 0 | 0 | 0 |

**Table 9: NNP training attendance and understanding of calculation strategies.**

| | | How well teachers understand the calculation strategies | | | | |
|---|---|---|---|---|---|---|
| | | Very well | Well | Partially | Do not understand | Total |
| How often teachers attend the NNP curriculum trainings | Always | 6 | 4 | 0 | 0 | 10 |
| | Sometimes | 2 | 3 | 2 | 0 | 7 |
| | Never | 0 | 0 | 0 | 3 | 3 |
| | Total | 8 | 7 | 2 | 3 | 20 |

From tables A and B, we see that there were almost the same number of female and male teachers, and most of them were in the age range of 41-50. In terms of class distribution, 6 teachers were teaching standards 1 and 2, while 4 teachers were teaching standards 3 and 4.When it comes to enrolment, none of the teachers have less than 40 students in their class. Most of the teachers (11) have an enrolment of between 81-100 students, with only four teachers having an enrolment of between 61-80 students, and five teachers having more than 100students in their class. In terms of teaching experience, 13 of the 20 the teachers have more than 10 years' experience of which 5 have more than 20 years. Experience of teaching mathematics is less for most of the teachers, however for the 3 teachers with experience of 16-20 and the 5 teachers with more than 20 years, the experience includes teaching mathematics. Most of the teachers (16) have 5 or less years' experience teaching mathematics in the current class, and none has been teaching the current class for more than 10 years. This suggests that teachers change teaching classes over the years. Regarding attendance at NNP curriculum training, half (10 out of 20) of the teachers attend the training always, while 7 attends sometimes and 3 never attend. A total of 8 teachers indicated that they understand the strategies very well and 7 teachers indicated that they understand well. This shows that 15 of the teachers, which is two thirds, indicated that they understand very well or well. Only 2 teachers indicated that they understand partially while 3 indicated that they do not understand. Teachers who always attend the NNP curriculum training have a better understanding of the calculation strategies. All 3 teachers that indicated do not understand are those that never attended training.

#### 4.2.2 Criteria used to select the eight teachers who participated in the main study

The following criteria were used to select teachers for classroom observation and interviews. I selected two teachers from each standard to ensure that all standards are represented in the study. I also selected teachers who have experience teaching mathematics for more than 10 years, as may have a better understanding of the subject matter, and teachers who always attend the NNP curriculum training, as they may have a better understanding of the NNP calculation strategies. I selected six teachers who always attend the training, and two teachers who sometimes attend the training teachers. Not only that, but I also selected teachers from different age ranges, as this may provide a diverse perspective on teaching strategies. I also selected an equal number of male and female teachers to ensure gender balance in the study. The eight selected teachers were coded S1TA, S1TB, S1TC, S1TD, S2TA, S2TB, S2TC, and S2TD.

### 4.3 RQ 1: What calculation strategies do teachers teach in the NNP curriculum?

In this sub-chapter, I present data that can help answer the research question on the calculation strategies that teachers teach in the NNP cuticulum. Below, I present findings of the data separately of the calculation strategies that teachers planned in the schemes of work and lesson plans, then the calculation strategies taught during the lesson and finally, the calculation strategies that teachers mentioned during the interviews.

#### 4.3.1 Findings from Lesson Plans

In this section, I present data and then findings the calculation strategies that were planned in the schemes of work and lesson plans for standards 1 and 2, and standards 3 and 4.

**Table 10: Calculation strategies in the lesson plans**

| Calculation strategy | Calculation strategies planned in the lesson plans |
|---|---|

| | Standards 1 and 2 | Standards 3 and 4 |
|---|---|---|
| Physical modelling | YES | NO |
| Count on | YES | NO |
| Doubling and halving | YES | NO |
| Number lines | NO | YES |
| Breaking down and building up | NO | YES |
| Complete 10s | NO | YES |
| Rounding and compensation | NO | YES |
| Commutativity | NO | YES |
| Number pyramid | NO | YES |
| Estimation | YES | YES |
| Repeated addition | YES | YES |
| repeated subtraction | YES | YES |
| Addition bubbles | YES | YES |

The findings show that some calculation strategies were planned for standards 1 and 2, and not for standards 3, and 4. These included physical modeling, counting on, and doubling and halving. While other calculation strategies including breaking down and building up, complete 10s, rounding and compensation, commutativity, and number pyramids, were planned for standards 3 and 4 and not standards 1 and 2. \in addition, there were calculation strategies that were used across the four standards, these include repeated addition/repeated subtraction and addition bubbles.

### 4.3.2 Findings from Lesson Observation

In this section, I present data of the calculation strategies taught during the lesson by S1TA, S1TB, S1TC, S1TD and S2TA, S2TB, S2TC, and S1TD for schools 1 and 2, respectively.

Below is part of the conversations that were used to teach the calculation strategies.

S1TA: open page 4 of your workbooks and count how many bananas are on each bunch

Learner: 5

| | |
|---|---|
| S1TA: | How many bunches of bananas are there? |
| Learner: | 12 |
| S1TA: | how many bananas are there altogether? |
| Leaner: | 60 |
| S1TA: | What calculation strategy did you use? |
| Learner: | I counted one by one |
| S1TA; | good, any other way |
| Learner 2: | I had one bunch with 5 bananas, then I added 5 to each number until I finished all the bunches and got 60. |
| S1TA: | What calculation strategy did you use? |
| Learner 2: | Repeated addition |
| S1TA: | good, so, the first strategy is called counting on strategy and the second one is called repeated addition. |
| S1TB, lesson 1: | writes this question on the chalkboard and read it aloud in vernacular, Thoko, Janet and Brenda share12 sweets. How many sweets will each one gets? |
| S1TB: | can you use to solve this question in groups please? |
| S1TB: | what strategy did you use? |
| Group 1: | repeated subtraction strategy, sharing one sweet at a time, then count how many does each have, so we got 4 |
| S1TB: | another way? |
| Group 2: we | we had all sweets together and we put them in groups of 3 then got 4 sweets |
| S2TA: | open your workbook on page number 14, term 2 |
| S2TA, Lesson 1: | estimate how many fish are there? |
| Learner 1: | 29 fish |

| | |
|---|---|
| S2TA: | any other estimation? |
| Learner 2: | 24 |
| Learner 3: | 26 |
| Learner 4: | 30 |
| S2TA: | Okay, good can you count in 2's then in 3's and find how many fish are there? |
| S2TA: | How many groups of fish did you make from the 2's,3's 5'ss. |
| Learner 5: | 15 groups of 2 fishes, 10 groups of 3 fish |
| S2TA: | which one has many groups? |
| Learners: | (answers together as a class) 15 groups of 2 fish |
| S2TA: | This method is called estimation, and you also were counting on |
| S2TC: | showing learners a pile of bottle tops and maize seeds and asked learners to estimate the number of bottle tops before counting. The conversation was as follows: |
| S2TC: | how many bottles do you think are there in the pile |
| Learners: | 200, 500, 490 |
| S2TC: | Okay, then make groups, then make groups of 10 from the groups, of 5', 10, 20, 50 and count |
| Learners: | (counting) |
| S2TC: | were your estimation less or more than the final amount, one at a time, please |
| Learner 1 | less |
| Learner 2 | less |
| Learner 3 | more |
| S2TC: | This is called estimation strategy. We were counting, and this is called counting on strategy |

S2TD: Writes a statement on the board" 12 boys share 720 marbles equally, how many marbles does each boy get?

S2TD: work in pairs and explain the strategies you used to solve it

Pair 1. We used to build up and breaking down strategy. We solved it as follows, 10 boys120 marbles, then 20 boys= 240 marbles, the last 20 boys =2marbles, therefore each one got 50 marbles

S2TD: okay, good, any pair with a different strategy?

Pair 2: We used repeated subtraction strategy as follows

20: 720 – 240 = 480

20: 480 – 240 = 240

20: – 240 – 240 =0

Therefore, each will get 60 marbles

S2TD: very good

**Table 11: A summary of the calculation strategies taught during the lesson**

| Calculation strategy | Calculation strategies taught during lessons | |
|---|---|---|
| | Standards 1 and 2 | Standards 3 and 4 |
| Physical modelling | YES | NO |
| Doubling and halving | YES | NO |
| Count on | YES | YES |
| Number lines | NO | YES |
| Breaking down and building up | NO | YES |
| Complete 10s | NO | YES |
| Estimation | NO | YES |
| Rounding and compensation | NO | YES |
| Commutativity | NO | YES |
| repeated subtraction | NO | YES |
| Number pyramid | NO | YES |
| Repeated addition/ | YES | YES |

| | | |
|---|---|---|
| Addition bubbles | YES | YES |

The data collected showed that some teachers focused on teaching specific strategies, while others taught other calculation strategies. for example, S1TA, S1TB, S2TA, S2TB who are standards 1 and 2 teachers from school 1 and 2 data showed that they taught physical modelling, count on, doubling and halving, addition bubbles, estimation, and repeated addition/repeated subtraction strategies in their lessons. S1TC, S1TD, S2TC. S2TD, standard 3 and 4 teachers from schools 1 and 2 respectively planned to teach, number lines, doubling and halving, breaking down and building up, complete 10s, estimation, rounding and compensation, commutativity, repeated subtraction, addition bubbles, and number pyramid strategies in their lessons. However, they did not teach physical modelling, or doubling and halving strategies for these standards the teachers taught some of the planned strategies.

The examples of conversations that took place during lessons demonstrate how teachers encouraged learners to use the different calculation strategies. For instance, S1TA asked learners to count the number of bananas in each bunch and then apply a calculation strategy to find the total number of bananas. In S2TA's lesson, learners were asked to estimate the number of fish in a drawing and then count in 2s and 3s to find the actual number of fish. In these conversations, learners were able to apply the strategies they learned and explain the reasoning behind their approach. The data suggests that teachers are providing a diverse range of strategies for learners to use in their calculations. By encouraging learners to apply different strategies, teachers are supporting the development of numeracy skills that enable learners to understand and solve mathematical problems.

### 4.3.3 Findings from the Interviews

This subchapter presents data and analysis of interviews with the eight teachers. In the first question, the teachers were asked teachers to mention the calculation strategies they teach. The second question was about how often they teach the calculation strategies mentioned. The data was collected and analysed based the MDI framework. It was observed that different calculation strategies are being taught. The most mentioned strategies are estimation, number lines, breaking down and building up, doubling, and halving, commutativity, compensation, physical modelling, and completing 10s. Some teachers mentioned that they follow the strategies suggested in the workbooks, while others use their own strategies that they find relevant to the tasks given.

