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Hard money and fiat money in an inflationary world

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ABSTRACT

The purpose is to determine whether a borrower prefers to borrow hard and fiat money from a bank to buy other assets from a seller, whether the seller wants to sell, how the nontraders are impacted, and whether the bank prefers to lend money and print or withdraw fiat money. The method is to compare the agents' and bank's Cobb Douglas utilities over two periods. The conclusions are that the bank prefers to print fiat money to a certain extent. Fiat money printing benefits the borrower/buyer which prefers inflation, benefits the bank if not excessive, and hurts the seller and nontraders. The seller and nontraders prefer a hard money economy or a fiat economy where the bank withdraws money to ensure deflation. More nontraders decrease inflation since the bank's money printing gets spread across more agents. The article provides further results illustrated by varying 64 parameters relative to a benchmark.

1. Introduction

This article introduces hard money and fiat money in a two-period economy. The big general idea in the article is to model three kinds of agents and a bank intended to capture a major part of what occurs in today's economies. The three kinds of agents are an agent which is a borrower and buyer, an agent which is a seller, and nontrading agents. The borrower borrows hard money and fiat money from the bank and buys other assets from the seller. The seller and nontrading agents hold hard money, fiat money and other assets. An agent's Cobb Douglas utility depends on its asset portfolio, that is, on whether the agent holds hard money, fiat money, other assets, loans in hard money, or loans in fiat money. The bank, which also has a Cobb Douglas utility, can lend hard money and fiat money to the borrower, earning interest, and can print and withdraw fiat money which may cause inflation or deflation which impacts the agents. In the model, the bank is a unified actor that represents a central bank and one or several commercial banks.

The article's research question and purpose are to determine how the three kinds of agents and bank are impacted in their Cobb Douglas utilities over two time periods when operating as specified, i.e. borrowing, selling, holding money and assets, printing and withdrawing fiat money, etc. For example, does the borrower prefer to borrow hard money or fiat money excessively or to a limited extent to acquire other assets? Does the seller want to sell other assets? How are the nontrading agents impacted by holding money and assets? How is the bank impacted by lending hard money and fiat money? Does the bank want to print or withdraw fiat money? The economic approach in the article, with three kinds of agents and one bank, is designed with the intention of being especially well

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equipped to match and answer these questions.

For hard money printing, withdrawal, inflation, and deflation are assumed to be infeasible. Gold does not have a fixed supply, i.e. the global gold supply increased by approximately 2% per year on average since 2013 (World Gold Council, 2023). The current annual growth rate of the Bitcoin supply is approximately 1.64% (Money Printer, 2023), which gradually decreases to zero over time until the maximum supply of 21 million Bitcoins is reached in ca 2140. Although the supply of gold and Bitcoin are currently not fixed, they are considered as two approximate examples of hard money. The opposite is assumed for fiat money. The US dollar is not hard money, but fiat money, per this article's definition.

A model is formulated to analyze the coexistence of hard and fiat money. Agent 1 borrows hard and fiat money from the bank and buys other assets from a seller. Agent 2 sells some of its other assets to agent 1 and does not borrow from the bank. Agent i, as a nontrading agent, $i = 3, \ldots, n$, does not borrow, lend, buy, or sell. Its asset portfolio remains unchanged over the two periods. The article introduces a theoretical model for studying the competition between hard and fiat money, analyzing the effects on agents and one bank of printing and withdrawing fiat money. It examines the implications on inflation and deflation of borrowing, lending, buying and selling.

Inflation, i.e. the rate at which the average price of goods or services increases over time, generally depends on the money supply (Gorton, 2023) and various other factors such as production, the logistics of making goods or services available, and consumer preferences. In this article's model, the adjustment of the money supply gets linked to other assets through the borrower/buyer buying other assets from the seller at a certain value, and through the nontraders holding other assets with a certain valuation. Hence in the agents' utilities, the price or value of these other assets changes depending on the adjustment of the money supply which causes inflation or deflation.

The impact of printing and withdrawing fiat money for the bank and the agents is examined. Comparisons are drawn between borrowing hard and fiat money. The article demonstrates how the utilities of the bank, of the agent which is a borrower and a buyer, of the agent which is a seller, and of the nontrading agents change as the values of the parameters for hard and fiat money vary. The resultant insights may enable central banks and individuals to develop a superior understanding of borrowing, buying, lending, selling, inflation, deflation, money printing, and withdrawal in a fiat economy and in a hard money economy.

The article analyzes inflation and deflation resulting from printing and withdrawing fiat money, abstracting away demand and supply shocks which require more extensive modeling. The nontrading agents are shown to be vulnerable in a fiat economy with money printing. A borrower and buyer benefit from borrowing fiat money. A seller benefits when the bank withdraws fiat money. The bank benefits from printing fiat money to a certain extent. The article explores hard and fiat money in a two-period economy. Inflation caused by fiat money printing or deflation caused by fiat money withdrawal are part of the article's research topic. The article analyzes the effects on one unified bank and multiple agents of printing and withdrawing fiat money. It compares the impacts of borrowing hard money versus fiat money. The article illustrates how the utilities of the bank, borrowing agents, buying agents, selling agents, and nontrading agents change with varying parameter values for hard and fiat money. It highlights the vulnerability of nontrading agents in an economy that employs fiat money printing. Advantages are discussed of borrowing fiat money for borrowers and buyers, while sellers benefit when the bank withdraws fiat money. The article posits that a bank within certain limits can derive benefits from printing fiat money.

The article more generally demonstrates how the bank, the borrower and buyer, the seller, and the nontrading agents get impacted by changes in parameter values. Features of hard money are illustrated, i.e. limited supply and outside the bank's control. Borrowing hard and fiat money is shown to have different impacts. The existence of hard money decreases the impact of inflation caused by printing fiat money. The article presents a benchmark where the bank prefers to lend hard money and fiat money to agent 1, agent 1 prefers to borrow hard and fiat money from the bank to buy other assets from agent 2, and agent 2 prefers to sell some of its assets to agent 1. The article illustrates how changing each of 64 parameter values relative to the benchmark impacts the preferences of the bank and the three kinds of agents. The article offers insights into questions such as the impacts of borrowing, buying, lending, selling, inflation, deflation, money printing, and withdrawal in both hard and fiat money economies on the bank's and agents' utilities. This article supplements the almost nonexistent analyses of the interaction between hard and fiat money. Overall, the article sheds light on the coexistence of hard and fiat money, providing valuable insights into their dynamics.

Section 2 presents the background. Section 3 reviews the literature. Section 4 presents the model. Section 5 analyzes the model. Section 6 illustrates the model. Section 7 provides an interpretation. Section 8 covers policy implications. Section 9 discusses the results. Section 10 covers limitations and avenues for future research. Section 11 concludes.

2. Background

Bitcoin (Nakamoto, 2008) is a decentralized digital currency which operates on a peer-to-peer network. It is not backed by any physical asset, government, or central authority. Historically, hard money approximated by gold, has been adopted widely. Bitcoin, without centralized parties or intermediaries, has different potential than physical gold related to censorship resistance, verifiability, portability, divisibility, convenience, and scarcity (Ikkurty, 2019). Central banks explore CBDCs (central bank digital currencies) to build efficient fiat payment systems and compete with cryptocurrencies. Hard money has real value and commands broad acceptance as a medium of exchange. Hard money is scarce, decentralized, has fixed supply, is difficult to counterfeit or manipulate, and cannot be printed. One further example is representative money (Nicholson, 1888; Steiner, 1941) that is backed by and redeemable for gold. Bitcoin's status as a hard currency is being debated. Although both gold and Bitcoin resemble hard money, current empirics illustrate differences. Long et al. (2021) apply the nonlinear autoregressive distributed lag model to show that gold can, while Bitcoin cannot, hedge against uncertainties to varying degrees. Wen et al. (2022) show during the Covid-19 pandemic that gold is, while Bitcoin is not,

Table 1
How printing and withdrawing fiat money impacts fiat money holders and borrowers. Downward arrow ↓ means negative impact. Upward arrow ↑ means positive impact.

	Fiat money holder	Fiat money borrower
Printing fiat money	↓	↑
Withdrawing fiat money	↑	↓

a safe haven for oil and stock markets. Some of this may be related to gold's market capitalization at ca $13 trillion[3] compared against Bitcoin's $0.6 trillion.[4] Boissay et al. (2022) describe the blockchain scalability and how high fees may fragment the crypto landscape, implying that, at least for now, cryptocurrencies cannot be a substitutive form of fiat money. This may change as the Lightning network and other innovations emerge.

Hard money has been used as a medium of exchange and as a store of value throughout history, partly because it is valuable and scarce. Precious metals, such as gold and silver, were adopted as money by historical civilizations around the world. Coins were frequently produced from metals, e.g. gold, silver, copper, which simplified transactions and promoted trading between various communities. Hard money is still recognized as a store of value in modern society, particularly during times of economic crises, because hard money is thought to retain value better than fiat money, which is susceptible to inflation. Hard money encompasses a form of currency or monetary system that relies on a commodity, i.e. a fixed asset with intrinsic value or decentralized consensus. Two approximate examples are Bitcoin and gold. Hard money is characterized by a fixed or limited supply, which sets it apart from fiat money which lacks physical asset backing and derives its value solely from trust in the government. Hard money provides a perceived sense of stability and limits the potential for inflation or devaluation, as the supply is constrained. In contrast, fiat money relies on trust in the government and central banks. Its value typically decreases over time due to fiat money printing or inflation.

The supply of Bitcoin is fixed at ca. 21 million. Bitcoin can be viewed as a form of hard money and is legal tender in two countries, i.e. El Salvador and the Central African Republic. More countries may adopt Bitcoin as legal tender in the future. Iwamura et al. (2019) discuss the potential competition between Bitcoin and central bank-issued fiat money. Ammous (2018) suggests a Bitcoin standard for nations.

Central banks are responsible for the issuance and governance of fiat money. Central banks can vary the supply of fiat money supply by printing it, for example by buying bonds and securities from the open market, or by withdrawing it, e.g. selling bonds and securities to the open market and thus destroying or burning the earned fiat money. However, central banks cannot print hard money since the supply of hard money is fixed. Hard money has the advantage of being a more reliable store of value than fiat money, which is not backed by tangible goods. Hard money is less vulnerable to inflation than fiat money due to its limited supply. Hard money thus provides stability for individuals and companies.

If fiat money is printed excessively, inflation and a decrease in the purchasing power of the currency may follow, which can disproportionately affect those who hold it, particularly those on fixed incomes or with savings in cash. Borrowers may benefit from expansions of the money supply if it causes lower interest rates and easier access to credit, but this can also contribute to inflation and a devaluation of the currency. The Cantillon effect (Murphy, 1986) is that the distribution of newly created money reaches different kinds of agents at different points in time which can affect the relative prices of goods and services disproportionately, also impacted by production and consumption patterns, market competition, and government policies. The withdrawal of fiat money from the economy may make it more expensive for borrowers to service their loans, but it can also lead to deflation and a decrease in economic activity, which can harm both borrowers and savers.

Printing fiat money may not necessarily entail confiscation or violation of property rights if done responsibly to maintain the stability and value of the currency. Obtaining such stability can be challenging and depends on supply, demand and other factors. With certain assumptions, printing fiat money does entail confiscation and violation of the property rights of those who hold it. Then borrowers benefit from expansions of the money supply, and withdrawing fiat money benefits savers and makes it more expensive for borrowers to service their pre-existing loans. Table 1 illustrates the negative ↓ and positive ↑ impact of printing and withdrawing fiat money on fiat money holders and borrowers. Since money printing dilutes the monetary value, fiat holders and borrowers are negatively and positively impacted, respectively. Money withdrawal has the opposite impact.

3. Literature

The limited literature on this topic covers five topics, namely hard money, competition between fiat currencies, competition between cryptocurrencies and fiat currencies, cryptocurrencies and CBDCs, inflation and currencies, and gametheoretic analyses.

3.1. Hard money

Fisher (1920) warns that "irredeemable paper money has almost invariably proved a curse to the country employing it." The world will experience unstoppable inflation unless the leading nations implement commodity or hard money standards. Cooper et al. (1982)

[3] https://8marketcap.com/metals/, retrieved August 7, 2023.
[4] https://coinmarketcap.com/, retrieved August 7, 2023.

point out that the primary motivation for reviving the gold standard is to eliminate inflation and to maintain a stable noninflationary environment. They propose a commodity standard that goes beyond gold. In their view, such a standard would stabilize general price levels. Friedman and Schwartz (1986) summarize the main pillars of monetary reform, namely the government monopoly on money creation, free banking, and the determination of units of account. They point out that Austrian economists support hard money and oppose discretionary money management. Ammous (2018) points out that individuals will gradually migrate from national money to hard money, which preserves value more effectively. Examples include seashells, glass beads, iron, copper, and other primitive forms of money, which were eventually replaced by gold and silver. Ammous and D'Andrea (2022) investigate the link between time preferences, money, and hard money. They point out that fiat money is expected to lose value over time due to inflation, which increases uncertainty, thus disincentivizing saving. However, forms of hard money such as Bitcoin are expected to maintain their value and purchasing power over time. Therefore, hard money reduces uncertainty and encourages savings. A hard money standard can lead to higher levels of social development. This article contributes to this literature by exploring the different impact of loans in hard and fiat money on various agents. Bibi (2023) explores the nature of money, focusing on cryptocurrencies such as Bitcoin and their potential impact on monetary systems. The author argues that state acceptance and citizens' adoption are crucial for Bitcoin to become money. The article highlights the potential influence of factors on the success and sustainability of Bitcoin, e.g. institutional pressures, convenience, environmental concerns, and the emergence of CBDCs. This article focuses mainly on the incentive of the bank to offer hard and fiat money loans and on the agents' incentives to apply for loans of the two kinds. The bank cannot print hard money, but it can create fiat money.

3.2. Competition between fiat currencies

Fernández-Villaverde and Sanches (2019) develop a model of competition between privately issued fiat currencies. They introduce entrepreneurs who can issue private currencies in a Lagos-Wright environment. They found that competing private currencies can coexist, but their coexistence does not necessarily result in efficiency or stability. Dowd and Greenaway (1993) analyze currency competition. They discover that network effects and switching costs cause agents to favor the use of a single currency. Mafi (2003) investigates the relationship between currency competition and inflation. She finds that countries in which citizens are legally allowed to hold foreign currencies tend to have lower average inflation rates. This result suggests that currency competition could lead to lower inflation. Eichengreen (2005) adopt a historical approach to competition between reserve currencies. He points out that competition for reserve-currency status is not a winner-takes-all game. Instead, it is likely that multiple currencies will continue to hold that status in the future. He predicts that the dollar and the euro will likely remain the dominant reserve currencies for the foreseeable future. Martin and Schreft (2006) challenge the traditional view that currencies cannot coexist. They demonstrate the existence of equilibria in which outside money is issued competitively. The findings show that it is unclear whether competing currency issuers can produce allocations superior to those that result from a monopolist issuer. Gawthorpe (2017) also shows that currency competition can lead to lower inflation rates than the exclusive use of a single fiat currency. Wang and Hausken (2021a) investigate competition between a national currency and a global currency across three different groups of agents, namely conventionalists, pioneers, and criminals. Currency features such as backing, convenience, confidentiality, transaction efficiency, financial stability, and security are represented in the model. The authors show how the three kinds of agents choose between the two currencies. Ron and Valeonti (2023) show during the US Civil War how more democratic governing institutions in the North impacted the legitimacy of tax policies and enabled more effective backing of the currency to cause moderate inflation, as opposed to the South which experienced hyperinflation. This article contributes to this literature by investigating competition between hard and fiat money. The bank can print and withdraw fiat money, causing increased inflation. The supply of hard money is fixed. The article shows how printing and withdrawing fiat money affects borrowers, non-borrowers, buyers, and sellers in an economy.

3.3. Competition between cryptocurrencies and fiat currencies

Almosova (2018) considers the calculation costs that private currencies entail, such as the expenses associated with mining and transaction verification. She finds that currency competition does not lead to price stability. However, the circulation of less costly private currencies exerts downward pressure on inflation. Schilling and Uhlig (2019) examine competition between a fiat currency that is used for daily payments and a cryptocurrency that can be used to avoid taxes, to maintain anonymity, and to resist repression. The results show that the substitution effect between fiat currencies and cryptocurrencies declines as asymmetries in trading costs and exchange fees become more pronounced. Senner and Sornette (2019) think that forms of fixed-supply money such as Bitcoin are negatively affected by their speculative and deflationary designs. The supply of stablecoins such as Tether can be varied. However, neither Bitcoin nor stablecoins are backed by governments or central banks. The authors contend that existing cryptocurrencies cannot replace fiat money. Jumde and Cho (2020) explore whether cryptocurrencies could eventually overtake fiat money. They employ the analytic hierarchy process method. Nine factors, namely accessibility, constant utility, value-common assets, stability, convertibility, divisibility, liquidity, volatility, and possibility of speculation, are introduced to analyze the performance of cryptocurrencies and fiat money. The findings show that fiat money is preferred to cryptocurrencies. Levulytė and Šapkauskienė (2021) explore the connections between cryptocurrencies and fiat money from the perspective of the three functions of money, i.e. medium of exchange, a unit of account, and store of value. They point out that cryptocurrencies such as Bitcoin and Ethereum are useful for cross-border transactions.

The results also show that fluctuations in cryptocurrency prices are affected by fluctuations in the prices of fiat currencies. Sissoko (2021) discusses the hypothetical scenario in which agents can buy goods at fixed rates by using various currencies. He emphasizes that a financial system can be established accordingly. The effectiveness of the banking system depends on its capacity to increase the money supply in response to societal needs. Wang and Hausken (2022a) explore how competition between a variable-supply currency, such as fiat money, and a fixed-supply currency, such as Bitcoin, impacts agents' choices of currency. They rely on a money-in-utility approach. They consider money printing and withdrawal, and an agent's support of money, i.e. backing, convenience, transaction efficiency, financial stability, confidentiality, and security. They analyze the dynamic volume fractions of transactions in two currencies over time. Yu (2023) adopts a search theoretic model to explore the conditions under which fiat money and cryptocurrencies coexist. For cryptocurrencies to exist, the inflation rate in a stationary monetary equilibrium must be zero. The growth rate of the money stock determines the inflation rate for fiat currencies. The findings show that cryptocurrencies can coexist with fiat money. In addition, under the zero-inflation equilibrium, bans on cryptocurrency may decrease social welfare due to the inflation tax. Helmi et al. (2023) apply a time-varying vector autoregressive model to examine the impact of CBDC news on financial and cryptocurrency markets. They find that CBDC uncertainty and volatility index shocks contribute significantly to cryptocurrency uncertainty and Bitcoin return shocks. This article considers competition between variable-supply fiat money and forms of fixed-supply hard money, such as Bitcoin. Agents gain utility by holding hard money, fiat money, other assets, and by borrowing. The article studies how the bank can lend hard or fiat money to the agents and the impact of that lending on the bank and the agents.

