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Dedication

This work is dedicated to my family for their unending support in kind. For their faith in me,

that I could pursue and complete this academic journey.

For their prayers and encouragement,

I love you.

God bless you.

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Abstract

The study investigated the opportunities to learn problem solving in Malawian grades 9 and 10 mathematics textbooks. To explore how problem-solving opportunities are provided and what factors influence the opportunities to learn problem solving in mathematics textbooks. The topics of Linear Equations and Simultaneous Equations were considered in the study. A total of 309 tasks from four textbooks were analyzed using the Mathematical Discourse in Instruction framework for textbook analysis (MDITx) to determine their levels of complexity. Levels 2 and 3 were considered potential problem-solving opportunities with a higher preference for level 3. The analysis revealed that there were 74% of level 2 tasks, 13% of level 3 tasks, and 11% of level 1 tasks. The results suggest that many tasks were similar to examples that students could refer to when solving the subsequent tasks. The preferred level 3 tasks were fewer than level 2 tasks, thus the textbooks provide few opportunities for students to learn problem solving. Of the few opportunities, many were presented in word form and were meant for the application of the learned methods and review. The dominance of level 2 tasks was due to three factors: 1) examples were detailed, 2) lessons were not introduced as challenges, and 3) questions were leading and not prompting. Due to the high textbook compliance and few qualified teachers in Malawi, the implications are that many students lack sufficient opportunities to learn problem solving. More problem solving should be included in the textbooks. In addition, textbook authors and curriculum developers need to define clearly what should count as problem solving in the context of Malawi. Recommendations and further study areas have been indicated.

Key words: Opportunity to learn, Problem solving, textbooks, Linear Equations, Simultaneous Linear Equations

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Abbreviations

CDSS Community Day Secondary Schools

JCE Junior Certificate of Education

MSCE Malawi School Certificate of Education

MANEB Malawi National Examinations Board

MDI Mathematical Discourse in Instruction

MDITx Mathematical Discourse in Instruction Analytical Tool for Textbook Analysis

MoEST Ministry of Education, Science and Technology

MIE Malawi Institute of Education

PSLCE Primary School Leaving Certificate of Education

SSCAR The Secondary School Curriculum and Assessment Framework

STEM Science, Technology, Engineering, Mathematics

TTI Teacher Training Institution

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CHAPTER 1: INTRODUCTION.

This study aimed at investigating the opportunities to learn problem solving provided in Malawian grades 9 and 10 mathematics textbooks. This chapter constitutes the background of the study, problem statement, purpose of the study, research questions, and significance of the study. Before focusing on the core of the study, below is a brief description of the Malawian education system.

The Malawian formal education structure is 8-4-4(5). There are 8 years of primary school whose entry age is 6. They are followed by 4 years of secondary school, then another 4–5 years of tertiary education. In the 8th year of primary school education, students write the Primary School Leaving Certificate of Education (PSLCE) examination to be enrolled into secondary school. Successful students are selected to either Conventional Secondary Schools or Community Day Secondary Schools (CDSS). There are junior secondary (grades 9 and 10) and senior secondary (grades 11 and 12) school levels. Students take the Junior Certificate of Education (JCE) examination at the end of grade 10, which graduates them into senior secondary. In grade 12 students take the Malawi School Certificate of Education (MSCE) examination. Successful students (with an aggregate number of credits of not more than 36 points) apply for University or Teacher Training Institutions (TTIs) (National Educational Sector Investment Plan 2020-2030, MoEST (2020). All selections are done on merit. Certification at both primary and secondary levels is done by Malawi National Examinations Board (MANEB) (MoEST, 2015). Repetition is not allowed at both primary and secondary schools (Maonga, 2020; MoEST, 2015).

1.1Background.

The Malawian curriculum considers mathematics as “a vehicle for the development and improvement of a person’s intellectual competence on logical reasoning, spatial visualization, analysis and abstract thought” (Ministry of Education, Science and Technology (MoEST), 2013, p. xi). Literature concurs that mathematics should enable students to think critically, analyze, reason well and solve a diversity of complex mathematical problems and real-life problems(Behlol et al., 2018; Fatima, 2021; Schoenfeld, 1983). MoEST also considers mathematics as the bedrock of science and technology whose purpose is to overcome challenges (solve problems) in the 21st century. It further points out problem solving as one of the skills that mathematics instills in students and anticipates the students to “use problem-solving techniques to solve practical problems” (MoEST, 2013, p. x). Problem solving is recognized in Malawian mathematics.

There are various arguments of a problem and problem solving in literature. Regardless, scholars agree that problem solving is finding a solution to a problem, situation, or challenge in which the solution path is not apparent to the problem solver (Behlol et al., 2018; Polya, 1945). Problem solving is a process that has got four stages in the following order; Understanding the problem, devising a plan, carrying out the plan, and lastly looking back (Polya, 1945). The first step considers what is unknown, the data given and the conditions. The second step considers the potential ways of solving the problem and the calculations and constructions that are involved in finding the unknown. The carrying out the plan step involves applying the procedure(s) devised in step 3. Lastly, in looking back, one must check the arguments and verify if the data was used correctly to find the unknown. Central in problem solving is that it constitutes unusual tasks whose procedure(s) is/are to be discovered (Ratnasari & Safarini TLS, 2020) and not recalled. Students could use operations like visualizing, associating, abstracting, comprehending, manipulating, reasoning and analyzing to find the solution (Nafees, 2011).

Scholars argue that problem solving and mathematics are inseparable (Ratnasari & Safarini TLS, 2020; Van Zanten & Van den Heuvel-Panhuizen, 2018). They consider problem solving to be the center of mathematics (Novotná et al., 2014; Szabo et al., 2020; Wilson et al., 1993). Moreover, mathematics is meant to stimulate thinking in students (Fatima, 2021). A better and more effective way to achieve this is through problem solving (Schoenfeld, 1983). Problem solving is important in various ways. Firstly, it is a good indicator of mathematical understanding (Gurat, 2018; Novotná et al., 2014). It enhances critical thinking, reasoning, and analysis capacity in students. When students are able to problem-solve it means they understand mathematics because that is the core of mathematics. Secondly, problem solving propels the solving of everyday real-life problems (Behlol et al., 2018; Fatima, 2021; Gurat, 2018; Isoda, 2012; Osman et al., 2018; Vongyai & Noparit, 2019; Wilson et al., 1993). As Fatima (2021) argued that mathematics is essential for dealing with our everyday needs. The process of problem solving, and the skills involved are applicable to problems in real life.

Thirdly, it motivates students to do more mathematics (Behlol et al., 2018; Wilson et al., 1993). Although problem solving might be tedious, Polya (1945) writes in the *Preface of the First Printing*: "…it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience the tension and the triumph of discovery" (p. v).

Successful problem solving gives students confidence and motivation while the need to solve a problem stirs their hunger to find the solution. Lastly, problem solving also enhances mathematics performance. According to Clark (n.d.), Singapore topped in the 1995, 1999 and 2003 Trends in International Mathematics and Science Study (TIMSS) in both 4th and 8th grade. They were also in the top 3 in 2007. Largely, these achievements were attributed to problem solving. Behlol et al. (2018) found that students who were taught through problem solving performed better than their counterparts. Even the low-performing students did well after being exposed to problem-solving approaches. Literature stresses that problem solving should be mastered by students (Sitompul et al., 2018). One of the ways through which this can be reinforced is through mathematics using the tasks given in the textbooks (Purwadi, 2020).

It is important to highlight the individualistic character of a problem. That is, problem to one individual might not be a problem to another individual (Wilson et al., 1993). When students encounter a mathematical task, to some it might be a problem while to some it might be a mere exercise (Buishaw & Ayalew, 2013; Van Zanten & Van den Heuvel-Panhuizen, 2018). Due to this individualistic character of a problem, it would be difficult to define a task in a textbook as an absolute problem or a straightforward task. Furthermore, a problem could be a specific task that could be solved through the application of previously learned algorithms (Van Zanten & Van den Heuvel-Panhuizen, 2018). Recent studies on problem solving also found that tasks that were considered as problems required the use of learned algorithms and not necessarily creating one (e.g., Jäder et al., 2020). Therefore, the current study considered tasks that required the application of learned algorithms as problems as well depending on how complex they are and how the framework defined a higher-level task.

1.2 Textbooks and learning mathematics.

Textbooks are defined as curricular representatives (Berisha et al., 2014). O'Keeffe (2013) defined textbooks as books that are written for teaching and learning. They are prepared in relation to the national goals in the intended curriculum and influence mathematical instruction in the classroom (Nirode & Boyd, 2021; Ulusoy & Incikabi, 2020). To ensure that problem solving is offered to students, textbooks should be analyzed since they are particularly purposed to assist students' learning. Textbooks are much closer to students daily as they solve different problems at school or home (Fan et al., 2013; Sunday, 2014). In addition, they are considered to be one of the most important elements for effective teaching and learning of mathematics (Fan et al., 2013; Fatima, 2021; Johansson, 2005; Lepik, 2015; O'Keeffe & O'Donoghue, n.d.; Sunday, 2014). Lepik (2015) further claims that textbooks influence school mathematics because they frame classroom instruction in the sense that teachers teach the contents of the textbook. What is not contained in the textbook is not taught and what is taught is what is contained in the textbooks.

Most of the mathematical explanations and exercises are from textbooks (Ulusoy & Incikabi, 2020). Few teachers use other resources to compensate for the gaps in the textbooks. Yet, sometimes textbooks could be deficient in supporting curriculum goals (Siegler & Oppenzato, 2021). On the other hand, students' acquisition of various mathematical skills depends on textbook affordances (Buishaw & Ayalew, 2013). Suggesting that textbooks could either limit or promote the learning of mathematics. Many authors have concurred that for students to do accurate computations it depends on the textbooks (e.g. Nirode & Boyd, 2021; Siegler & Oppenzato, 2021). Some authors (e.g. Qi et al., 2018) have found that despite the availability of other resources for teaching, like the internet, teachers still tend to rely on textbooks. It is in light of the influence of the textbook in mathematics instruction, that a textbook analysis study was carried out on the provision of problem-solving opportunities in grades 9 and 10 Malawian mathematics textbooks.

1.2.1 Textbook analysis.

The classroom setting is comprised of teachers, students and textbooks (Herbel-Eisenmann & Wagner, 2007). Research has shown that textbooks have a substantial impact on learning outcomes (Van Zanten & Van den Heuvel-Panhuizen, 2018). Therefore, it is important to analyze textbooks to ascertain their quality (Fatima, 2021) vis-à-vis achieving the intended mathematical objectives. Leshota (2020) argued impressively for textbook analysis that while it is important to strengthen teacher knowledge to improve the quality of teaching, it is equally important to look at curricular resources for teaching such as textbooks because they are critical in the teaching and learning of mathematics.

According to O'Keeffe (2013) textbook analysis is how features of the textbooks that are unknown to the author(s) can be identified and consequently, the effectiveness of the textbooks be established. The establishment of the effectiveness of the textbooks in this case is about the goals and objectives of the curriculum. These analyses seek to understand how certain topics and concepts are reflected in the textbooks (Fan et al., 2013) vis-à-vis the curriculum. Furthermore, the change(s) in pedagogical methodologies is of great concern as far as the quality of mathematics textbooks is concerned (O'Keeffe, 2013). Analogously, the curriculum reforms as happened in Malawi in 2013 – that includes a change in objectives and change of educational resources (textbooks inclusive) – also raise concerns about the quality of the mathematics in the textbooks. As such, there is a need to constantly and continuously analyze mathematics textbooks to check if they align with the current objectives, goals and visions of the mathematics curriculum (Fatima, 2021). In this study, it is imperative to ascertain if they are aligned to instill problem solving in Malawian grade 9 and 10 students.

Besides revealing the "mathematical content that is being served to students directly, textbook analysis can also reveal implicit or hidden choices that are made in textbooks" (Van Zanten & Van den Heuvel-Panhuizen, 2018, pp. 11–12). It is inevitable that authors may unintentionally structure the textbook content and incorporate some content that does not tally with the objectives of the curriculum. Textbook analysis brings into view such content (issues) so that they are addressed. In addition, textbook analysis helps in the undertaking of educational reforms since the strengths and weaknesses of textbooks are established vis-à-vis the promotion of the curricula vision (O'Keeffe, 2013; Rezat et al., 2021). Necessary steps are then taken to update the quality of the textbooks (Fatima, 2021). On the other hand, the objectives of the curriculum might be reviewed so that they align with what the textbooks are serving if viewed as important. Rezat et al. (2021) particularly underscored that in the 21st century, there is an increase on the recognition of mathematics textbooks as central in curriculum reforms. They contend that it is clear that "curriculum resources, especially textbooks, will likely continue to be seen and used as mediators of reform and change" (p. 1191). It implies that for mathematics change to be effective, it highly depends on textbook analysis.

1.3 Mathematics and Problem Solving in the Malawian curriculum.

Mathematics is one of the most prioritized subjects in Malawian secondary school education. The Secondary School Curriculum and Assessment Framework (SSCAR, 2015) indicates that seven periods per week are allocated for mathematics teaching and learning (see MoEST, 2015, p. 18). This is the highest number of periods as compared to other subjects. It is also a non-optional subject in both primary and secondary school. This means that Malawian students learn more mathematics in a school year. The expectation could be that Malawian students should be well knowledgeable in mathematics. Mathematics is considered a "vehicle for the development and improvement of a person's intellectual competence on logical reasoning, spatial visualization, analysis and abstract thought" (MoEST, 2013, p. xi). It is also underscored as the bedrock of science and technology to overcome 21st century challenges. The priority and importance placed on/in mathematics may imply that the mathematics that is being taught enhances the skills of critical thinking and problem solving among others.

MoEST (2013) and MoEST (2015) state that the rationale of secondary school mathematics is to "develop skills like computational, reasoning, critical thinking and problem solving…" (p. xi; p. 39). This concurs with the benefits of problem-solving indicated earlier. Both the SSCAR and the *Mathematics syllabus for forms 1 and 2* further expect students to be creative and critical and "use problem-solving skills to solve practical problems" (MoEST, 2013, p. x; MoEST, 2015, p. 18). Furthermore, problem solving is indicated as either a suggested teaching, learning, and assessment method or activity alongside discovery and practical work throughout the syllabus. These phrases indicate that, in Malawi, mathematics is considered a tool for learning problem solving. On the other hand, problem solving is also a way of learning mathematics.

The emphasis on problem solving in the Malawian mathematics curriculum is interesting as it leads to two assumptions and/or expectations. Firstly, secondary school students perform better in mathematics as problem solving enhances mathematical understanding (Gurat, 2018; Novotná et al., 2014). Secondly, problem solving opportunities are available in textbooks and are implemented in classrooms. This study focused on the second assumption because textbooks frame classroom instruction and learning outcomes (Lepik, 2015; Van Zanten & Van den Heuvel-Panhuizen, 2018). Teachers mostly teach what is contained in the textbook as indicated earlier. Malawi is no exception from this tendency regarding textbooks as it has been explained below. It was imperative to ascertain whether or not the mathematics textbooks contain opportunities for students to learn problem solving. Focusing on grades 9 and 10.

1.4 Textbooks and mathematics textbooks use in Malawian secondary schools.

In Malawi, the national curriculum determines the content of the textbooks. The curriculum is organized and approved by the MoEST after consulting other stakeholders in the society. Secondary school textbooks are authored and published by private publishers, but they are approved by the MoEST through the Malawi Institute of Education (MIE). It is only the approved textbooks that students, teachers and schools could choose from to use. No apparent directions are available on how schools choose their books to use (Maonga, 2020). Schools purchase from among the approved textbooks according to their economic capacity. However, if the books are at the teachers' disposal, the choice of which textbook(s) to use among the approved and purchased ones depends on the teachers' preference. Teachers may recommend the textbook(s) they are using to students. Students are also at liberty to use other textbooks among the approved ones to support their further learning. Currently, there is no research on individual textbook use by teachers and students. However, in Malawi, like in other countries, mathematics textbooks are a central tool in mathematics instruction for many teachers (Maonga, 2020; Mwadzaangati, 2019b).

Mwadzaangati (2019a) highlighted that Malawian mathematics teachers use textbooks to determine the type of tasks they present to students and how those tasks are implemented. Similar arguments are shared by other Malawian researchers (e.g., Chiyombo, 2020; Maonga, 2020; Phiri, 2018). Textbooks help to teachers organize the lessons that are delivered to students (Johansson, 2005). This is not a surprising scenario in Malawi because the textbook is the major tool of instruction that is readily available (Maonga, 2020). Many students also depend on textbooks for their mathematics practice and revisions. It is with this influence that textbooks have in mathematics instruction in Malawi, the study investigated the opportunities to learn problem solving provided in grades 9 and 10 textbooks. These grades constitute the lower secondary school section in Malawi and were chosen to ascertain the foundation of problem solving that is laid for the students.

1.5 Statement of the problem.

Schoenfeld (1983) pointed out that the primary function of mathematics is to make students think. He further asserted that a better and more effective way to achieve this is through the teaching and learning of problem solving. The Malawian curriculum and the *mathematics syllabus for forms 1 and 2* explicitly emphasize problem solving in many ways. Firstly, it is pinpointed as a learning method and activity that teachers could use in mathematics instruction (see MoEST, 2013, pp. 3–47). It is also stated in the rationale that students are expected to develop their critical thinking, reasoning, and problem-solving capacity through mathematics. Additionally, students are to use problem-solving techniques to solve practical problems (MoEST, 2013). These statements concur with Polya (1945) who described problem solving as a process and Wilson et al. (1993) who described it as a goal of mathematics and an instructional method for mathematics.

Internationally, the studies on problem solving in textbooks have focused on the presence and representation of problem-solving tasks and the use of problem-solving heuristics (e.g. Buishaw & Ayalew, 2013; Fan & Zhu, 2000, 2007; Jäder et al., 2020). In other studies, for instance, Brehmer et al. (2015) further studied the location of the problem-solving tasks and the form in which they are presented. All these studies indicated that many tasks in textbooks are not problems but mere exercises which promote rote memorization. Jäder et al. (2020) specifically highlighted that textbooks contain solution templates – from examples and previous topics – for students to refer to when solving tasks. Even the tasks that were considered problems required the templates to be solved. This finding encompasses many findings which suggest that there are similarities between examples and tasks or between the current tasks and previous tasks in mathematics textbooks. This makes many tasks to be mere exercises and not mathematical problems.

The current study took a similar direction like the other studies focusing on the following: First, to ascertain whether or not problem solving is present in Malawian secondary mathematics textbooks and how it is presented. Second, building on the task-example similarity highlighted by Jäder et al. (2020), this study investigated the factors that influence these similarities which affect the presence of problem-solving opportunities. The assumption was that, if tasks are found to be more similar to examples, then there must be factors that propel these similarities.

The study was also motivated by the revelation that no study on problem solving in textbooks has been conducted in Malawi according to my knowledge. In other words, it is not certain whether or not problem solving is incorporated in the textbook in the way that researchers describe it. Nevertheless, it could be anticipated that textbooks contain sufficient problem-solving opportunities given the emphasis that is indicated in the curriculum. The closest study to problem solving conducted in Malawian secondary textbooks is Maonga (2020), who explored the opportunities to learn mathematics in upper secondary mathematics textbooks. It was found that the opportunities were few.

Therefore, this study sought to first, fill the gap on the lack of problem solving in textbooks in Malawi. Secondly, highlight the factors that influence the presence of problem solving vis-à-vis example–task similarities. A tendency that has been implicit and explicit in the aforementioned studies.

1.6 Purpose of the study.

The purpose of the study was to investigate if opportunities to learn problem solving are available in Malawian grades 9 and 10 textbooks. The expectation was that there would be opportunities to learn problem solving in the textbooks, since it is emphasized in the curriculum. Furthermore, to find out how the textbooks responded to this call and what factors influence the problem-solving opportunities.

1.7 Research Questions.

The study was guided by one main objective and two subsidiary specific research questions. The main objective was to:

> Investigate the opportunities to learn problem solving provided in Malawian grades 9 and 10 mathematics textbooks.

To address this objective, two specific research questions were addressed:

i. How are problem-solving opportunities provided in Malawian grades 9 and 10 mathematics textbooks?
ii. What factors influence the opportunities to learn problem solving in grades 9 and 10 mathematics textbooks?

The topics of linear equations (to be called equations henceforth) and Simultaneous linear equations (to be called simultaneous equations) for grades 9 and 10 respectively were used as a case study. The choice of these topics was purposive since equations connect to many mathematical domains (Knuth et al., 2016). Andrews and Sayers (2012) substantiate that "learning to solve linear equations is a key component in school mathematics" (p. 2). They further assert that it is through solving equations that students connect their arithmetic skills to algebra. Equations are a connecting bridge from arithmetic to algebra and many other symbolisms involved in mathematics. In addition, the curriculum indicated problem-solving as a teaching method in the two topics, so it was expected that opportunities would be found. The literature reviewed in the next chapter focuses on algebraic linear equations and problem solving in textbooks.

1.8 Significance of the study.

The study might inform curriculum developers and textbook authors of the alignment of the curriculum and textbooks vis-à-vis the availability of problem-solving opportunities in the textbooks. That is if the understanding of problem solving described in the curriculum is the same as that is being implemented in the textbooks. Teachers might also benefit as they shall be made aware of the richness and deficiencies of the mathematics textbooks that might promote or deter deeper mathematical understanding so that they might make good use of it or adjust. Perhaps, teachers may work on how they implement the tasks in class. It might also reveal if the textbooks can propel the development of problem solving. Being one of the first studies on problem solving in Malawi, it might reveal opportunities for future research on this important topic. The study might contribute to the research field of mathematics education textbook analysis as there is a need for more textbook analysis globally (Fan et al., 2013), and in Malawi in particular (Maonga, 2020). Lastly, it might contribute to the field of research on equations and algebra focusing on problem solving.

1.9 Scope of the study.

The study focused on grades 9 and 10 mathematics textbooks. In addition, only the topics of Equations and Simultaneous equations were included. Furthermore, the study did not address in detail the extent to which the opportunities are available. It only reveals the presence of such opportunities. It does not compare the two grades included in the study. The grades are treated as a representation of the lower secondary school section in Malawi. Lastly, the conclusions are not generalizable rather they are just a glimpse of how problem solving is addressed in Malawian textbooks. In addition, the study is meant to influence further research and adjustments to be considered when coming up with other textbooks. Furthermore, the results are restricted to the analyzed textbooks and not any other textbooks in grades 9 and 10.

