International Review of Economics and Finance 92 (2024) 704–723

Contents lists available at ScienceDirect

International Review of Economics and Finance

journal homepage: www.elsevier.com/locate/iref

Debt crises between a country and an international lender as a two-period game[☆]

Kjell Hausken[a,*], Jonathan W. Welburn[b]

[a] *Faculty of Science and Technology, University of Stavanger, 4036 Stavanger, Norway*
[b] *RAND Corporation, 1776 Main St., Santa Monica, CA 90401, USA*

ARTICLE INFO

JEL classification:
C6
C7
G01

Keywords:
Game theory
Country
Debt
Loan
Borrow
Default
Consumption
Economic risk
Greece
EU
ECB
IMF
Sensitivity analysis

ABSTRACT

A two-period game between a country and an international lender is developed. In each period the country can repay debt, borrow from international credit markets, loan from an international lender, or default. The international lender can approve or deny the loan. The risk averse country maximizes a time-discounted utility incorporating its consumption. The international lender maximizes a time-discounted utility which values the country's consumption growth positively, and the debt-to-endowment ratio, the loan-to-endowment ratio, and the default penalty negatively. The subgame perfect equilibria are determined with backward induction. It is shown analytically that a country borrows less from international credit markets if it obtains a loan from an international lender and, intuitively, if its endowment increases. In contrast, the country borrows more if its future endowment (in period 2) can be expected to increase, or its initial borrowing is high. The model is simulated applying empirical data from the 2010 Greek crisis. The simulations illustrate how the country consumes in the two periods depending on whether the international lender approves or denies a loan. The impact is assessed of time discounting, risk aversion, default penalties, the country's endowment, interest rates, how the international lender values various characteristics of the country, and initial borrowing and consumption before the game starts.

1. Introduction

The 2010 Greek debt crisis caused the formation of a so-called troika, between the European Commission (EC), the European Central Bank (ECB) and the International Monetary Fund (IMF), to attempt resolving the crisis. To understand this common phenomenon, this article characterizes a two-player two-period game between a country and an International Lender (IL), such as the troika. By modeling the IL as a single player, we abstract away the collective action problem among potentially multiple ILs, acknowledging that the troika made numerous group decisions. In each period the country faces the decision to repay debt, borrow, loan, or default, which is a richer conceptualization than what has been common in the literature. If a loan from the IL is sought, a negotiation ensues. The IL can approve or deny the loan. Failed negotiations leave the country with the choice of either borrowing from credit markets or default. We formalize this as a repeated game with three rounds in each period. The conceptualization clarifies the options available to each player at each point in time, which enables planning period 2 in period 1 while accounting for the previous

☆ Author names were listed alphabetically by last name.

* Corresponding author.
E-mail addresses: kjell.hausken@uis.no (K. Hausken), jwelburn@rand.org (J.W. Welburn).

https://doi.org/10.1016/j.iref.2024.02.034
Received 9 October 2020; Received in revised form 16 October 2023; Accepted 13 February 2024
Available online 19 February 2024

period 1 in period 2. Our research question is how a country strikes a balance between debt repayment, borrowing, loaning, and defaulting, over the two periods, which impacts consumption and utilities in the two periods, and how the IL approves or denies loan applications. Analytical results are provided. The impact of 13 model parameters and three initial conditions in period 0 is illustrated with simulations.

The literature has not considered the research question in this article, but has focused on a variety of indirectly linked research questions. Araujo et al. (2017) consider a bargaining game between creditors and a debtor country. They find that for the 2010 Greece crisis, ex post, partial repayment avoids the cost of total default. Ex ante, however, expectations of debt relief increases the sovereign debt and delays fiscal adjustment. Eaton and Gersovitz (1981) consider agents forecasting the rate of return to capital assets in a model of sovereign debt, default, and repudiation. Arellano and Bai (2013) present a Nash bargaining model of debt renegotiation with linkages to sovereign debt markets.

Anevlavis et al. (2019) consider a game between two players responsible for monetary and fiscal policy within a country. They determine Nash equilibria accounting for the debt size and the rate of change for the debt. Finke and Bailer (2019) find that market pressure, more than formal rules, weakened debtor countries during the Eurozone financial crisis due to their difficulties in refinancing their public debt. Lopez and Nahon (2017) find for Argentina over 25 years that sustainable debt has been necessary for sustainable growth, and that austerity policies and a lax approach to debt have caused economic recession. To enable a fresh start, they recommend balanced and big-enough sovereign debt restructuring. Sopocko (2012) assesses a financial game involving stagnation or growth.

Pitchford and Wright (2013) question why countries repay their debts when no supranational institution enforces repayment, and why sovereign debt restructuring appears inefficient. They assess the role of self-enforcing contracts, the credibility of threats to punish, and how players may not commit to bargain and honor the terms of a bargain.

Baral (2013) develops a game between U.S. borrower banks, lender banks, and the Federal Reserve to model contagion. Acemoglu et al. (2013) determine Nash equilibria and assess how a densely connected interbank financial network market may be both resilient and fragile. Welburn and Hausken (2015, 2017) model the strategies of countries, central banks, banks, firms, households, and financial inter-governmental organizations in handling crises where countries may default.

Morris and Shin (2000, 2012) model contagion and adverse selection in a coordination game involving currency crises, bank runs, and debt. The importance of behavior is demonstrated by Chari and Kehoe (2004) who argue that models of herd behavior can explain financial crises. Broner (2008) argues that the private information in models of currency crises can create multiple equilibria and unpredictable currency devaluations. Many have argued that crises can be self-fulfilling. Obstfeld (1984) argues that speculative attacks can make balance-of-payments crises self-fulfilling. Lorenzoni and Werning (2013) create a model of self-fulfilling debt crises that follow from investor expectations on default. They argue that a crisis follows from a shift from a good equilibrium to a bad one.

Section 2 presents the conceptual background of the study. Section 3 develops the model. Section 4 solves the model. Section 5 simulates the Greek debt crises. Section 6 concludes.

2. Conceptual background of the study

Debt plays a major role in today's economy. Global debt in 2022 is $235 trillion and increasing. Global debt as a percentage of Gross Domestic Product is 238% in 2022 (increasing from ca 100% in 1950), consisting of 92% in public debt (weighted by each country's Gross Domestic Product) and 146% in private debt (household debt and non-financial corporate debt). See e.g. Gaspar et al. (2023) for more details. Some countries exceed while other countries subceed the high 92% of public debt divided by Gross Domestic Product. With globally increasing debt especially the countries exceeding the 92% number can be expected to face increasingly precarious economic circumstances and sometimes debt crises which may or may not be sustainable through time. Given this background, this article seeks to understand the phenomenon which is a prerequisite for solving precarious economic circumstances and debt crises.

A first step in understanding a phenomenon is to identify the key players which should be neither too few nor too many. Too many players cause too complex analyses and potentially unclear results. The first obvious player is a given country which may experience, and seeks to handle, a certain debt level. A second obvious player is a player equipped with a variety of different strategies for providing or regulating funding which is the source of the country's debt. This player is the international lender (IL) exemplified with the EC, ECB and IMF in the previous section. A potential third player is international credit markets which consist of a variety of subplayers offering loans with a plethora of characteristics and conditions. To avoid excessive complexity, this article considers this potential third player as parametric, but the country's borrowing from international credit markets is a non-parametric strategic choice.

Having identified the country and the IL as our two players, and international credit markets as non-parametric, the second step is to handle the time dimension. The time dimension is obviously needed since debt is assessed through time. More specifically, borrowing and incurring debt occur through time, may or may not be provided through time, may or may not be sustainable through time, may be paid back in various ways through time, and default may occur at various point in time. Too many time periods give too much complexity, and continuous time gives more complexity than discrete time. This article thus chooses two periods labelled as period 1 and period 2. The two time periods may occur several years apart, as interaction between a country and the IL typically occurs over several years. Additionally needed are initial conditions conceptualized as occurring in period 0, i.e. before entering period 1. The initial conditions in period 0 are the country's loan from the IL, the country's borrowing from international credit markets, and the country's consumption.

The third step is to determine what occurs within periods 1 and 2, after the three initial conditions in period 0. The country's and the IL's strategies related to debt cannot be made simultaneously. Instead, one player's strategy depends on the other player's earlier strategies. The various events within each period may occur several months, weeks, days or hours apart. The nature of debt is such that the country inevitably first chooses a strategy in round 1 in each of the two periods. We'll get to each player's strategies, but one obvious strategy for the country in round 1 is whether or not to seek or apply for a loan. Consequently, one obvious strategy for the IL in round 2 is whether to approve or deny the loan application. One might say that two rounds suffice. However, if the IL approves a loan application, loan applications come with terms and conditions which the country may or may not accept. Hence a round 3 is assumed where the country may accept or reject the IL's loan offer. Future research may model more than three rounds with continued back and forth loan applications, acceptances, conditional acceptances and rejections, offers and counteroffers, etc. This article assumes that three rounds capture the phenomenon sufficiently.

The fourth step is to determine the two players' strategic choices in each of the three rounds, in each of the two periods, exhaustively. In round 1 the country obviously can choose whether or not to seek a loan of a certain size. Secondly, the country can choose continuous levels of borrowing from international credit markets, ranging from maximum borrowing to paying off all debt. Third, the country can default which incurs a penalty. In round 2 the IL chooses two strategies. Then first is to approve or deny the loan application. If the loan application is approved, the second choice is the size of the loan offer ranging from some maximum level to zero, where zero means no loan approval. In round 3 the country cannot seek a loan anew. Instead the country can accept or reject the IL's loan offer. Additionally, as in round 1, the country can choose its borrowing from international credit markets ranging from maximum borrowing to paying off all debt, and the country can default.

The fifth step is to solve the game with backward induction, discuss the results and conclude.

3. Model

Nomenclature

Abbreviation.

IL	International Lender

Parameters.

β	Country's intertemporal discount factor, $\beta \in [0, 1]$
β_I	IL's intertemporal discount factor, $\beta_I \in [0, 1]$
ρ	Country's degree of relative risk aversion, $\rho \in [0, 1]$.
Φ_t	Country's default penalty in period t, $\Phi_t \geq 0$
Φ_{It}	IL's penalty if the he country defaults in period t, $\Phi_{It} \geq 0$
y_t	Country's endowment (which may be interpreted as Gross Domestic Product) in period t, $y_t \geq 0$
r_t	Country's interest rate on debt owed to international credit markets in period t, $r_t \in \mathbb{R}$
r_{It}	Country's interest rate on loan received from the IL in period t, $r_{It} \in \mathbb{R}$
α_1	Weight in IL's utility of positive impact of country's consumption growth c_t/c_{t-1} from period $t-1$ to period t, $0 \leq \alpha_1 \leq 1$
α_2	Weight in IL's utility of negative impact of country's debt-to-endowment ratio B_t/y_t in period t, $0 \leq \alpha_2 \leq 1$
$1 - \alpha_1 - \alpha_2$	Weight in IL's utility of negative impact of country's loan-to-endowment ratio L_t/y_t in period t, $0 \leq \alpha_1 + \alpha_2 \leq 1$.
T	Number of time periods, $T = 1, 2, \ldots$
t	Time expressed as period $t = 1, \ldots, T$
$\overline{L}$	IL's upper limit for loans over T periods, $\sum_{t=1}^{T} L_t \leq \overline{L}$
$\overline{B}_t$	Upper limit for country's borrowing from international credit markets in period $t = 1, \ldots, T$

Free choice variables.

A_t	IL's loan approval ($A_t = 1$) or loan denial ($A_t = 0$) for loan L_t in period $t = 1, \ldots, T$
X_t	Country's loan acceptance ($X_t = 1$) or loan rejection ($X_t = 0$) for loan L_t in period $t = 1, \ldots, T$
L_t	Negotiated loan L_t from the IL in period $t = 1, \ldots, T$ determined by both players, $L_t \in \mathbb{R}_{\geq 0}$
B_t	Country's borrowing from international credit markets in period $t = 1, \ldots, T$, $B_t \in [0, \overline{B}_t)$
D_t	Country's default ($D_t = 1$) or non-default ($D_t = 0$) in period $t = 1, \ldots, T$

Dependent variables.

c_t	Country's aggregate consumption in period $t = 1, \ldots, T$, $c_t \geq 0$
U_t	Country's utility in period $t = 1, \ldots, T$
U	Country's aggregate discounted utility over T periods
U_{It}	IL's utility in period $t = 1, \ldots, T$
U_I	IL's aggregate discounted utility over T periods

3.1. Game description

The interaction between a country and an IL can be viewed as a repeated game, continuing through time with potentially subsequent requests for IL-approved loans (henceforth loans), potentially renewed negotiations, potentially resulting in subsequent loan packages. We take this view by developing a two-player repeated game between a country and an IL.