3.4. Cryptocurrencies and central bank digital currencies (CBDCs)

Belke and Beretta (2020) recommend that central banks accept the technology that powers cryptocurrency, and develop a well-regulated two-tier system by engaging in innovation in the domain of payment infrastructures. Nabilou (2020) points out that cryptocurrencies such as Bitcoin may pose risks to the monopoly of central banks over the issuance of money, to price stability, to the smooth operation of payment systems, to the execution of monetary policy, and to the stability of financial institutions. Accordingly, central banks explore CBDCs. He notes that the European Central Bank must overcome several legal challenges before introducing CBDCs at the Eurozone level. Laboure et al. (2021) summarize the evolution of cryptocurrencies and CBDCs. They predict that cryptocurrencies and fiat money will coexist in the near future. They also note that numerous concerns, including ones that have to do with energy efficiency, transaction speed, identity problems, and regulation, must be addressed before cryptocurrencies can be accepted widely. Scharnowski (2022) explores market reactions to speeches on CBDCs from the perspective of cryptocurrency investors. He finds that cryptocurrency prices tend to react more strongly to positive speeches, while negative CBDC sentiment has a slight amplifying effect. The findings indicate that investors do not view CBDCs as a threat to cryptocurrencies. Benigno et al. (2022) examine competition between national currencies such as CBDCs and global cryptocurrencies such as Bitcoin in a two-country economy with complete markets. They conclude that national nominal interest rates must be equal in the two countries at the time when a global cryptocurrency is adopted. Deviations from interest rate equality indicate that there is a risk of the national currency being abandoned. They call this feature of the model "crypto-enforced monetary policy synchronization." Adrian and Mancini-Griffoli (2021) consider benefits and risks of digital money compared with traditional money, and assess digital money backed with central bank reserves as a private-public partnership. Ayadi et al. (2023) employ a Cross-Quantilogram model. They reveal a negative association between the CBDC uncertainty index and the returns of cryptocurrencies and stablecoins. This article adds to the literature by evaluating competition between forms of hard money approximated by Bitcoin, which are supported by a proof-of-work consensus mechanism, and forms of fiat money exemplified by CBDCs, paper money, and coins. CBDCs are one form of fiat money that central banks issue, support and supervise. Hard currencies, conversely, typically have a fixed supply because they are backed by assets such as commodities and gold or by consensus algorithms. This article contains a model that illustrates the effects of hard and fiat money lending and borrowing on the economy. It also discusses the conditions under which a bank is prepared to lend and those under which agents are willing to borrow from the bank to buy assets from other agents.

3.5. Inflation and currencies

Sakurai and Kurosaki (2023) find that major cryptocurrencies become slightly better inflation hedges after the reopening after the Covid-19 pandemic, regardless of whether they have a maximum supply cap. Xin and Jiang (2023) develop a dynamic stochastic general equilibrium model to show that CBDCs can stabilize the economic fluctuations caused by a negative interest rate policy implemented by interest rate adjustment to reach various economic objectives such as monetary stimulation, stable exchange rates, and desired inflation levels. Feres (2021) analyzes how the US Federal Reserve handles crises associated with fiat, debt and inflation. He recommends a transition to a monetary system backed by a finite commodity. Messay (2023) develops an idealized model as a thought experiment to show that an international fiat currency issued by one or several core countries is a main factor impacting national economic development, and that seigniorage accrued to developed countries by consuming more than they produce is at the expense of the developing countries in the Global South. This article analyzes how inflation relates to the coexistence of hard money and fiat money.

3.6. Gametheoretic analyses

Welburn and Hausken (2017) explore economic crises from a gametheoretic perspective. They introduce six kinds of agents, i.e. countries, central banks, intergovernmental financial organizations, banks, firms, and households. These agents can adopt various strategies, such as setting interest rates, lending, borrowing, and consuming. The authors use the model to illustrate the European debt crisis. Hart (2020) models the positive input consumption and the negative input pollution with a constant elasticity-of-substitution function. Since pollution has a negative impact, the corresponding exponent, which is the elasticity of the Cobb Douglas utility, is negative. This article uses a similar approach—loans are raised to a negative exponent in the borrower's Cobb Douglas utility because the borrower must pay interest to the bank. Mou et al. (2021) develop two gametheoretic models of CBDC adoption in different countries. The findings indicate that each country should issue a CBDC, regardless of the choices of other countries. The leading country needs to issue a CBDC to maintain its status. Other countries must also introduce a CBDC in order to avoid losing ground in the digital realm. Wang and Hausken (2022b) establish a game between a central bank and a household choosing between a CBDC, a non-CBDC such as Bitcoin, and consumption. The central bank determines the CBDC interest rate, which can be negative. The household chooses its portfolio while accounting for backing, transaction efficiencies, and costs. They demonstrate how the bank and the household choose their strategies. The outcome is determined analytically and illustrated numerically. This article relates to this literature by considering the interactions between a bank and the agents. A bank may choose to lend or not to lend hard or fiat money to an agent. An agent may choose to borrow or not to borrow hard or fiat money from the bank. The other agents may choose to sell or retain their assets or do nothing. The article shows the impact of these strategies.

4. The model

This section develops the model for n agents in Section 4.1, the one bank in Section 4.2, and the inflation rates in Section 4.3. The model is chosen to be minimally complex while simultaneously capturing reality. The model features one bank as a unitary actor, along with n agents consisting of one borrower and buyer, one seller, and $n-2$ nontraders over the two periods. The article establishes the Cobb Douglas utility function (objective function) for both the bank and the agents, following a step-by-step process as outlined in this section. The article employs a money-in-utility approach, where utility is derived from the possession of money or assets. This approach is commonly utilized in economic and financial research. The underlying conception is that the utility function captures an individual's preferences regarding a range of goods and services. Various studies have applied the money-in-utility approach, e.g. Ramsey (1928) and Sidrauski (1967). Recent examples include the research by Chen and Guo (2014), Mian et al. (2021), and Ferrari Minesso et al. (2022). The homogeneity of asset classes is determined by their Cobb Douglas utility elasticities. Appendix A shows the nomenclature.

4.1. The n agents

Subsection 4.1.1 considers period 1 for the three kinds of agents. Subsection 4.1.2 considers period 2 for the three kinds of agents. Agent i, $i=1,\ldots,n$, has a Cobb Douglas utility U_{it} with multiple inputs in period t, $t=1,2$. Agent i can be a household, or any agent, e.g. firm, institution, organization. In period t, agent i holds maximum three kinds of assets with value j_{it}, $j_{it}\geq 0$, $j=q,m,o$. The article employs a Cobb Douglas utility function and includes assets within the utility function. Other examples applying this approach are Ferrari Minesso et al. (2022); Syarifuddin and Bakhtiar (2022); Wachter and Yogo (2010). The agents assess their utilities across two periods and opt for trading in period 2 if the utility in that period surpasses the utility in period 1. That is a realistic description of an economy to some extent. Therefore, an intertemporal optimization approach is not employed in the article. These assets are hard money q_{it} and fiat money m_{it} deposited in the open market (e.g. in the stock, bond or decentralized finance markets), and other assets o_{it}. Examples of other assets o_{it} are anti-inflationary investments, non-fungible tokens, bonds, stocks, other financial assets, real estate, physical assets, and illegal assets. Holding asset j_{it} earns interest rate I_{jt}, $I_{jt}\in\mathbb{R}$, $j=q,m,o$, from the open market, as determined by the open market. Each Cobb Douglas input is raised to the Cobb Douglas elasticity α_{ijt}, $\alpha_{ijt}\geq 0$, $j=q,m,o$, which accounts for asset j's liquidity, backing, convenience, confidentiality, transaction efficiency, financial stability, and security.

4.1.1. Period 1

4.1.1.1. Agent 1. Assume, without loss of generality in choice of agent, that agent 1 in period 1 borrows L_{1q1} in hard money and L_{1m1} in fiat money from the bank and buys an asset valued as $L_{1q1}+L_{1m1}$. Agent 1's borrowing interest rate is r_{j1}, $r_{j1}\in\mathbb{R}$, $j=q,m$. Multiplying agent 1's loan L_{1j1} with $1+r_{j1}$ to account for the interest rate r_{j1}, and inverting since a loan L_{1j1} with interest rate r_{j1} is costly for agent 1 causing negative impact on agent 1's utility U_{11} (just as pollution is costly in Hart's, 2020 model, see Section 3.6), gives the input $\left(\frac{1}{(1+r_{j1})L_{1j1}}\right)^{\alpha_{1jL1}}=\left((1+r_{j1})L_{1j1}\right)^{-\alpha_{1jL1}}$, $j=q,m$, assuming the Cobb Douglas elasticity $\alpha_{ijL1}\geq 0$. Agent 1 uses its entire borrowing $L_{1q1}+L_{1m1}$ to buy other assets. For simplicity, assume that the borrower buys other asset o_{11} in period 1. Adding agent 1's loan $L_{1q1}+L_{1m1}$ to its other assets o_{11} gives $o_{11}+L_{1q1}+L_{1m1}$ which is multiplied with $1+I_{o1}$ to account for the interest rate I_{o1}, and raised to the Cobb Douglas elasticity α_{1o1} which gives the input $\left((1+I_{o1})(o_{11}+L_{1q1}+L_{1m1})\right)^{\alpha_{1o1}}$. Agent 1 holds neither hard money q nor fiat money m in period 1, i.e. $q_{11} = m_{11} = 0$. Requiring constant returns to scale gives $\alpha_{1o1}+\alpha_{1qL1}+\alpha_{1mL1} = 1$. Applying the $Max(1,\bullet)$ function for agent 1's loan L_{1j1}, $j = q,m$, agent 1's period 1 utility is

$$U_{11} = \left((1+I_{o1})\left(o_{11}+L_{1q1}+L_{1m1}\right)\right)^{\alpha_{1o1}}$$

$$\left(Max\left(1,\left(1+r_{q1}\right)L_{1q1}\right)\right)^{-\alpha_{1qL1}}\left(Max\left(1,\left(1+r_{m1}\right)L_{1m1}\right)\right)^{-\alpha_{1mL1}} \tag{1}$$

4.1.1.2. Agents $2,\ldots,n$. Assume that agent $i, i = 2,\ldots,n$, in period 1 does not borrow, i.e. $L_{ij1} = 0, j = q,m$, does not sell its other assets o_{i1}, and holds assets with value j_{i1}, $j = q,m,o$. Multiplying agent i's asset j_{i1} with $1+I_{j1}$ to account for the interest rate I_{j1}, and raising to the Cobb Douglas elasticity α_{ij1} gives the input $\left((1+I_{j1})j_{i1}\right)^{\alpha_{ij1}}$. Requiring constant returns to scale gives $\alpha_{iq1} + \alpha_{im1} + \alpha_{io1} = 1$. Hence agent i's period 1 utility is

$$U_{i1} = \left((1+I_{q1})q_{i1}\right)^{\alpha_{iq1}}\left((1+I_{m1})m_{i1}\right)^{\alpha_{im1}}\left((1+I_{o1})o_{i1}\right)^{\alpha_{io1}}, i = 2,\ldots,n \tag{2}$$

4.1.2. Period 2

4.1.2.1. Agent 1. In period 2 agent 1 borrows L_{1q2} in hard money and L_{1m2} in fiat money from the bank and buys an asset valued as $L_{1q2}+L_{1m2}$ from agent 2, without loss of generality. The assets are traded based on their value, regardless of whether they are traded in hard money or fiat money. Agent 1 retains its loans L_{1q1} and L_{1m1} from period 1 to period 2. Adding $L_{1q2}+L_{1m2}$ to agent 1's other assets, adding L_{1q2} and L_{1m2} to agent 1's loans, and applying the $Max(1,\bullet)$ function for agent 1's loans $L_{1j1}+L_{1j2}$, $j = q,m$, agent 1's period 2 utility is

$$U_{12} = \left((1+I_{o2})\left(o_{11}+L_{1q1}+L_{1m1}+L_{1q2}+L_{1m2}\right)\right)^{\alpha_{1o2}}$$

$$\left(Max\left(1,\left(1+r_{q2}\right)\left(L_{1q1}+L_{1q2}\right)\right)\right)^{-\alpha_{1qL2}}$$

$$\frac{\left(Max\left(1,(1+r_{m2})(L_{1m1}+L_{1m2})\right)\right)^{-\alpha_{1mL2}}}{(1+\pi_2)^{-\alpha_{1mL2}}} \tag{3}$$

Division with $(1+\pi_2)^{-\alpha_{1mL2}}$ for agent 1's fiat money loan $L_{1m1}+L_{1m2}$ is to account for the inflation rate π_2, $\pi_2 \in \mathbb{R}$. The inflation rate is positive if $\pi_2 > 0$, nonexistent if $\pi_2 = 0$, and negative, i.e. deflation if $\pi_2 < 0$. The negative signs on the Cobb Douglas elasticities α_{1jL2} correspond to the negative signs on α_{1jL1} in (1), due to inverting the base in the function since the loans L_{1q1} and L_{1m1} are costly. Requiring constant returns to scale gives $\alpha_{1o2}+\alpha_{1qL2}+\alpha_{1mL2} = 1$.

4.1.2.2. Agent 2. In period 2 agent 2 sells other assets valued at $L_{1q2}+L_{1m2}$ to agent 1, retaining $o_{21}-L_{1q2}-L_{1m2}$. Multiplying with $1+I_{o2}$ to account for the interest rate I_{o2}, and raising to the Cobb Douglas elasticity α_{2o2} gives the input $\left((1+I_{o2})\left(o_{21}-L_{1q2}-L_{1m2}\right)\right)^{\alpha_{2o2}}$. Agent 2's sale causes its hard money holding to increase from q_{21} to $q_{21}+L_{1q2}$ which is multiplied with $1+I_{q2}$ to account for the interest rate I_{q2}, and raised to the Cobb Douglas elasticity α_{2q2} which gives the input $\left((1+I_{q2})\left(q_{21}+L_{1q2}\right)\right)^{\alpha_{2q2}}$. Agent 2's sale causes its fiat money holding to increase from m_{21} to $m_{21}+L_{1m2}$ which is multiplied with $1+I_{m2}$ to account for the interest rate I_{m2}, raised to the Cobb Douglas elasticity α_{2m2}, and divided with $(1+\pi_2)^{\alpha_{2m2}}$ to account for the inflation rate π_2, which gives the input $\frac{((1+I_{m2})(m_{21}+L_{1m2}))^{\alpha_{2m2}}}{(1+\pi_2)^{\alpha_{2m2}}}$. Agent 2 neither buys nor borrows. Requiring constant returns to scale gives $\alpha_{2q2} + \alpha_{2m2} + \alpha_{2o2} = 1$. Multiplying the three inputs, agent 2's period 2 utility is

$$U_{22} = \left((1+I_{q2})\left(q_{21}+L_{1q2}\right)\right)^{\alpha_{2q2}}\frac{\left((1+I_{m2})(m_{21}+L_{1m2})\right)^{\alpha_{2m2}}}{(1+\pi_2)^{\alpha_{2m2}}}$$

$$\left((1+I_{o2})\left(o_{21}-L_{1q2}-L_{1m2}\right)\right)^{\alpha_{2o2}} \tag{4}$$

4.1.2.3. Agents $3,\ldots,n$. Assume that agent $i, i = 3,\ldots,n$, in period 2 neither borrows nor buys nor sells. That is, agent i does nothing, but is subject to the inflation rate π_2. Hence agent i's fiat money holding input is $\frac{((1+I_{m2})m_{i2})^{\alpha_{im2}}}{(1+\pi_2)^{\alpha_{im2}}}$, and agent i's period 2 utility is

$$U_{i2} = \left((1+I_{q2})q_{i2}\right)^{\alpha_{iq2}}\frac{\left((1+I_{m2})m_{i2}\right)^{\alpha_{im2}}}{(1+\pi_2)^{\alpha_{im2}}}\left((1+I_{o2})o_{i2}\right)^{\alpha_{io2}}, i = 3,\ldots,n \tag{5}$$

Requiring constant returns to scale gives $\alpha_{iq2} + \alpha_{im2} + \alpha_{io2} = 1$.

4.2. The bank

Three examples of articles assuming that the bank and central banks are one unitary actor are Chen et al. (2017); Gertler and Kiyotaki (2015); Wang and Hausken (2021b). The article employs a similar approach and assumes that the bank and central bank are one unitary actor, which holds an amount of asset j_t, and can lend L_{1jt} to agent 1, $j = q, m, t = 1, 2$. The bank holds no other assets. Therefore, the central bank's role is embedded by the bank actor. Banks have multifarious revenue streams. The bank earns an interest rate $I_{jt}, I_{jt} \in \mathbb{R}$, from the open market, analogously to the n agents. We exclude deposits by the n agents from the bank's utility since the n agents deposit their assets in the open market. Since the bank and the n agents earn the same interest rate I_{jt} in the open market, we may interpret the n agents as depositing their assets in the bank, which further deposits in the open market. The bank's utility U_t in period $t, t = 1, 2$, has two multiplicative inputs pertaining to holding asset $j_t, j = q, m$, and two multiplicative inputs pertaining to earning interest from lending L_{1jt} to agent $1, j = q, m$. Four other examples of articles assuming that the bank has a Cobb Douglas utility function are Goodfriend and McCallum (2007); Mullineaux (1978); Tsai (2013); Wang and Hausken (2022b).

4.2.1. Period 1

In period 1 the bank holds q_1 in hard money and m_1 in fiat money. The bank provides loans L_{1q1} in hard money and L_{1m1} in fiat money to agent 1. After providing the loans, the bank holds $j_1 - L_{1j1}$ in asset $j, j = q, m$, which is multiplied with $1 + I_{j1}$ to account for the interest rate I_{j1}, and raised to the Cobb Douglas elasticity $\beta_{j1}, \beta_{j1} \geq 0$, which gives the input $\left(\left(1 + I_{j1}\right)\left(j_1 - L_{1j1}\right)\right)^{\beta_{j1}}$. The bank does not print fiat money in period 1. Lending L_{1j1} to agent 1 gives an interest rate r_{j1}. Assume that when the bank lends L_{1j1} to agent 1, the bank retains the utility of the amount it lends out. Hence L_{1j1} is multiplied with $1 + r_{j1}$ instead of $r_{j1}, j = q, m$, and raised to the Cobb Douglas elasticity $\beta_{jL1}, \beta_{jL1} \geq 0, j = q, m$. Requiring constant returns to scale gives $\beta_{q1} + \beta_{m1} + \beta_{qL1} + \beta_{mL1} = 1$. Multiplying the four inputs, and applying the $Max(1, \bullet)$ function for the loans L_{1q1} and L_{1m1}, the bank's period 1 utility is

$$U_1 = \left(\left(1 + I_{q1}\right)\left(q_1 - L_{1q1}\right)\right)^{\beta_{q1}} \left((1 + I_{m1})(m_1 - L_{1m1})\right)^{\beta_{m1}}$$

$$\left(Max\left(1, \left(1 + r_{q1}\right)L_{1q1}\right)\right)^{\beta_{qL1}} \left(Max(1, (1 + r_{m1})L_{1m1})\right)^{\beta_{mL1}} \tag{6}$$

