Impediments to learning problem-solving in Malawian lower secondary mathematics textbooks

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Background: In a culture where teachers follow the textbook prescriptively, Malawian students perform low in mathematics, and no students reach the problem-solving levels.

Aim: To explore reasons for students' low performance, this study aims at investigating opportunities to learn problem-solving in Malawian mathematics textbooks.

Setting: This study focuses on Malawian mathematics textbooks in the lower secondary grades focusing on the areas of linear equations and simultaneous linear equations. These areas have a particular emphasis on problem-solving.

Methods: Four textbooks from two of the most widely used series of mathematics textbooks in Malawian secondary school were analysed. The Mathematics Discourse in Instruction framework was used to analyse examples and tasks in the four textbooks.

Results: Analysis indicates that the textbooks provide relatively few opportunities to learn problem-solving, and most of the opportunities are given through word problems. These word problems are typically presented towards the end of the chapters, and students are thus stimulated to apply already learned procedures to solve the problems rather than learn through problem-solving.

Conclusion: Limitations in opportunities to learn problem-solving are particularly challenging in a context like Malawi, where teacher–textbook compliance is high, where there is a shortage of qualified mathematics teachers and where few students have access to their own textbook.

Contribution: This study provides an overview of impediments to learning problem-solving in Malawian mathematics textbook, and knowledge about such impediments is necessary for change.

Keywords: mathematics; textbooks; problem-solving; algebra; Malawi.

Introduction

At some point, everyone has met a mathematical task that was hard to solve – or even get started with. Such tasks are often referred to as *problems*, and problem-solving is at the heart of mathematics (Van Zanten & Van den Heuvel-Panhuizen 2018). Most learners encounter mathematical tasks from a textbook, and mathematics textbooks influence their opportunities to learn mathematics. If textbooks are limited, more responsibility is placed on the teacher. In contexts with limited resources, like in many African countries, teachers tend to depend on mathematics textbooks (Leshota 2020). In Malawi, textbooks are often the only instructional tool (Mwadzaangati 2019b). Learners' performance in mathematics is generally low – even when compared to other countries in Southern Africa (Kazima 2014). According to results from the SACMEQ IV study (Awich 2021), few primary students in Malawi go beyond the basic numeracy level in mathematics, and no one gets to the problem-solving levels. Yet, problem-solving is foregrounded in the Malawian Mathematics curriculum. It is thus interesting to observe that there have been few previous studies of Malawian mathematics textbooks, and none of the studies have investigated the opportunities to learn problem-solving in the textbooks. This study aims at contributing to this by approaching the following research question: 'What opportunities and impediments to learn problem-solving are available in Malawian lower secondary mathematics textbooks?'

To answer this question, we have analysed a selection of Malawian mathematics textbooks from Grades 9 and 10. We focus on the areas of linear equations and simultaneous linear equations, which are two areas of the Malawian Mathematics curriculum for lower secondary school, in which problem-solving is presented as a suggested method of teaching and learning mathematics.

Before we approach the theoretical background, we must define two key concepts in our study: *problem-solving* and *opportunity to learn*. To define problem-solving, we draw on the classical work of Pólya (1945), where problem-solving can be described as the process of solving problems. Pólya described this process as *heuristic*, and his general four-step method has been widely used. A problem typically refers to a task for which the specific process for solving it is not immediately known to the problem solver (Buishaw & Ayalew 2013). *Opportunity to learn* is a classic concept in research on teaching, which refers to whether or not students have the opportunity to study or learn something. In our study, we consider the opportunity to learn in terms of content coverage in textbooks (cf. Wijaya, Van den Heuvel-Panhuizen & Doorman 2015). With these definitions, we continue in the next sections to provide an overview of research on mathematics textbooks and problem-solving in textbooks, and we present key issues concerning mathematics education in Malawi and previous research on Malawian mathematics textbooks. We then present the methodological considerations of our study, including the analytic framework, before we present and discuss the results of our analysis.

Theoretical background

Problem-solving in mathematics textbooks

Textbooks are resources written for teaching and learning (O'Keeffe 2013). Textbook resources have been more dominant in use among students and teachers than any other resource (Fan, Zhu & Miao 2013; Sunday 2014), and they appear to be particularly influential in mathematics instruction. Nowadays, with the presence of internet and the flourishing digital tools, the use of traditional textbooks is changing in many countries – particularly in more affluent countries (e.g. Golding 2023). Yet, teachers in many countries all over the world still depend on textbooks to guide their lessons (Qi et al. 2018). In a study of Estonian mathematics teachers, Lepik (2015) found that teachers relied heavily on textbooks. The textbooks were mostly used as a source of tasks and were mostly used in the planning phase. A more recent study from Turkey confirmed that teachers also relied on textbooks for planning their lessons (Ulusoy & Incikabi 2020). Where the Estonian teachers did not rely on the teaching methods suggested in the textbooks (Lepik 2015), the Turkish middle school teachers appeared to use textbooks to facilitate most of their instructional work (Ulusoy & Incikabi 2020). In a study of Malawian secondary teachers, Mwadzaangati (2019b) found that teachers used textbooks both to plan lessons and their delivery.