The game between a country and an IL often transpires over several years. We consider multiple time periods, each comprised of three rounds, shown in Fig. 1. We define the country's borrowing and debt in any given period $t \in [1, .., T]$ as B_t and d_t respectively, both of which are owed to international credit markets, where T is finite or infinite. The game is equivalent in each period, though with different initial conditions defined by the country's debt d_t, to the IL and credit market, at the start of period t. In round 1, the country acts upon its debt situation by choosing to borrow $B_t \in [0, \overline{B}_t)$, where $\overline{B}_t$ is the country's credit ceiling, seeking loan $L_{st} \in \mathbb{R}_{\geq 0}$, and seeking whether to default $D_t = 1$, or not default $D_t = 0$. Default $D_t = 1$ excludes the country from accessing credit, which precludes borrowing B_t and seeking loan L_{st}.[1] The bows in Fig. 1 express continuous action space. Borrowing comprises paying off all debt defined as $B_t = 0$, repaying past debt defined by borrowing such that $B_t \in [0, B_{t-1})$, rolling over debt defined by borrowing such that $B_t = B_{t-1}$, and increasing debt defined by borrowing such that $B_t \in (B_{t-1}, \overline{B}_t]$. Not seeking loan is expressed as $L_{st} = 0$.

In round 2 the IL observes L_{st} and responds by choosing two strategies. First, it chooses to approve $A_t = 1$ or deny $A_t = 0$ the loan L_{st}. Second, generally, it chooses a counter offer of loan $L_{It} \in \mathbb{R}_{\geq 0}$, which may be smaller, equal to, or higher than the country's loan application L_{st}. Thereafter negotiations occur causing a negotiated loan L_t which is a dependent variable determined in equilibrium. We assume $L_0 = 0$ before the crisis modeled as a game that starts in period 0. In round 3 the country may also default $D_t = 1$, which precludes borrowing B_t and accepting the loan L_t. If the country does not default, $D_t = 0$, it may choose to borrow $B_t \in [0, \overline{B}_t]$ on the open market, and it may either accept the negotiated loan L_t, expressed as $X_t = 1$, or reject the loan L_t, expressed as $X_t = 0$. The negotiated loan L_t is determined by[2]

$$L_t = A_t X_t L_{st}(1 - D_t) \tag{1}$$

In (1) only the country's loan application L_{st} matters, and the IL's counter offer L_{It} is irrelevant. If the country defaults in (1), i.e. $D_t = 1$, or rejects, i.e. $X_t = 0$, or the IL denies, i.e. $A_t = 0$, then $L_t = 0$.

Summing up, the country's strategy set consists of four strategic choice variables, i.e. borrowing B_t in rounds 1 or 3, seeking a loan L_{st} in round 1, defaulting D_t in rounds 1 or 3, and accepting or rejecting X_t the loan L_t in round 3, i.e. $\{B_t, L_{st}, D_t, X_t\}$. The IL's strategy set consists of two strategic choice variables, i.e. approval A_t and counter offer of loan L_{It} in round 2, i.e. $\{L_{It}, X_t\}$. The negotiated loan L_t follows from (1). All paths in Fig. 1 continue into the next period $t + 1$ except when the country defaults $D_t = 1$ or pays of all debt $B_t = 0$ in rounds 1 or 3.

3.2. Description of player utility functions

The country is assumed to have a benevolent government maximizing the utility of its households. Using the common approach in the sovereign debt literature, e.g. Aguiar and Gopinath (2006), the country's utility is isoelastic and defined recursively. Assuming constant relative risk aversion, the country's utility U_t in period t is functionally dependent on the country's aggregate consumption c_t in period t. Summing over T periods with time discounting, the country's utility is

$$U = \sum_{t=1}^{T} \beta^{t-1} U_t = \begin{cases} \sum_{t=1}^{T} \beta^{t-1} \left(\frac{c_t^{1-\rho} - 1}{1-\rho} - \Phi_t D_t \right) \; if \; \rho \in \left(0, 1 \right] \\ \sum_{t=1}^{T} \beta^{t-1} \left(Ln(c_t) - \Phi_t D_t \right) \; if \; \rho = 0 \end{cases} \tag{2}$$

where ρ is the country's degree of relative risk aversion which is positive if the country is risk averse, $\rho \in [0,1]$. The parameter β, $\beta \in [0, 1]$, is the country's intertemporal discount factor, and Φ_t is the country's default penalty in period t.[3] Consumption $c_t \geq 0$ in period t is determined by the sum of the country's endowment (which may be interpreted as the Gross Domestic Product) y_t, $y_t \geq 0$, in period t, and its net borrowing and loan conditional on default, i.e.

[1] While this assumption is not without loss of generality, it is justified by our focus on the interaction between the country and the IL, rather than the interaction between the country and external creditors.

[2] The IL's counter offer L_{It} is included in Fig. 1 for general illustration, and since at least two obvious alternatives to (1) exist. One alternative to (1) is $L_t = A_t X_t L_{It}(1 - D_t)$, where only the IL's counter offer L_{It} matters, and the country's loan application L_{st} is irrelevant. Another alternative to (1) is $L_t = A_t X_t (1 - D_t) min\{L_{st}, L_{It}\}$, where the minimum of the country's loan application L_{st} and the IL's counter offer L_{It} determines the negotiated loan L_t.

[3] The country receives default penalty Φ_t if it defaults $D_t = 1$ in period t. While penalties typically carry implications for reputation and future credit market access, the Greek debt crisis gave rise to the unique or, for some, ultimate penalty; "Grexit." That is, the common assumption throughout the Greek debt crisis is that if the country defaults on debt, it would be forced to exit the EU. The IL is penalized if the country defaults and exits the EU. This penalty would result from significant credit market losses in EU nations, harsh Euro devaluations, and a lack of economic and political confidence.

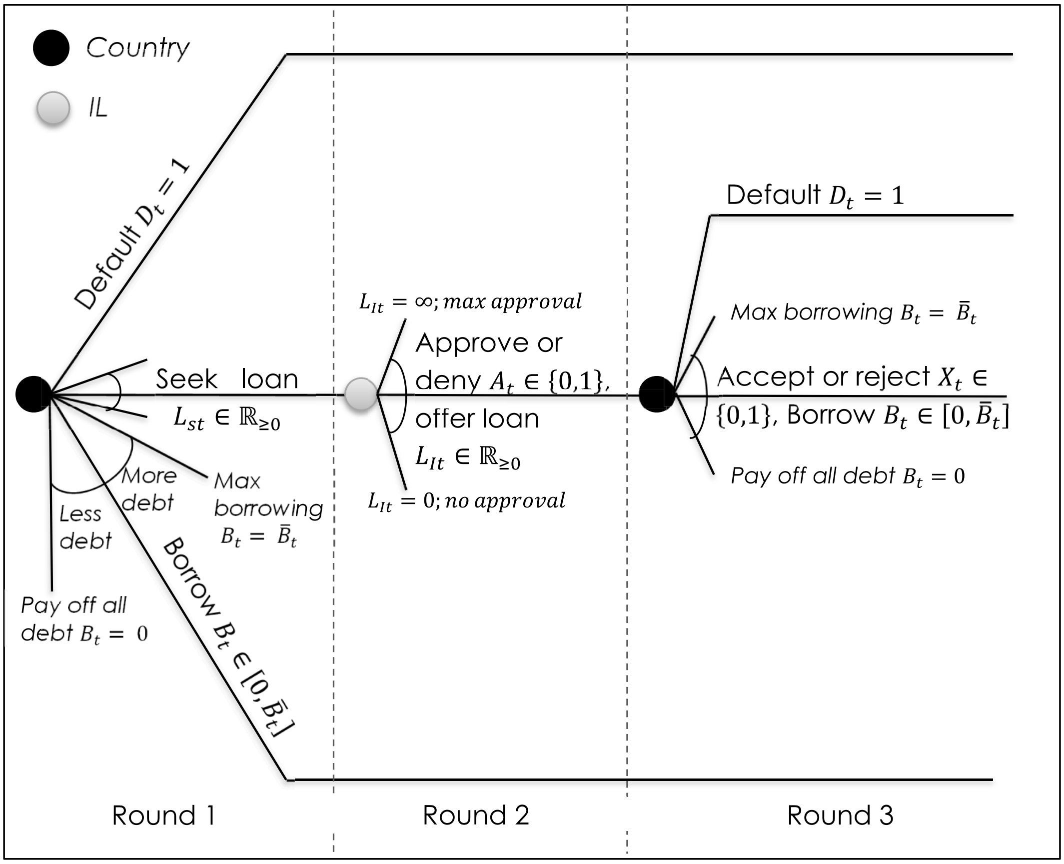

Fig. 1. Three rounds in the within-period game tree in period t, $t = 1, 2$, between the country and the IL.

$$c_t = Max(0, y_t + (q_t B_t - B_{t-1} + q_{It} L_t - L_{t-1})(1 - D_t)) \in \mathbb{R} \tag{3}$$

where q_t and q_{It} are prices for borrowing (by the country) and loan (offered by the IL), respectively, such that

$$q_t \equiv \frac{1}{1 + r_t}, q_{It} \equiv \frac{1}{1 + r_{It}} \tag{4}$$

where $r_t \in \mathbb{R}$ is the country's interest rate on debt owed to international credit markets and $r_{It} \in \mathbb{R}$ is the country's interest rate on loan received from the IL. In (3) the price q_t is multiplied with borrowing B_t in period t, and B_{t-1} is subtracted for the previous period $t-1$ to obtain net borrowing. Analogously for the IL, q_{It} is multiplied with L_t and L_{t-1} is subtracted. The whole parenthesis with four terms in (3) is multiplied with $1 - D_t$ causing zero and $c_t = y_t$ if default is chosen, and multiplication with 1 otherwise. For simplicity, and to avoid division with zero below, e.g. in (6) and in the simulation, in the remainder of the article we assume $c_t \geq \varepsilon > 0$, where ε is arbitrarily small and positive.

We impose the no Ponzi-scheme assumption such that

$$B_T = L_T = 0 \tag{5}$$

to require that no borrowing occurs in the final period T.

The IL seeks to establish long term stability by promoting the country's consumption, debt repayment, and averting default. Consequently, we define the IL's utility U_{It} in period t recursively where utility in each period t has four inputs. The first weighted with α_1, $0 \leq \alpha_1 \leq 1$, is the positive impact of the country's consumption growth c_t/c_{t-1} from period $t-1$ to period t. The second weighted with α_2, $0 \leq \alpha_2 \leq 1$, is the negative impact of the country's debt-to-endowment ratio B_t/y_t determined as borrowing B_t divided by endowment y_t. The third weighted with $1 - \alpha_1 - \alpha_2$, $0 \leq \alpha_1 + \alpha_2 \leq 1$, is the negative impact of the country's loan-to-endowment ratio L_t/y_t. The fourth is a penalty $\Phi_{It} D_t$ experienced as reduced welfare conditional on default. Summing over T periods with time discounting, the IL's utility is

$$U_I = \sum_{t=1}^{T} \beta_I^{t-1} U_{It} = \sum_{t=1}^{T} \beta_I^{t-1} \left(\alpha_1 \left(\frac{c_t}{c_{t-1}} \right) - \alpha_2 \left(\frac{B_t}{y_t} \right) - (1 - \alpha_1 - \alpha_2) \left(\frac{L_t}{y_t} \right) - \Phi_{It} D_t \right) \tag{6}$$

where $\Phi_{It} \geq 0$ is a penalty the IL experiences in period t if the country defaults, $\beta_I \in [0,1]$ is the IL's intertemporal discount factor, and $0 \leq \alpha_1 \leq 1$, $0 \leq \alpha_2 \leq 1$, $0 \leq \alpha_1 + \alpha_2 \leq 1$.

We assume that the sum $\sum_{t=1}^{T} L_t$ of all loans over the T periods cannot exceed the exogenous European Stability Mechanism budget $\overline{L}$, i.e.

$$\sum_{t=1}^{T} L_t \leq \overline{L} \tag{7}$$

All parameters are common knowledge.