CHAPTER 2: THEORETICAL BACKGROUND.

This chapter contains literature that focuses on the teaching and learning of equations. Other literature focuses on problem solving in mathematics textbooks. It also contains the theoretical framework and how it has been used in the study.

2.1 Literature Review.

The review of literature focused on two fields. The first sub-section of the literature expounds on the teaching and learning of equations. It combines literature for both equations and simultaneous equations because both are linear. The only difference is that the simultaneous equations comprise two linear equations. However, later in the same section, few studies for simultaneous equations have been reviewed. The second sub-section is a review of the literature on problem solving in mathematics textbooks. This section was designed as such because the study used the topics of Equations and Simultaneous equations to study problem solving in textbooks.

2.1.1 Algebra, equations and the equals symbol.

Algebra is an important subject in secondary education that determines one's post-secondary endeavors (Huntley et al., 2007). It involves factoring, substituting one expression for another, collecting like terms, expanding, and solving equations and inequalities. It also involves generating equations from word problems which have proven to be challenging for many students (Kieran, 2007). According to Kieran (2007), algebra is dominated by symbol manipulation and the use of formal procedures. Despite this fact, studies agree that algebra is challenging to many students while at the same time important in determining students' success in mathematics and science (Huntley et al., 2007; Star et al., 2015; Stephens et al., 2013). Understanding algebraic equations in algebra is a large part of understanding algebra.

An equation is "a mathematical sentence with the symbol «=»" (Thomo et al., 2015a, p. 108). Equations are one of the main components of learning algebra (Kiliç & Masal, 2019), and they constitute a core concept in secondary school mathematics (Andrews & Sayers, 2012; Huntley et al., 2007). They also appear to connect with "every mathematical domain" (Knuth, et al., 2016, p. 65). Studies on the learning and teaching of equations are many which agree that understanding algebraic equations is essential for students' success in secondary school mathematics and science (e.g. Huntley et al., 2007; Stephens et al., 2013; Supriadi et al., 2021; Xu et al., 2017). Some studies on equations that focused on the «=» symbol have revealed that the understanding of the equals symbol is inadequate among students (Knuth, et al., 2016). Consequently, students face substantial difficulties to solve equations (Sitompul et al., 2018; Supriadi et al., 2021).

Research has shown that most students consider the equals symbol as an operational or computational symbol (Huntley et al., 2007; Kiliç & Masal, 2019; Stephens et al., 2013). Students view the symbol as a command to compute or carry out an operation and not an equivalence between expressions. For example, in the equation $2 + 3 = x + 3$, one group could put 5 as the value of x by adding the left side. This group believes that when given an equation, you need to add the values on the left side and the result is the answer for the unknown. A second group would add the left side first then subtract 3 on both sides or take the 3 to the other side and change its sign. This group holds the notion that you collect numbers on one side and variables on the other while changing the signs when it crosses the equals symbol. Only a few would be quick to realize that the expressions are equal, and x is 2. This group understands that the expressions are equal, and, given that 3 is common, then what is missing is 2. This is the relational meaning (Stephens et al., 2013) of the “equals” symbol which is lacking in many students. As a result, the learning, understanding and solving of equations has been limited. Qetrani et al. (2021) argued that errors in solving equations are due to the misunderstanding of the equals symbol.

2.1.2 Teaching and learning of Linear equations and Simultaneous linear equations.

Equations are mainly of two kinds. These include conditional and identity equations (Huntley et al., 2007; Thomo et al., 2015a; Ugboduma, 2013). The former is where the equation is true for unique values while the latter is where the equation is true for all values of the unknown(s) (Ugboduma, 2013). Huntley and colleagues indicated another kind called contradiction equations which is less emphasized. In all these kinds, the direction of solving them seems to be common in literature.

Studies reveal that the solving of equations is highly dominated by the use of formal methods (Kieran, 2007). In highlighting these formal methods, Qetrani et al. (2021) described two approaches that students are taught to solve equations. For equations of the form $ax + b = 0$ where a and b are real numbers, students are asked move all constants to the right and remain with variables on the left by either adding, subtracting, multiplying and dividing the constant both sides to maintain equivalence (Kieran, 2007). This is also called the *balance method* (Thomo et al., 2015a). The other approach is called the *inverse operation*. This is where teachers ask students to move the constants to the other side of the equation and change their sign, which is a wrong generalization (Qetrani et al., 2021). For equations involving parenthesis, it involves four steps: (1) expanding the parenthesis, (2) collecting like terms, (3) moving constants to the right side and (4) dividing throughout by the coefficient of the unknown (Star & Seifert, 2006). These are the methods that many students are accustomed to when it comes to solving equations.

The dominance of formal procedures is supported by many studies. For example, Jäder et al. (2020) found that even in problem-solving tasks of equations, formal procedures were dominant as compared to geometry. Kiliç and Masal (2019) reported that students are computational thinkers when they work with equations. This might be attributed to the nature of equations or how they are taught. “School-level algebra usually emphasizes procedural skills over conceptual understanding for solving algebraic tasks” (Qetrani et al., 2021, p. 1). Yet, students do not understand the reasoning behind those procedures. Huntley et al. (2007) observed that high school students struggled with identity and contradicting equations as compared to conditional equations. Students used symbol manipulation on all equations even after observing that symbol manipulation worked only on equations with unique solutions. Suggesting that students had on the one hand, inadequate strategies and understanding of solving equations. On the other hand, they were accustomed to using formal procedures. Other researchers echo similar views about equations and standard procedures and symbol manipulation (e.g., Stephens et al., 2013).

Nevertheless, a substantial number of studies have revealed that students show reasoning when solving equations. Recently, Musdin and Wassahua (2021) underscored the ability of students to display reasoning as they solve equations with one variable. In their post-test interview with the student involved in their study, the student provided reasons for the arguments used in solving the equations. However, formal procedures were also used by the student. Xu et al. (2017) also observed that students were more comfortable using formal procedures of solving equations than creative ways. Yet, their study of practical and potential flexibility in linear equations revealed that students have creative methods of solving the equations when prompted to solve equations in other ways other than the standard method.

Star and Rittle-Johnson (2007) argued that there are multiple ways to solve linear equations. They also acknowledged that prompting students to solve an equation in two different ways enhances knowledge and use of multiple strategies. A study by Qetrani et al. (2021) with 61 Moroccan students revealed that the experimental group that was taught linear equations through the equivalence approach, 36% of the students used formal methods because they were easy and safe. The control group was taught through the traditional method of doing the same operation on both sides of the equation. The results indicated that 77% of the students used the formal methods with simply different structures. However, both groups used invented strategies when prompted although it was higher in the experimental group. This suggests that students are accustomed to formal procedures of solving equations. On the other hand, the way questions are posed could stimulate creative ways of solving equations. Good exercises should prompt students to engage in reasoning (Ulusoy & Incikabi, 2020).

Like Qetrani et al. (2021), other studies have also found that formal procedures are prominent among students (Huntley et al., 2007; Xu et al., 2017). Students have little understanding of methods like graphical to solve equations that do not work with symbol manipulation (Huntley et al., 2007). More recently, Star et al. (2022) found that middle and high school students from Sweden, Finland, and Spain, mostly used formal algorithms to solve linear equations. The tendency was dominant among middle school students. This suggests that students can be creative in equations if lessons in class and textbooks are designed in a discovery learning way. A study with eight experts in mathematics (two mathematics educators, two mathematicians, two secondary mathematics teachers and two engineers) also revealed the dominance in the use of formal procedures. When given the equation $7(n + 13) = 42$, experts' first method was applying the distributive property (Star & Newton, 2009). The experts said that distribution was the first solution procedure that came to their minds when such equations are given. However, they later changed and divided 42 by 7. Studies such as these indicate that there is a need to balance between teaching formal procedures and promoting creativity.

Studies specifically addressing simultaneous equations are rare (Johari & Shahrill, 2020). The above reviewed studies relate to both kinds of equations. Nevertheless, the few studies on simultaneous equations suggest that Simultaneous linear equations contain problem solving at junior level mathematics (Sitompul et al., 2018). Perhaps because it is usually a new concept to students especially in the context of Malawi. It is argued to be one of the challenging topics to teach and students find it challenging. Consequently, students tend to memorize the steps and methods (Johari & Shahrill, 2020; Ugboduma, 2013).

A more study by Johari and Shahrill (2020) has shown that students struggle with fraction equations. They further highlighted the common errors students make which include wrong substitution, complicating the subject and mathematical errors resulting from failed manipulation such as rearrangements and changing signs. They hinted that fraction equations are designed in such a way that the fraction can be get rid of and changed into much simpler fraction-less equation by LCM. Star et al. (2022) also argued that variables with fractional coefficients are well known areas of difficulties. A study with eight experts indicated earlier on how they solve linear equations also revealed their non preference to work with fractions (Star & Newton, 2009). The reason that they gave was that they are more likely to make errors when they work with fractions. Johari and Shahrill also addressed equations with variables that contain a negative coefficient. They argued that equations with negative coefficients have to be changed to positive to avoid mathematical errors.

Despite the errors in substitution methods highlighted by Johari and Shahrill (2020), Ugboduma (2013) found that students preferred the substitution method to the elimination method. Nevertheless, students were also able to determine the most effective method depending on the nature of the equations. This indicates that if the methods are presented simultaneously, it would give students the opportunity to think as compared to when they are presented separately whereby students a confined to one method at a particular moment. All the studies indicate the deficiency of creativity and the dominance of formal procedures in equations. They also emphasize that equations are prone to errors in some instances. Whether or not the equations contain problem solving tasks, it has not been explicit. They are simply challenging, especially the simultaneous equations.

Recent studies have advocated for problem-based teaching and learning of linear equations to enhance higher order thinking in linear equations (Kirana & Kholifah, 2020; Musdin & Wassahua, 2021). Through incorporating real life application problems, those that relate to other disciplines and demand thinking skills could propel problem solving. In Singaporean mathematics textbooks the levels were argued "… and on Level 3 questions that involve real-life applications, thinking skills, and questions that relate to other disciplines" (Kaur et al., 2020, p. 102). Purwadi (2020) also stressed that teaching equations through realistic mathematics and prompting students to model the equations evokes their thinking. Kieran (2007) also argued for the same more than a decade ago, implying that there should be less focus on symbol manipulation and provide more problem-based equations. Problem solving in mathematics is developed when the discipline is integrated with other disciplines. Students grasp a concept from a diverse perspective such that they are able to develop the skill of applying the mathematical skills in meaningful situations (Buishaw & Ayalew, 2013).

2.1.3 Problem solving in mathematics textbooks.

Globally, there has been a growing interest in problem solving in mathematics textbooks since the 1970s (Fan & Zhu, 2000). The studies have focused on the presence of problem solving, its representation and how textbooks in general support problem solving. The earliest 21st century study by Fan and Zhu (2000) focused on representation of problem solving and problem-solving procedures in Singaporean lower secondary school mathematics textbooks. They found that traditional and routine problems were dominant in the textbooks with 97% and 96% respectively. They also found that the heuristics were not used frequently. In addition, it was found that only the 'carrying out the plan' stage of Polya's (1945) four-stage model was applied in all the problems while the other three stages were applied sparingly. Despite the findings, the textbooks were said to be a good foundation for problem solving because the tasks were set up to develop higher order thinking. In addition, the textbooks exposed students to at least 14 heuristics.

Fan and Zhu (2007) also carried out a comparative analysis on the representation of problem-solving procedures in textbooks from China, Singapore, and USA. The study adopted Polya's four-stage problem-solving procedure to ascertain which stages are illustrated in textbooks. It also assessed the representation of problem-solving heuristics like acting it out, looking for a pattern, working backwards and other 14. All textbooks revealed that all textbooks illustrated the "carrying out the plan" stage, and few problems exhibited all stages. This suggests that problem solving is interpreted as applying an algorithm. Regarding heuristics, all textbooks used at least 10 out of the 17. However, only drawing the diagram, restating the problem and using an equation were dominant in all textbooks.

Berisha et al. (2014) in Kosovo replicated Fan and Zhu (2000)'s study and found that textbooks contained more routine problems than non-routine problems. Furthermore, the tasks were also in non-contextual form. Regarding the use of heuristics, the results were similar to Fan and Zhu's. In a recent study Jäder et al. (2020) analyzed 12 textbooks from 12 countries to ascertain how they support problem solving through the topics of equations and geometry. Few problem-solving tasks were found generally, but fewer were found in equations than in geometry. Most of the problem-solving tasks employed standard procedures which were dominant in equations. Brehmer et al. (2015) analyzed Swedish mathematics textbooks for upper secondary school. Their focus was on the quantity of tasks that were mathematical problems, their location, level of difficulty and the context of their presentation. All three analyzed textbooks included only a few mathematical problems. They were located at the end of the chapter with the highest level (level 3) of difficulty and presented in purely mathematical form. They also found that most of the tasks could be solved using a template provided earlier in the textbooks.

Van Zanten and Van den Heuvel-Panhuizen (2018) replicated a study by Kolovou et al. (2009). They analyzed grade 4 and 6 new mathematics textbooks for the opportunity to learn problem solving. They found that both textbooks contained few non-routine problems. These results were similar to those of Kolovou and colleagues a decade earlier. The results from Van Zanten and Van den Heuvel-Panhuizen (2018) imply that despite changing the textbooks, students were still being exposed to the same problems from a decade earlier. In another study, a comparative analysis was carried out between Singaporean and Spanish primary textbooks by Vicente et al. (2019). It revealed that Spanish textbooks contained few challenging tasks than Singaporean textbooks. In Africa, Buishaw and Ayalew (2013) evaluated grades 9 and 10 mathematics textbooks of Ethiopia on the inclusion of adequate problematic situations and problem-solving heuristics to enhance problem solving skills. The results were similar to the findings of Fan and Zhu (2000, 2007). Recently, Turkish middle school teachers revealed that the major weakness of textbooks was the deficiency of problem-solving activities (Ulusoy & Incikabi, 2020).

All studies on problem solving in textbooks point in a similar direction. They indicate a deficiency in problem-solving opportunities in textbooks. The general recommendation from the studies is the need to include more problem-solving tasks in textbooks. Specifically, including more real-life related problems could provide the opportunities to students to learn problem solving (Brehmer et al., 2015). Moreover, real-life related problems deepen mathematical understanding (Cady et al., 2015). However, this does not mean that low-level tasks should be neglected altogether (Glasnovic, 2018).

To my knowledge, no study on problem solving in Malawian mathematics textbooks has yet been conducted. However, studies on textbook analysis have been conducted (e.g., Chiyombo, 2020; Maonga, 2020; Phiri, 2018). All studies focused on the opportunities to learn mathematics that are available in upper primary school, upper secondary school, and lower primary school textbooks respectively. They generally found that there are limited opportunities to learn mathematics in textbooks. For instance, Maonga (2020) analyzed the opportunities to learn mathematics in grade 11 textbooks on the topic of quadratic equations and found that higher level (3 and 4) tasks ranged between 3% and 8% across the textbook; this is on the Mathematics Task analysis tool scale levels. In the same study the MDITx revealed that out of every 100 tasks the AMC (highest level) tasks ranged between 8 and 16 in each of the four textbooks that were analyzed.

Out of the studies done in Malawi, none of the studies narrowed it down to the concept of problem solving. Moreover, none of the studies focused on the lower secondary school. Hence this study could be of significance in addressing problem solving which is emphasized globally and in the Malawian curriculum.

2.2 Analytical framework

The study adopted a framework for mathematics textbook analysis as proposed by Ronda and Adler (2017), called the Mathematics Discourse in Instruction analytic framework for textbook analysis (MDITx). It reveals the mathematical opportunities available in mathematics textbook lessons for students to learn. It is an adaptation of the Mathematics Discourse in Instruction (MDI) framework by Adler and Ronda (2015). The MDI was developed to analyze opportunities offered to learners to learn mathematics in teachers' lessons (Ronda & Adler, 2017). Whereas the MDITx was adapted to make visible the opportunities to learn mathematics in textbook lessons.

2.2.1 The Mathematical Discourse in Instruction framework

The MDI is the original framework which is designed for a classroom setting to reveal the opportunities to learn mathematics in teachers' lessons. The framework is theoretically grounded in the social cultural theory and framed within the practices of secondary mathematics in relatively poor schools in contemporary South Africa (Adler & Ronda, 2015). This makes the framework suitable for the Malawian context as well since secondary schools in Malawi are equally poor or probably even poorer. Furthermore, the MDI framework is built within the theory of variation, which states that learning occurs when different patterns of variations are made available (Marton

& Pang, 2006). It contains four interacting components in the teaching of a mathematics lesson: exemplification, explanatory talk, learner participation and the object of learning. These can be treated as separate analytic constructs (Ronda & Adler, 2017). Figure 1 shows the components of the MDI framework.

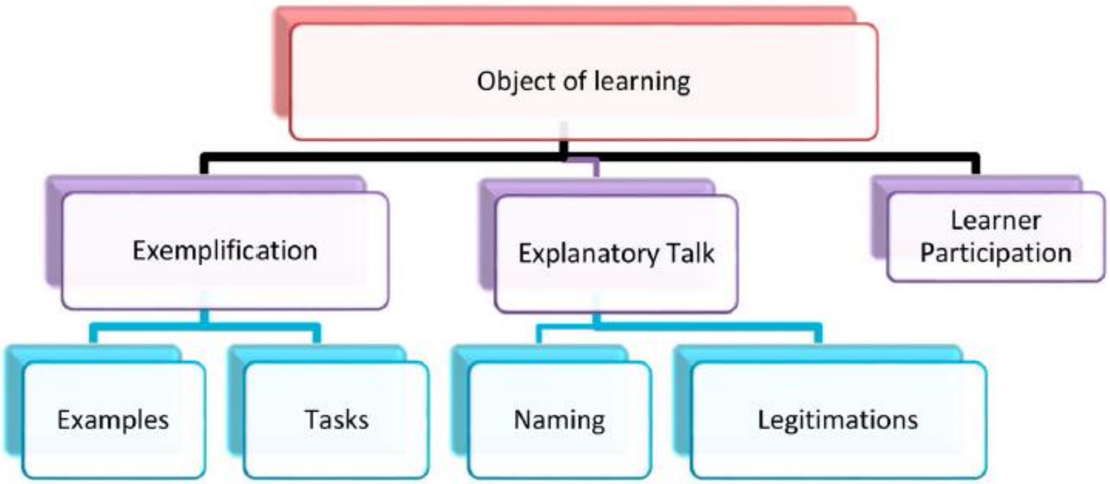


Figure 1: Elements of the Mathematical Discourse in Instruction framework (Adler & Ronda, 2015, p. 239)

The framework above was adapted to formulate the Mathematical Discourse in Instruction analytic framework for textbook analysis, to equally reveal the opportunities to learn mathematics in textbook lessons as in classroom lessons (Ronda & Adler, 2017). The section below describes the MDITx framework, which also reflects the original MDI that is based in the classroom.

2.2.2 The Mathematical Discourse in Instruction analytic framework for textbooks analysis.

The MDITx constitutes all components but learner participation. This is so because learner participation is not visible in the written curriculum and textbooks (Ronda & Adler, 2017). Briefly, the component of learner participation in the original framework (MDI) is used to determine what learners are invited to say apart from the tasks (Adler & Ronda, 2015). In the subsequent section I expound on the other three components.

OBJECT OF LEARNING

The object of learning as shown in Figure 1 is the core of the lesson or teaching (Adler & Ronda, 2015). It is the main focus of the lesson. This is what learners are expected to know and do at the end of the lesson. In other words, it is the goal of a mathematics lesson. Teachers' focus in a lesson needs to be on the object of learning. All mathematical instruction is driven toward this goal. An object of learning could be a concept, procedure or an algorithm, or meta-mathematical practice (Adler & Ronda, 2015). The object of learning is mediated through exemplification and explanatory talk. "What stands between (i.e. mediates) the object (and here of learning) and the subject (the learner) are a range of cultural tools such as examples and tasks, word use and the social interactions within which these are embedded" (Adler & Ronda, 2015, p. 238). This means that the other components of the framework facilitate the achievement of the object of learning. As seen in Figure 1, all three components are connected to the object of learning.

EXEMPLIFICATION

Exemplification consists of two subcomponents namely: examples and tasks as shown in Figure 1. These subcomponents make explicit the object of learning (Ronda & Adler, 2017). Students work with these to help the teacher determine whether or not the object of learning has been achieved. Below is a discussion of examples and tasks.

Examples

Examples are defined as a particular case of a large class used for drawing, reasoning, and generalization (Zodik & Zaslavsky, 2008). Examples mediate the object of learning so that students can apply the procedures on other tasks. The examples might either be worked or not worked. The former is meant to expose the procedure while the latter is a learners' exercise (Mwadzaangati, 2019b). A set of examples that Ronda and Adler (2017) call "example space" might lead to three patterns of variations which determine the opportunities available to learn. These variations are generalization, contrast, and fusion. (1) Generalization (G); this is when there is similarity in the examples; (2) Contrast (C); this is when there are differences between or among the examples; (3) Fusion (F); this is when an example space constitutes simultaneous variations. It means that an example space can be coded G, C, or F or a combination of two or all the variations.

When coding an example space, a level is assigned to an example space depending on the pattern of variations it is exhibiting. Level 1 is assigned if the examples only exhibit one form of variation, either C or G. Level 2 is assigned when two forms of variations are exhibited. The variations may be generalization and contrast or contrasts and fusion or generalization and fusion. Level 3 is assigned when all three variations are applied. If no patterns of variations are available, it is assigned "NONE". However, "NONE does not mean that the author did not provide any examples. It means that the author did not provide opportunities for learners to discern key features of the content" (Ronda & Adler, 2017, p. 1102).