Although textbooks are generally considered important, mathematics textbooks have also been criticised, and many studies that we reviewed on or related to problem-solving show that textbooks provide limited opportunities to learn problem-solving. In the Malawian context, studies have found that textbooks provide limited opportunities to learn mathematics in primary (Chiyombo 2020; Mwadzaangati 2019a) and in secondary school textbooks (Maonga 2020). Similarly, Buishaw and Ayalew (2013) found that Ethiopian mathematics textbooks for Grades 9 and 10 put little emphasis on developing problem-solving skills. Berisha, Thaqi and Jashari (2014) also found that textbooks in Kosovo contained more routine problems that were also not presented in context. The use of heuristics was also minimal. In their study, Van Zanten and Van den Heuvel-Panhuizen (2018) who replicated Kolovou, Van den Heuvel-Panhuizen and Bakker (2009) found Grades 4 and 6 Dutch primary textbooks limited in promoting problem-solving for containing few non-routine tasks. Vicente, Sánchez and Verschaffel (2019) also reported less problem-solving tasks in Spanish textbooks as compared to Singaporean textbooks. The Singaporean textbook tasks demanded more reasoning, while Spanish textbook tasks were superficial. More recently, Jäder, Lithner and Sidenvall (2020) concluded that the textbooks from 12 countries contained less problem-solving as many tasks could be solved using a template provided earlier in the textbooks. Brehmer, Ryve and Van Steenbrugge (2015) found similar results in Swedish upper secondary mathematics textbooks. These studies suggest the need to review textbooks for the inclusion of more problem-solving.

Even in a context like Singapore, where problem-solving has been emphasised in the Mathematics curriculum for decades, textbooks have been found to include mostly traditional and routine tasks. For instance, Fan and Zhu (2000) found that at least 96% of the mathematical problems in Singaporean lower secondary school textbooks were traditional and routine. Heuristics were also used sparingly. In a later study that compared textbooks from China, Singapore and the United States, Fan and Zhu (2007) found that textbooks from all three countries presented several problem-solving heuristics, but all textbooks had limitations. The studies further revealed the dominance of the 'carrying out the plan' stage of Pólya's model of problem-solving.

Malawian curriculum and textbooks

Mathematics is a core subject in Malawi, and it is allocated more textbooks as compared to other subjects (Ministry of Education Science and Technology [MoEST] 2022). It is also allocated seven periods per week, which is the highest number of periods as compared to other subjects except English (MoEST 2015). Furthermore, it is a compulsory subject in both primary and secondary schools. It is considered as the bedrock of science and technology and a propeller of one's intellectual competence on logical reasoning, spatial visualisation, analysis and abstract thought (MoEST 2013). The Ministry of Education, Science and Technology states in the curriculum that secondary school mathematics should 'develop skills like computational, reasoning, critical thinking and problem solving…' (MoEST 2015:39). It is expected of Malawian secondary school graduates to use problem-solving skills learned in mathematics to solve practical problems (MoEST 2013). According to the curriculum documents, it appears that Malawian mathematics is meant to promote problem-solving.

Textbooks in Malawi are authored by private authors and published by private publishers. Yet, the content of the textbooks is determined by the curriculum that is organised by the Ministry of Education, Science and Technology. The authored textbooks are approved by the MoEST through the Malawi Institute of Education (MIE) (Maonga 2020). It is the only approved textbooks that schools can choose from according to their financial capacity. Studies are unclear on how schools select the series of textbooks to use. However, anecdotally, teachers may use any of the approved books according to their preferences.

Like other developing countries, mathematics textbooks are a central instructional tool in mathematics classrooms in Malawi. There are not many studies on textbooks in Malawi, and hence, the information concerning textbooks is limited. However, Mwadzaangati (2019b) asserts that textbooks are an exclusive resource that Malawian teachers use for planning and delivering their lessons. Textbooks are a major tool of instruction that are readily available for teaching mathematics (Maonga 2020). Other Malawian scholars (e.g. Chiyombo 2020; Phiri 2018) share similar assertions. From these studies, it is apparent that textbooks are influential in Malawian mathematics classrooms.

Research methods and design

Research on problem-solving had not been extensively conducted in Malawi, as such we adopted a qualitative content analysis design to understand the problem in depth. Moreover, the text is qualitative (Krippendorff 2004), which made qualitative methods ideal for this textbook's analysis.