4. Solving for subgame perfect equilibria (SPE)

4.1. Country borrows $B_1 \in [0, \overline{B}_1]$

Starting with the bottom path $B_1 \in [0, \overline{B}_1]$ in Fig. 1,the borrower chooses optimal borrowing $B_1 \in [0, \overline{B}_1]$ in period $t = 1$ defined as B_1^* in Lemma 1.

Lemma 1. *The country's optimal borrowing when $\rho \in (0, 1]$ is*

$$B_1^* = \begin{cases} 0 \text{ if } b^* \leq 0 \\ b^* \text{ if } 0 < b^* < \overline{B}_1 \\ \overline{B}_1 \text{ if } b^* \geq \overline{B}_1 \end{cases};\ b^* \equiv \frac{y_2 + (B_0 - y_1)(\beta/q_1)^{1/\rho}}{1 + q_1(\beta/q_1)^{1/\rho}};\ \frac{\partial b^*}{\partial y_2} \geq 0, \frac{\partial b^*}{\partial B_0} \geq 0, \frac{\partial b^*}{\partial y_1} \leq 0, \frac{\partial b^*}{\partial \beta} \geq 0 \text{ if } B_0 \geq y_1 + q_1 y_2, \frac{\partial b^*}{\partial q_1} \leq 0 \text{ if } B_0$$
$$\geq y_1 + q_1 y_2(1-\rho), \frac{\partial b^*}{\partial \rho} \leq 0 \text{ if } B_0 \geq y_1 + q_1 y_2 \text{ and } Ln(\beta/q_1) \geq 0 \Leftrightarrow \beta \Big/ q_1. \tag{8}$$

Proof. Appendix A.

For the interior solution where $0 < b^* < \overline{B}_1$, Lemma 1, first, states that the country's borrowing B_1^* in period 1 increases as the endowment y_2 in period 2 (assuming it can be known or at least forecasted) increases. That follows since the country then becomes better equipped to pay back the borrowed amount in period 2. Second, borrowing B_1^* increases as the initial borrowing B_0 in period 0 increases, which increases the country's prior liability, sustained through continued borrowing. Third, borrowing B_1^* decreases as the country's endowment y_1 in period 1 increases, which enables the country to consume its endowment more directly, rather than relying on borrowing. Fourth, the country's borrowing B_1^* increases if its intertemporal discount factor β increases, given that its initial borrowing B_0 is high expressed as $B_0 \geq y_1 + q_1 y_2$. This means that if the future is important (β is high), and initial borrowing B_0 in period 0 is high, then more borrowing B_1^* in period 1 is required. Fifth, the country's borrowing B_1^* decreases if $q_1 = \frac{1}{1+r_1}$ increases, i.e. if the interest rate r_1 on borrowing decreases, given that its initial borrowing B_0 is high expressed as $B_0 \geq y_1 + q_1 y_2(1-\rho)$ (sufficient but not necessary condition, see Appendix A). Sixth, the country's borrowing B_1^* increases if its degree ρ of relative risk aversion increases, given that its initial borrowing B_0 is high expressed as $B_0 \geq y_1 + q_1 y_2$, and $Ln(\beta/q_1) \geq 0$. This means that if the country is risk averse, and its initial borrowing B_0 is high, and the future is important, then it borrows more.

4.2. Country seeks loan $L_{st} \in \mathbb{R}_{\geq 0}$

Next, following the middle path in Fig. 1, the country may choose to seek a loan $L_{st} \in \mathbb{R}_{\geq 0}$. In this path, the IL can choose to approve $A_1 = 1$ or not approve $A_1 = 0$ each loan. Following the IL's choice to approve or not approve the loan, the country has the additional choice of borrowing or defaulting.

We determine the optimal response of each player through the middle path by backwards induction. To do so, we consider the best response of the country given that L_t was negotiated over rounds 2 and 3 as determined by (1). Given the best response of the country associated with IL approval, we consider the best response of the IL as a function of the total loan amount in section 4.2.3. Finally, the optimal strategy for each player as determined by backwards induction is given in section 4.2.4.

4.2.1. IL approves, i.e. $A_1 = 1$, loan L_1

Starting with the bottom path of round 1 in Fig. 1, the country must choose optimal borrowing from international credit markets. The optimal level of borrowing is defined as $B_1^{**}(L_1)$ in Lemma 1.

Lemma 2. *The country's best response for borrowing $B_1^{**}(L_1)$ is a function of any given approved loan amount L_1 as follows:*

$$B_1^{**}(L_1) = \begin{cases} 0 \text{ if } b^{**}(L_1) \leq 0 \\ b^{**}(L_1) \text{ if } 0 < b^{**}(L_1) < \overline{B}_1 \\ \overline{B}_1 \text{ if } b^{**}(L_1) \geq \overline{B}_1 \end{cases};\ b^{**}(L_1) \equiv \frac{y_2 - L_1 \Big/ q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}}{1 + q_1(\beta/q_1)^{1/\rho}}; \frac{\partial b^{**}(L_1)}{\partial L_1} \leq 0 \tag{9}$$

Proof. Appendix B.

When $L_1 = 0$, $b^{**}(L_1 = 0)$ in (9) simplifies to b^* in (8). The negative sign before the two occurrences of L_1 in (9) expresses that an increased loan L_1 from the IL in period 1 enables the country to borrow less, i.e. lower $b^{**}(L_1)$ and lower $B_1^{**}(L_1)$, from international credit markets.

4.2.2. IL denies,i.e.$A_1 = 0$,loan L_1

The country can either borrow B_1 or default such that $D_t = 1$. If the borrower does not default, it is faced with the decision of how much to borrow. The borrowing decision is, in fact, the same as the borrowing decision faced in round 1, following the bottom path in Fig. 1. Therefore, if the IL chooses to deny the loan and the country chooses to not default, optimal borrowing is B_1^* as defined in (8). Consequently, the country will choose to default if and only if

$$D_1 = 1 \Leftrightarrow U(B_0 = 0, D_1 = 1) > U\left(B_0 = B_1^*, D_1 = 0\right) \tag{10}$$

4.2.3. The IL's best response to loan L_1

The IL can either approve, i.e. $A_1 = 1$, or deny, i.e. $A_1 = 0$, the country's loan application L_{s1}, which impacts the negotiated loan L_1 in (1). To determine the conditions for approval, consider an optimal loan region such that the IL approves all values of L_1 within a so-called acceptance region.

Lemma 3. *Acceptance Region*

An acceptance region R exists for which the IL is willing to lend:

$$\exists R : \forall L_t \in R, U_F(L_t) \geq 0 \tag{11}$$

Proof. Appendix C.

4.2.4. Optimal loan strategy

Lemma 4. *Optimal loan*

$$\exists \overline{L}_1 \in R : U_c\left(\overline{L}_1, B_1^{**}(\overline{L}_1)\right) \in \underset{L_1 \in R}{\operatorname{argmax}} U_c\left(L_1, B_1^{**}(L_1)\right)$$

Proof. Appendix D.

4.3. Algorithm

We construct an algorithm to solve the two-period game by backwards induction. The algorithm uses the analytical solutions for optimal borrowing and conditions for default while finding the IL's best response by looping through possible loan values. The algorithm solves the game in three steps.

First, the country's utility U_B associated with borrowing is determined. According to (8), we determine optimal borrowing B_1^*, the resulting levels of consumption c_t according to (3), and the resulting country utility according to (1).

Second, the country's utility U_D associated with default borrowing are determined. Given $D_1 = 1$ the resulting levels of consumption c_t according to (3) country utility according to (1) are determined.

Third, the algorithm evaluates the country's utility U_L associated with seeking a loan by evaluating the IL's best response for each possible loan value $L_1 \in [0, \overline{L}_1]$ (note, we initialize such that the loan cannot exceed the IL's budget $\overline{L}_1$). The best response of the IL is determined by calculating its utility according to (6) associated with loan approval, $A_1 = 1$, and denial $A_1 = 0$. That is given approval, optimal borrowing $B_1^{**}(\overline{L}_1)$ according to (9) and the resulting level of consumption are calculated to determine the utility of approval. Given denial, the default condition is used to determine whether the country borrows ($D_1 = 0, B_1 = B_1^{***}$) or defaults ($D_1 = 1, B_1 = 0$). The best response of the IL is then chosen according to which choice leads to the highest utility.

SPE (Subgame Perfect Equilibria) candidates are determined in each loan. For each value of $L_1 \in [0, \overline{L}_1]$, the highest country utility (borrowing U_B, default U_D, or seek loan U_L) is chosen and compared to a maximum utility value U_{max} (originally initialized to $-\infty$). If the utility is higher than the previous maximum, the new value is set as the maximum and each strategic choice variable is stored as SPE candidate values. After all values of $L_1 \in [0, \overline{L}_1]$ have been evaluated, the final SPE is returned. The algorithm is shown in Appendix E.

5. Simulating the Greek debt crises

Table 1 shows the empirics for the Greek debt crisis starting in 2010, with initial conditions in 2009 before the crisis. Table 1 contains 13 parameter values determined in the notes below Table 1, or by what we believe is plausible or conventional. Six of the parameters, $y_t, r_t, r_{It}, t = 1,2$, differ in periods $t = 1,2$. The seven other parameters, $\beta, \beta_I, \rho, \Phi_t, \Phi_{It}, \alpha_1, \alpha_2$ are the same in periods $t = 1$, 2. Table 1 contains one initial condition for borrowing B_0 in period 0, and one initial condition for consumption c_0 in period 0. The initial condition for the loan L_0 is set to $L_0 = 0$ before the crisis starts, as assumed in section 3.1. We also assume $B_2 = L_2 = 0$ according to the no Ponzi-scheme assumption in (5). The initial conditions and variables are determined in the notes below Table 1.

Table 1
Parameters, initial conditions and variables for Greece for 2009 (period 0), 2010 (period 1) and 2011 (period 2).

Period	0	1	2
Year	2009	2010	2011
Parameters			
β		0.9	
β_I		0.9	
ρ		0.5	
Φ_t		30	
Φ_{It}		30	
y_t		226.0314	207.0289
r_t		9.1%	16.69%
r_{It}		3.423%	3.173%
α_1		1/3	
α_2		1/3	
Variables			
A_t		1	1
X_t		1	1
L_t	0	110	0
B_t	301.0620	330.5700	0
D_t		0	0
c_t	217.205	206.992	189.782

Notes: y_t is in billion Euro; https://ec.europa.eu/eurostat/databrowser/view/tec00001/default/table?lang=en. B_t is in billion Euro; https://ec.europa.eu/eurostat/databrowser/view/sdg_17_40/default/table?lang=en. c_t is in billion Euro; https://data.worldbank.org/indicator/NE.CON.TOTL.CN?locations=GR. L_t is in billion Euro; https://www.consilium.europa.eu/media/25673/20100502-eurogroup_statement_greece.pdf. r_t in June 2010 is 9.1%, and r_t in June 2011 is 16.69%, https://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=229.IRS.M.GR.L.L40.CI.0000.EUR.N.Z&periodSortOrder=ASC. r_{It} June 15, 2010 is 3.423%, and r_{It} June 15, 2011 is 3.173%, https://ec.europa.eu/economy_finance/publications/occasional_paper/2011/pdf/ocp87_en.pdf.

At the benchmark parameter values in Table 1 and with the three initial conditions $L_0 = 0$, $B_0 = 301.0620$ and $c_0 = 217.205$, the SPE (subgame perfect equilibrium) is such that the country borrows from the IL ($L_{1SPE} = 135.94$) which approves the loan ($A_{1SPE} = 1$), the country does not borrow from international credit markets ($B_{1SPE} = 0$) which would be too expensive ($r_1 = 0.091 > r_{I1} = 0.03423$), the country does not default ($D_{1SPE} = 0$), the country consumes $c_{1SPE} = 56.41$ in period 1 and consumes $c_{2SPE} = 71.10$ in period 2, the country receives (aggregate discounted) utility $U_{SPE} = 26.40$, and the IL receives (aggregate discounted) utility $100U_{ISPE} = 26.42$. The loan $L_{1SPE} = 135.94$ is higher than $L_1 = 110$ which the country actually received from the IL in period 1. The country's consumption c_{1SPE} and c_{2SPE} in both periods is lower than the actual consumption in Table 1 which may account for other factors. This section focuses more on how the eight variables change as the 15 constants vary than on the many factors that may cause deviation from the empirical values in Table 1. That is, we focus on the dynamics of the model to understand its logic and operation as the various assumptions change. The simulations in Fig. 2 illustrate how the eight variables change as the 15 constants change relative to the benchmark.