The conversation was as follows.

| | |
|---|---|
| Interviewer: | What calculation strategies do you teach? |
| S1TA: | I normally teach the calculation strategies in the workbook and some that we were trained, for example, estimation, number lines and many more. Sometimes learners come up with their strategies, so I just emphasize and clarify the calculation strategies that they have suggested. |
| S1TB: | I teach the calculation strategies that are relevant to the questions that I want to discuss with learners for example, if it involves counting, I use counting on and counting down. If the numbers require regrouping, I use breaking down and building up and compensation. |
| S1TC: | There are calculation strategies that are suggested on each workbook page. So sometimes I follow and sometimes I use other strategies that are relevant to the tasks given. |
| S1TD: | I teach doubling and halving, completing 10s, commutativity, and others like bubbling and pyramid estimation and compensation. Mostly I teach those that are suggested in the workbooks for that day. |
| S2TA: | I teach all the calculation strategies suggested in the workbook like physical counting, count on, bubbling, doubling and halving estimation, and many more |
| S2TB: | I teach estimation, number lines, completing 10s, physical modeling, building up and breaking down, number pyramid ,and compensation. Sometimes we just discuss a calculation strategy. |
| S2TC: | I normally teach all the calculation strategies like a number line, breaking down and building up, number pyramid, commutativity and many more that are in the workbooks. I also use my own calculation strategies that seem relevant to the question. |

S2TD: Since these learners will be going to the senior classes, I do not have specific strategies. I teach any calculation strategy that will provoke the thinking capacity of learners, like compensation, commutativity, breaking down and building up, doubling, and halving. Teach the ones that I was taught b during the training that fits the tasks that I have planned and of course, I also teach some that are suggested in the workbooks

The frequency of teaching calculation strategies varies among the teachers. Some teach daily, while others teach two or three strategies per lesson. Some strategies require more time to explain, so they are not taught as often.

Below was the conversation that was there between the researcher and the teachers.

| | |
|---|---|
| Interviewer: | How often do you teach the calculation strategies**?** |
| S1TA: | Always only that some are difficult, and others are complicated for the learners |
| S1TB: | Mathematics is taught daily according to the timetable. So, I teach and explain the strategies daily. I encourage my learners to use the calculation strategies that fit the question |
| S1TC: | I teach two or three calculation strategies in every lesson, except physical modeling which is rarely taught. |
| ST1D: | I teach according to what is instructed in the workbooks, but some calculation strategies need enough time to be explained like the building up and breaking time, so I don’t often teach them. |
| S2TA: | I normally teach physical modeling and number lines daily because I want the learners to recognize and know the value of each number. |
| S2TB: | I want the learners to have a deep foundation of the value of numbers and perform basic operations. Most of the time and I follow the ones instructed in the workbooks |
| S2TC: | I teach them always. |
| S2TD: | I teach every mathematics lesson. |

The data collected from the interviews suggest that different calculation strategies are being taught. The most mentioned strategies are estimation, number lines, breaking down and building up, doubling, and halving, commutativity, compensation, physical modeling, and completing 10s. It was also observed that some teachers follow the strategies suggested in the workbooks, while others use their own strategies that they find relevant to the tasks given.

## 4.4 RQ 2: How Do Teachers Teach Calculation Strategies in the NNP Curriculum?

In this section, I present data on how teachers teach calculation strategies. The data starts with the findings from the lesson plans followed by the lesson observation and finally from the interviews. Data is rated in accordance with the exemplification, explanatory talk, and learner participation based on the MDI framework (Adler & Ronda, 2015).

### 4.4.1 Findings from Lesson Plans

#### Exemplification

The data showed that teachers planned the examples and tasks from the workbooks and sometimes formulated their own examples to teach various calculation strategies. Some teachers like S1TA, standard 1 teacher planned to select one example from workbook which was written as follows" *A farmer has 18 chickens, and her son gives her 2 more chickens, how many chickens does the farmer have*?" The teacher planned to use physical modelling and count on strategy. S2TA planned as example one question from the workbook which involved *the estimation of the number of fish*. The teacher planned to teach using estimation strategy. S2TB also planned to discuss one question 1 from workbook which was as follows, "*How many hands are there? How many fingers are there"* to teach doubling and halving, repeated addition as well as count on. S2TD, a standard 4 teacher, planned to an example from workbook as follows

*12 boys share marbles equally. How many marbles does each boy get?*

Furthermore, data showed that some teachers planned their own formulated examples and planned to select tasks from the workbooks. For example, S1TC planned to teach calculation strategies using a formulated example that involved *finding the total number of learners where there are 36 boys and 32 girls in a class* to teach building up and breaking down and count on strategies. |The teacher then planned tasks from the workbook. SITD planned a formulated example as follows; "*John buys 3 packets of sweets. Each packet has 25 sweets in it. How many sweets does he have*?" to teach breaking down and building up and repeated addition. The tasks were from the workbook. S2TC planned formulated example to teach estimation. She gave learners piles of bottle tops and maize seeds to estimate before counting. Then gave tasks from the workbook.

In addition to that, some teachers planned to use examples and tasks from the workbook that involved real life scenarios to help learners understand mathematical calculation strategies. S1TA, standard 1 teacher planned to use an example *of a farmer having 18 chickens and her son giving her 2 more chickens* to help learners understand count on and count all strategy, and physical modelling. S1TB, Standard 2 teacher planned to use an example of three girls sharing 12 sweets equally to help learners understand repeated addition, count down and physical modelling. S2TD, a standard 4 teacher planned to discuss an example from workbook which involved 12 boys sharing marbles equally and finding how many marbles each boy gets? This example also involved the real-life scenario.

Regarding the tasks planned, the data showed that all teachers planned to select tasks from the workbooks. The tasks were different from the examples, but the calculation strategies were either similar or different. For instance, S1TA, the example planned in the scheme of work was about a farmer who has 18 chickens and receives 2 more chickens from her son and used physical modelling, count on, and count all strategy. The task given to learners was for the learners to count the number of chickens and chicken legs in a picture and complete addition bubbles. The example and tasks involved different calculation strategies. While S1TC planned to teach calculation strategies using a formulated example. The example involved finding the total number of learners and used building up and breaking down and count on strategies. The tasks were from the workbook and used same calculation strategies. Some of the tasks planned by the teachers were different from the examples in terms of the type of questions asked and the level of difficulty. Some tasks involve simple counting and estimation, while others require learners to apply mathematical concepts to solve problems. For example, S1TA planned to give learners tasks from which involves simple addition. In contrast, S2TD planned to give learners tasks which involves more complex problem-solving skill.

Based on the data provided, it is analyzed that the teachers had different approaches in planning examples and tasks for teaching mathematical calculation strategies. Some teachers planned examples and tasks from the workbooks, while others formulated their own examples and tasks. The examples and tasks were aimed to help learners apply what they had learned in class and involved different calculation strategies, including physical modelling, counting on, doubling, and halving, breaking down and building up, and repeated addition. Additionally, some examples and tasks involved real-life scenarios such as counting chickens, sharing sweets or marbles hence, providing an opportunity for learners to experience forms of variation, similarity **(S),** and contrast **(C).** This is what Adler and Ronda (2015) classify level 2 examples. The tasks planned by the teachers varied in terms of the type of questions asked and the level of difficulty, ranging from simple counting and estimation to more complex problem-solving skills providing opportunity for learners to choose and applying what is known in relation to the MDI framework coded **(A)** classified as level 2 of tasks.

### Explanatory Talk

In this sub section, I present data on how teachers planned to explain calculation strategies. The teacher planned lessons with specific examples and tasks, to explain the calculation strategies which allowed learners to practice and apply the strategies taught. Teachers' explanations guided the learners through the calculation strategies. In some cases, some learners explained to other learners, which offered opportunities for peer-to-peer learning and feedback helped learners to apply the calculation strategies taught. For example, S2TA standard 1 teacher planned to use physical modeling to teach the learners how to solve problems using repeated addition. The teacher planned to ask four learners to come to the front and count their fingers, noticing the pattern and then asked the learners to solve a similar task from the workbook using the calculation strategies they had learned. The teacher also planned to use probing questions to encourage the learners to explain their reasoning and justify their choices of calculation strategies. S2TB planned to discuss with learners an example from the *workbook.* The teacher planned to explain by drawing a table on the chalkboard, complete the table with learners and notice the pattern. The teacher also planned to ask open-ended questions to encourage the learners to explain their reasoning and justify their choices of calculation strategies. The teacher planned to ask learners to present their work and the teacher to consolidate by explaining the repeated addition that could be noticed in the table. The teacher planned to conclude by explaining the repeated addition which led to multiplication.

The data presented shows that teachers carefully planned their lessons to explain calculation strategies to their learners. The teachers planned to use and name a variety of methods to teach calculation strategies, such as physical modeling, drawing tables on the chalkboard, and giving learners piles of bottle tops and maize seeds to estimate and count. This is coded **(MA)** level 3 based on the MDI framework of Adler and Ronda (205) They also asked probing questions to encourage learners to explain their reasoning and justify their choices of calculation strategies They planned to explain and justify the calculation strategies used when making corrections at the end of the lesson and reinforce the strategies taught through peer-to-peer learning and feedback which Adler and Ronda (2015) coded **(PF**) and level 2 of legitimization

## Learner Participation

The data showed that the teachers created a positive learning environment that encouraged learner participation and engagement. For example, S1TA, S1TB, S2TA, and S2TB, standard 1 and standard 2 teachers planned to have learners discuss the examples in groups and present the work to the whole class. Then have the learners do the tasks individually and share how solved the tasks to their peers.  S1TC and S2TC, the standard 3 teachers planned to discuss examples as a whole class and the tasks be done by learners in pairs and groups, and group leaders. S1TD, t standard 4 teacher, encouraged learner participation through group and pair work. The teacher planned that group learners should help each other if they had problems.

**Table 12: A summary of how teachers planned to invite learners to participate in the lessons.**

| How teachers planned to invite learners. | Schemes of work and lesson plan | |
|---|---|---|
| | Standards 1 and 2 | Standards 3 and 4 |
| Whole class | Y | Y |
| Group | Y | Y |
| Pair | Y | Y |
| Individual | Y | Y |

Based on the provided data, the teachers planned to discuss examples and tasks as a whole class, in groups, in pairs as well as individually. The use of group and pair work is particularly noteworthy, as it encourages collaboration and active engagement from the learners. Based on the provided data and the MDI framework developed by Adler and Ronda 2015, This is rated **(P)** level 2 of the MDI learner participation.

### 4.4.2 Findings of Lesson Observation

In this sub-chapter, I present data on how teachers taught calculation strategies. I present and analyse data on how teachers taught the calculation strategies using examples and tasks, and how teachers explained the calculation strategies. I also present data and analysis of how teachers invited learners to participate in the lesson. The data will be evaluated in accordance with the exemplification, explanatory talk, and learner participation of the MDI framework (Adler & Ronda, 2015).