Sampling

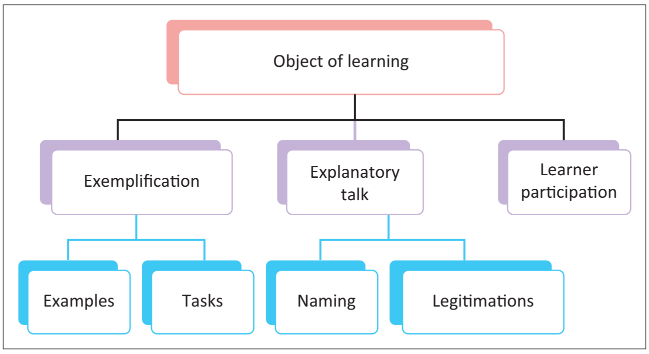
We purposively selected two textbook series that were mostly used in Malawian classrooms. The two series were common in both Grades 9 and 10, which means we had four textbooks altogether and two series for each grade. The selected series were the most sold according to legitimate book stores. We interpreted this to mean that they are the most used textbooks in schools and by students as no statistics are available about textbooks' use in Malawi. We further consulted some teachers who confirmed that they mostly used the same textbooks. Lastly, these textbooks had all the topics outlined in the curriculum; the topics were arranged according to the curriculum, and they had much content. The topics of equations and simultaneous linear equations for Grades 9 and 10, respectively, were the units of analysis. Firstly, problem-solving is indicated as an instructional method for these topics in the curriculum (MoEST 2013:18–20, 35–36). Secondly, algebra is central in mathematics education (Jäder et al. 2020), and equations are core topics in school mathematics (Andrews & Sayers 2012). Finally, as algebra is challenging (Star et al. 2015), it may promote higher-order thinking. We focused on examples and tasks that were the coding units (Krippendorff 2004).

Analytic framework (MDITx)

We employed the Mathematics Discourse in Instruction Analytic framework for Textbooks analysis (MDITx) by Ronda and Adler (2017), which was adapted from the Mathematical Discourse in Instruction framework by Adler and Ronda (2015). The MDITx is used to analyse textbooks to reveal the opportunities to learn that textbooks contain, where as the MDI is used to analyse classroom lessons and reveal the learning opportunities provided in the classroom lessons. It consists of the Objective of learning, Exemplification, Explanatory talk and Learner participation as shown in Figure 1. We focused on Exemplification to determine the similarities between examples and tasks in determining the complexity of the tasks. Examples and tasks constitute the example space (Ronda & Adler 2017). The examples and tasks are assigned levels according to the variations they exhibit and how demanding they are respectively. We elaborate on the levels in the analysis. Note that we defined examples as tasks that have been worked out in the textbooks through which a procedure being applied is performed (Liz et al. 2006). Tasks, on the contrary, were 'the various tasks that students should do…' (Glasnovic Gracin 2018:1005).

Analysis of data

Our analysis involved coding tasks and assigning levels according to the framework. The framework categorises task levels as follows. Level 1 involves tasks that demand already known procedures. Level 2 involves tasks that demand current topic procedures, while level 3 involves tasks that demand application and making connections (Adler & Ronda 2015). We considered levels 2 and 3 as potential problem-solving opportunities with more preference for level 3. We do not emphasise much on the levels of examples because it is mainly through tasks that students could be challenged and show their reasoning while examples model the method to be used in the tasks (Glasnovic Gracin 2018). We also deliberately use the word 'potential' because the MDITx was not originally meant to study problem-solving.



Source: Adler, J. & Ronda, E., 2015, 'A framework for describing mathematics discourse in instruction and interpreting differences in teaching', *African Journal of Research in Mathematics, Science and Technology Education* 19(3), 237–254. https://doi.org/10.1080/10288457.2015.1089677

FIGURE 1: Elements of the Mathematical Discourse in Instruction framework.

Ethical considerations

This article followed all ethical standards for research without direct contact with human or animal subjects.

Results

Overall tendencies

A total of 309 tasks were analysed across the four textbooks (see Table 1). A total of 33 tasks were coded on level 1, which indicates that they can be solved by known procedures or facts. Such tasks do not provide opportunities to learn problem-solving per se, but they invite to practise known procedures or facts, which can serve as prerequisites for problem-solving. The largest proportion of tasks were coded as level 2 tasks (n = 230), and these are tasks that require students to apply current topic procedures. Oftentimes, this implies applying procedures that have been illustrated in preceding examples. Finally, 41 tasks required students to make connections (level 3).

It is primarily among the level 3 tasks that we find opportunities to learn problem-solving, but such opportunities may also be found among level 2 tasks. The quantitative aspects of our analysis thus indicate that there might be opportunities to learn problem-solving in the textbooks, but we had to apply qualitative analysis to further investigate those opportunities. We elaborate on those results below, using some selected sets of examples and tasks to illustrate the findings. We first illustrate how potential opportunities to learn problem-solving might be impeded before we provide illustrations of tasks that might provide opportunities to learn problem-solving.