Tasks

Tasks are what learners are mandated to do with the examples given (Ronda & Adler, 2017). For example, learners might be asked to simplify, find the unknown and carry out various computations. The authors further highlight that examples and tasks are linked but different. As such the way they are coded and analyzed is linked as well but different. Tasks normally come after examples in many textbooks. Many textbooks are structured in a *rule-example-practice* way (Glasnovic, 2018). Like the examples, tasks have been put in variations. This description is in order of increasing cognitive levels. According to the framework, a task that involves a procedure learned previously can be coded KPF (Known Procedures/Facts) and is considered a level 1 task, because it only requires prior knowledge that is easily accessible. A task that requires the application of the current topic procedures is coded CTP (Current Topic Procedures) and is considered as a level 2, because the new method is not fully assimilated by the students, and it might prove to be challenging. Lastly, a task that stimulates learners to decide on the procedure to use and make connections among concepts is coded as AMC (Application Making Connections). This is considered a level 3 task because it is cognitively demanding. This increase is in terms of the extent of connections between and/or among concepts and procedures (Adler & Ronda, 2015).

EXPLANATORY TALK

This component also contains two subcomponents: naming and legitimization. Explanatory talk involves communicating to students what is vital in relation to the object of learning. That is, what is to be known or done, and how it is to be done. It mainly focuses on naming and legitimatizing the focus of the lesson, that is, related to examples (Adler & Ronda, 2015). The main focus of explanatory talk is to highlight the important parts of the lesson so that the objective is achieved.

On Naming, it is argued that the way mathematical concepts and procedures are named influences the focus of students in a lesson. Legitimization "examines the mathematical and non-mathematical criteria that are communicated to legitimize or substantiate the 'key' moves or steps in the procedures or in steps about the object of learning" (Ronda & Adler, 2017, p. 1105). In the context of textbook analysis, explanatory talk is considered as the authors' talk, that is, the written text.

2.2.3 The focus of the study

The study focused on exemplification because they make explicit the object of learning. Additionally, the study sought to establish how examples and tasks relate to influence the opportunities for problem solving in the textbooks. On the other hand, leaner participation was inapplicable because it is not visible in the written curriculum and the textbooks (Ronda & Adler, 2017). Explanatory talk was beyond the interest of the study. Therefore, the coding used in the study was that of the constructs of examples and tasks as described above. The summary of the coding system is in Table 1.

Table 1*:* The code system for analyzing textbook examples and tasks (adapted from Ronda & Adler, 2017, p. 1106).

Example	Task
Level 1 – at least one of the patterns of variation (C – Contrast, G – Generalization F – Fusion)	**Level 1** – carry out known procedures or use known concepts related to the object of learning (KPF)
Level 2 – any two of C, G, or F.	**Level 2** – carry out procedures involving the object of learning (includes CTP but no AMC codes)
Level 3 – all patterns of Variation	**Level 3** – carry out Level 2 tasks plus that involve multiple concepts and connections (includes CTP and AMC codes)

2.2.4 The framework as applied in the study.

The framework had not been used to analyze problem solving, as such it had to be adapted to fit the study. This involved defining examples and tasks in the context of the study. First, examples were defined as tasks that have been worked-out in the textbooks through which a procedure being applied is performed (Liz et al., 2006). These are meant to “offer students a model to be implemented in the subsequent exercises” (Glasnovic, 2018, p. 1005). The patterns of variations were found through the coding system in Table 1. The levels in the examples were not the focus because all that was needed were the features that determined the variations in the examples as defined in the study.

On the other hand, tasks were defined as unworked examples in the exercise section as indicated in the analyzed textbooks. Exercises “refer to the various tasks that students should do…” (Glasnovic, 2018, p. 1005). The levels of the tasks were determined as indicated in Table 1. The distinction between examples and tasks as defined in this study was to determine if the examples influence availability of problem solving in textbooks and what features contribute to that. While tasks were to determine if the opportunity to learn problem solving is provided in the textbooks. In the next section, I illustrate how this adaptation was applied in the study. Furthermore, I clarify the modified framework to befit the study.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter constitutes and discusses the research design and data collection techniques employed in the study. It expounds on the sample and sampling procedures undertaken. It highlights and clarifies how data analysis was conducted with a further explanation of how and why the framework was modified and used. Lastly, validity and reliability are addressed.

3.2 Research design

Research designs are procedures of inquiry that describe plans for data collection, sampling, analysis, interpretation and reporting in studies (Creswell & Creswell, 2018). A research design "spans the decisions from broad assumptions to detailed methods of data collection and analysis" (Creswell, 2009, p. 3). The study investigated *the opportunities to learn problem solving provided in Malawian grades 9 and 10 mathematics textbooks*. To address this problem, a qualitative content analysis design was adopted (Krippendorff, 2004).

Qualitative content analysis is a technique "for making replicable and valid inferences from texts to the contexts of their use" (Krippendorff, 2004, p. 18). It involves interpreting text according to their place of use and purpose. The design was suitable for the study for three reasons. (1) Text is qualitative (Creswell & Creswell, 2018; Krippendorff, 2004; Yin, 2011), as such the qualitative methods are best applicable in analyzing textbooks. (2) Meaning from texts is understood in detail through qualitative content analysis (Creswell & Creswell, 2018; Krippendorff, 2004; Schreier, 2014; Yin, 2011). (3) Qualitative methods are ideal for a study of a problem with little or no research and needs more understanding (Creswell & Creswell, 2018; Yin, 2011). In this case, problem solving has not yet been extensively studied in Malawi as indicated earlier.

Choosing a design depends on the nature of the problem being addressed, the researcher's personal experience and the audience of the study (Creswell 2009; Creswell & Creswell, 2018). Qualitative content analysis was chosen for this study to give a detailed picture of the state of problem solving in Malawian grades 9 and 10 mathematics textbooks. However, generally, research designs are meant for collecting, analyzing and interpreting data (Creswell, 2012).

3.3 Sampling procedures

3.3.1 Sample

The study targeted grades 9 and 10 mathematics textbooks to ascertain the problem-solving foundation laid by the textbooks on the concept of equations. Two textbook series were used: *Achievers Junior Secondary Mathematics student's Book* and *Excel and Succeed Junior Secondary Mathematics Student's Book*. Both series are common in both grades. That means two different series for each grade. Altogether, four textbooks were analyzed. The topics of Equations and Simultaneous equations were analyzed. The sampling process of the textbooks and the topics has been elaborated on later in the section.

Sampling/analysis units

According to Krippendorff (2004), sampling units are "units that are distinguished for selective inclusion in an analysis" (p. 98). In this study the sampling units were all topics in the selected textbooks for each grade. From each grade one topic was chosen to be the unit of analysis. The topic of Equations and Simultaneous equations were the units of analysis for grades 9 and 10 respectively.

Coding/recording units

These are "units that are distinguished for separate description, transcription, recording or coding" (Krippendorff, 2004, p. 99). This study investigated the opportunities to learn problem solving provided the textbooks through relating examples and tasks. Therefore, examples and tasks of the units of analysis were the coding units. That is to say, the examples are tasks in the two topics were the coding units. These are the units that were analyzed and discussed/described. Krippendorff (2004) hinted that sampling units contain coding units.

3.3.2 Sampling techniques

To end up with the two textbook series and topics, a two-stage purposive sampling process was employed. Purposive sampling is a deliberate selection of samples that the researcher believes would contribute data that might help in addressing the problem (Creswell & Creswell, 2018; Neuendorf, 2002; Yin, 2011). This was done for both grades 9 and 10.

Stage-1 purposive sampling – sampling textbook series

This stage aimed at selecting textbook series that: (1) are commonly used in Malawian secondary schools, (2) align with the curriculum, and (3) have much content for students. This could strengthen the results since the analyzed series potentially contribute to students' mathematical knowledge and implement the intended curriculum. Three steps were involved in this stage.

The first step aimed to establish the commonly used textbooks. I considered dominance and selling in the market as indicators of usage. That is if a series is dominant and selling most in the markets then it is the most used. Four textbook series (listed later) were dominant and selling most according to legitimate bookstores that the researcher inquired. Dominance and selling in the market implied dominance in schools and among students. A further random inquiry from my colleagues (mathematics teachers) revealed that they mostly used the four series that dominated the market.

The second step considered recent research on textbook analysis in Malawi and the textbook series that were used. The most recent study, by Maonga (2020), analyzed the same four (dominant) textbook series. Despite Maonga's study focusing on upper secondary school textbooks, anecdotal evidence asserts that a textbook series in Malawi dominates both lower and upper secondary school levels. The series were also recommended by Mercy Kazima, a professor in mathematics education at the University of Malawi and renown expert in the field. Consequently, physical copies of the four series were acquired. This led to the third step.

In the third step, I sought for textbook series with the following characteristics. (1) With content (topics) as indicated in the curriculum; (2) with much content (defined in the study as at least 10 tasks per exercise) to broaden the area of analysis. The assumption was that the more tasks, the more opportunities there could be; (3) with topics as arranged in the curriculum. The assumption was that the topics in the curriculum are purposively and systematically arranged to establish topical connections as textbooks progress from one topic to another. The four textbook series that were acquired and filtered were as follows presented in pairs of grades 9 and 10 (Book 1 and 2) respectively.

1. Achievers Junior Secondary Mathematics Student's Book 1 and 2 (Nyirenda et al., 2014a, 2014b), (series 1).

2. Arise with Mathematics Student's Book 1 and 2 (Chitera, 2013a, 2013b), (series 2).
3. Excel and Succeed Junior Secondary Mathematics Student's Book 1 and 2 (Thomo et al., 2015a, 2015b), (series 3).
4. Target in Junior Secondary Mathematics Student's Book 1 and 2 (Chikwakwa & Banda, 2015a, 2015b), (series 4).

The four textbook series were filtered based on their alignment with the curriculum vis-à-vis content and sequence of the topics. Then they were filtered based on quantity as defined above. Textbook series 2 and 4 failed to satisfy the criterion. For example, Series 4 did not follow the sequence as stipulated in the curriculum, and its textbooks contained almost eight tasks per exercise as in Figure 2. Whereas series 2 had less than 10 tasks in three exercises combining both grades (e.g., Figure 3). Finally, textbook series 1 and 3 were selected for analysis.

Exercise 1

Solve the following simultaneous equations, using substitution method:.

1. $x - 2y = 8$ and $2x + y = 21$
2. $x + 2y = 7$ and $2x + 3y = 11$
3. $x + 3y = 18$ and $x - 2y = 3$
4. $2x + 4y = 32$ and $2x - 3y = 11$
5. $a = 5b - 4$ and $a = 2b - 1$
6. $2x + y = 5$ and $x + 3y = 5$
7. $2m - n = 4$ and $2m + 3n = -4$
8. $x + 3y = 11$ and $5x + 4y = 22$

Figure 2*:* Exercise 1 illustrating few tasks per exercise for series 4 (Chikwakwa & Banda, 2015b, p. 47).

NB: The criterion used does not in any way refrain others from analyzing the left out textbooks since they were found to be dominant on the market. Yet, they fell short on some conditions that were put in this study.

b. $m + x = 3$
$m - x = 1$

c. $a - b = 8$
$a + 2b = 5$

d. $5x + y = 14$
$x + y = 6$

e. $5c - d = 3$
$3d - 8c = 5$

f. $3y - z = 11$
$2y - 3z = 5$

g. $x + y = 12$
$x - y = 2$

h. $3x + 6y = 48$
$5x - 6y = -32$

Note: Task ***"a"*** is not visible due to the layout of the textbook. However, it is similar to the tasks in the figure.

Figure 3: Exercise 7b illustrating few tasks per exercise for series 2 (Chitera, 2013b, p. 38)

Note: There was a case of a few tasks in three exercises of grade 10, series 1 textbook as in grade 10, series 2. In both textbooks, the tasks were falling under the same section of solving *Simultaneous linear equations by elimination* and solving *Simultaneous linear equations by graphical method* respectively. To rectify this, another condition for sampling was devised. I compared the variety of questions (tasks) in series 1 to those in series 2 under the same section. Series 1 included word problems and tasks that were not word problems, whereas series 2 only had tasks that were not word problems. This was the case in all sections that had less tasks. Therefore, series 1 was selected over series 2 with the impression that the variety of questions in series 1 would offer a diverse and rich sample for analysis.

The sampling criterion used is limited in the sense that it may have eliminated textbook series with relevant texts for the study. On the other hand, the criteria facilitated the effectiveness of the sampling procedure, which led to having a manageable and representative sample (Krippendorff,

2004) of textbook series and tasks for the study. Of course, representativeness is not an issue in qualitative research (Cohen et al., 2007).

The sampling led to the analysis of the following textbooks which henceforth will be referred to as books A, B, C and D. A summary of the sample is given in Table 1.

1. Achievers Junior Secondary Mathematics student’s Book 1 (Nyirenda et al., 2014a) (Textbook A).
2. Excel and Succeed Junior Secondary Mathematics Student’s Book (Thomo et al., 2015a) (Textbook B).
3. Achievers Junior Secondary Mathematics student’s Book 2 (Nyirenda et al., 2014b) (Textbook C).
4. Excel and Succeed Junior Secondary Mathematics Student’s Book 2 (Thomo et al., 2015b) (Textbook D).

Stage 2 purposive sampling – selecting topics.

This stage aimed at selecting topics for the study. The topics of Equations and Simultaneous equations were selected for the study for three reasons. First, problem solving is indicated as a suggested teaching, learning and assessment method for these topics in the curriculum (MoEST, 2013, pp. 18–20, 35–36). Secondly, the complexity of algebra in the equations could reinforce higher-order thinking. Algebra is also central in mathematics education (Huntley et al., 2007; Jäder et al., 2020). This meant studying a current issue with an impact on school mathematics. Lastly, equations are core topics in school mathematics (Andrews & Sayers, 2012; Huntley et al., 2007).

A summary of the sample used in the study has been given in Table 2.

Table 2: A summary of the sample in the study

Textbook	Series	Grade	Topic
Book A	1	9	Equations
Book B	3	9	Equations
Book C	1	10	Simultaneous Linear equations
Book D	3	10	Simultaneous Linear equations I Simultaneous Linear equations II

Note: Not all tasks under the topic of Equations were analyzed in Book B. See the Appendix for more details on tasks that were not analyzed.

3.4 Data collection

Data was collected from the topics of Equations and Simultaneous equations in grades 9 and 10 respectively. The data collection tool was the MDITx framework. The framework was used to collect qualitative data on the patterns of variations exhibited in the example spaces (examples and tasks). Its constructs under exemplification were used for coding examples and tasks depending on the patterns of variation they exhibited vis-à-vis descriptions of the variations of the MDITx. Data were collected from the two coding units, the examples and tasks as guided by the framework. This is one use of a theory, that is to provide "a lens that shapes what is looked at" (Creswell & Creswell, 2018, p. 90). It also involved calculating the frequencies of the patterns of variations. It means that the data was qualitative and descriptive.

3.5 Data Analysis

Data analysis involved choosing a suitable framework, modifying the framework, establishing the focus and direction of the analysis and coding examples and tasks. The subsection mentions other frameworks that were considered for the study but disregarded. It also explains the preference for the MDITx to guide the study. The names of the first three frameworks were made up by the researcher based on how they were used in their respective studies. The studies did not explicitly indicate the names of the frameworks as done here.

3.5.1 Analytical framework.

There are several frameworks that could be used to analyze textbooks for problem solving. For instance: (1) Task analysis framework (Van Zanten & Van den Heuvel-Panhuizen, 2018); (2) Task relatedness framework (Jäder et al., 2020); (3) Lithner's Imitative Reasoning and Creative Reasoning framework (Brehmer et al., 2015). These frameworks are suitable for analyzing tasks but have less emphasis on examples. The variations/features of examples and tasks on the presence on problem solving need to be emphasized and made explicit. For instance, features that make an example similar to a task make the task less of a problem. The Mathematics Discourse in Instruction analytic framework for textbook analysis (MDITx) (Ronda & Adler, 2017) is the framework that allowed such variations to be made explicit and enabled to establish the relationship between examples and tasks. That is why the MDITx framework was chosen to guide the analysis of the study.

The MDITx framework was designed to reveal the mathematical opportunities available in mathematics textbook lessons for students to learn (Ronda & Adler, 2017). This study investigated problem solving opportunities provided in grades 9 and 10 textbooks. It focused on exemplification which comprises examples and tasks. These reveal the intended and implicit objectives of the textbooks. An illustration of the analysis is given later in the section after modifying the framework.

3.5.2 Modifying the coding framework.

The constructs for analyzing examples in the MDITx are Generalization, Contrast and Fusion (Ronda & Adler, 2017). However, the study only used *Generalization* (G) and *Contrast* (C). *Fusion* was left out for two reasons. Firstly, there was no clear description of fusion in Ronda and Adler (2017). Therefore, its meaning was vague in the context of textbook analysis. Secondly, in more recent publications, Adler and colleagues have started to shift attention from fusion to juxtaposition (e.g., Adler & Pournara, 2020), and the framework thus appears to be in development with respect to this. Despite Ronda and Adler (2017) arguing that the topic of Inequalities they used allowed for the illustration of, "the full range of the indicators in the analytic tool" (p. 12), fusion was not illustrated. The current study assumed and concluded that G and C were sufficient to reveal the opportunities to learn problem solving. Moreover, the variation theory states that learning occurs when students see different patterns of variation (Marton & Pang, 2006). These were offered by G and C. Table 3 below shows the modified coding system as used in the study after withdrawing *Fusion* (F).

Table 3: The modified code system for analyzing textbook examples and tasks as used in the study (Descriptions adapted from Ronda and Adler, 2017, p. 1106)

Example	Task
Level 1 - At least one of the patterns of variations (C- Contrast or G- Generalization)	**Level 1** – Carry out known procedures or use known concepts related to the object of learning (KPF)
Level 2 - Both patterns of variations (C- Contrast and G-Generalization)	**Level 2** – Carry out procedures involving the object of learning (includes CTP but no AMC codes)
	Level 3 – Carry out Level 2 tasks plus that involve multiple concepts and connections (includes CTP and AMC codes)

On the one hand, withdrawing the *fusion* (F) construct might affect the interpretation of the results to a reader who is conversant with the framework and has used *fusion* before. On the other hand, it prevents the confusion that the analysis might have brought due to the vague description of the construct. However, the main purpose of examples in this study was to determine how related they are to their subsequent tasks. In other words, how the procedures they illustrated could influence the solution path of the tasks. Furthermore, the variations among/between examples were to determine if those variations would influence the opportunities to learn problem solving in tasks. With this focus and clarity on examples, the codes C and G were sufficient. This clarity hopes to rectify the occurrence of any misinterpretation of the results.

3.5.3 Establishing the focus and direction of the analysis.

First, the objectives of the topic were read to establish the object of learning. The objectives were then compared to the curriculum's objectives. Second, the topic was divided into blocks depending on the objectives. Each objective was a block. In each block there were/was an example space(s).

An example space includes examples and tasks (Ronda & Adler, 2017), which were coding units for the study. Example 7.3 and exercise 7.2 in Figure 4 illustrate an example space.

Within or outside an example space there were rules, procedures and guidelines meant to clarify the method(s) being taught. The researcher read and got familiarized with them. This familiarization of the objectives, rules, procedures and guidelines helped the researcher to establish the focus of the textbooks so that logical connections are made with and between the examples and tasks. Textbooks B and D did not indicate objectives at the beginning of a topic. However, this had no impact on the analysis because its content mirrored the objectives of the other series and the curriculum.

Example 7.3

Solve the simultaneous equations

$$\frac{x+y}{3} - \frac{x-y}{4} = \frac{2}{3} \quad \ldots\ldots\ldots\ldots(1)$$

$$\frac{2x-3}{3} - \frac{2y+3}{4} = \frac{{}^-19}{12} \quad \ldots\ldots\ldots\ldots(2)$$

Solution

Multiply (1) by 12 to remove the denominators.

$$4x + 4y - 3x + 3y = 8$$
$$x + 7y = 8 \ldots\ldots\ldots\ldots(3)$$

Multiply (2) by 12 to remove the denominators.

$$8x - 12 - 6y - 9 = {}^-19$$
$$8x - 6y = 2 \ldots\ldots\ldots\ldots(4)$$

Using (3), express x in terms of y:

$$x = 8 - 7y \ldots\ldots\ldots\ldots(5)$$

Substitute (5) in (4) to eliminate x:

$$\begin{aligned} 8(8-7y) - 6y &= 2 \\ 64 - 56y - 6y &= 2 \\ 64 - 62y &= 2 \\ 62y &= 62 \\ y &= 1 \end{aligned}$$

Substitute y = 1 in (5):

$$\begin{aligned} x &= 8 - 7(1) \\ &= 8 - 7 \\ &= 1 \end{aligned}$$

Substitute y = 1 and x = 1 in (1) or (2) to check that the solutions are correct.

Exercise 7.2

In Questions 1 to 4, find y in terms of x.

1. $4x - y = 12$ **2.** $2x + 5y = 10$

3. $\frac{1}{3}x - 4y = 16$ **4.** $3x - \frac{1}{4}y = 10$

In Questions 4 to 6, express x in terms of y.

5. $x - 5y = 3$ **6.** $9x + 4y = 0$

7. $\frac{1}{3}x = \frac{1}{6}(y-1)$ **8.** $\frac{1}{3}(x-2) - y = 2$

Use substitution method to solve the following pairs of simultaneous equations.

9. $a + b = 3$, $4a - 3b = 5$ **10.** $w - 2z = 5$, $2w + z = 5$

11. $x + y = 0$, $2y - 3x = 10$ **12.** $x = 5 - 2y$, $5x + 2y = 1$

13. $4x - 3y = 1$, $x - 4 = 2y$ **14.** $6a - b = {}^-1$, $4a + 2b = {}^-6$

15. $4m - n = {}^-3$, $8m + 3n = 4$ **16.** $5q + 2p = 10$, $3p + 7q = 29$

17. $\frac{1}{v} + \frac{1}{u} = \frac{1}{5}$, $\frac{1}{v} - \frac{1}{u} = \frac{2}{5}$ **18.** $2s - 4t = 8$, $3s - 2t = 8$

19. $2x - 4y + 10 = 0$, $3x + y - 6 = 0$ **20.** $\frac{1}{2}a - 2b = 5$, $\frac{1}{2}a + b = 1$

Figure 4*:* An example space in Book B (Thomo et al., 2015b, p. 52).