#### Exemplification

The data collected from the teachers indicated that the examples and tasks used to teach the calculation strategies were from the workbooks and some were formulated by the teachers themselves. Some teachers discussed the example from the workbooks and while others wrote the examples on a chart or on the chalkboard.

For example, S1TA discussed an example with learners from the workbook.

The conversation was as follows:

| | |
|---|---|
| S1TA | open page 4 of your workbooks and count how many bananas are on each bunch on question 1 |
| Learner: | 5 |
| S1TA: | How many bunches of bananas are there? |
| Learner: | 12 |
| S1TA: | how many bananas are there altogether? |
| Leaner: | 60 |

| | |
|---|---|
| S2TA: | open your workbook on page number 14 |
| S2TA: | estimate how many fish are there in question 1. |
| Learner 1: | 29 fish |
| S2TA: | any other estimation? |
| Learner 2: | 24 |
| Learner 3: | 26 |
| Learner 4: | 30 |

S2TA: Okay, good can you count in 2's then in 3's and find how many fish are there?

S2TA: How many groups of fish did you make from the 2's, 3's 5's.

S2TA discussed a formulated example with learners from the chalkboard as shown below.

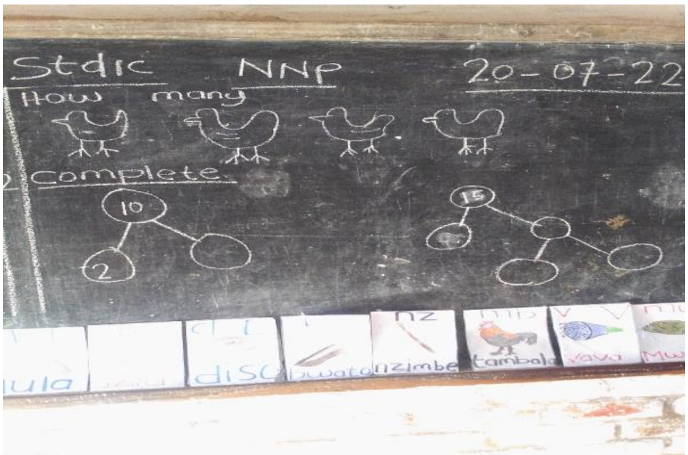


**Figure 5: Examples formulated by S2TA**

SITD discussed a formulated example using a chart as shown below:

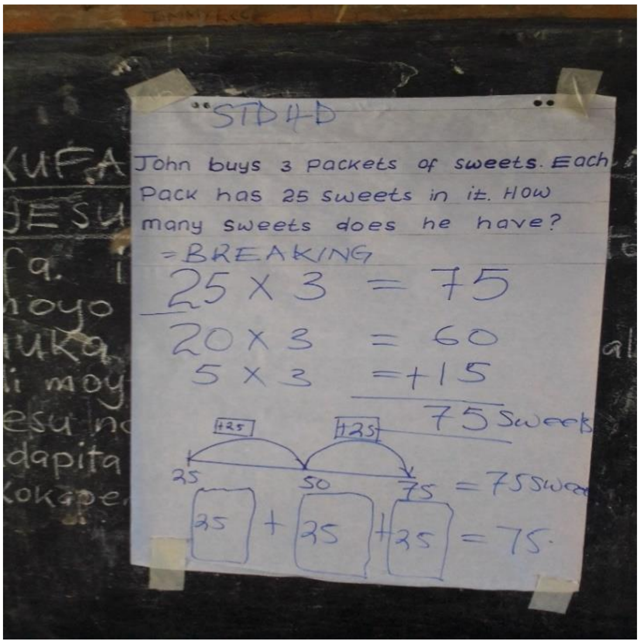


**Figure 6: Example formulated by S1TD**

On the tasks, teachers gave learners tasks from the workbooks, for example:

| | |
|---|---|
| S1TA:open your workbooks on page 1 term 3 and do questions1 up to | |
| | 3 |
| S1TC | open your workbook page 12 and do the work individually. 4 using any calculation strategy that you think fits the question. |
| S2TTC | open your workbooks on page 7 term 1 and do questions 2,3 and 4 individually. |
| S2TD | Turn to your workbooks on page 8 term 1 and do questions 2, 3and 4 using any calculation strategy that you think fits the question. |

Another thing that was noted between the examples and tasks was that tasks were different from the examples. The calculation strategies used to solve the questions were also different. For example, during the lesson, S1TB's example involved sharing sweets equally among three learners and the task includes working through questions related to groups of cassavas and completing a table and calculations involving groups of six. S1T3 examples involved finding the total number of learners in a class given the *number of boys and girls using building and breaking down strategy,* the tasks include finding *the total number of chickens Mrs. Jere has,* completing addition problems, and filling in addition bubbles. S1TD the example provided is about calculating the total number of sweets after buying 3 packets of sweets containing *25 sweets each, using the breaking down and building up, repeated addition and number line* strategy. The task assigned to the learners included three questions from workbook, which require them to compete the *number of packets and lemons using, how many books are needed in class of 53 learners and each learner need 6 books, which learners must use breaking down and building up, repeated addition, and number line calculation strategies. Finally, learners were filling in a multiplication table and completing division by 6 using repeated subtraction, and repeated addition.*

The data presented reveals that the examples and tasks used in the lessons were obtained from workbooks or formulated by teachers themselves. The examples and tasks were diverse and involved teaching a range of calculation strategies such as physical modelling, counting on and down, building up and breaking down, bubbling, and estimation. Furthermore, the data indicates that the examples and tasks were presented in different formats, including workbooks, charts, and chalkboards. Some teachers used examples from workbooks, while others wrote them on the chart or chalkboard.

Moreover, there was a contrast between the examples and tasks given to learners. While examples were used to model the problem and demonstrate different strategies, the tasks assigned to the learners were different and required the learners to use different strategies to solve the problem. For instance, one teacher used stones to represent chickens while another used sweet to teach the doubling and halving strategy. This was coded **(S, C)** level 2 because there was a variation of two features according to Adler and Ronda (2015) and learners were given the opportunity to choose and applying what is known in relation to the MDI framework coded **(A)** classified as level 2 of tasks.

The teachers asked questions to elicit learners' thinking, such as *"How can we solve this question?"* and *"What strategy can we use to solve this question*?" *Why did you use that strategy*? The teachers explained different calculation strategies, such as addition bubbles, doubling and halving, estimation, building up and breaking down, and count-on, and encouraged learners to justify their reasoning behind their answers.

Teachers’ explanations were as follows.

S1TA: (writes example on the chalkboard shown in figure A)

S1TA: we have 10, then what number can be added to 2 to make ten

Learner: 8

S1TA: What strategy dis you use to solve the question?

Learner: count on strategy

S1TA: How did you solve it?

Learner: I had 2 stones and then added on one number at a time and reached 10, then counted how many are the stones and got 8

S1TA: Why did you do that?

Learner: Because we have already have two stones, so when we count on by adding, we get to 10, so number of stones added to 2 to make 10 is the one that we put in the bubble

S1TA: any other way?

Learner 2: I had ten stones, then removed 2, and counted how many stones were remaining

S1TA: What calculation strategy did you use

Learner 2: Counting down strategy

S1TA: why did you do that?

Learner 2: Because we have 10 stones at first, so removing 2 stones will make the 10 stones less 2 which when we add, it will give 10

S1TA. Very good, The two strategies are called counting on and counting down strategy. And the way we have written here is a called bubbling. We have bubbles where we put the numbers.

S2TD used a chart to explain different strategies like breaking down and building up, repeated addition and commutativity as shown in figure B.

The teacher also used language that was clear, concise, and appropriate for the learners' level of understanding. The learners mentioned the count on strategy instead of counting all strategy. The teacher made clarification on this in the conversation below.

S1TB: how many cassavas is there?

Learner 1: 60

S1TB: How did you get the answer?

Learner 1: I counted one by one

S1TB: what name is given to that strategy?

Learner 1: count on strategy?

S1TB: Can you come in front and demonstrate

Learner: comes and counts one by one.

S1TB: any other way?

| | |
|---|---|
| Lerner 2: | I counted in 5s because each heap has 5 cassavas |
| S1TB: | what name is given to that strategy |
| Learner 2: | counting in 5s |
| S1TB.: | Repeated addition is also called multiplication. We are going to look at this later. |

The teacher also provided formal language to create meaning and understanding the concept of a variety of strategies and why they were useful for solving the problems. The following explanation was given:

| | |
|---|---|
| S1TB: | Counting the objects one by one is called Count all strategy. You can count on if you are adding and count down if you are subtracting. For the counting in 5s, this strategy is called the repeated addition. If the number to be added is the same, you rapidly add the same number. Repeated addition is also called multiplication. We are going to look at this later. |

Tasks given to learners were marked. However, there was no indication that the teachers provided opportunities for learners to reflect on the calculation strategies. The conversations were as follows.

| | |
|---|---|
| S1TB: | Most of you have used correct calculation strategies.<br>Those that I have marked wrong, you can<br>check with your group leaders. |
| S1TD: | most of you did the work better, but some of you had problems with number 2. |

The data also showed how teachers used different strategies and techniques to teach the learners. For example, in lesson 2, Teacher A used bubbling to explain the two strategies: counting on and counting down. In lesson 1, Teacher B used a chart with cassava on the board to explain different strategies like breaking down and building up, repeated addition, and commutativity. This is coded **(MA)** level 3 based on the MDI analytical framework. The teachers used language that was clear, concise, and appropriate for the learners' level of understanding. They also provided formal language to create meaning and understanding of the concept of a variety of strategies and why they were useful for solving the problems. Moreover, the data showed how the teachers clarified learners' understanding of the different strategies. For instance, when a learner used the count-all strategy instead of the count-on strategy, Teacher S1TB made clarification by explaining that counting all the objects one by one is called count-all strategy and not count-on. This showed that the explanation was full coded **(PF)** level 2 in the MDI analytical framework developed by Adler and Ronda (2015).