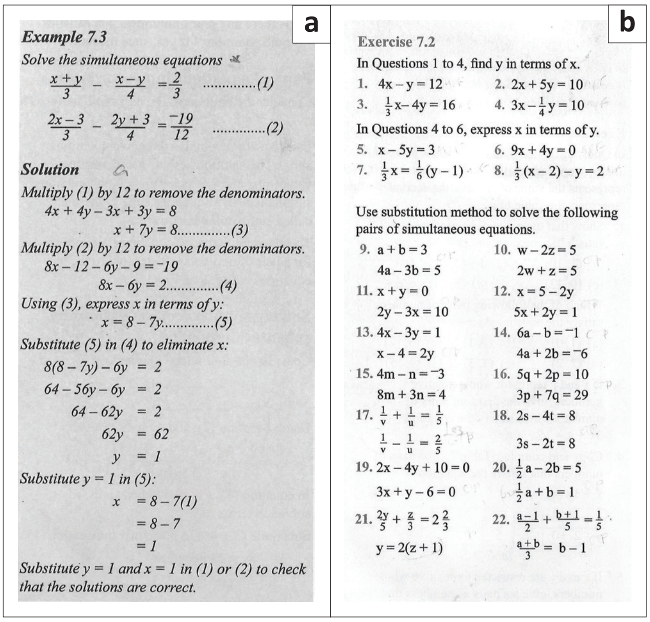
Impediments to problem-solving

In order to determine the opportunities to learn problem-solving, it is necessary to analyse the connection between examples and the sets of tasks that follow. From our analysis across the textbooks, we found that the details of an examples and the similarity between a given example(s) and the tasks that followed could impede opportunities to learn problem-solving. Oftentimes, there was a strong similarity between the given examples and the following tasks. An illustration of this can be found in Figure 2.

In Figure 2, we see an example space on the substitution method from Book D, followed by 22 tasks that followed (9–22 and 1–8 in Exercise 7.2). The eight tasks in Exercise 7.2 were all coded on level 1, because they only required knowledge of changing the subject of a formula, which the students had learned in the previous year. Tasks 9–16 and 18–21 were coded as level 2 tasks, and all those tasks required the application of procedures that were shown in the preceding example. Tasks 17 and 22 were coded on level 3 because they involved doubling of the fraction. These are known to be tough for students (Johari & Shahrill 2020).

When considering the example space, we noticed that there is only one example in the space, and we considered this as generalisation, because it can be interpreted as generalising the substitution method. The example space is also detailed for using a fraction equation instead of using a whole equation so that the fraction equation could challenge the students. Consequently, it highlights that students should use the method of least common denominator, thereby reducing the challenge for students. In addition, the example illustrates the substitution method, and the following prompt for tasks 9–22 ('Use substitution method to solve the following…') can thus be interpreted as a leading question, which points back to the given example.

As another illustration, Figure 3 displays an example space and a set of tasks that followed from Book C. The example in Figure 3 was given at the beginning of a textbook lesson. Introduction of textbook lessons typically starts by stating the objectives of the lessons, the methods to be discussed and then giving an


a

Example 7.3

Solve the simultaneous equations

$$\frac{x+y}{3} - \frac{x-y}{4} = \frac{2}{3} \quad \text{.............}(1)$$

$$\frac{2x-3}{3} - \frac{2y+3}{4} = \frac{^-19}{12} \quad \text{.............}(2)$$

Solution

Multiply (1) by 12 to remove the denominators.

$4x + 4y - 3x + 3y = 8$

$x + 7y = 8$.............(3)

Multiply (2) by 12 to remove the denominators.

$8x - 12 - 6y - 9 = {}^-19$

$8x - 6y = 2$.............(4)

Using (3), express x in terms of y:

$x = 8 - 7y$.............(5)

Substitute (5) in (4) to eliminate x:

$8(8 - 7y) - 6y = 2$

$64 - 56y - 6y = 2$

$64 - 62y = 2$

$62y = 62$

$y = 1$

Substitute y = 1 in (5):

$x = 8 - 7(1)$

$= 8 - 7$

$= 1$

Substitute y = 1 and x = 1 in (1) or (2) to check that the solutions are correct.

b

Exercise 7.2

In Questions 1 to 4, find y in terms of x.

1. $4x - y = 12$ 2. $2x + 5y = 10$
3. $\frac{1}{3}x - 4y = 16$ 4. $3x - \frac{1}{4}y = 10$

In Questions 4 to 6, express x in terms of y.

5. $x - 5y = 3$ 6. $9x + 4y = 0$
7. $\frac{1}{3}x = \frac{1}{6}(y - 1)$ 8. $\frac{1}{3}(x - 2) - y = 2$

Use substitution method to solve the following pairs of simultaneous equations.