Fig. 2 plots the four independent variables A_{1SPE}, L_{1SPE}, B_{1SPE}, D_{1SPE}, the two consumption dependent variables c_{1SPE} and c_{2SPE}, and the two players' aggregate discounted utilities (dependent variables) U_{SPE} and U_{ISPE} as functions of the 13 benchmark parameter values β, β_I, ρ, Φ_1, Φ_{I1}, α_1, α_2, y_1, y_2, r_1, r_2, r_{I1}, r_{I2} and the three initial conditions L_0, B_0 and c_0 in Table 1. Each of the 15 panels varies one of the parameters or initial conditions while the other 14 parameters or initial conditions are kept at their benchmark values in Table 1. The subscript SPE means subgame perfect equilibrium. Multiplication with 100 is for scaling purposes. The independent variable X_{1SPE} is not plotted since the country always accepts the loan, i.e. X_{1SPE}.[4]

Panel a plots the eight variables as functions of the country's intertemporal discount factor β. As β decreases below the benchmark $\beta = 0.9$, so that the future (period 2) becomes less important, the country borrows more as expressed with higher loan L_{1SPE} from the IL in period 1, which translates into higher consumption c_{1SPE} in period 1, lower consumption c_{2SPE} in period 2, slightly lower (aggregate discounted) utility U_{SPE} for the country, and lower (aggregate discounted) utility U_{ISPE} for the IL. Increasing β above the benchmark $\beta = 0.9$ has the opposite impact.

Panel b plots the eight variables as functions of the IL's intertemporal discount factor β_I. Seven of the eight variables are at the benchmark, while the eighth, the IL's utility U_{ISPE}, increases in β_I, which follows from (6) since the IL then gets more utility from period 2.

Panel c plots the eight variables as functions of the country's degree of relative risk aversion ρ. As ρ decreases below the benchmark $\rho = 0.9$ towards $\rho = 0.47$, the country receives increasing utility U_{SPE}, the IL receives slightly decreasing utility (from 0.270 when $\rho = 1$

[4] Plotting as functions of the country's default penalty Φ_2 in period 2 and as functions of the IL's penalty Φ_{I2} if the country defaults in period 2 is not done since Φ_2 is multiplied with D_2 in the country's utility in (2), and Φ_{I2} is multiplied with D_2 in the IL's utility in (6), where $D_2 = 0$ since the country does not default in period 2, and thus the variables are independent of Φ_2 and Φ_{I2}.

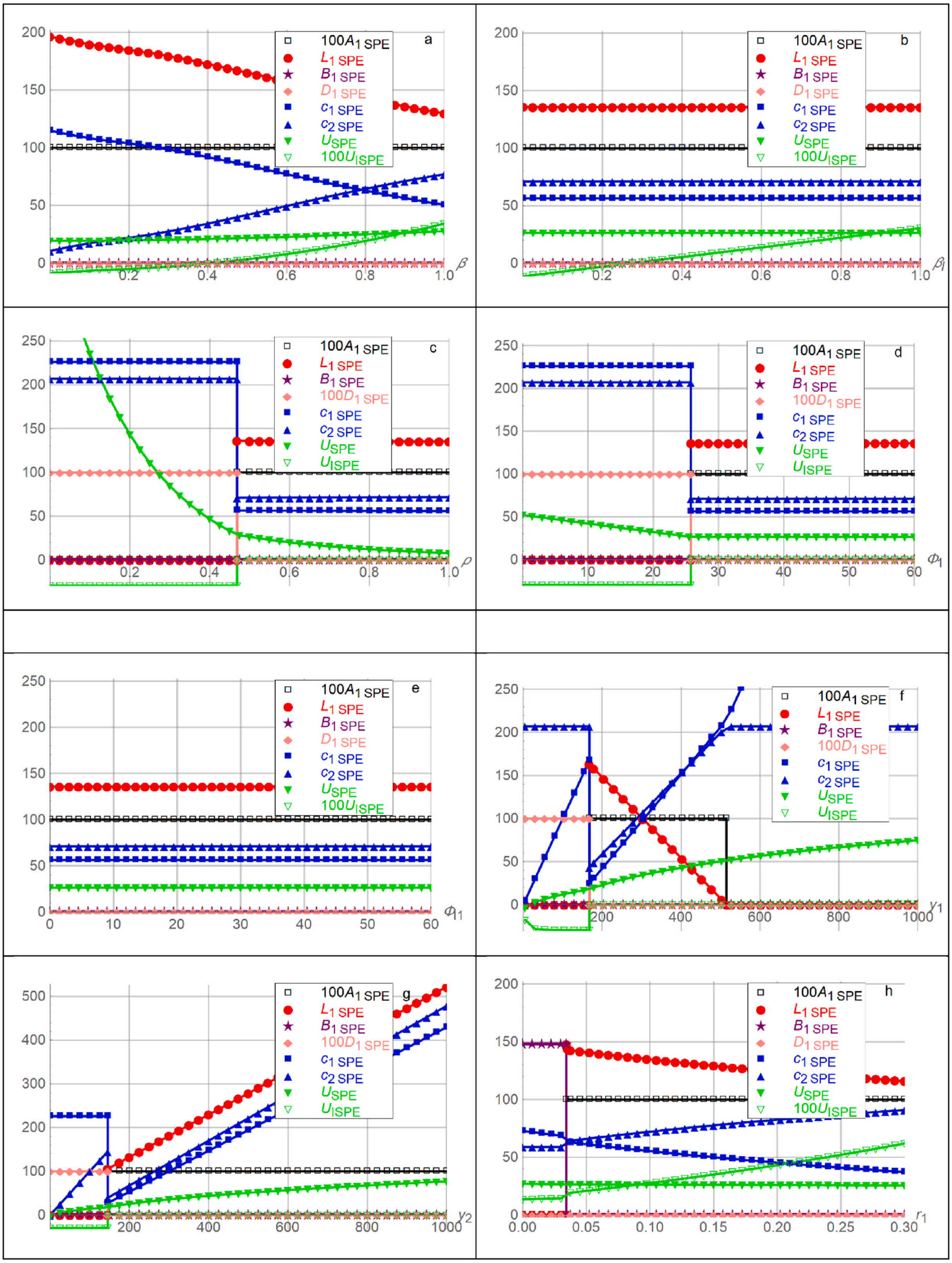
a
100A1 SPE
L1 SPE
B1 SPE
D1 SPE
c1 SPE
c2 SPE
USPE
100UISPE
β
b
100A1 SPE
L1 SPE
B1 SPE
D1 SPE
c1 SPE
c2 SPE
USPE
100UISPE
βI
c
100A1 SPE
L1 SPE
B1 SPE
100D1 SPE
c1 SPE
c2 SPE
USPE
UISPE
ρ
d
100A1 SPE
L1 SPE
B1 SPE
100D1 SPE
c1 SPE
c2 SPE
USPE
UISPE
ΦI
e
100A1 SPE
L1 SPE
B1 SPE
D1 SPE
c1 SPE
c2 SPE
USPE
100UISPE
Φ11
f
100A1 SPE
L1 SPE
B1 SPE
100D1 SPE
c1 SPE
c2 SPE
USPE
UISPE
y1
g
100A1 SPE
L1 SPE
B1 SPE
100D1 SPE
c1 SPE
c2 SPE
USPE
UISPE
y2
h
100A1 SPE
L1 SPE
B1 SPE
D1 SPE
c1 SPE
c2 SPE
USPE
100UISPE
r1

Fig. 2. The four independent variables A_{1SPE}, L_{1SPE}, B_{1SPE}, D_{1SPE}, the two consumption dependent variables c_{1SPE} and c_{2SPE}, and the two players' aggregate discounted utilities U_{SPE} and U_{ISPE} as functions of the 13 benchmark parameter values β, β_I, ρ, Φ_1, Φ_{I1}, α_1, α_2, y_1, y_2, r_1, r_2, r_{I1}, r_{I2} and the three initial conditions L_0, B_0 and c_0 in Table 1.

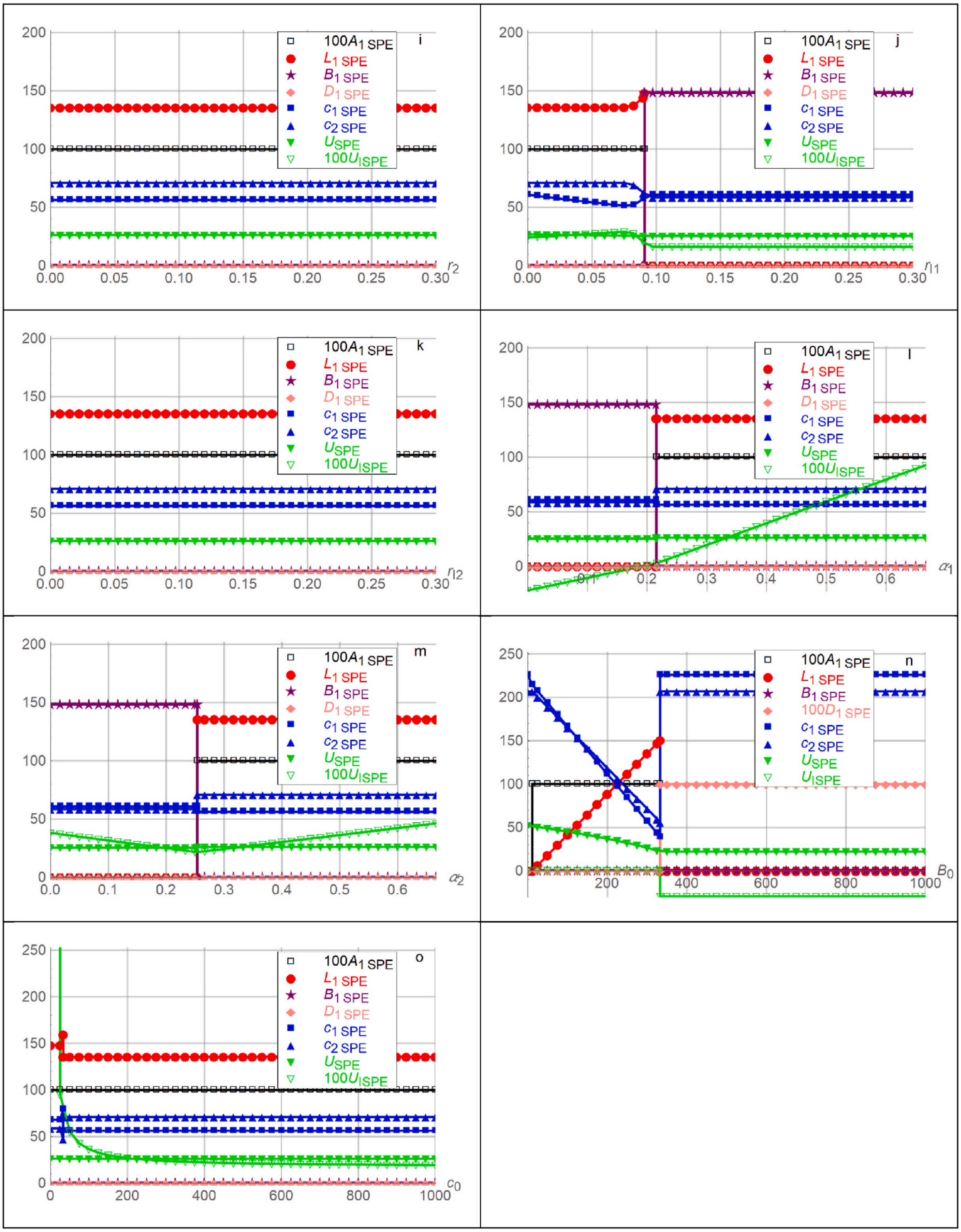

Fig. 2. (*continued*).

to 0.263 when $\rho = 0.47$), while the other variables remain at the benchmark. A low level of risk aversion below $\rho = 0.47$ causes the country to accept the default penalty $\Phi_1 = 30$ in period 1, since the positive term $\frac{c_t^{1-\rho}-1}{1-\rho}$ in (2) increases as ρ decreases, compensating for the negative default penalty term $\Phi_t D_t$. The country's default $D_1 = 1$ in period 1 when $\rho < 0.47$ means, according to (3), that the country's loan obligations vanish. Thus the country can consume its entire endowment $c_{1SPE} = y_1 = 226.0314$ in period 1, and consumes its entire endowment $c_{2SPE} = y_2 = 207.0289$ in period 2. Furthermore, the country's utility U_{SPE} increases as ρ decreases below $\rho = 0.47$ due to the logic of decreased risk aversion ρ in (3). The country's default when $\rho < 0.47$ means that the IL no longer offers a loan. Hence $A_{1SPE} = L_{1SPE} = B_{1SPE} = 0$ when $\rho < 0.47$. Also, the IL's utility U_{ISPE} decreases discontinuously to $U_{ISPE} = -29.38$ when ρ decreases below $\rho = 0.47$ due to the negative default penalty term $\Phi_t D_t$ in (6).