### Learner Participation

Regarding learner participation, the teacher used different modes of participation, such as whole class, group, pair, and individual, to engage learners in the lessons. The learners were also encouraged to present their work on the chalkboard and explain their reasoning behind their answers. The teachers encouraged the learners to participate in groups, as individuals and in pairs by asking open-ended questions such as *"Any other suggestion?"* and *"Can any pair demonstrate?"* The teachers also actively invited learners in the lesson by asking them to answer, solve the question, and explain their strategies to their peers. This provided an opportunity for learner participation, as learners were able to share their understanding and learn from each other. For example, S1TB in both lessons, the standard two teacher used different modes of learner participation. In Lesson 1, learners shared stones and explained their strategies for sharing. In Lesson 2, the teacher asked learners to count the number of people and eyes and explain the count-on strategy. Learners also used stones to double the given numbers. The teacher allowed learners to work on the task individually and then asked for volunteers to share their strategies. This provided an opportunity for learners to engage with their peers and share their strategies. For S1TD, in standard 4 Lesson 1, The learners were given tasks to work on individually, in pairs, and in groups. They were also encouraged to present their work on the chalkboard and explain their reasoning behind their answers. Lesson 2: The learners were given tasks to work on individually, in pairs, and in groups. They were also encouraged to present their work on the chalkboard and explain. S2TA, Standard 1 teacher promoted learner participation by asking learners to estimate the number of fish and bottle tops, count the total number of fish and dots, and count the number of hands and fingers. The teacher engaged the whole class, with some pair and individual activities. S2TB, in standard 2 engaged learners in activities such as counting the number of hands and fingers. The teacher promoted learner participation by asking questions and evaluating learners' responses. The teacher engaged the whole class, with some pair and individual activities. S2T3 standard 3 teacher promoted learner participation by asking open-ended questions to prompt their estimation and counting skills. The teacher engaged the whole class, with some pair and individual activities.

Part of the conversation was as follows:

| | |
|---|---|
| S1TB: | what name is given to that strategy? |
| Learner 1: | count on strategy? |
| S1TB: | Can you come in front and demonstrate |
| Learner: | comes and counts one by one. |
| S1TC: | The teacher presents this question, " |
| S1TC: | can you work out this example in groups |
| STTC: | which group is ready to explain how they solve it? |
| S1TC: | Another group with a different suggestion |
| Group 6: | we had 36 = 10+10+10 =6, then, 32= 10+10+10 +6. Then we added altogether to get 68 |
| S2TD: | writes a statement on the board" 15 boys share 750 marbles equally, how many marbles does each boy get? any pair which is ready to solve? |
| Pair 1. | We solved it as follows, 10 boys = 150 marbles, then 20 boys= 300 marbles, the last 20 boys = 300 marbles, therefore each one got 50 marbles |
| S2TD: | any pair with a different strategy? |
| Pair 2: | 10: 750 – 150 = 600 |
| | 20: 600 – 300 = 300 |
| | 20: 300 – 300 = 0 |
| | Therefore, each will get 50 marbles. |

The data presented describes the different modes of learner participation used by teachers in their lessons, and how they encourage learners to engage with their peers and share their strategies. The teachers use various techniques such as asking open-ended questions, allowing learners to work in pairs, groups, and individually, and inviting them to present their work on the chalkboard. The data suggests that the teachers are effective in promoting learner participation by creating an environment that encourages learners to share their understanding and learn from each other. The different modes of participation used by the teachers allow learners to work at their own pace, share their strategies, and collaborate with their peer. This coded as **(D)** level 2 of the MDI framework, learner participation.

### 4.4.3 Findings from Interviews

#### Exemplification

According to what the teachers said during interviews, S1TA starts by introducing the numbers that learners are going to use when teaching calculation stages. S1TB plans and formulates a question related to the previous work and asks learners to solve it. After solving, S1TB asks them to explain how they solved it and justify it and then comes in to highlight the strategies. S2TB's lessons start with the revision of the previous work. S2TB plans and formulates a question related to the previous work and asks learners to solve it. After solving, S2TB asks them to explain how they solved it and justify it.

When it comes to choosing examples to discuss calculation strategies with learners, S1TA chooses one of the questions in the learners' workbooks and plans it as an example. Sometimes S1TA formulates their own question related to that work page. S1TB chooses one of the questions in the learner's workbooks that has two or more strategies and plans it as an example. Sometimes S1TB formulates their question related to the work page and asks learners to discuss it in pairs or groups and use any calculation strategy they feel is appropriate for the question. S2TB normally uses one of the questions from the learners' workbook page, which can involve more than one calculation strategy, and uses it as an example and writes it on the chart or the board. Sometimes S2TB formulates their own example related to that work page. S2TD takes examples from the learners' workbooks. S2TD chooses one that has more calculation strategies. When introducing a new calculation strategy, S2TD formulates own example and makes sure that the question has two or more calculation strategies.

When it comes to selecting tasks to be given to learners, S1TA normally asks them to open their workbooks on that day's page. So, S1TA doesn't select but tells them to do what is there. However, depending on how much the work is, S1TA sometimes tells them to do the work in groups, individually, or in pairs. S1TA encourages learners to use any calculation strategy they feel fits the question. S1TB normally uses the tasks from their workbooks on that day's page. On Fridays, S1TB gives an assessment, so on this day, S1TB formulates own tasks related to the calculation strategies learned during that week. S2TB selects tasks from workbooks on that day's page, especially those that are related to what learners have been taught. S2TB formulates more tasks if the workbook tasks are not enough. S2TD uses tasks from the workbooks that have different calculation strategies, but if they are not challenging and do not allow the use of multiple strategies, then S2TD formulates their own tasks from the previous curriculum.

Below are some extracts of what the teachers said.

| | |
|---|---|
| Interviewer: | How do you start the teaching of the calculation strategies? |
| S1TA: | I first introduce numbers that learners are going to use when I am going to teach calculation stages. I feel this is important though it is not in the curriculum because most learners don't know the numbers. |
| S1TB: | I plan and formulate a question that is related to the previous work and ask learners to solve it, after solving, I ask them to explain how they solved it and justify it. Then I come in to highlight the strategies |
| S2TB: | My lessons normally start with revision of the previous work I plan and formulate a question that is related to the previous work and ask learners to solve it, after solving, I ask them to explain how they solved it and justify I |
| S2TD: | Normally, I introduce my lessons by going through the homework that I gave them. Leaners explain how they solved the work that they were given. |

**Examples**

| | |
|---|---|
| Interviewer: | How do you choose the examples that you discuss the calculation strategies with learners? |
| S1TA: | I choose one of the questions in the learner's workbooks and plan it as an example Sometimes I formulate my own question that is related to that work page |

S1TB: I choose one of the questions in the learner's workbooks that I feel has two or more strategies and plan it as an example Sometimes I formulate my own question that is related to the work page. I ask learners to either discuss it in pairs or groups and use any calculation strategy they fill is okay with the question

S2TB: Normally, I use one of the questions from the learners' workbook page which can involve more than one calculation strategies and use that one as an example and write it bon the chart or the board. Sometimes I formulate my own example. that is related to that work page.

S2TD: I take examples from the learners' workbooks. I choose one that has more calculation strategies. If I want to introduce, a new calculation strategy, then I formulate my own example. But I make sure that the question has two or more calculation strategies

**Tasks Selected**

Interviewer: How do you select the tasks to be given to learners?

S1TA: I normally ask them to open their workbooks on that day's page. So, I don't select but tell them to do what is there, however, depending on how much the work is, I sometimes tell them to do the work in groups, individually or in pairs for easy marking. I encourage them to use any calculation strategy they feel fits the question

S1TB: I normally use the tasks from their workbooks on that day's page. On Fridays, I give the assessment, so on this day I formulate my own tasks related to the calculation strategies learned during that week

S2TB: I select tasks from their workbooks on that day's page. Especially those that are related to what I have taught them. If I feel that they are not enough, I formulate some for them

S2TD: The workbooks have tasks that have different calculation strategies and I just use tasks from the workbooks. But if they are not challenging and do not allow the use of multiple strategies, then I formulate my own tasks from the previous curriculum.

The analysis of the data suggests that the teachers use various exemplification and task selection approaches to facilitate student learning of calculation strategies. The teachers used different approaches such as selecting examples from learners' workbooks, formulating their own questions related to the work page, and highlighting strategies used. They also encouraged learners to use any calculation strategy they feel fits the question. This strategy allows learners to engage with different strategies, understand their effectiveness, and develop a range of strategies for problem-solving. This is coded **(SC)**level 2 according to the MDI framework of Adler and Ronda (2015) Regarding task selection, the teachers used mainly two approaches; as using tasks from their workbooks, and formulating tasks related to the calculation strategies learned during the week. These approaches aim to provide learners with opportunities to apply their understanding of calculation strategies and to demonstrate their learning through different tasks. This is coded **(A)** level 2 of the MDI framework.

### Explanatory Talk

Regarding the explanation of calculation strategies, all teachers teach calculation strategies during the discussion of examples and tasks. S2TA names and teaches new calculation strategies and solves them for learners. S2TB, on the other hand, encourages learners to invent and explain strategies that they are comfortable with and justify why they used them. When correcting learners' work, the teachers name and explain the calculation strategies that should have been used. S2TC explains the calculation strategies by allowing learners to discuss and justify the strategies they have learned, invented, and applied, and then picking out important points and stressing them at the end as a reflection of what they have learned. S2TD, on the other hand, presents an example for learners to discuss and present the calculation strategy used to solve the question. After discussion, the teacher stresses and explains the calculation strategies, then after giving learners the task, marks and discusses the calculation strategies used to solve the questions, stressing the ones missed during reflection.

Below is what the teachers said:

Interviewer: How do you explain the calculation strategies to learners?

S1TA; Okay. So, I normally explain after the discussion of the example. Once the learners have explained and justified their strategies, I come in and emphasize that if they have not used a calculation strategy, I want to teach that day. That's how I do it.

S1TB: After the discussion of the example given, I normally explain clearly what learners have explained and justified, and sometimes make corrections where learners have made mistakes. I also teach calculation strategies that are new to the learners. Yea.

S1TD: Learners explain to their fellow learners. But I encourage them to use any calculation strategy they are familiar with; I just listen attentively and take note of the important points. I also take note of the mistakes. After that, I highlight and explain important points. I also name the strategies used for the earners to be familiar with their names.

S2TA: After the discussion of the example given, I teach calculation strategies that are new to the learners. I do this by bringing in another example, then I solve it for the learners at the end, I ask them if they have understood, and that they are free to ask questions if they have not

S2TC: Through learners' discussion and justification of the calculation strategies that they have used, I pick important points and stress them at the end as a reflection of what they have learned.

ST1B: Once I give them tasks to do, I mark and make corrections. When making corrections, I explain the calculation strategies used to solve the tasks

ST1TC: I normally teach the calculation strategies at during discussion of the tasks.

S1TD: I ask the best learners to present their work. Then I just come in and stress the important facts missed by learners during the presentation.

S2TA: When making corrections, I explain and teach the calculation strategies that are supposed to be used in the tasks'

ST2B: Through the reflection of what they have learnt, it is when I present the calculation strategies.

### Learner Participation

In this sub-section, I present data of how teachers responded on the way they invite learners to participate ii the lesson.

Interviewer: How do you invite learners to respond to the NNP calculation strategies?

S1TA: With the tasks in the workbooks, I invite the learners to do them in groups and finally ask each pair to come in front and explain or demonstrate what they have done.