9. $a + b = 3$, $4a - 3b = 5$
10. $w - 2z = 5$, $2w + z = 5$
11. $x + y = 0$, $2y - 3x = 10$
12. $x = 5 - 2y$, $5x + 2y = 1$
13. $4x - 3y = 1$, $x - 4 = 2y$
14. $6a - b = {}^-1$, $4a + 2b = {}^-6$
15. $4m - n = {}^-3$, $8m + 3n = 4$
16. $5q + 2p = 10$, $3p + 7q = 29$
17. $\frac{1}{v} + \frac{1}{u} = \frac{1}{5}$, $\frac{1}{v} - \frac{1}{u} = \frac{2}{5}$
18. $2s - 4t = 8$, $3s - 2t = 8$
19. $2x - 4y + 10 = 0$, $3x + y - 6 = 0$
20. $\frac{1}{2}a - 2b = 5$, $\frac{1}{2}a + b = 1$
21. $\frac{2y}{5} + \frac{z}{3} = 2\frac{2}{3}$, $y = 2(z + 1)$
22. $\frac{a-1}{2} + \frac{b+1}{5} = \frac{1}{5}$, $\frac{a+b}{3} = b - 1$

Source: Thomo, F., Gitu, D., Maina, L. & Ondera, J., 2015b, *Excel and Succeed Junior Secondary Mathematics Student's Book 2*, pp. 56-57, Longhorn Publishers Ltd, Nairobi

FIGURE 2: Examples and tasks illustrating the substitution method, (a) example 7.3, (b) exercise 7.2.

TABLE 1: Levels of tasks.

Task Levels	Book A (G9)		Book B (G9)		Book C (G10)		Book D (G10)		Total	
	N	%	*N*	%	*N*	%	*N*	%	*N*	%
Level 1 tasks	9	20.0	9	6.8	1	3.0	14	14.3	33	10.7
Level 2 tasks	26	57.8	103	77.4	29	87.9	72	73.5	230	74
Level 3 tasks	9	20.0	18	13.5	3	9.1	11	11.2	41	13.5
Outliers	1	2.2	3	2.3	0	0.0	1	1.0	5	1.6
Total	**45**	**100**	**133**	**100**	**33**	**100**	**98**	**100**	**309**	**100**

a

Elimination method

There are two rules governing elimination method.

1. If the coefficients of the variable to be eliminated are equal and have the same sign, use the rule **'Same Signs Subtract'.**
2. If the coefficients of the variable to be eliminated are equal but the signs are different, use the rule **'Different Signs Add.'**

Example 2

Solve the following simultaneous equations

(a) $\begin{cases} x + y = 1 \\ x + 2y = 2 \end{cases}$ (b) $\begin{cases} 2x + y = 1 \\ x + 2y = 2 \end{cases}$ (c) $\begin{cases} \frac{2}{3}x - \frac{y}{2} = 0 \\ \frac{x}{3} - \frac{3}{2}y = 0 \end{cases}$

Solution

b

Exercise 7.2

Solve the following simultaneous equations by elimination method.

1. $\begin{cases} x + y = 2 \\ x + y = 0 \end{cases}$ 2. $\begin{cases} 2a + b = 3 \\ a - 2b = -1 \end{cases}$ 3. $\begin{cases} 3e + 2f = 4 \\ e + f = 1 \end{cases}$

4. $\begin{cases} m = n - 2 \\ 3n + m = 7 \end{cases}$ 5. $\begin{cases} 3r + q - 9 = 0 \\ 2r + 3q - 13 = 0 \end{cases}$ 6. $\begin{cases} 3y - x = 8 \\ 2y + 3x 10 \end{cases}$

7. (a) Consider the equation $5x - y = c$. Find the value of c if the solution set is (1, –1).
   (b) Find the value of m and n if the solution set for the following simultaneous equation is (8, – 4).
   $\begin{cases} mx - y = 10 \\ nx + y = 2 \end{cases}$

Source: Nyirenda, S., Okumu, S. & Mbugua, C., 2014b, *Achievers Junior Secondary Mathematics student's Book 2*, pp. 75–77, East African Educational Publishers Ltd, Nairobi

FIGURE 3: Introducing a textbook lesson, (a) Elimination method, (b) Exercise 7.2.

example. Example 2 in Figure 3 illustrates the use of the elimination method for solving simultaneous equations, and the tasks that follow point back to this example. The prompting question in Exercise 7.2 is thus leading, and there is a close connection between solved examples and tasks. Instead of introducing the lesson by presenting a challenge or problem to be explored, the textbook lesson starts by illustrating a procedure (including rules 1 and 2 that state what to do without the why) and is then followed by tasks where students are told explicitly to use this procedure. Although the tasks, when seen in isolation, might have provided opportunities to learn problem-solving, the detailed examples, the introduction of the lesson, and the leading questions take away those opportunities.