4.2.2. Period 2

In period 2 the bank provides loans L_{1q2} in hard money and L_{1m2} in fiat money to agent 1. The bank continues in period 2 to hold the loans L_{1q1} and L_{1m1} that agent 1 incurred in period 1. The fiat money loan L_{1m2} is provided by money printing. After lending hard money L_{1q2} to agent 1, the bank holds $q_1 - L_{1q1} - L_{1q2}$ in hard money, which is multiplied with $1 + I_{q2}$ to account for the interest rate I_{q2}, and raised to the Cobb Douglas elasticity $\beta_{q2}, \beta_{q2} \geq 0$, which gives the input $\left(\left(1 + I_{q2}\right)\left(q_1 - L_{1q1} - L_{1q2}\right)\right)^{\beta_{q2}}$. After printing and lending fiat money L_{1m2} to agent 1, printing an amount $P_{m2}, P_{m2} \geq 0$, of fiat money, and withdrawing an amount $W_{m2}, W_{m2} \geq 0$, of fiat money, the bank holds $m_1 - L_{1m1} + P_{m2} - W_{m2}$ in fiat money, which is multiplied with $1 + I_{m2}$ to account for the interest rate I_{m2}, raised to the Cobb Douglas elasticity β_{m2}, $\beta_{m2} \geq 0$, and divided with $(1 + \pi_2)^{\beta_{m2}}$ to account for the inflation rate π_2, which gives the input $\frac{((1+I_{m2})(m_1 - L_{1m1} + P_{m2} - W_{m2}))^{\beta_{m2}}}{(1+\pi_2)^{\beta_{m2}}}$. The fiat money loan L_{1m2} to agent 1 is not subtracted in the previous expression since the bank prints the fiat money. Lending hard money L_{1q2} to agent 1 gives an interest rate r_{q2}. Adding agent 1's retained loan L_{1q1} from period 1, $L_{1q1} + L_{1q2}$ is multiplied with $1 + r_{q2}$ and raised to the Cobb Douglas elasticity β_{qL2}, $\beta_{qL2} \geq 0$, which gives the input $\left(\left(1 + r_{q2}\right)\left(L_{1q1} + L_{1q2}\right)\right)^{\beta_{qL2}}$. Lending fiat money L_{1m2} to agent 1 gives an interest rate r_{m2}. Adding agent 1's retained loan L_{1m1} from period 1, $L_{1m1} + L_{1m2}$ is multiplied with $1 + r_{m2}$ and raised to the Cobb Douglas elasticity β_{mL2}, $\beta_{mL2} \geq 0$, and divided with $(1 + \pi_2)^{\beta_{m2}}$ to account for the inflation rate π_2, which gives the input $\frac{((1+r_{m2})(L_{1m1}+L_{1m2}))^{\beta_{mL2}}}{(1+\pi_2)^{\beta_{mL2}}}$. $\beta_{q2} + \beta_{m2} + \beta_{qL2} + \beta_{mL2} = 1$. Multiplying the four inputs, the bank's period 2 utility is

$$U_2 = \left(\left(1 + I_{q2}\right)\left(q_1 - L_{1q1} - L_{1q2}\right)\right)^{\beta_{q2}}$$

$$\frac{((1 + I_{m2})(m_1 - L_{1m1} + P_{m2} - W_{m2}))^{\beta_{m2}}}{(1 + \pi_2)^{\beta_{m2}}}$$

$$\left(\left(1 + r_{q2}\right)\left(L_{1q1} + L_{1q2}\right)\right)^{\beta_{qL2}} \frac{((1 + r_{m2})(L_{1m1} + L_{1m2}))^{\beta_{mL2}}}{(1 + \pi_2)^{\beta_{mL2}}} \tag{7}$$

4.3. The inflation rates π_1 and π_2

The bank cannot print hard money q. Hence no inflation exists for hard money q. To create a reference standard with zero inflation rate $\pi_1 = 0$ in period 1, assume that the bank does not print fiat money m in period 1. Instead, the bank uses its fiat money holding for lending L_{1m1} to agent 1 in period 1. The inflation rate π_2 in period 2 equals a ratio. The numerator is the net increase from period 1 to period 2 in the amounts of hard money q and fiat money m. Since the amount of hard money q does not increase, which is the nature of hard money, the net increase from period 1 to period 2 is $L_{1m2} + P_{m2} - W_{m2}$, where L_{1m2} is what the bank prints to lend to agent 1, P_{m2} is what the bank prints to increase the fiat money circulating amount, and W_{m2} is what the bank withdraws to decrease the fiat money circulating amount. The denominator in the ratio is the amount $\sum_{i=1}^{n} q_{i1} + q_1$ of circulating hard money q in period 1 plus the amount $\sum_{i=1}^{n} m_{i1} + m_1$ of circulating fiat money m in period 1. Thus, the amount of hard money also impacts the inflation rate π_2 in period 2. Hence the inflation rate in period 2 is

$$\pi_2 = \frac{L_{1m2} + P_{m2} - W_{m2}}{\sum_{i=1}^{n} q_{i1} + q_1 + \sum_{i=1}^{n} m_{i1} + m_1} \tag{8}$$

5. Analyzing the model

The model conceptualizes one unitary bank and three kinds of agents, whose behaviors are driven by their respective utilities. The article assumes that the bank and agents act in a manner that maximizes their utilities and compares their utilities over the two periods. Factors that drive the bank's and agents' behavior include holdings of hard money, fiat money, and other assets, and borrowing in hard and fiat money. The bank's behavior is impacted by its holdings of hard money and fiat money, borrowing interest rates in hard money and fiat money, its fiat money printing and fiat money withdrawal.

5.1. Comparing periods 1 and 2

See Appendix B.

Property 1. Agents 1 and 2 prefer to trade if (13) and (14) are satisfied. Agent $i, i = 3,\ldots,n$, prefers the trade between agents 1 and 2 if (15) is satisfied. The bank prefers to lend to agent 1 if (16) is satisfied.

Proof: Eq. (13) implies that agent 1's utility U_{12} in period 2 is higher than its utility U_{11} in period 1, i.e. $U_{11} < U_{12}$. Thus, agent 1 prefers to buy other assets valued as $L_{1q2} + L_{1m2}$ from agent 2 in period 2. Analogously, (14) implies that agent 2's utility U_{22} in period 2 is higher than its utility U_{21} in period 1, i.e. $U_{21} < U_{12}$. Thus, agent 2 prefers to sell other assets valued as $L_{1q2} + L_{1m2}$ to agent 1 in period 2. It follows from (15) that agent $i, i = 3,\ldots,n$, prefers the trade between agents 1 and 2 since its utility U_{i2} in period 2 is higher than its utility U_{i1} in period 1, i.e. $U_{i1} < U_{i2}$. Agent $i, i = 3,\ldots,n$, is unaffected if $U_{i1} = U_{i2}$. Eq. (16) implies that the bank's utility U_2 in period 2 is higher than its utility U_1 in period 1, i.e. $U_1 < U_2$. Thus, the bank prefers to lend $L_{1q2} + L_{1m2}$ to agent 1 in period 2.

Property 1 states that agent 1 prefers to borrow $L_{1q2} + L_{1m2}$ from the bank and buy other assets from agent 2 when $U_{11} < U_{12}$. Agent 2 prefers to sell other assets $L_{1q2} + L_{1m2}$ to agent 1 when $U_{21} < U_{22}$. The bank prefers to lend $L_{1q2} + L_{1m2}$ to agent 1 when $U_1 < U_2$. Agent $i, i = 3,\ldots,n$, prefers the trade between agents 1 and 2 when $U_{i1} < U_{i2}$. Hence agent i is unaffected by the trade between agents 1 and 2 when $U_{i1} = U_{i2}$.

6. Illustrating the model

To illustrate the solution in Section 5, this section alters the model's 64 parameter values relative to the following plausible benchmark parameter values intended to capture the specificities of the context and how the three kinds of agents and bank operate within this context. The benchmark parameter values, and the ranges for the parameter values in the analysis, are chosen carefully with the following objectives in mind: 1. The analysis should capture the most interesting phenomena involved for the three kinds of agents and the bank. 2. The borrower should or should not prefer to borrow hard money and fiat money from the bank in order to buy other assets from the seller. 3. The seller should or should not prefer to sell other assets to the buyer. 4. The nontraders should or should not prefer the trade between the buyer and seller, and should or should not prefer the bank to lend to the borrower and print or withdraw fiat money, though without being able to impact the borrower, seller and bank. 5. The bank should or should not prefer to lend hard money and fiat money to the borrower, and should or should not prefer to print or withdraw fiat money. The analysis is intended to generate valuable insights shown below and believed not to be easily captured by alternative analyses.

Assume that agent 1 has no hard money and no fiat money in the two periods, i.e. $q_{11} = q_{12} = m_{11} = m_{12} = \$0$. Agent 1 also has no other assets before borrowing and buying other assets, i.e. $o_{11} = \$0$. This choice is made to test whether agent 1 may be willing to incur loans from the bank to acquire other assets. For agent 1 assume the loans $L_{1q1} = L_{1m1} = \$10$ in period 1, to enable buying other assets $L_{1q1} + L_{1m1} = \$20$ from agent 2, and the loans $L_{1q2} = L_{1m2} = \$15$ in period 2 to enable buying other assets $L_{1q2} + L_{1m2} = \$30$ from agent 2. Thus, agent 1 holds other assets $o_{11} + L_{1q1} + L_{1m1} = \$20$ after borrowing and buying in period 1, which equals the amount $o_{12} = \$20$ of other assets agent 1 holds before borrowing and buying other assets in period 2. Agent 1 holds other assets $o_{12} + L_{1q2} + L_{1m2} = o_{11} + L_{1q1} + L_{1m1} + L_{1q2} + L_{1m2} = \$50$ after borrowing and buying in period 2.

Assume that agent 2 in period 1 has hard money $q_{21} = \$100$ and fiat money $m_{21} = \$100$, after receiving payments $L_{1q1} = L_{1m1} = \$10$ from agent 1. Agent 2 in period 2 has hard money $q_{21} + L_{1q2} = \$100 + \$15 = \$115$ and fiat money $m_{21} + L_{1m2} = \$100 + \$15 = \$115$ after receiving payments $L_{1q2} = L_{1m2} = \$15$ from agent 1. Agent 2 in period 1 has other assets $o_{21} = \$400$ after selling $L_{1q1} + L_{1m1} = \$20$

to agent 1, chosen to be high to ensure that agent 2 may be willing to sell some of its other assets to agent 1. Agent 2 in period 2 has other assets $o_{21} - L_{1q2} - L_{1m2} = \$400 - \$15 - \$15 = \$370$ after selling other assets $L_{1q2} + L_{1m2} = \$30$ to agent 1.

For agent $i, i = 3, \ldots, n$, assume the benchmark $n = 3$ so that only one agent exists aside from agents 1 and 2, and $q_{i1} = q_{i2} = m_{i1} = m_{i2} = \$100$, $o_{i1} = o_{i2} = \$400$ so that agent 3 largely resembles agent 2. The differences are that agent 3 does not sell other assets, which agent 2 does, and does not buy other assets, as agent 1 does. Agent 3 is introduced to analyze how an agent can be impacted without buying and selling.

Assume for the benchmark that the bank's period 1 holding of hard money q_1 is the sum of the n agents' holding of hard money, i.e. $q_1 = \sum_{i=1}^{n} q_{i1} = (n-1)q_{i1} = \$200, i = 2, \ldots, n$. Analogously, the bank's period 1 holding of fiat money m_1 is the sum of the n agents' holding of fiat money, i.e. $m_1 = \sum_{i=1}^{n} m_{i1} = (n-1)m_{i1} = \$200, i = 2, \ldots, n$. Since the bank in period 1 lends $L_{1q1} = \$10$ in hard money and $L_{1m1} = \$10$ in fiat money to agent 1, the bank's hard money holding in period 1 is $q_1 - L_{1q1} = \$200 - \$10 = \$190$. Since the bank in period 1 does not print fiat money, and uses its fiat money holding for lending, the bank's fiat money holding in period 1 is $m_1 - L_{1m1} = \$200 - \$10 = \$190$. Analogously, Since the bank in period 2 lends $L_{1q2} = \$15$ in hard money and $L_{1m2} = \$15$ in fiat money to agent 1, the bank's hard money holding in period 2 is $q_1 - L_{1q1} - L_{1q2} = \$200 - \$10 - \$15 = \$175$. Assume the benchmark where the bank in period 2 prints $L_{1m2} = \$15$ in fiat money to furnish the loan to agent 1. The bank does not otherwise print money, i.e. $P_{m2} = \$0$, and does not withdraw money, i.e. $W_{m2} = \$0$. Hence the bank's fiat money holding in period 2 is $m_1 - L_{1m1} + P_{m2} - W_{m2} = \$200 - \$10 - \$0 - \$0 = \$190$.

Agent $i, i = 1, \ldots, n$, has the same Cobb Douglas elasticity for holding other assets in both periods, i.e. $\alpha_{io1} = \alpha_{io2} = 1/2$. Agent 1's Cobb Douglas elasticities of loans in hard money and fiat money are $\alpha_{1qL1} = \alpha_{1qL2} = \alpha_{1mL1} = \alpha_{1mL2} = 1/4$. Agent $i, i = 2, \ldots, n$ has the same Cobb Douglas elasticities for holding hard money and fiat money in both periods, i.e. $\alpha_{iq1} = \alpha_{iq2} = \alpha_{im1} = \alpha_{im2} = 1/4$. The bank has the same Cobb Douglas elasticities for holding hard money, fiat money, hard money lending, and fiat money lending, in both periods, i.e. $\beta_{j1} = \beta_{j2} = \beta_{jL1} = \beta_{jL2} = 1/4, j = q, m$. The inflation rate benchmark is $\pi_2 = 1.875\%$ based on (8), which is close to the common inflation rate target 2% in many fiat economies. The interest rates $I_{jt}, j = q, m, o, t = 1, 2$, for three kinds of assets determined by the open market in the two periods are equivalent, i.e. $I_{q1} = I_{q2} = I_{m1} = I_{m2} = I_{o1} = I_{o2} = 2\%$. The borrowing interest rates $r_{jt}, j = q, m, t = 1, 2$, for hard money q and fiat money m, determined by the bank in the two periods are also equivalent, i.e. $r_{q1} = r_{q2} = r_{m1} = r_{m2} = 5\%$. With these benchmark parameter values the benchmark solution is $U_{11} = 1.39$, $U_{12} = 1.40$, $U_{21} = 204.00$, $U_{22} = 209.43$, $U_{i1} = 204.00$, $U_{i2} = 203.06$, $U_1 = 45.11$, $U_2 = 69.23$, $\pi_2 = 1.875\%$. In the benchmark agents 1 and 2 and the bank prefer period 2 rather than period 1, while agent i prefers period 1 rather than period 2.

Figure 1 illustrates the agents' and the bank's utilities in response to variations in the 64 parameter values, relative to the plausible benchmark parameter values. The x-axis in each panel represents the labeled parameter, displaying the corresponding parameter values. The y-axis represents the utilities of both the agents and the bank. In Figure 1 each of the 64 parameter values is altered from its benchmark marked with vertical dashed lines in each panel, while the other 63 parameter values are kept at their benchmarks. Multiplication of π_2 with 10^4 and 10^2, and multiplication of U_{11} and U_{12} with 200, 20 and 10 are for scaling purposes. The 17 most interesting panels are interpreted in this section. The remaining 47 panels are interpreted in Appendix C.

In Figure 1a, as the number n of agents increases, which is intuitively beneficial for the bank, the bank's utilities U_1 and U_2 increase concavely toward infinity. Agent 1's utility U_{12} decreases slightly since the inflation rate π_2 decreases slightly, which hurts agent 1 because of agent 1's fiat money loans L_{1m1} and L_{1m2}. In contrast, agents 2 and i's utilities U_{22} and U_{i2} increase slightly because the inflation rate π_2 decreases slightly, which benefits agents 2 and i because of their fiat money holdings m_{22} and m_{i2}. The utilities U_{11}, U_{21}, U_{i1} remain constant since neither the number n of agents nor the inflation rate π_2 play a role in period 1. The inflation rate π_2 decreases convexly and asymptotically toward zero due to division with n in (8). The inflation impact of the bank's fiat money printing L_{1m2} to provide agent 1's loan L_{1m2} is spread across more agents. As the number n of agents increases, each agent and the bank experience a lower inflation rate π_2 according to (8).

In Figure 1d, as agent 1's borrowing L_{1q2} in hard money in period 2 increases, the bank's utility U_2 is inverse U shaped. That is, the bank prefers to lend an optimal amount L_{1q2} of hard money to agent 1. The maximum of U_2 is 85.12 when $L_{1q2} = \$90$. The bank's utility U_2 decreases concavely toward zero after the maximum. The bank prefers to lend hard money to agent 1 when $\$0 \leq L_{1q2} < \$185.98$. When $\$185.98 < L_{1q2} \leq \$190.00$, the bank's utility U_2 is less than U_1. That follows from the nature of the bank's inverse U shaped Cobb Douglas utility U_2, which is low when the bank lends excessively or minimally. Agent 2's utility U_{22} is also inverse U shaped. Agent 2 prefers to sell an optimal amount $L_{1q2} + L_{1m2}$ of its other assets $o_{21} - L_{1q1} - L_{1m1}$ to agent 1 in period 2. That follows from the nature of agent 2's inverse U shaped Cobb Douglas utility U_{22}, which is low when agent 2 sells its other assets excessively or minimally. The maximum of U_{22} is 213.18 when $L_{1q2} = \$61.67$. Agent 2's utility U_{22} decreases concavely after the maximum. Agent 2 wants to sell its other assets $L_{1q2} + L_{1m2}$ to agent 1 when $\$0 \leq L_{1m2} < \$143.96$. Hence, agent 2 prefers not to sell too much other assets to agent 1 in period 2. Agent 1's utility U_{12} increases since agent 1 benefits from buying other assets $L_{1q2} + L_{1m2}$ using its borrowing L_{1q2} in hard money. The utilities U_{i1}, U_1, and U_{11} are constant since agent 1's borrowing L_{1q2} in hard money plays no role in period 1. Agent i's utility U_{i2} is constant since L_{1q2} has no impact on agent i in period 2. The inflation rate π_2 remains constant since L_{1q2} plays no role in (8).

In Figure 1f, as agent 1's borrowing L_{1m2} in fiat money in period 2 increases, the inflation rate π_2 increases because L_{1m2} is added to the numerator in (8). Thus, the bank's utility U_2 increases concavely since it prints fiat money for lending in period 2. This implies that the benefit of printing L_{1m2} fiat money for lending overrides the negative impact of holding $m_1 - L_{1m1} + P_{m2} - W_{m2}$ of fiat money from the increasing inflation rate π_2. The bank always prefers to lend fiat money to agent 1 in period 2 since $U_2 > U_1$. Agent 1's utility U_{12} increases since the inflation rate π_2 increases, which benefits agent 1 because of agent 1's fiat money loans L_{1m1} and L_{1m2}. Agent 2's utility U_{22} is inverse U shaped since it prefers to sell an optimal amount L_{1m2} of its other assets o_{22} to agent 1. The maximum of U_{22} is 210.83 when $L_{1m2} = \$42.36$. Agent 2's utility U_{22} decreases concavely toward zero after the maximum. Agent 2 wants to sell other assets $L_{1q2} + L_{1m2}$ to agent 1 when $\$0 \leq L_{1m2} < \$110.96$. Agent 2 prefers not to sell too much other assets to agent 1 in period 2. Agent i's utility U_{i2} decreases from $U_{i2} = 204.00$ when $L_{1m2} = \$0$, to $U_{i2} = 203.06$ when $L_{1m2} = \$15$, and thereafter decreases further, because the inflation rate π_2 increases, which hurts agent i because of its fiat money holdings m_{i2}. Thus, agent i suffers from agent 1's borrowing in fiat money L_{1m2} in period 2 without doing anything. The utilities U_{11}, U_{21}, U_{i1}, and U_1 remain constant since neither agent 1's borrowing L_{1m2} in fiat money nor the inflation rate π_2 play a role in period 1.

In Figure 1s, as the bank's fiat money printing P_{m2} in period 2 increases, the inflation rate π_2 increases since P_{m2} is added to the numerator in (8). Interestingly, the bank's utility U_2 is inverse U shaped. It first increases toward a maximum $U_2 = 75.28$ when $P_{m2} = \$435$ and then decreases convexly and asymptotically toward zero. This implies that, before the maximum, the benefit of printing $L_{1m2} + P_{m2}$ fiat money overrides the negative impact of holding $m_1 - L_{1m1} + P_{m2} - W_{m2}$ of fiat money due to the increasing inflation rate π_2. After the maximum, the negative impact of holding $m_1 - L_{1m1} + P_{m2} - W_{m2}$ of fiat money due to the increasing inflation rate π_2 overrides the benefit of printing $L_{1m2} + P_{m2}$ fiat money. Hence the bank prefers to print an optimal amount of fiat money. When $\$0 \leq P_{m2} < \$17929.02$, $U_2 > U_1$. The bank prefers not to print more fiat money than \$17929.02 since $U_2 < U_1$ when $P_{m2} > \$17929.02$ in period 2. Agent 1's utility U_{12} increases concavely since the inflation rate π_2 increases, which benefits agent 1 because of its fiat money loans L_{1m1} and L_{1m2}. Agents 2 and i's utilities U_{22} and U_{i2} decrease convexly toward zero. Agents 2 and i are hurt by the increasing inflation rate π_2 due to their holdings m_{22} and m_{i2} of fiat money. Agents 2 and i suffer from the bank's fiat money printing $L_{1m2} + P_{m2}$ in period 2, without agent i doing anything. The utilities U_{11}, U_{21}, U_{i1}, and U_1 remain constant since neither the bank's fiat money printing $L_{1m2} + P_{m2}$ nor the inflation rate π_2 play a role in period 1.