S1TB: We discuss as a whole class, in groups, in pairs, and individually. I normally give learners tasks in groups so that they can share their ideas, sometimes I also ask them to do the tasks in pairs or individually and then explain to their friends.

S1TC: I invite them to work in pairs, in groups, and individually according to the work and time.

S1TD: I invite them to work in pairs, in groups, as a whole class, and individually according to the work and time. I also give them homework.

S2TA: I invite them to work in pairs and in groups but occasionally they work individually.

S2TB: It depends on the work. Sometimes, I invite them to work in pairs, in groups, and individually. In addition to that, I give them homework.

S2TC: It depends on the work. Sometimes, I invite them to work in pairs, in groups, and individually to demonstrate explain the strategies they have applied to the examples or tasks.

S2TD: most of the time, learners work in groups for easy marking because my class has more learners.

Based on the responses provided by the teachers, it seems that there is a variety of methods used to invite learners to participate in the lesson regarding the calculation strategies. S1TA and S1TB both mentioned that they use group work to facilitate discussion and encourage learners to share their ideas. Additionally, S1TA invites learners to come to the front of the class to demonstrate or explain their work. S1TD, S2TB, and S2TC also use a combination of pair work, group work, and whole-class discussions to encourage participation.

It is interesting to note that some teachers, such as S1TC and S2TA, indicate that the method of inviting learners to participate depends on the work or task at hand. This suggests that they tailor their approach to the specific needs of the lesson and the learners. On the other hand, S2TD mentions that learners work in groups most of the time due to the large class size. While this may facilitate easier marking, it is unclear if this approach is the most effective for encouraging active participation and engagement from learners.

The teachers are utilizing a range of strategies to engage learners in the lesson regarding NNP calculation strategies, with a focus on group work, discussion, and demonstration. However, the effectiveness of these strategies may vary depending on the specific context and learner needs. This is coded **(P)** level 2 based on the MDI framework developed by Adler and Ronda (2015)

## 4.5 RQ 3: How Do Teachers View the Teaching of Calculation Strategies

This subchapter presents data and analysis of how teachers view the teaching of calculation strategies in the NNP curriculum The MDI analytical framework developed by Adler and Ronda (2015) focusing on exemplification, explanatory talk, and learner participation was used to analyse the data provided by teachers to answer this research question.

### Exemplification:

This part focus on what responded related to the examples and used to teach the calculation strategies. The conversation between the teachers who gave their views in terms of examples and tasks and the researcher was as follows:

Interviewer: How do you view the teaching of calculation strategies in the NNP curriculum?

S1TA: I view the teaching of calculation strategies as a very important component as compared to the old curriculum. But it is difficult to select an example that has a variety of calculation strategies.

S1TC: The teaching of calculation strategies in the NNP curriculum has made the teaching simple. When I present an example, learners select a variety of calculation strategies with which they are comfortable. I as a teacher also learn from them.

S2TB: The teaching of calculation strategies is good because some strategies are already indicated in the workbooks. For example, doubling and halving, count on. So, when selecting an example, I choose the one that fits my learners and the example. These learners that are learning the calculation strategies right away from standard one will develop their thinking and reasoning capacity.

S2TD: As compared the OBE curriculum, this curriculum is good because the calculation strategies that are included make learners think. They think and reason their thinking and the thinking of their friends. However, some calculation strategies are difficult and complicated so to formulate an example that fits the calculation strategies becomes difficult. As such I normally choose example and teach the ones that I am familiar with and leave the complicated ones.

**On views of tasks, this was the conversation**:

S1TB: The inclusion of calculation strategies is very good because learners are given the opportunity to apply the calculation strategy that they think is applicable to the tasks... However, there is too much work and more tasks to be done on a day which also require a variety of calculation strategies. I suggest that there should be one or two tasks per page for the learners to maximise the calculation strategies learnt, S1TD: The thinking capacity of learners is improving. Learners are fully participating in the lesson because different tasks involve different calculation strategies as well. This means learners are supposed to apply the calculation strategies that they learnt.

S2TB: The teaching of mathematics is now simple because you just pause the task and ask learners to discuss the strategy, they feel is good to solve that task. Moreover, the tasks are already in the workbooks, so learners just apply the calculation strategies that they are they are comfortable with to solve the tasks. I as a teacher, I also learn from the learners. However, the work is too much, for example, from the learners' workbook, there 3 or more tasks that learners are supposed to solve in a day and in 30 minutes. I suggest, the work should be reduced to may be 2 tasks per work page.

Based on the conversations between the teachers and the researcher, it can be inferred that the teaching of calculation strategies in the NNP curriculum is viewed as an important component that helps learners improve their thinking and reasoning capacity. The inclusion of calculation strategies in the curriculum also allows learners to apply the strategy they feel is appropriate for the given task, making the learning process more interactive and engaging. However, there are some challenges associated with selecting appropriate examples and tasks that fit the calculation strategies and reducing the workload for learners.

S1TA highlighted the difficulty of selecting an example that has a variety of calculation strategies. S1TC, on the other hand, found the teaching of calculation strategies to be simple as learners select a variety of calculation strategies that they are comfortable with. S2TB views the teaching of calculation strategies as beneficial to learners, and S2TD believes that learners' thinking capacity is improving due to the inclusion of calculation strategies in the curriculum. However, some strategies are difficult and complicated, which makes formulating examples that fit the calculation strategies challenging. In terms of tasks, S1TB noted that the workload is too much, and learners are supposed to do many tasks that require a variety of calculation strategies in a day. S1TD believes that the inclusion of different tasks involving different calculation strategies helps learners fully participate in the lesson. S2TB also found the tasks in the workbooks to be beneficial, and the learners can apply the calculation strategies they are comfortable with. However, the work is too much, and S2TB suggests reducing the workload by having fewer tasks per work page. This could be coded **(S, C)** and **(A)** of the MDI exemplification.

### Explanatory Talk

In this subsection, I present teachers' views on the explanation of the examples and selected tasks.

S1TA: As a standard 1 teacher, my learners are not familiar with some of the numbers that are in the workbooks, so to explain them first, it needs more time, I suggest learners should first be taught the numbers, learners should know the numbers, then in the second term, we should concentrate on the calculation strategy.

S1TB: our work has been made simple. Learners can explain to their fellow learners. But because the calculation strategies are many at each page, I suggest the work per page should be reduced, for example, there should be one or two strategies per page. The numbers used for calculation should fit the level of learners.

S1TC: the diagrams in the workbooks for the tasks are self-explanatory. This is good I follow them and understand easily, this makes me explain my selected example easily. To make them more effective, I suggest there must be two or three calculation strategies per day.

S2TD: NNP calculation strategies are quite okay because they help the learners in thinking critically to explain how and why they solved a mathematics problem in the way they have explained. However, the tasks in the workbooks are too much and they need more time to be worked on. I suggest, mathematics should be allocated double periods.

S2TA: The inclusion of calculation strategies in standard 1 is not good because in standard 1, learners do not know the numbers. So cannot start calculating without knowing the numbers. I suggest the teaching of calculation strategies should start in standard 2 because at this level, at least learners know numbers.

From the data presented, it appears that there are both positive and negative views on the teaching of calculation strategies and the explanation of examples and selected tasks. Overall, it seems that the teachers find the inclusion of calculation strategies in the NNP curriculum to be beneficial for the learners in terms of developing critical thinking and problem-solving skills. This is coded as (**MA)** level 2 of the MDI framework.

However, there are concerns about the level of difficulty of some tasks and the amount of time allocated for mathematics lessons. Some teachers suggest that the numbers used for calculation should fit the level of learners, and that there should be one or two strategies per page to make it easier for learners to follow. Others suggest that the time allocated for mathematics lessons should be increased, and that teachers should receive more in-service training on calculation strategies. This is coded **(PF)** Level 2 of the MDI framework.

### Learner Participation

In this subsection data was collected on teachers views on how learners are invited to participate in the teaching of calculation strategies in the NNP curriculum. The data collected is presented below:

S1TC: The teaching of calculation strategies in the NNP curriculum has made the teaching simple. Learners are invited to participate either in groups, in pairs even as a whole class. Learners share ideas and as a result, they have many ways of solving mathematical questions that I as a teacher also learn from them.

S1TB: The engagement of learners to select the calculation strategy that fits the question is good. Learners are encouraged to think. However, I suggest, the numbers used for calculation should fit the level of learners.

S2TB. Learners are confident and they feel happy when they are sharing what they have done with their friends. Their friend also listens attentively to find out their friend is saying the truth. I as a teacher, am really learning. This is a good way of teaching.

S2TD: The teaching of calculation strategies is building a very good foundation for our learners in the senior classes. Learners are engaged in the lesson and are encouraged to think and share their thinking to their peers either as a whole class, in pairs or in groups.

The teachers' responses show that the teaching of calculation strategies has a positive impact on learner participation. Teachers reported that learners are invited to participate in the lesson either individually, in pairs, or in groups. This collaborative approach allows learners to share their ideas and strategies with their peers and learn from each other. Teachers have also reported that learners are confident, happy, and engaged in the lesson, and that they actively participate in discussions and share their thinking. This is coded **(D)** level 3 in terms of the MDI framework learner participation.

### 4.6 General views of NNP calculation strategies

There is also a difference in opinion regarding when the teaching of calculation strategies should start. Some teachers like standard 1 teacher feel that it is appropriate to start teaching these strategies in standard 1, while others suggest that it should begin in standard 2 when learners have a better understanding of numbers

Furthermore, teachers noted that this approach has resulted in learners developing a variety of ways to solve mathematical problems, which has contributed to their thinking and reasoning capacity. However, some teachers have raised concerns about the workload and the level of difficulty of some calculation strategies, which may require more time to be taught. They also suggested that the numbers used in calculations should fit the level of learners to ensure their understanding and engagement.

### 4.7 Summary of the MDI coding

Analysis of data from the three different sources - lesson plans, lesson observations, and interviews on how teachers teach the calculation strategies was based on the MDI analytical framework developed by Adler and Ronda (2015). On exemplification, he data analysis from the lesson plans and lesson observation, both showed similarity, **(S)** and contrast **(C),** level 2 of examples and application, **(A),** level 2 of tasks. In terms of explanatory talk, data from both the lesson plan and lesson observation showed **(MA)** level 2 of naming and **(PF)** level 2 of legitimization.in learner participation, data analysis showed **(P**) level 2 of learner participation from the lesson plan but level **(D)** level 3 of learner participation during the lesson observations. In the interviews, all categories were coded level 3.