Although the tasks are normally similar to the preceding examples, there are some instances where the connection is not so strong. For instance, Figure 4 displays two examples (Examples 5 and 6) of changing the subject of the formula from Book B in Grade 9, followed by a set of tasks (9.3). Most of the following tasks are similar to the examples in that they require the students to do some manipulations to identify the unknown x. All tasks involve other numbers and unknowns in addition to the one x that is to be found, but tasks 8 and 10 include the unknown x in two places. This not only requires students to figure out what they must add, subtract, multiply or divide on both sides to find the unknown, but they must also use factoring before they can proceed. As the students are thus required to make connections to other procedures and not only use current topic procedures, these tasks were coded as level 3 tasks. Similar tendencies were also found in Thomo et al. (2015a), another grade 9 textbook that were analysed in this article.

a

Example 5

Given that $\frac{a(x + b)}{c} = 2$, find the expression for x.

Solution

$\frac{a(x + b)}{c} = 2$

$a (x + b) = 2c$

$ax + ab = 2c$

$ax = 2c - ab$

$x = \frac{2c}{a} - \frac{ab}{a}$

$x = \frac{2c}{a} - b$

b

Example 6

Given that $3p + q = s$, find p.

Solution

$3p + q = s$

subtract q from both sides

$3p = s - q$

Divide both sides by 3

$p = \frac{s - q}{3}$

Exercise 9.3

Find the expression for x in the following equations.

1. $x + 5 = y$ **2.** $5x - 3 = c$

3. $bx = 2a + 3$ **4.** $ax + b = c$

5. $2ax + 5 = c$ **6.** $\frac{2x + b}{a} = c$

7. $\frac{4xc + 5a}{3b} = 2$ **8.** $3xa + 5bx = 7$

9. $\frac{ax + b}{3} = \frac{c}{a} + 1$ **10.** $\frac{9}{x} + \frac{b}{x} = 1$

Source: Nyirenda, S., Okumu, S. & Mbugua, C., 2014a, *Achievers Junior Secondary Mathematics student's Book 1*, pp. 124–125. East African Educational Publishers Ltd, Nairobi

FIGURE 4: Similarities between examples and tasks, (a) Example 5, (b) Example 6.

Opportunities for problem-solving

In the previous section, we reported on how tasks that vary from the given examples are more challenging as they might require students to make connections. Still, such tasks cannot necessarily be classified as genuine problems because the general approach to solving them is already given in the preceding examples. Throughout the textbooks, we found that the tasks that provide opportunities to learn problem-solving were mostly given as word problems. The following task from Book C is an illustration of this:

> Two numbers are such that the sum of half of the first number and the second number is 6. But twice the second number added to three times the first number is 24. Find the two numbers. If two lines, $2y + 5x = 1$ and $y + p = 3x$ meet at (q,1), find p and q. (Nyirenda et al. 2014b:82)

This task was coded as a level 3 task for a couple of reasons. The first part of the task requires students to generate equations before using the graphical method, and the requirement to make connections indicates level 3 (Ronda & Adler 2017). In the second part of the task, it is not made explicit that (q,1) are the solutions, and students are thus required to think and make connections – again indicating level 3. In addition, previous examples and tasks mostly

Exercise 7.5

1. The sum of two numbers is 10, and their difference is 6. Make a pair of equations and solve them simultaneously to find the numbers.
2. Mary is one year older than June, and their ages add up to 15. Form a pair of equations and solve them to find the ages of the girls.
3. Two books have a total of 500 pages. One book has 350 pages more than the other. Find the number of pages in each book.
4. A bag contains K 5 coins and K 10 coins. There are 14 coins in all, and their value is K 105. Find the number of each type of coin.
5. Two numbers are such that twice the larger number differs from thrice the smaller number by four. The sum of the two numbers is 17. Find the numbers.
6. If 5 is added to both the numerator and denominator of a fraction, the result is $\frac{4}{7}$. If 1 is subtracted from both the numerator and denominator, the result is $\frac{2}{5}$. Find the fraction.

11. The sum of the number of edges and faces of a solid is 20. The difference between the number of edges and faces is 4. Find the number of edges and faces.
12. The velocity in km/h of a car after t hours is given by the formula $v = u + at$, where u and a are constants. Given that $v = 50$ when $t = 2$ and $v = 140$ when $t = 5$, find
    (a) the constants u and a.
    (b) the velocity when $t = 7$ hours.
    (c) the time at which $v = 260$ km/h.
13. The sum of the digits in a three digit number is nine. The tens digit is half the sum of the other two and the hundreds digit is half the units digit. Find the number.

14. Asale and Mbiya collected a number of stones each to use in an arithmetic lesson. If Asale gave Mbiya 5 stones, Mbiya would have twice as many as Asale. If Asale had five stones less than Mbiya, how many stones did each have?
15. A student invested K 50 000 in two different savings accounts. The first account pays an annual interest rate 3%. The second account pays an annual interest rate of 4%. At the end of the year, she had earned K 1 850 in interest. How much money did she invest in each account?