In Figure 1t, conversely, as the bank's fiat money withdrawing W_{m2} in period 2 increases, the inflation rate π_2 decreases and becomes negative when $W_{m2} > \$15$, caused by W_{m2} being subtracted from the numerator in (8). Interestingly, the bank's utility U_2 decreases concavely toward zero. It implies that the negative impact of withdrawing money overrides the benefits of holding $m_1 - L_{1m1} + P_{m2} - W_{m2}$ of fiat money due to the decreasing inflation rate π_2. The bank prefers not to withdraw more fiat money than $W_{m2} = \$168.44$ since $U_2 < U_1$ when $P_{m2} > \$168.44$ in period 2. Agent 1's utility U_{12} decreases sightly since the inflation rate π_2 decreases, which hurts agent 1 because of its fiat money loans L_{1m1} and L_{1m2}. Agents 2 and i's utilities U_{22} and U_{i2} increase. Thus, agents 2 and i benefit in period 2 from the decreasing inflation rate π_2 due to their holdings m_{22} and m_{i2} of fiat money. That is, agents 2 and i benefit from the bank's fiat money withdrawal W_{m2} in period 2, without agent i doing anything. More specifically, agent i's period 2 utility increases from $U_{i2} = 203.06$ when $W_{m2} = \$0$ to $U_{i2} = U_{i1} = 204.00$ when $W_{m2} = \$15$, which exactly matches the bank's money printing $L_{1m2} = \$15$. Thereafter agent i's period 2 utility increases to $U_{i2} = 216.99$ when $W_{m2} = \$190$. The utilities U_{11}, U_{21}, U_{i1}, and U_1 remain constant since neither the bank's fiat money printing $L_{1m2} + P_{m2}$ nor the inflation rate π_2 play a role in period 1.

In Figure 1w and Figure 1z, as the interest rate I_{j1} for holding money $j, j = q, m$, in period 1 increases, which is intuitively beneficial to the bank and agent $i, i = 2, \ldots n$, the three utilities U_1, U_{21}, and U_{i1} increase concavely toward infinity. The bank only wants to lend money $L_{1q2} + L_{1m2}$ to agent 1 in period 2 when $0 \leq I_{j1} < 4.66$ in period 2. If the interest rate is too high, $I_{j1} > 4.66$, the bank prefers to hold money j rather than lending it out. The inflation rate π_2 is constant since I_{j1} plays no role in (8). Agent 1's utilities U_{11} and U_{12} are constant since agent 1 holds no money j in the two periods. The utilities U_2, U_{22}, and U_{i2} remain constant since I_{j1} plays no role in period 2.

In Figure 1af and Figure 1ai, as the borrowing interest rate r_{j1} for money $j, j = q, m$ in period 1 increases, which is intuitively beneficial to the bank, the bank's utility U_1 increases concavely toward infinity. The bank only wants to lend money L_{1j2} to agent 1 in period 2 when r_{j1} is sufficiently low, i.e. $0 \leq r_{j1} < 4.82$. Agent 1's utility U_{11} decreases convexly toward zero since a higher borrowing interest rate r_{j1} for money j is costly. The inflation rate π_2 is constant since r_{j1} plays no role in (8). The utilities U_{21} and U_{i1} are constant since agents 2 and i do not borrow money j in period 1. The utilities U_2, U_{12}, U_{22}, and U_{i2} remain constant since r_{j1} plays no role in period 2.

In Figure 1am, as agent 1's Cobb Douglas elasticity α_{1o2} for holding $o_{11} + L_{1q1} + L_{1m1} + L_{1q2} + L_{1m2}$ of the other assets in period 2 increases, which is intuitively beneficial to agent 1, the utility U_{12} increases concavely. Agent 1 wants to borrow $L_{1q2} + L_{1m2}$ from the bank in period 2 when α_{1o2} is not too low, i.e. $0.5 < \alpha_{1o2} \leq 1$. The inflation rate π_2 is constant since α_{1o2} plays no role in (8). Agent 1's utility U_{11} is constant since α_{1o2} plays no role in period 1. The utilities U_1, U_2, U_{21}, U_{22}, U_{i1}, and U_{i2} remain constant since α_{1o2} has no impact on the bank, agents 2 and i.

In Figure 1as and Figure 1av, as agent 2's Cobb Douglas elasticity α_{2j2} for holding money $j_{21} + L_{1j2}, j = q, m$ in period 2 increases, its utility U_{22} in period 2 decreases convexly because holding other assets $o_{21} - L_{1q2} - L_{1m2}$ becomes less beneficial for agent 1 with decreasing Cobb Douglas elasticity $\alpha_{2o2} = 1 - \alpha_{2q2} - \alpha_{2m2}$. Hence, in contrast to Figure 1as and Figure 1av, agent 2 wants to sell its other assets valued as $L_{1q2} + L_{1m2}$ when α_{2j2} is sufficiently low, i.e. $0 \leq \alpha_{2j2} \leq 0.27, j = q, m$. The eight variables U_{11}, U_{12}, U_{21}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1ay and Figure 1bb, as agent i's Cobb Douglas elasticity α_{ij2} for holding money $j_{i2}, j = q, m$ in period 2 increases, its utility U_{i2} in period 2 decreases convexly because holding other assets o_{i2} becomes less beneficial for agent 1 with decreasing Cobb Douglas elasticity $\alpha_{io2} = 1 - \alpha_{iq2} - \alpha_{im2}$. Agent i prefers the trade between agents 1 and 2 when α_{ij1} is sufficiently high, i.e. $0.25 \leq \alpha_{iq2} \leq 1, j = q, m$. The eight variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i2}, U_1, U_2, π_2 remain constant.

Table 2
Comparing this article's approach and results with those in the literature.

Literature	Comparing this article's approach and results with those in the literature
Adrian and Mancini-Griffoli (2021)	They assess the benefits and risks of digital money compared with traditional money.
Almosova (2018)	She proposes that private currencies exert downward pressure on inflation.
Ammous (2018)	He argues that hard money will eventually replace fiat money while this article assesses coexistence.
Ammous and D'Andrea (2022)	They suggest that hard money maintains value over time and that a hard money standard fosters higher levels of social development.
Ayadi et al. (2023)	They show that the CBDC uncertainty index impacts the return on cryptocurrencies negatively.
Belke and Beretta (2020)	They argue that central banks should embrace the technology of hard money.
Benchimol and Fourçans (2012)	They separate central banks and commercial banks as different players.
Benigno et al. (2022)	They propose a crypto-enforced monetary policy synchronization when hard money and fiat money coexist.
Boissay et al. (2022)	They suggest that hard money currently cannot substitute fiat money, while this article allows coexistence.
Chen et al. (2017)	They assume that the commercial banks and central banks are one unitary actor.
Chen and Guo (2014)	They adopt a money-in-utility approach, as this article also does, where utility is obtained from holding assets.
Cooper et al. (1982)	They propose that a hard money standard aims to reduce inflation, consistently with this article.
Dowd and Greenaway (1993)	They argue that network effects and switching costs are driving forces for players to use one currency.
Eichengreen (2005)	He suggests that multiple reserve currencies will continue to coexist.
Feres (2021)	He proposes a hard money based monetary system.
Fernández-Villaverde and Sanches (2019)	They point out that competing private currencies can coexist.
Ferrari Minesso et al. (2022)	They adopt a money-in-utility approach, as this article also does, where utility is obtained from holding assets.
Fisher (1920)	He suggests a hard money standard to control the unstoppable inflation associated with a fiat money standard, which is a finding compatible with this article.
Friedman and Schwartz (1986)	They support hard money standards and oppose the government monopoly on fiat money creation.
Gawthorpe (2017)	He suggests that currency competition causes lower inflation rates.
Gertler and Kiyotaki (2015)	They assume that the commercial banks and central banks are one unitary actor.
Goodfriend and McCallum (2007)	They assume that banks have Cobb Douglas utility functions, which this article also assumes.
Gorton (2023)	He argues that inflation generally depends on the fiat money supply, consistently with this article.
Hart (2020)	He proposes a negative exponent for the elasticity of the Cobb Douglas utility for pollution as a negative impact factor, which this article also does for the borrower's hard money and fiat money loans.
Helmi et al. (2023)	They find that CBDC uncertainty and volatility index shocks significantly impact the volatility of hard money approximated by Bitcoin.
Engelhardt (1996)	He considers the resource constraints for the players and banks.
Iacoviello (2005)	He assesses the resource constraints for the players and banks.
Ikkurty (2019)	He argues that hard money approximated by Bitcoin has features such as censorship resistance, verifiability, portability, divisibility, convenience, and scarcity.
Iwamura et al. (2019)	They believe that hard money is unlikely to replace fiat money such CBDC, which to some extent differs from this article which illustrates coexistence.
Jumde and Cho (2020)	They suggest that hard money will eventually overtake fiat money.
Kadiyala (1972)	He suggests that a Cobb Douglas utility is appropriate for even distributions of multiple assets.
Laboure et al. (2021)	They claim that cryptocurrencies and fiat money will coexist.
Levulytė and Šapkauskienė (2021)	They highlight that hard money is advantageous for international transactions.
Long et al. (2021)	He contends that hard money approximated by gold can, while hard money approximated by Bitcoin cannot hedge against uncertainties to varying degrees.
Mafi (2003)	She argues that currency competition causes lower inflation.
Messay (2023)	She suggests that an international currency issued by one or several major countries is the driving factor that impacts national economic development at the expense of the Global South.
Martin and Schreft (2006)	They demonstrate the existence of competing currencies.
Mian et al. (2021)	They adopt a money-in-utility approach, as this article also does, where utility is obtained from holding assets.
Mou et al. (2021)	They argue that central banks need to issue their fiat money as CBDCs.
Mullineaux (1978)	He assumes that banks have Cobb Douglas utility functions.
Murphy (1986)	He proposes that the Cantillon effect, i.e. the uneven distribution of wealth and purchasing power that occurs as a result of changes in the fiat money supply, benefits those who receive the new money first at the expense of others.
Nabilou (2020)	He argues that hard money approximated by Bitcoin poses risks to fiat money.
Nakamoto (2008)	He/she/they propose a hard money currency.
Nicholson (1888)	He studies examples of hard money approximated by representative money, which is backed by and redeemable for gold.
Ramsey (1928)	They adopt a money-in-utility approach, which this article also does, where utility is obtained from holding assets.
Ron and Valeonti (2023)	They point out that democratic governing institutions tend to have moderate inflation with fiat money.
Sakurai and Kurosaki (2023)	They find that major cryptocurrencies become slightly more effective safeguards against inflation after the Covid-19 pandemic.
Scharnowski (2022)	He suggests that investors do not view fiat money CBDCs as a threat to cryptocurrencies.
Schilling and Uhlig (2019)	They reveal that as trading cost and exchange fee disparities increase, the substitution effect between fiat money and hard money diminishes.
Schuster and Sigmund (1983)	They propose a replicator dynamics model.
Senner and Sornette (2019)	They argue that hard money cannot replace fiat money.
Sidrauski (1967)	He uses a money-in-utility approach, where utility is obtained from holding assets.
Sissoko (2021)	He suggests that a financial system can be established based on competing currencies, which is compatible with this article.
Steiner (1941)	He studies examples of hard money approximated by representative money, which is backed by and redeemable for gold.
Syarifuddin and Bakhtiar (2022)	They employ a Cobb Douglas utility function for holding assets.
Tsai (2013)	He assumes that banks have Cobb Douglas utility functions, which this article also assumes.

(*continued on next page*)

Table 2 (*continued*)

Literature	Comparing this article's approach and results with those in the literature
Wachter and Yogo (2010)	They employ a Cobb Douglas utility function for holding assets.
Wang and Hausken (2021a)	They show how conventionalists, pioneers, and criminals choose between two currencies.
Wang and Hausken (2021b)	They assume that the commercial banks and central banks are one unitary actor.
Wang and Hausken (2022a)	They explore competition between hard money and fiat money, focusing on money printing and withdrawal, accounting for how an agent supports the two kinds of money.
Wang and Hausken (2022b)	They assume that banks have Cobb Douglas utility functions.
Welburn and Hausken (2017)	They analyze financial crises assuming fiat money.
Wen et al. (2022)	They argue that hard money approximated by gold serves as a safe haven for oil and stock markets, while hard money approximated by Bitcoin does not provide the same level of safety.
Xin and Jiang (2023)	They argue that fiat money such as CBDC can stabilize economic fluctuations arising from a negative interest rate policy.
Yu (2023)	He suggests that fiat money and cryptocurrencies can coexist, which is compatible with this article.

In Figure 1be and Figure 1bh, as the bank's Cobb Douglas elasticity β_{j2} for holding money $j_1 - L_{1j1}, j = q, m$ in period 2 increases, its utility U_2 in period 2 increases convexly because holding hard money $q_1 - L_{1q1} - L_{1q2}$ and fiat money $m_1 - L_{1m1} + P_{m2} - W_{m2}$ becomes more beneficial for the bank with increasing Cobb Douglas elasticity β_{j2}, which overrides the negative impact of decreasing Cobb Douglas elasticity $\beta_{mL2} = 1 - \beta_{q2} - \beta_{m2} - \beta_{qL2}$ for fiat money loans. The bank always wants to give money loans $L_{1q2} + L_{1m2}$ to agent 1 in period 2 since $U_1 < U_2$ for Figure 1be and Figure 1bh. The eight variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, π_2 remain constant.

In Figure 1bk, as the bank's Cobb Douglas elasticity β_{qL2} for hard money lending L_{1q2} in period 2 increases, its utility U_2 increases slightly. The reasons are as follows. According to (7), the borrowing interest rates $r_{q2} = r_{m2}$ for hard money and fiat money are the same in period 2, but the bank's loans $L_{1m1} + L_{1m2}$ in fiat money are impacted by the positive inflation rate $\pi_2 = 1.875\%$ in period 2. Thus, the increase in the bank's utility U_2 from holding the hard money loan $L_{1q1} + L_{1q2}$ is higher than the decrease from holding the fiat money loan $L_{1m1} + L_{1m2}$ in period 2 due to the decreasing Cobb Douglas elasticity $\beta_{mL2} = 1 - \beta_{q2} - \beta_{m2} - \beta_{qL2}$. The bank always wants to lend fiat money $L_{1q2} + L_{1m2}$ to agent 1 in period 2 since $U_1 < U_2$. The eight variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, π_2 remain constant.

In Figure 1bl, as the bank's Cobb Douglas elasticity $\beta_{qL1} = \beta_{qL2}$ for hard money lending L_{1q1} and $L_{1q1} + L_{1q2}$ in the two periods increases, its utility U_2 increases slightly. The net impact of increasing $\beta_{qL1} = \beta_{qL2}$ is different for the bank in periods 1 and 2. In period 1, according to (6), the bank's decreasing utility from hard money lending L_{1q1} is offset by the bank's increasing utility U_1 from fiat money lending L_{1m1} due to the decreasing Cobb Douglas elasticity $\beta_{mL1} = 1 - \beta_{q1} - \beta_{m1} - \beta_{qL1}$ for fiat money lending L_{1m1}. Thus, the bank's utility U_1 remains constant. In contrast, in period 2 according to (7), the borrowing interest rates $r_{q2} = r_{m2}$ for hard money and fiat money are equivalent, but the bank's fiat money loans $L_{1m1} + L_{1m2}$ are impacted by the positive inflation rate $\pi_2 = 1.875\%$. Thus, the bank's increasing utility U_2 from holding the hard money loan $L_{1q1} + L_{1q2}$ is higher than the decrease from holding the fiat money loan $L_{1m1} + L_{1m2}$ due to the decreasing Cobb Douglas elasticity $\beta_{mL2} = 1 - \beta_{q2} - \beta_{m2} - \beta_{qL2}$. Thus, the bank's utility U_2 increases slightly. The bank always wants to lend fiat money $L_{1q2} + L_{1m2}$ to agent 1 in period 2 since $U_1 < U_2$. The eight variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, π_2 remain constant.

Table 2 compares this article's approach and results with those in the literature.

7. Interpreting the model

The authors have identified 24 insights in the previous section.

1. More agents benefit the bank and cause less inflation since the bank's money printing to provide agent 1's loans gets spread across more agents. That causes lower utility for agent 1 which borrows and buys and prefers high inflation, higher utility for agent 2 which sells and prefers low inflation, and higher utility for the nontrading agent i, $i = 3, \ldots, n$ which prefers low inflation.
2. As agent 1's borrowing of hard money increases in period 1, agent 1 benefits from buying other assets. The bank's utility is inverse U shaped. The bank prefers to lend to a certain degree to benefit from agent 1's interest rate payment, but prefers not to lend excessively which depletes its hard money holding.
3. As agent 1's borrowing of hard money increases in period 2, it benefits from buying other assets. The selling agent 2's period 2 utility is inverse U shaped, because it prefers to sell some of its other assets, which are abundant, without, however, depleting its stock. The utility is inverse U shaped as in the previous point.
4. As agent 1's borrowing of fiat money in period 1 increases, it benefits from buying other assets. Analogously to the case of hard money, the bank's utility is inverse U shaped. The bank prefers to lend to a certain degree to benefit from agent 1's interest payments, but prefers not to lend excessively which depletes its holdings of fiat money.
5. As agent 1's borrowing of fiat money increases in period 2, it benefits from buying other assets. The bank's utility increases concavely because it prints fiat money for lending and because it benefits from agent 1's interest payments. The utility of agent 2, a seller, in period 2 takes the shape of an inverted U, as described in point 3. The bank prints fiat money for lending which hurts the nontrading agent i.