Defining examples and tasks

An example was defined as a solved problem that is labeled as *Example(s)* in the textbook. A task was defined as an unsolved problem that students are expected to do, and these were labeled as *Exercise* and came after examples. A single task was defined as each numbered or lettered question in the exercise section. Refer to Figure 4 for *examples* and *tasks* (e.g., Example 7.3 and Exercise 7.2 respectively). If questions were dependent or connected, they were treated as one task (e.g., Figure 5). Tasks (a), (b), (c) and (d) in the figure were all treated as one task.

5. Fig 9.2 represents the scale drawing of an athletics track. The dimensions are given in meters, and the enclosed area is made up of a rectangle and two semi-circles. The area is A m^2, and its perimeter is p m.

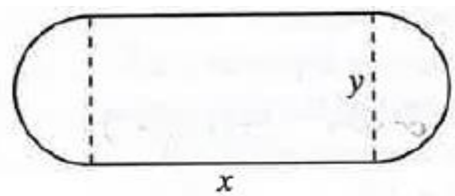


Fig. 9.2

Find:

(a) a formula for p in terms of x and y.
(b) a formula for A in terms of x and y.
(c) (i) y in terms of x and p.
(ii) x in terms of y and A.
(d) the value of (i) p and (ii) A given that $x = 100$ m and $y = 63.6$ m.

Figure 5*:* A task illustrating dependent questions in Book B (Thomo et al., 2015a, p. 118)

3.5.4 The coding processes

Coding examples

All examples were coded vis-à-vis the objective of the topic or lesson. The objective would either be on presentation, method or a type of reasoning relayed by examples. For a specified objective, if all the examples in the example space are similar, it was coded **G (Generalization).** If there were differences between/among the examples in an example space, it was coded **C (Contrasting).** Those with both similarities and differences were coded **CG.**

Two cases of **G** were considered special. First case, example spaces constituting only one example were automatically coded **G.** The assumption was that the example aimed to show that there is only one general way. Second case, example spaces with more than one example, but each presented and solved differently to exhibit distinct methods explained within the example space were coded **G**. This held if and only if there were no other similar examples in the same example space to compare to. Figure 6 illustrates such a case.

The two examples in Figure 6 emphasized the distinct general rules in Figure 7 for changing the subject of the formula. The interpretation was that each example in the figure was the only one in that example space (relating to special case 1) illustrating a single method of changing the subject of the formula in Figure 7.

Example 5

Given that $\frac{a(x+b)}{c} = 2$, find the expression for x.

Solution

$\frac{a(x+b)}{c} = 2$

$a(x+b) = 2c$

$ax + ab = 2c$

$ax = 2c - ab$

$x = \frac{2c}{a} - \frac{ab}{a}$

$x = \frac{2c}{a} - b$

Example 6

Given that $3p + q = s$, find p.

Solution

$3p + q = s$

subtract q from both sides

$3p = s - q$

Divide both sides by 3

$p = \frac{s-q}{3}$

Figure 6*:* A second special case of generalization in Book A (Nyirenda et al., 2014a, p. 124)

Summary

Take note of the following.

(a) Where the wanted letter is:

(i) multiplying, we divide by the unwanted letter of figure

$xy = 2ab$

- divide both sides by y

$x = \frac{2ab}{y}$

(ii) dividing, we multiply by the unwanted letter or figure e.g. $\frac{x}{2y} = 2a$

- multiply both sides by 2y

$x = 4ay$

(iii) adding, we subtract the unwanted letter or figure

e.g. $x + 2 = ab$

- subtract 2 from both sides

$x = ab - 2$

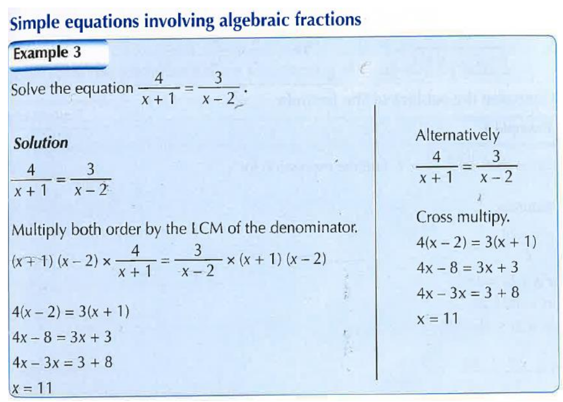
(iv) Subtracting, we add the unwanted letter of figure e.g. $x - y = 2a$

- add y to both sides

Figure 7*:* General rules for changing the subject of the formula in Book A (Nyirenda et al., 2014a, p. 125)

An example of coding examples

Figure 8 shows an example space with two examples whose objective is to illustrate the solving of fraction equations. In this case the object of learning is solving fraction equations using the LCM and cross-multiplication method. This example space was coded CG because students could see contrast between example 3 and example 4 in the simplification. In the first example uses the simple distribution law while the second one uses the FOIL (First-Outer-Inner-Last) method to distribute the expressions after cross multiplication. Second, the numerators of example 3 are monomials while of example 4 are binomials. Therefore, students are able to see at least the two contrasting features in solving the equation and the structures which are not the learning objectives. They could also see similarity in how both equations were solved since the method is the same.



Simple equations involving algebraic fractions

Example 3

Solve the equation $\frac{4}{x+1} = \frac{3}{x-2}$.

Solution

$\frac{4}{x+1} = \frac{3}{x-2}$

Multiply both order by the LCM of the denominator.

$(x+1)(x-2) \times \frac{4}{x+1} = \frac{3}{x-2} \times (x+1)(x-2)$

$4(x-2) = 3(x+1)$

$4x - 8 = 3x + 3$

$4x - 3x = 3 + 8$

$x = 11$

Alternatively

$\frac{4}{x+1} = \frac{3}{x-2}$

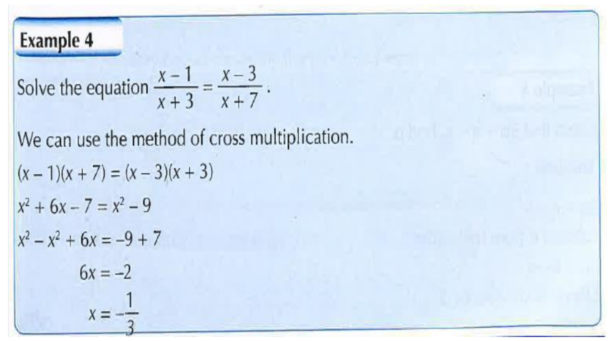
Cross multipy.

$4(x-2) = 3(x+1)$

$4x - 8 = 3x + 3$

$4x - 3x = 3 + 8$

$x = 11$



Example 4

Solve the equation $\frac{x-1}{x+3} = \frac{x-3}{x+7}$.

We can use the method of cross multiplication.

$(x-1)(x+7) = (x-3)(x+3)$

$x^2 + 6x - 7 = x^2 - 9$

$x^2 - x^2 + 6x = -9 + 7$

$6x = -2$

$x = -\frac{1}{3}$

Figure 8: Example coded CG in Book A (Nyirenda et al., 2014a, Book 1, p. 123)

Coding tasks

Tasks were coded as follows. A task demanding already known procedures or facts was coded **KPF** (level 1). This construct was viewed to be a case where the solution path is apparent to the student. In other words, the student would easily connect the task to a certain solution procedure that they already know without any struggles. Therefore, the task was categorized to be of less cognitive demand and qualified to be a level 1 task.

A task that demanded already known procedures and/or the current (new) procedures, was coded **KPF + CTP** (level 2). This included tasks that were either similar to the solved examples or could easily be solved by the rules given within the same section of the task. In addition, the current topic was viewed to be a new concept such the students have not fully assimilated, and it would be challenging to some to apply it. At some point, they might be required to modify the problem a bit so that it fits the current procedure. Therefore, it becomes more complex than KPF tasks. Specifically, for word problems, it included tasks that could easily follow the sequence of solving them as outlined in the rules, and connections are clear between and/or among the variables in a task.

Finally, a task demanding making connections, applications and the current procedures was coded **AMC** (level 3). For word problems, it also included tasks with obscure connections between and/or among variables, and do not follow the sequence as outlined in the rules. Tasks with multiple ways of solving were also coded level 3**.** Central in this construct is that a student has to make connections among concepts and come up with a solution procedure or select a proper procedure from a variety of possible procedures. Making it more complex than level 2 tasks.

All level 2 and 3 tasks were considered potential opportunities for learning problems with level 3 having the highest potential. Level 1 tasks were not considered as potential problem solving opportunities. Table 4 below indicates the conditions for coding tasks.

Table 4*:* Conditions for the coding system of tasks

CODE	INTERPRETATION IN THE STUDY
KPF (level 1)	1. A task whose procedure was already learned, and students are well conversant with it. 2. It is easy to recall, apply or locate the procedure in the book.
KPF + CTP (level 2)	1. A task whose method is the current one. 2. A task similar to the solved example(s) 3. Could use KPF methods but are not enough. **Note:** The current method is not fully assimilated hence the task is considered challenging as compared to level 1.
AMC (level 3)	1. A task with multiple connections 2. A task with multiple ways of solving or solutions. 3. Apply other concepts to solve the task. 4. Task not similar to solved examples

An example of coding tasks

Figure 10 shows Exercise 9.1 under forming and solving equations from word problems in grade 9. The exercise illustrated all the variations for coding tasks, which is why it was used here. Figure 9 shows the rules to be followed when working on Exercise 9.1

In Figure 10, task 5 exemplifies a level 1 task because the wording in this task was similar to tasks done in chapter 3 in the same book such that students could easily recall and come up with an equation. In addition, they could use knowledge from lines and angles in chapter 4 to solve the formed equation. This is a simple task that could be solved without applying the new procedures. At this school level, students already know symbol manipulation (Huntley et al., 2007).

In Figure 10, tasks 3 and 4 illustrate level 2 tasks. These tasks could easily be solved by implementing the rules outlined in Figure 9. For instance, task 4 demands, first, representing Jane's age **(x)** algebraically. Then subtract 2 from **x** to have Mike's age as **(x – 2),** followed by adding the two expressions and equating them to 28. Then solve them as illustrated in the solved example. Students could also use prior knowledge to solve these tasks. For instance, symbol manipulation from lines and angles like in level 1. Furthermore, the tasks have clear and straightforward connections between the variables involved.

An instance of a level 3 task is task 1. Firstly, this task is not similar to any of the examples. Secondly, the task demands more reasoning than the examples. It is not straightforward as it seems. Thirdly, the task has multiple ways of connecting the expressions to get to the solution. Three ways are: *(1)* $3(x + 7) - 5x = 6$; *(2)* $3(x + 7) = 5x + 6$ *and (3)* $3(x + 7) - 6 = 5x$. This allows students to appreciate different connections and the reasoning behind the task. Other tasks that were coded level 3 required the application of other concepts. For instance, task 8 needed the knowledge of perimeter while tasks 11 and 13 required knowledge of linear motion from physics.

Linear equations formation from word problems

Let us consider certain word problems, which can be expressed in terms of equations. Take the following steps to translate a statement in words into an equation.

(i) Read the problem carefully and decide what you are expected to find.

(ii) Represent the unknown number or numbers algebrically by a letter or letters respectively.

(iii) If you are required to find only one number, represent this unknown by a letter.

(iv) Read the problem and find the conditions under which the two expressions are equal. Write down the equation and solve for the unknown.

Figure 9: Rules for formulating linear equations from word problems in Book A (Nyirenda et al., 2014a, p. 121)

Exercise 9.1

1. Three times $(x + 7)$ exceeds $5x$ by 6. Find x.
2. The perimeter of a square whose side is 52 m. Find the area of the square.
3. One number is 6 times another number with their sum being 49. Find the two numbers.
4. Mike is 2 years younger than Jane and the sum of their ages is 28. What is Jane's age?
5. Find two consecutive integers whose sum is 55.
6. The three angles of a triangle are $3x°$, $(4x - 12)°$ and $(2x - 15)°$. Find the value of x and hence the size of the angles.
7. The perimeter of a rectangle is x cm. It is $\frac{3}{8}$ cm long. Find its width in terms of x.
8. In triangle ABC, what fraction of the perimeter of the triangle is the shortest side? If the perimeter of the triangle is 65 cm, find the length of the shortest side.

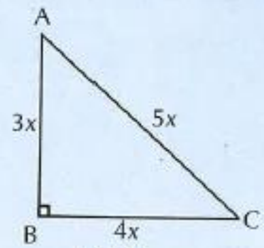


Fig 9.1

9. A school contains 585 students. If there are 29 more girls than boys, find the number of boys.
10. Find two consecutive integers such that one fifth of the larger exceeds one-sixth of the smaller by 3.
11. One man cycles at x km/h and another at $(x - 2)$ km/h. They both start at 2 p.m. from places 51 km apart and ride towards one another and meet at 3.30 p.m., find x.
12. At a picnic party there were 10 more girls than boys. The boys paid Mwk. 200 each and the girls Mwk. 150 for the total cost of Mwk. 8 500. How many boys and girls were there at the party?
13. A man drives $3x$ km at 90 km/h and $8x$ km at 60 km/h and takes 40 minutes to complete the journey. Find x.

Figure 10*:* Exercise 9.1 Illustrating coding of tasks in Book A (Nyirenda et al., 2014a, p. 122)

3.6 Validity and reliability

Two kinds of validity were considered in the study. First, theoretical validity, which requires giving a clear description of the pre-defined theoretical construct (Cohen et al., 2007). In this case, the theoretical constructs are those of the MDITx. The MDITx was used to analyze examples and tasks. This was impossible with other frameworks (see the analysis for the constructs). Furthermore, its constructs and their descriptions clear-cut the characteristics of level 1, 2 and 3 examples and tasks. This characteristic of MDITx facilitated the determination of potential opportunities for problem solving with level 3 as the preferred potential. Therefore, with these attributes of the MDITx, it was considered an appropriate instrument for collecting and analyzing the data required for the study (Cohen et al., 2007).

The second type of validity was content validity (Cohen et al., 2007). Content validity is concerned with the representativeness of the content. To ensure this type, the chosen textbook series for the study were sampled from those that were proven to be in use in Malawian secondary schools (see sampling section). Secondly, it was confirmed that the content in all four textbook series was similar despite varied arrangement and quantity. In essence, the studied textbooks were a good representation of the content that is being learned in Malawian grades 9 and 10 mathematics. Lastly, examples and tasks were analyzed in the study. These are elements that were meant to be studied (Cohen et al., 2007).

Reliability was ensured through an explicit explanation of the methodology. Justification for the selection of the design and how it fitted the intent of the study has been provided. In addition, a step-by-step sampling procedure has been clarified and reasons have been given for every decision involved in sampling. In addition, relevant information concerning the textbook series is provided for verification if need be. Both the strengths and the weaknesses of the sampling technique have been highlighted including the assumptions that the researcher held when applying the technique. Furthermore, an illustration of the analysis has been provided substantiated with figures from the original copies of the analyzed textbooks. In addition, the conditions under which the coding of word problems is done are explicit. On the framework, it has been explained how and why it was modified for the study. Under similar conditions, the study could be replicated (Cohen et al., 2007).

In addition, to ensure the stability of the codes (Cohen et al., 2007), examples and tasks were constantly revisited after the first coding. Tasks and examples with uncertain codes were discussed with the supervisor and other mathematics teachers to ascertain the complexities of the tasks and examples and check if they matched the assigned code(s). This should not be interpreted as inter-rater reliability (Cohen et al., 2007) or intercoder agreement (Creswell & Creswell, 2018). Rather it is a confirmation of codes on selected tasks. The mathematics teachers included two Master's students in mathematical sciences and one Master's student in mathematics education. Two had at least 3 years of teaching experience while one had a year of teaching experience and was a student assistant in the mathematics and physics department at the time of this study.

CHAPTER 4: FINDINGS

This chapter presents the findings of the four textbooks that were analyzed. It focuses on the distribution of the tasks that are considered as problem-solving opportunities and the factors that influence the availability of problem-solving opportunities. It is organized as follows. It begins with an introduction, followed by the general results and then the factors influencing problem-solving. Lastly, outliers and the summary of the chapter are presented.

4.1 Introduction

The Malawian mathematics curriculum emphasizes learning for and through problem solving. It also indicates problem solving as a suggested teaching, learning and assessment method (MoEST, 2013). The study investigated the opportunities to learn problem solving provided in Malawian grades 9 and 10 mathematics textbooks. The MDITx framework by Ronda and Adler (2017) was used to reveal the opportunities to learn problem solving afforded by the textbooks.

4.1.1 Focus of the findings

The findings focus on the qualitative aspects that led to the distributions presented in the tables. The distribution helped to establish if the pattern of variations is similar across the textbooks. In the examples, the variations were meant to establish if the examples are different or similar to determine if these variations might affect the availability of problem-solving opportunities in subsequent tasks.

The tasks were analyzed vis-à-vis the features in the examples to establish the similarities and differences between the tasks and the examples. The focus here is also on the features that led to the distribution of the levels in the tasks.

4. 2 General findings

The general findings dwell on the presence of problem-solving opportunities according to the levels of the MDITx in response to the main research objective. The main objective was: *Investigating the opportunities to learn problem solving in Malawian grade lower secondary mathematics textbooks*. The results aim at answering the questions of *(1) How are problem-solving opportunities provided in Malawian grades 9 and 10 mathematics textbooks?* and *(2) What factors influence the opportunities to learn problem solving in Malawian grades 9 and 10 mathematics* *textbooks?* The results also include other textbook characteristics that influence the problem-solving opportunities.

Generally, the results show that textbooks offer the opportunities for students to learn problem solving mainly through word problems. However, many tasks are similar to examples, and there are some features in the examples and textbooks that influence problem solving opportunities. The features include: detailed examples, leading questions and lack of prompting questions. Lastly, not introducing lessons as challenges. It was also found that textbooks do not test higher-order concepts even though acknowledged. This limits the opportunities to learn problem solving. The following sections expound on these points.

4.2.1 Findings from Examples

Table 5 shows that 24 example spaces which comprised 53 examples were analyzed. Out of the 24, 12 example spaces exhibited generalization, while six exhibited contrast and the other six showed both contrast and generalization. The levels show that there were 18 level 1 example spaces (combining G and C) and six level 2 example spaces (CG). The levels suggest that 18 example spaces exhibited one variation while six example spaces exhibited two variations. However, the focus of the study in these examples was on the features in these examples which were compared to the tasks to determine the levels of the tasks. The tasks then determined if problem solving was available in the textbooks.

Table 5: Distribution of variations in examples from all textbooks

Book	G	C	CG	Total E. S	Examples
A	2	1	1	4	7
B	3	3	2	8	21
C	2	1	1	4	10
D	5	1	2	8	15
Total	**12**	**6**	**6**	**24**	**53**

Key*: G – Generalization, C – Contrast, E.S – Example spaces, L-Level*

To establish if there is an association between the textbooks and the observed patterns, a Fisher exact test was performed in R using the data in Table 5, yielding a p-value of 0.96. This indicates that there is no association between the textbooks and the observed patterns in solved examples.

4.2.2 Findings from Tasks

Table 6 shows that 309 tasks were analyzed. There were 33 level 1 tasks. These could be solved by known procedures or facts while 230 tasks were level 2 tasks that required the application of the current topic procedures. Lastly, 41 were level 3 tasks that required making connections and application of other concepts and related to real life. The results indicate that there are more level 2 tasks than level 1 and 3 tasks. In general, out of the 309 tasks, 74% of the tasks were similar to the solved examples and used the solved examples as templates to be solved while 13% of the tasks were challenging such that they required making connections and applying other concepts. Lastly, 11% of tasks were simple and could be solved using already-known procedures that were easily accessible. Figure 11 visualizes this distribution.

Many of the level 3 tasks in all textbooks were found in word problems that involved generating equations from word problems and were located at the end of the topic. Out of the 41 level 3 problems in Table 6, only three were not word problems. These three problems have been highlighted later.

Table 6: Distribution of levels in tasks from all textbooks

Book	L1	L2	L3	OL	TOTAL
A	9	26	9	1	**45**
B	9	103	18	3	**133**
C	1	29	3	0	**33**
D	14	72	11	1	**98**
Total	**33**	**230**	**41**	**5**	**309**

Key: *L- Level, OL- Outlier*

To establish if there is an association between the textbooks and the observed patterns in tasks, a Fisher exact test was run in R using the data in Table 6. A p-value of 0.04 was found, which meant there was an association between the textbooks and the observed patterns.

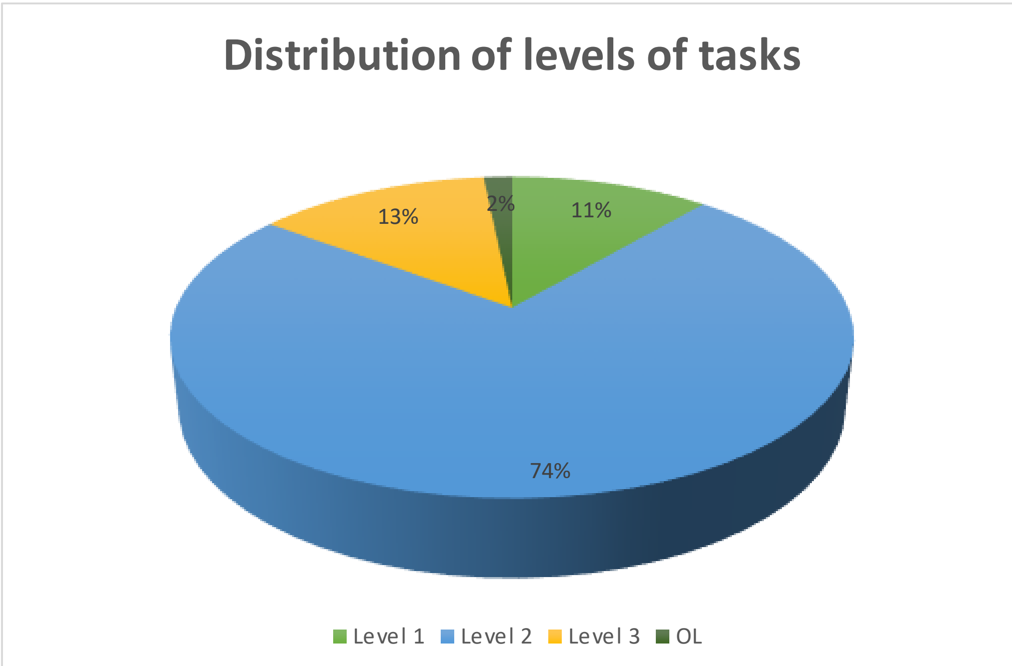


Figure 11: The distribution of levels of tasks

4.2.3 Findings as responses to the curriculum

First, all textbooks contained the objectives as stipulated in the curriculum. In textbooks A and C, all objectives were stated at the beginning of the chapter while in textbooks B and D they were stated as lesson topics. In other words, they were stated separately. All textbooks could contain a section of detailed explanations of procedures related to the objective. Yet, they were minimal in A and C.