Source: Thomo, F., Gitu, D., Maina, L. & Ondera, J., 2015b, *Excel and Succeed Junior Secondary Mathematics Student's Book 2*, pp. 56-57, Longhorn Publishers Ltd, Nairobi

FIGURE 5: Word problems from Book D.

involve pairs of solutions as numbers only, but the students here are faced with a task that relates q to the value of x. This task can be described as an example of working backwards, starting with a solution, which is a heuristic for problem-solving (Schoenfeld 1985). Finally, this task will eventually lead to a situation where the students must decide if the elimination method or the substitution method is more efficient for solving the problem. This is thus a type of task that might provide students with opportunities to learn problem-solving.

Figure 5 illustrates sets of tasks that were given as word problems from Book D. Tasks 1–3 and 5–6 were coded as level 2 tasks, because the connection between the elements of the question is clear and enables students to easily come up with the equations. In tasks 4 and 12–13, however, the connection is not straightforward, and these tasks were coded on level 3. Tasks 12 and 15 involved making connections to linear motion and commercial mathematics, respectively, and they were also coded as level 3 tasks.

Summary of results

From our analysis of examples and tasks in four lower secondary mathematics textbooks in Malawi – focusing on the units of linear equations and simultaneous linear equations – we have identified some opportunities to learn problem-solving, but we have also identified some impediments to learning problem-solving. A first impediment lies in the details of the examples. When worked examples are detailed, and when there is a close connection between the examples and the following tasks, opportunities to learn problem-solving are impeded. A second impediment lies in the very structure and design of the textbooks, where textbook lessons are introduced by presenting worked examples – sometimes preceded by a presentation of the learning objective and a presentation of methods to be discussed – and then providing a list of tasks. The opposite would be to introduce lessons as challenges. A third impediment is the number of leading questions in tasks. Throughout the textbooks, there were numerous examples of leading questions. These provided explicit or implicit indications of the methods to apply to solve the tasks. There were no examples of prompting questions that invited students to solve tasks in their own way. In sum, these are impediments to the students' opportunities to learn problem-solving.

Discussion

Based on our analysis of opportunities to learn problem-solving in Malawian mathematics textbooks for lower secondary school, we are going to develop three claims in the following discussion. The first claim is that opportunities to learn problem-solving are few in Malawian mathematics textbooks. Based on this, the second claim is that limited opportunities to learn problem-solving in textbooks can be an impediment in a context like Malawi, where low teacher qualifications accompany high teacher–textbook compliance (Mwadzaangati 2019b). The third claim is that relying on word problems as the main opportunities to learn problem-solving can be problematic.

Firstly, our study indicates that Malawian mathematics textbooks for lower secondary school provide limited opportunities to learn problem-solving. There are only 13% level 3 tasks in the units of linear equations and simultaneous linear equations. A large portion of these tasks were word problems. The textbooks seem to define problem-solving as the application of learned procedures to solve word problems. We appreciate this provision because word problems are challenging for students (Kieran 2007). Yet, this provides a narrow and insufficient view of problem-solving. Problem-solving then becomes a matter of extracting the necessary information from a given text – often with no distracting elements in the Malawian textbooks – and converting this information into mathematics, which can then be solved by already presented algorithms. When these word problems are placed towards the end of the chapters, like in the Malawian textbooks, it provides a view of learning mathematics *for* problem-solving rather than learning mathematics *through* problem-solving. Problem-solving should be used to promote mathematical understanding and not simply a measure of mathematical understanding. Textbook authors should consider incorporating problem-solving tasks of all forms throughout a topic to promote learning through problem-solving.

Secondly, we found that at least 74% and 11% of the tasks were level 2 and level 1, respectively. These tasks demand current topic procedures and already known procedures, respectively. Having a large portion of level 2 tasks means that the focus of the textbook is on procedural fluency which could promote rote memorisation. This tendency is not surprising, and studies of mathematics textbooks in other countries also indicate that textbooks provide few opportunities to learn problem-solving. For instance, Brehmer et al. (2015) found few high-level tasks in their analysis of Swedish mathematics textbooks, and Berisha et al. (2014) also found that there were more routine tasks than non-routine tasks in mathematics textbooks from Kosovo. Even in a context like Singapore, where the curriculum and textbooks are considered to emphasise problem-solving, most textbook tasks were routine tasks (Fan & Zhu 2000). Our findings also agree with Maonga (2020) who found that Malawian mathematics textbooks offer few opportunities for learning mathematics. We suggest that textbooks have a well-balanced combination of tasks that promote both mastery of procedures and problem-solving. Much as this might be challenging to implement, it could be achieved through proper distribution and incorporation of more challenging tasks throughout a topic. In other words, considering the approach of learning through problem-solving at every stage of the topic could support the achievement of this balance.