6. As the bank prints more fiat money in period 2, its utility is inverse U shaped. The bank prefers to print to a certain degree to benefit from holdings, but prefers not to print excessively which causes extremely high inflation. Agent 1 benefits from buying other assets and prefers high inflation. However, the selling agent 2's utility decreases because it prefers low inflation. Analogously, the nontrading agent *i*'s utility decreases because it suffers from high inflation.
7. Conversely, as the bank's withdrawal of fiat money increases in period 2, the inflation rate decreases, and the bank's utility decreases concavely. Interestingly, the bank prefers to withdraw fiat money to a certain degree to benefit from the decrease in inflation due to its fiat money holding. However, it also strives to avoid excessive withdrawal, which may cause extremely low inflation. Agent 1 suffers a detriment because it buys other assets and thus prefers high inflation. The utilities of the selling agent 2 and the nontrading agent *i* increase because they prefer low inflation.
8. As the interest rate for hard and fiat money increases in period 1, agents 2, *i*, and the bank benefit from holding money. The bank prefers not to lend money to agent 1 in period 2 if the interest rate for holding money in period 1 is excessively high. That is so because the bank benefits from holding money in period 1. Thus, the bank is uninterested in lending in period 2.
9. As the borrowing interest rate for hard or fiat money increases in period 1, the bank's utility increases concavely. The bank prefers not to lend money to agent 1 in period 2 when the borrowing interest rate in period 1 is too high because it benefits from lending in period 1. Thus the bank is uninterested in lending in period 2. Agent 1 intuitively suffers from a high borrowing interest rate.
10. As agent 1's Cobb Douglas elasticity of holding other assets increases in period 1, it benefits from buying other assets. Agent 1 wants to borrow money from the bank in period 2 when its Cobb Douglas elasticity of holding other assets is low, because it benefits from buying other assets in period 1. Hence agent 1 prefers not to buy other assets in period 2.
11. As agent 1's Cobb Douglas elasticity of holding other assets increases in period 2, it benefits from buying other assets. Agent 1 wants to borrow money from the bank in period 2 when its Cobb Douglas elasticity of holding other assets is not low.
12. As agent 2's Cobb Douglas elasticity for holding hard or fiat money in period 1 increases, its utility decreases due to the corresponding decrease in the Cobb Douglas elasticity of holding other assets. Agent 2 wants to sell its other assets when its Cobb Douglas elasticity of holding moneyis sufficiently high.
13. As agent 2's Cobb Douglas elasticity of holding hard or fiat money increases in period 2, its utility decreases due to the decrease in the Cobb Douglas elasticity of holding other assets. In contrast to the previous point, agent 2 wants to sell its other assets when its Cobb Douglas elasticity of holding money is sufficiently low.
14. As agent 2's Cobb Douglas elasticity of holding hard money increases over the two periods, agent 2's utilities decrease convexly, as described in the previous two points. Agent 2 wants to sell other assets when its Cobb Douglas elasticity of holding hard money over the two periods is sufficiently high.
15. As agent 2's Cobb Douglas elasticity of holding fiat money over the two periods increases, agent 2's utilities in the two periods decrease convexly, as in points 11 and 12. Analogously to the previous point, agent 2 wants to sell other assets when its Cobb Douglas elasticity of holding fiat money is sufficiently high.
16. As agent *i*'s Cobb Douglas elasticity of holding hard or fiat money increases in period 1, its utility decreases due to the decrease in the Cobb Douglas elasticity of holding other assets. Interestingly, agent *i* prefers the trade between agents 1 and 2 when its Cobb Douglas elasticity of holding money is sufficiently high.
17. As agent *i*'s Cobb Douglas elasticity of holding hard or fiat money in period 2 increases, its utility decreases convexly, as in the previous point. Agent *i* prefers the trade between agents 1 and 2 when its Cobb Douglas elasticity of holding money is sufficiently high.
18. As the bank's Cobb Douglas elasticity of holding hard or fiat money in period 1 increases, its utility increases convexly. That follows since the bank benefits more from holding money than from lending it due to the decrease in the Cobb Douglas elasticity of money loans. The bank prefers to lend to agent 1 in period 2 when its Cobb Douglas elasticity of holding money is sufficiently low.
19. As the bank's Cobb Douglas elasticity of holding money in period 2 increases, its utility in period 2 increases convexly, as in the previous point. The bank always wants to lend money to agent 1 in period 2.
20. As the bank's Cobb Douglas elasticity of holding hard money over the two periods increases, its utilities increase convexly, as in point 17. The bank wants to provide money loans to agent 1 in period 2 when its Cobb Douglas elasticity of holding hard money is sufficiently low.
21. As the bank's Cobb Douglas elasticity of holding fiat money increases over the two periods, its utilities increase convexly, as in the previous point. The bank wants to give money loans to agent 1 in period 2 when its Cobb Douglas elasticity of holding fiat money is sufficiently low.
22. As the bank's Cobb Douglas elasticity of lending money in period 1 increases, its utility remains constant. Thus, the bank's benefit from the increase in the Cobb Douglas elasticity of holding hard money is offset by the decrease in the Cobb Douglas elasticity of lending hard money in period 1. The bank always wants to lend fiat money to agent 1 in period 2.
23. As the bank's Cobb Douglas elasticity of lending hard money in period 2 increases, its utility increases slightly. That follows since the bank then benefits more from lending hard money than from lending fiat money.
24. As the bank's Cobb Douglas elasticity of lending hard money increases over the two periods, its utility in period 1 remains constant as in point 21, and its utility in period 2 increases slightly, as in point 22.

8. Policy implications

Money plays an essential role in an economy by serving as a medium of exchange, as a unit of account, and as a store of value. Modern society cannot operate without money. This article investigates an economy with both hard and fiat money. The findings offer insights to traders, such as borrowers and sellers, nontraders, policymakers, central banks, and others.

The model incorporates certain aspects of monetary policy, e.g. fiat money printing, borrowing interest rates, and deposit interest rates. First, the article contains insights that may be useful to central banks adjusting the money supply, monetary policy, and the inflation rate. Central banks are commonly responsible for issuing and managing fiat money. Central banks fully control fiat money, but do not control the supply of hard money.

Second, the fixed supply of hard money means that inflation and deflation cannot be manipulated by varying its supply. Thus, the effectiveness of monetary policy in the context of hard money is limited. It is beneficial for central banks to account for the existence of hard money when they design monetary policies.

Third, the results have potential implications for understanding the impact of borrowing hard and fiat money. Borrowers benefit from borrowing both hard and fiat money. Notably, borrowing hard money has no impact on nontrading agents. Fiat money borrowing harms nontrading agents due to inflation following money printing.

Fourth, printing fiat money might boost the economy and increase the amount of fiat money that is available for lending, buying, and other financial activities. The analysis shows that central banks benefit from printing fiat money. However, its utility decreases when printing too much fiat money. Therefore, it is reasonable for central banks to limit the supply of fiat money to a certain degree.

Fifth, the inflation that the printing of fiat money causes is spread across all nontrading agents. The impact of inflation diminishes as the number of nontrading agents increases. Thus, in an economy with many agents, central banks can print more fiat money without causing excessive inflation.

Sixth, central banks benefit from withdrawing fiat money to a limited degree since it decreases causes decreasing inflation. Reducing the amount of fiat money in circulation curbs inflation. However, withdrawal discourages borrowing, buying, selling, and other financial activities. As a whole, withdrawing fiat money is not conducive to economic growth. Prudent implementation is recommended when implementing deflationary monetary policies such as withdrawing fiat money.

Seventh, nontrading agents suffer as a result of fiat money printing, and benefit from fiat money withdrawal. Therefore, as inflation increases, it becomes more sensible for nontrading agents to consider becoming borrowers and buyers of other assets.

Eighth, the findings provide insights to the spread effect of money printing, withdrawal, borrowing, lending, buying, selling, inflation, and deflation, which account for most of the financial activities that unfold in an economy.

Nineth, researchers, individuals, firms, financial analysts, investors, business owners, and others may find the findings informative as they attempt to understand hard money, fiat money, borrowing, buying, and selling.

9. Discussion

The bank's withdrawal of fiat money in period 2 is the only scenario in which the nontrading agent $i, i = 3, \ldots, n$ prefers period 2 over period 1. This shows how vulnerable agent i is or can be in a fiat economy. More generally, the model shows how agent i is negatively affected by changes in parameter values. The negative impact decreases with the number of nontrading agents. In contrast, agent i is unaffected in a hard money economy. That agent 1 borrows hard money does not influence agent i's utility. In a hard money economy, financial activities, e.g. borrowing, lending, buying, and selling, only affect agents as a result of trading. Inflation has no influence on them.

Agent 2, as a seller, also benefits from the bank's withdrawal of fiat money. Analogous to agent i, agent 2 suffers from fiat money printing. In contrast, agent 1, being a borrower and a buyer, prefers the bank to print fiat money and not to withdraw it. The bank favors printing over withdrawal. Specifically, since the bank prints fiat money to lend to agent 1 in period 2, its utility is higher in period 2 than in period 1 except if it prints or withdraws fiat money excessively.

For simplicity, while retaining the key ingredients, the article assumes only one agent which borrows and buys, i.e. agent 1, only one seller, i.e. agent 2, and arbitrarily many nontrading agents, i.e. agent i, $i = 3, \ldots, n$. The notional agent 1 can represent an aggregate of many borrowers and buyers. The seller can be an aggregate of many sellers.

In a fiat economy, the impact of the inflation that printing fiat money causes is split across all agents. Specifically, agent 1 benefits and agent 2 suffers. Agent i, which does not borrow, lend, buy, or sell, also experiences the undesirable impacts of the printing of fiat money. Its asset holdings depreciate as inflation increases. Beyond agent 1, the bank, as an issuer and controller, also benefits from printing fiat money. That benefit stems from the inflation costs that are borne by sellers and nontrading agents. However, the benefit of printing fiat money is limited. The bank cannot increase its utility by printing fiat money continuously, which may cause hyperinflation and harm both the bank and the economy.

In a hard money economy, the bank cannot print hard money to lend to agent 1. Lending and borrowing thus have no impact on inflation, and the utilities of the nontrading agents remain unchanged. Hence the bank cannot transfer costs through inflation like in a fiat economy. The impact of fluctuations in the fiat money supply, which results in inflation or deflation, is diminished by the existence of hard money.

The bank benefits from lending both hard and fiat money because it receives interest payments from agent 1. However, the utility curve of the bank takes the shape of an inverted U, which indicates that the bank prefers to lend to agent 1, up to a certain point. The excessive lending of hard money causes the holdings of the bank to decrease significantly. The excessive lending of fiat money causes hyperinflation. Both affect the utility of the bank adversely.

Agent 1 holds no money or assets before borrowing from the bank and buying assets from agent 2. Therefore, agent 1 is a poor agent compared with agents 2 and *i*. Agent 1 benefits from buying other assets using its borrowing from the bank. That follows both for hard and fiat money. Agent 1, as a borrower, prefers high inflation, which results in lower interest payments. However, only borrowing fiat money can cause inflation to increase if the bank prints fiat money. Agent 1 prefers fiat money to hard money. Fiat money is favored by borrowers and buyers, but it harms sellers and nontrading agents. Agent 2 possesses abundant other assets and benefits from selling some of its other assets in exchange for hard or fiat money. However, agent 2's willingness to sell its other assets is limited. Therefore, the agent 2's utility takes the shape of the letter U.

When the bank and agents 2 and *i* suddenly become rich in period 1, i.e. their holdings of hard or fiat money increase in period 1, intuitively, their utilities increase. Therefore, a money airdrop in period 1 is beneficial to the economy. In addition, the inflation rate decreases in period 2 if such an airdrop has occurred in period 1. The foregoing indicates that an increase in holdings of hard and fiat money in period 1 diminishes the inflation in period 2. The impact of a money airdrop in period 1 is analogous to that of an increase in the number of agents in an economy. An other-asset airdrop in period 1 also benefits the economy, but it has no impact on inflation in period 2.

When the bank benefits excessively in period 1, which may occur as a result of an increase in the deposit interest rate, in the borrowing interest rate, or in the Cobb Douglas elasticities of holding or lending hard or fiat money, the bank loses interest in lending to agent 1 in period 2. That follows because the bank benefits significantly in period 1. Analogously, when agent 1 benefits excessively in period 1, for instance due to a dramatic increase in its Cobb Douglas elasticity of holding other assets, it loses interest in borrowing and in buying other assets in period 2.

10. Limitations and future research

One limitation of this article pertains to the nature of a Cobb Douglas utility. Limited amounts of one kind of assets combined with abundant amounts of another kind of assets causes low utility. Kadiyala (1972) suggests that a Cobb Douglas utility is more suitable for even distributions of multiple assets. In the present article, the issue is mitigated by introducing a Max function. Future studies may identify and formulate alternative utility functions to account for other phenomena. The proposed model provides some mathematical development followed by Property 1. Mathematical development, e.g. in the sense of equilibrium determination, is not analyzed in the article. Future research may adopt a gametheoretic approach and examine the equilibrium between the bank and the agents. Future research may explore extensions to the model concerning hard money, e.g. where the hard money supply increases, but the growth rate decreases over time, or the burning of hard money causing a decreased available amount of money. Future research may incorporate real-world data as a supplementary source to verify the model's findings. Another potential limitation is that the article does not examine the agents' and bank's resource constraints (Engelhardt, 1996; Iacoviello, 2005). Future studies may introduce wages, limits on borrowing and selling, maximum lending amounts, capital adequacy requirements, and other regulatory prescriptions. Future research may reduce the number of nontraders and assume that each buyer, seller, and nontrader are represented by a [0, 1]-continuum, formulating a representative agent's problem for each type. In addition, future research may combine models and incorporate more structure on preferences and constraints of the agents' problem, e.g. a Lagos-Wright monetary model, a money-in-utility function mode, and a cash-in-advance-constraint model (Benigno et al., 2022). Another limitation is that inflation is solely attributed to changes in the fiat money supply. Future research may enhance the modeling of inflation by incorporating other relevant factors, e.g. the money velocity, quantity of produced goods, and transaction efficiency. It would be valuable to explore the influence of agents' expectations, such as how a seller's willingness to sell debt in fiat money may be driven by its expectations regarding central banks' fiat money printing.

While hard money is less susceptible to inflation due to its limited supply, the lack of flexibility in adjusting the money supply can cause economic instability and crises. In a fixed supply hard money economy, demand and supply shocks can cause price fluctuations, creating economic instability. This suggests that fiat money economies may continue to exist, since they allow for greater flexibility in managing the money supply to support economic growth and stability. The model accounts for this by modeling how the agents and bank weigh hard money against fiat money in their Cobb Douglas utility functions. Future research can analyze how demand and supply shocks impact inflation, and how governmental agencies and central banks can regulate. Future research may explore the issue of pricing in trading assets and analyze how the prices of assets are determined.

Future research may also introduce multiple borrowers, buyers, and sellers with different preferences and beliefs, which may enable more robust analyses, and generalize this article's aggregation of agents into the specific agent kinds assumed in this article. The bank may be split into a central bank and commercial banks. Several banks and governments may be introduced. Risk averse agents and banks may be modeled, see e.g. Benchimol and Fourçans (2012). This article divides agents into three kinds, i.e. borrower and buyer, seller, and nontrader. In the real world, an agent may choose to borrow, to buy, and to sell. Restricting the analysis to hard money, fiat money, and other assets is a limitation because other assets have different characteristics, e.g. stocks, bonds, and financial derivatives. There are also different kinds of hard money, approximated by e.g. Bitcoin and gold, and different kinds of fiat money, e.g. paper money, coins, CBDCs. Future research may analyze portfolios and competition between multiple kinds of assets. Future research may expand the model to cover more than two time periods. Techniques such as replicator dynamics (Schuster and Sigmund, 1983) may be applied to capture dynamic evolutionary patterns and determine the potential of the stationary coexistence of hard money and fiat money. A more sophisticated analysis of the competition between hard and fiat money would account for factors other than supply and inflation, e.g. transaction efficiency, convenience, security, and monetary policy. Empirical analyses can be employed to support the theoretical and simulation results.

11. Conclusion

A two-period economy is analyzed with one borrower/buyer (which can be an aggregate of many borrowers/buyers), one seller (which can be an aggregate of many sellers), and arbitrarily many nontraders. The article focuses on the actions of one unitary bank and multiple agents, comparing their utilities over the two periods. They choose their actions, e.g. borrow, buy, sell, lend, to maximize their utilities. Period 1 is a benchmark where the bank neither prints nor withdraws fiat money, causing ceteris paribus neither inflation nor deflation. In period 2 the bank prints fiat money to lend to the borrower/buyer, which causes inflation, and it can additionally print and withdraw fiat money. That impacts the fiat money supply causing inflation or deflation. The adjustment of the money supply gets linked to other assets through the borrower/buyer buying other assets from the seller at a certain value, and through the nontraders holding other assets with a certain valuation, which causes inflation or deflation and impacts the agents' utilities. The bank cannot print or withdraw hard money. Periods 1 and 2 are compared to analyze the impact on the agents and the bank. Instead of determining equilibria gametheoretically through maximizing behavior, the article assesses and compares the agents' and the bank's utilities in the two periods. If an agent's or the bank's utility in period 2 exceeds the agent's or the bank's utility in period 1, the agent or the bank prefers trading based on the higher utility in period 2.

Fiat money printing benefits the borrower/buyer which prefers inflation, benefits the bank if not excessive, and hurts the seller and nontraders. Sellers and nontraders bear the costs of inflation. The seller and the nontraders prefer fiat money withdrawal which causes deflation. Fiat money borrowing causes inflation because the bank prints to lend. The nontraders are vulnerable in a fiat economy with money printing, but unaffected in a hard money economy. More nontraders decrease inflation since the bank's money printing gets distributed across more agents. That benefits the seller, nontraders and the bank, and hurts the borrower/buyer. A hard or fiat money airdrop in period 1 decreases the inflation in period 2. The bank prefers not to lend to the borrower/buyer in period 2 if it benefits excessively in period 1. The borrower/buyer prefers not to borrow and buy other assets in period 2 if it benefits excessively in period 1.

In a fiat economy, inflation and deflation impact all agents. In a hard money economy the bank cannot transfer the costs of inflation to the agents. In a hard money economy with borrowing and lending, ceteris paribus, neither inflation nor deflation occur. Hence the nontraders holding hard money and other assets are not impacted. The borrower/buyer, the seller, and the bank are impacted in a hard money economy by their portfolio changes between hard money, other assets, loans, and the associated interest rates.

The borrower/buyer benefits from buying other assets using its hard and fiat money borrowing from the bank if two conditions are met. First, the borrower/buyer must value other assets more than the interest payment of the loan. Second, the borrower/buyer must ensure that the fiat money loan is sufficiently high compared with the hard money loan so that the borrower/buyer benefits sufficiently from the inflation caused by the bank's money printing to provide the loan.

The seller benefits from selling some of its other assets for hard and fiat money if two conditions are met. First, the seller must value hard and fiat money more than the other assets that it sells. Second, the seller must ensure that it receives sufficiently little fiat money relative to hard money for the other assets that it sells so that it does not suffer excessively from the inflation caused by the bank's fiat money printing to provide the loan to the borrower/buyer of the other assets.

As lending increases, the borrower/buyer's, the seller's and the bank's utilities take the shape of an inverted U. Excessive lending of hard or fiat money does not benefit the bank which prefers a balanced portfolio between money holdings and lending which earns interest payment from the borrower/buyer. The borrower/buyer prefers a balanced portfolio between other assets earning interest and loans incurring interest payments. The seller prefers a balanced portfolio between money holdings and other assets. The seller and nontraders prefer not to be hurt by inflation. Thus they prefer a hard money economy or a fiat economy where the bank withdraws money to ensure deflation. The article provides further results illustrated by varying 64 parameters relative to a benchmark. Supplementing the general understanding of debtors desiring inflation to reduce the value of their debt and creditors being averse to inflation, the article provides a more nuanced analysis and sheds light on specific aspects of this relationship. By examining the dynamics and interplay between debtors, creditors, and banks, the article contributes to the existing literature by providing empirical evidence and a deeper understanding of how inflation expectations impact their decision-making processes. The findings provide insights into the complex motivations and strategic considerations of these actors, which have implications for policymaking and risk management in the financial sector.

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Data Availability

No data was used for the research described in the article.

Appendix A. Nomenclature

General parameters

n	Number of agents, $n \geq 1$.
t	Time period, $t = 1, 2$.
q	Hard money, $q \geq 0$.
m	Fiat money, $m \geq 0$.
o	Other assets, $o \geq 0$.
I_{jt}	Interest rate for asset j determined by the open market in period t, $j = q, m, o$, $I_{jt} \in \mathbb{R}$.

Parameters for agent 1

α_{1jLt}	Agent 1's Cobb Douglas elasticity for borrowing L_{jt} of asset j in period t, $j = q, m$, $\alpha_{1jLt} \geq 0$.

Parameters for agent i, i=1,…,n

α_{ijt}	Agent i's Cobb Douglas elasticity for holding j_{it} of asset j in period t, $j = q, m, o$, $\alpha_{ij} \geq 0$.

Parameters for the bank

β_{jt}	The bank's Cobb Douglas elasticity for holding j_t of asset j in period t, $j = q, m$, $t = 1, 2, \beta_{jt} \geq 0$.
β_{jLt}	The bank's Cobb Douglas elasticity for lending L_{1jt} to the agent 1 in period t, $j = q, m$, $\beta_{jLt} \geq 0$.