Second, the curriculum indicated problem solving as a teaching method. However, the textbooks presented the examples and tasks in a rote-memorization way with no challenging activity attached. Furthermore, not a single instruction or indicator was given for a teacher or a student to independently come up with a problem-solving activity. Neither did any of the books propose challenges for students to accomplish. The only method portrayed in the textbooks was *written exercises.*

Lastly, the textbooks presented all the success criteria as stipulated in the curriculum concerning Equations and Simultaneous equations. These have been outlined in the subsequent analysis of individual grades.

4.3 Analysis of Grade 9 textbooks

This section presents results from textbooks A and B. According to MoEST (2013), students in grade 9 are supposed to learn and be assessed (objects of learning) in the following: (1) *solving linear equations;(2) solving linear equations involving simple fractions;(3) formulating linear equations from word problems;(4) identifying the subject of the formula* and *(4) changing the subject of the formula.* All the textbooks reflected these assessment activities. Textbook A incorporated objective *(1)* in *(2)* and *(3).* Textbook B clearly stated the *cover-up technique* and *balance method* as methods of *solving linear equations* (Thomo et al., 2015a). Nevertheless, both textbooks fulfilled the objectives for grade 9.

Table 7: Distribution of levels from Grade 9 tasks

Level	Number of tasks
L1	18
L2	129
L3	27
OL	4
Total	**178**

Key: *OL – Outliers,* L-Level.

Table 7 shows the distribution of levels of tasks for the two grade 9 textbooks. Out of the 178 analyzed tasks, 129 of the tasks were level 2 while 27 were level 3 tasks. Only 18 of the tasks belonged to level 1. The results suggest that 73% of the tasks are either similar to the solved examples or require the current procedures to be solved. On the other hand, 15% of the tasks are challenging and require application and making connections. Lastly, only 10% of the tasks were simple – in the sense that they could be solved using already known procedures. A visual representation of the distribution is shown in Figure 12.

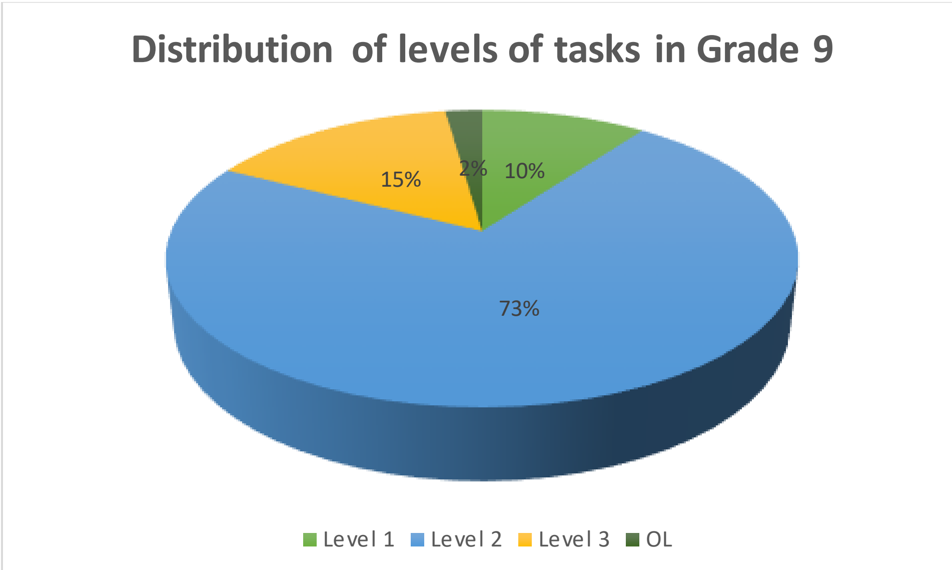


Figure 12*:* Distribution of levels of tasks in Grade 9 textbooks

To exemplify this distribution is Figure 13 and Figure 14. In Figure 13 the objective is to solve equations. On the left side are examples that are given just before the tasks. On the right side are tasks that follow directly after the examples.

Example 9.5 *Solve* $\frac{2}{5}x = 10$ ***Solution*** $\frac{2}{5}x = 10$ *Multiplying both sides by 5:* $\frac{2x \times 5}{5} = 10 \times 5$ $2x = 50$ *Dividing both sides by 2:* $\frac{2x}{2} = \frac{50}{2}$ $\therefore\ x = 25$	**2.** State whether the following are true or false. (a) $\frac{x}{4} = 5$ when $x = 24$ (b) $x - 2 = 9$ when $x = 7$ (c) $20 + x = 28$ when $x = 8$ (d) $4x = 20$ when $x = {}^{-}5$ (e) $12 - 3x = 0$ when $x = 4$ (f) $8 = 9 - x$ when $x = 1$.

Thus, if the two sides balance, they will still do so if **what is done on one side is also done on the other.**

Example 9.3

Solve the equation $8x - 6 = 5x + 9$.

Solution

Imagine a pair of scales with $8x - 6$ on one side balanced by $5x + 9$ on the other (Fig. 9.1 (a)).

$8x - 6 = 5x + 9$

Add 6 to both sides: $8x - 6 + 6 = 5x + 9 + 6$

$$8x = 5x + 15$$

Subtract $5x$ from both sides:

$$8x - 5x = 5x + 15 - 5x$$

$$3x = 15$$

(see Fig. 9.1 (b) and (c))

Divide both sides by 3:

$$\frac{3x}{3} = \frac{15}{3}$$

i.e. $x = 5$ (Fig. 9.1 (d))

Exercise 9.3

Solve the following equations using the balance method and stating the steps as in Examples 9.4 and 9.5.

1. (a) $x + 113 = 153$ (b) $24 = x + 13$
(c) $x + 8 = {}^{-}12$ (d) $x + {}^{-}7 = {}^{-}19$

2. (a) $x - 17 = 37$ (b) $11 = x - 7$
(c) $x - 2 = {}^{-}4$ (d) $x - {}^{-}2 = {}^{-}7$

3. (a) $x - 72 = {}^{-}24$ (b) ${}^{-}x + 72 = {}^{-}30$
(c) $4 = x - 5$ (d) $15 + 2x = {}^{-}3x$

4. (a) $24x = 72$ (b) $+10 = {}^{-}5x$
(c) ${}^{-}25x = 200$ (d) ${}^{-}7 = {}^{-}84x$

5. (a) $6x - 5 = 25$ (b) $9x + 8 = 35$
(c) ${}^{-}5x + 5 = {}^{-}15$ (d) ${}^{-}7x - 6 = {}^{-}20$

Figure 13: Illustrating the distribution of levels in grade 9, Book B (Thomo et al., 2015a, pp. 110 –111)

Task 2 on the top right under Exercise 9.2 was coded as level 1 because it did not require the methods illustrated in the examples. It could be solved by simple substitution which was illustrated earlier in another topic in the book. Therefore, students could use KPF. Tasks under exercise 9.3 were coded level 2 because they were similar to the solved examples. Moreover, the question indicated that they could be solved as illustrated in Examples 9.4 and 9.5. Which means the tasks and the examples were similar. No level 3 tasks were found in this section. Out of the 26 tasks in the figure, six belonged to level 1 while 20 belonged to level 2. Similar results were found in Book A under the same topic in which all tasks were coded level 2 because they were similar to the solved examples.

Figure 14 exemplifies level 3 tasks that were found in word problems. In Figure 14, the tasks are explicitly introduced as problems. The impression that this introduction gives is that the section contains challenging problems. It was not surprising to find problem solving tasks in the word problems sections in all textbooks.

Exercise 9.7

Find an answer to each of the following problems by forming an equation and solving it.

1. When I double a number and add 17, the result is 59. What is the number?
2. When a number is added to 4 times of itself, the result is 30. Find the number.
3. The difference of two numbers is 5 and their sum is 19. Find the two numbers.
4. Mr Ali has 7 marbles less than Mohammed and they have 29 between them. How many does each boy have?
5. When a number is doubled and 4 added, the result is the same as when it is trippled and 9 subtracted. Find the number.

11. The result of adding one third of a number to itself is 28. What is the number?
12. From a certain number, subtract 3, multiply the result by 5, and then add 9. If the final result is 54, find the number.
13. Find two consecutive even numbers such that the sum of 3 times the smaller and 5 times the larger is 106.
14. Two tanks contain diesel. The first tank contains 5 times as much as the second. When 20 litres of diesel are allowed to flow from the first tank into the second, the first contains 3 times as much as the second. What were the original contents of the tanks?
15. A woman earns three times as much as her husband earns. After spending three fifths of their combined income, the couple have K 20 000 left. How much does the husband earn?

Figure 14: Illustrating Level 3 tasks in grade 9 word problems in Book B (Thomo et al., 2015a, pp. 115–116)

Figure 14 shows word problems in grade 9 that require students to generate equations from a given text and solve them. There were 20 word problems of which 11 were coded level 3 and nine were level 2. No level 1 tasks were found. Tasks whose connections were easy for instance, tasks 1–3, 5, 11, and 12 were coded level 2 because the presentation is straightforward such that students could easily decipher the relationship among variables. Tasks that had no straightforward presentation, complex language and required more thinking like tasks 4, 13–15 were coded level 3.

Book A had similar tasks since they are of the same grade and were coded similarly. It contained 15 word problems of which eight were coded level 3 while 7 were coded level 2. Only one level 3 task which was not a word problem was found in grade 9. See Figure 22, task 22 from Book B and the explanation is given.

4.4 Analysis of Grade 10 textbooks

This section presents the results from grade 10 textbooks. It combines results from textbooks C and D. The topic that was analyzed in this grade is Simultaneous linear equations. MoEST (2013) outlined the following as assessment areas (objects of learning) in this topic. *(1) Solving* *simultaneous equations by substitution; (2) Solving simultaneous equations by elimination; (3) formulating simultaneous equations from practical problems; (4) solving simultaneous equations; (5) completing tables of values; (6) drawing graphs* and *(7) solving simultaneous linear equations graphically.* All textbooks demonstrated these assessment areas.

Table 8: Distribution of levels from grade 10 tasks.

Levels	Number of tasks
L1	15
L2	101
L3	14
OL	1
Total	**131**

Key*:* *OL – Outlier, L-Level*

Table 8 shows the distribution of levels from grade 10 tasks. It shows that 131 tasks were analyzed. Out of the 131 tasks, 101 were coded level 2 while 14 were coded level 3. In addition, 15 tasks were coded level 1. The results translate into that 77% of the tasks were similar to examples and could be solved using the current topic procedures, while 11% of the tasks require an application and making connections. Lastly, another 11% of the tasks require already known procedures. Figure 15 is a pie chart to visualize the distribution.

Since the simultaneous equations in the Malawian textbooks are designed to be taught in three different lessons like the methods; substitution, elimination and graphical methods, the explanation of the distribution of the levels is presented in these categories.

It is worth noting that the objective *formulating simultaneous equations from practical problems* relates to the expectation by the curriculum that students should use "problem-solving techniques to solve practical problems" (MoEST, 2013, p. x). Suggesting that in word problems from which the equation are formulated ought to contain problem-solving tasks.

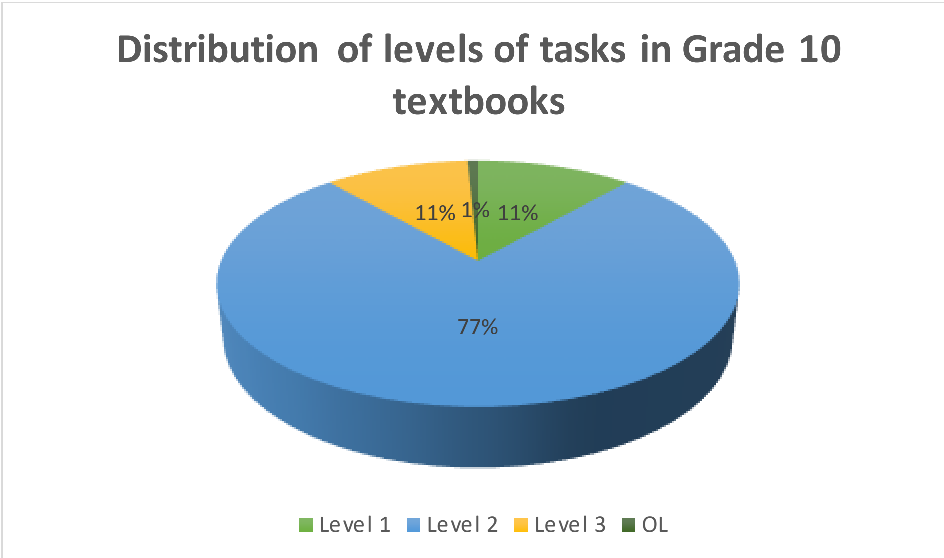


Figure 15: Distribution of levels of tasks in Grade 10 textbooks

Analysis of the substitution method

To exemplify the distribution in the substitution method is Figure 16. In the figure is an example space from Book D. There is one solved example on the left and 22 tasks on the right and below that follow just after the example. Out of the 22 tasks, eight (1–8) were coded level 1 because they require the knowledge of changing the subject of formula from grade 9. These tasks came before the simultaneous equations. They were more of a recap on how to express one letter in terms of another to help the students understand the concept behind the substitution method.

There were 12 level 2 tasks: tasks 9–16 and 18–21 in Figure 16.These tasks are similar to the examples. For instance, tasks 9–15 contained a variable with a coefficient of 1 as in the example, and students could easily make that variable the subject. For tasks like 18–21 , they can use the LCM to simplify the fractions into whole equations (Johari & Shahrill, 2020) as done in the examples. Moreover, students at this level have simplified fraction equations using the LCM in grade 9.

There were only two level 3 tasks under the substitution method. Tasks 17 and 22 were coded level 3 because they could involve having a double fraction. In task 17 students might have this kind of equation $\frac{1}{\frac{5u}{u-5}} - \frac{1}{u} = \frac{2}{5}$ if they make either v or u the subject. The same applies to task 22 if students decide to make *b* in equation 2 the subject where they will have b = $\frac{a+3}{2}$. Johari and Shahrill (2020) call this complicating the subject.

Example 7.3

Solve the simultaneous equations

$$\frac{x+y}{3} - \frac{x-y}{4} = \frac{2}{3} \quad \ldots\ldots\ldots\ldots(1)$$

$$\frac{2x-3}{3} - \frac{2y+3}{4} = \frac{^{-}19}{12} \quad \ldots\ldots\ldots\ldots(2)$$

Solution

Multiply (1) by 12 to remove the denominators.

$4x + 4y - 3x + 3y = 8$

$x + 7y = 8 \ldots\ldots\ldots\ldots(3)$

Multiply (2) by 12 to remove the denominators.

$8x - 12 - 6y - 9 = {}^{-}19$

$8x - 6y = 2 \ldots\ldots\ldots\ldots(4)$

Using (3), express x in terms of y:

$x = 8 - 7y \ldots\ldots\ldots\ldots(5)$

Substitute (5) in (4) to eliminate x:

$$\begin{aligned} 8(8-7y) - 6y &= 2 \\ 64 - 56y - 6y &= 2 \\ 64 - 62y &= 2 \\ 62y &= 62 \\ y &= 1 \end{aligned}$$

Substitute y = 1 in (5):

$$\begin{aligned} x &= 8 - 7(1) \\ &= 8 - 7 \\ &= 1 \end{aligned}$$

Use substitution method to solve the following pairs of simultaneous equations.

9. $a + b = 3$, $4a - 3b = 5$

10. $w - 2z = 5$, $2w + z = 5$

11. $x + y = 0$, $2y - 3x = 10$

12. $x = 5 - 2y$, $5x + 2y = 1$

13. $4x - 3y = 1$, $x - 4 = 2y$

14. $6a - b = {}^{-}1$, $4a + 2b = {}^{-}6$

15. $4m - n = {}^{-}3$, $8m + 3n = 4$

16. $5q + 2p = 10$, $3p + 7q = 29$

17. $\frac{1}{v} + \frac{1}{u} = \frac{1}{5}$, $\frac{1}{v} - \frac{1}{u} = \frac{2}{5}$

18. $2s - 4t = 8$, $3s - 2t = 8$

19. $2x - 4y + 10 = 0$, $3x + y - 6 = 0$

20. $\frac{1}{2}a - 2b = 5$, $\frac{1}{2}a + b = 1$

21. $\frac{2y}{5} + \frac{z}{3} = 2\frac{2}{3}$, $y = 2(z + 1)$

22. $\frac{a-1}{2} + \frac{b+1}{5} = \frac{1}{5}$, $\frac{a+b}{3} = b - 1$

Exercise 7.2

In Questions 1 to 4, find y in terms of x.

1. $4x - y = 12$ **2.** $2x + 5y = 10$

3. $\frac{1}{3}x - 4y = 16$ **4.** $3x - \frac{1}{4}y = 10$

In Questions 4 to 6, express x in terms of y.

5. $x - 5y = 3$ **6.** $9x + 4y = 0$

7. $\frac{1}{3}x = \frac{1}{6}(y - 1)$ **8.** $\frac{1}{3}(x - 2) - y = 2$

Figure 16: Illustrating the distribution of levels in substitution method, Book D (Thomo et al., 2015b, p. 52)

The results show that the level 1 tasks under the substitution method are not simultaneous. Suggesting that there are no level 1 tasks in the substitution method of simultaneous equations. The observations were similar in Book C. Only that there were no tasks that involved double fractions in Book C and simply changing the subject of the formula. Hence no level 3 and level 1 tasks were found under the substitution method in Book C.

Analysis of the elimination method

Figure 17 below illustrates an example space under the elimination method of simultaneous equations from Book C. Similar examples and tasks were found in Book D. In the figure, there are three examples, ***a, b,*** and ***c,*** and exercise 7.2 on the bottom right. Exercise 7.2 was to be solved using the elimination method only. Tasks 1– 6 under the Exercise were coded level 2 because they were similar to the examples. For example, tasks 1 and 4 are similar to example ***a*** because their pair of equations share a variable with the same coefficient of 1. While tasks 2–3 and 5–6 were similar to example ***b*** because they also require the student to make a common variable in the pair of equations to have the same coefficient before eliminating. Tasks 7a and 7b were coded level 1 because they require students to substitute the values into the equations and solve the equations as linear equations from grade 9. At this level, students have learned the concept of x-y solutions from linear graphs in grade 9 so it could be easy for them to figure it out. For instance, that 1 is for x and -1 for y in task 7a.

The results indicate that no level 3 tasks were available under the elimination method of simultaneous equations. The same was observed in Book D. Furthermore, no level 1 task was found in Book D because all tasks were similar to tasks 1–6 which are level 2 tasks.

Example 2

Solve the following simultaneous equations

(a) $\begin{cases} x + y = 1 \\ x + 2y = 2 \end{cases}$ (b) $\begin{cases} 2x + y = 1 \\ x + 2y = 2 \end{cases}$ (c) $\begin{cases} \frac{2}{3}x - \frac{y}{2} = 0 \\ \frac{x}{3} - \frac{3}{2}y = 0 \end{cases}$

Solution

(a) $x + y = 1$.... *equation (i)*

$x + 2y = 2$ .... *equation (ii)*

Let x be eliminated. Since its coefficients are equal and have the same sign, subtract equation (ii) from equation (i).

$x + y = 1$

$x + 2y = 2$

$-y = -1$

$-(-y) = -(-1)$ (multiply each side by –ve sign)

$\therefore\ y = 1$

To find the value of x, eliminate y by simply replacing it with its numerical value, as obtained above.

$x + 1 = 1$ or $x + 2(1) = 2$

$x = 1 - 1$ $x + 2 = 2$

$= 0$ $x = 2 - 2$

Hence $x = 0$, $y = 1$.

(c) $\frac{2}{3}x - \frac{4y}{62} = 0$.... *equation (i)*

$\frac{x}{3} - \frac{3}{2}y = 1$ .... *equation (ii)*

Remove the fractions by multiplying them with the LCM of 3 and 2 in equation (i) on each side, and the LCM of 2 in equation (ii) on each side.

$6\left(\frac{2}{3}x - \frac{y}{2}\right) = 0$ .... *equation (i)*

$6\left(\frac{x}{3} - \frac{3}{2}y\right) = 1 \times 6$... *equation (ii)*

$4x - 3y = 0$ ... *equation (iii)*

$2x - 9y = 6$... equation (iv)

Multiply equation (iii) by 3 and equation (iv) by 1, using the rule, "Same Signs Subtract":

(b) $2x - y = 1$... *equation (i)* $x + 2y = 2$... *equation (ii)* First, ensure the coefficients of y in both equations are equal. To do so, multiply equation (i) by 2 on each side and equation (ii) by 1 on each side. $2 \ (2x - y) = 2 \ (1)$ .... *equation (i)* $1 \ (x + 2y) = 1 \ (2)$ .... *equation (ii)* $4x - 2y = 2$ ....*equation (iii)* $x + 2y = 2$ .... *equation (iv)* Using the rule 'Different Signs Add', add equation (iii) to equation (iv). $\begin{array}{r} 4x - 2y = 2 \\ +\ \ x + 2y = 2 \\ \hline 5x \quad\ = 4 \end{array}$ $\frac{5x}{5} = \frac{4}{5}$ (divide each side by 5) $\therefore\ x = \frac{4}{5}$ To find the value of y, replace x in equations (i) or (ii) with its numerical value. $(2 \times \frac{4}{5}) - y = 1 \quad \Rightarrow \quad -y = 1 - \frac{8}{5}$ $-y = \frac{5-8}{5} = \frac{-3}{5} \qquad y = \frac{3}{5}\ \therefore\ x = \frac{4}{5},\ y = \frac{3}{5}$	**Exercise 7.2** Solve the following simultaneous equations by elimination method. 1. $\begin{cases} x + y = 2 \\ x + y = 0 \end{cases}$ 2. $\begin{cases} 2a + b = 3 \\ a - 2b = -1 \end{cases}$ 3. $\begin{cases} 3e + 2f = 4 \\ e + f = 1 \end{cases}$ 4. $\begin{cases} m = n - 2 \\ 3n + m = 7 \end{cases}$ 5. $\begin{cases} 3r + q - 9 = 0 \\ 2r + 3q - 13 = 0 \end{cases}$ 6. $\begin{cases} 3y - x = 8 \\ 2y + 3x = 10 \end{cases}$ 7. (a) Consider the equation $5x - y = c$. Find the value of c if the solution set is (1, −1). (b) Find the value of m and n if the solution set for the following simultaneous equation is (8, − 4). $\begin{cases} mx - y = 10 \\ nx + y = 2 \end{cases}$

Figure 17: Illustrating the distribution of levels in the elimination method (Nyirenda et al., 2014b, pp. 75–77).