Panel d plots the eight variables as functions of the country's default penalty Φ_1 in period 1. The benchmark default penalty $\Phi_1 = 30$, and all default penalties $\Phi_1 \geq 25.77$, give the benchmark values for the eight variables. However, as the default penalty decreases below $\Phi_1 = 25.77$, the country accepts the penalty and defaults. The six independent variables and the IL's utility are then the same as when $\rho < 0.47$ in panel c, i.e. $A_{1SPE} = L_{1SPE} = B_{1SPE} = 0, D_1 = 1, c_{1SPE} = y_1 = 226.0314, c_{2SPE} = y_2 = 207.0289, U_{ISPE} = -29.38$. As the default penalty Φ_1 decreases below $\Phi_1 = 25.77$, the country's utility U_{SPE} intuitively increases.

Panel e plots the eight variables as functions of the IL's penalty Φ_{I1} if the country defaults in period 1. Since the country does not default in period 1 at the benchmark, i.e. $D_1 = 0$ causing $\Phi_{I1} D_1 = 0$, the eight variables remain constant at their benchmark for all $\Phi_{I1} \geq 0$.

Panel f plots the eight variables as functions of the country's endowment y_1 in period 1. As y_1 increases above the benchmark $y_1 = 226.0314$, the country gets a lower need to loan money from the IL, and thus the loan L_{1SPE} decreases, reaching $L_{1SPE} = 0$ causing no loan acceptance $A_{1SPE} = 0$ when $y_1 > 514.82$. The country's increased endowment from $y_1 = 226.0314$ to $y_1 = 514.82$ gives increased consumption c_{1SPE} and c_{2SPE} in both periods, and increased utility U_{SPE}. As y_1 increases above $y_1 = 514.82$, the country's period 1 consumption c_{1SPE} increases more abruptly, the country's period 2 consumption c_{2SPE} equals its period 2 endowment $c_{2SPE} = y_2 = 207.0289$, the country's utility U_{SPE} increases, and the IL's utility increases from $U_{ISPE} = 0.622$ when $y_1 = 514.82$ to $U_{ISPE} = 1.161$ when $y_1 = 1000$. In contrast, as y_1 decreases below the benchmark $y_1 = 226.0314$, two noteworthy events occur. First, the country requests a higher loan from the IL, and thus the loan L_{1SPE} increases, reaching a maximum $L_{1SPE} = 163.34$ when $y_1 = 167.60$. Second, despite the increased loan L_{1SPE} from the IL, the country's decreased period 1 endowment y_1 causes decreased consumption reaching minima $c_{1SPE} = 24.48$ and $c_{2SPE} = 43.68$ when $y_1 = 167.60$. Decreasing y_1 below $y_1 = 167.60$ causes the country to default, $D_1 = 1$. Thus when $y_1 < 167.60$ and y_1 decreases, the country receives decreasing utility U_{SPE}, consumes its entire period 2 endowment $c_{2SPE} = y_2 = 207.0289$, and consumes its entire period 1 endowment $c_{1SPE} = y_1$ which decreases linearly to $c_{1SPE} = 0$ when $y_1 = 0$. Also, when $y_1 < 167.60$, the IL no longer offers a loan. Hence $A_{1SPE} = L_{1SPE} = B_{1SPE} = 0$, and the IL's utility U_{ISPE} is negative because of the default penalty term $\Phi_t D_t$ in (6).

Panel g plots the eight variables as functions of the country's endowment y_2 in period 2. When y_2 is high, the country loans substantially (high L_{1SPE}) and consumes substantially in both periods (c_{1SPE} and c_{2SPE} are high). As y_2 decreases, and eventually decreases below the benchmark $y_2 = 207.0289$, the country is forced to loan less (L_{1SPE} decreases), since the loan L_{1SPE} gets justified by lower period 2 endowment y_2. Thus the country's consumption c_{1SPE} and c_{2SPE} in both periods decrease. The decreasing c_{1SPE} and c_{2SPE} corresponds to the decreasing c_{1SPE} and c_{2SPE} in panel f when y_1 decreases, but the decreasing L_{1SPE} is opposite of the increasing L_{1SPE} in panel f when y_1 decreases. The reason for this difference is that in panel g decreasing period 2 endowment y_2 cannot justify increasing period 1 loan L_{1SPE}, which are events in two different periods, whereas in panel f decreasing period 1 endowment y_1 can justify increasing period 1 loan L_{1SPE}, which gets are events in the same period 1. As y_2 decreases to $y_2 = 145.36$, the country's consumption in the two periods reach their minima $c_{1SPE} = 27.40$ and $c_{2SPE} = 39.43$, supported by the loan $L_{1SPE} = 105.92$. Decreasing y_2 below $y_2 = 145.36$ causes the country to default, $D_1 = 1$. Thus when $y_2 < 145.36$ and y_2 decreases, the country receives decreasing utility U_{SPE}, consumes its entire period 1 endowment $c_{1SPE} = y_1 = 226.0314$, and consumes its entire period 2 endowment $c_{2SPE} = y_2$ which decreases linearly to $c_{2SPE} = 0$ when $y_2 = 0$. Also, when $y_2 < 145.36$, the IL no longer offers a loan. Hence $A_{1SPE} = L_{1SPE} = B_{1SPE} = 0$, and the IL's utility U_{ISPE} is negative because of the default penalty term $\Phi_t D_t$ in (6).

Panel h plots the eight variables as functions of the country's interest rate r_1 on debt owed to international credit markets in period 1. As r_1 increases above the benchmark $r_1 = 0.091$, the initial loan $B_0 = 301.0620$ in period 0 becomes more expensive to maintain. Thus the country can afford to loan less from the IL at the interest rate $r_{I1} = 0.03423$ in period 1, so L_{1SPE} decreases, which causes the country's consumption c_{1SPE} in period 1 to decrease. The decreased loan L_{1SPE} in period 1, which is subtracted from c_2 in period 2 in (3), causes increased consumption c_2 in period 2. The country's utility U_{SPE} decreases marginally. The IL's utility U_{ISPE} increases since $\frac{c_2}{c_1}$ in (6) increases and $\frac{L_1}{y_1}$ in (6) decreases. In contrast, as r_1 decreases below the benchmark $r_1 = 0.091$, the loan L_{1SPE} to the IL reaches its maximum $L_{1SPE} = 148.03$ when $r_1 = r_{I1} = 0.03423$ where the country is indifferent between loaning from the IL and from international credit markets. As r_1 decreases below $r_1 = r_{I1} = 0.03423$, the country's loan L_{1SPE} from the IL decreases discontinuously to $L_{1SPE} = 0$, and $A_{1SPE} = 0$, which holds for $0 \leq r_1 < 0.03423$, while the country's loan B_{1SPE} from international credit markets in period 1 increases discontinuously to $B_{1SPE} = 148.03$, and decreases marginally to $B_{1SPE} = 148.03$ as r_1 decreases to $r_1 = 0$. The loan B_{1SPE} does not increase to infinity as the interest rate r_1 in period 1 decreases to $r_1 = 0$ since a loan has to be repaid, and if the loan is consumed, it cannot be paid back in the two-period model. As r_1 decreases below $r_1 = 0.03423$, the country's consumption c_{1SPE} in period 1 increases, the country's consumption c_{2SPE} in period 2 and its utility U_{SPE} increase marginally, while the IL's utility decreases marginally.

Panel i plots the eight variables as functions of the country's interest rate r_2 on debt owed to international credit markets in period 2. Since all occurrences of $q_2 = \frac{1}{1+r_2}$, from (4), in the model are multiplied with $B_2 = 0$, the eight variables remain constant at their benchmark for all $r_2 \geq 0$.

Panel j plots the eight variables as functions of the country's interest rate r_{It} on loan received from the IL in period 1. As r_{I1} decreases below the benchmark $r_{I1} = 0.03423$, the country's loan $L_{1SPE} = 135.94$ remains constant at its benchmark value. The loan does not increase since it has to be paid back. However, the lower interest rate r_{I1} causes the country's consumption c_{1SPE} in period 1 to increase, while its consumption c_{2SPE} in period 2 remains at its benchmark $c_{2SPE} = 71.10$, and its utility U_{SPE} increases marginally. The IL's utility U_{ISPE} decreases marginally. In contrast, as r_{I1} increases above the benchmark $r_{I1} = 0.03423$, and approaches the interest rate r_1 on debt owed to international credit markets in period 1, the loan L_{1SPE} eventually increases marginally to its maximum $L_{1SPE} = 148.31$ when $r_{I1} = r_1 = 0.091$, where the country is indifferent between lending from the IL and from the international credit markets in period 1. Increasing r_{I1} above $r_{I1} = r_1 = 0.091$ causes the loan L_{1SPE} to decrease discontinuously to $L_{1SPE} = 0$, and thus $A_{1SPE} = 0$, while the loan B_{1SPE} to international credit markets in period 1 increases discontinuously to $B_{1SPE} = 148.31$, where it remains constant for $r_{I1} > 0.091$. The consumption $c_{1SPE} = 60.91$ and $c_{2SPE} = 58.72$, and the utilities $U_{SPE} = 25.60$ and $100U_{ISPE} = 16.40$, also remain constant when $r_{I1} > 0.091$, caused by no loan, $L_{1SPE} = 0$, to the IL.

Panel k plots the eight variables as functions of the country's interest rate r_{I2} on loan received from the IL in period 2. Since all occurrences of $q_{I2} = \frac{1}{1+r_{I2}}$, from (4), in the model are multiplied with $L_2 = 0$, the eight variables remain constant at their benchmark for all $r_{I2} \geq 0$.

Panel l plots the eight variables as functions of the weight α_1 in the IL's utility of the positive impact of the country's consumption growth c_t/c_{t-1} from period $t-1$ to period t. As α_1 increases above the benchmark $\alpha_1 = 1/3$, the IL's utility $100U_{ISPE}$ increases since the positive term c_t/c_{t-1} in (6) is assigned higher weight. The other seven variables remain constant since the country's utility U_{SPE} is not impacted by α_1. In contrast, as α_1 decreases below the benchmark $\alpha_1 = 1/3$, a point is eventually reached, at $\alpha_1 = 0.215$, below which the weight assigned to the country's consumption ratio c_t/c_{t-1} in (6) is so low that the IL prefers not to provide the loan L_{1SPE}. Observe in (6) the negative term $(1-\alpha_1-\alpha_2)\left(\frac{L_t}{y_t}\right)$, where the weight $1-\alpha_1-\alpha_2$ increases when α_1 decreases, which causes a cost for the IL. Hence when α_1 decreases below $\alpha_1 = 0.215$, the IL does not loan to the country, L_{1SPE} decreases discontinuously to $L_{1SPE} = 0$, $A_{1SPE} = 0$, and the IL's utility U_{ISPE} decreases. The country is not impacted by α_1 and still prefers a loan. Hence the country borrows $B_{1SPE} = 148.31$ from the international credit markets when $0 \leq \alpha_1 < 0.215$. The slightly higher loan $B_{1SPE} = 148.31 > 135.94$, than the $L_{1SPE} = 135.94$ benchmark in period 1, enables the country to consume more in period 1, i.e. higher c_{1SPE}, while the higher interest rate $r_1 = 0.091 > r_{I1} = 0.03423$ causes the country to consume less in period 2, i.e. lower c_{2SPE}, causing slightly lower utility U_{SPE}.