Agent 1's parameter or free choice variable

L_{1jt}	Agent 1's borrowing of hard or fiat money j in period t, $j = q, m$, $L_{1jt} \geq 0$.

Agent i's parameter or free choice variable i, i=1,…,n

j_{it}	Agent i's holding of three kinds of assets in period t, $j = q, m, o$, $j_{it} \geq 0$.

The bank's parameters or free choice variables

j_t	The bank's holding of two kinds of assets j in period t, $j = q, m$, $j_t \geq 0$.
r_{jt}	The n agents' borrowing interest rate for hard money and fiat money j in period t, $j = q, m$, $r_{jt} \in \mathbb{R}$.
P_{m2}	The bank's printing of fiat money m in period 2, $P_{m2} \geq 0$.
W_{m2}	The bank's destruction of fiat money m in period 2, $W_{m2} \geq 0$.

Dependent variable

π_t	Inflation rate in period t, $\pi_t \geq 0$.

Agent i's dependent variable i, i=1,…,n

U_{it}	Agent i's Cobb Douglas utility in period t, $U_{it} \geq 0$.

The bank's dependent variable

U_t	The bank's Cobb Douglas utility in period t, $U_t \geq 0$.

Appendix B. Comparing periods 1 and 2

Dividing (3) by (1), agent 1 prefers to borrow $L_{1q2} + L_{1m2}$ if

$$\frac{\left((1+I_{o2})(o_{11}+L_{1q1}+L_{1m1}+L_{1m2}+L_{1q2})\right)^{\alpha_{1o2}}}{\left((1+I_{o1})(o_{11}+L_{1q1}+L_{1m1})\right)^{\alpha_{1o1}}}$$

$$\frac{\left((1+r_{q2})\left(L_{1q1}+L_{1q2}\right)\right)^{-\alpha_{1qL2}}\left((1+r_{m2})(L_{1m1}+L_{1m2})\right)^{-\alpha_{1mL2}}}{\left((1+r_{q1})L_{1q1}\right)^{-\alpha_{1qL1}}\left((1+r_{m1})L_{1m1}\right)^{-\alpha_{1mL1}}(1+\pi_2)^{-\alpha_{1mL2}}} > 1 \tag{9}$$

Dividing (4) by (2), agent 2 prefers to sell an amount $L_{1q2} + L_{1m2} = o_{21} - o_{22}$ of its other assets if

$$\frac{\left((1+I_{q2})(q_{21}+L_{1q2})\right)^{\alpha_{2q2}}\left((1+I_{m2})(m_{21}+L_{1m2})\right)^{\alpha_{2m2}}}{\left((1+I_{q1})q_{21}\right)^{\alpha_{2q1}}\left((1+I_{m1})m_{21}\right)^{\alpha_{2m1}}(1+\pi_2)^{\alpha_{2m2}}}$$

$$\frac{\left((1+I_{o2})(o_{21}-L_{1q2}-L_{1m2})\right)^{\alpha_{2o2}}}{\left((1+I_{o1})o_{21}\right)^{\alpha_{2o1}}} > 1 \tag{10}$$

Dividing (5) by (2), agents $3, \ldots, n$ prefer the trade between agents 1 and 2 if

$$\frac{\left((1+I_{q2})(q_{i1}+L_{iq2})\right)^{\alpha_{iq2}}\left((1+I_{m2})m_{i2}\right)^{\alpha_{im2}}\left((1+I_{o2})o_{i2}\right)^{\alpha_{io2}}}{\left((1+I_{q1})q_{i1}\right)^{\alpha_{iq1}}\left((1+I_{m1})m_{i1}\right)^{\alpha_{im1}}(1+\pi_2)^{\alpha_{im2}}\left((1+I_{o1})o_{i1}\right)^{\alpha_{io1}}} > 1 \tag{11}$$

Dividing (7) by (6), the bank prefers to lend $L_{1q2}+L_{1m2}$ to agent 1 if

$$\frac{\left(\left(1+I_{q2}\right)\left(q_1-L_{1q1}-L_{1q2}\right)\right)^{\beta_{q2}}\left(\left(1+I_{m2}\right)\left(m_1-L_{1m1}+P_{m2}-W_{m2}\right)\right)^{\beta_{m2}}}{\left(\left(1+I_{q1}\right)\left(q_1-L_{1q1}\right)\right)^{\beta_{q1}}\left(\left(1+I_{m1}\right)\left(m_1-L_{1m1}\right)\right)^{\beta_{m1}}\left(1+\pi_2\right)^{\beta_{m2}}}$$

$$\frac{\left(\left(1+r_{q2}\right)\left(L_{1q1}+L_{1q2}\right)\right)^{\beta_{qL2}}\left(\left(1+r_{m2}\right)\left(L_{1m1}+L_{1m2}\right)\right)^{\beta_{mL2}}}{\left(\left(1+r_{q1}\right)L_{1q1}\right)^{\beta_{qL1}}\left(\left(1+r_{m1}\right)L_{1m1}\right)^{\beta_{mL1}}\left(1+\pi_2\right)^{\beta_{mL2}}}>1 \tag{12}$$

Since agent1 has the same three inputs in periods 1 and 2, we set $\alpha_{1j1}=\alpha_{1j2},\ j=o,qL,mL$. Thus, (9) is simplified as

$$\left(\frac{\left(1+I_{o2}\right)\left(o_{11}+L_{1q1}+L_{1m1}+L_{1m2}+L_{1q2}\right)}{\left(1+I_{o1}\right)\left(o_{11}+L_{1q1}+L_{1m1}\right)}\right)^{\alpha_{1o2}}$$

$$\left(\frac{\left(1+r_{q2}\right)\left(L_{1q1}+L_{1q2}\right)}{\left(1+r_{q1}\right)L_{1q1}}\right)^{-\alpha_{1qL2}}\left(\frac{\left(1+r_{m2}\right)\left(L_{1m1}+L_{1m2}\right)}{\left(1+r_{m1}\right)L_{1m1}\left(1+\pi_2\right)}\right)^{-\alpha_{1mL2}}>1 \tag{13}$$

Since agent 2 has the same three inputs in periods 1 and 2, we set $\alpha_{2j1}=\alpha_{2j2},\ j=q,m,o$. Thus, (10) is simplified as

$$\left(\frac{\left(1+I_{q2}\right)\left(q_{21}+L_{1q2}\right)}{\left(1+I_{q1}\right)q_{21}}\right)^{\alpha_{2q2}}\left(\frac{\left(1+I_{m2}\right)\left(m_{21}+L_{1m2}\right)}{\left(1+I_{m1}\right)m_{21}\left(1+\pi_2\right)}\right)^{\alpha_{2m2}}$$

$$\left(\frac{\left(1+I_{o2}\right)\left(o_{21}-L_{1q2}-L_{1m2}\right)}{\left(1+I_{o1}\right)o_{21}}\right)^{\alpha_{2o2}}>1 \tag{14}$$

Since agent $i,\ i=3,\ldots,n$ have the same three inputs in periods 1 and 2, we set $\alpha_{ij1}=\alpha_{ij2},\ j=q,m,o$. Thus, (11) is simplified as

$$\left(\frac{\left(1+I_{q2}\right)\left(q_{i1}+L_{iq2}\right)}{\left(1+I_{q1}\right)q_{i1}}\right)^{\alpha_{iq2}}\left(\frac{1+I_{m2}}{\left(1+I_{m1}\right)\left(1+\pi_2\right)}\right)^{\alpha_{im2}}\left(\frac{1+I_{o2}}{1+I_{o1}}\right)^{\alpha_{io2}}>1 \tag{15}$$

Since the bank has the same four inputs in periods 1 and 2, we set $\beta_{j1}=\beta_{j2},\ j=q,m,qL,mL$. Thus, (12) is simplified as

$$\left(\frac{\left(1+I_{q2}\right)\left(q_1-L_{1q1}-L_{1q2}\right)}{\left(1+I_{q1}\right)\left(q_1-L_{1q1}\right)}\right)^{\beta_{q2}}\left(\frac{\left(1+I_{m2}\right)\left(m_1-L_{1m1}+P_{m2}-W_{m2}\right)}{\left(1+I_{m1}\right)\left(m_1-L_{1m1}\right)\left(1+\pi_2\right)}\right)^{\beta_{m2}}$$

$$\left(\frac{\left(1+r_{q2}\right)\left(\left(L_{1q1}+L_{1q2}\right)\right)}{\left(1+r_{q1}\right)L_{1q1}}\right)^{\beta_{qL2}}\left(\frac{\left(1+r_{m2}\right)\left(L_{1m1}+L_{1m2}\right)}{\left(1+r_{m1}\right)L_{1m1}\left(1+\pi_2\right)}\right)^{\beta_{qL2}}>1 \tag{16}$$

Appendix C. Interpretation of 41 of the panels in Figure 1

In Figure 1b, as agent 1's holding o_{11} of other assets in period 1 increases, its utilities U_{11} and U_{12} increase concavely toward infinity. Agent 1 prefers not to borrow $L_{1q2}+L_{1m2}$ of money from the bank since $U_{12}>U_{11}$. The utilities U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, and U_2 remain constant since agent 1's holding o_{11} of other assets has no impact on agents 2 and i, and the bank. The inflation rate π_2 is constant since o_{11} plays no role in (8).

In Figure 1c, as agent 1's borrowing L_{1q1} in hard money in period 1 increases, the bank's utilities U_1 and U_2 are inverse U shaped. The bank prefers to lend an optimal amount L_{1q1} of hard money to agent 1 in period 1. The maximum of U_2 is 85.12 when $L_{1q1}=\$85$. The maximum of U_1 is 68.33 when $L_{1q1}=\$100$. The bank's utilities U_1 and U_2 decrease concavely toward zero after the maximum. The bank prefers to lend hard money L_{1q2} to agent 1 when $\$0\le L_{1q1}<\$175.71$. The bank prefers not to lend too much hard money L_{1q1} to agent 1 in period 1, since then it has a limited amount of hard money q_1-L_{1q1} available for lending in period 2. The nature of the bank's Cobb Douglas utility is such that if it lends excessively in both periods, its utility U_2 is low. Agent 1's utilities U_{11} and U_{12} increase with L_{1q1} since agent 1 benefits from buying other assets using its borrowing L_{1q1}. Agents 2 and i's utilities U_{21}, U_{22}, U_{i1}, and U_{i2} are constant since agent 1's borrowing L_{1q1} in hard money has no impact on agents 2 and i. The inflation rate π_2 is constant since L_{1q1} plays no role in (8).

In Figure 1e, as agent 1's borrowing L_{1m1} in fiat money in period 1 increases, the bank's utilities U_1 and U_2 are inverse U shaped. That is, the bank prefers to lend an optimal amount of fiat money to agent 1 in period 1. The maximum of U_2 is 86.46 when $L_{1q1}=\$92.50$. The maximum of U_1 is 68.33 when $L_{1q1}=\$100$. The bank's utilities U_1 and U_2 decrease concavely after their maxima. Agent 1's utilities U_{11} and U_{12} increase with L_{1m1} since agent 1 benefits from borrowing L_{1m1} in fiat money. Agent 1 prefers not to borrow $L_{1q2}+L_{1m2}$ from the bank since $U_{12}>U_{11}$. Agent 2 and i's utilities U_{21}, U_{22}, U_{i1}, and U_{i2} are constant since agent 1's borrowing L_{1m1} in fiat money has no impact on agent 2 and agent i. The inflation rate π_2 is constant since L_{1m1} plays no role in (8).

In Figure 1g, Figure 1h, and Figure 1i, as agent 2's assets holdings j_{21}, $j = q, m, o$, in period 1 increases, its utilities U_{21} and U_{22} increase toward infinity. The inflation rate π_2 decreases convexly and asymptotically toward zero due to division with j_{21} in (8). Agent 1's utility U_{12} decreases slightly since the inflation rate π_2 decreases slightly, which hurts agent 1 because of agent 1's fiat money loans L_{1m1} and L_{1m2}. In contrast, agent i's utility U_{i2} increases slightly because the inflation rate π_2 decreases slightly, which benefits agent i because of its fiat money holding m_{i2}. The bank's utility U_2 increases slightly since the inflation rate π_2 decreases slightly, which benefits the bank because the benefit of the bank's fiat money holding $m_1 - L_{1m1} + P_{m2} - W_{m2}$ of from the decreasing inflation rate π_2 overrides the negative impact of fiat money lending L_{1m2} from the decreasing inflation rate π_2. The utilities U_1, U_{11}, U_{21}, and U_{i1} remain constant since the inflation rate π_2 plays no role in period 1.

In Figure 1j and Figure 1m, as agent i's money holding j_{i1}, $j = q, m$, in period 1 increases, its utility U_{i1} increases concavely toward infinity. The inflation rate π_2 decreases convexly and asymptotically toward zero due to division with j_{i1} in (8). Agent i's utility U_{i2} increases slightly since agent i benefits from the decreasing inflation rate π_2. Agent 1's utility U_{12} decreases slightly since the inflation rate π_2 decreases slightly, which hurts agent 1 because of its fiat money loans L_{1m1} and L_{1m2}. In contrast, agent 2's utility U_{22} increases slightly because the inflation rate π_2 decreases slightly, which benefits agent 2 because of its fiat money holdings m_{22}. The bank's utility U_2 increases slightly since the inflation rate π_2 decreases slightly, which benefits the bank because the benefits of the bank's fiat money holding of $m_1 - L_{1m1} + P_{m2} - W_{m2}$ from the decreasing inflation rate π_2 override the negative impact of fiat money lending L_{1m2} from the decreasing inflation rate π_2. The utilities U_1, U_{11}, and U_{21} remain constant since the inflation rate π_2 plays no role in period 1.

In Figure 1k and Figure 1n, as agent i's period 2 money holding j_{i2}, $j = q, m$, increases above the benchmark $j_{i2} = \$100$, its period 2 utility U_{i2} increases concavely from $U_{i2} = 203.06$, reaching $U_{i2} = U_{i1} = 204$ when $j_{i2} = \$101.88$, and proceeds concavely toward infinity. The inflation rate π_2 is constant since j_{i2} plays no role in (8). The utilities U_1, U_2, U_{11}, U_{21}, and U_{i1} remain constant since agent i's money holding j_{i2} of plays no role in period 1. The utility U_2 remains constant since the inflation rate π_2 is constant in period 2.

In Figure 1l and Figure 1o, analogously to Figure 1j and Figure 1m, as agent i's money holding $j_{i1} = j_{i2}$, $j = q, m$, in the two periods increases, its utilities U_{21} and U_{22} increase concavely toward infinity. The inflation rate π_2 decreases convexly and asymptotically toward zero due to division with j_{i1} in (8). Agent 1's utility U_{12} decreases slightly since the inflation rate π_2 decreases slightly, which hurts agent 1 because of its fiat money loans L_{1m1} and L_{1m2}. In contrast, agent 2's utility U_{22} increases slightly because the inflation rate π_2 decreases slightly, which benefits agent 2 because of its fiat money holdings m_{22}. The bank's utility U_2 increases slightly since the inflation rate π_2 decreases slightly, overriding the negative impact of fiat money lending L_{1m2}, which benefits the bank because of its fiat money holding $m_1 - L_{1m1} + P_{m2} - W_{m2}$. The utilities U_1, U_{11}, U_{21}, and U_{i1} remain constant since the inflation rate π_2 plays no role in period 1.

In Figure 1p, as agent i's holding o_{i1} of other assets in period 1 increases, its utility U_{i1} increases concavely toward infinity. The inflation rate π_2 is constant since o_{i1} plays no role in (8). The utilities U_1, U_{11}, and U_{21} remain constant since agent i's holding o_{i1} of other assets has no impact on the bank and agents 2 and i. The utility U_{i2} is constant since agent i's holding o_{i2} of other assets is constant in period 2. The utility U_2 remains constant since the inflation rate π_2 is constant in period 2.

In Figure 1q, analogously, as agent i's holding o_{i2} of other assets in period 2 increases, its utility U_{i2} increases concavely toward infinity. The inflation rate π_2 is constant since o_{i2} plays no role in (8). The utilities U_1, U_{11}, U_{21}, and U_{i1} remain constant since agent i's holding o_{i2} of other assets plays no role in period 1. The utility U_2 remains constant since the inflation rate π_2 is constant in period 2.

In Figure 1r, analogously, as agent i's holding $o_{i1} = o_{i2}$ of other assets in the two periods increase, its utilities U_{i1} and U_{i2} increase concavely toward infinity. The inflation rate π_2 is constant since o_{i1} and o_{i2} play no role in (8). The utilities U_1, U_{11}, and U_{21} remain constant since agent i's holding $o_{i1} = o_{i2}$ of other assets plays no role in period 1. The bank's utility U_2 remains constant since the inflation rate π_2 is constant in period 2.

In Figure 1u and Figure 1v, as the bank's money holding j_1, $j = q, m$ in period 1 increases, its utilities U_1 and U_2 increase concavely to infinity. The period 2 inflation rate π_2 decreases convexly and asymptotically toward zero due to division with j_1 in (8). Agent 1's utility U_{12} decreases slightly since the inflation rate π_2 decreases slightly, which hurts agent 1 because of its fiat money loans L_{1m1} and L_{1m2}. In contrast, the utilities U_{22} and U_{i2} increase slightly because the inflation rate π_2 decreases slightly, which benefits agents 2 and i because of their fiat money holdings m_{22} and m_{i2}. More specifically, agent i's utility U_{i2} approaches U_{i1} asymptotically from below as j_1 approaches infinity, i.e. $\lim_{q_1 \to \infty} U_{i2} = U_{i1} = 204.00$. The utilities U_{11}, U_{21}, and U_{i1} remain constant since neither q_1 nor the inflation rate π_2 impact agents 1, 2 and i in period 1.

In Figure 1x and Figure 1aa, as the interest rate I_{j2} for holding money j, $j = q, m$, in period 2 increases, which is intuitively beneficial to the bank and agent i, $i = 2, \ldots n$, the three utilities U_2, U_{22}, and U_{i2} increase concavely toward infinity. The inflation rate π_2 is constant since I_{j2} plays no role in (8). Agent 1's utilities U_{11} and U_{12} are constant since agent 1 holds no money j in the two periods. The utilities U_1, U_{21}, and U_{i1} remain constant since I_{j2} plays no role in period 1.

In Figure 1y and Figure 1ab, as the interest rate $I_{j1} = I_{j2}$ for holding money j, $j = q, m$, in the two periods increases, which is intuitively beneficial to the bank and agent i, $i = 2, \ldots n$, the six utilities U_1 U_2, U_{21}, U_{22}, U_{i1}, and U_{i2} increase concavely toward infinity, equivalently to the three concave increases in Figure 1w and the three concave increases in Figure 1x. The inflation rate π_2 is constant since I_{j1} and I_{j2} play no role in (8). Agent 1's utilities U_{11} and U_{12} are constant since agent 1 holds no money j in the two periods.

In Figure 1ac, as the interest rate I_{o1} for holding other assets o in period 1 increases, which is intuitively beneficial to all the agents, the utilities U_{21}, U_{21}, and U_{i1} increase concavely toward infinity. The inflation rate π_2 is constant since I_{o1} plays no role in (8). The bank's utilities U_1 and U_2 are constant since the bank holds no other assets o in the two periods. The utilities U_2, U_{22}, and U_{i2} remain constant since I_{o1} plays no role in period 2.

In Figure 1ad, as the interest rate I_{o2} for holding other assets o in period 2 increases, which is intuitively beneficial to all the agents, the utilities U_{12}, U_{22}, and U_{i2} increase concavely toward infinity. The inflation rate π_2 is constant since I_{o2} plays no role in (8). The bank's utilities U_1 and U_2 are constant since the bank holds no other assets o in the two periods. The utilities U_1, U_{11}, U_{21}, and U_{i1} remain constant since I_{o2} plays no role in period 1.