### 4.8 Chapter Summary

In this chapter data and findings from the questionnaires, teaching, and the interviews were presented. The data from the questionnaire helped the researcher to select the eight participants who participated in this study. The Data and analysis of teaching and interviews for all the eight teachers were then presented. The data has been analysed using the MDI framework developed by Adler and Ronda (2015). Data from teaching involved the analysis of the lesson plans that were used to teach the calculation strategies and the lesson presentation. A summary of data analysis for each data source is presented separately. A summary of the MDI coding is presented at the end.

The next chapter presents the discussion of the findings, conclusion, implications, and recommendations.

# CHAPTER 5: DISCUSSION AND CONCLUSION

## 5.1 Introduction

This study sought to investigate the teaching of the calculation strategies in the NNP curriculum in standards 1 to 4 in Malawi. The study proposed to answer the following research questions.

a. What calculation strategies do mathematics teachers teach in the NNP curriculum?
b. How do mathematics teachers teach calculation strategies in the NNP curriculum?
c. How do mathematics teachers view the teaching of calculation strategies in the NNP curriculum?

In this chapter, I provide the summary of findings as presented in Chapter Four and discuss the findings. Lastly, I present the conclusion, and the implications of the study.

## 5.2 Summary of Findings

### 5.2.1 Calculation Strategies That Teachers Teach in the NNP Curriculum

Data analysis of the three sources, the lesson plan, the lesson observation, and the interviews revealed that the teachers planned and taught a variety of calculation strategies, with some variations in frequency and focus among different teachers and standards. The identified calculation strategies include building up and breaking down, number lines, physical modelling, counting on and counting down, addition bubbles, estimation, number pyramid, doubling and halving, commutativity, completing 10s, rounding and compensation. However, the frequency of teaching these strategies varied among the teachers. Some teachers taught these strategies consistently, while others taught them occasionally. Some strategies that required more time to explain were also taught less frequently.

The findings also showed that certain strategies were planned and taught for specific standards. For example, physical modelling, counting on, and doubling and halving were planned and taught  for standards 1 and 2 but not for standards 3 and 4. On the other hand, breaking down and building up, completing 10s, rounding and compensation, commutativity, and number pyramids were planned and taught  for standards 3 and 4 but not for standards 1 and 2. There were also strategies that were used across all four standards, such as repeated addition/repeated subtraction and addition bubbles. The data also revealed that calculation strategies taught in Standards 3 and 4 appeared to be more advanced than those taught in Standard 1 and 2 for example, number lines, breaking down and building up, doubling, and halving, number pyramid, and rounding and compensation due to the complexity of the problems.

### 5.1.2 The Teaching of Calculation Strategies in the NNP Curriculum

The data analysis from the lesson plans, lesson observation, and the interviews revealed that teachers used all aspects of the MDI framework; exemplification, explanatory talk, and learner participation to introduce and teach the calculation strategies.

In exemplification, the data findings revealed that the examples and tasks planned and used in the lessons were obtained from workbooks or formulated by teachers themselves. They were well structured and analysed to meet the intended objective. From the workbooks, teachers selected one question and engaged it as an example. The remaining tasks were given to learners to solve either individually, in groups, in pairs or as a whole class, and involved applying what was known and taught. In addition to that, the examples and tasks were presented in different formats including referring to learners' workbooks, written on charts, and written on chalkboards. The teachers planned and taught using the workbooks, charts, tables, and physical models to help students understand the calculation strategies. However, the data findings also revealed that there was contrast between the examples and tasks given to learners. While examples were used to model the problem and demonstrate different strategies, the tasks assigned to the learners were different and required the learners to use different strategies to solve the problem. The examples and the tasks required learners to apply what they had learned that day or previously. When solving the examples or tasks, learners were given the freedom to choose and use any calculation strategy that they think fits the question.

In the explanatory talk, data showed that teachers used examples and tasks to guide learners through the calculation strategies. First, teachers presented the example on the chalkboard, charts, or workbooks as earlier stated and asked learners to discuss the example in groups, in pairs, as a whole class or individually. Afterwards learners were asked open ended questions to explain how they did the work to their peers and the whole class. The teachers then came in to highlight the work done by the leaners to the whole class. On how teachers encouraged learners to use various calculation strategies, data showed that Learners were given examples and tasks that required them to apply the strategies they learned and explain their reasoning. The teachers employ various questioning techniques to elicit learners' thinking and encourage them to justify their reasoning behind their answers. This allowed learners to reflect on their approaches and explain their thinking. They named and different calculation strategies such as addition bubbles, doubling and halving, estimation, building up and breaking down, and count-on. The teachers explained and used clear, concise, and appropriate language to ensure learners' understanding. They emphasised why the calculation strategies were useful for solving problems. The data also showed  that teachers clarified misconceptions and provided formal language to create meaning and understanding of the concepts. However, most of the learners' work was not presented and marked due to time.

In learner participation, the data revealed that teachers used the different modes of learner participation in their lessons such as pairs, groups, individuals, and the whole class. The teachers created an environment that encouraged learners to engage with their peers and share their strategies. It is also interesting to note that teachers did not dominate the classroom in terms of talking and doing but asked open ended question for learners to actively participate in the classroom. Teachers did not ask learners to recall the information or procedures, but rather to describe an alternative strategy for solving that question, for example *"any group with a different strategy?"*

### 5.2.3 Teachers Views on the Teaching of Calculation Strategies in the NNP Curriculum

Data analysis from the interviews revealed that the teaching of calculation strategies in the NNP curriculum is considered important for enhancing learners' thinking and reasoning abilities. The inclusion of calculation strategies allows learners to choose and apply strategies that they are comfortable with, promoting interactive and engaging learning experiences. To improve the teaching of calculation strategies, some of the teachers' comments, especially standard 1 and 2 teachers, suggested starting with teaching numbers before moving on to calculation so that learners can have a solid understanding of numbers before, they could effectively use calculation strategies. Some teachers also suggested reducing the workload on each workbook page or increasing the time allocation for mathematics for them to cover all the work.

## 5.3 Discussion of the Findings

In this subchapter, I present the discussion of the findings based on the components of MDI framework.to answer the research questions.

### 5.3.1 Examples and Tasks

The data findings regarding the examples and tasks used by teachers indicate that they played a crucial role in teaching and helping learners understand the calculation strategies. This aligns with the perspective of Ronda and Adler (2017), who emphasize the importance of using examples in the teaching process to facilitate learners' comprehension of specific learning objectives. Teachers had the option to either select examples from workbooks or create their own.

Interestingly, the data revealed that when selecting examples from workbooks, teachers typically chose only one question from the suggested tasks on each work page as an example, while the remaining questions were considered as tasks. This finding contrasts with the study conducted by Rittle-Johnson and Star (2009), who found that providing learners with multiple examples is more beneficial than a single example. According to Rittle-Johnson and Star, multiple examples allow learners to identify similarities and differences, enabling them to generalize mathematical concepts or processes more effectively. While the current data findings suggest that teachers predominantly used a single example, it is worth considering the potential benefits of incorporating multiple examples in the teaching process. Using a single example may limit learners' exposure to different problem-solving scenarios while incorporating multiple examples, as suggested by Rittle-Johnson and Star offer opportunities for students to observe different instances of the calculation strategies in action. This exposure to diverse examples enhances learners' understanding and promote the development of generalization skills

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Despite using only one example, the data findings indicated that teachers carefully selected and formulated examples that were analyzed and structured to effectively achieve the intended objectives, as emphasized by Adler and Ronda (2015). This aligns with the perspective of Rittle-Johnson and Star (2009), who highlight the importance of providing learners with well-structured examples and tasks. By doing so, teachers can highlight the key features and relationships of the concept being taught, fostering a robust understanding of the concept, and enhancing problem-solving abilities.

Furthermore, the findings revealed that the selected examples involved multiple applications of calculation strategies, which was also reflected in the tasks given to the learners. This practice aligns with the recommendation of Stein et al. (1996), who encourage teachers to present examples and tasks that allow for multiple calculation strategies. By providing such opportunities, teachers create a learning environment where students can demonstrate their understanding in various ways. This approach is in line with the Common Core State Standards Initiative (2015), which emphasizes that examples and tasks supporting multiple strategies foster classroom discourse by enabling students to construct viable arguments and critique the reasoning of others. Additionally, Adler and Ronda (2015) suggest that the examples chosen by teachers should show variation to enable learners to develop a comprehensive understanding of the various concepts, including the calculation strategies discussed in this study. By presenting diverse examples, teachers expose students to different problem-solving approaches and promote a deeper understanding of the subject matter.

In addition to that, the data findings also showed that the examples and tasks that were used provided the opportunity to learners to use any calculation strategy that they think fits the question. This approach aligns with the recommendations of Math Perspectives (2007) and Piggott (2011), who emphasize the need for teachers to present examples and tasks that require the use of multiple calculation strategies. By doing so, all students are able to access the problem and select a strategy that aligns with their thinking and problem-solving abilities. Introducing tasks that allow for the use of different strategies has several benefits. Primarily, it promotes inclusivity and equity in the classroom, as students with diverse mathematical backgrounds and abilities can approach the problem in a way that makes sense to them. This helps to create an inclusive learning environment where all students have the opportunity to succeed and contribute.

Moreover, such tasks stimulate classroom discourse and promote the development of critical thinking and reasoning skills. When students are encouraged to select and defend their chosen strategies, they engage in constructing viable arguments and evaluating the reasoning of their peers. This promotes active participation, collaboration, and the development of mathematical communication skills. This approach aligns with the principles outlined by the Common Core State Standards Initiative (2015), which emphasize the importance of fostering mathematical practices, including constructing arguments and critiquing the reasoning of others. By providing examples and tasks that allow for the use of different strategies, teachers create a rich learning environment that encourages students to think deeply, analyze various approaches, and support their mathematical thinking with evidence.

The data findings also highlighted that teachers purposefully selected examples and tasks that incorporated real-life scenarios to teach calculation strategies. These real-life contexts were chosen to enhance learners' understanding of mathematical concepts and strategies. This practice aligns with the recommendations of Marton and Pang (2006), who advocate for the use of rich mathematical examples and tasks that connect to real-world situations, as they can support students' reasoning and engagement in the learning process. The selection of examples and tasks involving real-world scenarios serves multiple purposes. Firstly, it provides students with a meaningful and relatable context in which to apply calculation strategies. By presenting problems that resemble real-life situations, teachers demonstrate the relevance and practicality of these strategies, thereby increasing students' motivation and interest in learning them. Furthermore, real-life examples and tasks encourage students to communicate and discuss mathematical ideas and concepts. According to Jackson et al. (2012), rich tasks that incorporate real-world scenarios facilitate students' communication about specific mathematical topics, such as calculation strategies. By engaging in discussions and explanations related to these tasks, students have the opportunity to articulate their understanding, clarify their thinking, and exchange ideas with their peers. This not only strengthens their conceptual understanding but also nurtures their ability to express mathematical reasoning and argumentation. Moreover, using real-life contexts helps students see the connection between mathematics and the world around them. It enables them to recognize that calculation strategies are not isolated procedures but tools that can be applied to solve practical problems in various domains, such as finance, science, and everyday life. This broader perspective enhances students' appreciation for the relevance and utility of mathematics beyond the classroom.