Analysis of the graphical method

The graphical method was in two parts. The first part involved plotting two graphs on the same axis. This part was found in Book D only. Tasks under this part were coded as level 1 because students already know how to plot graphs. That is why this part was not available in Book C. It can be assumed that the author expected students to know about plotting graphs at this level. To demonstrate this part is Figure 18, Exercise 8.1 from Book D which require students to draw the graphs of the pair of the equations on the same axis.

Exercise 8.1

In each of the following cases draw the graphs of the given equations using the same axis.

1. $2x + 5y = 0$, $3x - 2y = 0$
2. $3x - 7 = 4y$, $6x + 2y = 5$
3. $y = 3x + 6$, $7 - 2x = 4y$
4. $4x + y - 8 + 0$, $2y + 7 = 4$
5. $2x + 3y = 0$, $4x + 6y = 0$
6. $2x + 3y = 1$, $2x + 3y = 4$

Figure 18: Illustrating a level 1 task under the graphical method (Thomo et al., 2015b, p. 59)

The second part involved solving simultaneous equations using the graphical method. All 16 tasks in Figure 19 in this part were coded level 2 because, they all involved plotting the graphs on the same axis and locating the coordinates where the graphs intersect as solutions to the pair of equations. The same was found in Book C. To exemplify this is Figure 19.

Example 8.2

Solve graphically the simultaneous equations

$$x - 2y = {}^{-}1$$
$$2x - y = 4$$

Solution

Step 1:

Make a table of values for each equation. Three pairs of values are sufficient for each (Table 8.5 (a) and (b)).

(a) $x - 2y = {}^{-}1$ *(b) $2x - y = 4$*

(a)

x	0	1	3
y	$\frac{1}{2}$	1	2

(b)

x	0	1	2
y	$^{-}4$	$^{-}2$	0

Table 8.5

Step 2:

Choose a suitable scale and plot the points. Draw the lines. Extend if necessary, so that they intersect. Fig. 8.5 shows the graphs of the two lines.

Exercise 8.2

Solve graphically the following pairs of simultaneous equations.

1. $x + y = 3$; $y = 3x - 1$ CTP
2. $2x = 3 + y$; $7x + 2y = 16$ CTP
3. $2x - y = 3$; $2y = -x + 14$
4. $3x + 2y = 0$; $5x + y = 7$
5. $y + 1 = 2x$; $2y + x + 7 = 0$
6. $3y - x = 4$; $2x - 5y = {}^{-}7$
7. $3x + 4y = 3.5$; $7x - 6y = 0.5$
8. $2y + 3x = {}^{-}5$; $3y + 2 = x$
9. $4x - y = 2$; $6x + 4y = 25$
10. $4x - 2y = 4$; $2x - 3y = 0$
11. $2x - y = {}^{-}1$; $x - 2y = 4$
12. $12x + 6y = 12$; $2x - 3y = {}^{-}2$
13. $10x - 10y = 3$; $2x + 3y = 3.1$
14. $x - y = {}^{-}1$; $4x - 8y = 4$
15. $2y + 2 = 3x - 6$; $\frac{y-1}{2} + \frac{x-3}{2} = \frac{x+3}{3}$
16. $5x - 2y = 1$; $4x + 3y = {}^{-}10.7$

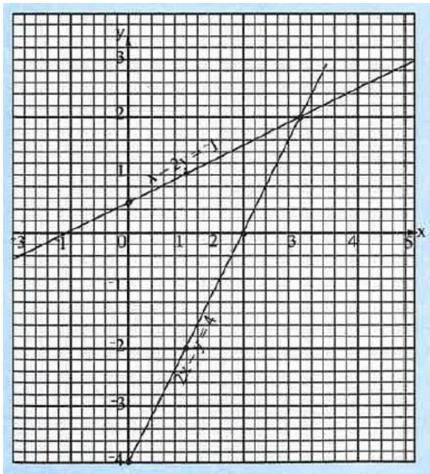


Fig. 8.5

Step 3:

Read the coordinates of the point of intersection.

From the graph, the lines intersect at (3, 2). The solution of the simultaneous equations is

$\therefore x = 3,\ y = 2$

Figure 19: Illustrating level 2 tasks under the graphical method (Thomo et al., 2015b, p. 61).

In Figure 19, each pair of equations were intersecting when plotted on the same axis. This further makes the tasks similar to the equations.

A Level 3 task was found in Book C illustrated in Figure 20. In the figure, there are two tasks presented in word problems under the graphical method. The first part was coded level 3 because it requires students to generate equations before using the graphical method. The second part was coded level 3 because the question does not mention that (q,1) are the solutions. Eventually, the students are made to think. Also, students are used to the pair of solutions being numbers only. In this case, students might be challenged to relate q to the value of x. The task is considered as an example of working backward where one starts with a solution. Polya (1945) states that working backward is a problem-solving method. Lastly, the task will lead a student to use the elimination or substitution method. Therefore, students are allowed to choose which method is effective for the problem.

3. Two numbers are such that the sum of half of the first number and the second number is 6. But twice the second number added to three times the first number is 24. Find the two numbers. If two lines, $2y + 5x = 1$ and $y + p = 3x$ meet at (q,1), find p and q.

Figure 20: Illustrating a level 3 task under the graphical method, Book C (Nyirenda et al., 2014b, p. 82)

Other level 3 tasks were found in word problems in both books. In Figure 21 are word problems that require students to form simultaneous equations and solve them. The section had 15 tasks. Out of the 15 tasks, seven tasks were coded level 3 while eight were level 2. Tasks like 1–3, 5–6, and 11 are level 2 tasks because the connection between the elements of the question is clear such that students can easily come up with the equations. Tasks like 4 and 12–15 were coded level 3 because the connection among the variables is not straightforward. In addition, tasks 12 and 15 are connected to linear motion and commercial mathematics respectively, making them challenging and connected to real life. In Book C, many tasks were similar to the level 2 tasks in the Figure 21. Only three level 3 tasks were found. However, in general, word problems were found to be potential opportunities for problem solving.

Exercise 7.5

1. The sum of two numbers is 10, and their difference is 6. Make a pair of equations and solve them simultaneously to find the numbers.
2. Mary is one year older than June, and their ages add up to 15. Form a pair of equations and solve them to find the ages of the girls.
3. Two books have a total of 500 pages. One book has 350 pages more than the other. Find the number of pages in each book.
4. A bag contains K 5 coins and K 10 coins. There are 14 coins in all, and their value is K 105. Find the number of each type of coin.
5. Two numbers are such that twice the larger number differs from thrice the smaller number by four. The sum of the two numbers is 17. Find the numbers.
6. If 5 is added to both the numerator and denominator of a fraction, the result is $\frac{4}{7}$. If 1 is subtracted from both the numerator and denominator, the result is $\frac{2}{5}$. Find the fraction.

11. The sum of the number of edges and faces of a solid is 20. The difference between the number of edges and faces is 4. Find the number of edges and faces.
12. The velocity in km/h of a car after t hours is given by the formula $v = u + at$, where u and a are constants. Given that $v = 50$ when $t = 2$ and $v = 140$ when $t = 5$, find
    (a) the constants u and a.
    (b) the velocity when $t = 7$ hours.
    (c) the time at which $v = 260$ km/h.
13. The sum of the digits in a three digit number is nine. The tens digit is half the sum of the other two and the hundreds digit is half the units digit. Find the number.

14. Asale and Mbiya collected a number of stones each to use in an arithmetic lesson. If Asale gave Mbiya 5 stones, Mbiya would have twice as many as Asale. If Asale had five stones less than Mbiya, how many stones did each have?
15. A student invested K 50 000 in two different savings accounts. The first account pays an annual interest rate 3%. The second account pays an annual interest rate of 4%. At the end of the year, she had earned K 1 850 in interest. How much money did she invest in each account.

Figure 21: Illustrating level 3 tasks under word problems in grade 10, Book D (Thomo et al., 2015b, pp. 56–57).

4.5 Summary of the general findings.

The findings from all grades show that problem-solving opportunities are available in the textbooks under the topics of equations and simultaneous equations. They are mainly presented in word problems. However, the opportunities are few due to the high similarity between examples and tasks. Suggesting an emphasis on procedural fluency (Kilpatrick et al., 2001) in the textbooks. In addition, the word problems seem to be treated as application sites for the learned procedures and review since they are located at the end. In the next section, I focus on the factors that influence the presence of problem solving in the textbooks. The main emphasis is on what propels the similarities between examples and tasks.

4.6 Factors influencing Problem solving in Textbooks.

The preceding results illustrated the presence and distribution of problem-solving opportunities. The distribution is influenced by the similarities between examples and tasks. In this section, I present the three features in the examples and the textbooks that influence these similarities according to the analysis.

1. Detailed examples

In the study, this refers to when an example uses and/or adds a complex structure to illustrate a method instead of using a simple one. The second instance is when extra information is given separately on how to solve a certain structure of a problem that has not been illustrated through an example just to simplify it for students. These instances were observed in both grades as elaborated on below.

Grade 9

Figure 22 shows rules (on the top left), solved examples (from bottom left to top right) and an exercise (on the bottom right) on solving equations involving brackets. Example 9.8 is a whole equation that illustrated rules (iv) and (ii) while Example 9.9 is a fraction equation that illustrated rule (i) using the concept of LCM. The solved examples were meant to generalize the rules of working with the brackets. At the same time, the structures of the equations were different such that students could see the contrast. Hence it was coded CG. Using whole equations on both examples would still achieve the object of learning. However, the inclusion of the fraction equation was considered too much detail to be illustrated in an example because fraction equations are challenging to students (Johari & Shahrill, 2020; Xu et al., 2017) since they require the application of concepts like LCM. In the end, the similarity between the examples and tasks was widened. Consequently, tasks 1–5 and those similar were coded level 2 because they were similar to Example 9.8. Similarly, tasks 19–21 and those similar were coded level 2 because they could use LCM as in Example 9.9. Only task 22 was coded level 3 because it had a double equation which made it different from the rest. This task could require a student to realize that it could simply be solved by any two parts of the equation.

Equations involving brackets

Recall that:

(i) $a + (b + c) = a + b + c$

(ii) $a + (b - c) = a + b - c$

(iii) $a - (b + c) = a - b - c$

(iv) $a - (b - c) = a - b + c$

Use these rules to remove brackets before solving equations involving brackets.

Multiply both sides by 12 (LCM of 4, 2 and 3)

$$12\left\{\left(\frac{5x+2}{4}\right) - \frac{3}{2}\right\} = 12\left(\frac{7x+1}{3}\right)$$

i.e. $12 \times \left\{\left(\frac{5x+2}{4}\right) - \frac{3}{2}\right\} = 12 \times \left(\frac{7x+1}{3}\right)$

i.e. $3(5x + 2) - 6 \times 3 = 4(7x - 1)$

Removing brackets:

$15x + 6 - 18 = 28x - 4$

Simplifying LHS:

$15x - 12 = 28x - 4$

Subtracting 28x from both sides and adding 12 to both sides:

$15x - 28x = {}^{-}4 + 12$

i.e. ${}^{-}13x = 8$

Dividing both sides by ⁻13:

$\frac{{}^{-}13x}{{}^{-}13} = \frac{8}{{}^{-}13}$

i.e. $x = \frac{8}{{}^{-}13}$

Example 9.8

Solve the equation:

$9x - (4x - 3) = 11 + 2(2x - 1).$

Solution

$9x - (4x - 3) = 11 + 2(2x - 1)$

Removing brackets:

$9x - 4x + 3 = 11 + 4x - 2$

Simplifying both sides:

$5x + 3 = 4x + 9$

Subtracting 4x and 3 from both sides:

$5x - 4x = 9 - 3$

i.e. $x = 6$

Example 9.9

Solve the equation $\frac{5x+2}{4} - \frac{3}{2} = \frac{7x+1}{3}$

Solution

$\frac{5x+2}{4} - \frac{3}{2} = \frac{7x+1}{3}$

Exercise 9.6

Solve the following equations.

1. $4d + (5 - d) = 17$
2. $12m + (1 - 7m) = 23$
3. $23 = 7 - (3 - 4t)$
4. $(8w - 7)(5w + 13) = 0$
5. $24 - (5 + 3x) = 8x + (4 - 5x)$

19. $\frac{2}{3}(4 - 2y) - \frac{3}{5}(2 - 4y) = 2$
20. $\frac{3}{4}(8 - 4z) - \frac{2}{3}(3 - 2z) = 1$
21. $\frac{x}{5} + 1\frac{1}{2}x - \frac{11}{20} = \frac{5x-1}{3}$
22. $\frac{7e-5}{5} - \frac{2e-1}{10} = \frac{4e-3}{15} = 0$

Figure 22: An example of a detailed example space in grade 9, Book B (Thomo et al., 2015b, pp. 113–114).

Grade 10

Refer to Figure 16, which shows an example and tasks under the substitution method. The example was coded G because it was the only example in the space. According to how MDITx was adapted for the study, it was interpreted as generalizing the substitution method. The example is a fraction equation which was complex to illustrate the method. A whole equation would be better so the fraction equations should serve as challenges to students. Yet, this example simplified the work for students and highlighted that they should use the LCM. In the end, only tasks 17 and 22 (two out of 22 tasks) were coded as level 3 because students would have to work with double fractions which are extremely challenging (Johari & Shahrill, 2020), unlike the other tasks whose equations students have encountered in grade 9 and could be solved as in the example. Thus, they were coded level 2. See Figure 16 for an explanation of this example space.

See also Figure 17 as another example for detailed examples in which a fraction equation was used to illustrate the elimination method instead of a simple whole equation. In the end, no fraction equation was assessed. And if it were assessed it would not be complex as it would be since a templated is given.

The second point of being too detailed was giving extra information separately that students could easily figure out. This information is given in the textbooks to simplify students' work. This has been discussed below.

Grade 9.

Figure 23 shows an example (from the left to top right) of how to solve algebraic fractions and their tasks (on the bottom right). The example demonstrated how to solve regular fraction equations and did not include mixed fractions. The solved examples were coded as G because they were all similar as they were simply regular fractions. Students could generalize that when you have a fraction you simplify using the LCM. However, extra information was given through the ***note*** just before the tasks (Exercise 9.5) to demonstrate how to work with mixed fractions. This information limited the opportunity to challenge students because students could easily work out the mixed fractions as in the note and relate the result to the examples to apply the LCM rule. As a result, all the tasks were coded level 2. The questioning as well in Exercise 9.5 gives directions to the students on how they could solve them. The instruction points at Example 9.6 as a reference point, suggesting that the solved examples and the tasks are similar. See also Figure 13 Exercise 9.3 with a similar direction on tasks.

Solving linear equations involving fractions

When solving equations involving fractions, simplify the fractions using the LCM.

(a) $\frac{2x+7}{3} - \frac{5x+6}{4} = 0$

$\frac{4(2x+7) - 3(5x+6)}{12} = 0$ *(Use LCM to get same denominator)*

$4(2x+7) - 3(5x+6) = 0$ *(Multiply both sides by 12)*

$8x + 28 - 15x - 18 = 0$ *(Open brackets)*

$-7x + 10 = 0$

$-7x = -10$

$\therefore\ x = \frac{10}{7} = 1\frac{3}{7}$

(b) $\frac{2a+36}{a} - \frac{4}{5} = 0$

$\frac{5(2a+36) - 4a}{5a} = 0$

$5(2a+36) - 4a = 0$

$10a + 180 - 4a = 0$

$6a + 180 = 0$

$6a = {}^-180$

$a = {}^-30$

Solution

$\frac{2}{5}y + \frac{3}{4} = 10 + \frac{y}{2}$

LCM of 5,4 and 2 = 20

Multiplying both sides by 20:

$20\left(\frac{2}{5}y + \frac{3}{4}\right) = 20\left(10 + \frac{y}{2}\right)$

$8y + 15 = 200 + 10y$

Subtracting 10y from both sides:

$8y + 15 - 10y = 200 + 10y - 10y$

${}^-2y + 15 = 200$

Subtracting 15 from both sides:

${}^-2y + 15 - 15 = 200 - 15$

${}^-2y = 185$

Dividing both sides by ⁻2

$\frac{{}^-2y}{{}^-2} = \frac{185}{{}^-2}$

$\therefore\ y = -92\frac{1}{2}$

Note: Should there be a term like $1\frac{1}{2}x$ in the equation, always write it in the improper fraction form, as $\frac{3}{2}$x, and then proceed as in Example 9.7.

Exercise 9.5

Solve the following equations using the balance method and stating the steps as in Example 9.6.

1. (a) $\frac{x}{6} - 2 = 10$ (b) $9 - \frac{x}{2} = 5$
   (c) $\frac{k}{5} = 0$ (d) $p - 2\frac{1}{2} = 6\frac{1}{2}$
2. (a) ${}^-3\frac{3}{4} = x + 1\frac{2}{5}$ (b) $4\frac{1}{2} = 5q - -$
   (c) $2p - 8 = p - 3$ (d) $t + 7 = 17 - 4t$
3. (a) $3\frac{1}{2} + 2 - f = 17\frac{1}{2} - 1 - f$
   (b) $1\frac{1}{2}x + \frac{1}{4} = 1\frac{1}{2}x + 3\frac{1}{4}$
   (c) $\frac{0.1}{x} + \frac{3.9}{x} = 12$

Figure 23*:* An example of giving out extra information in grade 9, Book B (Thomo et al., 2015b, pp. 112–113)

2. Not introducing lessons as challenges

This tendency was to do with the design of the textbooks. All the analyzed textbooks introduced a lesson through one of the following layouts. The first layout began with an introduction to state the objective(s), which included the method to be discussed. It was followed by solved examples and then tasks. See Figure 24, which illustrates this layout. In Figure 24, the textbook introduces the lesson of elimination method for solving Simultaneous equations in grade 10 by getting straight to the rules and then examples, ending with tasks. No challenge has been given to the students.

Elimination method

There are two rules governing elimination method.

1. If the coefficients of the variable to be eliminated are equal and have the same sign, use the rule **'Same Signs Subtract'**.
2. If the coefficients of the variable to be eliminated are equal but the signs are different, use the rule **'Different Signs Add.'**

Example 2

Solve the following simultaneous equations

(a) $\begin{cases} x + y = 1 \\ x + 2y = 2 \end{cases}$ (b) $\begin{cases} 2x + y = 1 \\ x + 2y = 2 \end{cases}$ (c) $\begin{cases} \frac{2}{3}x - \frac{y}{2} = 0 \\ \frac{x}{3} - \frac{3}{2}y = 0 \end{cases}$

Exercise 7.2

Solve the following simultaneous equations by elimination method.

1. $\begin{cases} x + y = 2 \\ x + y = 0 \end{cases}$ 2. $\begin{cases} 2a + b = 3 \\ a - 2b = -1 \end{cases}$ 3. $\begin{cases} 3e + 2f = 4 \\ e + f = 1 \end{cases}$

4. $\begin{cases} m = n - 2 \\ 3n + m = 7 \end{cases}$ 5. $\begin{cases} 3r + q - 9 = 0 \\ 2r + 3q - 13 = 0 \end{cases}$ 6. $\begin{cases} 3y - x = 8 \\ 2y + 3x = 10 \end{cases}$

7. (a) Consider the equation $5x - y = c$. Find the value of c if the solution set is $(1, -1)$.

(b) Find the value of m and n if the solution set for the following simultaneous equation is $(8, -4)$.

$\begin{cases} mx - y = 10 \\ nx + y = 2 \end{cases}$

Figure 24: Exemplifying how lesson were introduced, Book C (Nyirenda et al., 2014b, pp. 75–77)

The second way of introducing a lesson would begin with an example, explaining the method and then tasks. This was observed in Book D under the same lesson on the elimination method of solving Simultaneous equations. Similar presentations were observed in grade 9 (see Figure 23). This suggests that the textbooks are designed to teach students the standard methods of solving equations and not stimulate their creative thinking. In this regard, the textbooks confined the students to tasks similar to those exemplified.