In Figure 1ae, as the interest rate $I_{o1} = I_{o2}$ for holding other assets o in the two periods increases, which is intuitively beneficial to all the agents, the utilities U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, and U_{i2} increase concavely toward infinity. The inflation rate π_2 is constant since I_{o1} and I_{o2} play no role in (8). The bank's utilities U_1 and U_2 are constant since it holds no other assets in the two periods.

In Figure 1ag and Figure 1aj, as the borrowing interest rate r_{j2} for money $j, j = q, m$ in period 2 increases, which is intuitively beneficial to the bank, the bank's utility U_2 increases concavely toward infinity. Agent 1's utility U_{12} decreases convexly toward zero since a higher borrowing interest rate r_{j2} for money j is costly. The inflation rate π_2 is constant since r_{j2} plays no role in (8). The utilities U_{22} and U_{i2} are constant since agents 2 and i do not borrow money j in period 2. The utilities U_1, U_{11}, U_{21}, and U_{i1} remain constant since r_{j2} plays no role in period 2.

In Figure 1ah and Figure 1ak, as the borrowing interest rate $r_{j1} = r_{j2}$ for money $j, j = q, m$ in the two periods increases, which is intuitively beneficial to the bank, the bank's utilities U_1 and U_2 increase concavely toward infinity. These two concave increases are equivalent to the concave increases in Figure 1af, Figure 1ai, Figure 1ag and Figure 1aj. Agent 1's utilities U_{11} and U_{12} decrease convexly toward zero since higher borrowing interest rate $r_{j1} = r_{j2}$ for money j is costly. These two convex decreases are equivalent to the convex decreases in Figure 1af, Figure 1ai, Figure 1ag and Figure 1aj. The inflation rate π_2 is constant since r_{j1} and r_{j2} play no role in (8). The utilities U_{21}, U_{22}, U_{i1} and U_{i2} are constant since agents 2 and i do not borrow money j in the two periods. Thus, r_{j1} and r_{j2} play no role for agents 2 and i.

In Figure 1al, as agent 1's Cobb Douglas elasticity α_{1o1} for holding $o_{11} + L_{1q1} + L_{1m1}$ of other assets in period 1 increases, which is intuitively beneficial to agent 1, its utility U_{11} increases concavely. Agent 1 wants to borrow $L_{1q2} + L_{1m2}$ from the bank in period 2 when α_{1o1} is sufficiently low, i.e. $0 \leq \alpha_{1o1} < 0.50$. The inflation rate π_2 is constant since α_{1o1} plays no role in (8). Agent 1's utility U_{12} is constant since α_{1o1} plays no role in period 2. The utilities U_1, U_2, U_{21}, U_{22}, U_{i1}, and U_{i2} remain constant since α_{1o1} has no impact on the bank, agents 2 and i.

In Figure 1an, as agent 1's Cobb Douglas elasticity $\alpha_{1o1} = \alpha_{1o2}$ for holding $o_{11} + L_{1q1} + L_{1m1}$ and $o_{11} + L_{1q1} + L_{1m1} + L_{1q2} + L_{1m2}$ of other assets in the two periods increases, its utilities U_{11} and U_{12} increase concavely, and equivalently to the concave increases in Figure 1al and Figure 1am. Agent 1 wants to borrow $L_{1q2} + L_{1m2}$ from the bank in period 2 when $\alpha_{1o1} = \alpha_{1o2}$ is not too low, i.e. $0.5 \leq \alpha_{1o1} = \alpha_{1o2} < 1$. The inflation rate π_2 is constant since α_{1o1} and α_{1o2} play no role in (8). The utilities U_1, U_2, U_{21}, U_{22}, U_{i1}, and U_{i2} remain constant since α_{1o1} and α_{1o2} have no impact on the bank, agents 2 and i.

In Figure 1ao, as agent 1's Cobb Douglas elasticity α_{1qL1} for borrowing L_{1q1} in hard money in period 1 increases, its utility U_{11} in period 1 is constant since the bank does not print money to lend L_{1m1} to agent 1. Hence decreasing Cobb Douglas elasticity $\alpha_{1mL1} = 1 - \alpha_{1o1} - \alpha_{1qL1}$ due to increasing Cobb Douglas elasticity α_{1qL1} has no impact since $L_{1q1} = L_{1m1} = \$10$. The nine variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1ap, as agent 1's Cobb Douglas elasticity α_{1qL2} for borrowing L_{1q2} in hard money in period 2 increases, its utility U_{12} in period 2 decreases slightly because the positive inflation rate $\pi_2 = 1.875\%$ becomes less beneficial for agent 1 when lower Cobb Douglas elasticity $\alpha_{1mL2} = 1 - \alpha_{1o2} - \alpha_{1qL2}$ is assigned to borrowing $L_{1m1} + L_{1m2}$ in fiat money. The eight variables U_{11}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1aq, as agent 1's Cobb Douglas elasticity $\alpha_{1qL1} = \alpha_{1qL2}$ for borrowing L_{1q1} and L_{1q2} in hard money in the two periods increases, the results are as in Figure 1ap where only α_{1qL2} changes while α_{1qL1} is constant. The reason follows from Figure 1ao where the changing Cobb Douglas elasticity α_{1qL1} does not impact the nine variables.

In Figure 1ar and Figure 1au, as agent 2's Cobb Douglas elasticity α_{2j1} for holding money $j_{21}, j = q, m$ in period 1 increases, which means decreasing Cobb Douglas elasticity $\alpha_{2o1} = 1 - \alpha_{2q1} - \alpha_{2m1}$ for holding other assets o_{21}, agent 2's utility U_{21} in period 1 decreases because holding other assets o_{21} becomes less beneficial. Hence agent 2 prefers period 1 when $\alpha_{2j1} < 0.23$ and prefers period 2 when $0.23 \leq \alpha_{2j1} \leq 1$. That is, agent 2 wants to sell its other assets valued as $L_{1q2} + L_{1m2}$ when α_{2j1} is sufficiently high, i.e. $0.23 \leq \alpha_{2j1} \leq 1, j = q, m$. The eight variables U_{11}, U_{12}, U_{22}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1at, as agent 2's Cobb Douglas elasticity $\alpha_{2q1} = \alpha_{2q2}$ for holding hard money q_{21} and $q_{21} + L_{1q2}$ in the two periods increases, its utilities U_{21} and U_{22} decrease convexly, and equivalently to the convex decreases in Figure 1ar and Figure 1as. Agent 2 wants to sell other assets valued as $L_{1q2} + L_{1m2}$ when $\alpha_{2q1} = \alpha_{2q2}$ is sufficiently high, i.e. $0.13 \leq \alpha_{2q1} = \alpha_{2q2} \leq 1$. The seven variables U_{11}, U_{12}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1aw, as agent 2's Cobb Douglas elasticity $\alpha_{2m1} = \alpha_{2m2}$ for holding fiat money m_{21} and $m_{21} + L_{1m2}$ in the two periods increases, its utilities U_{21} and U_{22} decrease convexly, and equivalently to the convex decreases in Figure 1au and Figure 1av. Agent 2 wants to sell other assets valued as $L_{1q2} + L_{1m2}$ when $\alpha_{2m1} = \alpha_{2m2}$ is sufficiently high, i.e. $0.12 \leq \alpha_{2m1} = \alpha_{2m2} \leq 1$. The seven variables U_{11}, U_{12}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1ax and Figure 1ba, as agent i's Cobb Douglas elasticity α_{ij1} for holding money $j_{i1}, j = q, m$ in period 1 increases, its utility U_{i1} in period 1 decreases convexly because holding other assets o_{i1} becomes less beneficial for agent i with decreasing Cobb Douglas elasticity $\alpha_{io1} = 1 - \alpha_{iq1} - \alpha_{im1}$. Agent i prefers the trade between agents 1 and 2 when α_{ij1} is sufficiently high, i.e. $0.25 \leq \alpha_{iq1} \leq 1, j = q, m$. The eight variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i2}, U_1, U_2, π_2 remain constant.

In Figure 1az and Figure 1bc, as agent i's Cobb Douglas elasticity $\alpha_{ij1} = \alpha_{ij2}$ for holding money $j_{it}, j = q, m, t = 1, 2$ in the two periods increases, its utilities U_{i1} and U_{i2} decrease convexly, and equivalently to the convex decreases in Figure 1ax and Figure 1ay, Figure 1ba and Figure 1bb. Agent i does not prefer the trade between agents 1 and 2 since $U_{i2} < U_{i1}$ holds for Figure 1az and Figure 1bc. The seven variables U_{11}, U_{12}, U_{21}, U_{22}, U_1, U_2, π_2 remain constant.

In Figure 1bd and Figure 1bg, as the bank's Cobb Douglas elasticity β_{j1} for holding money $j_1 - L_{1j1}, j = q, m$ in period 1 increases, its utility U_1 in period 1 increases convexly because holding money $j_1 - L_{1j1}$ becomes more beneficial for the bank with increasing Cobb Douglas elasticity β_{j1}, which overrides the negative impact of the decreasing Cobb Douglas elasticity $\beta_{mL1} = 1 - \beta_{q1} - \beta_{m1} - \beta_{qL1}$ for fiat money loans. The bank wants to give money loans $L_{1q2} + L_{1m2}$ to agent 1 in period 2 when β_{j1} is sufficiently low, i.e. $0 \le \beta_{j1} \le 0.4, j = q, m$. The eight variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_2, π_2 remain constant.

In Figure 1bf, as the bank's Cobb Douglas elasticity $\beta_{q1} = \beta_{q2}$ for holding hard money $q_1 - L_{1q1}$ and $q_1 - L_{1q1} - L_{1q2}$ in the two periods increases, its utilities U_1 and U_2 in the two periods increase convexly because holding money $q_1 - L_{1q1}$ and $q_1 - L_{1q1} - L_{1q2}$ becomes more beneficial for the bank with increasing Cobb Douglas elasticity $\beta_{q1} = \beta_{q2}$, which overrides the negative impact of the decreasing Cobb Douglas elasticity $\beta_{mL1} = 1 - \beta_{q1} - \beta_{m1} - \beta_{qL1} = \beta_{mL2} = 1 - \beta_{q2} - \beta_{m2} - \beta_{qL2}$ for fiat money loans. The bank wants to give money loans $L_{1q2} + L_{1m2}$ to agent 1 in period 2 when $\beta_{q1} = \beta_{q2}$ is sufficiently low, i.e. $0 \le \beta_{q1} = \beta_{q2} \le 0.69$. The seven variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, π_2 remain constant.

In Figure 1bi, as the bank's Cobb Douglas elasticity $\beta_{m1} = \beta_{m2}$ for holding fiat money $m_1 - L_{1m1}$ and $m_1 - L_{1m1} + P_{m2} - W_{m2}$ in the two periods increases, its utilities U_1 and U_2 in the two periods increase convexly because holding money $m_1 - L_{1m1}$ and $m_1 - L_{1m1} + P_{m2} - W_{m2}$ become more beneficial for the bank with increasing Cobb Douglas elasticity $\beta_{m1} = \beta_{m2}$, which overrides the negative impact of decreasing Cobb Douglas elasticity $\beta_{mL1} = 1 - \beta_{q1} - \beta_{m1} - \beta_{qL1} = \beta_{mL2} = 1 - \beta_{q2} - \beta_{m2} - \beta_{qL2}$ for fiat money loans. The bank wants to give money loans $L_{1q2} + L_{1m2}$ to agent 1 in period 2 when $\beta_{m1} = \beta_{m2}$ is sufficiently low, i.e. $0 \le \beta_{m1} = \beta_{m2} \le 0.72$. The seven variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, π_2 remain constant.

In Figure 1bj, as the bank's Cobb Douglas elasticity β_{qL1} for hard money lending L_{1q1} in period 1 increases, its utility U_1 remains constant because the benefit of increasing Cobb Douglas elasticity β_{qL1} is offset by the negative impact of decreasing Cobb Douglas elasticity $\beta_{mL1} = 1 - \beta_{q1} - \beta_{m1} - \beta_{qL1}$. The bank benefits from lending money $L_{1q2} + L_{1m2}$ to agent 1 in period 2 because $r_{j2} > I_{j2}, j = q, m$. The bank always wants to lend fiat money $L_{1q2} + L_{1m2}$ to agent 1 in period 2 since $U_1 < U_2$. The nine variables U_{11}, U_{12}, U_{21}, U_{22}, U_{i1}, U_{i2}, U_1, U_2, π_2 remain constant.

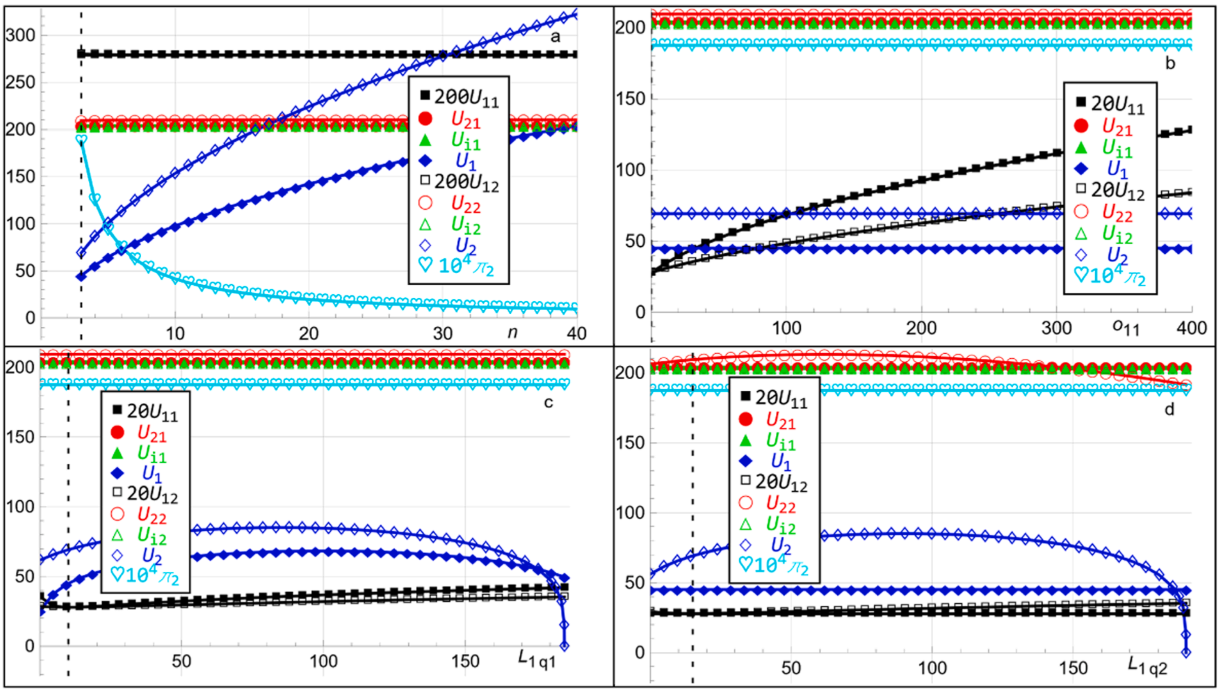


Fig. 1. Agent 1's utilities U_{11} and U_{12}, agent 2's utilities U_{21} and U_{22}, agent i's utilities U_{i1} and U_{i2}, the bank's utilities U_1 and U_2, and the inflation rate π_2, respectively, relative to the benchmark parameter values $q_{11} = q_{12} = m_{11} = m_{12} = o_{11} = \$0$, $L_{1q1} = L_{1m1} = \$10$, $L_{1q2} = L_{1m2} = \$15$, $q_{21} = m_{21} = \$100$, $n = 3$, $q_{i1} = q_{i2} = m_{i1} = m_{i2} = \$100$, $o_{i1} = o_{i2} = \$400$, $q_1 = m_1 = \$200$, $P_{m2} = W_{m2} = \$0$, $\alpha_{io1} = \alpha_{io2} = 1/2$, $i = 1, \ldots, n$, $\alpha_{1qL1} = \alpha_{1qL2} = \alpha_{1mL1} = \alpha_{1mL2} = 1/4$, $\alpha_{iq1} = \alpha_{iq2} = \alpha_{im1} = \alpha_{im2} = 1/4$, $\beta_{q1} = \beta_{q2} = \beta_{qL1} = \beta_{qL2} = 1/4$, $\beta_{m1} = \beta_{m2} = \beta_{mL1} = \beta_{mL2} = 1/4$, $\pi_2 = 1.875\%$, $I_{q1} = I_{q2} = I_{m1} = I_{m2} = I_{o1} = I_{o2} = 2\%$, $r_{q1} = r_{q2} = r_{m1} = r_{m2} = 5\%$.