Finally, findings revealed that there was contrast between the examples and tasks given to learners. The examples were designed to serve as models for learners, illustrating how to approach a particular problem using specific calculation strategies. These examples helped learners understand the calculation strategies and process involved in solving the problem.

They provided guidance and support, highlighting different strategies and techniques that can be employed. On the other hand, the tasks assigned to learners differed. The tasks were designed to assess learners' understanding and application of the concepts and strategies taught. They required learners to independently apply what they have learned, either on the same day or in previous lessons, to solve the given problem. The tasks  also varied in complexity and involved different contexts or scenarios. This is similar to what Ronda and Adler (2017) explain that examples and tasks must engage learners in a variety of content experiences that allow learners to make connections between features of the mathematical content. This approach of encouraged learners to explore different approaches, compare strategies, and make informed decisions based on the problem at hand.

### 5.3.2 What Calculation Strategies Do Teachers Teach in the NNP Curriculum?

The findings of this study indicated that teachers employed a range of calculation strategies to teach students. These strategies included building up and breaking down, number lines, physical modeling, counting on and counting down, addition bubbles, estimation, number pyramid, doubling and halving, commutativity, completing 10s, and rounding and compensation. This approach aligns with the third principle of teaching calculation strategies outlined by Ofsted (2008), which emphasizes the importance of teaching a variety of strategies to prevent learners from becoming overly reliant on a single approach for each operation. By exposing learners to different strategies, teachers promote critical thinking, problem-solving skills, and the ability to select appropriate methods for different mathematical situations.

The study further revealed that learners had the opportunity to select from the variety of calculation strategies taught and apply the most suitable strategy to unfamiliar tasks. This approach encourages learners to think critically and choose strategies that are reasonable and appropriate for solving different types of problems. The teaching of a range of calculation strategies was observed to begin from the first grade (Standard 1) and continued up to the fourth grade (Standard 4). This finding supports the recommendation of Torbeyns (2009) that the teaching of various calculation strategies should commence early in students' education, enabling them to develop a solid foundation in these strategies. This approach ensures that all students have the advantage of experiencing and mastering different calculation methods.

Additionally, the data revealed that the calculation strategies taught in Standards 3 and 4 appeared to be more advanced compared to those taught in Standards 1 and 2. Strategies such as number lines, breaking down and building up, doubling, and halving, number pyramid, and rounding and compensation were introduced at a later stage due to the complexity of the problems involved. This progression aligns with the explanations provided by Fuson (2004) and Torbeyns and Verschaffel (2013), who suggest that as children develop their understanding and proficiency in foundational calculation strategies, they naturally progress to more advanced strategies such as compensation strategy in later years. This approach allows students to build a solid foundation and gradually advance to more complex strategies as they develop their mathematical understanding and proficiency.

### 5.3.3 How Do Teachers Teach the Calculation Strategies in the NNP Curriculum?

The findings of this study indicate that teachers employed various teaching resources, such as workbooks, chalkboards, charts, tables, and physical models, to support students' understanding of calculation strategies. These resources served as tools to engage learners and guide them towards achieving the curricular goal of understanding calculation strategies. This approach aligns with the perspectives of Bruce (2007) who emphasize the role of teaching resources in motivating students to participate in mathematics lessons and facilitating effective instruction.

Teachers carefully selected examples and tasks that required complex thinking, aiming to introduce and teach multiple calculation strategies. Furthermore, learners were encouraged to apply the calculation strategies they had learned, either that day or in previous lessons. This practice aligns with the principles of effective teaching discussed by Watson and Mason (2006), Marton and Tsui (2004), and Lo (2012). When introducing a new concept, examples should provide learners with opportunities to observe variations and discern critical features. Additionally, learners should be prompted to apply what they have learned to solve new problems. This approach helps learners connect prior knowledge with new concepts and promotes a deeper understanding of the calculation strategies.

The data also revealed that learners were given the freedom to choose and utilize any calculation strategy they deemed appropriate for solving the examples or tasks. Threlfall (2009) supports this approach, emphasizing the importance of instilling confidence in learners and encouraging them to select strategies that work for them, without excessive concern for how others solve the problems. This approach promotes student autonomy, experimentation, and the development of problem-solving skills.

The teaching process involved presenting examples and tasks to learners, followed by group discussions, pair work, or individual attempts. Learners were then asked to name the calculation strategy used, explain their approach, and justify their solutions to their peers and the whole class. The teacher played a facilitative role by highlighting the students' work to the entire class. This instructional sequence aligns with the recommendations of Walshaw and Anthony (2008), who advocate for teachers to monitor students' initial participation in mathematical discussions and provide appropriate support based on their observations.

Contrary to the procedural teaching approach observed by Kazima and Jakobsen (2013) in Malawi where teachers begin teaching by working and showing learners examples from the textbooks, followed by practicing tasks that are similar to the example which also comes from the textbook. The findings of this study revealed a different teaching approach. In this study, teachers adopted a more interactive and exploratory teaching method. Teachers first presented the example and asked learners to attempt by discussing the in groups, or in pairs or work individually. Then, the teacher asked learners to name the calculation strategy used to solve the question, explain, and justify how and why they solved the question the way they did to their peers and the whole class. The teachers then came in to highlight the work done by the leaners to the whole class. This was then followed by assigned learners tasks from workbooks that were different from the examples, encouraging learners to apply their understanding of calculation strategies to new problems. This approach promotes problem-solving skills and the ability to generalize strategies to different contexts.

However, the study also revealed that due to time constraints, most of the learners' work was not presented and marked. This finding contrasts with the recommendation of Bruce (2007), who suggests that students should be given sufficient time to answer questions that require higher-level thinking. Allowing adequate preparation time can lead to more detailed explanations and increased confidence among students.

During the explanatory talk, the study findings revealed that teachers utilized everyday mathematics language to explain calculation strategies, aligning with the MDI framework. They employed open-ended questions to encourage students to express their thoughts, engage in discussions, and reason through unfamiliar problems. This approach resonates with the perspective of Anderson et al. (2015), who highlight that open-ended questions stimulate critical thinking, analysis, and problem-solving skills among students. Teachers explained the calculation strategies by explicitly mentioning and justifying the relevance of a chosen strategy to the given question. Requesting students to explain their strategy choices, how they worked, and analyzing alternative strategies promotes deeper mathematical understanding, as advocated by Hiebert and Wearne (1993). Moreover, teachers encouraged students to demonstrate their problem-solving approach on the chalkboard and justify their reasoning to their peers. This practice aligns with the recommendations of DfES (2006) in the United Kingdom, emphasizing the importance of providing learners with opportunities to explain and justify their thinking, allowing them to articulate the stages of their calculation strategies and the reasoning behind their decisions. Importantly, findings revealed that teachers did not focus on mere recall of information or procedures; instead, they prompted students to describe alternative strategies for solving the given questions. Engaging students in discussions and verbal explanations not only enables them to articulate their thinking but also allows teachers to gain insights into students' thinking processes and the potential development of their calculation strategies in the future, as highlighted by Ofsted (2008).

Regarding learner participation, the data revealed that teachers actively involved students in the lesson through various modes such as pairs, groups, individuals, and whole-class interactions. Encouraging students to work collaboratively in pairs, small groups, or as a class, as recommended by Bruce (2007), creates opportunities for students to learn from each other, share strategies, and collectively develop solutions. It is noteworthy that teachers did not dominate the classroom in terms of talking and taking charge of the learning process. Instead, they fostered an environment that encouraged students to engage with their peers and share their strategies. This approach aligns with the recommendation made by Van de Walle et al. (2014) that teachers should create learning situations and environments that promote active learner participation, facilitating the development and fluency of calculation strategies. By empowering students to effectively choose from a range of strategies and adapt them flexibly, teachers support students in their current and future mathematics.

### 5.3.4 How Do Teachers View the Teaching of Calculation Strategies?

The data from the interviews showed that teachers expressed positive views regarding the inclusion of calculation strategies in the NNP curriculum. They believed that these strategies were beneficial for learners as they promoted critical thinking and the ability to justify their solutions. However, many teachers, particularly those teaching standard 1, suggested that calculation strategies should be introduced after teaching number sense. This perspective aligns with the principles for teaching numeracy strategies, which emphasize the importance of establishing a solid foundation in place value, as highlighted by Ofsted (2006).

Data also revealed that teachers  expressed concerns about the limited time allocated for mathematics lessons. They believed that the current time constraints hindered their ability to fully explore calculation strategies with their students. This viewpoint  is similar to  the principle advocated by Bruce (2007), who emphasizes the need to provide learners with sufficient time to answer questions, especially those requiring more complex thinking. Bruce suggests that learners often require additional time to process and respond to questions that involve higher-level cognitive processes. By allowing learners adequate thinking time, a wider range of calculation strategies can be fostered, leading to more detailed explanations and a deeper understanding among students.

## 5.4 Conclusion of the Study

Based on the MDI framework Adler and Ronda (2015), it is concluded that teachers teach the calculation strategies in the NNP curriculum by emphasizing exemplification, explanatory talk, and learner participation and it is effective in promoting critical thinking and reasoning among learners.

**What calculation strategies do teachers teach in the NNP curriculum?**

The conclusion drawn from the findings is that the teachers in the study teach a variety of calculation strategies. such as building up and breaking down, number lines, physical modelling, counting on and counting down, addition bubbles, estimation, number pyramid, doubling and halving, commutativity, completing 10s, rounding and compensation Teachers use either formulated or selected example and tasks suggested in workbooks. Calculation strategies taught in Standards 3 and 4 are more advanced than those taught in Standard 1 and 2 for example, number lines, breaking down and building up, doubling, and halving, number pyramid, and rounding and compensation. However, calculation strategies like compensation, commutativity, and completing 10s are not taught frequently.

**How do mathematics teachers teach the calculation strategies in the NNP curriculum?**

Based on the findings, it is concluded that in Exemplification, the selection and use of examples and tasks by teachers in teaching calculation strategies play a crucial role in the implementation of the NNP curriculum. Teachers employ different approaches in selecting these examples and tasks, either drawing from workbooks or creating their own. Typically, one question from the workbook page is chosen as an example, while the remaining questions are considered tasks for learners to solve. The examples used by teachers serve as models to demonstrate different strategies, while the tasks assigned to learners require the application of various strategies to solve problems. Teachers adopt a non-procedural approach to teaching calculation strategies, integrating them into their instructional methods. They utilize different techniques to introduce and teach these strategies, including questioning learners and referencing the strategies covered in the lesson. Teachers employ visual aids such as charts, tables, and physical models to facilitate learners' understanding of calculation strategies. They incorporate real-life examples and open-ended tasks, inviting learners to participate in groups, pairs, or as a whole class. This participatory approach allows learners the opportunity to explain and justify their thinking and reasoning, promoting a deeper understanding of the concepts.