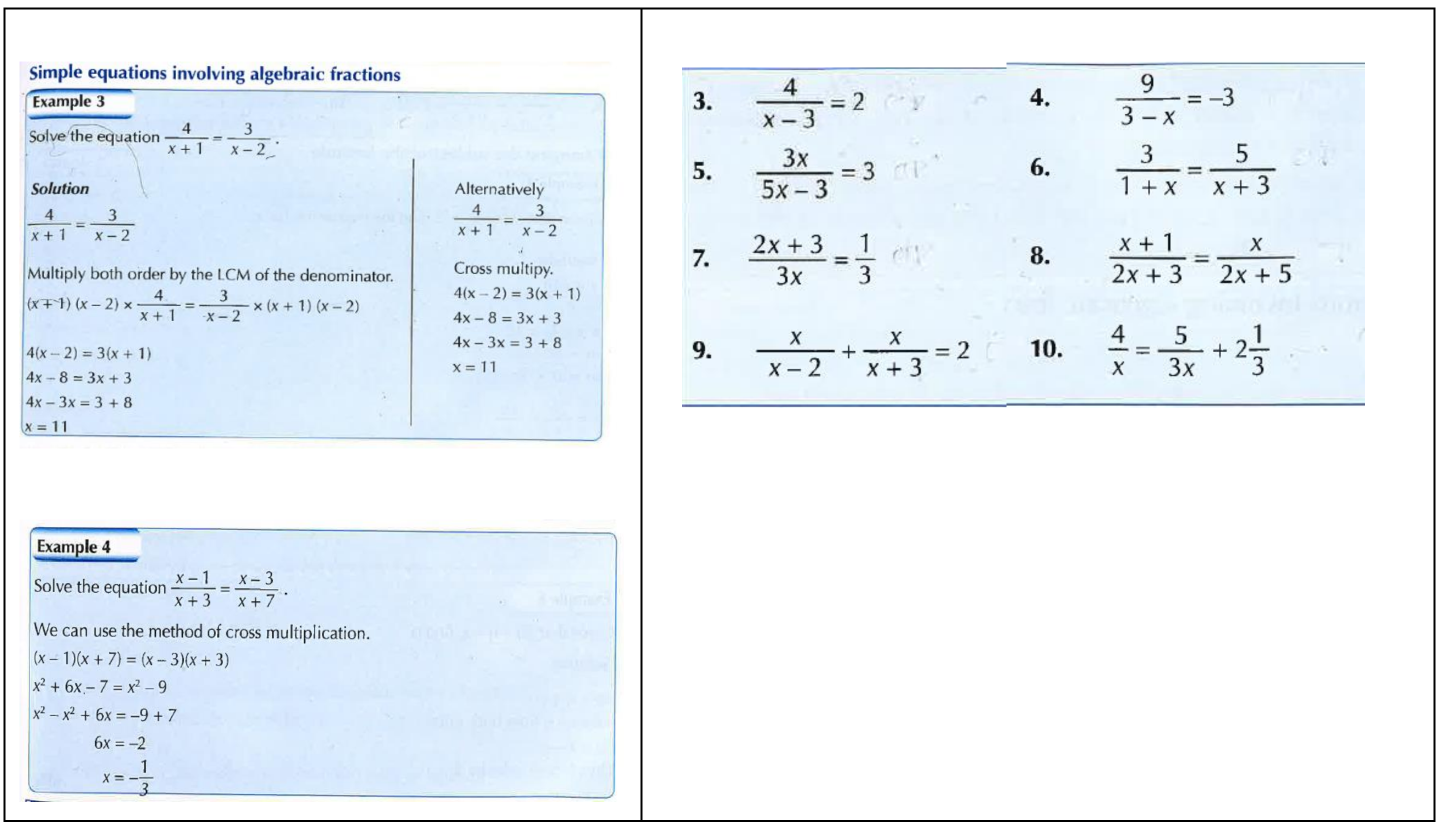
3. Leading questions versus lack of prompting questions

This finding is to do with how the questions were presented. Textbooks B and D had leading questions that explicitly indicated the method to be used on a task. These were presented as headings of the exercise. The same was implicit in A and C. Yet, both cases signal to students the similarity between solved examples and tasks. This has been exemplified below.

Grade 9

Refer to Figure 13 Exercise 9.3 and Figure 23 Exercise 9.5 which show examples and tasks for solving equations. The questions explicitly state that students should use *the balance method* as illustrated in the preceding examples. Meaning students are confined to this method only such that it does not give them room for creativity. This explicit indication of the method is a sign that the equations a similar to the examples (e.g., Example 9.4) and become less challenging. Consequently, they were coded level 2.

In Book A of grade 9, there were no leading questions, and neither did it contain prompting questions. However, in both books, the tasks were of the same structure as in the solved examples. Compare structures of tasks with their examples in Figure 23 and Figure 25 under the same lesson of solving fraction equations**.** This could be interpreted as a deliberate arrangement for students to use the current procedures. Hence, the structure is leading.


Simple equations involving algebraic fractions

Example 3

Solve the equation $\frac{4}{x+1} = \frac{3}{x-2}$.

Solution

$\frac{4}{x+1} = \frac{3}{x-2}$

Multiply both order by the LCM of the denominator.

$(x+1)(x-2) \times \frac{4}{x+1} = \frac{3}{x-2} \times (x+1)(x-2)$

$4(x-2) = 3(x+1)$

$4x - 8 = 3x + 3$

$4x - 3x = 3 + 8$

$x = 11$

Alternatively

$\frac{4}{x+1} = \frac{3}{x-2}$

Cross multipy.

$4(x-2) = 3(x+1)$

$4x - 8 = 3x + 3$

$4x - 3x = 3 + 8$

$x = 11$

Example 4

Solve the equation $\frac{x-1}{x+3} = \frac{x-3}{x+7}$.

We can use the method of cross multiplication.

$(x-1)(x+7) = (x-3)(x+3)$

$x^2 + 6x - 7 = x^2 - 9$

$x^2 - x^2 + 6x = -9 + 7$

$6x = -2$

$x = -\frac{1}{3}$

3. $\frac{4}{x-3} = 2$ **4.** $\frac{9}{3-x} = -3$

5. $\frac{3x}{5x-3} = 3$ **6.** $\frac{3}{1+x} = \frac{5}{x+3}$

7. $\frac{2x+3}{3x} = \frac{1}{3}$ **8.** $\frac{x+1}{2x+3} = \frac{x}{2x+5}$

9. $\frac{x}{x-2} + \frac{x}{x+3} = 2$ **10.** $\frac{4}{x} = \frac{5}{3x} + 2\frac{1}{3}$

Figure 25*:* Structure of examples and tasks in Book A (Nyirenda et al., 2014a, pp. 123–124)

Figure 25 shows examples on the left and tasks on the right under solving fraction equations. The examples were meant to illustrate the LCM and cross-multiplication methods for solving fraction equations. The tasks came right after the examples. Tasks 1 and 2 that are not appearing in the figure are similar to tasks 3–5.

Grade 10

In grade 10 the questioning was predominantly leading because the three methods of solving Simultaneous equations were placed as different lessons in all textbooks. As such, the questions indicated the method according to the lesson in that section. That is why there were few level 3 and level 1tasks in grade 10 since the tasks strictly employed the current procedure(s). Figure 16, Figure 17 and Figure 19 illustrate the leading questions in grade 10. However, the analysis interpreted this as non-influential on the presence of problem solving because it is the nature of the simultaneous equations to be solved using the three methods. The design to teach the methods as separate lessons limited the opportunity for students to decide which method works better for a specific problem. The skill to choose and apply a suitable method is the third step of problem solving (Polya, 1945).

Under this factor, both grades contained leading questions explicitly and implicitly. On the other hand, they did not contain prompting questions to allow the students to solve the equations in their own way. In the end, many of the tasks were solved with the aid of the examples.

4.7 Other findings

Apart from the above findings, other four interesting observations were made. First, textbooks acknowledged higher-order concepts of equations, yet did not give tasks to students. For instance, textbook B mentioned identity equations (Thomo et al., 2015a) and Textbook D mentioned simultaneous graphs that coincide and those that are parallel as possible solutions to Simultaneous equations (Thomo et al., 2015b). However, these were not given as tasks to students in the textbooks. Huntley et al. (2007) consider these as higher-order concepts of equations that challenge students.

Second, the results from Table 6 signal that more tasks imply more level 3 tasks. For instance, grade 10 textbooks C and D show that D has 98 tasks with 11 level 3 tasks, while C has 33 tasks with three level 3 tasks. This finding agrees with the assumption made in the sampling that more quantity could increase the opportunities to learn problem solving.

Third, level 3 tasks were found in the middle or at the end of the topic. An exception was textbook A, which had level 3 tasks at the beginning because its first tasks were word problems. In the same book, level 1 tasks were also found at the end of the topic. This was a surprising finding as compared to the other books. Level 2 topics were spread throughout the topic.

Lastly, level 3 tasks were mainly found in word/practical problems in all textbooks. The tasks required students to formulate equations and solve them. They incorporated tasks that required the application of other concepts so that students could formulate the equations. Only three tasks that were not word problems were found. One in book B and two in book D.

4.8 Outliers

Table 6 showed that the study had outliers. These are tasks that had unclear and/or confusing presentations. In addition, some were considered to be for senior secondary school level. These are presented in Table 9. It also includes the reason(s) for the consideration. Note that these tasks are a part of the 309 tasks analyzed in this study.

Table 9*:* A presentation of outliers in the study

Task	Book	Page	Reason
The perimeter of a square whose side is 52. Find the area of the square.	A	122	The grammar was not clear.
$4\frac{1}{2} = 5q - -$	B	113	Double negative at the end
$(8w - 7)(5w + 13) = 0$	B	114	Turned into a quadratic equation after expanding the brackets. Not for grade 9.
$7(5x - 3)10 = 2(3x - 5)3(5 - 7x)$	B	114	Turned into a quadratic equation after expanding the brackets. Not for grade 9.
$4x + y - 8 + 0{,}2y + 7 = 4$	D	59	The first equation had no equal sign.

3.8 Chapter summary

This chapter presented the findings of the study in response to the question of the presence of problem-solving opportunities in the textbook. It shows that the textbooks offer opportunities for students to learn problem solving mainly through word problems. The distribution of the levels shows that many tasks are similar to the solved examples such that many tasks belong to level 2 and fewer to level 1 and level 3.

Table 6 shows that out of the 309 tasks analyzed, 74% were level 2 while 13% were level 3. Only 11% of the tasks were level 1. Level 1 and level 3 tasks were fewer in grade 10 than in grade 9 due to the nature of simultaneous equations which were arranged in a method-specific lesson order such that the questions were leading.

The problem-solving opportunities were mainly limited due to detailed exemplification, leading questions and lack of prompting questions and not introducing lessons as challenges. It was also found that higher-order concepts were not tasked even when recognized.

Lastly, all the textbooks responded well in presenting the objectives as stipulated in the curriculum. Nevertheless, teaching and learning through problem solving were not well presented because only the written exercise was an explicit method of teaching and learning that was applied in the textbooks.

CHAPTER 5: DISCUSSION

5.1 Introduction

The study investigated the problem-solving opportunities provided in lower Malawian secondary school mathematics textbooks. This objective was achieved by answering two research questions: *(1) How are problem-solving opportunities provided in Malawian grade 9 and 10 mathematics textbooks?* and *(2) What factors influence the opportunities to learn problem-solving in mathematics textbooks?* Four textbooks were analyzed, and this chapter discusses the findings presented in Chapter 4. The focus is on the presence of problem-solving opportunities in textbooks and how they are presented, and on the factors that influence the opportunities of problem solving.

5.1.1 The opportunities to learn problem-solving provided in textbooks and how they are presented.

The results in Figure 11 show that at least 74% of the tasks were level 2 while 13% were level 3, and 11% were level 1. It was indicated earlier that level 2 and level 3 tasks were considered as potential opportunities for problem solving with more preference to level 3. The results reveal that the lower secondary school mathematics textbooks contain the opportunities to learn problem solving. The presence of 13% of level 3 tasks indicates a limitation to genuine problem solving since students are mostly given level 2 tasks (74%), which are more akin to examples compared to level 3 tasks. It can be concluded that textbooks contain opportunities for problem solving, yet more on a low level. This limits the opportunity for students to learn problem solving at a higher level of 3. Many other studies on problem solving in textbooks found that problem-solving opportunities are generally few in mathematics textbooks. Brehmer et al. (2015) found that level 3 tasks were few in Swedish mathematics textbooks. Contradicting the finding in the current study that many are of a lower level. Berisha et al. (2014) also found that textbooks in Kosovo contained more routine problems than non-routine problems. Other studies also found limited opportunities for problem solving in textbooks (e.g., Buishaw & Ayalew, 2013; Fan & Zhu, 2000, 2007).

It can be concluded that problem solving is limited in textbooks in many countries, and there is a need to include more problem-solving tasks in textbooks. Otherwise, the findings from this study confirm that textbooks might be designed to teach students methods on how to get answers right. Yet, the sole purpose of mathematics is to help students think (Schoenfeld, 1983). The study also revealed that few tasks belonged to level 1. This finding signal that the textbooks a geared toward higher order thinking despite containing more level 2 tasks than level 3. This is a good sign for the possibility of having higher level problematic tasks. There is a need to take careful consideration when including tasks in the textbooks to achieve the inclusion of more level 3 tasks.

Regarding equations specifically, Jäder et al. (2020) also found them to contain less problem-solving tasks as they required the use of standard procedures than creating new methods, which means the current study's results are not surprising. Many studies also revealed the dominance of using standard procedures among students when solving equations. Kieran (2007) also found the same when reviewing studies on algebra. Huntley et al. (2007) observed that students used standard procedures even on equations that did not require such. As already highlighted, mathematics experts also demonstrated the use of standard procedures as a first response to solving equations (Star & Newton, 2009). A recent study with middle and high school students from Sweden, Finland and Spain found similar tendencies among middle school students from all three countries (Star et al., 2022). Many studies, including the current one, suggest that it is difficult to find problem-solving tasks in equations unless adjustments are made in classroom instruction and in textbooks. However, this assumption was beyond the scope of this study. An interesting finding was that all level 3 tasks also required the application of procedures learned in the textbook, but they were complex in their manipulation at some stage. For example, tasks 19 and 22 in Figure 16 were problem-solving tasks but still used the LCM method. This is consistent with Jäder et al.'s (2020) findings of their analysis of textbooks from 12 countries in which problem-solving tasks required the application of already known procedures. This suggests that equations require the application of procedures than creating new ones.

The results also revealed that many of the tasks that were found to involve problem solving were presented in word form. In Figure 14 and Figure 21 there are more level 3 tasks as compared to all other figures containing exercises presented in CHAPTER 4: FINDINGS. The level 3 tasks were presented in mathematical and contextual form. Only three out of the 41 level 3 tasks that were not word problems were found to be problem-solving tasks. They were fraction equations that involved a double fraction in the simplification and another one had two equal signs to connect three expressions (see Figure 22 task 22). This particular finding agrees with research findings indicating that fraction equations are challenging to students (e.g., Johari & Shahrill, 2020; Xu et al., 2017). Furthermore, they suggest that problem solving is considered as solving word problems in the Malawian context. As indicated in the grade 9 word problems in Figure 14, the tasks were clearly introduced as problems.

However, not all word problems were challenging enough to be problematic. Some tasks were easy to generate relationships between variables. In addition, some tasks had similar wording as in previous tasks done in algebraic expressions. And some were similar to examples. It was found in Book C that there were more level 2 tasks than level 3 tasks in the word problems section. This implies that word problems are not automatically problem-solving tasks; it depends on how the question has been framed and what it requires of the students. For instance, all the word problems in the study were framed in such a way that every element (variable) in the question was used in solving the problem. No distracting elements were in the questions which lessened the problematic nature of the questions. This suggests another way of making word problems more challenging, that is, including distractions in the questions. These are details that are not relevant to solving the problem but would challenge the student to think and select the relevant information. Nevertheless, going by the definition of word problems as problem solving as revealed in the textbooks, the presence of simple word problems could be a quest to provide opportunities to all students. Including those with low cognitive abilities. A call that was made by Brehmer et al. (2015) after observing that the problem-solving tasks in Swedish textbooks were of higher level only.

The findings further align with Kieran's (2007) assertion that students find word problems challenging as they struggle with generating equations. This difficulty might also be attributed to the English language effect in mathematics. But this discussion is beyond this study. Kieran also argued that word problems are designed for students to apply the learned algorithms for solving equations. In all textbooks except book A, word problems were located at the end of the topic. This could be a deliberate location to challenge the students on what they have learned. Consequently, it means that the textbooks support learning *for* problem solving more than learning *through* problem solving. Much as learning for problem solving is crucial, it is equally vital to learn through problem solving. Brehmer et al. (2015) also found that problematic tasks in Swedish textbooks were located at the end of the chapter with a level 3 of difficulty. The findings are also consistent with recent studies that agree that word problems that incorporate real life applications, relate to other disciplines and demand thinking skills, could propel problem solving (Kirana & Kholifah, 2020; Musdin & Wassahua, 2021; Purwadi, 2020). Kaur et al. (2020) reported that

"…and on Level 3 questions that involve real-life applications, thinking skills, and questions that relate to other disciplines" (p. 102). In addition, word problems give students the opportunity to formulate or find the best strategy to use (Purwadi, 2020), thereby enhancing their problem-solving ability. It is commendable that the Malawian textbooks incorporated these tasks; it means that problem solving is being considered.

From the results, it can be concluded that the Malawian textbooks regard problem as solving applying mathematics to solve word problems. The objective is to learn for and not through problem solving since the word problems are placed at the end for students to apply the procedures that they have learned. This aligns with the rationale of the curriculum for students to use problem solving strategies to solve practical problems. In this case, the strategies appear to be the various methods illustrated and the practical problems could be the word problems. Yet, this could be regarded as a shallow or perhaps a different understanding of problem solving and strategies on the one hand. Because according to (Fan & Zhu, 2000, 2007; Polya, 1945; Schoenfeld, 1983), problem-solving strategies are heuristics and not formal procedures. On the other hand, the word problems do give the students the opportunity to choose strategies from among those that have been taught, which is Polya's third stage of problem solving. However, this might be untrue if only one method has been learnt in a topic. Which is the case of Malawian textbooks. This scenario needs further research to understand how students could determine their methods to conclude if there is problem solving involved. The results could also mean that every task in the textbook is regarded as a mathematical problem. This could be true on the individualistic level of a problem in the context of textbooks analysis (Fan & Zhu, 2007). Yet, the textbooks contain fewer problem solving tasks and more mere exercises in the topics of equations and simultaneous equations.

The conclusion could be attributed to the nature of equations and the importance of equations. First, equations in secondary school connect with almost every concept (Knuth, et al., 2016), such that examples and tasks are presented in such a way that they emphasize standard procedures to be used in other topics. Equations are more or less a guide to solving other concepts as such that methods need to be underscored. Understanding the equation is argued to be key in the success of secondary school mathematics (Andrews & Sayers, 2012). Second, and specifically for grade 10, the nature of simultaneous equations is limiting with regards to problem solving. They are solved through three standard methods, and the textbooks were designed to present each method as a separate lesson. This means having more similar examples and tasks in an example space focusing on one method in a single lesson, as a result, students would only use the same method in an example space to solve all the tasks. However, this does not dispute the fact that students in class find simultaneous equations challenging(Johari & Shahrill, 2020; Ugboduma, 2013).

5.1.2 Factors that influence problem solving in textbooks.

The results indicated that many tasks belonged to level 2. Three factors restricted the tasks to the current procedures or to be similar to the examples. Some tasks that could have been level 3 were reduced by these factors. These have been discussed below.

1. Detailed examples

The results revealed that some examples that were used to illustrate procedures were complex. Consequently, some tasks that could be complex were rendered simple because such tasks were similar to the complex examples. For example, in the substitution method in Figure 16, Example 7.3, and the elimination method in Figure 17 Example 2c. in these examples, fraction equations were used as examples to illustrate the methods. Similarly, on working with equations involving brackets in Figure 22, fraction equations were used in Example 9.9. Yet, simpler equations as used in Example 9.8 could have been used to illustrate the same methods so that the fraction equations could serve as challenges to students. In the end, the fraction equation although are known to be challenging (Johari & Shahrill, 2020; Xu et al., 2017) were not, because students could refer to the examples. For example, in **Figure 16** tasks 16–22 were fraction equations, yet rendered uncomplex because the example revealed students could use the LCM to solve the equations. A concept that could have been left for students to discover for themselves. At this level, students have encountered LCM and could figure out how to apply it. At the same time, this revelation ruled out the possibility of students to use other alternatives. For example, adding the fractions first to have a single expression then using the LCM or cross-multiplication. This opportunity to decide what best works for the fraction equation was taken away through the use of complex structures which forced the authors to reveal the strategy on how to work them out. The same applied to the tasks in Figure 22.

Another form of being detailed was textbooks giving out extra information on how to manipulate some structures that students could figure out. In Figure 23 before Exercise 9.5, the textbook hinted that when students are working with mixed fractions, they should first simplify it into an improper fraction then proceed using the LCM. Giving this hint indicates that the mixed fractions were considered challenging in the textbooks. Yet, the textbook included a hint. This could propel memorization. It was sufficient to exemplify the LCM method when dealing with fraction equations since that was the objective, and reserve information on mixed fraction as a challenge for the students. At this level, students have tackled [mixed] fractions since primary school. Some might still struggle, but it benefits the students to discover for themselves that the first step is to change the mixed fraction into an improper one. The inclusion of the information weakened the probability of mixed fraction being problem-solving tasks. This factor provided a template for students to refer to when solving the equations.

This is consistent with Jäder et al.'s (2020) findings that when tasks are similar to examples or earlier tasks in the textbook, their nature to be problems reduces. They become mere exercises because the examples and earlier tasks act as templates.

The results suggest the need to consider the kind of examples that are given to students when illustrating methods. Using simple structures could give room for other structures to pose as challenging tasks which could propel problem solving. In other words, textbook authors should strive to strike a balance between providing students with necessary information and allowing them to acquire other information through their own efforts. This suggests that tasks in textbooks could consist of simple structure that have been clearly explained for students understanding. Meanwhile, more complex structures could be presented as opportunities for students to engage in problem solving. Furthermore, the results caution the inclusion of extra information that students could discover or recall by themselves. Textbooks authors should appreciate the cognitive abilities of students by offering less hints and providing more room for discovery learning. The whole essence of conceptual understanding, procedural fluency and strategic competence is for students to know what, when and why to apply a concept (Kilpatrick et al., 2001) without being told what and how to do it. That is what promotes problem solving. If anything, extra information should be presented in teachers' guides for teachers to offer help when students are challenged, or tasks that require extra information could serve as classroom challenges to enhance problem solving and group discussions. Textbooks are designed while considering students' cognitive characteristics (Qi et al., 2018), and authors ought to assume that every concept that students encountered already could be tackled later on. As such, students should be left to discover for themselves the methods they are to use.