Panel m plots the eight variables as functions of the weight α_2 in the IL's utility of the negative impact of the country's debt-to-endowment ratio B_t/y_t in period t. As α_2 increases above the benchmark $\alpha_2 = 1/3$, the IL's utility $100U_{ISPE}$ increases since the negative term $\frac{L_t}{y_t}$ in (6) is assigned lower weight $1-\alpha_1-\alpha_2$. The other seven variables remain constant since the country's utility U_{SPE} is not impacted by α_2. In contrast, as α_2 decreases below the benchmark $\alpha_2 = 1/3$, a point is eventually reached, at $\alpha_2 = 0.254$, below which the weight $1-\alpha_1-\alpha_2$ assigned to the country's ratio $\frac{L_t}{y_t}$ of loan to endowment is so high, and it impacts the IL's utility U_{ISPE} in (6) negatively, that the IL prefers not to provide the loan L_{1SPE}. Hence when α_2 decreases below $\alpha_2 = 0.254$, the IL does not loan to the country, L_{1SPE} decreases discontinuously to $L_{1SPE} = 0$, $A_{1SPE} = 0$, and the IL's utility U_{ISPE} increases. The country is not impacted by α_2 and still prefers a loan. Hence the country borrows $B_{1SPE} = 148.31$ from the international credit markets when $0 \leq \alpha_2 < 0.254$. The slightly higher loan $B_{1SPE} = 148.31 > 135.94$, than the $L_{1SPE} = 135.94$ benchmark in period 1, enables the country to consume more in period 1, i.e. higher c_{1SPE}, while the higher interest rate $r_1 = 0.091 > r_{I1} = 0.03423$ causes the country to consume less in period 2, i.e. lower c_{2SPE}, causing slightly lower utility U_{SPE}.

Panel n plots the eight variables as functions of the country's borrowing B_0 from international credit markets in period 0, which is an initial condition for 2009. As B_0 decreases below the benchmark $B_0 = 301.0620$, the country becomes less burdened by interest payments on its initial borrowing B_0 in period 0. It thus increases its consumption c_{1SPE} and c_{2SPE} in both periods, and decreases its loan L_{1SPE} to the IL which reaches 0 when $0 \leq B_0 < 11.69$, which increases its utility U_{SPE} and increases the IL's utility U_{ISPE}. In contrast, as B_0 increases above the benchmark $B_0 = 301.0620$, the country becomes more burdened by its initial borrowing B_0 in period 0. Thus, when $B_0 = 332.98$, the country's consumption reaches minima $c_{1SPE} = 38.97$ and $c_{2SPE} = 56.11$, the loan from the IL reaches its maximum $L_{1SPE} = 150.80$, and both players' utilities decrease to their minima where they remain constant for $B_0 > 332.98$. As B_0 increases above $B_0 = 332.98$, the country becomes so burdened by its initial borrowing B_0 in period 0 that it defaults, $D_{1SPE} = 1$, enabling it to consume its entire endowment $c_{1SPE} = y_1 = 226.0314$ and $c_{2SPE} = y_2 = 207.0289$ in both periods, causing negative utility for the IL due to the default penalty term $\Phi_t D_t$ in (6).

Panel o plots the eight variables as functions of the country's aggregate consumption c_0 in period 0, which is an initial condition for 2009. As c_0 increases above the benchmark $c_0 = 217.205$, the IL's utility U_{ISPE} decreases because of the positive term $\frac{c_1}{c_0}$ in the IL's utility U_{ISPE} when $t = 1$ in (6). The other seven variables remain constant since they are not impacted by c_0. In contrast, as c_0 decreases below the benchmark $c_0 = 217.205$, the IL's utility U_{ISPE} increases, while the other seven variables are constant, until $c_0 = 32.73$. This low value of c_0 causes $\frac{c_1}{c_0}$ in (6) to be high, and eventually approaches infinity as c_0 approaches zero, which indirectly impacts which loan L_{1SPE} is optimal for the IL in (6). This high value of $\frac{c_1}{c_0}$ assigned positive weight in (6) impacts the optimal value of $\frac{L_t}{y_t}$ assigned negative weight in (6). More specifically, the negative weight of $\frac{L_t}{y_t}$ in the IL's utility U_{ISPE} in (6) becomes negligible compared with $\frac{c_1}{c_0}$, and the optimal value of the loan L_{1SPE} increases slightly, which occurs discontinuously when $c_0 = 32.73$.

6. Conclusion

The article develops a two-period game between a country and an IL (international lender). In each period the country can repay debt, borrow from international credit markets, seek loan from an IL (e.g. various Financial Intergovernmental Organizations), or default. The IL can approve or deny the loan. The risk averse country maximizes an isoelastic utility with time discounting, depending on its consumption in the two periods, with a default penalty if it defaults. The IL maximizes a utility with time discounting which values the country's consumption growth positively, and the debt-to-endowment ratio, the loan-to-endowment ratio, and the default penalty negatively. The focus on two periods enables focusing on which strategies are optimal at one point in time, weighed against optimal strategies at a subsequent point in time.

The subgame perfect equilibria are determined with backward induction. Analytical results are developed for a country's borrowing from international credit markets, which decreases if a loan is obtained from the IL. For example, a country borrows more in period 1 if its endowment (gross domestic product) in period 2 is expected to be high and if its initial borrowing in period 0 is high. In contrast, a country borrows less in period 1 if its endowment in period 1 is high which enables more direct consumption in period 1.

To illustrate the solution further, an algorithm is developed and simulated applying empirical data from the 2010 Greek debt crisis. The country's borrowing, loan, and default in period 1, and consumption in both periods, the IL's loan approval, and both players utilities, are plotted as functions of 13 parameters and three initial conditions, relative to a plausible benchmark. Some findings are as follows. As a country's time discount parameter decreases, it loans more, consumes more in period 1, consumes less in period 2, and both players earn lower utilities. As a country's degree of relative risk aversion decreases, its utility increases, eventually encouraging it to default if the default penalty is not too high. If the country's endowment in period 1 decreases, the country loans more, consumes less, and eventually defaults, earning lower utility. In contrast, and intuitively, if the country's endowment increases, the country loans less, eventually does not loan, and consumes more. The dependence on the country's endowment in period 2 is similar, except that the loan in period 1 is proportional to the period 2 endowment. If the country's interest rate to the international credit markets increases, it loans less and consumes less in period 1, consumes more in period 2, and earns lower utility. In contrast, if the interest rate to the international credit markets decreases, and unrealistically becomes lower than that of the IL, the country switches to borrowing from the international credit markets, earning higher utility, while the IL earns lower utility. If the IL values the positive impact of the country's consumption growth less, and the negative impact of the country's loan-to-endowment ratio more, eventually a point is reached where it no longer offers a loan, causing the country to borrow from international credit markets instead. If the country's initial borrowing from international credit markets before the game starts in period 0 increases, the country loans more from the IL, and consumes less, and eventually it defaults because the initial debt burden is too high. Future research should extend to more than two periods which gets substantially more complicated if the complexity in the current model is kept. Empirical support should be furnished for other crises than the 2010 Greek crisis.

Funding

No funding was received.

Data availability

The article contains no associated data. All data generated or analyzed during this study are included in this published article.

Declaration of competing interest

On behalf of all authors, the corresponding author states that no conflict of interest exists.

Appendix A. Proof of Lemma 1 on optimal borrowing

Using (2), (3), and (5) for $T = 2$, $L_0 = L_1 = L_2 = B_2 = D_1 = D_2 = 0$, the country chooses borrowing B_1 to maximize

$$\begin{aligned}&\max_{B_1 \in [0,\overline{B}_1]} \{U_1 + \beta U_2\}\\ &\text{s.t.}\\ &c_1 = y_1 + q_1 B_1 - B_0\\ &c_2 = y_2 - B_1\\ &0 \le B_1 \le \overline{B}_1\end{aligned} \tag{12}$$

The Lagrangian is

$$\mathscr{L} = U_1 + \beta U_2 + \pi_1(c_1 - y_1 - q_1 B_1 + B_0) + \pi_2(c_2 - y_2 + B_1) - \mu_1 B_1 + \mu_2(\overline{B}_1 - B_1) \tag{13}$$

where $\pi_1, \pi_2, \mu_1, \mu_2$ are the Lagrangian multipliers. Differentiating the Lagrangian $\mathscr{L}$ with respect to $c_1, c_2, B_1, \pi_1, \pi_2$ and equating with zero gives the first order equations

$$\begin{aligned}&\frac{\partial \mathscr{L}}{\partial c_1} = U_1' + \pi_1 = 0,\\ &\frac{\partial \mathscr{L}}{\partial c_2} = \beta U_2' + \pi_2 = 0,\\ &\frac{\partial \mathscr{L}}{\partial B_1} = q_1\pi_1 - \pi_2 - \mu_1 - \mu_2 = 0,\\ &\frac{\partial \mathscr{L}}{\partial \pi_1} = c_1 - y_1 - q_1 B_1 + B_0 = 0,\\ &\frac{\partial \mathscr{L}}{\partial \pi_2} = c_2 - y_2 + B_1 = 0\end{aligned} \tag{14}$$

which is solved to yield

$$\begin{aligned}&\frac{U_1'}{\beta U_2'} = \frac{\pi_1}{\pi_2},\\ &q_1\pi_1 = \pi_2,\\ &c_1 = y_1 + q_1 B_1 - B_0,\\ &c_2 = y_2 - B_1,\\ &\mu_1 B_1 = 0,\\ &\mu_2(\overline{B}_1 - B_1) = 0\end{aligned} \tag{15}$$

which causes three cases:

$$\text{Case 1} : \mu_1 \ne 0, \mu_1 B_1 = 0 \Rightarrow B_1 = 0 \tag{16}$$

$$\text{Case 2} : \mu_1 = 0, \mu_2 \ne 0, \overline{B}_1 - B_1 = 0 \Rightarrow B_1 = \overline{B}_1 \tag{17}$$

$$\text{Case 3} : \mu_1 = 0, \mu_2 = 0 \tag{18}$$

For case 3, differentiating (2) with respect to c_1 and c_2 when $\rho \in (0, 1]$, and using the first two equations in (15), gives

$$\frac{1}{q_1} = \frac{U_1'}{\beta U_2'} = \frac{c_1^{-\rho}}{\beta c_2^{-\rho}} = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\rho} \Rightarrow \frac{c_2}{c_1} = \left(\frac{\beta}{q_1}\right)^{1/\rho} \tag{19}$$

Solving (19) together with the last three equations in (15) gives (8). Differentiating (8) gives

$$\frac{\partial b^*}{\partial y_2} = \frac{1}{1+q_1(\beta/q_1)^{1/\rho}}, \frac{\partial b^*}{\partial B_0} = -\frac{\partial b^*}{\partial y_1} = \frac{1}{q_1+(\beta/q_1)^{-1/\rho}},$$
$$\frac{\partial b^*}{\partial \beta} = \frac{(B_0-y_1-q_1y_2)(\beta/q_1)^{1/\rho}}{\beta\rho\left(1+q_1(\beta/q_1)^{1/\rho}\right)^2},$$
$$\frac{\partial b^*}{\partial q_1} = -\frac{\left(B_0-y_1-q_1y_2(1-\rho)+q_1(B_0-y_1)(\beta/q_1)^{1/\rho}\rho\right)(\beta/q_1)^{1/\rho}}{q_1\rho\left(1+q_1(\beta/q_1)^{1/\rho}\right)^2}, \tag{20}$$
$$\frac{\partial b^*}{\partial \rho} = -\frac{(B_0-y_1-q_1y_2)Ln(\beta/q_1)(\beta/q_1)^{1/\rho}}{\rho^2\left(1+q_1(\beta/q_1)^{1/\rho}\right)^2},$$
$$Ln(\beta/q_1)\geq 0 \Leftrightarrow \beta \geq q_1 = \frac{1}{1+r_1}$$

□

Appendix B. Proof of Lemma 2 on optimal borrowing given approved loan amount

Using (2) and (3) for $T=2$, $L_0=0, L_2=D_1=D_2=0$, the country chooses borrowing B_1 to maximize

$$\begin{aligned}&\max_{B_1\in[0,\overline{B}_1]}\{U_1+\beta U_2\}\\ &\text{s.t.}\\ &c_1=y_1+L_1+q_1B_1-B_0,\\ &c_2=y_2-(1+r_1)L_1-B_1,\\ &0\leq B_1\leq \overline{B}_1\end{aligned} \tag{21}$$