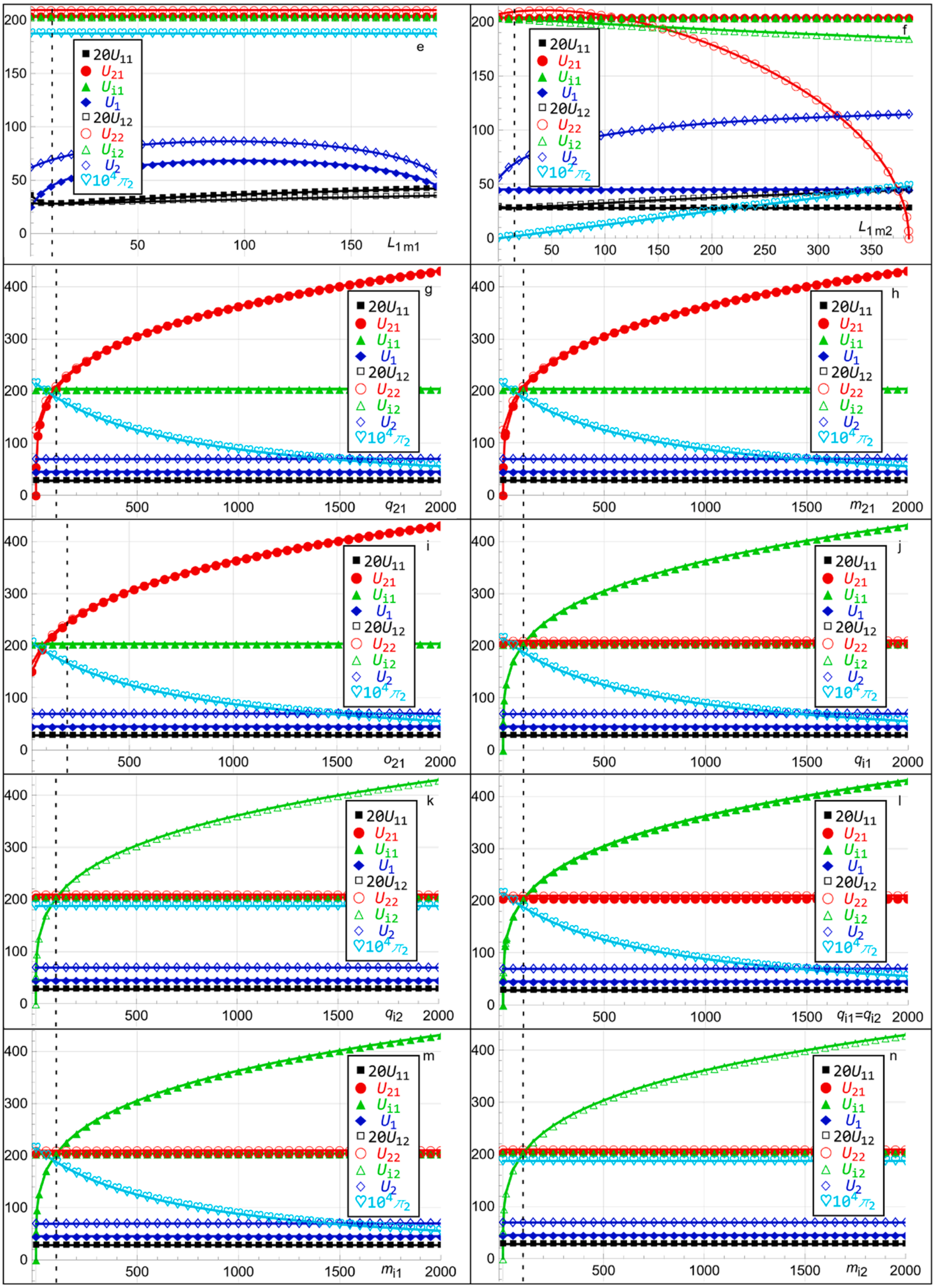
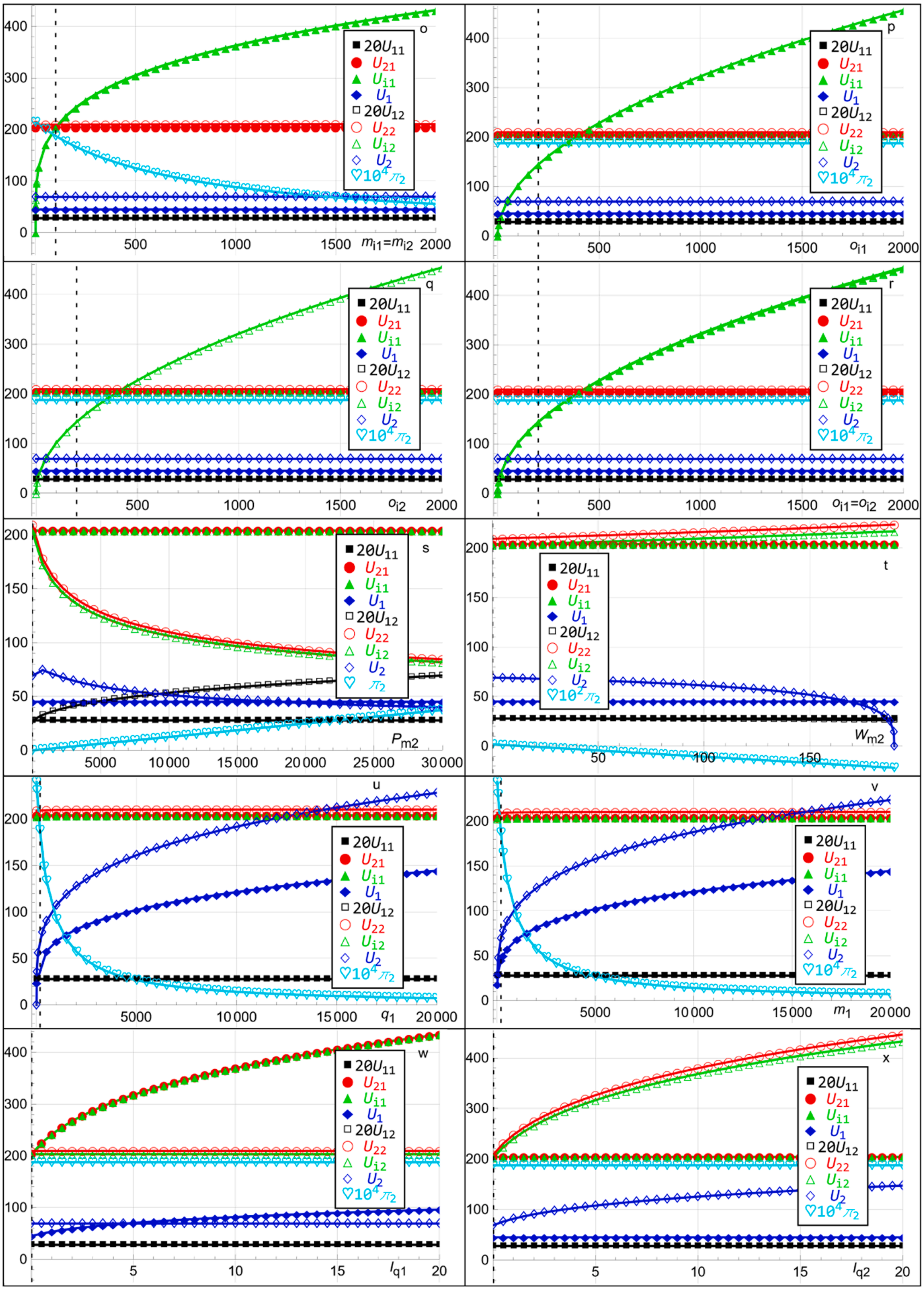


Fig. 1. (*continued*).



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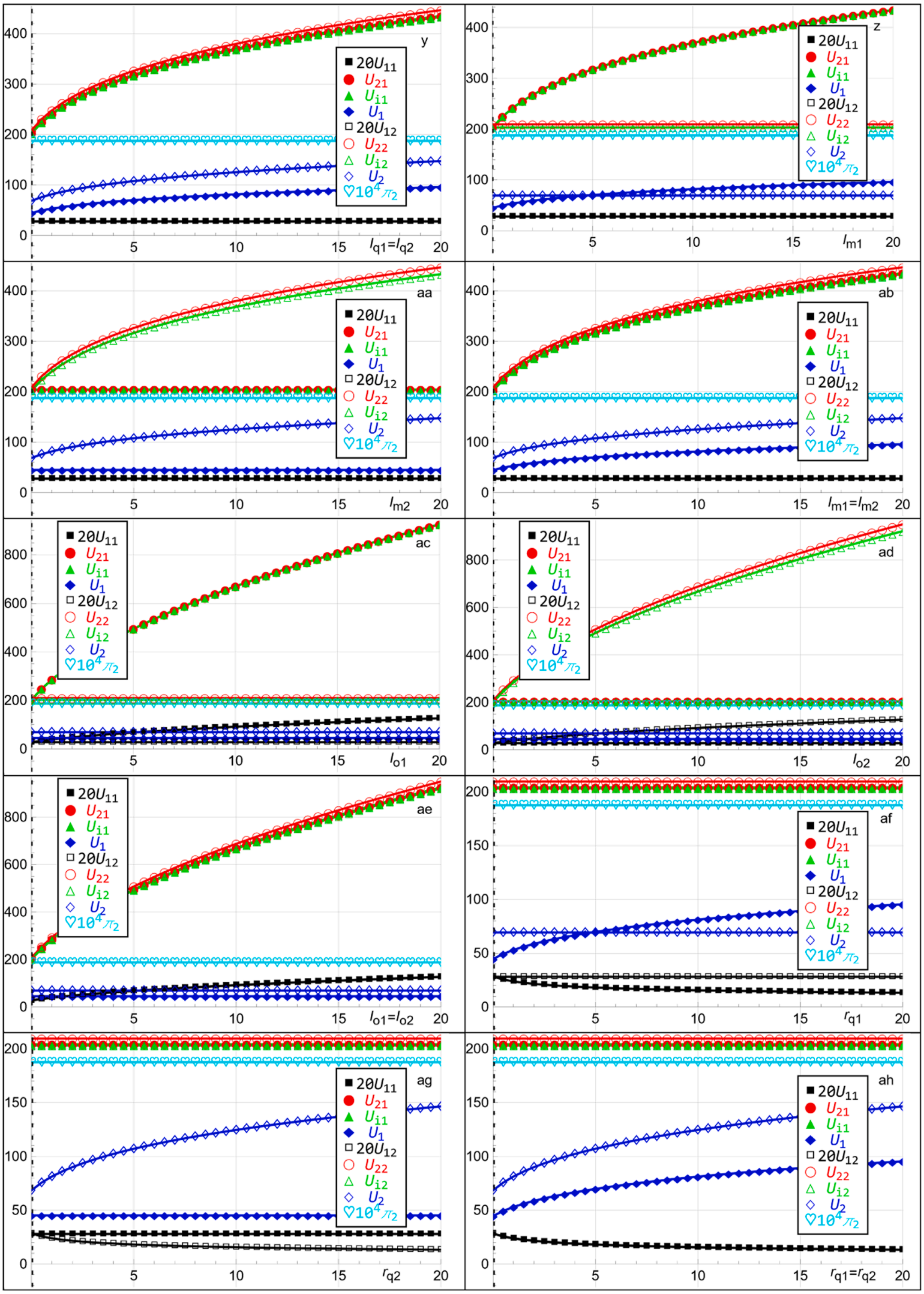

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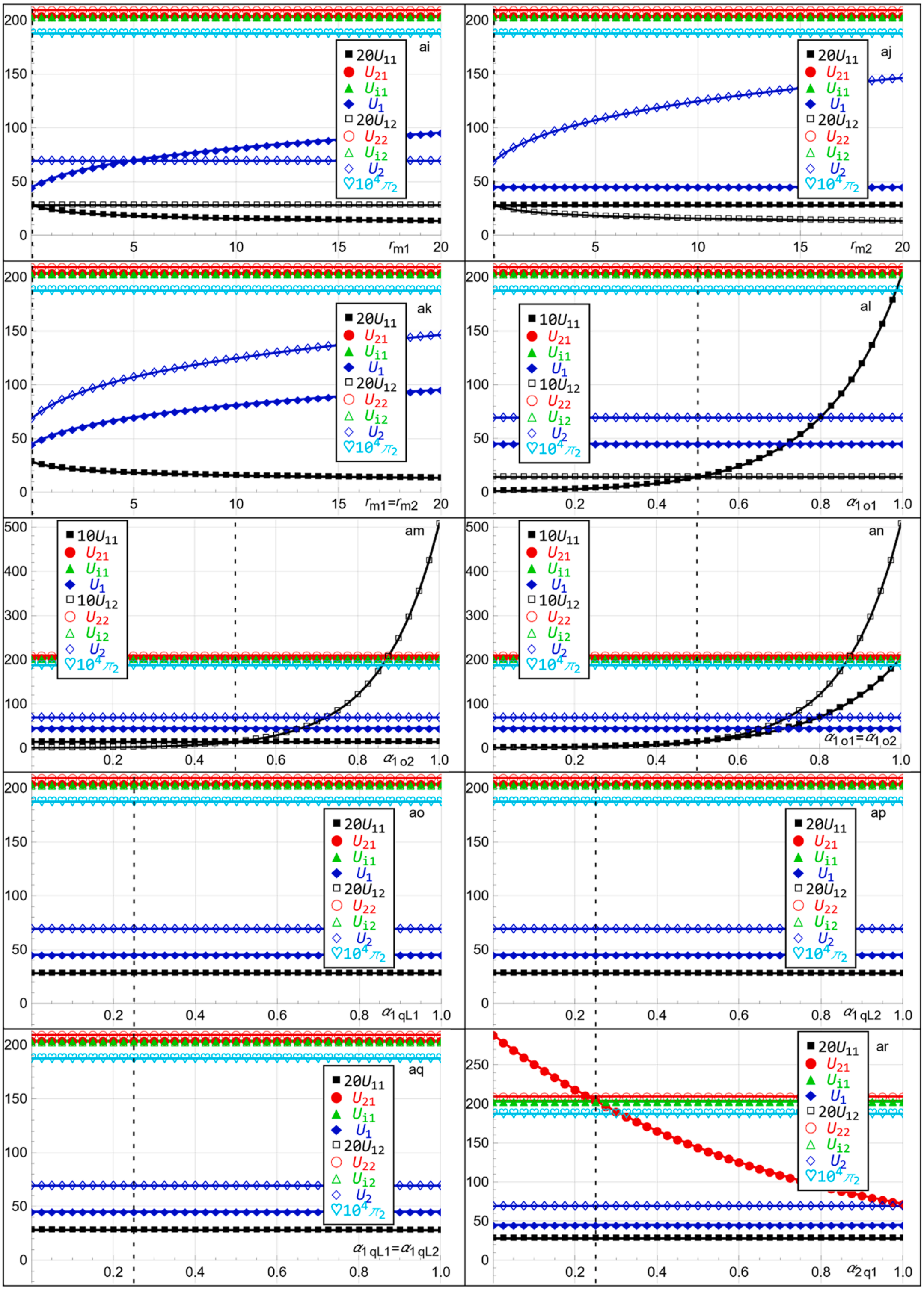

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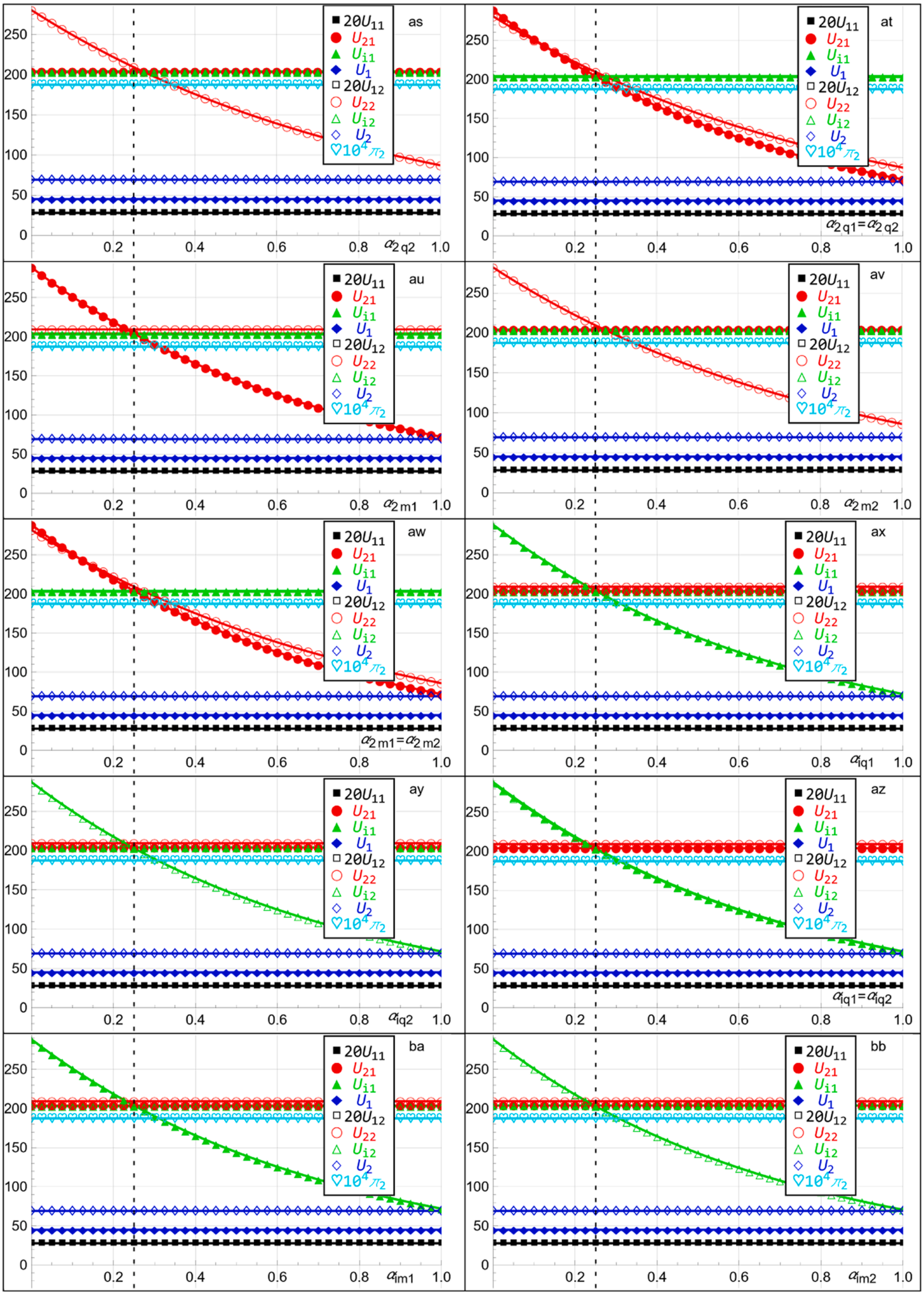

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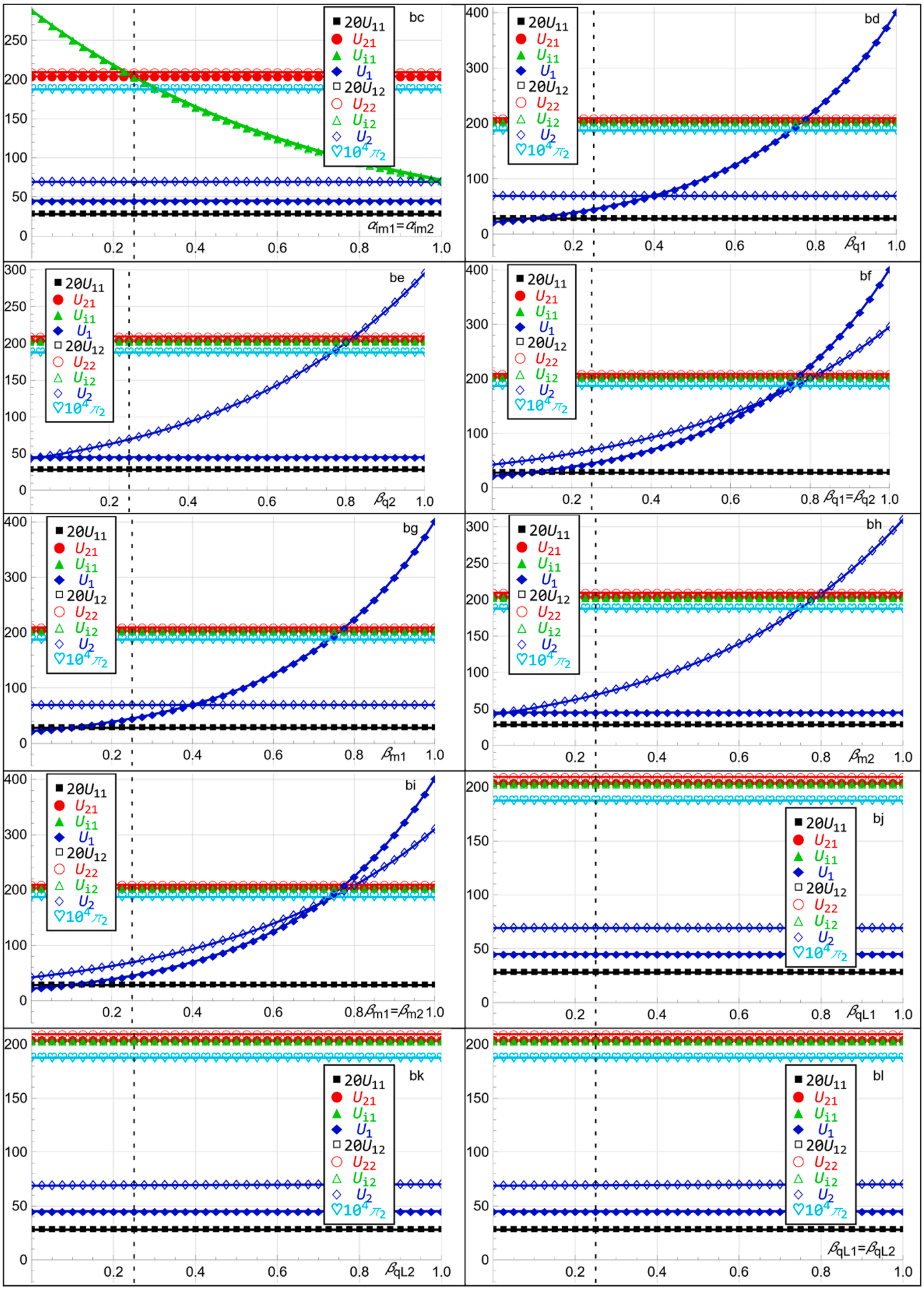

Fig. 1. (*continued*).

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