2. Not introducing lessons as challenges

The results show that the lessons were introduced and presented in such a way that students are not given the opportunity to recall or reflect on what they already know. Textbooks present lessons in a rote memorization way. The way a textbook lesson is organized could promote or deter problem-solving skill (Buishaw & Ayalew, 2013). In the textbooks of this study, the main components illustrated in a lesson's example space were objectives, examples, and then tasks (see Figure 23 and Figure 24). Gracin (2018) called this the *rule-example-practice* structure. No challenges are incorporated. This outline considers students as not having any prior knowledge. As a result, students memorize what has been presented without considering other options.

Introducing or presenting topics as challenges that would elicit students' prior knowledge could be essential in prompting the students to devise their own ways of tackling problems by connecting with their prior knowledge. Such provisions are opportunities for students to practice problem solving. It also helps to make thinking visible and judge whether or not students are able to problem-solve. It is hard for students to get the right methods and answers at the beginning of the lesson if provided as a challenge. Yet, it is even more detrimental to present the topic in a rote memorization way. Discovery learning should be encouraged through challenges to promote problem solving. "In order to develop the problem-solving capacity of students, textbooks should also give students an opportunity to discover by themselves rather than pouring on ready-made factual information" (Buishaw & Ayalew, 2013, p. 1315). Moreover, some of the suggested methods of teaching and learning in the curriculum are discussions and pair work. I assume that these are based on posing challenges for students to brainstorm and try to solve the challenge.

3. Leading questions vs lack of prompting questions

This finding is two-fold. First, it deals with the presence of leading questions which stress the use of standard procedures illustrated in the examples. Second, the absence of prompting questions that would explicitly demand students to use other ways other than the standard procedures to solve the equations. In sections of the topics analyzed, the textbooks referred students to the examples that they should use when solving the tasks. This directive rules out any possibility of students being creative. Much as it could be appreciated that such kind of questioning is aimed at helping the students to achieve the lesson objective, it unfortunately limits the students' ability to reason beyond the current methods. As a result, it may propel students not to use innovative strategies to solve the equations even when there is a possibility and they are capable (Xu et al., 2017). When such a type of questioning is given, it is equally important to provide another section that would demand the students to use other creative methods for solving tasks other than the one illustrated.

Some questions were not leading, however, because they came after some examples and there were no prompts, students could easily refer to them. Prompting is where students are demanded to use other methods other than the standard one (Qetrani et al., 2021; Star & Rittle-Johnson, 2007). I recommend that prompting should not always be done by the teacher; textbooks should also contain prompts. The textbooks appear to have been designed in such a way that the tasks would demand the students to refer to the examples. For instance, in grade 9 under solving fraction equations, textbooks A and B present different structures of fractions. Refer to the examples in Figure 25 and Figure 23 respectively. Their respective tasks resemble their examples. None of the textbooks' s tasks resemble the examples of the other textbook despite being the same lesson. The textbooks direct the students to see tasks that are strictly similar to the examples. The tasks in textbook A were designed for students to apply both the cross-multiplication and LCM method, while in textbook B they were designed for students to easily apply the LCM method only. Thus, leading the students to a template which reduces the opportunity to problem-solve.

To put the grade 9 case in context, it is as follows. In textbook A Figure 25, it was observed that under solving equations involving fractions, mixed fractions were not tackled in the examples. No explanation was given on how to solve them. None of the 10 tasks that followed included mixed fractions. Similarly, in book B, despite not formally tackling mixed fractions in the examples, a description and a short illustration of the initial step were given. In the end, they were dominant in the exercise section (see Figure 23). Both textbooks seem to align the tasks with the methods that have been illustrated. This finding suggests the need for textbooks to include tasks that demand more than just what has been exemplified without deviating from the main objective.

The findings align with findings from classroom practice as an influence of textbooks. Qetrani et al. (2021) argued that school algebra emphasizes procedural skills. They also confirm that students are more comfortable and feel safe when using standard procedures (Huntley et al., 2007; Qetrani et al., 2021; Xu et al., 2017). Procedures are emphasized and students are led towards them in textbook tasks. Studies show that when students are prompted, they show creativity. Otherwise they use standard procedures which are subject to memorization and not understanding the reasoning behind them (Huntley et al., 2007; Qetrani et al., 2021). Buishaw and Ayalew (2013) found that many tasks in grade 9 and 10 textbooks were promoting memorization and not problem solving because they were similar to examples. Fortunately, many studies with experts and students have proven that equations can be solved through other ways other than the standard methods (e.g., Musdin & Wassahua, 2021; Star & Rittle-Johnson, 2007; Xu et al., 2017). But these methods are usually elicited after prompting subjects to solve the equations in other ways other than the standard procedures. It means that if the prompting is implemented in textbooks and students are demanded to solve the tasks in their own ways, then it could enhance creativity in students which may promote problem solving in equations. Perhaps such tasks could be located at the end of the textbooks to test the creativity of the students and not for examination purposes.

5.1.3 Other discussions

It was also found that the tasks seemed to emphasize that the solution to a linear equation is unique. This is despite mentioning in book B that a solution could also be an identity. Similarly, in textbook D for grade 10 mentioned three possible solutions of the graphical method: (1) Where lines intersect; (2) Where lines are parallel and (3) Where the lines meet and there is one common line. Surprisingly, it is only (1) that was given as tasks for students to practice the graphical method. This limited the opportunity to learn problem solving. Huntley et al. (2007) argued that the graphical method is creative way of solving a pair of equations. Their study revealed that when students found that two graphs were parallel or coinciding, it challenged them to think more. What the textbooks in this study portray is that in the graphical method, all graphs should intersect, which is wrong generalization. Including graphs (3) and (2) could be essential for problem solving. In both grades the textbooks recognize such higher order properties of equations but do not implement them. This could be referred to as *limitation by choice*. Whether or not this limitation is due to grade level is uncertain. When textbooks recognize higher order mathematics it could be beneficial to include them in textbooks for students to practice problem solving and not for grading purposes. For instance, Swedish textbooks contained a section comprised of problems to offer problem-solving opportunities to students (Brehmer et al., 2015).

The results also show that more level 3 tasks were found in textbooks that had more tasks. In Table 6 textbooks B and D had more level 3 tasks than textbooks A and C. This finding means that there is a high chance of including more problem solving tasks if there are as many tasks as possible. When tasks are many, each level could have enough tasks to provide ample opportunities for practice. However, this is not a default occurrence. Textbook authors still need to be critical on the kind of tasks they include and strike a balance between various levels of the tasks. Much as the distribution cannot be equal, it is possible to include sufficient problem-solving tasks in the textbooks. There is a need to find out if there is any relationship between the size of a book and the presence of problem-solving opportunities.

The results further indicate that the level 1 tasks under simultaneous equations are not simultaneous. It appears that simultaneous equations are challenging, and many authors have highlighted this (e.g., Johari & Shahrill, 2020; Supriadi et al., 2021; Ugboduma, 2013). Many tasks were saturated on level 2 and less on level 3. Perhaps this arrangement was to not portray simultaneous equations hard as students perceive them. However, there is a need to have a fair distribution of level 3 tasks as well to enhance problem solving. Despite that the equations are inherently challenging, still their methods are direct, and the most probable challenges are simply errors (Johari & Shahrill, 2020). More consideration should be given to making simultaneous equations contain problem solving opportunities.

Lastly, the results show that the textbooks responded well to the curriculum demands in terms of content. All the objectives stated under the topics of Equations and Simultaneous equations were addressed. However, the learning of mathematics through problem solving was not clear. The curriculum stated that students should use problem-solving skills to solve practical problems. Whether these practical problems are meant to be real life problems or those embedded in the tasks is unclear. It could be because of this vagueness that the textbooks could not be so clear as well. Despite the curriculum indicating group discussions, questions and answers, discussions, pair work and many other collaborative methods of teaching and learning, the textbooks could not show these demarcations. This task seems to be left for the teacher. The teaching method that is clear is *written exercise.* Clearly, the textbooks focus more on exercises than problem solving. To this end, it could be concluded that the textbooks regard every tasks as a problem (Fan & Zhu, 2000, 2007) and that problem solving is solving any kind of task regardless of the availability of a solution path or not. In addition, it is regarded as the application of learned procedures on word problems.

5.2 Summary of discussion

The chapter discussed the results of the study and addressed two questions. *(1) How are problem-solving opportunities provided in Malawian grades 9 and 10 mathematics textbooks?* This question addressed the presence of problem-solving opportunities and the nature of presentation in the textbooks. In general, the results of the study show that textbooks have limited opportunities for problem solving. The results concur with other studies on problem solving in textbooks that there are few opportunities that promote problem solving. The results also indicated that there is a huge similarity between many tasks and examples such that students do not have to be creative in solving the tasks. Out of the limited tasks that offer problem-solving opportunities, many are presented in word form as compared to those that are not presented in word form. This could mean that problem solving in Malawian textbooks is largely thought of as solving word problems. Furthermore, perhaps a problem is regarded as every task in the textbook (Fan & Zhu, 2000, 2007) regardless of the availability of the solution path or not.

The second question was: *What factors influence the opportunities to learn problem solving in grades 9 and 10 mathematics textbooks?* The study found three factors that seem to influence the opportunities to learn problem-solving in textbooks. These include: (1) Detailed examples that included using complex structures to illustrate examples and giving extra information to simplify students' work. (2) Not introducing lessons as challenges, which means the topics not including any activity for students to use their prior knowledge or create their own methods. (3) Leading questions versus lack of prompting questions, which means textbooks giving directions on what method(s) to use in a task. On the other hand, it does not demand students to use other method(s) apart from the one(s) that has been taught.

It was also discovered that some higher-order concepts that the textbooks recognized were not implemented which is limitation by choice. What is clear in the textbooks is mere exercises as compared to problem solving.

CHAPTER 6: CONCLUSION

The study investigated the opportunities to learn problem solving in Malawian Grades 9 and 10 mathematics textbooks. Altogether, four textbooks were analyzed which constituted 309 tasks under the topics of Equations and Simultaneous equations. The MDITx framework which reveals the opportunities to learn on a scale level of 1–3 was used. Level 2 and level 3 were considered as opportunities for problem-solving with more preference for level 3. To address the objective of the study, two research questions were addressed: *(1) How are problem-solving opportunities provided in Malawian grades 9 and 10 mathematics textbooks?* and *(2) What factors influence the opportunities to learn problem solving in grades 9 and 10 mathematics textbooks?* The overall results revealed that there are fewer tasks that could be considered as genuine problem-solving tasks. Many problem-solving opportunities were presented in word problems. Only three problem-solving tasks were not presented in word problems. Interestingly, in grade 9, the word problems were explicitly introduced as problems.

The study indicated that grade 9 and 10 textbooks offer the opportunity to students to learn problem solving. However, more provision is on a lower level (level 2) than on a higher level (level 3). According to the MDITx, level 2 tasks require the application of the current procedures which means many of the tasks in the textbooks are similar to the examples. They further required the application of easily accessible methods from the textbooks. Therefore, going by the preference of level 3 tasks as the best opportunities for problem solving, the study concluded that the opportunities are limited. More opportunities are needed to enhance problem solving. Otherwise, the textbooks could promote rote-memorization.

Other studies also found the same in textbooks from other countries (e.g., Brehmer et al., 2015; Buishaw & Ayalew, 2013; Jäder et al., 2020). Furthermore, the study found that problem solving in Malawian textbooks primarily means working with word problems. The presence of more level 3 tasks in the word problem-sections revealed this. Kieran (2007) argued that word problems are challenging to students in terms of generating equations from them. They are designed for students to apply the algorithms they have learnt in a topic. In the Malawian textbooks, the notion is similar. The word problems are presented to be difficult tasks at the end of the topic. Yet not all tasks were difficult. Another view of problem solving revealed in the textbooks is that of solving any problem in the textbook. This could be attributed to the individualistic nature of a problem (Fan & Zhu, 2007).

The study further revealed three factors that influence problem-solving opportunities. The first factor is detailed examples. This is when in the examples, complex structures are used instead of simple ones. For instance, in the study, fraction equations were used to illustrate the substitution method which rendered the subsequent tasks less of problem-solving tasks. Other details included giving extra information on how to tackle other problems whose structures were not solved but could potentially be problem-solving tasks if the details were not given. Second, not introducing lessons as challenges. This factor revealed that the lessons were introduced in such a way that students were not given an activity to test their prior knowledge. In addition, the lessons were structured in a teacher centered way, thus, giving no room for discovery learning and inquiry (Buishaw & Ayalew, 2013). They began with objectives, examples, then tasks. In these sections only extra information was included and no challenges. The last factor is leading questions and lack of prompting questions. It was found that the questions in some tasks indicated the method that should be used for solving it. In some tasks, though the method was not stated, yet the structures of the tasks were similar to those of examples hence they could lead the students to refer to the examples. No prompts to solve tasks in other ways were given. This is despite studies revealing that prompting enhances creativity in equations (Qetrani et al., 2021; Star et al., 2022; Xu et al., 2017).

In general, textbooks offer less opportunities for students to learn problem solving. Many tasks are similar to examples and require the students to use standard procedures and not create one. Even those considered as problem-solving tasks require the application of already known procedures. Therefore, the foundation of problem-solving that is laid by lower secondary school mathematics textbooks is not sufficient but on the right path. It is insufficient because, firstly it is unbalanced since more problems are in word form hence there is a need to include more that are not in word form. Secondly, those tasks that are problem-solving tasks require the use of standard procedures and are not cognitively demanding. Most importantly, the teaching methodological approaches that are listed in the SSCAR (2015) meant to promote problem solving, are not reflected in the textbooks. These include: teaching and learning that encourages active participation of all students, students discover and use individual learning techniques, encourage independent study and research among students, among others. There is a need for the incorporation of tasks that would promote these approaches.

6.1 Implications to teaching and learning of mathematics.

The study revealed that there are less opportunities to learn problem solving at a higher level. At the lower level of 2 there are numerous opportunities, however the complexity is not demanding. Of the problem-solving tasks, only three were not word problems while the rest were word problems. It implies that in Malawi, the implementation of problem solving is left for the teacher. They need to formulate tasks that would offer these opportunities to students. Unfortunately, there are "inadequate qualified teachers especially in STEM subjects. As a result, there are teachers who teach subjects in which they are not specialized" (MoEST, 2020, p. 22). According to the 2022 Education Statistics Report, there are 1222 qualified mathematics teachers against 7337 secondary schools. In fact the "Pupil qualified Teacher Ratio has been declining from 2018 at 41 to 32.3 in 2022 (MoEST, 2022, p. 103)". This means the quality of teachers who could engage in problem solving on their own is low since this is a higher order concept. It could be hard for teachers who have not been trained as mathematics teachers to realize the lack of the concept in textbooks. As a result, students shall remain exposed to mere exercises in the textbooks. In some instances, teachers might shun away from the already available challenging tasks due to lack of knowledge.

As already revealed in literature, textbooks are a major mathematics instruction tool in Malawian mathematics classroom. The textbook compliance by teachers is high as they use it to plan and implement tasks (Mwadzaangati, 2019a). Consequently, the limited opportunities for problem solving in textbooks directly limit the presence of the opportunities in classroom. So, the findings of this study are even more problematic in Malawi than in affluent countries like Singapore and China who share similar results. Such is the case because the western countries have other alternative instruction tools like technology, such that reliance on textbooks has been decreased in recent years (Qi et al., 2018).

The study also showed that the three factors that influence the availability of problem-solving opportunities tend to increase the similarities between tasks and examples. Consequently, the work assigned to students is simplified and does not propel higher-order thinking. The implication of this is that students may not develop the ability to solve difficult problems when they encounter one. Furthermore, the students' ability to formulate their own solution procedures is not supported since many of the tasks have got solution templates or guides in the textbooks. In addition, the textbooks do not prompt the students to come up with their own solution procedures, hence they are confined to the standard procedures. This does not undermine the importance of the solution templates; they are essential to aid the students in understanding the concepts being presented. Moreover, it is procedural fluency, conceptual understanding which promote strategic competence (Kilpatrick et al., 2001). Nevertheless, the textbooks promote students to mainly be good at solving exercises and not at problem solving. This might promote rote memorization in the long run.

The study also showed that problem solving is understood as solving word problems which are mostly located at the end. It appears that these word problems are meant for students to apply the methods that have been learned and for review. No wonder not all word problems were as challenging as problems ought to be. This does not reflect what is contained in the curriculum concerning mathematics as a propeller of critical thinking, reasoning and problem solving.

6.2 Recommendations and further research

The study revealed the deficiency of problem-solving opportunities in the textbooks. It is recommended that more problem-solving tasks are incorporated to provide more opportunities for students to learn problem solving. Mathematics is one of the subjects that need the incorporation of problematic situations (Buishaw & Ayalew, 2013). In addition, there should be sufficient problem solving tasks that are not in word form to curb the language effect that might hinder other students from executing the mathematical problems. Furthermore, the curriculum should define problem solving and what is expected of the textbooks in promoting the same. There appears to be a discrepancy between the experts' definition of problem solving and what is presented in the textbooks. For Malawi, the discrepancy might be due to the lack of a section in the curriculum which discusses textbooks regarding problem solving. The only discussion about textbooks is the textbook policy about approval (MoEST, 2015). Lastly, it is recommended that teachers should implement textbooks tasks in a way that would challenge students so that they learn problem solving.

Further research could focus on replicating this study but on all textbook series for individual grades. The study might also include other topics. A comparative analysis between grade 9 and 10 could be carried out and find if there are any differences in terms of the opportunities and why. A quantitative study replication for the study could also be done to find out the extent to which the textbooks each offer the opportunities for problem solving. Another study could involve a comparative analysis with textbooks of upper secondary school to ascertain if problem solving opportunities are increasing as students move to upper grades.

6.3 Limitations of the study

The first limitation stems from the sample. Only two topics were analyzed – Equations and Simultaneous linear equations – which do not represent the whole lower secondary school mathematics content. In addition, only the concept of algebra was tackled, and just two textbook series were analyzed. As acknowledged earlier, the left-out textbooks might have given different results. Nevertheless, this study could be replicated such that all mathematical concepts could be analyzed. It might also include more textbook series from the junior section. Alternatively, a different sampling technique could be used.

Related to the first limitation, the sampling was built on anecdotal evidence, assumptions and little consultation. Although the selected textbooks are certified by the MoEST, schools and students might prefer using one selected series to another series. If such information was available, the relevance of the textbooks vis-à-vis usage would be certain.

The study's framework was also a limitation because the construct of fusion was unclear and therefore not included in the analysis. The researcher believes that if fusion had been clear and included, the results may have been different, potentially spreading to level three cognitive levels. However, until the construct is clarified, the results should be interpreted in light of the modified framework. Furthermore, this framework was adapted for study as such some modifications might need further improvement.

The last limitation is that the focus was on the textbook's representation of the tasks and not on how the tasks are implemented in the classroom. In practice, teachers could present tasks in a way that might cognitively vary as presented in the textbooks. Similarly, the interpretation of the examples and tasks was based on a theoretical point of view of the researcher. In practice, the same examples and tasks would have a different code or level if responses are given by students themselves.

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Appendix

Figure 26 shows the left out content from Book B and the reasons are given below.

2. State whether the following are true or false.

(a) $3 + 6 = 7 + 2$ (b) $7 - 3 = 6 - 2$
(c) $6 + 2 = 6$ (d) $5 + 6 + 1 = 11$
(e) $10 + 7 + 4 = 13 + 4$ (f) $5 + 6 - 3 = 9$
(g) $3 + 5 - 2 = 8$

3. State whether the following are true, false or open.

(a) $2 - 5 = 3$ (b) $8 + 13 = 23$
(c) $^{-}2 + 9 = 7$ (d) $7 - x = 0$
(e) $2 = 2x - 3$ (f) $13 - 17 = {}^{+}7 + {}^{-}3$
(g) $(^{-}24) \div (^{+}3) = (^{-}8)$ (h) $\frac{(-6) \times (-5)}{10} = 3$

4. Copy and complete the following to make them true.

(a) $4 + 7 =$ (b) $5 + 1 =$
(c) $4 - 1 =$ (d) $6 - 6 =$

Exercise 9.2

1. Copy and complete the following to make them true.

(a) $18 \div 3 = \square$ (b) $5 + \square = 7$
(c) $9 - \square = 7$ (d) $3 \times \square = 6$
(e) $6 + \square = 9$ (f) $24 \div \square = 6$
(g) $4 \times \square = {}^{-}12$ (h) $7 + 4 + \square = 13$
(i) $3 + 4 + \square = 10$ (j) $8 + 2 + \square = 6$
(k) $4 + 5 - \square = 2$ (l) $4 + 6 - \square = 2$
(m) $6 + \square + 4 = 3$
(n) $3 + 7 + 1 = 9 - \square$
(o) $8 + 3 + 2 = 10 + \square$
(p) $6 + 5 = 11 - \square$
(q) $6 + 6 - 3 = 12 - \square$
(r) $3 + 12 - 8 = 15 + \square$

Exercise 9.1

1. Which of the following are equations?

(a) $3 - 5 + 1$ (b) $6 - 5 = 1$
(c) $5 + 2 - 6$ (d) 4×6
(e) $3 - 9$ (f) $3 \times 8 = 24$
(g) $16 - 9 = 3 + 4$ (h) $4 + 17 - 3$
(i) $6 \times 4 = 3 \times 8$ (j) $17 = 13 + 4$

Figure 26: Left out content (Thomo et al., 2015, pp. 108–109)

The study primarily as indicated earlier was focused on algebra. The tasks above are not algebraic. Specifically, the study focused on solving equations. Whereas the above tasks were meant to distinguish equations from expressions. The tasks are more of acting as a reminder about the concept of equations in general that students had learned in primary school. This meant referring back to primary school textbooks for a proper analysis. That was beyond the scope of this study. Thirdly, it appears that this is the only book that included such an introductory part among all books. This might imply that the curriculum does not consider this as an objective. Alternatively, it could be done in class by teachers when introducing the topic together with students just to elicit their prior knowledge. In support of the preceding point, it seems that the curriculum treats this as something that students already know and have completely assimilated. It cannot be challenging at this level, as such it should not be tested. Therefore, these tasks are the author's choices. The example spaces also are incomplete due to lack of solved examples. Therefore, they do not meet the definition of an example space according to Ronda and Adler (2015). This occurrence makes it difficult to code without any reference to a set of examples so that sound distinctions are made to establish the focus of both the examples and tasks.