Defining $\Lambda_1\equiv y_1+L_1$ and $\Lambda_2\equiv y_2-L_1(1+r_1)$, the Lagrangian is

$$\mathscr{L}=U(c_1)+\beta U(c_2)+\pi_1(c_1-\Lambda_1-qB_1+B_0)+\pi_2(c_2-\Lambda_2+B_1)-\mu_1B_1+\mu_2(\overline{B}_1-B_1) \tag{22}$$

where π_1,π_2,μ_1,μ_2 are the Lagrangian multipliers. Note that substituting Λ_t for y_t equates the Lagrangian in (22) with the Lagrangian in (13). Thus, it can be shown by following the steps shown in the proof of Lemma 1, that differentiating the Lagrangian in (22) and solving the resulting the first order equations gives the optimal level of borrowing as a function of loan L_1 as gives (9). □

Appendix C. Proof of Lemma 3 on acceptance region

Assume $D_t=0$ and that, due to perfect information, $B_t=B_t^{**}(L_t)$. From the no-Ponzi scheme rule in (5), $B_2=0$ follows.

$$\begin{aligned}U_I(x_I,x_{-I})=U_{I1}+\beta_IU_{I2}=&\left(\alpha_1\left(\frac{c_1}{c_0}\right)-\alpha_2\left(\frac{B_1^{**}(L_1)}{y_1}\right)-(1-\alpha_1-\alpha_2)\left(\frac{L_1}{y_1}\right)-D_1\Phi\right)+\beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)-\alpha_2\left(\frac{B_2}{y_2}\right)-(1-\alpha_1-\alpha_2)\left(\frac{L_2}{y_2}\right)\right.\\ &\left.-D_2\Phi\right)\geq 0\end{aligned} \tag{23}$$

$$\Leftrightarrow \alpha_1\left(\frac{c_1}{c_0}\right)-\alpha_2\left(\frac{B_1^{**}(L_1)}{y_1}\right)-(1-\alpha_1-\alpha_2)\left(\frac{L_1}{y_1}\right)+\beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\geq 0$$
$$\Leftrightarrow \alpha_1\left(\frac{c_1}{c_0}\right)+\beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\geq \alpha_2\left(\frac{B_1^{**}(L_1)}{y_1}\right)+(1-\alpha_1-\alpha_2)\left(\frac{L_1}{y_1}\right)$$
$$\Leftrightarrow y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right)+\beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right)\geq \alpha_2B_1^{**}(L_1)+(1-\alpha_1-\alpha_2)L_1 \tag{24}$$

Case 1. $B_1^{**}(L_1)=0$.

$$B_1^{**}(L_1)=0\Leftrightarrow b^{**}(L_t)\leq 0$$

$$\Leftrightarrow \frac{y_2 - L_1 \Big/ q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}}{1 + q_1(\beta/q_1)^{1/\rho}} \leq 0$$

$$\Leftrightarrow y_2 - L_1 \Big/ q_1 - (y_1 + L_1)(\beta/q_1)^{1/\rho} + B_0(\beta/q_1)^{1/\rho} \leq 0$$

$$\Leftrightarrow y_2 + B_0(\beta/q_1)^{1/\rho} - y_1(\beta/q_1)^{1/\rho} \leq L_1 \Big/ q_1 + L_1(\beta/q_1)^{1/\rho}$$

$$\Leftrightarrow L_1 \geq \frac{y_2 + (B_0 - y_1)(\beta/q_1)^{1/\rho}}{1 \Big/ q_1 + (\beta/q_1)^{1/\rho}} \equiv L^{'} \tag{25}$$

Case 2. $B_1^{**}(L_1) = b^{**}(L_t)$.

$$U_{It}(A_t) \geq 0 \Leftrightarrow y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right) + \beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right) \geq \alpha_2 B_1^{**}(L_1) + (1 - \alpha_1 - \alpha_2)L_1$$

$$\Leftrightarrow y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right) + \beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right) \geq \frac{\alpha_2\left(y_2 - L_1\Big/q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}\right) + (1 - \alpha_1 - \alpha_2)L_1}{1 + q_1(\beta/q_1)^{1/\rho}}$$

$$\Leftrightarrow y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right) + \beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right)\left(1 + q_1(\beta/q_1)^{1/\rho}\right) \geq \alpha_2\left(y_2 - L_1 \Big/ q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}\right) + (1 - \alpha_1 - \alpha_2)L_1$$

$$\Leftrightarrow y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right) + \beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right)\left(1 + q_1(\beta/q_1)^{1/\rho}\right) \geq \alpha_2\left(y_2 - y_1(\beta/q_1)^{1/\rho} - L_1\left(1 \Big/ q_1 + (\beta/q_1)^{1/\rho}\right) + B_0(\beta/q_1)^{1/\rho}\right) + (1 - \alpha_1 - \alpha_2)L_1$$

$$\Leftrightarrow y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right) + \beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right)\left(1 + q_1(\beta/q_1)^{1/\rho}\right) \geq \alpha_2\left(y_2 - y_1(\beta/q_1)^{1/\rho} + B(\beta/q_1)^{1/\rho}\right)$$
$$- L_1\left(\alpha_2\left(1 \Big/ q_1 + (\beta/q_1)^{1/\rho}\right) + (1 - \alpha_1 - \alpha_2)\right)$$

$$\Leftrightarrow L_1 \geq \frac{\alpha_2\left(y_2 + (B_0 - y_1)(\beta/q_1)^{1/\rho}\right) - y_1\left(\alpha_1\left(\frac{c_1}{c_0}\right) + \beta_I\left(\alpha_1\left(\frac{c_2}{c_1}\right)\right)\right)\left(1 + q_1(\beta/q_1)^{1/\rho}\right)}{\left(\alpha_2\left(1\Big/q_1 + (\beta/q_1)^{1/\rho}\right) + (1 - \alpha_1 - \alpha_2)\right)} \equiv L^{''} \tag{26}$$

Case 3. $B_1^{**}(L_1) = \overline{B}_1$.

$$B_1^{**}(L_1) = \overline{B}_1 \Leftrightarrow \overline{B}_1 \leq b^{**}(L_t)$$

$$\Leftrightarrow \overline{B}_1 \leq \frac{y_2 - L_1\Big/q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}}{1 + q_1(\beta/q_1)^{1/\rho}}$$

$$\Leftrightarrow \overline{B}_1\left(1 + q_1(\beta/q_1)^{1/\rho}\right) \leq y_2 - L_1 \Big/ q_1 - (y_1 + L_1)(\beta/q_1)^{1/\rho} + B_0(\beta/q_1)^{1/\rho}$$

$$\Leftrightarrow \overline{B}_1\left(1 + q_1(\beta/q_1)^{1/\rho}\right) + y_1(\beta/q_1)^{1/\rho} - y_2 - B_0(\beta/q_1)^{1/\rho} \leq -L_1\left(1 \Big/ q_1 - (\beta/q_1)^{1/\rho}\right)$$

$$\Leftrightarrow L_1 \leq \frac{\overline{B}_1\left(1 + q_1(\beta/q_1)^{1/\rho}\right)y_1(\beta/q_1)^{1/\rho} - y_2 - B_0(\beta/q_1)^{1/\rho}}{\left(1\Big/q_1 - (\beta/q_1)^{1/\rho}\right)} \equiv L^{'''} \tag{27}$$

Define the lower bound on lending as $L^{low} = \min\{0, L^{'}, L^{''}\}$ and the upper bound as $L^{up} = \min\{L^{'''}, y_{I1}\}$ where y_{I1} is the IL's budget at $t = 1$. Thus,

$$R = \left[L^{low}, L^{up}\right] \tag{28}$$

Thus, the best response function $A_1^*(L_1)$ is given as follows

$$A_1^*(L_1) = \begin{cases} 1, L_1 \in R \\ 0, L_1 \notin R \end{cases} \tag{29}$$

Appendix D. Proof of Lemma 4 on optimal loan

We solve with backwards induction. L_1 is determined in round 2. $B_1^{**}(L_1)$ is determined in round 3. Hence, mathematically, B_1^{**} gets determined first accounting for all possible L_1. Thereafter L_1 is determined. This implies

$$\max_{B_1 \in [0, \overline{B}_1]} \{U_1 + \beta U_2\}$$
s.t.
$$c_1 = y_1 + L_1 + q_1 B_1^{**}(L_1) - B_0$$
$$c_2 = y_2 - L_1 / q_1 - B_1^{**}(L_1)$$
$$L_1 \in R$$

Case 1, $b^{**} \leq 0$:

$$\mathscr{L} = U_1(c_1) + \beta U_2(c_2) + \pi_1(c_1 - y_1 - L_1 + B_0) + \pi_2(c_2 - y_2 + L_1 / q_1) \tag{30}$$

where $\pi_1, \pi_2, \mu_1, \mu_2$ are the Lagrangian multipliers. Differentiating the Lagrangian $\mathscr{L}$ with respect to $c_1, c_2, B_1, \pi_1, \pi_2$ and equating with zero gives the first order equations

$$\frac{\partial \mathscr{L}}{\partial c_1} = U_1^{'}(c_1) + \pi_1 = 0, \frac{\partial \mathscr{L}}{\partial c_2} = \beta U_2^{'}(c_2) + \pi_2 = 0, \frac{\partial \mathscr{L}}{\partial L_1} = -\pi_1 + \pi_2 \Big/ q_1 = 0, \tag{31}$$

which is solved to yield

$$\frac{U_1^{'}(c_1)}{\beta U_2^{'}(c_2)} = \frac{\pi_1}{\pi_2}, \pi_1 = 1 \Big/ q_1 \tag{32}$$

which gives

$$\frac{U_1^{'}(c_1)}{\beta U_2^{'}(c_2)} = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\rho} = 1 \Big/ q_1 \Rightarrow \frac{c_2}{c_1} = (\beta/q_1)^{\frac{1}{\rho}} \Rightarrow c_2 = c_1(\beta/q_1)^{\frac{1}{\rho}}$$

$$\Rightarrow y_2 - L_1 \Big/ q_1 = (y_1 + L_1 - B_0)(\beta/q_1)^{\frac{1}{\rho}}$$

$$\Rightarrow y_2 - (y_1 - B_0)(\beta/q_1)^{\frac{1}{\rho}} = L_1\left[1 \Big/ q_1 + (\beta/q_1)^{\frac{1}{\rho}}\right]$$

$$\Rightarrow L_1^* = \frac{y_2 - (y_1 - B_0)(\beta/q_1)^{\frac{1}{\rho}}}{1 \Big/ q_1 + (\beta/q_1)^{\frac{1}{\rho}}}$$

Case 2, $b^{**} \in (0, \overline{B}_1)$:

$$\mathscr{L} = U_1(c_1) + \beta U_2(c_2) + \pi_1(c_1 - y_1 - L_1 + B_0) + \pi_2(c_2 - y_2 + L_1 / q_1) \tag{33}$$

where $\pi_1, \pi_2, \mu_1, \mu_2$ are the Lagrangian multipliers. Differentiating the Lagrangian $\mathscr{L}$ with respect to $c_1, c_2, B_1, \pi_1, \pi_2$ and equating with zero gives the first order equations

$$\frac{\partial \mathscr{L}}{\partial c_1} = U_1^{'}(c_1) + \pi_1 = 0, \frac{\partial \mathscr{L}}{\partial c_2} = \beta U_2^{'}(c_2) + \pi_2 = 0, \frac{\partial \mathscr{L}}{\partial L_1} = -\pi_1 + \pi_2 \Big/ q_1 = 0, \tag{34}$$

which is solved to yield

$$\frac{U_1^{'}(c_1)}{\beta U_2^{'}(c_2)} = \frac{\pi_1}{\pi_2}, \pi_1 = 1 \Big/ q_1 \tag{35}$$

which gives

$$\frac{U_1^{'}(c_1)}{\beta U_2^{'}(c_2)} = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\rho} = 1 \Big/ q_1 \Rightarrow \frac{c_2}{c_1} = (\beta/q_1)^{\frac{1}{\rho}} \Rightarrow c_2 = c_1(\beta/q_1)^{\frac{1}{\rho}}$$

$$\Rightarrow y_2 - L_1 \Big/ q_1 - \left[\frac{y_2 - L_1 \big/ q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}}{1 + q_1(\beta/q_1)^{1/\rho}}\right] = \left(y_1 + L_1 + q_1\left[\frac{y_2 - L_1 \big/ q_1 + (B_0 - y_1 - L_1)(\beta/q_1)^{1/\rho}}{1 + q_1(\beta/q_1)^{1/\rho}}\right] - B_0\right)(\beta/q_1)^{\frac{1}{\rho}}$$