Our study further found three impediments to learning problem-solving in textbooks. These constraints are foregrounded on the major finding in this study and other studies that many tasks offered are similar to the examples offered or other details given in a textbook.

The first impediment is *detailed examples*. Van Zanten and Van den Heuvel-Panhuizen (2018) state that whether or not a task is a routine task or a problem depends on the given examples, and not only on the task itself. We noticed that the Malawian textbooks provide very detailed examples. The worked examples often introduced the textbook lessons, which were then followed by similar exercises. In other instances, a complex worked example was used to illustrate a method. For instance, in Figure 2, a fraction equation was used to illustrate the substitution method instead of using a whole equation. Consequently, all fraction equations, though known to be tough (Johari & Shahrill 2020), become unchallenging because their simplification has been exemplified. Textbooks ought to use simple structures for worked examples to illustrate a method and leave the complex structures to pose challenges to students. In this way, students would be compelled to reason on how to solve such tasks.

The second impediment is *leading questions*. We observed that the questions in tasks were often pointing back to the given examples. As a result, many tasks that could have provided opportunities to learn problem-solving in and of themselves instead became routine tasks. Thus, the textbooks promote procedural fluency that may propel rote memorisation. For instance, in Figure 3, the question explicitly points that elimination method should be used. This kind of questioning deprives students the opportunity to think and create their own ways of solving the tasks. It provides a template for students to use when solving the tasks. Templates for tasks provided mostly within the same topic limit the ability of students to learn problem-solving (Jäder et al. 2020). We understand that templates shall always be there to propel students' understanding of procedures. Therefore, we suggest that textbooks' authors must also consider including a kind of questioning that might encourage critical thinking. For instance, 'solve the task in your own ways other than the ones shown in the example'. This kind of questioning could switch the focus of students from the templates given to their creative minds.

The last constraint is *not introducing a lesson as a challenge*. The structure of the textbooks was typically *rule-example-practice* (Glasnovic Gracin 2018). Implicitly, it implies that the textbooks teach the students what to do and not why it is done. For example, in Figure 3, the students are introduced to the two rules of the elimination method, then examples followed by an exercise. Consequently, students will simply memorise the rules not knowing why that is done. If the textbooks had introduced the rules through an example where students would figure out for themselves on how to eliminate one variable, then it would have propelled their ability to solve the problem. We believe that problem-solving is knowing both what to do and why that is so. Therefore, it is imperative that textbooks' topics are arranged in a way that students discover various procedures for themselves.

Although textbooks in many countries appear to provide few opportunities to learn problem-solving, we claim that such limitations are particularly problematic in a context like Malawi. Our claim draws on Golding's (2023) emphasis on considering any analysis of curriculum resources considering their social context. The first reason is that teacher–textbook compliance is high in Malawi (Mwadzaangati 2019b), and teachers typically follow the textbook prescriptively. Thus, only what is in the textbook is taught. High teacher compliance does not have to be a problem if the textbooks are good. However, Malawian textbooks provide limited opportunities to learn problem-solving, so this places more responsibility on the teachers. Sadly, and this is the other reason, Malawi is short on qualified mathematics teachers. A recent report shows that there are only 1222 qualified mathematics teachers in the 7337 secondary schools in Malawi (MoEST 2022). Problem-solving is a high-level topic in mathematics, and it cannot be expected that unqualified teachers will go beyond what is in the textbook. It can be expected that these teachers might skip the more challenging tasks from the textbooks and thereby provide even fewer opportunities to learn problem-solving than the textbooks provide. Seen in combination, these two issues thus indicate that the textbooks will often represent the upper limits of the opportunities to learn problem-solving. In a context like Malawi with limited resources and qualified teachers, the limitations in the opportunities to learn problem-solving can thus be particularly detrimental.

Conclusion

Problem-solving is strongly emphasised in the Malawian curriculum, and leading mathematics textbooks for lower secondary school provide some opportunities for learning problem-solving. Yet, the textbooks also provide some limitations. In a context where the overall qualifications of teachers are low, and where teachers are prone to only rely on textbooks, these limitations might constitute impediments to learning problem-solving. We identified three impediments in the textbooks, which include detailed examples, leading questions and not introducing lessons as challenges. These strengthened the similarities between examples and tasks. Word problems offered more opportunities for problem-solving, but representing opportunities to learn problem-solving mainly in word problems provides a limited view of problem-solving. We believe that providing equally more opportunities through tasks that are not word problems would be essential for propelling problem-solving at every stage of the topic and in all forms because word problems were mostly placed at the end of the topic. The overall implication is that students in Malawi are provided with limited problem-solving opportunities through textbook tasks. However, we also believe that how teachers implement the tasks may provide different opportunities. Further research on other topics of the Malawian textbooks is needed to ascertain the extent to which the opportunities for learning problem-solving are provided and if more impediments could be discovered.

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Data availability

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