Regarding the explanatory talk, it is concluded that teachers effectively utilize mathematical language to name, explain, and justify the chosen calculation strategies relevant to the given problem. They encourage learners to actively participate by asking them to demonstrate and justify their solutions, fostering a deeper understanding of the strategies employed. Teachers emphasize the importance of learners explaining their reasoning behind the chosen strategies, creating an environment that promotes critical thinking and mathematical discourse. By encouraging learners to use calculation strategies that are suitable for each question, teachers support the development of problem-solving skills and mathematical proficiency

In terms of learner participation, the study indicates that teachers employ various modes such as group work, pairs, and individual work. This approach caters to the diverse learning needs and preferences of the students, providing them with opportunities to engage with their peers and learn from each other.

**How do mathematics teachers view the teaching of calculation strategies in the NNP curriculum?**

This study establishes that all teachers involved acknowledge the significance of teaching calculation strategies to their learners. They understand the value of equipping learners with effective calculation strategies to solve mathematical problems. One key suggestion put forth by the teachers is to reduce the workload per page in teaching materials. They propose focusing on one or two strategies per page. This aims to enhance comprehension and facilitate learning for students. By presenting a few calculation strategies per page and at a given time, teachers believe that learners will find it easier to grasp and apply these techniques successfully. Furthermore, the standard 1 teacher emphasizes the importance of introducing numerical concepts before diving into complex calculation strategies. They recommend that learners should first develop a solid foundation in numbers. This sequential approach ensures that students have a comprehensive understanding of numbers before progressing to more advanced problem-solving techniques.

## 5.5 Implications for further Study

The implications of this study suggest several areas for future research. Firstly, there is a need for in-depth exploration of the specific calculation strategies taught in the NNP curriculum. Further investigation into the effectiveness and outcomes associated with each strategy, as well as ways to enhance or expand upon them, can provide valuable insights for improving teaching practices. The study also revealed differences in the teaching frequency of certain calculation strategies. Future research could aim to uncover the factors influencing why some strategies are taught more frequently than others. Understanding these factors would enable educators to adopt a more balanced and comprehensive approach to teaching calculation strategies. The use of workbooks as a resource for selecting examples and tasks was highlighted in the study. Future research could explore how teachers effectively utilize these workbooks in their lesson preparation and classroom instruction. Investigating different workbook approaches and examining how teachers adapt and incorporate workbook materials would offer valuable insights for curriculum development and instructional design. Additionally, the study indicated that some teachers develop their own calculation strategies in addition to those suggested in the workbooks. Further research could investigate these teacher-generated strategies, examining their nature, effectiveness, and impact on student learning. Understanding the rationale and outcomes of these strategies would contribute to a more comprehensive understanding of effective teaching practices. The suggestion from the standard 1 teacher to teach numbers before focusing on calculation strategies warrants further investigation. Future research could explore the benefits and effectiveness of adopting a sequential approach, where learners first develop a solid foundation in numerical concepts before progressing to more complex calculation strategies. This research could provide insights into the optimal sequence for teaching mathematical concepts and strategies, leading to improve the teaching practices.

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# APPENDECES

## Appendix 1. 1 NSD Approval form

Assessment of processing of personal data

| Reference number | Assessment type | Date |
|---|---|---|
| 208353 | Standard | 16.09.2022 |

**Project title**

Investigating mathematics teachers understanding and implementation of calculation strategies of the National Numeracy Program curriculum in Malawi

**Data controller (institution responsible for the project)**

Universitetet i Stavanger / Fakultet for utdanningsvitenskap og humaniora / Institutt for grunnskolelærerutdanning, idrett og spesialpedagogikk

**Project leader**

Arne Jakobsen

**Student**

Getrude Kazembe

**Project period**

15.09.2022 - 01.08.2023

**Categories of personal data**

General

**Legal basis**

Consent (General Data Protection Regulation art. 6 nr. 1 a)

The processing of personal data is lawful, so long as it is carried out as stated in the notification form. The legal basis is valid until

01.08.2023.

Notification Form

**Comment**

ABOUT OUR ASSESSMENT

Data Protection Services has an agreement with the institution where you are carrying out research or studying. As part of this agreement, we provide guidance so that the processing of personal data in your project is lawful and complies with data protection legislation.

We have now assessed the planned processing of personal data in this project. Our assessment is that the processing is lawful, so long as it is carried out as described in the Notification Form with dialogue and attachments.

IMPORTANT INFORMATION

You must store, send, and secure the collected data in accordance with your institution's guidelines. This means that you must use online survey, cloud storage, and video conferencing providers (and the like) that your institution has an agreement with. We provide general advice on this, but it is your institution's own guidelines for information security that apply.

TYPE OF DATA AND DURATION

The project will process general categories of personal data until 01.08.2023.

LEGAL BASIS SAMPLE 1

The project will gain consent from data subjects to process their personal data. We find that consent will meet the necessary requirements under art. 4 (11) and 7, in that it will be a freely given, specific, informed, and unambiguous statement or action, which will be documented and can be withdrawn.

The legal basis for processing general categories of personal data is therefore consent given by the data subject, cf. the General Data Protection Regulation art. 6.1 a).

LEGAL BASIS THIRD PERSONS

The project will gain consent from parents of the data subject to process their personal data. We find that consent will meet the necessary requirements under art. 4 (11) and 7, in that it will be a freely given, specific, informed, and unambiguous statement or action, which will be documented and can be withdrawn.

The legal basis for processing general categories of personal data is therefore consent given by parents of the data subject, cf. the General Data Protection Regulation art. 6.1 a).

PRINCIPLES RELATING TO PROCESSING PERSONAL DATA

We find that the planned processing of personal data will be in accordance with the principles under the General Data Protection Regulation regarding:

- lawfulness, fairness, and transparency (art. 5.1 a), in that data subjects will receive sufficient information about the processing and will give their consent
- purpose limitation (art. 5.1 b), in that personal data will be collected for specified, explicit and legitimate purposes, and will not be processed for new, incompatible purposes
- data minimisation (art. 5.1 c), in that only personal data which are adequate, relevant, and necessary for the purpose of the project will be processed
- storage limitation (art. 5.1 e), in that personal data will not be stored for longer than is necessary to fulfil the project's purpose

THE RIGHTS OF DATA SUBJECTS

We find that the information provided to data subjects about the processing of their personal will meet legal requirements for form and content, cf. art. 12.1 and art. 13.

So long as data subjects can be identified in the collected data, they will have the following rights: access (art. 15), rectification (art. 16), erasure (art. 17), restriction of processing (art. 18) and data portability (art. 20).

We remind you that if a data subject contacts you about their rights, the data controller has a duty to reply within a month.

FOLLOW YOUR INSTITUTION'S GUIDELINES

Our assessment presupposes that the project will meet the requirements of accuracy (art. 5.1 d), integrity and confidentiality (art. 5.1 f) and security (art. 32) when processing personal data.

When using a data processor (questionnaire provider, cloud storage, video call etc.), the processing must meet the requirements for the use of a data processor, cf. art. 28 and art. 29. Use suppliers with whom your institution has an agreement.

To ensure that these requirements are met you must follow your institution's internal guidelines and/or consult with your institution (i.e., the institution responsible for the project).

NOTIFY CHANGES

If you intend to make changes to the processing of personal data in this project, it may be necessary to notify us. This is done by updating the information registered in the Notification Form. On our website we explain which changes must be notified. Wait until you receive an answer from us before you carry out the changes.

FOLLOW-UP OF THE PROJECT

We will follow up the progress of the project at the planned end date to determine whether the processing of personal data has been concluded.

Good luck with the project!

Contact person: Markus Celiussen

## Appendix 2 .1 Questionnaire

**Teacher's teaching of NNP Calculation strategies**

Name of school___________________________ Male [] Female []
Age range: 20-30 [] 31-40 [ 41-50 [] Over 50 []
Class you are teaching ___________________ Enrolment in Your class ______________

1. How long have you been teaching? ____________________________
2. How long have you been teaching mathematics? ___________________________
3. How long have you been teaching mathematics in this class? _______________
4. How well do you understand the NNP calculation strategies? Please tick
   a) Very well []
   b) Well [
   c) Partially []
   d) Do not understand []

5, How often did you attend the NNP curriculum trainings? Please tick

a) Always []

b) Sometimes []

c) Never []

## Appendix 3 .1 Lesson Observation

Name of the teacher: ______________________________ Male [] Female []
Age range: 20-30 [] 31-40 [ 41-50 [ Over 50 []
Date________________________Name of school _________________________________
Class _______________________ Time: From ________To: _______________
Number of students: Boys ______ Girls______ Total ________________
Unit/ Topic ___________________________________________________________________

1. Comments on the general learning environment:

□ Classroom arrangement

_______________________________________________________________________________

___

□ Classroom space

_______________________________________________________________________________

2. Main ways in which learners are invited to participate in the lesson.

Whole group [] Small groups [] Pairs [ Individuals []

3. Calculations strategies taught in the lesson

a) Physical modelling [ ]
b) Number lines [ ]
c) Breaking down and building up numbers [ ]
d) Doubling and halving [ ]
e) Estimation [ ]
f) Rounding and compensating [ ]
g) Commutatively [ ]
h) Other (specify)___________________________________________________

Comment

4. Calculation strategies indicated in the schemes of work and lesson plan

a) Physical modelling [
b) Number lines []
c) Breaking down and building up numbers []
d) Doubling and halving []
e) Estimation []
f) Rounding and compensating []
g) Commutatively []
h) Other (specify)_____________________________________________________________

5. Is the lesson based on workbook page? _______ If yes, page number______________ and
term__________________

6. Examples planned in the schemes of work and lesson plans-
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

7.Tasks planned in the schemes of work and lesson plan_______________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________
___________________________________________________________________________

## Appendix 4 .1 Interview Guide

1. What calculation strategies do you teach in the NNP curriculum?
2. How often do you teach the calculation strategies
3. How do you choose the examples and tasks used to teach the calculation strategies?
4. How do you explain the calculation strategies?
5. How do you invite learners to respond to the NNP calculation strategies that you teach?
6. How do you view the teaching of the NNP calculation strategies in the NNP Curriculum?
7. How best can the teaching of calculation strategies be improved to be successful?

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