Case 3, $b^{**} \geq \overline{B}_1$:

$$\mathscr{L} = U_1(c_1) + \beta U_2(c_2) + \pi_1(c_1 - y_1 - L_1 + B_0) + \pi_2(c_2 - y_2 + L_1 / q_1) \tag{36}$$

where $\pi_1, \pi_2, \mu_1, \mu_2$ are the Lagrangian multipliers. Differentiating the Lagrangian $\mathscr{L}$ with respect to $c_1, c_2, B_1, \pi_1, \pi_2$ and equating with zero gives the first order equations

$$\frac{\partial \mathscr{L}}{\partial c_1} = U_1^{'}(c_1) + \pi_1 = 0, \frac{\partial \mathscr{L}}{\partial c_2} = \beta U_2^{'}(c_2) + \pi_2 = 0, \frac{\partial \mathscr{L}}{\partial L_1} = -\pi_1 + \pi_2 \Big/ q_1 = 0, \tag{37}$$

which is solved to yield

$$\frac{U_1^{'}(c_1)}{\beta U_2^{'}(c_2)} = \frac{\pi_1}{\pi_2}, \pi_1 = 1 \Big/ q_1 \tag{38}$$

which gives

$$\frac{U_1^{'}(c_1)}{\beta U_2^{'}(c_2)} = \frac{1}{\beta}\left(\frac{c_2}{c_1}\right)^{\rho} = 1 \Big/ q_1 \Rightarrow \frac{c_2}{c_1} = (\beta/q_1)^{\frac{1}{\rho}} \Rightarrow c_2 = c_1(\beta/q_1)^{\frac{1}{\rho}}$$

$$\Rightarrow y_2 - L_1 \Big/ q_1 - \overline{B}_1 = (y_1 + L_1 + q_1\overline{B}_1 - B_0)(\beta/q_1)^{\frac{1}{\rho}}$$

$$\Rightarrow y_2 - \overline{B}_1 - (y_1 + q_1\overline{B}_1 - B_0)(\beta/q_1)^{\frac{1}{\rho}} = L_1\left[1 \Big/ q_1 + (\beta/q_1)^{\frac{1}{\rho}}\right]$$

$$\Rightarrow L_1^* = \frac{y_2 - \overline{B}_1 - (y_1 + q_1\overline{B}_1 - B_0)(\beta/q_1)^{\frac{1}{\rho}}}{1 \big/ q_1 + (\beta/q_1)^{\frac{1}{\rho}}}\Bigg]$$

Thus, the optimal loan strategy x^L is

$$x^L = \begin{cases} \left\{L_1 = \overline{L}_1, A_1 = 1, B_1 = B_1^{**}\left(\overline{L}_1\right), D_1 = 0\right\}, U^A \geq \left\{U^B, U^D\right\}^+ \\ \left\{L_1 = 0, A_1 = 0, B_1 = B_1^*, D_1 = 0\right\}, U^B \geq U^D \\ \left\{L_1 = 0, A_1 = 0, B_1 = 0, D_1 = 1\right\}, U^L \geq U^B \end{cases} \tag{39}$$

such that $U^A \equiv U_c(L_1 = \overline{L}_1, B_1 = B_1^{**}(\overline{L}_1), D_1 = 0)$, $U^B \equiv U_c(L_1 = 0, B_1 = B_1^*, D_1 = 0)$, $U^D \equiv U_c(L_1 = 0, B_1 = 0, D_1 = 1)$, $U^L \equiv U_c(x^L)$.

Appendix E. Algorithm

#Initialize variables in period 0.

Set $L_1 = A_1 = 0, U_{max} = U_{Lmax} = U_{ILmax} = -\infty$.

Start periods 1 and 2. Rounds 1,2,3 refer to period 1. No decisions are made in period 2; c_2 *as a dependent variable in period 2 follows from period 1.*

#Determine the country's utility U_B *associated with borrowing in rounds 1 and 3*

Calculate B_1^* *according to (8),* c_{1B} *and* c_{2B} *as a function of* B_1^* *according to (3).*

Calculate $U_B = U(L_1 = 0, B_1 = B_1^*, D_1 = 0, c_t = c_{tB})$ *according to (2).*

#Then, determine the country's utility U_D *associated with default in rounds 1 and 3*

Calculate c_{1D} and c_{2D} given $D_1 = 0$ according to (3).
Calculate $U_D = U(L_1 = 0, B_1 = 0, D_1 = 1, c_t = c_{tD})$ according to (2).
#Finally, determine the country's and IL's utilities U_L and U_{IL} associated with seeking a loan by looping through all loan values L_1, ..., $\bar{L}_1$, in increments of Δ_L, in rounds 1–3.

While { $L_1 \le \bar{L}_1 - \Delta_L$,
$L_1 = L_1 + \Delta_L$
#Determine IL's best response in round 2 through three steps
#First, calculate IL's utility U_{IL} in round 2 given approve $A_1 = 1$
Calculate $B_1^{**}(\bar{L}_1)$ in round 3 according to (9), $c_{tapprove}$ according to (3), and
$U_{IL} = U_I\left(A_1 = 1, L_1, B_1^{**}(\bar{L}_1), c_{tapprove}\right)$ according to (6),
#Second, calculate IL's utility U_{IL} in round 2 given deny $A_1 = 0$
$L_{1deny} = 0$
If{$U_D < U_B$,
Set $B_{1deny} = B_1^*, D_{1deny} = 0$
Calculate c_{tdeny} according to (3)
Else,
Set $B_{1deny} = 0, D_{1deny} = 1$
According to (3), $c_{1deny} = y_1, c_{2deny} = y_2$}
Calculate $U_{INL} = U_I\left(L_1 = 0, B_1 = B_{1deny}, D_1 = D_{1deny}, c_t = c_{tdeny}\right)$ according to (6)

#Third, determine IL's best response in round 2
If{$U_{INL} < U_{IL}$,
Set $A_{1L} = 1$, $L_1 = L_1, B_{1L} = B_1^{**}(\bar{L}_1), D_{1L} = 0, c_{tL} = c_{tapprove}$
Else,
Set $A_{1L} = 0$, $L_1 = 0, B_{1L} = B_{1deny}, D_{1L} = D_{1deny}, c_{tL} = c_{tdeny}$}
Calculate $U_{IL} = U_I(A_1 = A_{1L}, L_1 = L_{1L}, B_1 = B_{1L}, D_1 = D_{1L}, c_t = c_{tL})$ according to (6)
Calculate $U_L = U(A_1 = A_{1L}, L_1 = L_{1L}, B_1 = B_{1L}, D_1 = D_{1L}, c_t = c_{tL})$ according to (2).

#Determine if loan L_1 leads to the highest utility
If {$U_{IL} > U_{ILmax}$and $U_L > U_{Lmax}$,
Set $U_{ILmax} = U_{IL}$, $U_{Lmax} = U_L$,
Set-$L_{1loan} = L_1, B_{1loan} = B_{1L}, c_{tloan} = c_{tL}$} }

#Determine the country's best response given IL's best response
If{$U_D > max\{U_B, U_{Lmax}\}$,
Set $A_{1SPE} = 0, L_{1SPE} = 0, B_{1SPE} = 0, D_{1SPE} = 1, c_{1SPE} = c_{1D}, c_{2SPE} = c_{2D}$
ElseIf{$U_B > U_{Lmax}$,
Set $A_{1SPE} = 0, L_{1SPE} = 0, B_{1SPE} = B_1^*, D_{1SPE} = 0, c_{1SPE} = c_{1B}, c_{2SPE} = c_{2B}$}
Else
Set $A_{1SPE} = 1, L_{1SPE} = L_{1loan}, B_{1SPE} = B_{1loan}, D_{1SPE} = 0, c_{1SPE} = c_{1loan}, c_{2SPE} = c_{2loan}$}
Calculate $U_{SPE} = U(A_{1SPE}, L_{1SPE}, B_{1SPE}, D_{1SPE}, c_{1SPE}, c_{2SPE})$ according to (2)
Calculate $U_{ISPE} = U_I(A_{1SPE}, L_{1SPE}, B_{1SPE}, D_{1SPE}, c_{1SPE}, c_{2SPE})$ according to (6)

References

Acemoglu, D., Ozdaglar, A., & Tahbaz-Salehi, A. (2013). *Systemic risk and stability in financial networks*. National Bureau of Economic Research Working Paper Series, No. 18727. https://doi.org/10.3386/w18727

Aguiar, M., & Gopinath, G. (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics, 69*(1), 64–83. Retrieved from http://www.sciencedirect.com/science/article/pii/S0022199605000644.

Anevlavis, T., Papavassilopoulos, G., Engwerda, J., & van Aarle, B. (2019). Debt stabilization in the presence of endogenous risk premia: A dynamic game approach. *Macroeconomic Dynamics, 23*(7), 2616–2648. https://doi.org/10.1017/s1365100517000906

Araujo, A., Leon, M., & Santos, R. (2017). Bargained haircuts and debt policy implications. *Economic Theory, 64*(4), 635–656. https://doi.org/10.1007/s00199-016-0981-4

Arellano, C., & Bai, Y. (2013). *Linkages across sovereign debt markets*. National Bureau of Economic Research Working Paper Series, No. 19548. https://doi.org/10.3386/w19548

Baral, P. (2013). *Essays on formation of networks and its applications*. Indiana University.

Broner, F. A. (2008). Discrete devaluations and multiple equilibria in a first generation model of currency crises. *Journal of Monetary Economics, 55*(3), 592–605. Retrieved from http://www.sciencedirect.com/science/article/pii/S030439320800041X.

Chari, V. V., & Kehoe, P. J. (2004). Financial crises as herds: Overturning the critiques. *Journal of Economic Theory, 119*(1), 128–150. Retrieved from http://www.sciencedirect.com/science/article/pii/S0022053103002254.

Eaton, J., & Gersovitz, M. (1981). Debt with potential repudiation: Theoretical and empirical analysis. *The Review of Economic Studies, 48*(2), 289–309. https://doi.org/10.2307/2296886

Finke, D., & Bailer, S. (2019). Crisis bargaining in the European union: Formal rules or market pressure? *European Union Politics, 20*(1), 109–133. https://doi.org/10.1177/1465116518814949

Gaspar, V., Poplawski-Ribeiro, M., & Yoo, J. (2023). *Global debt is returning to its rising trend*. Retrieved from https://www.imf.org/en/Blogs/Articles/2023/09/13/global-debt-is-returning-to-its-rising-trend.

Lopez, P. J., & Nahon, C. (2017). The growth of debt and the debt of growth: Lessons from the case of Argentina. *Journal of Law and Society, 44*(1), 99–122. https://doi.org/10.1111/jols.12016

Lorenzoni, G., & Werning, I. (2013). *Slow moving debt crises*. National Bureau of Economic Research Working Paper Series, No. 19228. https://doi.org/10.3386/w19228

Morris, S., & Shin, H. (2000). *Global games: Theory and applications*. New Haven, Connecticut: Cowles Foundation for Research in Economics.

Morris, S., & Shin, H. (2012). Contagious adverse selection. *American Economic Journal: Macroeconomics, 4*(1), 1–21.

Obstfeld, M. (1984). *Rational and self-fulfilling balance-of-payments crises*. National Bureau of Economic Research Working Paper Series, No. 1486. https://doi.org/10.3386/w1486

Pitchford, R., & Wright, M. L. J. (2013). On the contribution of game theory to the study of sovereign debt and default. *Oxford Review of Economic Policy, 29*(4), 649–667. https://doi.org/10.1093/oxrep/grt023

Sopocko, A. (2012). Financial game: Stagnation or growth of the rich world? *Ekonomista*, (4), 449–486. Retrieved from <Go to ISI>://WOS:000309088600003.

Welburn, J. W., & Hausken, K. (2015). A game theoretic model of economic crises. *Applied Mathematics and Computation, 266*, 738–762.

Welburn, J. W., & Hausken, K. (2017). Game theoretic modeling of economic systems and the European debt crisis. *Computational Economics, 49*(2), 177–226. https://doi.org/10.1007/s10614-015-9542-3